New Development for Galactic Cosmic-Ray Propagation

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We have developed a new model and a corresponding computer code to study cosmic ray propagation and interactions. The model depends on the expansion of the time backward stochastic solution of the general diffusion transport equation [4] starting from an observer position to solve a group of diffusion transport equations each of which represents a particular element or isotope of cosmic ray nuclei. In this paper we are focusing on key abundace ratios such as B/C, sub-Fe/Fe. The effect of inhomogeneousity in the interstellar medium is investigated. The contribution of certain cosmic ray nuclei to the production of other nuclei is addressed.

1. Introduction

In this work we are introducing a numerical model that allows the study of cosmic rays production and propagation in the Galaxy. Using the backward stochastic solution of the general diffusion transport equation starting from an observer's position described by [4], we can calculate the elemental or isotopic abundance for a single cosmic ray nuclei. The same technique is applied to calculate the abundances for a certain number of nuclei by solving a group of diffusion transport equations, each represent a single nucleus. In this study we will focus on the calculation of B/C, sub-Fe/Fe to validate our model. The effect of the low density Local Super Bubble on the elemental and isotopic ratios observed at the interplanetary space is investigated. Using this model we can also investigate the contribution of some cosmic rays nuclides to the production of other nuclides



Figure 1. B/C and subFe/Fe ratios calculated for $z_h = 4$ kpc and dV/dz = 0. Solid curve (lower) local interstellar medium, dashed-dotted curve (upper) modulated $\phi = 500$ MV

2. Method

The general diffusion transport equation for the cosmic rays density distribution function $N_i(t, q)$ has the form [1]

$$\frac{\partial N_i}{\partial t} = f(\mathbf{\tilde{r}}, p) + \nabla \cdot (k_{\mathbf{xx}} \nabla N_i) - \mathbf{\tilde{v}} \cdot \nabla N_i + \frac{\partial}{\partial p} \left[\left(b_i - \frac{p}{3} (\nabla \cdot \mathbf{\tilde{V}}) \right) N_i \right] \\ + k_{pp} \frac{\partial^2 N_i}{\partial p^2} - \frac{1}{p^2} \frac{\partial}{\partial p} (k_{pp} p^2) N_i - nv \sigma_i N_i - \frac{1}{\tau_i} N_i + \sum_{j < i} nv \sigma_{ij} N_j + \sum_{j < i} \frac{1}{\tau_{ij}} N_j.$$
(1)

Here $f(\mathbf{\tilde{r}}, p)$ is the source term; $k_{\mathbf{xx}}$ is the spatial diffusion coefficent; $b_i(\mathbf{\tilde{r}}, p)$ characterizes momentum (energy) loss rate dp/dt; V is the convection velocity; $P(\nabla \cdot V)/3$ describes the adiabatic momentum (energy) losses; $\sigma_i(p)$ is the inelastic scattering cross section of a nucleus of type *i* with nuclei of the interstellar gas; $n(\mathbf{\tilde{r}})$ is the density of the interstellar gas; *v* is the velocity of the nucleus; σ_{ij} is the production cross section for a nuclei of type *j* from heavier nuclei of type *i* where j < i; τ_i is the life time of a nucleus of type *i* with respect to radioactive decay; τ_{ij} is the mean lifetime for the production of species *j* as a daughter nucleus in the radioactive decay of species *i*. The spatial position and momentum of a particle in diffusion process are given as in [4].

Equation (1) has a stochastic solution in the form:

$$N(t,\mathbf{q}) = E \int_0^T f(\mathbf{Q}_t^{t,\mathbf{q}}) exp\left(-\int_0^t c(\mathbf{Q}_s^{t,\mathbf{q}}) ds\right) dt$$
(2)

where $\mathbf{Q} = {\{\mathbf{\tilde{x}}, p\}}$ is 4-d stochastic process, E[Y] is defined as the expectation value of a random variable or a function of random variables [Y] with respect to the distribution space of all stochastic processes, T is the time needed for the stochastic process to run backward from q at time t till it gets to the boundary ∂D . These particles come through various stochastic paths from locations in the galaxy at time 0. The notation $\mathbf{Q}_{s}^{t,\mathbf{q}}$, describes the path of the particles starting at time t = 0. The exponential term contains the integration of $c(t, \mathbf{q})$ along the stochastic path allows the stochastic process to be destroyed at an exponential rate as a function of time. The parameter $c(t, \mathbf{q})$ contains terms describing the inelastic and production cross sections; density and mean life time.

Solving individual diffusion equations for each element is a long and inefficient processes. In this work we describe a new method that enables the solution of only one diffusion equation that describes all nuclei under consideration. In this case equation (1) is written in the form

$$\frac{\partial N}{\partial t} = F + \nabla \cdot (k_{\mathbf{x}\mathbf{x}} \nabla N) - \mathbf{\tilde{v}} \cdot \nabla N + \frac{\partial}{\partial p} \left[\left(b - \frac{p}{3} (\nabla \cdot \mathbf{\tilde{V}}) \right) N \right] \\ + k_{pp} \frac{\partial^2 N}{\partial p^2} - \frac{1}{p^2} \frac{\partial}{\partial p} (k_{pp} p^2) N - CN$$
(3)

where

where the vector N represents the calculated abundances at the solar system, F represents the source abundances where $f(\tilde{\mathbf{r}}, p)$ is given as in [2]. The composition of the source is assumed to be uniform. The 1 -d $c(\mathbf{Q}_s^{t,\mathbf{q}})$ term in equation (3) is replaced by the $((M \times M), M = 87)$ C matrix in the multidimensional solution where M is the number of nuclei under investigation

$$C = \begin{pmatrix} c_{11} & 0 & 0 & \dots & 0 \\ c_{21} & c_{22} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ c_{M1} & c_{M1} & \dots & \dots & c_{MM} \end{pmatrix},$$

where the components c are defined as

$$c_{ii} = nv\sigma_i + \frac{1}{\tau_i}$$
$$c_{ij} = -nv\sigma_{ij} - \frac{1}{\tau_{ij}}$$

The solution of equation (3) is given as:

$$N = E \int_0^T exp\left[-\int_0^t Cds\right] F(t)dt \tag{4}$$

Each of the C diagonal elements represents a loss of a certain nuclei either by spallation and / or decay, on the other hand, the off diagonal elements describe the production of the cosmic rays nuclei. The simulation includes a few thousand stochastic trajectories and the average abundance for each element at the solar system is estimated. We allow free escape at galactic radius R_h and the halo size $\pm z_h$. We take $R_h = 30$ kpc and $z_h = 4$ kpc. The He/H ratio is 0.11 by number and $d\mathbf{V}/dz = 0$. We assume a distance of 8.5 kpc from the solar system to the galactic center.

3. Results

We mainly investigated the B/C and subFe/Fe ratios for testing our model as they have the most accurately measured ratios covering a wide energy range and well established cross sections. The results show good agreement with the observational data as shown in figure (1). Figure 2 shows the effect of the low-density 0.06 particles/cm³ and an average radius of 200 pc region, which simulates the Local Super Bubble surrounding the solar system on the production of the ¹⁰Be/⁹Be. The ratio apparently decreased at lower energies due to the decay of the radioactive ¹⁰Be with lifetime of 3.1 million years which decrease its diffusion distance and make the effect of the Bubble significant; however the effect of the low-density region will be negligible at very high



Figure 2. Left. The effect of the low-density Local Bubble surrounding the solar system with radius of 200 pc and average density of 0.06 particles cm-3 on the ¹⁰Be/⁹Be ratio, Right. Elements percent contribution to the production of ¹⁰B at 1 GeV/nucleon

energies due to the relativistic lifetime of the nuclei at these energies. Using the new propagation model we can determine which nuclides make the most contribution to the production of other element or isotope at certain energies. By using equation (3) we can calculate the contribution of any element to the production of another element by knowing the production cross section and the source distribution. Carbon was found as the main contributer to the production of Boron ~ 55%, Oxygen is contributing by ~ 30%, and the rest ~ 15% from the participation of other nuclei as shown in figure 3.

4. Conclusions

We introduced a new numerical model to calculate the elemental and isotopic abundances of cosmic rays. The model depends on solving a group of diffusion transport equations each representing a particular element or isotope using the backward Markov Stochastic technique staring at an observer location in the solar system and stoping at the galaxy boundaries. Diffusion coefficient, halo size and rigidity are the main parameters contributing to this model and are chosen to best fit the observational data. Primary to secondary ratio have been used to validate the model. The results show good agreement with the B/C and subFe/Fe ratios. The effect of the inhomogeneities in the interstellar space is investigated as the model allows us to address small scale structure. We also calculated the source contribution to a particular cosmic ray species observed at the solar system.

References

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