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Updating the BTFM of the PSB

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1. Preamble

The aim here is to reconsider a measurement tool (Beam Transfer Function Measurement) developed by H. Schönauer and M. Sjöström [1] that was using obsolete hardware. The software and hardware interfaces need to be modified to make use of a new instrument (vector analyzer) that is now available.

2. Some definitions:

The **BTF** (Beam Transfer Function) $= \dfrac{I_{Beam}}{V_{RF}}$ is the beam current response over voltage excitation (longitudinal or transversal) ratio. This is the primary calculation carried out by the equipment.

The **LSD** (Longitudinal Spectral Density) is a 2D graph showing particle density versus revolution frequency or momentum. This plot is obtained by integrating the BTF using Vlasov equation.

The **Shottky** power spectrum gives a direct measurement of the LSD, but at PSB injection, the remaining rf structure inherited from the Linac (bubbles in phase space) makes it unreliable to use it as a tool.

3. New measurement set:
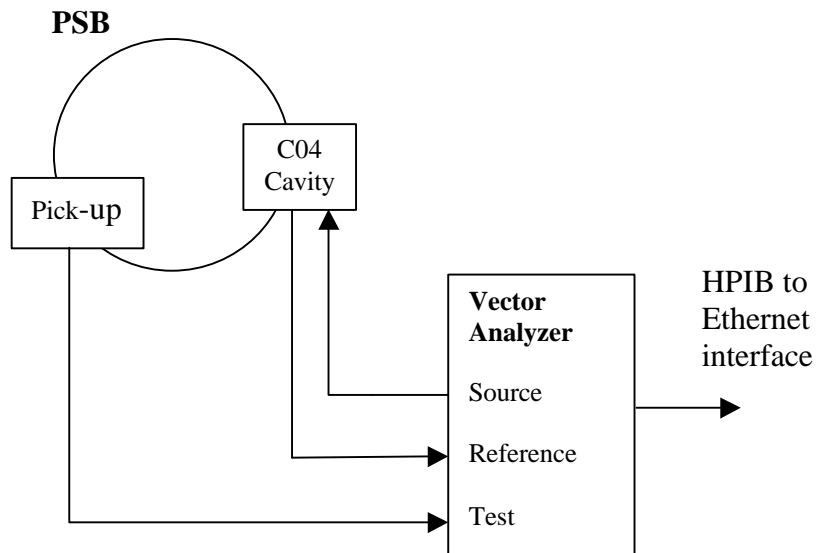
Some useful parameters

Physical parameter	Symbol	PSB value	unit
Particle charge	e	$1.60219 . 10^{-19}$	As
Proton rest energy	E_0	938.2796	MeV
Mean radius	R	25	M
Mean kinetic energy (inj)	E_k	49.89	MeV
Mean revolution frequency	ω_{rev}	$2\pi.598.73$	Krad/s
Relative velocity factor	β	0.3137	
Relative mass factor	γ	1.0532	
Off-momentum parameter	η	-0.8412	
Relative momentum spread	$\Delta p_s/p_0$	$\approx 1.5 . 10^{-3}$	
Number of particles	N_p	$\rightarrow 10^{13}$	
Excitation harmonic number	h	6	
Cavity – PU azimuth	θ_m	249.2	degree
Coupling impedance	Z_{long}	$\approx 50 - i\,1500$	Ω

4. Basic measurements

4.1 Beam Transfer Function (BTF):

$$\mathrm{BTF}(\acute{u}) = \frac{I_{Beam}(\omega)}{V_{RF}(\omega)} = \frac{test}{reference}$$

As the BTF is the ratio of two complex rf signals, it has an amplitude and a phase. To obtain proper results, care has to be taken to equalize delays on both signal and to compensate the phase lag of beam pick-up with respect to cavity pick-up.

4.2 Longitudinal Spectral Density (LSD):

Vlasov gives an expression linking BTF to Density Distribution $F_0(\omega)$ in the longitudinal case:

$$BTF_{long}(\omega) = \frac{\hat{I}_{Beam}(\omega)}{\hat{V}_{rf}(\omega)} = -i \cdot \frac{e^2 \cdot \omega_{rev0}^3 \cdot \eta \cdot N_p}{(2\pi)^2 \cdot \beta^2 \cdot \gamma \cdot E_0 \cdot h} \cdot \int_{-\infty}^{\infty} \frac{\dfrac{dF_0}{d\omega_{rev}}(\omega_{rev} - \omega_{rev0})}{\dfrac{\omega}{h} - \omega_{rev}} d\omega_{rev}$$

ω_{rev0} is the mean revolution frequency
ω_{rev} is the revolution frequency of each individual particle

The integral is calculated on an infinite domain, but is practically limited to where the response is significant.

To simplify the notation the first factor will be gathered in a constant C_m.

$$C_m = \frac{e^2 \cdot \mathsf{w}_{rev0}^3 \cdot \mathsf{h} \cdot N_p}{(2\mathsf{p})^2 \cdot \mathsf{b}^2 \cdot \mathsf{g} \cdot E_0 \cdot h}$$

The integral can be split into principal value (PV) and residue at the pole. This gives:

$$BTF_{long}(\mathsf{w}) = \frac{\hat{I}_{Beam}(\mathsf{w})}{\hat{V}_{rf}(\mathsf{w})} = C_m \cdot \left(\pm \mathsf{p} \cdot \frac{dF_0}{d\mathsf{w}_{rev}}\left(\frac{\mathsf{w}}{h} - \mathsf{w}_{rev0}\right) + i \cdot PV \int \frac{\dfrac{dF_0}{d\mathsf{w}_{rev}}(\mathsf{w}_{rev} - \mathsf{w}_{rev0})}{\dfrac{\mathsf{w}}{h} - \mathsf{w}_{rev}} d\mathsf{w}_{rev} \right)$$

The + or – sign in front of the residue depends on the initial conditions. Without excitation prior to the measurement it should be +. Else it has to be -.

$$\Rightarrow \mathrm{Re}\{BTF_{long}(\mathsf{w})\} = \mathsf{p} \cdot C_m \cdot \left(\frac{dF_0}{d\mathsf{w}_{rev}}\left(\frac{\mathsf{w}}{h} - \mathsf{w}_{rev0}\right) \right)$$

When working at base band zero we take $\mathsf{w} = h \cdot \mathsf{w}_{rev0} + \mathsf{e}$

$$\Rightarrow \mathrm{Re}\{BTF_{long}(\mathsf{e})\} = \mathsf{p} \cdot C_m \cdot \left(\frac{dF_0}{d\mathsf{w}}\left(\frac{\mathsf{e}}{h}\right) \right)$$

This equation means that the BTF response is wider when h increases.

By integrating this relation we get the density distribution:

$$\Leftrightarrow F_0(\mathsf{w}) = \frac{1}{\mathsf{p} \cdot C_m} \cdot \int_{-\infty}^{\mathsf{w}} \mathrm{Re}\{BTF(\mathsf{e})\} d\mathsf{e}$$

This formula will be used to obtain the density distribution.

As the measurements are made at harmonic h of the revolution, they need to be transposed. The measured frequency spread needs thus to be divided by h.

The momentum spread can be obtained by applying the following relation:

$$\frac{\Delta p}{p_0} = \frac{\Delta w}{w_{rev0}} \cdot \frac{-1}{h}$$

w_{rev0} is the mean revolution frequency
p_0 is the mean momentum

When the $F_0(\omega)$ distribution (LSD) is obtained, $\Delta p/p_0$ can be expressed in terms of standard deviation or Full Width at Half Maximum value (FWHM).

4.3 Schottky Spectrum

The coasting beam is composed of a set of particles (delta pulses) with different revolution frequencies.

$$i_{schottky}(t) = q \cdot \sum_{n,i} \frac{d(t - n \cdot T_{rev\_i})}{T_{rev\_i}}$$

The Fourier transform is:

$$i_{schottky}(w) = q \cdot \sum_{n,i} \frac{d(w - n \cdot w_{rev\_i})}{T_{rev\_i}}$$

Spectral (frequency) lines can be transformed into momentum lines by applying:

$$w_{rev\_i} = -\frac{w_{rev\_mean}}{p_{mean}} \cdot h \cdot p_i$$

In each spectral band, the power spectrum is equal to the mean square current $\overline{i^2(t)}$ over this band.

$$\text{Power per Shottky band} = \overline{i^2(t)} = 2e^2 f_{rev0}^2 N_{p0}$$

f_{rev0} is the central frequency of the Shottky band.
N_{p0} number of particle affecting the Shottky band.

This formula is used to obtain the spectral density from the beam current measurement.

5. Computation operated by the system (until now)

5.1 Empirical Transfer Function Estimate (ETFE)

The aim is to divide the Discrete Fourier Transform (DFT) of beam response and excitation.

$$ETFE(e^{jw}) = \frac{I_{beam}(w)}{V_{rf}(w)}$$

Property: its variance doesn't decay with the number of samples

Some hint

The momentum spread of particles leads to a revolution frequency spread:
The initial BTF measurement was made using a tracking filter; it is now carried out using a FFT.

5.2 Black-box Prediction Error Method (PEM)

In this method the transfer function is assumed to be:

$$BTF(z) = z^{-k} \cdot \frac{b_0 + b_1 \cdot z^{-1} + ... + b_{n-1} \cdot z^{-(n-1)}}{1 + a_1 \cdot z^{-1} + ... + a_m \cdot z^{-m}} = \frac{B(z)}{A(z)}$$

where k is the pipeline delay in terms of clock periods

Assuming a noise model $W(z)$ driven by a white noise process $w(t)$ the beam response is supposed such as:

$$I_{Beam}(t) = BTF(z) \cdot V_{rf}(t) + W(z) \cdot w(t)$$

More generally, a black box model can be expressed such as:

$$A(z) \cdot I_{Beam}(t) = \frac{B(z)}{F(z)} \cdot V_{rf}(t) + \frac{C(z)}{D(z)} \cdot w(t)$$

A loss function is then defined and a minimization algorithm is applied to obtain the vector containing all the coefficients of the z polynomials to be estimated:

$$J = [a_1 ..., b_0 ..., c_1 ..., d_1 ..., f_1]^T$$

5.3 Coupling impedance compensation and stability diagram

The Beam experiences the cavity voltage but also the self-induced voltages in the beam pipe. Both Schottky and BTF measurements are modified by the effect of these coupling impedances. They should be compensated for.

$$Z_{Coupling} = \frac{V_{induced}}{I_{Beam}} = \frac{\oint E(w,s)ds}{I_{Beam}}$$

There are four major contributors to the coupling impedance:

1. Space charge forces
2. Vacuum wall impedance
3. Vacuum chamber discontinuities (resonators)
4. RF cavities (resonators)

Space charge is the most dominating effect in low energy rings as the PSB. It acts as a pure negative inductance.

$$Z_{SC-long} = \frac{-jw}{w_{rev0}} \cdot \frac{Z_0 \cdot g_0}{2bg}$$

$g_0 = 1+2\log_e(b/a)$: coupling coefficient with a : beam radius and b : vacuum tube radius
$Z_0 = m_0.c = 377\ \Omega$ = free space impedance

As the beam response is normally in a narrow bandwidth compared to the revolution frequency w/w_{rev0} can be approximated by h.

At 50 MeV in the PSB $|Z_{SC\text{-}long}|$ is in the order of 1-1.6 kΩ/h

Vacuum chamber impedance can be expressed as:

$$Z_{VCI-long} = \frac{R}{s \cdot b \cdot d_w} \quad \text{at DC}$$

R = machine mean radius
s, b, d_w are the vacuum pipe conductivity, radius and thickness

But due to skin effect the impedance at high frequencies has to be noted:

$$Z_{VCI-long} = \sqrt{\frac{w}{w_{rev0}}} \cdot (1+j) \cdot \frac{b}{b} \cdot \sqrt{\frac{Z_0 c}{2sw_{rev0}}}$$

$Z_{VCI-long}$ is in the order of a few tens of Ω/h.

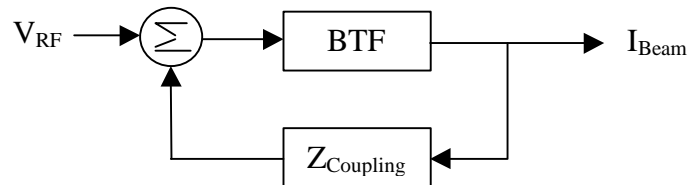The two other resonating effects are described by:

$$Z_{Res-long} = \frac{R_S}{1 + jQ\left(\frac{w}{w_{res}} - \frac{w_{res}}{w}\right)}$$

R_S is the shunt impedance
Q the quality factor

The order of magnitude of these impedances is 0.2 to 50 Ω/h.

The coupling impedance effect can be described in the following block-diagram:

Analytically this gives:

$$BTF_{Measured} = \frac{BTF}{1 - BTF \cdot Z_{Coupling}}$$

This arrangement becomes instable when $BTF_{Measured}$ tends to infinity, or in other words when $\left(\frac{1}{BTF} - Z_{Coupling}\right)$ tends to zero

1/BTF$_{meas}$ plotted in the complex plane is called the **stability diagram**. The curve shouldn't approach the origin.

The Shottky spectrum is also influenced by collective effects:

$$P_{S-measured} = \frac{P_S}{1 - BTF \cdot Z_{Coupling}}$$

6 BTFM working set (used until now)

File		
	Save settings	
	Load settings	
	Save raw data	
	Load raw data	

View		
	Ring 1	
	Ring 2	
	Ring 3	
	Ring 4	
	Horizontal	
	Vertical	
	Longitudinal	
	Table	
(the following parameters appear on specialist mode only)	DAC content	
	Raw data	Beam Reference
	Complex FFT	Beam Reference
	Check denominator	
	BTF: Raw	
	BTF: Averaged	
	BTF: angle comp.	
	BTF: time comp.	
	Time gated signal	
	BTF: time gated	
	BTF: Coupling impedance	
	Magnitude/Phase	
	Stability Diagram	
	Table	

Options		
	Show ETFE	
	Show PEM	
	Dp/p	Definition FWHM Gaussian Parabolic Quadratic 5-Quadratic
	Energy axis	
	Smooth	
	Log Scale (Magn)	
	Zoom X all	
	Zoom X none	
	Zoom Y all	
	Zoom Y none	
	Specialist mode	

Control		
	Ring 1	
	Ring 2	
	Ring 3	
	Ring 4	
	Horizontal	
	Vertical	
	Longitudinal	
	Settings (Spec)	
	Measure ADC Offsets	
	Reset ADC Offsets	
	Pulse	

When opening Control-Setting (Spec):	
Settings	
Measure:	Beam/test (with ccv)
Noise:	Max length Random phase Sine Sweep No noise
Excitation:	On / Off (with ccv)

Sine harmonic (using 4 digits)	
Damp. Width (using4 digits)	
Damp. Min: (**0.** followed by 2 digits`)	
Cavity detuning (R1,2,3,4) with ccv	
Start train:	C / B with 4 digits aqn
Start time (pulses) using 5 digits	
Interburst time (pulses) using 5 digits	
Interburst train	B / C / fastrf with 4 digits ccv
Overhead time (Smpl) using 5 digits and aqn	
Burst phase (smpl) using 5 digits and aqn	
Averages:	1 / 2/ 4/ 8/ 16 / 32 with aqn
Burst length (Smpl) using 4 digits and aqn	
Flat top (MeV) 3digits +_2 after coma	
Window:	Rect Sine Hamm Hann
PLS line with aqn	
Attenuation (dB): 3 times: one for each plane	Excitation using 2 digits + 1 after coma Reference using 2 digits + 1 after coma Beam using 2 digits + 1 after coma
Down mixer gains (dB): 3 times: one for each plane	Reference Cos using 2 digits + 1 after coma Reference Sin using 2 digits + 1 after coma Beam Cos using 2 digits + 1 after coma Beam Sin using 2 digits + 1 after coma
Frequency, Bandwidth 3 times: one for each plane	Spectral line using 5 digits Bandwidth (kHz):50/100/200/400 with aqn Synt. Frequency (kHz): 4 digits with aqn Q value: 2 digits + 2 after coma Measure time (ms) using 9 digits display
Signal processing: 3 times: one for each plane	ΔT ring 1,2,3,4 (ns) using 5 digits θ_m (degrees) using 5 digits Time gate win using 4 digits Re{Z/n}(Ohm) using 5 digits Im{Z/n}(Ohm) using 5 digits
Parametric estimation: 3 times: one for each plane	Data window Na using 2 digits Nb using 2 digits Nc using 2 digits Nd using 2 digits Nf using 2 digits Nk using 2 digits

7 Implementation

7.1 General environment

During BTFM measurement:
- Beam and C04 cavity signals have to be connected to the instrument – they must not be during other pulses because the induced voltage may kill the instrument.
- The rf excitation voltage should be applied to the summing point at cavity amplifier level.
- A DC voltage should be applied to the tuning offset control input of C04 so to set the resonant frequency at the proper harmonic number.
- The control voltage – or voltage program - of C02, C04, C16 should remain at zero Volt during the measurement. If unused, C02 and C16 should even be short-circuited by applying "rf act OFF".

During Shottky measurement:
- All cavities should be programmed to zero volt or short-circuited.
- The rf excitation should be disconnected from the cavity.

7.2 Timing

6 pulses have to be supplied (TG8):
- One pulse to reset the equipment that should disconnect all BTF signals from cavities and from the vector analyzer. This pulse should come by default at the beginning of each cycle.
- Four different pulses for ring selection: BAXi.SBTFM
- One pulse to disable the rf excitation in case of a Shottky measurement: BAX.SSHOTTKY.

7.3 Tuning offset

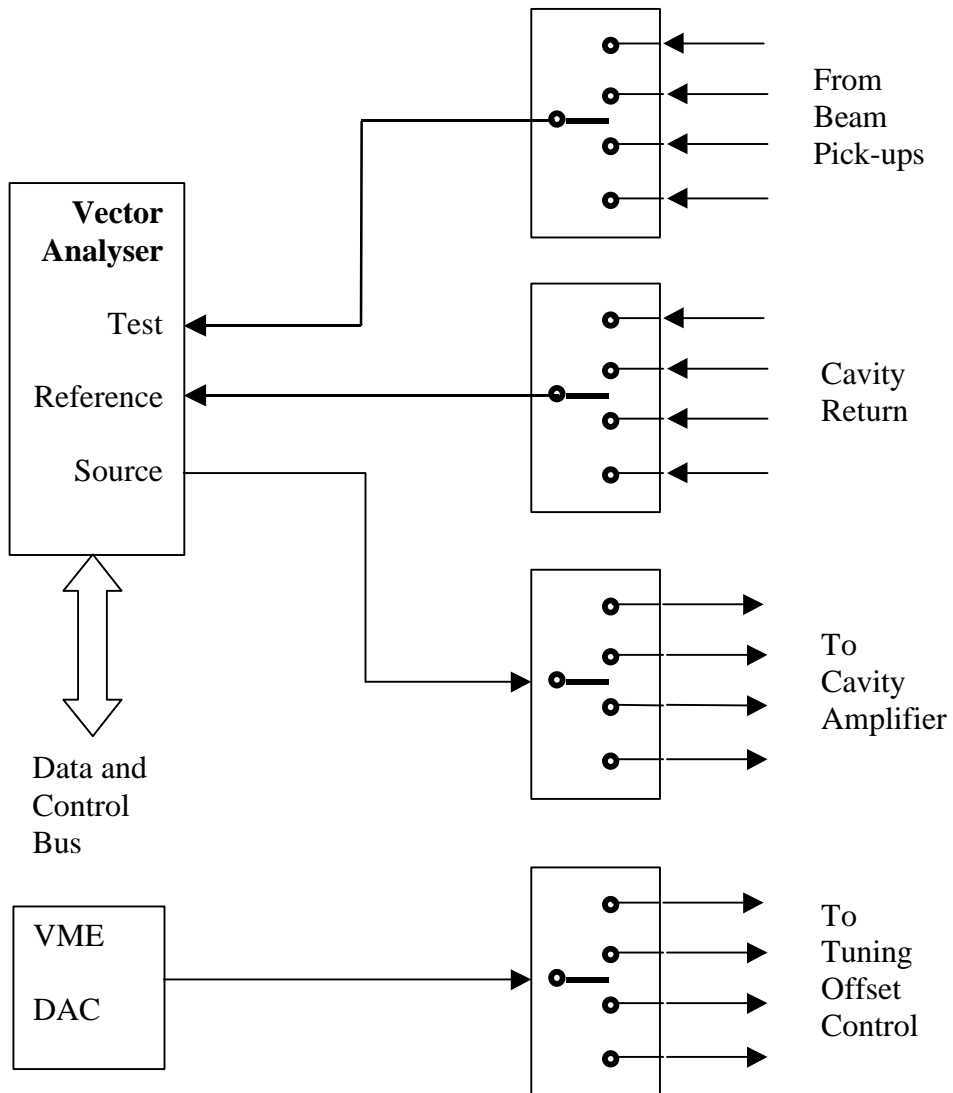Four VME DACs (one for each ring) should be supplied to set the tuning frequency

7.4 Measurement data

The vector analyzer will sample the cavity and beam pick-ups voltages. These samples could be transferred to any computing tool but as it represents a big data flow it seem more reasonable to use the FFT tool supplied with the instrument and send only the computed complex values (only a few tens points).
At this point we would need some mathematical tools:
- A phase addition to compensate any phase offset (mainly cavity – beam PU)
- A delay compensation

7.5 Hardware setup

Vector
Analyser

Test

Reference

Source

Data and
Control
Bus

VME

DAC

From
Beam
Pick-ups

Cavity
Return

To
Cavity
Amplifier

To
Tuning
Offset
Control

7.6 Instrument setup

The HP vector analyzer (HP 89410A) has to be properly configured to supply the FFT of the two main signals: beam and rf. About 400 points per shot, with frequency, real and imaginary parts have to be sent through the network. We need to be able to set the start and stop frequencies of the analysis and also the amplitude of the excitation signal. All the computation will be operated on a dedicated PC using an adequate mathematical package (mathematica^{TM}).

7.7 Very basic computation to be supplied

- Cavity transfer function   FFT of "cavity return/VA source"
- Schottky Spectrum   FFT of Beam PU
- BTF   FFT of "Beam/Cavity return"
- LSD $\cdot \int_{-\infty}^{w} \text{Re}\{BTF(e)\} de$
- Delay difference (beam return-cavity return) compensation algorithm
- Phase difference (Beam PU – cavity) compensation algorithm

The results should appear as a Bode or Nyquist plot and also optionally as a table.