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Analytical Model of Thermo-electrical Behaviour in Superconducting Resistive Core Cables

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High field superconducting Nb_3Sn accelerators magnets above 14 T, for future High Energy Physics applications, call for improvements in the design of the protection system against resistive transitions. The longitudinal quench propagation velocity (vq) is one of the parameters defining the requirements of the protection. Up to now vq has been always considered as a physical parameter defined by the operating conditions (the bath temperature, cooling conditions, the magnetic field and the over all current density) and the type of superconductor and stabilizer used. It is possible to enhance the quench propagation velocity by segregating a percent of the stabilizer into the core, although keeping the total amount constant and tuning the contact resistance between the superconducting strands and the core. Analytical model and computer simulations are presented to explain the phenomenon. The consequences with respect to minimum quench energy are evidenced and the strategy to optimize the cable designed is discussed.

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Analytical model of thermo-electrical behaviour in Superconducting Resistive Core Cables

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Abstract— **High field superconducting Nb_3Sn accelerators magnets above 14T, for future High Energy Physics applications, call for improvements in the design of the protection system against resistive transitions. The longitudinal quench propagation velocity (vq) is one of the parameters defining the requirements of the protection. Up to now vq has been always considered as a physical parameter defined by the operating conditions (the bath temperature, cooling conditions, the magnetic field and the over all current density) and the type of superconductor and stabilizer used. It is possible to enhance the quench propagation velocity by segregating a percent of the stabilizer into the core, although keeping the total amount constant and tuning the contact resistance between the superconducting strands and the core. Analytical model and computer simulations are presented to explain the phenomenon. The consequences with respect to minimum quench energy are evidenced and the strategy to optimize the cable designed is discussed.**

***Index Terms*—Superconducting Cables, Quench Propagation Velocity, Stability.**

I. Introduction

THE addition of copper as separate strands at the cabling step has been utilized in the past as a method for grading conductors, in order to provide the normal metal shunt path for magnet protection. Recently, another incentive for utilizing this approach was realized as a result of conductor cost studies performed as part of the HEP Conductor Development Program. The labor cost factor for wire fabrication depends directly on the volume of wire being produced. Thus, if the copper necessary for magnet protection can be added after wire fabrication is complete, wire costs will be reduced significantly.

Several alternative methods have been proposed for adding copper at the cabling stage. These include adding pure Cu strands to the cable, adding Cu as a core in the cable, or wrapping Cu strip around the finished cable. However, a number of questions must be answered before this approach is adopted for use in accelerator magnets. The manufacturability and overall conductor cost study have been already investigated and the results presented in [1]. In the following the analysis on the thermo-electrical dynamics of the cable during a resistive transition is presented by means of analytical models, to better point out the interesting properties of such cables (to be compared to the work already done on super-stabilized cables [2]). The results are compared with more sophisticated numerical simulations [3], carried out both with simplified equivalent circuits and full scale cables [4]. Finally the procedure for a possible design optimization of the cable is presented, based on protection, stability and cost issues.

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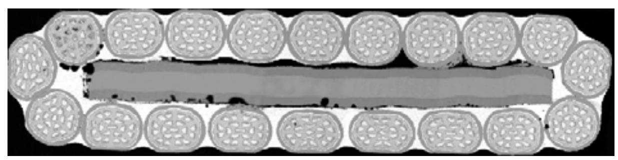

Figure 1 The picture of a prototype cable developed at LBNL. This is the cable design used as a reference for the thermo-electrical analysis.

II. Cable Description

The most promising cable design is sketched in Figure 1. It consists of Nb_3Sn superconducting strands wrapped on a rectangular core of pure copper. The electrical and thermal contact between the central core and the superconducting strands is tuned by means of two thin layers of stainless-steel. The current redistribution induced by a resistive transition between the superconducting strands and the core is a very effective mechanism to enhance the quench propagation velocity. The electrical contact (G) and the ratio between the overall copper and the one segregated in the core (μ) are the key parameters to tune this effect to have the most suitable value of quench propagation velocity needed for magnet protection against quenches.

III. The Analytical Model

The model implemented to estimate the quench propagation velocity in resistive core cables consists of two coupled partial differential equations, one for describing the diffusion of the current among the two components and the other for estimating the evolution of the temperature along the cable. The main approximations introduced to simplify the mathematics are:

1. neglecting the current sharing regime, thus reducing T_c to T_{cs} and vice versa;
2. neglecting the temperature dependence of the material parameters and evaluating their values in the temperature range around the propagating front

3. considering the superconductor in thermal equilibrium with the resistive core in any cross section (this hypothesis is not essential to analytically solve the equations, see conclusions);

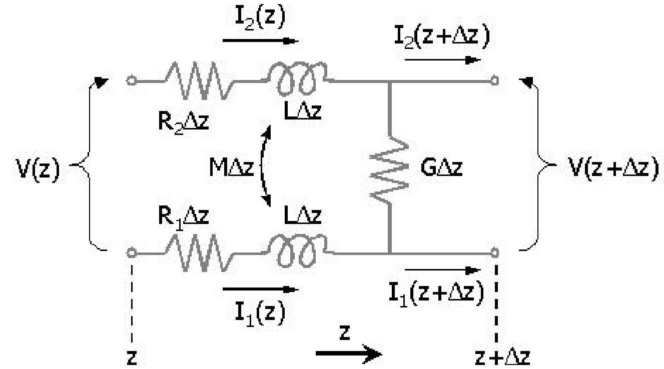

Figure 2 The electrical network implemented for the estimation of the current redistribution between the superconducting strands (R2) and the central core (R1).

A. The electrical equations

To evaluate the current distribution in between the copper core and the superconducting elements, a distributed two wires electrical circuit has been considered (see Figure 2). Writing the equations of the voltage and current for $\Delta z \rightarrow 0$, the final equation is the following

$$L\dot{I} = \frac{1}{G}\frac{\partial^2 I}{\partial z^2} - \left(R_1(z) + R_2(z)\right)I + R_2(z)I_0, \tag{1}$$

where I is the current in the core ($I=I_1$) and I_0 is the total current in the cable ($I_1+I_2=I_0$). L is the linear inductance (L_1+L_2-2M), G is the transversal electrical conductance per unit length between the superconducting and the resistive core. R_1 and R_2 are the linear resistances of the resistive core and of the superconducting elements respectively. Being interested in the propagation of the normal conducting zone, the solutions of the (1) are expected to have this form $I(z,t)=I(z-v_q t)$. Substituting into the (1)

$$\frac{\partial^2 I}{\partial z^2} + LGv_q\frac{\partial I}{\partial z} - G\left(R_1(z) + R_2(z)\right)I + GR_2(z)I_0 = 0\cdot \tag{2}$$

Considering two separate spatial domains, one for $z<0$ where temperature of the cable, T, is above T_c and one for $z>0$, $T<T_c$, and assuming R_1 and R_2 temperature independent, eq. (2) gets

$$\begin{cases} \dfrac{\partial^2 I}{\partial z^2} + \dfrac{1}{\lambda_0}\dfrac{\partial I}{\partial z} - G(R_1 + R_2)I + GR_2 I_0 = 0 & z<0 \\ \dfrac{\partial^2 I}{\partial z^2} + \dfrac{1}{\lambda_0}\dfrac{\partial I}{\partial z} - GR_1 I = 0 & z>0 \end{cases} \tag{3a}$$

where

$$\lambda_0 = \frac{1}{LGv_q}\cdot \tag{3b}$$

Looking for a bounded function and imposing the continuity and the derivability of $I(z)$ in $z=0$, the problem is fully defined. The current may be expressed with the following formula:

$$I(z) = \begin{cases} \Delta I_0\left[1 - \dfrac{\lambda_n}{\lambda_n + \lambda_s}\exp(z/\lambda_n)\right] & z<0 \\ \Delta I_0\dfrac{\lambda_s}{\lambda_n + \lambda_s}\exp(-z/\lambda_s) & z>0 \end{cases}, \tag{4a}$$

where

$$\Delta I_0 = \frac{R_2}{(R_1 + R_2)}I_0, \tag{4b}$$

$$\frac{1}{\lambda_n} = \frac{1}{2}\left(-\frac{1}{\lambda_0} + \sqrt{\frac{1}{\lambda_0^2} + 4G(R_1 + R_2)}\right), \tag{4c}$$

$$\frac{1}{\lambda_s} = \frac{1}{2}\left(\frac{1}{\lambda_0} + \sqrt{\frac{1}{\lambda_0^2} + 4GR_1}\right)\cdot \tag{4d}$$

λ_n and λ_s are respectively the characteristic length of the current distribution in the normal and superconducting side of the cable. The total power dissipated per unit length can be evaluated with the following general expression:

$$P(z) = \begin{cases} R_1 I^2 + R_2(I_0 - I)^2 + \dfrac{1}{G}\left(\dfrac{\partial I}{dz}\right)^2 & z<0 \\ R_1 I^2 + \dfrac{1}{G}\left(\dfrac{\partial I}{dz}\right)^2 & z>0 \end{cases}, \tag{5}$$

where $R_1 I^2$ is the power dissipated by Joule effect in the copper core, $R_2(I_0-I)^2$ is the power dissipated in the superconductor when $T>Tc$ and $(dI/dz)^2/G$ is the power dissipated by current redistribution from the superconducting elements into the copper core through the electrical contacts. Introducing the expression of $I(z)$ evaluated in the (4a) into the (5) the power dissipated per unit length gets:

$$P(z) = \begin{cases} p_0 + p_n\exp(2z/\lambda_n) & z<0 \\ p_s\exp(-2z/\lambda_s) & z>0 \end{cases} \tag{6a}$$

where

$$p_0 = R_{12}I_0^2,\ p_n = \alpha\left(1 + GR\lambda_n^2\right),\ p_s = \alpha\left(1 + GR_1\lambda_s^2\right) \tag{6b}$$

$$\alpha = \frac{I_0^2 R_2^2}{GR^2\lambda_e^2} \tag{6c}$$

where $\lambda_e = \lambda_n + \lambda_s$, $R_{12}=R_1//R2$ and $R=R_1+R_2$.

Due to the current redistribution the total dissipated power is

$$\Delta P = \tfrac{1}{2}\left(p_n\lambda_n + p_s\lambda_s\right)\cdot \tag{7}$$

The knowledge of the power dissipated in each cross section as a function of the steady propagation of the normal conducting front, is the information needed to estimate the actual v_q developing the thermal associated problem.

B. The thermal equations

The heat balance equation, which describes the temperature evolution in any cross section of the cable can be in general expressed as follow,

$$Ac(\theta)\dot{\theta} = A\frac{\partial}{\partial z}\left(k(\theta)\frac{\partial\theta}{\partial z}\right) + P(z) \tag{8}$$

θ is the mean temperature in a cable cross section, c and k are respectively the average linear heat capacity and the heat conductivity of the materials composing the cable cross section. $P(z)$ is the power dissipated per unit length, previously evaluated in (6). Considering again the material parameters as temperature independent, the (8) simplifies into

$$Ac\dot{\theta} = Ak\frac{\partial^2\theta}{\partial z^2} + P(z)\cdot \tag{9}$$

Looking again for stationary solution like $\theta(z, t)=\theta(z - v_q t)$, the equation gets finally

$$\frac{\partial^2\theta}{\partial z^2}+\frac{cv_q}{k}\frac{\partial\theta}{\partial z}+\frac{P(z)}{Ak}=0\cdot \quad (10)$$

Substituting the (6) into the (10) one can obtain

$$\begin{cases}\dfrac{\partial^2\theta}{\partial z^2}+\dfrac{1}{\lambda_T}\dfrac{\partial\theta}{\partial z}+\tilde{p}_0+\tilde{p}_n\exp(2z/\lambda_n)=0 & z<0\\ \dfrac{\partial^2\theta}{\partial z^2}+\dfrac{1}{\lambda_T}\dfrac{\partial\theta}{\partial z}+\tilde{p}_s\exp(-2z/\lambda_s)=0 & z>0\end{cases} \quad (11)^1$$

where

$$\lambda_T=\frac{k}{cv_q} \quad (12)$$

is the characteristic length of the temperature profile. The solution of the (11) has the following general form

$$\theta(z)=\begin{cases}\theta_0\exp(-z/\lambda_T)+\theta_1+\theta_P^-(z) & z<0\\ \theta_2\exp(-z/\lambda_T)+\theta_3+\theta_P^+(z) & z>0\end{cases}, \quad (13a)$$

where the particular solutions are

$$\theta_P^-(z)=b_0z+b_1\exp(2z/\lambda_n), \quad (13b)$$

$$\theta_P^+(z)=b_2\exp(-2z/\lambda_s). \quad (13c)$$

Injecting the expression of the (13) inside the (11), the following values are obtained for the coefficient b_0, b_1, b_2,

$$b_0=-\tilde{p}_0\lambda_T,\; b_1=-\frac{\tilde{p}_n\lambda_n^2\lambda_T}{2(\lambda_n+2\lambda_T)},\; b_2=\frac{\tilde{p}_s\lambda_s^2\lambda_T}{2(\lambda_s-2\lambda_T)} \quad (14)$$

To have a bounded solution θ_0 should be null. The following conditions are applied:

$$\theta(0^-)=\theta(0^+)=T_c \quad (15a)$$

$$\theta(+\infty)=T_b\cdot \quad (15b)$$

Equations (13a) both define the value of $\theta(z)$ in $z=0$ and imply its continuity. Finally $\theta_1=-b_1+T_c$ and $\theta_2+\theta_3=T_c-b_2$. With the (13b) it gets $\theta_3=T_b$ and $\theta_2=T_c-b_2-T_b$. Implying the continuity of the derivative of $\theta(z)$ in $z=0$, the following equation is derived:

$$b_0+\frac{2b_1}{\lambda_n}+\frac{2b_2}{\lambda_s}+\frac{\theta_2}{\lambda_T}=0\cdot \quad (16)$$

The only free parameter left in (16) is the quench propagation velocity. Solving (16) gives the values of v_q as a function of the whole set of parameters previously introduced. The results are presented in Figure 3. As already anticipated, the quench propagation velocity increases when the electrical contacts between the core and the superconducting strands increases. This is due to the increasing power dissipated around the propagating front, see Figure 4. Also, the larger is the ratio between the copper in the core with respect to the total amount of copper (μ), the larger is the quench propagation velocity. This is simply related to the increased amount of current redistributed (R_{12}) and consequently power dissipated around the front. The current profile inside the core gets more spread as the electrical conductance decreases. It is interesting to note that for infinite electrical conductance even if the profile gets sharper, in the normal conducting part the redistribution length does not get to zero and its value is related to the inductive term $\lim_{G\to+\infty}\lambda_n=\frac{Lv_q}{R}$. While in the superconducting part $\lim_{G\to+\infty}\lambda_s=0$ gets to zero. This is the reason why the total power dissipated by the redistribution is not zero, even considering a perfect electrical contact. The function (6) which describes the power dissipated along the cable has a discontinuity in the front (z=0) and this should not surprise since the hypothesis includes a sharp discontinuity between the super-conducting phase and the normal one.

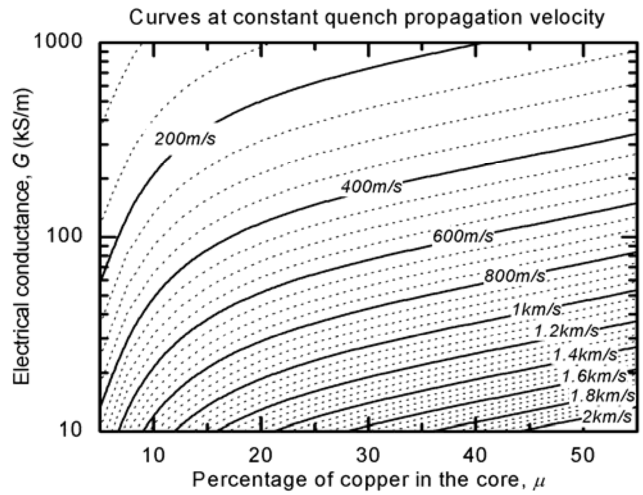

Figure 3 The quench propagation velocity in copper core cable for different electrical contacts and different copper core dimensions, keeping the total amount of copper constant.

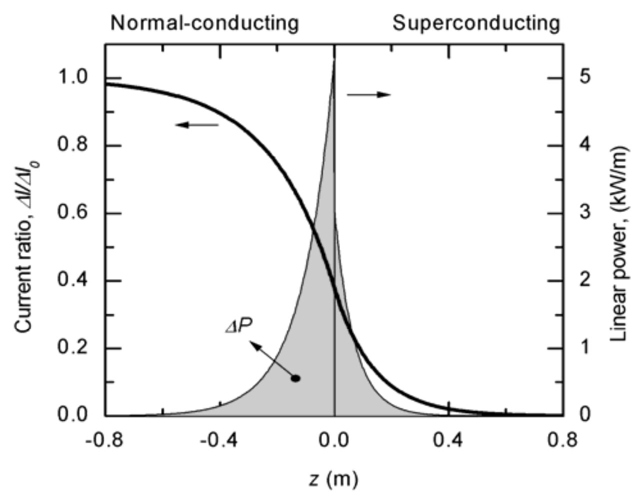

Figure 4 The current profile (on the left axis) and the linear power dissipated (right axis) induced by the redistribution of current.

TABLE 1 CALCULATION PARAMETERS

number of strands	22	
strand cross section	0.3848	mm^2
Copper to non copper ratio	0.81	
Core cross section	1.95	mm^2
Tb	4.2	K
Copper core *RRR*	250	
Copper strands *RRR*	50	
Current	20	kA
Magnetic field	8	T

[1] $\tilde{p}_i\equiv p_i/(Ak)$

IV. COMPARISON WITH COMPUTER SIMULATIONS

Computer simulations have been performed to validate the analytical model, see Figure 5. The details of the computation can be found in [4]. Computer simulations have the advantage of also taking into account the non linearity of the system and several details that we neglected in the analytical approach. On the contrary the simulation turn over is more time consuming and the results must be validated by a careful sensitivity study to the integrated parameters like the time step and the mesh.

Despite the high non linearity present in such system the main phenomenon, i.e. the enhancement of the quench propagation velocity, is well described by the linear part of the equation. This explains the good agreement between the analytical and simulation approaches.

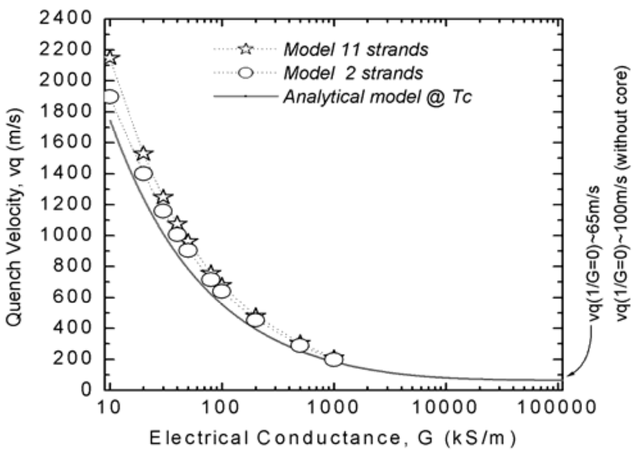

Figure 5 Comparison between the simulated [4] quench propagation velocity as a function of the electrical conductance and the value provided by the analytical model

V. TOWARDS DESIGN OPTIMIZATION

The parameters chosen for the cable design optimization are the quench propagation velocity, the stability margin and the overall cost of the cable. The energy density (*Qe*) to quench such a cable in a regime of high current (70%-90% of the short sample limit) is not affected by the contact resistance between the strands and the c ore and nor by the ratio between the core and the strand overall dimension. On the contrary the quench energy (*QE*) is proportional to the enthalpy of the superconducting strands. This can be explained simply as the copper in the core does not play an active role for stability because the current cannot be efficiently shared with the core to prevent a quench. The cost (c) of the cable is defined by the amount of superconductor (c_0) and the ratio (μ) between the copper segregated in the core and the over all copper. The cost of the segregated copper is negligible with respect to the one co-processed with the superconductor while it has a much higher quality (i.e. elevated *RRR*). All these evidences can be formalized with the following systems of scaling laws:

$$v_q \propto \mu, 1/G, \qquad (17a)$$

$$QE \propto (1-\mu), \qquad (17b)$$

$$c = c_0 + \beta(1-\mu) + \gamma\mu \quad (\beta >> \gamma). \qquad (17c)$$

Equation (17) gives the essential knowledge to tune the resistive core-cable towards the most suitable design for the specific application.

TABLE 2 LIST OF SIMBOLS

Simbol	*Description*
A	Total cable cross section (m^2)
G	Electrical conductance per unit length (S/m)
I_0	Total current in the cable (A)
$I_1(z)$	Current in the superconducting component (A)
$I_2(z)$	Current in the core (A)
L	Linear inductance (Henry/m)
λ_s , λ_n	Characteristic length of current redistribution in the superconducting side ($z>z_0$) and in the normal conducting one ($z<z_0$) (m)
λ_T	Characteristic length of temperature profile (m)
$P(z)$	Linear power dissipated (W/m)
R_1, R_2	Liner resistivity (Ω/m)
Tc,Tcs	Critical temperature, current sharing temp. (K)
$\theta(z)$	Temperature profile (K)
v_q	Quench propagation velocity (m/s)

VI. CONCLUSIONS

An analytical model to predict the quench propagation velocity in resistive core cable has been introduced and its validity have been checked against sophisticated numerical simulations [4]. The enhancement of the quench propagation velocity has been demonstrated to be mainly related to the current redistribution between the superconducting strands and the core which can be well described by the linear part of the system. The cost of the cable and the requirements of the protection system are reduced while the core dimension is increased and the contact resistance enhanced. At the same time the perturbation spectrum, which characterizes the operation of the cable should be taken into account. These two requirements should lead to an optimum design.

Further investigation is ongoing to model the impact of the thermal conductance (G_{th}) between the superconducting strands and the copper core which can introduce limitation to the efficiency of the described propagating mechanism. Simplified analytical formulas of the quench propagation velocity are expected, mainly in the approximation of high/low velocity regimes and negligible inductance.

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