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Fixed points of quantum gravity in higher dimensions

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Abstract. We study quantum gravity in more than four dimensions by means of an exact functional flow. A non-trivial ultraviolet fixed point is found in the Einstein–Hilbert theory. It is shown that our results for the fixed point and universal scaling exponents are stable. If the fixed point persists in extended truncations, quantum gravity in the metric field is asymptotically safe. We indicate physical consequences of this scenario in phenomenological models with low-scale quantum gravity and large extra dimensions.

Keywords: Quantum gravity, Extra dimensions

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Introduction

In this contribution, we discuss new ultraviolet fixed points for quantum gravity in higher dimensions [1].

Gravity in more than four space-time dimensions has received considerable interest in recent years. The possibility that the fundamental Planck mass – within a higher dimensional setting – may be as low as the electroweak scale [2, 3, 4] has stimulated extensive model building and numerous investigations aiming at signatures of extra spatial dimensions ranging from particle collider experiments to cosmological and astrophysical settings. Central to these scenarios is that gravity lives in higher dimensions, while standard model particles are often confined to the four dimensional brane. In part, these models are motivated by string theory, where additional spatial dimensions arise naturally. Then string theory would, at least in principle, provide for a short distance definition of these theories which presently have to be considered as effective rather than fundamental ones. In the absence of an explicit ultraviolet completion, gravitational interactions at high energies including low scale gravity can be studied with effective field theory or semi-classical methods, as long as quantum gravitational effects are absent, or suppressed by some ultraviolet cutoff of the order of the fundamental Planck mass, *e.g.* [5].

One may then wonder whether a quantum theory of gravity in the metric degrees of freedom can exist in four and more dimensions as a cutoff-independent, well-defined and non-trivial local theory down to arbitrarily small distances. It is generally believed that the above requirements imply the existence of a non-trivial ultraviolet fixed point under the renormalisation group, governing the short-distance physics. The corresponding fixed point action then provides a microscopic starting point to access low energy phenomena of quantum gravity. This ultraviolet completion does apply for quantum gravity in the vicinity of two dimensions, where an ultraviolet fixed point has been identified with ϵ -expansion techniques [6, 7, 8]. In the last couple of years, a lot of efforts have been put forward to access the four-dimensional case, and a number of independent studies have detected an ultraviolet fixed point using functional and renormalisation group methods in the continuum [9, 10, 11, 12, 13, 14, 15, 16, 17, 18] and Monte Carlo simulations on the lattice [19, 20].

Continuity in the dimension suggests that a non-trivial fixed point – if it exists in four dimensions and below – should persist at least in the vicinity and above four dimensions. Furthermore, the critical dimension of quantum gravity – the dimension where the gravitational coupling has vanishing canonical mass dimension – is two. For any dimension above the critical one, the mass dimension of the gravitational coupling is negative. Hence, from a renormalisation group point of view, four dimensions are not special. More generally, one expects that the local structure of quantum fluctuations, and hence local renormalisation group properties of quantum theories of gravity, are qualitatively similar for all dimensions above the critical one, modulo topological effects in specific dimensions.

In this talk, we summarise our results for ultraviolet fixed points of gravity in more than four dimensions, also extending the results given previously in [15, 1, 17]. For technical details, see [27, 28].

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Flows of quantum gravity

We perform a fixed point search for quantum gravity in more than four dimensions [15] (see also [13]). An ultraviolet fixed point, if it exists, should already be visible in the purely gravitational sector, to which we confine ourselves. Matter degrees of freedom and gauge interactions can equally be taken into account. We employ a functional renormalisation group based on a cutoff effective action Γ_k for the metric field [9, 10, 11, 12, 13, 14, 15, 16, 17, 21], see [22] and [23] for reviews in scalar and gauge theories. The functional Γ_k comprises momentum fluctuations down to the momentum scale k , interpolating between Γ_Λ at some reference scale $k = \Lambda$ and the full quantum effective action at $k \rightarrow 0$. The variation of the effective action with the cutoff scale ($t = \ln k$) is given by an exact functional flow

$$\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k. \quad (1)$$

The trace is a sum over fields and a momentum integration, and R_k is a momentum cutoff for the propagating fields. The flow relates the change in Γ_k with a loop integral over the full propagator. By construction, the flow (1) is well-defined (finite, no poles), and, together with the boundary condition Γ_Λ , defines the theory. In renormalisable theories, the cutoff Λ can be removed, $\Lambda \rightarrow \infty$, and $\Gamma_\Lambda \rightarrow \Gamma_*$ remains well-defined for arbitrarily short distances. In perturbatively renormalisable theories, Γ_* is given by the classical action, *e.g.* in QCD. In perturbatively non-renormalisable theories, proving the existence (or non-existence) of a short distance limit Γ_* is more difficult. A fixed point action qualifies as a fundamental theory if it is connected with the correct long-distance behaviour by a well-defined renormalisation group trajectory Γ_k .

To solve the flow (1), we restrict Γ_k to a finite set of operators, which can systematically be extended. Highest reliability and best convergence behaviour is achieved through an optimisation of the momentum cutoff [24, 25, 26, 27, 29]. We employ the Einstein-Hilbert truncation where the effective action, apart from a classical gauge fixing and the ghost term, is given as

$$\Gamma_k = \frac{1}{16\pi G_k} \int d^D x \sqrt{g} [-R(g) + 2\bar{\lambda}_k]. \quad (2)$$

In (2), g denotes the determinant of the metric field $g_{\mu\nu}$, $R(g)$ the Ricci scalar, G_k the gravitational coupling constant, and $\bar{\lambda}_k$ the cosmological constant. For k -independent G_k and $\bar{\lambda}_k$, (2) reduces to the standard Einstein-Hilbert theory. The dimensionless renormalised gravitational and cosmological constants are

$$g = k^{D-2} G_k \equiv k^{D-2} Z_{N,k}^{-1} \bar{G}, \quad \lambda = k^{-2} \bar{\lambda}_k, \quad (3)$$

where \bar{G} and $\bar{\lambda}_\Lambda$ denote the couplings at some reference scale $k = \Lambda$, and $Z_{N,k}$ the wave function renormalisation factor for the newtonian coupling. Their flows are given by

$$k \partial_k g \equiv \beta_g = [D - 2 + \eta_N] g, \quad k \partial_k \lambda \equiv \beta_\lambda \quad (4)$$

with $\eta_N(\lambda, g) = -k \partial_k \ln Z_{N,k}$ the anomalous dimension of the graviton. Fixed points correspond to the simultaneous vanishing of (4) at values (λ_*, g_*) of the couplings. Explicit expressions for (4) and η follow from (1) by projecting onto the operators in (2), using background field methods. We employ momentum cutoffs with tensorial structures introduced in [9] (Feynman gauge) and in [11] (harmonic background field gauge), and optimised scalar cutoff function (see below). For explicit analytical flow equations, see [15, 17]. The ghost wave function renormalisation is set to unity. We neglect the running of the gauge fixing parameter α and study the α -dependence of the results. It is known that $\alpha = 0$ is a non-perturbative fixed point of the flow (1), independently of the truncation [30]. Diffeomorphism invariance is controlled by modified Ward identities [9], similar to those employed for non-abelian gauge theories [31].

Three comments are in order. Firstly, the cosmological constant λ obeys $\lambda < \lambda_{\text{bound}}$, where $2\lambda_{\text{bound}} = \min_{q^2/k^2} [(q^2 + R_k(q^2))/k^2]$ depends on the momentum cutoff $R_k(q^2)$ and $q^2 \geq 0$ denotes (minus) covariant momentum squared. For gauge fixing parameters $\alpha > 1$, we have $\lambda_{\text{bound}} \rightarrow \alpha \lambda_{\text{bound}}$, see [15, 17]. Elsewise the flow (1), (4) could develop a pole at $\lambda = \lambda_{\text{bound}}$. The property $\lambda < \lambda_{\text{bound}}$ is realised in any theory where $\Gamma_k^{(2)}$ develops negative eigenmodes, and simply states that the inverse cutoff propagator $\Gamma_k^{(2)} + R_k$ stays positive (semi-) definite [32]. Secondly, we detail the momentum cutoffs used in the numerical analysis. We introduce $R_k(q^2) = q^2 r(y)$, where $y = q^2/k^2$. Within a few constraints these regulators can be chosen freely [24]. Presently, we use $r_{\text{mexp}} = b/((b+1)^y - 1)$, $r_{\text{exp}} = 1/(\exp c y^b - 1)$, $r_{\text{mod}} = 1/(\exp[c(y + (b-1)y^b)/b] - 1)$, with $c = \ln 2$, $r_{\text{pow}} = y^{-b}$ and $r_{\text{opt}} = b(1/y - 1)\theta(1 - y)$,

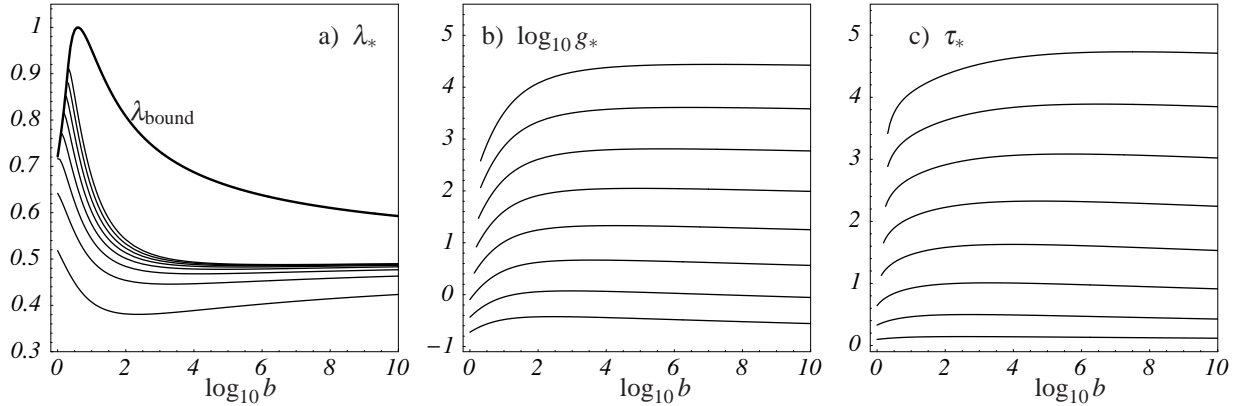


FIGURE 1. Fixed points in $D = 4 + n$ dimensions with $n = 0, \dots, 7$ (thin lines from bottom to top) as a function of the cutoff parameter b for the momentum cutoff $r_{\text{mexp}}(b)$ in Feynman gauge [9]; a) the cosmological constant λ_* and λ_{bound} (thick line); b) the gravitational coupling g_* ; c) the dimensionless product $\tau_* = \lambda_*(g_*)^{2/(D-2)}$.

with $b > 0$ for r_{mexp} and r_{opt} and $b \geq 1$ for the others. These cutoffs include the sharp cutoff for ($b \rightarrow \infty$) and asymptotically smooth Callan-Symanzik type cutoffs $R_k \sim k^2$ as limiting cases. The larger the parameter b , for each class of cutoffs, the ‘sharper’ the corresponding momentum cutoff, i.e. the narrower the window of momentum modes contributing at each infinitesimal integration step in the renormalisation flow. Thirdly, we note that physical quantities, derived from the full, integrated flow (1), are independent of the cutoff R_k . Truncations, however, may induce artificial cutoff dependences. It is vital to establish which cutoffs lead to the most reliable results within a given expansion. We employ a criterion based on the stability of the flow (1), which provides optimised choices for R_k (or the parameter b). In the present case, stability is controlled by the parameter $1 - \lambda_k/\lambda_{\text{bound}}$, which reads $1 - \lambda_*/\lambda_{\text{bound}}$ on a fixed point (see [27] for details).

Results

Next we summarise our results for non-trivial ultraviolet fixed points $(g_*, \lambda_*) \neq (0, 0)$ of (4), the related universal scaling exponents, trajectories connecting the fixed point with the perturbative infrared domain, the graviton anomalous dimension, cutoff independence, gauge-fixing independence and the stability of the underlying expansion. We restrict ourselves to $D = 4 + n$ dimensions, with $n = 0, \dots, 7$.

Existence. A real, non-trivial, ultraviolet fixed point exists for all dimensions considered. Fig. 1 shows our results for λ_* and $\log_{10} g_*$ based on a momentum cutoff in Feynman gauge [9] and scalar cutoff $r_{\text{mexp}}(b)$, with parameter b up to 10^{10} . For small b , their numerical values depend strongly on b , while for large b , they become independent thereof. For $D > 6$, the fixed point approaches the boundary value $\lambda_* \rightarrow \lambda_{\text{bound}}$ for some $b_{\text{bound}} > 1$, beyond which the momentum cutoff is no longer applicable. Results similar to Fig. 1 are found for all momentum cutoffs indicated above [27].

Continuity. The fixed points λ_* and g_* , as a function of the dimension, are continuously connected with their perturbatively known counterparts in two dimensions [10, 11, 15, 17].

Uniqueness. This fixed point is unique in all dimensions considered.

Positivity of the gravitational coupling. The gravitational coupling constant only takes positive values at the fixed point. Positivity is required at least in the deep infrared, where gravity is attractive and the renormalisation group running is dominated by classical scaling. Since the flow β_g in (4) is proportional to g itself, and stays finite for small g , it follows that renormalisation group trajectories cannot cross the line $g = 0$ for any finite scale k . Therefore the sign of g is fixed along any trajectory, and positivity in the infrared requires positivity already at an ultraviolet fixed point.

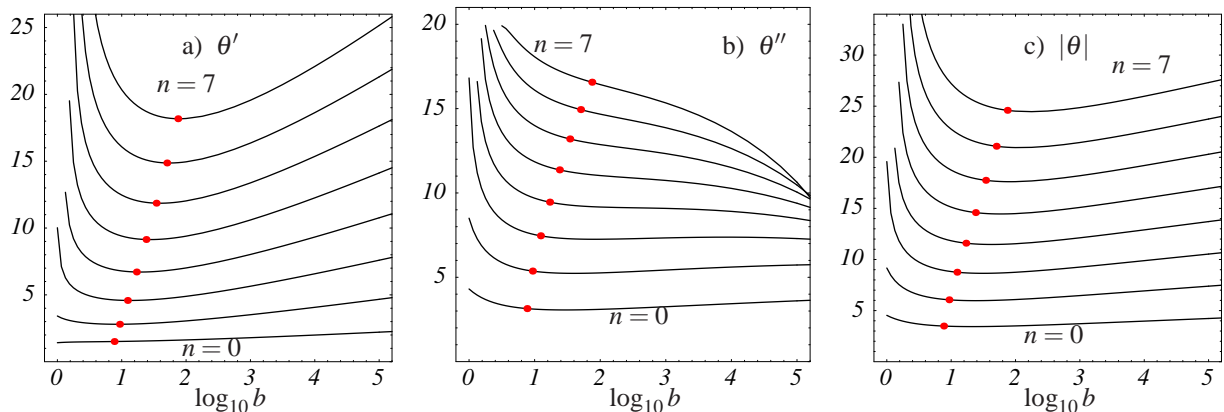


FIGURE 2. Eigenvalues of the stability matrix at criticality in $D = 4 + n$, $n = 0, \dots, 7$ (from bottom to top). The results obtaining from optimised cutoffs are indicated by the red dots. a) Real part θ' ; b) imaginary part θ'' ; c) modulus $|\theta|$.

At the critical dimension $D = 2$, the gravitational fixed point is degenerate with the gaussian one $(g_*, \lambda_*) = (0, 0)$, and, consequently, takes negative values below two dimensions.

Positivity of the cosmological constant. At vanishing λ , β_λ is generically non-vanishing. Moreover, it depends on the running gravitational coupling. Along a trajectory, therefore, the cosmological constant can change sign by running through $\lambda = 0$. Then the sign of λ_* at an ultraviolet fixed point is not determined by its sign in the deep infrared. We find that the cosmological constant takes positive values at the fixed point, $\lambda_* > 0$, for all dimensions and cutoffs considered. In pure gravity, the fixed point λ_* takes negative values only in the vicinity of two dimensions. Once matter degrees of freedom are coupled to the theory, the sign of λ_* can change, *e.g.* in four dimensions [14]. We expect this pattern to persist also in the higher-dimensional case.

Dimensional analysis. In pure gravity (no cosmological constant term), only the sign of the gravitational coupling is well-defined, while its size can be rescaled to any value by a rescaling of the metric field $g_{\mu\nu} \rightarrow \ell g_{\mu\nu}$. In the presence of a cosmological constant, however, the relative strength of the Ricci invariant and the volume element can serve as a measure of the coupling strength. From dimensional analysis, we conclude that

$$\tau_k = \bar{\lambda}_k (G_k)^{\frac{2}{D-2}} \quad (5)$$

is dimensionless and invariant under rescalings of the metric field [33]. Then the on-shell effective action is a function of τ_k only. In the fixed point regime, τ_k reduces to $\tau_* = \lambda_* (g_*)^{2/(D-2)}$. In Fig. 1c), we have displayed τ_* using the cutoff r_{mexp} for arbitrary b . In comparison with the fixed point values in Figs. 1a,b), τ_* only varies very mildly as a function of the cutoff parameter b , and significantly less than both g_* and λ_* . This shows that (5) qualifies as a universal variable in general dimensions. In Tab. 1, we have collected our results for (5) at the fixed point. For all dimensions shown, τ_* displays only a very mild dependence on the cutoff function.

Universality. The fixed point values are non-universal. Universal characteristics of the fixed point are given by the eigenvalues $-\theta$ of the Jacobi matrix with elements $\partial_x \beta_y|_*$ and x, y given by λ or g , evaluated at the fixed point. The Jacobi matrix is real though not symmetric and admits real or complex conjugate eigenvalues. For all cases considered, we find complex eigenvalues $\theta \equiv \theta' \pm i\theta''$. The real and imaginary part and the modulus are displayed in Fig. 2 as functions of the cutoff parameter b for $r = r_{\text{mexp}}$. The results for the other cutoff functions are very similar. In the Einstein-Hilbert truncation, the fixed point displays two ultraviolet attractive directions, reflected by $\theta' > 0$. Complex scaling exponents are due to competing interactions in the scaling of the volume invariant $\int \sqrt{g}$ and the Ricci invariant $\int \sqrt{g} R$ [17]. The eigenvalues are real in the vicinity of two dimensions, and in the large- D limit, where the fixed point scaling is dominated by the $\int \sqrt{g}$ invariant [15]. θ' and $|\theta|$ are increasing functions of the dimension, for all $D \geq 4$ [17]. For the dimensions shown here, θ'' equally increases with dimension.

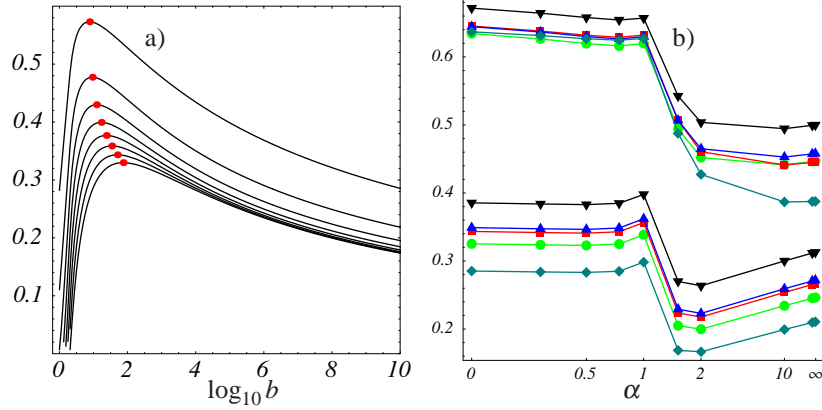


FIGURE 3. Stability of renormalisation group flows. a) The parameter $1 - \lambda_*/\lambda_{\text{bound}}$ as a function of the cutoff parameter b for the momentum cutoff r_{mexp} in Feynman gauge [9] (for more details, see [1]); the red dot indicates the maximum. b) The maximum of $1 - \lambda_*/\lambda_{\text{bound}}$ for different cutoff functions and gauge fixing parameters in a covariant background field gauge [11]. We note that $\alpha \leq 1$ leads to more stable flows than $\alpha > 1$.

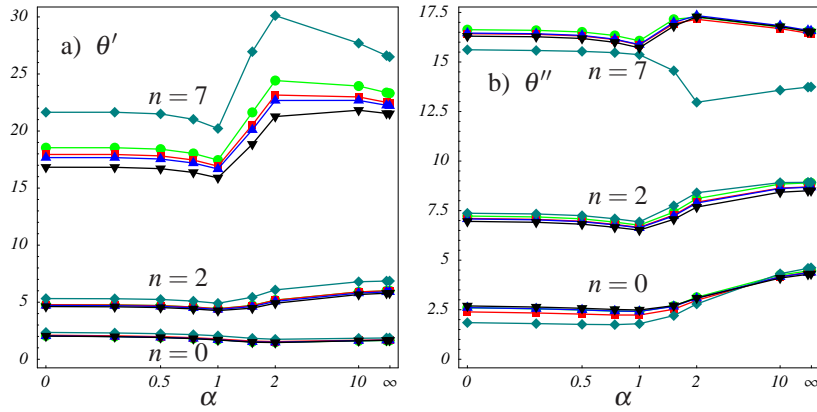


FIGURE 4. Universal eigenvalues a) θ' and b) θ'' for various gauge fixing parameter and cutoffs, with $n = 0, 2, 7$ extra dimensions. The five sets of data obtain from r_{mexp} \bullet , r_{exp} \blacksquare , r_{mod} \blacktriangle , r_{pow} \blacklozenge and r_{opt} \blacktriangledown .

Convergence. The convergence of the results is assessed by comparing different orders in the expansion. The fixed point persists in the truncation where the cosmological constant is set to zero, $\lambda = \beta_\lambda = 0$. Then, $\beta_g(g_*, \lambda = 0) = 0$ implies fixed points $g_* > 0$ for all dimensions and cutoffs studied. The scaling exponent $\theta = -\partial\beta_g/\partial g|_*$ at g_* is real and of the order of $|\theta|$ given in Tab. 1. The analysis can be extended beyond (2), e.g. including $\int \sqrt{g}R^2$ invariants and similar. In the four-dimensional case, R^2 interactions lead to a mild modification of the fixed point and the scaling exponents [11, 12]. It is conceivable that the underlying expansion is well-behaved also in higher dimensions.

Stability. The stability of the flow is controlled by the parameter $1 - \lambda_*/\lambda_{\text{bound}}$, displayed in Fig. 3 (see [27]). As a function of b , it displays a unique maximum for all classes of cutoffs studied. In Fig. 3 b), we have indicated the maximum values for $1 - \lambda_*/\lambda_{\text{bound}}$ as a function of the gauge fixing parameter. We note that the stability of flows is larger for gauge fixing parameters $\alpha \leq 1$. This had to be expected, given that $\alpha = 0$ is a non-perturbative fixed point of the flow [30].

TABLE 1. Range of scaling exponents θ' , θ'' and $|\theta|$, obtained from optimised momentum cutoffs $r_{\text{mexp}}, r_{\text{exp}}, r_{\text{mod}}$ and r_{opt} in Feynman or harmonic background field gauge with $0 \leq \alpha \leq 1$ (see text).

$n = D - 4$	0	1	2	3	4	5	6	7
θ'	1.48 – 2.76	2.69 – 3.11	4.26 – 4.78	6.43 – 6.89	8.19 – 9.34	10.5 – 12.1	13.1 – 15.2	15.9 – 18.5
θ''	2.23 – 3.14	4.54 – 5.16	6.52 – 7.46	8.43 – 9.46	10.3 – 11.4	12.1 – 13.2	13.9 – 15.0	15.7 – 16.6
$ \theta $	2.85 – 3.49	5.31 – 6.06	7.79 – 8.76	10.4 – 11.6	13.2 – 14.7	16.1 – 17.9	19.1 – 21.3	22.4 – 24.9

Gauge-fixing independence. In the harmonic background field gauge [11], the flow and its solutions may depend spuriously on the gauge fixing parameter α [28]. We found a non-trivial fixed point and universal eigenvalues for arbitrary gauge fixing parameter $\alpha \in [0, \infty)$, and all classes of momentum cutoffs considered. Hence, the fixed point appears not to be an artefact of the gauge fixing. Fig. 4 shows our results for θ' and θ'' . For each α , we first identified the most stable flow parameters as in Fig. 3, and then deduced the corresponding values for θ' and θ'' . From Fig. 3 b), we conclude that the stability of the flow is higher for $\alpha \leq 1$ as opposed to $\alpha > 1$ [17]. Furthermore, the extremely weak α -dependence of $1 - \lambda_*/\lambda_{\text{bound}}$ in Fig. 3 b), for $\alpha \leq 1$, implies that physical observables should be independent of α in this regime. This is indeed confirmed by our analysis in Fig. 4.

Cutoff independence. Non-trivial fixed points are found independently of the momentum cutoff, *e.g.* Fig. 1, and independently of the gauge-fixing parameter *e.g.* Fig. 4. The scaling exponents θ , however, depend spuriously on R_k due to the truncation. This dependence strictly vanishes for the full, untruncated flow. For best quantitative estimates of scaling exponents we resort to an optimisation, following [24, 25, 27], and use optimised values for b , for each class of cutoffs given above. Optimised flows have best stability properties and lead to results closer to the physical theory [25]. In Tab. 1, we show our results for θ' , θ'' and $|\theta|$ obtained from the cutoffs $r_{\text{mexp}}, r_{\text{exp}}, r_{\text{mod}}$ and r_{opt} in Feynman gauge with $\alpha = 1$, and harmonic background field gauge with $0 \leq \alpha \leq 1$. The remaining spread in the numerical values indicates the remaining uncertainty due to the truncation. On a quantitative level, it is found that r_{opt} with $b = 1$ leads to flows with best stability. The variation in θ' , θ'' and $|\theta|$ is reasonably small, and significantly smaller than the variation with b [27]. With increasing n , the variation slightly increases for θ' and $|\theta|$, and decreases for θ'' . The expected error due to the truncation (2) is larger than the variation in Tab. 1. In this light, our results in the Einstein-Hilbert truncation are cutoff independent.

Anomalous dimension. The non-trivial fixed point implies a non-perturbatively large anomalous dimension for the gravitational field, due to (4), which takes negative integer values $\eta = 2 - D$ at the fixed point.¹ The dressed graviton propagator $\mathcal{G}(p)$, neglecting the tensorial structure, is obtained from evaluating $1/(Z_{N,k} p^2)$ for $k^2 = p^2$. Therefore, the graviton propagator scales as

$$\mathcal{G}(p) \sim 1/p^{2(1-\eta/2)}, \quad (6)$$

which reads $\sim 1/(p^2)^{D/2}$ in the deep ultraviolet and should be contrasted with the $1/p^2$ behaviour in the perturbative regime. The anomalous scaling in the deep ultraviolet implements a substantial suppression of the graviton propagator. We verified the crossover behaviour of the anomalous dimension from perturbative scaling in the infrared to ultraviolet scaling by numerical integration of the flow (4). More generally, higher order vertex functions should equally display scaling characterised by universal anomalous dimensions in the deep ultraviolet. This is due to the fact that a fixed point action Γ_* is free of dimensionful parameters.

UV-IR connection. A non-trivial ultraviolet fixed point is physically feasible only if it is connected to the perturbative infrared domain by well-defined, finite renormalisation group trajectories. Elsewise, it would be impossible to connect the known low-energy physics of gravity with the putative high energy fixed point. A necessary condition is $\lambda_* < \lambda_{\text{bound}}$, which is fulfilled. Moreover, we have confirmed by numerical integration of the flow that the fixed points are connected to the perturbative infrared domain by well-defined trajectories in higher dimensions.

Running gravitational coupling. The physically relevant trajectories connect the UV fixed point with the vicinity of the gaussian fixed point in the infrared limit, and the running gravitational coupling displays a cross-over between

¹ Integer anomalous dimensions are known from other gauge theories at a fixed point away from their canonical dimension, *e.g.* abelian Higgs [34] below or Yang-Mills [35] above four dimensions.

them. In the deep ultraviolet, the non-trivial fixed point implies that the gravitational coupling scales as

$$G_k = g_*/k^{D-2}. \quad (7)$$

Hence, and in contrast to the regime of standard perturbation theory where $G_k \approx \text{const.}$, the coupling becomes dimensionally suppressed at a non-trivial ultraviolet fixed point ($k \rightarrow \infty$). Then gravity becomes weakly coupled in the approach to the fixed point. This “weakly coupled” regime should be contrasted with asymptotic freedom of four-dimensional QCD, where the weak coupling in the UV limit is due to a gaussian fixed point rather than to the dimensional suppression (7).

Discussion

In summary, we have found a unique and non-trivial ultraviolet fixed point with all the right properties for quantum gravity in more than four dimensions. Within the present truncation, the fixed point exists independently of the cutoff and the gauge fixing. Furthermore, for sufficiently stable flows, we have established that universal scaling exponents are even quantitatively independent of the cutoff and the gauge fixing parameter. This constitutes a highly non-trivial consistency check for the result. Together with the renormalisation group trajectories which connect the fixed point with the perturbative infrared domain, our findings provide a viable realisation of Weinberg’s asymptotic safety scenario in terms of the metric degrees of freedom.

In future work, these studies should be extended to include higher powers in the Riemann tensor, the Ricci scalar, derivatives thereof, and matter degrees of freedom. In the four-dimensional case, R^2 interactions lead only to minor modifications of the fixed point and its scaling properties [12]. It is important to confirm this pattern in higher dimensions, where R^2 interactions are relevant already in perturbation theory. If the fixed point persists in extensions, quantum gravity can well be formulated as a fundamental theory in the metric degrees of freedom. Furthermore, it would be useful to have an independent verification of the fixed point based on *e.g.* lattice simulations. Recent progress in the four-dimensional case seems to suggest that Monte-Carlo studies of higher-dimensional Einstein gravity within Regge calculus or causal dynamical triangulation is in reach [19, 20].

Finally, we highlight main implications of the above fixed point for phenomenological particle physics models with compact extra dimensions and low scale quantum gravity. If realised in Nature, quantum gravity at the TeV scale could be accessible experimentally in hadron colliders. In these models, standard model particles live on a four-dimensional brane whereas gravity propagates in a $D = 4 + n$ dimensional bulk. Under the assumption that standard model particles do not spoil the fixed point, we can neglect their presence for the following considerations. Without loss of generality, we consider n extra spatial dimensions with compactification radius L . The four-dimensional Planck scale M_{Pl} is related to the D -dimensional (fundamental) Planck mass M_D and the radial length L by the relation $M_{\text{Pl}}^2 \sim M_D^2 (M_D L)^n$, where L^n is a measure for the extra-dimensional volume. A low fundamental Planck scale $M_D \ll M_{\text{Pl}}$ therefore requires the scale separation $1/L \ll M_D$ which states that the radius for the extra dimensions has to be much larger than the fundamental Planck length $1/M_D$. For momentum scales $k \ll 1/L$, where $\eta \approx 0$, the hierarchy implies that the running couplings scale according to their four-dimensional canonical dimensions. At $k \approx 1/L$, the size of the extra dimensions is resolved and, with increasing k , the couplings display a dimensional crossover from four-dimensional to D -dimensional scaling. Still, the graviton anomalous dimension stays small and gravitational interactions $G_k \approx \text{const.}$ are perturbatively weak. This dimensional crossover is insensitive to the fixed point in the deep ultraviolet. In the vicinity of $k \approx M_D$, however, the graviton anomalous dimension displays a classical-to-quantum crossover from the gaussian fixed point $\eta \approx 0$ to non-perturbative scaling in the ultraviolet $\eta \approx 2 - D$. This crossover takes place in the full D -dimensional theory. In the transition regime, the propagation of gravitons is increasingly suppressed. In addition, the gravitational coupling becomes weak, following (7). Therefore, the onset of the fixed point scaling cuts off gravity-mediated processes with characteristic momenta at and above M_D , and provides dynamically for an effective momentum cutoff of the order of M_D . This dynamical suppression, and a decreasing anomalous dimension $\eta < 0$ in the transition regime, can be seen as signatures for the non-trivial fixed point. We conclude that the gravitational fixed point could be detectable in experimental setups sensitive to the TeV energy range, *e.g.* in hadron colliders, provided that the fundamental scale of gravity is as low as the electroweak scale.

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