Monte Carlo simulations and semianalytical parameterisations of the atmospheric muon flux controlled by muon charge ratio measurements performed with WILLI detector

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The atmospheric muon flux have been simulated using the CORSIKA code for two different geographical positions (Bucharest: 44 deg N, 26 deg E and Hiroshima:34 deg N, 132 deg E). The simulations have been done for different angles of incidence between 0 deg and 70 deg. The comparison between the simulations and the experiment have been done using the measurements of the muon charge ratio with the WILLI detector in Bucharest. The results of the Monte Carlo simulations of the muon flux for the geographical positions of Hiroshima and Bucharest are compared with the semi-analytical formulae of Judge and Nash, and of Gaisser for different angles of incidence between 0 deg and 70 deg and with experimental results of the Bess experiment (vertical incidence). Various sensitivities of the approach of Judge and Nash, in particular to variations of the pion and kaon production spectra have been studied.

1. Introduction

The muon belongs to the family of elementary particles known as leptons. Like the electron it may be positively or negatively charged and has a spin $\frac{1}{2}$. However its mass is about 100 MeV, more than two orders of magnitude larger than that of the electron, and about one order of magnitude less than of the proton. It is produced mainly by the decay of pions and kaons generated by high-energy collisions of cosmic rays with the atoms of the Earth atmosphere. Muons are unstable decaying to electrons and positrons and neutrinos (electron (ν_e) and muon (ν_{μ}) neutrinos) with a half - life of $\tau_{\mu} = 2.2\mu s$.

2. The air shower simulation program CORSIKA

The simulation tool CORSIKA has been originally designed for the four dimensional simulation of extensive air showers with primary energies around 10^{15} eV. The particle transport includes the particle ranges defined by the life time of the particle and its cross-section with air. The density profile of the atmosphere is handled as continuous function, thus not sampled in layers of constant density.

Ionization losses, multiple scattering, and the deflection in the local magnetic field are considered. The decay of particles is simulated in exact kinematics, and the muon polarization is taken into account.

In contrast to other air shower simulations tools, CORSIKA offers alternatively six different models for the description of the high energy hadronic interaction and three different models for the description of the low energy hadronic interaction. The threshold between the high and low energy models is set by default to $E_{Lab} = 80 \text{ GeV/n.}$

3. Calculation of atmospheric muon flux

The calculation of muon flux proceeds by a full 3D-simulation (CORSIKA). The simulations have been done using for the primary particle's spectrum the expression: $J_p(E) \sim E^{-2.78}$.

The differential particle flux

$$J_{\mu} = \frac{dN}{dt \cdot dA \cdot d\Omega \cdot dP} \qquad (cm^{-2} \cdot s^{-1} \cdot sr^{-1} \cdot (GeV/c)^{-1}) \tag{1}$$

resulting from the simulation was calculated by dividing the number of particles detected by the surface of the particle collection area (cm^2) , solid angle, momentum bin size, and equivalent sampling time of the CR flux.



Figure 1. Differential flux of the muons for $0^{\circ} \le \theta < 10^{\circ}$

4. Semi-analytical approaches

There are several empirical approximations describing the fluxes in by analytical expressions like power-law distributions (see P.Grieder[4]). Recent approaches by T.K. Gaisser[1] display explicitly the dependence on primary energy, but with complicated mathematical procedures and valid only for muon energies above 10 GeV. This holds also for the simplification given in Gaisser's Book:

$$\phi_{\mu} = \frac{0.14}{cm^2 \cdot s \cdot sr \cdot GeV} \cdot (E/GeV)^{-2.7} \left| \frac{1}{1 + \frac{E \cdot cos\theta}{110GeV}} + \frac{0.37}{1 + \frac{E \cdot cos\theta}{760GeV}} \right|$$
(2)

used for example by Unger[5]. In Fig.2 this formula is compared with the the results of the Monte Carlo simulations, displaying the disaggreement in particular at lower energies.

The approach by Judge and Nash[2] uses as input the production spectra of parent pions and kaons and calculates the flux resulting from pion and kaon decay by:

$$D_{\pi}(E_{\mu},\theta) = \frac{A_{\pi} \cdot W_{\mu} \cdot E_{\pi}^{-\gamma_{\pi}} \cdot H_{\pi}}{E_{\pi} \cdot \cos\theta + H_{\pi}}$$
(3)

$$D_k(E_\mu, \theta) = \frac{A_k \cdot W_\mu \cdot E_k^{-\gamma_k} \cdot H_k}{E_k \cdot \cos\theta + H_k}$$
(4)

There $H_{\pi,k}$ and H_{μ} are parameters accounting for the propagation of the particles in the atmosphere. The parameters $A_{\pi,k}$ are the normalisations of the pion and kaon production spectra. There are several other parameters entering in the approximation: the absorption lengths of the primary particle λ_p , of the pions λ_{π} and kaons λ_k . There is clearly some influence, but in the present investigation the values have fixed along the original proposal. Only the $A_{\pi,k}$ values have been changed in order to adjust the calculated fluxes to the the results of the Monte Carlo simulations and to experimental data from the BESS[3] experiment.



Figure 2. Comparison of the results of Monte Carlo simulations and BESS data with predictions of simplified semianalytical formulae. In the approach of Jugde and Nash $A_{\pi} = 0.373$ and $A_{k} = 1.0$ (Fig. 1.a) and $A_{\pi} = 0.373$ and $A_{k} = 0.373$ (Fig. 1.b) is used.

5. Test of the simulation program by experimental results

The Monte Carlo program outlined in chapt. 2 and to be used for calculations of the muon fluxes at various locations has been checked by experimental results of accurate muon charge ratio measurements[6]. Fig.3 displays the comparison between measured results of the East -West effect performed with WILLI and with CORSIKA simulations. The values are compiled in tab.1.



Figure 3. Comparison of the muon charge ratio and the East -West effect with the CORSIKA predictions

Azimuth	Momentum(GeV/c)	$< \theta_p >$	R_{WILLI}	$R_{CORSIKA}$
	0.36	35	0.89	0.68
	0.50	35	0.95	0.88
EAST	0.65	35	0.85	0.91
	0.87	35	0.84	0.95
	0.36	35	1.50	1.4
	0.50	35	1.43	1.46
WEST	0.65	35	1.06	1.32
	0.87	35	1.23	1.36

 Table 1. Compilation of the numerical values displayed in fig.3

6. Concluding remarks

Semi-analytical approaches[7] are able to reproduce globally the results of Monte Carlo simulations and experimental data, and in particular the approach of Judge and Nash does account for muon energies < 10GeV. However, these approaches can be hardly modified in order to take into account also finer effect like the influence of the geomagnetic field. For that detailed Monte Carlo simulations have to be invoked.

References

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