

Revisiting Bremsstrahlung emission associated with Light Dark Matter annihilations

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We compute the single bremsstrahlung emission associated with the pair annihilation of spin-0 particles into electrons and positrons, via the t -channel exchange of a heavy fermion. We compare our result with the work of Beacom et al. [1]. Unlike what is stated in the literature, we show that the Bremsstrahlung cross section is not necessarily given by the tree-level annihilation cross section (for a generalized kinematics) times a factor related to the emission of a soft photon. Such a factorization appears only in the soft photon limit or in the approximation where the masses of the particles in the initial and final states are negligible with respect to the mass of the internal particle. However, in the latter case, we do not recover the same factor as for $e^+e^- \rightarrow \mu^+\mu^-\gamma$. Numerically the difference, in the hard photon limit, is as large as a factor 3.6. However the effect on the upper limit of the dark matter mass is not significant. Using gamma ray observations, we obtain an upper limit on the dark matter mass of 30 or 100 MeV (depending on the region that is considered for the 511 keV analysis) while Beacom et al. found 20 MeV.

I. INTRODUCTION AND SET UP

The Milky Way radiation has been mapped extensively at different wavelengths for many decades. Photon emission at low and ultra-high energy is still currently studied. All the latest observations appear extremely exciting and shed new light on the physics of the galactic centre. Among them, there is the observation of a 511 keV emission line by SPI, a spectrometer on board of the INTEGRAL satellite launched in 2002. The 511 keV line has been detected by many experiments since the seventies (see Ref. [2, 3, 4]), but the origin of the line could not be established unambiguously. With the new results from SPI [5, 6, 7], one could determine that it is due to (para-)positronium formation, thus confirming the presence of anti matter in our galaxy.

A very striking feature of this line is that it is mostly emitted in the bulge of the Milky Way (the distribution is well approximated by a sphere; the full width at half maximum is about 10° of diameter). No significant emission was detected in the disk. This characteristic tends to exclude astrophysical sources such as hypernovae/Wolf-Rayet stars, pulsars, and cosmic ray interactions. On the other hand, this observation could point towards an old galactic population [8].

A blind source analysis showed that more than 8 point sources could explain the emission [7]. A diffuse source provides a very good fit too. None of these sources has been found as yet. This makes the origin of the galactic positrons even more mysterious.

Low Mass X-ray Binaries (LMXB) could perhaps be an explanation if the positrons emitted in the disk escape into the bulge. Their characteristics are poorly known though (e.g. flux, jet content). No 511 keV emission has been seen so far from the brightest of these objects, but more observations will be dedicated to this kind of candidates. Type 1a supernovae (SN1a) may also explain the line emission if the positron escape fraction and the supernovae explosion rate are

large enough to maintain a steady flux. At present various observations contradict with each other and it is quite difficult to know whether these two conditions are indeed satisfied in our galaxy [39]. Dedicated observations of SN1a remnants are also scheduled with INTEGRAL and should help answering these questions [8, 9].

Another possible source of low energy positrons could be dark matter annihilations [10]. This scenario requires a dark matter mass between a few MeV and a few hundred MeV, two kinds of interactions, and a cuspy dark matter halo profile (likely to be as cuspy as a Navarro-Frenk-White model [11]). This dark matter profile corresponds to theoretical predictions from numerical simulations, but it has not been confirmed as yet observationally [12, 13].

Light dark matter (LDM) particles were initially proposed in the context of the so-called cold dark matter crisis. These particles were meant to experience a new damping effect, including one that is half-way between the collisional damping and the free-streaming. They were meant to inherit from the damping of relativistic species ($X = \gamma, \nu$) with which they weakly interact either directly or indirectly [14, 15, 16, 17, 18]. In the indirect case, the damping of the species X is communicated to dark matter through the dark matter coupling to other particles in contact with X . For example, before recombination, dark matter indirectly experiences the Silk damping, until it thermally decouples from electrons. This new effect (called induced damping) is simply the generalization of the Silk damping effect to dark matter and has been confirmed numerically [19]. In the direct case, LDM may suffer from the mixed damping effect [14, 15, 17], which corresponds to the situation where, due to a weak but large enough neutrino-DM interaction rate, dark matter inherits from the neutrino free-streaming. This occurs when LDM stays coupled to neutrinos after their thermal decoupling. For some specific values of the DM- ν elastic scattering cross section, these interactions affect the DM fluid but do not change the neutrino fluid properties. This effect was stressed in Refs. [14, 15] for neutralinos when they are strongly mass degenerated with sneutrinos and for MeV particles having weak interactions [14, 15, 18].

Light dark matter particles therefore have this interesting property: on the one hand, they look like cold dark matter candidates because of their mass and weak interactions. On

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the other hand, their linear matter power spectrum is cut-off at a cosmological scale (due to these new damping effects), which is a well-known characteristic of warm dark matter candidates.

To avoid an overproduction of gamma-rays and satisfy simultaneously the relic density criterion, the LDM total annihilation cross section at early times should be larger than that in the primordial universe or in virialized objects [20]. This leads to a scenario in which the LDM interactions, and hence the damping, are suppressed [18] (the damping mass in the linear matter power spectrum was estimated to be about $100 M_\odot$).

Following up Ref. [10], there have been many other dark matter models proposed to explain the 511 keV emission, e.g. Refs. [21, 22, 23, 24, 25, 26]. Here we focus on the annihilating case. In this scenario, there is a light extra gauge boson Z' (needed for the relic density) and heavy charged particles (for the 511 keV emission) [20]. This new set of particles is quite similar to that expected in $N = 2$ supersymmetry.

The model is described in detail in Ref. [27]. The constraints on the couplings are given in Ref. [28, 29, 30]. We disregard the possibility that dark matter is fermionic [40], following up Ref. [30].

Heavy dark matter particles, i.e. $100 \text{ MeV} > m_{\text{dm}} > 10 \text{ MeV}$, are in danger of yielding a too large contribution to the anomalous magnetic moment of the electron [29, 30]. Also they would make the observed fraction of para and orthopositronium very hard to explain [6] and would eventually yield a too large e^+e^- inflight annihilation contribution [31].

Any other way of reducing the mass range which is not too model dependent and which could be related to the 511 keV flux is very interesting. In that context, the Beacom et al. proposal [1], to constrain the dark matter mass from the comparison between observations and the gamma-ray flux originating from the Bremsstrahlung, looks promising even if it gives a smaller contribution than the inflight annihilations.

This suggestion is complementary to a previous estimate made before INTEGRAL's publication [20]. In that work, it was assumed that each electron and positron produced by dark matter annihilations spontaneously generates one photon after their emission. The authors assumed that the total annihilation cross section was either velocity-dependent or velocity-independent. After comparison with the gamma-ray observations (and requiring the correct relic density), they concluded that only a velocity-dependent cross section was possible for $m_{\text{dm}} < 100 \text{ MeV}$ while for greater masses either a velocity-dependent or independent cross section could be allowed.

However, the model that fits INTEGRAL/SPI data requires two kinds of interactions simultaneously. One is velocity-dependent and is needed for fitting the relic density. The other one is velocity-independent and is quite mandatory for fitting the 511 keV emission morphology. Such a scheme is not compatible with Ref. [20] in which it was assumed that only one process (either velocity dependent or independent) was contributing to the total annihilation cross section. One therefore has to estimate the gamma ray production again and derive the corresponding upper bound on the dark matter mass, assuming that LDM is at the origin of all the 511 keV emission mapped by SPI.

By doing so the authors in Ref. [1] concluded that m_{dm} could not exceed 20 MeV. Their estimate of $\sigma_{\text{dm dm}^* \rightarrow e^+ e^- \gamma}$ is based on an old observation made by Berends et al. [32] according to which, in QED or $SU(N)$ gauge theories, the cross sections associated with the single emission of a hard bremsstrahlung photon becomes remarkably simple in the ultrarelativistic limit. Using specific examples, Berends et al. showed that a factorization into two terms can be observed. One factor is the squared amplitude associated with the 2-to-2 process (i.e. without the additional emission of the photon), and evaluated for a generalized kinematics. The second factor is related to the infrared factor, which describes the soft photon limit. Such a property shows up for example in $e^+e^- \rightarrow \mu^+\mu^-\gamma$. The expressions for the initial and final-state radiation cross sections associated with this process can be found for example in the review of Berends and Böhm [33].

Beacom et al. thus argued that the final-state radiation cross section of dark matter particles annihilating while they are almost at rest can be written as the product of the two-to-two process times an universal factor. In a next step they assumed that this factor can be obtained from the known results for the final-state radiation associated with the $e^+e^- \rightarrow \mu^+\mu^-\gamma$ process in the ultrarelativistic limit, where the electron mass is neglected.

However, in the case of dark matter, the ultrarelativistic limit is not appropriate. Furthermore for chiral dark matter couplings it is also obvious that the electron mass cannot be neglected. Also $e^+e^- \rightarrow \mu^+\mu^-$ and $\text{dm dm}^* \rightarrow e^+e^-$ are intrinsically different. $e^+e^- \rightarrow \mu^+\mu^-$ is due to a photon exchange in the s -channel; it involves only vectorial couplings. In contrast, $\text{dm dm}^* \rightarrow e^+e^-$ proceeds through a t -channel exchange of a heavy fermion and can involve chirality flipping couplings. Thus writing the Bremsstrahlung cross section as the product of two factors may not be correct.

Here we demonstrate that the approach followed in Ref. [1] fails to correctly describe the Bremsstrahlung process in the hard photon limit.

II. ANNIHILATION AND BREMSSTRAHLUNG CROSS SECTION

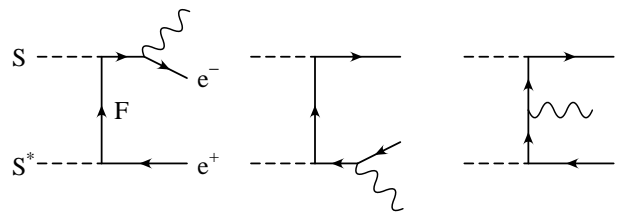


FIG. 1: Feynman diagrams contribution to $S + S^* \rightarrow e^+ + e^- + \gamma$.

In what follows we study the process

$$S(p_{\text{dm}}) + S^*(p_{\text{dm}^*}) \rightarrow e^-(p_{e^-}) + e^+(p_{e^+}) + \gamma(p_\gamma), \quad (1)$$

where S denotes a spin-0 dark matter particle.

Since dark matter is neutral, the emission of a photon necessarily comes from the final state particles and the particle that is exchanged during the annihilation process (F). The corresponding Feynman diagrams are shown in Fig. 1. It is convenient to use the scaled energies as independent variables:

$$x = \frac{2k \cdot p_{e^-}}{s}, \quad \bar{x} = \frac{2k \cdot p_{e^+}}{s}, \quad x_\gamma = \frac{2k \cdot p_\gamma}{s}, \quad (2)$$

where $k = p_{\text{dm}} + p_{\text{dm}^*} = p_{e^-} + p_{e^+} + p_\gamma$ and $s = (p_{\text{dm}} + p_{\text{dm}^*})^2$. Due to energy momentum conservation, x, \bar{x} , and x_γ are not independent of each other:

$$2 = x + \bar{x} + x_\gamma. \quad (3)$$

In terms of x, \bar{x} and x_γ , the scalar products between final state momenta read:

$$p_{e^+} \cdot p_\gamma = \frac{s}{2}(1-x), \quad (4)$$

$$p_{e^-} \cdot p_\gamma = \frac{s}{2}(1-\bar{x}), \quad (5)$$

$$p_{e^-} \cdot p_{e^+} = \frac{s}{2}(1-x_\gamma-2z), \quad (6)$$

where $z = m_e^2/s$ and m_e is the electron mass. In the case of dark matter it is legitimate to assume that it is almost at rest when it annihilates. At Earth position, the dark matter velocity dispersion is about three orders of magnitude smaller than the speed of light ($v \sim 10^{-3}c$). It is even smaller below 1 kpc (say, inside the galactic centre). Thus any scalar product that involves the four-momenta of both initial and final state particles can be expressed in a very simple way, without involving any undefined angles. In particular we obtain:

$$p_{\text{dm}} \cdot p_{e^+} \approx \frac{s}{4}\bar{x}, \quad (7)$$

$$p_{\text{dm}} \cdot p_{e^-} \approx \frac{s}{4}x, \quad (8)$$

$$p_\gamma \cdot p_{\text{dm}} \approx \frac{s}{4}x_\gamma. \quad (9)$$

A. The annihilation cross section

For later use we reproduce here the annihilation cross section associated with the process

$$S(p_{\text{dm}}) + S^*(p_{\text{dm}^*}) \rightarrow e^-(p_{e^-}) + e^+(p_{e^+}) \quad (10)$$

at tree-level [27]. Neglecting the P-wave annihilation cross section because of the suppression of the dark matter velocity in the 511 keV region, we find that the corresponding cross section σ_0 is given by:

$$\begin{aligned} \sigma_0 &= \frac{1}{16\pi} \frac{1}{s} \frac{\beta_e}{\beta_i} |M_0|^2 \\ &= \frac{1}{32\pi} \frac{\beta_e^3}{\beta_i} \frac{(2c_r c_l m_F + m_e(c_l^2 + c_r^2))^2}{(m_F^2 + m_{\text{dm}}^2 - m_e^2)^2}, \end{aligned} \quad (11)$$

with

$$\beta_e = \sqrt{1-4z} \quad (12)$$

and

$$\beta_i = \sqrt{1 - \frac{4m_{\text{dm}}^2}{s}}, \quad (13)$$

where m_{dm} denotes the mass of the annihilating scalar and m_F the mass of the heavy charged fermion that is exchanged in the t -channel. From inspection of Eq. (11), one can easily see that in general it is not appropriate to neglect the electron mass. If the ratio between the chiral couplings c_l, c_r is of the order of m_e/m_F then the term that is formally suppressed through the electron mass has the same importance as the term in m_F . Such a scenario arises naturally in 2-Higgs doublet models with flavour changing neutral couplings. I.e. the lagrangian

$$\mathcal{L} = -g_l \frac{m_e}{v} \bar{e}_R F_L S - g_r \frac{m_F}{v} \bar{F}_R e_L S + h.c. \quad (14)$$

leads immediately to $\frac{c_l}{c_r} = \frac{m_e}{m_F}$ if we identify $c_l = g_l \frac{m_e}{v}$ and $c_r = g_r \frac{m_F}{v}$. Such a case does not fit SPI data unless one considers extremely large couplings of a few units. Since the particle F is charged, the mass m_F must be larger than 100 GeV, given LEP limits [34] restricting the range of the couplings that can be considered.

In the limit where m_{dm} and $m_e \ll m_F$, the cross section becomes:

$$\sigma_0 = \frac{1}{32\pi} \frac{\beta_e^3}{\beta_i} \frac{1}{m_F^2} \kappa(c_l, c_r, m_e, m_{\text{dm}}, m_F) + O\left(\frac{1}{m_F^5}\right), \quad (15)$$

with

$$\begin{aligned} \kappa(c_l, c_r, m_e, m_{\text{dm}}, m_F) &= \left(2c_l c_r + (c_l^2 + c_r^2) \frac{m_e}{m_F}\right)^2 \\ &+ 8c_r^2 c_l^2 \frac{m_e^2 - m_{\text{dm}}^2}{m_F^2}. \end{aligned} \quad (16)$$

Keeping only the leading term in the $1/m_F$ expansion, we obtain for σ_0

$$\frac{c_l^2 c_r^2 \beta_e^3}{8\pi \beta_i m_F^2} + O\left(\frac{1}{m_F^3}\right). \quad (17)$$

To compute the annihilation rate and therefore the flux of positrons and/or photons that are emitted during the dark matter annihilations, one must consider $\sigma_0 v_r$ (where v_r is the dark matter relative velocity) instead of σ_0 . v_r is twice the dark matter velocity in the centre-of-mass frame (written here as β_i). We therefore recover the results of Ref. [27].

B. Bremsstrahlung cross section

1. General formulae

In terms of the squared matrix element M_γ , the single differential bremsstrahlung cross section reads

$$\frac{d\sigma_\gamma}{dx_\gamma} = \frac{1}{256\pi^3 \beta_i} \int_{x_-}^{x_+} dx |M_\gamma|^2, \quad (18)$$

where the bounds x_-, x_+ are given by

$$x_{\pm} = \frac{1}{2} \left(2 - x_{\gamma} \pm x_{\gamma} \sqrt{1 - \frac{4z}{1 - x_{\gamma}}} \right). \quad (19)$$

The squared matrix element is obtained from the evaluation of the Feynman diagrams shown in Fig. 1. Since there are many different mass scales, the complete result – keeping the full mass dependence – is rather lengthy. We find:

$$\begin{aligned} |M_{\gamma}|^2 = & \frac{-e^2}{\mathcal{E}} \left\{ m_F^4 \mathcal{A} \left[\left(2c_l c_r m_F + (c_l^2 + c_r^2) m_e \right)^2 \right. \right. \\ & \left. \left. - 8c_l^2 c_r^2 \left((2x_{\gamma} - 1) m_{\text{dm}}^2 + m_e^2 \right) \right] \right. \\ & + 8c_l c_r (c_l^2 + c_r^2) m_e m_F^3 \mathcal{B} \\ & + 2m_F^2 \left(m_e^2 (c_l^4 + c_r^4) \mathcal{C} + 2c_l^2 c_r^2 m_{\text{dm}}^2 \mathcal{D} \right) \\ & - 4c_l c_r (c_l^2 + c_r^2) m_e m_F \left(m_e^2 + m_{\text{dm}}^2 (2x_{\gamma} - 1) \right) \mathcal{C} \\ & \left. + \mathcal{N} \right\}, \quad (20) \end{aligned}$$

where e denotes the electric charge and where the explicit results for the functions $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{N}$ are given in the appendix. We note that in addition to the symmetry in c_l, c_r , there is also a symmetry in $x \leftrightarrow \bar{x}$, when using \bar{x} instead of x_{γ} . Furthermore we have checked that we reproduce the correct factorization when the emitted photon becomes soft:

$$\begin{aligned} |M_{\gamma}|^2 \xrightarrow{x_{\gamma} \rightarrow 0} & e^2 \left(\frac{2(p_{e^-} \cdot p_{e^+})}{(p_{e^-} \cdot p_{\gamma})(p_{e^+} \cdot p_{\gamma})} - \frac{m_e^2}{(p_{e^-} \cdot p_{\gamma})^2} - \frac{m_e^2}{(p_{e^+} \cdot p_{\gamma})^2} \right) |M_0|^2. \quad (21) \end{aligned}$$

For arbitrary hard photons no such factorization can in general be observed.

There are many extra terms in the squared amplitude with respect to that of the annihilation. Some are particularly relevant when $m_F/2 \lesssim m_{\text{dm}} \lesssim m_F$ (e.g. $\propto x_{\gamma} \mathcal{A}, \mathcal{D}, \mathcal{N}$) or in the case of very large couplings, i.e. $(c_l^2 + c_r^2) m_e \gtrsim 2c_l c_r m_F$.

Hence from Eq. (20) one readily sees that – unless one makes simplifying assumptions – the cross section for the emission of an additional photon cannot be written as the tree-level cross section for a generalized kinematics times a factor related to the soft photon emission.

The integration of the squared matrix element $|M_{\gamma}|^2$ over x to obtain the photon spectrum is straightforward. In the following we only present analytic results in the approximation that the dark matter mass is small with respect to the mass of the particle F .

2. Results in the limit $m_F \gg (m_{\text{dm}}, m_e)$

Expanding Eq. 20 in terms of $1/m_F$ and keeping the dominant terms up to $1/m_F^4$, we obtain:

$$|M_{\gamma}|^2 = \frac{e^2 \kappa(c_l, c_r, s, m_e, m_F)}{s^2 m_F^2}$$

$$\begin{aligned} & \times \left\{ 8m_e^2 (m_e^2 - m_{\text{dm}}^2) \left(\frac{1}{(1 - x - x_{\gamma})^2} + \frac{1}{(1 - x)^2} \right) \right. \\ & - \frac{16}{x_{\gamma}} \left(m_e^4 - (3 - 2x_{\gamma}) m_{\text{dm}}^2 m_e^2 + (x_{\gamma}^2 - 2x_{\gamma} + 2) m_{\text{dm}}^4 \right) \\ & \left. \times \left(\frac{1}{1 - x - x_{\gamma}} - \frac{1}{1 - x} \right) \right\} + O\left(\frac{1}{m_F^5}\right). \quad (22) \end{aligned}$$

Integrating over x we obtain the photon spectrum:

$$\begin{aligned} \frac{d\sigma_{\gamma}}{dx_{\gamma}} = & \sigma_0 \frac{\alpha}{\pi} \frac{1}{\beta_e^3} \frac{1}{s^2} \\ & \times \left\{ 2s(-s + 4m_e^2) \frac{1}{x_{\gamma}} \sqrt{(1 - 4z - x_{\gamma})(1 - x_{\gamma})} \right. \\ & + \frac{16}{x_{\gamma}} \left(m_e^4 - (3 - 2x_{\gamma}) m_{\text{dm}}^2 m_e^2 + (x_{\gamma}^2 - 2x_{\gamma} + 2) m_{\text{dm}}^4 \right) \\ & \left. \times \ln\left(\frac{1 - x_-}{1 - x_+}\right) \right\} + O\left(\frac{1}{m_F^5}\right). \quad (23) \end{aligned}$$

Keeping the electron mass only in the logarithm so as to regularize the mass singularity, the above result can be simplified further:

$$\frac{d\sigma_{\gamma}}{dx_{\gamma}} \approx \sigma_0 \frac{\alpha}{\pi} \frac{1}{x_{\gamma}} \left\{ \left(1 + \frac{s'^2}{s^2} \right) \ln\left(\frac{s'}{m_e^2}\right) - 2\frac{s'}{s} \right\}, \quad (24)$$

where we introduced

$$s' = (p_{e^-} + p_{e^+})^2 = s(1 - x_{\gamma}). \quad (25)$$

For $e^+ + e^- \rightarrow \mu^+ + \mu^- + \gamma$, keeping only the final-state radiation, the formula is given by [33]:

$$\begin{aligned} \frac{d\sigma_{e^+e^- \rightarrow \mu^+\mu^- + \gamma}}{dx_{\gamma}} = & \sigma_{e^+e^- \rightarrow \mu^+\mu^-} \\ & \times \frac{\alpha}{\pi} \frac{1}{x_{\gamma}} \left(1 + \frac{s'^2}{s^2} \right) \left(\ln\left(\frac{s'}{m_{\mu}^2}\right) - 1 \right) \quad (26) \end{aligned}$$

Comparing Eq. (24) and Eq. (26), it is easy to see that the bremsstrahlung cross section associated with the annihilation of two scalars cannot be obtained from the final-state radiation in muon pair production. However, in the soft photon limit ($s' \rightarrow s$) the two results agree after the identification $m_{\mu} \rightarrow m_e$, as it must be. We are thus led to the conclusion that the bremsstrahlung correction to the annihilation of two scalars cannot be evaluated as it was done in Ref. [1].

III. DISCUSSION AND UPPER LIMIT ON THE DARK MATTER MASS

In the upper plot in Figure 2, we show the Bremsstrahlung cross section as given in Ref. [1] (dashed lines) versus our expression (dotted lines) for 2, 20, 30 and 100 MeV. All our plots are normalized to $\sigma_0 \frac{\alpha}{\pi} \frac{1}{x_{\gamma}}$. To show more explicitly the difference between these two expressions, we show in the lower

plot the ratio:

$$R = \frac{\left(\frac{d\sigma}{dx_\gamma}\right)_{\text{Eq. 24}}}{\left(\frac{d\sigma}{dx_\gamma}\right)_{\text{Ref. [1]}}}$$

for 1, 10 and 100 MeV.

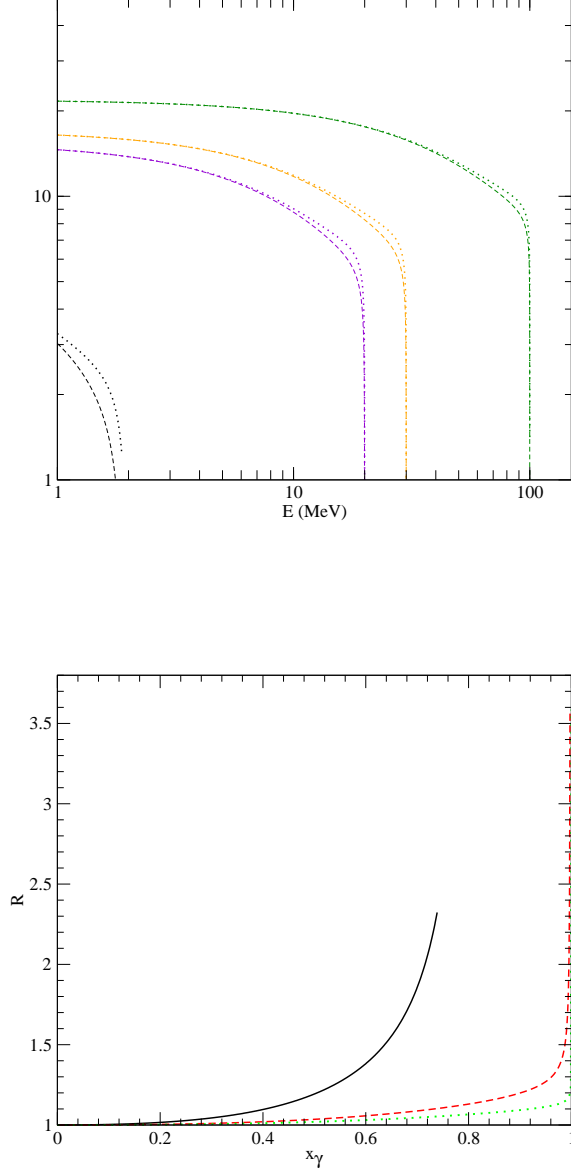


FIG. 2: Upper plot: our Bremsstrahlung cross section (dotted lines) versus the Bremsstrahlung cross section used in Ref. [1] (dashed lines) for 2, 20, 30 and 100 MeV. Lower plot: ratio of the two expressions for 1, 10 and 100 MeV.

The lower plot in Figure 2 shows that the result obtained in this paper is up to a factor 3.6 larger than the result obtained

in Ref. [1]. For maximal photon energy ($x_\gamma = 1 - 4z$), we obtain:

$$R|_{x_\gamma=1-4z} = 1 + \frac{(1-4z)^2}{(1+16z^2)(\ln(4)-1)}. \quad (27)$$

In the limit $z \rightarrow 0$, Eq. 27 tends to 3.6. Although the difference between Ref. [1] and the results presented in this letter becomes larger for large m_{dm} , the phenomenological relevance might be larger for smaller m_{dm} . For large m_{dm} , the ratio is peaked at the phase-space boundary in contrast with small m_{dm} where a larger phase space region is affected.

The upper limit on the dark matter mass comes from the hard photon region. Fig. 3 shows the expected flux of Bremsstrahlung photons for dark matter particles with a mass of 20, 30 and 100 MeV and for $|b| < 5^\circ$, $|l| < 5^\circ$ (dashed lines) and $|b| < 5^\circ$, $|l| < 30^\circ$ (solid lines) with b the latitude and l the longitude. The expected flux in Ref. [1] is represented by the dotted lines for $m_{\text{dm}} = 30$ MeV.

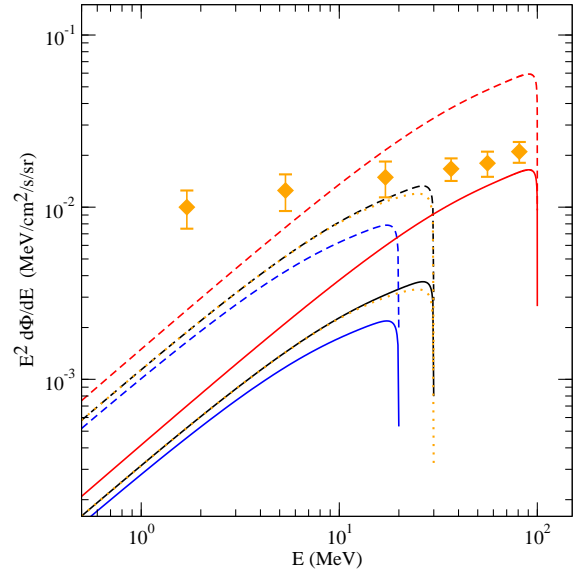


FIG. 3: Bremsstrahlung flux for 20, 30 and 100 MeV, in the limit ($m_{\text{dm}} \ll m_F$), for $b < 5^\circ$ and $l < 5^\circ$ (dashed lines) and $b < 5^\circ$ and $l < 30^\circ$ (solid lines). The data are represented by diamonds and the result from Ref. [1] is plotted for comparison (dotted lines).

Despite the sizeable difference in the hard photon limit the upper limit on the dark matter mass is not changed significantly due to the limited precision of the data. We obtain the same result as in [30], i.e. $m_{\text{dm}} < 30$ or 100 MeV, depending on whether one uses SPI data from regions defined by $|b| < 5^\circ$ and $|l| < 5^\circ$ (for which the 511 keV line intensity is about 0.018 ph/cm²/s) or $|l| < 30^\circ$ (for which the intensity is 0.005 ph/cm²/s) respectively.

Many papers, e.g. Refs. [35, 36, 37] use the expression that is given in [1]. It might be worth checking whether their results remain unchanged using the exact formula presented in this work.

IV. CONCLUSIONS

In light of Ref. [1], we computed the Bremsstrahlung emission associated with the annihilation of spin-0 particle dark matter particles and compared it with the result for $e^+e^- \rightarrow \mu^+\mu^-\gamma$. We find that, in general, one cannot write the Bremsstrahlung cross section as the tree-level annihilation cross section times a factor related to soft photon emission. In the limit where the mass of the new charged fermion F (which is exchanged in the t -channel) is much heavier than the dark matter mass m_{dm} and the electron mass m_e , we observe a factorization similar to that assumed by Beacom et al. However, the factor we find from the explicit calculation is different from that given in Ref. [1]. We thus conclude that the bremsstrahlung cross section associated with the annihilation of spin-0 particles into an electron-positron pair cannot be obtained from final-state radiation in $e^+ + e^- \rightarrow \mu^+ + \mu^-$.

The difference between Ref. [1] and our result is sizeable. However, the analysis presented in Ref. [30] remains valid. For $b < 5^\circ$ and $l < 5^\circ$, we find that the upper limit on the dark matter mass is about 30 MeV. For $b < 5^\circ$ and $l < 30^\circ$, it is rather 100 MeV.

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Appendix

The squared amplitude for the Bremsstrahlung process is given by:

$$|M_\gamma|^2 = \frac{|M_{e^-}|^2}{D_{e^-}^2} + \frac{|M_{e^+}|^2}{D_{e^+}^2} + \frac{|M_F|^2}{D_F^2} + 2 \frac{\text{Re}(M_{e^-} M_{e^+}^*)}{D_{e^-} D_{e^+}} + 2 \frac{\text{Re}(M_F M_{e^-}^*)}{D_F D_{e^-}} + 2 \frac{\text{Re}(M_F M_{e^+}^*)}{D_F D_{e^+}}, \quad (28)$$

where $M_i = e\mathcal{E}^{*\mu} \bar{u}_e M_i v_e$ and

$$M_{e^-} = \gamma_\mu (\not{p}_\gamma + \not{p}_{e^-} + m_e) (c_l P_l + c_r P_R) \times (\not{p}_{e^-} - \not{p}_\gamma - \not{p}_{\text{dm}} + m_F) (c_l P_R + c_r P_l), \quad (29)$$

$$M_{e^+} = (c_l P_l + c_r P_R) (\not{p}_{e^+} - \not{p}_{\text{dm}} + m_F) \times (c_l P_R + c_r P_l) (-\not{p}_\gamma - \not{p}_{e^+} + m_e) \gamma_\mu, \quad (30)$$

$$M_F = (c_l P_l + c_r P_R) (\not{p}_{e^-} - \not{p}_{\text{dm}} + m_F) \times \gamma_\mu (\not{p}_{\text{dm}^*} - \not{p}_{e^+} + m_F) (c_l P_R + c_r P_l), \quad (31)$$

with

$$D_{e^-} = \left((p_\gamma - p_{\text{dm}} + p_{e^-})^2 - m_F^2 \right) \left(2p_\gamma \cdot p_{e^-} \right),$$

$$D_{e^+} = \left((p_{e^+} - p_{\text{dm}})^2 - m_F^2 \right) \left(2p_\gamma \cdot p_{e^+} \right),$$

$$D_F = \left((p_{e^-} - p_{\text{dm}})^2 - m_F^2 \right) \left((p_{e^+} - p_{\text{dm}^*})^2 - m_F^2 \right).$$

$$\mathcal{A} = -m_e^4 x_\gamma^2 - 2m_{\text{dm}}^4 (1-x)(1-\bar{x})(2-2x_\gamma+x_\gamma^2) + m_{\text{dm}}^2 m_e^2 [4x\bar{x}(1-x_\gamma) - (2-x_\gamma)(2-3x_\gamma)],$$

$$\mathcal{B} = m_e^6 x_\gamma^2 - 2m_{\text{dm}}^2 m_e^4 (1-x_\gamma)(2x\bar{x} - 2 + 2x_\gamma + x_\gamma^2) + m_{\text{dm}}^4 m_e^2 [-8 + 24x_\gamma - 26x_\gamma^2 + 9x_\gamma^3 - x\bar{x}(-8 + 16x_\gamma - 11x_\gamma^2)] + m_{\text{dm}}^6 [-4x^4(1-x_\gamma) + 8x^3(2-3x_\gamma+x_\gamma^2) + x^2(-20 + 36x_\gamma - 18x_\gamma^2 + x_\gamma^3) + x_\gamma(-4 + 10x_\gamma - 9x_\gamma^2 + 3x_\gamma^3) + x(8 - 12x_\gamma + 8x_\gamma^3 - 3x_\gamma^4)],$$

$$\mathcal{C} = m_e^6 x_\gamma^2 - 2m_{\text{dm}}^2 m_e^4 (1-x_\gamma) [-2 - 2x^2 + 2x(2-x_\gamma) + 2x_\gamma + x_\gamma^2] + 2m_{\text{dm}}^6 (1-x)(1-x_\gamma) [-2 + 4x^3 + 2x_\gamma + x_\gamma^2 - x_\gamma^3 + 4x^2(-3 + 2x_\gamma) + x(10 - 12x_\gamma + 3x_\gamma^2)] + m_{\text{dm}}^4 m_e^2 [-8 + 24x_\gamma - 27x_\gamma^2 + 10x_\gamma^3 - 4x^2(2 - 4x_\gamma + 3x_\gamma^2) - 4x(-4 + 10x_\gamma - 10x_\gamma^2 + 3x_\gamma^3)],$$

$$\mathcal{D} = m_e^6 (1-2x_\gamma)x_\gamma^2 + 2m_{\text{dm}}^6 (1-x)(1-2x_\gamma)^2(1-x-x_\gamma)(2-2x_\gamma+x_\gamma^2) + 2m_{\text{dm}}^2 m_e^4 [2 - 8x_\gamma + 8x_\gamma^2 - 2x_\gamma^4 + x^2(2 - 6x_\gamma + 3x_\gamma^2) + x(-4 + 14x_\gamma - 12x_\gamma^2 + 3x_\gamma^3)] + m_{\text{dm}}^4 m_e^2 [-16 - 8x^4(1-x_\gamma) + 64x_\gamma - 97x_\gamma^2 + 68x_\gamma^3 - 18x_\gamma^4 + 16x^3(2 - 3x_\gamma + x_\gamma^2) + 2x^2(-28 + 64x_\gamma - 49x_\gamma^2 + 15x_\gamma^3) + 2x(24 - 76x_\gamma + 90x_\gamma^2 - 51x_\gamma^3 + 11x_\gamma^4)],$$

$$\mathcal{E} = 2m_{\text{dm}}^4 [(1-2x)m_{\text{dm}}^2 + m_e^2 - m_F^2]^2 (1-x)^2 \times [(1-2\bar{x})m_{\text{dm}}^2 + m_e^2 - m_F^2]^2 (1-\bar{x})^2,$$

$$\mathcal{N} = -2c_l^2 c_r^2 m_e^2 [m_e^2 + m_{\text{dm}}^2(1-2x)] \times [m_e^2 + m_{\text{dm}}^2(1-2\bar{x})] \mathcal{R} + (c_l^4 + c_r^4) \mathcal{V}$$

$$\mathcal{R} = m_e^4 x_\gamma^2 + 2m_{\text{dm}}^4 (1-x)(1-\bar{x}) [2 + x_\gamma(-6 + 5x_\gamma)] + m_{\text{dm}}^2 m_e^2 [4 - 4x\bar{x}(1-x_\gamma) + x_\gamma(-8 + x_\gamma(3 + 2x_\gamma))],$$

$$\mathcal{V} = -m_e^{10} x_\gamma^2 - 2m_{\text{dm}}^8 m_e^2 (1-x)(1-\bar{x})(1-2x_\gamma) \times [-6 + 8x\bar{x}(1-x_\gamma) + (10 - 3x_\gamma)x_\gamma] - 16m_{\text{dm}}^{10} (1-x)^2(1-\bar{x})^2(1-x_\gamma) [(1-x)^2 + (1-\bar{x})^2] + m_{\text{dm}}^2 m_e^8 [-4 + 4x\bar{x}(1-x_\gamma) - x_\gamma(-8 + x_\gamma + 4x_\gamma^2)] + m_{\text{dm}}^4 m_e^6 [12 - 2x\bar{x}(6 + x_\gamma(-14 + 11x_\gamma))]$$

$$\begin{aligned}
& - x_\gamma \left(40 + x_\gamma (-47 + 2x_\gamma(7 + 2x_\gamma)) \right) \\
& + m_{\text{dm}}^6 m_e^4 \left[4 - 16x^4(-1 + x_\gamma) - 32x^3(2 - x_\gamma)(1 - x_\gamma) \right. \\
& \left. + x_\gamma \left(8 + x_\gamma(-51 + 8(8 - 3x_\gamma)x_\gamma) \right) \right. \\
& \left. + 4x^2 \left(21 + x_\gamma(-37 + x_\gamma(14 + 3x_\gamma)) \right) \right. \\
& \left. - 4x(2 - x_\gamma) \left(5 + x_\gamma(-5 + x_\gamma(-6 + 7x_\gamma)) \right) \right]. \quad (32)
\end{aligned}$$

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