Track-Based Alignment of Composite Detector Structures

Veikko Karimäki, Tapio Lampén, and Frank-Peter Schilling

***Abstract*—An iterative algorithm for track based alignment is presented. The algorithm can be applied to rigid composite detector structures or to individual modules. The iterative process involves track reconstruction and alignment, in which the χ^2 function of the hit residuals of each alignable object is minimized. Six alignment parameters per structure or per module, three for location and three for orientation, can be computed. The method is computationally light and easily parallelizable. The performance of the method is demonstrated with simulated tracks in the CMS pixel detector and tracks reconstructed from experimental data recorded with a test beam setup.**

***Index Terms*—Alignment, calibration, HIP, particle tracking, position measurement, position sensitive particle detectors.**

I. Introduction

MODERN silicon tracking detectors such as the CMS tracker [1], [12] are composed of a large number of modules assembled in a hierarchy of support structures [2]. The sensor modules are assembled in ladders or petals. Ladders and petals are in turn assembled in cylindrical or disc-like layers which further constitute higher-level structures.

Sophisticated geometrical calibration is essential in such large detector systems to fully benefit from the high intrinsic resolution of the silicon sensors. For instance the CMS tracker consists of approximately 16 000 individual sensors, which have to be position-calibrated with an accuracy comparable to their intrinsic resolutions of 10–50 μm [1], [12]. The corresponding assembly precisions range from 100 μm to a few hundreds of μm for individual sensors, and in addition the rotational misalignments of the sub-detectors are of the order of 10 μrad around the beam axis [3]. Therefore the position information must be improved by an order of magnitude with calibration procedures.

A laser alignment system and track based alignment algorithms will be used to align the CMS tracker [4]. Infrared laser beams monitor positions of selected detector modules, and can be used to align the corresponding support structures. The laser alignment system does not, however, cover all parts of the CMS tracker. In addition, it cannot be used to align individual detector modules independently of each other. Track based alignment is needed to complement the laser alignment system. In particular, the alignment of the pixel detector is carried out solely with particle tracks.

Trajectories of high momentum particles are useful for track based alignment, since they are continuous and smooth. The hit residuals, i.e., the spatial differences between the reconstructed track and the recorded hit positions, provide constraints such that the position and orientation of the modules can be optimized with a large sample of tracks.

This paper presents an effective method by which individual sensors in a detector setup can be aligned to a high precision with respect to each other [5]. This track based "Hits and Impact Points" (HIP) method has a long history [6]–[8], [13]. The formalism has been recently extended in [9] to the case of the alignment of composed hierarchical tracker structures, for example ladders or layers.

The algorithm involves iteration over the event sample. During each iteration the particle trajectories are kept static, which makes the calculation of alignment corrections computationally easy. After each pass over the event sample, the alignment corrections are computed and used in the next iteration over the event sample and the tracks are refitted with the alignment corrections.

II. Alignment Transformations

The following conventions are used in the formulations: Lower case, bold face characters $\mathbf{r}$, $\mathbf{g}$, and $\mathbf{q}$ denote 3-D vectors in global, composite and module ('local') coordinate systems, respectively. The upper case, bold face characters $\mathbf{R}$ and $\mathbf{G}$ denote rotations from global to local and from global to composite coordinate systems, respectively.

The local coordinates $\mathbf{q} = (u, v, w)$ are defined such that w is normal to the sensor and u and v are the measured coordinates (for single sided strip modules only u is measured). The global coordinates are denoted as $\mathbf{r} = (x, y, z)$. The transformations are then

$$\mathbf{q} = \mathbf{R}(\mathbf{r} - \mathbf{r}_0) \tag{1}$$

$$\mathbf{g} = \mathbf{G}(\mathbf{r} - \mathbf{g}_0). \tag{2}$$

A composite misalignment (unknown small translation and rotation) would be corrected by a rotation matrix $\Delta\mathbf{G}$ and a translation vector $\Delta\mathbf{g}$ which would be common to a group of sensors, e.g., belonging to the same support structure. The correction for alignment is: $\mathbf{g} \rightarrow \mathbf{\Delta G^T g} + \Delta\mathbf{g}$ so that the corrected transformations are defined by

$$\mathbf{G} \rightarrow \mathbf{G}_C = \Delta\mathbf{G}^T\mathbf{G} \tag{3}$$

$$\mathbf{g}_0 \rightarrow \mathbf{g}_c = \mathbf{g}_0 - \mathbf{G}^T\Delta\mathbf{G}\Delta\mathbf{g}. \tag{4}$$

Manuscript received June 1, 2006; revised August 26, 2006.

V. Karimäki and T. Lampén are with Helsinki Institute of Physics, FIN-00014, Finland.

F.-P. Schilling is with CERN Physics Department, CH-1211, Geneva 23, Switzerland.

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TNS.2006.884384

The correction of $\mathbf{g}$ is to be expressed in terms of the transformation between the local and global systems. It follows that the corrected rotations and their centres are

$$\mathbf{R}_C = \mathbf{R}\mathbf{G}^T\boldsymbol{\Delta}\mathbf{G}^{\mathbf{T}}\mathbf{G} \tag{5}$$

$$\mathbf{r}_c = \mathbf{g}_0 - \mathbf{G}^T\Delta\mathbf{G}\Delta\mathbf{g} + \mathbf{G}^T\Delta\mathbf{G}\mathbf{G}(\mathbf{r}_0 - \mathbf{g}_0). \tag{6}$$

Notice that the corrective rotation $\Delta\mathbf{G}$ and translation $\Delta\mathbf{g}$ are common to a group of sensors to be aligned collectively.

III. HIP Alignment Algorithm

A basic feature of the HIP algorithm is that particle trajectories are kept static during a pass over the events. The benefit is that the formalism only requires small matrices. The algorithm involves inversion of only up to 6×6 matrices. A consequence is that the algorithm requires iteration over the event sample—at each pass the tracks are refitted and new alignment corrections calculated. The iteration continues until no statistically significant improvement is obtained for the alignment. Another basic feature is that a particle trajectory is approximated as a straight line in the vicinity of the impact point.

It is necessary provide the alignment algorithm reference measurements, which constrain global translations, rotations as well as the scale of the space. This can be done by fixing some sensors.

The algorithm can easily be run in a parallel environment by processing a fraction $1/N$ of the event sample on N machines in parallel. The alignment parameters $\bar{\mathbf{p}}$ are calculated using the combined information before the next iteration is started.

A key formula of the algorithm is the variation of the trajectory impact point $\mathbf{q}_\times$ as a function of the corrections $\Delta\mathbf{G}$ and $\Delta\mathbf{g}$. The derivation of the formula is a small linear algebra exercise with the following result:

$$\mathbf{q}_\times(\Delta\mathbf{G}, \Delta\mathbf{g}) = \mathbf{R}_C[\mathbf{r}_\times - \mathbf{r}_c + h_\times(\Delta\mathbf{G}, \Delta\mathbf{g})\hat{\mathbf{s}}] \tag{7}$$

where $\mathbf{r}_\times$ is the uncorrected impact point, $\hat{\mathbf{s}}$ is the trajectory direction at $\mathbf{r}_\times$ and the scalar function $h_\times$ is defined as

$$h_\times(\Delta\mathbf{G}, \Delta\mathbf{g}) = -\frac{\mathbf{R}_C(\mathbf{r}_\times - \mathbf{r}_c)\cdot\hat{\mathbf{w}}}{\mathbf{R}_C\hat{\mathbf{s}}\cdot\hat{\mathbf{w}}}. \tag{8}$$

For verification one can readily see that the third component of (7) is identically zero.

The determination of the tilt and translation parameters $(\Delta\alpha, \Delta\beta, \Delta\gamma)$ and $(\Delta g_1, \Delta g_2, \Delta g_3)$ takes place by the χ^2 minimization method. The terms of the χ^2 sum are of the form $\boldsymbol{\varepsilon}_i^T\mathbf{V}_i^{-1}\boldsymbol{\varepsilon}_i$, where $\mathbf{V}_i$ is the sum of hit and impact point covariance matrices. The χ^2 function is nonlinear in terms of the parameters $\bar{\mathbf{p}} = (\Delta g_1, \Delta g_2, \Delta g_3, \Delta\alpha, \Delta\beta, \Delta\gamma)$ so the linearized χ^2 minimization method is used for solution. This method employs the first derivatives of the residuals $\boldsymbol{\varepsilon} = \mathbf{q}_\times - \mathbf{q}_m$. The hit measurements in the local system $(\mathbf{q}_m)$ do not depend on the alignment parameters $\bar{\mathbf{p}}$ so that the derivatives of $\boldsymbol{\varepsilon}$ revert to the derivatives of $\mathbf{q}_\times(\Delta\mathbf{G}, \Delta\mathbf{g})$ (7).

The derivatives of (7) with respect to the translation parameters $(p_1, p_2, p_3) = (\Delta g_1, \Delta g_2, \Delta g_3)$ are

$$\frac{\partial\mathbf{q}_\times}{\partial p_j} = \mathbf{R}\mathbf{G}^T\hat{\mathbf{e}}_j - [\mathbf{R}\mathbf{G}^T\hat{\mathbf{e}}_j]_3\frac{\mathbf{R}_C\hat{\mathbf{s}}}{[\mathbf{R}_C\hat{\mathbf{s}}]_3}, \quad j = 1,2,3 \tag{9}$$

where $\hat{\mathbf{e}}_j$ are the unit vectors (1,0,0), (0,1,0), and (0,0,1). The derivatives with respect to the tilt angles $(p_4, p_5, p_6) = (\Delta\alpha, \Delta\beta, \Delta\gamma)$ are

$$\frac{\partial\mathbf{q}_\times}{\partial p_j} = \mathbf{D_j}\mathbf{r}(\bar{\mathbf{p}}) - [\mathbf{D_j}\mathbf{r}(\bar{\mathbf{p}})]_3\frac{\mathbf{R}_C\hat{\mathbf{s}}}{[\mathbf{R}_C\hat{\mathbf{s}}]_3}, \quad j = 4,5,6, \tag{10}$$

where $\mathbf{D_j}$ are the derivatives of the matrix $\mathbf{R}_C$ and the vector $\mathbf{r}(\bar{\mathbf{p}})$ is defined as: $\mathbf{r}(\bar{\mathbf{p}}) = \mathbf{r}_\times + h_\times(\bar{\mathbf{p}})\hat{\mathbf{s}} - \mathbf{g}_0$.

It is interesting to note that in case the "structures" are composed of only one module (i.e., $\mathbf{G} = \mathbf{R}$), the above formalism reduces to the module by module alignment formalism described in the earlier paper [8], [13]. Another special case is when the composite coordinate system is the same as the global system (i.e., $\mathbf{G} = \mathbf{I}, \mathbf{g}_0 = \mathbf{0}$). This may be the case, for example, for barrel layers.

IV. Alignment Studies

A. Stand-Alone Alignment of CMS Pixel Barrel Modules With Simulated Tracks

The algorithm is applied to the CMS pixel barrel detector. It has been implemented within the CMS reconstruction software ORCA [10] using a common alignment software framework. Silicon sensors and composite structures can be misaligned at the reconstruction level with a dedicated software tool [3].

To misalign the pixel barrel, independent random shifts sampled from a uniform distribution between ± 300 μm were applied to the pixel barrel modules in x, y, and z. Details of the study are described in [9].

Half a million fully simulated and reconstructed $Z^0 \rightarrow \mu^+\mu^-$ events were used with 19 iterations. The result is shown in Figs. 1 and 2. The alignment corrections have been obtained only for 504 pixel barrel modules (720 in total) since tracks are required to have hits in all three pixel barrel layers. One pixel barrel module was kept fixed to avoid shifts and deformations of the entire pixel barrel.

To avoid a bias originating from the possibly misaligned strip tracker, the procedure refits the track with pixel detector hits only. In addition, the two muon tracks from the $Z^0 \rightarrow \mu^+\mu^-$ events are fitted to a common vertex. The p_T estimate was obtained from the full track fit. This improved significantly the convergence of the standalone pixel alignment.

A special data format containing only tracks used in the alignment was utilized. In addition, the refit of already reconstructed tracks was made using hits already found by the pattern recognition. In this configuration, the CPU time needed is dominated by the time needed to read in the events and to refit the tracks used for alignment. One iteration could be processed in approximately 20 min in a parallel environment of 20 CPUs in Intel Xeon 3.06 GHz nodes.

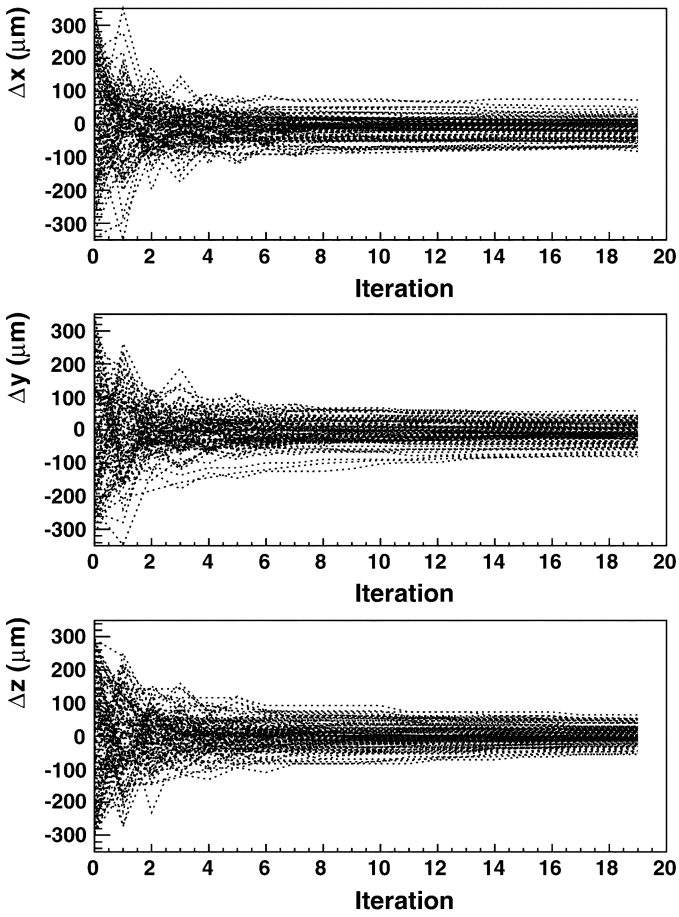

Fig. 1. Convergence of alignment of 504 pixel barrel modules. The residuals in global coordinates are shown as a function of the iteration number for 100 randomly chosen modules.

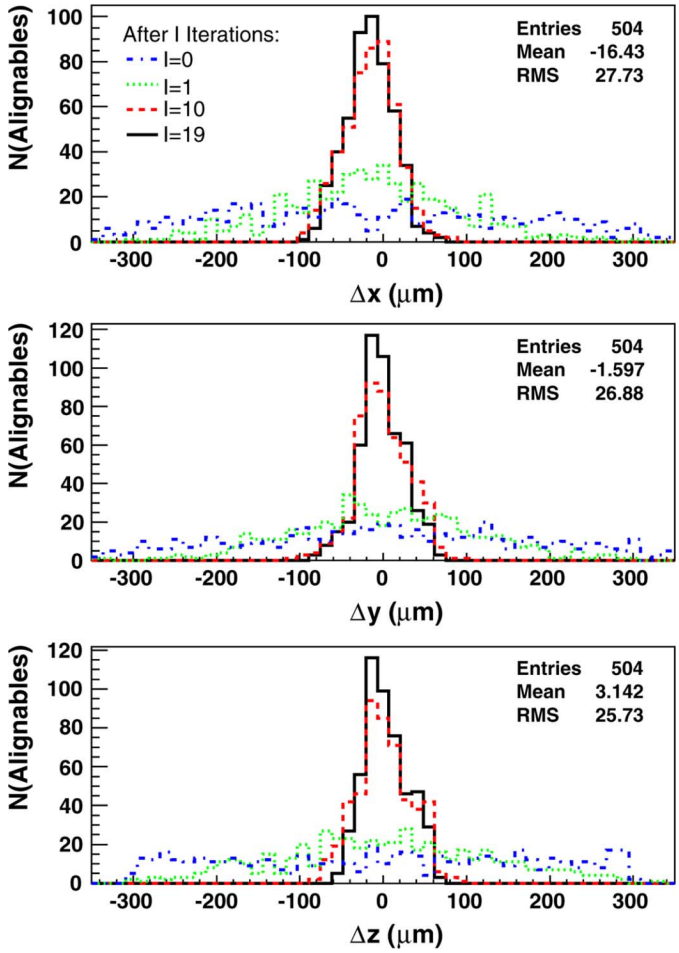

Fig. 2. Residual distributions of alignment of 504 pixel barrel modules. The residuals in global coordinates are shown for the initial misalignment (iteration 0) and after 1, 10, and 19 iterations. The statistical parameters refer to the iteration 19.

A good convergence is obtained for the alignment parameters. The residual RMS values are around 25 μm for all three coordinates. Although this is not yet a sufficiently precise result considering the intrinsic resolution of the pixel modules, it demonstrates that the method for the standalone alignment of the pixel detector works. The precision of the alignment can be improved by making use of a larger track sample.

B. Alignment With Data From a Test Beam Setup

The algorithm was applied also to test beam data recorded with a test beam setup called the Cosmic Rack, which mimicks the outer barrel of the CMS Tracker. The Cosmic Rack can be equipped with a maximum of 20 rods, the carbon fiber structures holding the detector modules in the CMS Tracker. Two kind of detector modules with pitch of 122 and 183 μm were used, corresponding to binary resolutions of 35 or 53 μm. A full chain of genuine CMS hardware and reconstruction software was used. The Cosmic Rack is illustrated in Fig. 3.

The Cosmic Rack was equipped with six rods, each holding two detector modules. The setup was tilted and placed to a beam of 120 GeV pions. The outermost rods provided reference measurements in two dimensions, whereas modules of the four innermost rods were measuring only one direction in which they were also aligned. Details of the setup can be found in [11].

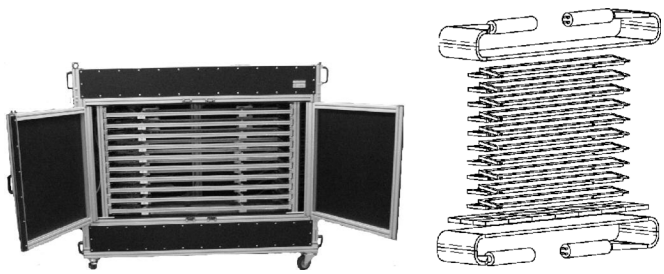

Fig. 3. Left: Photograph of the TOB Cosmic Rack; Right: Schematic view of rods and scintillators in the fully equipped Cosmic Rack. The width of the apparatus is approximately 2 m.

The convergence of the mean χ^2 values for the tracks is shown in Fig. 4. Also the convergence of the alignment correction is shown for one particular module. As can be expected in this simple test setup, the algorithm converges very quickly, reaching approximately the final level already after two iterations.

The RMS of the hit residuals of different detectors were initially in the range of 170–310 μm, and with alignment improved to a range of 64–170 μm, which is somewhat larger than the binary resolutions.

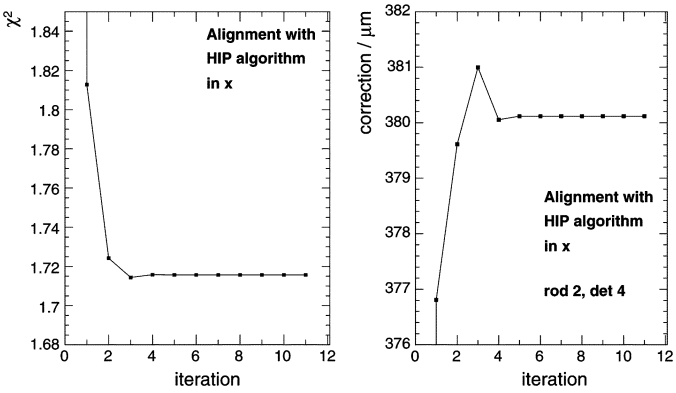

Fig. 4. Left: The convergence of the mean χ^2 value of the test beam data when only x is aligned. The initial value of 61 at iteration zero is not shown. Right: The corresponding convergence of one particular module (initial value 0 not shown).

V. Prospects

The results of the stand-alone alignment for CMS pixel detector are encouraging. Results obtained from the alignment of the Cosmic Rack serve as a small-scale proof-of-principle for both software and hardware. More detailed studies of realistic alignment scenarios are needed as well as studies of how to benefit from special events: minimum bias events, cosmic muons etc., which can be very beneficial in the early operation of the CMS experiment. The invariant mass constraint from muon pairs of Z or J/ψ would also be beneficial for the alignment.

Acknowledgment

The authors would like to express thanks to E. Anttila, O. Buchmüller, A. Dierlamm, T. Mäenpää, and M. Weber for fruitful discussions regarding this topic.

References

[1] CMS Collaboration, CMS Tracker Project Technical Design Report, 1998, CERN/LHCC 1998/06.
[2] A. Dierlamm, "Status of the production of the CMS silicon tracker," *IEEE Trans. Nucl. Sci.*, vol. 52, pp. 3206–3210, Dec. 2005.
[3] I. Belotelov *et al.*, Simulation of Misalignment Scenarios for CMS Tracking Devices CMS NOTE 2006/008, 2006.
[4] A. Ostaptchouk *et al.*, The Alignment System of the CMS Tracker CMS NOTE 2001/053, 2001.
[5] V. Karimäki, "Track based alignment of composite detector structures," in *Proc. Int. Conf. Comput. High Energy and Nuclear Physics CHEP06*, Mumbai, India, Feb. 13–17, 2006.
[6] A. Heikkinen and V. Karimäki, Fine Calibration of Detector Positions by Tracks in Helsinki Silicon Beam Telescope CMS NOTE 1999/029, 1999.
[7] K. Banzuzi *et al.*, "Performance and calibration studies of silicon strip detectors in a test beam," *Nucl. Instr. and Meth.*, vol. A453, p. 536, 2000.
[8] V. Karimäki *et al.*, "Sensor alignment by tracks," in *Proc. CHEP2003—Int. Conf. Comput. in High Energy and Nuclear Physics*, La Jolla, CA, Mar. 24–28, 2003, CMS CR-2003/022.
[9] V. Karimäki *et al.*, The HIP Algorithm for Track Based Alignment and Its Application to the CMS Pixel Detector CMS NOTE 2006/018, 2006.
[10] CMS Reconstruction Software ORCA, CMS Collaboration [Online]. Available: http://cmsdoc.cern.ch/orca/
[11] T. Lampén *et al.*, Alignment of the Cosmic Rack with the HIP Algorithm CMS NOTE 2006/006, 2006.
[12] CERN/LHCC 2000/016 2000, Addendum.
[13] 2003, arXiv:physics/0306034.