Design of Large Scale Transportation Service Networks with Consolidation: Models, Algorithms and Applications

by

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Abstract

The primary focus of our research has been to develop models and algorithms to optimize large scale service network design problems for transportation providers. Service network design problems arise at airlines (passenger and cargo), trucking companies, railroads, etc., wherever there is a need to determine cost minimizing routes and schedules, given constraints on resource availability and level of service. We have developed models and a novel decomposition technique to solve large scale service network design problems with time windows and demand consolidation. We apply our models and algorithms to design the service network of a key player in the express shipment delivery industry. Our approach results in savings in total operating costs and provides a valuable tool for making decisions at strategic and tactical levels.

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Biographical Note

Niranjan Krishnan (b.1973)

Niranjan Krishnan joined Indian Institute of Technology - Madras, in 1991 to study Civil Engineering. He graduated from the Institute's undergraduate program with top honors in 1995. The basic focus of Krishnan's work at MIT has been on applying OR/MS techniques for modeling transportation and logistics systems. At MIT, Krishnan worked as the Teaching Assistant to a core graduate course in Transportation Systems Analysis in the Fall of 1997. Krishnan's primary research interests include network algorithms, large scale optimization techniques, transportation and logistics analysis, and Wodehousian analysis of aristocracy in Edwardian England.

Acknowledgments

Dedication

To the memory of two people whose company I will miss for eternity:

My father, for more reasons than I could possible accomodate on a thesis

and,

Annu, my mentor, whom I could always turn to.

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Chapter 1

Introduction

1.1 Network Design Problem

Network Design Problem requires the determination of facility locations and the routing of demand on the network of facilities such that the sum of fixed cost associated with locating the facilities facilities and the variable cost associated with the flow of demand is minimized. We motivate the Network Design Problem with an example of supply chain planning (see Sheffi [82]). In a typical supply chain operation, different raw materials from suppliers are fed into different plants, which manufacture products for consumption (commodities). The products from manufacturing plants are stored in regional warehouses or distribution centers (DCs) that cater to the needs of different customer zones. In designing such a system the questions that arise are:

- How many facilities such as suppliers for raw materials, plants and DCs are needed, where should they be located and at which capacity should they operate?

- How much of each product from each plant should be routed through each DC to each customer zone so that all the demand is met ?

The overall objective of such a planning exercise is to minimize total cost, that is a sum of fixed costs and variable costs. Such a planning problem belongs to a general class of mixed integer programming problems called the Network Design Problem

(NDP). We present below a mathematical formulation of NDP and a survey of recent literature on this subject.

1.1.1   Baseline Network Design Problem Formulation

Let $G = (N, A)$ be a directed network, where N is the node set and A is the arc set. For our purpose, we will assume throughout the thesis that by default, a commodity k is identified with a specific origin and destination. We will let $K(\ni k)$ be the set of commodities and b^k be demand for commodity k that is to be transported from its origin denoted by $O(k)$ to its destination $D(k)$. The network design model contains two types of decision variables - one modeling integer design decisions and the other modeling continuous flow decisions. Let u^f be the capacity of asset type f. Let y_{ij}^f indicate the number of times asset type f is deployed on arc (i, j) in the solution. Let x_{ij}^k represent the fraction of b^k on arc (i, j). Let h_{ij}^f be the fixed cost of deploying asset type f on arc (i, j) once and c_{ij}^k be the cost per unit flow of commodity k on arc (i, j). The mathematical program for the network design problem can be written as follows:

$$Minimize \quad \sum_{f \in F} \sum_{(i,j) \in A} h_{ij}^f y_{ij}^f + \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k b^k x_{ij}^k \tag{1.1}$$

such that:

$$\sum_{k \in K} b^k x_{ij}^k \leq \sum_{f \in F} u^f y_{ij}^f \quad \forall (i, j) \in A \tag{1.2}$$

$$\sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{ji}^k = \begin{cases} 1 & \text{if } i = O(k) \\ -1 & \text{if } i = D(k) \quad \forall i \in N, \forall k \in K \\ 0 & \text{otherwise} \end{cases} \tag{1.3}$$

$$x_{ij}^k \geq 0, \quad \forall k \in K, \forall (i, j) \in A \tag{1.4}$$

$$y_{ij}^f \geq 0 \text{ and integer}, \quad \forall (i, j) \in A, \forall f \in F \tag{1.5}$$

The objective (1.1) is to find the cost minimizing deployment of assets and routing of flows on the network constituted by the assets subject to constraints (1.2) - (1.5). Constraints (1.2), the *forcing constraints* limit the amount of flow on any arc to its

capacity, as determined by the value of design variables. Constraints (1.3) are the *flow conservation constraints* that ensure that each commodity is fully serviced from its origin to destination. Constraints (1.4) ensure non-negativity of commodity flows and constraints (1.5) ensure the integrality and non-negativity of design variables.

1.1.2 Literature

A survey of literature on network design can be found in Minoux [66], Magnanti and Wong [64] and Kim [52]. For a course on the design of survivable networks under connectivity constraints in telecommunications, we refer the reader to Stoer [84]. We outline some of the recent research in the area below:

- Magnanti et al. [63] demonstrate how to tailor Benders' decomposition to the uncapacitated network design problem. The uncapacitated network design problem is a variant of the general network design problem where there are no forcing constraints (1.2).

- Gendron and Crainic [38] analyze classical relaxation methods applied to several formulations of a fixed charge multicommodity network design problem using resource-decomposition based solution techniques.

- Clarke and Gong [24] contrast link-based and path-based formulations of the capacitated telecommunication network design problem. They strengthen the model using valid inequalities developed by Magnanti et al. [62] and propose SOS constraints to take advantage of SOS branching in MINTO (the Mixed INTeger Optimizer [69]).

- Li et al. [57] consider the computational complexity of point-to-point delivery problems and closely related point-to-point connection problems. They prove that all variations of both the problems are $NP-hard$, but there are polynomial algorithms for special cases.

- Medhi and Tipper [65] present different solution approaches to a multi-hour communication network design problem. They compare the approaches based

on a genetic algorithm and Lagrangean relaxation.

- Balakrishnan et al. ([5] and [6]) present models and algorithms for the multi-level network design problem that addresses topological design trade-offs in hierarchical networks.

- Balakrishnan et al. [8] develop and test a decomposition algorithm to generate cost-effective expansion plans with performance guarantees, for local access networks.

- Magnanti et al. ([61] and [62]) study two core subproblems of a specialized capacitated network design problem called the Network Loading Problem (NLP). They develop families of facets and completely characterize the convex hull of feasible solutions to the integer programming formulation of the problems.

- Balakrishnan et al. [7] study a class of models, called overlay optimization problems, composed of "base" and "overlay" subproblems, linked by the requirement that the overlay solution be contained in the base solution. They describe a heuristic procedure and establish worst-case performance guarantees for the uncapacitated multicommodity network design problem.

Some of the recent developments in the area have been in approximation algorithms.

- Goemans and Williamson [40] demonstrate how the primal-dual method of solving linear programs can be modified to provide good approximation algorithms for a wide variety of NP-hard problems. They also provide a good summary of developments in primal-dual approximation algorithms for network design problems.

- Agrawal, Klein and Ravi [2] introduce the use of approximation schemes for network design problems without reference to linear programming.

- Goemans and Williamson [41] expand the approach of Agrawal, Klein and Ravi [2] and make explicit use of linear programming to provide an approximation scheme for network design problems.

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- Williamson et al. [88] present the first polynomial-time algorithm for a class of network design problems including the Steiner network problem (see Ahuja et al. [3]) and the survivable network design problem (see Stoer [84] for details) that arises in telecommunication.

- Gabow et al. [37] improve the approximation algorithm presented by Williamson et al. [88] for the survivable network design problem.

- Goemans et al. [39] study a class of network design problems where one needs to find a minimum cost network satisfying certain connectivity requirements and present an approximation algorithm with a performance guarantee that is harmonic with respect to the requirement function.

- Hochbaum and Naor [46] use the results of Goemans et al. [39] and provide an approximation scheme for network design problems with some special connectivity requirements.

The major disadvantage of approximation algorithms has been the fact that the bounds given by approximation schemes are very loose and the analyses done to arrive at the bounds are usually very tight. Also the treatment of approximation algorithms in the literature has been rather theoretical in nature and there is a dearth of computational testing on practical applications.

1.2 Contributions

The contributions of our research are three-fold:

- **Modeling Contributions**

  We have developed an iterative modeling framework for large scale transportation service network design problems with time windows. Within our framework, we divide the problem into two subproblems. The first subproblem involves routing decisions and the second involves shipment flow decisions. We use an approximate service network design model for routing decisions. For

the shipment flow decisions, we have developed a novel variant of mixed integer multicommodity flow models called the *Location Elimination Model* (LEM). We present two equivalent formulations of LEM based on path-based and tree-based definition of flow variables. LEM enables the overall framework to provide insight out of an infeasible routing plan and enables decisions to be made that subsequently increase the ease of subproblem solution future iterations. Since the amount of memory needed is the key stumbling block, the exactness of the modeling framework increases if the memory availability is increased or if the problem size is reduced, thereby making apparent the trade-off between computer memory and exactness of the solution.

- **Algorithmic Contributions**

We have contributed a decomposition solution algorithm for service network design that is particularly amenable for large scale problems. The overall solution algorithm involves solution of two kinds of subproblems, one for each type of decision variable. The route generation subproblem is a large scale general integer program and is solved using a Branch-and-Price-and-Cut approach, where both route variables and violated constraints are generated on an "as needed" basis. We use a Branch-and-Price strategy to solve the shipment flow subproblem, LEM, where the shipment flow variables are generated on an "as needed" basis by solving a series of shortest path subproblems.

We provide a proof-of-concept of the efficacy of our solution approach by solving the service network design problem of a large carrier in the express shipment delivery industry. We are unable to generate optimal IP solutions due to the huge size of the carrier's operations and NP-completeness that typifies service network design problems. However, our solution approach provides IP feasible solutions that result in annual cost savings measuring in tens of millions of dollars, with run times acceptable for strategic planning.

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- **Applied Contributions**

  We have developed a modeling framework and a decomposition solution approach for service network design problems that arise at airlines, trucking companies, railroads, supply chains, etc. The common characteristic of these service network design problems is the need to determine cost minimizing routes and schedules, given constraints on resource availability and level of service. Depending on the specific operating characteristics of the application, additional constraints may be necessary. For our express shipment delivery application, we illustrate how to model fleet balance constraints, fleet size constraints, fleet capacity constraints, hub landing capacity constraints and connectivity constraints. These specific constraints may be either relaxed or interpreted differently for other applications. For example, the hubs in the express shipment delivery operations are analogous to distribution centers (DCs) in supply chain operations in that they serve the same purpose of demand consolidation. Hence hub capacity constraints can be envisaged as constraints on storage space at DCs and the landing capacity constraints would similarly correspond to the limitations on number of vehicles in the loading/unloading area.

  Our decomposition approach for service network design has the capability to solve large scale real-life problems in a reasonable time frame, and thereby supplanting cumbersome manual planning processes. As a decision support system, it enables planners to focus on analyzing relevant scenarios at strategic and tactical levels, and translating the results into recommendations for operations planning.

1.3   Outline of the Thesis

The rest of the thesis is outlined as follows:

In Chapter 2, we describe the Transportation Service Network Design Problem and present three equivalent model formulations. We review the literature specific to the Service Network Design Problem and motivate the need for our solution approach.

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We also present a high-level description of our decomposition solution approach. In Chapter 3, we present routing models for Service Network Design and outline solution techniques for large scale problems. In Chapter 4, we detail shipment flow models and their applications. In Chapter 5, we apply our models and algorithms to solve the Service Network Design Problem of a large carrier in the express shipment delivery industry. In Chapter 6, we conclude the thesis with some final remarks and directions for future research.

Chapter 2

Transportation Service Network Design

2.1 Problem Description

In a typical transportation service operation, the service provider carries customer demand from origins to destinations using the assets that are deployed on various transportation legs. The Service Network Design Problem (SNDP) requires the determination of a set of routes for the assets, that satisfies all customer demand at a minimum cost without violating the capacities of the service legs. The SNDP has an added degree of complexity over the Network Design Problem (NDP) described in Chapter 1 in that, the assets need to be balanced at the end of the planning period for continuity in the service cycle. Such problems include the following:

- $Less - than - Truckload$ (LTL) $Operations$ $Planning$. Motor carriers carry freight from origin $end - of - line$ (EOL) terminals to destination EOL terminals, through consolidation centers (CCs), on a daily basis. The objective is to determine minimum cost routes and schedules for tractors and trailers so that all the demand can be conveyed with the available fleet size and capacity. The tractors and trailers must reach their starting terminal at the end of the day, to be available for service the following day.

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- *Airline Scheduling.* Airlines need to determine a revenue maximizing set of routes and schedules for their fleet of aircraft. Since passengers are flown on a daily basis, aircraft must be repositioned to allow repetition of the schedule on the following day.

- *Express Package Delivery.* This is similar to airline scheduling where packages, instead of passengers are transported from origins to destinations on a daily basis. Timing constraints are more stringent in this case, since level of service guarantees are often in place.

2.2   Service Network Design Problem Formulations

In this section we present three equivalent models for the service network design problem. The models may differ in the number of variables and the number of constraints they contain.

2.2.1   Node-Arc Formulation

$$Minimize \quad \sum_{f \in F} \sum_{(i,j) \in A} h_{ij}^f y_{ij}^f + \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k b^k x_{ij}^k \tag{2.1}$$

such that,

$$\sum_{k \in K} b^k x_{ij}^k \leq \sum_{f \in F} u^f y_{ij}^f \quad \forall (i,j) \in A \tag{2.2}$$

$$\sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{ji}^k = \begin{cases} 1 & \text{if } i = O(k) \\ -1 & \text{if } i = D(k) \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in N, \ \forall k \in K \tag{2.3}$$

$$\sum_{j \in N} y_{ij}^f - \sum_{j \in N} y_{ji}^f = 0 \quad \forall i \in N, \ \forall f \in F \tag{2.4}$$

$$x_{ij}^k \geq 0, \quad \forall k \in K, \ \forall (i,j) \in A \tag{2.5}$$

$$y_{ij}^f \geq 0 \text{ and integer}, \quad \forall (i,j) \in A, \ \forall f \in F \tag{2.6}$$

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The objective (2.1) is to find the cost minimizing deployment of assets and routing of flows on the network constituted by the assets subject to constraints (2.2) - (2.6). Constraints (2.2), the *forcing constraints* limit the amount of flow on any arc to the capacity of that arc, as determined by the value of design variables. Constraints (2.3) are the *flow conservation constraints* that ensure that each commodity is fully serviced from its origin to destination. Constraints (2.4) are *design balance* constraints that distinguish service network design problems from conventional network design problems. Constraints (2.5) ensure non-negativity of commodity flows and constraints (2.6) ensure the integrality and non-negativity of design variables.

2.2.2 Path Formulation

For the path and tree formulations, we let design route r be a sequential set of design variables of some type f in the Node-Arc formulation, that is balanced everywhere except possibly at the start and the end of the sequence. Any individual aircraft route can start and terminate at any location. So the starting point of the aircraft route may not always coincide with the ending point and this could result in an imbalance of fleet types. Our requirement is that the schedule be repeatable or cyclic. So we need to impose balance only by aircraft *type* and not by *aircraft*.

Notations

We define some notations before presenting the path formulation.

SETS

$K(\ni k)$: the set of all O-D commodities

R^f : the set of all design routes for fleet type f

$P^k(\ni p)$: the set of all feasible paths from origin $O(k)$ to destination $D(k)$ for eack $k \in K$

PARAMETERS

$h_r^f = \sum_{(i,j) \in A} h_{ij}^f \alpha_{ij}^r$: the cost of design route r of type f

$c_p^k = \sum_{(i,j) \in A} c_{ij}^k \delta_{ij}^p$: the cost of flowing one unit of commodity k from $O(k)$ to

$D(k)$ along path $p \in P^k$

INDICATOR VARIABLES

$$\alpha_{ij}^r = \begin{cases} 1 & \text{if design variable for } (i,j) \text{ is included in design route } r \\ 0 & \text{otherwise} \end{cases}$$

$$\beta_i^r = \begin{cases} 1 & \text{if } i \in N \text{ is the start node of design route } r \\ -1 & \text{if } i \in N \text{ is the end node of design route } r \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{ij}^p = \begin{cases} 1 & \text{if arc } (i,j) \text{ belongs to path } p \\ 0 & \text{otherwise} \end{cases}$$

DECISION VARIABLES

y_r^f : number of assets of type f deployed on design route r

x_p^k : fraction of b^k on path $p \in P^k$ for all $k \in K$

With these notations we present the path formulation below (see Ahuja et al. [3] for demonstration of equivalence between Node-Arc and Path formulations).

SNDP-Path

$$Minimize \quad \sum_{r \in R^f} h_r^f y_r^f + \sum_{k \in K} \sum_{p \in P^k} (c_p^k b^k) x_p^k \tag{2.7}$$

such that,

$$\sum_{k \in K} \sum_{p \in P^k} (\delta_{ij}^p b^k) x_p^k \leq \sum_{f \in F} \sum_{r \in R^f} u^f y_r^f \alpha_{ij}^r \quad , \forall (i,j) \in A \tag{2.8}$$

$$\sum_{p \in P^k} x_p^k = 1 \quad , \; \forall k \in K \tag{2.9}$$

$$\sum_{r \in R^f} \beta_i^r y_r^f = 0 \quad , \; \forall i \in N, \; \forall f \in F \tag{2.10}$$

$$x_p^k \geq 0 \quad \forall p \in P^k, \; \forall k \in K \tag{2.11}$$

$$y_r^f \geq 0 \ and \ integer, \ \forall r \in R^f, \ \forall f \in F \tag{2.12}$$

The objective (2.7) is to find the cost minimizing deployment of assets and routing of flows on the network constituted by the assets. Constraints (2.8) - (2.12) correspond to constraints (2.2) - (2.6) in the Node-Arc formulation.

2.2.3 Tree Formulation

If arc costs remain the same for all commodities, then the flow variables can be represented on origin-based or destination-based trees (see Jones et al. [48]) to arrive at an equivalent tree formulation. Here the idea is to aggregate all O-D commodities with the same origin (destination) into a single *super commodity* so that we have one commodity for each origin (destination) location.

Notations

We additionally use the following notations for the origin-based tree formulation of SNDP.

    SETS

    $O(\ni o)$: set of all origin locations

    $Q_o(\ni q)$: the set of all trees at origin o, for all $o \in O$

    K_o : the set of all O-D commodities with origin o

    p_q^k : the unique path from $O(k)$ to $D(k)$ in tree q

    PARAMETERS

    $c_o^q = \sum_{k \in K_o} \sum_{(i,j) \in A} \Gamma_{ij}^{qk} b^k$: the cost of flowing the entire portion of all O-D commodities with $O(k) = o$ from $O(k)$ to $D(k)$ along path p_q^k in tree q.

    INDICATOR VARIABLES

$$\Gamma_{ij}^{qk} = \begin{cases} 1 & \text{if arc } (i,j) \text{ belongs to the path } p_q^k \text{ from } O(k) \text{ to } D(k) \text{ in tree } q \\ 0 & \text{otherwise} \end{cases}$$

    DECISION VARIABLES

    w_o^q: fraction of b^k flown on the path p_q^k from $O(k) = o$ to $D(k)$ in tree q

SNDP-Tree

$$Minimize \quad \sum_{r \in R^f} h_r^f y_r^f + \sum_{o \in O} \sum_{q \in Q_o} c_o^q w_o^q \qquad (2.13)$$

such that,

$$\sum_{o \in O} \{ \sum_{q \in Q_o} (\sum_{k \in K_o} \Gamma_{ij}^{qk} b^k) w_o^q \} \leq \sum_{f \in F} \sum_{r \in R^f} u^f y_r^f \alpha_{ij}^r \ , \quad \forall (i,j) \in A \qquad (2.14)$$

$$\sum_{q \in Q_o} w_o^q = 1, \quad \forall o \in O \qquad (2.15)$$

$$\sum_{r \in R^f} \beta_i^r y_r^f = 0 \ , \quad \forall i \in N, \ \forall f \in F \qquad (2.16)$$

$$w_o^q \geq 0 \ , \quad \forall q \in Q_o, \ \forall o \in O \qquad (2.17)$$

$$y_r^f \geq 0 \ and \ integer, \ \forall r \in R^f, \ \forall f \in F \qquad (2.18)$$

The objective (2.13) is to find the cost minimizing deployment of assets and routing of flows on the network constituted by the assets. Constraints (2.14) - (2.18) correspond to constraints (2.2) - (2.6) in the Node-Arc formulation.

2.2.4   Comparison of Formulations

Compared to $SNDP - Node - Arc$ the number of *flow conservation constraints* in $SNDP - Path$ is reduced from $|N| \times |K|$ to $|K|$. In the tree formulation this number is further reduced to $|O|$, the number of origin locations. The reduction in the number of constraints makes the path and tree formulations particularly amenable to large scale problems. We refer the reader to Ahuja et al. [3], Jones et al. [48] and Kim [52] for a discussion on the advantages of path and tree formulations over the node-arc formulation. However, the number of variables increases exponentially in the path and tree formulations. In Chapter 3, we outline some solution methods for problems with a large number of decision variables. Table (2.1) summarizes the differences in the number of *flow conservation constraints* and the number of decision variables among the three formulations.

	SNDP-Node-Arc	SNDP-Path	SNDP-Tree
Flow Balance Constraints	$|N| \times |K|$	$|K|$	$|O|$
Number of Variables	$|A| \times |K|$	$O(2^N)$	$O(2^{O(2^N)})$

Table 2.1: Comparison of SNDP Formulations

2.3 Literature

In this section we summarize some of the recent research in transportation service design.

- Chestler [22] studies the express shipment delivery industry in the early stages of development. Qualitative analyses of network configuration, competition and hub location are presented.

- Chan and Ponder [21] review the air freight industry with special reference to the Federal Express Corporation. They outline the characteristics of the industry and present a survey of different managerial practices.

- O'Kelly [72] develops several models for the location of interacting hub facilities. An empirical example is used to demonstrate the relevance of a single-hub model for an understanding of contemporary express delivery networks.

- Kanafani and Ghobrial [50] analyze the phenomenon of hubbing in airlines. They examine the economics behind hubbing and conclude that there are significant potential benefits to the airports to be gained from some sort of hub pricing.

- Hall [43] examines the impact of overnight restrictions and time zones on the configuration of an air freight network. The effect of the location of a hub terminal on the arrival pattern of aircraft at the terminal is studied. Practical restrictions that tend to favor eastern terminal locations over western locations are reported.

- Kuby and Gray [54] compare the cost-effectiveness of hub-and-spoke networks with stopovers and feeders to that of pure hub-and-spoke networks and present a case study on Federal Express. They assume a single sorting hub and a relatively small market covering only the western United States and develop a mixed integer program to design the least-cost air network.

- Barnhart and Schneur [18] develop a model and algorithm for an express shipment service network design problem. In their model, (1) there is only one hub, (2) transfer of shipments between aircraft at gateways in disallowed and (3) only one type of aircraft is allowed to serve each gateway location. The upshot is that shipment routings are completely determined by aircraft routes.

- Kamoun and Hall [49] analyze the express mail delivery problem for the courier services industry. These companies operate like taxi companies, but transport mail and packages instead of people. Kamoun and Hall propose two new designs without using linear programming techniques and provide results based on simulation.

- Kim [52] develops generic models and algorithms for large-scale transportation service network design problems and illustrates an application in the express package delivery industry. By exploiting special problem structure and applying novel problem reduction techniques, a dramatic decrease in problem size is achieved without compromising exactness of the model.

- If we consider the NDP with a single source node e and a fixed capacity $u_{ij} = U$ on all arcs and add assignment constraints,

$$\sum_{j \in N} y_{ij} = 1 \quad , \forall i \in N \setminus \{e\}$$

$$\sum_{i \in N} y_{ij} = 1 \quad , \forall j \in N \setminus \{e\}$$

as well as the constraint,

$$\sum_{i \in N} y_{ej} \leq n,$$

the NDP becomes a *vehicle routing problem* (VRP) for a homogeneous fleet of n vehicles each domiciled at depot e and each having a capacity U. Comprehensive surveys by Magnanti [59], Magnanti and Wong [64], Golden and Assad [42] Desauliniers et al. [30] and Desrosiers et al. [32] summarize the developments in this field.

- Talluri and Gopalan [85] survey various mathematical models in airline schedule planning. Barnhart et al. [17] and Shenoi [83] present integrated models and solution techniques for airline planning including integrated fleet assignment and maintenance routing, and integrated crew scheduling and deadhead selection. Daskin and Panayotopoulos [29] analyze the probem of assigning aircraft to scheduled routes to maximize profits in passenger hub and spoke networks. A Lagrangian relaxation of the formulation is outlined together with heuristics for converting Lagrangian solutions into primal solutions and for improving on the solutions.

- In the railroad industry, Ziarati et al. [91] develop models for assigning loco-motives to trains to operate a given schedule. They solve the models using a Dantzig-Wolfe decomposition technique, where subproblems are formulated as constrained or unconstrained shortest path problems. Newton [71] and Barn-hart et al. [14] study network design problem with budget constraints with an application to railroad blocking problems.

- Powell [74] models the load planning problem for Less-Than-Truckload (LTL) motor carriers as a Service Network Design Problem. A local improvement heuristic is proposed which adds and drops links to and from the network in an intelligent sequence. After each change, the routing of freight over the net-work is approximately reoptimized. Farvolden and Powell [33] present local-improvement heuristics for a SNDP encountered in LTL common carrier appli-cations. The add/drop heuristics are based upon subgradients derived from the optimal dual variables of the shipment routing subproblem that is modeled as a multicommodity network flow problem. The basis of the multicommodity net-

work flow problem is partitioned to facilitate the calculation of dual variables, reduced costs and subgradients (see Farvolden et al. [34]). Powell and Sheffi [75] also use add/drop heuristics for LTL motor carrier applications.

- Wong [89] raises some algorithmic and computational questions in transportation network research. Possible advances in improving the quality of solutions, increasing the size of problems that can be handled and use of approximate procedures are suggested.

Most of the previously developed models and solution techniques have been used to solve small and medium scale real-life problems (see Kim [52] for details). In the next section we identify the needs to solve very large scale real-life problems and present an overview of our solution approach.

2.4   Decomposition Solution Approach

We recall that the service network design problem has two kinds of decision variables, one for the deployment of transportation assets onto network paths and the other for the routing of shipments over the network. Simultaneous solution for both these types of variables for many transportation applications requires hardware capabilities that are far greater than those available to most transportation service providers. As the size of the problem increases, it might take hours of runtime just to discover that there is not enough memory in the system to solve the problem. This time consuming process often yields very little insight into the problem. Hence there is a pressing need for a procedure that allows us to solve very large-scale transportation applications. Our solution approach strives to satisfy this need by using two models, each dealing with a subproblem for one kind of decision variable, in an iterative framework that integrates results from both the models. We use an approximate service network design model, called $SNDP-Approx$, to generate routes for transportation assets and a variant of integer multicommodity flow models, called LEM, to generate shipment flows.

2.4.1 Decomposition Algorithm

We first generate routes for transportation assets by using the route generation model, $SNDP - Approx$. On the network constructed from these routes, we flow shipments in an intelligent manner using the shipment flow model, LEM, and based on these shipment flows we fix some of the routes and strip out the shipments that can be serviced by the routes selected so far. We repeat this procedure with the shipments remaining to be serviced. This way the number of shipments that need to be serviced is successively reduced so that in the end we either have a set of selected routes that have enough capacity to service all the shipments or a small enough number of shipments that the problem can be solved using exact models without computational burden.

Our solution approach can be outlined as follows:

0. Initial Input: Input all commodity demand data, locations, capacities, costs and initialize the set of selected routes to a null-set. Go to Step 1.

1. Route Generation: Solve the model $SNDP - Approx$ with the input set of commodities and generate "promising" routes for transportation assets.

2. Shipment Flow Generation: Solve LEM and arrive at a set of commodity flows over the service network generated in the *Route Generation* step.

3. Variable Fixing : Augment the set of selected routes by fixing some of the promising routes generated in the *Route Generation* step. The selected routes are those that are very likely to be in the optimal solution and are identified based on the flows from the *Shipment Flow Generation* step. We provide more details in Chapter 5.

4. Feasibility Check: Verify using LEM if the service network formed by the current set of selected routes has enough capacity to carry all demand. If yes, STOP; else go to the *Problem Size Reduction* step.

5. Problem Size Reduction: Eliminate commodities that can be moved on the service network formed by the set of selected routes. If the service design problem for the remaining commodities is small enough to solve exactly with available hardware, solve using the service network design model (detailed in Section 2). Otherwise, go to the *Route Generation* step with the remaining set of commodities as input.

A key idea of our approach is that we keep selecting routes and flowing commodities at each step until the size of the problem with the remaining commodities is small enough to be solved exactly with current memory availability or we have a feasible solution. This greatly enhances the tractability of our solution technique. The flow chart corresponding to our algorithm is shown in Figure 2-1.

2.4.2 Shipment Flow Model Description

We now introduce a variant of integer multicommodity flow models called the *Location Elimination Model* (LEM) to flow shipments in such a way as to reduce the size of the subproblem at the next iteration. In Chapter 4, we describe LEM in detail. The motivation for LEM arises from the fact that the routes generated in the *Route Generation* step of our solution approach may not have enough capacity to move the entire demand for shipments from their origins to their destinations. Hence there is a need for a model that can flow *some of the shipments* on such a network that does not have enough capacity and aid the overall solution algorithm in making some decisions based on the shipment flow patterns. LEM establishes a set of commodity flows on the fleet network that maximizes the number of origin and destination locations for which all demand originating at or destined for that location, as the case may be, is assigned to network paths. If all origin and destination locations do not have their entire demands assigned to the network, we can fix only the fleet routes that carry the entire demand of origin or destination locations they visit. This results in the following:

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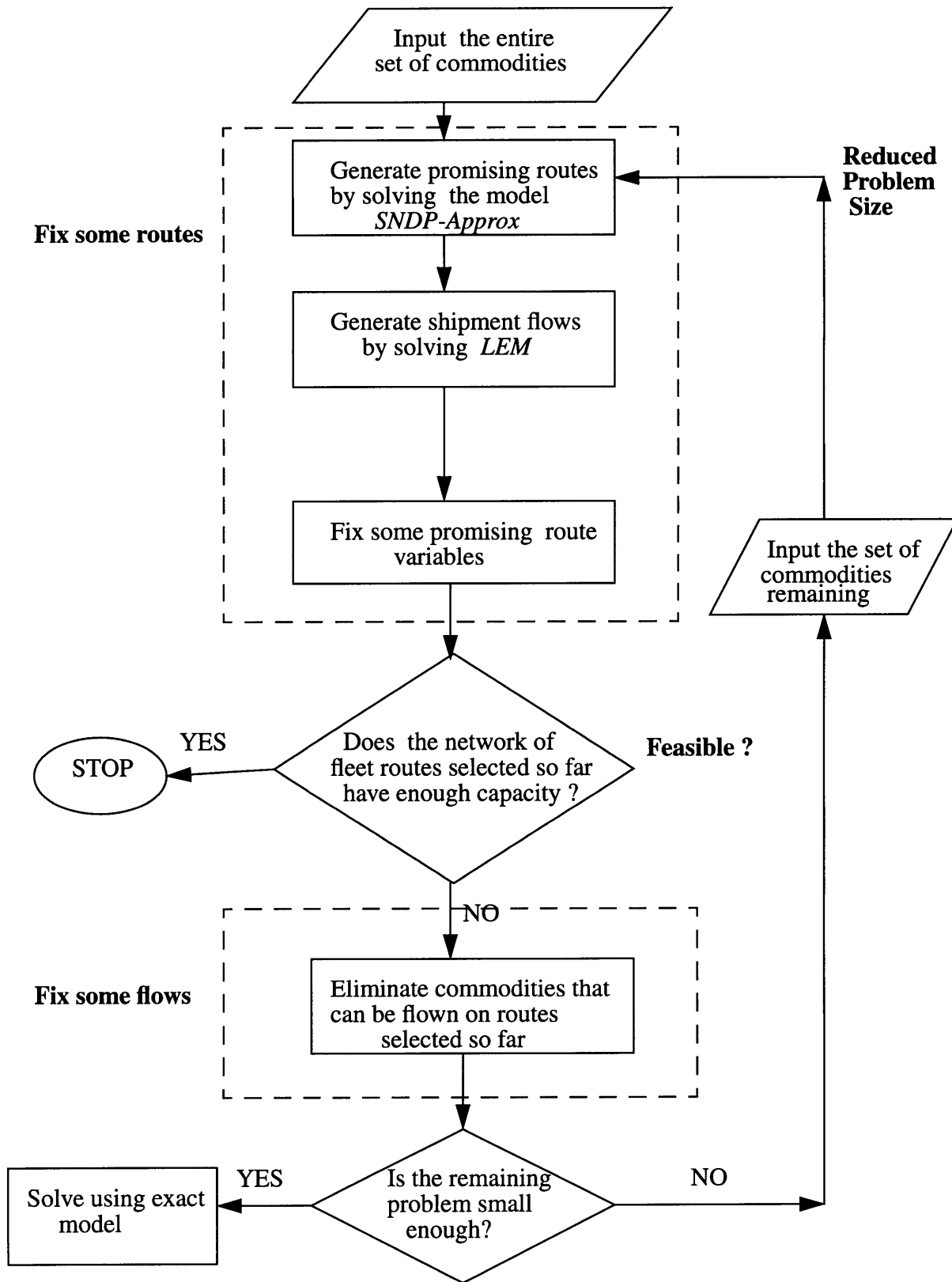
Figure 2-1: Decomposition Algorithm

- The number of O-D commodities, whose demand is satisfied is reduced. This translates into a reduction in the size of input data for the *Route Generation* problem in the next iteration.

- There is an elimination of some origin and destination locations whose demand has been entirely moved. This results in a corresponding reduction in the size of the fleet network in the *Route Generation* step.

LEM can be used either in a stand-alone fashion or embedded in an iterative framework for solving large scale problems. If all origin and destination locations have their entire demands assigned to the network, we can infer that there is enough capacity in the network to carry all O-D demand. Thus LEM can be used as a stand-alone model to check the feasibility of any solution to the NDP with respect to demand.

The elimination of some origin and destination locations might result in a considerable reduction in the size of the fleet network in the *Route Generation* step, since many transportation applications involve time as a factor and every single physical location will correspond to multiple nodes in a network in space and time. Thus LEM is intelligent because it flows shipments in a way that reduces problem size for future consideration.

Chapter 3

Routing for Transportation Service Network Design: Models and Solutions

In this chapter we describe the first type of subproblem that deals with arriving at a routing plan for the transportation assets or fleet. We use approximate service network design models to solve this problem. We first present approximate SNDP models after providing some necessary background. We then present a brief review of literature followed by some solution techniques for solving large scale problems.

3.1   Valid Inequalities

Given some values for the design variables y, thereby determining the capacities on the arcs, the SNDP has a feasible flow of b^k from $O(k)$ to $D(k)$ only if the capacity of every $O(k) - D(k)$ cutset is at least b^k. In general, for any feasible solution to SNDP, the aggregate capacity across the cutset must be no less than the demand across the cutset. These *aggregate capacity demand inequalities* (see Magnanti et al. [62]) are

expressed as:

$$\sum_{f \in f} u_f Y_{S,T}^f \geq D_{S,T} \quad , \quad for \ all \ O - D \ cutsets \ \{S, T\} \tag{3.1}$$

where we define cutset $\{S, T\}$ to be a partition of the node set N into two mutually exclusive and collectively exhaustive non-empty subsets S and $T = N \setminus S$. We let $Y_{S,T}^f = \sum_{(i,j) \in A} y_{ij}^f$ in the Node-Arc formulation and $Y_{S,T}^f = \sum_{r \in R^f} \sum_{(i,j) \in \{S,T\} \cap r} y_r^f$ in the route-based formulation. We let $D_{S,T}$ be the total demand of all commodities with origin in subset S, i.e. $O(k) \in S$ and destination in subset T $(D(k) \in T)$.

Inequalities (3.1) are knapsack inequalities that can be strengthened in many ways. One way is to use rounding that results in $Chv\acute{a}tal - Gomory \ cuts$. Alternatively we could generate *cutset inequalities* as explained in Magnanti et al. ([62] and [61]) and Magnanti and Mirchandani [60]. Also see Stoer [84] for details on valid inequalities.

3.1.1 Chvatal-Gomory Cuts

The aggregate capacity demand inequalities (3.1) can be lifted by applying simple integer rounding to produce $Chv\acute{a}tal - Gomory$ (C-G) cuts. If we let l be a particular asset or service type then the following C-G cuts are valid for each O-D cutset $\{S, T\}$.

$$\sum_{f \in F} \left(\left\lceil \frac{u^f}{u^l} \right\rceil Y_{S,T}^f \right) \geq \left\lceil \frac{D_{S,T}}{u^l} \right\rceil \quad , \forall \ l \in F, \ for \ all \ O - D \ cutsets \ \{S, T\} \tag{3.2}$$

We refer the reader to Nemhauser and Wolsey [70] for further details. The difficulty in using $C - G \ cuts$ is that there are exponentially many. For instance, in the case of N origin locations and F fleet types, the total number of $C - G \ cuts$ including the aggregate capacity demand inequalities is $\{2(2^N - 1)\} \times (|F| + 1)$.

3.1.2 Cutset Inequalities

The aggregate capacity demand inequalities (3.1) can be alternatively strengthened by using *cutset inequalities*. Cutset inequalities, extensively researched by Magnanti, Mirchandani and Vachani ([62] and [61]) and Magnanti and Mirchandani [60], may

provide tighter LP bounds than C-G cuts. We assume that there are only two service or asset types for ease of exposition. However, they can be generated for more than two types. The cuts (3.1) can be written as:

$$u^1 Y^1_{S,T} + u^2 Y^2_{S,T} \geq D_{S,T} \tag{3.3}$$

and the two types of cutset inequalities can be written as:

$$Y^1_{S,T} + \frac{u^2}{r(D_{S,T}, u^1)} \geq \left\lceil \frac{D_{S,T}}{u^1} \right\rceil \tag{3.4}$$

$$\frac{u^1}{r(D_{S,T}, u^2)} Y^1_{S,T} + Y^2_{S,T} \geq \left\lceil \frac{D_{S,T}}{u^2} \right\rceil \tag{3.5}$$

where $r(D_{S,T}, u) \equiv D_{S,T} - \left(\left\lceil \frac{D_{S,T}}{u} \right\rceil \right) u$

The generalized version of cutset inequalities can be found in Magnanti et al. [62]. As with the $C - G$ cuts, the number of cutset inequalities grows exponentially with the number of locations. For example, in the case of N origin locations and F fleet types, the total number of cutset inequalities including the aggregate capacity demand inequalities is $\{2(2^N - 1)\} \times (|F| + 1)$, which is the same as that for $C - G$ *cuts*.

3.2 The Approximate Service Network Design Cutset Model

When the costs of flow variables are negligible compared to fixed design variable costs, we can approximate the network design problem without considering flows explicitly. When there is only one commodity, we can show using a $max - flow\ min - cut$ argument that aggregate capacity demand inequalities are both necessary and sufficient, to guarantee a feasible single-commodity solution (see Ahuja et al. [3] for details). For multicommodity problems, aggregate capacity demand inequalities are a necessary, but not a sufficient, condition to guarantee a feasible multicommodity flow solution (for example, see Mirchandani [67]). Because SNDP involves multiple

commodities, we approximate $SNDP$ with $SNDP-Approx$. Correspondingly we call the approximation of $SNDP-Node-Arc$ as $SNDP-Approx-Node-Arc$ and that the approximation of $SNDP-Path$ and $SNDP-Tree$ as $SNDP-Approx-Route$. $SNDP-Approx$ models enable us to solve SNDP approximately, considering only the design variables and ignoring the huge number of flow variables and large number of constraints (flow balance and forcing constraints) associated with the flow variables.

3.2.1   Approximate SNDP Formulations

We present below the approximate node-arc and route-based formulations.

SNDP-Approx-Node-Arc

$$Minimize \quad \sum_{f \in F} \sum_{(i,j) \in A} h_{ij}^f y_{ij}^f \tag{3.6}$$

such that

$$\sum_{f \in f} u_f Y_{S,T}^f \geq D_{S,T} \quad , \; \forall \; O-D \; cutsets \; \{S,T\} \tag{3.7}$$

$$\sum_{j \in N} y_{ij}^f - \sum_{j \in N} y_{ji}^f = 0 \quad \forall i \in N, \; \forall f \in F \tag{3.8}$$

$$y_{ij}^f \geq 0 \; and \; integer, \quad \forall (i,j) \in A, \; \forall f \in F \tag{3.9}$$

SNDP-Approx-Route

$$Minimize \quad \sum_{r \in R^f} h_r^f y_r^f \tag{3.10}$$

such that

$$\sum_{f \in f} u_f Y_{S,T}^f \geq D_{S,T} \quad , \; \forall \; O-D \; cutsets \; \{S,T\} \tag{3.11}$$

$$\sum_{r \in R^f} \beta_i^r y_r^f = 0 \quad , \; \forall i \in N, \; \forall f \in F \tag{3.12}$$

$$y_r^f \geq 0 \; and \; integer, \; \forall r \in R^f, \; \forall f \in F \tag{3.13}$$

3.3 Literature

Cutset based formulations have been used in the literature to model variants of NDP such as the survivable network design problem, Steiner tree problems, location-design and location-routing problems, point-to-point routing problems etc. Goemans and Williamson [40] outline several of these problems and summarize the research in approximation algorithms for solution to these problems. Stoer [84] presents a study on existing cutset models, decomposition solution techniques, valid inequalities and lifting theorems for the design of survivable networks. In transportation, Kim [52] presents an application of cutset models for express shipment delivery.

3.4 Solution Approach

In this section we present a generic solution to large-scale integer programs with a prohibitively huge number of variables and constraints. Direct solution to these problems is usually not possible even with state-of-the-art LP/IP solvers (such as CPLEX [26] , MINTO [69] , OSL [47]). We first present a solution technique for large-scale linear programs that is based on variable restriction and constraint relaxation. We then present a solution technique for integer programs that is based on variable restriction and constraint relaxation embedded within a Branch-and-Bound framework. We provide details of specific techniques for our application in Chapter 5.

3.4.1 LP solution

Column Generation

When a linear program contains an exponential number of variables (or columns) rendering a direct solution very difficult, we resort to Dantzig-Wolfe decomposition [28] or Column Generation solution techniques. The key idea is to start with a very small number of decision variables from the original formulation called the *Master Problem* (MP) to form a smaller problem called the *Restricted Master Problem* (*RMP*), that is solved to optimality. To check the optimality of a solution to RMP

with respect to MP, a subproblem called the *pricing problem* is solved. The result is that either optimality is proved or new variables whose inclusion into the RMP might improve the solution quality are identified. If such variables are found, the RMP is re-optimized. The entire process is repeated until optimality of the MP is proved.

The ease with which the pricing subproblem is solved determines tractability of the column generation procedure. Column generation can be either *implicit* or *explicit*. Implicit column generation refers to solving the pricing subproblem without evaluating all variables (e.g., by solving an optimization problem of some kind such as the shortest path problem), whereas explicit column generation refers to solving the pricing subproblem by calculating the reduced cost of all variables.

Row Generation

Row generation is the dual analogue of column generation for LPs with a huge number of constraints. The idea is to start with a very small number of constraints from the MP to form a smaller problem called the relaxed problem (*RP*), that is solved to optimality. To check the optimality of a solution to RP with respect to MP, a subproblem called the *separation problem* is solved, that identifies violated constraints (rows) to add to the current RP. If such constraints are found, the RP is re-optimized. The entire process is repeated until no violated constraints are found. Row generation can be either *explicit* or *implicit*. Explicit row generation is necessary when there is no efficient algorithm to solve the separation problem and violated constraints may be identified by evaluating each constraint. Implicit row generation refers to the case when there is an algorithm to solve the separation problem.

When there is a single commodity, the task of identifying the most violated cutset inequalities, called the *cutset separation problem*, by max-flow min-cut duality, reduces to identifying the min-cut. For the case with multiple commodities, there is no efficient procedure to solve the cutset separation problem (see Balakrishnan et al. [5]). Leighton and Rao [56] present an approximate max-flow min-cut theorem for a special kind of multicommodity flow problem called the *uniform multicommodity flow problem*. Linial et al. [58] present an approximate max-flow min-cut theorem

for a general multicommodity flow problem and provide an approximation algorithm for the cutset separation problem. They use the technique of embedding metrics for the multicommodity cut problem. Williamson et al. [88] and Gabow et al.[37] solve a cutset separation problem as an intermediate step in an approximation algorithm for the survivable network design problem. They demonstrate how to solve the separation problem efficiently for special cases that could arise in telecommunication.

Synchronized Column and Row Generation

To solve LPs with both a huge number of variables and a huge number of constraints, it may be necessary to use both column and row generation. The idea is to start with a small number of columns (variables) and rows (constraints) from the original master problem MP, to form the RMP. One approach is to perform column generation until all columns have non-negative reduced costs. Then this is followed by row generation until no violated constraints are identified. After completing row generation, the entire process is repeated. The algorithm terminates when there are both no columns and no violated constraints to add. The challenge in implementing such a procedure is in adding constraints that do not change the structure of the pricing subproblem and adding columns that do not change the structure of the separation subproblem. This is averted however, if we use explicit column generation and explicit row generation. The difficulty is that explicit generation may be computationally expensive or even, impractical. The growth of the size of RMP is illustrated in Figure 3-1.

3.4.2 IP solution

Branch − and − Bound is a *divide − and − conquer* solution technique (refer to Cormen et al. [25]) to solve IPs by solving a series of LP subproblems (see Nemhauser and Wolsey [70] and Bradley et al. [20]). When the problem contains a huge number of decision variables, a technique called *Branch − and − Price* that integrates features of both Column Generation and Branch-and-Bound, is used (see Figure 3-2). The major challenge of Branch-and-Price is to devise branching rules that maintain an
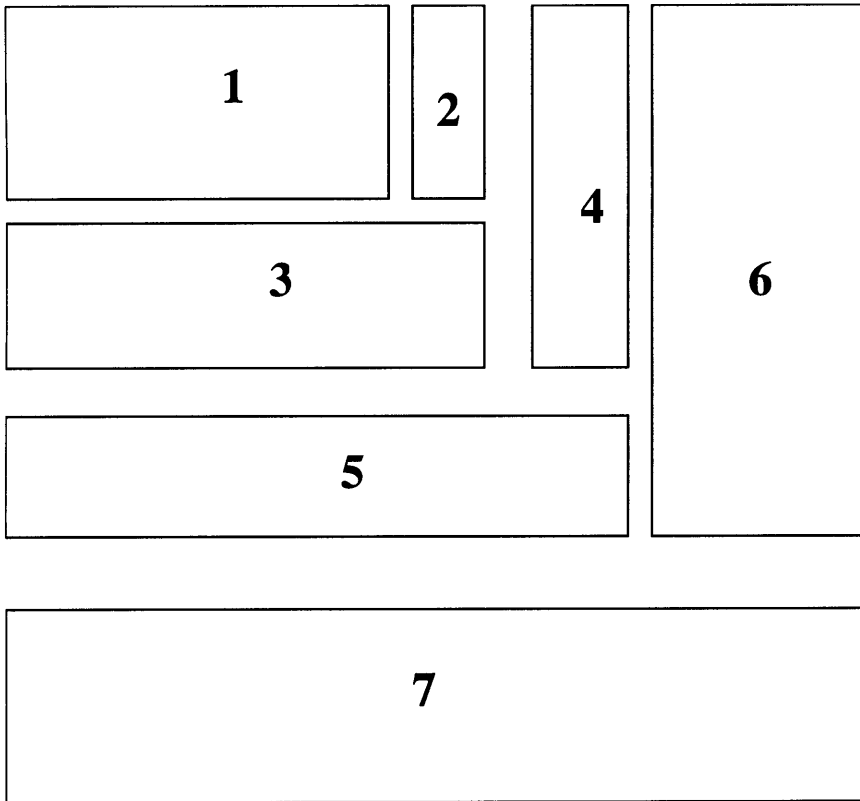
Figure 3-1: Illustration: Synchronized Colummn and Row Generation

efficient structure of the pricing subproblem (see Barnhart et al. [16]). We refer the reader to Parker and Ryan [73], Desrosiers et al. [32], Vance et al. [87], Barnhart et al. [16] and Desrochers and Soumis [31] for alternative branching strategies for different types of problems.

When the problem contains a huge number of constraints, a $Branch - and - Cut$ technique can used. Branch-and-Cut integrates Branch-and-Bound and Row Generation by using Row Generation (or Cut Generation) in solving LPs at each node of the Branch-and-Bound tree. Branching occurs when no violated constraints are found. The difficulty is in developing an algorithm to solve the separation problem.

For problems with huge number of both columns and rows, Branch-and-Price and Branch-and-Cut can be integrated to form a $Branch - and - Price - and - Cut$ solution technique. However, incompatibility between row and column generation steps often make it difficult to solve IPs using Branch-and-Price-and-Cut. Barnhart et al. [16] present heuristic approaches that may be able to provide a good integer solution.
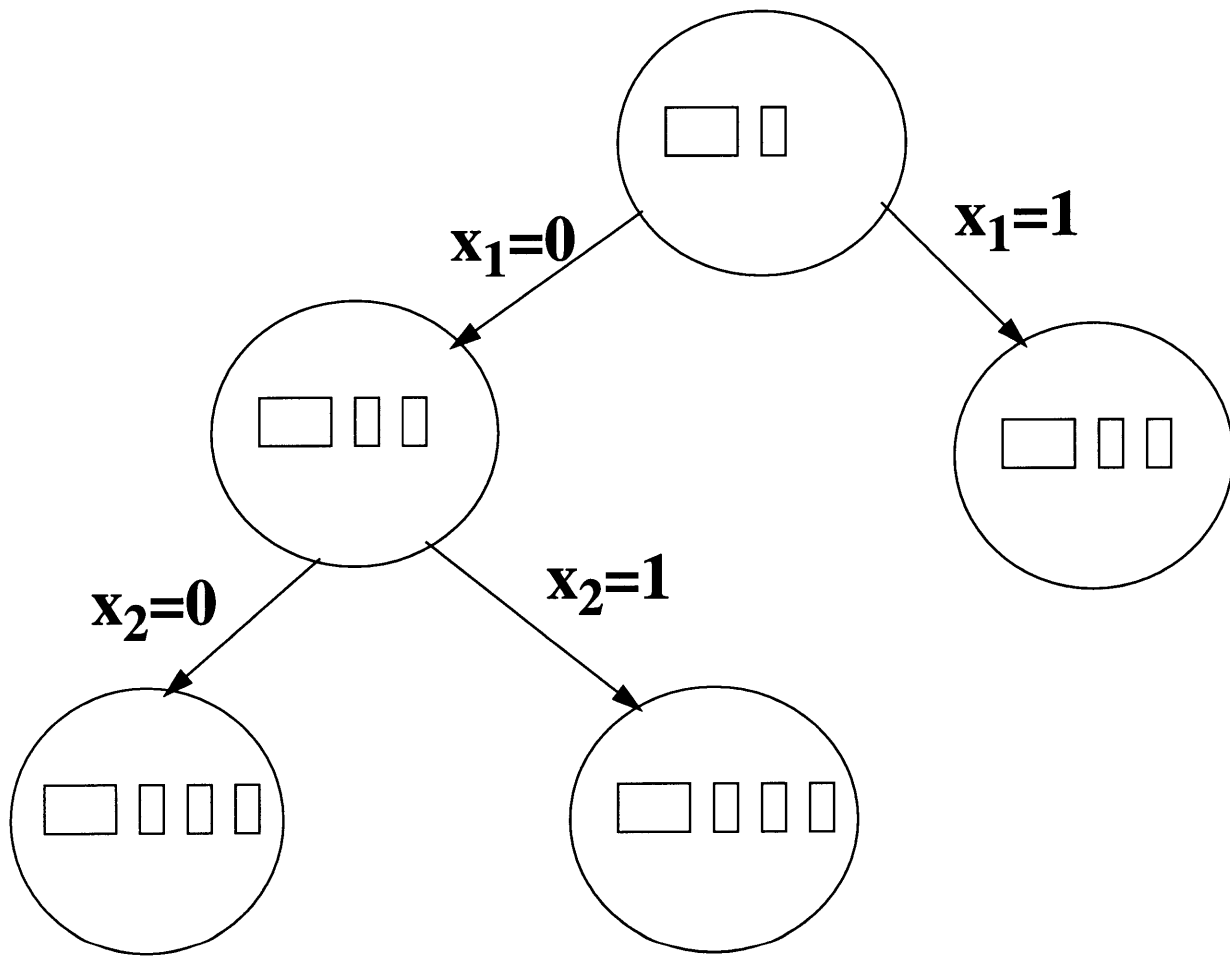
Figure 3-2: Illustration: Branch-and-Price

Chapter 4

Shipment Flow Problem: Models and Applications

In this chapter we describe the second type of sub-problem that deals with the decision of commodity routing over a given fleet network with finite arc capacities. Such problems belong to a general class of network flow problems called Multicommodity Flow (MCF) problems. Because of their interesting mathematical properties and wide applicability, MCF problems have been extensively studied. We first outline some generalities of the MCF problem before presenting our model.

4.1 Baseline Multicommodity Flow Problem Formulation

The multicommodity flow problem is a subproblem embedded in the Network Design Problem. When the design variables in the NDP model (as presented in Chapter 1) are fixed, only decisions on the flow variables remain and the problem reduces to finding a minimum cost routing of commodities from their origins to destinations over the network formed by design variables. If we assume u_{ij} to be the capacity on arc $(i, j) \in A$ after fixing the design variables, then the resulting MCF problem can be formulated as:

$$\text{Minimize} \quad \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k b^k x_{ij}^k \tag{4.1}$$

such that

$$\sum_{k \in K} b^k x_{ij}^k \leq u_{ij} \quad \forall(i,j) \in A \tag{4.2}$$

$$\sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{ji}^k = \begin{cases} 1 & \text{if } i = O(k) \\ -1 & \text{if } i = D(k) \quad \forall i \in N, \; \forall k \in K \\ 0 & \text{otherwise} \end{cases} \tag{4.3}$$

$$x_{ij}^k \geq 0, \quad \forall k \in K, \; \forall(i,j) \in A \tag{4.4}$$

The objective (4.1) is to find a minimum cost routing of commodities from their origins to destinations over the network. Constraints (4.2) are called *bundle constraints* that limit the amount of flow on any arc to its capacity. Constraints (4.3) are *flow conservation constraints* that ensure that each commodity is fully serviced from origin to destination. Constraints (4.4) guarantee non-negativity of flow variables.

4.2 Applications of Multicommodity Flow Models

Multicommodity flow problems have a number of applications in various areas like transportation, logistics, telecommunication, production, etc. Some of the applications are presented in this section.

4.2.1 Transportation and Logistics

Some applications of MCF problems in transportation and logistics are described below.

- *Routing vehicles in Traffic networks(Dynamic Traffic Assignment)*. This involves the determination of minimum delay routes for vehicles from their origins to their respective destinations over the traffic network. The allowable

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congestion levels determine the arc capacities. Alternatively, there are no capacities but the cost on an arc is a function of the amount of flow on the arc.

- *Airline Fleet Assignment.* Given a time table of flight arrivals and departures, the expected demand on the flights and a set of aircraft, the objective is to arrive at a minimum cost assignment of aircraft to the flights. This problem has been extensively studied (see Abara [1] and Hane et al. [44]).

- *Aircraft Maintenance Routing.* Given an assignment of flights to fleets, the task is to determine a sequence of flights, or routes, starting and ending at maintenance stations or to find rotations (cycles) in the network, to be flown by individual aircraft so that all the flight legs are covered. This problem has been studied by Barnhart et al. [11] Clarke et al. [23], Shenoi [83] and Feo and Bard [35].

- *Airline Crew Scheduling.* This problem deals with minimum cost pairing of crews and aircraft. Factors such as aircraft type compatibility, hours of work limitations and Federal Aviation Administration (FAA) regulations must be taken into account while solving the problem. For an in-depth study the reader is referred to Anbil et al. [4] and Barnhart et al. [15].

- *Distribution systems planning.* In this problem there are different commodities produced at several plants with known production capacities. Each commodity has a certain demand in each customer zone. The demand is satisfied by shipping via regional distribution centers (DCs) with finite storage capacities. The problem of routing the commodities from the manufacturing plants to the customer zones through the DCs can be formulated as a MCF problem.

- *Import and export models.* One of the factors that may affect export is handling capacity at ports. Barnett, Binkley and McCarl [9] uses a MCF model to analyze the effect of US port capacities on the export of wheat, corn and soybean.

- *Optimization of freight operations.* Crainic, Ferland and Rousseau [27] develop a MCF-based routing and scheduling optimization model that considers

the planning issues for the railroad industry. More recently, Newton [71] and Barnhart et al. [14] study the railroad blocking problem using multicommodity based formulations.

- *Freight Assignment in the Less − than Truck load(LTL) industry.* An LTL carrier has to consolidate many shipments to make economic use of the vehicles. This requires the establishment of a large number of terminals to sort freight. Trucking companies use forecasted demands to define routes for each vehicle to carry freight to and from the terminals. Once the routes are fixed, the problem is to deliver all the shipments with minimum total service time or cost. This problem can be formulated as a MCF problem.

- *Express Shipment Delivery.* Kim [52] models the shipment delivery problem faced by express carriers like Federal Express, Unites States Postal Service, United Parcel Service etc. as a MCF problem on a network in space and time.

4.2.2   Other Applications

We present below some other applications of MCF models:

- *Routing messages in a communication or computer network.* The network consists of transmission lines. Each message request is a commodity. The problem is to route the messages from origins to the respective destinations at a minimum cost.

- *Long − term hydro − generation optimization.* The task in this case is to determine the amount of hydro-generation at a reservoir in an interval of time, that minimizes the expected cost of power generation over a period of time, divided into several intervals. Nabonna [68] showed that this problem can be modeled as a MCF problem with inflows given as probabilistic density functions.

- *Forest Management.* For each planning period, forest managers have to make decisions concerning the land areas to be harvested, the volume of timber to be harvested from these areas, the land areas to be developed for recreation

and the road network to be built and maintained in order to support both the timber haulers and recreationists. This problem has been formulated as a MCF problem by Helgason, Kennington [51] and Wong [45].

- *Street planning.* Foulds [36] introduced this problem and modeled it as a MCF problem. The objective is to identify a set of two-way streets such that making these streets one-way minimizes the total congestion cost in the network.

- *Spatial price equilibrium(SPE) problem.* This problem requires modeling consumer flows within a general network. The SPE problem determines the optimum levels of production and consumption at each market and the optimal flows satisfy the equilibrium property. Segall [81] models and solves the SPE problem as a MCF problem.

For a more comprehensive description of applications we refer the reader to Schneur [79], Ahuja et al. [3] and Kennington [51].

4.3 Multicommodity Flow Solution Techniques

The *linear* MCF problem is the most studied instance of MCF problems. A fairly comprehensive survey of linear multicommodity flow models and solution methods is presented in Ahuja et al. [3]. We describe below some of the recent results in this area.

- Saviozzi [78] uses subgradient techniques on the Lagrangean relaxation of the bundle constraints and proposes a method of arriving at an advanced starting basis for the minimum cost multicommodity flow problem. Depending on whether the basis is primal feasible or dual feasible, further iterations can be carried out with primal or dual simplex method, as the case may be.

- Barnhart and Sheffi [19] present a network-based heuristic solution strategy in a primal-dual framework for MCF problems. This procedure is designed for large-scale problems, whose size is excessive for the simplex or barrier methods.

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- Barnhart et al.[13] present a cycle-based formulation of the MCF problems. They also describe a partitioning solution procedure for large-scale MCF problems based on both column generation and constraint relaxation.

- Schneur [79] describes a procedure that uses the concepts of ϵ-optimality (see Ahuja et al. [3]) and scaling, together with a quadratic penalty function for the capacity constraint, to solve the linear MCF problems.

- Barnhart [10] provides a dual-ascent heuristic for MCF problems that gives a valid lower bound on the optimal primal objective value and an advanced starting solution for primal-based solution methodologies. A heuristic to generate an approximate primal-optimal solution is also described.

- Jones et al. [48] compare and contrast the different formulations of MCF problems. They present empirical evidence that the path-based formulation by decomposition yields lower CPU times than the equivalent tree-based formulation.

- Radzik [76] considers one kind of MCF problem called the *maximum concurrent flow problem*, where the objective is to design a set of commodity flows with minimum possible congestion. The *congestion* of a given flow of commodities is defined as the minimum number having the property that the flow is feasible when all edge capacities are multiplied by this number. Radzik demonstrates how approximate solutions to this problem can be computed deterministically using a number of single-commodity minimum cost flow computations.

- Klein et al. [53] develop approximation algorithms for the concurrent flow problem with uniform capacities. Leighton et al. [55] present generalized algorithms that are valid for problems with arbitrary capacities.

- Interior Point methods provide polynomial time algorithms for the MCF problems. The best time bound is due to Vaidya [86].

- Schultz and Meyer [80] provide an interior point method with massive parallel computing to solve multicommodity flow problems.

- Zenios [90] presents an algorithm for nonlinear optimization problems with multicommodity flow constraints. Empirical results for a parallel implementation are reported for quadratic programs with approximately 10 million columns and 100,000 rows.

4.4 Integer Multicommodity Flow Problems

4.4.1 Integer MCF Applications

In many applications of MCF models, each commodity is identified with a specific origin and destination. In many cases, the demand for any commodity cannot be split and assigned to multiple paths. Since MCF problems do not obey the *integrality* property (see Ahuja et al. [3]), unlike pure network flow problems, it is necessary to impose integrality requirements. These applications include the following (see Barnhart et al. [16]):

- *Bandwidth packing.* This involves optimal allocation of bandwidth in telecommunication networks and routing of calls from their points of origins to their destinations. In case of video teleconferencing, calls cannot be split and all O-D demand should be routed on a single path.

- *Express Package Delivery.* In this problem that is mentioned before, often it is desirable for all O-D demand to be routed on a single path to facilitate operational ease and satisfy all the level of service requirements.

- *Aircraft Maintenance Routing.* In this problem mentioned before, each aircraft is a commodity and aircraft should be assigned to paths such that each flight is covered by exactly one aircraft.

4.4.2 Integer MCF Solution Techniques

Barnhart et al. [16] present a general solution strategy called Branch-and-Price, for solving large-scale IPs. Branch-and-Price involves features from both Branch-and-

Bound and Column Generation. Researchers have tailored Branch-and-Price solution techniques for solving different integer multicommodity flow models as follows:

- Barnhart et al. [12] study the large-scale Integer Multicommodity Flow Problem in which the entire demand of any commodity is to be assigned to the same path. They present a column generation model and Branch-and-Price-and-Cut algorithm with specialized branching rules.

- Parker and Ryan [73] describe a Branch-and-Price algorithm for the bandwidth packing problem in telecommunication networks. Bandwidth packing involves the selection of a set of commodities to maximize revenue.

- Ziarati et al. [91] consider the problem of assigning railway locomotives to trains. They model the problem as an integer multicommodity flow problem with side constraints and solve using a Dantzig-Wolfe decomposition technique, where subproblems are formulated as constrained or unconstrained shortest path problems.

- Raghavan and Thompson [77] illustrate the use of randomized algorithms to solve some integer multicommodity flow problems. They use randomized rounding procedures that give provably good solutions in the sense that they have a very high probability of being close to optimality.

4.5   Location Elimination Model (LEM)

In this section we present a variant of a mixed integer multicommodity flow model, called the Location Elimination Model (LEM), that was introduced in the context of our decomposition solution approach in Chapter 2. We recall that LEM establishes a set of commodity flows on the fleet network that maximizes the number of origin and destination locations such that all demand originating at or destined for those locations, as the case may be, is assigned to network paths. We let $G = (N, A)$ be the given fleet network over which commodities are to be flown, O the set of all

origin locations and D the set of all destination locations. Again, each commodity is identified by a specific origin and destination pair. We let y_o be a zero-one variable for each origin $o \in O$, that is equal to 1 if all the O-D commodities with $O(k) = o$ are assigned to network paths and equal to zero otherwise. Similarly we let z_d be a zero-one variable for each destination $d \in D$, that is equal to 1 if all the origin-destination (O-D) commodities with $D(k) = d$ are assigned to network paths and equal to zero otherwise. The objective is to maximize

$$\sum_{o \in O} y_o + \sum_{d \in D} z_d \ .$$

As with the multicommodity flow problem, the flow variables can be represented on arcs and paths. Since we will be dealing with commodity independent arc costs, we can also represent the flows on origin-based or destination-based trees, as has been demonstrated by Jones et al. [48]. Since an arc based formulation has huge memory requirements, we will consider only path-based and tree-based formulations.

4.5.1 LEM Path Formulation

Notations

Before presenting the path formulation, we first define some notations.

SETS

$K(\ni k)$: the set of all O-D commodities

$P^k(\ni p)$: the set of all feasible paths from origin $O(k)$ to destination $D(k)$ for eack $k \in K$

K_o : the set of all O-D commodities with origin o

K_d : the set of all O-D commodities with destination d

O_d : the set of all origins o such that there exists a commodity with $O(k) = o$ and $D(k) = d$, for each $d \in D$

D_o : the set of all destinations d such that there exists a commodity with $O(k) = o$ and $D(k) = d$, for each $o \in O$

PARAMETERS

b^k : the demand for commodity for each $k \in K$

u_{ij} : the capacity of arc $(i,j) \in A$

INDICATOR VARIABLES

$$\delta_{ij}^p = \begin{cases} 1 & \text{if arc } (i,j) \text{ belongs to path } p \\ 0 & \text{otherwise} \end{cases}$$

DECISION VARIABLES

x_p^k : fraction of b^k on path $p \in P^k$ for all $k \in K$

The path formulation is as follows:

$$Maximize \quad \sum_{o \in O} y_o + \sum_{d \in D} z_d \tag{4.5}$$

such that,

$$\sum_{k \in K_o} \sum_{p \in P^k} x_p^k - y_o |D_o| \geq 0, \quad \forall o \in O \tag{4.6}$$

$$\sum_{k \in K_d} \sum_{p \in P^k} x_p^k - z_d |O_d| \geq 0, \quad \forall d \in D \tag{4.7}$$

$$\sum_{p \in P^k} x_p^k \leq 1, \quad \forall k \in K \tag{4.8}$$

$$\sum_{k \in K} \sum_{p \in P^k} x_p^k (\delta_{ij}^p b^k) \leq u_{ij}, \quad \forall (i,j) \in A \tag{4.9}$$

$$x_p^k \geq 0 \tag{4.10}$$

$$y_o, z_d \in \{0, 1\} \tag{4.11}$$

Constraints (4.6) ensure that a location cannot have all of its originating demand served unless each commodity originating there is fully assigned to an origin-destination path. Similarly we have constraints (4.7) for the destination locations. Constraints (4.8) are generalized upper bounding constraints to ensure that the total flow of any commodity k from $O(k)$ to $D(k)$ does not exceed its demand. Constraints

(4.9) represent the arc capacity constraints. Constraints (4.10) ensure that the path-flows are non-negative. Constraints (4.11) imply that a location has either all of its commodities served or it does not.

4.5.2 LEM Tree Formulation

Notations

Before presenting the tree formulation, we first define some more notations.

    SETS

    $Q_o(\ni q)$: the set of all trees at origin o, for all $o \in O$

    O_d : the set of all origins o such that there exists a commodity with $O(k) = o$ and $D(k) = d$, for each $d \in D$

    p_q^k : the unique path from $O(k)$ to $D(k)$ in tree q

    INDICATOR VARIABLES

$$\Gamma_{ij}^{qk} = \begin{cases} 1 & \text{if arc } (i,j) \text{ belongs to the path } p_q^k \text{ from } O(k) \text{ to } D(k) \text{ in tree } q \\ 0 & \text{otherwise} \end{cases}$$

    DECISION VARIABLES

w_o^q: fraction of b^k flown on the path p_q^k from $O(k) = o$ to $D(k)$ in tree q

The tree formulation is as follows:

$$Maximize \quad \sum_{o \in O} y_o + \sum_{d \in D} z_d \tag{4.12}$$

such that,

$$\sum_{q \in Q_o} w_o^q - y_o \geq 0, \quad \forall o \in O \tag{4.13}$$

$$\sum_{o \in O_d} \sum_{q \in Q_o} w_o^q - z_d |O_d| \geq 0, \quad \forall d \in D \tag{4.14}$$

$$\sum_{q \in Q_o} w_o^q \leq 1, \quad \forall o \in O \tag{4.15}$$

$$\sum_{o \in O} \{ \sum_{q \in Q_o} (\sum_{k \in K_o} \Gamma_{ij}^{qk} b^k) w_o^q \} \leq u_{ij}, \quad \forall (i,j) \in A \qquad (4.16)$$

$$w_o^q \geq 0 \ , \forall q \in Q_o, \ \forall o \in O \qquad (4.17)$$

$$y_o, z_d \in \{0, 1\} \qquad (4.18)$$

Constraints (4.13) through (4.18) in the tree formulation correspond to constraints (4.6) through (4.11) in the path formulation.

4.6 LEM LP Relaxation Solution

We will first consider solution to the linear programming relaxation of LEM. Our approach to both the path and tree formulations will be based on *Dantzig-Wolfe decomposition* [28] or *Column Generation*. The general approach is to start with an initial subset of variables and generate more variables as needed by solving a *pricing subproblem*. In our case, since the y_o and z_d variables are not too many in number, we will start with all of them and generate only the flow variables by solving the pricing subproblem.

4.6.1 Solution to the Path Formulation LP

We will generate the path flow variables x_p^k's by solving the pricing problem. In order to formulate the pricing problem, we will use the following vectors for the dual variables:

λ : duals corresponding to constraints (4.6)

μ : duals corresponding to constraints (4.7)

σ : duals corresponding to constraints (4.8)

π : duals corresponding to constraints (4.9)

The pricing problem then can be written as follows:

$$\rho* = \max_{p \in P^k} \{ -\lambda_{O(k)} - \mu_{D(k)} - \sigma_k - \sum_{(i,j) \in A} (\delta_{ij}^p b^k) \pi_{ij} \}, \quad \forall k \in K \qquad (4.19)$$

or equivalently,

$$\rho* = -\min_{p \in P^k}\{\lambda_{O(k)} + \mu_{D(k)} + \sigma_k + \sum_{(i,j) \in A} (\delta_{ij}^p b^k)\pi_{ij}\}, \quad \forall k \in K \qquad (4.20)$$

The reduced costs of all the path variables are non-positive if the following set of conditions is satisfied:

$$-\{\lambda_{O(k)} + \mu_{D(k)} + \sigma_k + \sum_{(i,j) \in A} (\delta_{ij}^p b^k)\pi_{ij}\} \leq 0, \quad \forall p \in P^k, \quad \forall k \in K \qquad (4.21)$$

Equivalently, optimality is reached if the following set of conditions is satisfied:

$$\min_{p \in P^k}\{\sum_{(i,j) \in A} (\delta_{ij}^p b^k)\pi_{ij}\} \geq -(\lambda_{O(k)} + \mu_{D(k)} + \sigma_k) , \quad \forall k \in K \qquad (4.22)$$

The problem of determining the path that minimizes $\sum_{(i,j) \in A}(\delta_{ij}^p b^k)\pi_{ij}$ is a shortest path problem for each $k \in K$ over the fleet network with modified arcs costs of π_{ij} (see Ahuja et al [3]). That is, the pricing problem for the path formulation is solved by finding a solution to an origin-destination shortest path problem for each commodity $k \in K$.

4.6.2 Solution to the Tree Formulation LP

We use the same notation for duals as the path formulation. The pricing problem is formulated as:

$$\rho* = \max_{q \in Q_o}\{-\lambda_o - \sum_{d \in D_o} \mu_d - \sigma_o - \sum_{k \in K_o}\sum_{(i,j) \in A} (\Gamma_{ij}^{qk} b^k)\pi_{ij}\}, \quad \forall o \in O \qquad (4.23)$$

or equivalently,

$$\rho* = -\min_{q \in Q_o}\{\lambda_o + \sum_{d \in D_o} \mu_d + \sigma_o + \sum_{k \in K_o}\sum_{(i,j) \in A} (\Gamma_{ij}^{qk} b^k)\pi_{ij}\}, \quad \forall o \in O. \qquad (4.24)$$

Optimality is reached if the following set of conditions is satisfied:

$$\sum_{k \in K_o} b^k (\min_{q \in Q_o} \{ \sum_{(i,j) \in A} (\Gamma^{qk}_{ij}) \pi_{ij} \}) \geq -(\lambda_o + \sum_{d \in D_o} \mu_d + \sigma_o) \ , \quad \forall o \in O. \qquad (4.25)$$

The problem of finding the path that minimizes $\sum_{(i,j) \in A} (\Gamma^{qk}_{ij}) \pi_{ij}$ over all trees $q \in Q_o$ is a shortest path problem for each origin $o \in O$ and for each commodity $k \in K_o$ over the fleet network with modified arcs costs of π_{ij}. That is, the pricing problem for the tree formulation is solved by finding a solution to a single-origin single-destination shortest path problem for the combination of each origin $o \in O$ and each O-D commodity $k \in K_o$.

4.7   LEM IP Solution

Although integer multicommodity flow problems have been studied by many researchers (see section 4.4), LEM is a very different kind of integer MCF problem in that the demand for any commodity *can* be split between paths. Hence previously developed Branch-and-Price techniques do not apply. We will use a variant of Branch-and-Price in which simple branching is done on origin and destination variables, y_o and z_d, and the shortest path pricing subproblem is solved for the generation of flow variables (x^k_p and w^q_o).

Chapter 5

Service Network Design for Express Shipment Delivery: A Case Study

In this chapter we demonstrate a successful application of our models and solution techniques in the express shipment delivery industry, where consolidation occurs when shipments are sorted at the hubs. We first describe the express shipment delivery operation and the practical constraints that need to be considered while modeling the decision problem. We then present a routing model and solution algorithm and describe how we use our decomposition technique to solve this service network design problem. We conclude this chapter with computational results.

5.1 Express Shipment Delivery Operation

In this section we describe the service network design problem of a large carrier in the express shipment delivery industry. Our description is abstracted from Kim [52]. The objective is to determine the cost minimizing movement of shipments (or packages) from their origins to their destinations using a limited amount of resources, such that all level of service (LOS) requirements are satisfied. There are multiple products or service types, depending on the LOS requirements. Premium service that occurs

overnight is referred to as *Next Day Service*, while guaranteed service within 48 hours is called *Second Day Service* and service within 3-5 days is called *Deferred Service*.

5.1.1 Problem Description

This service network design problem, like others, can be envisaged to consist of two types of decisions: the first is to determine the *service network* formed by the routes of transportation assets and the second is to determine *shipment flows* over the service network. The service network is defined by the movements in space and time of the transportation assets, in this case, aircraft and ground vehicles. The routes for individual shipments from origins to destinations determine the flows on each link in the service network.

Service Network and Shipment Flows

Shipments originate and terminate at locations, that could either be airports called *gateways* or non-airport *all ground* locations, where they are loaded, unloaded and transferred between aircraft and/or ground vehicles. From its initial gateway, a shipment may be flown to at most one additional gateway before it is flown to a hub. Hubs are specialized gateways or *all ground* locations, where shipments are consolidated as they are sorted based on their respective destinations. At a hub, a shipment is unloaded from an inbound vehicle, sorted and loaded on an outbound vehicle. Again it may be flown to at most one intermediate gateway before being flown to its final destination.

Associated with the gateways are *earliest pickup times (EPT's)* and *latest delivery times (LDT's)*. An EPT denotes the time at which shipments will be available for pickup at a gateway. Each gateway's EPT is scheduled as late as possible to allow customers enough time to prepare their shipments, but early enough so that delivery service standards can be met. A gateway's LDT denotes the time by which all shipments must be delivered to the location in order to satisfy delivery standards.

In setting EPTs and LDTs, we also consider hub sort capacities and time windows designating the start and end sort times. An aircraft route can be decomposed into two distinct components, a *pickup route* and a *delivery route*. A pickup route typically departs from some gateway in the early evening and is restricted to contain at most one intermediate stop before its final stop at a hub. A delivery route begins at a hub, typically departing in the early morning, and stops at most at two gateways. The number of stops on a pickup or delivery route is restricted to three to limit the potential of schedule problems arising in this hub-and-spoke network. Fewer take-offs and landings result in a reduction in the expected schedule slippage. Given the time sensitive nature of the express shipment delivery operation, robustness of operation is critical.

Costs

The costs of an express shipment operation can be expressed as the sum of vehiucle costs and shipment handling costs. For a given flight leg and aircraft type, aircraft operating costs are the sum of associated fuel cost, crew cost, cycle cost (cost of take-off and landing), maintenance cost, etc. Depending on the nature of the model, aircraft ownership cost may or may not be included. Ownership cost is not included for near-term planning because the company already owns the aircraft, however a fixed cost per aircraft is included if the model is to be used in a more strategic context, e.g., if the model is to be used to determine future fleet composition. Shipment handling costs are expressed per unit of shipment for each gateway and hub location.

5.1.2 The Planning Process

For most express package operations, there are four types of planning activities, characterized by their time horizon. The first, termed, *strategic planning* looks several years into the future. This type of planning is focused on problems of aircraft acquisition, hub capacity expansion, new facility location, etc. The planners are not constrained by existing resources. As a matter of fact, determining the required

resources under future operating conditions is one of the objectives of this type of planning activity. Although the data used in this type of planning exercise are often imprecise, relying heavily on forecasts, the planners must construct an operating plan to assess various scenarios.

The second type of planning is *tactical planning*. This planning activity, that considers anything from one month to several months into the future, generates a plan to be executed in the operation. Components of short-range *operations planning* such as flight crew planning and maintenance planning are based on the output of this process. In tactical planning, the freedom to change existing resources is very limited. However, it is common to use these models to analyze different scenarios, like determining the incremental operating costs for a set of volume changes. The results are used to direct marketing efforts over the next one to two years. We call this type of analysis *market planning*.

The other type of planning activity called *contingency planning* includes preparing the system to adapt to sudden volume changes, recovering from weather disruptions, etc.

While our primary objective is to facilitate strategic planning, we design our models to be also applicable for tactical planning, operations planning and market planning as well.

In the case of the company we study, planning is still done primarily manually, with limited automation. In the case of tactical planning, a database system is used to generate reports to compare operations and plan, but no automated decision support *per se* is available. As a result, it takes an entire year to develop a long range plan. The models we develop will shorten this planning cycle and allow extensive analyses to be performed.

Scope of the Problem

The focus of our research is on solving the next day problem over a single day planning horizon. We determine both the service network and the shipment flows over the service network. The output specifies the schedule of every vehicle route, the vehicle

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type assigned to each route and the shipment routings in the resulting network.

The model inputs include shipment movement requirements, a network of potential ground movements (called *legs*), fleet composition and characteristics, and sort capabilities and characteristics. The fleet composition is given with an option available to lease additional aircraft. All aircraft, gateway and hub operating characteristics, such as range, speed, runway length, sort capacity, etc. are assumed fixed.

Although we have a myopic focus on a shortened planning horizon and only one kind of product, our approaches allows us to develop modeling techniques and decomposition solution algorithms for the ultimate problem by solving relatively smaller self-contained problems.

5.2 Express Shipment Service Network Design

5.2.1 Design Variables

In the case of the Express Shipment Service Network Design Problem (ESSNDP), the design variables in the general SNDP presented before represent the movement of vehicles (aircraft and trucks) and the flow variables represent the movement of shipments. We use route-based design variables for the following reasons (see Kim [52]):

1. Fixed costs and nonlinearities in the cost structure can be captured since each vehicle is assigned to one design variable.

2. Complicated restrictions on vehicles and shipment routes cannot be represented using other types of design variables.

3. The node-arc formulation could result in a problem size prohibitively large to solve.

5.2.2 Side Constraints

The SNDP must be tailored to represent the constraints under which express carriers operate, including fleet balance, fleet capacity, hub sort capacity, hub landing capacity and network connectivity (see Kim [52]).

Fleet Balance

Fleet balance constraints force the number of routes into a gateway location for each fleet type $f \in F$ to be equal to the number out of it:

$$\sum_{r \in R_P^f \cup R_D^f} \beta_i^r y_r^f = 0 \ , \quad \forall i \in N, \quad \forall f \in F \tag{5.1}$$

where β_i^r is equal to 1 if route r ends at node i, is equal to -1 if route r begins at i, and is equal to zero otherwise.

Fleet Size

There is a limitation on the number of vehicles available for each fleet type. We model this by restricting the number of pickup routes *or* the number of delivery routes for fleet type f to be less than the fleet size n^f, for all $f \in F$:

$$\sum_{r \in R_P^f} y_r^f \leq n^f \ , \quad \forall f \in F \tag{5.2}$$

or,

$$\sum_{r \in R_D^f} y_r^f \leq n^f \ , \quad \forall f \in F \tag{5.3}$$

There is no need to impose both sets of constraints because fleet balance is ensured by constraints (5.1).

Hub Sort Capacity

Each hub can sort only a limited number of shipments per unit time. To model these hub sort capacity constraints, we divide the total time during which sorting is performed into equal intervals $t = \{1, 2, ..., T\}$. We let P_i^t to be the set of package routes with the earliest arrival time at hub i, that is at or after the start time of the interval $t \in \{1, 2, .., T\}$ and e_i^m be the sort capacity of hub i during the interval $m \in T$. Then the hub sort capacity constraints are written as follows:

- Path Formulation

$$\sum_{k \in K} \sum_{p \in P^k \cap P_i^t} b^k x_p^k \leq \sum_{m=t}^{T} e_i^m , \quad \forall i \in H, \ \forall t \in \{1, 2, ..T\} \tag{5.4}$$

- Tree Formulation

$$\sum_{o \in O} \sum_{q \in Q_o} \{ \sum_{k \in K} \sum_{p \in P^k \cap P_i^t} \delta_{ij}^p b^k \} w_o^q \leq \sum_{m=t}^{T} e_i^m , \quad \forall i \in H, \ \forall t \in \{1, 2, ..T\} \tag{5.5}$$

Hub Landing Capacity

Each hub cannot have more than a certain number of aircraft land in an interval of time. The hub landing capacity constraint for each hub i is modeled by dividing the total time during which aircraft arrive, into equal intervals $t = \{1, 2, ..., T\}$. We let L_i^t be the set of pickup routes with earliest arrival time at hub i no earlier than the start time of interval t and a_i^m be the number of aircraft that can land at hub i during interval $m \in T$. Then the hub landing capacity constraints are represented as follows:

$$\sum_{f \in F} \sum_{r \in R_p^f \cap L_i^t} y_r^f \leq \sum_{m=t}^{T} a_i^m , \quad \forall i \in H, \ \forall t \in \{1, 2, ..T\} \tag{5.6}$$

Network Connectivity

Many carriers offering express delivery services operate with a major hub and one or more regional hubs. For example Federal Express has a major hub at Memphis, United Parcel Service at Louisville and Airborne Express at Wilmington. For operational reasons, some carriers require *all-point service* to and from their regional hubs. Hence there should be one pickup route from each origin location to the major hub and one delivery route from the major hub to each destination location. We let V_P be the set of pickup routes ending at the major hub and V_D be the set of delivery routes beginning at the major hub. The Network Connectivity constraints or the *all-point service* constraints are represented as follows:

$$\sum_{f \in F} \sum_{r \in R_P^f \cap V_P} y_r^f \geq 1 \ , \quad \forall i \in N \tag{5.7}$$

$$\sum_{f \in F} \sum_{r \in R_D^f \cap V_D} y_r^f \geq 1 \ , \quad \forall i \in N. \tag{5.8}$$

5.3 Routing Model for Express Shipment Delivery

In this section we present a routing model that is used in our decomposition approach for solving the ESSNDP. Since the routing model is only a part of the decomposition approach and not an end in itself, we will only use an approximate model. Since we would like to consider only the routes and not the flows, we will not impose hub sort capacity constraints. However, we will impose hub landing capacity constraints to serve as a kind of proxy for hub sort capacity constraints.

Since do not consider shipment flows, we do not explicitly include flow conservation constraints in our model. We use *cutset inequalities* to model the capacity requirements in our problem.

5.3.1 Cutset Inequalities

The motivation behind these inequalities is the requirement that the total capacity provided by the design variables must be greater than or equal to the total demand, for any O-D cutset $\{S, T\}$. An O-D cutset $\{S, T\}$ is defined as a partition of the node set N into two mutually exclusive and collectively exhaustive non-empty subsets S and T such that S contains o and T contains d for some shipment commodity k with origin $O(k) = o$ and destination $D(k) = d$. Arc (i, j) belongs to O-D cutset $\{S, T\}$ if nodes i and j belong to different sets S and T. We will let $D_{S,T}$ be the total demand of all commodities with origin in subset S and destination in subset T. Assuming $Y_{S,T}^f = \sum_{r \in R_P^f \cup R_D^f} \sum_{(i,j) \in \{S,T\} \cap r} y_r^f$, the aggregate capacity demand inequalities are written as follows:

$$\sum_{f \in f} u_f Y_{S,T}^f \geq D_{S,T} \quad , \quad forall \ O - D \ cutsets \ \{S, T\}. \tag{5.9}$$

We strengthen the inequalities (5.9) by lifting and creating *Chvátal − Gomory cuts* or *cutset inequalities* as illustrated in Chapter 2. We let inequalities (5.9) represent both the original aggregate capacity demand inequalities plus the lifted inequalities and we refer to them collectively as *cutset inequalities*.

5.3.2 ESSNDP-Approx Model

For the route generation step in our decomposition, we would ideally like to consider only the route decision variables and not the shipment flows, to limit problem size. Hence we do not impose hub sort capacity constraints. However, we do impose hub landing capacity constraints to serve as a proxy for hub sort capacity constraints.

The approximate model is written as follows:

$$Minimize \sum_{r \in R_P^f \cup R_D^f} h_r^f y_r^f \tag{5.10}$$

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such that:

$$\sum_{r \in R_P^f \cup R_D^f} \beta_i^r y_r^f = 0 \ , \quad \forall i \in N, \quad \forall f \in F \tag{5.11}$$

$$\sum_{r \in R_P^f} y_r^f \leq n^f, \quad \forall f \in F \tag{5.12}$$

$$\sum_{f \in F} \sum_{r \in R_P^f \cap L_i^t} y_r^f \leq \sum_{m=t}^{T} a_i^m \ , \quad \forall i \in H, \ \forall t \in \{1, 2, ..T\} \tag{5.13}$$

$$\sum_{f \in F} \sum_{r \in R_P^f \cap V_P} y_r^f \geq 1 \ , \quad \forall i \in N \tag{5.14}$$

$$\sum_{f \in F} \sum_{r \in R_D^f \cap V_D} y_r^f \geq 1 \ , \quad \forall i \in N \tag{5.15}$$

$$\sum_{f \in f} u_f Y_{S,T}^f \geq D_{S,T} \ , \quad \forall \ O - D \ cutsets \ \{S, T\} \tag{5.16}$$

$$y_r^f \geq 0 \ and \ integer \ , \quad \forall r \in R_P^f \cup R_D^f, \ \forall f \in F. \tag{5.17}$$

5.3.3 ESSNDP-Approx Solution Algorithm

We solve ESSNDP-Approx by applying synchronized column and row generation (the procedure is described in Chapter 3) as follows:

Step 1: Solve the LP relaxation of ESSNDP-Approx, given a fixed set of constraints, using explicit column generation of route variables.

Step 2: Arrive at a feasible IP solution for ESSNDP-Approx, given a fixed set of variables (columns) and constraints, using branch-and-bound.

Step 3: Explicitly generate violated inequalities, given the current feasible IP solution. If any violated inequalities are found, add them to the current basis and go to Step 1. Otherwise, go to Step 4.

Step 4: Collect all the positive route variables in the current ESSNDP-Approx IP solution. These routes will be input to LEM.

5.4 Decomposition Solution Algorithm for Express Shipment Delivery

We use the decomposition solution algorithm outlined in Chapter 2 to solve the ESSNDP. We can outline the solution approach in the context of the ESSNDP as follows:

0. Initial Input: Input all commodity demand data, locations, capacities, costs and initialize the set of selected routes to a null-set. Go to Step 1.

1. Route Generation: Solve the cutset model $ESSNDP - Approx$ with the input set of commodities and generate "promising" routes.

2. Shipment Flow Generation: Solve LEM and arrive at a set of commodity flows over the service network generated in the *Route Generation* step.

3. Variable Fixing : Augment the set of selected routes by fixing some of the promising routes generated in the *Route Generation* step. We use the following rule to fix the routes:

If any pickup (delivery) route generated in the *Route Generation* step carries all the flow out of (into) the locations it visits, fix this route and add it to the set of selected routes.

4. Feasibility Check: Verify using LEM if the service network formed by the current set of selected routes has enough capacity to carry all demand. If yes, STOP; else go to the *Problem Size Reduction* step.

5. Problem Size Reduction: Eliminate commodities that can be moved on the service network formed by the set of selected routes. If the service design problem for the remaining commodities is small enough to solve exactly with available hardware, solve using the service network design model. Otherwise, go to the *Route Generation* step with the remaining set of commodities as input.

5.5 Computational Results

5.5.1 Data Description

We use three different data sets provided by the express shipment delivery company to test our solution procedure (Table 5.1). DS3 represents the company's entire operation while DS1 and DS2 represent portions of their operation. We use DS1 and DS2 primarily for computational testing and for gaining insight about the larger DS3 problem. For DS3, we have eight fleet types, including one type for ground vehicles and eight sorting hubs including one *all ground* hub, meaning the hub can service only ground vehicles.

We utilize the following characteristic of the company's cost structure to reduce the size of the data set:

The cost of transporting shipments using ground vehicles is negligible when compared to the cost of transport using aircraft. Also, the number of ground vehicles available is very large (on the order of hundreds of thousands) when compared to the number of available aircraft (less than 200). So if any shipment can be transported from its origin to its destination through the *all ground* hub, using only ground transport without violating any of the LOS requirements, we will do so and fix these vehicle and shipment routes *a priori*. Also, we will ground feed the demand originating at an *all ground* location to the gateway nearest the origin and similarly ground feed the demand destined for an *all ground* location to the gateway nearest the destination. This results in reductions in the size of input data for our decomposition solution approach. The size of the input data after this *ground consolidation* is given in Table 5.2.

5.5.2 Results

We implemented our decomposition algorithm on a SGI Power Challenge workstation, using CPLEX 4.0 [26] with 2 processors and 256 MBs RAM. Recall that the number of cutset inequalities in the route generation model are exponentially many (see Chapter

	DS1	DS2	DS3
Number of Locations	31	91	258
Number of Hubs	4	7	7
Number of O-D Commodities	863	6157	54563
Number of Fleet Types	4	6	8

Table 5.1: Data Description (before ground consolidation)

	DS1	DS2	DS3
Number of Locations	24	50	84
Number of Hubs	3	5	7
Number of O-D Commodities	552	2070	6606
Number of Fleet Types	4	6	8

Table 5.2: Data Description (after ground consolidation)

3). We therefore reduce memory requirements for the route generation model by considering only cutsets with $|S| \leq 3$ and $|T| \leq 3$. (Even with $|S| \leq 3$ and $|T| \leq 3$, the number of possible inequalities exceeds 5 million.) Also, we will not solve LEM optimally for an integer solution. While applying Branch-and-Price, we perform column generation only at the root node of the Branch-and-Bound tree.

We evaluate the performance of our decomposition approach by comparing our solution cost for data sets DS1, DS2, and DS3 with the corresponding cost of the solutions generated by the planners of the express shipment delivery company. The costs we consider are ownership cost, cycle cost, and operating cost (maintenance cost plus fuel cost). Our results are summarized in Table 5.3. Our solution approach has resulted in a 16% reduction in total annual costs. This cost does not include the reduction in crew costs that results from using fewer aircraft.

The run times for DS1, DS2 and DS3 are about 5 minutes, 1 hour and 6 hours respectively.

Since DS3 represents the carrier's entire scale of operation for a future forecast, we examine its results in greater detail. In Table 5.4, we compare the Planners' solution

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with our solution, contrasting the number of legs flown by aircraft, distance traveled and aircraft utilization. The comparison of individual cost components, distance, number of legs, and the number of aircraft for each fleet type is given in Table 5.5. The figures corresponding to the Planners' solution are given in parentheses. From these results, we observe the following:

- When compared to the Planners' solution, our solution uses more legs. However, fewer aircraft are used resulting in a considerable reduction in aircraft ownership costs that more than offset the increase in the leg-based cycle costs. The reason we have fewer aircraft is that in our solution, aircraft do a lot of double stopping because the model has intelligently identified two-legged routes that satisfy LOS requirements.

- Our solution utilizes cheaper fleet types more effectively. That is, cheaper fleet types, especially type 4, fly more legs and a longer distance. In our solution, aircraft of type 4 fly 231 legs and 98,481 miles whereas, in the Planners' solution, they fly only 165 legs and 76,277 miles. Our solution, with more legs per aircraft on average, results in a higher utilization of aircraft capacity. The capacity utilization for our solution is about 66%, compared to 58% for the Planners' solution.

- In our solution, aircraft do not always trace back their pickup routes on the delivery side, unlike the Planners' solution. The possible reason for this is that the shipment demand originating at any gateway location need not be proportional to that destined for that location and sometimes it is more cost-effective to use different routes for pickup and delivery. Consider the following example in which there are two types of aircraft, the first with 200 units of capacity and the second with 100 units. Supposing we have two locations (1 and 2) with 50 units of demand to be sent from 1 to 2 and 130 units of demand to be sent from 2 to 1. If the aircraft trace back their pickup routes on the delivery side, in the best solution, the first aircraft would travel four legs: location 1 to location 2, location 2 to hub, hub to location 2, and location 2 to location 1.

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	DS1	DS2	DS3
ESSNDP	194,929	488,462	4,143,722
Planners'	267,027	603,039	4,971,022
Cost Savings	27%	19%	16.6%

Table 5.3: Results

	ESSNDP	Planners'
Number of Aircraft	109	140
Number of Legs	362	336
Number of Legs per Aircraft	3.32	2.40
Distance traveled by aircraft	152,198	188,448
Aircraft Capacity Utilization	66%	58%

Table 5.4: Analysis for DS3

The second aircraft would travel two legs: location 2 to hub and hub to location 2. However, our algorithm gives a solution where the first aircraft travels only three legs: location 1 to location 2, location 2 to hub and hub to location 1. For illustration, see Figure 5-1.

We highlight below an attractive feature of our solution:

The schedules generated by our models are flexible in that there is a lot of leeway in moving the aircraft arrival and departure times within time windows without violating other constraints. To illustrate this, we compare the arrival pattern of aircraft at the sorting hubs for our solution with that of the Planners'. We divide the duration of sorting at the hubs into three time slots and enumerate the number of aircraft arriving in each time slot. The results are shown in Table 5.6. The numbers corresponding to those of the Planners' solution are given in parentheses. It can be noticed that, all the aircraft could arrive in the first time slot in our solution. This feature makes our solution more robust since we have a lot of slack in our schedule. Our schedule may be able to absorb delays without disruption to the delivery route schedules.

Aircraft Type	Aircraft	Distance	Operating Cost	Legs	Cycle Cost	Ownership Cost
1	9 (19)	7,949 (17,877)	21,987 (49,447)	26 (47)	28,600 (51,700)	166,500 (351,500)
2	2 (7)	790 (4,454)	3,362 (18,953)	5 (18)	6,000 (21,600)	38,000 (133,000)
3	2 (3)	10,828 (11,942)	82,973 (91,510)	4 (6)	10,000 (15,000)	64,000 (96,000)
4	61 (61)	98,481 (76,277)	251,977 (195,165)	231 (165)	231,000 (165,000)	1,830,000 (1,830,000)
5	15 (15)	18,443 (30,656)	55,329 (91,968)	56 (32)	56,000 (32,000)	720,000 (720,000)
6	15 (18)	8,801 (23,048)	35,922 (94,073)	28 (34)	36,400 (44,200)	337,500 (405,000)
7	5 (17)	6,866 (24,194)	25,222 (88,876)	12 (34)	18,000 (51,000)	125,000 (425,000)
Total	109 (140)	152,198 (188,448)	476,722 (630,022)	362 (336)	386,000 (380,500)	3,281,000 (3,960,500)

Table 5.5: Cost Distribution for DS3

Hub	Time Slot 1	Time Slot 2	Time Slot 3
1	7 (9)	0 (0)	0 (0)
2	8 (11)	0 (2)	0 (0)
3	7 (9)	0 (0)	0 (0)
4	10 (13)	0 (7)	0 (2)
5	12 (19)	0 (0)	0 (0)
6	63 (58)	0 (11)	0 (0)

Table 5.6: Aircraft Arrival Pattern at the Hubs for DS3
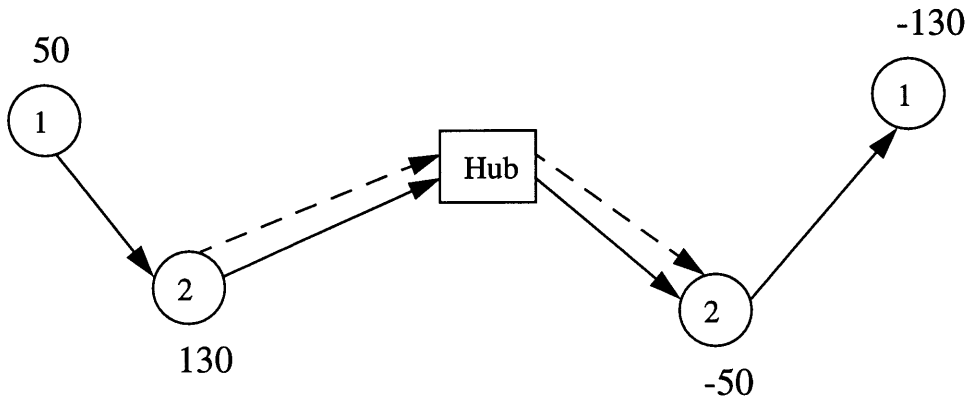
Best Solution with Mirroring Pickup and Delivery Routes

Model Solution

Aircraft with 200 units of capacity

Aircraft with 100 units of capacity
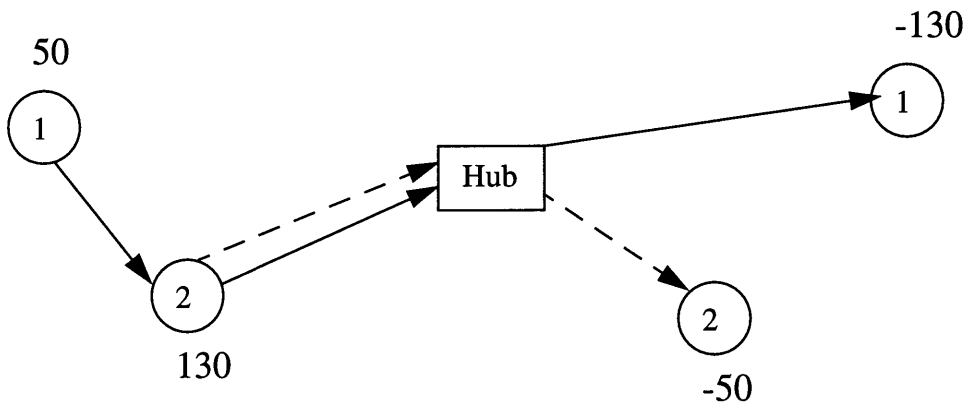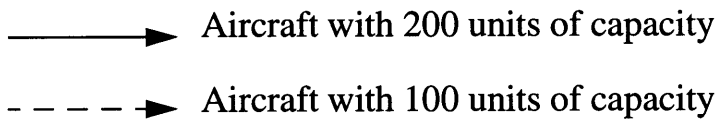
Figure 5-1: Illustration for Disproportionate Demand Distribution

5.5.3 Scenario Analysis

One more advantage of our approach is that we can relatively quickly examine a number of scenarios and perform "what-if" analyses. We use our models and decomposition solution technique to evaluate different scenarios that are of strategic importance to the express shipment delivery company. The key strategic decisions are the ones concerning the number of hubs and fleet composition. We consider six relevant scenarios for DS3 corresponding to the company's entire scale of operation for a future forecast and compare each of our solutions to the Planners' solution. The overall results are juxtaposed in Table 5.7. The first five scenarios are different combinations of the number of hubs and the fleet composition. For the sixth scenario we consider a different operating model and present its evaluation.

Baseline Scenario

The baseline scenario with respect to which we evaluate our new scenarios corresponds to the company's operating plan with seven aircraft types and seven air hubs. The eighth fleet type in the data description corresponds to ground vehicles and the eighth hub is an *all ground* hub.

Scenario 1: Single Hub Operation Model with All Seven Aircraft Types

We consider a single hub model of operation where consolidation of shipments serviced by aircraft occurs *only* at the company's main sorting hub, i.e., hub 6. We will consider the entire spectrum of seven aircraft types and impose the same fleet size restrictions as the baseline scenario. The results are given in Table 5.8. It can be seen that all available aircraft of types 4 and 5 are used in our solution and no aircraft of types 1, 2 and 7 are used, demonstrating the cost-effectiveness of aircraft types 4 and 5. The resulting cost savings for this scenario are about 30%.

Scenario 2: Multiple Hub Operation Model with Aircraft Types 4 and 5

The second scenario we consider differs from the baseline scenario in terms of the number of aircraft types. Instead of using all seven aircraft types, we investigate the possibility of using only two aircraft types. Narrowing the range of aircraft types may ensure more uniformity in crew compatibility requirements and maintenance requirements. However, such a strategy is implementable only if all the gateway locations have operating characteristics, such as, runway length, that are compatible with the chosen aircraft types. Our results for the baseline scenario and the preceeding scenario 1 indicate that all available aircraft of types 4 and 5 are used in our solutions. Aircraft types 4 and 5 are cost-effective in general (based on the cost structure and the company's past experience) and they form the backbone of the company's existing operations. Also, they represent the state-of-the-art in terms of aviation technology. These factors motivate us to choose aircraft types 4 and 5 for our current analysis. While solving the *ESSNDP-Approx* model in the *Route Generation Step*, we do not impose fleet size constraints (5.12) because one of our objectives is to determine future fleet composition. The results corresponding to this scenario are given in Table 5.9. Since it is not possible to satisfy the LOS requirements for the demand for some locations with only aircraft types 4 and 5, we allow our model to use a minimal number of higher speed aircraft of type 3. The results indicate savings beyond the base case of at least 7% for this multiple hub model of operation with primarily two aircraft types.

Scenario 3: Single Hub Operation Model with Aircraft Type 4

As the third scenario, we consider a single hub model of operation and evaluate the impact of homogenizing the aircraft mix by allowing only aircraft of type 4 to be used. As before, we allow a minimal use of aircraft of type 3. The cost distribution is given in Table 5.10. The resulting cost savings are about 10%.

Scenario 4: Single Hub Operation Model with Aircraft Type 5

The fourth scenario is similar to the third except that we allow the use of aircraft of type 5 instead of type 4. The cost distribution for this scenario is given in Table 5.11. The resulting cost savings are about 20%. The savings resulting from the use of only type 5 aircraft are about twice as much as those resulting from the use of only type 4 aircraft. A possible explanation based on the fact that aircraft of type 5 have a higher capacity than those of type 4, is that there are significant economies of scale resulting from the use of aircraft of type 5. The total distance traveled, the total number of legs, and the total number of aircraft are all less when compared to Scenario 3 resulting in an overall reduction in costs. This is because the total capacity of aircraft in our solution is 1,012,516 units which is less when compared to 1,041,404 units of capacity for Scenario 3 implying a lesser degree of unused capacity in the system. Also, the average number of legs per aircraft is higher resulting in an effective use of two leg routes. For type 5 aircraft, it is possible to have more legs per aircraft and yet satisfy LOS requirements because, they have higher speed when compared to type 4 aircraft.

Scenario 5: Single Hub Operation Model with Aircraft Types 4 and 5

We consider a single hub operating model as in the two preceeding scenarios. However, instead of homogenizing the aircraft mix, we allow the use of two aircraft types (types 4 and 5). The results for this strategy are shown in Table 5.12. We can see that for the single hub *modus operandi*, the strategy allowing the use of both type 4 and type 5 aircraft is more economical. The reason for this phenomenon is that the distribution of demand between locations is not uniform and hence two different aircraft capacities are more effectively used to accomodate these differences in demand. The total distance traveled, the total number of legs, and the total number of aircraft are all less when compared to Scenario 3 but they are more when compared to Scenario 4. The average number of legs per aircraft is greater than that for Scenario 3 but less than that for Scenario 4. This is because the average aircraft

speed is less than that for Scenario 4 and it is not possible for aircraft to satisfy all LOS requirements while using as many two-legged routes as Scenario 4. The reduction in ownership cost is much greater than the increase in operating cost and cycle cost when compared to Scenario 4, resulting in greater cost savings. Although the average number of legs per aircraft is more (when compared to Scenario 4), cost savings are possible because the total aircraft capacity is 886,500 units which is less than 1,012,516 units of capacity for Scenario 4. Also, the range of feasible solutions for scenarios 3 and 4 is contained within the range of feasible solutions for scenario 5 and hence the solution for scenario 5 cannot be worse than the solution for either of the scenarios 3 and 4.

Scenario 6: Regional Hub Operation Model with Aircraft Types 4 and 5

We consider a new model of operation with aircraft types 4 and 5. At the core, we essentially use a single hub operating model. However, rather than having no demand consolidation at the regional hubs, we consider a different configuration of the fleet network and allow the use of satellite sorting facilities at the regional hubs. We allow consolidation to occur in two degrees: *primary consolidation* occurs when the shipments are sorted at the company's main hub, i.e., hub 6 and *secondary consolidation* occurs whenever shipments are sorted at the regional hubs, i.e., hubs 1, 2, 3, 4 and 5.

We design a fleet network in which the allowable routes for aircraft are the only the ones connecting the regional hubs and the allowable routes for ground vehicles are the ones connecting gateway locations and the regional hubs. If the demand for any O-D commodity can be serviced from its origin to its destination on such a fleet network while satisfying all LOS requirements, we will fix the shipment routes as such. All such shipments bypass the main hub and are subject to secondary consolidation at the satellite sorting facilities at the regional hubs. A typical origin-to-destination shipment route will use three legs: one ground leg from the origin to a regional hub on the pickup side, one aircraft leg from the regional hub on the pickup side to a regional hub on the delivery side and one ground leg from the regional hub on the delivery side to the destination. Under such a setting, shipments are sorted twice,

once at each regional hub.

We first heuristically design the fleet network described above and fix some of the shipment flows. Then, we use our decomposition approach to design the network for the shipment demand remaining to be serviced. The cost figures for this new *modus operandi* are given in Table 5.13. We see that the cost savings are about 45% more than the best single hub model as in scenario 5.

Summary

We summarize below the results of our scenario analyses:

- In general, a single hub model of operation is more cost-effective than a multiple hub model because maximum possible degree of demand consolidation is attained. The savings from using a single hub model of operation are enhanced if a new model consisting of main hub consolidation and regional hub consolidation is adopted.

- Aircraft types 4 and 5 are indeed cost-effective. This inference follows from the fact that all available aircraft of these two types are used by our solutions in both the baseline scenario corresponding to a multiple hub model and the second scenario corresponding to a single hub model.

- For the single hub model, using a heterogeneous aircraft mix with two types (with different capacities) is less expensive than using a homogeneous aircraft mix because the service network can be more effectively tailored to account for non-uniform distribution of demand between locations.

Attribute	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6
Cost	3,480,493	4,603,559	4,444,387	3,939,254	3,641,495	3,037,893
Cost Savings	30%	7.4%	10.6%	20.8%	26.7%	38.9%
Number of Aircraft	83	114	116	69	82	71
Number of Legs	312	394	428	261	306	247
Legs per Aircraft	3.46	3.76	3.69	3.78	3.73	3.48
Distance	138,999	166,014	184,129	113,925	133,362	115,865

Table 5.7: Results of Scenario Analysis

Aircraft Type	Aircraft	Distance	Operating Cost	Legs	Cycle Cost	Ownership Cost
3	2	10,828	82,973	4	10,000	64,000
4	61	96,674	247,353	239	239,000	1,830,000
5	15	23,661	70,983	60	60,000	720,000
6	5	7,836	31,984	9	11,700	112,500
Total	83	138,999	433,293	312	320,700	2,726,500

TOTAL COST	3,480,493

Table 5.8: Cost Distribution for Scenario 1

Aircraft Type	Aircraft	Distance	Operating Cost	Legs	Cycle Cost	Ownership Cost
3	2	10,828	82,973	4	10,000	64,000
4	96	128,809	329,575	333	333,000	2,880,000
5	16	26,377	79,011	57	57,000	768,000
Total	114	166,014	491,559	394	400,000	3,712,000

TOTAL COST	4,603,559

Table 5.9: Cost Distribution for Scenario 2

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Aircraft Type	Aircraft	Distance	Operating Cost	Legs	Cycle Cost	Ownership Cost
3	2	10,828	82,973	4	10,000	64,000
4	114	173,301	443,414	424	424,000	3,420,000
Total	116	184,129	526,387	428	434,000	3,484,000

TOTAL COST	4,444,387

Table 5.10: Cost Distribution for Scenario 3

Aircraft Type	Aircraft	Distance	Operating Cost	Legs	Cycle Cost	Ownership Cost
3	2	10,828	82,973	4	10,000	64,000
5	67	103,097	309,281	257	257,000	3,216,000
Total	69	113,925	392,254	261	267,000	3,280,000

TOTAL COST	3,939,254

Table 5.11: Cost Distribution for Scenario 4

Aircraft Type	Aircraft	Distance	Operating Cost	Legs	Cycle Cost	Ownership Cost
3	2	10,828	82,973	4	10,000	64,000
4	55	79,481	203,363	206	206,000	1,650,000
5	25	43,053	129,159	96	96,000	1,200,000
Total	82	133,362	415,495	306	312,000	2,914,000

TOTAL COST	3,641,495

Table 5.12: Cost Distribution for Scenario 5

Aircraft Type	Aircraft	Distance	Operating Cost	Legs	Cycle Cost	Ownership Cost
3	2	10,828	82,973	4	10,000	64,000
4	53	79,735	204,014	182	182,000	1,590,000
5	16	25,302	75,906	61	61,000	768,000
Total	71	115,865	362,893	247	253,000	2,422,000

TOTAL COST	3,037,893

Table 5.13: Cost Distribution for Scenario 6

Chapter 6

Closure

6.1 Conclusions

In this thesis, we present an iterative modeling framework and a decomposition solution algorithm for large scale transportation service network design problems with time windows. We provide a proof-of-concept of the efficacy of our solution approach by solving the service network design problem of a large carrier in the express shipment delivery industry.

6.2 Future Research Directions

We present below some directions for future research:

- The immediate task is to evaluate more thoroughly the solutions generated by our models and arrive at a better quantification of the benefits. For instance, in our express shipment delivery application we did not measure the savings in crew costs resulting from our solution.

- The models we have developed are deterministic in nature and assume a constant market demand. However, the demand varies cyclically with the day of the week as well as stochastically. There is a need to test the robustness of our solution with respect to changing demand patterns. The usage of our models

and algorithms within a stochastic optimization framework should be explored to consider variations in demand.

- We need to develop efficient branching strategies for use within a Branch-and-Price-and-Cut solution framework for the Route Generation problem.

- Finding an efficient, or an approximate, algorithm for the cutset separation problem would lessen the computational burden of solving the Route Generation problem.

- While there exist approximation algorithms for telecommunication network design, approximation schemes that exploit the hub-and-spoke structure of transportation service network design problems need to be developed so that feasible integer solutions with theoretically known bounds can be achieved.

- From an applied perspective, we need to enhance our models to consider *service differentiation*. For the express shipment delivery application, we need to plan for different levels of service such as the Next Day Service, Second Day Service, Deferred Service and so on.

Appendix A

Notations

A.1 SETS

$N(\ni i)$: the set of all nodes in the service network

$A(\ni (i,j))$: the set of all arcs in the service network

$G = (N, A)$: the service network with set of nodes N and set of arcs A

$K(\ni k)$: the set of all O-D commodities

$F(\ni f)$: the set all asset types available

$R^f(\ni r)$: the set of all design routes for fleet type f

$R_P^f(\ni r)$:the set of all design pickup routes for fleet type f

$R_D^f(\ni r)$:the set of all design delivery routes for fleet type f

$P^k(\ni p)$: the set of all feasible paths from origin $O(k)$ to destination $D(k)$ for eack $k \in K$

$O(\ni o)$: set of all origin locations

$D(\ni d)$: set of all destination locations

$Q_o(\ni q)$: the set of all trees at origin o, for all $o \in O$

K_o : the set of all O-D commodities with origin o

K_d : the set of all O-D commodities with destination d

O_d : the set of all origins o such that there exists a commodity with $O(k) = o$ and $D(k) = d$, for each $d \in D$

D_o : the set of all destinations d such that there exists a commodity with $O(k) = o$

and $D(k) = d$, for each $o \in O$

p_q^k : the unique path from $O(k)$ to $D(k)$ in tree q

A.2 PARAMETERS

b^k : the demand for commodity for each $k \in K$

u_{ij} : the capacity of arc $(i, j) \in A$

$h_r^f = \sum_{(i,j) \in A} h_{ij}^f \alpha_{ij}^r$: the cost of design route r of type f

$c_p^k = \sum_{(i,j) \in A} c_{ij}^k \delta_{ij}^p$: the cost of flowing one unit of commodity k from $O(k)$ to $D(k)$ along path $p \in P^k$

$c_o^q = \sum_{k \in K_o} \sum_{(i,j) \in A} \Gamma_{ij}^{qk} b^k$: the cost of flowing the entire portion of all O-D commodities with $O(k) = o$ from $O(k)$ to $D(k)$ along path p_q^k in tree q.

A.3 INDICATOR VARIABLES

$$\Gamma_{ij}^{qk} = \begin{cases} 1 & \text{if arc } (i,j) \text{ belongs to the path } p_q^k \text{ from } O(k) \text{ to } D(k) \text{ in tree } q \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_{ij}^r = \begin{cases} 1 & \text{if design variable for } (i,j) \text{ is included in design route } r \\ 0 & \text{otherwise} \end{cases}$$

$$\beta_i^r = \begin{cases} 1 & \text{if } i \in N \text{ is the start node of design route } r \\ -1 & \text{if } i \in N \text{ is the end node of design route } r \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_{ij}^p = \begin{cases} 1 & \text{if arc } (i,j) \text{ belongs to path } p \\ 0 & \text{otherwise} \end{cases}$$

A.4 DECISION VARIABLES

y_r^f : number of assets of type f deployed on design route r

x_p^k : fraction of b^k on path $p \in P^k$ for all $k \in K$

w_o^q: fraction of b^k flown on the path p_q^k from $O(k) = o$ to $D(k)$ in tree q

$$y_o = \begin{cases} 1 & \text{if if all the O-D commodities with } O(k) = o \text{ are assigned to network paths} \\ 0 & \text{otherwise} \end{cases}$$

$$z_d = \begin{cases} 1 & \text{if if all the O-D commodities with } D(k) = d \text{ are assigned to network paths} \\ 0 & \text{otherwise} \end{cases}$$

Appendix B

Model Formulations

B.1 Baseline Network Design Problem Formulation

$$Minimize \quad \sum_{f \in F} \sum_{(i,j) \in A} h_{ij}^f y_{ij}^f + \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k b^k x_{ij}^k \tag{B.1}$$

such that,

$$\sum_{k \in K} b^k x_{ij}^k \leq \sum_{f \in F} u^f y_{ij}^f \quad \forall (i,j) \in A \tag{B.2}$$

$$\sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{ji}^k = \begin{cases} 1 & \text{if } i = O(k) \\ -1 & \text{if } i = D(k) \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in N, \ \forall k \in K \tag{B.3}$$

$$x_{ij}^k \geq 0, \quad \forall k \in K, \ \forall (i,j) \in A \tag{B.4}$$

$$y_{ij}^f \geq 0 \ and \ integer, \quad \forall (i,j) \in A, \ \forall f \in F \tag{B.5}$$

B.2 Service Network Design Problem Formulations

B.2.1 Node-Arc Formulation

$$Minimize \quad \sum_{f \in F} \sum_{(i,j) \in A} h_{ij}^f y_{ij}^f + \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k b^k x_{ij}^k \qquad (B.6)$$

such that,

$$\sum_{k \in K} b^k x_{ij}^k \leq \sum_{f \in F} u^f y_{ij}^f \quad \forall (i,j) \in A \qquad (B.7)$$

$$\sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{ji}^k = \begin{cases} 1 & \text{if } i = O(k) \\ -1 & \text{if } i = D(k) \quad \forall i \in N, \ \forall k \in K \\ 0 & \text{otherwise} \end{cases} \qquad (B.8)$$

$$\sum_{j \in N} y_{ij}^f - \sum_{j \in N} y_{ji}^f = 0 \quad \forall i \in N, \ \forall f \in F \qquad (B.9)$$

$$x_{ij}^k \geq 0, \quad \forall k \in K, \ \forall (i,j) \in A \qquad (B.10)$$

$$y_{ij}^f \geq 0 \ and \ integer, \quad \forall (i,j) \in A, \ \forall f \in F \qquad (B.11)$$

B.2.2 Path Formulation

$$Minimize \quad \sum_{r \in R^f} h_r^f y_r^f + \sum_{k \in K} \sum_{p \in P^k} (c_p^k b^k) x_p^k \qquad (B.12)$$

such that,

$$\sum_{k \in K} \sum_{p \in P^k} (\delta_{ij}^p b^k) x_p^k \leq \sum_{f \in F} \sum_{r \in R^f} u^f y_r^f \alpha_{ij}^r \quad, \forall (i,j) \in A \qquad (B.13)$$

$$\sum_{p \in P^k} x_p^k = 1 \quad, \forall k \in K \qquad (B.14)$$

$$\sum_{r \in R^f} \beta_i^r y_r^f = 0 \quad, \forall i \in N, \ \forall f \in F \qquad (B.15)$$

$$x_p^k \geq 0 \quad \forall p \in P^k, \ \forall k \in K \qquad (B.16)$$

$$y_r^f \geq 0 \ and \ integer, \ \forall r \in R^f, \ \forall f \in F \qquad (B.17)$$

B.2.3 Tree Formulation

$$Minimize \quad \sum_{r \in R^f} h_r^f y_r^f + \sum_{o \in O} \sum_{q \in Q_o} c_o^q w_o^q \tag{B.18}$$

such that,

$$\sum_{o \in O} \{ \sum_{q \in Q_o} (\sum_{k \in K_o} \Gamma_{ij}^{qk} b^k) w_o^q \} \leq \sum_{f \in F} \sum_{r \in R^f} u^f y_r^f \alpha_{ij}^r \quad , \quad \forall (i,j) \in A \tag{B.19}$$

$$\sum_{q \in Q_o} w_o^q = 1, \quad \forall o \in O \tag{B.20}$$

$$\sum_{r \in R^f} \beta_i^r y_r^f = 0 \quad , \quad \forall i \in N, \ \forall f \in F \tag{B.21}$$

$$w_o^q \geq 0 \ , \ \forall q \in Q_o, \ \forall o \in O \tag{B.22}$$

$$y_r^f \geq 0 \ and \ integer, \ \forall r \in R^f, \ \forall f \in F \tag{B.23}$$

B.3 The Approximate Service Network Design Cutset Model

B.3.1 Node-Arc Formulation

$$Minimize \quad \sum_{f \in F} \sum_{(i,j) \in A} h_{ij}^f y_{ij}^f \tag{B.24}$$

such that

$$\sum_{f \in f} u_f Y_{S,T}^f \geq D_{S,T} \quad , \quad \forall \, O - D \; cutsets \; \{S, T\} \tag{B.25}$$

$$\sum_{j \in N} y_{ij}^f - \sum_{j \in N} y_{ji}^f = 0 \quad \forall i \in N, \; \forall f \in F \tag{B.26}$$

$$y_{ij}^f \geq 0 \; and \; integer, \quad \forall (i,j) \in A, \; \forall f \in F \tag{B.27}$$

B.3.2 Route Formulation

$$Minimize \quad \sum_{r \in R^f} h_r^f y_r^f \tag{B.28}$$

such that

$$\sum_{f \in f} u_f Y_{S,T}^f \geq D_{S,T} \quad , \quad \forall \, O - D \; cutsets \; \{S, T\} \tag{B.29}$$

$$\sum_{r \in R^f} \beta_i^r y_r^f = 0 \quad , \quad \forall i \in N, \; \forall f \in F \tag{B.30}$$

$$y_r^f \geq 0 \; and \; integer, \; \forall r \in R^f, \; \forall f \in F \tag{B.31}$$

B.4   Shipment Flow Models

B.4.1   Baseline Multicommodity Flow Problem Formulation

$$Minimize \quad \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^k b^k x_{ij}^k \tag{B.32}$$

such that

$$\sum_{k \in K} b^k x_{ij}^k \leq u_{ij} \quad \forall (i,j) \in A \tag{B.33}$$

$$\sum_{j \in N} x_{ij}^k - \sum_{j \in N} x_{ji}^k = \begin{cases} 1 & \text{if } i = O(k) \\ -1 & \text{if } i = D(k) \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in N, \ \forall k \in K \tag{B.34}$$

$$x_{ij}^k \geq 0, \quad \forall k \in K, \ \forall (i,j) \in A \tag{B.35}$$

B.4.2   Location Elimination Model Path Formulation

$$Maximize \quad \sum_{o \in O} y_o + \sum_{d \in D} z_d \tag{B.36}$$

such that,

$$\sum_{k \in K_o} \sum_{p \in P^k} x_p^k - y_o |D_o| \geq 0, \quad \forall o \in O \tag{B.37}$$

$$\sum_{k \in K_d} \sum_{p \in P^k} x_p^k - z_d |O_d| \geq 0, \quad \forall d \in D \tag{B.38}$$

$$\sum_{p \in P^k} x_p^k \leq 1, \quad \forall k \in K \tag{B.39}$$

$$\sum_{k \in K} \sum_{p \in P^k} x_p^k (\delta_{ij}^p b^k) \leq u_{ij}, \quad \forall (i,j) \in A \tag{B.40}$$

$$x_p^k \geq 0 \tag{B.41}$$

$$y_o, z_d \in \{0, 1\} \tag{B.42}$$

B.4.3 Location Elimination Model Tree Formulation

$$Maximize \quad \sum_{o \in O} y_o + \sum_{d \in D} z_d \qquad (B.43)$$

such that,

$$\sum_{q \in Q_o} w_o^q - y_o \geq 0, \quad \forall o \in O \qquad (B.44)$$

$$\sum_{o \in O_d} \sum_{q \in Q_o} w_o^q - z_d |O_d| \geq 0, \quad \forall d \in D \qquad (B.45)$$

$$\sum_{q \in Q_o} w_o^q \leq 1, \quad \forall o \in O \qquad (B.46)$$

$$\sum_{o \in O} \{ \sum_{q \in Q_o} (\sum_{k \in K_o} \Gamma_{ij}^{qk} b^k) w_o^q \} \leq u_{ij}, \quad \forall (i,j) \in A \qquad (B.47)$$

$$w_o^q \geq 0 \, , \forall q \in Q_o, \, \forall o \in O \qquad (B.48)$$

$$y_o, z_d \in \{0, 1\} \qquad (B.49)$$

B.5 Routing Model for Express Shipment Delivery

$$Minimize \sum_{r \in R_P^f \cup R_D^f} h_r^f y_r^f \tag{B.50}$$

such that,

$$\sum_{r \in R_P^f \cup R_D^f} \beta_i^r y_r^f = 0 \ , \quad \forall i \in N, \quad \forall f \in F \tag{B.51}$$

$$\sum_{r \in R_P^f} y_r^f \leq n^f, \quad \forall f \in F \tag{B.52}$$

$$\sum_{f \in F} \sum_{r \in R_P^f \cap L_i^t} y_r^f \leq \sum_{m=t}^{T} a_i^m \ , \quad \forall i \in H, \ \forall t \in \{1, 2, ..T\} \tag{B.53}$$

$$\sum_{f \in F} \sum_{r \in R_P^f \cap V_P} y_r^f \geq 1 \ , \quad \forall i \in N \tag{B.54}$$

$$\sum_{f \in F} \sum_{r \in R_D^f \cap V_D} y_r^f \geq 1 \ , \quad \forall i \in N \tag{B.55}$$

$$\sum_{f \in f} u_f Y_{S,T}^f \geq D_{S,T} \ , \quad \forall \ O - D \ cutsets \ \{S, T\} \tag{B.56}$$

$$y_r^f \geq 0 \ and \ integer \ , \quad \forall r \in R_P^f \cup R_D^f, \ \forall f \in F. \tag{B.57}$$

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