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The Price of WMAP Inflation in Supergravity

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Abstract

The three-year data from WMAP are in stunning agreement with the simplest possible quadratic potential for chaotic inflation, as well as with new or symmetry-breaking inflation. We investigate the possibilities for incorporating these potentials within supergravity, particularly of the no-scale type that is motivated by string theory. Models with inflation driven by the matter sector may be constructed in no-scale supergravity, if the moduli are assumed to be stabilised by some higher-scale dynamics and at the expense of some fine-tuning. We discuss specific scenarios for stabilising the moduli *via* either D- or F-terms in the effective potential, and survey possible inflationary models in the presence of D-term stabilisation.

1 Introduction

The dominant paradigm for explaining the great size, age and flatness of the Universe, as well as the absence of unwanted primordial relics such as magnetic monopoles or gravitinos, is cosmological inflation. According to this hypothesis, very early in its history, the Universe underwent an epoch of near-exponential expansion, dubbed cosmological inflation. During this period, the energy density of the Universe is hypothesized to have been dominated by a near-constant vacuum energy density. Within the general context of quantum field theory, the favoured origin of this vacuum energy density is the potential energy V of some elementary scalar field ϕ, called the inflaton. Consistency with the magnitude of primordial density perturbations inferred from the pioneering observations by the COBE satellite, as well as subsequent experiments, suggests that this potential energy density $V \ll m_P^4$.

In order for the inflationary epoch to last long enough, and for the density perturbations to remain small, the inflationary potential must obey certain slow-roll conditions:

$$\epsilon \equiv \frac{m_P^2}{2}\left(\frac{V'}{V}\right)^2, \; \eta \equiv m_P^2\left(\frac{V''}{V}\right) \ll 1. \tag{1}$$

In principle, both the slow-roll parameters ϵ and η may be extracted from the spectra of density perturbations, in particular *via* the scalar spectral index $n_s = 1 - 6\epsilon + 2\eta$ and the amplitude ratio of tensor and scalar perturbations $r = 16\epsilon$. Also observable in principle is the running of the scalar spectral index, $dn_s/d\ln k$, but this is expected to be very small in slow-rolling inflationary models.

Important constraints on the slow-roll parameters have been obtained in the past, using the first-year data from WMAP and other observational inputs. Qualitatively new insight has recently been provided by the new three-year WMAP data (WMAP3), which are strikingly consistent with simple models of inflation based on an elementary scalar inflaton field whose potential is just a low-order polynomial [1] [1]. In particular, the data are consistent with the simplest model for chaotic inflation with a quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$, as seen in Fig. 1. They also constrain severely possible modifications, e.g., of the form:

$$V(\phi) = \frac{1}{2}m^2\phi^2\left(1 + \frac{1}{2}\kappa_c^2\phi^2\right). \tag{2}$$

As seen in the right panel of Fig. 1, the supplementary parameter κ_c cannot exceed a few per mille, with the exact number depending of the number of e-foldings during the inflationary epoch. As seen in Fig. 1, the WMAP3 data [1] are also consistent with another very simple potential:

$$V(\phi) = \lambda m_P^4\left(1 - \kappa_s^2\frac{\phi^2}{m_P^2}\right)^2, \tag{3}$$

with the requirement, seen also in the right panel of Fig. 1, that κ_s^2 cannot exceed one per cent if inflation is to start with the small initial value of the inflaton field [2], i.e., on the branch

[1] Note that this statement is also supported by the WMAP3 data analysis given in [2], which yields a larger allowed region.

[2] Note, however, that this potential also allows inflation with a large value of the inflaton field.

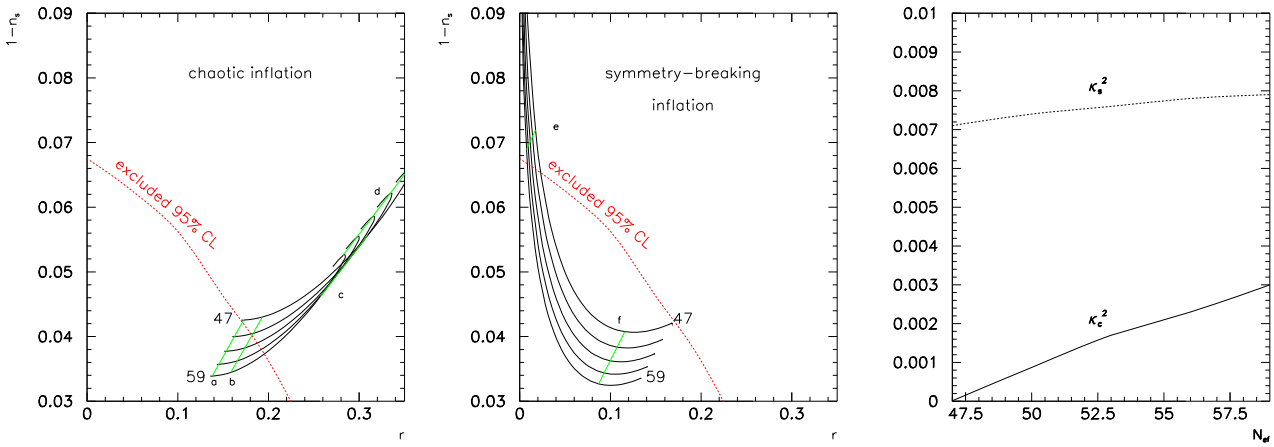

Figure 1: *Confrontation of low-order polynomial inflationary potentials with the three-year data from WMAP [1], in combination with other data [3]. The deviation from scale invariance of the scalar spectral index,* $1 - n_s$*, is shown as a function of the tensor-to-scalar ratio* r *for chaotic inflation (left) and new inflation (centre). The lines correspond to* $N = 47, 50, 53, 56, 59$ *e-foldings, as indicated. The green (light gray) lines denoted by* $\mathsf{a} - \mathsf{d}$ *correspond to* $\kappa_c^2 = 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}$ *in (2), and the green (light gray) lines denoted by* e, f *correspond to* $\kappa_s^2 = 10^{-2}, 10^{-3}$ *in (3). The dashed curves show the regions of the parameters allowed by WMAP3 and other data [3] at the 95% CL. Right panel displays the corresponding 95% upper limits on* κ_c^2 *and* κ_s^2 *as functions of the numbers of inflationary e-foldings in the chaotic and new inflationary models, respectively.*

$|\phi| \leq m_P/\kappa_s$. This scenario is known as new or symmetry-breaking inflation. These and other models make characteristic predictions for the spectral index of scalar density perturbations, n_s, and for the tensor-to-scalar ratio r. The left and central panels of Fig. 1 display the constraints on n_s and r inferred from a combination of WMAP3 [1] with other data [3]. These are confronted with the different predictions of the potentials given in (2) and (3), assuming negligible running of the scalar spectral index, as predicted in these models.

It has long been a challenge to embed inflation in realistic models of particle physics. There are two aspects to this problem: identifying a suitable candidate for the inflaton, and ensuring that it has a suitable effective potential. We take the point of view that realistic particle models should incorporate supersymmetry [4], which incidentally provides many scalar fields that one might hope to exploit as an inflaton. None of the MSSM fields fits the bill, but the scalar partner of one of the heavy singlet neutrinos in a seesaw model of neutrino masses might be suitable [5]. In the framework of global supersymmetry, such a sneutrino would naturally possess a simple quadratic potential without significant perturbative corrections, and its mass would fit naturally with the estimate of $m \sim 2 \times 10^{13}$ GeV required for the simplest chaotic inflation model (2).

However, a more appropriate framework for describing inflation is presumably that offered by local supersymmetry. The low-energy limit of any supersymmetric theory of quantum gravity

is necessarily some such supergravity theory. Moreover, since inflation is a fundamental property of the evolution of space-time, it must involve the gravitational sector of the microscopic theory, in which case supergravity is likely to play an essential role. The support of the data for the simplest inflationary models is then highly non-trivial information, since the effective potentials yielded by supergravity theories generically have forms that are more complicated than (2) or (3). Indeed, many supergravity potentials have minima with vacuum energy densities that are negative and $\mathcal{O}(m_P^4)$.

The general term for this troublesome feature of supergravity is the η problem, namely the statement that supergravity with simple forms for the Kähler potential for scalar superfields, such as minimal supergravity, naturally predict large mass terms for all the scalars, destroying the flatness of any candidate inflationary direction in the effective potential. One possible way out of this problem is to postulate a Kähler potential whose prefactor e^K dependens on the scalar fields in a manner softer than an exponential. The non-minimal form of the Kähler potential which is particularly well motivated from the point of view of string theory is the no-scale structure that appears naturally in compactifications of higher-dimensional superstring models. The great advantage of the no-scale structure is that it leads to a semi-positive definite scalar potential, very similar to a globally supersymmetric one [6].

Many superstring theories contain moduli fields T_i whose effective potentials are flat classically. These no-scale models are characterized by effective low-energy supergravity theories containing terms in the Kähler potential K of the characteristic logarithmic form: $K \ni -\log(T_i + \bar{T}_i)$. Since the effective potentials of these moduli fields vanish classically, one might expect them to remain small even when corrections are calculated, so that they might be candidates for the inflaton field. Efforts to implement modular inflation continue and provide interesting and encouraging results. However, here we prefer to explore the other possibilities offered by the matter sectors of string theories, which are typically rather rich in the structures of their superpotentials and Kähler potentials, potentially offering ample opportunities to generate inflationary behaviour.

Specifically, in this paper we examine in some detail conditions necessary for matter-driven inflation to take place in a locally supersymmetric model. For simplicity, we consider here a no-scale model with a single modulus T and a matter field Φ, with a no-scale Kähler potential of the form:

$$K \;=\; -3\log(T + \bar{T} - 2|\Phi|^2) \tag{4}$$

and a matter superpotential to be discussed below. We find that the matter sector in such a simplified no-scale supergravity may indeed lead to acceptable inflation, provided that the T modulus is stabilised appropriately. One must also apply specific fine-tuning conditions on the inflationary superpotential, which depend on the precise form of the constraint on the modulus T. In at least some such models, the naturalness of these fine-tuning requirements is not obvious from the point of view of the effective low-energy theory.

We then go on to consider explicit models for the stabilization of the modulus T. One is D-type stabilisation made possible by supersymmetric D-terms, and the other is F-type stabilisation, where the superpotential depends on a T modulus. As we show below, model building is much easier in the case of D-stabilisation, as the scales in the sector of the matter-type inflaton Φ and in the T-modulus sector can decouple from each other.

2 No-scale Structure and Inflation from Matter Fields

As already mentioned, minimal supergravity models with canonical kinetic terms for the matter fields have a serious disadvantage when it comes to inflationary scenarios. The trouble is known as the η problem, and is due to the common prefactor $e^K = e^{|\Phi|^2}$ in the scalar potential, where we denote by Φ the matter-type inflaton [3]. However, it is possible to modify the Kähler potential, [7], so as to obtain simple models that are very similar to those of the globally supersymmetric case [4]. These are no-scale models of the type that arises naturally in compactifications of string models [9], [10], [11], in which case

$$K = -n_1 \log(T_1 + \bar{T}_1 - 2|\Phi_1|^2) - n_2 \log(T_2 + \bar{T}_2 - 2|\Phi_2|^2) - n_3 \log(T_3 + \bar{T}_3 - 2|\Phi_3|^2), \qquad (5)$$

where $n_1 + n_2 + n_3 = 3$ and $n_i \geq 0$ for $i = 1, 2, 3$.

As already mentioned in the Introduction, for simplicity, we consider the minimal case $K = -3\log(T + \bar{T} - 2|\Phi|^2)$, for which the kinetic part of the Lagrangian for the fields T and Φ reads:

$$\mathcal{L}_{\rm kin} = \frac{1}{(T + \bar{T} - 2|\Phi|^2)^2} \left(\partial_\mu \bar{T}, \partial_\mu \bar{\Phi}\right) \begin{pmatrix} 3 & -6\bar{\Phi} \\ -6\Phi & 6(T + \bar{T}) \end{pmatrix} \begin{pmatrix} \partial^\mu T \\ \partial^\mu \Phi \end{pmatrix}. \qquad (6)$$

This expression can easily be written in an equivalent form:

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\left[\partial_\mu \sqrt{\frac{3}{2}} \ln 2(t - |\Phi|^2)\right]^2 + \frac{3\left[\partial_\mu t' + {\rm i}(\bar{\Phi}\partial_\mu \Phi - \Phi \partial_\mu \bar{\Phi})\right]^2}{4(t - |\Phi|^2)^2} + \frac{3\partial_\mu \bar{\Phi} \partial^\mu \Phi}{(t - |\Phi|^2)}, \qquad (7)$$

where $t \equiv {\rm Re}\, T$, $t' \equiv {\rm Im}\, T$. Note, that the above kinetic term is positive definite as long as $|\Phi| < \sqrt{t}$. Assuming an arbitary superpotential [5] $W(\Phi)$ which does not depend on T, one obtains the scalar potential

$$V_F(\Phi) = \frac{|\partial_\Phi W|^2}{6(T + \bar{T} - 2|\Phi|^2)^2}. \qquad (8)$$

In a complete model, T is a dynamical degree of freedom, and one can readily convince oneself that in fact it is necessary to stabilise T. To see this, let us compute the η and ϵ parameters along the direction of the real part of T:

$$\eta_T = \frac{1}{2V}\frac{\partial}{\partial t}\left((K^{-1})^{\bar{T}T}\frac{\partial V}{\partial t}\right), \qquad \epsilon_T = \frac{1}{4}(K^{-1})^{\bar{T}T}\left(\frac{1}{V}\frac{\partial V}{\partial t}\right)^2. \qquad (9)$$

Because of the particular structure of the scalar potential (8), where the dependence on the real part of T comes only from the denominator, one finds, independently of the form of W,

[3] Here and subsequently, we use the system of units with $m_P = 1$.

[4] The η problem itself can also be cured in other ways, for instance by using a phase of a field with canonical Kähler potential, and a Kähler potential of the form $K = 1/2(M + \bar{M})^2$ which makes the factor e^K insensitive to the imaginary part of M [8]. Nevertheless, the models closest in their forms to globally supersymmetric Lagrangians are offered by the no-scale Kähler potentials.

[5] Addressing the possible origins of non-trilinear terms in the effective superpotential goes beyond the scope of this paper, but we note that various mechanisms for generating them may exist, see for instance [12].

the results $\eta_T = \epsilon_T = 4/3$. This means that the curvature of the potential, and hence the driving force acting on T along the direction of $\mathrm{Re}\,T$, is large everywhere, making it impossible to generate any inflation in the Φ sector.

We therefore discuss inflation from matter fields, assuming that the only modulus T in this simple model is frozen by some, as yet unspecified, mechanism [6]. Then the part of the supergravity Lagrangian involving the Φ field can be obtained by combining (6) with $T = \mathrm{const}$ and (8):

$$\mathcal{L}_\Phi = \frac{6(T+\bar{T})}{(T+\bar{T}-2|\Phi|^2)^2}\partial_\mu\Phi\partial^\mu\bar{\Phi} - V_F(\Phi) \qquad . \tag{10}$$

We can now rewrite the kinetic term for the modulus $|\Phi|$ coming from (10) in terms of the canonically normalised field ϕ (we freeze the phase of Φ for the moment): $|\Phi|/\sqrt{t} = \tanh(\phi/\sqrt{6})$. The effective potential $V_F(\Phi)$ may therefore be written in the form

$$V_F(\phi) \;=\; \frac{1}{24t^2}\left[|\partial_\Phi W|^2\right]_{|\Phi|=\sqrt{t}\tanh(\phi/\sqrt{6})} \times \cosh^4(\phi/\sqrt{6}). \tag{11}$$

Assuming the simplest forms

$$W(\Phi) = \frac{1}{2}M\Phi^2 \text{ or } W(\Phi) = \frac{1}{3}\lambda\Phi^3 \tag{12}$$

for the superpotential, corresponding to

$$\left[|\partial_\Phi W|^2\right]_{|\Phi|=\sqrt{t}\tanh(\phi/\sqrt{6})} = M^2\tanh^2(\phi/\sqrt{6})t \text{ or } \left[|\partial_\Phi W|^2\right]_{|\Phi|=\sqrt{t}\tanh(\phi/\sqrt{6})} = \lambda^2\tanh^4(\phi/\sqrt{6})t^2, \tag{13}$$

we find the resulting scalar potential

$$V_F(\phi) = \frac{M^2}{96t}\sinh^2\left(\sqrt{\frac{2}{3}}\phi\right) \qquad \text{or} \qquad V_F(\phi) = \frac{\lambda^2}{24}\sinh^4\left(\sqrt{\frac{1}{6}}\phi\right), \tag{14}$$

respectively, which do not satisfy the slow-roll conditions.

Hence, with T stabilised one should construct models which go beyond the quadratic superpotential for Φ. From the point of view of an effective low-energy theory with a polynomial superpotential in Φ, obtaining an inflationary potential that is a simple quadratic in the canonically-normalized inflaton field ϕ, as apparently preferred by the WMAP3 data, would seem quite unnatural. Nevertheless, a potential that is purely quadratic in ϕ may in principle be achieved by postulating a non-polynomial superpotential $W(\Phi)$, which gives

$$\frac{\partial W}{\partial \Phi} \;\propto\; \mathrm{artanh}\left(\frac{|\Phi|}{\sqrt{t}}\right) \times \left(1 - \frac{|\Phi|^2}{t}\right). \tag{15}$$

However, this form looks all the more fine-tuned because the value of t is to be fixed dynamically by some unknown mechanism.

[6] We note that fluxes and non-perturbative gaugino condensates may determine many or all moduli in semi-realistic models.

Another simple choice would be

$$W = W_0 + \mu^2\Phi + \frac{1}{3}d\Phi^3 \ , \tag{16}$$

from which one obtains the scalar potential

$$V_F(\Phi) = \frac{\mu^4}{24(t-|\Phi|^2)^2}\left[\left(1-\frac{|\Phi|^2}{\Phi_0^2}\right)^2 + \frac{4|\Phi|^2}{\Phi_0^2}\cos^2\theta\right], \tag{17}$$

where $\theta \equiv \arg\Phi$ and $\Phi_0^2 = \mu^2/d$. For $|\Phi|^2 < t, \Phi_0^2$, the contributions to the mass of $|\Phi|$ coming from the numerator and the denominator of (17) can approximately cancel each other, making this degree of freedom much lighter than θ and hence a good candidate for the inflaton field. It is, therefore, reasonable to assume that θ takes the value at its minimum, namely $\pi/2$, already at the onset of inflation, and the potential (17) can then be effectively rewritten in terms of $|\Phi|$ only. For $|\Phi| \ll t$, one can expand the potential (17) in $|\Phi|^2/t$, obtaining an effective potential of the type (3) with $\kappa_s^2 = t/\Phi_0^2 - 1$. Hence, symmetry-breaking inflation is possible, provided that the parameters of the matter superpotential and the value t of the fixed modulus are such that:

$$0 < 1 - \frac{\Phi_0^2}{t} \lesssim 10^{-2}. \tag{18}$$

We note that the potential (17) reduces to the effective potential (3) only in the limit $|\Phi|^2 \ll t$ and, unlike (3), it does not allow for chaotic initial conditions, as $\epsilon \geq 4/3$ and $\eta \geq 4/3$ for $|\Phi| > \Phi_0$.

Alternatively, instead of T stabilisation one could postulate stabilisation of the combination $t-|\Phi|^2$, which appears in the Kähler potential. In this case, the canonically-normalized inflaton field ϕ is given by

$$\phi \ = \ \frac{\sqrt{6}|\Phi|}{\sqrt{t-|\Phi|^2}} \tag{19}$$

and the effective inflationary potential is then simply proportional to $|\partial W/\partial\Phi|^2$. Contrarily to the previous case, if $W = \frac{1}{2}M\Phi^2$ the effective potential reduces to the simplest quadratic form allowed by WMAP3:

$$V_F = \frac{1}{2}m^2\phi^2 \qquad \text{where} \qquad m^2 = \frac{M^2}{72(t-|\Phi|^2)} = \text{const.} \tag{20}$$

In order to go beyond the fixed-t or fixed-$(t-|\Phi|^2)$ approximation, one needs to solve the equations of motion for all relevant fields. Since the kinetic terms are complicated functions of the original fields T, Φ and their derivatives, and the potential is a very complicated function of the canonically normalised fields, we shall use the following degrees of freedom: $|\Phi|$, θ, $\tau = \ln(2t - 2|\Phi|^2)$ and ζ, where $t' = \mathrm{Im}\,T$ and $\mathrm{d}\zeta = \mathrm{d}t' - 2|\Phi|^2\mathrm{d}\theta$. The potential is, in general,

a function of τ, $|\Phi|$ and θ. In terms of these fields, the equations of motion read:

$$\ddot{\tau} + 3H\dot{\tau} + 4\dot{\zeta}^2 e^{-2\tau} + 2|\dot{\Phi}|^2 e^{-\tau} + 2|\Phi|^2\dot{\theta}^2 e^{-\tau} = -\frac{2}{3}\frac{\partial V}{\partial \tau} \tag{21}$$

$$\ddot{\zeta} + 3H\dot{\zeta} - 2\dot{\zeta}\dot{\tau} = -\frac{e^{2\tau}}{6}\left(\frac{\partial V}{\partial t'} - \frac{1}{2|\Phi|^2}\frac{\partial V}{\partial \theta}\right) \tag{22}$$

$$|\ddot{\Phi}| + 3H|\dot{\Phi}| - |\dot{\Phi}|\dot{\tau} + |\Phi|\dot{\theta}^2 = -\frac{e^{\tau}}{12}\frac{\partial V}{\partial |\Phi|} \tag{23}$$

$$\ddot{\theta} + 3H\dot{\theta} + 2\frac{|\dot{\Phi}|}{|\Phi|}\dot{\theta} - \dot{\theta}\dot{\tau} = -\frac{e^{\tau}}{12|\Phi|^2}\frac{\partial V}{\partial \theta}, \tag{24}$$

where the dot denotes differentiation with respect to cosmic time and the Hubble parameter is:

$$H^2 = \frac{1}{4}\dot{\tau} + \dot{\zeta}^2 e^{-2\tau} + 2|\dot{\Phi}|^2 e^{-\tau} + 2|\Phi|^2\dot{\theta}^2 e^{-\tau} + \frac{1}{3}V. \tag{25}$$

The equations of motion (22) and (24) show that, for τ approximately fixed and for the potential depending only on $|\Phi|$, the kinetic energy associated with the quantity ζ parametrising the imaginary part of T and with the phase θ is redshifted away with the expansion of the Universe, and it is legitimate to restrict one's attention to τ and $|\Phi|$ in the discussion of inflation. In our numerical analyses of the models presented in the following sections, we integrate the full set of the equations of motion (21)-(25), assuming that the initial values of θ, $\dot{\theta}$ and $\dot{\zeta}$ vanish. We neglect the isocurvature perturbations, generically present in the multi-field inflationary models, assuming that we can choose the parameters of the potential so that the curvature of the potential in the directions transverse to the classical trajectory is much larger than that in the longitudinal direction [7].

3 Modulus Stabilisation through the D-terms

We now describe the forms of the D-terms in various cases, and the ways in which they may stabilize the modulus [13]-[18].

We first consider gauging the imaginary shift of the modulus T. We assume that $K = K(T + \bar{T})$, and consider the imaginary shift $T \to T + i\delta/2\Lambda$, which is generated by the Killing vector $P_T = i\delta/2$ where δ is real. The prepotential D fulfills the Killing equation $K_{T\bar{T}}\bar{P}_T = i\partial D/\partial T$, and generates the scalar potential $\delta V = 1/2g^2 D^2$. In the case of a pseudo-anomalous U(1) [8], the gauge coupling depends on the modulus T as follows: $g^{-2} = \mathrm{Re}(T)$. One finds, upon solving the Killing equation, that $D = -\frac{\delta}{2}\frac{\partial K}{\partial T} + \xi$, where ξ is a genuine constant. If $\xi \neq 0$, the gauged symmetry acts on the gravitino and becomes a local U(1) R-symmetry. To simplify the discussion, we set $\xi = 0$, unless stated otherwise. To be consistent with the global supersymmetry algebra, the superpotential must be invariant under any local symmetry which is not an R-symmetry. The invariant superpotential which is a function of the modulus T alone must be a constant, which is insufficient to stabilize the modulus T.

[7] We thank D. Langlois for drawing our attention to this point.

[8] I.e., one for which the sum of the charges of all fields on which the symmetry is realised linearly does not vanish.

The solution is to introduce more charged fields:

$$\delta\phi = i\Lambda q_\phi \phi; \quad P_\phi = i q_\phi \phi, \tag{26}$$

in which case the general D-term becomes

$$D = -\frac{\delta}{2}\frac{\partial K}{\partial \bar{T}} - q_\phi \frac{\partial K}{\partial \phi}\phi \ . \tag{27}$$

in order to begin the discussion of D-term stabilisation, we assume initially that the superpotential does not depend on the modulus T. Then the only possibility for stabilizing this modulus is to rely on the D-terms, which give positive contributions to the scalar potential, as seen in [16]. One obvious case is to make the gauge coupling T-dependent: $g^2 = 1/t$. However, if this is the only source of T dependence, there is no stabilization, as the resulting contribution to the scalar potential is monotonic in t: $V_D = 1/(2t)D^2$. To have a better chance for stabilization, one needs to introduce T inside the D-term. One option is to make T charged under the U(1). According to the discussion from the previous subsection, there are two options. First, the U(1) may be an R-symmetry, in which case there appears an additional constant term in the D-term: $D \to D + \xi$, which indeed aids in stabilization. However, the superpotential must then be charged under the U(1), which means that (a) Φ must be charged, or (b) one introduces yet another field, say M, which is charged and mixes with Φ in the superpotential via a term of the form $M^\alpha \Phi^\beta$, so as to give the proper charge to terms containing Φ. Both options are disastrous for Φ inflation, since either they restrict very strongly the allowed form of the superpotential for Φ [case (a)] or lead to complicated multi-inflaton models [case (b)]. Hence we give up on the R-symmetric models.

Instead, we continue with a symmetry which leaves the superpotential invariant. However, in this case we need yet another T-dependent contribution to the D-term, so there must exist other fields charged under the U(1), which we denote by X_i. Let the relevant terms in the Kähler potential be $K(X_i, \bar{X}_i) = \frac{|X_i|^2}{(T+\bar{T})^{n_i}}$. The D-term contribution to the scalar potential then reads

$$V_D = \frac{g^2}{2}\left(\frac{3\delta}{T+\bar{T}-2|\Phi|^2} + \sum_i Q_{X_i}\frac{|X_i|^2}{(T+\bar{T})^{n_i}}\right)^2 . \tag{28}$$

It is obvious that, given the right assignment of charges (δ and Q_{X_i}), one can stabilize the real part of T once the spectator fields X_i assume non-zero expectation values, [16]. This conclusion is independent of the possible T-dependence of g^2. Of course, the question then arises of the source of the v.e.v. of the X_i. One needs to assume that there exists a sector in the theory, independent of the X_i, which stabilizes the X_i and decouples above the scale of Φ inflation. This type of assumption is often made in inflationary model building, and does not constrain the discussion in a significant manner. It may be viewed as an additional fine-tuning, which is in any case present in inflationary models.

An interesting question concerns the issue of supersymmetry breaking after inflation. As the vacuum energy vanishes in the post-inflationary minimum, all D-terms and F-terms must become zero at that minimum with the exception of the F^T[9], whose expectation value is

[9]The $\langle F^T \rangle$ is not bound to vanish, since, as long as the no-scale structure is not disturbed, $K_{T\bar{T}}|F^T|^2 = 3e^K|W|^2$ identically.

$\langle F^T \rangle = m_{3/2}\langle T + \bar{T} - 2|\Phi|^2\rangle = e^{K/2}\langle W\rangle\langle T + \bar{T} - 2|\Phi|^2\rangle$. In all models we consider, this could be zero only if the expectation value of the complete superpotential, $\langle W\rangle$, vanishes. This is generically not the case, so supersymmetry is broken in the post-inflationary minimum. However, the scale of $\langle W\rangle$ is set by the expectation values of the fields Φ and X_i, which in turn determine the height of the inflationary potential. The additional freedom which helps to match the post-inflationary gravitino mass and WMAP normalisation lies in the fact that the WMAP normalisation constrains the ratio of the height of the potential and the ϵ parameter, and, if the initial conditions are set by the saddle point, it is just the constraint on the precision of initial localization of the inflaton near the saddle point (where ϵ goes towards zero). To study this issue reliably one needs to specify the model in more details than are necessary to study the inflationary epoch alone, hence we do not pursue this discussion any further in the present paper.

4 Inflation in the Presence of D-term Stabilisation

Assuming that we are able to stabilise the modulus T with the help of a D-term, we have a number of options to create an inflationary epoch. The available scenarios can be grouped into three broad classes.

Model A: The simplest and, in some sense, the cleanest version of D-term stabilisation corresponds to a charged modulus T and a single spectator with weight $n = 0$. This choice avoids the kinetic mixing between the spectator and the modulus. In this case, the scalar D-term potential (28) is a function of τ only and, if the coupling g^2 turns out to be T-independent, then the D-terms would approach a constant value as $\tau \to \infty$, thus allowing for a τ-driven inflation of the chaotic type even without matter fields. Alternatively, one may utilise (28) to stabilize τ, and the choice of the superpotential $W = \frac{1}{2}M\Phi^2$ leads then to a scalar F-term potential of the form (20). Note that, in stringy models, the potential stabilising t should vanish in the decompactification limit $t \to \infty$. In the context of (28) with $n = 0$, this is possible when g^2 depends on t in such a way that $g^2 \to 0$ when $t \to \infty$. As a result, the full scalar potential can be written as:

$$V(\tau, |\Phi|) = \frac{M^2}{6}e^{-2\tau}|\Phi|^2 + \frac{\tilde{g}^2}{2}\left(e^{-\tau} - e^{-\tau_0}\right)^2, \tag{29}$$

where $e^{-\tau_0} = |Q_X||X|^2/3\delta$ and we have chosen $\tilde{g}^2 = 9\delta^2 g^2/(e^{\tau}/2 + |\Phi^2|)$ as a T-independent rescaled coupling. The surface plot of the potential (29) generically exhbits a valley in the $|\Phi|$ direction. More detailed predictions of this model are presented in Figure 2, where we set $\tau_0 = \ln 2$ and $M^2 = 10^{-4}\tilde{g}^2$.

Model B: Another simple possibility consists of using a charged T field and a single spectator field with the weight $n > 0$. One might wish to stabilise T by the D-term potential and to use the superpotential (16) to build a model of new inflation, as outlined in Section 2. However, the presence of $|\Phi|$ in the D-term potential generically leads to a large contributions to η_Φ, which can be suppressed only if the second term in (28) is much smaller than the first term for t at its minimum. In other words, there would be a triple fine-tuning to achieve a small inflaton mass: between terms from V_D, from e^K and $\partial_\Phi W$. Since all these terms have in principle different origins, we discard this scenario as relatively improbable.

Model C: At this point one may note that, even if the modulus T is neutral, two spectator fields X_1 and X_2 with different n_1 and n_2 and charges of opposite signs can stabilise t. Thus D-term stabilization looks like a generic phenomenon in a wide class of models. In the following, we focus on this simple case. For simplicity, we parametrise the supersymmetric D-term potential by

$$V_D = V_0 \left(\frac{t_0}{t}\right)^{2n_1} \left(1 - \left(\frac{t_0}{t}\right)^{\beta}\right)^2 , \tag{30}$$

where we have taken

$$\beta = n_2 - n_1 > 0, \qquad t_0 = \frac{1}{2}\left(-\frac{Q_{X_2}|X_2|^2}{Q_{X_1}|X_1|^2}\right)^{\frac{1}{n_2-n_1}} , \qquad V_0 = \frac{g^2|Q_{X_1}|^2|X_1|^2}{2^{2n_1+1}t_0^{2n_1}}. \tag{31}$$

There may be some further constraints due to the anomaly cancellation conditions, but there is enough freedom in the model to satisfy these conditions without spoiling inflation. For instance, one can easily add more charged fields with vanishing expectation values. In order to specify completely the form of the scalar potential, we have to choose the superpotential $W(\Phi)$. Since V_D has been devised primarily to stabilise t, following the considerations in Section 2, we shall choose W as in (16). Given that $V_0 \gg \mu^4$, the stabilisation of t is insensitive to the presence of the inflaton potential and the effective potential for the canonically normalised inflaton field is of the form (3).

We note that inflation from the t field is still impossible, since the slow-roll parameters ϵ_T and η_T cannot be small simultaneously. More precisely, V_D has a local maximum corresponding to $\epsilon_T = 0$ but $\eta < -4/3$. Away from the local maximum, η_T changes sign, but ϵ_T grows large. Similarly, with the coupling being T-dependent, T-driven inflation becomes difficult due to the η problem that cannot be avoided in this case.

Characteristic predictions of the potential (30) for inflationary observables are depicted in Figure 2, where we set $n_1 = \beta = 1$, $t_0 = 1$ and $\Phi_0^2 = 0.99$. We also chose $V_0 = \mu^4$ (thin lines) to make some features of the potential better visible. The upper part of the regions of initial conditions with a large number of e-foldings corresponds to fields rolling to infinity in the runaway direction (decompactification limit), while the lower part corresponds to the field evolution to the minimum of the potential at finite (τ_0, Φ_0). Increasing the D-term contribution, *e.g.* changing V_0 to $10\mu^4$ (thick lines), raises the barrier between the valley and the runaway-roll-down region and the allowed region of the initial conditions reduces to a narrow strip corresponding to a single-field inflation described by the potential (17).

5 Modulus Stabilisation through the F-Terms

It follows from the discussion in Section 2 that inflation from matter fields is, in principle, possible if one assumes that T or $t - |\Phi|^2$ is fixed. In this Section, we present two scenarios of T stabilisation through F-terms and discuss whether the presence of these additional terms in the scalar potential can be compatible with inflation.

We note in advance that, in general, two tunings are necessary: one to obtain inflation with the correct properties, and the second is to lift the vacuum energy to make the inflationary

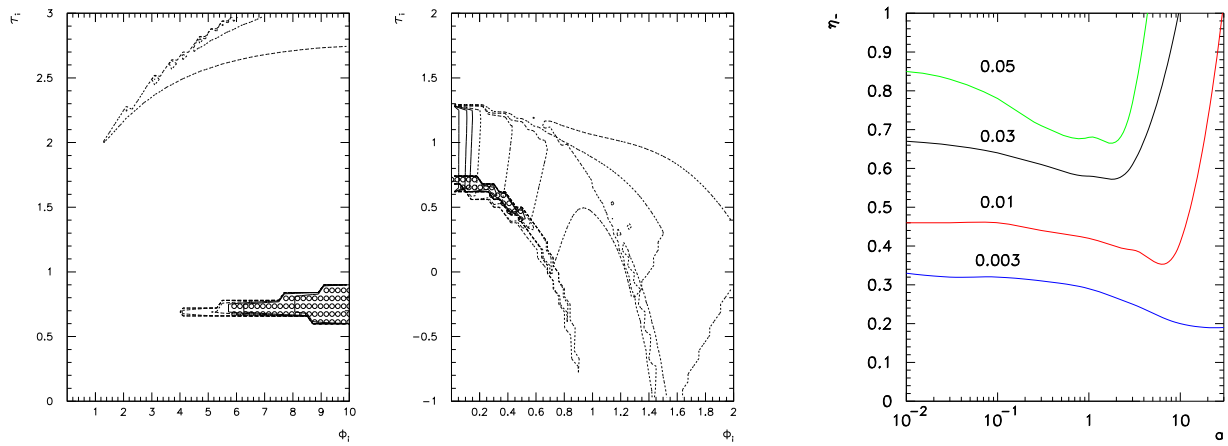

Figure 2: *Left and centre: Predictions for the number of e-foldings and the spectral index as functions of the initial conditions for inflation for the Model A (left) and Model C (centre). Isocontours of* $N_{\rm ef} = 5, 10, 20, 40$ *(dashed) and* $N_{\rm ef} = 50, 60, 100$ *(solid) are shown. The red circles correspond to the spectral index consistent with the WMAP data at 95% CL (the running of the scalar spectral index is negligible). Right: Moduli of negative eigenvalues of* η_{ij} *defined in (37) as a function of* a *for different values of* $1 - \Phi_0^2/t_0$.

energy density positive and sufficiently large, and to end in the flat space after inflation. The big question is: to what extent are these tunings independent? If the cancellation of the vacuum energy is to be supersymmetric, then it is rather obvious that in general the two tunings must be correlated, as the F^2 terms and the $-3e^K|W|^2$ term in the effective potential both depend on the same superpotential W.

In models of flux compactification, [11], [14], [19], one may obtain the following superpotential for the modulus field:

$$W_1(T) = \frac{1}{\sqrt{6}}\left(Ae^{-aT} + B\right). \tag{32}$$

For $B/A < 0$, the resulting scalar potential has a minimum for a finite value of t in a single-field scenario. However, this minimum corresponds to a negative value of the vacuum energy, which would result in an anti-de-Sitter Universe after inflation. This problem cannot be circumvented by adding a simple, monomial superpotential for the matter field, $W_2(\Phi) \propto \Phi^n$, $n = 1, 2, 3$. In the following, we choose $W_2(\Phi) = \tilde{A}\Phi^3$, which has the advantage that the contributions from W_2 to the scalar potential decouple from those coming from an arbitrary $W_1(T)$, thereby ensuring that the resulting scalar potential:

$$V(t, |\Phi|) = \frac{1}{(t - |\Phi|^2)^2}\left[\frac{A^2 a}{12}\left(e^{-2at}(1 + \frac{1}{3}at) + \frac{B}{A}e^{-at}\cos(at')\right) + \frac{3}{8}\tilde{A}^2|\Phi|^4\right] \tag{33}$$

is bounded from below. The slow-roll parameter $\eta_{t'}$ associated with the imaginary part of T is proportional to at and typically large. We can, therefore, assume that t' sits already at its

minimum, $at' = 0, 2\pi, \ldots$ for $B/A < 0$, when inflation starts. The terms originating from W_2 cannot lift the minimum of the scalar potential to non-negative values, since, due to the presence of the $|\Phi|^2$ term in the prefactor of (33), the minimum of the one-dimensional potential $V(t, 0)$ is a saddle point of the full scalar potential. The values t_0, Φ_0 of the fields at the minimum are given by the relations

$$\frac{3}{8}\tilde{A}^2 = \frac{A^2 a^2 e^{-2at_0}}{36\Phi_0^2} \qquad \text{and} \qquad \frac{B}{A} = -e^{-at_0}\left(1 + \frac{2}{3}at_0\right), \tag{34}$$

and the negative value of the potential for this field configuration is

$$V(t_0, \Phi_0) = \frac{A^2 a^2}{36(t_0 - \Phi_0^2)} e^{-2at_0} \equiv -V_0. \tag{35}$$

At this level, there is no potential for $\theta = \arg \Phi$, which supposedly arises at the loop level through Yukawa interactions of Φ.

If the necessary uplifting does not destroy the saddle point at $(t_0, 0)$, one could impose the initial conditions for inflation in its vicinity. The value of t would not then change significantly, since this field is already at its minimum and this model would realise single-field inflation.

A possibility for uplifting a negative minimum of the scalar potential consists in adding a brane-antibrane potential, as arises naturally in flux compactification models, [20]. In this scenario, we replace V_0 by $V_0 \left(\frac{t_0}{t}\right)^{2n}$. The relations (34) and (35) then change to:

$$\frac{3}{8}\tilde{A}^2 = \frac{A^2 a^2 e^{-2at_0}}{36\Phi_0^2(1-\alpha)}, \ \frac{B}{A} = -e^{-at_0}\left(1 + \frac{2-\alpha}{3(1-\alpha)}at_0\right) \text{ and } V_0 = \frac{A^2 a^2 e^{-2at_0}}{36(t_0 - \Phi_0^2)(1-\alpha)}, \tag{36}$$

where $\alpha = \frac{2n}{2+at_0}(1 - \Phi_0^2/t_0)$. However, utilising the brane-antibrane potential typically shifts the saddle point away from $\Phi = 0$ and the trajectory linking the saddle point with the minimum of the potential involves a substantial change in t, which spoils inflation, as the curvature of the potential is typically large in the T direction. This statement is illustrated in Figure 2, where we present the values of the negative eigenvalue η_- of the slow-roll matrix:

$$\eta_{ij} = \frac{1}{2V}\frac{\partial}{\partial \phi_i} K^{-1}_{jk} \frac{\partial V}{\partial \phi_k} \tag{37}$$

calculated at the saddle point for $n = 1$ and for various values of a and $1 - \frac{\Phi_0^2}{t_0}$. All the negative eigenvalues are of the order of unity and generally decrease with $1 - \frac{\Phi_0^2}{t_0}$, but one would need an extreme fine-tuning to get successful inflation.

6 Summary and Outlook

We have investigated in this paper several possible embeddings of simple inflationary potentials in supergravity theories, in manners consistent with the recent WMAP3 data. We have concentrated on models with the no-scale structure Kähler potential in the inflaton sector, so as to avoid the η problem and to take the advantage of the semipositivity of the potential which

holds in the exact no-scale structure limit of the models that we have considered. We have found that the no-scale matter sector may lead to acceptable inflation, however the modulus T must be stabilized by some additional mechanism which does not spoil the no-scale structure too violently, and some apparent fine-tuning is necessary.

We have considered D-type stabilisation which in the leading approximation keeps intact the semi-positivity of the potential, and F-type stabilisation, which breaks this feature. With D-stabilisation, the model building becomes unexpectedly simple, as the scales in the Φ-sector (by Φ we denote the matter-type inflaton) and in the T-modulus sector decouple. Depending on the assignments of charges and the specific form of the Kähler potential for charged fields, both chaotic and symmetry-breaking inflation are possible in this case. In particular, slow-roll inflation starting from the saddle points in the domain of sub- or near-Planckian field strengths With F–stabilisation, the post–inflationary minimum is naturally of the AdS type but, after suitable uplifting of the vacuum energy, the structure necessary for an inflationary epoch may appear. However, with a brane-antibrane-type potential for the modulus, the requirement that the uplifted vacuum energy is zero curves the potential too strongly to allow for inflation, at least if one stays with a reasonable level of fine tuning between expectation values of the modulus and the matter scalar, say up to one part per mille. In fact, in all the cases when Φ inflation takes place, an interesting fine-tuning between the parameters of the superpotential for the Φ field and the parameters of the sector which stabilises the T modulus is necessary.

In summary, we have demonstrated that inflation from matter fields is a nontrivial feature of supergravity models, not only because of the η problem, but also because it is subtly intertwined with the issue of moduli stabilisation. Nevertheless, models with inflation driven by the matter sector may be constructed in no-scale supergravity, if the moduli are stabilised by some higher-scale dynamics and at the expense of a certain level of fine-tuning.

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