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Goudsmit-Saunderson distribution with ionization described with Molière parameters

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Molière theory is inferior in accuracy than Goudsmit-Saunderson theory due to the small angle approximations, although we have accumulated far much analytical results in the former theory. We find Goudsmit-Saunderson distribution can also be obtained using Moliére's characteristic parameters, B and $\theta_{\rm M}$, based on Lewis theory. This new method will help us deriving Goudsmit-Saunderson angular distribution easily, rapidly, and accurately, taking account of ionization loss.

1. Introduction

Although Molière theory [1, 2, 3] has limit in accuracy so that inferior to Goudsmit-Saunderson theory [4, 5] due to the small angle approximation [6], we have obtained many valuable results about the transport properties of charged particles penetrating through materials from the Molière theory, e.g. the angular and the lateral distributions taking account of ionization loss [7, 8] and others. We have found another excellent property of Molière theory to apply on Goudsmit-Saunderson formulation of the multiple scattering theory, that the Goudsmit-Saunderson distribution with ionization can also be expressed by the characteristic parameters B and $\theta_{\rm M}$ of Molière based on the Lewis theory [9]. We confirm the fact by comparing our results with the Monte Carlo results.

2. Goudsmit-Saunderson angular distribution with ionization expressed by the Molière parameters

According to Lewis, the probability density of Goudsmit-Saunderson angular distribution $f_{\rm GS}(\theta,t)2\pi\sin\theta d\theta$ is expressed as

$$\begin{aligned} 2\pi f_{\rm GS}(\theta,t) &= \sum_{l=0}^{\infty}(l+\frac{1}{2})P_l(\cos\theta)\exp\{-\int_0^x dx\int_0^\pi 2\pi\frac{N}{A}\sigma(\theta)[1-P_l(\cos\theta)]\sin\theta d\theta\} \\ &\equiv \sum_{l=0}^{\infty}(l+\frac{1}{2})P_l(\cos\theta)\exp\{-\int_0^x \kappa dx\}. \end{aligned} \tag{1}$$

Evaluating the exponent as Lewis did by substituting the single-scattering formula

$$\frac{N}{A}\sigma_{\rm R}(\theta)2\pi\sin(\theta)d\theta ds = \frac{1}{\pi\Omega}\frac{K^2}{E^2}\frac{1}{4(1-\cos(\theta)+\chi_{\rm a}^2/2)^2}2\pi\sin(\theta)d\theta dt \tag{2}$$

with the screening angle

$$\chi_{\rm a} = \frac{K}{E}e^{-\frac{\Omega}{2}+\frac{1}{2}-C}, \tag{3}$$

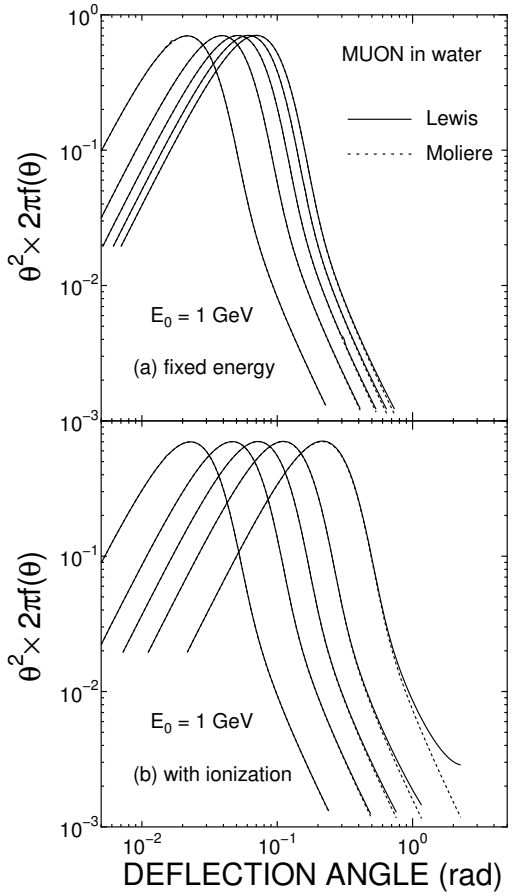

Figure 1. Comparison of expected angular distributions multiplied by θ^2 for muons derived by Goudsmit-Saunderson-Lewis theory (solid curves) and by Molière theory (dot curves), assuming the rest energy negligible. Curves in (b) show the distributions after traverse of thickness at 10, 30, 50, 70, and 90 percent dissipations of the incident energy from left to right, and curves in (a) show the distributions without energy loss after traverse of the same thickness.

then we have

$$
\begin{aligned}
-\int_0^x \kappa dx &= -\int_0^t \frac{2K^2z^2dt}{\Omega p^2v^2}\int_{-1}^{1}\frac{1-P_l(\mu)}{4(1-\mu+\chi_a^2/2)^2}d\mu \\
&\simeq -\int_0^t \frac{K^2z^2dt}{4\Omega p^2v^2}l(l+1)\big(1-\ln\frac{\chi_a^2}{4}-2\sum_{m=1}^{l}m^{-1}\big) \\
&= -\int_0^t \frac{K^2z^2dt}{4p^2v^2}l(l+1)\big(1-\frac{1}{\Omega}[\ln\frac{\beta'^2K^2}{4p^2v^2}-2C+2\sum_{m=1}^{l}m^{-1}]\big) \\
&= -\frac{\theta_{\rm G}^2}{4}l(l+1)\big(1-\frac{1}{\Omega}[\ln\frac{\beta'^2\theta_{\rm G}^2}{4\nu z^2t}+2\psi(l+1)]\big) \\
&= -\frac{\theta_{\rm M}^2}{4}l(l+1)\big(1-\frac{1}{B}[\ln\frac{\theta_{\rm M}^2}{4}+2\psi(l+1)]\big), \qquad (4)
\end{aligned}
$$

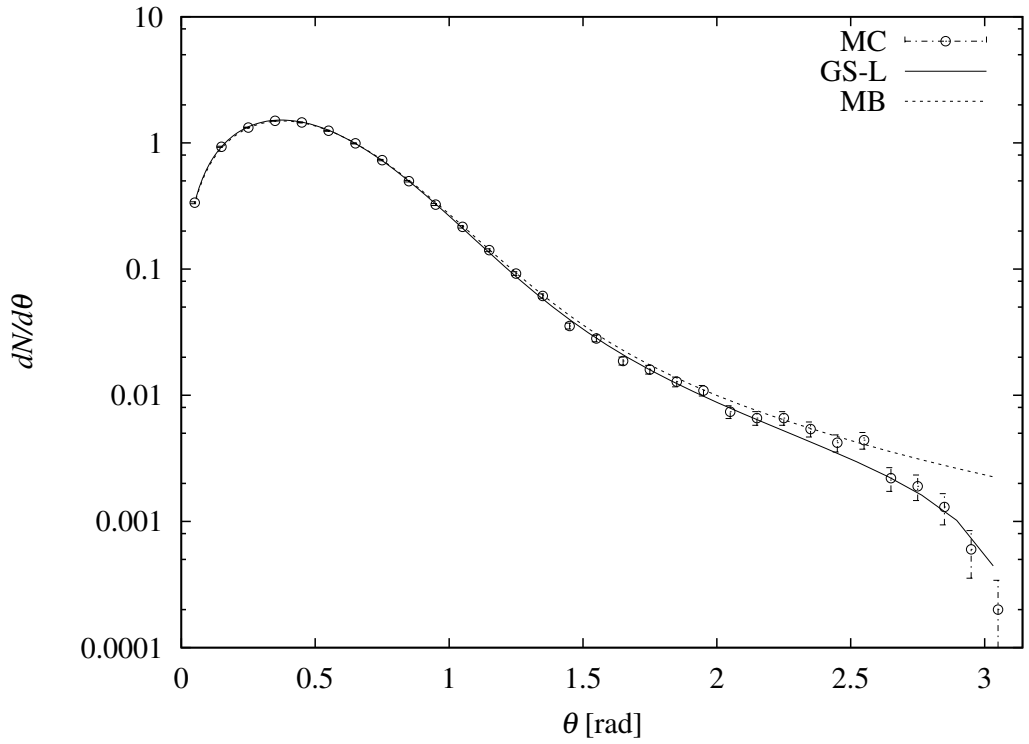

Figure 2. The probability densities of deflection angle, analytically expected by Molière-Bethe theory (dot line) and by Goudsmit-Saunderson theory (solid line), are compared with that evaluated by the Monte Carlo simulation (open circle) for 10 MeV electron, after dissipating half of its energy by ionization loss of 2.5 MeV in unit path-length of g/cm^2, neglecting the rest energy. The Monte Carlo results agree with Goudsmit-Saunderson results at the angular range where the small-angle approximations are no more satisfied.

where

$$\psi(z) = (d/dz)\ln\Gamma(z) \tag{5}$$

denotes the psi function [10], the gaussian mean-square angle $\theta_{\rm G}^2$, the contraction factor ν, the expansion parameter B, and the scale angle $\theta_{\rm M}$ are those derived before under the Molière theory [7, 11]. So we obtain the Goudsmit-Saunderson angular distribution with ionization

$$2\pi f_{\rm GS}(\theta,t) \;=\; \sum_{l=0}^{\infty}(l+\frac{1}{2})P_l(\cos\theta)\exp\{-\frac{\theta_{\rm M}^2}{4}l(l+1)\big(1-\frac{1}{B}[\ln\frac{\theta_{\rm M}^2}{4}+2\psi(l+1)]\big)\}, \tag{6}$$

corresponding to the Molière angular distribution $f_{\rm M}(\theta,t)2\pi\theta d\theta$ with ionization

$$2\pi f_{\rm M}(\theta,t) = \int_0^{\infty}\zeta d\zeta J_0(\theta\zeta)\exp\{-\frac{\theta_{\rm M}^2\zeta^2}{4}\big(1-\frac{1}{B}\ln\frac{\theta_{\rm M}^2\zeta^2}{4}\big)\}, \tag{7}$$

using the same characteristic parameters B and $\theta_{\rm M}$.

For singly-charged extreme relativistic particles, the characteristic parameters B and $\theta_{\rm M}$, thus the Goudsmit-Saunderson angular distribution, with ionization loss of a constant rate are derived from the results without ionization by only replacing the thickness t and the energy E with the effective thickness νt and energy $\sqrt{\nu E_0 E}$. Goudsmit-Saunderson angular distribution thus derived are compared with Molière angular distribution in Fig. 1 both under the fixed-energy and the ionization processes. We cannot find almost any difference between them within angular ranges to satisfy the small angle approximation.

3. Discussion

The Goudsmit-Saunderson distributions with and without ionization loss are compared with the Molière-Bethe distributions, in Fig. 1. The differences begin to appear at angles greater than about 1 radian.

We compare our Goudsmit-Saunderson distribution with ionization expressed by the Molière parameters B and $\theta_{\rm M}$ with the Monte Carlo result based on Rutherford cross-section (2) with (3) without the small-angle approximation. We derived the angular distribution of 10 MeV electrons dissipating half of their energies by ionization loss of 2.5 MeV in each actual path-length in cm, neglecting the rest energy. The results are indicated in Fig. 2. The both agree very well.

4. Conclusions

We have found the simple and convenient method to obtain Goudsmit-Saunderson angular distribution with ionization based on the Lewis formulation, using the Molière parameters of B and $\theta_{\rm M}$. We have compared the distribution derived by our method with a full Monte Carlo result and have got good agreement between the both.

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References

[1] G. Molière, Z. Naturforsch. **2a**, 133(1947).
[2] G. Molière, Z. Naturforsch. **3a**, 78(1948).
[3] H.A. Bethe, Phys. Rev. **89**, 1256(1953).
[4] S.A. Goudsmit and J.L. Saunderson, Phys. Rev. **57**, 24(1940).
[5] S.A. Goudsmit and J.L. Saunderson, Phys. Rev. **58**, 36(1940).
[6] W.T. Scott, Rev. Mod. Phys. **35**, 231(1963).
[7] T. Nakatsuka, *Proc. of the 26th ICRC*, Salt Lake City, **1**, 486(1999).
[8] T. Nakatsuka and K. Okei, *Proc. of the 29th ICRC*, Pune, in this conference.
[9] H.W. Lewis, Phys. Rev. **78**, 526(1950).
[10] *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1965), Chap. 6.
[11] T. Nakatsuka, *Proc. of the 27th ICRC*, Hamburg, **1**, 284(2001).