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# Universality of the lateral distribution of energy deposit in extensive air showers

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Using the CORSIKA air shower simulation package, the spatial distribution of ionization energy deposited by high energy showers in the atmosphere is calculated. The fraction of total energy deposit versus distance from the shower axis is derived. If the lateral distance is measured in Molière units the energy deposit is, to a good approximation, independent of primary energy, primary particle type and zenith angle. It depends only on the shower age and can well be parametrized as a function of an effective shower age parameter only.

## 1. Introduction

One of the methods of extensive air shower (EAS) detection is recording fluorescence light emitted by nitrogen molecules in the air along the shower path. For very high energies of the primary particle, enough fluorescence light is produced so that the shower can be recorded from a distance of many kilometers by an appropriate optical detector system [1, 2]. As the amount of fluorescence light is proportional to the ionization energy deposit in air, it provides a calorimetric measure of the primary energy.

Given an optical imaging system for recording the light emitted by the shower, the size of shower image is defined as the minimum angular diameter $2\alpha$ of the image spot containing a certain fraction $F(\alpha)$ of the total light recorded by the detector. The intensity distribution of light in this image, $f_\gamma(\alpha)$, is proportional to the lateral distribution of the emitted fluorescence light around the shower axis [2, 3] . Therefore the fraction of light recorded $F(\alpha)$ can be obtained from the corresponding fraction of light emitted around the shower axis

$$F(\alpha) \equiv \int_0^\alpha f_\gamma(\alpha^{'})2\pi\alpha^{'}d\alpha^{'} \sim \int_0^r f(r^{'})2\pi r^{'}dr^{'} \equiv F(r), \tag{1}$$

where $f(r)$ is the (normalized) lateral distribution of fluorescence light emitted. The main task is therefore to derive $f(r)$ and calculate the corresponding fraction of light $F(r)$.

## 2. Method

In this paper, we study the lateral distribution of energy deposit density in air showers, as it is directly proportional to the number of expected fluorescence photons. The amount of light can be obtained by using the energy deposit $dE(X)/dX$ as a function of atmospheric slant depth interval $dX$ together with a density- and temperature-dependent fluorescence yield $Y(\rho,T)$ [4]. In this approximation the distribution of photons emitted around the shower axis is proportional to the lateral distribution of energy deposit, $f(r) \sim \frac{dE(X,r)}{dX_v}$ at a given stage of shower evolution, where $dX_v = dX\cos(\theta)$ is the vertical depth interval and $\theta$ is the shower zenith angle. The distribution of energy deposit $dE(X,r)/dX_v$ is calculated with the CORSIKA shower simulation package [5, 6] as the sum of the energy released by charged particles with energies above the simulation threshold and the releasable energy fraction of particles discarded due to the simulation energy threshold [6].

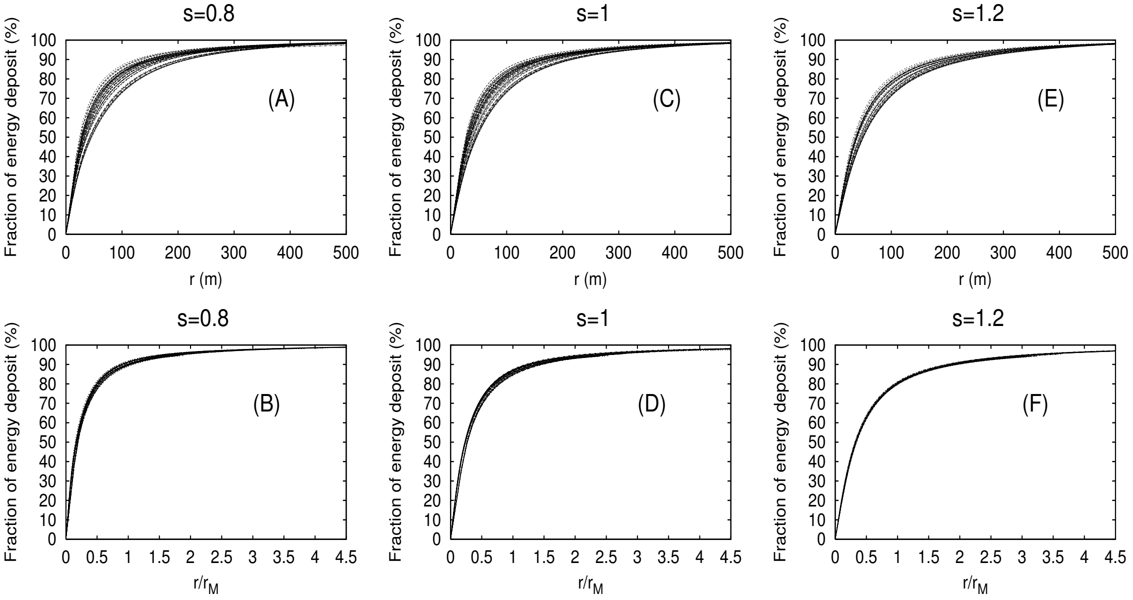

**Figure 1.** *Integral of the energy deposit density for different age parameters and for 10 individual proton and 5 individual iron showers with different zenith angles ($\theta = 0°, 45°, 60°$) and energy 10 EeV; (A), (C), (E) Integral of energy deposit density versus distance from shower axis; (B), (D), (F) Integral of energy deposit density versus distance measured in Molière units.*

Using CORSIKA, a two-dimensional energy deposit distribution around the shower axis is stored in histograms during the simulation process for 20 different vertical atmospheric depths. Each of the 20 horizontal layers has a thickness of $\Delta X_v = 1$ g/cm$^2$ and corresponds to a certain atmospheric depth: the first one to $X_1 = 120$ g/cm$^2$ and the last one to $X_{20} = 870$ g/cm$^2$. Linear interpolation between the observation levels is performed in order to get the lateral distribution at a given vertical depth $X_n$ located between two CORSIKA observation levels $X_k$ and $X_{k+1}$. The fraction of energy deposit $F(r)$ is calculated by numerically integrating the histograms up to the lateral distance $r$.

## 3. Discussion

In the following we study the dependence of the lateral distribution of energy deposit density on energy, primary particle and zenith angle. A natural transverse scale length in air showers, which proves to be useful for obtaining a universal parameterization of the lateral distribution, is given by the Molière radius [7]

$$r_M \equiv E_s \frac{X_l}{\epsilon_0}, \tag{2}$$

where $E_s \simeq 21$ MeV is the scale energy, $\epsilon_0 = 81$ MeV the critical energy and $X_l = 37$ g/cm$^2$ the radiation length in air. The local Molière radius in units of length at a given atmospheric depth (at altitude $h$) can be obtained by dividing Eq. (2) by the air density, $\rho(h)$, and is approximately given by $r_M = 9.6\mathrm{gcm}^{-2}/\rho(h)$.

It is also well known that the distribution of particles in a shower at a given depth depends on the history of the changes of $r_M$ along the shower path rather than on the local $r_M$ value at this depth. To take this into

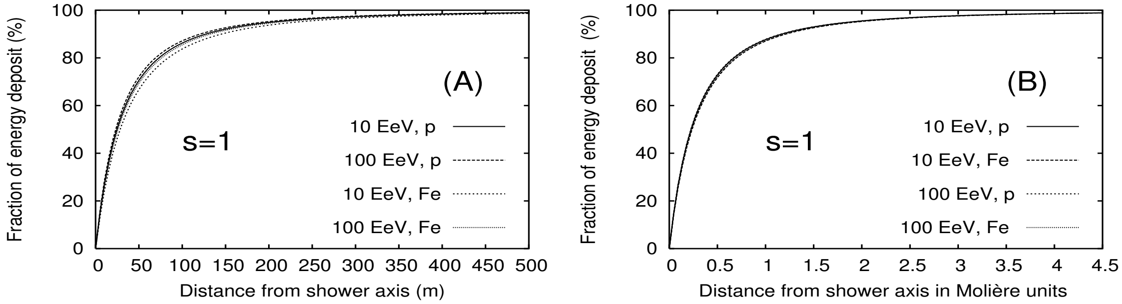

**Figure 2.** *(A) Integral of energy deposit density versus distance from shower axis; (B) The integral profiles versus distance measured in Molière units; The profiles are shown for vertical showers (at s=1) with different primary particle type and energy.*

account, the $r_M$ value is calculated at 2 radiation lengths above the considered depth [7]. Using the value of the Molière radius calculated based on the atmospheric profile (the US Standard Atmosphere) for vertical depth $X_n - 2X_l \cos(\theta)$, the fraction of energy deposit density $F(r^*)$ versus the distance in Molière units $r^* = r/r_M$ is found. The knowledge of $F(r^*)$ gives a possibility to study the variation of the shape of energy deposit density due to properties of the atmosphere. The variation of the density of the atmosphere along the path of a shower affects the Molière radius and consequently also the radial particle distribution. To characterize the development stage of a shower, we use the shower age parameter $s \equiv 3X/(X + 2X_{max})$, where $X_{max}$ is the atmospheric depth of shower maximum extracted from simulated data[1]. A shower reaches its maximum at $s = 1$.

In Figure 1 we present the integral of the energy deposit density for different age parameters for 10 individual proton and 5 individual iron showers with different zenith angles ($\theta = 0°, 45°, 60°$) and energy 10 EeV. The shower-to-shower fluctuations are strongly reduced for a given age when we correct $F(r)$ profiles for the atmospheric effect, i.e. consider $F(r^*)$. Also, there are very little differences in the shape of $F(r^*)$ for showers with different zenith angles and primary particle type. The analysis of Figs. 1 and 2 leads to the following conclusion: *the lateral shape of the energy deposit density versus distance from shower axis measured in Molière units is independent of the primary energy, primary particle type and zenith angle. It depends, to a good approximation, only on the shower age.*

This means that it is possible to find a universal function which describes the shape of the energy deposit density as a function of shower age only. Following our earlier work [8] we will use the function

$$F(r^*) = 1 - (1 + a(s)r^*)^{-b(s)}, \tag{3}$$

where the parameters $a(s)$ and $b(s)$ are assumed to be functions of shower age. Fits of this functional form to the integral of energy deposit density were performed for the data from Figures 1B, D, F. The values of the parameters $a(s)$ and $b(s)$ for different shower ages are presented in Figure 3. The age dependence of $a(s)$ and $b(s)$ is well described by

$$a(s) = 5.151s^4 - 28.925s^3 + 60.056s^2 - 56.718s + 22.331, \tag{4}$$

$$b(s) = -1.039s^2 + 2.251s + 0.676. \tag{5}$$

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[1] $X_{max}$ was determined by fitting a Gaisser-Hillas type function to the CORSIKA longitudinal profile of energy deposit.

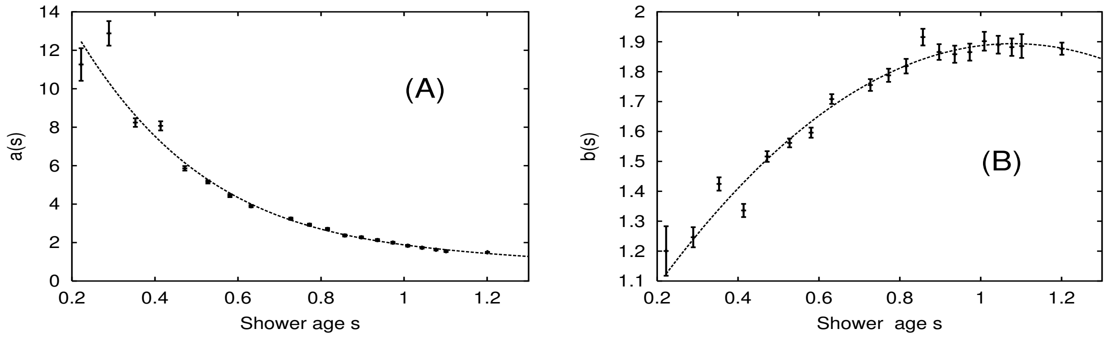

**Figure 3.** *Values of parameters $a(s)$ and $b(s)$ of Eqs. (4) and (5) obtained based on integral of CORSIKA energy deposit density for vertical showers at energy 10 EeV.*

Thus, Eqs. (3), (4) and (5) provide a model to describe the fraction of energy deposit within a specified distance from the shower axis for different energies, zenith angles and primary particles. Similarly, a dependence of the lateral distribution of particles on shower age was found in [9].

## 4. Conclusions

In this work, the distribution of light in the shower optical image is analyzed, based on the lateral distribution of energy deposited by the shower as derived from CORSIKA simulations. The lateral distribution of deposited energy is parameterized with a functional form inspired by the NKG distribution. The angular distribution of photons arriving simultaneously at the detector (i.e. the intensity distribution of light in the instantaneous image of the shower) is obtained. The shape of this distribution can be approximated by a universal function that depends on the shower age only. The results presented here are described in more detail in [10].

## 5. Acknowledgements

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