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CRITERIA FOR PROGRAMMING INVESTMENT PROJECT SELECTION

H. Martin Weingartner

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CRITERIA FOR PROGRAMMING INVESTMENT PROJECT SELECTION¹

H. Martin Weingartner

In their recent article in this Economic Journal,² Professors Baumol and Quandt have presented some interesting insights into a difficult problem. However, they have confined themselves entirely to a purely economic framework, and by so doing, have fallen short of their desired goal of a construct "which is usable directly in the computations required for optimal investment project selections."³ This paper will examine the problem of the objective in programs for investment and the Baumol and Quandt solution; it will also offer a more operational approach.

The model which forms the point of departure for their analysis is

Maximize

$$(1a) \quad \Pi = \sum_{j=1}^n \sum_{t=0}^h [a_{jt}/(1+i)^t] x_j$$

subject to

$$(1b) \quad \sum_{j=1}^n b_{jt} x_j \leq M_t, \quad (t = 0, 1, \dots, h)$$

$$\text{all } x_j \geq 0$$

Where \underline{i} represents the rate of interest;

\underline{a}_{jt} represents the net cash flow obtained from a unit of project \underline{j} during period \underline{t} ;

\underline{b}_{jt} is the net amount of cash used up by a unit of project \underline{j} during period \underline{t} ;

\underline{x}_j is the number of units of project \underline{j} constructed; and

\underline{M}_t is the amount of cash made available from outside sources during \underline{t} .

The troublesome issue is, of course, that given capital rationing, what meaning is to be attached to the discount rate \underline{i} in the objective func-

tion (la), or, alternatively, what is the appropriate form for the objective function? To seek answers to these questions it is necessary, however, to go behind the formal statement of the economic problem and to look at its origin in a business organization. Thus, if the analysis is to be of use to decision-makers, it must do more than to describe conditions which an optimum must satisfy. This distinction may be categorized as one between capital theory and managerial economics, and is one which Professors Baumol and Quandt do not make.

The investment decisions which are "made under conditions which closely approximate capital rationing"⁴ are indeed encountered in practice. That is, investment projects are selected within a budgeting process, and expenditure ceilings are frequently imposed by company managements. It must be recognized, however, that these budgets fulfill different purposes, and hence require different models for their analysis. The apparently most preponderant use of budgets is for the two distinct functions of planning and control. When used for planning the budgets themselves serve only as coordinating devices, and the ceilings should formally be regarded only as parameters, to be varied in the course of the exploratory analysis. That is, quantities \underline{M}_t are not "amounts of cash made available from outside sources during t ," having been established by the operation of the capital market. Indeed, there is considerable doubt about the existence of this kind of capital rationing which would in any case, require explicit recognition of uncertainty in the analysis.⁵

When budgeting serves the primarily administrative purpose of control, i.e., it is used to motivate behavior in the direction of economizing

and to encourage or discourage project formulation, it is not appropriate to make use of the equation of Baumol and Quandt,⁶

$$(2) \quad a_{jt} = -b_{jt}.$$

That is, the quantities to be controlled by the expenditure ceiling are outlays on capital account, and the cash inflows will not, in general, automatically be made available for reinvestment without passing through the same control channels which set the quantities \underline{M}_t in the first place. By this interpretation, and contrary to what Baumol and Quandt state, the constraints would be considered as not "meaningful ... if \underline{b}_{jt} is interpreted as the net input of cash needed by project \underline{j} in period \underline{t} ."⁷ Without this equality, of course, we are back to the original Lorie and Savage problem⁸ with the proper form for the objective function still to be determined.

The difficulty is that if strict economic capital rationing is denied as the introduction by Baumol and Quandt appears to do, then it must also be admitted that the setting of the quantities \underline{M}_t incorporates the management's or entrepreneur's set of preferences. By restricting the employment of funds, the firm's opportunities are similarly limited. This also is shown by the objective function of the dual problem,

$$(3) \quad \text{Minimize} \quad \sum_t \rho_t M_t$$

which incorporates the M_t . The question then arises as to the origin of these quantities, which Baumol and Quandt take as given and outside the optimization which they propose to do.

Two answers occur to this writer. First, to the extent that the budgets limit expenditures which would be larger in their absence, they may reflect the judgment that sources of equity capital are temporarily costly, and expenditures should be limited to internally generated funds and to debt financing. Such a policy may be promulgated due to a depressed stock market or because of the belief that there are information lags which will lead to more favorable prices for common stock once the firm's prospects are more widely known. Even in this case, of course, it is not necessary to treat the limitations in the form of absolute spending limits. Instead, as has been detailed elsewhere, it is possible to utilize a rising supply curve for outside funds.⁹ With respect to the discount rate to be employed in the objective function, it is clear that the market discount rate, more appropriately termed the cost of capital, will serve the purpose. The effect of expenditure limitations for a few years is to impose additional premiums on the expenditure of curtailed funds which are expressed by the dual prices. Making use of these, it is possible to see whether the saving from postponed equity financing is worth the foregone investment opportunities. To make this analysis realistic requires then also the explicit introduction of the mutually exclusive alternatives of projects now and the same ones postponed with possible different cash flows. The resulting solution represents then not a maximization of the utility of the income stream from the investments, but it rather gives effect to a changing cost of capital, an entirely different matter.

The Baumol and Quandt utility model, which they regard as operational, will now be examined on its own terms. In doing so it will also be possible to indicate by example the difference between the approach of managerial economics (or management science) vs. that of traditional economists.

To substitute for the discounted net value of the investment projects of expression (1a), Baumol and Quandt propose the sum of weighted withdrawals in each of the future periods made possible by the projects undertaken. The weights themselves are the subjective utility of the withdrawal. Formally, their model is

Maximize

$$(4a) \quad \sum_t U_t W_t$$

subject to

$$(4b) \quad - \sum_j a_{jt} x_j + W_t \leq M_t$$

$$(4c) \quad x_j \geq 0$$

$$W_t \geq 0$$

where \underline{W}_t is the amount the entrepreneur plans to withdraw in the t th period to be used for consumption, and the terms \underline{a}_{jt} , \underline{M}_t and \underline{x}_t are as previously defined. \underline{U}_t is the (fixed) utility of a dollar in period t .

The utility function employed (4a) fits well into the programming formulation in that it is linear in the quantities \underline{W}_t . Unfortunately, this convenience is purchased at too high a price. First, it assumes an identity between the firm and the (single?) owner-entrepreneur, and it requires an assignment of the utility index in advance of information about the consumption possibilities.¹⁰ Second, it makes the utility of consumption in one period independent of the amount available for consumption in another. While it is true that a more general utility function $U=f(W_0, W_1, \dots, W_n)$ would not substantially alter the economic implications of the dual problem,¹¹ it would alter feasibility of the approach by several orders of magnitude.

The solution then is not to substitute a more general utility function, which would also add a substantial burden to the computational problems,¹² but to utilize a parametric method for the purpose of exploring the set of consumption possibilities, at the same time taking into account the distinction between the firm and its owners. To that end an extension of the author's previously formulated models will go further toward solving the problems of application.

In Mathematical Programming and the Analysis of Capital Budgeting Problems the "basic horizon model" was formulated as follows.¹³

Maximize

$$(5a) \quad \sum_j \hat{a}_j x_j + v_T - w_T$$

subject to

$$(5b) \quad \sum_j -a_{1j} x_j + v_1 - w_1 \leq D_1,$$

$$(5c) \quad \sum_j -a_{tj} x_j - (1 + r_{t-1})v_{t-1} + v_t + (1 + r'_{t-1})w_{t-1} - w_t \leq D_t, \quad t = 2, \dots, T$$

$$(5d) \quad 0 \leq x_j \leq 1, \quad j = 1, \dots, n$$

$$(5e) \quad v_t, w_t \geq 0, \quad t = 1, \dots, T$$

with notation to be defined below.

Formally, this model and that of equations (1a and b) are quite similar. They differ, however, in a number of important respects. First, the right-hand-side terms, \underline{D}_t , represent not amounts of funds made available from outside sources but the cash throw-off from the assets of the firm before project selection begins. These quantities, like the \underline{M}_t , may be negative. The objective function

contains basically two components, the residual value of all physical assets acquired up to the horizon, and the net amounts borrowed or lent at that date. The former, strangely omitted in all the models of Baumol and Quandt, is required to express the fact that the firm is not necessarily liquidated at the horizon.¹⁴ Thus \hat{a}_j is the residual value of project j at the horizon. How it is to be determined will be discussed below.¹⁵ The quantities \underline{v}_t are the amounts of cash available which are invested in outside financial investments, assumed to earn interest at \underline{r}_t per year. Similarly, the quantities \underline{w}_t are amounts borrowed from outside the firm, at an interest cost of \underline{r}'_t per year. Borrowing can then also be limited, if appropriate, by borrowing limits in the form of constraints on the \underline{w}_t , or alternatively, a rising supply curve for funds can be included in the model.¹⁶ The objective function then included also the term $(\underline{v}_T - \underline{w}_T)$, the net amount of financial assets accumulated at the horizon.

As asserted in connection with the development of these models¹⁷ the difficulty presented by the inclusion of dividends is not in their effect on the model, but rather in the evaluation of alternative solutions.¹⁸ Rather than to impose a utility index (with the limitations already pointed out) it may be preferable to explore the opportunity set for dividends within a framework of a dividend policy. Such a policy can be formulated to weigh the preferences for distributions of diverse owners, or to attain as high a market value for the stock of the company, so that the consumption decision by the owners can be separated from that of investment in the way Fisherian analysis permits in the presence of perfect capital markets.

Although there is considerable dispute about the role of dividends, given an investment policy, in the determination of market value,¹⁹ it has been suggested that some payout policies are to be preferred to others.²⁰

To illustrate, in the following model it is supposed that dividends denoted by \underline{d}_t , must be of some minimum amount, \underline{d}_{\min} . Further, dividends must be nondecreasing over time, so that in each period the amount paid out is at least as large as the amount paid out in any earlier period, i.e., $\underline{d}_t \geq \underline{d}_{t-1}$. At the horizon, it is required that the value of remaining assets, physical and financial, be sufficiently large to maintain the dividend rate in the horizon year. The objective of the investment policy is then to maximize the growth of dividends, which here is equivalent to maximization of dividends at the horizon. The model thus becomes

Maximize

$$(6a) \quad d_T$$

subject to

$$(6b) \quad \sum_j a_{1j} x_j + v_1 - w_1 + d_1 \leq D_1$$

$$(6c) \quad \sum_j a_{tj} x_j - (1 + r_{t-1})v_{t-1} + v_t + (1 + r'_{t-1})w_{t-1} - w_t + d_t \leq D_t$$

$t = 2, \dots, T$

$$(6d) \quad d_1 \geq \underline{d}_{\min}$$

$$(6e) \quad d_t \geq d_{t-1}, \quad t = 2, \dots, T$$

$$(6f) \quad r \left\{ \sum_j \hat{a}_j x_j + \hat{D} + v_t - w_t \right\} \geq d_T$$

$$(6g) \quad 0 \leq x_j \leq 1, \quad j = 1, \dots, n$$

$$(6h) \quad v_t, w_t \geq 0, \quad t = 1, \dots, T,$$

plus borrowing limits or other conditions on the supply of outside funds. Restriction (6f) requires amplification. The expression in braces represents the capital value of the assets of the firm as of the horizon, and consists of three components. First, the net amount of financial assets, as before, is denoted by the difference $\underline{v}_T - \underline{w}_T$. The physical assets as of the horizon include two parts, the assets resulting from the investments undertaken, and those which existed as the start. Cash flows of the latter were denoted by \underline{D}_t , the right-hand-side terms of restriction (6 b and c). The residual value as of the horizon was not needed in the earlier model, being a constant, but is required here, and is denoted by \hat{D} . The investments undertaken may also have residual values at the horizon, these being denoted, as before, by \underline{a}_j .

The problem, which gives rise to a circularity, is that the residual values must be determined by discounting their underlying cash flows back to the horizon at some rate of interest. How is this rate to be determined? While an infinite horizon would avoid this difficulty (implicitly what Baumol and Quandt appear to have done), an operational solution to the investment problem requires a more direct answer. It would, of course, be possible to substitute estimated market values for these assets, as of the horizon. On the other hand, if the value to the firm of the assets is substantially larger than the market value, the cash flows must be discounted. Fortunately, the investment decisions which the model is to determine are not likely to be sensitive to the actual rate utilized for discounting these residual values. Hence, the estimated cost of capital in the absence of additional constraints will serve if the horizon is not very near.²¹ The same rate, denoted by \underline{r} in restriction (6f) is used to obtain the periodic income generated by the residual

assets in perpetuity, which is to be at least as large as the dividends at the horizon.²² This model, by maximizing dividends at the horizon, though subject to the constraint that this level of dividends can be maintained (or increased by further investments subsequent to the horizon), also maximizes the rate of growth of dividends.²³ However, this model contains two drawbacks. First, maximization of the growth rate of dividends up to the horizon will produce a stream of dividends which is small over a large portion of the time to the horizon, but rises sharply at the end. Second, it yields a single solution rather than a set of alternatives from which the optimal pattern of dividends, within the policy limits, can be chosen.

Both of these objections can readily be met. In place of restriction (6e) we substitute

$$(7) \quad d_t \geq \alpha d_{t-1} \quad \text{or} \quad d_t / d_{t-1} = \alpha$$

By making restriction (6d) and equation, viz., $d_1 = \underline{d}_{\min}$, (7) can be made a valid linear constraint of the form

$$(8) \quad d_t / \underline{d}_{\min} \geq \alpha^t$$

The term α is then varied parametrically,²⁴ and the set of solutions can be exhibited as functions of the horizon level of dividends and the minimal growth rate of dividends, α . This solution set can be viewed as an efficiency frontier, as in Figure 1. That is, no other solutions exist which have a higher average growth rate for dividends for a given minimum rate; or, alternatively, none has a higher minimum growth rate for a given average growth rate.

FIGURE 1



The procedure just outlined contains the considerable advantage over the subjective utility index required by Baumol and Quandt in that, as in the model for portfolio selection by Markowitz,²⁵ the decision maker can examine his alternatives before expressing his preferences. In addition, he does not have to consider all possibilities but only those which conform to certain time shapes, expressed as efficient sets of alternatives within the stated dividend policy.

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12. H. M. Markowitz, Portfolio Selection, (New York: John Wiley and Sons), 1959.

Footnotes

1. Work on this paper was supported by the Ford Foundation's Grant to the Alfred P. Sloan School of Management, Massachusetts Institute of Technology, for research in business finance.
2. See [1].
3. See [1], p. 329.
4. See [1], p. 327.
5. See the discussion in [2, 3, 4, and 5], especially with respect to the question whether the point will be reached beyond which a lender will not agree to increase his loan, even with unlimited compensation in the form of higher interest rates.
6. See [1], p. 321.
7. See [1], p. 321.
8. See [6], and also [7], Chap. 2 and 3.
9. See [7], Chap. 8 and 9.
10. Presumably, if the complete set of consequences of all alternative investment programs could readily be envisaged, there would be no need to resort to programming models, whose advantage resides in their computability.
11. See [1], p. 326, footnote 4.
12. The relevant utility function is not only nonlinear, but more important, also not separable in the quantities W_t .
13. See [7], Chap. 8 and 9.
14. Failure to include such a term makes the solution extremely sensitive to the choice of the horizon. In fact, in their formulation, with investments having constant returns to scale without limit, the maximum number of projects that will be selected is h , the number of periods to the horizon. See [7], Sec. 4.3. Here, the project definition is treated by imposing the upper bounds on the x_j in constraints (5c).
15. The residual value of the existing plant is a constant, and hence need not enter into the objective function. This will be the case even if partial or complete liquidation of assets is an alternative to be considered.
16. These constraints were analyzed in detail in [7], Sec. 9.1 and 9.3, and will not be gone into here. It is necessary to point out, however, that the Baumol and Quandt model for the Carry Over of Funds [1], p. 328, implicitly assumes that surplus funds will be held in the form of cash, ignoring the possibility of financial investments.
17. See [7], p. 174, n. 31.

18. The term dividends is substituted for withdrawals for consumption since in the modern corporation the distinction between the firm and its owners must be maintained.

19. See [8] and [9].

20. See, for example, [10] and [11]. This issue is by no means settled. The purpose of what follows is only to exemplify the use of the investment model in conjunction with a given dividend policy.

21. Thus, $\hat{a}_j = \sum_{t=T+1}^{\infty} a_{tj} (1+r)^{T-t}$ $\hat{D} = \sum_{t=T+1}^{\infty} D_t (1+r)^{T-t}$

22. Dividends growing perpetually at some positive rate could be handled within the model, but are clearly an impossibility in the real world.

23. Actually, the rate of growth, g , depends also on \underline{d}_1 : $\underline{g} = (\underline{d}_T / \underline{d}_1)^{1/T}$.
 However, assuming $\underline{d}_1 = \underline{d}_{\min}$ (or is close to it), \underline{g} is maximized by maximization of \underline{d}_T , as can be seen from the expression for $\log \underline{g} = 1/T (\log \underline{d}_T - \log \underline{d}_{\min})$
 or $\log \underline{g} = 1/T \log \underline{d}_T - C$.

24. See [7], Sec. 7.4.

25. See [12].

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