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# Probing Brane-World Scenarios with Vacuum Refraction of Light Using Gamma-Ray Bursts

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## Abstract

We argue that in fat brane-world scenarios the light propagating in vacuum will, because of massive “Kaluza–Klein” (KK) excitations, experience a refraction. The motion of a photon inside a fat brane can be decomposed in the longitudinal and transverse directions with respect to the surface of the brane. Since the light observable propagation is related only with the longitudinal motion, the observed speed of light depends on the value of the momentum transverse fraction contributing as the massive KK excitations. This is directly connected with the energy of the particles emitting the light, and hence with the frequency of the light itself. Using recent results on the arrival times of radiation of different energies from the measurements of gamma-ray bursters with known redshifts, we establish the limit  $M > 620$  TeV on the inverse thickness of the brane, and thus on the masses of the KK excitations. This limit exceeds by at least one order of magnitude the typical energy scale currently in use to characterize brane phenomena in the realm of future colliders.

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# 1 Introduction

The constancy of the velocity of light is one of the tenets of modern physics. In particular, the special and general theories of relativity postulate a single universal velocity of light. However, there is a general expectation of a need, in many theoretical efforts to find a synthesis of general relativity and quantum mechanics [1], for great sophistication in the discussion of the propagation of light in vacuum.

There are several theoretical models that consider the modification of electrodynamics at high energies. A hypothetical Lorentz symmetry deformation, considered as an explanation of the observed ultra-high energy cosmic rays (and possibly neutrino oscillations), may introduce an energy-dependent photon mass that would thus change the speed of light at high energies [2]. Changes of the photon speed are also predicted in quantum gravity theories [3]. Within these approaches, the propagation of light in modified space-time exhibits a non-trivial dispersion relation in vacuum, corresponding to Lorentz violation through an energy-dependent velocity of light. It was also pointed out [3] that one powerful way to probe this possibility may be provided by some distant astrophysical sources of energetic photons that exhibit significant and rapid variations in time, such as gamma-ray bursters (GRBs). However, in quantum-gravity models, this modification is expected to be significant at photon energies close to the Planck scale.

In the meantime, in brane-world scenarios [4], the energy dependence of the photons' speed can appear at lower energies, since the brane scale is assumed to be much smaller than Planck's. Taking this advantage, in the present paper we discuss the modification of the velocity of light with respect to the energy of a photon in the framework of such brane-world scenarios setups. In particular, we consider the propagation of photons on a fat brane [5], with Standard Model (SM) particles localized gravitationally on the brane. Because of the relatively low scale ( $\mathcal{O}(1)$  TeV) of the massive extra-dimensional "Kaluza-Klein" (KK) excitations currently under analysis [6–8] for the experimental signatures of brane-world scenarios, it is expected that there is a significant delay of the arrival time of lower-energy photons relative to the higher-energy ones, propagating through large cosmological distances from remote sources.

Similar effects have been proposed in quantum-gravity models [3] which have recently been studied in detail in [9,10], where the robust limit on the violation of Lorentz invariance has been established.

In this paper, we first consider the photon propagation on the brane and derive the dependence of the observed longitudinal velocity on the mass of the KK excitations, and then find the relation with experimental measurements through the light refractive index. The method elaborated in [9,10], using wavelet techniques, is applied to the available data on GRBs to estimate the limit on the mass of the KK excitations. This estimate is based on the analysis of the correlation with the redshift of the time-lags between the arrival times of sharp transients in GRB light curves, observed in higher- and lower-energy bands, due to the photon propagation in the expanding brane-world Universe. We must stress that a set of GRBs is studied, and not a single source, since, as was argued in [9–11], one cannot rely on a single GRB [12], for which it would be impossible to distinguish between

intrinsic time-lag at the origin and a delay induced by the propagation in vacuum. This is particularly important if the observation of the source is uncertain in a crucial energy band, and this is why such an estimate may be misleading. A large sample of 35 GRBs, with different redshifts and in measured different energy bands by BATSE, HETE and SWIFT space instruments, is systematically analysed. Using the most robust spectral features of the GRBs light curves, we estimate the 95% C.L. lower limit on the energy scale of the effective refractive index, and thus on the inverse thickness of the brane, or on the mass scale of the KK excitations.

## 2 Photon on the brane

The brane-world idea [4] provides possible new solutions to such long-standing problems of particle physics beyond the SM and cosmology as the hierarchy problem, the smallness of the cosmological constant, the nature of flavour, the hierarchy of fermion masses and mixings, etc. The key ingredient of the brane-world scenarios is that Newtonian gravity and the SM fields are constrained to be localized on a brane in  $n$  extra dimensions, while the gravitons, and perhaps other SM particles, are free to propagate in the bulk of extra dimensions. Since the gravity has the unique feature to be universally coupled with all kinds of SM fields, the most economic way to trap these fields on the brane would be to consider models with a mechanism of purely gravitational localization. Such models, with gravitational trapping of zero modes of all SM fields, including photons, have been investigated in [13].

In this paper for simplicity in this paper we limit ourselves to a construction with only one extra dimension, where the matter fields are trapped by some potential well within a fat brane of width  $\epsilon = M^{-1}$  along the extra (space-like) coordinate  $z$ . Since the SM gauge fields can only propagate inside the brane, the  $\epsilon$  effectively acts as the size of the extra dimension for them. Therefore, at energies above  $M$ , the KK excitations (the higher harmonics of a particle in a box) of the gauge fields can be produced. The current bound, owing to the lack of capability to excite the KK modes in different reactions, is close to  $\mathcal{O}(1)$  TeV, as imposed by LEP [8].

In the following we do not consider any specific case of the trapping potential on the brane, while just introducing some general assumptions about the structure of space-time. To be able, in principle, to localize the Newtonian gravity on the brane, we consider the metric to have the standard structure with a warp factor. In the other words, the 4-dimensional part of the multidimensional metric depends conformally upon  $z$ . To avoid gravitational singularities we also assume that the warp factor is represented by an even function of  $z$ . Then one can choose the following toy-ansatz for the metric

$$ds^2 = e^{-z^2/\epsilon^2} dl^2 - dz^2, \quad (1)$$

where

$$dl^2 = g_{\alpha\beta}(x^\nu) dx^\alpha dx^\beta \quad (2)$$

is the metric of the 4-dimensional space ( $\alpha, \beta, \dots = 0, 1, 2, 3$ ) and  $e^{-z^2/\epsilon^2}$  is the gravitational warp factor, which mimics the dependence of the gravitational potential on the extra coordinate inside the brane. When  $z$  goes to 0, where the brane is assumed to be centred in the transverse direction, the ansatz (1) describes the 5-dimensional Minkowski space. As soon as the metric (1) does not provide gravitational localization of photons on the brane, one can introduce, in 5-dimensional models, different mechanisms of photon trapping such as, for example, adding both bulk- and brane-localized mass terms [14], or considering models with higher dimensions [13]. For the present consideration, the only important condition is that SM fields, placed within a fat brane at different points along the transverse direction, are exposed to different gravitational potentials, as follows from (1).

It is clear that in our 3-dimensional world the propagation of light is associated only with the longitudinal direction inside the brane. This means that, once a photon is placed by emission in any reaction at some distance from  $z = 0$ , it then propagates in constant gravitational potential. Therefore, it is instructive to use an interpretation that can be rephrased as if it were applied to the light propagation in static brane potential. Namely, following [15], one can expect that the frequency  $\omega$  of the photon, and hence its energy  $E = \omega\hbar$ , do not depend upon the gravitational potential, while the momentum, and thus the velocity, are expected to be different for photons propagating at different transverse distances from the centre of the brane. We shall use below the system of units where  $\hbar = 1$ , while  $c \neq 1$  in standard units of  $c_0$ , the speed of light.

A massless photon in five dimensions obeys the equation

$$g^{AB}P_AP_B = 0, \quad (3)$$

where  $P_A$  is the 5-momentum and  $g^{AB}$  defines the covariant components of the metric (1) with capital Latin indexes running over 0, 1, 2, 3 and 5. The momentum of a photon on the brane can be represented as the superposition of two components, namely, one along the longitudinal direction and another one in the transverse direction relative to the brane. Therefore, the dispersion relation (3) for the ansatz (1) takes the form

$$\frac{E^2}{v^2} - P^2 - e^{-z^2/\epsilon^2}P_z^2 = 0. \quad (4)$$

Here  $v$  is the velocity of the photon on the brane, and  $P$  and  $P_z$  are the components of the momentum in the directions parallel and perpendicular to the brane, respectively.

Note that since we assume that photons can only propagate inside the brane their wave functions are quantized in the potential well of the brane, therefore there is always a mass gap in their spectrum. The appearance of the transverse component  $P_z$  of the momentum for the brane observer is equivalent to the occurrence of the massive KK excitations of the photon. So, there is no already the massless zero mode in the model and the first quantum level for the quantized in the brane's potential photon responds to a mass, which depends on the parameters of the model, for example, upon the width of the brane  $\epsilon$ , trapping and gravitational potentials, etc. This means that the effective *longitudinal* speed  $v$  on the brane should be smaller than that in the bulk,  $c$ . Once the transverse component of

the total momentum, and hence the KK excitations, are fixed to 0, the usual dispersion relation of the "zero-mass" photon is restored from (4):

$$\frac{E^2}{c^2} = P^2 . \quad (5)$$

Note that the introduction of the effective mass of the photon for the brane observer, unlike the case where the 4-dimensional rest-mass term is postulated, does not lead to a loss of gauge invariance.

Further on, the longitudinal and the transverse components of the momentum of a particle which sees 5-dimensional Minkowski space-time should be of the same order:

$$P \sim P_z . \quad (6)$$

However, the transverse component in (4) is effectively suppressed by the gravitational warp factor  $e^{-z^2/\epsilon^2}$ . Then, it follows that the effective longitudinal velocity of the photon is

$$v^2 = \frac{c^2}{1 + e^{-z^2/\epsilon^2}} \approx \frac{c^2}{2} \left( 1 + \frac{z^2}{\epsilon^2} \right) , \quad (7)$$

and that it now depends on the position of the photon with respect to the extra coordinate  $z$  inside the brane.

Let us repeat here that our metric (1) represents just a "toy-ansatz" and the shape of the warp factor and the potential, by which the photons and other SM particles are trapped, does not play a significant role. The important only is that in the linear approximation we use the odd  $z$ -terms do not appear in the expansion (7), so it starts from terms of the  $z^2$  dependence. It turns out from (7) that the velocity of a photon at the centre of the brane ( $z = 0$ ) is reduced by a factor  $\sqrt{2}$  with respect to the speed  $c$  of light in the bulk and becomes equal to  $c$  only if the photon propagates in the transverse direction at the distance  $z = \epsilon$  from the centre. The latter means that, for a photon placed almost out of the brane, the transverse component of the velocity tends to 0, owing to the trapping caused by the gravitational potential, the latter increasing from the centre of the brane toward the bulk.

We can estimate the potential energy  $U$  of a particle of mass  $m$ , at a distance  $z$  from the centre of the source of the static gravitational field, as

$$U \sim maz , \quad (8)$$

where  $a \sim \Gamma_{00}^z$  represents the  $z$ -component of the particle gravitational acceleration. The particle with higher energy could penetrate to a larger distance along the extra coordinate. Therefore, there should exist also the escape energy,

$$E_\epsilon \sim Mc^2 , \quad (9)$$

where  $M \sim 1/(c\epsilon)$  is the scale at which the brane, viewed as a topological defect in higher-dimensional space-time, was formed. Once the potential energy  $U$  of a particle exceeds  $E_\epsilon$ , the particle escapes from our world to the bulk [16].

Then, the energy  $E$  of a photon is proportional to the energy of the particle by which it is emitted. Therefore, as follows from (8), the characteristic distance at which the emitted photon gets placed relative to the centre of the brane is given by

$$z \sim E . \quad (10)$$

Now, it is clear from (7) that photons with higher energy should propagate faster than those with lower energies. Using (9) and (10), we can express the transverse coordinate of the photon  $z^2/\epsilon^2$  in (7) via

$$\frac{z^2}{\epsilon^2} \sim \frac{E^2}{E_c^2} \sim \frac{E^2}{M^2 c^4} . \quad (11)$$

Finally, (7) reads

$$v = \frac{c}{\sqrt{2}} \left( 1 + \frac{E^2}{M^2 c^4} \right)^{1/2} . \quad (12)$$

The latter means that, because of the influence of the massive KK excitations on the fat brane, the photons with higher energy should have higher observable (longitudinal in the brane) velocity.

Let us consider a classical analogy to the propagation of photons as discussed here, which can be found as photon propagation along a waveguide. An ideal waveguide imposes a “cut-off frequency” on a propagating electromagnetic wave based on the geometry of the tube, and will not sustain waves of any lower frequency. The group velocity in a waveguide is always less than the velocity of light in vacuum [17]. In our consideration a new ingredient such as gravity is introduced, which changes universally the transverse component of the photon momentum being responsible for its mass. This would correspond to the fact that the photons of different energies propagate along waveguides with different “cut-off frequencies”.

To relate the result (12) with experimental measurements, one generally requires the propagating photon to have the energy  $E$  much smaller than the mass scale  $M$  of the KK excitations, which may be of the order of a TeV, in order to take brane-world scenarios accessible for the probe on present and future colliders. This, according to Eq. (12), implies the following energy dependence of the velocity  $v$  of light:

$$v = c_0 n(E) , \quad (13)$$

where

$$n(E) = 1 + \frac{E^2}{8M^2} \quad (14)$$

is the effective vacuum refractive index<sup>1</sup>.

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<sup>1</sup>We use the relation  $c = \sqrt{2}c_0$  between the bulk and the standard speed of light  $c_0$ .

### 3 Light Propagation in the Expanding Brane-World Universe

To search for the signature of brane-world scenarios, we compare the propagation of photons with energies much smaller than  $M$ , characterizing the divergence of the vacuum refractive index from unity. A small difference between the velocities of two photons with an energy difference  $\Delta E$ , emitted simultaneously by a remote cosmological source, would lead to a time-lag between the arrival times of the photons.

In what follows we take into account that the propagation of photons from a remote object is affected by the expansion of the Universe and depends upon the cosmological model. Present cosmological data motivate the choice of a spatially-flat Universe:  $\Omega_{\text{total}} = \Omega_{\Lambda} + \Omega_M = 1$  with cosmological constant  $\Omega_{\Lambda} \simeq 0.7$ . The corresponding differential relation between time and redshift is

$$dt = -\frac{1}{H_0} \frac{dz}{(1+z)h(z)}, \quad (15)$$

where  $H_0$  is the Hubble constant and

$$h(z) = \sqrt{\Omega_{\Lambda} + \Omega_M(1+z)^3}. \quad (16)$$

Thus, a particle of velocity  $v$  travels an elementary distance

$$vdt = -\frac{1}{H_0} \frac{v dz}{(1+z)h(z)}, \quad (17)$$

giving the following difference between the distances covered by two particles of velocities differing by  $\Delta v$ :

$$\Delta L = \frac{1}{H_0} \int_0^z \frac{\Delta v dz}{(1+z)h(z)}. \quad (18)$$

We can consider two photons travelling with velocities very close to  $c$ , whose present-day energies are  $E_1$  and  $E_2$ , where  $E_1 > E_2$ . At earlier epochs, their energies would have been blue-shifted by a factor  $1+z$ . Defining  $\Delta E \equiv E_2 - E_1$ , we infer from Eq. (12) that

$$\Delta v = \frac{\Delta E(E_1 + E_2)(1+z)^2}{8M^2}. \quad (19)$$

Inserting the last expressions into (18) one finally finds that the brane-world scenario induced differences in the arrival times of the two photons of energy difference  $\Delta E$  is

$$\Delta t = \frac{\Delta E(E_1 + E_2)}{8H_0M^2} \int_0^z \frac{(1+z)dz}{h(z)}. \quad (20)$$

To look for such a refractive effect induced by the brane-world setup, we need a distant, transient source of photons of different energies, preferably as high as possible. One may

then measure the differences in the arrival times of sharp transitions in the signals in different energy bands. GRBs are at cosmological distances, as inferred from their redshifts, and exhibit many transient features in their time series in different energy bands. In comparison, the observed active galactic nuclei (AGNs) have lower redshifts and broader time structures in their emissions, but have the advantage of higher photon energies [18]. Observable pulsars have very well defined time structures in their emissions, but are only at galactic distances [19]. Moreover, the key issue in all such probes is to distinguish the effects of the vacuum refraction from any intrinsic delay in the emission of photons of different energies by the source. It is obvious from Eq. (20) that the effect of the brane-induced time delay should increase with the redshift of the source, whereas source effects would be independent of the redshift in the absence of any cosmological evolution effects [9–11]. Therefore, in order to disentangle source and propagation effects, it is preferable to use transient sources of high energy radiation with a broad spread in known redshifts  $z$ . Thus, one of the most model-independent ways to probe the time-lags that might arise from the brane-world setup is to use the GRBs with known redshifts, which range up to  $z \sim 6$ .

In the present paper, we study the brane-world-induced light refraction in vacuum, compiling the results from [10] on time-lag measurements for a sample of 35 GRBs with known redshifts, including 9 GRBs detected by the Burst And Transient Source Experiment (BATSE) aboard the Compton Gamma Ray Observatory (CGRO), 15 detected by the High Energy Transient Explorer (HETE) satellite and 11 detected by the SWIFT satellite. The GRBs currently in use are listed in Table 1 of [10], together with their redshifts and the time-lags; the latter were extracted from their light curves by using the special wavelet techniques pioneered in [9] and developed further in [10]. The gamma ray light curves in [10] are from BATSE [20], HETE [21] and SWIFT [22] public archives, and the information on the redshifts is from [23].

## 4 GRB Constraint on Mass of “Kaluza–Klein” Excitations on a Fat Brane

As just discussed above, Eq. (20) may be accompanied by *a priori* unknown intrinsic energy-dependent time-lags, caused by unknown properties of the sources. To take this into account we use the fit  $\Delta t(z)$  of the measured time-lags where we include a term  $b_{\text{sf}}$  specified in the rest frame of the source, so that the resulting observed arrival time delays  $\Delta t_{\text{obs}}$  are fitted by two contributions:

$$\Delta t_{\text{obs}} = \Delta t_{\text{KK}} + b_{\text{sf}}(1 + z), \quad (21)$$

one,  $\Delta t_{\text{KK}}$ , reflecting the possible effects of the brane-world setup, and another one representing intrinsic source effects. Rescaling (21) by a factor  $(1 + z)$ , we arrive at a simple linear fitting function

$$\frac{\Delta t_{\text{obs}}}{1 + z} = a_{\text{KK}}K + b_{\text{sf}}, \quad (22)$$



where

$$K \equiv \frac{1}{1+z} \int_0^z \frac{(1+z)dz}{h(z)} \quad (23)$$

is a non-linear function of the redshift  $z$  related to the measure of the cosmic distance in (20), and the slope coefficient

$$a_{\text{KK}} = H_0^{-1} \frac{\Delta E(E_1 + E_2)}{8M^2} \quad (24)$$

in  $K$  is connected to the mass scale of KK excitations. The energy difference in (20) is taken in a way that the negative slope parameter  $a_{\text{KK}}$  in the fit would correspond to the refractive index (14) predicted in brane-world scenarios. The GRB data [10] used here allows looking for spectral time-lags in the light curves recorded at the energy  $E_1 = 400$  keV, relative to those at lowest  $E_2 = 30$  keV energy.

Before proceeding further and finding the estimate on the mass of the KK excitations, we want to underline that the procedure applied here follows that of [9, 10] and allows arriving at firm conclusions. Let us dwell on the main statements of the procedure, given in detail in [10]:

- To disentangle the most significant variations of time profiles, the GRB data light curves undergo a complex handling procedure, using discrete and continuous wavelet transforms. In this way the most singular, or “genuine” sharp points in the higher- and the lower-energy spectral band counterparts are determined to be used in (22).
- The obtained points are carefully checked for statistical stability. In each energy band, artificial noise has been generated and added to the original light curves, and the above wavelet search has been repeated a couple of thousand times per band, to provide reasonable convergence of the noise iteration process.
- The contamination of the real data with artificial noise is applied to allow statistically significant estimates of the errors in the determination of the signals positions.
- Particular attention has been devoted to figuring out how sensitive the fit results are to the unknown uncertainties due to the intrinsic properties of the sources we use. Several procedures of weighting the estimated time-lags and their (systematic) errors have been considered. It was revealed that the most conservative way to estimate the time-lags and the errors is to compare the arrival time of those photons coming from a given GRB when the highest parts of emission are progressing while a universal stochastic spread in the intrinsic time-lag at the source is allowed.

As seen, (22) depends linearly on  $K$ , and therefore, in order to probe the energy dependence of the velocity of light that might be induced by the brane-world, we perform a linear regression analysis of the rescaled by  $(1+z)$  time-lags (20) of the data studied here. These data, representing a set of measurements from 35 GRBs and compiled from Table

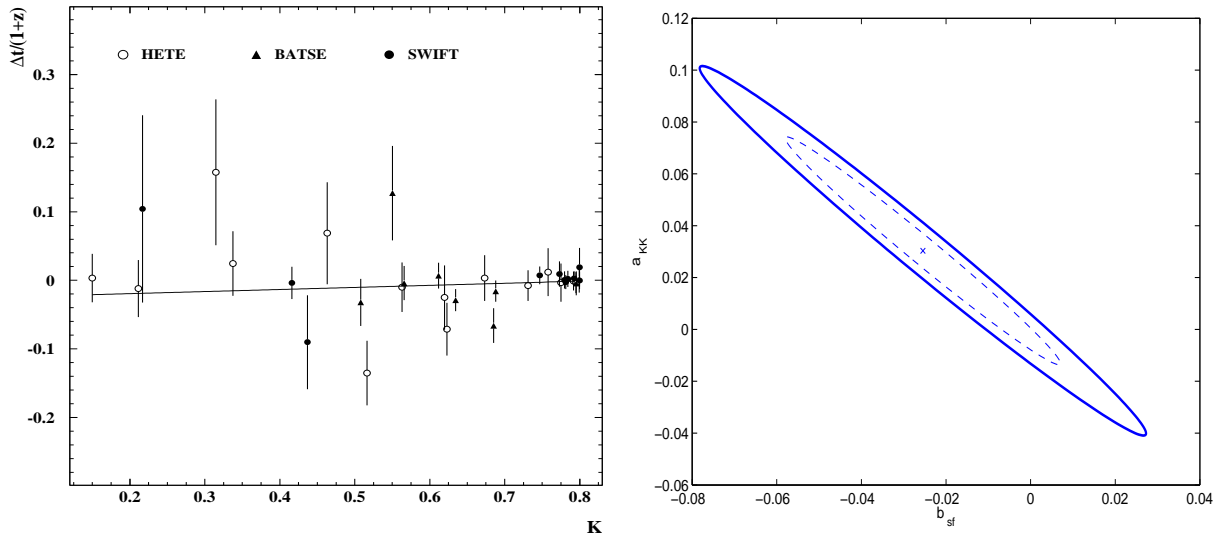


Figure 1: **Left panel:** The rescaled spectral time-lags at highest pulses between the arrival times of pairs of genuine sharp features, wavelet-extracted [10] from the light curves of the full set of 35 GRBs, with measured redshifts observed at time resolutions of 64 ms (BATSE, SWIFT) and 164 ms (HETE), and a linear fit (25) to these data with  $\chi^2/\text{d.o.f.} = 32.8/33$ . The data are normalized to the difference between the energies  $E_1 = 400$  keV and  $E_2 = 30$  keV of the third and first HETE spectral bands. The errors in the redshifts and hence in  $K$  are negligible: the errors in the time-lags are estimated by the wavelet analysis [10] and increased by 35 ms, modelling a possible stochastic spread at the sources (see text). **Right panel:** The error ellipse in the slope-intercept plane for the fit (25). The 68% and 95% confidence-level contours are represented by the dashed and solid lines, respectively.

1 of [10], as described above, are shown here in the left panel of Fig. 1 as a function of  $K$  (23).<sup>2</sup>

The result of the straight-line fit (22) leads to

$$\left(\frac{\Delta t_{\text{obs}}}{1+z}\right) = (0.030 \pm 0.029) K - (0.025 \pm 0.021), \quad (25)$$

and is also shown in the left panel of Fig. 1. To take systematics into account, we allow for a universal spread in the intrinsic time-lag at the source by adding in quadrature, for all the GRBs, a universal source error whose normalization is then fixed so that  $\chi^2/\text{d.o.f.} = 1$ . The corresponding universal source error is estimated in this way to be 35 ms,<sup>3</sup> which has been added to the errors of time-lags from [10]. The fit (25) yields a positive value of the slope parameter  $a_{KK}$  at the  $1\sigma$  level, while the slope in the model we wish to explore must

<sup>2</sup>Since the data from the HETE and SWIFT instruments are made available in slightly different energy bands, we have rescaled the time-lags and errors from Table 1 of [10] by the ratios of the energy differences of the HETE and SWIFT data relative to those of the BATSE instrument.

<sup>3</sup>This universal source error is well within the resolution of all the instruments whose data are used for the analysis.

be negative semi-definite. Moreover, the slope and intrinsic time-lag parameters are highly correlated, as seen in the right panel of Fig. 1. This certainly cannot be considered as evidence for the absence of vacuum refractive index induced by brane-world scenarios and will be used to quote a limit on the mass scale  $M$  of the KK excitation mode, which is supposed to be one of the main parameters to be addressed in studies of the brane-world phenomenology at future colliders [24].

To estimate the lower limit of the energy required to excite the zero KK mode on a fat brane, we use the marginal distribution of the slope parameter in the fit (25), as obtained by integrating over the intercept parameter. Taking into account the correlation matrix as described in [25], we rescale by a factor  $\sqrt{1 - \rho^2}$  the Gaussian-like shape of this marginal distribution, where  $\rho$  is the correlation coefficient of the bivariate slope-intercept distribution. The mean value is still unchanged at  $a_{\text{KK}} = 0.030$ , whereas the variance (defined as the width at half-maximum) is  $\sigma_{a_{\text{KK}}} = 0.025$ , which is still  $1.2\sigma$  above zero.

We quote a limit on  $M$  in the Bayesian manner proposed in [26], where the confidence range was constructed for a Gaussian distribution with positive mean, which is physically constrained to be negative. For the measured positive mean of the marginalized slope distribution at  $1.2\sigma$  above zero, we calculate the 95% confidence limit on the mass scale of the KK excitations, assuming a random variable obeying Gaussian statistics with a boundary at the origin. The corresponding upper limit on the negative value of the slope parameter is  $a_{\text{KK}}^{\text{min}} = -0.024$ .<sup>4</sup> Inserting  $a_{\text{KK}}^{\text{min}}$  into (24) and solving it for  $M$ , one gets

$$M = \sqrt{H_0^{-1} \frac{\Delta E (E_1 + E_2)}{8 a_{\text{KK}}}}, \quad (26)$$

which, with the energies  $E_1 = 400$  keV and  $E_2 = 30$  keV one deals with, leads to the lower limit

$$M \geq 620 \text{ TeV} \quad (27)$$

on the masses of the KK excitations, due to the light refraction in vacuum of the brane-world Universe. A similar result is obtained from using the likelihood method as an alternative to this Bayesian approach, as well as with another way of scaling the errors. This is similar to what has also been obtained in [10], where details on the likelihood approach and additional way of error scaling can be found.

## 5 Conclusions

In summary, we have investigated possible non-trivial refractive properties of vacuum induced by models with a fat brane-world embedded into a higher-dimensional space-time. This feature can appear for those setups where the SM particles are localized gravitationally on the brane. Then, as shown, the energy of the particle emitting a photon defines

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<sup>4</sup>This value is obtained by multiplying the upper edge of the 95% confidence interval ( $-0.97$ ) with  $\sigma_{a_{\text{KK}}} = 0.025$  listed in Table X of [26], at the line corresponding to  $x_0 = -1.2$ .

the frequency of the latter, and thus its momentum transverse component contributing as the massive KK excitations in the spatial extra dimension. We have derived the effective vacuum refractive index for photons propagating on a fat brane as a function of the mass of the KK excitations.

To search for observable signatures of brane-world scenarios, we have related the obtained refractive index formula with the arrival time-lags of two photons with different energies emitted simultaneously by a remote cosmological source. The expression found has been applied to recent compilation results [10], obtained with wavelet techniques exploiting data from 35 GRBs with known redshifts, to estimate correlation with redshift of the time-lags between the arrival times of sharp transients in GRB light curves observed in higher- and lower-energy bands.

The analysis does not show any significant correlation of the measured time-lags with the cosmological redshift to indicate any deviation of the vacuum refractive index from unity. This fact allows us to establish 95% C.L. lower limit on the inverse thickness of the brane and hence on the mass scale of the KK excitations at the level of 620 TeV. The constraint obtained is at least an order of magnitude higher than the energy achievable at future high energy colliders and is of great interest in the ongoing discussion on experimental scale probes of the considered class of brane-world scenarios.

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