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A Monte Carlo simulation of underground muon transport

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We have been developing a method for sampling multiple Coulomb scattering which can sample angular and lateral deviations simultaneously and yield the Molière distribution. Using the method, we performed Monte Carlo simulations for underground muons and results are compared with ones from simulations with the Gaussian approximation method for sampling multiple Coulomb scattering.

1. Introduction

The Monte Carlo simulation of muon transport through a large amount of matter is important for underground physics and one of the most important information obtained from the simulation is the lateral spread of muons. Though the angular deviation of a muon is dominated by multiple Coulomb scattering, the process is usually treated in the Gaussian approximation [1, 2, 3, 4]. Hence, the lateral deviation may be underestimated due to the neglect of the single scattering tail.

We have been developing a method for sampling multiple Coulomb scattering (see [5, 6] and references therein). If we use the screened Rutherford cross section with the small angle approximation, our method yields the Molière distribution[7, 8, 9]. Since the method doesn't require auxiliary numerical tables, it can be implemented as simple as, and as fast as the Gaussian approximation method. The most important feature of our new method is the simultaneous sampling of the deflection angle and the lateral displacement. The method also can take constant energy loss into account.

Using our method, we performed the Monte Carlo simulation of muon propagation in standard rock. Results are compared with ones from the simulation with the Gaussian approximation.

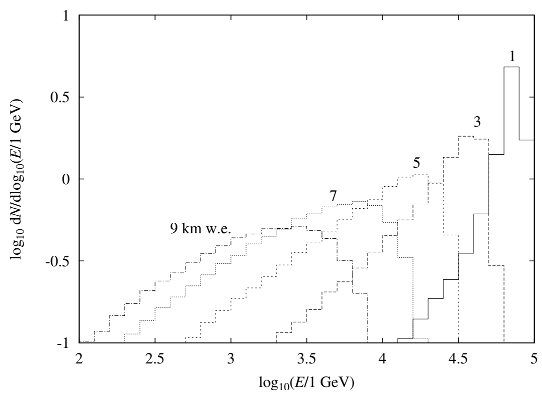

Figure 1. Energy distribution of muons of 10^5 GeV after propagation in standard rock.

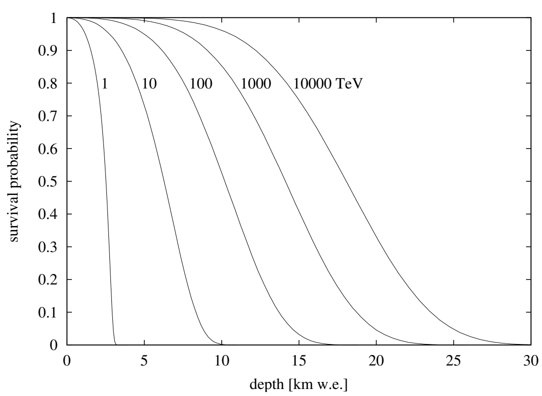

Figure 2. Survival probabilities of muons of energy from 1 to 10^4 TeV in standard rock.

2. Simulation

Muons have been followed down to the threshold energy of 1 GeV. We have treated muon energy losses due to ionization, bremsstrahlung, pair production and photonuclear interaction. The latter three processes have been simulated discretely if the muon fractional energy loss v exceeded $v_{\rm th} = 10^{-3}$, otherwise treated as continuous energy loss. The cross sections described in ref. [10] have been used.

Figure 1 shows the energy distributions at depth 1, 3, 5, 7 and 9 km w.e. for muons with initial energy of 10^5 GeV. Figure 2 shows the survival probabilities for muons with initial energy of 1, 10, 100, 1000 and 10000 TeV. Figure 3 shows the energy distribution of muon at 3 and 10 km w.e. when initial energy has been sampled according to a power spectrum with index $\gamma = 3.7$. Comparing these figures with the corresponding figures of refs. [1] and [2], we can see the reliability of our simulation.

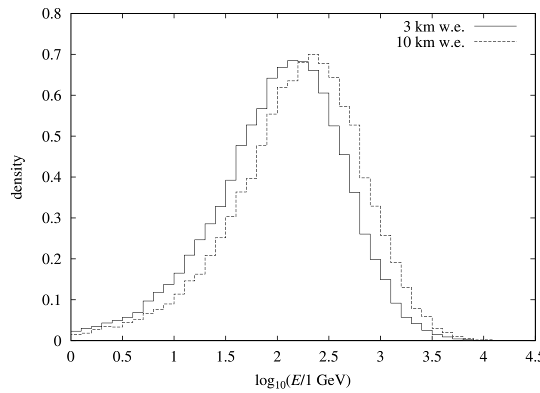

Figure 3. Energy distribution of muons from a power spectrum with index $\gamma = 3.7$ at 3 km w.e. (solid histogram) and 10 km w.e. (dashed histogram).

The angular and lateral distributions of muons from a power spectrum with index $\gamma = 3.7$ at 3 km w.e. are shown in figure 4. The histograms and solid circles show the distributions from the simulations with the Molière and Gaussian multiple scattering respectively.

Some results of the simulation are summarized in table 1. It can be seen that the angular distributions from the simulations with the Molière and Gaussian multiple scattering are consistent each other but the lateral distributions.

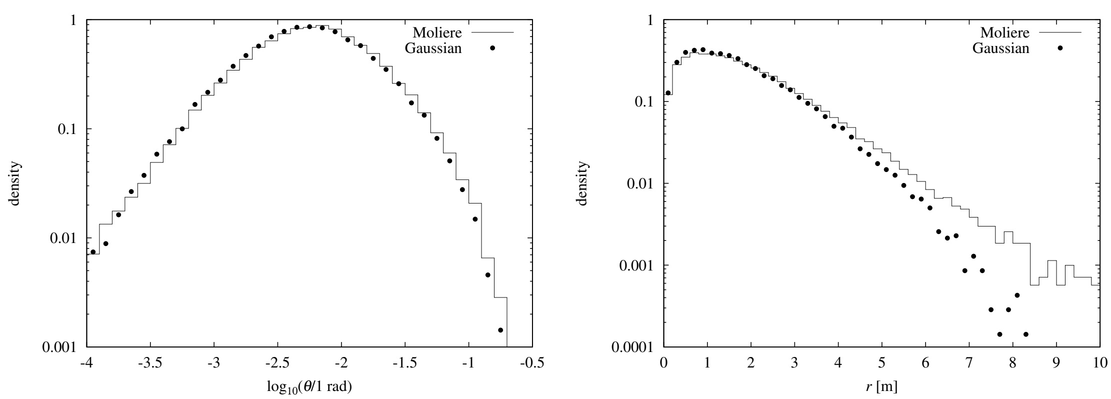

Figure 4. Angular (left) and Lateral (right) distributions of muons from a power spectrum with index $\gamma = 3.7$ at 3 km w.e. (histograms: Molière, solid circle: Gaussian)

Table 1. The mean energy, angle, lateral displacement of muons from a power spectrum with index $\gamma = 3.7$ at 3 km w.e. and 10 km w.e. The numbers of survived and simulated muons and CPU times (with Intel Pentium II processor 400 MHz) are also shown.

	E [GeV]		θ [degree]		r [m]		survived #/simulated #		CPU time [s]	
depth [km w.e.]	3	10	3	10	3	10	3	10	3	10
Molière	248	364	0.578	0.470	1.85	2.25	$35187/10^5$	$43833/10^6$	69	1907
Gaussian	243	355	0.534	0.426	1.66	1.96	$35032/10^5$	$43499/10^6$	57	1539
ref.[1](Gaussian)	225	325								
ref.[2](Gaussian)	250	367	0.56	0.45	1.56	1.87				

3. Discussion and Conclusions

Since the RMS angle of multiple Coulomb scattering of a muon is inversely proportional to energy, the angular distribution depends on the energy distribution. That is, the fluctuation of energy loss plays dominant role.

For the lateral displacement, however, this argument can not be entirely true since the large angle scattering of a muon at the initial part of the trajectory is important. To see this, we show the trajectories of 3 TeV muons in standard rock. Figure 5 and 6 show the trajectories of muons which have projected scattering angle of larger than 0.01 radian and projected lateral displacement of larger than 2 m at 3km w.e. respectively. It can be seen that the muons with the large scattering angle have had large angle deflections at the final part of their trajectories and their lateral displacements are not always large. On the other hand, the muons with the large lateral displacement have had relatively large angle deflections (single scattering) at the initial part of their trajectories. Finally, we show the correlations between the residual energy of 3 TeV muons and scattering angle or lateral displacement at 3 km w.e. in figure 7. The weak dependence of the lateral displacement on the residual energy can be seen.

Exploiting our newly developed method for sampling multiple Coulomb scattering, we have performed the Monte Carlo simulation of muon propagation in standard rock. We can conclude that to obtain accurate lateral distributions of muons after propagation through thick layers of matter, the single scattering tail of multiple Coulomb scattering must not be neglected.

References

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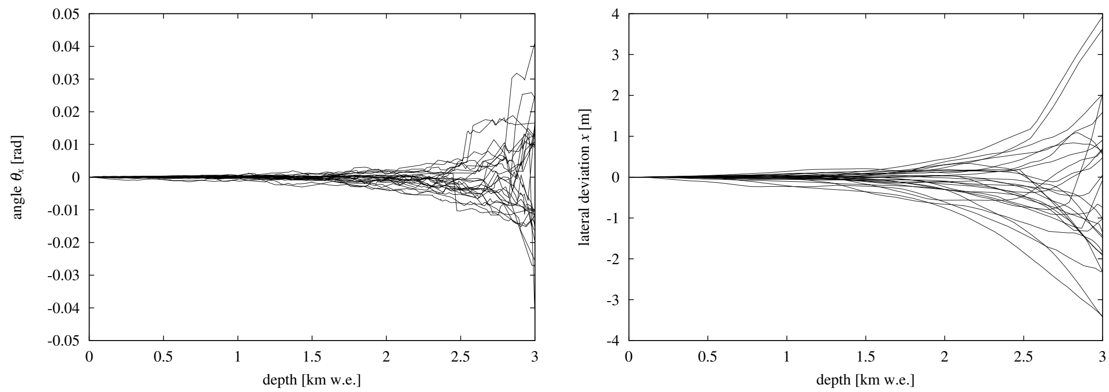

Figure 5. The trajectories of 3 TeV muons which have projected scattering angle of larger than 0.01 radian at 3 km w.e. Left:depth-angle relation, Right:depth-lateral displacement relation.

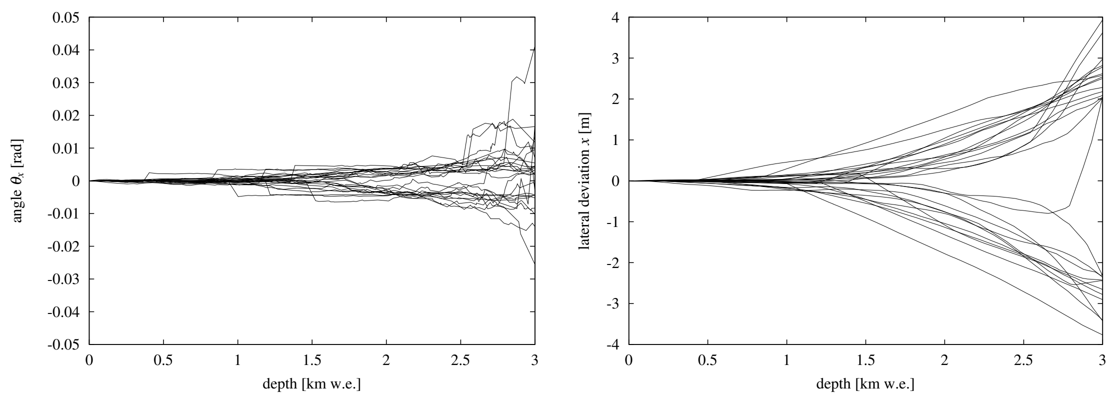

Figure 6. The trajectories of 3 TeV muons which have projected lateral displacement of larger than 2 m at 3 km w.e. Left:depth-angle relation, Right:depth-lateral displacement relation.

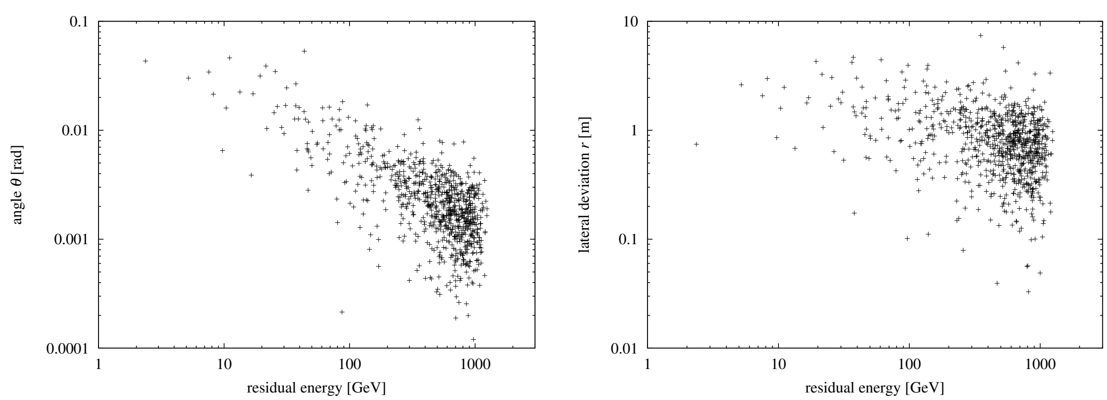

Figure 7. The scatter plots of the residual energy of 3 TeV muons versus scattering angle (left panel) and lateral displacement (right panel) at 3 km w.e.