CORRELATIONS IN INCLUSIVE EXPERIMENTS

Luca CANESCHI

1. INTRODUCTION

The continuing interest of experimentalists and theorists in inclusive reactions has recently focused on two (or more) particle distributions. Actually, single particle inclusive distributions (ID) offer only a rough picture of what is going on, whereas it is likely that at the level of two particle distributions some insight into the details of the production processes can be gained. For instance, physically very different models like the multiperipheral model [1] (MPM) and the diffractive excitation model [2] (DEM) lead both to expect a limiting distribution [3] for single particle spectra: the MPM actually predicts the stronger property of scaling [3], with the consequent logarithmic behaviour of $<n>$, but also the DEM can accommodate this stronger feature. However, the two models differ sharply at the level of two particles ID: the MPM predicts a regular behaviour at $x_1 \simeq x_2 \simeq 0$, whence a $\ell n^2 s$ behaviour of $<n^2>$, whereas in the DEM one expects a singular behaviour and $<n^2> \simeq s^{\frac{1}{2}}$ [4].

Another reason for the focusing of interest on several particles ID was the presumption that little new could be learned from single particle spectra. Let us remember that some months ago the small and intermediate angle ISR experiments [5], that could cover values of x down to about 0.1, had given results in good agreement with limiting distribution. Also, the only experiment [6] on cosmic rays interactions with protons supported a logarithmic behaviour of the charged multiplicity, with a coefficient of $\ln s$ quite compatible with the value of

$$\int f(0, p_\perp, \bar{s}) \; d^2 p_\perp \qquad \text{at} \quad \bar{s} \simeq 50 \ \text{GeV}^2 \qquad ;$$

it was, therefore, natural to conclude [7] that scaling was valid, and that the single particle I D at accelerator energies already yielded the asymptotic value of $f(x,p_\perp)$ in the whole x range. In the MPM this conclusion was a priori rather surprising. In fact, scaling in the MPM can be understood in terms of the hypothesis of short range correlations that forms the basis of the model: at fixed x the difference of rapidity between the observed particle and the incoming one that has opposite longitudinal momentum in the c.m. system grows like $\ln s$: when this distance exceeds the assumed correlation length, no further s dependence can be exhibited by $f(x,p_\perp)$. On this basis one expects that low values of x scale later: assuming a correlation length equal to two, as it is fashionable, we expect that

$$\frac{1}{2} \ln s/m^2 + \ln \sqrt{\frac{x^2 s}{4(m^2+p_\perp^2)} + 1} \tag{1}$$

should exceed four in order to obtain scaling at the 10% level, but at $x=0$ this happens only for $s \simeq 3000 \ \text{GeV}^2$. Also, at $x=0$ scaling takes place when the observed particle falls outside the correlation length with respect to <u>both</u> incoming particles: hence, when the $x=0$ region reaches its limit, the rapidity distribution should start exhibiting the famous plateau, but no such plateau was seen at $s \simeq 50 \ \text{GeV}^2$.

These argumentations have been vindicated by the recent large angle ISR experiments [8], which have shown that the yield of particles at 90° in the c.m.s. is about twice higher at

ISR energies that it is at 50 GeV^2, also, within the ISR energy range there is a clear, if not conclusive, trend to increase. The data are quite consistent with an s dependence of the form $A + B\, s^{-1/4}$, as suggested by the MPM [9], but the relative size (and sign) of A and B $(B/A \simeq -2)$ is intriguing.

In conclusion we think that the emphasis goes back to one-particle features, i.e., single particle ID and $<n>$. Experimentally we need to know whether the yield at 90^o approaches a limit or keeps increasing with s, and whether $<n>$ is logarithmic, theoretically it would be very interesting to see if the above value of B/A is consistent with the MPM.

It is interesting to remark that the same large angle ISR experiments [8] show the existence of the plateau in rapidity predicted by a scaling theory. One might wonder whether this is compatible with an indefinite increase of $f(x,s)$ at $x = 0$, i.e., with a limiting function $f(x)$ continuous but singular at $x = 0$. The answer is yes. In fact, since $f(x,s)$ is finite at fixed s by definition, we can choose a constant c and define $x(s)$ as the smallest value of x for which $f(x,s) + c > f(x)$. Obviously $x(s) \neq 0$ for any s. There will be a plateau in rapidity of length $L(s)$ whenever $L(s) = \ln s + \ln x(s)$ does not decrease to zero asymptotically.

2. KINEMATICAL CORRELATIONS

After this rather lengthy introduction let us come to correlations. Since the basic definitions have been clearly stated by Le Bellac in the previous talk, and since the following one by Peccei covers most of the dynamical part of the subject, we will concentrate on the necessary correlations forced by conservation laws. We will start by examining momentum conservation in a theory of identical bosons with no quantum numbers. It is by now well known that the conservation of momentum imposes the following integral constraint between neighbouring ID [10]:

$$\left(P_\mu - \sum_{i=1}^{k} p_\mu^i\right) f^{(k)}(p^i, s) = \int \frac{d^3q}{E_q}\, q_\mu\, f^{(k+1)}(p^i, q, s) \tag{2}$$

where

$$f^{(k)}(p^i, s) = \frac{1}{\sigma(s)}\, E_1 \ldots E_k \, \frac{d\sigma\,(\text{inclusive})}{d^3p^1 \ldots d^3p^k}$$

We can rewrite (2) in terms of the correlation functions $\rho^{(k)}$ defined as usual:

$$\left(\rho^{(1)}(p,s) = f^{(1)}(p,s)\ ; \quad \rho^{(2)}(p,q,s) = f^{(2)}(p,q,s) - f^{(1)}(p,s)\, f^{(1)}(q,s), \right.$$

$$\left. \text{etc} \right)$$

$$-\left(\sum_{i=1}^{k} P_\mu^{(i)}\right) \rho^{(k)}(p^i, s) = \int \frac{d^3q}{E_q}\, q_\mu\, \rho^{(k+1)}(p^i, q, s) \tag{3}$$

A straightforward consequence of (3) is [11]

$$\int \rho^{(k+1)}(q, p^i, s)\, \frac{d^3q}{\sqrt{s}}\, \frac{d^3p^1}{E_1} \ldots \frac{d^3p^k}{E_k} = (-)^k\, k! \tag{4}$$

3. TRANSVERSE CORRELATIONS

Choosing for simplicity $k = 1$, let us consider (2) with $\mu = 1$ or 2, multiply by p_1 or p_2 and sum to obtain

$$-p_\perp^2\, \rho^{(1)}(p,s) = \int \frac{d^3q}{E_q}\, (q_\perp \cdot p_\perp)\, \rho^{(2)}(p,q,s) \tag{5}$$

In a scaling theory the left-hand side approaches a finite limit when $s \to \infty$, and so does $\rho^{(2)}(p,q,s)$. The integration over d^3q/E_q, however, can in principle produce an unwanted $\ell n\, s$

behaviour of the right-hand side. It is instructive to examine the mechanism that prevents this from happening in different models [remember that (5) has to hold in any model that conserves momentum!].

In the MPM there is a local compensation between transverse components of the momentum, i.e., if the k^{th} particle of the chain has $p_x > 0$, the $(k-1)^{th}$ and $(k+1)^{th}$ have in average $\langle p_x \rangle < 0$, and the perturbation caused by, say, an abnormally large value of some p^k is absorbed in a few steps around the k^{th} position. The mechanism that causes this effect is the momentum transfer cut-off: t_k contains terms

$$\left(p_\perp^{k} - p_\perp^{k+1} \right)^2 \quad \text{and} \quad \left(p_\perp^{k} - p_\perp^{k-1} \right)^2$$

and is minimum when the azimuthal angles between the k^{th} particles and the neighbours are around π.

This statement can be neatly translated in the Müller language: the twofold inclusive cross-section is related to an analytic continuation of mass discontinuity of the forward $4 \to 4$ scattering amplitude. Assuming for this function a Regge expansion in the relative energy $s_{pq} = (p+q)^2$ of the two detected particles, it is easy to see that contributions to the azimuthal correlations come only from terms with Toller quantum number $M \neq 0$. The terms with $M = 0$ have no dependence on $q_\perp \cdot p_\perp$ and integrate to zero in (5). In the MPM the leading term of the Pomeron trajectory has $M = 0$, therefore, contributions of the Pomeron to (5) can come only from the non-leading term that behaves like s_{pq}^{-1}. Bassetto and Toller [12] have estimated that this contribution is important, of the order of one half of the left-hand side. The rest can come from lower lying trajectories with intercept α_{NL} and with $M \neq 0$ or from cuts. In the former case the correlations are short range: they behave like $(s_{pq})^{-\beta}$ with $\beta = 1 - \alpha_{NL}$, i.e., like $e^{-\beta \Delta \eta}$ where $\Delta \eta = \ln s_{pq}$ is the difference in rapidity between the two detected particles.

In the latter case the azimuthal correlations die much slower: a behaviour of the form $(\Delta\eta)^{-p}$ is expected. Remark that Eq. (5) keeps giving troubles (of the $\ln s^{1-p}$ form) if $p \leq 1$. Therefore, if cuts which behave like, say $1/\ln s$ are present, their contribution to $\rho^{(2)}$ should again be independent of $(q_\perp \cdot p_\perp)$. The experimental observation of azimuthal correlations and of their dependence on $\Delta\eta$ has been proposed as a sensitive way to study the nature of the Pomeron [13]. The presence of slowly decreasing terms of the type $(\Delta\eta)^{-p}$ would reveal the presence and the nature of cuts.

In conclusion the reason why in the MPM the right-hand side of (5) does not grow like $\ln s$ is that the integration over dq_L/E_L, i.e., over the rapidity of the second detected particle η_q, does not run over the whole range, of length $\ln s$, but is confined to an energy independent range around the rapidity of the particle of momentum p. Let us now turn our attention to the uncorrelated jet model [14] (UJM) that we take for our purpose in the simplest form, i.e., one in which the square of the amplitude for producing n particles is given by

$$|A_{2n}(P,p^i)|^2 = \delta^4\left(P - \sum_{i=1}^{n} p^i\right) \prod_{i=1}^{n} \frac{d^3p^i}{E^i} \, e^{-(p_\perp^i)^2} \tag{6}$$

It is obvious that a completely different structure is present in this model: the longitudinal and transverse models decouple in (6) (if we change variable for p_L to rapidity). Therefore, the azimuthal correlations (necessarily present in the model because of momentum conservation) are expected to be independent of the relative rapidity of the two detected particles. However, it is also clear that a non-zero value of $p_\perp$ of one particle influences the transverse distribution of the other particles in a way that is proportional to $1/n$, since each particle will have to supply an average value of transverse momentum equal to $-p_\perp/n$. Therefore, azimuthal correlations are expected to behave in inclusive reactions like $1/\bar{n}$, i.e., like $1/\ln s$.

These expected features can be easily checked: defining a function [14]

$$\phi(P_\mu) = \sum_n \frac{(2g)^n}{n!} \int \prod_{i=1}^{n} \frac{d^3p^i}{E_i} \; e^{-(p_\perp^i)^2} \; \delta^4\Big(P_\mu - \sum_{i=1}^{n} p_\mu^i\Big) \tag{7}$$

the inclusive distributions are given by

$$f^{(1)}(p,s) = 2g \; e^{-p_\perp^2} \; \frac{\phi(P_\mu - p_\mu)}{\phi(P_\mu)} \quad (\text{with } P^2 = s)$$

$$f^{(2)}(p,q,s) = 4g^2 \; e^{-(p_\perp^2 + q_\perp^2)} \; \frac{\phi(P_\mu - p_\mu - q_\mu)}{\phi(P_\mu)}$$

and so on.

The leading behaviour for $P_o \to \infty$ of (7) is of the form [14]

$$\frac{(P^2)^{2g-1}}{\ln P^2} \; e^{-p_\perp^2/\ln P^2}$$

with $g = 1$ to obtain (almost) constant cross-sections.

Hence,

$$f^{(1)}(p,s) \simeq 2\,(1-|x|)\,\frac{\ln s}{\ln((1-|x|)\,s)}\; e^{-p_\perp^2(1 + 1/\ln s)} \underset{s\to\infty}{\longrightarrow} 2\,(1-|x|)\,e^{-p_\perp^2} \tag{8}$$

$$f^{(2)}(p,q,s) \underset{s\to\infty}{\simeq} 4\,\big(1-|x|-|y|+|xy|\,\theta(-xy)\big)\; e^{-(p_\perp^2+q_\perp^2)-(p_\perp+q_\perp)^2/\ln s} \quad \Big(y = \frac{2q_L}{\sqrt{s}}\Big) \tag{9}$$

Restricting ourselves for the moment to the region $x \simeq 0$, $y \simeq 0$ [the potentially dangerous one for (5)], we compute the correlation function

$$\rho^{(2)}(p,q,s) \simeq e^{-(p_\perp^2+q_\perp^2)} \left(-1 + e^{-(p_\perp+q_\perp)^2/\ln s} \right)$$

$$\simeq - e^{-(p_\perp^2+q_\perp^2)} \quad \frac{(p_\perp+q_\perp)^2}{\ln s} \tag{10}$$

Insertion of (8) into (5) yields an identity [with $\rho^{(1)}$ given by (8) in the $x \simeq 0$ region], the explicit $1/\ln s$ in (10) cancelling the $\ln s$ obtained from the dq_L/Eq integration.

Therefore, the MPM and the UJM predict sharply different behaviour of the azimuthal correlations. In the MPM the correlations decrease fast (or not so fast, in the presence of cut) with $\Delta\eta$ but are s independent at fixed $\Delta\eta$ (as it must be, since any s dependence violates short range), in the UJM, the correlation is independent of $\Delta\eta$, but decreases with s at fixed $\Delta\eta$.

What happens in the D.E.M.? In the approximation that the transverse momentum of the two diffractively produced "fireballs" is considerably smaller than the average transverse momentum of the decay products, the fragments of each fireball should add to zero transverse momentum independently of each other: therefore, no azimuthal correlations are expected between a $x > 0$ and a $x < 0$ particles. Correlations are expected amongst the decay products of the same fireball. Let us remember that events with many particles populate smaller x regions, therefore, if the x of particle 1 is kept fixed, we expect the transverse correlation to increase with y, the scaling variable of particle 2. However, detailed predictions can only be made on the basis of a specific choice of the decay mechanism of the fireball.

In conclusion the study of azimuthal correlations offers a very interesting test of the various models. The results of some preliminary (not inclusive) phenomenological investigation are embarrassing for the MPM [15]. We hope that good inclusive data will be available soon.

4. LONGITUDINAL CORRELATIONS

Correlations between longitudinal momenta (or better, energies) in inclusive reactions are especially interesting because they determine the multiplicity distributions. Consequently, indirect information on correlations can be obtained from experiments in which no momenta are measured. However, in these "indirect" measurements it can be hard to disentangle the kinematical effects (i.e., correlations imposed by energy momentum conservation only) from dynamics. We will devote ourselves especially to this point.

Through this paragraph we will assume that inclusive distributions are limiting [3] (if not scaling), and use scaling variables to describe them. We will also implicitly perform all the $p_\perp$ integrations and use the notation

$$\overline{x} = \sqrt{x^2 + \frac{4}{s}\left(m^2 + \langle p_\perp^2 \rangle\right)}$$

The $\mu = 0$ component of Eq. (3) becomes in these notations:

$$- \sum_{i=1}^{k} |x_i| \quad \rho^{k}(x_i) = \int dy \ \rho^{(k+1)}(x_i, y) \tag{11}$$

and Eq. (4) reads

$$\int \rho^{(k+1)}(y, x_1 \dots x_k) \ dy \ \frac{dx_1}{\overline{x}_1} \dots \frac{dx_k}{\overline{x}_k} = (-)^k 2(k!) \tag{12}$$

Let us concentrate for the moment on $k = 2$. A few trivial observations to start.

1) An obvious region in the x, y plot in which we know $\rho^{(2)}$ is for $|x+y| > 1$: here

$$\rho^{(2)}(x,y) = -\rho^{(1)}(x)\ \rho^{(1)}(y)$$

2) This region does not saturate (11) unless $\rho^{(1)}(x) = 1$. In this case it is easy to show that the trivial set of $\rho^{(k)}$, i.e., the ones constructed with Θ functions [e.g., $\rho^{2}(x,y) = -\Theta(|x+y|-1)$] satisfies (11).

3) If $\rho^{(1)}(x)$ is an increasing function of x, the region $|x+y| > 1$ oversaturates (11), and kinematics forces an average positive value of $\rho^{(2)}$ in the remaining region. In the more familiar case in which $\rho^{(1)}(x)$ decreases $\rho^{(2)}(x,y)$ is in average negative also in the non-trivial region.

4) Combining longitudinal momentum and energy conservation, it is easy to see that $\rho^{(2)}(x,y)$ is in average vanishing for $x \,.\, y < 0$.

5) All these features are exhibited by the UJM, in which from (8) and (9), $\rho^{(1)} \simeq 2(1-|x|)$, and

$$\rho^{(2)}(x,y) = \begin{cases} -4\,(1-|x|)(1-|y|) & \text{for } |x+y| > 1 \\ -4\,|x|\,|y| & \text{for } xy > 0,\ |x+y| < 1 \\ 0 & \text{for } xy < 0 \end{cases} \qquad (13)$$

Of course, the sum rule (11) is satisfied.

Indirect information on the correlations can be obtained by studying the departures from the Poisson distribution [16),17)]. In fact, for instance

$$\Delta_2 = \langle n(n-1)\rangle - \langle n\rangle^2 = \int \rho^{(2)}(x,y) \frac{dx}{\bar{x}} \frac{dy}{\bar{y}} \tag{14a}$$

$$\Delta_3 = \langle n(n-1)(n-2)\rangle - \langle n\rangle^3 - 3\langle n\rangle \Delta_2 = \int \rho^{(3)}(x,y,z) \frac{dx}{\bar{x}} \frac{dy}{\bar{y}} \frac{dz}{\bar{z}} \tag{14b}$$

and so on.

Let us remark now that from (11) and (12)

$$I_2 = \int \rho^{(2)}(x,y)\, dx \frac{dy}{\bar{y}} = -2 \tag{15a}$$

$$I_3 = \int \rho^{(3)}(x,y,z)\, dx \frac{dy}{\bar{y}} \frac{dz}{\bar{z}} = 4 \qquad \text{etc.} \tag{15b}$$

For moderate values of s, such that $\bar{x}$ is larger than a suitable constant in the whole x integration, the behaviour of Δ_n is determined by the behaviour of I_n. As s increases the region $x_i \simeq 0$ of $\rho^{(k)}(x_i)$ is more and more emphasized in the integral, and dynamical features take over [18)].

Remember now that in the short range correlations hypothesis, i.e., if the correlation functions vanish when the produced particles are not within a fixed range of rapidity with respect to each other

$$\Delta_n \xrightarrow[s\to\infty]{} a_n \ln s + b_n \tag{16}$$

This is obvious since in the n fold integration over rapidities that define Δ_n only the over-all rapidity (the rapidity of the "centre-of-mass" of the particles) can span the whole $\ln s$ range [19]. However, we see that longitudinal kinematical correlations are short range <u>with respect to the incoming particles</u> [e.g., the non-trivial part of $\rho^{(2)}$ is in rapidity $e^{-|\eta_a-\eta_x|}\ e^{-|\eta_a-\eta_y|}$]. Therefore, in a model with kinematical correlations only (like the UJM) all $\Delta_n \xrightarrow[s\to\infty]{} b_n$. For instance, from (13)

$$\Delta_2 \to -8\int_0^1 \frac{dx}{x}\left[\int_0^{1-x}\frac{dy}{y}\,xy + \int_{1-x}^{1}\frac{dy}{y}(1-x)(1-y)\right] = 8 - \frac{4\pi^2}{3}$$

In a model with short range correlations, if the functions

$$f(g) = \exp\left(\sum (g-1)^m\, b_m/m!\right)$$

and

$$\alpha(g) = \sum (g-1)^m\, a_m/m!$$

exist, (here $\Delta_1 = \langle n\rangle = a_1 \ln s + b_1$), the generating function

$$S(g,s) = \sum g^m\, \sigma(m,s)\ \sigma_{Tot}^{-1}(s)$$

takes the form [20] (very suggestive of multiperipheralism)

$$S(g,s) = f(g)\ s^{\alpha(g)} \tag{17}$$

The fact that kinematical correlations do not affect $\alpha(g)$ is very natural. Since they are end effects, they should all be contained in $f(g)$ (in multiperipheral language the position of the pole $\alpha(g)$ is determined by the dynamics, not the kinematics).

Keeping in mind all this, let us try to interpret the experimental data on Δ_2, Δ_3 and Δ_4 as collected by Białas and Zalewski, Ref. 21). Before we get started we must cope with the fact that most produced particles are π, and that only charged multiplicities are usually observed. Therefore, we have a choice to study the deviations Δ_k from the Poisson distribution in the number of charged particles n_c or in the number of particles of a given charge, say, negative, n_-. The choice is crucial in view of the non-homogeneity of Δ_k in n. Several models [22] suggest that the latter is a more sensible choice, since charge conservation in a sense nails the production of a charged particle to the production of its antiparticle. Also phenomenologically the Δ_k turn out to be smaller in terms of n_-. Figures 1, 2, 3 show Δ_2, Δ_3 and Δ_4 computed from the n_- distribution as a function of $\langle n_- \rangle$, which, in turn, is a monotonic function of s. It is obvious that the data at accelerator energies are compatible with kinematical correlations only. They show the expected alternate sign pattern, and also the order of magnitude is the expected one, if we insert a factor $(3)^{-n}$ in the kinematical correlations due to the presence of three charges. The high energy points are obtained from the Echo Lake results, hence they should be considered with caution. However, also preliminary data from Serpukhov (and some considerations further on in this paragraph) indicate a change of sign of the Δ_k at $s = 100\ \mathrm{GeV}^2$, showing the existence of a dynamical effect that, as expected, takes over at large s. Let us now try to interpret this "dynamical" high energy part of Δ_k. We remark first that the data are compatible with anything, in particular with the $\Delta_k < a_k \ln s$ bound expected in a short range model. We also remark that such a behaviour is hard to disprove experimentally if no upper bound on the value of a_k is established. Therefore, it would again be nice to have a quantitative estimate of the size of the a_k expected in the MPM. At least the pattern followed by the a_k is, however, the one expected in the MPM. The data hint that $a_2 > 0$ and that the a_k have alternating signs. The first fact is expected in the MPM since the two particle longitudinal correlation is produced by the (positive) non-leading term of an out of the mass shell total cross-section. Also, the alternating sign pattern is expected. Let us remember that the a_k are given by

$$a_k = \frac{d^k}{dg^k} \alpha(g) \Big|_{g=1}$$

In the MPM $\alpha(g)$ is defined implicitly as the position of the leading zero of the Fredholm determinant [17], i.e., by an equation of the type $u(\alpha) = 1/g$. The function u has the form (e.g., in the multi-Regge model [23])

$$\mu(\alpha) = \int_{-\infty}^{0} \frac{e^x}{\alpha - x} \, dx \tag{18}$$

and the a_k so defined alternate in sign [24].

5. ASSOCIATED MULTIPLICITIES

Since the knowledge of

$$\Delta_k^{(s)} = \int \rho^{(k)}(x_1 \ldots x_k) \frac{dx_1}{\bar{x}_1} \cdots \frac{dx_k}{\bar{x}_k}$$

gives only a rough insight in the structure of the correlation functions $\rho^{(k)}$ it is tempting to look for some more detailed information that still depends on few variables. An obvious possibility is to consider the functions

$$\Delta_k(x,s) = \int \rho^k(x, x_2 \ldots x_k) \frac{dx_2}{\bar{x}_2} \cdots \frac{dx_k}{\bar{x}_k}$$

i.e., the deviations from the Poisson distribution in the associated multiplicities [25]. This type of measurement is very suitable for bubble chamber analysis, in which the number of prongs always provides a convenient label, and in which to consider orthodox inclusive distribution obviously means to throw away too much information. By separating the inclusive single particle distribution in contributions from events with a given multiplicity:

$$f^{(1)}(x) = \sum f^{(1)}_{(n)}(x,s)$$

we can construct a generating function for the associated multiplicities:

$$M(x,g,s) = \sum g^{n} \; f^{(1)}_{(n)}(x,s)$$

Since energy is conserved independently for each multiplicity, we must have that

$$\int_0^1 M(x,g,s)\, dx = S(g,s) \tag{19}$$

By definition the associated multiplicity $\bar{n}(x,s)$ is given by:

$$\bar{n}(x,s) = \frac{d}{dg} \ln M(x,g,s)\Big|_{g=1} - 1$$

therefore, it follows that

$$\int \Delta_2(x,s)\, dx = \int dx \; f^{(1)}(x) \left[\bar{n}(x,s) - \bar{n}(s) \right] =$$

$$= -2 + \int^1 dx \; \frac{d}{dg}\Big[M(x,g,s) - S(g,s) \Big]_{g=1} = -2$$

in accordance with (12).

In general $M(x,q,s)$ is given by

$$M(x,q,s) = q\, S(q,s) \quad \Sigma \frac{(q-1)^n}{n!} \Delta_{n+1}(x,s)$$

Hence $\Delta_{n+1}(x,s) = \frac{d^n}{dq^n} \left. \frac{M(x,q,s)}{q\, S(q,s)} \right|_{q=1}$ and the constraint (19) is sufficient to enforce (12) : this is not too surprising, since (12) and (19) both stem from energy conservation. Some experimental data [26] on $\Delta_k(x)$ are shown in Figs. 4, 5, 6. Again, in order to try to separate kinematical effects remark that in the UJM

$$\bar{n}(x,s) = \bar{n}(M^2) = \bar{n}((1-|x|)s)$$

where M^2 is the missing mass. Since $\bar{n}(s) = 2\ln s + c$, we get $\Delta_2(x,s) = 4\ln(1-|x|)(1-|x|)$, that obviously satisfies (12). An indication on the shape of $\Delta_k(x,s)$ for $k>2$ can be obtained by keeping only the leading term (in $\ln s$) in the UJM for $S(g,s)$ and $M(x,g,s)$, i.e., assuming

$$S(g,s) = f(g)\; s^{2g-2} \tag{20}$$

$$; \quad M(x,g,s) = 2g\, f(g)\, ((1-|x|)s)^{2g-1} s^{-1} \tag{21}$$

We obtain from (20) and (21)

$$\Delta_k(x,s) \,/\, f^{(1)}(x) = [\,2 \ln(1-|x|)\,]^{k-1}$$

It is easy to check that the sum rules (12) are satisfied by this set of Δ_k. It is obvious since (20) and (21) comply with the requirement (19).

A glance at Fig. 4 and 5 shows that again the bulk of the data are easily interpreted in terms of kinematical correlation. There is, however, an important effect in the $x \simeq 0$ region (by definition, the dynamical one) to which we will devote our attention in a moment. Before leaving the subject of kinematical correlations, let us remark that the knowledge of $\Delta_k(x,s)$, which in our assumptions (20) and (21) is independent of the choice of $f(g)$, is, on the contrary sufficient to determine $f(g)$ through the relation

$$f(g) = \exp 2\left[\sum_{k=2}^{\infty} \frac{(g-1)^k}{k!} \int_0^1 \Delta_k(x) \frac{dx}{x} + 2\gamma(g-1)\right]$$

In our approximation to the UJM [27] we obtain

$$\frac{d^k}{dg^k} \ln f(g)\Big|_{g=1} = -2^{k+1} \frac{d^k}{dg^k} \ln \Gamma(g)\Big|_{g=2} \Rightarrow f(g) = \left[\Gamma(2g)\right]^{-2}$$

Also, we see that $\Delta_k(x,s)$ do not depend on s in the model that we are considering. It is easy to show that this is the case also in a model in which the correlations are short range and scaling. Going back to the data shown in Fig. 6, in which data on $\Delta_2(x,s)$ at different s are compared, we see that the $x > 0.1$ region is consistently dominated by roughly energy independent kinematical effects, but the $x \simeq 0$ region changes substantially. One more indication that scaling has not yet set in at accelerator energies (or that long range correlation exists after all).

The positive value of $\Delta_2(x,s)$ around $x = 0$ for high s, is certainly a dynamical effect, since kinematical effects vanish at $x = 0$ and are negative elsewhere. This positive value is in agreement with a value $a_2 > 0$ in $\Delta_2(s)$, as suggested by the cosmic ray points. We see that $\Delta_2(x,s)$ offers a more detailed information than $\Delta_2(s)$. From data on $\Delta_2(x,s)$ at accelerator energy it is possible to hint a positive dynamical

$\ell n\, s$ term in $\Delta_2(s)$, whereas $\Delta_2(s)$ is still compatible with a purely kinematical interpretation in the same energy range.

6. CHARGE AND ISOSPIN

Up to now we have considered for simplicity a world of identical bosons: in the actual world a summation over all possible kinds of detected particles has to be added to the integral constraint that follows from momentum conservation [28].

In the actual world of particles with quantum numbers, more sum rules for ID of the type considered until now can be derived as a consequence of the existence of additively conserved quantum numbers. Let us focus our attention on the charge (keeping in mind that similar considerations apply to strangeness, baryon number, etc.). In a reaction $A + B \rightarrow C + X$, charge conservation imposes that

$$(Q_A + Q_B)\,\sigma_{AB} = \sum_c Q_c \int \frac{d^3\sigma^{(c)}}{d^3p}\, d^3p \tag{22}$$

To understand the meaning of (17) let us consider a simplified world in which only pions exist. Then, in any event

$$n_{\pi^+} - n_{\pi^-} = Q_A + Q_B \tag{23}$$

hence (22) is an identity.

Sum rules of the type (22) can be derived for higher order ID, for instance

$$- Q_c\, E_p \frac{d^3\sigma^{(c)}}{d^3p} = \sum_d Q_d \int \frac{d^3q}{E_q}\, \rho^{(2)}_{cd}(p, q, s) \tag{24}$$

whence

$$- Q_c \langle n_c \rangle = \sum_d Q_d \int \frac{d^3q}{E_q} \frac{d^3p}{E_p} \rho^{(2)}_{cd}(p, q, s) \tag{25}$$

or also

$$-(Q_A + Q_B) = \sum_{c,d} Q_d \int \frac{d^3q}{E_q} \frac{d^3p}{E_p} \rho^{(2)}_{cd}(p, q, s) \tag{26}$$

Insertion of (23) into (25) or (26) leads to trivial identities.

From (24) also relations of the form (4) can be obtained

$$- Q_c \int f^{(1)}_c(p, s)\, d^3p = \sum_d Q_d \int d^3p \frac{d^3q}{E_q} \rho^{(2)}_{cd}(p, q, s)$$

What is interesting about (22) is again the way in which the potential logarithmic behaviour of $\int d^3q/Eq$ is overcome. The discussion and the results are strictly parallel to the transverse momentum correlations analysis of Chapter 3, and so are the conclusions. In a short range correlation model like the MPM there is no divergence on the right-hand side because

$$f^{(1)}_c(x) = E \frac{d^3\sigma^c}{d^3p}$$

and

$$f^{(1)}_{\bar{c}}(x) = E \frac{d^3\sigma^{\bar{c}}}{d^3p}$$

($\bar{c}$ being the antiparticle of c: in our example $c = \pi^+$, $\bar{c} = \pi^-$), approach each other at small x, in such a way that

$$\delta(x) = f^{(1)}_c(x) - f^{(1)}_{\bar{c}}(x) \simeq x^P$$

$$\delta^{c}(x) = f^{(1)}_{c}(x) - f^{(1)}_{\bar{c}}(x) \simeq x^{p}$$

with p expected to be $1-\alpha_R \simeq \frac{1}{2}$. Here α_R is the intercept of the leading Regge trajectory with non-vanishing isospin. On the contrary in long range models like the DEM on the UJM $\delta^c(x)$ is not expected to depend on x, but the leading term should cancel at all x between $f_c(x)$ and $f_{\bar{c}}(x)$ and $\delta(x)$ should have an explicit $1/\ln s$ behaviour. Model testing in this sense is possible and strongly suggested [4].

For the moment we have exploited only charge conservation. To exploit the full isospin conservation seems harder: the only obvious thing to do is to use the fact that $\vec{I}$ is limited in the A + B state to perform a rotation in $\vec{I}$ space and conclude that also between, say, π^+ and π^0 the same type of cancellation between the $\ln s$ terms in (22) has to occur, i.e., $\lim_{s\to\infty}(\langle n_+\rangle - \langle n_0\rangle) = \text{constant}$, where the constant depends on the value of I^2 of the initial state. Hence, also

$$\delta_{+0}(x) = f^{(1)}_{\pi^+}(x) - f^{(1)}_{\pi^0}(x) \simeq x^{1/2}$$

in a MPM (as it is obvious, since the Pomeron in trajectory has not only $I_Z = 0$ but also $I = 0$) or δ_{+0} has an explicit $(\ln s)^{-1}$ behaviour at fixed x in the long range models.

More detailed prediction can be obtained only in the framework of specific models. Let us remember that explicitly s channel isospin conserving models can be built in the MPM by considering definite t channel isospin exchanges in the production amplitudes. This program has to be carried out in collaboration with Schwimmer [22]. Let us list some of the results here.

1) Cancellations of the $\ln s$ term in $\langle n_+\rangle - \langle n_0\rangle$ is independent of the model, and we conclude that it is a consequence of s channel isospin conservation only.

2) In "sensible" models a Poisson distribution in n_- (hence not in $n_c = n_+ + n_-$) can be recovered in some limits. This is in agreement with phenomenology. On this basis it is more sensible to consider, as we did in Chapter 4, correlations between negative particles than between charged particles.

3) The asymptotic behaviour of topological cross-sections

$$\tilde{\sigma}(n_c, s) = \sum_{n_0} \sigma(n_c, n_0, s)$$

can be very different from the behaviour of $\sigma(n_c, n_0, s)$, and is model dependent [4].

4) In the kinematical approximations in which the various models were considered in Ref. 22) identical particles would follow Poisson distributions. Nevertheless, $\Delta_c = (n_c(n_c-1)) - \langle n_c\rangle^2$ turns out to be non-vanishing, and of the canonical short range form. $a \cdot \ln s + b$, in the models under consideration. The simplest non-trivial model in this sense is the A model, in which dominance of alternate $I = 0$ and $I = 1$ exchange along the multiperipheral chain is assumed. In this model the generating function

$$S(g_c, g_0, s) = \sum g_c^{n_c} \, g_0^{n_0} \, \sigma(n_c, n_0, s)$$

takes the canonical form

$$f(g_0, g_c) \quad s^{\alpha(g_0, g_c)}$$

with,

$$\alpha(g_0, g_c) = 3^{-1/2} \left(g_0^2 + 2 g_c^2\right)^{1/2}$$

The coefficient of $\ln s$ in Δ_c is

$$a = \frac{d^2}{dg_c^2} \alpha(g_0, g_c)\Big|_{g_0 = g_c = 1} = \frac{2}{9} > 0$$

Remark between that Δ_2 as defined in Chapter 4 is given by

$$\Delta_2 = \langle n_-(n_- - 1)\rangle - \langle n_-\rangle^2 = \frac{1}{4}\Delta_c - \frac{1}{2}\langle n_-\rangle = -\frac{1}{9}\ln s$$

Hence a positive value of a_2 can be detailed only by the addition of explicit short range interaction terms. The calculation in the other models (I and R) considered in Ref. 22) is less straightforward and has not yet been performed.

7. HIGHER ORDERS, LONGER RANGES, ETC.

The big advantage of inclusiveness is that it provides tests of production models without having to deal with the enormous complexity of multiparticle production kinematics. However, if a more and more detailed insight is required, we have to increase the number of particles inclusively detected, going back to the same kind of difficulties that we tried to avoid. From this point of view quantities like the distribution in missing momentum [29)], i.e., in the over-all momentum of the neutral particles, are interesting, since they offer one dimensional measurable quantity that depends for its properties (e.g., scaling) on the properties of inclusive distributions of all orders, and also deals with charged and neutral particles on equal footing.

Let us come now to the tests of short range correlations; even the most convinced multiperipheralist cannot expect long range correlations to be absent. On one side, as we have seen diffractive processes are not compatible with short range. Furthermore, absorptive corrections to multiperipheralism, a first necessary step towards the enforcing of unitarity, introduce in general long range correlation. In a recent investigation of the problem [30)] (again in collaboration with Schwimmer) we have examined what features of multiperipheralism are most likely to be changed by absorptive corrections, and they turn out to be, as expected, the cancellation of the leading $\ell n\, s^k$ term in $\langle n^k \rangle - \langle n \rangle^k$, and the geometrical interpretation of multiparticle production as a random walk of fixed step in impact parameter space. On the contrary, other features like logarithmic multiplicity and scaling, that in the MPM are obvious consequences of short range, seem to be able to survive this kind of corrections.

I am indebted to Antonio Bassetto for several helpful conversations on the subject of the present talk.

FOOTNOTES AND REFERENCES

1) D. Amati, S. Fubini and A. Stanghellini, Nuovo Cimento 26, 817 (1962);
L. Caneschi and A. Pignotti, Phys.Rev.Letters 22, 1219 (1969).

2) R.C. Hwa, Phys.Rev.Letters 26, 1143 (1971).

3) In the following we define by "limiting distribution" the hypothesis that for a fixed value p_{lab} of the momentum of the detected particle in the target rest system the ID approaches a finite limit when $s \to \infty$. With "scaling" we intend the stronger hypothesis that

$$E \, \frac{d^3\sigma}{dp_\perp^2 \, dp_L} (p, s) \xrightarrow[s\to\infty]{} f(x, p_\perp)$$

with f limited, continuous and rapidly decreasing in $p_\perp$ in the whole x range.

$(x = 2 p_L^{c.m.} / \sqrt{s}$ as usual $)$.

4) L. Caneschi, Nuovo Cimento Letters 2, 1261 (1971).

5) L.G. Ratner et al., "Inelastic proton-proton scattering at very high energy", American Physical Society Meeting, Rochester, 30 Aug. 1971;
G. Giacomelli, Talk at the present meeting.

6) L.W. Jones et al., Phys.Rev.Letters 25, 1679 (1970).

7) N. Bali, L.S. Brown, R. Peccei and A. Pignotti, Phys.Rev. Letters 25, 557 (1970).

8) G. Barbiellini et al., Phys.Letters, in press;
M. Breidenbach, private communication;
G. Neuhofer et al., Phys. Letters 38B, 51 (1972).

9) At $x = 0$ the relative energy of the detected particle (c) with respect to either incoming one (a and b) is of order $\sqrt{s}$. Hence, a correction term of the form $s_{ac}^{-\frac{1}{2}}$ or $s_{bc}^{-\frac{1}{2}}$ behaves like $s^{-1/4}$.

10) The physical meaning of (2) is transparent: integrating over the full phase space of the $(k+1)^{th}$ particle with the weight q_n we collect the balance of the momentum that has not been accounted for by the k particles that we keep fixed. For an elegant formal derivation see
L.S. Brown, Phys.Rev., in press, who also shows the sufficiency of the set of equations (2) to ensure momentum conservation.

11) S.H.M. Tye, Nuovo Cimento Letters 2, 1271 (1971).

12) A. Bassetto and M. Toller, Nuovo Cimento Letters 2, 409 (1971).

13) D.Z. Freedman, C.E. Jones, F.E. Low and J.E. Young, Phys.Rev.Letters 26, 1197 (1971).

14) L. Van Hove, Revs.Mod.Phys. 36, 655 (1964);
E.M. de Groot and Th. Ruijgrok, Nuclear Phys. B27, 45 (1971);
A. Bassetto, M. Toller and L. Sertorio, Nuclear Phys. B34, 1 (1971).

15) J.H. Friedman, C. Risk and D.B. Smith, Phys.Rev.Letters 28, 191 (1972).

16) A.H. Muller, Phys.Rev. D4, 150 (1971).

17) L. Caneschi, Nuclear Phys. B35, 406 (1971).

18) Obviously x and y are not the right variables to understand (14). For instance, the limit $x \to 0$, $y \to 0$ can be reached with the relative energy of the two particles $(p+q)^2$ arbitrary. The right variables are the rapidities η_x and η_y, in terms of which (14) reads:

$$\Delta_2 = \int \rho^{(2)}(\eta_x, \eta_y) \quad d\eta_x \quad d\eta_y$$

and (15) becomes

$$\int \rho^{(2)}(\eta_x, \eta_y)\, d\eta_x\, d\eta_y\, e^{-|\eta_x - \eta_a|} = -1$$

where η_a is the rapidity of one of the incoming particles.

19) Equation (16) implies that

$$\sum_n (n - \langle n \rangle)^2 \sigma(n,s) < c \ln s$$

obviously incompatible with a finite asymptotic limit of any $\sigma(n,s)$. Considering higher moments it is easy to show that in a strictly short range theory $\sigma(n,s)\, \ln^p s \to 0$ for arbitrary p. This point is discussed thoroughly in Le Bellac's talk and bears much resemblance to the footnote on diffraction dissociation in the first page of Ref. 28).

20) Remark, however, that $f(g)$ and $\alpha(g)$ are in general not analytic around $g = 0$. Hence (17) cannot be used, for instance, to compute properties of $\sigma(n,s)$. See Ref. 17) for a full discussion of this point.

21) A. Białas and K. Zalewski, Preprint TPJU 22-71, Cracow University.

22) L. Caneschi and A. Schwimmer, Phys.Rev. D3, 1588 (1971).

23) L. Caneschi and A. Pignotti, Phys.Rev. 180, 1525 (1969).

24) Actually, I have only checked that from (18) it follows that $a_2 > 0$, $a_3 < 0$, $a_4 > 0$. I believe that the alternation keeps going.

25) H.T. Nieh and J.J. Wang, Phys.Rev. in press. By associated multiplicity we mean the multiplicity of the remaining particles in events in which one particle has a prescribed value of x.

26) A. Białas, K. Fielkowski and R. Wit, Preprint TPJU 24-71, Cracow University.

27) The same result has been obtained by E.H. de Groot (Oxford University Preprint) and B. Webber (DAMPT, Cambridge Preprint) by direct calculation.

28) The unfortunate fact that neutral particles are hard to detect makes it sometimes hard to explcit phenomenologically some interesting consequences of the sum rules [see for instance, L. Caneschi, Phys.Letters 37B, 288 (1971)].

29) L. Caneschi, H. de Groot and A. Schwimmer, Oxford University preprint (1972).

30) L. Caneschi and A. Schwimmer, Weizmann Institute preprint (1972).

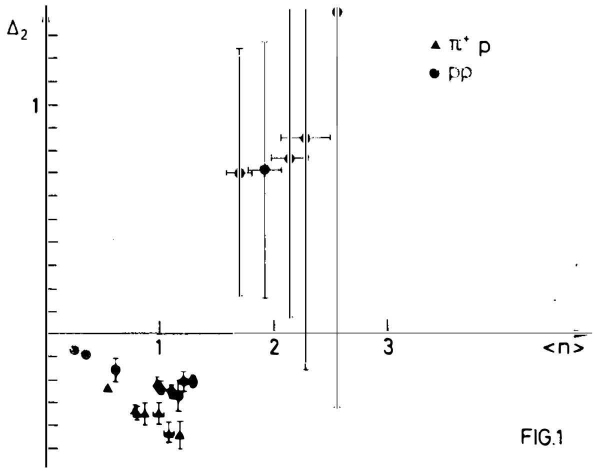

FIG.1

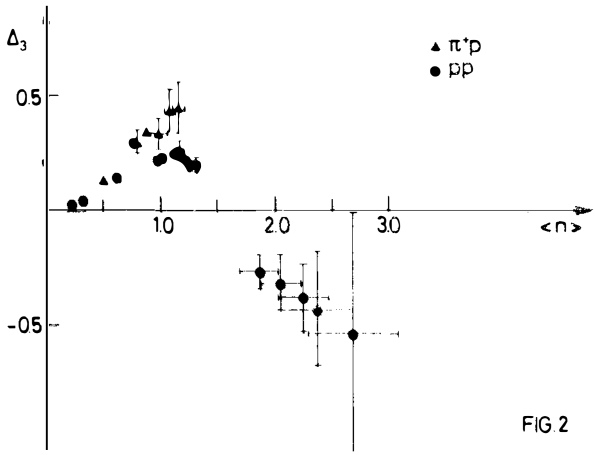

FIG.2

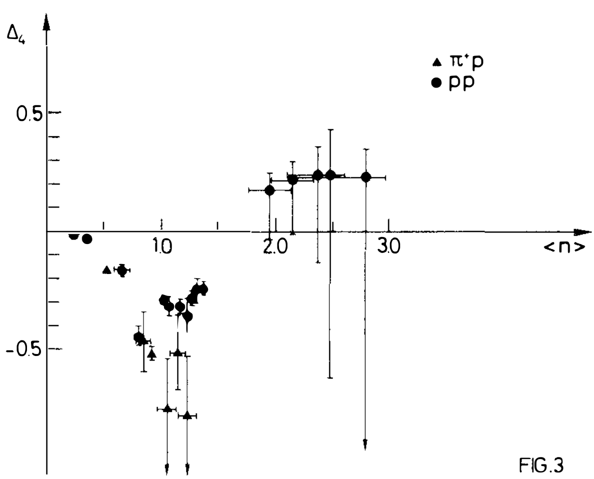

FIG.3

TWO PARTICLE CORRELATIONS
$\pi^+ p \rightarrow \pi^- + \pi^- +$ ANYTHING
at 18.5 GeV/c
NOTRE DAME DATA

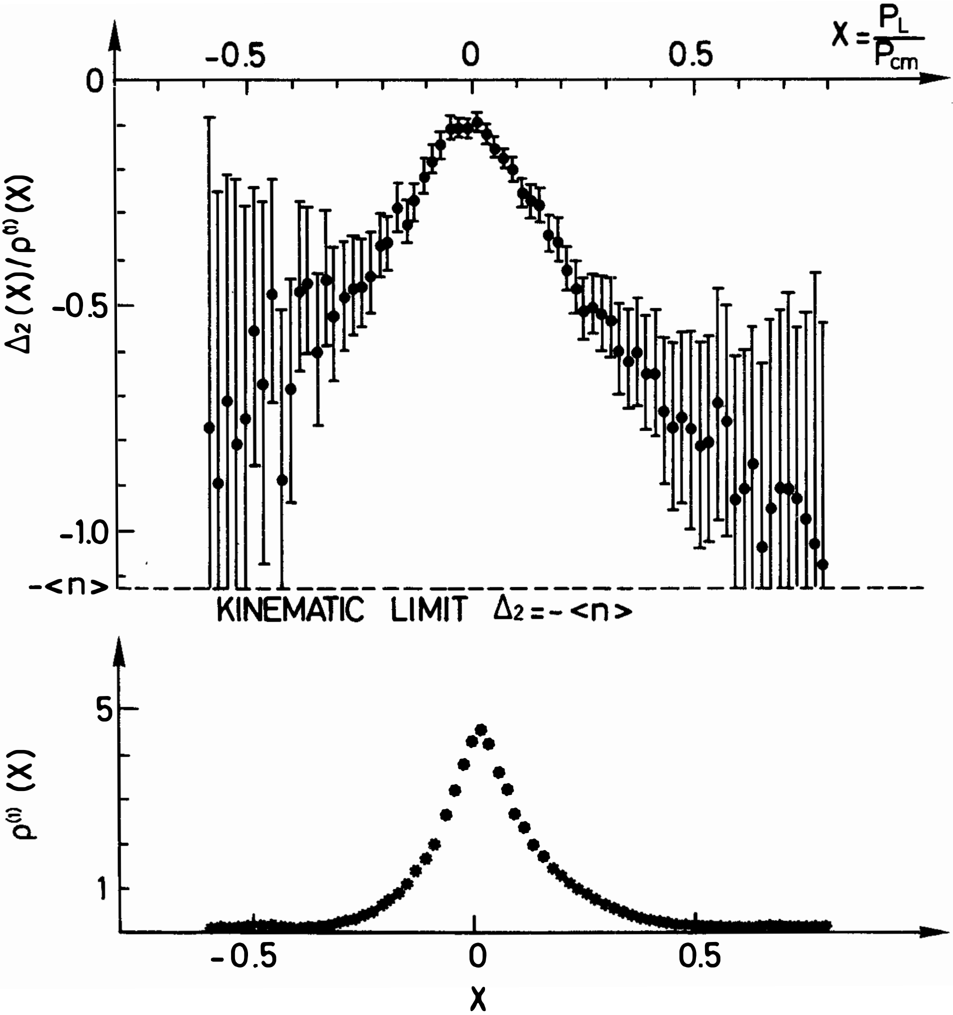

FIG.4

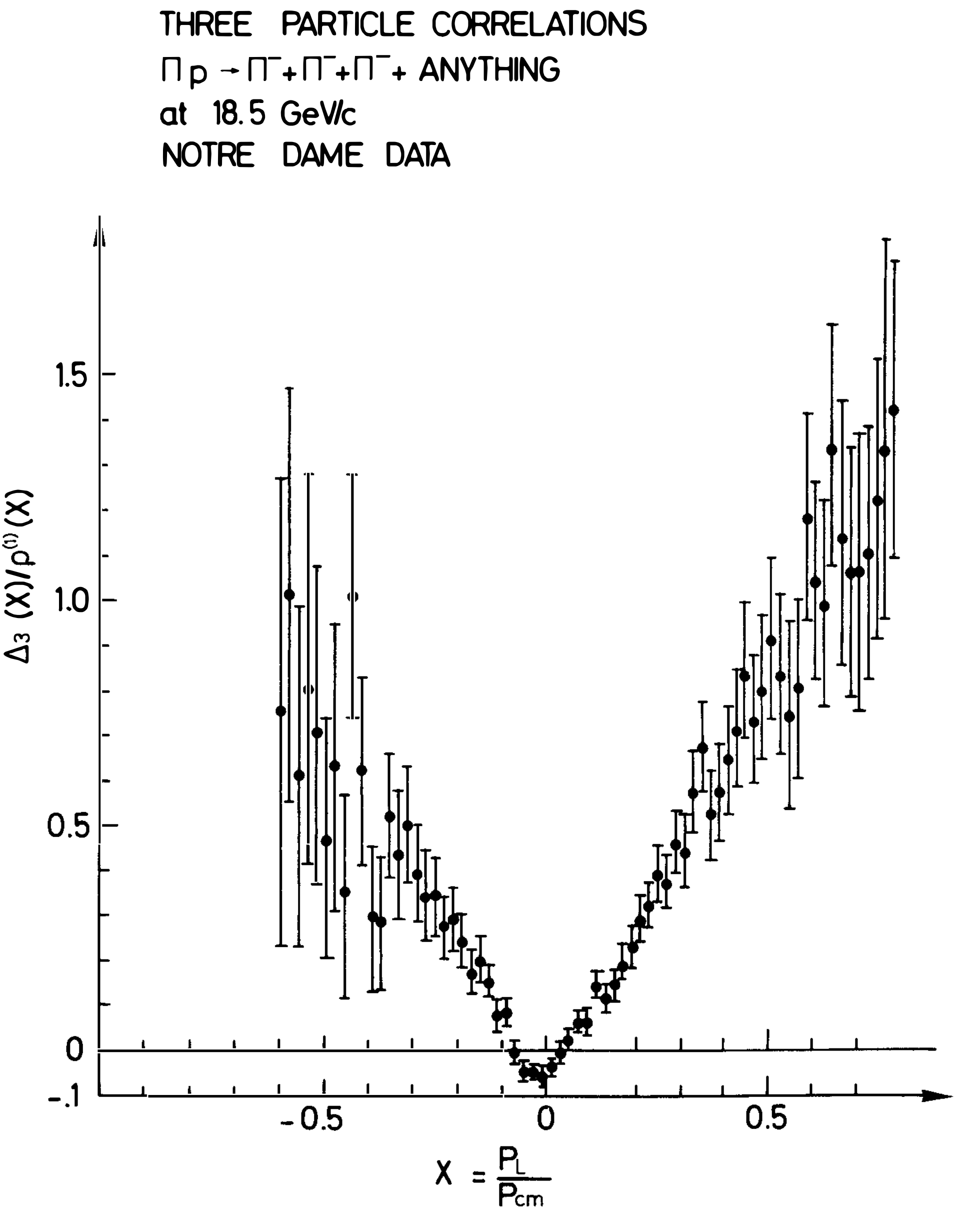
THREE PARTICLE CORRELATIONS
Π p → Π⁻+Π⁻+Π⁻+ ANYTHING
at 18.5 GeV/c
NOTRE DAME DATA
1.5
1.0
0.5
0
-.1
Δ3 (X)/ρ(1) (X)
-0.5
0
0.5
X = PL/Pcm

FIG.5

TWO PARTICLE CORRELATIONS

FIG.6

pp → π^- + π^- + ANYTHING

SMITH DATA