SEMI-NUMERICAL EVALUATION OF ONE-LOOP CORRECTIONS

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We present a semi-numerical method to compute one-loop corrections to multi-leg processes. We apply the method to the study of Higgs plus four parton and six gluon amplitudes.

1 Motivation

At the moment, there is no striking discrepancy between Standard Model (SM) predictions and data. The only missing cornerstone of the SM is the Higgs boson. However, many hints indicate that the SM is only a low-energy effective theory. In view of this, the main tasks of the LHC will be to discover the Higgs and measure its properties, to (stress-)test the quantum structure of the SM at the TeV scale and to unravel possible physics beyond the SM. At the LHC dominant corrections will be due to higher orders in QCD. Our ability to accomplish the aforementioned tasks will thus be limited by the quality of our understanding of QCD. Next-to-leading order (NLO) predictions are in this respect important as they allow one to establish cross-section normalization and shapes and to reduce the dependence on unphysical factorization and renormalization scales. In searches for new physics it is important to have a good understanding of backgrounds, especially for small signal to background ratios. Finally, loop-induced corrections allow one to get indirect informations about sectors not directly accessible by experiments. For instance electroweak precision data lead to an indirect bound on the Higgs mass.

Despite the fact that at the LHC and ILC most processes/backgrounds involve multi-particle final states, today very few NLO calculations of multi-leg processes exist. A remarkable exception is the NLO calculation of $e^+e^- \to 4$ fermions [1].

A full N-particle NLO calculation requires the tree level prediction of (N+1) parton amplitudes, the pure virtual correction to the N parton process, and, since both of the above are divergent, one needs to compute a set of subtraction terms as well. While the evaluation of

leading-order (LO) amplitudes has been largely automated and also the computation of one-loop subtraction terms is well understood, as it requires only the knowledge of the divergent part, the complexity of the *analytical* evaluation of the virtual terms of multi-leg amplitudes is the limiting factor in NLO calculations. We therefore developed a method to compute such virtual corrections semi-numerically.

2 The method

Our semi-numerical method is very simple and general. It is based on the following three steps:

A: we one use a combination of Qgraf/Form/Mathematica to generate the amplitude for the specific process and to write it as a sum of contraction between tensor integrals and coefficients which depend on the kinematic of the process (momenta p_i and polarizations ε_i)

$$\mathcal{A}(p_1,\dots p_N) = \sum K_{\mu_1\cdots\mu_M}(p_1,\dots p_N;\varepsilon_1,\dots\varepsilon_N) I^{\mu_1\cdots\mu_M}(D;\nu_1,\dots\nu_N), \tag{1}$$

where $I^{\mu_1\cdots\mu_M}(D;\nu_1,\dots,\nu_N)$ denotes the generic scalar integral [a]

$$I^{\mu_1\cdots\mu_M}(D;\nu_1,\dots,\nu_N) \equiv \int \frac{d^{\,D}l}{i\pi^{D/2}} \, \frac{l^{\mu_1}\dots l^{\mu_M}}{d_1^{\nu_1} d_2^{\nu_2}\cdots d_N^{\nu_N}}, \quad d_i \equiv (l+q_i)^2\,, \quad q_i \equiv \sum_{j=1}^{i} p_j\,. \tag{2}$$

Apart from trival manipulations, the critical point here is to cancel all quadratic terms in the loop-momentum in $I^{\mu_1\cdots\mu_M}$, so that $K_{\mu_1\dots\mu_M}$ does not depend on the D-dimensional metric tensor;

N1: using the Davydychev reduction [2], we reduce tensor integrals to a combination of higher dimensional scalar integrals with shifted exponents;

N2: using a complete set of recurrence relations [3,4], we reduce each scalar integral to a combination of analytically known, basis integrals. A sample relation reads

$$I(D;\{\nu_l\}) = \frac{1}{B(D-1-\sigma)} \left(I(D-2;\{\nu_l\}) - \sum_{i=1}^{N} b_i I(D-2;\{\nu_l-\delta_{li}\}) \right), \tag{3}$$

where $S_{ij} = (q_i-q_j)^2$, $b_i \equiv \sum_{j=1}^{N} S_{ij}^{-1}$ and $B \equiv \sum_{j=1}^{N} b_i$.

While step A is done analytically only once for each process, steps N1 and N2 are repeated numerically for each phase space point. Note that since recursion relations generally involve $D = 4-2\epsilon$, results for intermediate scalar integrals are stored as Laurent expansions in ϵ. Since one uses analytical expressions for basis integrals, unless there is a specific numerical instability, no loss of accuracy in the semi-numerical evaluation is to be expected. However, it is well known that for so-called exceptional momentum configurations normal recursion relations become unstable, e. g. eq. (3) is undefined for $B = 0$ (other ones for $\det(S) = 0$). These phase space points correspond to either accidental degeneracies (such as planar configurations) or to physical thresholds. They have vanishing phase space measure but recursions become numerically unstable close to those points. The solution we adopt here is an extension [4] of the method suggested in [5] and tested there with the simple forward $\gamma\gamma \to \gamma\gamma$ scattering. The idea is to exploit the existence of a small parameter (B or $\det(S)$) to define expanded relations. For instance, for small B eq. (3) can be expanded in B to give

$$I(D-2;\{\nu_l\}) = \sum_{i=1}^{N} b_i I(D-2;\{\nu_l-\delta_{li}\}) + (D-1-\sigma)\; B\, I(D;\{\nu_l\})\,. \tag{4}$$

[a] Note that we consider here only the case of massless internal propagators although the method is more general.

The structure of the expanded relations must be such that the integral to the l. h. s. side is expressed as a sum of integrals with coefficients $\mathcal{O}(1)$, which have a simpler kinematical matrix and will generally not give rise to further exceptional expansions (though in some cases they might) and a sum of integrals whose coefficients are suppressed by a small parameter but who have exactly the same kinematical matrix and will therefore be computed by applying the same expanded relation iteratively. Similarly, if $\det(S) \ll 1$ one expands around the eigenvector corresponding to the smallest eigenvalue. Since exceptional points are easily detected numerically, there is no need to understand analytically their origin. This is particularly important if one considers cases with internal masses in the loop. In this respect, the method [6] used to compute treat exceptional regions for instance in the calculation of $e^+e^- \to 4$ fermions is very similar to ours, but relies on a different tensor reduction.

3 Applications

3.1 Higgs plus four partons

As a first application of the method we considered the NLO corrections to Higgs plus four parton amplitudes in the large m_t limit. Gluon fusion Higgs plus dijet production is to be considered a background to vector boson fusion Higgs plus dijet process. Indeed the latter is the most promising channel to measure the Higgs couplings at the LHC, while the former is plagued by large QCD uncertainties. It was therefore important to compute this process at NLO in QCD. We first computed the virtual corrections numerically [7]. As a check of the calculation, we computed also the analytical amplitude for amplitudes with four external quarks. In the two-quark two-gluon and four-gluon amplitudes we could check the poles, the Ward identities, as well as relations involving various color amplitudes (cyclicity, reflection and decoupling identities [10]). For all non-exceptional points examined [b] the relative accuracy turns out to be of $\mathcal{O}(10^{-13})$. We also examined the stability close to those exceptional points. Here one defines a target accuracy ρ (we choose $\rho = 10^{-6}$) and stops the iterative expansion once this accuracy reached. Our results turned out to be always within the target accuracy. To complete the calculation of cross section for this process one needs to implement real radiation and compute subtraction terms. Preliminar results have been presented in [8].

3.2 Six gluon amplitudes

As a next application we considered six gluon amplitudes. One important difference is that starting with six parton process, after eliminating one momenutm due to momentum conservation the remaining partons are no longer independent. One expects therefore the Davydychev decomposition to becomes rapidly inefficient with increasing number of legs, since one decomposes in a redundant set of momenta. We also found a quite large loss in numerical precision using Davydychev reduction starting from rank four six-point tensor integrals. Therefore we choose to use a method which explicitly uses the (over-)completeness of the set of momenta [9]. We performed a standard color decomposition of the amplitude (h_i and a_i denote helicity and color of gluon i) [10]

$$\mathcal{A}_n(\{p_i,h_i,a_i\}) = g^n \left(\sum_{\sigma\in Z_n} N_c \mathrm{Tr}(T^{\sigma(a_1)}\cdots T^{\sigma(a_n)})\, A_{n;1} \right. \tag{5}$$

$$\left. + \sum_{c=2}^{\lfloor n/2\rfloor+1} \sum_{\sigma\in S_n/S_{n;c}} \mathrm{Tr}(T^{\sigma(a_1)}\cdots T^{\sigma(a_{c-1})})\,\mathrm{Tr}(T^{\sigma(a_c)}\cdots T^{\sigma(a_n)})\, A_{n;c} \right) . \tag{6}$$

[b] After rescaling the hard event to the hard scale of the process, we choose to deem an event to be exceptional if B or $\det(S)$ is less then ε and played with values of ε in the range $10^{-3} - 10^{-6}$.

Subleading color amplitudes $A_{n;c}$ ($c > 1$) are completely determined by the leading color ones $A_{n;1}$, therefore we considered only $A_{n;1}$ in the basic ordering of momenta. One can then consider a helicity decomposition of the amplitude. Out of 64 possible amplitudes only 8 are independent, the others can be obtained by parity and cyclicity. Specifically we choose to consider the two finite amplitude $\{6+\}$ and $\{1-5+\}$, the three MHV amplitudes $\{2-4+\}$ and the three NMHV amplitudes $\{3-3+\}$. In order to compare with the literature we considered a supersymmetric decomposition of the amplitude, i. e. apart from the spin 1 ($A^{[1]}$) and spin 1/2 ($A^{[1/1]}$) terms, we considered also the contribution of a complex scalar in the loop ($A^{[0]}$) and used this to construct amplitudes with $\mathcal{N} = 4$ and $\mathcal{N} = 1$ supersymmetric multiplets in the loop

$$\begin{aligned} A^{\mathcal{N}=4} &= A^{[1]} + 4A^{[1/2]} + 3A^{[0]}\,, \\ A^{\mathcal{N}=1} &= A^{[1/2]} + A^{[0]}\,. \end{aligned} \tag{7}$$

After the revision of some analytical calculations we now agree with all public available results (for details about the comparison with analytical results see [9]). The relative accuracy is now of the order of $\mathcal{O}\left(10^{-8} - 10^{-9}\right)$. The main new results are 5 out of 8 scalar amplitudes, which were still unknown analytically. This completes the virtual calculation of six-gluon amplitudes.[c] This calculation illustrates the complementarity between analytical and numerical results: the scalar contribution is numerically the easiest one, while it is by far the hardest contribution to compute analytically. Additionally, these results show that numerical methods provide very useful independent checks of analytical calculations.

Despite the fact that virtual corrections to six gluon amplitudes are now fully know numerically, the way to go to obtain cross sections is still long: one should extend the expanded relations to cases with six external particles similarly to what has been done for five-point amplitudes (alternatively, one could use a more brute-force approach of interpolating around the exceptional points), one should consider amplitudes with external quarks, real radiation from a six parton ensemble and compute subtraction terms. Finally, one should merge all these elements in an efficient phase space integrator.

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References

1. A. Denner, S. Dittmaier, M. Roth and L. H. Wieders, Phys. Lett. B **612** (2005) 223 and Nucl. Phys. B **724** (2005) 247.
2. A. I. Davydychev, Phys. Lett. B **263**, 107 (1991).
3. W. T. Giele and E. W. N. Glover, JHEP **0404**, 029 (2004).
4. R. K. Ellis, W. T. Giele and G. Zanderighi, Phys. Rev. D **73**, 014027 (2006).
5. W. Giele, E. W. N. Glover and G. Zanderighi, Nucl. Phys. Proc. Suppl. **135**, 275 (2004).
6. A. Denner and S. Dittmaier, Nucl. Phys. B **734**, 62 (2006).
7. R. K. Ellis, W. T. Giele and G. Zanderighi, Phys. Rev. D **72**, 054018 (2005).
8. See talk of K. Ellis at Loops and Legs 2006.
9. R. K. Ellis, W. T. Giele and G. Zanderighi, arXiv:hep-ph/0602185.
10. Z. Bern and D. A. Kosower, Nucl. Phys. B **362** (1991) 389.
11. R. Britto, B. Feng and P. Mastrolia, Phys. Rev. D **73** (2006) 105004.
12. C. F. Berger, Z. Bern, L. J. Dixon, D. Forde and D. A. Kosower, hep-ph/0604195.

[c] We note that contemporary to our paper in [11] all cut-constructable parts to six gluon amplitudes have been computed and that very recently one more NMHV scalar six-gluon amplitude has been computed [12].