29th International Cosmic Ray Conference Pune (2005) 7, 187-190

A new parameterisation of the longitudinal shower development

M.Giller[a], A.Kacperczyk[b], J.Malinowski[a] and W.Tkaczyk[a]
(a) Department of Experimental Physics, University of Łódź, Poland
(b) Department of Theoretical Physics, University of Łódź, Poland
Presenter: M. Giller (mgiller@kfd2.fic.uni.lodz.pl), pol-giller-M-abs1-he14-oral

Using CORSIKA simulations of the highest energy extensive air showers we show that the total number of particles in a shower can be described as a function of age by two halves of a Gaussian function with two different widths, σ_1 and σ_2 fluctuating from one shower to another. Thus, a shower cascade curve can be fitted by 4 parameters: X_{max} (to determine $s(X)$), N_{max} , σ_1 and σ_2 . The both sigmas are quite well correlated with X_{max}, and with each other. This makes the choice of initial parameters for a fitting procedure (in shower reconstruction from fluorescence data) quite easy.

1. Introduction

We have shown in [1] (see also Nerling et al [2]) that the shower age parameter s, as defined by Hillas [3] (in analogy to that for a pure electromagnetic cascade):

$$s(X) = \frac{3X}{X + 2X_{max}} \tag{1}$$

where X is the slant depth (in g cm^{-2}) of the shower level in the atmosphere, is the only parameter determining the shape of the energy spectrum of electrons. As a consequence of this the electron angular distribution is a function of the shower age as well [5,6]. Also the lateral distribution of electrons (when the lateral distance is expressed in Moliere units) depends on the shower age only [4]. As the fluorescence light emission, caused by an electron in a path element of the atmosphere, is believed to be proportional to its energy deposit in this path, the light flux depends (although weakly) on the electron energy spectrum. In all the above mentioned distributions it was their shape that is s-dependent. The absolute numbers depend, of course, on the total number of particles N. In this paper we study how the total number of charged particles N behaves as a function of s.

2. Total number of particles *N*(*s*)

The total number of particles N(X), which are mainly electrons (in the largest showers, here in consideration, the fraction of muons is about 2-3%) has been usually described by the analytical (gamma) function of the slant depth X in the atmosphere, proposed by Gaisser and Hillas [7] with four (or six) free parameters. As the age parameter s describes so well the energy, angular and lateral distributions of electrons, we thought it worth to check how the total number of particles depends on it. It is not so that the shape of $N(s)$ dependence is unique; it does fluctuate from shower to shower. However, we have found that for any shower (primary proton or iron, E=10^{19}, 10^{20} eV) $N(s)$ can be described very well by two halves of Gaussian distributions:

$$N(s) = \begin{cases} N_{max} \exp\left(-\frac{(s-1)^2}{2\sigma_1^2} \right) & \text{for } s < 1 \\ N_{max} \exp\left(-\frac{(s-1)^2}{2\sigma_2^2} \right) & \text{for } s > 1 \end{cases} \tag{2}$$

Thus, the longitudinal development of any shower can be described again by 4 parameters: N_{max}, X_{max} (to determine $s(X)$, σ_1 and σ_2. A comparison of our new parameterization with that of the 4- and 6-parameter Gaisser-Hillas is shown in Figure1, where we have drawn the ratios of simulated $N(s)$ to the fitted values, for four more or less typical showers, each one for primary proton or iron, both with energies 10^{19} eV and 10^{20} eV. The differences between the fitted and simulated $N(s)$ are seen only there where the number of particles is very small. Our two-Gaussian fit seems to be a little better for primary iron nuclei than the 4-parameter Gaisser-Hillas, whereas for protons it is the opposite. Of course, the 6-parameter Gaisser-Hillas is the best.

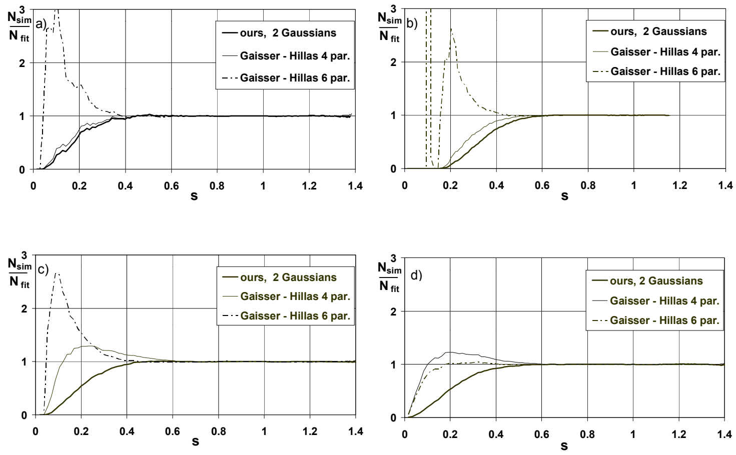

Figure 1. Ratio of simulated number of particles to that fitted (see the inset) for four typical showers: (a) primary proton, 10^{19} eV; (b) proton, 10^{20} eV; (c) Fe, 10^{19} eV; (d) Fe, 10^{20} eV.

The observed light fluxes fluctuate much more than the simulated number of particles, so that fitting a 6-parameter $N(X)$ is not always effective. One has to use as few parameters as possible, and the two-Gaussian form seems to be promising. Moreover, we studied the correlations between X_{max} and the both sigma's. The

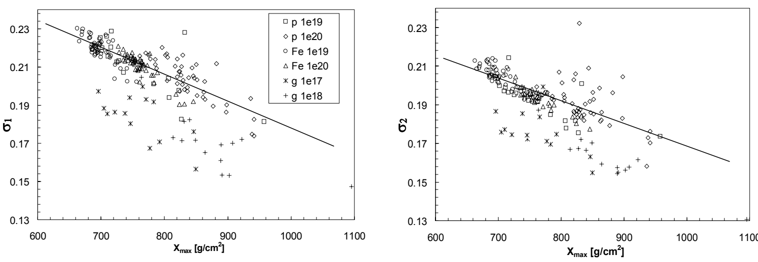

Figure 2. Correlation of σ_1 (left) and σ_2 (right) with X_{max}. g denotes the primary photon. The lines are best linear fits to hadronic showers only

both sigma's are negatively correlated with X_{max}. Figure2 shows values of the fitted σ_1 and σ_2 for several proton and iron initiated showers versus X_{max} . The deeper a shower develops the narrower is the dependence N(s), as the two sigma's are positively correlated with each other, see Figure3. In Figure2 and 3 there are also values (denoted as g) for pure electromagnetic cascades with lower energies (10^{17} and 10^{18} eV). These are the components of the higher energy showers constituted of many cascades. At first we thought that the scatter in the correlation plots might be produced mainly by fluctuations in the hadronic processes. However, as one can see from the result for the gamma initiated cascades this is not quite so, as the fluctuations in these pure electromagnetic cascades are not much smaller than those in the hadronic showers. The correlations between X_{max}, σ_1 and σ_2 can be used in the reconstruction procedure of showers from the fluorescence detectors measurements.

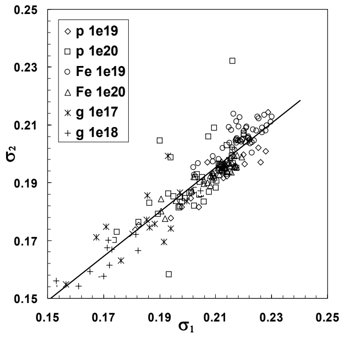

Figure 3. Correlation between σ_1 and σ_2 in individual showers.

3. Application to shower reconstruction

In a fluorescence detector the telescope camera consists of many pixels (PMT's), each having a small field of view and observing a small element of a shower track (440 pixels, with ~1.5° diameter in the Auger experiment). A distant shower is then seen as a line of 'fired' pixels on the camera, each detecting the light flux from its field of view. Usually, most of the light is fluorescence, with some admixture of the Cherenkov which, however, can not be neglected. The fluorescence light emitted by any small path element of a single electron is isotropic, so that the light emitted by all electrons in a shower track element will also be isotropic, independently of the angular distribution of electrons at this point. With the Cherenkov (Ch) light, however, the situation is different. As it is well known, it is emitted at very small angles with respect to the particle direction (<1°in the air). The main Ch problem is that this light is scattered by the atmosphere aside, so that it adds to the fluorescence light observed at large angles to the shower axis. To reconstruct a shower, i.e. to find $N(X)$, one has to be able to predict the amount of the Ch contribution for any track element of a shower. Thus, the number of photons Δn_i emitted towards the camera *i*-th pixel, seeing the shower track element ΔX_i at depth X_i, collecting light from the (small) solid angle $\Delta\Omega$ at an angle θ_i to the shower axis, consists of two components: the fluorescence and the Cherenkov light:

$$\Delta n_i = \Delta n_{i,fl} + \Delta n_{i,Ch} \quad with \quad \Delta n_{i,fl} = k\, N(X_i)\langle dE/dX\rangle \Delta\Omega(\theta_i)/4\pi \qquad (3)$$

where k is the proportionality constant between the number of fluorescence photons emitted per unit path (in g cm^{-2}) and the energy loss rate for ionization and $\langle dE/dX \rangle$, the mean energy deposit rate per unit shower track length, is taken for the age s_i of the shower level X_i. The second term in equation (3) consists again of two components: the direct and the scattered Ch light discussed in some detail in [5,6]. The procedure proposed here to find the shower cascade curve $N(X(s))$, having as data Δn_i (assuming that the atmosphere scattering properties are known one can deduce these values from the number of photons arriving to the individual pixels) and assuming that the shower geometry is known, i.e. the distance of its core to the detector and the zenith and azimuth angles (knowing atmosphere and shower geometry are separate experimental problems), is the following:

1. first guess the initial values of N_{max} and X_{max} for the minimizing procedure: X_{max} - as the depth of the *m*-th pixel with the maximal signal, and N_{max} calculated from (3), with $i=m$ and $<dE/dX>$ taken for $s=1$ (we neglect first Ch light),
2. having X_{max} the dependence $X(s)$ is determined from (1),
3. from the straight line $\sigma_1(X_{max})$ in Figure2 (fitted to the points without any weights): $\sigma_1 = 0.319 - 0.000141X_{max}$ we find the initial value of σ_1,
4. from the line in Figure3: $\sigma_2 = 0.0340 + 0.7682\ \sigma_1$ we find the initial value of σ_2,
5. and finally, having all the necessary initial parameters, one can calculate the expected number of photons, to be detected by the individual pixels, and compare them with the measured ones [5,6]. Using a minimising procedure the four best fitting parameters can be found.

In principle the reconstruction possibilities of the above procedure should not depend on the fraction of Ch admixture, as it can be very well predicted (its both components - direct and scattered), once the adopted shower age is correct. So, it can be used to the showers with a large Ch fraction, i.e. to their parts deep in the atmosphere and/or to those inclined by (relatively) small angles to the telescope line of sight. So, the Ch light, treated so far as a nuisance in the fluorescence experiments (to get rid of it an iteration procedures have been applied, working properly only if that fraction is small), can be an additional information, helping in finding shower cascade curves.

4. Conclusions

We have found that each shower curve $N(s)$ can be very well described by a 4-parameter analytical curve- two halves of the Gaussian distributions with different widths. Finding the correlations between the parameters (σ_1 with X_{max}, and σ_2 with σ_1) we are able to propose a procedure for reconstructing large showers observed by telescope detectors. Thanks to the similarity of the showers [5], both the fluorescence and the direct and scattered Cherenkov light fluxes can be accurately predicted for any adopted $N(X(s))$ and compared with experimental results in order to find the best $N(X)$.

5. Acknowledgment

This work has been supported by the grant of the Łódź University no. 696.

References

[1] Giller M et al., J.Phys. G: Nucl. Phys., 30, 97 (2004)
[2] Nerling F et al., Proc. 28th ICRC, Tsukuba (2003)
[3] Hillas A M, J. Phys G: Nucl. Phys. 8, 1475 (1983)
[4] Giller M, Stojek H, Wieczorek G, Int. Mod. J. Phys. in print (2005)
[5] Giller M, Kacperczyk A, Malinowski J, Tkaczyk W, Wieczorek G, J. Phys. G: Nucl. Part. Phys. 31, 947 (2005)
[6] Giller M and Wieczorek G, 29th ICRC, Pune (2005)
[7] Gaisser T K and Hillas A M, Proc. 15th ICRC, Plovdiv 8, 353 (1977)