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Propagation of Cosmic Ray Electrons Including the Source Region II : Diffusion Model

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Galactic supernova remnants(SNRs) are currently considered to be the source of galactic electrons. In SNRs it is supposed that electrons escape from the acceleration site, diffuse in the magnetic field and lose energy through the synchrotron radiation. Such propagation and the energy losses certainly affect the observed electron spectrum. We treat the electron propagation in the diffusion model and estimate the electron spatial distribution in SNRs and its effects to the escape rate from SNRs into the interstellar space. The result shows that the location of the acceleration site influences the escape rate. In particular, if the energy dependence of the diffusion coefficient is small, the escape rate for plerions has an abrupt decrease in the high energy region above TeV.

1. Introduction

The supernova remnants(SNRs) are most likely sources of cosmic ray electrons because the non-thermal radio and X-ray emission from SNRs indicate the existence of high energy electrons in SNRs. In this paper the source region is approximately treated as the diffusion space; electrons ejected from the shock front of blast waves or pulsar winds, diffuse in a SNR and escape from it at some rate. The electron propagation and its escape rate from SNRs may be distinguished by shell-type SNRs or plerions. Assuming some simple spatial distributions of the acceleration site, we calculate the electron distribution in SNR and the escape rate from it. The distribution is calculated at various energies which reflects observed radiation profile with corresponding frequency. Thus the consistency with the observed radiation data will decide the validity of the model.

2. Formulation

The electron spectrum in the source region is given by $f(\mathbf{r}, E)$ with the space variable $\mathbf{r}$. If the acceleration site has the source distribution $q(\mathbf{r}, E)$ and ejected electrons diffuse with the diffusion coefficient $D(\mathbf{r}, E)$, the propagation equation in SNRs is represented by

$$-\mathrm{div}(D\nabla f(\mathbf{r}, E)) + \frac{\partial}{\partial E}(\frac{dE}{dt} \cdot f(\mathbf{r}, E)) = q(\mathbf{r}, E) \ . \tag{1}$$

In this paper the spherical equation with the radial variable $x = r/r_N$ (r_N : SNR radius, $x = (0, 1)$) is solved. The electron spectrum $f(x, E)$ and its source spectrum $q(x, E)$ are Fourier transformed as

$$\{ \ f(x, E), \ \ q(x, E) \ \} = \sum_{n=0}^{\infty} \{ \ a_n(E), \ \ q_n(E) \ \} \cos[(n + 1/2)\pi x$$

with the boundary condition $f(1, E) = 0$ and $q(1, E) = 0$. The Fourier expressions are substituted into Eq. (1) and consequently the coefficient a_n satisfies the leaky box equation,

$$-\frac{a_n}{\tau_n} + \frac{d}{dE}(bE^2 \cdot a_n) + q_n = 0 \ , \tag{2}$$

in which τ_n is defined as

$$D(E)[(n+1/2)\pi]^2 \equiv 1/\tau_n \ ,$$

and the diffusion coefficient $D(E) = D_0 E^{\delta_d}$ ($D_0 = D(1\text{GeV})$) is assumed to be spatially uniform.

Next, we calculate the electron flux escaping from the source region. Integrating the Eq. (1) over the whole source region with the volume V, we get

$$\int_V -\text{div}(D\nabla f(\mathbf{r},E))dV + \frac{\partial}{\partial E}(\frac{dE}{dt} \cdot N(E)) = Q(E) \ . \tag{3}$$

The first term represents the electron spectrum $S(E)$ escaping from the source region into the interstellar space.

$$S(E) = \int_V -\text{div}(D\nabla f(\mathbf{r},E))dV \ .$$

In the spherical case, $S(E)$ is obtained from the solution a_n of Eq. (2),

$$S(E) = -D\frac{df}{dx}|_{x=1} = D\sum_{n=0}^{\infty}(-1)^n(n+1/2)\pi \cdot a_n(E) \ .$$

The electron spectrum $N(E)$ and the source spectrum $Q(E)$ in Eq. (3) are given by

$$\{ \ N(E), \ \ Q(E) \ \} = \int_0^1 \{ \ f(x,E), \ \ q(x,E) \ \} dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1/2)\pi} \{ \ a_n(E), \ \ q_n(E) \ \} \ .$$

The escape rate from the source region is calculated from

$$\frac{1}{\tau_s(E)} = \frac{S(E)}{N(E)} = D(E) \cdot \frac{\sum_{n=0}^{\infty}(-1)^n a_n(E) \cdot (n+1/2)\pi}{\sum_{n=0}^{\infty}(-1)^n a_n(E)/[(n+1/2)\pi]} \ . \tag{4}$$

In the next section. the distribution $f(x,E)$ and the escape rate $1/\tau_s(E)$ are calculated in some simple cases, in which $q(x,E) = Q_0 E^{-\gamma} \cdot Q(x)$ is assumed.

3. Calculations

Spatial distribution in SNRs : In two types of SNRs, a plerion and a shell-type SNR, the spatial distribution of electrons is calculated. A plerion has the acceleration site at the pulsar wind shock. If we adopt the typical SNR, Crab nebula, the shock radius of 8" and the nebular size of 200" [1] give the source function described as

$$Q_p(x) \ = \ 1, \ \ 0 \le x < 0.04, \quad \ = \ 0, \ \ 0.04 \le x \le 1 \ \ .$$

The result of $f(x,E)$ ($E = (\nu[\text{MHz}]/16.1/B[\mu\text{G}])^{1/2}$) is shown in Fig. 1(a). The diffusion coefficient has the value of $D_0 = 1 \times 10^{25}\text{cm}^2/\text{s}$ for $B = 1.7 \times 10^{-4}$G[2] and the distance of 2kpc.

As shown in Fig 1(a), the energy dependence δ_d significantly affects the high-energy distribution. The calculated curves with $\delta_d = 0.3$ are consistent with the data larger than equipartition (r/r_N>0.3). Inner region(<0.3) probably has a different propagation process and increasing magnetic field distribution, so that the optical and X-ray curves do not fit the data. More precise calculations are needed. The value of 0.6 does not reproduce the data except radio data because 0.6 means weak confinement and electrons escape too rapidly.

The distribution $f(x, E)$ of a typical shell-type SNR, SN1006, is demonstrated in Fig 1(b). The data are so complicated that we present the calculation compared with just one slice of the northeast limb(Long 2003, Fig.4 A)[3], in which the radius of 1000" and one source close to the boundary is assumed.

$$\begin{aligned} Q_s(x) &= 0, \quad 0 \leq x < 0.92, \quad 0.94 < x \leq 1 \\ &= 1, \quad 0.92 \leq x \leq 0.94\,, \end{aligned}$$

The large decrease of the radio intensity toward the inner region[4] indicates a small value of the diffusion coefficient or a rather strong magnetic field. The curves shown in Fig. 1(b) roughly satisfy the data and have the parameter set ($B\mu$G, D_0cm^2/s) = (10, 1 × 10^{22}) or (100, 1 × 10^{24}) for the distance of 2kpc.

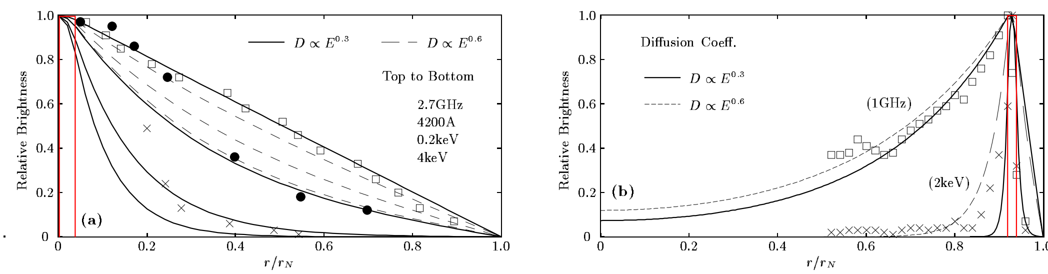

Figure 1. (a) **Plerion** : The spatial distribution $f(x, E)$ is shown with the observed radio(open square), optical(filled circle) and X-ray profile (cross) data of Crab Nebula given in DeJager et al.(1992)[1]. The curves have the corresponding frequency as shown in the figure. (b) **Shell-type SNR** : $f(x, E)$ is shown with the data of Long et al., 2003, Fig. 4 A [3]. The calculation of radio profile at 1GHz and X-ray of 2keV are indicated. In both cases of (a) and (b), the red line indicates the source distribution and $\delta_d = 0.3$(straight line), $\delta_d = 0.6$(dashed line) are shown.

Escape rate from SNRs : For three distributions of Q_p , Q_s and a linearly decreasing $Q_l = |1 - x|$, the escape rate $1/\tau_s$ of Eq. (4) is calculated and shown in Fig. 2. The parameters adopted in the calculation are the source spectral index $\gamma = 2.2$ and the diffusion coefficient $D = b_1 r_N^2 / 0.05 \cdot E^{\delta_d}$cm^2/s with $\delta_d = 0.3,\ 0.6$ (b_1: energy loss coefficient in SNR).

In the case of $\delta_d = 0.3$, we notice the slope of the curves deviates from the value of δ_d. The escape rate of plerions(Q_p) drastically decreases in the high energy region because high energy electrons lose the most part of energy during the propagation in a plerion and cannot escape from it. On the other hand the slope of the shell-type SNRs(Q_s) gradually increases than 0.3 as energy increases. The linear decrease q_l maintains the slope of 0.3, so that the escape rate has no effect of the spatial distribution in the source region. In the case of $\delta_d = 0.6$, that means easily escape of high energy electrons, the escape rate has little changes from 0.6 for each curve. It means that the spatial distribution hardly affects the escape rate and the observed spectrum may have no energy cut-off. The results indicate that the spatial distribution of the acceleration site gives the large influence to the escape rate only with a small energy dependence of the diffusion coefficient. These estimates of escape rate in Fig. 2 just present the value of $b_1 r_N^2 / D_0 = 0.05$(Fig. 1(a)). A precise investigation including different values will be required.

Interstellar spectrum : The interstellar spectrum of electrons which originates in plerions or shell-type SNRs is shown in Fig. 3. In the calculation propagation parameters are the same as in Fig. 1, and the interstellar space is treated as a leaky box model that assumes the uniform distribution of space and time; the resulting curves just present the effectiveness of electron propagation in SNRs to the observed spectrum. In Fig. 3, both curves do not exceed the data above several hundreds GeV, even if the source index is 2.2 , since δ_d in SNRs is assumed to be small. In the case that shell-type SNRs are major contributors, they can cover the whole observed spectrum.

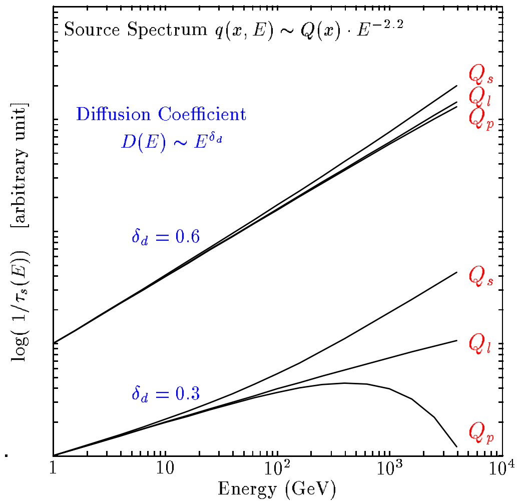

Figure 2. Escape rate $1/\tau_s$ from SNRs: The source spectrum $Q(x)E^{-2.2}$ has three types of a shell-type SNR(Q_s), a linear decrease(Q_l) and a plerion(Q_p), which expression is indicated in the text. $D(E)$ is $b_1 r_N^2/0.05 \cdot E^{\delta_d} \mathrm{cm}^2/\mathrm{s}$ with $\delta_d = 0.3,\ 0.6$.

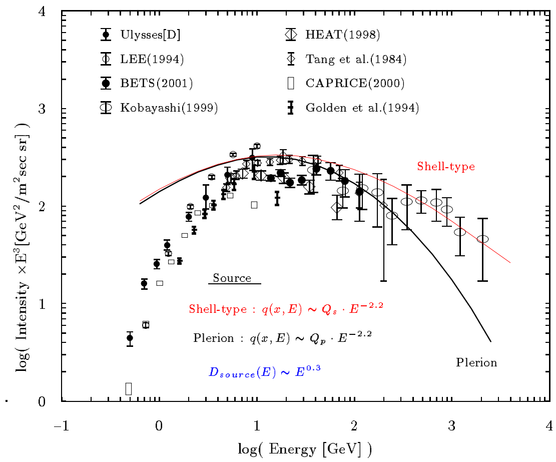

Figure 3. Interstellar spectrum from two types of SNRs: shell-type SNRs and plerions. $\delta_d = 0.3$, the source index of 2.2 are assumed and other parameters are given in Fig. 1. These curves indicate the effectiveness of spatial distribution of electrons in SNRs to the interstellar spectrum.

4. Conclusions

The electron propagation including the source region as the diffusion space has been discussed. The electron spatial distribution in SNRs and the escape rate from them have been calculated in some cases with the simple spatial distribution of the acceleration site. In the case of Crab nebula and SN1006, the radiation profiles have roughly been estimated and it became clear that the X-ray profile determines the energy dependence of the diffusion coefficient. The escape rate from SNRs is largely influenced by the spatial distribution in SNRs when the energy dependence of the diffusion coefficient is small. In particular, high-energy electrons lose major energy in a plerion, so that the escape rate has a drastic decrease above TeV. The interstellar electron spectrum is influenced whether major contributors are plerions or shell-type SNRs. If the shell-type SNRs are major contributors, it is possible to cover the observed spectrum up to a few TeV[5].

References

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