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A POSSIBLE TYPE OF ELECTROSTATIC LENSES

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1. INTRODUCTION

The multipole coefficients of a beam can be measured directly by means of suitably shaped electrodes cut in the walls of a chamber of arbitrary cross section. Under certain restrictive conditions these same electrodes can be used as electrostatic lenses of any chosen multipolarity.

2. GENERAL DERIVATION OF EFFECTIVE FIELD

Using the notations of Fig. 1 we make the following assumptions:

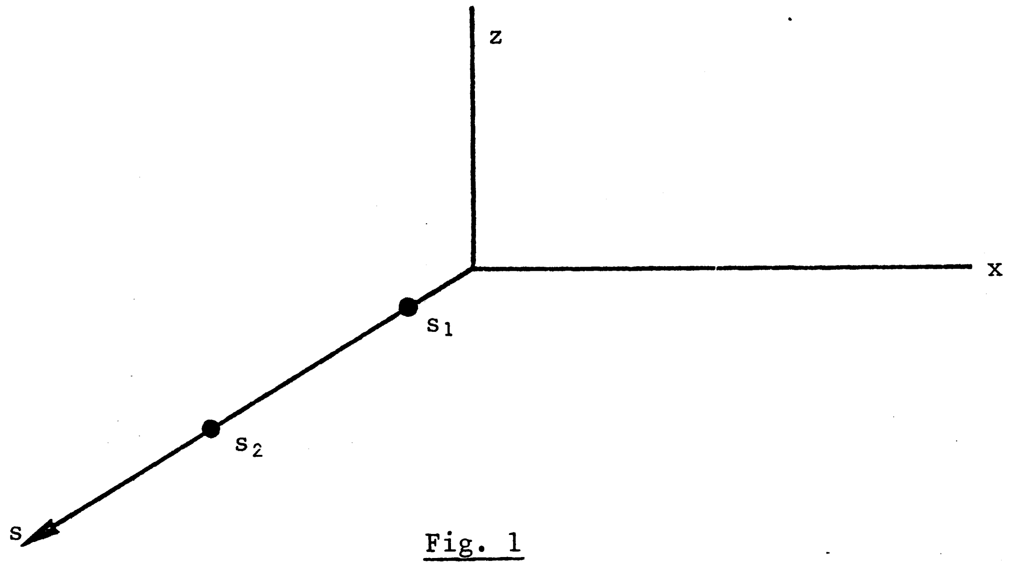

Fig. 1

a) Chamber cross section independent of s,

b) Potentials on electrodes very small compared to that corresponding to the energy of the particles,

c) Particle trajectories essentially parallel to the axis of the vacuum chamber,

d) The electric fields are limited to the region extending from s_1 to s_2.

The x component of the transverse impulse received by a particle of velocity v

$$\Delta p_x = e \int_{t_1}^{t_2} E_x(x,z,s)\,dt$$

becomes under our restrictions:

$$\Delta p_x = \frac{e}{v} \int_{s_1}^{s_2} E_x(x,z,s)\,ds = \frac{e}{v}\, \ell\, \overline{E_x}(x,z)$$

where $\ell = s_2 - s_1$ and x,z are constant for a given particle. The dependence of $\overline{E_x}$ on x determines the type of effective field, as seen by the particle.

$\overline{E_x}$ = const. dipole field

$\overline{E_x} = Kx$ quadrupole field

....

$\overline{E_x} = Kx^{n-1}$ 2n pole field

We suppose that we have an electrode system that measures a_n, the x component of the multipole coefficient of order n, as shown in Fig. 2.

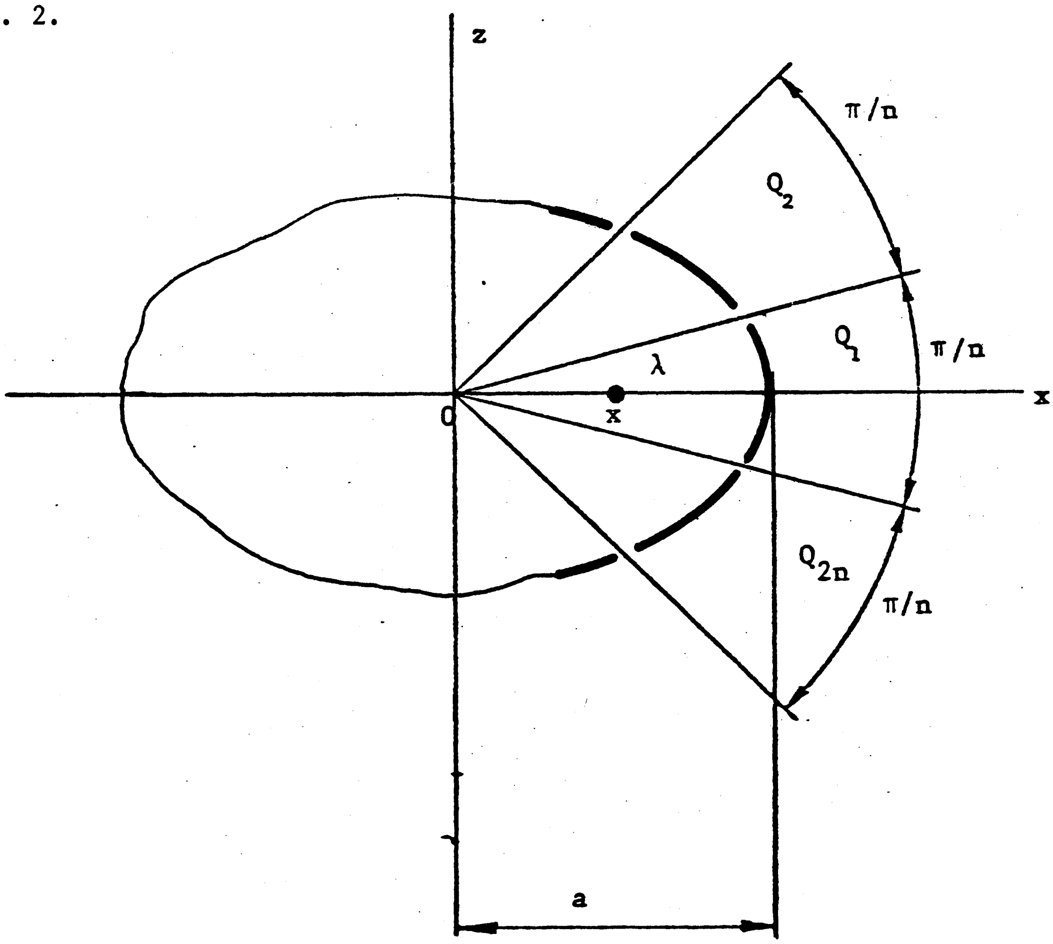

Fig. 2

We have[1])

$$a_n = \frac{1}{n}\frac{a^n}{L}\sum_1^{2n} (-1)^{i+1} Q_i$$

where a = chamber radius along x axis

L = length of electrode at (a,o)

Q_i = charge induced on individual electrode.

We now imagine that these electrodes are raised to the potentials $V_i = (-1)^{i+1} V$ and calculate the work done by moving a line charge λ from the origin to (x,o).

The work done against the field is

$$W_1 = \int_o^x \int_{s_1}^{s_2} \lambda E_x(x,o,s)\,ds\,dx = \lambda \ell \int_o^x \overline{E_x}(x,o)\,dx$$

The work done by the generators supplying the electrodes will be

$$W_2 = \sum_1^{2n} (Q_{ix} - Q_{io})V_i$$

$$= V\left[\sum_1^{2n} (-1)^{i+1} Q_{ix} - \sum_1^{2n} (-1)^{i+1} Q_{io}\right]$$

$$= \frac{nL}{a^n}(a_{nx} - a_{no})V$$

For a line charge λ

$$a_{nx} = \frac{1}{n}\lambda\, x^n \qquad a_{no} = 0$$

so that

$$W_2 = \frac{L}{a^n}\lambda\, V\, x^n \,.$$

Equating W_1 and W_2 and differentiating with respect to x we find

$$\ell \overline{E_x} (x,o) = \frac{nL\ V}{a^n} x^{n-1}$$

i.e. the radial impulse experienced by a particle traversing a structure that measures the multipole coefficient of order n is proportional to x^{n-1}.

3. EXAMPLES

3.1 For n = 1

$$\ell \overline{E_x} = \frac{VL}{a}$$

we have a uniform deflecting field. This case has recently been discussed by Goldin and Skachkov[2)], based on the "straight cut" type of position monitor. The various generalisations of Ref. 1 are also applicable here. A possible form of combined horizontal and vertical deflecting system for a circular chamber is illustrated in Fig. 3.

3.2 For n = 2

$$\ell \overline{E_x} = \frac{2VL}{a^2} x$$

we have a quadrupole lens, one possible form of which is shown in Fig. 4. It must of course be remembered that in order for the behaviour to be strictly correct, the space between the electrodes should be filled by the continuation of the chamber wall.

For n = 3, 4, ... higher order lenses can be obtained in a similar manner.

4. CONCLUSION

One must note that unlike their classical counterparts, lenses of this type will only function correctly if the particle trajectories are parallel to the axis.

For weak lenses where this condition is satisfied and where a large area of good field relative to chamber dimension is required, these devices could be of interest.

ACKNOWLEDGEMENTS

I am indebted to K.H. Reich for drawing my attention to the work of L. Goldin and S. Skachkov.

REFERENCES

1. G. Nassibian, The measurement of the multipole coefficients of a cylindrical charge distribution, SI/Note EL/70-13.

2. L.L. Goldin, S.V. Skachkov, Bending condensators for beams of charged particles, Report No. 651 of the Institute for Theoretical and Experimental Physics, Moscow (1969).

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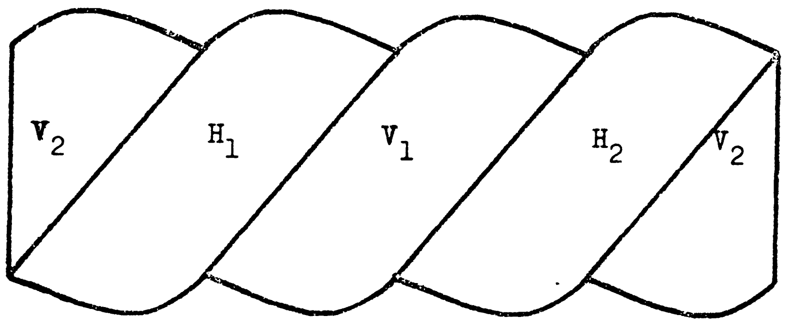
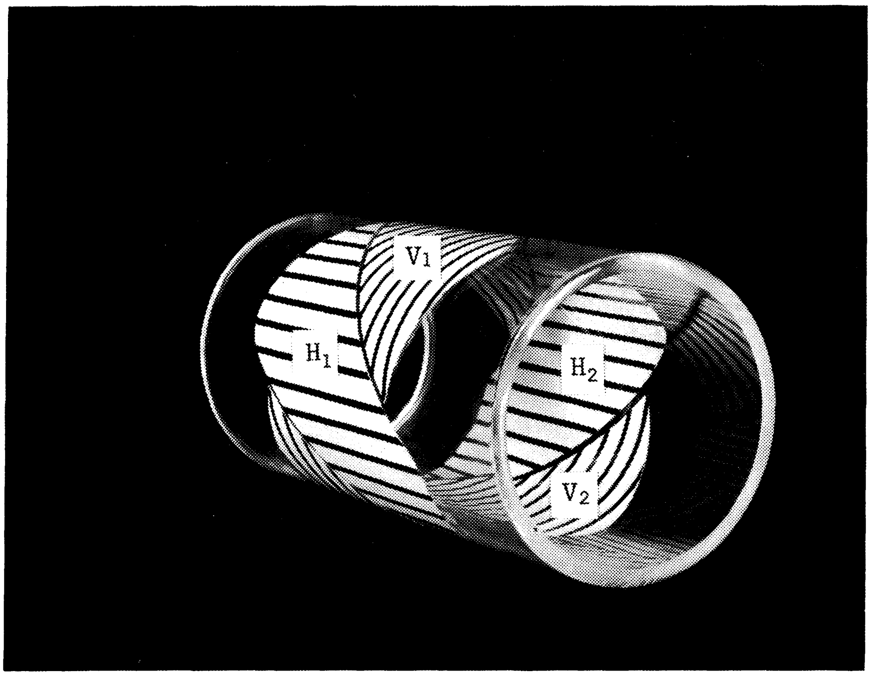


a)



b)

Fig. 3

Combined horizontal and vertical deflecting system

a) Development
b) Photograph of model

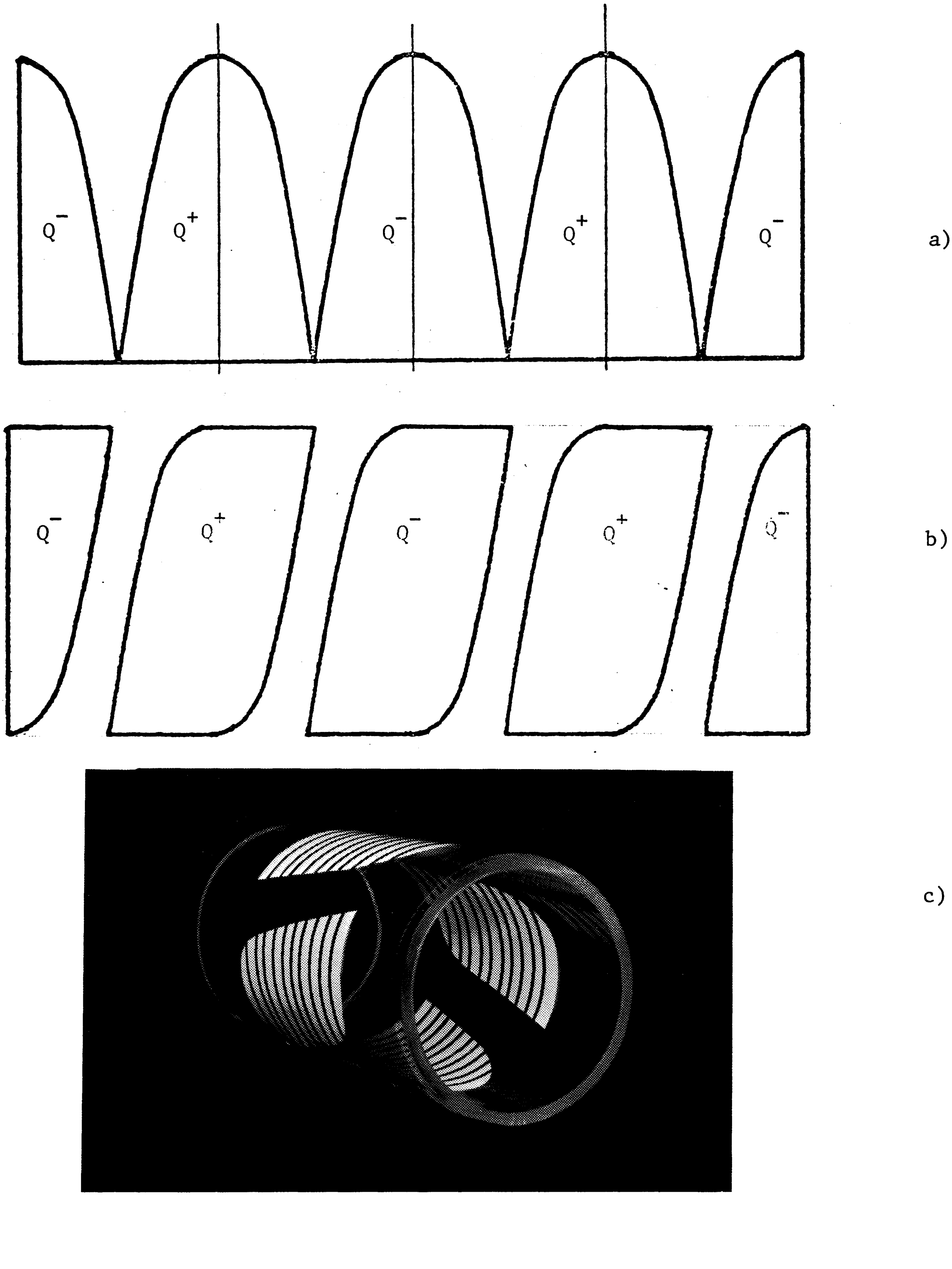


Fig. 4

Quadrupole lens

a) Development of elementary form
b) Alternative arrangement with improved voltage holding possibilities
c) Model of arrangement b)