Urban Congestion and the Development of Employment Subcenter

by

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Submitted to the Department of Civil and Environmental Engineering in Partial Fulfillment
of the Requirements for the Degree of

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Abstract

This thesis extends the previous literature on urban subcenter formation by introducing congestion cost of transportation. A new model is constructed to analyze the efficiency of private decision about firm decentralization with the presence of unpriced transportation congestion. The model features a linear city with provision of transportation infrastructure oriented toward carrying workers to the CBD. Congestion cost of transportation is modeled by a function of traffic density, that is, the ratio of traffic volume to transportation capacity. The introduction of congestion costs creates mutual dependence between private decision about residential location and the transportation cost even in a simple monocentric model. The simultaneity problem becomes much more complicated in the decentralized scenario. The standard urban land market theory is applied to the model to analyze equilibrium condition of the decentralized city, and the optimal level of decentralization is analyzed by using the aggregate household utility of the whole city as a criteria. Two scenarios of firm decentralization that are investigated include when technology of transportation infrastructure is non-uniform and when land market failure due to unpriced congestion externality is not considered. Complex computer algorithm is required for simulation of the model. Under various scenarios, the simulations show that only the technology of transportation infrastructure affects the efficiency of private decision about decentralization. If transportation infrastructure serving the whole city is uniform in technology, private decision about decentralization will be socially efficient. However, if the transportation infrastructure serving the central area is technologically less congestable than that serving the subcenter, the private decision will lead to too many firms decentralizing to the subcenter. These results suggest that subsidy for firms located in the central area may be required in order to ensure the socially desirable level of firm decentralization.

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Chapter 1

Introduction

1.1 Transportation and the Development of Employment Subcenter

In a long history of urban development, changes in urban structure have been closely intertwined with transportation technology and infrastructure. When the dominant mode of transportation was to walk, employment and residential areas were located close together. Later, when motorized modes, such as subways and automobiles in particular, became common, households took advantage of improvement in transportation technology by decentralizing to suburban areas. As a result, the most prevalent urban form in the era was a city with firms concentrating in a single, central location, and residential areas around it. This urban form was termed a monocentric city. The monocentric phenomenon may be explained by the fact that firms need to locate near transportation terminal such as port, through which they can transport their product to the market, and raw materials to their plants.

In recent decades, however, the trend of urban development has changed again. Rationally, retail trade and personal service firms decentralized to be close to their household clienteles. Following were manufacturing firms, which decentralized because of two major reasons. The first was the change in manufacturing technology, such as horizontal assembly line, which requires firms to use more land per unit of output than before. Since land in the central area is expensive, firms were forced to relocate of suburban area where land is cheaper. The second reason for decentralization of production firms was the improvement of highway system, which made trucking an effective mode of transporting firms' product. Consequently, firms no longer need to locate close to the central area to be near transportation terminal because trucks behave like moving terminal that firms can use wherever they are located. More recently, there have been many evidences that financial and service firms that are office-based also decentralize. This could probably be explained by the improvement in communication technology, which allows interactions between firm employees and clients to take place without physical proximity. Again, this trend is related to transportation in that communication technology has reduced the need for physical movement of transaction agents.

With the changes mentioned above, firms have increasingly decentralized out of most metropolitan areas over years. However, it is evident that firms locations do not randomly scatter throughout the area, but rather cluster together to form smaller centers, which is termed employment subcenter. This is a result of the benefit from firm agglomeration as well as the advantage of easy access to highway system. The most important reason that firms do not completely decentralize from the central business district (CBD) is probably because building stock and infrastructure are durable. Old cities that were developed during the era of monocentric city or earlier have huge amount of office space in the CBD, and transportation infrastructure oriented toward serving traffic flow to the CBD. Examples of these include Boston, New York, and Philadelphia. On the other hand, newer cities that emerged in the age of highway and automobile are much more decentralized, and transportation infrastructure is provided more uniformly than their older counterparts. Modern cities with these characteristics include Los Angeles, Dallas, and Phoenix. In any case, these evidences reflect the evolution of urban structure from monocentric to multicentric one, resulting from transportation technology change.

1.2 Transportation Congestion and Optimal Subcenter Size

The impact of transportation on the development of urban employment center as well as residential areas is significant. Under the monocentric framework, Solow [1973] shows that with the presence of transportation congestion, failure to price the transportation cost properly, i.e., internalize the congestion cost, would lead to land market failure, that is the value of land is too low from social viewpoint. Using market price of land for benefit-cost analysis of transportation infrastructure project would lead to too much land being used for the infrastructure. Wheaton [1996] further argues that even when transportation infrastructure is provided optimally, land market still fails to function properly, given the unpriced congestion. These results show that congestion has substantial impact in urban development, and unpriced congestion will lead to suboptimal land allocation even in the simple monocentric framework.

Effect of congestion on firm decentralization has not yet been examined, however. In order to illustrate the relationship between transportation congestion and optimal employment subcenter size, consider a simple city where there exist two identical employment centers that are served by two separate and different transportation infrastructure. Suppose the number of household and worker in the city is N, N1 workers commute to work in center 1, and N-N1 workers commute to center 2. Also, transportation cost of using infrastructure to commute to work in center i is a function of traffic density, i.e., the ratio of the number of commuters to the transportation capacity provided by the infrastructure.

First, consider the equilibrium condition of decentralization of this city. To keep every household at the same level of utility, the level of decentralization must be such that transportation costs of all households in both employment centers are equal, i.e.,

$$T_1 = T_2$$
$$T_1 = \left(\frac{N_1}{C_1}\right)^{\alpha_1}$$
$$T_2 = \left(\frac{N - N_1}{C_2}\right)^{\alpha_2}$$

where: T_1: individual transportation cost of center 1
T_2: individual transportation cost of center 2
C_1: capacity of transportation infrastructure serving center 1
C_2: capacity of transportation infrastructure serving center 2

Next, consider the optimal decentralization, which would lead to minimization of total transportation costs. The total transport costs of the city can be determined as follows:

$$TT = N_1 T_1 + (N - N_1) T_2$$

where: TT: total transportation cost of the city

To minimize transport cost, the first order condition requires that:

$$\frac{\partial TT}{\partial N_1} = \left(\frac{N_1}{C_1}\right)^{\alpha_1} + \frac{\alpha_1 N_1}{C_1}\left(\frac{N_1}{C_1}\right)^{\alpha_1 - 1} - \left(\frac{N - N_1}{C_2}\right)^{\alpha_2} - \frac{\alpha_2 (N - N_1)}{C_2}\left(\frac{N - N_1}{C_2}\right)^{\alpha_2 - 1}$$

$$= T_1(1+\alpha_1) - T_2(1+\alpha_2) = 0$$

$$\frac{T_1}{T_2} = \frac{(1+\alpha_2)}{(1+\alpha_1)}$$

This condition means that if transportation cost function of infrastructure serving both centers are identical ($\alpha_1 = \alpha_2$), and the transportation cost depends only on the *ratio* of number of household in each center and capacity, then the optimal allocation of employment is achieved when commuting costs to both centers are equal. Implication of this result is that when transportation technology is uniform over the city, the optimal employment decentralization of the city is the same as the equilibrium one. On the other hand, if transportation technology is non-uniform, for instance, $\alpha_1 > \alpha_2$, the optimal allocation of employment will be different from the equilibrium one. While the equilibrium decentralization will equalize transportation cost, the optimal decentralization will lead to reallocation of households in the center with more congestable transportation infrastructure (α_1) to the other center (α_2).

The simple example shows that the private decision about decentralization is efficient if the technology of transportation infrastructure is uniform. However, due to the unrealistic assumptions, it cannot be concluded that this result is true in the real world. This thesis will therefore investigate whether this conjecture holds under the more realistic assumptions of a decentralized land use model.

1.3 Objectives and Methodology

1.3.1 Objectives

With emergence of urban employment subcenters in many leading metropolitan areas around the world, it is important to realize the possibility of inefficient development of those centers due to the land market failure and inappropriate pricing of transportation cost. Therefore, the main objective of the thesis is to provide a theoretical framework to analyze the efficiency of firm decentralization under such conditions. More specifically, the thesis attempts to answer the following questions:

(1) With the presence of transportation congestion, are decisions that firms make about decentralization socially efficient?

(2) What are the major factors that affect the optimal development of employment subcenter?

(3) If the provision of transportation infrastructure is among those factors, how does it affect firms' decisions and the social optimality?

(4) If the firm decentralization by private decision is not socially efficient. How can we bring it to the socially efficient level?

(5) How do firm decisions about location affect transportation congestion? Is it possible to influence the employment decentralization so that the transportation congestion is mitigated?

1.3.2 Methodology

To answer the questions outlined above, the thesis combines methodological approaches used by three related groups of theories and researches on urban land use. These include the traditional land market theory [e.g., Alonso, 1960; Muth, 1969; Mills, 1972], the research on employment subcenter formation [e.g., White, 1976; Ogawa and Fujita, 1980; Sivitanidou and Wheaton, 1992], and the research on urban congestion and land use [i.e., Solow, 1973; Arnott, 1979; Wheaton, 1996]. The first group provides general framework for modeling of simple urban formation. The second group addresses the issue of firm decision to decentralize to the subcenter. The last group provides practical approach to modeling of transportation congestion in the urban land use model. Under the theoretical framework, four simplified models of a city with an employment subcenter were developed to investigate the issue of private decentralization efficiency under the condition of unpriced transportation congestion.

Simulations of the four models were done in order to analyze the optimal level of firm decentralization under different modeling assumptions and exogenous factors. Results of the simulation examples were summarized and exhibited in various tables and graphs. From the analysis of the simulation results, conclusions about efficiency of private decision on firm decentralization would be drawn. Based on these conclusions, the thesis will finally suggest the proper public policies on the development of urban employment subcenters, and how they might help alleviate transportation congestion problem.

1.4 Organization and Outline

The thesis is organized in five chapters. Chapter 1, which is this chapter, introduces the general issues of firm decentralization and transportation congestion, as well as objectives and methodologies of the thesis. In chapter 2, which is an introductory to the theoretical modeling, various types of models in the literature on urban land use are reviewed. These models include traditional models of urban land use, models of firm decentralization and formation of employment subcenter, and models of urban land use with explicit consideration of transportation congestion. In addition, the research gap that has not yet been investigated is pointed out. Chapter 3 describes the models created to analyze the central problem of efficiency of firm decentralization by private decision. Theories and assumptions of the monocentric models are first discussed in order to provide basic knowledge, and followed by those of the more general decentralized model. The formulations of problems for model simulations are also provided. Chapter 4 presents special characteristics of the four alternative models. The values of common exogenous variables and exogenous variables that are specific to each model are discussed. The examples of simulation results are summarized in tables and graphs later in the chapter. These results are analyzed, and the answers to the central questions of the thesis are drawn. The important findings and the public policies about urban land use that are implicated by the findings then conclude the thesis in chapter 5.

Chapter 2

An Overview of Existing Urban Land Use Models: Congestion and Employment Decentralization in the Literature

The theory of urban land use has a long history of development. Initially, the theory was based on traditional land market models, with the most important assumption of monocentricity [i.e., Alonso, 1960; Muth, 1969; Mills, 1972; Wheaton, 1977]. That assumption is important, since it means that all jobs are located at one pre-specified city center. Because of limitations in real world application, the monocentric model was later extended to consider a city with multiple employment centers [i.e., White, 1976; Ogawa and Fujita, 1980; Wieand, 1985; Helsley and Sullivan, 1992].

The model of urban land use has also been developed in other directions besides the relaxation of monocentric assumption. The most notably development is probably when transportation congestion cost was incorporated in the standard monocentric model by Solow [1973]. The follow-up research on urban congestion was presented by Arnott[1979], Arnott, de Palma, and Lindsey [1993], and Wheaton[1996]. Unfortunately, the urban land use models that consider transportation congestion are all monocentric.

This chapter explores a series of urban land use models that have been developed over years, including both monocentric and decentralized models. Also, the literature on urban congestion models will be reviewed. At the end of this chapter, the research space that has not yet been explored before will be pointed out.

2.1 The Monocentric Models of Urban Land Use

The monocentric models were created to explain the theory of urban land use, that is, how residential and land market operate in urban areas. A series of monocentric models that have been developed includes the models of Alonso [1960], Muth [1969], Mills [1972], and Wheaton [1977]. The models generally feature a well-defined linear or circular city. The city has a single and pre-specified employment center where all firms are located. This is the most important assumption of monocentric models since it implies that the analysis of firm equilibrium is ignored, and the models concentrate on only the analysis of the household equilibrium and the way residential land rents vary across location.

Assumptions under which most monocentric models were developed are listed by category as follows:

Assumptions about characteristics of an abstract city

- The city is located on a flat plain.
- The city is monocentric, with a pre-specified center where all jobs are located.
- The city is closed, i.e., no movement of household into or out of the city.
- The city owns the whole land. It collects rent and redistributes dividend to the residents.

Assumptions about firms

- Firms consume negligible amount of land compared with households.
- Firms are immobile. Their only location is the center of the city.
- Firms' products are homogeneous and are marketed outside the city.
- Firms employ all of the city's labor force.

Assumptions about households

- Households are homogeneous. The number of worker per household, household income, and household type are identical.
- All workers commute to work in the center of the city.
- Households are mobile.
- Household utility function is identical for each household. Only land consumption and consumption of other goods and services enter the utility function.
- Household income is spent on housing, transportation cost, and other goods and services.
- Households maximize the identical utility function over location.

Other Assumptions

- Residential land market is perfectly competitive. Housing is occupied by household that pays the highest rent.
- Transportation cost is a linear function of distance. There is no transportation congestion.

An array of assumptions makes the analysis of household equilibrium in the monocentric model particularly tractable. The most common and elementary result from various analyses is that land rents vary with locational amenities. Originally, the only amenity at each location that was considered in the models was the accessibility to the CBD. This is represented by cost of commuting from that location to the CBD. It was also assumed that the commuting cost is a function of only distance. With this assumption, the amenity of each location is simply the distance from that location to the CBD.

All models suggest that variation in transportation cost, as a locational amenity, is perfectly capitalized by land rent. According to the monocentric model by Muth [1969], the slope of rent gradient with respect to distance from the CBD is given by:

$$\frac{\partial R}{\partial t} = -\frac{T'(t)}{q(t)} \tag{2.1}$$

where:
t: distance from the CBD
T(t): transportation cost at t
q(t): land consumption at t

In certain models [e.g. DiPasquale and Wheaton, 1996], land consumption is assumed to be fixed. Hence, according to equation (2.1), slope of rent gradient will be proportional to that of transportation cost gradient. If it is assumed further that transportation cost is a linear function of the distance from the CBD, i.e.,

$T(t) = k \cdot t$

where k is a positive constant, then

$T'(t) = k$

The slope of transportation cost k is a constant and can be interpreted as the unit cost of traveling a distance of one mile. Consequently, the slope of the rent gradient in this case is given by equation (2.2).

$$\frac{\partial R}{\partial t} = -\frac{k}{q} \tag{2.2}$$

This means that the rent gradient of a simple monocentric model, which assumes a linear transportation cost function and fixed land consumption is linear with the slope equal to the ratio of unit cost of transportation and lot size.

In other models, however, the land consumption is allowed to vary across location. Thus, the household utility function is solved for the land consumption (density) gradient. (See the derivation of this in section 3.1.2.) In this case, the rent gradient will no longer be linear, but will have a convex shape. The slope of rent gradient will be steep near the CBD where land consumption is small, and flat near the border where land consumption is large.

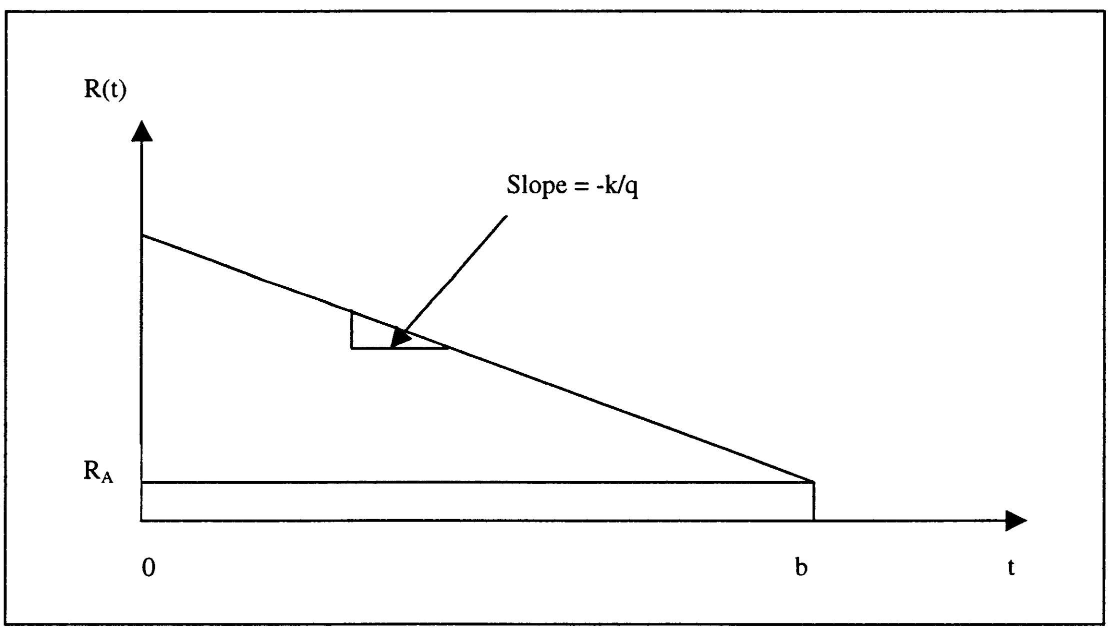


Figure 2-1 Rent gradient of a simple monocentric model with fixed land consumption.

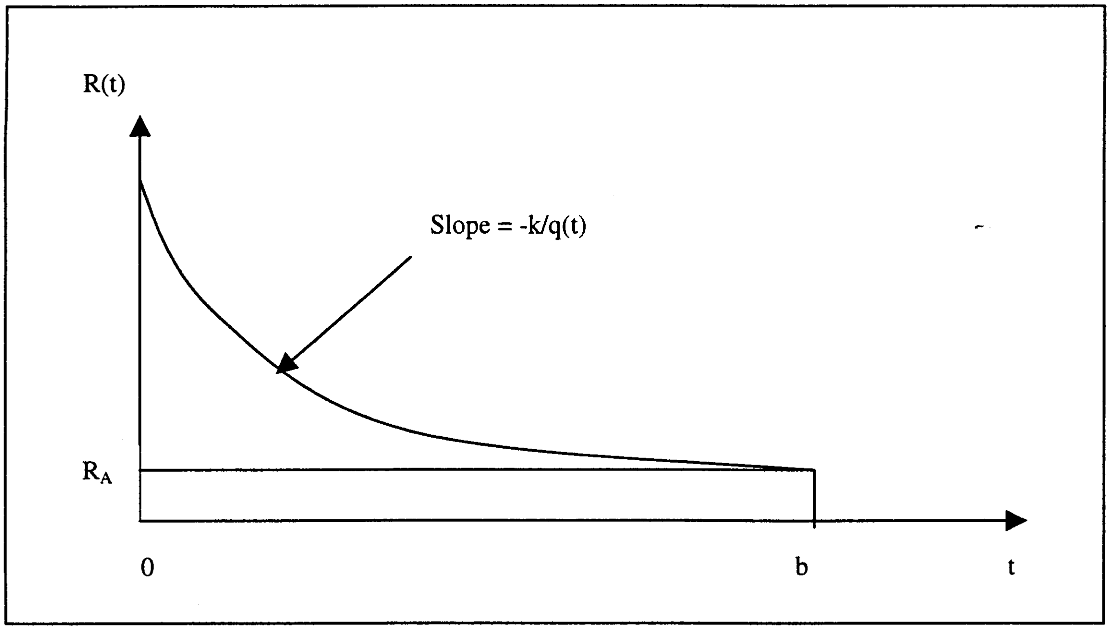


Figure 2-2 Rent gradient of a simple monocentric model with variable land consumption.

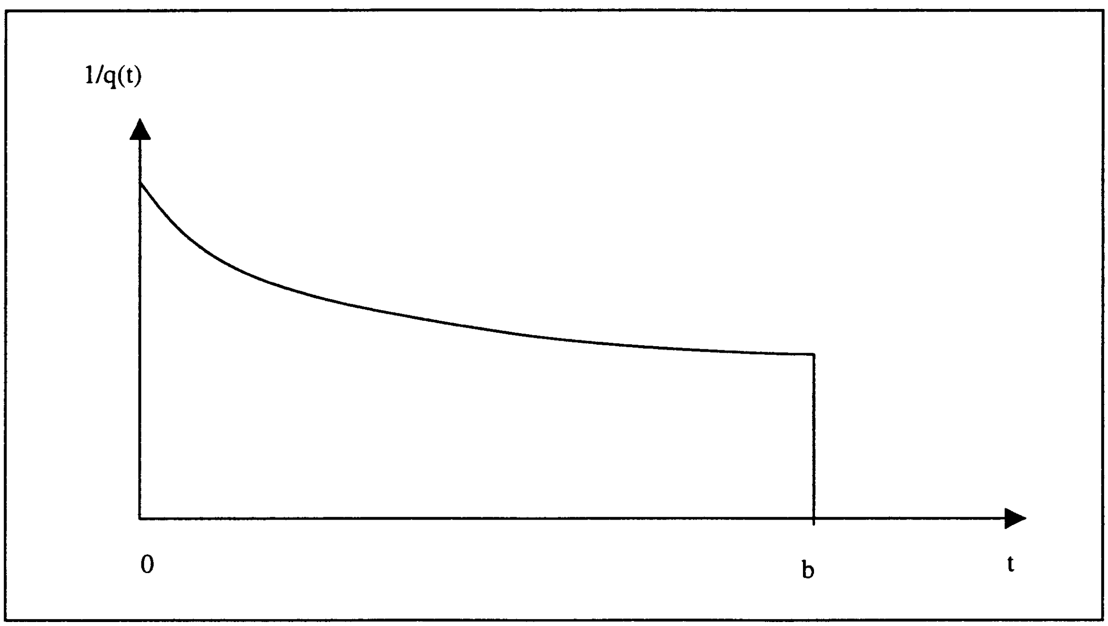


Figure 2-3 Density gradient of a simple monocentric model with variable land consumption.

Figure 2-1 and Figure 2-2 illustrate the rent gradient of the fixed land consumption model and variable land consumption model, respectively. At the center of the city, rent is the highest, while moving outward from the center, it decline. This is so because rents capitalize transportation cost saving resulting from living closer to employment location. Note that at the edge of the city where transportation cost is greatest, the rent capitalize zero saving, and therefore it must be equal to the agricultural land rent.

In the real world, however, locational variations are not limited to only variation in transportation cost. There are many other kinds of variations in locational amenities. For example, the quality of school, the amount of air pollution, and the beauty of scenery almost always vary across location. To account for these variations, Polinsky and Shavell [1976] created the more advance monocentric model. The major different between this new model and standard monocentric model is that a vector of various locational amenities is used instead of only variation in transportation cost.

Whether locational amenity is represented by only transportation cost or an array of many variables does not affect the principle of household equilibrium. According to Alonso [1964], household equilibrium requires that all households achieve the same level of utility. If this condition did not hold, some households would move in order to increase their utility. Therefore, the difference in land rent due to locational amenity in equilibrium represents the compensating variation, which keep the level of utility of households equal even though their locations differ in amenity level [Sivitanidou, 1991].

Despite the strong assumptions, the standard monocentric model gives many useful explanations about residential location and land market. There have been several attempts to make the model more complete by relaxing some assumptions or including some extensions. In the next section, for example, the assumption that firms are immobile will be relaxed. This will lead to a much more complicated analysis than the monocentric model.

2.2 Employment Decentralization Models

With the continuing trend of increasing decentralization and declining importance of CBD as an employment center, the monocentric assumption is deemed inadequate for analyzing the modern cities [White, 1976, Wheaton, 1979, Ogawa and Fujita, 1980, and

Helsley and Sullivan 1991]. According to White [1976], the monocentric assumption and the assumption of homogeneous household income could prevent an application of the land use model in analysis of many important policy issues. She therefore developed an employment decentralization model, which consider a city in which firms in the CBD can relocate to the new subcenter. To reduce complexity of the analysis, she assumed that firms relocate to a ring of land at fixed distance from the CBD. She argued that firms might benefit from decentralization through paying lower wage, but a problem that might arise is the scarcity of labor at the new suburban location. From her analysis, she showed that the firms' decentralization under labor scarcity condition would cause the household utility to increase, and the city size to expand. However, the change in land value due to the decentralization is inconclusive.

Fujita and Ogawa [1980] argued that location and size of the employment center of a city in the urban land use model should not be pre-specified. Rather the model should provide a framework in which residential and employment location can be determined simultaneously. By relaxing the assumption of monocentricity, they developed a model in which spatial interactions among economic activities are considered explicitly. Each firm is allowed to choose location in accordance with its transaction activities with other firms. Each household location is chosen optimally with regards to its commuting trip to employment location. From their analysis, they concluded that the monocentric assumption could be defended only under special circumstances, particularly when commuting volume is small.

Helsley and Sullivan [1991] developed a series of dynamic model of urban subcenter formation. In each model, different assumptions about technology and economies of scale in production at each employment location were used. They showed that the development of employment subcenter arises from the tradeoff between economies of scale in production and diseconomies of scale in transportation. The development of the subcenter is only a part of the three-phase development of multicentric city, which include an exclusive development of CBD, an exclusive development of subcenter, and a simultaneous development of both centers. They concluded that according to assumptions about technology and scale economies in production, the models' predictions differ in the two aspects, the relative size of the CBD after the development is completed and the duration of the exclusive development of subcenter.

Sivitanidou and Wheaton [1992] focused their study on different issues, that is, wage and rent capitalization in a decentralized city. In their paper, a two-center city was developed to explain how wage and rent capitalization works under certain circumstances, namely, when employment centers differ in productivity advantages and when there are some kinds of land market regulations. They showed how differences in spatial amenities, particularly transportation cost, are capitalized in rent in the long-run equilibrium framework. However, the transportation cost that they used was only the distance cost. The cost of congestion was ignored for simplicity. They concluded that differences in locational advantages of employment centers are capitalized mainly by wage in a competitive land market. Only when the land market of the advantageous center is regulated will rents capture more locational value.

In most employment decentralization models, the major departure from the standard monocentric model is the relaxation of the monocentric assumption. In many cases, most of other assumptions as described in section 2.1 are still kept in order to preserve the analytical tractability of the model. Models in the next section, however, depart from the standard monocentric model in a different way. They relaxed the assumption that there is no transportation congestion in the city. This change has led to a significant improvement in urban land use theory.

2.3 Urban Land Use Model with Transportation Congestion

The use of only transportation distance cost to represent the location's accessibility to the employment center is inadequate with the presence of transportation congestion. In a large city where transportation congestion is intense, commuting cost due to congestion is in fact more significant than the distance cost. Because of the congestion, the rent gradient will no longer be linear even with the assumption of fixed land consumption, but will become convex since transportation congestion cost will increase exponentially as one moves closer to the employment center.

Solow [1973] generalized the basic model in the previous section by introducing congestion cost. He pointed out that the incorporation of congestion cost will generate the rent gradient with more curvature than those generated by earlier models (He already assumed that land consumption varies across location.), and that this rent gradient represents market value of land. Further, he argued that without congestion pricing, the market value of land reflects only private transportation costs, which is the sum of distance and congestion costs, but not the full social cost, which must also include the cost of congestion externality. Hence, the land market value is too low without congestion pricing. Lastly, by using numerical examples, he showed that if market value of land were used for cost-benefit analysis of street construction, more land would be used for street than the socially optimal amount.

Wheaton [1996] extended the study of Solow. He maintained that internalizing congestion externality could be achieved by two equivalent approaches, congestion pricing and optimal land use (density) regulation. The question in his research was whether these two approaches could be substitute when transportation capacity is provided optimally. Following Solow's approach, he simulated a city where transportation capacity is optimally provided. Even with the optimal transportation capacity, he found that when congestion is present, the density gradient must be adjusted differentially upward in order to improve social welfare. The density adjustment would be especially large near the employment center, while the adjustment near the border would be small. He concluded that transportation capacity policy is not a perfect substitute for land use regulation policy.

Since incorporation of transportation congestion cost introduces mutual dependence between transportation cost and household location, the analysis of employment decentralization will be particularly difficult even if the linear form of the city is assumed. This is probably why there has not yet been any employment decentralization model that considers transportation congestion explicitly.

2.4 Research Gap

While research about the effect of congestion on land use has been brief, research on employment decentralization has been far more explored. However, there has been an obvious research gap between the two strands of literature. While the urban land use models that consider transportation congestion explicitly are all monocentric, the employment decentralization models neglected to consider transportation congestion. Although negligence of congestion is acceptable when commuting rate is small, it is inadequate when a large metropolitan area with significant transportation congestion is in question. Unfortunately, in the real world, the employment decentralization is caused in part by the growing traffic congestion in a large city. Therefore, in real world application the analysis given by the models outlined in section 2.2 would suffer from failure to recognize transportation congestion.

The work in this thesis is at the cross between the two groups of literature mentioned in sections 2.2 and 2.3. It extends the employment decentralization models by introducing transportation congestion cost as used in the urban land use models with congestion. In this way, the new model is used to study the effect of transportation congestion on employment decentralization. In addition, the efficiency of private decision about decentralization will be examined. It is hoped that the work in this thesis would lead to better understanding about the formation of employment subcenter with the presence of congestion, and new way to improve social welfare by influencing such process.

Chapter 3

Modeling Firm Decentralization with Transportation Congestion

Decisions that firms make about decentralization are affected by transportation cost and vice versa. The central question of this thesis is whether the firms' decisions are socially efficient with the presence of uninternalized costs of transportation congestion. In order to answer that question, a set of theoretical models was developed under various assumptions about characteristics of the city, households, transportation infrastructure, and transportation congestion. The most basic model that was created is a general monocentric model, which features a linear city with a pre-specified city center. This model was then extended to a set of two-center models. Under various assumptions, these models were used to study the long-run effect of firm decentralization on social welfare.

Since the models were built in attempt to address the issues of firm decentralization, it is not necessary that the models created for this purpose strictly replicate the real city. One might argue that a circular model of urban location better replicates a real city than a linear model does thanks to its two dimensionality. However, the process of firm decentralization is very difficult to model in this framework since commuting pattern will be very complicated. As a result, a linear model was usually chosen in order to reduce the complexity in most previous work on modeling employment decentralization. This is also the case for the models in this thesis.

The problem of the equilibrium of urban location is very complex when transportation congestion is taken into consideration because of mutual dependence among many relationships. The most difficult problem is the simultaneous determination of household location and transportation congestion. The household location decision is affected by income and rent. These two factors are dependent upon transportation cost for the reason that will be explained in this chapter. Further, transportation cost itself is determined by residential location. Because of this difficulty, many simplifications of the model were made to allow such a complex problem to be solvable. It would be shown later in this chapter that even with all the simplifications, to solve the problem analytically is virtually impossible, and the only practical way is to solve the problem numerically, which is by itself not very easy.

3.1 The General Monocentric Model

3.1.1 Characteristics of a monocentric city

The general monocentric model features a linear city of finite width, t_w and length, 2e, as shown in Figure 3-1. The city is assumed to own the whole land. It collects land rent and redistributes rent dividend to the residents. The city's land is allocated among commercial sector (firms), residential sector, and transportation infrastructure. All firms are located in the Central Business District (CBD), of which a location is pre-specified at the center of the city. Firms are assumed to consume negligible amount of land relative to the residential sector. Transportation infrastructure is not uniformly provided, but rather oriented toward moving workers to the main employment center (CBD). Equation (3.1) shows the relationship between transportation capacity and the distance from the CBD.

$$v(t) = c_0 - \frac{c_0 t}{e} \tag{3.1}$$

where:

t : distance from the CBD
v(t) : fraction of land devoted to transportation infrastructure at distance t from the CBD
c_0 : fraction of land devoted to transportation infrastructure at the CBD
e : the end of transportation infrastructure provision

The amount of land devoted to transportation infrastructure is maximum at the CBD, and decreases linearly as one moves away from the CBD. Note that the amount of residential land increases as the transportation capacity decreases.

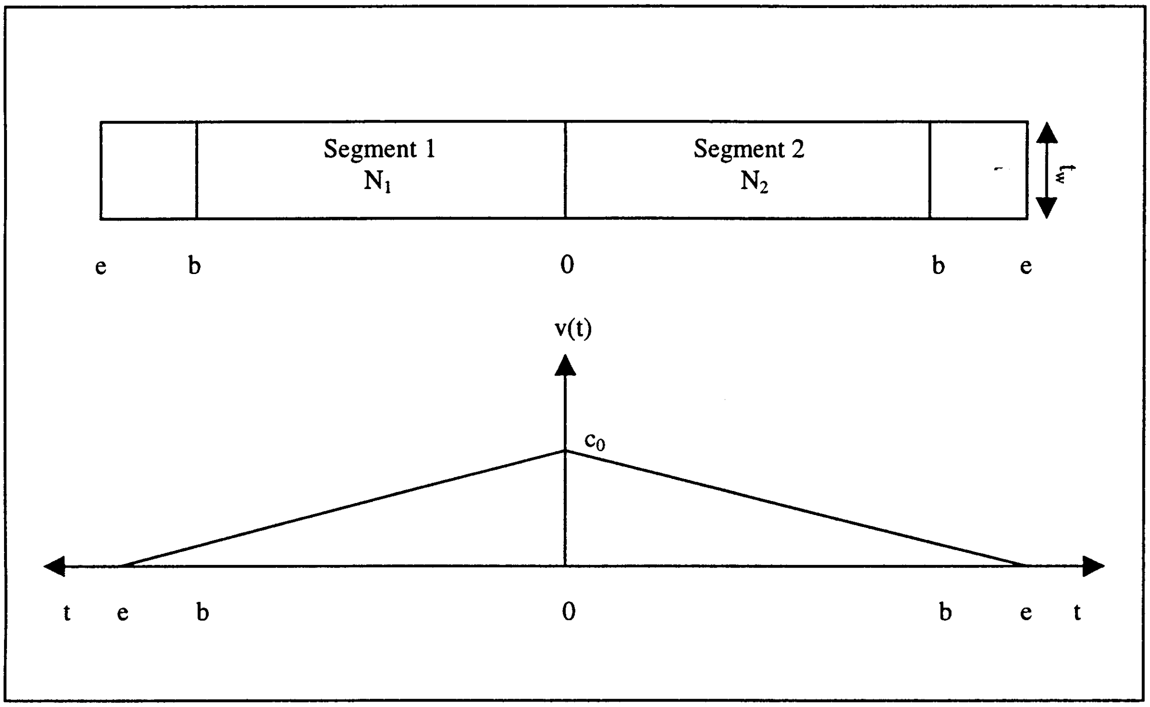


Figure 3-1 The shape of the monocentric city and the transportation capacity function.

3.1.2 Assumptions about households

The city's inhabitants reside in N identical households. Every household has one worker who commutes to work in the CBD. Household income is composed of two components: wage and rental income. Wage income is the annual salary that the worker earns. Rental income is the rent dividend that was collected and redistributed by the city. Four components of household expenditure include housing expenditure, commuting cost, and expenditure for aggregate consumption of other goods and services. Each household faces the utility function and budget constraint as shown in the following expression:

$$\begin{aligned} & U = X \cdot q^{\alpha} \\ & \text{Subject to:} \\ & y + ry = R \cdot q + T + X \end{aligned} \tag{3.2}$$

where:

y : annual salary
ry : rental income
R : land rent
q : land consumption
T : commuting cost
X : aggregate expenditure on consumption of other goods and services

Household utility function has a simple Cobb-Douglas form. When the exponent of land consumption α is small, it could be interpreted as a fraction of income devoted to housing expenditure. Each household attempts to maximize utility by choosing its housing location and amount of land consumption. This is important since at different locations, transportation cost and land rent vary. Thus, each household must adjust land consumption and aggregate expenditure of other goods and services according to its location.

The household utility maximization problem can be solved by Lagrangian method as follows:

$$\text{Max} : U = Xq^{\alpha}$$

$$\text{s.t.} : y + ry = Rq + T + X \qquad : \lambda$$

$$L = Xq^{\alpha} - \lambda(y + ry - Rq - T - X)$$

$$\frac{\partial L}{\partial X} = q^{\alpha} + \lambda = 0, \ -\lambda = q^{\alpha} \qquad \text{(i)}$$

$$\frac{\partial L}{\partial q} = \alpha X q^{\alpha-1} - \lambda R = 0, -\lambda R = \alpha X q^{\alpha-1} \qquad \text{(ii)}$$

Dividing (ii) by (i) yields:

$$R = \frac{\alpha X}{q} \qquad \text{(iii)}$$

From budget constraint,

$$X = y + ry - Rq - T$$

Substituting X into (iii) yields:

$$R = \frac{\alpha(y + ry - Rq - T)}{q}$$

$$R + \alpha R = \frac{\alpha(y + ry - T)}{q}$$

Thus,

$$R = \frac{\alpha}{1+\alpha} \cdot \frac{y + ry - T}{q} \tag{3.3}$$

, and

$$X = \frac{1}{1+\alpha} \cdot (y + ry - T) \tag{3.4}$$

Equation (3.3) and (3.4) are the conditions for the utility maximization for individual household. Let t be the location of the city, which is defined by the distance from the CBD. Since the values of rent, transportation cost, and land consumption vary over location t, the utility maximization problem can be written as:

$$\begin{aligned} \text{Max}: U(t) &= X(t) \cdot (q(t))^{\alpha} \\ &= (y + ry - R(t) \cdot q(t) - T(t)) \cdot (q(t))^{\alpha} \end{aligned}$$

If there is no congestion, then transportation cost and land consumption will be independent. In other words, the transportation cost function T(t) is exogenous, while land consumption q(t) is endogenous to the problem. (This will be explained more in section 3.1.3.) In the spatial equilibrium of household, the household utility must be maximized over location. This problem can be solved as follows:

$$\frac{dU}{dt} = \left(-R\frac{dq}{dt} - q\frac{dR}{dt} - \frac{dT}{dt} \right) \cdot q^{\alpha} + \alpha q^{\alpha-1}\frac{dq}{dt}(y + ry - Rq - T)$$

$$\text{from equation (3.4)}, \; y + ry - Rq - T = X = \frac{1}{1+\alpha} \cdot (y + ry - T),$$

$$\begin{aligned} \frac{dU}{dt} &= \left(-R\frac{dq}{dt} - q\frac{dR}{dt} - \frac{dT}{dt} \right) \cdot q^{\alpha} + \alpha \cdot \frac{1}{1+\alpha} \cdot (y + ry - T) q^{\alpha-1}\frac{dq}{dt} \\ &= \left(-R\frac{dq}{dt} - q\frac{dR}{dt} - \frac{dT}{dt} \right) \cdot q^{\alpha} + R \cdot q \cdot q^{\alpha-1}\frac{dq}{dt} \\ &= \left(-R\frac{dq}{dt} - q\frac{dR}{dt} - \frac{dT}{dt} \right) \cdot q^{\alpha} + R \cdot q^{\alpha}\frac{dq}{dt} \\ &= \left(-q\frac{dR}{dt} - \frac{dT}{dt} \right) \cdot q^{\alpha} \end{aligned}$$

First order condition : $\frac{dU}{dt} = 0$,

therefore,

$$\left(-q\frac{dR}{dt} - \frac{dT}{dt}\right) \cdot q^{\alpha} = 0$$

$$\frac{dR}{dt} = -\frac{1}{q} \cdot \frac{dT}{dt} \tag{3.5}$$

The spatial equilibrium of household requires that household utility be uniform over location. This condition is guaranteed by the differential equation (3.5). The right-hand-side of the equation can be interpreted as transportation cost saving per unit of land. Therefore, equation (3.5) implies that without congestion, the change in land rent with respect to location is compensated by the change in transportation cost saving with respect to location.

3.1.3 Transportation cost and congestion function

According the relationship in equation (3.3) and (3.4), transportation cost is an important determinant of rent and land consumption behavior. The transportation cost is composed of two components, operational cost, which is dependent on the distance traveled and congestion cost, which is proportional to traffic density. In the model of congested city, the congestion cost is large, and the operational costs are assumed to be small. Following Solow [1973], the marginal cost of congestion can be expressed mathematically as follows:

$$T'(t) = k_1 \left(\frac{\int_b^t \frac{t_w (1 - v(z))}{q(z)} dz}{k_2 t_w v(t)} \right)^g \tag{3.6}$$

where:

q(t), q(z): land consumption at point t, z
v(t): fraction of land devoted to transportation infrastructure at point t
k_1, k_2, and g: parameters of the marginal cost function

The marginal cost of congestion is proportional to traffic density, which is defined by the ratio of traffic volume passing a given point and the transportation capacity at that point. The traffic volume is assumed to be proportional to the number of households located beyond point t, which is equal to the nominator of equation (3.4). This introduces the mutual dependence between transportation cost and land consumption, and really complicates the matter of solving the equilibrium condition because both transportation cost and land consumption are now endogenous to the problem. The definition of transportation capacity function v(t) was given in section 3.1. The parameter k_1 and k_2 of the function are chosen to imitate the marginal congestion cost in the real world. The exponent g of the function is more important, as it represents the level of congestability of transportation infrastructure. For the same traffic density, the higher value of g, the larger marginal cost of congestion. This results from the fact that different kinds of transportation infrastructure exhibit different level of congestability. The more intense the congestability of the transportation infrastructure, the larger marginal cost of congestion. For example, it may be shown that marginal cost of using highway is much larger than marginal cost of using public transit, provided that the same volume of traffic is served by the same amount of transportation capacity. In other words, the marginal cost of congestion of an automobile entering a congested highway is far greater than that of an additional passenger using a subway.

The complexity of the transportation cost function results from its integral term. To facilitate the analysis, a new variable n(t) is created with the following definition:

$$n(t) = \int_b^t \frac{t_w(1-v(z))}{q(z)} dz$$

Hence, equation (3.6) can be written as:

$$T'(t) = k_1 \left(\frac{n(t)}{k_2 t_w (1 - v(t))} \right)^g \tag{3.7}$$

By construction of n(t),

$$n'(t) = \frac{-t_w(1-v(t))}{q(t)} \tag{3.8}$$

Substituting this into equation (3.3) and (3.5) yields the following differential equations respectively:

$$R'(t) = \frac{T'(t)}{n'(t)} \cdot t_w (1 - v(t)) \tag{3.9}$$

$$n'(t) = -\frac{\alpha}{1+\alpha} \cdot \frac{y + ry - T(t)}{R(t)} \cdot t_w (1 - v(t)) \tag{3.10}$$

Because of the symmetry of the monocentric model, transportation capacity function v(t) and the boundary b are identical in the left and the right segments. Hence, the system of differential equations (3.7), (3.9), and (3.10) is valid for both segments of the city.

3.1.4 Labor market

It is assumed that the whole labor force of the city is fully employed by the city's commercial sector. In the case of monocentric city, all workers commute to the CBD. Let N_1 be the number of households located in the left segment, and N_2 be the number of households located in the right segment. (See figure 3.1) Then, the following conditions must be true:

$$n(0) - n(b) = N_1 = N_2$$

$$N = N_1 + N_2$$

Due to the symmetry of the city, the transportation capacity function v(t) and the boundary b of each segment are identical. Thus, half of the city's households are located in the left segment of the city, and the second half in the right segment.

3.1.5 Boundary conditions of a monocentric city

In equilibrium, the following boundary conditions must hold for a monocentric city:

$$T(0) = 0$$

$$R(b) = R_A$$

$$n(0) = N_1 = N_2 = \frac{N}{2}$$

$$n(b) = 0$$

The interpretation of boundary conditions is straightforward. The first condition implies that the transportation cost of traveling within (at) the CBD is assumed to be zero. The second condition requires that the household located at the border of the city pay land rent equal to the agricultural land rent R_A. This condition is essential since it guarantees the equilibrium condition of the city. If the household located at the border paid more than agricultural land rent, there would be incentive to develop land just beyond the border, and the border would be extended. The third and fourth conditions follow the definition of household distribution variable n(t). The number of household located outside the CBD equals the total number of household, and the number of household located outside the border equals zero.

3.1.6 Formulation of problem for the equilibrium monocentric model

The equilibrium condition for a monocentric model can be solved from the system of differential equations and the boundary conditions developed in previous section. The full problem summarized here:

The system of differential equations:

$$T'(t) = k_1 \left(\frac{n(t)}{k_2 t_w (1 - v(t))} \right)^g \tag{3.7}$$

$$R'(t) = \frac{T'(t)}{n'(t)} \cdot t_w (1 - v(t)) \tag{3.9}$$

$$n'(t) = -\frac{\alpha}{1+\alpha} \cdot \frac{y + ry - T(t)}{R(t)} \cdot t_w (1 - v(t)) \tag{3.10}$$

The boundary conditions:

$$T(0) = 0$$
$$R(b) = R_A$$
$$n(0) = N_1 = N_2 = \frac{N}{2}$$
$$n(b) = 0$$

The system of equations, (3.7), (3.9), and (3.10) contains three unknown variables, R, n, and T. Theoretically, solving this system of differential equations with the boundary conditions above would yield rent, density, and transportation cost gradients (R(t), q(t), and T(t)) of the equilibrium city. However, the simultaneity problem is complex, and therefore the system of differential equations could not be solved analytically. The numerical solution, which is a practical way to solve the problem, along with its algorithm, will be presented in the next Chapter.

3.2 The General Model of Employment Decentralization

3.2.1 Characteristics of a decentralized city

In general, the characteristics of a decentralized city are not different from the monocentric one. However, for convenience of the analysis, the city is divided into four segments, as shown in Figure 3-2. Segment 1 and 3 are parts of the old monocentric city, while segment 2 and 4 compose the new subcenter. The distance T is an inner border that divides segment 3 and 4, and thus the CBD and the subcenter. The assumption about commuting patterns has also changed from the case of monocentric model. Workers in households located in the odd-numbered segments commute to work in the CBD, while those in the even-numbered segments commute to the subcenter. The location of the subcenter is given by the distance S from the CBD.

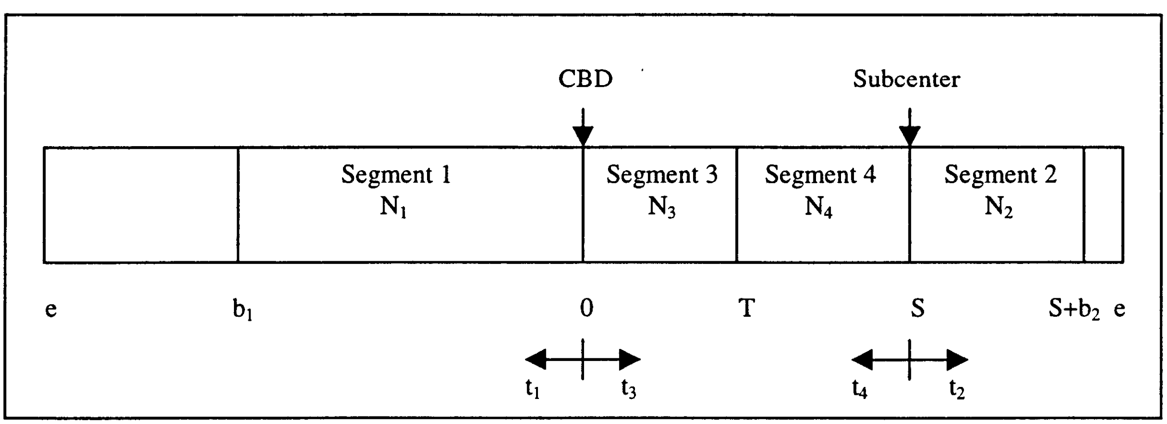


Figure 3-2 The segmentation and location of employment centers of a decentralized city.

3.2.2 Assumptions about households

The assumptions about households in the case of decentralized city are identical to those of the monocentric one, except for the commuting pattern in each segments of the city as mentioned in the previous section. Thus, household utility function and budget constraint remain unchanged, and so does the utility maximization condition. However, due to the change in commuting pattern, the rent and density gradients will be different for each segment. Thus, there are eight separate equations of utility maximization conditions, two equations for each segment of the city:

$$R_i'(t_i) = \frac{T_i'(t_i)}{n_i'(t_i)} \cdot t_w(1 - v_i(t))$$

$$n_i'(t_i) = -\frac{\alpha}{1+\alpha} \cdot \frac{y + ry - T_i(t_i)}{R_i(t_i)} \cdot t_w(1 - v_i(t))$$

$$\text{where } i: \text{segment of the city}$$

The variable t_i stands for the distance from each employment center. For example, t_1 is the distance from the CBD measured to the left, while t_2 is the distance from the subcenter measured to the right. Note that the subscript of t corresponds with the city segment of interest.

3.2.3 Transportation cost and congestion function

Because of the segmentation of the city, there are four transportation capacity functions v(t), one for each segment of the city:

$$v_1(t_1) = c_0 - \frac{c_0 t_1}{e}, 0 \le t_1 \le b_1 \qquad v_2(t_2) = c_0 - \frac{c_0(S + t_2)}{e}, 0 \le t_2 \le b_2$$

$$v_3(t_3) = c_0 - \frac{c_0 t_3}{e}, 0 \le t_3 \le T \qquad v_4(t_4) = c_0 - \frac{c_0(S - t_4)}{e}, 0 \le t_4 \le S\text{-}T$$

The definition of t_i is given in the previous section. Since the borders of segment 3 and 4 are S and S-T. (See section 3.2.1.), the values of t_3 and t_4 are bounded by the following ranges: [0, T] and [0, S-T].

According to the four transportation capacity functions, there will be four different marginal transportation cost functions for the four segments of the city:

$$T_i'(t_i) = k_1 \left(\frac{n_i(t_i)}{k_2 t_w (1 - v_i(t))} \right)^{g_i}$$

where i : segment of the city

3.2.4 Labor market

With the commuting pattern described in section 3.2.1, and assuming that there is no cross commuting, the labor market for each segment of the city is mutually exclusive, and the following conditions must hold (See Figure 3-2.):

$$n_i(0) - n_i(b_i) = N_i$$

$$N = \sum_{i=1}^{4} N_i$$

where i : segment of the city

Since it is assumed that the city is closed, the number of household before and after the decentralization are equal.

3.2.5 Boundary conditions of a decentralized city

In equilibrium, the following boundary conditions must hold for a decentralized city:

Segment 1: $T_1(0) = 0$, $R_1(b_1) = R_A$, $n_1(0) = N_1$, $n_1(b_1) = 0$

Segment 2: $T_2(0) = 0$, $R_2(b_2) = R_A$, $n_2(0) = N_2$, $n_2(b_2) = 0$

Segment 3: $T_3(0) = 0$, $n_3(0) = N_3$, $n_3(T) = 0$

Segment 4: $T_4(0) = 0$, $n_4(0) = N_4$, $n_4(S\text{-}T) = 0$

$R_1(0) = R_2(0)$

$R_3(T) = R_4(S\text{-}T)$

Interpretations of these conditions are straightforward and similar to the one given in section 3.1.5. The last two boundary conditions, however, did not appear in the monocentric case. The first condition implies that in the long run equilibrium, rent at each employment center must be equal; otherwise, firms in the center with higher rent would have incentive to relocate to the other center. The second condition implies that the rents at the common boundary of segment 3 and segment 4 must be equal in order to insure the equilibrium land market.

3.2.6 Formulation of problem for the equilibrium decentralized model

The full problem of equilibrium decentralized model is summarized here:

The system of differential equations:

$$T_i'(t_i) = k_1\left(\frac{n_i(t_i)}{k_2 t_w(1-v_i(t))}\right)^{g_i}$$

$$R_i'(t_i) = \frac{T_i'(t_i)}{n_i'(t_i)}\cdot t_w(1-v_i(t_i))$$

$$n_i'(t_i) = -\frac{\alpha}{1+\alpha}\cdot\frac{y+ry-T_i(t_i)}{R_i(t_i)}\cdot t_w(1-v_i(t))$$

The boundary conditions:

$$n_i(0)-n_i(b_i) = N_i$$

$$N = \sum_{i=1}^{4} N_i$$

Segment 1: $T_1(0) = 0$, $R_1(b_1) = R_A$, $n_1(0) = N_1$, $n_1(b_1) = 0$

Segment 2: $T_2(0) = 0$, $R_2(b_2) = R_A$, $n_2(0) = N_2$, $n_2(b_2) = 0$

Segment 3: $T_3(0) = 0$, $n_3(0) = N_3$, $n_3(T) = 0$

Segment 4: $T_4(0) = 0$, $n_4(0) = N_4$, $n_4(S\text{-}T) = 0$

$R_1(0) = R_2(0)$

$R_3(T) = R_4(S\text{-}T)$

where i : segment of the city, i = 1,2,3, and 4

The employment decentralization model contains a system of 12 equations and 12 unknown variables, R_i, n_i, and T_i. Again, theoretically, solving this system of differential equations with the boundary conditions would yield rent, density , and transportation cost gradients for each segment of the city. However, the problem of simultaneity is so serious that solving it analytically is virtually impossible. The numerical solution is therefore the last resort for the decentralized case. The algorithm of the numerical solution will be presented in the next Chapter.

3.2.7 Firm decentralization process by private decision

In section 3.1, the city is assumed to be monocentric, and the firms are assumed to be immobile. In the firm decentralization model, however, the assumption that firms are immobile is relaxed. If there is no barrier to relocation, firms in the CBD will attempt to reduce their production costs by relocating to a new subcenter. In a rational forward-looking model, firms would select a site (distance from the CBD) to set up a subcenter such that when that center was at its completed size, the firm costs would be minimized. In a myopic model, however, firms would locate the center at the edge of the city since that is where the rent would be minimum for the first firm relocating. Figure 3-4 shows the firm decentralization in a transitional period when rents at the CBD and the subcenter are unequal.

The firms' relocation to the new subcenter continues as long as the firms' costs there are less than at the CBD. Therefore, if wage is assumed to be uniform, firms will move as long as rents at the two centers are different. A long-run equilibrium occurs when rent at the CBD equals the rent at the subcenter (See Figure 3.5) since no firm can lower its costs by moving to the other location. This means the market solution is characterized by the condition of rent equality between the two employment centers.

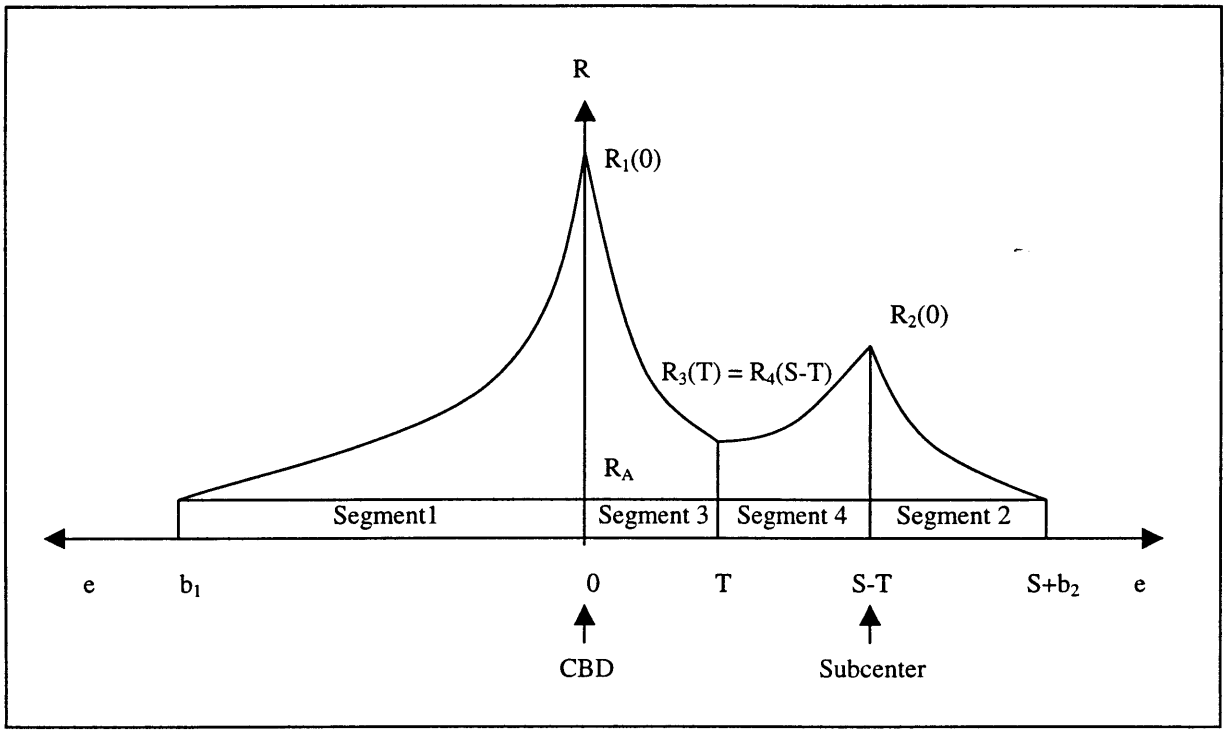


Figure 3-3 Rent gradient of a city with firm decentralization in a transitional period.

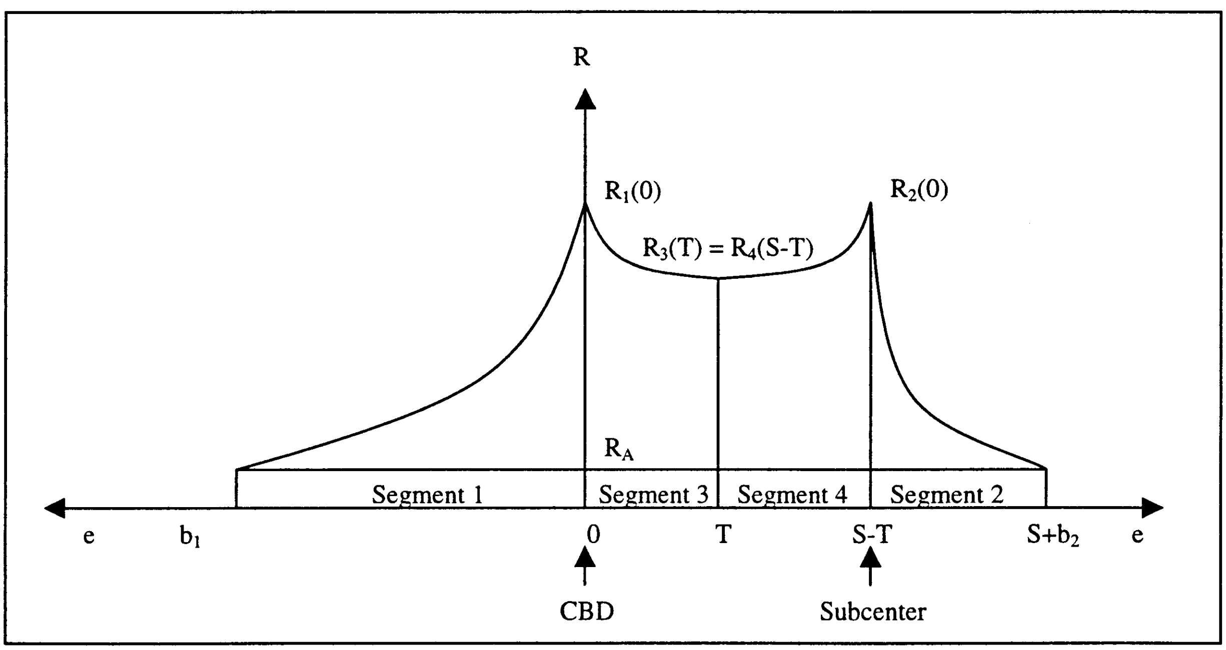


Figure 3-4 Rent gradient of a city with firm decentralization in equilibrium condition.

It is important to note that the provisions of transportation infrastructure serving the two centers are quite different; i.e., the CBD is served by greater transportation capacity than the subcenter. According to equation (3.1), the farther the distance from the CBD, the smaller the amount of land is devoted to transportation infrastructure. The asymmetry of transportation infrastructure will create very different patterns of transportation congestion and land consumption behaviors between the two centers as well as among the city's segments. To illustrate, consider segment 3 and 4 of the decentralized city in Figure 3-2. In segment 3, a large amount of the land is devoted to transportation infrastructure and a relatively small amount of land is left for residential area. Therefore, congestion in segment 3 will be modest relative to segment 1 and 2, and the rent gradient will be relatively flat. In segment 4, transportation capacity provision is rather absurd; i.e., more capacity is provided near the border than near the center. In this case, congestion will be more severe, and rent gradient will be steeper than segment 3, but less than segment 1 and 2 because of the smaller amount of available residential land.

3.2.8 Social optimality of firm decentralization

The social optimality of firm decentralization is characterized by the level of decentralization that leads to maximum aggregate utility of all households. In the case of a monocentric city and a decentralized city in market equilibrium, the household utility does not vary with location; hence, the aggregate utility is simply the product of the common household utility and the number of household in the city:

$$U^{e}_{aggregate} = N \cdot U_0$$

where:

$U^{e}_{aggregate}$: aggregate utility of a city in market equilibrium

U_0: level of household utility that prevails in the city

N: number of household

However, in the transition period of a decentralized city, the utility of household in one center can be different from the other because of the difference in rent at the centers. Thus, the aggregate utility in this case is the sum of the product of the individual household utility and the number of household in each center:

$$U^{t}_{aggregate} = \sum_{i} N_i \cdot U_i$$

where:
$U^{t}_{aggregate}$: aggregate utility of a decentralized city in transition period
U_i: level of household utility that prevails in center i
N_i: number of household in center i

To analyze the social optimality, first consider the monocentric city and the decentralized city in market equilibrium. The decentralization will reduce congestion near and around the CBD because the number of household in segment 1 will decrease. Thus, transportation cost and rent at the CBD of the decentralized city will be smaller than those of the monocentric city. Reduction in transportation cost and rent will lead to higher level of utility and aggregate utility of the decentralized city, and the decentralized city will therefore be more efficient than the monocentric city.

Now, consider the decentralized city in the period of transition. As firms decentralize, transportation cost and rent of the CBD will decline, while those of the subcenter will grow. Assuming fixed income, the worker who works for the first firm that decentralizes will enjoy the highest level of utility since he pays zero transportation cost and low rent. The level of utility of the worker of the decentralized firm represents the level of utility prevailing in the subcenter. However, this level of utility will decline as firms decentralize more to the subcenter since rent and transportation cost will grow. The aggregate utility of the city can be calculated as shown above.

There is no obvious solution, at which level of decentralization the aggregate utility will be maximized. In other words, there is no guarantee that the market decision will lead to maximization of aggregate utility. It is possible that the aggregate utility is maximized during the transition period of firm decentralization. If it is so, the market solution of firm decentralization will not be a social optimal solution, and the social optimality can be achieved when firm costs (rent) and household utility are allowed to be different between the two centers. This, however, would necessitate locational taxes or subsidies for firms in one or both of the centers.

3.3 Summary

This chapter presents a basic approach to modeling a decentralized city when transportation congestion is considered explicitly. First, the basic model of a monocentric city was presented along with modeling assumptions about city characteristics, household, household utility, transportation and congestion cost, commuting-pattern and labor market. The equilibrium conditions of the monocentric city were then discussed, and the formulation of the problem for solving these conditions was shown. Second, the monocentric assumption of the basic model was relaxed. Some modifications of the modeling assumptions due to this change were then described. The equilibrium of conditions of a decentralized city were shown, and the formulation of this problem was presented. Lastly, the process of firm decentralization was illustrated and the social optimality of firm decentralization was discussed in detail.

Chapter 4

Alternative Models of Firm Decentralization and Simulations

Inefficiency of firm decentralization can be divided into two parts. The first part is the inefficiency associated with land market failure due to uninternalized transportation congestion costs. The second part is the inefficiency resulting from the firms' decisions about relocation, which are greatly influenced by the provision of transportation infrastructure. Therefore, to study the efficiency of firm decentralization, four alternative models were created based on assumptions about land market and provision of transportation infrastructure.

Land market failure results from uninternalized externality created by transportation congestion. Since land rent reflects transportation cost, unpriced transportation congestion means lower transportation cost and flatter rent and density gradients than when congestion is priced. Figure 4-1 illustrates the difference between optimal (priced congestion) and market (unpriced congestion) density gradients [Wheaton, 1996]. It should be noted that the effect of unpriced congestion is more pronounced near CBD where congestion is more intense.

Two methods, imposing congestion toll and policy-controlling the density to imitate the optimal density gradient can correct the land market failure. Theoretically, however, there exists a special case when land market functions properly without congestion pricing. If each household is assumed to consume the same amount of land q, congestion pricing will not be necessary since the only way that households can change behavior when congestion toll is imposed is to adjust land consumption. Thus, in this special case, the inefficiency of firm decentralization stems only from the firms' decisions about relocation.

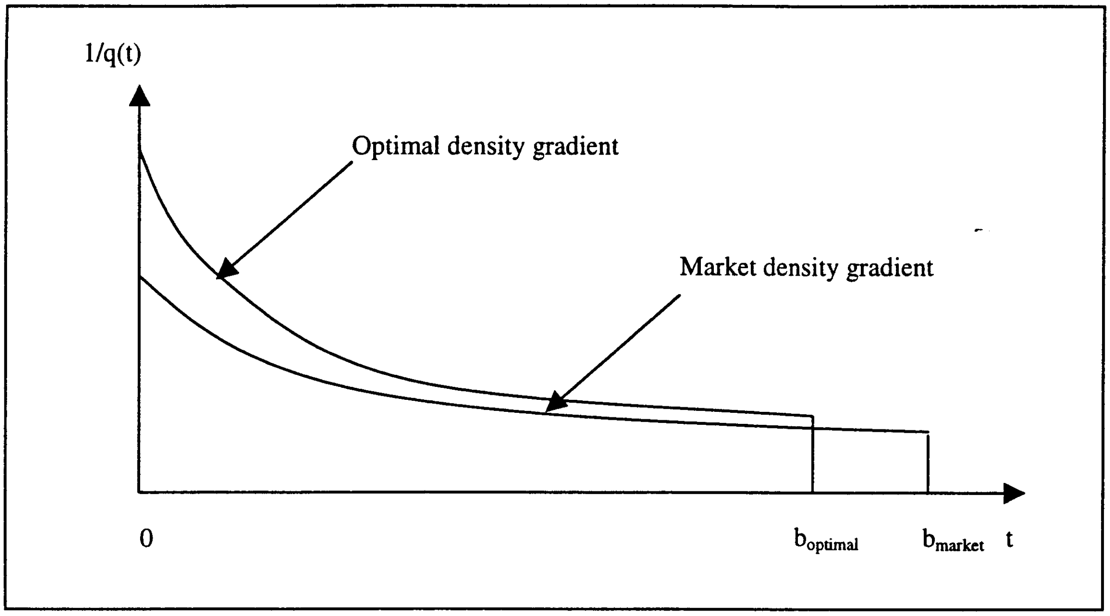


Figure 4-1 Market density gradient vs. optimal density gradient.

Provision of transportation infrastructure greatly affects land market and firms' decisions about relocation. Rent at the CBD of a monocentric city, for example, is far greater than at the edge of the city because transportation (commuting) cost at the CBD is effectively zero, while commuting cost from the city edge to the CBD is very large. In the case of decentralized city, the difference of provision of transportation infrastructure is usually present between the CBD and the subcenter. Such difference will affect commuting costs to each center and lead to the difference in center sizes.

The difference in provision of transportation infrastructure can be distinguished to two types, namely, capacity difference and technology difference. It is usually the case that capacities of highways and roads leading to the CBD are much larger than those leading to a suburban city are. An example of transportation technology difference is the difference in congestability of highway and mass transit. It can be argued that highway is more susceptible to congestion than mass transit with the same amount of capacity. In this aspect, the difference in provision of transportation infrastructure is that the CBD is usually served by extensive and radial mass transit, while the suburban city's transportation infrastructure is usually limited to few bus routes and highways.

According to the assumptions about land market and transportation technology, four models were built to test the efficiency of firms' decision about decentralization. These include a fixed land consumption model with uniform transportation technology, a fixed land consumption model with non-uniform transportation technology, a variable land consumption model with uniform transportation technology, and a variable land consumption model with non-uniform transportation technology. The variation in assumptions has led to different degree of difficulty in simulation of each model. While the basic model requires only a simple algorithm to solve the equilibrium condition, special treatments are required for the more complicated model. These will be discussed in detail later on in this chapter.

In the next section, general characteristics of fixed land consumption models will be discussed, and simulation algorithm will be presented. Then, general characteristics and the simulation algorithm of variable land consumption models will be presented in section 4.2. The generic exogenous variables and exogenous variables specific to each model will described in section 4.3. Lastly, the simulation examples will be presented and discussed in section 4.4.

4.1 Fixed land consumption models

These models focus on the inefficiency of firms' decisions about relocation and ignore the effect of land market by assuming that the amount of land consumption is constant for every household. To solve for an equilibrium city, recall the utility maximization problem presented in section 3.1.2:

$$\begin{aligned}&\text{Max}: U = X \cdot q^{\alpha}\\&\text{Subject to}:\\&y + ry = X + R(t) \cdot q + T(t)\end{aligned}$$

In this special case, households cannot adjust their land consumption. The utility maximization problem above is therefore reduced to only the problem of solving budget constraint. However, equilibrium condition requires that utility be equalized for every household. From the budget constraint, this is equivalent to requiring aggregate consumption X to be identical across households. This can be expressed mathematically in equation (4.1).

$$X = y + ry - R(t) \cdot q - T(t) \tag{4.1}$$

Since wage income (y) and rental income (ry) are identical for every location, equation (4.1) states that the sum of land rent expenditure and commuting cost is uniform across households.

4.1.1 Model I: Fixed land consumption model with uniform transportation technology

This is the simplest model of all four firm decentralization models. The land consumption is assumed to be fixed for all households. As mentioned earlier in section 3.1.1, the capacity of transportation infrastructure is oriented toward carrying commuters to the CBD; i.e., the capacity is the largest at the CBD and is decreasing with the distance from the CBD. Additionally in this case, the transportation infrastructure is provided with identical technology across the city. For example, the city may be served by only road network, which is dispersed across the city in the manner described above, and no major transit service is provided in the CBD. This means the transportation congestability level is uniform everywhere in the city.

The uniform transportation technology means that the exponent g of marginal transportation cost functions are similar for every segment (See definition of segment in section 3.2.1.) in of the city, both the CBD and the subcenter. Hence, the problem formulated in section 3.2.6 is reduced to:

$$T_i'(t_i) = k_1 \left(\frac{n_i(t_i)}{k_2 t_w (1 - v_i(t))} \right)^g$$
$$R_i'(t_i) = \frac{T_i'(t_i)}{n_i'(t_i)} \cdot t_w (1 - v_i(t))$$
$$n_i'(t_i) = -\frac{\alpha}{1+\alpha} \cdot \frac{y + ry - T_i(t_i)}{R_i(t_i)} \cdot t_w (1 - v_i(t))$$

, with unchanged boundary conditions.

Note that the exponent g no longer has subscript i. Because land consumption is fixed and exogenous, $n_i'(t_i)$ and hence $T_i'(t_i)$ and $R_i'(t_i)$ are all determined only by the capacity function $v_i(t_i)$. As a result, most of the gradients are linear, and hence the boundary conditions determine the solution to the equilibrium condition of the decentralized city.

With these simplifications, a rent gradient of each segment of the city can be solved. Consider the households located at the CBD and at the edge of segment 1 of the city, the utility equalization condition in mentioned earlier in this section requires that:

$$R_1(0) + T_1(0) = R_1(b_1) + T_1(b_1)$$

According the assumptions that the rent at the edge of the city equals the agricultural land rent and that the transportation cost at the CBD is zero, the above equation is reduced to:

$$R_1(0) = R_A + T_1(b_1) \tag{4.2}$$

Now consider the households located at the CBD and at the distance t from the CBD, again the utility equalization condition can be satisfied by:

$$R_1(t_1) + T_1(t_1) = R_1(0) + T_1(0)$$

From equation (4.2),

$$R_1(t_1) = R_A + T_1(b_1) - T_1(t_1) \tag{4.3}$$

The same analysis can be applied to segment 2 and the results are:

$$R_2(0) = R_A + T_2(b_2) \tag{4.4}$$

$$R_2(t_2) = R_A + T_2(b_2) - T_2(t_2) \tag{4.5}$$

As for the internal segment 3, consider the households located at distance t from the CBD and at the CBD,

$$R_3(t_3) + T_3(t_3) = R_3(0) + T_3(0)$$

Since $R_3(0) = R_1(0)$, from equation (4.2),

$$R_3(t_3) = R_A + T_1(b_1) - T_3(t_3) \tag{4.6}$$

Similarly, the rent gradient of segment 4 can be determined:

$$R_4(t_4) = R_A + T_2(b_2) - T_4(t_4) \tag{4.7}$$

To guarantee the equilibrium of land market, the rents at the edge of internal segments must be equal. Recall the boundary condition for equilibrium that was discussed earlier in section 3.2.6:

$$R_3(T) = R_4(S - T) \tag{4.8}$$

According to equation (4.8), the internal border T is obviously endogenous. The external border b_1 and b_2 are determined simultaneously, i.e., given the value of one border, the other border can be calculated because of the assumption of fixed land consumption. It should also be noted that due to the symmetry of the city and transportation infrastructure, the length of the land that is occupied (b_1+b_2) is constant for any combination of b_1 and b_2.

4.1.2 Model II: Fixed land consumption model with non-uniform transportation technology

It is typical that the transportation infrastructure serving the CBD differs from that serving suburban employment center, especially in the capacity. However, the difference is not only limited to the capacity, but also the degree that the transportation infrastructure can be congested, which is termed here "the non-uniform transportation technology". Model II was therefore built to analyze the effect of the non-uniformity.

The non-uniform transportation technology can be accounted for by the different exponent g of the marginal transportation cost function. The higher the exponent the more congestable the transportation infrastructure can become. In this model, the transportation infrastructure in the CBD is assumed to be less congestable than that serving the subcenter. Therefore, the city's segment 1 has smaller exponent than all other segments, i.e.:

$$T_i'(t_i) = k_1 \left(\frac{n_i(t_i)}{k_2 t_w v_i(t_i)} \right)^{g_i}$$

where: i:segment of the city

$g_i > 0$, for all i

$g_1 < g_2 = g_3 = g_4$

The derivations of rent gradients in section 4.2.1 (equation (4.2) to (4.8)) are still valid in this model, although the transportation cost function $T_i(t_i)$ of each segment of the city must be changed in accordance with the marginal cost function $T_i'(t_i)$, which varies with the exponent g_i.

4.1.3 Exogenous parameters for fixed land consumption models

Table 4.1 summarizes the exogenous parameters and their default values for fixed land consumption models. The default values of exogenous parameters in reflect approximate characteristics of large metropolitan areas in the U.S. The number of household of 1,000,000 represents an approximate population of 3,000,000. The city width of 2.0 miles was chosen to avoid complexity in commuting pattern. The city edge of 40.0 miles is far enough to assume that transportation capacity there approaches zero. Transportation capacity function is linear, with maximum capacity at the CBD and zero capacity at the edge. The parameter c_0 is the fraction of land devoted to transportation capacity at the CBD. The value of 0.35 is chosen for c_0, as it is about the average value in most U.S. cities [Wheaton, 1996].

The household utility function has a Cobb-Douglas form. The exponent α is the fraction of income spent on housing (land rent). Each household is assumed to consumed fixed amount of land, q. Land consumption is therefore another exogenous parameter, and its value is set to 0.0001 square mile per household. The household income of $40,000 is wage income from a single worker of each household. The total income is the sum of wage income and rental income, which must be determined endogenously from the simulation.

Table 4.1 Summary of exogenous parameters for fixed land consumption models

City Characteristics	*parameter name*	*default value*
Number of households	N	1,000,000
City width (mi.)	t_w	2.0
City edge (mi.)	e	40.0
Distance between centers (mi.)	S	20.0
Transportation capacity function, $v(t) = c_0(e-t)/e$	c_0	0.35
Parameters for transportation cost function		
Common parameters	k_1	80
	k_2	40,000
Parameters specific to city segment		
Model I, for all segment i	g_i	1.0
Model II, for segment 1	g_1	1.0
for all other segments	g_2, g_3, g_4	2.0
Household Characteristics		
Exponent of household utility function, $U=Xq^{\alpha}$	α	0.1
Land consumption (mi.^{2})	q	0.0001
Household income ($/year)	y	40,000
Agricultural land rent ($/mi.^{2})	R_A	4,000,000

Note the difference in the parameters for transportation cost function for model I and model II, which are set as shown. In model I, transportation technology is uniform. Therefore, the transportation congestion functions share similar exponent for every segment of the city, and the value of the exponent is set to 1.0. In model II, however, the transportation technology in each segment of the city is non-uniform. The exponent value of 1.0 is chosen for segment 1, while the value of 2.0 for segments 2, 3, and 4. The exponent value reflects the congestability of transportation infrastructure. The exponent value of 2.0 reflects the congestability level of urban highway, while the value of 1.0 reflects the congestability level of mixed transportation infrastructure, in which both highway and public transit are provided.

4.1.4 Simulation algorithm for fixed land consumption models

Simulation algorithms for model I and II are similar. The only difference is the value of exogenous variables that represent uniformity in transportation technology as mentioned in the previous section. The algorithm of the simulation for model I and II is described as follows:

1. Assign the values of exogenous variables, which are listed in table 4.1.
2. Initialize the value of border of segment 1, $b_1 = e$.
3. From the given values of N, q, $v_1(t_1)$, and $v_2(t_2)$, compute the following: number of household in segment 1, N_1, border of segment 2, b_2, number of household in segment 2, N_2
4. Compute transportation cost and rent gradient of segment 1, using the following routine:
   4.1 Initialize distance from the center (CBD), $t_1 = 0$. From boundary conditions in section 3.2.5, $n_1(t_1) = N_1$, $T_1(t_1) = 0$. (See definition of these variables in section 3.2.)
   4.2 Initialize the total transportation cost of segment 1, $TT_1 = 0$.
   4.3 With a small increment Δt_1, compute the following:
   $$T_1(t_1 + \Delta t_1) = T_1(t_1) + T_1'(t_1) \cdot \Delta t_1$$
   $$n_1(t_1 + \Delta t_1) = n_1(t_1) + n_1'(t_1) \cdot \Delta t_1$$
   $$TT_1(t_1 + \Delta t_1) = TT(t_1) + \frac{T_1(t_1) \cdot (1 - v_1(t_1)) \cdot t_w}{q}$$
   where $T_1'(t_1)$ and $n_1'(t_1)$ can be computed from equation (3.7) and (3.8). Continue until $t_1 = b_1$. Note that q and $v_1(t_1)$ are exogenous, and $v_1(t_1)$ can be determined from the equation given in section 3.2.3.
   4.4 Use equation (4.3) to determine the rent gradient of segment 1.
   4.5 Store the values of rent and density gradients, and the total transportation cost TT_1.
5. Compute transportation cost and rent gradient of segment 2, using the same algorithm as in step 4.
6. Find the equilibrium inner border T and the total transportation cost and rent gradient of segment 3 and 4, using the following routine.
   6.1 Initialize T = S/2.
   6.2 Given the value of T in step 6.1, determine the number of household in segment 3 and 4, N_3 and N_4 from q, $v_3(t_3)$, and $v_4(t_4)$.
   6.3 Use the same algorithm as in step 4.1 through 4.3 to compute the transportation cost gradient of segment 3 and 4.

6.4 Use equation (4.6) and (4.7) to determine the rent gradient of segment 3 and 4, respectively.

6.5 Check the equilibrium condition, $R_3(T) = R_4(S-T)$. If successful, store the values of rent and density gradients, and the total transportation costs TT_3, and TT_4, and go to step 7; otherwise:

6.5.1 If $R_3(T) < R_4(S-T)$, decrease T by a small increment ΔT.

6.5.2 If $R_3(T) > R_4(S-T)$, increase T by a small increment ΔT.

6.5.3 Go to step 6.2. Reiterate until the equilibrium condition is met.

7. Check the market equilibrium condition, $R_1(0) = R_2(0)$. If successful, stop. The center size given by borders, b_1, b_2, and T is the market equilibrium solution. Otherwise, decrease b_1 by as small increment, and go to step 3. Reiterate until the market equilibrium condition is satisfied.

The algorithm given above is for the determination of market equilibrium solution. To determine the optimal solution, few adjustments are needed. The test in step 7 must be replaced by the condition of whether the aggregate transportation cost $(\sum_{i=1}^{4} TT_i)$ is minimized. If it is, then the center size given by borders, b_1, b_2, and T is the optimal solution. Otherwise, increment b_1 and reiterate until the minimum total transportation cost is found.

4.1.5 Simulation examples of fixed land consumption models and discussion

As a basis of comparison, the monocentric model with fixed land consumption is first simulated. The results from simulation are summarized in Table 4.2, and the rent gradient of the monocentric city is displayed in figure 4-2. As mentioned in section 4.1.1, the rent gradient is relatively linear since land consumption is fixed.

Simulation of Model I yields the results that are summarized in Table 4.3. As can be seen rents at the CBD and the subcenter are equalized, and aggregate utility maximized. Further, total transportation cost drops significantly from $21.93 billion for the monocentric city to $10.15 billion for the decentralized one. Also, the aggregate utility increases substantially.

From the results of many simulations with various sets of exogenous parameters, it was concluded that the level of decentralization that represents market equilibrium and the optimal level firm decentralization are identical under the assumptions of Model I. This is a very important finding as it indicates that the private decision about firm decentralization is socially efficient, given an important modeling assumption of Model I, that is, the uniformity of transportation technology across the city.

Figure 4.3 shows the rent gradient as a result of simulation I. As can be seen, the rent gradients of segment 1 and 2 are much steeper than segment 3 and 4. This results from the fact that the ratios of traffic volume to capacity and hence transportation cost of segment 1 and 2 are much greater than those of segment 3 and 4. Note, however, that the rent gradient of segment 4 is steeper than that of segment 3 near the center because the transportation capacity near the subcenter is much smaller than that near the CBD, and therefore, the transportation cost of the subcenter is greater than that of the CBD. For the same reason, this result will be typical for every model simulation.

Simulation II-a and II-b are the simulation of Model II. As summarized in Table 4.3 and 4.4, the results of Simulation II-a represent the market equilibrium of firm decentralization, while those of Simulation II-b represent the optimal firm decentralization. Unlike the results of Simulation I, these simulation results cannot be compared with the monocentric simulation since the transportation infrastructure is provided differently. However, from the results of simulation I, it is quite clear that the decentralized city is much more efficient than the monocentric one because it exploits the capacity in segment 3 and 4 to reduce the transportation congestion that would have occurred in segment 1 and 2.

The different level of firm decentralization in market equilibrium and the socially optimal level indicate the inefficiency of firm decentralization by private decision. From the results of Simulation II-a and II-b, the optimal CBD size is 745,800, while the equilibrium size is 712,320. It is concluded, from many other simulation runs, that there exists the difference between market equilibrium and socially optimal level of decentralization in Model II. On the contrary to Model I, this means that firm decentralization by private decision is not socially efficient given the assumption that transportation technology serving each center is non-uniform.

Figure 4.4 and 4.5 illustrate the rent gradient from Simulation II-a and II-b. As can be seen, the rent gradients of segment 2, 3, and 4 are much more convex than that of segment 1. This can be explained by the difference in transportation technology, that is, transportation infrastructures serving segment 2, 3, and 4, which are much more congestatble than that of segment 1. As a result, the transportation cost and rent in these segments rise more sharply as one moves toward the employment center than those of segment 1 does.

Table 4.2 Summary of results of monocentric simulation-a: fixed density model

Distance t	Rent R(t)/1,000	Density 1/q(t)
0	364,450	10,000
5	307,714	10,000
10	253,445	10,000
15	199,439	10,000
20	143,873	10,000
25	85,212	10,000
30	23,618	10,000
31.7	4,000	10,000

Results:
Border = 31.7 miles
Rental income = $12,251/household
Aggregate utility = 6,292 million
Total transportation cost = $ 21.93 billion

Figure 4-2 Rent gradient: Monocentric simulation with fixed land consumption

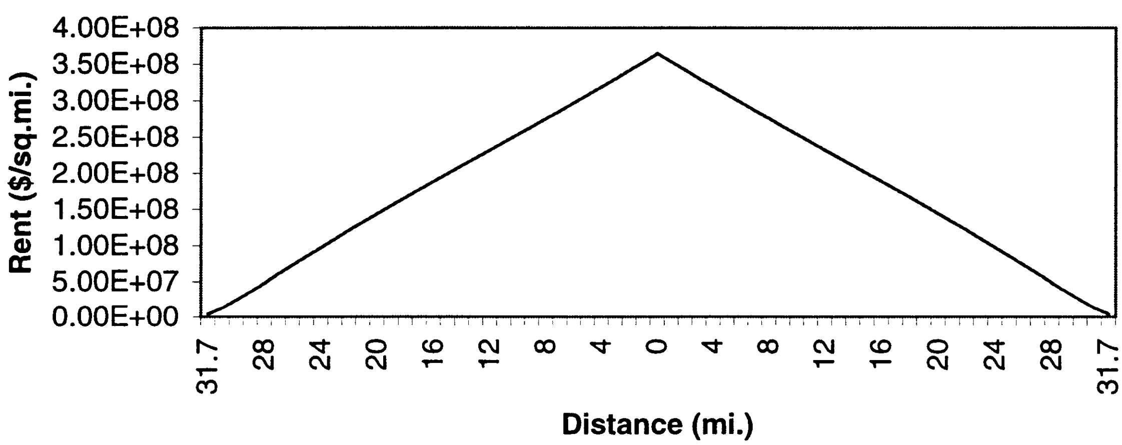


Table 4.3 Summary of results from simulation I

Center	Segment	Population	Border	Utility	Total cost	R(0)	R(b)
CBD	1	408240	26.6	10775.4	5.69E+09	2.26E+08	4.00E+06
	3	159570	11.4	10775.4	3.06E+08	2.26E+08	1.97E+08
Subcenter	2	296760	16.5	10804.8	3.88E+09	2.25E+08	4.00E+06
	4	135430	8.6	10804.8	2.67E+08	2.25E+08	1.97E+08

Other results:
CBD size = 567,810
Subcenter size = 432,190
Rental income = $12,435/household
Aggregate utility = 10,788 million
Total transportation cost = $10.15 billion

Figure 4-3 Rent gradient of Simulation I: Equilibrium and Optimal Firm Decentralization

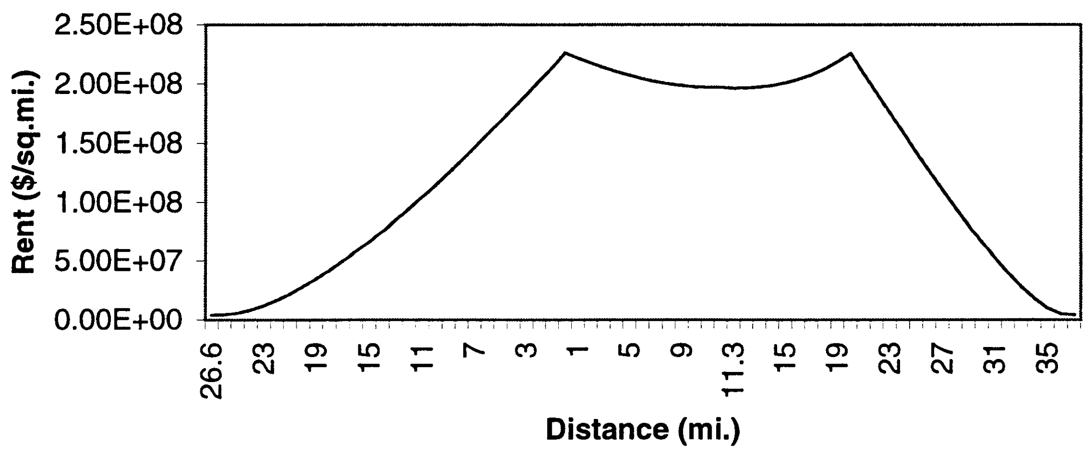


Table 4.4 Summary of results from simulation II-a

Center	Segment	Population	Border	Utility	Total cost	R(0)	R(b)
CBD	1	543720	34.0	6065	1.45E+10	4.50E+08	4.0E+06
	3	168600	12.0	6065	1.73E+09	4.50E+08	3.1E+08
Subcenter	2	161280	9.3	6065	5.12E+09	4.50E+08	4.0E+06
	4	126400	8.0	6065	1.38E+09	4.50E+08	3.1E+08

Other results:
CBD size = 712,320
Subcenter size = 287,680
Rental income = $ 22,355/household
Aggregate utility = 6,065 million
Total transportation cost = $ 22.73 billion

Figure 4-4 Rent Gradient of Simulation II-a: Firm Decentralization in Market Equlibrium

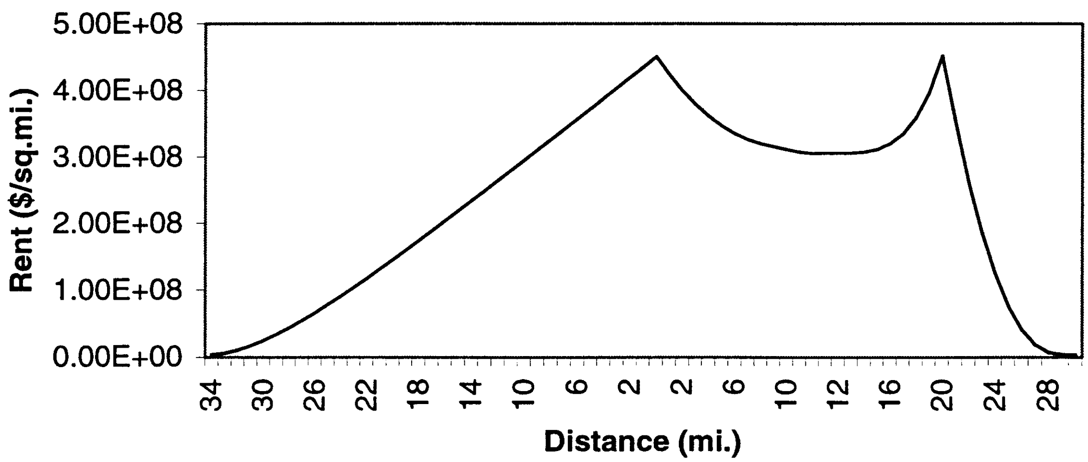
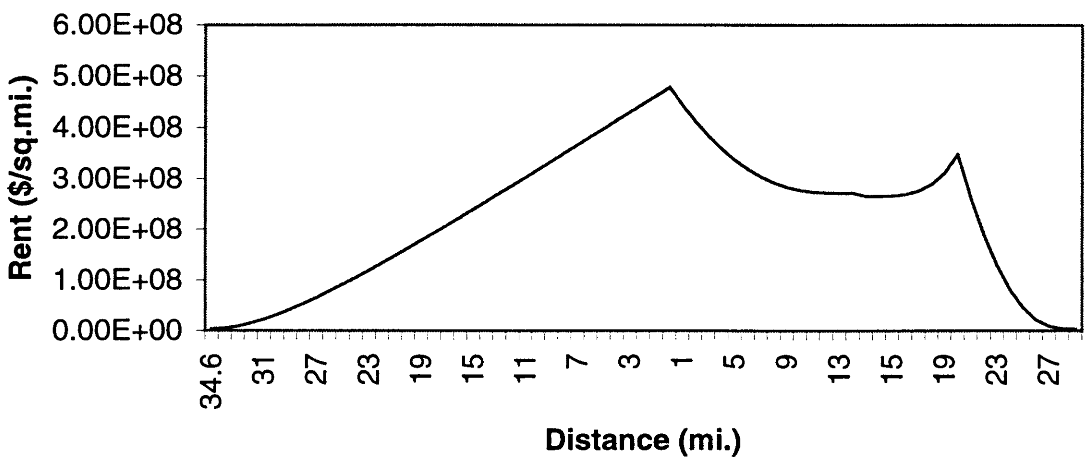


Table 4.5 Summary of results of simulation II-b

Center	Segment	Population	Border	Utility	Total cost	R(0)	R(b)
CBD	1	555890	35.0	4960	1.56E+10	4.78E+08	4.0E+06
	3	189910	13.4	4960	2.84E+09	4.78E+08	2.7E+08
Subcenter	2	149110	8.7	10159	3.65E+09	3.47E+08	4.0E+06
	4	105090	6.6	10159	6.74E+08	3.47E+08	2.7E+08

Other results:
CBD size = 745,800
Subcenter size = 254,200
Rental income = $ 21,717/household
Aggregate utility = 6,275 million
Total transportation cost = $ 22.78 billion

Figure 4-5 Rent gradient of Simulation II-b: Optimal Firm Decentralization



4.2 Variable land consumption models

4.2.1 Model III: Variable land consumption model with uniform transportation technology

This model is a more general version of model I, with the assumption of fixed land consumption relaxed. As in model I, transportation technology for model III is homogeneous and the exponent of marginal transportation cost function is the same for each segment of the city. The major difference is that households can now adjust their land consumption, consumption of other goods, and location to maximize their utility. Hence, the derivations of rent gradient in section 4.2 are no longer valid. The system of equations presented in section 3.2.6 with the given boundary conditions must be solved simultaneously for each segment of the city. However, because of simultaneity problems, the problem cannot be solved analytically. It must therefore be solved numerically with the aid of computer program. Simulation algorithm and examples of this model are provided in the section 4.2.3.

4.2.2 Model IV: Variable land consumption model with non-uniform transportation technology

This model is an extension of model III. In addition to variable land consumption, the technology of transportation infrastructure serving the CBD and the subcenter is also assumed to be different, as in the case of model II. Hence, the exponent of each marginal transportation cost function needs to be adjusted according to the modeling assumption of the difference in transportation technology. According to this additional assumption, the inefficiency of firm decentralization arising from this model will be the result of not only land market failure and private decision about firm relocation but also the asymmetric provision of transportation infrastructure in the two employment centers. However, the problem formulation and the algorithm for simulation of this model will not be different from model III.

4.2.3 Exogenous parameters for variable land consumption models

Table 4.6 summarizes the exogenous parameters and their default values for fixed land consumption models. All default values of exogenous parameters for the variable land consumption models are identical to those for the fixed land consumption models, with one exception, that is, the amount of land consumption per household, q, which is no longer an exogenous variable, but must be determined endogenously from model simulation. See the discussion about the values of these parameters in section 4.1.3.

Table 4.6 Summary of exogenous parameters for variable land consumption models

City Characteristics	*parameter name*	*default value*
Number of households	N	1,000,000
City width (mi.)	t_w	2.0
City edge (mi.)	e	40.0
Distance between centers (mi.)	S	20.0
Transportation capacity function, $v(t) = c_0(e-t)/e$	c_0	0.35
Parameters for transportation cost function		
Common parameters	k_1	80
	k_2	40,000
Parameters specific to city segment		
Model I, for all segment i	g_i	1.0
Model II, for segment 1	g_1	1.0
for all other segments	g_2, g_3, g_4	2.0
Household Characteristics		
Exponent of household utility function, $U=Xq^{\alpha}$	α	0.1
Household income ($/year)	y	40,000
Agricultural land rent ($/mi.^{2})	R_A	4,000,000

4.2.4 Simulation algorithm for variable land consumption models

As in the case of fixed land consumption models, simulation algorithms for model III and IV are not different. Only the values of exogenous variables that represent uniformity in transportation technology need to be adjusted. The algorithm of the simulation for model I and II is described as follows:

1. Set the values of exogenous variables, which are listed in below:
2. Initialization of trial values.

2.1 Initialize the values of number of household in each city segment, N_1, N_2, N_3, and N_4 according to the condition, $\sum_{i=1}^{4} N_i = N$.

2.2 Initialize the values of household utility in each city segment, U_1, U_2, U_3, and U_4.

2.3 Initialize the value of rental income, RY.

3. Compute transportation cost and rent gradient of segment 1, using the following algorithm:

3.1 Initialize distance from the center (CBD), $t_1 = 0$. From boundary conditions in section 3.2.5, $n_1(t_1) = N_1$, $T_1(t_1) = 0$.

3.2 Compute the initial value of $q_1(t_1) = \left[\frac{1}{(1+\alpha)(y + RY - T_1(t_1))}\right]^{\alpha}$.

3.3 Initialize the total transportation cost and total rent of segment 1, $TT_1 = 0$ and $TR_1 = 0$.

3.4 With a small increment Δt_1, compute the following:

$$n_1(t_1 + \Delta t_1) = n_1(t_1) + n_1'(t_1) \cdot \Delta t_1$$
$$T_1(t_1 + \Delta t_1) = T_1(t_1) + T_1'(t_1) \cdot \Delta t_1$$
$$q_1(t_1 + \Delta t_1) = -\frac{\Delta t_1 \cdot (1 - v_1(t_1)) \cdot t_w}{n_1(t_1 + \Delta t_1) - n_1(t_1)}$$
$$R_1(t_1 + \Delta t_1) = \frac{\alpha}{1+\alpha} \cdot \frac{(y + RY - T_1(t_1))}{q_1(t_1 + \Delta t_1)}$$
$$TT_1(t_1 + \Delta t_1) = TT_1(t_1) + \frac{T_1(t_1) \cdot (1 - v_1(t_1)) \cdot t_w}{q_1(t_1 + \Delta t_1)}$$
$$TR_1(t_1 + \Delta t_1) = TR_1(t_1) + R_1(t_1) \cdot (1 - v_1(t_1)) \cdot t_w \cdot \Delta t_1$$

where $T_1'(t_1)$ and $n_1'(t_1)$ can be computed from equation (3.7) and (3.8). Note that $v_1(t_1)$ is exogenous, and can be determined from the equation given in section 3.2.3. Continue until $R_1(t_1) = R_A$.

3.5 If $n_1(t_1) = 0$, stop. Store the values of border $b_1 = t_1$, rent and density gradients, TT_1, and TR_1, and go to step 4. Otherwise,

3.5.1 If $n_1(t_1) > 0$, increase U_1 by a small increment.

3.5.2 If $n_1(t_1) < 0$, decrease U_1 by a small increment.

3.5.3 Go to step 3.1. Reiterate until the condition in step 3.5 is satisfied.

4. Compute transportation cost and rent gradient of segment 2, using the same algorithm as in step3.

5. Find the equilibrium inner border T and the total transportation cost and rent gradient of segment 3 and 4, using the following routine.

5.1 Initialize T = S/2.

5.2 Compute the rent gradient of segment 3.

5.2.1 Initialize distance from the center (CBD), $t_3 = 0$. From boundary conditions in section 3.2.5, $n_3(t_3) = N_3$, $T_3(t_3) = 0$.

5.2.2 Compute the initial value of $q_3(t_3) = \left[\frac{1}{(1+\alpha)(y+RY-T_3(t_3))}\right]^{\alpha}$.

5.2.3 Initialize the total transportation cost and total rent of segment 1, $TT_3 = 0$ and $TR_3 = 0$.

5.2.4 With a small increment Δt_3, compute the following:

$$n_3(t_3+\Delta t_3) = n_3(t_3) + n_3'(t_3)\cdot\Delta t_3$$

$$T_3(t_3+\Delta t_3) = T_3(t_3) + T_3'(t_3)\cdot\Delta t_3$$

$$q_3(t_3+\Delta t_3) = -\frac{\Delta t_3\cdot(1-v_3(t_3))\cdot t_w}{n_3(t_3+\Delta t_3)-n_3(t_3)}$$

$$R_3(t_3+\Delta t_3) = \frac{\alpha}{1+\alpha}\cdot\frac{(y+RY-T_3(t_3))}{q_3(t_3+\Delta t_3)}$$

$$TT_3(t_3+\Delta t_3) = TT_3(t_3) + \frac{T_3(t_3)\cdot(1-v_3(t_3))\cdot t_w}{q_3(t_3+\Delta t_3)}$$

$$TR_1(t_3+\Delta t_3) = TR_3(t_3) + R_3(t_3)\cdot(1-v_3(t_3))\cdot t_w\cdot\Delta t_1$$

where $T_3'(t_3)$ and $n_3'(t_3)$ can be computed from equation (3.7) and (3.8). Note that $v_1(t_1)$ is exogenous, and can be determined from the equation given in section 3.2.3. Continue until $t_3 = T$.

5.2.5 If $n_3(T) = 0$, stop, store the values of rent at inner border $R_3(T)$, rent and density gradients, TT_3, and TR_3, and go to step 5.3. Otherwise,

5.2.5.1 If $n_3(t_3) > 0$, increase U_3 by a small increment.

5.2.5.2 If $n_3(t_3) < 0$, decrease U_3 by a small increment.

5.2.5.3 Go to step 3.1. Reiterate until the condition in step 5.2.5 is satisfied.

5.3 Compute the rent gradient of segment 4, using the same algorithm as in step 5.2.

5.4 Using the stored values of the rents at inner borders, check the equilibrium condition, $R_3(T) = R_4(S-T)$. If successful, store the values of rent and density gradients, and the total transportation costs TT_3, and TT_4 , and go to step 6; otherwise:

5.4.1 If $R_3(T) < R_4(S-T)$, decrease T by a small increment ΔT.

5.4.2 If $R_3(T) > R_4(S-T)$, increase T by a small increment ΔT.

5.4.3 Go to step 5.3. Reiterate until the equilibrium condition is met.

6. Check the aggregate rent with the trial value. If $\sum_{i=1}^{4} \frac{TR_i}{N} = RY$, go to step 7; otherwise, set the value of $RY = \sum_{i=1}^{4} \frac{TR_i}{N}$, and go to step 3. Reiterate until the condition is met.

7. Check the market equilibrium condition, $R_1(0) = R_2(0)$. If successful, stop. The center size given by borders, b_1, b_2, and T is the market equilibrium solution. Otherwise,

7.1 If $R_1(0) > R_2(0)$, decrease the CBD size, $N_1 + N_3$, and increase the subcenter size N_2+N_4.

7.2 If $R_1(0) < R_2(0)$, increase the CBD size, $N_1 + N_3$, and decrease the subcenter size, N_2+N_4.

7.3 Go to step 3. Reiterate until the market equilibrium condition is satisfied.

The algorithm provided above is for the determination of market equilibrium solution. To determine the optimal solution, few adjustments are needed. The test in step 7 must be replaced by the condition of whether the aggregate transportation cost $(\sum_{i=1}^{4} TT_i)$ is minimized. If it is, then the center size given by borders, b_1, b_2, and T is the optimal solution. Otherwise, adjust the center size of the CBD and subcenter, and reiterate until the minimum total transportation cost is found.

4.2.5 Simulation examples of variable land consumption models and discussion

The monocentric model with variable land consumption is first simulated for comparison. The results from the simulation are summarized in Table 4.7, and the rent gradient of the monocentric city is illustrated in figure 4.6. The rent gradient is convex because transportation congestion intensifies as one moves closer to the CBD, and therefore the rent rise slowly further from the CBD and sharply near the CBD. The border of the city shrinks slightly from the case of fixed land consumption. This means that the average amount of land occupied by each household is about the same for the two models. In the case of variable land consumption model, however, the density of household near the CBD (73,392 households/sq.mi.) is much higher than that near the border (1,492 households/sq.mi.).

Simulation of Model III yields the results that are summarized in Table 4.3. As can be seen, the level of firm decentralization from the simulation equalizes rents at the CBD and the subcenter, maximizes aggregate utility, and reduces total transportation over 30 percent, compared with the monocentric city. It was concluded, from many simulation runs with numerous sets of exogenous parameters, that the level of decentralization that represents market equilibrium and the optimal level firm decentralization are identical under the assumptions of Model III. This result substantiates the conclusion of Model I; that is, the private decision about firm decentralization is socially efficient, given the uniformity of transportation technology across the city.

The rent gradient as a result of Simulation III is illustrated in Figure 4.7. Unlike that of Simulation I, the steepness of rent gradients of all segments of the city are comparable. Due to the variation of land consumption, the number of households in the segment where large transportation capacity is available can be adjusted, and hence the ratios of traffic volume to capacity and transportation cost are somewhat similar for every segment of the city.

Model IV is simulated in Simulation IV-a and IV-b. The results of Simulation IV-a represent the market equilibrium of firm decentralization, while those of Simulation IV-b represent the optimal firm decentralization. Because of the difference in technology of transportation infrastructure, these simulation results cannot be compared with those of the monocentric simulation.

The simulation results show that the optimal CBD size is 719,300, while the equilibrium size is 695,700. As in the case of Model II, it is concluded, from many other simulation runs, that the level of firm decentralization in market equilibrium and the socially optimal level are different under the assumptions of Model IV. This result also substantiates the conclusion that firm decentralization by private decision is not socially efficient given the assumption that transportation technology serving each center is non-uniform.

The rent gradients from Simulation IV-a and IV-b are displayed in Figure 4.8 and 4.9. As can be seen, the rent gradients from both simulations look like those of two separate cities. Rents near the inner border between segment 3 and 4 are very low. The rents in those segments rise very slowly near the inner border, but grow very rapidly near the centers. This can be explained by the difference in transportation technology, that is, transportation infrastructures serving segment 2, 3, and 4 are much more congestatble than that serving segment 1.

Table 4.7 Summary of results of monocentric simulation-b: variable density model

Distance t	Rent R(t)/1000	Density 1/q(t)
0	289,618	73,392
5	74,683	21,445
10	29,883	9,321
15	15,070	5,001
20	8,788	3,062
25	5,733	2,077
30	4,199	1,564
31.2	4,000	1,492

Results:
Border = 31.2 miles
Rental income = $3,838/household
Aggregate utility = 12,870 million
Total transportation cost = $ 5.28 billion

Figure 4-6 Rent gradient of monocentric simulation with variable land consumption

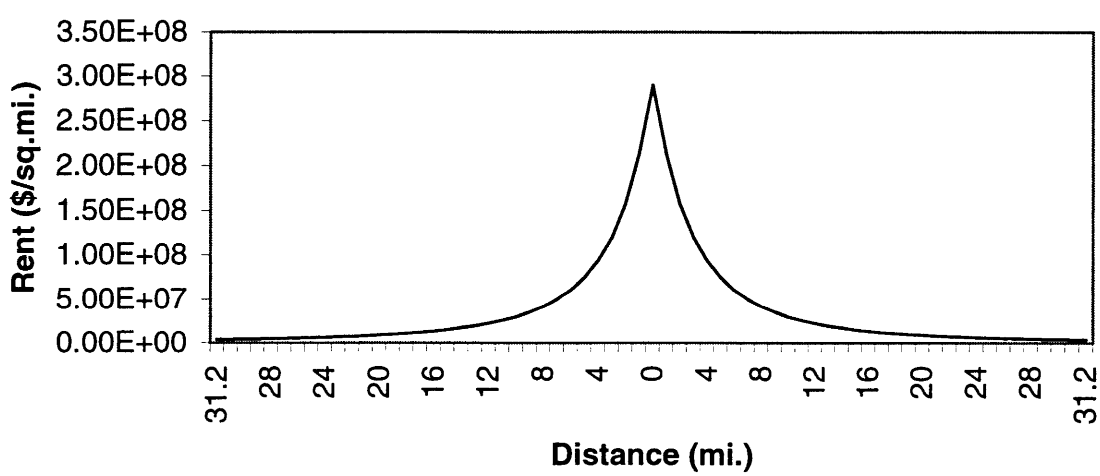


Table 4.8 Summary of results from simulation III

Center	Segment	Population	Border	Utility	Total cost	R(0)	R(b)
CBD	1	302400	33.0	14220	1.44E+09	1.13E+08	4.0E+06
	3	251350	11.5	14220	5.92E+08	1.13E+08	4.1E+07
Subcenter	2	232350	18.9	14220	1.02E+09	1.13E+08	4.0E+06
	4	213900	8.5	14220	5.01E+08	1.13E+08	4.1E+07

Other results:
CBD size = 553,750
Subcenter size = 446,250
Rental income = $3,618/household
Aggregate utility = 14,220 million
Total transportation cost = $3.55 billion

Figure 4-7 Rent gradient of Simulation III: Equilibrium and Optimal Firm Decentralization

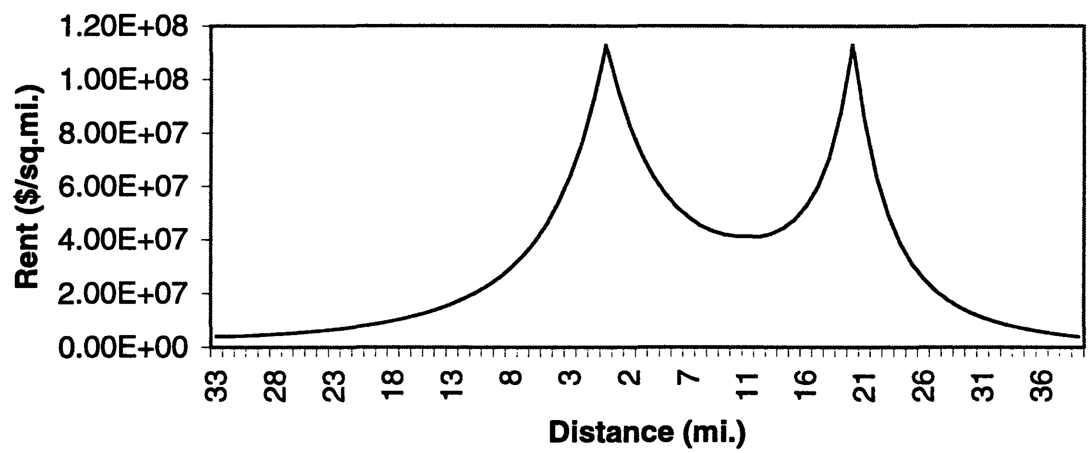


Table 4.9 Summary of results from simulation IV-a

Center	Segment	Population	Border	Utility	Total cost	R(0)	R(b)
CBD	1	481900	33.1	12960	2.61E+09	2.70E+08	4.0E+06
	3	213800	12.0	12960	1.22E+09	2.71E+08	1.9E+07
Subcenter	2	150200	13.1	12960	9.93E+08	2.72E+08	4.0E+06
	4	154100	8.0	12960	9.23E+08	2.72E+08	2.0E+07

Other results:
CBD size = 695,700
Subcenter size = 304,300
Rental income = $3,428/household
Aggregate utility = 12,960 million
Total transportation cost = $5.74 billion

Figure 4-8 Rent gradient of Simulation IV-a: Firm Decentralization in Market Equilibrium

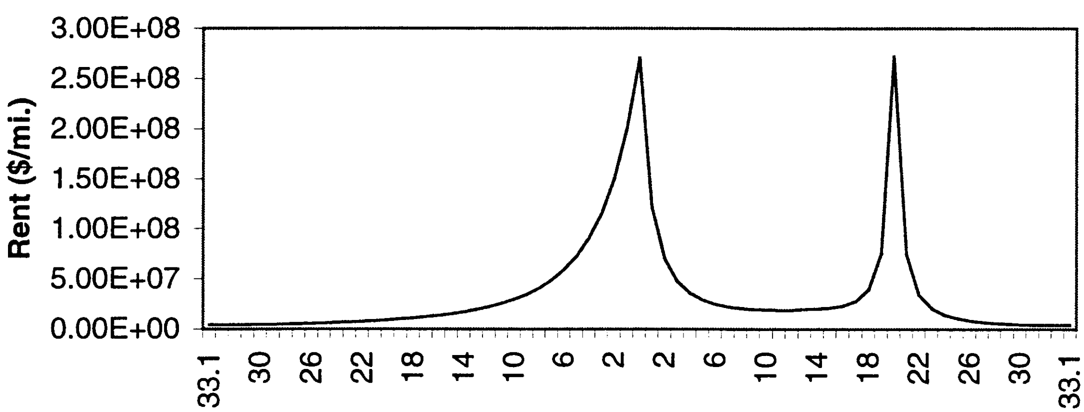
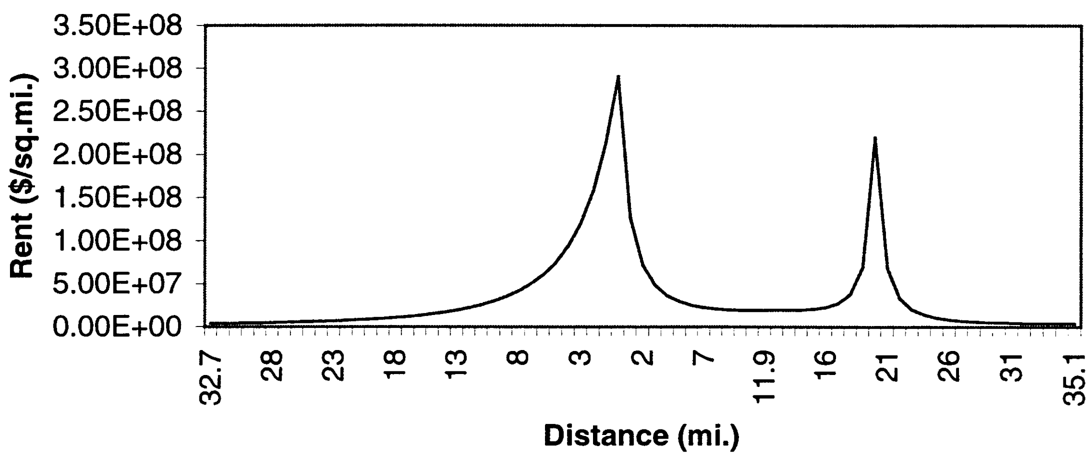


Table 4.10 Summary of results from simulation IV-b

Center	Segment	Population	Border	Utility	Total cost	R(0)	R(b)
CBD	1	500000	32.7	12885	2.71E+09	2.90E+08	4.0E+06
	3	219300	11.9	12885	1.27E+09	2.89E+08	1.9E+07
Subcenter	2	138500	15.1	13247	9.57E+08	2.20E+08	4.0E+06
	4	152200	8.1	13247	8.09E+08	2.19E+08	1.9E+07

Other results:
CBD size = 719,300
Subcenter size = 290,700
Rental income = $3,451/household
Aggregate utility = 12,988 million
Total transportation cost = $5.74 billion

Figure 4-9 Rent gradient of Simulation IV-b: Optimal Firm Decentralization



Chapter 5

Conclusion

5.1 Summary of Important Findings and Their Implications on Public Policy

The results from model simulations show that a decentralized city is more efficient than a centralized one. This is so because the decentralization increases the utilization of land that is available further out from the central area where available amount of land is limited. However, the decentralization will result in dispersion of congestion throughout a wider area than in the case of centralized city although this will not significantly affect transportation cost as the decentralized city also utilizes transportation infrastructure in the outer area. On the other hand, households consume more land because of the greater availability of land in the outer area. With a slight change in transportation cost, increase in household land consumption will lead to higher level of aggregate utility of the city. Therefore, it could be concluded that a decentralized city is more socially efficient than the monocentric one.

The efficiency of private decision about decentralization depends on how transportation infrastructure is provided. The private decision, which is affected by transportation congestion and land rent, is efficient when the technology of transportation infrastructure that is provided across the city is uniform. This condition means that the whole metropolitan area is served by only a single predominant mode of transportation, which is most likely highway network and automobile.

If technology of the infrastructure serving the CBD and the suburban areas is non-uniform, however, the private decision about decentralization would not be efficient, given the presence of unpriced transportation congestion. An example of non-uniform transportation technology is a city of which the CBD is served by both highway network and extensive rapid transit system, and the subcenter is served by only private mode of transportation, i.e. automobile. In this case, the simulation results show that too many firms will decentralize to employment subcenters, and the equilibrium subcenter size will be larger than the optimal one.

It was also found that when there is no land market failure due to unpriced transportation congestion (Fixed land consumption models), the utility gained from optimal decentralization is more significant than when there exists unpriced congestion (Variable land consumption models). In other words, the inefficiency of private decision about decision is more observable. This could probably be explained by the following reasons. When there is no land market failure, the utility gained from optimal decentralization, that is the reallocation of households from the subcenter to the CBD, is purely the effect of decentralization itself. However, when there is unpriced congestion, the reallocation of households from the subcenter to the CBD would worsen the uninterni\alized congestion externality in the subcenter. Thus, the utility gained from optimal decentralization is offset by the worsening congestion externality, and the magnitude of utility gained is smaller in this case than the previous one.

The public policy implied by the findings of this thesis is very important, particularly in the cities with radial public transit system intended to serve mainly the CBD, such as Boston. According to the finding, this type of provision of transportation infrastructure would lead to the level of decentralization that is not socially desirable since too many firms would decide to relocate to suburban areas. Therefore, in order to bring the level of decentralization to optimality, the local government needs to influence the private decision by subsidizing firms that are located in the CBD for their higher cost to keep them from decentralizing. On the contrary, in the cities where public transit system is of little importance, such as Los Angeles, the local government needs not to interfere with private decision about relocation because the level of decentralization in market equilibrium and the optimal level are the same.

5.2 Further research

There are many possible ways to generalize the models created in this thesis in order to obtain a more refined analysis. Several strong and unrealistic assumptions could be relaxed; for example,

- The assumption of fixed household income is difficult to defend and could greatly affect the simulation results. Wage capitalization of locational value was proved to be important especially in the case of employment decentralization; hence it should also be considered explicitly besides rent capitalization.
- The assumption of homogeneous household and perfect job-match. This assumption results in the absence of cross commuting, which should be allowed to a certain extent.
- The modeling of transportation cost function could be improved by considering also the distance cost explicitly. In addition, the transportation capacity function could be adjusted to replicate the provision of transportation infrastructure in the city.
- The assumption about availability of land in suburban area is not so realistic. The current assumption is that the amount of land available for residential development increases linearly with the distance from the CBD. The limitation of transportation infrastructure should somehow affect the availability of land.
- The assumption of linear shape of the city could be generalized to consider circular the city. This would replicate real cities better, but it would be difficult to model commuting pattern.
- Decisions of firms about decentralization could be better modeled by introducing the production cost function of firms, which would be the function of land rent and wage. This would require relaxing the assumption that firm consume spaceless land and that household income is fixed.
- The current model can not give the exact optimal level of firm decentralization. After all the generalizations, however, it might be possible to determine the actual level of decentralization that is socially optimal for a real city.

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