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Spatial Distribution of the Magnetic Field Amplified in Young Supernova Remnants

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The spatial distribution of the magnetic field generated by a cosmic ray streaming instability in supernova remnants is determined. An alfvénic turbulence generated upstream supernova shock is amplified at the shock and advected downstream where its level rapidly approach to the value determined by the cosmic ray streaming downstream. Applications for an interpretation of X-ray observations are considered.

1. Introduction

It is evident now that a magnetic field in young supernova remnants (SNRs) is amplified in comparison with a shock compressed interstellar field [10]. One of the possible explanations is the Rayleigh-Taylor instability at a discontinuity which separates a dense supernova ejecta shocked at a backward shock and a circumstellar medium shocked at a forward shock (Gull [6]). However, the observations of the thin X-ray filaments (Bamba et al. [2]) which presumably correspond to a forward shock position are interpreted as an evidence of strong magnetic fields just behind the forward shock . This result is difficult to explain by the action of the Raleigh-Taylor instability. The most plausible explanation is the magnetic field amplification by a cosmic ray streaming instability [3, 4] upstream the forward shock. The maximum energy of cosmic rays accelerated at supernova shocks increases also. This permits to explain a cosmic ray spectrum up to the "knee" energy $E_k = 3\cdot 10^{15}$ eV [8].

In this paper we investigate this mechanism in more detail. We shall calculate a spatial distribution of the magnetic field amplified downstream the supernova shock. This is done in the next Section. The Sect. 3 contains the applications of the results obtained and conclusions.

2. The amplified field distribution

High energy cosmic ray particles accelerated at the forward supernova shock are concentrated in the shock front vicinity at distances smaller then κR from the shock front (cf. e.g. Berezhko et al. [5]). Here R is the forward shock radius and $\kappa \sim 0.1$. We shall use a plane shock approximation for this reason. Let us assume that a plane shock moves in the negative direction of the x axis with velocity u. We are interested in the case of a strong magnetic field amplification and neglect effects of a weak regular field. Assuming an equipartition between kinetic and magnetic energies of the magnetohydrodynamic (MHD) turbulence we use the following steady state equation for the magnetic field B written in the shock front frame:

$$u\frac{\partial}{\partial x}\frac{B^2}{4\pi} = V_a\left|\frac{\partial P_c}{\partial x}\right| - \eta\frac{V_a}{r_g}\frac{B^2}{4\pi}. \qquad (1)$$

Here $V_a = B/\sqrt{4\pi\rho}$ is the Alfvén velocity in the fully ionized medium with density ρ upstream the shock. The energy density of the MHD turbulence is given by $\varepsilon = B^2/4\pi$. The first term in the left hand side describes the MHD turbulence generation by the gradient of a partial cosmic ray pressure P_c. It is the pressure of highest energy particles accelerated at the shock. The gyroradius r_g of these particles is of the order of the turbulence scale. The nonlinear damping of the turbulence is determined by the second term in the right-hand side of this equation. The numeric factor $\eta \sim 0.1 \div 1.0$ is not well determined by a modern theory of the MHD turbulence. The simulations of the incompressible MHD turbulence give $\eta = C_K^{-3/2} = 0.15$, where C_K is the so-called Kolmogorov's constant (Verma et al. (1996)). The two other terms look similar to ones found in a framework of the weak Alfvénic turbulence theory. A similar equation was used by Bell and Lucek [3] for a qualitative consideration of the magnetic field amplification at supernova shocks. We should underline here that in the case of a strong magnetic field amplification it is not reasonable to treat Alfvén turbulence as waves propagating in different directions as was done by Bell and Lucek [3].

The equation for the cosmic ray partial pressure can be written as

$$u\frac{\partial P_c}{\partial x} = \frac{\partial}{\partial x} D \frac{\partial P_c}{\partial x}. \tag{2}$$

Here D is the cosmic ray diffusion coefficient.

Neglecting the nonlinear damping upstream the shock we find from Eq. (1):

$$\frac{B^2}{4\pi} = \frac{P_c^2}{4\rho u^2}. \tag{3}$$

In order to obtain the condition when the nonlinear damping is negligible we should compare the terms in the right-hand side of Eq.(1). The cosmic ray gradient can be estimated from Eq. (2). Assuming the Bohm diffusion coefficient $D = cr_g/3$ we find the condition

$$\eta \frac{P_c}{\rho u^2} << \frac{12u}{c}. \tag{4}$$

Since the partial cosmic ray pressure P_c hardly can be larger than $0.1\rho u^2$, this condition is fulfilled for shock velocities $u >> 0.01\eta c$.

At the shock front the medium velocity drops a factor of σ, where σ is the shock compression ratio. In addition, the medium compression results in the amplification of the turbulent energy by a factor of $f_m \sim 10$. Since the diffusion dominates advection for highest energy particles, we find from Eq. (2) $\partial P_c/\partial x = -P_c B/(\kappa R B_\infty)$, where B_∞ is the asymptotic value of the magnetic field produced by the cosmic ray streaming instability downstream. Now the solution of Eq. (1) downstream is

$$B^2 = \frac{B_0^2 B_\infty^2 \exp(x/l)}{B_0^2(\exp(x/l) - 1) + B_\infty^2}, \tag{5}$$

where B_0 is the magnetic field strength downstream just behind the shock and the length l is $l = \kappa R u B_\infty \sqrt{\rho/(4\pi\sigma)}/P_c$. The gyroradius of particles with maximum energy is determined by the condition $u\kappa R/\sigma = cr_g/3$ downstream the shock. Using also the Eq. (1) we find that

$$\frac{B_\infty^2}{4\pi} = \frac{3u}{c\sigma\eta} P_c. \tag{6}$$

Now the length l can be written as

$$l = \frac{\kappa R}{\sigma}\sqrt{\frac{3\rho u^3}{\eta c P_c}} \tag{7}$$

Let us compare B_0 and B_∞. Using Eqs. (3) and (6) we find

$$\frac{B_0^2}{B_\infty^2} = f_m \sigma \frac{\eta P_c c}{12\rho u^3}. \tag{8}$$

Although the ratio in the right-hand side of this expression is smaller than unity according to the condition (4), the magnetic field just downstream the shock B_0 can be larger than the asymptotic value B_∞ because of the factor $f_m\sigma \sim 40$. In this case the solution (5) describes the damping of the turbulent magnetic field advected from upstream to downstream down to the value B_∞. We should note here that the characteristic damping scale just behind the shock is a factor of $\frac{B_0^2}{B_\infty^2} - 1$ smaller than l:

$$l_d = \frac{l}{\frac{B_0^2}{B_\infty^2} - 1} = \kappa R \frac{4}{\sigma^2 f_m}\left(\frac{3\rho u^3}{c\eta P_c}\right)^{3/2}\left(1 - \frac{12\rho u^3}{\eta c P_c \sigma f_m}\right)^{-1},\ B_0 > B_\infty. \tag{9}$$

In the opposite case $B_0 < B_\infty$ the magnetic field is amplified further downstream up to the value B_∞. This is possible for SNR shock velocities larger than 15000 km s^{-1} for $\eta \sim 0.15$ and $P_c/\rho u^2 = 0.1$.

3. Discussion

The scale l_d found in the previous Section should be compared with the scale of synchrotron losses of high-energy electrons downstream the shock

$$l_{syn} = 5.6 \cdot 10^{19}\mathrm{cm}\frac{u}{\sigma c}\left(\frac{100\mu\mathrm{G}}{B}\right)^{3/2}\left(\frac{1\ \mathrm{keV}}{\epsilon}\right)^{1/2}. \tag{10}$$

Here ϵ is the energy of synchrotron X-rays.

The Eq. (9) can be rewritten in the case $B_0 >> B_\infty$

$$l_d = 6.5 \cdot 10^{19}\mathrm{cm}\left(\frac{u}{c}\right)^{3/2}\frac{\kappa}{0.1}\ \frac{R}{3\ \mathrm{pc}}\left(\frac{\eta}{0.15}\right)^{-3/2}\left(\frac{0.1\ \rho u^2}{P_c}\right)^{3/2}\frac{160}{\sigma^2 f_m}. \tag{11}$$

For the fast shocks with $u/c \sim 0.01$ both these quantities are about 10^{17} cm. We confirm the previous result of Pohl et al. [7] that the dissipation of the MHD turbulence may influence on the production of synchrotron X-rays in young supernova remnants. If the thickness of X-ray filaments is determined by the dissipation of MHD turbulence downstream supernova shocks, the inferred magnetic field may be smaller than in the case without dissipation [10]. The radio observations with a high angular resolution can help to distinguish between these possibilities, since the magnetic filaments may be observable in the radio.

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