Reliability Improvement and Assessment of Safety Critical Software

by

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Abstract

In order to allow the introduction of safety-related Digital Instrumentation and Control (DI&C) systems in nuclear power plants, the software used by the systems must be demonstrated to be highly reliable. The most widely used and most powerful method for ensuring high software quality and reliability is testing. An integrated methodology is developed in this thesis for reliability assessment and improvement of safety critical software through testing. The methodology is based upon input domain-based reliability modeling and structural testing method. The purpose of the methodology is twofold: Firstly it can be used to control the testing process. The methodology provides path selection criteria and stopping criteria for the testing process with the aim to achieve maximum reliability improvement using available testing resources. Secondly, it can be used to assess and quantify the reliability of the software after the testing process. The methodology provides a systematic mechanism to quantify the reliability and estimate uncertainty of the software after testing.

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Chapter 1

Introduction

1.1 Motivation of the Study

In Nuclear Power Plants (NPPs), the current Instrumentation and Control (I&C) system is based upon analog technologies. Analog devices are prone to aging problems, need to be calibrated regularly, are becoming obsolescent and their replacement parts are more and more difficult to find as the outside industries are quickly digitized. In safety critical systems, use of digital computers provides many potential advantages including improved availability, easier maintenance, reduced installation cost, ease of modification and potential for reuse. Many nuclear utilities are trying to replace their obsolescent analog control systems with Digital Instrumentation and Control (DI&C) systems. The next generation of NPPs in the United States is expected to be completely computerized and have an integrated DI&C system.

Despite of all the advantages the DI&C system may offer, currently the acceptance of DI&C system in the nuclear industry has been greatly retarded. The reason is mainly due to the difficulties of verification and validation (V&V) of the digital control software, and the difficulties of assessment of the reliability of the digital control software. The DI&C system introduces new failure modes (i.e., software failures), into the control system. Unlike that of the hardware, software failures are exclusively caused by design errors. They are common-cause failures and can not be reduced by adopting redundancy principles. The most widely used and most powerful method for attacking software design errors is testing. The software is tested against its design specification, which is a part of the verification and validation process. Comparing results of the test cases with the oracle can reveal errors in the software. If the software can be exhaustively tested, its reliability can be proven to be unity. However, exhaustive testing can seldom be achieved in practice due to the complexity of the software, the limited time

and resources available. Consequently two problems exist: How to control the testing process to achieve maximum testing efficiency, i.e., to achieve maximum reliability improvement using the limited testing resources; How to assess the reliability of the improved, while still not perfect software after testing? Assessment and quantification of software reliability play a central role in ensuring high software quality, which is especially important for the safety-critical control software in nuclear industry applications. If the reliability of the software can be assessed and quantified with high confidence, it can then be compared with that of the hardware, which can be readily quantified using current technologies, to show that new digital systems are sufficiently reliable. Wide acceptance of DI&C system and substantial improvements in nuclear power safety and economics would be likely.

In the current practice, the most widely used models for software reliability assessment and qualification are time domain-based software reliability growth models. For a given software, an appropriate reliability growth model is first chosen for use, the parameters in the model are determined by fitting testing data into the model, the fitted model is then used for reliability assessment and prediction. Reliability growth models do not require any structural information of the target software. Only testing data is needed for their application, which makes them applicable for almost all types of software. They have been applied to safety-related software in many fields, notably aerospace applications. However, their predictions had high uncertainties and were not satisfactory. The reason is mainly due to the inherent limitations of growth models, e.g., they take no consideration of the structure of the target software, and they have no rigorous mechanism of evaluating the uncertainties associated with their predictions. The target software to which the method had been applied typically had very high complexity, which made accurate assessment of the software reliability even more difficult.

Another type of model for software reliability assessment and qualification is input domain-based. These models consider the software as a mapping from inputs to outputs. The software reliability is defined as the ratio of the number of "good" inputs (i.e., the inputs that are

mapped to the correct outputs), to the number of total inputs. As we see in **Chapter 2**, input domain-based model testing using certain testing strategies (e.g., path testing) has several advantages over reliability growth models and is better suited for application to safety critical software. However, the complexity of their implementation is typically much higher than that of growth models, which hinders their wide and successful applications.

Typical nuclear control software is small in size, and does not have complex data or control structures. The greater simplicity of nuclear software offers the potential for greater success. Software reliability quantification may be achieved with higher accuracy and higher certainty with the introduction of new methodologies, which is explored in the work reported here.

1.2 Goal of the Study

The goal of this work is to develop an integrated methodology for reliability assessment and improvement of safety critical software. The methodology is based upon an input domain-based reliability model using a path testing strategy. It takes the detailed structures of the target software into consideration aiming to provide accurate estimate of software reliability. The purpose of the methodology is twofold: First, it can be used to control the testing process. The methodology provides path selection criteria and stopping criteria for the testing with the aim to achieve maximum reliability improvement using limited testing resources. Secondly, it can be used to assess and quantify the reliability of the software after testing. We wish to show that, at least for some simple software, which is typical of nuclear control software, high reliability can be achieved using the methodology we developed.

1.3 Organization of the Thesis

Chapter 2 presents a literature survey of the currently available software reliability models, namely, the reliability growth models, the fault seeding models, and the input domain-based models. For each class of the models, a few representative models are first briefly discussed, and then references are given for some other models in the class. Finally the advantages and limitations associated with each class of models are discussed.

Chapter 3 presents the elements of the proposed integrated methodology. First the underlying reliability model of the integrated methodology is presented. Then for each element of the methodology, its implementation, the advantage and limitations of its implementation, the difficulties associated with its implementation, and the possibility for its automation are discussed in detail. In the summary section, an overall evaluation of the methodology is made. The potential applications, advantages, and limitations of the methodology are discussed.

In **Chapter 4**, a case study is performed applying the integrated methodology to a sample program to show its appicability. Due to the limited time and resources available, only part of the methodology is applied in the case study. Results of the case study are assessed and implications discussed.

Chapter 5 serves as a summary of the thesis. Major findings in this research are summarized and conclusions are drawn. Directions for future work are presented.

Chapter 2

Study of the Current Practice

2.1 Introduction

This chapter summarizes results from a literature search on the current practices in software reliability assessment. **Section 2.2** presents some fundamental concepts of reliability theory as an introductory step. The following three sections discuss three classes of reliability models that have been developed for software reliability estimation, i.e., software reliability growth models, fault seeding models, and input domain-based reliability models, respectively. For each class, a few representative models in the class are first briefly discussed, and then references are given for some other models in the class. It is not intended to be an exhaustive survey, and its main purpose is to help the reader understand the underlying process. Finally the advantages and limitations associated with each class of models are discussed.

2.2 Fundamentals of Reliability Theory

Before we discuss software reliability estimation methods, it is necessary to introduce many terms and definitions related to reliability theory. Reliability theory is essentially the application of probability theory to the modeling of failures and the prediction of success probability.

2.2.1 Reliability Mathematics

Modern probability theory is based upon random variables, the probability density functions, and the cumulative probability distribution functions. In the case of reliability, the random variable of interest is the time to failure, **t**. The probability that the time to failure, **t**, lies in some interval $(t_1, t_1 + \Delta t)$ is:

$$P(t_1 \le t \le t_1 + \Delta t) = f(t_1)\Delta t = F(t_1 + \Delta t) - F(t_1) \quad , \tag{2.1}$$

where

$f(t_1)$ = value of the failure probability density function at time t_1,

$F(t_1)$ = value of the cumulative probability distribution function at time t_1.

If we divide by Δt in Eq. (2.1) and let $\Delta t \rightarrow 0$, we obtain the result:

$$f(t) = \frac{dF(t)}{dt} \quad . \tag{2.2}$$

From Eq. (2.2), the result follows:

$$F(t) = \int_0^t f(x)dx \quad . \tag{2.3}$$

Now, we can define the reliability, $R(t)$, as the probability of no failure occurring at time, t, or sooner:

$$R(t) = 1 - F(t) = 1 - \int_0^t f(x)dx \quad . \tag{2.4}$$

A useful quantity that provides a synthetic index of reliability is the so-called *Mean Time To Failure (MTTF)*. It is defined as:

$$MTTF = \int_0^{\infty} t * f(t)dt \quad . \tag{2.5}$$

Another frequently used concept is the failure rate function (hazard function), $z(t)$. It is defined as the probability that a failure occurs in some interval $(t_1, t_1 + \Delta t)$, given that system has survived up to time, t:

$$P(t_1 \le t \le t_1 + \Delta t | t > t_1) = z(t_1)\Delta t \quad . \tag{2.6}$$

After some manipulations we can obtain the result:

$$z(t) = \frac{f(t)}{R(t)} = -\frac{1}{R(t)}\frac{dR(t)}{dt} \quad , \tag{2.7}$$

and:

$$R(t) = e^{-\int_0^t z(x)dx} \quad . \tag{2.8}$$

2.2.2 Some Useful Definitions in Software Reliability Theory

A widely accepted definition of *software reliability* is the probability that the program performs successfully for a given time period in a specified environment. The random variable in this definition is the operating time, **t**, of the software. The choice of time as the argument of the random variables of interest reflects the assumption that failures occur due to random traversing of paths in the program which contain errors for some values of the input parameters. As the operating time increases, more and more paths are traversed and the probability of encountering at least one error increases (Shoo83). This definition fits into the general reliability theory in **Section 2.2.1**. All the concepts discussed in **Section 2.2.1** can be applied to software reliability as well.

A *failure* is a departure of the external results of program operation from program requirements on a run. A *fault* is a defective, missing, or extra instruction or set of related instructions that is the cause of one or more actual or potential failure types (Musa87). In this thesis another term *error* is often used interchangeably with the term *fault*.

An *operational profile* is a frequency distribution of input data that represents the typical usage of the program. An *oracle* is any means that provides sufficient information about the (correct) expected behavior of a program that it is possible to determine whether the software produced the correct output corresponding to a set of input data (Beiz90).

2.3 Software Reliability Growth Models

Software reliability growth models are time-based reliability models. They are so called because they try to predict the software's reliability evolution in the future. Software reliability growth models take failure data from a period when faults are being detected and fixed, during testing or operational use, and use this information to estimate the reliability of the program under study and predict how this reliability will change in the future.

Usually, a software growth model contains three stages. First, the model structure is selected according to some preliminary general assumption or specific hypothesis concerning the software characteristics and the testing environment. Second, the free parameters in the model are estimated by fitting the data from testing to the model. Finally, a rule is deduced for the fitted model to be used for predictive purposes (Goos88).

Software reliability growth models can be grouped into two categories: Time Between Failure Models and Failure Count Models.

2.3.1 Time Between Failure Models

In this class of models the variable under study is the time between failures. The models are based upon statistical characterization of the time intervals between successive observed failures. Let a random variable T_i denote the time between the *(i-1)*st and the *i*th failures. The most common approach is to assume that T_i follows a known distribution, whose parameters depend upon the number of faults remaining in the program after the *(i-1)*st failure. The parameters are determined from the observed values of times between failures in the testing phase. The fitted model can then be used to estimate the software reliability, mean time to failure, etc. (Goel85)

One of the earliest and probably the most widely used model in this class is the Jelinski/Moranda (JM) De-eutrophication Model (Jeli72, Goos88). The JM model assumes that $t_1, t_2, \ldots$ are independent random variables with exponential probability density functions (t_i denotes the time between the *(i-1)*st and *i*th failures), where:

$$P(t_i | z(t_i)) = z(t_i) e^{-z(t_i)t_i}, t_i > 0 \ , \tag{2.9}$$

where $z(t_i)$ is the failure rate at time, t_i:

$$z(t_i) = \phi[N - (i-1)] \ , \tag{2.10}$$

where ϕ is a proportional constant (a model parameter),

N is the total number of faults originally in the program (a model parameter).

The rationale for the model is as follows: The model assumes that the initial fault content in the software is N before testing, each fault is independent of the others and is equally likely to cause a failure during testing. During each of the interval between the *(i-1)*st and *i*th failures, a fault is detected and removed with certainty and no new faults are introduced during the

debugging process (i.e., perfect debugging). The failure rate at any time is assumed to be proportional to the current fault content, $N-(i-1)$, in the program.

The model parameters, N and ϕ, are estimated by maximum likelihood method. Predictions are made by the "plug-in" rule: substitution of the maximum likelihood estimates, $\hat{N}$ and $\hat{\phi}$, into the appropriate model expressions. For example, when $t_1, t_2, ..t_{i-1}$ are the observed data, the predicted (current) reliability is:

$$\hat{R}_i(t) = e^{-(\hat{N}-(i-1)\hat{\phi}t} \quad , \tag{2.11}$$

where $\hat{R}_i(t)$ is an estimate of $R_i(t) = P(t_i < t)$.

The most serious criticism of this model is that it assumes perfect debugging and that all faults contribute the same amount to the overall failure rate. It is argued by Littlewood (Litt81) that the different frequencies of execution of different portions of code will in themselves lead to different rates of fault occurrence, all other factors being equal.

Many other models in this class are extensions of the JM model with some modifications. The Schick/Wolverton model (Schi78) is based upon the same assumptions as that of the JM model except that the failure rate function is assumed to be proportional to the current fault content of the program as well as to the time elapsed since the last failure:

$$z(t_i) = \phi[N-(i-1)]t_i \quad . \tag{2.12}$$

The model implies a linearly increasing time interval between failures, which is difficult to justify.

Another extension of the JM model is the Goel/Okumoto imperfect debugging model (Goel78). In this model, the number of faults in the system at time t, $X(t)$, is treated as being governed by a Markov process where the software can change its failure population state randomly. In this process these transition probabilities are governed by the probability of imperfect debugging. Times between the transitions of $X(t)$ are taken to be exponentially

distributed with rates dependent upon the current fault content of the system. The failure rate function during the interval between the *(i-1)*st and the *i*th failures is given by the relationship:

$$z(t_i) = [N - p(i-1)]\lambda \quad , \tag{2.13}$$

where p is the probability of imperfect debugging,

λ is the failure rate per fault.

The Littlewood/Verrall Bayesian Model (Litt73) takes a different approach from the JM model. In this model, the times between failures are assumed to follow an exponential distribution (Eq. (2.9)), which is the same as that of the JM model. However, the parameter of this distribution, $z(t_i)$, is treated as a random variable obeying a gamma distribution:

$$f(z(t_i)|\alpha, \Psi(i)) = \frac{[\Psi(i)]^{\alpha} z(t_i)^{\alpha-1} e^{-\Psi(i)z(t_i)}}{\Gamma\alpha} \quad , \tag{2.14}$$

where α and $\Psi(i)$ are model parameters.

2.3.2 Failure Count Models

In this class of models the variable under study is the number of failures seen or faults detected during given testing intervals. The time intervals may be a fixed priori, and the number of failures in each interval is treated as a random variable. Most of the models in this class assume that faults in the program are independent so that the failures have a Poisson distribution, whose parameters take different forms for different models.

One of the earliest and simplest Poisson model is the Goel/Okumoto Nonhomogenous Poisson Process (NHPP) Model (Goel79). The model assumes that software is subject to failures at random times caused by faults present in the system. Let $N(t)$ denote the cumulative number of failures observed by time, t. The model treats $N(t)$ as a nonhomogeneous Poisson process, i.e., as a Poisson process with a time dependent failure rate:

$$P(N(t) = y) = \frac{(m(t))^y}{y!} e^{-m(t)}, y = 0,1,2,\ldots \quad , \tag{2.15}$$

where $m(t)$ is the mean value function, it gives the expected number of failures observed by time, t, as:

$$m(t) = a(1 - e^{-bt}) \quad . \tag{2.16}$$

The failure rate is given by the relationship:

$$z(t) \equiv m'(t) = abe^{-bt} \quad . \tag{2.17}$$

The parameter a represents the expected number of failures eventually to be observed. The parameter b can be interpreted as the occurrence rate of an individual fault and is a constant of proportionality relating the failure rate to the expected number of faults remaining in the program.

The model is similar to the JM model in assuming a direct proportionality between the failure rate and the number of remaining faults, but differs in modeling this proportionality continuously, rather than by discrete steps when faults are removed. The model permits imperfect debugging and the introduction of new faults during debugging, however, the interpretation of b will be different in such cases.

The mean value function, $m(t)$, in the Goel/Okumoto Nonhomogenous Poisson Process Model is exponential on the time, t. Using a function, $m(t)$, of this type assumes that software quality continues to improve as testing progresses, and results in a monotonically decreasing failure rate vs. time. Some other NHPP models use $m(t)$ functions of different forms in order to capture different realism in software testing.

In practice, it is observed that in many testing situations, the failure rate first increases and then decreases. In order to model this behavior, Goel (Goel82) proposed a generalization of the Goel/Okumoto Nonhomogenous Poisson Process Model. The model uses a mean value function of the form:

$$m(t) = a(1 - e^{-bt^c}) \quad , \tag{2.18}$$

where a has the same meaning as that in the Goel/Okumoto NHPP model,

b, c are constants that reflect the quality of testing.

The failure rate function, $z(t)$, is given by the relationship:

$$z(t) \equiv m'(t) = abce^{-bt^c} t^{c-1} \quad . \tag{2.19}$$

In practice, there is a delay between the fault detection and the fault removal. The testing process in this case consists of two phases: fault detection and fault isolation/removal. S-shaped NHPP models are proposed to capture this realism (Ohba84, Gokh96). The mean value function, $m(t)$, is given by the relationship:

$$m(t) = a(1-(1+bt)e^{-bt}) \quad , \tag{2.20}$$

where a,b have the same meanings as that in the Goel/Okumoto NHPP model .

The failure rate is given by the relationship:

$$z(t) \equiv m'(t) = b^2 te^{-bt} \quad . \tag{2.21}$$

Some other widely referenced models in this class include the Shooman Model (Shoo72), the Musa Model (Musa71), the Generalized Poisson Model (Angus80), etc. During recent years many models have been developed to model imperfect debugging and the introduction of new errors during the debugging process. Examples include a Poisson type model (Bust94) and a hyper-geometric distribution model (Hou95). These models are also based upon Poisson process with new random variables introduced to model imperfect debugging. Complicated mathematics is involved in these newly developed models and their details are omitted here.

2.3.3 Advantages and Limitations of Software Reliability Growth Models

Software reliability growth models have a long development history, which can be traced up to the 1970's. Until now several growth models have been developed. Many of them have been extensively studied, and have had real applications for various types of software in many

fields. By choosing the appropriate form of the model, many typical behaviors of the software during the testing phase (e.g., the first increasing then decreasing failure rate) can be modeled. Many practical concerns during the software testing (e.g., imperfect debugging) can be incorporated into the models. Only testing data are required for the application of the growth models, which makes them easy to implement and applicable to essentially all types of software, ranging from a small module to a complex flight control system consisting of millions of lines of code. The automation of the application of the growth models is relatively easy. Many automated tools have been developed to facilitate the applications of the growth models and the comparison of their results (Li95, Vall94, Vall95). For software without a very high reliability requirement, satisfactory results can often be obtained if the appropriate growth model is used.

Despite of all these advantages, the growth models also have many limitations. First, most of the models are based upon the assumption that all the faults in the program contribute equally to the unreliability of the software, which typically is not realistic because the occurrence frequencies of different types of faults may vary significantly. Secondly, the growth models assume a typical operational profile of the software is available and base their testing upon such a profile. If there are big errors or changes in the operational profile, the models may not produce good results. Redoing the testing based upon the new operational profile may even be necessary. Much research has been done to analyze the sensitivity of the model predictions to errors in the operational profile (Chen94A, Cres96). No conclusive remarks can be made, though, due to the limited set of case studies performed and discrepancies seen in the results. Finally, the predictions of the growth models are typically not very accurate. The uncertainties associated with the growth model estimations are high and there is no effective mechanism to assess the uncertainties. The difficulty in validating the growth models makes them not well suited for applications to software with a very high reliability requirement.

As a summary, the growth models treat the software as a black box and do not take the structures of the target software into consideration. The overly simplified nature of the models

prevents the models from producing reliable estimations. Although many types of growth models with different assumptions have been developed aiming to capture different characteristics of the target software and the testing process, experiments have shown that the model whose assumptions appear to best match these characteristics of the target software and the testing process is not guaranteed to be the most appropriate model for use (Abde86, Niko95). Experimental application of different models "..has shown that there is commonly great disagreement in predictions, while none of them has been shown to be more trustworthy than others in terms of predictive quality in all applications..." (Lu93). It is not a surprise to see that application of reliability growth models to safety critical software, e.g., space shuttle applications, failed to produce satisfactory results due to high uncertainties in their estimation (Dunn86, Schn92).

2.4 Fault Seeding Models

The basic approach in this class of models is to "seed" a known number of faults in the program, which is assumed to contain an unknown number of indigenous faults. The program is then tested and the exposed number of seeded and indigenous faults is recorded. From the recorded data, an estimate of the fault content of the program prior to seeding can be obtained and used to estimate the software reliability.

The most popular fault seeding model is the Mill's Hypergeometric Model (Mill72). The model randomly seeds a number of faults in the target program. The program is then tested for a certain amount of time. The number of indigenous faults can be estimated from the number of seeded and indigenous faults uncovered by the testing by using a hypergeometric distribution. Some other models in this class include the Lipow Model (Lipow72), and the Basin Model (Basin74).

The models in this class are economical and easy to implement. However, they are based upon implicit assumptions that the indigenous faults are independent of one another, and that all of the seeded and indigenous faults are detected with equal probability. The latter assumption is so unrealistic that the models typically do not produce satisfactory results. Another serious limitation of the seeding models is that there is no way to obtain the failure rate function. The seeding models can only work as error count models. During recent years studies on the seeding models have ceased. Although seeding is still an active technique in other applications (e.g., the validation of reliability models), seeding models are rarely applied.

2.5 Input Domain-based Reliability Models

Before we discuss input domain-based reliability models, it is necessary to introduce some related terms. The *input domain* of a given program is the set of all relevant inputs to the program. The *input subdomain* is a subset of the input domain. A *partition* is a division of the input domain into disjoint subdomains. An input subdomain is *homogeneous* if either all of its members cause the program to succeed or all cause it to fail. Thus, any member in a homogeneous subdomain is a good representation of the entire subdomain. A *path domain* is the subdomain corresponding to a given path in a program, i.e., all the inputs in the subdomain cause the same path to be traversed.

The basic approach taken by input domain-based models is to sample a set of test cases from the input domain according to an input distribution (i.e., the operational profile), which is representative of the operational usage of the program. An estimate of the program reliability is obtained from the failures observed during the execution of the test cases. Since the input distribution is very difficult to obtain, the input domain is often divided into subdomains and test cases are generated randomly in each subdomain based upon a uniform distribution. Based upon

different ways to divide the input domain, the models in this class can be grouped into two categories: random testing, and subdomain/partition testing.

2.5.1 Random Testing

Random testing method randomly and uniformly selects test cases from the entire input domain. An earliest model in this category is the Nelson Model (Nels78). It is then extensively studied by many other authors, and its performance is compared to the other testing strategies (e.g., subdomain testing and partition testing) (Duran84, Chen94B, Chen96). For a program P, denote its input domain as D and the size of D as d (>0). Let m $(0 \leq m \leq d)$ denote the number of failure-causing inputs (i.e., the elements of D which produce incorrect outputs) in D. The failure rate of the program, θ, is defined as:

$$\theta = \frac{m}{d} . \tag{2.22}$$

Let n denote the total number of inputs selected for testing. Let n_e denote the number of failures observed during the execution of the n inputs. In other words, n_e denotes the number of inputs that resulted in execution failures. Then an estimate of θ can be obtained by the relationship:

$$\hat{\theta} = \frac{n_e}{n} . \tag{2.23}$$

If the program is executed for a long period of time using a given input distribution, then the actual failure rate of the program will converge to the probability that the program will fail to execute correctly on an input case chosen from the particular input distribution. An unbiased estimate of the software reliability per execution, $\hat{R}$, can be obtained by the relationship:

$$\hat{R} = 1 - \hat{\theta} = 1 - \frac{n_e}{n} . \tag{2.24}$$

An estimate of the software reliability on K successive executions is given by the relationship:

$$\hat{R} = (1-\hat{\theta})^{K} \quad . \tag{2.25}$$

A useful quantity for assessing the effectiveness of random testing and comparing its performance to that of other testing methods is P_r, the probability of finding at least one error in n tests.

$$P_r = 1-(1-\theta)^{n} \quad . \tag{2.26}$$

Another significant quantity is θ^*, the $(1-\alpha)$ upper confidence limit on θ. θ^* is the largest value of θ such that:

$$\sum_{i=1}^{n_e} \binom{n}{i} \theta^{i} (1-\theta)^{n-i} > \alpha \quad . \tag{2.27}$$

If the number of errors found $n_e = 0$, then:

$$\theta^* = 1-\alpha^{1/n} \quad . \tag{2.28}$$

In this case θ^* decreases as the total number of tests n increases. $\theta^* \rightarrow 0$ as $n \rightarrow \infty$.

Eq. (2.27) and Eq. (2.28) provide an effective mechanism to assess the uncertainty associated with the model estimate.

2.5.2 Subdomain/Partition Testing

Both subdomain testing and partition testing divide the input domain into subdomains, one or more test cases from each subdomain are selected to test the program. The term *subdomain testing* is used for the general case when the subdomains may or may not be disjoint. The term *partition testing* is used for the special case when all subdomains are disjoint (Chen96). In this section we will focus on partition testing because it is more extensively studied. However, most of the results can also be applied to subdomain testing.

Partition testing refers to any test data generation method that partitions the input domain and forces at least one test case to be selected from each subdomain. Suppose the input domain D is partitioned into k subdomains, which are denoted by D_i, where $i=1,2,..k$. Each subdomain D_i has size d_i, contains m_i $(0 \le m_i \le d_i)$ failure-causing inputs, and has failure rate:

$$\theta_i = \frac{m_i}{d_i} \quad . \tag{2.29}$$

Let p_i denote the probability that a randomly selected input comes from the subdomain D_i. The failure rate of the entire program is given by the relationship:

$$\theta = \sum_{i=1}^{k} p_i \theta_i \quad . \tag{2.30}$$

Use n_i (≥ 1) to denote the number of test cases selected from the subdomain D_i. Use n_{ei} to denote the number of test cases select from the subdomain D_i which result in program failures. All the random selections are also assumed to be independent, with replacement, and based upon a uniform distribution. This means that when a test case is selected from D_i, the probability that it is a failure-causing input will be exactly θ_i. An estimate of the overall failure rate of the program, θ, can be obtained using Eq. (2.23). Another estimate of θ is given by Lipow (Lipow75):

$$\hat{\theta}_2 = \sum_{i=1}^{k} p_i \hat{\theta}_i = \sum_{i=1}^{k} p_i \left(\frac{n_{ei}}{n_i}\right) \quad . \tag{2.31}$$

Then Eq. (2.24), (2.25) can be used to estimate software reliability.

P_p, the probability of finding at least one error in n tests is given by the relationship:

$$P_p = 1 - \prod_{i=1}^{k} (1-\theta_i)^{n_i} \quad . \tag{2.32}$$

The confidence limit of the estimate of partition testing strategies takes more complex form than Eq. (2.27). It varies for different strategies.

Research has been conducted to compare the performance of partition testing with that of random testing (Duran84, Weyu91, Chen94B). Results show that using the same number of test cases, partition testing in general is not more cost-effective than random testing unless some conditions are meet, such as the proportional rule produced by Chen (Chen94B). A special case occurs when the subdomains are homogeneous. In this case only one test case is needed for each subdomain. The total number of test cases required by partition testing will be greatly reduced and become much less than that required by random testing. Nevertheless, a truly homogeneous subdomain is very difficult to obtain in practice. Most of the partition testing strategies developed by now attempt to produce more or less homogeneous subdomains (i.e., the inputs in the subdomain share some similarities in error revealing capability) with manageable effort. Examples include the data flow testing criteria, a family of testing strategies developed by Rapps and Weyuker (Rapps82, Rapps85). The strategies require the exercising of path segments determined by combinations of variable definitions and variable uses. Popular code coverage strategies can be considered to be partition testing strategies. The statement/branch/path testing requires that sufficient test cases be selected so that every statement/branch/path in the program is executed at least once. Note that only path testing divides the input domain into disjoint subdomains. Mutation testing (DeMi78, Offu96) can also be viewed as a partition strategy.

2.5.3 Advantages and Limitations of Input Domain-based Reliability Models

Input domain-based models have many advantages over the reliability growth models. First, input domain-based models do not depend upon the unrealistic assumption that that all the faults in the program contribute equally to the unreliability of the software. By selecting the test cases based upon a given input distribution, input domain-based models implicitly weigh the contribution of a fault to the software unreliability by p_i, the probability that a randomly selected

input belongs to an equivalent class which exercises the fault (i.e., the occurrence rate of the fault). Secondly, by choosing appropriate testing strategies (e.g., the code coverage strategies), more information of the target program (e.g., the control structure and the data usage) can be incorporated into the models. The white box treatment of the program can be expected to be superior to the black box treatment by reliability growth models and able to generate better results. Thirdly, input domain-based models provide an effective mechanism to assess the uncertainties in the reliability estimation (e.g., Eqs. (2.27) and (2.28) for random testing). Assuming detecting no errors, which is typical in the later testing phase of safety critical software, the uncertainty associated with the reliability estimation decreases as the total number of test cases n increases. Theoretically it is possible to obtain any accuracy in reliability estimation after performing sufficiently large number of test cases. Thus input domain-based models are particularly suited for applications to safety critical software, in which high accuracy and high certainty is required for reliability estimation. Finally, input domain-based models can easily accommodate changes in the operational profile. Input domain-based models randomly select test cases from each subdomain using a uniform distribution. Changes in the operational profile only have effect on the estimation of θ through the changes in the p_i's (Eq. (2.31)). Only the p_i's need to be recomputed using the modified operational profile, test cases need not to be regenerated.

Input domain-based models also have several limitations. First, an underlying assumption made by both random testing and partition testing is that no fault is removed and the software reliability remains unchanged during the testing process. In practice faults are identified and removed from the program during testing and the software reliability is expected to improve during the testing process. Input domain-based models are too conservative in such sense. Secondly, the application of input domain-based models is much more complex than that of growth models. In order to produce more or

less homogeneous subdomains, structural information of the target program is often required. Typically complex testing strategies have to be applied to control the partition process, the generation of test cases, and the testing process. Automation of the modeling is often necessary for relatively small programs (compared with what can be handled by growth models), while very few automated tools of this kind are available now. Finally, input domain-based models randomly select test cases form each subdomain using a uniform distribution rather than using the actual input distribution, which introduces errors into the reliability estimation. One exception is that if the subdomains are truly (or very close to) homogeneous, either all inputs in the same subdomain cause the program to fail or none of the inputs causes a failure (i.e., θ_i in Eq. (2.31) is either one or zero). The input distribution inside the subdomain has no effect on the reliability estimation. Only the p_i of the subdomain needs to be estimated with high certainty.

In recent years input domain-based models are receiving more and more interest as a promising technique for reliability estimation for safety critical software, especially after the unsuccessful experience of applications of reliability growth models (Dunn86, Schn92). Nevertheless, input domain-based models are still at the beginning of their development stage. Many difficulties have to be overcome before their wide and success applications. Much work needs to be done in this area.

Chapter 3

The Integrated Methodology

This chapter formulates an integrated methodology for reliability estimation of safety critical software. The methodology is built upon an input domain-based reliability model using a path testing (partition) strategy. **Section 3.1** introduces the control flowgraph and some other related concepts that are used extensively in the thesis. **Section 3.2** presents some fundamental definitions and equations that comprise the theoretical foundation of the methodology. Elements of the methodology are discussed in **Section 3.3** through **Section 3.6**. A summary of the advantages and limitations of the methodology is given in **Section 3.7**.

3.1 The Control Flowgraph

The proposed integrated methodology is based upon the concept of structured path testing. Before we introduce the definition of the path, it is first necessary to introduce the control flowgraph concept. In general, the control flowgraph is a graphical representation of the control structure of a program.

3.1.1 Elements of the Control Flowgraph

The control structure of a program is composed of three different types of elements, namely, process block, decision, and junction. The process block is composed of a group of statements that must be executed in a sequential order from the entrance to the exit. The length of the process block can vary from one to hundreds of statements. The decision is a section of code where control flow can diverge into different paths. The standard decision construct is the

if..then..else statement which relies upon the value of the predicate, a Boolean function of certain variables, in order to decide which path will be executed. The junction is a section of code where control flows can merge together. For example, when the separate true and false paths end after an *if..then* statement, the flow can merge together to form a junction. All computer programs can be broken down into some combination of these three elements.

The control flowgraph is composed of two types of components: circles and lines (with arrows). A circle is called a node and a line is called a link. A node with only one link entering and only one link exiting is a process node, which is used to represent the process block structure (Fig. 3.1).

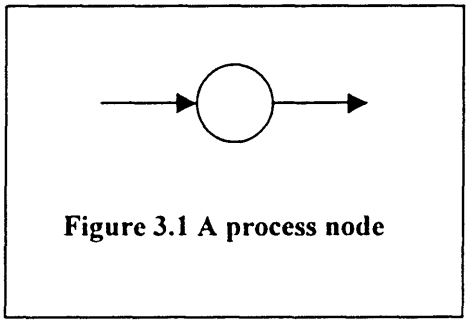


Figure 3.1 A process node

A node with more than one link exiting is a decision node, which is used to represent a decision structure (Fig. 3.2).

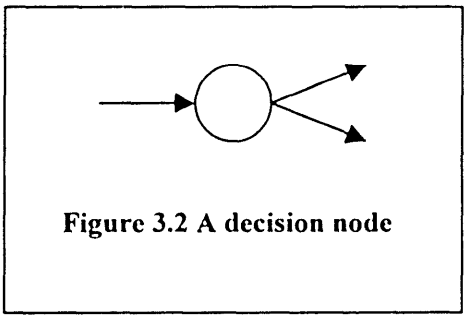


Figure 3.2 A decision node

A node with more than one link entering is a junction node, which is used to represent a junction structure (Fig. 3.3).

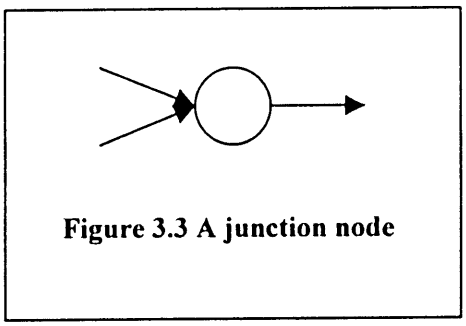


Figure 3.3 A junction node

In some cases statements can be both decisions and junctions, such as loop control statements. The corresponding nodes have more than one link entering and more than one link exiting.

From the above all control structures can be expressed using the control flowgraph.

3.1.2 An Illustrative Example

In this section an example program will be analyzed and its control flowgraph constructed in order to facilitate the understanding of the control flowgraph concept. The example program used here is taken from Lunglhofer's report (Lung96). The readers can refer to **Section 2.2.1.2** in Lunglhofer's report for the original statements. The language used is a pseudo programming language based upon that developed by Rapps and Weyuker in 1985 (Rapps85). The purpose of using this language is to break down any language barriers other high level

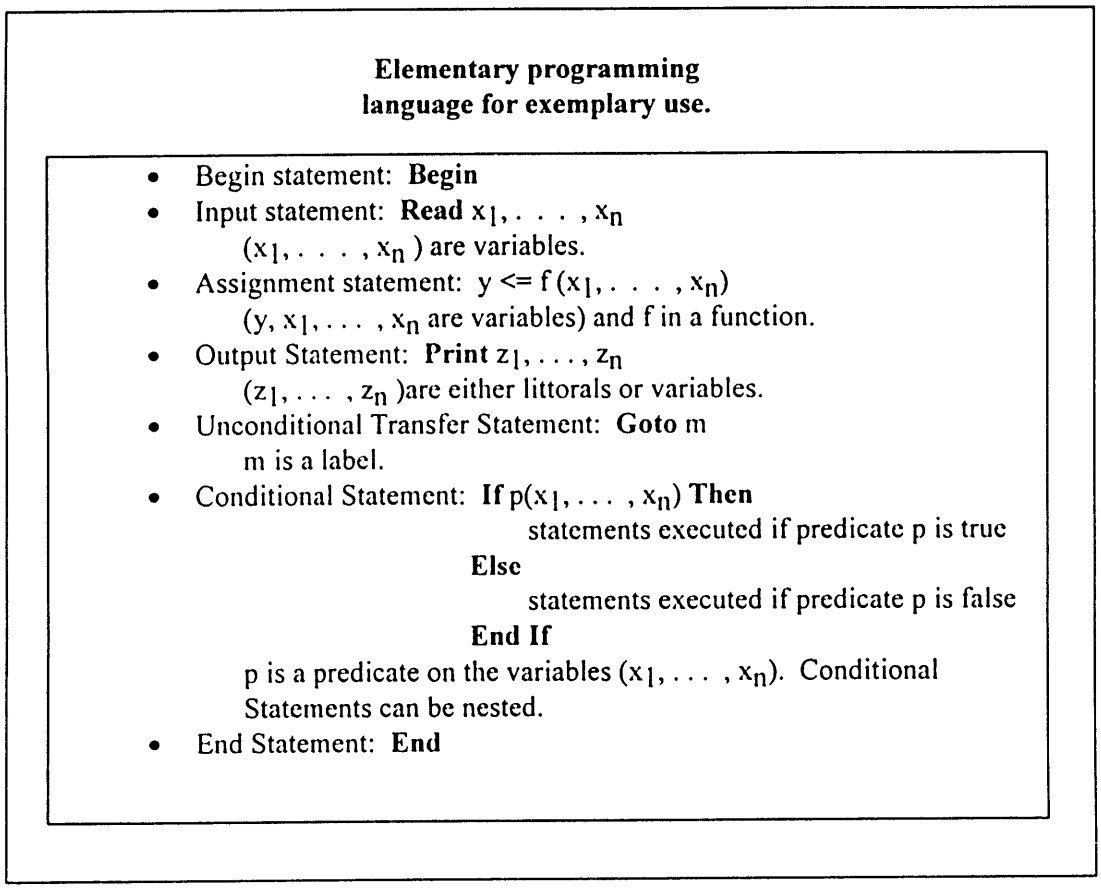


Figure 3.4 The pseudo programming language

(Duplicated from Fig. 2-4 in Lung96)

languages might cause. Figure 3.4 shows the statement types that are included in the language. The example program is given in Figure 3.5. The program simulates the operation of an Automatic Teller Machine (ATM). It reads the customer's bank account balance (b) and then queries the customer for an amount to be withdrawn (w). It then checks to see if the withdrawal amount is larger than the account balance. If so, no withdrawal is made. If the withdrawal amount is equal to the account balance, then the user is notified that the final account balance will be zero. If the withdrawal is less than the balance, then the balance is reduced by the withdrawal amount.

```
Begin
Read (b)

Read (w)
withdraw <= true
If (w > b) Then

   withdraw <= false
Else
   If (w < b) Then

   Else

   End If
End If
If (withdraw)=true Then
   b <= b - w
Else
   b <= b
End
```

This code is written in the language specified in **Figure 3.4** and is used as an example in order to further the understanding of control flowgraphs.

Figure 3.5 Automatic Teller Machine (ATM) example code

(Duplicated from Figure 2-5 in Lung96)

The control flowgraph of the example program in Figure 3.5 is constructed in Figure 3.6. In Figure 3.6 there are nine nodes, *A-I*, in the control flowgraph of the example program. Each

consists of a single statement or a block of statements. In Figure 3.6, *B*, *D*, *E*, *G* and *H* are process nodes; *A*, *C* and *F* are decision nodes; *F* and *I* are junction nodes.

ATM Code Control Flowgraph Breakdown

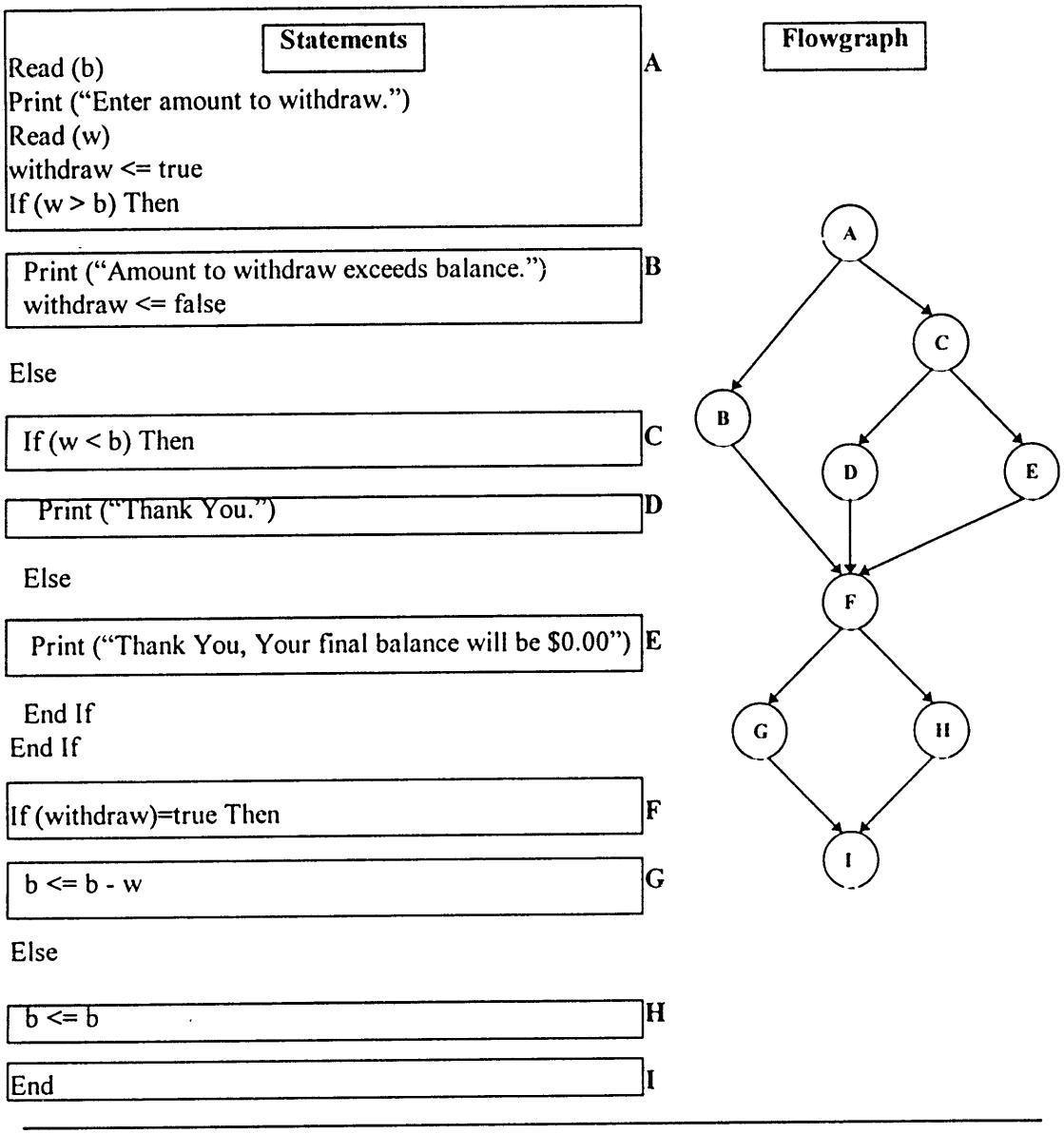


Figure 3.6 ATM example code and its flowgraph

(Duplicated from Figure 2-6 in Lung96)

3.1.3 Some Useful Terms

Using the control flowgraph of the example program in Figure 3.6, we can introduce some terms that are used extensively in this thesis (Rapps85, Lung96). The links in the control flowgraph are also known as *edges*. An edge from node j to node k is given the notation *(j, k)*. Node *j* is called a *predecessor* of node *k*, and node *k* a *successor* of node *j*. The first node of a program is called the *start node* and has no predecessors, e.g., node *A* in Fig. 3.6. The last node is called the *exit node* and has no successors, e.g., node *I* in Fig. 3.6. A *path* or *flow path* is a finite sequence of nodes connected by edges. A path is designated by a sequence of nodes. For instance, a path in Fig. 3.6 is Path *1=(A+B+F+G+I)*. A path is a *complete path* if its first node is the start node and its last node is the exit node. A path is *a loop free path* if none of its nodes is repeated in the sequence. If any node in a path is repeated, the path contains a looping structure. Following these definitions Path *1=(A+B+F+G+I)* is both a complete path and a loop free path. A path is a *feasible path* if some inputs exist that will cause the path to be traversed during program execution. For instance, a feasible path in Fig. 3.6 is *Path 2=(A+B+F+H+I)*, which will be executed if we assign input variables to be *w=200, b=100*. A path is an *infeasible path* if no input exists that will cause the path to be traversed during program execution. For instance, Path *1=(A+B+F+G+I)* is infeasible. Whenever block *B* is executed, the variable "withdraw" will be set to *false*, and block *G* will not be executed thereafter.

3.2 The Underlying Software Reliability Model

For a given software, use N_{total} to denote the total number of paths in the program. Use N_{iden} to denote the total number of paths identified in the software through structural analysis.

Use N_{tested} to denote the number of paths that are tested in the testing phase. Use $N_{untested}$ to denotes the number of paths that are identified but not tested. Clearly we have:

$$N_{iden} = N_{tested} + N_{untested} \quad . \tag{3.1}$$

For any path, i, in the software, use D_i to denote the corresponding path subdomain. Use θ_i to denote the probability of failure upon a single traverse of the path i. From Eq. (2.29) in **Section 2.5.2**:

$$\theta_i = \frac{m_i}{d_i} \quad , \tag{3.2}$$

where m_i $(0 \le m_i \le d_i)$ is the number of failure-causing inputs in D_i,

d_i is the total number of inputs in D_i.

Use p_i to denote the probability that the path, i, will be traversed in a single program execution, averaged over the program lifetime. It is equivalent to the p_i defined in **Section 2.5.2**. From Eq. (2.30) in **Section 2.5.2**, the failure rate of the entire program is:

$$\theta = \sum_{i=1}^{N_{tested}} p_i \theta_i \quad . \tag{3.3}$$

An estimate of the failure rate, θ, is given by:

$$\hat{\theta} = \sum_{i=1}^{N_{tested}} \hat{p}_i \hat{\theta}_i \quad . \tag{3.4}$$

Use P to denote the probability that an identified path will be traversed in a single program execution.

$$P = \sum_{i=1}^{N_{iden}} p_i \quad . \tag{3.5}$$

P is equal to unity if all the paths in the program are identified. In reality, it is not always possible to identify all the paths in a program, especially when the program contains looping structures. In

such cases P is less than unity. Use Q_T to denote the unreliability contribution of the N_{tested} tested paths. Use Q_U to denote the unreliability contribution of the $N_{untested}$ untested paths:

$$Q_T = \sum_{i=1}^{N_{tested}} p_i \theta_i \quad , \tag{3.6}$$

$$Q_U = \sum_{i=N_{tested}+1}^{N_{total}} p_i \theta_i \quad . \tag{3.7}$$

Another estimate of the failure rate, θ, can be obtained as:

$$\hat{\theta} = \frac{1}{\hat{P}} \sum_{i=1}^{N_{taken}} \hat{p}_i \hat{\theta}_i = \frac{1}{\hat{P}} (\hat{Q}_T + \hat{Q}_U) \quad . \tag{3.8}$$

An estimate of software reliability per execution is given by the relationship:

$$\hat{R} = 1 - \hat{\theta} \quad . \tag{3.9}$$

An estimate of software reliability on K successive executions is given by the relationship:

$$\hat{R} = (1 - \hat{\theta})^K \quad . \tag{3.10}$$

The methodology is essentially a partition testing method using a path testing strategy. The input domain D is partitioned into path subdomains, D_i's, in such a way that, all the inputs in a subdomain lead to the execution of the same path, i, in the software. The product, $p_i \theta_i$, corresponds to the unreliability contribution of the path subdomain, D_i. Since the inputs in a path subdomain lead to the execution of the same path in the software, it is very likely that they will reveal the same errors in the software. In other words, the path subdomain can be expected to be more or less *homogeneous*. Based upon this partition strategy, the methodology we proposed is expected to produce better results than that of random testing.

Using the above definitions, the estimation of the software reliability is reduced to the estimation of p_i and θ_i for each path in the software. p_i is closely related to the operational profile of the software. The estimation of p_i is discussed in **Section 3.4**. θ_i is closely related to

the fault content in the path. The estimation of θ_i is discussed in **Section 3.5**. If both p_i and θ_i can be estimated with high certainty, the quantitative value of the reliability of the software, R, can be obtained using the above equations with high certainty.

3.3 The Feasible Structured Path Testing Method

Since in the methodology the definition of the software reliability is based upon paths in the software, the first step in the methodology is to identify all the possible paths in the software. In reality, it is not always possible to identify all the paths in a software, especially when the software contains looping structures. In such case, we wish to identify most of the paths in the software in order to assure high accuracy in the reliability estimation. The testing method we employed in this methodology for path identification is called the *Feasible Structured Path Testing (FSPT)* method. The method is developed by Jon R. Lunglhofer (Lung96). It is chosen for our use because it has several desirable features such as the ability to identify all the executable paths in certain cases, not overly time consuming, and the ability to identify and remove infeasible paths. Using this method, most (sometimes all) of the executable paths in a software can be identified.

Although we chose the Feasible Structured Path Testing (FSPT) method as the testing method for our methodology, it needs not to be the only choice. Any other path testing method that is able to identify some of the paths in the program is also applicable to our methodology. After the paths are identified, all the other elements of the methodology can be applied in the same manner independent of the testing method. Since the FSPT method is used in our case study in **Chapter 4**, elements of the FSPT method will be presented in this section to help the readers understand the process. Interested readers can find more details of the testing method in Lunglhofer's report (Lung96).

3.3.1 Method Overview

The feasible structured path testing method is based upon the concept of structured path testing method. Structured path testing method attempts to test every path in the software, i.e., to achieve path coverage. It differs from the complete path testing in that it limits the number of loop iterations to a value, *k*. If a looping structure exists in a section of code, the loop is only executed *k* times through at most. This strategy sacrifices completeness, however, it produces a finite and manageable set of paths, which is necessary for the software analysis and testing to be practical. The feasible structured path testing method adds a step to the structured path testing method to determine and eliminate infeasible paths in the control flowgraph, which greatly simplifies the control flowgraph and reduces the number of test cases required.

3.3.2 Method Details

"The feasible structured path testing method consists of a five step formal process. The goal of the process is to take a program, and through analysis to systematically produce test cases, which achieve structured path testing coverage involving no infeasible test cases." (Lung96) The five steps are discussed in detail in this section, some are quoted from Lunglhofer's report, and some are modified and reformulated in this thesis.

Step 1: The first step of the Feasible Structured Path Testing (FSPT) method examines the program being analyzed and constructs the program's control flowgraph. The construction process has been illustrated in **Section 3.1.3**. In the case that looping structures are present in the code, one expands the flowgraph to include at most *k* iterations of any looping structure. An

example of the expansion *(k=2)* is given in Figure 3.7. There are several factors in selecting the appropriate value of *k*. When *k* is small, the number of paths that are included in the control flowgraph is small, and the number of test cases required is small. As *k* increases, more and more paths will be included in the control flowgraph. The effort needed in analyzing and testing the program will increase, while the probability of detecting more errors will also increase. A complete test occurs when *k* reaches infinity or the maximum value of loop iterations specified in the program. Based upon the requirement of the analysis and the time and resources available, the analyzer can choose the value of *k* that is best suited for his own application.

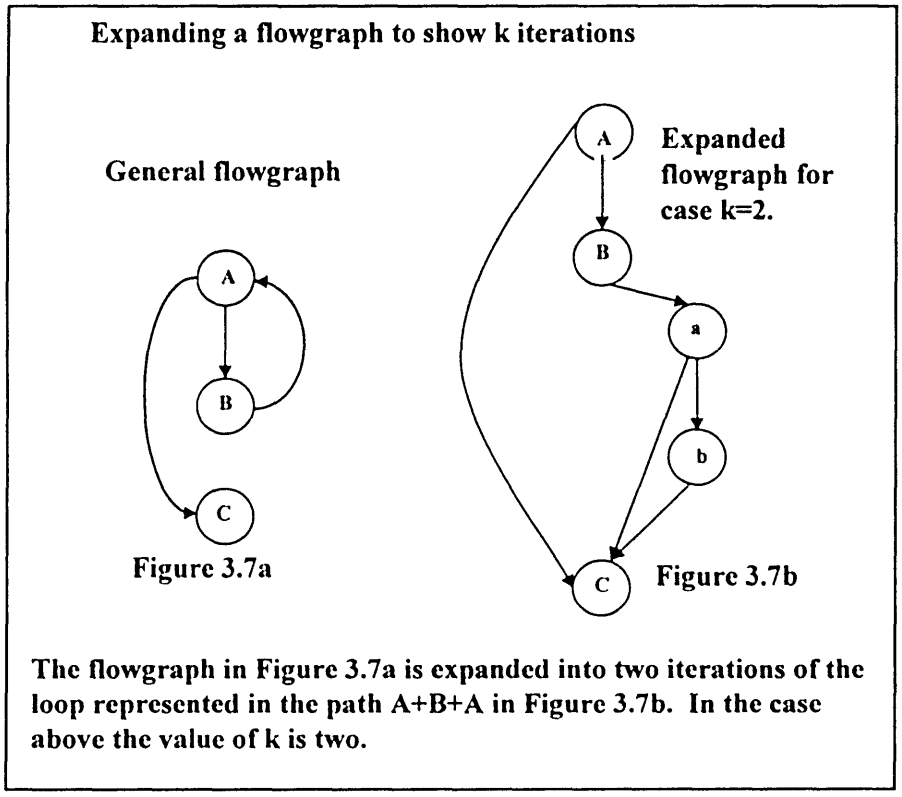


Figure 3.7 Flowgraph expanded to k iterations

(Duplicated from Figure 3-2 in Lung96)

The first step assumes that no *horrible loops* exist in the code. Horrible loops are described by Beizer as ". . . code that jumps into and out of loops, intersecting loops, hidden

loops, and cross-connected loops . . ." (Beiz90). Figure 3.8 shows an example of horrible loops. Horrible loops in a program are troublesome in expanding the looping structures and generating the control flowgraph. They make the program difficult to understand and complicate the analysis and debugging of the program. They are often resulted from poor programming experience and should be avoided by all means in high quality software. The FSPT assumes that the program contains no horrible loops.

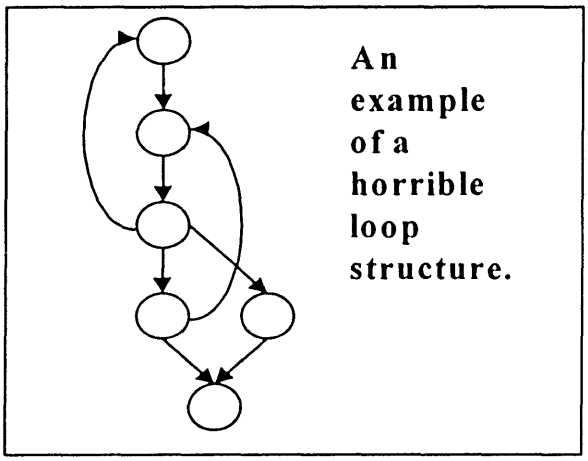


Figure 3.8 Horrible Loops

(Duplicated from Figure 3-1 in Lung96)

Step 2: The second step identifies the *transitional variables* in the program. The *transitional variables* will then be used in the third step. The next a few paragraphs are quoted from Lunglhofer's report (Lung96) to show the meaning of the transitional variables and the way to identify them.

"2.1 The second step involves an examination of the data flow of the program, and results in a list of variable transitions. These transitions relate the input variables to the output variables. The first step of the process is recognizing the input and output variables of the code. Any variable or data object that attains its value from outside the piece of code being examined is considered an input variable. Those variables which facilitate the ultimate use of the program are

known as output variables. To recognize the output variables, one must have some understanding of the program's function. Output variables are produced in several ways. A function can be performed and its products presented on an output device (e.g. a monitor), data can be stored in a memory device (e.g. RAM) for later use, or in the case of testing a program module, variables can attain values for use in other modules. Once the tester recognizes both input and output variables of the code, the remainder of the second step can begin.

2.2 A list of every variable which directly affects the flow of data from the input to the output variables is generated. This list contains the transitional variables. For a variable to be included in the list of transitional variables, it must meet one of two requirements. The variable can directly attain its value as a function of an input variable, or of another transitional variable. Also, other variables are designated transitional variables by directly affecting the value of the output variables, or by affecting the values of other transitional variables which affect the output variables. Recall from the exemplary language, detailed in **Figure 2-4** (Figure 3.4 in this thesis), the assignment statement: $y <= f(x_1, \ldots, x_n)$ where $(y, x_1, \ldots, x_n)$ are variables. In order for a variable to be included in the transitional variable list, it must be created from an assignment statement involving either input variables or other transitional variables already assigned to the list. The relationship also works in reverse. If y is the output variable or another transitional variable, then $x_1, \ldots, x_n$ are added to the list of transitional variables. When all flow branches have been examined including as many as k iterations of any looping structure, one should have a complete list of every transitional variable leading from the input to the output variables. When this list is complete, one can move on to the third step of the process." (Lung96)

Step 3 The third step further expands the control flowgraph obtained in step 1 with respect to the *transitional variables* obtained in step 2. The next a few paragraphs are quoted from Lunglhofer's report (Lung96) to show the expansion.

"3.2 Starting from the top node, one systematically examines the state of the transitional variables that exist in each node. If a node contains a transitional variable that has not

reached the end of its development, the transitional variable affects other transitional variables or the output variables at some point further down the execution path. It is necessary to expand the flowgraph into a parallel structure reflecting each possible unique state of that transitional variable. The number of states produced depends upon the number of possible paths reaching the node in question. For instance, in **Figure 3-2** (Figure 3.7 in this thesis), if at node C a transitional variable x exists that affects the program's output variables, node C is expanded. If a unique value of the variable x arises from each of the three paths leading into node C, node C expands into three separate nodes and a new exit node is added." (Lung96)

Figure 3.9 shows an illustration of the expansion.

Expanding a flowgraph to show unique transitional variable states.

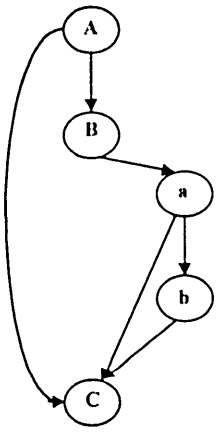


Figure 3.9a

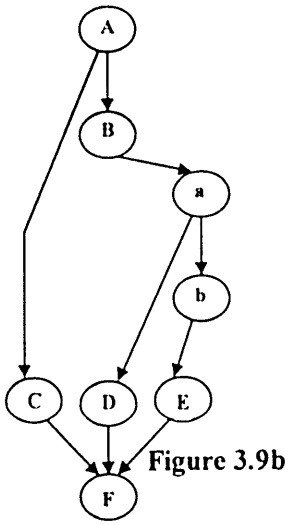


Figure 3.9b

The flowgraph on the left is expanded at node C to reflect the three unique states of the transitional variable x which leads to the program output at node F.

Figure 3.9 Flowgraph expansion with respect to transitional variables

(Duplicated from Figure 3-3 in Lung96)

The expansion of the flowgraph in step 3 can also be performed in another equivalent, yet more straightforward way. For each node in the flowgraph, if it has more than one, say, *l*, links entering, one expands the node into *l* seperate nodes and makes *l* copies of all the nodes below the node (i.e., the successors of the node, the successors of the successors of the node..). This is a naive way to achieve path coverage. More complex algorithms may be employed to reduce the total number of paths needed for path coverage to improve efficiency. It is not a focus of our work and is not discussed here.

Step 4 In **Section 3.2.2** in Lunglhofer's report (Lung96), the fourth step in the FSPT finds the value of the McCabe's metric (McCa76) of the flowgraph altered in step 3. "This value is then used to determine the number of test cases needed to completely test the code. As described in **Section 2**, a base test path is selected, and a set of linearly independent paths is chosen from the code. These paths represent the complete set of the FSPT test cases." (Lung96) After a closer look at the expanded flowgraph obtained after step 3 (e.g., Figure 3.9b), we can see that after the expansion in step 3, the number of independent paths in the control flowgraph is simply the total number of paths in the flowgraph. Thus the calculation of the McCabe's metric and the identification of the independent paths is not necessary. From the expanded flowgraph obtained after step 3 (e.g., Figure 3.9b), we can also see that the expanded flowgraph has such a property that the path coverage is equivalent to the branch coverage. Finding all the paths in the expanded flowgraph is equivalent to covering all the branches in the program. This is a big advantage that can be obtained by performing step 3 because automation of the branch coverage is much easier to achieve in practice than that of the path coverage. Thus the step 4 in the modified FSPT is reduced to finding all the paths in the expanded flowgraph, or equivalently, covering all the branches in the flowgraph.

Step 5: The fifth step in the FSPT determines the values of the input variables that cause each test path to be executed. The step also determines which test paths are infeasible. These can be done through predicate analysis.

3.3.3 Method Automation

Automation of the Feasible Structured Path Testing (FSPT) method requires automatic generation of the control flowgraph, expansion of the flowgraph, and generation of the test cases. Automatic generation of the control flowgraph for a given program is relatively easy. It is incorporated in most symbolic execution tools (Clark76, Rama76, Kemm85, Coen90, Jasp94), and many other Computer Aided Software Engineering (CASE) tools (Webb87, Heis93, Bhat94, Harr95). The expansion of looping structures in the flowgraph to allow for k iterations is a simple task and can be easily implemented. The expansion of the flowgraph with respect to the transitional variables requires data flow analysis of the program, which is incorporated in most of software reverse engineering tools (Fran85, Webb87, Heis93, Bhat94, Rich94, Lyu94, Harr95). The most difficult step in automating the FPST method is the identification of infeasible paths and the generation of test cases.

The most widely used method for path-driven test data generation (i.e., generation of test data for a given path) is the symbolic execution technique. A common approach to symbolic execution is to construct a control flowgraph of a given program. Each path in the flowgraph is represented by a series of input variables, condition predicates and assignment statements. A set of paths in the flowgraph are selected for testing, either by the user or by the symbolic executor based upon certain criteria. Each of the selected paths is then traversed symbolically, i.e., each input variable is given a symbol in place of an actual value, each assignment statement is evaluated so that it is expressed in terms of symbolic values of input variables and constants. At the end of the symbolic execution, the output variables of the path will be represented by expressions in terms of symbolic values of input variables and constants, subjecting to certain constraints. The constraints, often called the path conditions (PC), are provided by the set of symbolic representations of each condition predicate along the path.

The feasibility of the path can be determined by checking the consistency of the constraints. If there is any contradiction, the path is infeasible. This is often done by a theorem prover embedded into the symbolic executor.

The generation of test case for the path reduces to finding a set of input variables that make all the constraints satisfied. Then the outputs of the path execution can be obtained by substituting the actual values of the input variables into the symbolic expressions of the output variables. The feasible inputs are often found using linear programming (for linear constraints) and nonlinear programming (for nonlinear constraints) technique. The technique attempts to optimize an arbitrary objective function using the path constraints with the aim to find a feasible solution satisfying all the constraints, making use of the fact that an optimal solution is also a feasible solution. The technique has been extensively studied and many packages are available which facilitate its implementation. Many existing symbolic executors employ this technique, such as ASSET(Clar76), CASEGEN(Rama76), DISSECT(Howd77), Fortran Testbed (Henn83). However, the complexity (both time and space) associated with its application is still very high and it can not guarantee finding a feasible solution even if such a solution exists, especially when nonlinear constraints are present.

Another method appearing in the literature is the trial-and-error method (Inam89). The method randomly generates inputs and checks if the inputs satisfy all the constraints. Since the efficiency of the method is problematic and no real applications have been found, the method is not discussed in detail here.

The symbolic execution method has great difficulties in handling dynamic data structures, array references, and modular calls. In order to overcome these difficulties, recently an alternative approach of test data generation has been proposed by Korel (Korel90, Korel92, Korel96). The approach ".. is based on actual execution of the program under test, function minimization methods, and dynamic data flow analysis. Test data are developed for the program using actual values of input variables. When the program is executed, the program execution flow

is monitored. If during program execution an undesirable execution flow is observed (e.g., the "actual" path does not correspond to the selected control path) then function minimization search algorithms are used to automatically locate the values of input variables for which the selected path is traversed. In addition, dynamic data flow analysis is used to determine those input variables responsible for the undesirable program behavior, leading to significant speeding up of the search process." (Korel90) The approach is based upon actual execution of the target program, thus having great advantages over the symbolic execution method in handling dynamic data structures, array references, and modular calls. The effectiveness of test data generation can be significantly improved. A limitation of the approach is that it has very limited ability to detected path unfeasibility. A combination of the approach with the symbolic execution method is suggested to overcome this difficulty (Korel90). Although the approach seems to be promising, no real applications have been found in the literature.

Some other methods for automatic test case generation include the Genetic Algorithm Approach (Hunt95, Rudn97, Mich97) which uses a combinatorial optimization technique; and the Chaining Approach (Ferg95, Ferg96) which uses data dependence analysis. These methods are not path-driven and can not be employed for our applications. Their details are omitted here.

As a summary, automatic identification of infeasible paths and generation of test cases for given paths are very difficult to implement in practice. The tools available now have limited capability on this aspect and can not always produce correct results. Due to limited time and resources, we do not attempt to solve this problem, rather leave it for future research. Instead of developing path computation and test case generation algorithms, the focus of this work is setting up a framework for software reliability quantification.

3.4 The Estimation of p_i

After a set of paths is identified using the testing method discussed in **Section 3.3**, the estimation of p_i for each of the paths can be done in two steps. First the operational profile of the software has to be estimated. Then p_i can be estimated based upon the operational profile. As we will see in the following two sections, both steps are very difficult to implement in practice, which makes the estimation of p_i the most difficult element of the methodology.

3.4.1 The Estimation of the Operational Profile

The operational profile is essential to the estimation of p_i and has to be obtained in some way. The operational profile generally is not determined by the software itself but by some other systems providing inputs to the software. A higher level of knowledge of the operational environment of the software is required. The acquisition of the operational profile is very difficult in practice. The software may have millions of input variables and the complexity in acquiring the operational profile can be unmanageable. The software may have applications in a variety of environments and the averaged operational profile is hard to obtain. The application environment of the software may be highly dynamic and unpredictable and a "typical" operational profile does not exist. All these factors contribute to difficulties in obtaining the operational profile in practice. However, for nuclear control software, things are somewhat easier. Nuclear control software is special purpose software and has a specific application environment, which is typically predictable and not highly dynamic. During most of the software application lifetime the input variables vary smoothly and over narrow ranges. For simple nuclear control software with relatively few input variables, it is possible to accumulate enough knowledge to obtain an estimate of the operational profile with certain accuracy. Due to limited time and resources we do

not go any further in handling this difficulty in the thesis and assume we can obtain the operational profile in some way.

3.4.2 The Estimation of p_i Based upon the Operational Profile

Even if the operational profile of the software can be obtained, it is still very difficult to estimate p_i based upon the operational profile. There are two possible solutions, namely, the direct integration method and the Monte Carlo method.

3.4.2.1 The Direct Integration Method

The operational profile can be expressed as a joint probability distribution function, $f(x_1, x_2, ..x_n)$, of the input variables $x_1, x_2, ..x_n$. The joint probability distribution function, $f(x_1, x_2, ..x_n)$, satisfies the normalization constraint:

$$\int_D f(x_1, x_2, ..x_n) dx_1 dx_2 .. dx_n = 1 \quad , \tag{3.11}$$

where D is the input domain.

p_i, by definition, is the integration of the operational profile, $f(x_1, x_2, ..x_n)$, over the path subdomain, D_i, corresponding to the path, *i*:

$$p_i = \int_{D_i} f(x_1, x_2, ..x_n) dx_1 dx_2 .. dx_n \quad . \tag{3.12}$$

If the input variables are independent of one another, the equations are simplified to:

$$f(x_1, x_2, ..x_n) = f_1(x_1) * f_2(x_2) * .. * f_n(x_n) \quad , \tag{3.13}$$

$$\int_D f_j(x_j) dx_j = 1, \quad j = 1..n \quad , \tag{3.14}$$

$$p_i = \prod_{j=1}^{n} \int_{D_j} f(x_j) dx_j \quad . \tag{3.15}$$

The direct integration method thus consists of three steps: Obtain the operational profile in the form of $f(x_1, x_2, ..x_n)$; identify the path subdomain, D_i; then integrate $f(x_1, x_2, ..x_n)$ over the subdomain, D_i.

Although mathematically elegant, the complexity associated with the method is so high that the method is not feasible in practice. Firstly the operational profile may not be available in the elegant form of $f(x_1, x_2, ..x_n)$. In practice the operational profile is typically obtained by consulting system users. It is often available in a partially integrated form, e.g., the probabilities that the input variables have values in certain ranges *[a,b]*. Much effort is needed to convert the operational profile into the appropriate form of $f(x_1, x_2, ..x_n)$.

Secondly, the path subdomain corresponding to a particular path is very difficult to obtain. As we have seen in **Section 3.3.3**, even finding a single input point in a path subdomain is a very hard problem and can not be solved effectively by the currently available methods, e.g., the symbolic execution method. Identifying the entire path subdomain is a much more difficult problem than finding a single point in the path subdomain. It requires identification of all the boundaries of the subdomain. In nuclear control software most of the predicates are linear (i.e., the predicates are linear Boolean functions of their variables). The boundaries of the corresponding input subdomains are linear on their variables and the subdomains are generalized polygons. No method was found in the literature survey to solve this problem even in this simplified case. For nonlinear predicates the subdomains have much more complex shapes and the difficulties in solving the problem increase even further.

Thirdly, the integration of f over a path subdomain is a multidimensional integration and is very computational intensive. From a search on the Guide to Available Mathematical Software (GAMS) on the web, many mathematical packages are available now which contain subroutines

to evaluate a multidimensional integral numerically on special regions, e.g., hyper-rectangular regions and n-dimensional spherical regions. Examples include the package NAG developed by The Numerical Algorithms Groups, Inc., and the package TOMS developed by the Association of Computing Machinery. For an integration region of an arbitrary shape, the numerical integration becomes much more complex and no packages having such a functionality exist. Even if such a package can be developed, its running time will be exponential on n, the number of input variables. The integration computation will soon become intractable as n increases.

Due to the prohibitive difficulties in implementing the direct integration method, the method is not feasible in practice. In the next section the Monte Carlo method is proposed as an alternate method for the estimation of p_t.

3.4.2.2 The Monte Carlo Method

3.4.2.2.1 Application of the Monte Carlo Method

The Monte Carlo method is a numerical method that provides approximate solutions to a variety of mathematical problems by performing statistical sampling experiments on random variables using a computer. The input variables to the Monte Carlo method are random variables with certain probability density distributions. In each single trial of the Monte Carlo method, the values of input variables are generated randomly with respect to the probability density distributions, the system responses to the generated input variables are computed and the outcomes recorded. After a huge number of trials an approximate solution can be obtained by averaging over the outcomes from all the trials. The absolute error of the estimate is inversely proportional to the square root of N, the total number of trials. (Fish96, Sobo94)

The application of the Monte Carlo method to the estimation of p_i is straightforward. The input variables to the Monte Carlo method are the input variables to the target program. The

probability density distributions of the input variables can be obtained from the operational profile. In each single trial, the values of the input variables are generated randomly, the target program is run with the generated input variables and the path executed is identified and recorded. After a number of trials, p_i can be estimated by the relationship:

$$\hat{p}_i \approx \frac{N_i}{N} \quad , \tag{3.16}$$

where

N_i = the number of trials in which the path, i, is executed,

N = the total number of trials performed.

The absolute error of the estimate is given by the relationship:

$$\tilde{p}_i = |p_i - \hat{p}_i| \propto \frac{1}{\sqrt{N}} \quad . \tag{3.17}$$

3.4.2.2.2 Advantages and Limitations

The biggest advantage in applying the Monte Carlo method is its simple implementation and manageable effort. Monte Carlo method always produces an answer, which is guaranteed to converge on a large number of trials. The nature of Monte Carlo method is well suited for parallel computation, which provides the opportunity to further decrease its running time.

The Monte Carlo method also has several serious limitations. The convergence rate of the Monte Carlo method is slow, a huge number of trials may be needed to achieve a sufficiently high estimate accuracy. Although qualitatively the absolute error of the estimate decreases as the total number of trials increases according to Eq. (3.17), the quantitative value of the error is very difficult to obtain (i.e., the implicit proportional constant in Eq. (3.17) is hard to quantify). The application of the Monte Carlo method will introduce new uncertainties into the software reliability estimation.

3.4.2.2.3 Method Automation

The Monte Carlo method requires repetitions of a large number of trials and automation is essential. The ideal tool for the automation of the Monte Carlo method should have the abilities to: (1) construct the flowgraph of the target program and identify/store all the paths of interest; (2) instrument the target program so that which path is traversed during a single program execution can be identified and recorded; (3) randomly generate test cases according to a given input distribution; (4) execute the target program with the randomly generated test cases. The construction of the flowgraph has been shown in **Section 3.3.3** to be a simple task. The instrumentation of the target program is an elementary step in software analysis and is incorporated in most of the software reverse engineering tools and test coverage tools. An illustration of the instrumentation process is given in Gall95. Random sampling according to a certain input distribution is a standard technique and has been extensively studied. Monitored execution of the target program is also easy to realize. There is no fundamental difficulties in automating the Monte Carlo method.

3.5 Testing

3.5.1 The General Testing Strategy

After a set of paths is identified using the testing method discussed in **Section 3.3** and the p_i for each of the paths is obtained using the Monte Carlo method discussed in **Section 3.4**, we are ready for the testing phase. The total number of paths to be tested, N_{tested}, can be determined based upon the time and resources available for testing. All the identified paths are ranked by their p_i values and the first N_{tested} paths with the largest p_i's are selected for testing. The

rationale in applying this simple path selection criterion has been given in **Section 3.2**. With the assumption that that once an error is found, debugging will be performed to remove the error, maximum reliability improvement can be achieved by reducing the error content of the paths with the largest p_i's through testing and debugging, making use of the fact that the unreliability contribution of a path is weighted by the p_i of the path (Eq. (3.4)). The N_{tested} selected paths are then tested and the number of errors detected is recorded for use for the estimation of θ_i in the next section. A discussion of the stopping criteria of the testing process can be found in the end of **Section 3.7.1**.

One subtlety in the testing strategy is to determine the number of test cases needed for each path. As mentioned previously in **Section 3.5.2**, for a truly homogeneous subdomain, only one test case needs to be generated for each subdomain. The path subdomain, although being more homogeneous than a randomly partitioned subdomain, may not be truly homogeneous. If the path of interest contains no errors, all inputs in the corresponding path subdomain will cause the program to execute successfully and the subdomain is truly homogeneous. However, if the path does contain errors of certain types, some of the inputs in the corresponding path subdomain may still produce correct outputs (which is often called *coincidentally correct*), a single test case selected from the path subdomain may not be able to reveal the errors. For example, *domain errors* (errors in the control flow of the program that will cause a wrong path to be traversed for some inputs (White80)) cause shifts in the path subdomain boundaries. If the shifts are small, a majority of inputs in the path subdomain still causes the correct path to be traversed and produces correct outputs. A single test case randomly selected from the path domain has a low probability of being the failure-causing and error-revealing inputs, which constitute only a small portion of the path subdomain. It is worth to note that in this case the unreliability contribution of the errors is much smaller than the p_i of the path subdomain. A domain strategy has been proposed for the detection of domain errors (White80, Clar82, Jeng94). The strategy basically generates several

test cases close to the boundaries of a path subdomain aiming to detect the shifts in the boundaries. Due to difficulties in locating the subdomain boundaries (discussed in **Section 3.4.2.1**), the implementation of the domain strategy is rather complicated and no practical applications have been found. A practical alternative to the domain strategy is to randomly generate several test cases for each path. This method increases the probability of detecting both *domain errors* and *computation errors* (errors in some assignment statement that cause a wrong function to be computed for one or more of the output variables (White80)) with manageable effort.

Another problem needed to be solved for testing is the oracle problem. An *oracle* is any means that provides information about the (correct) expected behavior of a program. It can be obtained by three means: It can be automatically generated from software specification documents (Brown92, Luqi94, Pete94). Two or more versions of the same program can be written by independent groups and run in parallel so that each one acts as an oracle to the others. Any discrepancies in their outcomes indicate errors in at least one version. As a last choice, the tester can act as the oracle and decide if the outputs of the testing are correct. In this case the tester must have a deep understanding of the structure and functionality of the program. The first two methods provide opportunity for automated oracle generation and automated testing, while the third requires tester intervention. Our methodology assumes an oracle is available in one of the three ways.

3.5.2 Testing Automation

The automation of the testing strategy requires an automated oracle and automated test case generation for the selected paths. The automated oracle has been discussed in **Section 3.5.1**. The automatic test cases generation has been discussed in **Section 3.3.3**. The other components of the testing strategy can be implemented without difficulty.

3.6 The Estimation of θ_i

3.6.1 The Average Error Content per Path, *e*

Before we begin discussions on the estimation of θ_i, we wish to introduce another term, *e, the average error content per path,* which is given by the relationship:

$$e = \frac{E}{N} \quad , \tag{3.18}$$

where E is the total number of errors in the program,

N is the total number of paths in the program.

e can also be interpreted as the probability that an arbitrarily chosen path in the program contains at least one error. Any path in a program either contains some errors or not, thus having a deterministic error content. However, since typically we can only analyze a portion of all the paths in the program, we have to use a probabilistic approach for the remaining paths. This is why the probabilistic term e is introduced.

By definition, θ_i of a path, *i,* is the probability of failure upon a single execution of the path. It can be related to the error content of the path, e_i, by the relationship:

$$\theta_i = e_i * f_i \quad , \tag{3.19}$$

where f_i is a factor in the range *[0,1]*, which is used to account for the *coincidental correctness* discussed in **Section 3.5.1**. Loosely speaking, for a single execution of the program, the probability that the path being executed has at least one error is e. Even if the path being executed contains some errors, a failure may not always occur because the input used to exercise the path may not be a failure-causing input. Thus the failure probability, θ_i, is not more than the error-containing probability, e_i. Whether θ_i is equal to e_i, or close to e_i, or much less than e_i depends upon the types of errors in the path. If the path contains domain errors that cause shifts in

the path subdomain boundaries, f_i may be much less than unity due to the coincidental correctness. The errors may not be revealed and corrected on the first test of the path. This is the main reason why multiple test cases are generally needed for each path. If the path only contains computation errors that cause a wrong function to be computed, the coincidental correctness has a low probability to occur and f_i is very close to unity. The computation errors are very likely to be revealed and removed on the first test of the path and further tests are not necessary. Since different types of errors have different characteristics, they may be treated separately to achieve better results.

The failure rate of the program, θ, is an average of θ_i's over all the paths in the program, weighted by the p_i's (Eq. (3.8)). Ignoring the subtlety associated with the p_i factors, we claim that in general:

$$\theta \leq e \quad . \tag{3.20}$$

As we will see in the following sections, sometimes an estimate of e is easier to obtain than that of θ. In such cases the estimate of e can server as a conservative estimate of θ.

In the following sections the *error content* is often used as an abbreviation of the term *the average error content per path*.

3.6.2 The General Strategy

Suppose we are given a program as a black box. The only structural information of the program we have is that there are N paths in the program, and there is no way to distinguish between them. In this case the best we can do is to estimate the failure rate, θ, of the program, and use it as an estimate of θ_i of all the paths. If we assume that faults are uniformly distributed over the paths, and that the faults are of the same type, the failure rate of all the paths will be the same, θ will be a good estimate of all the θ_i's. Since the assumption may not be valid in

practice (It may be more realistic to assume that faults are uniformly distributed over the program statements.), the strategy proposed in this section needs validation and has to be used with cautiousness.

In the following sections θ will be used exclusively as the failure rate of the program **before** testing and debugging. **Section 3.6.2.1** and **3.6.2.2**, show how the estimate of θ_i of the tested paths and the untested paths can be obtained from the estimate of θ. **Section 3.6.3** discusses the methods for the estimation of θ.

3.6.2.1 The Estimation of the θ_i of the Tested Paths

Use e_b, e_t to denote the error content of the program before and after testing, respectively. We have the relationship:

$$E[e_t] = (1 - c_t c_d) E[e_b] \quad , \tag{3.21}$$

where $E[]$ is the expectation operator,

c_t is the error-revealing effectiveness of the testing process,

c_d is the error-removing effectiveness of the debugging process.

The variable, c_t, can be interpreted as the probability that, if the path under testing contains some errors, the errors will be revealed during the testing process. Let n, n_t denote the number of errors originally in the tested paths and the number of errors revealed by the testing, respectively. Then c_t is given by the relationship:

$$c_t = E[\frac{n_t}{n}] \quad . \tag{3.22}$$

The variable, c_t, always lies in the range *[0,1]*. As discussed in **Section 3.5.1**, since the path subdomains are not truly homogeneous, a randomly selected input from a path subdomain may

not reveal errors in the path even if they exist, i.e., the output may be *coincidentally correct.* By increasing the number of test cases generated for each subdomain, the probability of revealing errors if they are present (i.e., c_t) increases monotonically. However, it will never reach unity unless all the inputs in the subdomain have been tested. This is a problem inherent to all structured testing methods and is very hard to solve.

The variable, c_d, is the error-removing effectiveness of the debugging process. Let n_t, n_d denote the number of errors revealed by testing and the number of errors remaining in the program after debugging, respectively. Unrevealed errors are not counted for either n_t or n_d. Then c_d is given by the relationship:

$$c_d = 1 - E[\frac{n_d}{n_t}] \quad . \tag{3.23}$$

c_d is unity for perfect debugging and less than unity for imperfect debugging. c_d may even be negative if new errors are introduced in the debugging process.

Use θ_t to denote the failure rate of the tested paths after testing and debugging. Analogous to Eq. (3.21), we can write:

$$\hat{\theta}_t \approx (1 - c_t c_d)\hat{\theta} \quad . \tag{3.24}$$

The estimate of c_t and c_d can be obtained from experimental data. Provided with c_t and c_d, the estimate of θ_i of the tested paths can be calculated using Eq. (3.24) from the estimate of θ_i of the paths before testing, i.e., the estimate of θ.

A special case occurs under the assumptions that: (1) If there are any errors in a path, they will be revealed with a probability of unity during the testing phase. (2) Once an error is revealed, debugging will be performed to remove the error from the program. The debugging is perfect, errors are removed without introducing new errors. Under these assumptions

$c_t = c_d = 1$. From Eq. (3.24) we have $\hat{\theta}_t = 0$. In this case the tested paths are error-free and their unreliability contribution is zero.

It may be argued that since the tested paths are actually somewhat "visible" to us during testing and it is possible to distinguish between the them, why we are still using a black box treatment of the tested paths. The answer to the argument is that in order to obtain a good estimate of θ_i for a particular tested path, a large number of test cases typically needs to be generated for the same path subdomain. This circumvents the main advantage we are expecting from our path testing strategy, i.e., to make the subdomain homogenous so as to reduce the number of test cases required for each subdomain. Although it is mentioned in **Section 3.5.1** that multiple test cases may be needed for each path subdomain to increase the probability of detecting errors, there is a significant difference between the effort and test cases needed to determine whether $\theta_i > 0$ (error revealing) and to determine the actual value of θ_i. Thus even for the tested path we typically can not afford to estimate their θ_i's individually, and have to use a black box treatment as that for the untested paths.

3.6.2.2 The Estimation of the θ_i of the Untested Paths

Although untested paths are not tested and debugged, their failure rate may still be changed by the debugging process. The reason is that untested paths may share with tested paths some program statements, which may be modified during debugging. A factor, c_c, can be used to account for this correlation. Let n_u, n_{ud} denote the number of errors in untested paths before and after debugging, respectively. c_c is given by the relationship:

$$c_c = E[\frac{n_{ud}}{n_u}] \quad . \qquad (3.25)$$

An estimate of θ_i of untested paths after testing and debugging, $\hat{\theta}_u$, is given by the relationship:

$$\hat{\theta}_u = c_c \hat{\theta} \quad . \tag{3.26}$$

Eq. (3.26) can be simplified under the assumption that the debugging has no effect on the failure rate of untested paths. In this case $c_c = 1$ and $\hat{\theta}_u = \hat{\theta}$. The estimate is conservative as long as the debugging process has an overall positive effect on untested paths (i.e., the number of errors removed from untested paths is not less than the errors introduced during debugging).

3.6.3 The Estimation of θ

3.6.3.1. The Estimation of θ Using Software Metric

Software metric is a quantitative measure of software complexity. Some well-known software metric include the Lines of Code (LOC), the Halstead's Metric (Hals77), and the McCabe's Metric (McCa76). Interested readers can find a survey of software metric in Beiz90.

It is generally believed that the number of errors in a program is correlated to the software complexity. Many models have been developed trying to model this correlation and use software metric to predict the number of errors in a program (Otte81, Lipow82, Gaff84, Craw85, Khos90). The general form of the models is:

$$B = f(M_1, M_2, .. a_1, a_2, ..) \quad , \tag{3.27}$$

where B is the number of errors in the program,

$M_1, M_2, ..$ are software metric,

$a_1, a_2, ..$ are model parameters.

The model parameters are determined by fitting experimental data into the model. The fitted model can then be used to estimate B, the error content of the program. A specific example is given by Khos90:

$$B = a_1 + a_2 LOC^{a_3} \quad , \tag{3.28}$$

where LOC is the number of lines in the program.

The *average error content per path, e,* can be obtained by the relationship:

$$\hat{e} = \frac{B}{N} \quad , \tag{3.29}$$

where N is the total number of paths in the program.

As discussed in **Section 3.6.1**, $\hat{e}$ can serve as a conservative estimate of $\hat{\theta}$.

Although many models of this kind have been developed and they occasionally produce good results, none of them can be applied reliably to all applications. The uncertainty associated with their predictions is high and there is no effective way to quantify it. This method is not preferable for the estimation of θ for safety critical software.

3.6.3.2 The Estimation of θ Using the Bayesian Updating Method

Since a failure history of the target program can be observed during the testing, a better estimate of θ can be obtained by making use of the testing data. In this section a Bayesian updating method is proposed for the estimation of θ using testing data.

3.6.3.2.1 Mathematical Foundation

The Bayesian updating method treats θ as a random variable in the range $[0,1]$. Before the testing phase is conducted, θ is assigned a *prior distribution*, which represents a prior knowledge of θ. During the testing phase a number of paths is tested and the number of errors found is recorded. The testing data is then used as an evidence to update the prior distribution of θ. The resulting distribution (i.e., the *posterior distribution*) represents a posterior knowledge of

θ and can be used for the estimation of θ. The updating is based upon the Bayes' Theorem (Flor90):

$$f'(\theta) = f(\theta|E) = \frac{P(E|\theta)f(\theta)}{\int_0^1 P(E|\theta)f(\theta)d\theta} \quad , \tag{3.30}$$

where E denotes the evidence from testing,

$f(\theta)$ is the prior distribution of θ,

$f'(\theta) = f(\theta \mid E)$ is the posterior distribution of θ,

$P(E \mid \theta)$ is the probability that E will happen given θ.

Suppose in the testing phase N paths are tested and M failures occur. Assume that all the M failures are independent of one another. Also assume $M << N$, which is valid for high quality software. Then we have:

$$P(E|\theta) = \binom{N}{M}\theta^M(1-\theta)^{N-M} = \frac{N!}{M!(N-M)!}\theta^M(1-\theta)^{N-M} \quad . \tag{3.31}$$

Eq. (3.30) and (3.31) constitute the mathematical foundation of the Bayesian updating method.

The prior distribution represents a prior knowledge of θ before testing. If nothing is known about the program beforehand, the prior can be set to be a uniform distribution over a range to which θ is expected to belong.

Given a prior, $f(\theta)$, and an evidence, E (i.e., N and M), the posterior distribution, $f'(\theta)$, can be computed using Eq. (3.30) and (3.31). Various estimates of θ can be obtained using $f'(\theta)$. For example, the expectation of θ is given by the relationship:

$$\mu = E[\theta] = \int_0^1 \theta f'(\theta)d\theta \quad . \tag{3.32}$$

The variance of θ is given by the relationship:

$$\sigma^2 = \int_0^1 \theta^2 f(\theta)d\theta - \mu^2 \quad . \tag{3.33}$$

The $(1-\alpha)$ one sided upper confidence limit θ_u is the largest θ such that:

$$\int_{\theta}^{1} f'(x)dx > \alpha \quad , \tag{3.34}$$

where x is a dummy integration variable.

The $(1-\alpha)$ one sided lower confidence limit θ_l is the smallest θ such that:

$$\int_{0}^{\theta} f'(x)dx > \alpha \quad . \tag{3.35}$$

The interval $[\theta_l, \theta_u]$ is the $(1-2\alpha)$ confidence interval. Use δ to denote the size of the interval:

$$\delta = \theta_u - \theta_l \quad . \tag{3.36}$$

These quantities defined in the above equations provide various measures of θ. $E[\theta]$can serve as an estimate of θ:

$$\hat{\theta} = \mu = E[\theta] \quad . \tag{3.37}$$

Both σ^2 and δ can serve as measures of the uncertainty associated with $\hat{\theta}$. $\theta_u$can serve as another measure of the estimate uncertainty, when people are more interested in an upper bound (e.g., for safety critical software, people always wish to be conservative) rather than the entire interval.

3.6.3.2.2 Performance Analysis

Generally speaking, the posterior distribution depends on both the prior distribution and the evidence. In this section we analyze how the posterior distribution is affected by variations in the prior distribution and the evidence. Experiments showed that the typical behavior (e.g., the shape) of the posterior distribution is mostly affected by three factors: whether any failures occur during the testing (whether *M=0*); the number of paths tested (*N*); and the prior distribution ($f(\theta)$). The three factors are discussed in the next three sections, respectively. The performance

of the Bayesian updating method under different conditions is analyzed and the implications are discussed. Two measures used for the performance analysis are the expectation $E[\theta]$, which serves as an estimate of θ, and the size of the $1-2\alpha$ ($\alpha = 0.025$) confidence interval, δ, which serves as a measure of the estimate uncertainty.

3.6.3.2.2.1 Failure Vs. No Failure

Experiments showed that the posterior distribution takes different shapes depending upon whether any failures occur during the testing (whether *M=0*).

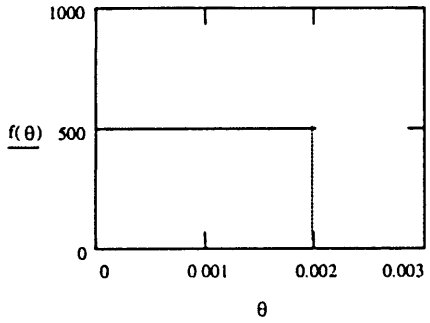


Figure 3.10a The prior (uniform)

$$E[\theta] = 0.001$$

$$\delta = 0.0019$$

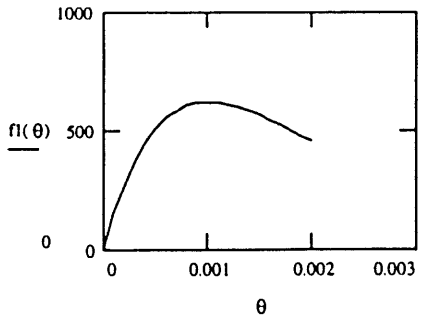


Figure 3.10b The posterior *(M=1)*

$$E[\theta] = 0.00109$$
$$\delta = 0.00176$$

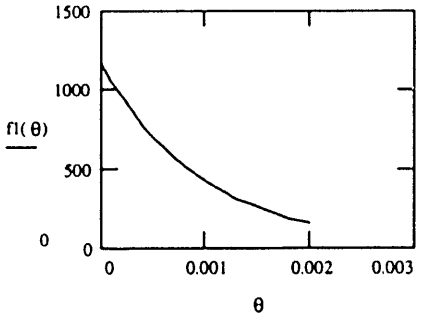


Figure 3.10c The posterior *(M=0)*

$$E[\theta] = 0.00068$$
$$\delta = 0.00183$$

Figure 3.10 Comparison of failure vs. no failure cases *(N=1000)*

Generally speaking, with a uniform prior distribution, if $M>0$, the posterior has a "$\cap$" shape with a peak at approximately $\frac{M}{N}$. If $M=0$, the posterior has a decaying shape representing a posterior belief that θ is expected to be very small. Note that $E[\theta] \neq 0$ even when $M=0$. This is an advantage of the Bayesian updating method, i.e., we can always obtain a nonzero estimate of θ.

3.6.3.2.2.2 Number of Tested Paths

In this section the evolution of the posterior as the number of tested path N increases is analyzed. Intuitively, the uncertainty of the estimate should decrease as the number of tested paths increases and more knowledge is accumulated. Experiments showed that this intuition is correct.

3.6.3.2.2.2.1 No Failure Case

The prior distribution is the same as that in Figure 3.10a.

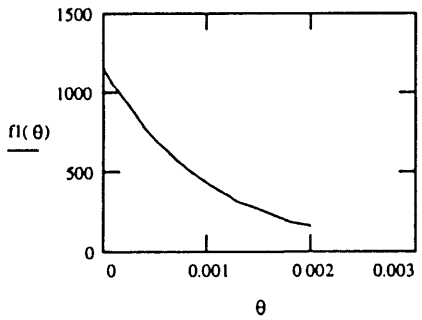


Figure 3.11a The posterior
(N=1,000, M=0)

$E[\theta] = 0.00068$
$\delta = 0.00183$

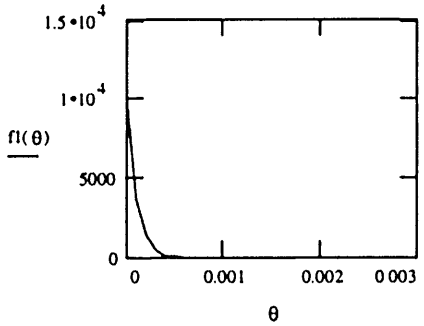


Figure 3.11b The posterior
(N=10,000, M=0)

$E[\theta] = 0.00010$
$\delta = 0.00037$

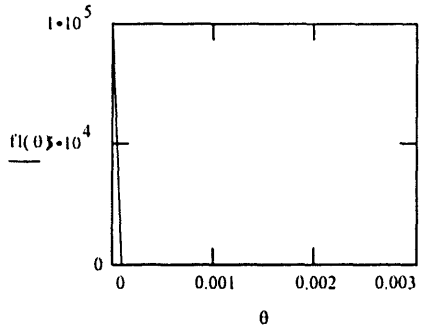


Figure 3.11c The posterior
(N=100,000, M=0)

$$E[\theta] = 1.1 * 10^{-6}$$
$$\delta = 3.7 * 10^{-5}$$

Figure 3.11 Posterior evolution when no failures occur

From Figure 3.11 we can see that in no failure case, both $E[\theta]$ and δ decrease as N increases.

3.6.3.2.2.2.2 With Failure Case

The prior distribution is the same as that in Figure 3.10a.

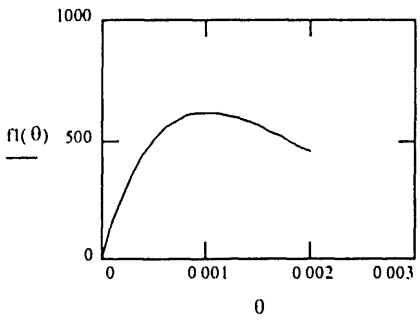


Figure 3.12a The posterior
(N=1000, M=1)

$$E[\theta] = 0.00109$$
$$\delta = 0.00176$$

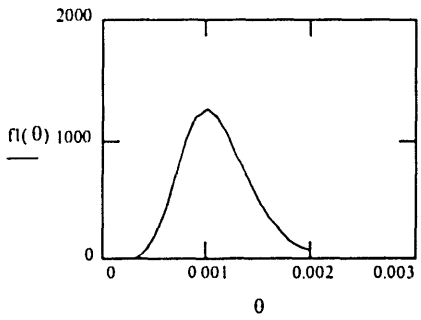


Figure 3.12b The posterior
(N=10000, M=10)

$$E[\theta] = 0.00109$$
$$\delta = 0.00122$$

Figure 3.12 Posterior evolution when failures occur

From Figure 3.12 we can see that for these two experiments, as N increases, $E[\theta]$ remains unchanged (provided that $\frac{M}{N}$ remains unchanged), while δ decreases. The decrease rate of δ is much slower than that in no failure case. A ten-fold increase in N only decreases δ by a third.

3.6.3.2.2.3 The Prior Distribution

Experiments showed that the prior distribution also has an effect on the posterior distribution, while the magnitude of the effect depends upon if any failures occur.

3.6.3.2.2.3.1 No Failure Case

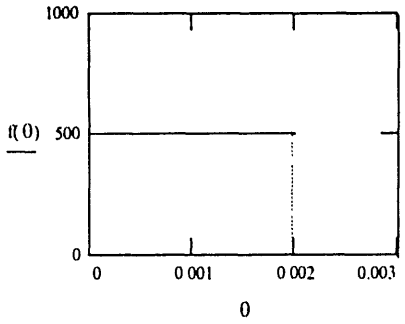


Figure 3.13a The prior (uniform)

$E[\theta] = 0.001$

$\delta = 0.0019$

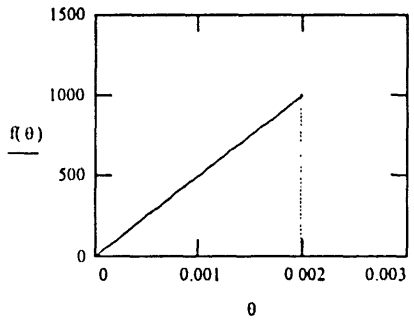


Figure 3.13b The prior (nonuniform)

$E[\theta] = 0.00133$

$\delta = 0.00166$

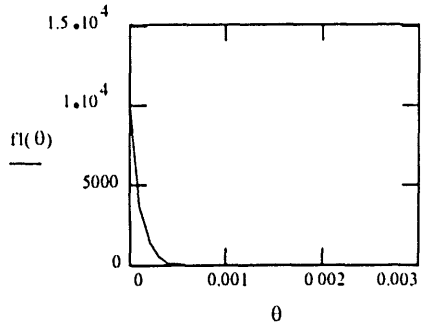


Figure 3.13c The posterior
(N=10000, M=0)

$E[\theta] = 0.00010$

$\delta = 0.00037$

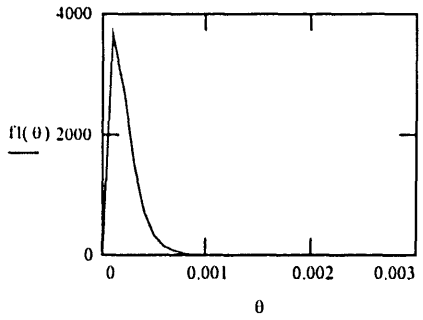


Figure 3.13d The posterior
(N=10000, M=0)

$E[\theta] = 0.00020$

$\delta = 0.000246$

Figure 3.13 Sensitivity analysis when no failures occur

In no failure case, there are big differences between both the $E[\theta]$'s and the δ's of the posterior. The posterior is very sensitive to changes in the prior, despite of the fact that *10,000* paths have been tested.

3.6.3.2.2.3.2 With Failure Case

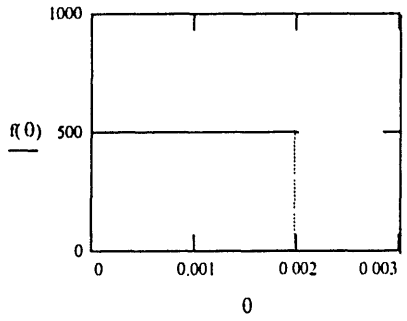


Figure 3.14a The prior (uniform)

$E[\theta] = 0.001$

$\delta = 0.0019$

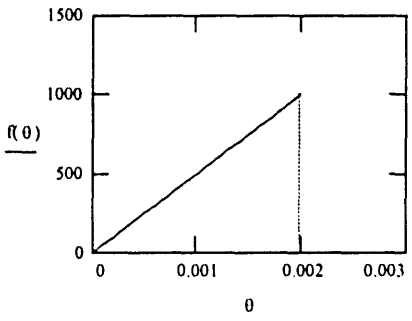


Figure 3.14b The prior (nonuniform)

$E[\theta] = 0.00133$

$\delta = 0.00166$

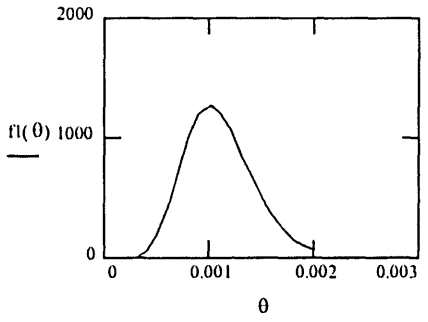


Figure 3.13c The posterior
(N=10000, M=10)

$E[\theta] = 0.00109$
$\delta = 0.00122$

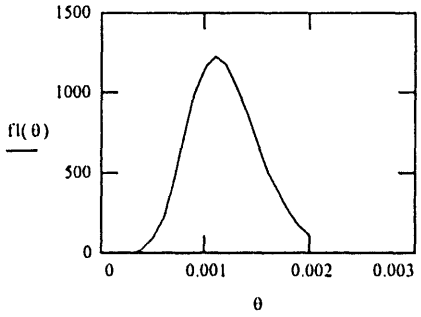


Figure 3.13d The posterior
(N=10000, M=10)

$E[\theta] = 0.00118$
$\delta = 0.00122$

Figure 3.14 Sensitivity analysis when failures occur

If some failures occur during the testing process, the posterior is not sensitive to changes in the prior distribution.

3.6.3.2.2.4 A Brief Summary

(1) The posterior distribution takes different shapes depending upon whether any failure occurs during the testing (whether *M=0*).

(2) As *N* increases, the uncertainty of the estimate decreases. If failures occur during the testing process, the rate of the decrease is much slower than that of no failure case.

(3) The posterior distribution is sensitive to the prior if no failure occurs during the testing. It is not sensitive to the prior if any failure does occur.

3.6.3.2.3 Advantages and Limitations

The Bayesian updating method has a solid mathematical foundation and a simple implementation. It does not require a detailed failure history of the program during testing. As long as there are *M* failures in *N* tests, it does not matter how these M failures are distributed, the Bayesian updating method always produces the same result. The Bayesian updating method

provides an effective mechanism to quantify the uncertainty of its estimate, which is especially important for applications to safety critical software. The uncertainty of its estimate decreases as the number of tests, N, increases, and the rate of decrease is very high if no failures occur during the testing. Since in the later testing phase of safety critical software it is typical to have no failures on a large number of successive tests, it can be expected that the uncertainty of the estimate will diminish very quickly. A relatively small number of tests may be sufficient to achieve desired accuracy and certainty. Another advantage of the Bayesian updating method is that it always produces a nonzero estimate of the software reliability no matter failures occurred in the testing process or not.

The most serious limitation of the Bayesian updating method is associated with the prior. For safety critical software we are most interested in the no failure case. However, as we have seen in **Section 3.6.3.2.2.3.1**, in this case the posterior is sensitive to the prior. Large uncertainties can be introduced into the estimate of θ from uncertainties in the prior estimation. One way to mitigated this is to increase the number of tests, which results in an increase in the method complexity and effort needed. Secondly, since the Bayesian updating method always produces a nonzero reliability estimate (even for no failure cases), which decreases as the total number of tested paths increases, the number of paths available for testing should be sufficiently large in order to obtain ultra-high reliability estimate. **Figure 3.11** gives a good illustration. Assume that no debugging is performed during the testing process and the software reliability is unaffected by the testing process. With a given uniform prior distribution in **Figure 3.10a**, if the reliability requirement of the software is in the order of $10^{-3} \sim 10^{-4}$ (per execution), a total of $10^{3} \sim 10^{4}$ paths needs to be tested. For software with ultra high reliability requirement, say, in the order of 10^{-6} (per execution), at least 10^{5} paths have to be tested if no failure occurs. Even more (but not much more) paths are required when failures are observed. Considering the

complexity in identifying and testing the paths, 10^5 tests are very difficult to implement in practice.

3.7 Summary of the Methodology

3.7.1 The General Application Strategy

For a given program, first the control flowgraph of the program needs to be constructed and expanded as discussed in **Section 3.3**. From the expanded flowgraph, a number (N_{iden}) of feasible paths can be identified. Based upon the given operational profile of the program, the p_i's of the identified paths can be estimated (e.g., using the Monte Carlo method discussed in **Section 3.4.2.2**). Using the p_i's, a number (N_{tested}) of paths can be selected for testing (as discussed in **Section 3.5.1**). Testing is then conducted and debugging performed when errors are found. After the testing and debugging phase, an estimate of the failure rate of the program before testing (i.e., $\hat{\theta}$) can be obtained using methods discussed in **Section 3.6.3**. Then estimates of the θ_i's of the tested paths (i.e., $\hat{\theta}_t$) and the untested paths (i.e., $\hat{\theta}_u$) after testing can be obtained using Eq. (3.24) and (3.26), respectively. The reliability (per execution) of the program after testing is given by the relationship:

$$\hat{R} = 1 - \frac{1}{\hat{P}}[(\sum_{i=1}^{N_{tested}} \hat{p}_i)\hat{\theta}_t + (\sum_{i=N_{tested}+1}^{N_{iden}} \hat{p}_i)\hat{\theta}_u] = 1 - \frac{1}{\hat{P}}(\hat{P}_t\hat{\theta}_t + \hat{P}_u\hat{\theta}_u) \ , \qquad (3.38)$$

where

$$\hat{P} = \sum_{i=1}^{N_{iden}} \hat{p}_i \ , \qquad (3.39)$$

$$\hat{P}_t = \sum_{i=1}^{N_{tested}} \hat{p}_i \ , \qquad (3.40)$$

$$\hat{P}_u = \sum_{i=1}^{N_{untested}} \hat{p}_i \ . \tag{3.41}$$

If we do not distinguish between the paths that are not identified and the paths that are identified but not tested and assume that they all have the same failure rate $\hat{\theta}_u$, an estimate of the reliability can be directly obtained from Eq. (3.4) and Eq. (3.9):

$$\hat{R} = 1 - (\sum_{i=1}^{N_{tested}} \hat{p}_i)\hat{\theta}_t - (\sum_{i=N_{tested}+1}^{N_{total}} \hat{p}_i)\hat{\theta}_u = 1 - \hat{P}_t\hat{\theta}_t - (1 - \hat{P}_t)\hat{\theta}_u \ . \tag{3.42}$$

The uncertainty associated with the reliability estimate $\hat{R}$ can be obtained if the uncertainties associated with the estimates $\hat{p}_i$'s and $\hat{\theta}_i$'s can be quantified. For illustration purpose, consider the general equation:

$$\hat{R} = 1 - \sum_i \hat{p}_i\hat{\theta}_i \ . \tag{3.43}$$

The correlation between the uncertainties associated with R, p_i, and θ_i is given by Barf69:

$$s^2(R) = \sum_i (s^2(p_i)\hat{\theta}_i^{\,2} + s^2(\theta_i)\hat{p}_i^{\,2}) \ , \tag{3.44}$$

where $s^2(x)$ is the *adjusted mean square deviation* of measurements on x, an estimate of the variance σ^2 of x. It can be interpreted as an uncertainty measure.

3.7.2 Some Useful Observations

With the assumptions that the debugging is perfect for the tested paths and has no effect on the failure rates of the untested paths and the unidentified paths (i.e., $\hat{\theta}_t = 0$ and $\hat{\theta}_u = \hat{\theta}$), Eq. (3.42) is simplified to:

$$\hat{R} = 1 - (1 - \hat{P}_t)\hat{\theta} \ , \tag{3.45}$$

where $\hat{\theta}$ is the failure rate of the program before testing.

What Eq. (3.45) tells us is that in order to improve the reliability of a given software, we need to render the value of $\hat{\theta}$ as small as possible, and render the value of $\hat{P}_t$ as large as possible. The former can be realized by various means of software quality control methods, such as the use of formal methods in software development process. The latter can be achieved by using appropriate testing strategies that maximize $\hat{P}_t$. This justifies the path selection criteria proposed in **Section 3.5.1**, i.e., selecting the paths with the largest p_i's for testing.

In the later testing phase of safety critical software it is typical to have no failures upon a large number of successive tests. In this case the testing has two positive effects upon the software reliability estimate. The first effect is that $\hat{P}_t$ increases as more paths are tested, the second effect is that $\hat{\theta}$ decreases as more paths are tested without failures. For typical nuclear control software, the input variables vary smoothly and over narrow ranges during most of the software application lifetime. It can be expected that only a small fraction of all the paths in the program is frequently demanded and its demand probability (the sum of the p_i's) is close to unity. If we select these paths for testing and debugging, we can obtain a high reliability estimate of the resulting program. For example, if we can make $\hat{P}_t \sim 0.999 = 1 - 10^{-3}$, and make $\hat{\theta} \sim 10^{-3}$ (This is relatively easy to achieve. According to the analysis in **Section 3.6.3.2**, only $\sim 10^3$ paths need to be tested without failures to obtain this result), the reliability estimate of the program after testing will be $\hat{R} \sim 1 - 10^{-6}$, which fulfills typical ultra-high reliability requirement. However, it should be noted that in order to also obtain satisfactory accuracy in the reliability estimate, much more efforts have to be devoted to reduce the uncertainties in estimates of θ and the p_i's. The former can be achieved by testing more paths, the latter can be achieved by performing more trials in the Monte Carlo methods. Another issue is that some paths may have very small p_i values and do not make big contributions to the unreliability of the program.

Nevertheless, they still deserve special care and need to be extensively tested because errors in these paths may lead to very severe consequences. Examples include the paths in emergency handling procedures. Error consequence analysis is beyond our research scope and is not addressed here.

In order to assure high reliability of the program after testing, $\hat{P}_t$ should be made very close to unity. In practice, if a number of paths has been tested, yet the reliability estimate or the estimate uncertainty does not fulfill requirements, more paths should be identified and tested and the reliability estimation should be repeated. The process iterates until both the reliability estimate and the estimate uncertainty become satisfactory. This determines the criteria for stopping the testing process.

3.7.3 Underlying Assumptions and Potential Applications

The methodology has several underlying assumptions, which limit its applicability. First, the methodology is based upon structural testing method. The target program should not have complex control or data structures (e.g., array pointers) which hinder control flowgraph generation and analysis. Secondly, the complexity in applying the methodology is considerably higher than that of the other widely used testing methods, such as reliability growth models and random testing. Automation is essential for its practical implementation. Although there are no fundamental difficulties in automating the methodology except the automatic test case generation, it is conceivable that even provided with an automated tool, the size of program that can be handled is still limited by available computational resources, such as the memory and CPU time. The target program should not be very big, or more precisely, should not contain a huge number of paths in order for the methodology to be applicable with manageable effort. Finally, the target program should not be very small. As we have seen in **Section 3.6.3.2**, in order to obtain a

reliability estimate in the order of 10^{-6} (per execution), at least 10^5 paths have to be tested (The number may be reduced, though, based upon discussions in **Section 3.7.2**). If the total number of paths in the program is much less than 10^5, the only way to obtain a satisfactory reliability estimate is to completely test the program. Thus the potential application of the methodology is for medium size program. The lower bound on the number of paths in the program is in the order of $10^4 \sim 10^5$. The upper bound depends on the implementation and automation of the methodology. With the advances in computer hardware and software, we can conceive that more and more complex program can be handled by the automated methodology.

3.7.4 Advantages and Limitations

The methodology is based upon input domain-based reliability modeling and structural testing method. It possesses the advantages of both input domain-based reliability models (discussed in **Section 2.5.3**), and structural testing method (e.g., ease in error revealing and debugging). The methodology can be used to control the testing process, providing path selection criteria and stopping criteria. It can also be used to quantify the reliability of the program after testing, producing both reliability estimate and estimate uncertainty. Since the path subdomains are expected to be more or less homogeneous, the estimate of the methodology should not be sensitive to changes in the operational profile (for the reasons discussed in **Section 2.5.3**).

The most serious limitation of the methodology is the high complexity involved in its implementation, which limits its applicability (discussed in **Section 3.7.2**). Since the methodology is based upon structural testing methods, it also has several inherent limitations of structural testing methods, such as the difficulties in dealing with coincidental correctness and missing path problems. The methodology is based upon an arguable assumption that errors are uniformly distributed over the paths. The validity of the assumption needs to be verified in

practice. The estimation of p_i is a very difficult step in the methodology. Although the Monte Carlo method proposed in the methodology can produce estimates of the p_i's, it has difficulties in quantifying the estimate uncertainties. Its probabilistic nature may hinder its acceptance by the nuclear industry, or some other industries with ultra-high reliability requirements. Some other alternatives may be needed.

Chapter 4

A Case Study Using the Integrated Methodology

4.1 Introduction

This chapter presents the results of a case study in which the designed integrated methodology in **Chapter 3** is applied to a program used in a nuclear power plant's reactor coolant system. The program is the generic Signal Validation Algorithm (SVA) used for calculating and validating the values of parameters of the reactor coolant system. Because we did not use any automated tool and had to perform the control flowgraph analysis and testing manually, we were unable to follow the general application strategy of the methodology discussed in **Section 3.7.1**. The estimation of the p_i's of the program requires the operational profile of the target program, which is not readily available. The proposed method for the estimation of the p_i's (i.e., the Monte Carlo method) requires automation and can not be realized by hand. For these reasons the case study bypassed the estimation of the p_i's and focused upon program testing and the estimation of θ using the testing data. The purpose of the case study is not to assess the effectiveness of the methodology, rather to show the applicability of the methodology. **Section 4.2** presents descriptions of the SVA program. **Section 4.3** and **Section 4.4** discuss the testing procedures and the testing results, respectively. Conclusions are drawn in **Section 4.5**.

4.2 The Signal Validation Algorithm (SVA)

4.2.1 Signal Validation Algorithm Features

In the nuclear power industry, many efforts have been devoted to improve plant performance and availability. An important factor for the success of these efforts is the improved access to reliable information for the entire plant, especially for safety critical systems. Many nuclear utilities are installing or upgrading their plant computer systems to provide such a capability to handle plant data and provide operators and other personnel with data interpretations that are accurate, extensive, reliable, and useful. The application of these computer systems provides operators with a ready access to a variety of sensor signals. It permits operators to use an increased number of redundant, independent measurements in the decision-making process during normal and emergency plant operations. However, simply displaying all these redundant measurements is not desirable because it may adversely complicate the operator's mental processing tasks of data evaluation and analysis. One way to reduce the operators' work load in comparing, analyzing, and evaluating sensor signals is to apply automatic signal validation algorithms as a pre-filter to reduce the amount of information passed to the operator and to the other systems in the plant.

The Signal Validation Algorithm (SVA) of this case study is a generic algorithm that performs signal validation for the plant processes that contain multiple sensors measuring the same or closely related process parameters. The algorithm is called generic because it is generally applicable to all types of parameters. Only slight modifications need to be made for each particular parameter according to different quantities of the sensors used.

4.2.2 Implementation of the SVA

The specifications and design documents of the SVA were provided by ABB-Combusting Engineering, a commercial reactor manufacturer firm. A graduate student in MIT, M. Ouyang, developed formal specifications of the SVA using an Integrated Formal Approach (IFA) in the OO1 language (Ouya95). The primary concern of Ouyang's work is to find errors and ambiguities in the specifications using formal methods and software development tools. In other words, it is mainly concerned with producing error-free specifications rather than error-free code. Since the OO1 tool has the capability of generating executable code automatically based upon the formal specifications written in OO1 language, we treated the SVA formal specifications as a target program and used it for our case study. The owner of the SVA, ABB-Combusting Engineering, generously permitted us to use the SVA and its supporting documentation in our case study, however, on a confidential basis. Thus we may not disclose the SVA code itself. It is worth noting that the formal specifications were developed in order to find errors in the original specifications of the SVA as part of its developmental process. This example is not one of the high quality commercial code that is actually used in nuclear power plants. The OO1 tool suite is a CASE tool supporting the Development Before The Fact (DBTF) formal method. Interested readers can find descriptions and applications of the OO1 tool in Ouya95 and Lung96.

The SVA formal specifications written in OO1 language have 18 independent modules and 416 primitive functional statements. Primitive functions are those elementary functions supported by the OO1 tool. Some examples of the primitive functions can be found in **Figure 4.1**. Looping structures are extensively used in the program representing the manipulations on identical elements in a set, i.e., the redundant sensor measurements for the same process parameter. A typical looping structure is shown in the module *AverageGood* in **Figure 4.1**. The

number of paths in the program is considerably much larger than the number of statements. Based upon a rough estimate, when the number of iterations for each loop is limited to $k=2$, the total number of feasible paths in the program is in the thousands. When the number of iterations for each loop is limited to $k=3$, the total number of feasible paths in the program in tens of thousands. It is a desirable feature for our case study because the more paths are available for test, the better accuracy may be achieved.

4.3 The Testing Process

The testing of the formal specifications is performed in two phases: a modular testing phase in which each of the modules is completely tested, and an integrated testing phase in which the entire program is tested.

4.3.1 Modular Testing

Each of the modules in the program is small and contains a small number of paths. Complete testing is performed for each module to ensure that it has the correct functionality. In order to show how the complete testing is done, we use a module *AverageGood* as an illustrative example. The Fmap (function map in OO1 language which represents the relationships between the primitive functions) of the module *AverageGood* is shown in **Figure 4.1**. The input to the module is *Set01*, a pointer to a set of identical sensor measurements. Each sensor measurement has a quality field to store the quality of the measurement and a value field to store the actual value of the measurement. The output of the module is *output*, the average of the sensor measurements which have *good* qualities. The module starts from the first element in the set (statements *a-d*), checks the quality of the element (statements *g-j*). If the quality is *good*, the

value of the sensor measurement is added to the sum of the measurements and the number of *good* sensors is incremented by one (statements *n-p*). Otherwise, the sensor measurement will be ignored (statements *k-m*). Statement *e* checks if the program has reached the end of the set. If it has not, the Boolean variable *AtNull:Sensor(Getit1)* has a value of *FALSE*. The module goes to the next element in the set and performs the same manipulation (statements *f-r*). If the end of the set has been encountered, the Boolean variable *AtNull:Sensor(Getit1)* has a value of *TRUE*. The module exits from the loop, divides the sum of the good sensor measurements by the number of good sensor measurements, and outputs the average (statements *t-v*).

Figure 4.2 shows the control flowgraph of the AverageGood module and its expansion using the FSPT method discussed in **Section3.3**. Each node in the flowgraph corresponds to one or more statements in **Figure 4.1**. The correspondence is shown on the upper right corner of the figure. **Figure 4.1a** shows the unexpanded flowgraph. **Figure 4.2b** shows the expanded flowgraph after limiting the number of iterations to *k=2* (Note that node *C* is the exit node so that the maximum number of iterations in **Figure 4.2b** is two). Since node *B* in **Figure 4.2b** has two incoming edges, each of the nodes *B* in **Figure 4.2b** is duplicated resulting in the expanded flowgraph in **Figure 4.3c**.

In **Figure 4.3c**, there are seven independent paths in the expanded flowgraph. The paths and their sequences are listed in **Table 4.1**. For each of the paths, the values of the input variables are generated based upon program predicate analysis. The expected outputs corresponding to these inputs are obtained based upon analysis of the specifications and design documentation of the SVA. The path is then tested using the input values and its outputs are compared with expected outputs. Discrepancies in the comparison indicate the presence of errors and debugging is then performed. Testing is redone whenever debugging is performed to ensure the correctness of the modified code. The test case generation, oracle generation, and debugging process are omitted here because they require deep understanding of the target program.

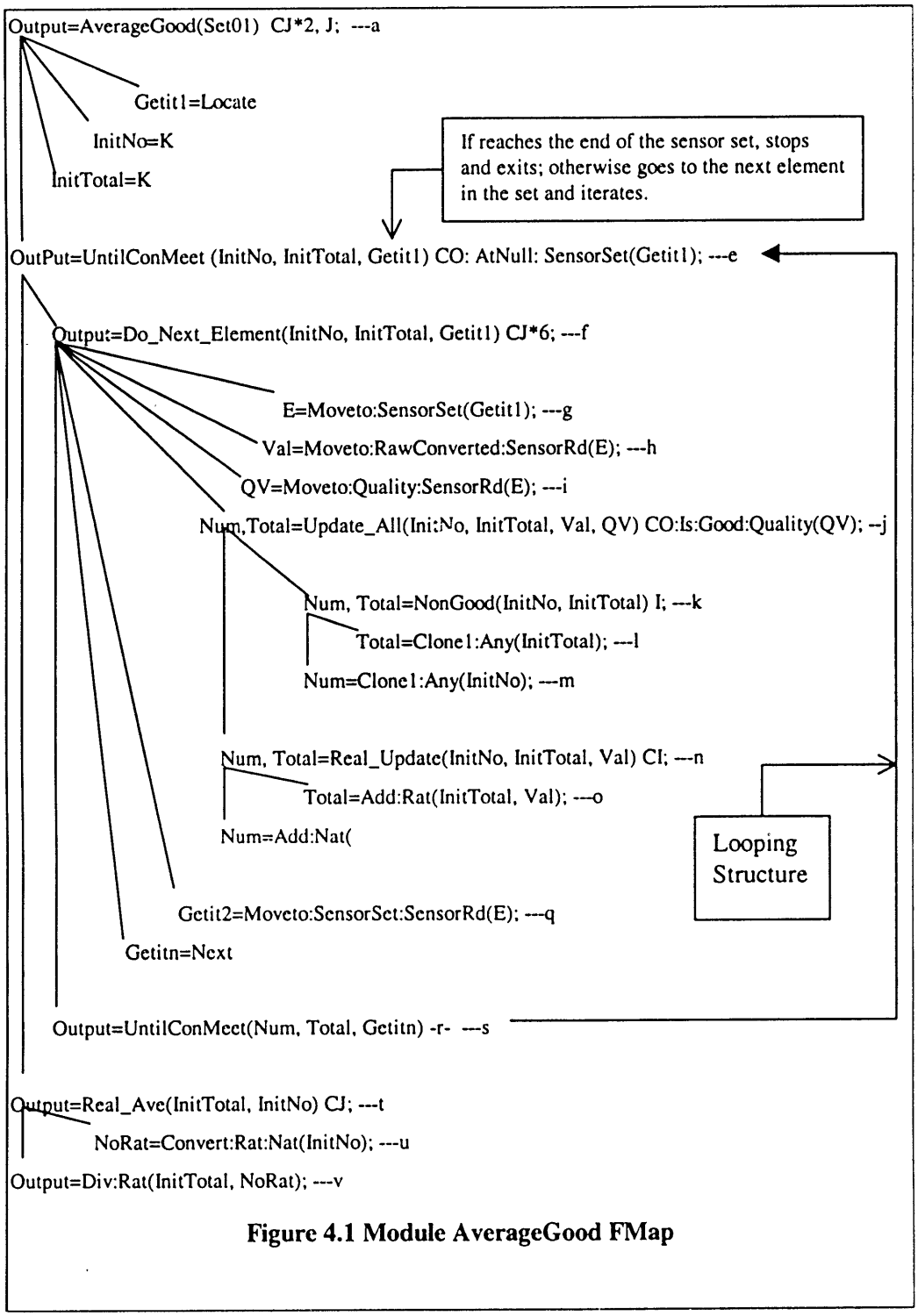


Figure 4.1 Module AverageGood FMap

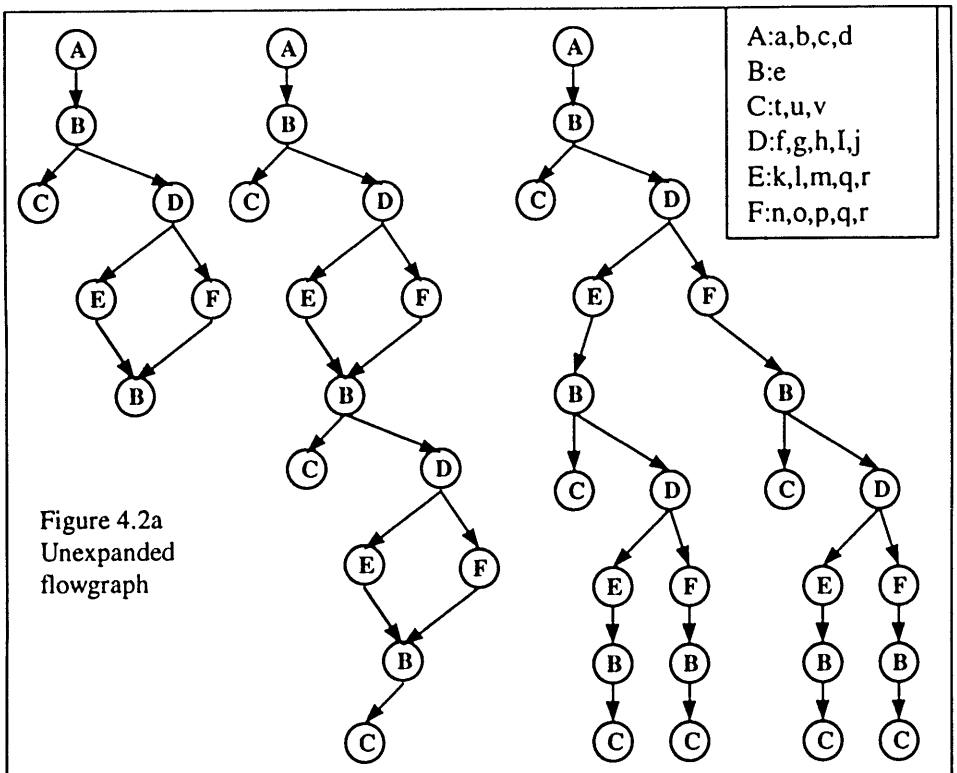


Figure 4.2a Unexpanded flowgraph

Figure 4.2b Expanded flowgraph with limit k=2

Figure 4.2c Expanded flowgraph with node B duplicated

Figure 4.2 Control flowgraph of the module AverageGood and its expansion

Path Number	Sequence
1	A+B+C
2	A+B+D+E+B+C
3	A+B+D+E+B+D+E+B+C
4	A+B+D+E+B+D+F+B+C
5	A+B+D+F+B+C
6	A+B+D+F+B+D+E+B+C
7	A+B+D+F+B+D+F+B+C

Table 4.1 Paths in module AverageGood

The testing process for all the other modules follows the same procedure. In order to improve error-revealing capability, multiple test cases are generated for each path whenever possible. This is achievable in the modular testing phase because the total number of paths in each module is small enough. In the modular testing phase a total number of *171* paths are tested and *10* errors are found. *6* errors are syntax errors that are readily detected by OO1 consistency checking capability. *4* errors are control errors that result in errors in the control flow of the program and are detected by comparing the actual outputs of the module to the expected outputs by specifications.

4.3.2 Integrated Testing

After the modular testing phase, each module has been extensively tested and is expected to have fairly low failure rate. Then integrated testing is performed, which treats the entire program as an integrated system. Our first attempt for integrated testing is to use the same approach as that in modular testing phase, or that, generates and expands the control flowgraph, identifies all the paths, generates test cases and expected outputs for each path, then performs the testing and debugging. However, this approach turns out to be intractable. In integrated testing the number of predicates along each path increases dramatically, which increases the complexity of both test case generation and oracle generation. Since we do not have any automated tools and have to do the testing all by hands, solving a combinations of constraints, even when they are all linear, turns out to be very tedious. In order to reduce the complexity, we used a different approach in integrated testing phase. Instead of specifying a path and trying to find the input values that cause the path to be traversed, we randomly specify some input values and try to find which path in the program is traversed with these input values. The overall effect of using this

approach is close to a random selection of paths in the program. Since we do not have the p_i's of the paths, we have to assume that each path is of the same importance (has the same p_i's), the random selection does not have much adverse effect. Although the new approach helps to avoid the test case generation problem, we still have the oracle problem, i.e., we still need to generate the expected outputs for the specified inputs. This makes integrated testing a tedious and time-consuming process. Our original goal in integrated testing was to test a majority of the paths in the program so as to analyze the evolution of the tested software reliability estimate as the number of paths increases. However, as the testing proceeds it becomes clear that the goal is unachievable because the time and efforts needed are far more than what we can afford. The total number of paths we managed to test in integrated testing phase is *390*, which is only a small fraction of all the paths in the program (estimated to be in the tens of thousands). The number of errors found is zero, which is a much better result than that in modular testing phase. The large discrepancy may be explained by the assumption that modular testing has effectively eliminated most of the errors in the program and the reliability of the program has been greatly improved.

4.4 Testing Results Analysis

In this section we will use the Bayesian Updating method discussed in **Section 3.6.3.2** to obtain an estimate of the reliability of the target program using the testing data. The testing data from modular testing gives a rough estimate of the error content in the program and can be used to set the prior distribution. The testing data from integrated testing can serve as an evidence to update the prior distribution. The resulting posterior distribution can then be used for reliability estimation.

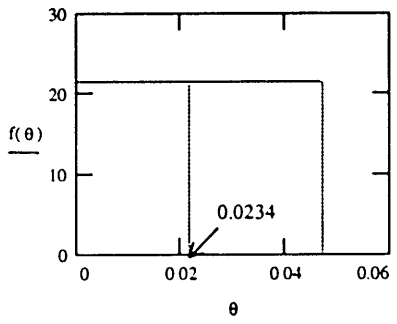
In our analysis we use a uniform prior distribution with a mean at:

$$\mu = E[\theta] = \frac{M_m}{N_m} = \frac{4}{171} = 0.0234 \quad , \tag{4.1}$$

where M_m is the number of errors found in modular testing,

N_m is the number of paths tested in modular testing.

Note that we only counted the 4 control errors found in the modular testing phase. The 6 syntax errors were ignored, because they were detected by consistency checking in the OO1 system before the generation of the executable code. They do not exist in the executable code and do not contribute to the unreliability of the executable code. The prior distribution is shown in **Figure 4.3**.

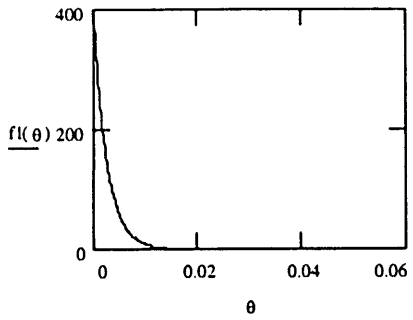


$$E[\theta] = 0.0234$$

$$\delta = 0.0445$$

Figure 4.3 The prior distribution of the case study

The posterior distribution is shown in **Figure 4.4**.



$$E[\theta] = 0.00255$$
$$\delta = 0.00934$$

Figure 4.4 The posterior distribution of the case study
(N=390, M=0)

From **Figure 4.4** we see that the estimate of the failure rate of the program before testing is $\hat{\theta} \approx 3*10^{-3}$, which corresponds to a failure probability of $3*10^{-3}$ per program execution. From Eq. (3.44), the failure rate of the program after testing, $\hat{\theta}_a$, is given by the relationship:

$$\hat{\theta}_a = (1 - \hat{P}_t)\hat{\theta} \quad . \tag{4.2}$$

The reliability (per execution) of the program after testing is given by the relationship:

$$\hat{R} = 1 - (1 - \hat{P}_t)\hat{\theta} \quad . \tag{4.3}$$

Because we only tested a small fraction of all the paths in the program, $\hat{P}_t$ is expected to be very small. We approximately have $\hat{\theta}_a \cong \hat{\theta}$, and $\hat{R} \cong 1 - \hat{\theta} = 0.997$. Although it is far from meeting the ultra-high reliability requirement, it is a fairly good result considering the fact that we only tested a few hundred of paths, a small fraction of all the paths in the program.

4.5 Conclusions

The primary conclusion drawn from this case study is that automation is essential for practical application of the proposed methodology. Even for a relatively small program (416 functional statements) the complexity in identifying, analyzing, and testing all the paths soon becomes intractable by hand. The second conclusion is that the estimation of the p_i's is very important for the application of the methodology. Without the p_i's we lose the opportunity of improving the efficiency of the testing process (maximizing $\hat{P}_t$), and can not obtain an accurate reliability estimate ($\hat{P}_t$ can not be obtained with high accuracy). Finally, although we were not able to follow the general application strategy of the methodology, we did apply part of the methodology to a sample program and obtained a reasonable result. We believe that after further refinement and provided with necessary supporting tools, much better results can be obtained using the proposed methodology.

Chapter 5

Conclusions and Future Work

5.1 Conclusions

Safety critical software is required to have ultra-high reliability. The most widely used and most powerful method for ensuring high software quality and reliability is testing. An integrated methodology is developed in this study for reliability improvement and assessment of safety critical software through testing. The methodology is based upon input domain reliability modeling and structural testing method. It possesses the advantages of both input domain-based reliability models and structural testing method. The methodology can be used to control the testing process, providing path selection criteria and stopping criteria. It provides a systematic mechanism to quantify the reliability and estimate uncertainty of the software after testing. In the thesis the underlying reliability model of the methodology is first presented. Then each element in the model is discussed. Feasible methods for quantifying each of the elements are presented. Their advantages and limitations are addressed. The general application strategy of the methodology is then formulated. The potential applications, advantages and limitations of the methodology are summarized. Finally a case study is performed applying the methodology to a sample program to show its applicability.

Software testing and reliability quantification is a very complex process. The primary concern of the study is not to provide an immediate solution to all practical difficulties (such as the automatic path generation and test case generation), rather to set up a framework of a new software reliability quantification methodology. The goals of the study are to reveal what are involved in software reliability quantification, to show where the difficulties are and how they might be overcome, and to give guidance for future improvement. Due to limited time and

resources, we were not able to explore in detail many important issues in software testing and reliability assessment, such as imperfect debugging, error consequences, and domain testing strategy. We did not actually automate the proposed methodology although we claim that automation is essential for its application. All these are left for future research. Despite of all the limitations, we believe that the methodology constitutes a new and promising technique for software reliability assessment, especially for safety critical software. After further refinement and automation, the methodology should be able to produce high reliability estimate with high accuracy.

5.2 Future Work

First, efforts need to be devoted to automate the entire methodology so that it can actually be applied to medium size programs. Secondly, Monte Carlo method has several undesirable features and alternative methods for the estimation of p_i need to be explored. Thirdly, although the methodology models the debugging process using Eq. (3.23) and Eq. (3.25) and can account for imperfect debugging, the two equations may be overly simplified and may not represent the reality. More complex mechanism may be needed to model the effect of imperfect debugging. Fourthly, error consequence analysis can be introduced into the methodology so that the software can be treated in a similar way as the hardware in Probabilistic Risk Assessment (PRA). Finally, the performance of the Bayesian updating method needs to be analyzed and the underlying assumptions of the Bayesian updating method need to be verified in practical applications.

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