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Propagation of Cosmic Ray Electrons Including the Source Region I : Leaky Box Model

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We treat the propagation of Galactic electrons including the source region, which is currently considered to be supernova remnants(SNRs). The simplest model is formulated in which the source region is considered as a leaky box and is characterized by the escape rate of electrons into the interstellar space. It becomes clear that the energy index of the escape lifetime in SNRs influences the high energy part of the interstellar spectrum, and makes it possible to explain the observed data when the source spectral index is smaller than 2.3 that is expected from the radio spectral index in SNRs. The escape lifetime of electrons in SNRs is also discussed using the ratio of radio flux from SNRs and the Galactic background radio flux. The result shows that the electron lifetime in SNRs is $\sim 10^4$yr around 1GeV, which corresponds to the SNR age in the Sedov phase.

1. Introduction

The supernova remnants(SNRs) are most likely sources of cosmic ray electrons because the non-thermal radio and X-ray emission from SNRs indicate the existence of high energy electrons in SNRs. The average radio index of 0.5∼0.6 in SNRs[1] implies the source index of 2.0∼2.2. For a global view of the cosmic ray propagation the source region is regarded as a leaky box and distinguished from the interstellar space in this paper. The propagation parameters in a SNR include the energy loss coefficient of electrons $dE/dt = -bE^2$, $b = 1.02 \times 10^{-16}(W_{photon} + B^2/8\pi)[\mathrm{eV/cm^3}]$ in the average magnetic field B and the escape lifetime (inversion of the escape rate). The average escape rate from SNRs is estimated from the ratio of the total radio flux from SNRs to the Galactic background radio emission. Duric et al.[2] have investigated the escape rate in M33 and concluded that SNRs are enough to be the candidate of cosmic ray sources.

2. Formulation

If each SNR is specified by the subscript i and has the source spectrum $Q_i(E) = Q_{0i}E^{-\gamma_i}$, the electron lifetime $\tau_i(E) = \tau_{0i}E^{-\delta_i}$ and the energy loss coefficient b_i, the electron spectrum $N_i dE$ satisfies the leaky box model(LBM) equation,

$$\frac{N_i(E)}{\tau_i(E)} + \frac{d}{dE}[\frac{dE}{dt}N_i(E)] = Q_i(E)\ , \tag{1}$$

and the solution[3] is given by

$$N_i(E) = \int_0^1 Q_i(\frac{E}{x})\exp[-\int_x^1 \frac{1}{\lambda_i(E/x')}\frac{dx'}{x'}]\frac{dx}{b_iEx^2} \simeq \frac{Q_{0i}}{b_iE^{\gamma+1}}\,\frac{1}{\gamma-1+\lambda_i^{-1}}\ , \tag{2}$$

where $\lambda_i \equiv b_iE\tau_i(E)$ means the (escape/energy-loss) lifetime ratio in the SNR(i). The spectral index changes from $-(\gamma+\delta_i)$ to $-(\gamma+1)$ at the break energy E_i satisfying $\gamma - 1 = \lambda_i^{-1}$, namely $\tau_i(E_i) \sim 1/b_iE_i$, $E_i = [(\gamma_i-1)b_i\tau_{0i}]^{-1/(1-\delta_i)}$. The approximate expressions are estimated as

$$\begin{aligned} N_i(E) \sim Q_i(E)\tau_i(E) \quad &\propto \quad E^{-(\gamma_i+\delta_i)} \qquad E < E_i\ ,\\ N_i(E) \sim Q_i(E)\ /\ b_iE(\gamma_i-1) \quad &\propto \quad E^{-(\gamma_i+1)} \qquad E_i < E\ . \end{aligned} \tag{3}$$

There are many SNRs in the Galaxy from which we suppose that Galactic electrons are injected into the interstellar space. Thus the source spectrum in the Galaxy $Q_g(E)$ is represented by the summation of the solution N_i of Eq. (2) multiplied by the escape rate of $1/\tau_i(E)$,

$$Q_g(E) = \sum_i \frac{N_i(E)}{\tau_i(E)} = \sum_i \frac{1}{\tau_i(E)} \cdot \frac{Q_{0i}}{b_i E^{\gamma_i+1}} \int_0^1 x^{\gamma_i-2} \exp[\frac{-1+x^{1-\delta_i}}{\lambda_i(E)(1-\delta_i)}]dx \; .$$

The spectral index changes from $-\gamma_i$ to $-(\gamma_i+1-\delta_i)$ at each break energy E_i.

If the Galaxy is treated as one leaky box with the lifetime $\tau_g(E) = \tau_0 E^{-\delta_g}$, the interstellar electron spectrum $N_g(E)dE$ also satisfies the LBM equation and is given by the solution of Eq. (2) with the source spectrum Q_g. Substituting the above expression, we obtain the interstellar spectrum,

$$\begin{aligned} N_g(E) &= \int_0^1 \sum_i \frac{N_i(E/x)}{\tau_i(E/x)} \exp[-\int_x^1 \frac{dx'/x'}{\lambda_g(E/x')}] \frac{dx}{b_g E x^2} \\ &= \sum_i \frac{Q_{0i}}{\lambda_i(E) b_g E^{\gamma_i+1}} \int_0^1 dx \int_0^1 dx' x^{\gamma_i-1-\delta_i} x'^{\gamma_i-2} \cdot \exp[\frac{x^{1-\delta_i}(-1+x'^{1-\delta_i})}{\lambda_i(E)(1-\delta_i)}] \exp[\frac{-1+x^{1-\delta_i}}{\lambda_g(E)(1-\delta_g)}] \quad (4) \\ &\simeq \sum_i \frac{Q_{0i}}{\lambda_i(E) b_g E^{\gamma_i+1}} \frac{1}{\gamma_i - 1 + \lambda_i^{-1}} \frac{1}{\gamma_i - \delta_i + \lambda_g^{-1}} \; , \quad (5) \end{aligned}$$

in which the energy-loss coefficient in the interstellar space is assumed to be $b_g = 2.0 \times 10^{-16}[\mathrm{GeV}^{-1}\mathrm{sec}^{-1}]$ [4] and the lifetime ratio becomes $\lambda_g \equiv \tau_g(E) b_g E$. The solution is easily calculated using the numerical integration technique.

The interstellar spectrum N_g has two break energy: $E_c = [(\gamma_i - \delta_i) b_g \tau_0]^{-1/(1-\delta_g)}$ is given by $\gamma_i - \delta_i = \lambda_g^{-1}$ and $E_i = [(\gamma_i - 1) b_i \tau_{0i}]^{-1/(1-\delta_i)}$ is given by $\gamma_i - 1 = \lambda_i^{-1}$. The break energy in the interstellar space E_c is almost independent of i and has the value of 11 ± 3GeV when $\gamma_i = 2.0 \sim 2.2$, $\delta_i = 0.3 \sim 0.6$ and $\tau_g(1\mathrm{GeV}) = 2 \times 10^7$yr (=9.4g/cm^2[5] for hydrogen density $n_H = 0.3\mathrm{cm}^{-3}$). On the other hand, E_i depends on the SNR type, age and so on, and is larger than E_c because of $b_i \tau_i < b_g \tau_g$. Thus the spectrum changes as

$$\begin{array}{llll} N_g(E) \sim & \sum_i Q_i(E) \tau_g(E) & \propto E^{-(\gamma_i+\delta_g)} & E < E_c \; , \\ N_g(E) \sim & \sum_i Q_i(E) \,/\, b_g E(\gamma_i - \delta_i) & \propto E^{-(\gamma_i+1)} & E_c < E < E_i \; , \\ N_g(E) \sim & \sum_i Q_i(E) \,/\, b_i b_g E^2 (\gamma_i - 1)(\gamma_i - \delta_i) \tau_i(E) & \propto E^{-(\gamma_i+2-\delta_i)} & E_i < E \; . \end{array} \quad (6)$$

The spectral index in the highest energy region depends on the SNR parameter δ_i and is steeper than the index $\gamma_i + 1$ of the simple LBM by $1 - \delta_i$. It is important to notice that when $\delta_i \to 1$, $E_i \to \infty$ and Eq. (4) is reduced to the simple LBM solution with the single break energy E_c. Another point to notice is that if $\delta_i = \delta_g = \delta$, the integral in Eq. (4) is reduced to the difference of two simple LBM solutions as
$\int_0^1 dx\, x^{\gamma_i-2}/(\lambda_g^{-1} - \lambda_i^{-1}) \cdot \{\exp[(-1+x^{1-\delta})/(\lambda_i(1-\delta))] - \exp[(-1+x^{1-\delta})/(\lambda_g(1-\delta))]\}$.

3. Calculations

We investigate the electron lifetime in SNRs from the electron number ratio of two regions: interstellar space and SNRs. The ratio $\sum_i N_i/N_g$ is calculated from Eq. (2) divided by Eq. (5). If major SNRs, namely shell-type SNRs, have the same parameters of τ_s and b_s, the ratio is given by

$$\frac{\sum_i N_i(E)}{N_g(E)} = \frac{\tau_s(E)}{\tau_g(E)} (1 + (\gamma - \delta_s)\lambda_g) \; .$$

At the low energy around 1GeV ($E < E_c$), the Galactic parameter λ_g(1GeV) becomes ~ 0.1 with the values of b_g and τ_g given in Sec. 2. Therefore, the electron number ratio approximates the lifetime ratio, namely $\sum_i N_i(E)/N_g(E) = \tau_s(E)/\tau_g(E)$. In the case that Galactic SNRs have different types and ages, this relation is also derived from the definition of the average rate $1/\tau_s(E)$ ($\tau_s(E) = \tau_0 E^{-\delta_s}$), namely

$$1/\tau_s(E) \equiv \frac{\sum_i N_i(E)/\tau_i(E)}{\sum_i N_i(E)} .$$

If we use $N_g = \tau_g \sum_i Q_i = \tau_g \sum_i N_i/\tau_i$ obtained from Eq. (3) and Eq. (6), we get $\sum_i N_i/N_g = \tau_s/\tau_g$.

If the observed radio flux $S(\nu)$ has the spectral index $-\beta$ around the frequency ν, the electron spectrum producing this flux in the magnetic field B is expected to be $N(E)dE = \frac{S(\nu)}{Y(\beta)\nu^{-\beta}} B^{-(\beta+1)} E^{-(2\beta+1)} dE$, where $Y(\beta)$ only includes β and does not sensitive to β. This formula gives the relationship between the escape lifetime ratio τ_s/τ_g and the radio flux ratio S_1/S_2, in which $S_1(\nu) \propto \nu^{-\beta_s}$ is the total radio flux from Galactic SNRs, and $S_2(\nu) \propto \nu^{-\beta_g}$ is the whole background radio flux in the Galactic disk.

$$\frac{\tau_s}{\tau_g}(1\mathrm{GeV}) = \frac{S_1(\nu_s)}{S_2(\nu_g)}(\frac{B_s}{B_g})^{-1} = \frac{S_1(\nu_s)}{S_2(\nu_s)}(\frac{B_s}{B_g})^{-(\beta_g+1)} \simeq 0.02 \cdot (\frac{B_s}{B_g})^{-(1.6\sim1.7)} , \tag{7}$$

where 1GeV electrons emit the radiation of the frequency ν_s and ν_g in the magnetic field B_s of SNRs, and in the interstellar magnetic field B_g respectively. In addition we used the relationship of $\nu_s/\nu_g = B_s/B_g$ and assume that β_s and β_g do not change between ν_s and ν_g. The Galactic radio index $\beta_g = 0.6 \sim 0.7$.

The flux ratio $S_1/S_2 = 0.02$ is estimated in the following way. $\underline{S_1 = S_{Av} \times n_s}$: S_{Av} is the average flux of 51 SNRs with the known distance in the Green's catalog[1] and has the value of $S_{Av} = (1.7 \pm 0.4) \times 10^{17}\mathrm{W/Hz}$ at 1GHz. The total number of SNRs, n_s, has the expression of $n_s = n_0 \pi R^2$, $(n_0 = 1.3)$ within the distance R[kpc] from the solar system as shown in Fig. 1. It is estimated from the accumulated number of 54 SNRs in Green's catalog[1] and of 125 SNRs in Milne's catalog [6]. Green's and Milne's data sets are consistent within 3kpc. 231 SNRs are currently detected[1], however we consider that almost nearby SNRs within several kpc are counted in 54 SNRs. $\underline{S_2 = I_p(\nu)(4\pi/L) \times V_g = 8\pi^2 R^2 I_p(\nu)}$: $I_p(1\mathrm{GHz}) = 3.9 \times 10^{-22}\mathrm{W/m^2 sr\ Hz}$ is the intensity in the polar direction [7][4] with the distance of the line of sight, L[kpc]. The volume of the propagation region $V_g \simeq \pi R^2 \times 2L$ is substituted. The flux ratio finally has the following expression and the value.

$$\frac{S_1}{S_2}(1\mathrm{GHz}) = \frac{S_{Av} \times n_0}{8\pi I_p} = 2.3 \times 10^{-2} .$$

At the different frequency of 408MHz, the average flux $S_{Av} = (2.8 \pm 0.7) \times 10^{17}\mathrm{W/Hz}$ [8] and the Galactic radio flux $I_p(408\mathrm{MHz}) = 7.6 \times 10^{-22}\mathrm{W/m^2 sr\ Hz}$ give the ratio $S_1/S_2(408\mathrm{MHz}) = 1.9 \times 10^{-2}$.

The value of B_s is not well known, however, the strength for several SNRs are investigated from the measurements of X-ray and gamma ray emissions. In the case of typical shell-type SNR, SN1006, HESS has recently reported the lower limit of 25μG[9]. The arguments of particle acceleration using the X-ray morphology require 10-100μG[10][11]. As B_s represents the value in radio emission region, it may be smaller than that in acceleration site. Since 80% of Galactic SNRs are shell-type, B_s seems to be below 100μG and $B_s/B_g \leq 10$. If the magnetic field ratio is $B_s/B_g = 10$, Eq. (7) gives the lifetime ratio $\tau_s/\tau_g = (5 \pm 1) \times 10^{-4}$. Thus the mean lifetime of 1GeV electrons in SNRs becomes $\tau_s = 1 \times 10^4$yr as the lifetime $\tau_g = 2 \times 10^7$yr. Consequently, the electrons around 1GeV stay in SNRs for the rather long period of $\sim 10^4$yr which corresponds to the SNR age in the Sedov phase.

Using the result of Eq. (7), we compare the interstellar solution of Eq. (4) with the observed data. The curves of Eq. (4) with different energy index δ_s are shown in Fig. 2. It is clearly shown that the value of δ_s influences the high energy spectrum and is consistent with the ECC data [12] when $\delta_s = 0.2 \pm 0.1 (\gamma = 2.2)$. These small values of δ_s indicates that the high-energy electrons in SNRs are intensively confined.

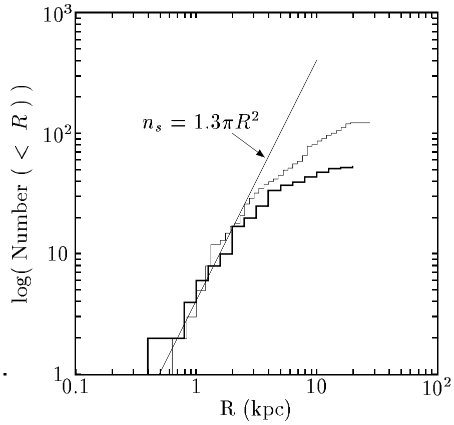

Figure 1. Accumulated number of SNRs within the distance R from the sun. Two data sets are given: the thickline is made up of 54 SNRs in Green's catalog [1], the thinline is early data set of 125 SNRs in Milne's[6]. Both data sets are consistent within 3kpc. The regression line is assumed to be proportional to R^2.

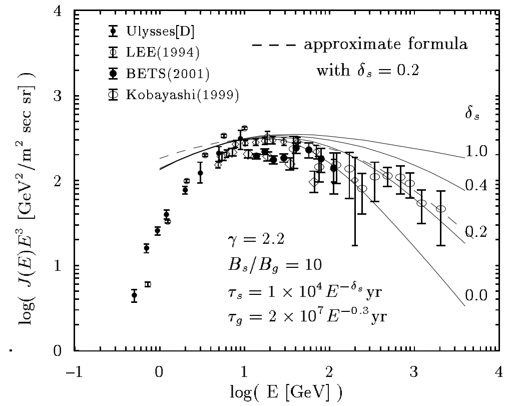

Figure 2. Calculated local spectra from the model with different values of δ_s. Those lines are compared with the observed data[12]. The best fit curve has the break energy of $E_c = 10$GeV, and $E_i = 500$GeV. The dashed line normalized at 10GeV shows the approximate expression of Eq (5).

4. Conclusions

We have discussed the propagation of Galactic electrons treating each SNR as a leaky box. This is the global modeling of electron propagation including the source region. The key parameter, the electron escape lifetime in SNRs has been estimated from the radio flux from SNRs and the Galactic background radio emission. If the magnetic field in SNRs is nearly 10 times the strength of the Galactic magnetic field, the lifetime of 1GeV electrons in SNRs becomes $\sim 10^4$yr, which corresponds to the SNR age in the Sedov phase. The escape lifetime from SNRs influences the interstellar electron spectrum above a few hundred GeV. The observed data indicate that the energy index of the escape rate is small, if the source spectral index is $2.0 \sim 2.2$ that is expected from the average radio index of SNRs.

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