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STOCHASTIC MODELS OF DYNAMIC BALANCE STATE FOR THE MORPHOLOGICAL PATTERNS OF CRYOLITHOZONE LANDSCAPES

ABSTRACT. The paper deals with mathematical modeling of a morphological pattern for a broad spectrum of cryolithozone landscapes in a state of a dynamic balance. The state of the dynamic balance means that all the elements of this morphological pattern are in continuous changing while its general parameters as a whole are stable. Two contra-directional processes at the same territory is a precondition for a state of dynamic balance.

We developed a morphological pattern model for lacustrine thermokarst plains with fluvial erosion on the base of the mathematical morphology of landscape using the random process theory. The contra-directional processes here include thermokarst lakes appearing and increasing in size from one side and drainage of the lakes by fluvial erosion, from the other. Thus, the regularities of the structure and dynamics of each landscape morphological pattern are theoretically substantiated. The results of the mathematical modeling were empirically verified at some key sites.

KEY WORDS: Morphological pattern; mathematical modeling; dynamic balance; thermokarst lake; khasyrei (alas), landscapes of the cryolithozone

INTRODUCTION

Different researchers studied a dynamic balance state in the cryolithozone, including natural hazard enhancing due to disturbing of the dynamic balance. For instance, these studies include disturbance of tundra landscapes under man-caused impact such as thermal erosion along track lines at bogs, disturbance of vegetation cover, flooding (Novikova et al. 1998; Vinogradov 1999; Mamai 2005); some others dealt with the thermodynamic balance in the permafrost soil. Some researchers examined changing shares of thermokarst lakes due to climatic change; that can also be regarded as a change of the dynamic balance (Polishuk, Polishuk 2014; Muster et al. 2019; Dneprovskaya et al. 2009; Kipotin et al. 2009; Burn, Smith 1990; Kravtsova, Bystrova 2009). At the same time, we have a lack of works specially devoted to the dynamic balance state of landscape morphological patterns in the cryolithozone.

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A broad spectrum of cryolithozone landscapes has the same type of morphological pattern with a possibility of the dynamic balance. Our research shows that this type of landscapes includes lacustrine thermokarst plains with fluvial erosion, alluvial plains, and plains with the extensive development of the landslide process.

The state of the dynamic balance means that all the elements of this morphological pattern, such as thermokarst lakes, khasireis (drained thermokarst lakes), ridges and inter-ridge depressions of alluvial plains, as well as landslide series, are in continuous changing while general parameters of the morphological pattern as a whole are stable.

In the case of these landscapes, the usual methods of analysis and stationary observation do not provide the necessary information about the trends of their evolution. Our research aims to consider the possibility of dynamic balance in the course of development of the morphological pattern of cryolithozone landscapes and assessment techniques for corresponding natural hazards. We analyzed the problem for a case of thermokarst plains with fluvial erosion as an example.

MATERIALS AND METHODS

The area under consideration is a slight wavy subhorizontal area covered by tundra vegetation, interspersed with lakes and khasyreis and rare fluvial erosion network (Melnikov 2012). Isometric, often roundish shaped lakes are randomly scattered across the plain. Khasyreis are also isometric flat-bottomed and flattened peaty depressions covered with meadow or bog vegetation similar to lakes randomly scattered across the plain. Majority of researchers expect khasyreis to be the result of thermokarst lake drainage, usually due to fluvial erosion (Fig.1).

Thermokarst, thermo-abrasion, and thermo-erosion processes perform in complex interrelations and determine the type of the area (Günther et al. 2013; Liu et al. 2013). Thermokarst depressions separately appear and grow as lakes due to thermos-abrasion after being filled with water. Their growth depends on random factors associated with a meteorological situation of a particular layer and soil condition. At last, at a random point in time, a lake can be lowered by erosion processes and turned into khasyrei, overgrown with meadow and bog vegetation with separate relict lakes; thus the depression stops growing because of lack of water. At the same time, permafrost can reappear within a khasyrei. The whole area appears to be a complex
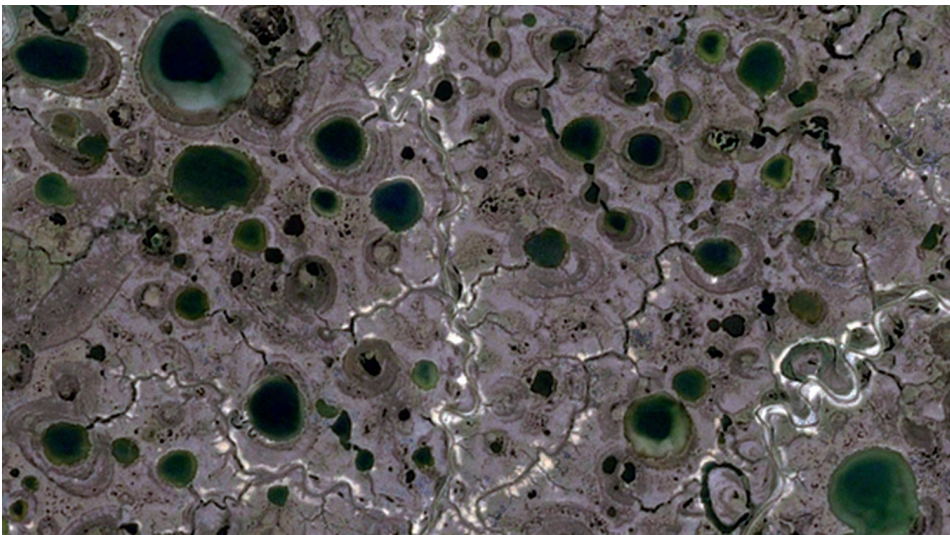

Fig. 1. A typical space image of a thermokarst plain with fluvial erosion

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mosaic of sites, which were lakes and khasyreis at a different time. Thus, the morphological pattern of these landscapes changes under two contra-directional processes

- thermokarst lakes appearing and increasing in size,

- drainage of the lakes by fluvial erosion, their turn into khasireis and stop of their growth.

Two contra-directional processes at the same territory is a precondition for a state of dynamic balance. We made morphological pattern modeling for lacustrine thermokarst plains with fluvial erosion on the base of the mathematical morphology of landscape using the random process theory.

RESULTS

There are two variants for developing the morphological pattern of the thermokarst plain with fluvial erosion:

• synchronous start of thermokarst lake appearance;

• asynchronous start of thermokarst lake appearance.

In the case of a synchronous start, we suggest that primary thermokarst depressions were appearing within a relatively short period compared with their further development. The model for lacustrine thermokarst plains with fluvial erosion in uniform nature environment in case of the synchronous start is based on the following assumptions:

1. Thermokarst depressions were appearing within a relatively short period (i.e., «synchronous start») independently across the different non-adjacent landscapes; the probability of a new depression appearance within a sample plot was dependent exclusively on the plot area:

$$p_1 = \tau_l^0 \Delta s + o(\Delta s) ,$$

where τ_l^0 is an average density lake location (the initial one for the «synchronous start»).

2. The radius of an appeared thermokarst depression is a random variable being a time function; it is undependable of other lakes, and the growth rate is directly proportional to heat losses through the side surface of the lake basin.

3. In the course of its growth, a lake can turn into a khasyrei after draining by the erosion network; the probability of this does not depend on the development of other lakes; if it happens, the depression stops growing.

4. The appearance of new sources of fluvial erosion within a randomly selected area is a random event, and the probability of this depends only on the site area.

The mathematical analysis of the model gives us a set of results (Victorov 1995; Victorov et al. 2017). The simplest of them are the following:

• A quantity of primary thermokarst depressions within as trial plot obeys a Poisson distribution:

$$P(k) = \frac{(\tau_l^0 S)^k}{k!} e^{-\tau_l^0 S} ,$$

where τ_l^0 is a parameter, S is the plot area.
• Change of lake size (area, diameter) in case of unlimited growth without the possibility of lake drainage taken into account should obey the lognormal distribution (probability density):

$$f_0(x,t) = \frac{1}{\sqrt{2\pi}\sigma x \sqrt{t}} e^{\frac{[\ln x - at]^2}{2\sigma^2 t}} ,$$

where a, σ are parameters, t is time.
• The lake drainage due to fluvial erosion and their transformation into khasyreis changes lake size distribution because the lakes have different probabilities of being dried depending on their size.

More sophisticated results deal with the case of the morphological pattern developing for a long time $(t \rightarrow +\infty)$.
At any time, the khasyrei radius distribution is determined by the distance to the nearest source of a stream, which stops its growth (the first member under the integral), and the probability that in the

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course of its growth the lake reaches the size exceeding this distance. The following formula is used:

$$f_h(x,t) = \frac{2\pi\gamma x e^{-\pi\gamma x^2}\left[1 - F_0(x,t)\right]}{\int\limits_0^{+\infty} 2\pi\gamma u e^{-\pi\gamma u^2}\left[1 - F_0(u,t)\right]du} \quad ,$$

where $F_0(x,t)$ is the radius distribution for a free growing thermokarst lake at the moment **t**.

With time increasing $(t \to +\infty)$ the khasyrei radii distribution tends to specific limit distribution. It is easy to see that as far as at any x the lake radii distribution $F_0(x,t)$ tends to 0 in case of free growth, the limit distribution density of the khasyrei radii after a long time of development is given by an expression:

$$f_h(x,\infty) = 2\pi\gamma x e^{-\pi\gamma x^2},$$

and the distribution itself is given by a formula:

$$F_h(x,\infty) = 1 - e^{-\pi\gamma x^2},$$

In other words, after a long time of development khasyrei radii distribution should obey the Rayleigh distribution. Hence, the khasyrei areas distribution (**sh**) obeys the exponential distribution:

$$F_{sh}(x,\infty) = 1 - e^{-\frac{x}{q}},$$

where $\bar{q}$ is an average khasyrei area.

Let us examine the radii distribution of thermokarst lakes. At any moment the radii distribution of the thermokarst lakes is determined by the corresponding radii distribution in case of the free growth, but under the condition that the lake has not turned into khasyrei up to this moment, i.e., the distance to the source of a stream is longer than the lake radius. Thus, the distribution density of the thermokarst lake radii is given by the expression:

$$f_l(x,t) = \frac{f_0(x,t)e^{-\pi\gamma x^2}}{\int\limits_0^{+\infty} f_0(x,t)e^{-\pi\gamma x^2}dx} \quad .$$

Using the expression for free growth and simplifying the same terms in the

numerator and denominator depending only on time we get

$$f_l(x,t) = \frac{x^{\frac{a}{\sigma^2}-1}e^{-\pi\gamma x^2}e^{\frac{ln^2 x}{2\sigma^2 t}}}{\int\limits_0^{+\infty} x^{\frac{a}{\sigma^2}-1}e^{-\pi\gamma x^2}e^{\frac{ln^2 x}{2\sigma^2 t}}dx},$$

where a,σ are the distribution parameters.

After a long time of development $(t \to +\infty)$ this expression tends to a limit distribution:

$$f_l(x,\infty) = \frac{x^{\frac{a}{\sigma^2}-1}e^{-\pi\gamma x^2}}{\int\limits_0^{+\infty} x^{\frac{a}{\sigma^2}-1}e^{-\pi\gamma x^2}dx},$$

in which we can see the known chi-distribution. Taking a roundish shape of the lakes into account, we get that the gamma-distribution is the limit distribution for the lake area (**sl**):

$$f_{sl}(x,\infty) = \frac{\gamma^{\frac{a}{2\sigma^2}}x^{\frac{a}{2\sigma^2}-1}e^{-\gamma x}}{\Gamma\left(\frac{a}{2\sigma^2}\right)},$$

where Γ(x) is the gamma function.

The spatial distribution of lakes is the Poisson one, as well as in the model of the lacustrine thermokarst plains during the whole development period. It results from the first assumption of the model. However, the average density of lake location has been continuously decreasing due to their turn into khasyreis. It depends on the probability that the lake does not reach a size more significant than the distance to the nearest source of a stream and is equal to

$$\tau_l(t) = \tau_l^0\int\limits_0^{+\infty} \frac{1}{\sqrt{2\pi\sigma}\sqrt{t}}x^{\frac{a}{\sigma^2}-1}e^{-\pi\gamma x^2}e^{\frac{ln^2 x}{2\sigma^2 t}}dx.$$

Thus, the mathematical analysis shows that in this case, we get stable shares between quantitates of thermokarst lakes and khasyreis of different size:
- the khasyrei radii distribution after a long enough time of the development should obey the Rayleigh distribution:

$$F_h(x,\infty) = 1 - e^{-\pi\gamma x^2},$$

- the khasyrei area distribution after a long enough time of the development should obey the exponential distribution:

$$F_{sh}(x,\infty) = 1 - e^{-\frac{x}{q}},$$

The spatial distribution of lakes obeys the Poisson law during the whole time of their development; however, the average location density of lake continually decreases due to their turn into khasyreis.

Let us consider the second variant, i.e., the variant of the asynchronous start. In this case, we expect the continuous appearance of new initial thermokarst depressions (germs of thermokarst lakes) with the constant density of appearance at that. This approach is based on the fact that we can observe new thermokarst lakes within khasyreis. The roundish shape of these lakes indicates that they are not relict parts of the previously drained thermokarst lake but new thermokarst foci.

In case of the asynchronous start the model for lacustrine thermokarst plains with fluvial erosion in uniform nature environment is based on the following assumptions:

1. The appearance of initial thermokarst depressions during non-overlapping time intervals (Δt) and at non-overlapping sites (Δs) are independent random events; the probability for a depression to appear depends only on the duration of the interval and the site area:

$$p_1 = \lambda \Delta s \Delta t + o(\Delta s \Delta t),$$

where λ is a parameter.

2. New lakes do not appear within the already existing thermokarst lakes.

3. The radius of an appeared thermokarst depression is a random variable being a time function; it does not depend on other lakes, and the growth rate is directly proportional to heat losses through the side surface of the lake basin.

4. In the course of its growth, a lake can turn into a khasyrei after draining by the erosion network; the probability of this does not depend on the development of other lakes; if it happens, the depression stops growing.

5. The appearance of new sources of fluvial erosion within a randomly selected area is a random event, and the probability of this depends on the area only.

These assumptions differ from that of the previous model because:
* the constant average rate of new lake generation
* new lakes do not appear within the already existing lakes

It is evident that in the case of an asynchronous start, we also have preconditions for dynamic balance resulting from the development of the landscape morphological pattern under consideration.

The mathematical analysis of this situation is more sophisticated than the previous one, but it gives us a set of conclusions (Victorov 2005):
• after a sufficiently long period $(t \to +\infty)$ since the start of the process a particularly stable state appears under a broad spectrum of conditions;
• a dynamic equilibrium is established between processes of thermokarst lake origination and their turn into khasyreis. Stable quantitative ratios characterize this state of a dynamic balance:
• the limit value for the lake spacing density is

$$\eta(\infty) = -\frac{\lambda}{2a} Ei(-\pi\gamma),$$

where a, λ are parameters, γ is an average spacing density of erosion sources; $Ei(x)$ is an integral exponential function.
• the limit value of the thermokarst impact is

$$P(\infty) = 1 - \exp\left[-\frac{\lambda}{2a\gamma}\right]e^{-\pi\gamma};$$

• the limit radii distribution for khasyreis is

$$F_h(x) = 1 - e^{-\pi\gamma x^2}.$$

The spatial distribution of lakes obeys the Poisson distribution throughout the whole course of the development. However, the average lakes location densityvaries because of their constant generation and turning into khasyreis.

DISCUSSION

Empirical testing of the models

We empirically tested some of the modeling results at specific key sites located in a different natural environment and permafrost condition (Fig. 2).

The key sites are within different natural and permafrost environments; they are usually characterized by various marine and

alluvial–moraine deposits with different ice content and temperature of permafrost soil.

We have tested the conformity of the khasyrei area empirical distribution to the theoretical exponential distribution (Fig. 3, Table 1) at all the three key sites with sample volumes from 73 to 122. We verified the results of the comparison using Pearson's fitting criterion. Table 1 demonstrates an accord between empirical data and the exponential distribution at the significance level 0.99.
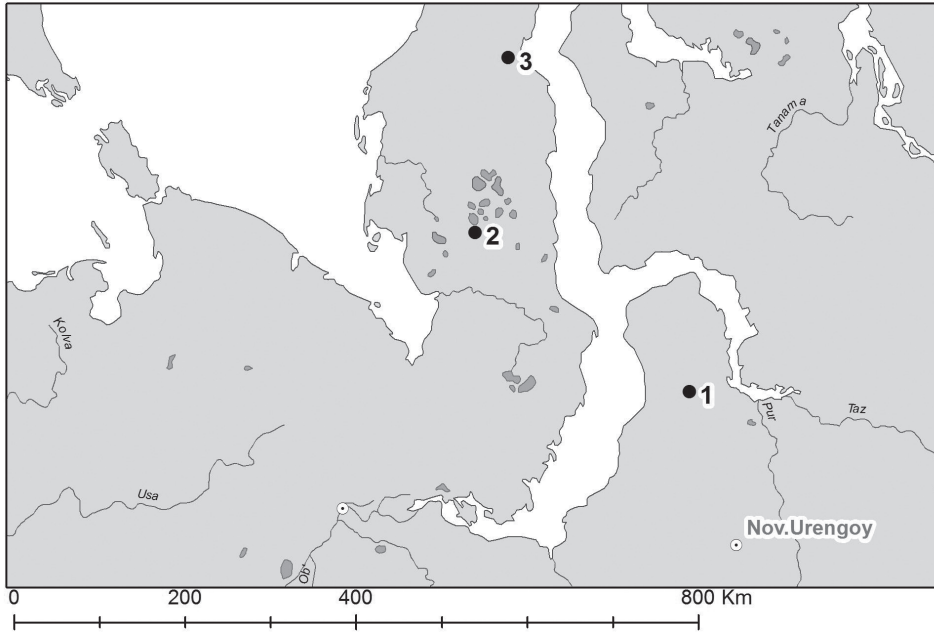

Fig. 2. The overview map of the location of key sites within thermokarst plains with fluvial erosion
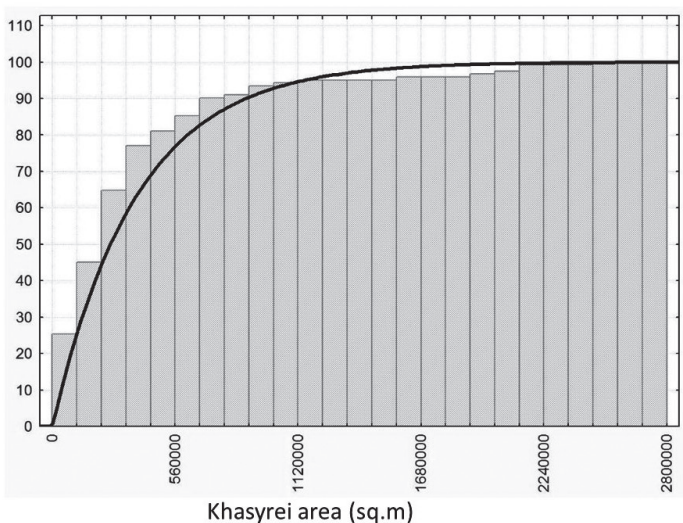
Number of observations
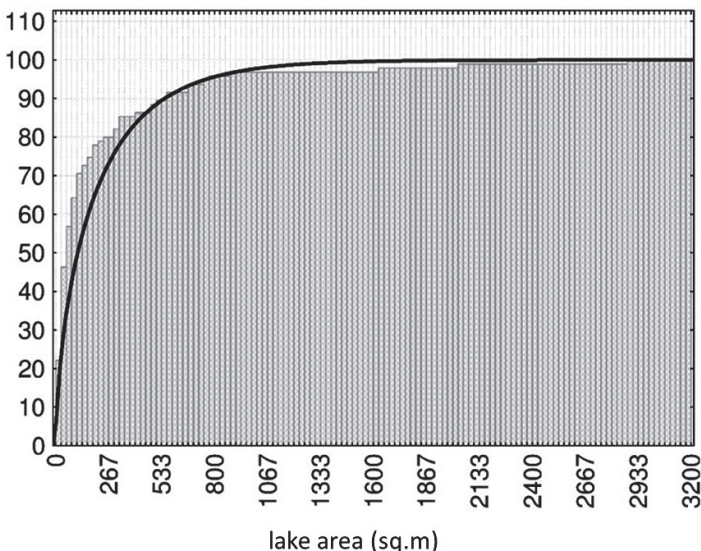

Khasyrei area (sq.m)

Fig. 3. The correspondence of the distribution of the khasyrei area (sq.m) to the exponential distribution (an example)

Table 1. The correspondence of the distribution of the khasyrei area to the theoretical distributions (Pearson's criterion) for thermokarst plains with fluvial erosion

Key sites	The volume of the sample	The theoretical distribution	P value
Site 1	73	The exponential distribution The lognormal distribution The normal distribution	0.191 0.044 0.000
Site 2	122	The exponential distribution The lognormal distribution The normal distribution	0.396 0.353 0.000
Site 3	76	The exponential distribution The lognormal distribution The normal distribution	0.723 0.012 0.000

At these key sites with sample volumes from 53 to 95 we have also investigated the accord between empirical data on the distribution of thermokarst lake areas and gamma distribution, as it is correct for the synchronous start. The obtained data (Fig. 4, Table 2) show that the empirical data do not contradict the suggested model at the significance level 0,99. At the same time, there is a similarity with the lognormal distribution. We regard this situation as pointing on the synchronous start because in this case, an area develops at the first stage as a lacustrine thermokarst plain due to little influence of fluvial erosion until the lakes have not grown in size.

When the lakes become large enough, the situation changes and the lake area distribution tends to the limit gamma distribution, and the appearing khasyreis tends to the exponential distribution, but due to the finite time, the distribution of the lake and khasyrei areas are not exactly equal to them. It should be stressed that according to our research the area distribution of thermokarst lakes within the lacustrine thermokarst plains (without fluvial erosion) usually corresponds to the lognormal distribution while only a few measurements agree with the gamma distribution (Table 3).

Number of observations

lake area (sq.m)
Fig. 4. The correspondence of the distribution of the thermokarst lake area (sq.m) to the gamma-distribution (an example)

Table 2. The correspondence of the distribution of the thermokarst lake area to the gamma-distribution (an example)

Key sites	The volume of the sample	The theoretical distribution	P value
Site 1	95	The gamma-distribution The lognormal distribution The normal distribution	0.016 0.241 0.000
Site 2	53	The gamma-distribution The lognormal distribution The normal distribution	0.051 0.206 0.000
Site 3	93	The gamma-distribution The lognormal distribution The normal distribution	0.082 0.012 0.001

Table 3. The correspondence of the thermokarst lake area distribution to the theoretical distributions (Pearson's criterion) for lacustrine thermokarst plains (without fluvial erosion)

Key site	The volume of the sample	p-value For the lognormal distribution	For the gamma-distribution
Alaska 2	108	0.112	0.000
Taimyr1	345	0.112	0.007
Taimyr2	209	0.631	0.330
Kolyma 1	154	0.216	0.023
Kolyma 2	576	0.000	0.000
Ust'-Lena1-1	145	0.011	0.000
Ust'-Lena1-2	91	0.155	0.006
Ust'-Lena1-3	383	0.00	0.000
Ust'-Lena2	167	0.006	0.000
Yamal1	209	0.00	0.259
Yamal2	176	0.012	0.005
Alaska 1	100	0.023	0.004
West-Siberian2	84	0.088	0.000
Canadian	154	0.127	0.009
West-Siberian1	78	0.587	0.000
Gydansky	74	0.517	0.000

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CONCLUSIONS

Thus, the described research gave us the following results:

- After a long enough time of the development thermokarst plains with fluvial erosion come to the state of dynamic balance. For an asynchronous start, we have confirmed it under a specific broad range of conditions. It reveals in establishing and preserving stable quantitative relationships among elements of the morphological pattern of this landscape.

- This state characterizes with the correspondence of the probabilistic distribution of khasyrei area to the exponential distribution, and their radii to the Rayleigh one despite synchronous or asynchronous start.

- The probabilistic distribution for areas of the thermokarst lakes obeys the gamma-distribution in the case of the synchronous start and a particular distribution in the case of the asynchronous start.

- In case of the asynchronous start the state of dynamic balance results in the stabilization of both the lake location density and the share of lake area within the landscape.

- We are aware that the obtained results need additional empirical testing, but we regard them to be a promising approach.

- The same approach, which we used in the analysis of the lacustrine thermokarst plains, can be involved for assessment of the impact on a linear engineering structure, for instance, at thermokarst plains with fluvial erosion.

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