Decomposition Algorithms for Global Solution of Deterministic and Stochastic Pooling Problems in Natural Gas Value Chains

by

Emre Armagan

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Abstract

In this thesis, a Benders decomposition algorithm is designed and implemented to solve both deterministic and stochastic pooling problems to global optimality. Convergence of the algorithm to a global optimum is proved and then it is implemented both in GAMS and C++ to get the best performance. A series of example problems are solved, both with the proposed Benders decomposition algorithm and commercially available global optimization software to determine the validity and the performance of the proposed algorithm. Moreover, a two stage stochastic pooling problem is formulated to model the optimal capacity expansion problem in pooling networks and the proposed algorithm is applied to this problem to obtain global optimum. A number of example stochastic pooling problems are solved, both with the proposed Benders decomposition algorithm and commercially available global optimization software to determine the validity and the performance of the proposed algorithm applied to stochastic problems.

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Chapter 1

Introduction

1.1 Pooling Problems

The pooling problem is a planning problem that arises in blending materials to produce products; an example might be the blending of petroleum or natural gas. Pooling occurs whenever streams are mixed together, often in a storage tank, and the resulting mixture is distributed to several locations. Pooling and blending of raw materials and stored products is an important step in the synthesis of end products having different quality specifications. Products possessing different attribute qualities are mixed in a series of pools in such a way that the attribute qualities of the blended products of the end pools must satisfy given requirements. Pooling also occurs in distillation and other separation processes. The mathematics of the pooling problem applies to such processes and their applications. In a pooling problem, each material has a set of attributes with associated qualities, such as percentage of sulfur or carbon dioxide percentage. Pool qualities are defined by a flow-weighted average of the source qualities and product qualities are similarly defined by a flow-weighted average of the pool qualities. Product qualities are constrained to lie in specified ranges. The pooling problem is to maximize the total profit, subject to flow and quality constraints.

The pooling problem is a bilinear optimization problem because the output stream qualities, which are unknown, depend on the flowrates, which is also unknown, and on the quality of the input streams. Because of the bilinear terms, the process of pooling introduces nonlinearities and nonconvexities into optimization models leading to the possibility of several locally optimal solutions some of which may be suboptimal. Naturally, it takes more effort to solve a problem to guaranteed global optimality than it takes to find a locally optimal solution and one must often weigh the benefits against the costs. However, it is apparent that global optimization of the pooling and blending process could lead to substantial savings in cost, resulting in higher profits as in the case of the petroleum industry.

Numerical algorithms for solving pooling problems have included sensitivity and feasibility analysis and local optimization techniques. However, because of the benefits of solving pooling problems to guaranteed global optimality as explained above, more recently deterministic global optimization algorithms (which use Branch-and-Bound, Benders Decomposition (BD) or Generalized Benders Decomposition (GBD), etc.) have also been proposed. However, the application of global optimization algorithms to the pooling problem continues to be a challenge because of the slow convergence speed of the proposed algorithms. Since the nonconvexities and nonlinearities of a pooling problem come from the bilinear terms, a BD or GBD based algorithm looks as a promising approach in order to find the global optimal solution of the problem. Moreover, decomposition algorithms are often regarded as better candidates to solve stochastic infrastructure development problems in the natural gas value chains, which is the ultimate objective of this project. However, as explained in the later sections of this thesis, until now in the literature, in order to solve pooling problems to global optimality with GBD algorithms (in the literature, a BD algorithm has not yet been proposed for the solution of pooling problems), only one of the variables appearing in the bilinear terms was taken as the complicating variable (detailed information about BD and GBD algorithms is provided in *Chapter 4*) and with this approach even for relatively simple pooling problems, the proposed GBD algorithms converge to suboptimal solutions, even non-KKT points, and therefore does not guarantee a global optimum.

1.2 Importance of Pooling Problems in the Natural Gas Value Chain

Natural gas is a vital component of the world's supply of energy and its importance has been increasing as a fossil fuel in recent years because of different factors. First of all, unlike other fossil fuels, natural gas is a relatively clean fuel since it emits low levels of potentially harmful byproducts such as sulphur particulates, carbon dioxide and nitrogen oxides, as it burns. In addition, from the geographical perspective, natural gas is more uniformly distributed than oil. Moreover, since it is relatively easy, cheap and clean to convert it into hydrogen, natural gas is considered to be one of the most important elements in the transition to a hydrogen economy.

Raw natural gas typically consists primarily of methane (CH_4), the shortest and lightest hydrocarbon molecule. It also contains varying amounts of heavier gaseous hydrocarbons (ethane (C_2H_6), propane (C_3H_8), butane (C_4H_{10}), etc.), acid gases (carbon dioxide (CO_2), hydrogen sulfide (H_2S), etc.), nitrogen (N_2), helium (He) and water vapor. All of those gases except methane are called the impurities and the raw natural gas must be purified to meet the quality standards specified by the contractual agreements between production companies and major pipeline transmission and distribution companies. Those quality standards vary from pipeline to pipeline and are usually a function of a pipeline systems design and the markets that it serves. In general, the standards specify that the natural gas be within a specific range of heating value (For example, in the United States, it should be about 1,035 ± 5% Btu per cubic foot of gas at 1 atmosphere and 60 °F); be delivered at or above a specified hydrocarbon dew point temperature; be free of particulate solids and liquid water to prevent erosion, corrosion or other damage to the pipeline; be dehydrated of water vapor sufficiently to prevent the formation of methane hydrates within the gas processing plant or pipeline; contain no more than trace amounts of components such as hydrogen sulfide, carbon dioxide, nitrogen, and water vapor [10].

If natural gas is transported in the form of liquefied natural gas (LNG), during LNG production, the liquefaction process involves condensation of natural gas into liquid form by cooling it to approximately −163 °C (−260 °F). The natural gas fed into the LNG plant has to be treated to remove water, hydrogen sulfide, carbon dioxide and other components that will freeze under the low temperatures needed for storage or be destructive to the liquefaction facility.

Moreover, with the advent of sustained higher natural gas prices and declining reserves, and as technology and geological knowledge advances, so-called "unconventional" natural gas sources are coming to market. Although, there are many different sources of "unconventional" natural gas today, one common characteristic of all is the higher concentration of acid gases compared to "conventional" natural gas sources. Therefore, as technology advances, large amounts of off-spec natural gas becomes available that does not meet pipeline quality without some sort of adjustment. This off-spec gas has to be processed to meet the requirements before being pumped into pipelines.

Hence, to transport natural gas from fields to consumers by any means or to make "unconventional" natural gas available to consumers, it is required to reduce the concentration of undesired molecules. Two methods exist to achieve this goal: purification and blending. The process of purification of natural gas to pipeline gas quality levels is quite complex, highly capital-intensive and usually involves four main processes to remove the various impurities: oil and condensate removal, water removal, separation of natural gas liquids, sulfur and carbon dioxide removal. These processes become more complex and therefore more expensive, as the concentration of the impurities increases in the natural gas being purified and increases the cost of the natural gas for consumers while reducing the profits of the production companies. Detailed information about purification processes can be found in Guo and Ghambalor (2005) [17].

Blending of natural gas from different fields or wells with different different concentrations of hydrocarbons, carbon dioxide, hydrogen sulfide and nitrogen is a cheaper method. However, it does not always guarantee achievement of the desired concentration levels. But, blending can be utilized to reduce the concentrations of undesired molecules before purification processes in order to reduce the cost of the purification. The opportunity for blending different sources of natural gas comes into the picture especially when the natural gas production infrastructure is being developed. When new wells or fields are being developed, it is possible to construct the pipeline system such that the gas from different wells are mixed together to satisfy the requirements for different qualities. However to develop the pipeline system optimally, a stochastic version of the pooling problem where the quality parameters in the wells are not known exactly has to be solved. Although advancing technology provides the necessary tools to predict the gas content of natural gas in different fields during the exploration stage, the impurities in the natural gas are still uncertain before drilling the well. Thus, stochastic programming principles have to be used to achieve an optimum solution to the infrastructure development problem. As mentioned, in this study, one of the important reasons to develop a BD algorithm to solve pooling problems is the adaptability of decomposition algorithms to stochastic programming. More information about the stochastic pooling problem is given in *Chapter 5* of this thesis.

1.3 Benders Decomposition for the Global Solution of Pooling Problems

As explained in *Section 4.1*, BD and GBD algorithms are proposed to solve multi-variable nonlinear programs and take at least one of the variables appearing in bilinear terms as fixed to solve the problem. In the literature, in order to solve pooling problems to global optimality with GBD, only one of the variables appearing in the bilinear terms was taken as the complicating variable. With this approach, even for relatively simple pooling problems, the GBD algorithm tends to generate suboptimal points and does not guarantee to attain a global optimum. For instance, Floudas & Aggarwal (1990) [11] use the GBD algorithm to solve pooling problems by fixing the pool quality variables as the complicating variables and decompose the original pooling problem into a primal problem and a relaxed master problem. But, their strategy is only successful for Haverly's pooling problem (which is a very simple problem) and in general, it offers no guarantee for global optimality. This GBD algorithm may converge to a local minimum, a local maximum, or even a non-KKT point.

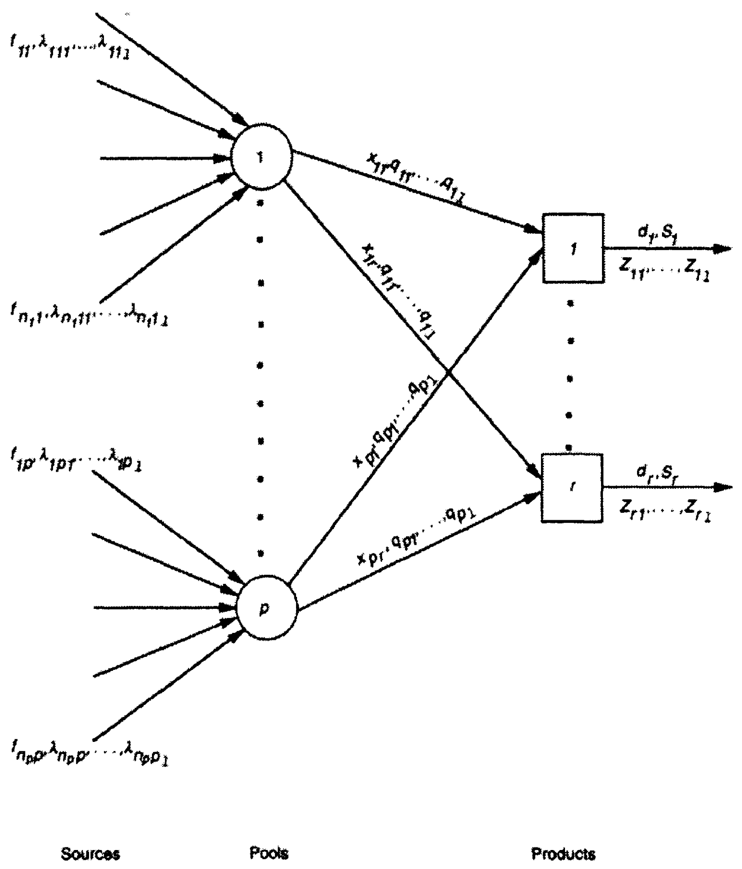
In this study, both of the variables appearing in bilinear terms are treated as the complicating variables and by doing so the problem can be formulated such that the Benders Decomposition algorithm can be used instead of Generalized Benders Decomposition and hence satisfaction of the Property P becomes unnecessary and, unlike Floudas & Aggarwal (1990) [11], convergence is guaranteed and can be proven directly from Benders (1962) [5]. Fixing both variables in the bilinear terms provides a linear program in the first stage of the algorithm and smaller (and hence easier to solve) bilinear second stage problems. For comparison of the proposed algorithm with a well known and respected global solver, The Branch And Reduce Optimization Navigator (BARON) is selected [42]. BARON is a computational software developed by Nikolaos Sahinidis and Mohit Tawarmalani for solving nonconvex optimization problems to global optimality. Purely continuous, purely integer, and mixed-integer nonlinear problems can be solved with this software. BARON combines constraint propagation, interval analysis, range (domain) reduction and duality with enhanced Branch-and-Bound (B+B) concepts to solve optimization problems globally. In general, BARON is a nonconvex optimization solver using range reduction methods integrated into the B+B algorithm with advanced relaxation techniques [39]. In this study, in order to check global optimality and validity of the approach, various example pooling problems are solved with both the proposed BD algorithm and BARON. In addition, the solution times of the BD algorithm and BARON are compared to study the overall performance of BD for pooling problems.

Chapter 2

Problem Definition

In general, the pooling problem can be stated in a general way as follows: given several streams with different qualities, what quantities of each must be mixed in intermediate pools in such a way that the quality and quantity requirements of all demands are satisfied. A pooling network consists of several source nodes, pools and end-product nodes. Each source node has a unique quantity of available supply and quality components. Sources are linked to pools and each pool represents a blend from various source nodes and the quality component of a pool is a function of the levels of in-flows from sources and their qualities. Pools are linked to product nodes and each pool's total in-flow is equal to its total out-flow (mass balance). The quality component of a product node is also a function of the levels of in-flows from sources and pools and their qualities. Product nodes are subject to specific demand and quality requirements. In practice, because of the presence of a large number of supply nodes, pools, qualities and end-products, pooling problems are more complicated than expected. Usually, each stream into a pool can have more than one quality component. The pooling problem then becomes a problem with multiple component qualities and every end product has to be in the range of expected quality specifications for each of the quality components. The existence of multiple pools, products and qualities creates hundreds of bilinear terms even for a relatively small problem and therefore a large number of suboptimal local minima can also exist, hence the need for a global optimization approach increases.

Figure 2-1 shows a general pooling problem with n sources, p pools, r end-products and l quality parameters. In this representation, i is the index for sources, j is the index for pools, k is the index for products and w is the index for qualities. In addition, f_{ij} is the variable for the total flow from the i^{th} source into pool j; q_{jw} is the variable for the w^{th} quality component of pool j and x_{jk} is the variable for the total flow from the j^{th} pool to product k. Also parameters in this representation are listed in Table 2.1.



$f_{11}, \lambda_{111}, \ldots, \lambda_{11l}$
$f_{n_1 1}, \lambda_{n_1 11}, \ldots, \lambda_{n_1 1l}$
$f_{1p}, \lambda_{1p1}, \ldots, \lambda_{1pl}$
$f_{n_p p}, \lambda_{n_p p1}, \ldots, \lambda_{n_p pl}$
1
p
$x_{11}, q_{11}, \ldots, q_{1l}$
$x_{1r}, q_{11}, \ldots, q_{1l}$
$x_{p1}, q_{p1}, \ldots, q_{pl}$
$x_{pr}, q_{p1}, \ldots, q_{pl}$
1
r
d_1, S_1
$z_{11}, \ldots, z_{1l}$
d_r, S_r
$z_{r1}, \ldots, z_{rl}$
Sources
Pools
Products

Figure 2-1: Graphical representation of a general pooling problem.

Parameter	Definition
c_{ij}	cost of the flow from the i^{th} source into pool j
d_k	unit price of product k
l	total number of component qualities
N_j	set of sources entering pool j
p	total number of pools
r	total number of end-products
S_k	demand requirement for product k
Z_{kw}	w^{th}quality requirement for product k
λ_{ijw}	w^{th} quality component of the flow from the i^{th} source into pool j

Table 2.1: Parameters of the pooling problem and corresponding definitions

Then, a mathematical representation of the general pooling problem that is represented in Figure 2-1 becomes:

$$\min_{f,x,q} \quad \sum_{j=1}^{p} \sum_{i \in N_j} c_{ij} f_{ij} - \sum_{k=1}^{r} d_k \sum_{j=1}^{p} x_{jk} \tag{2.1}$$

$$\text{s.t.} \quad \sum_{i \in N_j} f_{ij} - \sum_{k=1}^{r} x_{jk} = 0, \qquad j = 1, \ldots, p \tag{2.2}$$

$$q_{jw} \sum_{k=1}^{r} x_{jk} - \sum_{i \in N_j} \lambda_{ijw} f_{ij} = 0, \qquad j = 1, \ldots, p; \;\; w = 1, \ldots, l \tag{2.3}$$

$$\sum_{j=1}^{p} x_{jk} - S_k \leq 0, \qquad k = 1, \ldots, r \tag{2.4}$$

$$\sum_{j=1}^{p} q_{jw} x_{jk} - Z_{kw} \sum_{j=1}^{p} x_{jk} \leq 0, \qquad k = 1, \ldots, r; \;\; w = 1, \ldots, l \tag{2.5}$$

$$f_{ij}^L \le f_{ij} \le f_{ij}^U, \qquad i=1,\ldots,n_j;\ j=1,\ldots,p$$

$$q_{jw}^L \le q_{jw} \le q_{jw}^U, \qquad j=1,\ldots,p;\ w=1,\ldots,l$$

$$x_{jk}^L \le x_{jk} \le x_{jk}^U, \qquad j=1,\ldots,p;\ k=1,\ldots,r$$

In this formulation, the objective function represents the difference between the cost of the flow from the source nodes and the returns from selling the end-products. (2.2) represents the mass balances for each pool. (2.3) expresses the mass balance for each quality component. (2.4) ensures that the flows to each end-product node do not exceed the demands. (2.5) enforces that the quality requirements are satisfied at each end-product node. More information about the formulation can be found in Audet et. al. (2004) [3].

In addition, in the literature there are some widely known and solved pooling problem formulations which are just special cases of this general representation. These problems are solved in numerous papers about the pooling problem and hence their global optimal solutions are known and there are different global optimization algorithms, which have already been proven to converge, available for them, which can be used for comparison with the BD algorithm. Thus, these problems can be used as examples to check the validity and performance of the proposed BD algorithm. The pooling problem was first investigated by Haverly (1978-1979) [19, 20]. Therefore, Haverly's pooling problem is one of these widely known pooling problems and it consists of only 3 source nodes, 1 pool and 2 demand nodes as shown in Figure 2-2. Figure clearly represents that three feed streams are available (f_1, f_2 and f_3), with the costs of \$6, \$16 and \$10 (per unit) respectively. There are also two output streams with the prices of \$9 and \$15 (per unit) respectively.

In Haverly's pooling problem, there is a single pool which receives supplies from two different sources which have different sulfur qualities. A third supply is not connected to

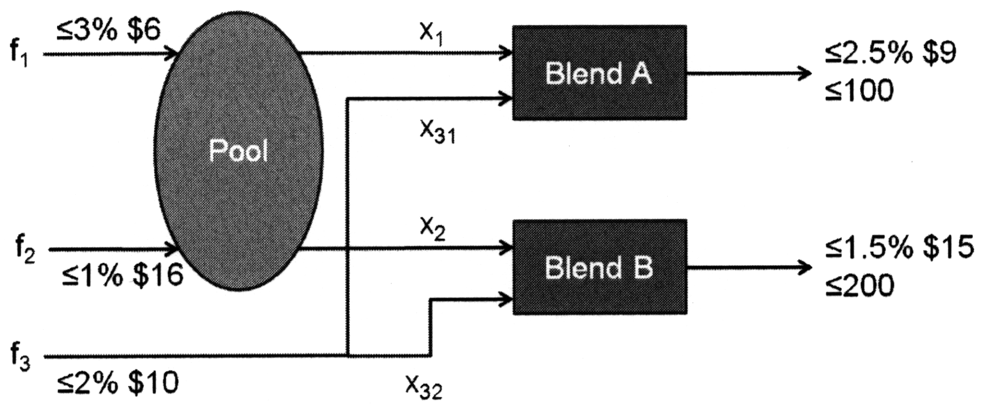


Figure 2-2: Haverly's pooling problem

the pool but is directly feeding the two end-product nodes. The quality parameters for the streams going into the pool are 3% for the first source node, 1% for the second and 2% for the third node. The blending of flows from the pool and from the third supply node produces products 1 and 2, which are subjected to sulfur quality requirements of 2.5% and 1.5% respectively. The maximum demands for products 1 and 2 are 100 and 200 respectively. Then the mathematical formulation of Haverly's pooling problem is:

$$\min_{f,x,q} \quad 6f_{11} + 16f_{21} + 10f_{12} - 9(x_{11} + x_{21}) - 15(x_{12} + x_{22}) \tag{2.6}$$

$$\text{s.t.} \quad f_{11} + f_{21} - x_{11} - x_{12} = 0 \tag{2.7}$$

$$f_{12} - x_{21} - x_{22} = 0 \tag{2.8}$$

$$q(x_{11} + x_{12}) - 3f_{11} - f_{21} = 0 \tag{2.9}$$

$$qx_{11} + x_{21} - 2.5(x_{11} + x_{21}) \leq 0 \tag{2.10}$$

$$qx_{12} + 2x_{22} - 1.5\,(x_{12} + x_{22}) \leq 0 \tag{2.11}$$

$$x_{11} + x_{21} \leq 100$$

$$x_{12} + x_{22} \leq 200$$

where q is the sulfur quality of the pool output, f_{ij} are the quantities of supplies, x_{11} and x_{12} are the magnitude of flows from pool to end-products and x_{21} and x_{22} are the magnitude of flows from the third source node to end-products. Similar to the general formulation, the objective function represents the difference between the cost of the flow from the source nodes and the returns from selling the end-products. (2.7) and (2.8) represent the mass balance. (2.9) represents the sulfur mass balance. (2.10) and (2.11) expresses the quality restrictions on the products; (2.12) and (2.13) ensure that the flows to each end-product node do not exceed the demands. GAMS implementation of Haverly's pooling problem is provided in *Appendix A*.

As it can be realized, although the objective function is linear, the bilinear terms in (2.9), (2.10) and (2.11) introduce nonconvexities in the problem (which are enough to make this problem nonconvex) causing multiple local optima. Therefore, local nonlinear programming (NLP) solution algorithms (well known examples are SNOPT, MINOS, CONOPT, etc.) may provide suboptimal solutions which are usually not useful in any practical sense and hence it is necessary to explore global optimization techniques in pooling problems.

In this study, also Adhya's [1] and Foulds' [12] pooling problems are solved to test the BD algorithm. Since they are just special versions of the general formulation, it is not necessary to give explicit formulations for those problems, just the numbers of pools, sources, qualities and end-products should be enough to produce an explicit formulation by using the general problem formulation. For Adhya's problem, the number of pools is 7; the number of sources is 8, the number of qualities is 4 and the number of end-products is 4; in Foulds' problem, the number of pools is 8; the number of sources is 14, the number of qualities is 1 and the number of end-products is 6. More information for both of these example problems including quality specs, demand requirements, cost coefficients and GAMS implementations are given in *Appendix A*.

2.1 The p-, q- and pq-Formulations

The formulation of the pooling problem given above was first proposed by Haverly (1978) [19] and referred to as the p-formulation. A distinct, but equivalent formulation is proposed by Ben-Tal et al. (1994) [6] which is called the q-formulation. Ben-Tal et al. (1994) [6] derives the q-formulation of the pooling problem by introducing new variables t_{ij}satisfying the relationship: $f_{ij} = t_{ij} \sum_{k=1}^{K} x_{jk}$.

It can be easily shown that the p- and q-formulations are equivalent. However, the main advantage of the q-formulation is that, in many applications, the number of extreme points of the simplex containing the variables t_{ij} is much smaller than the number of extreme points of the hypercube q_{jw}. This advantage is exploited algorithmically by Ben-Tal et al. (1994) [6]. Figure 2-3 shows the q-formulation of the Haverly's pooling problem.

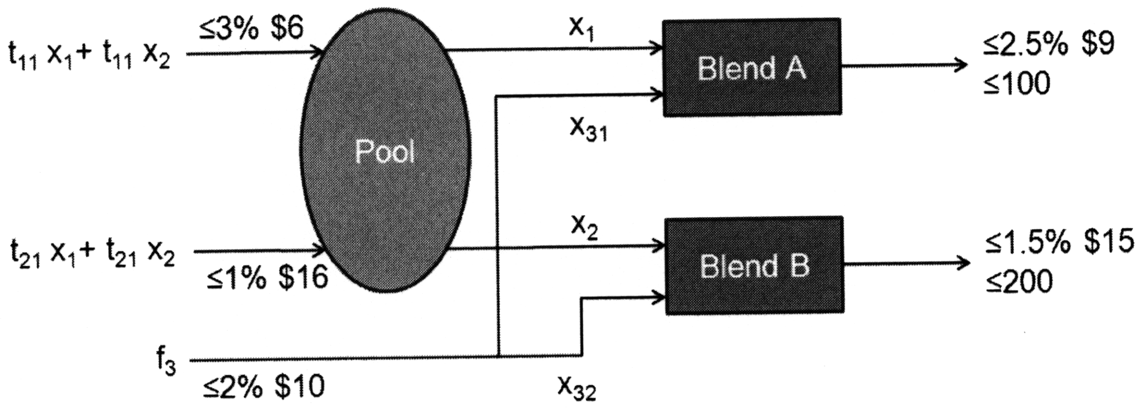


Figure 2-3: The q-formulation of the Haverly's pooling problem

Tawarmalani & Sahinidis (2002) [45] constructs the pq-formulation by adding the following constraint to the q-formulation:

$$\sum_{i=1}^{I} t_{ij} x_{jk} = x_{jk}, \qquad j = 1, ..., p; \; k = 1, ..., r \tag{2.12}$$

Figure 2-4 illustrates the pq-formulation of the Haverly's pooling problem. As can be realized from this example, the newly added constraints are redundant and don't change the feasible region. However, the main point of interest in the pq-formulation is the tightness of the convex relaxations relative to the other two formulations. Tawarmalani&Sahinidis (2002) [45] prove that the pq-formulation provides much tighter convex relaxations compared to the p- and q-formulations.

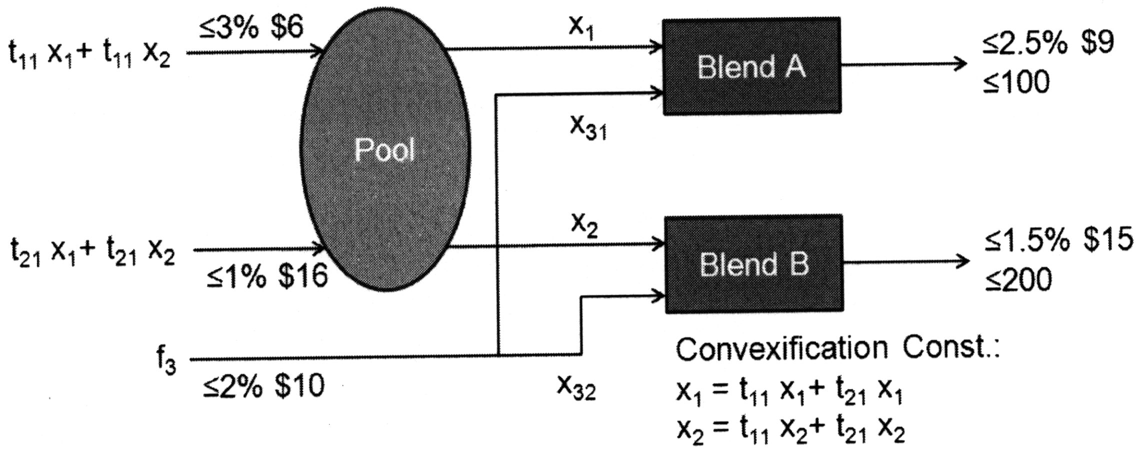

$t_{11}\, x_1 + t_{11}\, x_2$
≤3% $6
$t_{21}\, x_1 + t_{21}\, x_2$
≤1% $16
f_3
≤2% $10
Pool
x_1
x_{31}
x_2
x_{32}
Blend A
Blend B
≤2.5% $9
≤100
≤1.5% $15
≤200
Convexification Const.:
$x_1 = t_{11}\, x_1 + t_{21}\, x_1$
$x_2 = t_{11}\, x_2 + t_{21}\, x_2$

Figure 2-4: The pq-formulation of the Haverly's pooling problem

Tawarmalani & Sahinidis (2002) [45] claim that for all example pooling problems, the pq-formulation decreases solution times drastically and solution times of example pooling problems (solved with BARON) presented in [45] to prove this statement. However, in [45] the chart provided for comparison of three formulations in terms of solution times does feature solutions from different references and therefore with different processors and hence the validity of their claims can be questioned. Therefore, it is decided to model three example pooling problems (Haverly's [18], Foulds' [12] and Adhya's [1]) with all

Problem	**p-**	**q-**	**pq-**
Haverly	0.02	0.016	0.01
Foulds	1.89	1.46	1.25
Adhya	9.27	7.71	6.12

Table 2.2: Solution times for the p-,q- and pq- formulations in example problems (in seconds).

three different formulations, and these three example problems are solved in GAMS 22.5 [13] with BARON 7.8 [42] used as the global optimization solver. When comparisons are done with a computer having Intel 3.20 GHz Xeon processor, results show that the solution times do not differ immensely as presented in Table 2.2. But still the pq-formulation has the lowest solution times, hence the pq-formulation is featured in this study to formulate the pooling problems to be solved.

Chapter 3

Literature Review

3.1 Deterministic Pooling Problem

Various optimization procedures for the pooling problem have been proposed in the literature. These solution procedures can be classified based on their convergence to either a local or a global optimum. The first algorithm for the pooling problem was suggested by Haverly (1978-1979) [19, 20]. Haverly's approach was based on the idea of using recursion to solve the pooling problem. A recursive approach guesses the value of the pool qualities. These values for the pool qualities converts the pooling problem into a linear program in the flow variables. The actual values of the pool qualities can then be calculated from the values of the flow variables that are obtained by solving the linear program. The process continues until the actual values of the qualities are within a range of tolerance from the guessed values. The main drawback in using any form of recursive method for the pooling problem is that often the algorithm does not converge to a solution, and when it converges, it converges only to a local minimum, a local maximum, or even a non-KKT point. In addition, as the number of pools and end-products increases, recursive methods tend to become more unstable, resulting in computational difficulties.

Successive Linear Programming (SLP) approaches which solve nonlinear problems as a sequence of linear programs are also widely used. Lasdon (1979) [31] proposes an algorithm based on SLP technique. These approaches also do not guarantee global optimal solutions and may converge to even a non-KKT point .

As in the case of GBD, decomposition methods are based on the observation that a difficult problem can be converted to an easier problem by fixing values of certain variables. In the case of the pooling problem, for example, fixing the pool quality variables converts it into a linear program. By using this approach, Floudas & Aggarwal (1990) [11] suggest an algorithm based on fixing the pool quality variables as the complicating variables and decomposing the original pooling problem into a primal problem and a relaxed master problem and iterating between these problems based on the GBD algorithm until the termination conditions are satisfied. Although their decomposition strategy is successful for the problems suggested by Haverly, in general it offers no guarantee for global optimality. This GBD algorithm may converge to a local minimum, a local maximum, or even a non-KKT point. Visweswaran & Floudas (1996) [50] propose a GOP algorithm for solving the pooling problem. The algorithm was proven to terminate finitely with a global optimum. Using this algorithm, the authors were able to solve three cases of the Haverly problem. It is also reported that a single pool, five-product problem, with each stream having two quality components is solved to global optimality using this algorithm. Large-scale pooling problems, generated randomly, having up to 5 pools, 5 products, and 30 qualities, were solved by Androulakis et al. (1996) [2] using a different implementation of the GOP algorithm.

Branch-and-bound (B+B) methods for pooling and blending problems have been suggested by different authors. These methods usually differ in the relaxations used to provide valid lower bounds to the global optimum. Foulds et al. (1992) [12] use a procedure which involves replacing the bilinear terms in the pooling problem by their McCormick (1983) [35] concave and convex envelopes. The nonlinear pooling problem can be relaxed to a linear programming problem, the solution of which provides a lower bound on the global optimal solution. The B+B procedure proceeds by partitioning the feasible set and relaxing on each partition. It is quite obvious that by replacing each bilinear term by its concave or convex envelope introduces a relaxation, but this relaxation also tends to zero as the partitions get finer and the algorithm converges to the global optimal solution. Using this approach, Foulds et al. (1992) [12] were able to solve single-quality problems, with the largest problem having 8 pools and 14 products. The constraints which provide the convex and concave envelopes of the problem at a specific node of the B+B tree are not in general valid for other nodes of the tree. Thus, the convex and concave envelopes have to be generated at each node of the B+B tree. However, the McCormick relaxation requires four linear constraints to provide the envelopes for each bilinear term in the problem. Hence, as the number of pools, products, or component qualities increase, the size of the linear program to be solved at each node of the B+B tree also increases.

Ben-Tal et al. (1994) [6] propose another lower-bounding procedure based on Lagrangian relaxation of another formulation of the pooling problem (explained in the previous chapter as the q-formulation). In this paper, a B+B algorithm which partitions the feasible set of the pooling problem is provided and it is shown that this approach can reduce the duality gap between a nonconvex problem and its dual. Later it is also proven that for partially convex problems such as the pooling problem, under certain regularity conditions, this approach can reduce the duality gap between the primal and the dual to zero.

Adhya et al. (1999) [1] use yet another formulation of the pooling problem (explained in the previous chapter as the pq-formulation). The authors provide a new Lagrangian relaxation approach for developing lower bounds for the B+B to solve the pooling problem and it is proven that the Lagrangian relaxation approach provides tighter lower bounds than the standard linear-programming relaxations used in global optimization algorithms and hence guarantees faster convergence speeds.

3.2 Infrastructure Development and the Stochastic Pooling Problem

For the infrastructure development problem, most of the literature is on oil production planning and unfortunately there is only small amount of literature dealing specifically with natural gas production planning, but usually modeling and solution strategies for oil and gas infrastructure development problems are very similar. Hence, no distinction is made between the oil and gas production planning literature, and the literature for oil production planning is also included to this review.

Most of the available literature for planning of oil and gas field infrastructures uses a deterministic approach without considering how uncertainty affects the overall system (Iyer, Grossmann, Vasantharajan & Cullick (1998) [22]; Van den Heever & Grossmann (2000) [47]; Van den Heever & Grossmann (2001) [48]; Barnes, Linke & Kokossis (2002) [4]; Lin & Floudas (2003) [32]; Ortiz-Gomez, Rico-Ramirez & Hernandez-Castro (2002) [37]). For a recent review of the existing literature on deterministic approaches for these problems, refer to Van den Heever & Grossmann (2001) [48]. Recently, there has been some work that considers uncertainty in the infrastructure development problem. Jonsbraten (1998) [24] presents an MILP model for optimizing the investment and operation decisions for an oil-field under uncertainty in oil prices. The author uses the Progressive Hedging Algorithm to solve the problem. Jonsbraten (1998ii) [25] presents an implicit enumeration algorithm for the sequencing of oil wells under uncertainty in oil reserves. The decision models for both these papers include investment and operational decisions for one field only. Jornsten (1992) [27] uses Lagrangian relaxation of constraints to solve a stochastic program for the sequencing of gas fields under uncertainty in future demands. The author assumes that production profiles and capacities of platforms have already been fixed. Haugen (1996) [18] proposes a single parameter representation for uncertainty in the size of reserves and incorporates it into a Stochastic Dynamic Programming model for scheduling of petroleum fields. This work also assumes that the only decisions that need to be made are regarding the scheduling of fields. Meister, Clark, and Shah (1996) [36] present a model to derive exploration and production strategies for one field under uncertainty in reserves and future oil price. The model is analyzed using stochastic control techniques. Lund (2000) [33] presents a stochastic dynamic programming model for evaluating the value of flexibility in offshore development projects under uncertainty in future oil prices and in the reserves of one field. Jonsbraten (1998iii) [26] discusses an interesting problem dealing with planning of oil field development. A situation is considered where two surface lease owners with access to the same oil reservoir bargain their shares of production. The author assumes a mixed-integer optimization model and uses game theory. Recently, there has also been some work using real options based approaches (Dias, 2001 [9]) for planning of oil and gas field developments under uncertainty.

Based on the dependence of the stochastic process on the decisions, Jonsbraten (2001) [27] and Goel & Grossmann (2004) [15] classify uncertainty in planning problems into two categories: project exogenous uncertainty and project endogenous uncertainty. Problems where the stochastic process is independent of the project decisions are said to have project exogenous uncertainty. For these problems, the scenario tree is fixed and does not depend on the decisions. Hence the most relevant characteristic of this kind of stochastic programming model is that its formulation assumes a given scenario tree. The uncertainty in gas prices in a planning problem similar to the one described here is an example of project exogenous uncertainty. For recent reviews on models and solution techniques for stochastic programs with project exogenous uncertainty, please refer to Kall and Wallace (1994) [29] and Birge and Louveaux (1997) [7]. Problems where the project decisions influence the stochastic process are said to possess project endogenous uncertainty. A gas production planning problem with uncertainty in gas reserves is included in this category. This is because the uncertainty in gas reserves of a field is resolved only if, and when, exploration or investment is done at the field. If no action is taken, the uncertainty in the field does not resolve at all. For problems with project endogenous uncertainty, the scenario trees are decision-dependent. This leads to difficulties in defining the model because, traditionally, the stochastic programming literature has relied on the assumption of given scenario trees. Hence, there is very little literature dealing with problems having process endogenous uncertainty. An intensive literature search provides only four papers (Pflug, (1990) [38]; Jonsbraten, Wets & Woodruff, (1998) [24]; Jonsbraten, (1998ii) [25]; Goel & Grossmann, (2004) [15]) which deal with project endogenous uncertainty.

A Literature review clearly shows that none of the literature about the infrastructure development problem considers the concentrations of the impurities in the natural gas produced as a source of uncertainty, but as mentioned in the first chapter, because of the contractual agreements, regulations and the pipeline requirements, the production company has to adjust the composition of the gas within some limits to sell it, and the composition of gas is unknown when infrastructure is being developed. To blend gas from different fields, while the infrastructure is being developed, the pipeline system has to be constructed to allow the gas from different wells to be mixed to satisfy the requirements. Therefore, to develop the value chain optimally, a stochastic version of the pooling problem where the quality parameters in the wells are unknown has to be solved. Therefore, gas quality uncertainty in the infrastructure development problem is selected in this study as the first step to construct and solve a realistic model of the whole infrastructure development problem with more realistic or less assumptions than the literature until now.

Another important assumption in the literature is that the effect of the contractual framework is not considered. However, in most fields natural gas cannot be produced unless a contractual demand exists and in addition the rules given in contracts and also in governmental regulations need to be taken into account to reach a realistic model of the system. In addition, there are other important assumptions: no expansion in capacity of a platform is considered; in most of the references production rate decreases linearly in time; flow models and reservoir models are assumed as linear and more importantly effects of contractual framework are neglected.

Chapter 4

BD Algorithm for Deterministic Pooling Problem

4.1 Introduction of Benders Decomposition Algorithm

The Benders Decomposition algorithm was originally proposed by Benders in 1962 [5] for nonlinear, nonconvex mixed variables programming problems of the form:

$$\max_{x,y} \quad c^{\mathrm{T}}x + f(y) \tag{4.1}$$

$$\text{s.t.} \quad Ax + F(y) \leq 0 \tag{4.2}$$

$$x \in X \subset \mathbb{R}^{n_x}, y \in U \subset \mathbb{R}^{n_y}$$

where y is a vector of complicating variables, since the problem above can be solved more easily when y is fixed constant. In other words, for fixed y, this problem separates into a number of smaller problems each having only subsets of x as variable or the problem assumes a special structure, such as a linear program as in the case of the pooling problem or the problem is converted into a convex program. In these cases, by fixing y, a simpler primal problem can be solved and a relaxed master problem is solved to generate valid lower bounds and the algorithm converges to the global optimum by iterating between these problems. In practice, the BD algorithm decomposes problem into two smaller problems: primal problem (linear program) and relaxed master problem (nonlinear program in bilinear problems). The primal problem is used to find the upper bound (UBD); the relaxed master problem is used to find the lower bound (LBD). When LBD$\geq$UBD, algorithm terminates.

On the other hand, the Generalized Benders Decomposition algorithm is first proposed by Geoffrion (1972) [14] and also based on Benders Decomposition, but it is proposed to solve more general form of nonconvex nonlinear programs of the form:

$$\max_{x,y} \quad f(x,y) \tag{4.3}$$

$$\text{s.t.} \quad g(x,y) \leq 0 \tag{4.4}$$

$$x \in X \subset \mathbb{R}^{\mathrm{n_x}}, y \in U \subset \mathbb{R}^{\mathrm{n_y}}$$

where y is a vector of complicating variables, again, in the sense that it is much easier to solve in x when y are held fixed. However, the problem to be solved has to satisfy a property called "Property P", unlike Benders Decomposition. Basically, the problem to be solved has to be formulated such that for every $\lambda \geq 0$, (where λs are the Lagrange multipliers), the infimum of $f(x,y) + \lambda^T g(x,y))$ over X can be taken essentially independently of y, so that the constraints in the relaxed master problem can be obtained explicitly with little or no more effort than is required to evaluate it at a single value of y.

As it is known, bilinear terms are formed by the multiplication of two variables of the problem and these bilinear terms introduce nonconvexities to the problem. If the nonconvexities in the problem are only introduced by the bilinear terms, as in the case of pooling problems, it is possible to treat the whole bilinear terms as a complicating variable in the BD algorithm as opposed to fixing only one of the variables in bilinear terms as the complicating variable. Fixing the bilinear terms yields constant parameters. Then, the general formulation of the pooling problem can be written (consistently to the notation given in *Chapter 2*) as:

$$\max_{f,y} \quad c^{\mathrm{T}} f + d^{\mathrm{T}} y \tag{4.5}$$

$$\text{s.t.} \quad Af + F(y) \leq 0 \tag{4.6}$$

$$f \in F \subset \mathbb{R}^{\mathrm{n_f}}, y \in U \subset \mathbb{R}^{\mathrm{n_y}}$$

where c is the cost vector, d is the price vector, f is the input flow vector and y is the vector for the bilinear terms which is equal to $q^{\mathrm{T}}x$ (q is the vector of quality variables and x is the vector for flow from the pools to demands as explained in *Chapter 2*).

Therefore, the BD algorithm can be applied to pooling problems and is guaranteed to converge to the global optimum (as proved in the next section) when both of the bilinear terms are taken as the complicating variables. Obviously, in the BD implementation, the primal problem becomes a linear program which is obviously convex and the relaxed master problem is a nonconvex NLP where a global solver such as BARON can be used to obtain global optimal solutions. Using these global optimal solutions to iterate, it is possible to generate valid cuts that converge. Hence, this approach is expected to converge to the global optimum of the pooling problem with Benders Decomposition reliably.

Then, for instance, in Haverly's pooling problem, the primal problem can be formulated as:

$$\min_{f,x} \quad 6f_{11} + 16f_{21} + 10f_{12} - 9\left(x'_{11} + x_{21}\right) - 15\left(x'_{12} + x_{22}\right) \tag{4.7}$$

$$\text{s.t.} \quad f_{11} + f_{21} - x'_{11} - x'_{12} = 0 \tag{4.8}$$

$$f_{12} - x_{21} - x_{22} = 0 \tag{4.9}$$

$$q'\left(x'_{11} + x'_{12}\right) - 3f_{11} - f_{21} = 0 \tag{4.10}$$

$$q'x'_{11} + x_{21} - 2.5\left(x'_{11} + x_{21}\right) \leq 0 \tag{4.11}$$

$$q'x'_{12} + 2x_{22} - 1.5\left(x'_{12} + x_{22}\right) \leq 0 \tag{4.12}$$

where q', x'_{11} and x'_{12} are constant parameters which are assigned as the fixed complicated variables. Therefore, bilinearities in the primal problem disappear and it becomes a linear program and therefore, it is convex. However, the relaxed master problem is still a bilinear program and it is obviously a nonconvex NLP. Hence, still the relaxed master problem has to be solved with a global solver such as BARON. But, the potential benefit of utilizing BD algorithm might be to solve number of smaller problems (the relaxed master problems) with the B+B procedure (such as BARON) instead of solving one huge problem with the B+B. B+B based algorithms are exponential-time algorithms. In other words, as the problem size increases, solution times of B+B algorithms increases exponentially. Therefore, instead of solving a problem with large number of variables, solving number of problems with small number of variables can be quicker in terms of the solution times.

As mentioned, the primal problem becomes a linear program and general formulation of the primal problem becomes:

P:

$$\min_{f,x} \quad \sum_{j=1}^{p} \sum_{i\in N_j} c_{ij} f_{ij} - \sum_{k=1}^{r} d_k \sum_{j=1}^{p} x'_{jk} \tag{4.13}$$

$$\text{s.t.} \quad \sum_{i\in N_j} f_{ij} - \sum_{k=1}^{r} x'_{jk} = 0 \qquad j = 1,\ldots,p \tag{4.14}$$

$$q'_{jw} \sum_{k=1}^{r} x'_{jk} - \sum_{i\in N_j} \lambda_{ijw} f_{ij} = 0, \qquad j = 1,\ldots,p;\ \ w = 1,\ldots,l \tag{4.15}$$

$$\sum_{j=1}^{p} x'_{jk} - S_k \le 0 \qquad k = 1,\ldots,r \tag{4.16}$$

$$\sum_{j=1}^{p} q'_{jw} x'_{jk} - Z_{kw} \sum_{j=1}^{p} x'_{jk} \le 0, \qquad k = 1,\ldots,r;\ \ w = 1,\ldots,l \tag{4.17}$$

$$f_{ij}^{L} \le f_{ij} \le f_{ij}^{U} \qquad i = 1,\ldots,n_j;\ \ j = 1,\ldots,p$$

where q'_{jw}, x'_{jk} are the fixed parameters. And as it is seen, also in a general pooling problem formulation, the primal problem is a linear program and therefore, it is convex.

In addition, the relaxed master problem can be formulated as:

R:

$$\min \quad \eta \tag{4.18}$$

$$\text{s.t.} \quad \eta \ge \inf(F + \lambda^{\mathrm{T}} g_i) \tag{4.19}$$

$$\mu^{\mathrm{T}} g_i \le 0 \tag{4.20}$$

where λ is the vector of Lagrange multipliers, μ is the vector of multipliers for the feasibility problem, F is the objective function and g_i are the constraint functions, which means:

$$F = \sum_{j=1}^{p} \sum_{i \in N_j} c_{ij} f_{ij} - \sum_{k=1}^{r} d_k \sum_{j=1}^{p} x_{jk} \tag{4.21}$$

$$g_1 = \sum_{i \in N_j} f_{ij} - \sum_{k=1}^{r} x_{jk}, \qquad j = 1, \dots, p \tag{4.22}$$

$$g_2 = q_{jw} \sum_{k=1}^{r} x_{jk} - \sum_{i \in N_j} \lambda_{ijw} f_{ij}, \qquad j = 1, \dots, p; \;\; w = 1, \dots, l \tag{4.23}$$

$$g_3 = \sum_{j=1}^{p} x_{jk} - S_k \leq 0 \qquad k = 1, \dots, r \tag{4.24}$$

$$g_4 = \sum_{j=1}^{p} q_{jw} x_{jk} - Z_{kw} \sum_{j=1}^{p} x'_{jk} \leq 0, \qquad k = 1, \dots, r; \;\; w = 1, \dots, l \tag{4.25}$$

Then, the proposed BD algorithm for pooling problems is presented in Algorithm 1 and also flowchart of the algorithm is provided in Figure 4-1. As Figure 4-1 represents, basically, The primal problem provides the upper bound value (UBD) whereas the relaxed master problem provides the lower bound value (LBD) and when LBD$\geq$UBD, algorithm terminates.

By using this algorithm, different pooling problems from the literature are solved and validity and speed of this approach is tested versus algorithms which guarantees global optimal solution such as BARON. However, before testing the algorithm, the first step is to prove its convergence to global optimum.

4.2 Proof of Convergence

To prove the convergence of the proposed algorithm, the first step is to show that the pooling problem formulation satisfies the form given by Benders (1962) [5]:

Algorithm 1 Benders decomposition algorithm for global solution of pooling problems

```
{INITIALIZATION}
i (iteration) := 1, UBD := INF, LBD := -INF, p := 0, r := 0;
Select an initial configuration for the variables: $q(i) = q'(i)$ and $x(i) = x'(i)$
{STEP 1: LP PRIMAL PROBLEM}
Solve Problem (P) with $q(i) = q'(i)$ and $x(i) = x'(i)$,
{FEASIBLE PRIMAL}
if Problem (P) with $q(i) = q'(i)$ and $x(i) = x'(i)$ is feasible then,
    Let the solution be $f^\star(i)$, let $p = p+1$ and $\hat{\lambda} = \lambda^p$. ($\hat{\lambda}$ is the corresponding duality multiplier.)
    if $z^*(i) \leq UBD$ then, (where $z^*(i)$ is obj. value of the LP Primal Problem at iteration $i$.)
        {RECORD BETTER SOLUTION}
        $UBD := z^\star(i), x^* := x'(i), f^* := f^\star(i), q^* := q'(i)$. end if
{INFEASIBLE PRIMAL}
if Problem (P) with $q(i) = q'(i)$ and $x(i) = x'(i)$ is infeasible then, $r = r+1$ and $\hat{\mu} = \mu^r$.
end if
{STEP 2: NLP RELAXED MASTER PROBLEM}
if $p=0$ then, solve the feasibility version of the NLP Master Problem.
else, solve
    $\min_{x,q,\eta} \eta$
    s.t $\eta \geq \inf_{x,q}(h(f,x) + (\lambda^j)^T g_i(f,x,q)), \forall j = 1,\ldots,p$
    $(\mu^j)^T g_i(f,x,q) \leq 0, \forall j = 1,\ldots,r$
    where $h(f,x)$ is the objective function and $g_i(f,x,q)$ are the constraints.
    Let the solution be $q^{mp}(i)$ and $x^{mp}(i)$, then $q'(i+1) = q^{mp}(i)$ and $x'(i+1) = x^{mp}(i)$. end if
if $\eta^* \geq LBD$ then,
    {RECORD BETTER SOLUTION}
    $LBD := \eta^\star(i)$. end if
if $LBD \geq UBD$ then, STOP.
else, $i := i+1$, Go to STEP 1. end if
{END OF ALGORITHM}
```

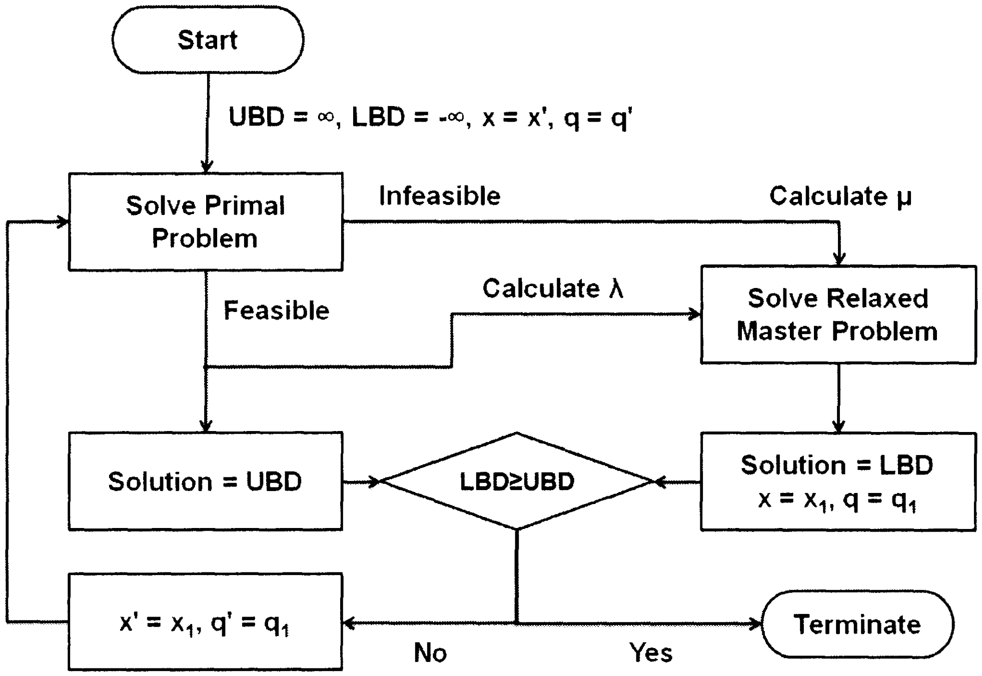


Figure 4-1: Flowchart of the proposed BD algorithm

$$\max_{x,y} \quad c^{\mathrm{T}}x + f(y) \tag{4.26}$$

$$\text{s.t.} \quad Ax + F(y) \leq 0 \tag{4.27}$$

$$x \in X \subset \mathbb{R}^{n_x}, y \in U \subset \mathbb{R}^{n_y}$$

Then, the convergence can be directly proved from Benders (1962). The pooling problem in *Chapter 2* can be reformulated as:

$$\max_{x,f} \quad c^{\mathrm{T}}f + d^{\mathrm{T}}x \tag{4.28}$$

$$\text{s.t.} \quad Af + F(x,q) \leq 0 \tag{4.29}$$

$$f \in F \subset \mathbb{R}^{\mathrm{n_f}}, x \in X \subset \mathbb{R}^{\mathrm{T_x}}, q \in Q \subset \mathbb{R}^{\mathrm{T_q}}$$

The crucial point in satisfying Benders (1962) [5] formulation and hence proving convergence is when the complicating variables are fixed, the resulting formulation has to be a linear program. Since in the proposed algorithm both x and q (bilinear terms) are fixed as complicating variables. The resulting formulation in the pooling problem is:

$$\max_{x,f} \quad c^{\mathrm{T}} f + B \tag{4.30}$$

$$\text{s.t.} \quad Af + C \leq 0 \tag{4.31}$$

$$f \in F \subset \mathbb{R}^{\mathrm{n_f}}$$

where $B = d^{\mathrm{T}}\bar{x}$, $C = F(\bar{x}, \bar{q})$ and $\bar{x}$ and $\bar{q}$ are fixed parameters. It is obvious that the resulting formulation is a linear program and hence it can be concluded that proof of convergence for the proposed BD algorithm can be derived directly from the proof of convergence of Benders original algorithm.

Benders (1962) [5] states that the problem given in the form of (4.28) and (4.29) can be written in the equivalent form by introducing a scalar variable f_0:

$$\max \left\{ f_0 | f_0 - c^{\mathrm{T}} f - d^{\mathrm{T}} x \leq 0, Af + F(x,q) \leq 0, x \geq 0 \right\} \tag{4.32}$$

In other words, $(\bar{f_0}, \bar{f}, \bar{x}, \bar{q})$ is an optimum solution of problem if and only if $\bar{f_0} = c^{\mathrm{T}}\bar{f} + d^{\mathrm{T}}\bar{y}$ and $(\bar{f}, \bar{x}, \bar{q})$ is an optimum solution of the problem.

Theorem 3.1 (Partitioning Theorem for mixed-variables) of Benders (1962) [5] proves that (a) $(\bar{f}, \bar{x}, \bar{q})$ is an optimum solution of problem denoted by (4.29) and (4.30) if and only if $(\bar{f_0}, \bar{f}, \bar{x}, \bar{q})$ is an optimum solution of (4.33). In addition, this theorem shows that (b) if

$(\bar{f},\bar{x},\bar{q})$ is an optimum solution of (4.32), and $\bar{f_0} = c^{\mathrm{T}}\bar{f} + d^{\mathrm{T}}\bar{y}$ then $(\bar{f_0},\bar{x},\bar{q})$ is an optimum solution of (4.32) and $\bar{f}$ is an optimum solution of the linear programming problem:

$$\max\left\{c^{\mathrm{T}}f | Af \leq -F(x,q),\, x \geq 0\right\} \tag{4.33}$$

Also, the same theorem proves that (c) if $(\bar{f_0},\bar{x},\bar{q})$ is an optimum solution of (4.32), then (4.33) is feasible and the optimum value of the objective function in this problem is equal to $\bar{f_0} - F(\bar{x},\bar{q})$. If $\bar{f}$ is an optimum solution of (4.33), then $(\bar{f},\bar{x},\bar{q})$ is an optimum solution of the original problem.

(a), (b) and (c) of the Partitioning Theorem for mixed variables show that a two stage algorithm fixing x and q as complicating variables converges to the global optimum the mixed variable problem in the form of (4.28) and (4.29).

4.3 Implementation

After convergence is proved, the next step is to implement the algorithm. The GAMS language is powerful enough for reasonably complex algorithms. Hence, at first GAMS is chosen to implement the proposed BD algorithm. GAMS Version 22.5 [13] is used as the implementation language and as mentioned before both BARON (Version 7.8) [42] and the BD algorithm is implemented as the global solvers for the example pooling problems. However, because of the reasons explained in the next section, the algorithm is reimplemented in C++ with first using a custom B+B solver to solve the relaxed master problem in the BD implementation, then using a callable BARON C++ library and the results are compared with BARON alone as the global solver of the pooling problem.

4.3.1 GAMS Implementation

In GAMS, both problem specific formulations and the general formulation are implemented in order to check if there is a problem with the general formulation. Fortunately, the implementation of the general problem shown is not different from the problem specific implementations. In this project, Haverly's pooling problem and also Adhya's [1] and Foulds' [12] pooling problems are solved to test the proposed BD algorithm.

The GAMS implementation of the BD algorithm is provided in *Appendix A* in addition to the GAMS implementations of the example problem formulations. It is quite well known that the optimal objective value of Haverly's pooling problem -400. This value is also confirmed by the BARON implementation and the proposed BD algorithm gives the same objective value as the solution. In addition, the BD implementation is tested with several different starting points and it is observed that for all tested starting points, it converges to the global optimal solution (only the number of iterations changes, hence solution times also change slightly). Hence, it can be stated that, the BD algorithm is working for Haverly's pooling problem without any problem and converges to a global optimum.

The algorithm is also tested with Fould's [12] pooling problem with 8 pools, 14 sources, 1 quality and 6 end-products. BARON converges to -52 as the optimal objective value and also the proposed BD algorithm gives the same optimal objective value. Again, the BD implementation is tested with several different starting points for Fould et al.'s pooling problem and it is observed that for all tested starting points, it converges to the global optimal solution.

Another test problem is Adhya's [1] pooling problem with 7 pools, 8 sources, 4 qualities and 4 end-products. BARON converges to -1185 as the optimal objective value and also the proposed BD algorithm gives the same optimal objective value. Again, the BD implementation is tested with several different starting points for Adhya et al.'s pooling problem and it is observed that for all tested starting points, it converges to the global optimal solution.

In addition, 4 example pooling problems (which were created by the author) are also solved. More information for both of these example problems including quality specs, demand requirements, cost coefficients and GAMS implementations are given in *Appendix A*. Example 1 has 14 pools, 18 sources, 1 quality and 9 end-products. The BD solver

Problem	BARON	BD
Haverly	-400	-400
Foulds	-52	-52
Adhya	-1185	-1185
Example 1	-894	-894
Example 2	-1225	-1225
Example 3	-726	-726
Example 4	-2745	-2745

Table 4.1: Optimal objective values in GAMS

gives the optimal objective value as -894 which is the same as BARON. Example 2 has 14 pools, 18 sources, 6 qualities and 9 end-products. The BD solver converges to the same optimal objective value as BARON. Example 3 and Example 4 is also solved with both BD and BARON. Example 3 has 16 sources, 10 pools, 6 end-products and 1 qualities whereas Example 4 has 16 sources, 10 pools, 6 end-products and 8 qualities and in all examples the proposed BD algorithm and BARON converge to same optimal objective value for all tested starting points regardless of the size of the problem.

The optimal objective values for all of the example problems are shown in Table 4.1.

It can be concluded that, the proposed BD algorithm works without any problem and converges to the global optimal solution for all tested starting points regardless of the size of the problem. An important point to mention is that when a local solver (e.g. SNOPT, MINOS, CONOPT, etc.) is used to solve the relaxed master problem in the BD implementation; if the pooling problem to be solved has only one quality variable, the BD algorithm with local solver for the relaxed master problem converges to the global optimum. However, if the problem has more than one quality variables, the proposed BD implementation does not converge to global optimal solution when the local solver is used (converges to suboptimal points) and also the solution value returned by the algorithm changes dramatically with different starting point values. In other words, algorithm converges to local optimal values. The possible reason is that when a local solver is used, invalid cuts are generated from a local solution and as a result the algorithm does not converge to the global

Problem	BARON	BD
Haverly	0.01	0.03
Foulds	1.25	3.61
Adhya	6.12	17.13
Example 1	4.27	21.46
Example 2	21.43	85.52
Example 3	2.08	8.41
Example 4	36.36	181.6

Table 4.2: Solution times in GAMS (in seconds)

optimal solution.

We can also compare the solution times of the proposed BD implementation and BARON Version 7.8 as shown in Table 4.2. It is necessary to note that, in both the BARON and the BD implementation, all example problems are solved with a computer having an Intel 3.20 GHz Xeon processor.

Table 4.2 shows that the solution times of the BARON implementation are lower than the ones of the BD solver. In general, solution times in BARON are three to four times lower than the solution times in the BD algorithm and for the problems having more than one quality variables, the difference between solution times of BARON and the BD is more than the problems having only one quality variable. It is obvious that, when the number of quality variables increase, the number of bilinear terms also increases, and Table 4.2 clearly shows that as the number of bilinear terms increases solution times for both implementations increase, but also the difference between solution times of the BARON and the BD solvers also increases. The reason of this problem is the extra bilinear terms, and therefore extra nonconvexities, introduced by the quality variables. As discussed in *Chapter 2*, in pooling problems, the sole source of bilinearities is the mass balance equation for each quality variable and therefore as the number of quality variables increase, the total number of mass balance equations also increases and as a result the total number of bilinear terms rises.

One may think that since the number of bilinear terms in the problem affects the solution times of the proposed BD solver drastically, this proposed BD algorithm can be useful to solve problems with smaller number of bilinear terms such as the gas network problems. The gas network problems are a special kind of pooling problems where pools can be modeled as mixers and splitters. Modeling pools as mixers and splitters gives the opportunity to write mass balances for each quality separately.

For mixers, mass balances can be written as the sum of each flow regardless of the quality variables, therefore mass balance equations for mixers do not include any bilinear terms. In other words, for a selected quality, mass balance can be written as the output volume flow rate equals to the sums of input volume flow rates and it is a linear equation. However, for splitters, writing mass balances separately still introduces bilinear terms. However, now since bilinear terms are only coming from the splitters instead of all of the pools, the number of bilinear terms reduces and therefore the complexity of the problem reduces greatly. Thus, one can expect lower solution times from the BD solver in gas network problems. In order to test the performance of the proposed BD algorithm in a gas network problem, an example problem shown in Figure 4-2 is studied. As shown in the figure this problem has 10 pools, 8 sources, 3 qualities and 4 end-products. Detailed definition of this problem is provided in *Appendix B*. The example gas network problem is formulated both as a network with mixers and splitters and as a classical pooling problem for comparison purposes. Both formulations are solved with both BARON and the BD solver. Results confirm that both of the algorithms (BARON and the BD) converge to the global optimum.

The solution times of both the BD and BARON implementation is shown in Table 4.3 for the gas network example. This problem is also solved with a computer having an Intel 3.20 GHz Xeon processor. As shown in Table 4.3, BARON has still lower solution times than the BD algorithm even in a problem with less number of bilinear terms than a comparable pooling problem. However, as expected the difference between solution times of BARON and the BD decreases as the number of bilinear terms decrease in the problem with the mixer-splitter formulation.

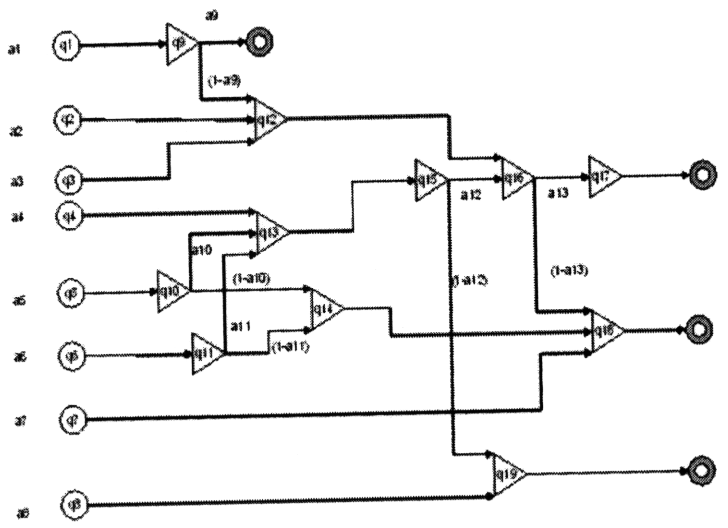


Figure 4-2: The gas network example

Formulation	BARON	BD
Gas Network	11.28	38.63
Pooling	13.72	42.91

Table 4.3: Solution times for the gas network problem (in seconds)

It can be seen from Table 4.3, solution times with BARON are lower than the ones with the proposed BD solver in both formulations. However, an important point to mention is the decrease in the solution times of the BD algorithm with two different formulations which confirms the expectations. This example clearly shows that the performance of the BD algorithm depends on the number of bilinear terms. In other words, as it is realized in Table 4.2, as the number of bilinear terms increases in the problem, the solution time difference between BARON and the BD algorithm increases, because as the problem complexity increases the number of iterations required by the BD solver to converge to the global optimal point increases.

However, when the output and log files of the problems solved in GAMS are inspected, another important problem affecting the performance of the BD implementation is observed. Since in the BD implementation, to iterate between the primal and master problem, there is a loop and in every iteration for both primal and master problem GAMS executes compilation and problem generation phases, in other words, in every iteration GAMS executes 2 compilations and 2 problem generations, and considering that the BD algorithm iterates around 5-6 times to solve an average pooling problem, it incurs a total 10 to 12 compilations and problem generations. In addition, in each iteration we should call BARON to solve the relaxed master problem globally, these calls also cause executions of compilations and problem generations in GAMS. Moreover, when the number of bilinear terms increase, the number of iterations of the BD algoritm also increases and as a result the number of compilations, problem generations and the number of BARON calls to solve the relaxed master problem in each iteration increases. However, BARON does both compilation and problem generation only once in GAMS. This fact can explain the huge differences in terms of the solution times between BARON and the BD algorithm when the number of quality variables increase. Therefore, it is obvious that the proposed BD algorithm which uses BARON at each iteration cannot compete against BARON in the sense of solution times. However, CPU times of the BD can still be considered close to BARON's solution times and it is quite reasonable to assume that by preventing the executions of compilations and problem generations in each iteration (i.e. changing the implementation such that the problem generation and compilation occurs only once in the beginning of the execution) plus with some tweak in the code, it is possible to get lower solution times from the BD algorithm.

4.3.2 C++ Implementation

Since, GAMS executes compilation and problem generation in each iteration and there is no way to prevent these executions in GAMS; it is decided to reimplement the BD algorithm by using C++. The algorithm is implemented in Linux and G++ (version 4.2) is used as the compiler. As LP solver for the primal problem a subroutine that calls CPLEX 10.2 [21] as the LP solver is used. However, the main issue is to write a custom B+B solver that can handle the relaxed master problems. In GAMS, BARON is used to solve bilinear relaxed master problems, in C++ to solve them, there are two methods, one is to implement a B+B code and the other one is to use the callable BARON library. The B+B solver implemented is based on a B+B algorithm developed and implemented by Chaukun Lee in the Process Systems Engineering Laboratory (PSEL) before.

However, BARON is a mature, commercially available and advanced software. Therefore, a simple B+B code for the relaxed master problem cannot compete with it even if the problem generation and compilation repetitions are omitted. There are two advantages of BARON against a simple B+B algorithm. First one is the range (domain) reduction and the second one is the tighter convex relaxations. Since range reduction has been presented as the major feature of the branch and reduce algorithm, it is believed having more direct effect in terms of solution times, therefore at first range reduction is applied.

Basically, range reduction is the process of eliminating regions from the feasible space such that the removal does not affect the convergence of the algorithm to a global optimum. Various techniques for range reduction have been proposed in the literature (Mangasarian & McLiden (1985) [34], Thakur (1990) [46], Lamar (1993) [30], Savelsbergh (1994) [43], Ryoo & Sahinidis (1996) [39], Shectman & Sahinidis (1998) [44] and Zamora & Grossmann (1999) [51]), but in this study, in order to be as close to BARON's methods as possible, a range reduction algorithm proposed in Tawarmalani & Sahinidis (2002) [45] is used.

Table 4.4 shows the effect of the implementation of the range reduction. Although the range reduction reduces the solution times almost half comparing to simple B+B algorithm, it is still slower than BARON which shows that BARON has more weapons to reduce the solution times and as mentioned one of them is the implementation of tighter convex relaxation techniques. The next step would be to implement the tighter relaxation techniques in the literature to the BD algorithm but then it is decided to use BARON library instead for convenience and to get quicker results.

In order to use all the advantages of BARON, a callable BARON library is obtained and

Problem	BARON	BD with B+B	BD with B+B (+Ran. Red.)
Haverly	0.01	0.04	0.018
Foulds	1.25	4.12	2.38
Adhya	6.12	19.03	10.25
Example 1	4.27	9.67	5.81
Example 2	21.43	78.11	44.51
Example 3	2.08	5.29	3.13
Example 4	36.36	135.21	82.06

Table 4.4: Solution times in C++ with and without Range Reduction (in seconds)

Problem	BARON	BD with B+B (+Ran. Red.)	BD with BARON lib.
Haverly	0.01	0.018	0.01
Foulds	1.25	2.38	1.42
Adhya	6.12	10.25	7.63
Example 1	4.27	5.81	4.96
Example 2	21.43	44.51	33.42
Example 3	2.08	3.13	2.37
Example 4	36.36	82.06	58.57

Table 4.5: Solution times in C++ (in seconds)

implemented to the C++ code. The main advantages of using the BARON library besides having all weapons of BARON are convenience and quicker implementation and it still does not have the problem generation and compilation problem in GAMS. The solution times and the related discussion of the BD with BARON library to solve the relaxed master problem are given in the next section.

4.3.3 Results

The GAMS implementation shows that convergence is achieved, hence in the C++ implementation, only the solution times are taken into discussion. Solution times of the example problems in the C++ implementation with both the custom B+B code (with range reduction) and BARON library is given in Table 4.5 with solution times of BARON itself. Again a computer with Intel 3.20 GHz Xeon processor is used.

Table 4.5 illustrates that BARON still gives better solution times than the BD with both the custom B+B code (with range reduction) and BARON library. This proves that BARON is a very powerful software to solve bilinear problems and even by using all the strategies available, it is almost impossible to get better solution times with the BD algorithm. However, as mentioned in the first chapter, decomposition algorithms traditionally work more efficiently than the B+B based algorithms in stochastic programming and the goal of this study is to model the infrastructure development problem in terms of gas quality variables and therefore the results in deterministic pooling problems are not that important at this stage. The real objective of this stage is to develop and implement a working BD algorithm in order to use it to solve stochastic pooling problems. The reasons behind the author's expectations about the better efficiency of the BD in stochastic programming are explained in the next chapter.

Chapter 5

Application to the Stochastic Pooling Problem

5.1 Infrastructure Development Problems in Natural Gas Value Chain

The prime objective is to solve long-term infrastructure development problems in the natural gas production industry considering all possible uncertainties and to develop new optimization methods, decision support tools for the infrastructure development problem. Other research objectives are to develop methodology for analysis of robustness, flexibility and risk in long-term infrastructure investments in gas production and to study how operational flexibility should be incorporated in long term investments and infrastructure analysis. In addition, it is necessary to demonstrate the methodology for analysis in important industrial cases.

The complexity of the problem requires to develop new methodology and mathematical models for the design, development and operation of infrastructure in natural gas production under uncertainty and the main subsystems involved in the model will be the gas field, the surface processing facilities, the transportation facilities and the markets.

The optimization model thus generated will be a large-scale optimization problem that will involve a large number of nonconvex functions. Therefore, most probably this problem will be unsolvable by commercial solvers. Thus, better solution methods should be explored to solve this problem to global optimality. Hence, it is possible that some theoretical work will be required in optimization theory in the process of solving this problem.

Possible uncertainties in long-term infrastructure development problems in natural gas production are production profiles and the amount of gas in natural gas fields; gas quality in terms of heating value, NGL content, LPG content, CO_2content etc. and the demands and prices for the products of the gas value chain. Some important operational characteristics of natural gas production networks such as blending and pooling possibilities, contractual constraints, multiple routing and pipeline options and pressure constraints should also be considered and incorporated into the production planning problem.

Then, it can be stated that there are three major research challenges that should be addressed in the long-term infrastructure development problem: uncertainty and decision flexibility; reaching global optimality of the overall system and combining economical modeling with the production planning problem.

Uncertainty and decision flexibility is probably the most difficult challenge in a long term production planning problem. Strategic decision support models for investment analysis need to capture the long term uncertainty and in addition be able to value short term operational flexibility since the operational characteristics of initial investments, capacity expansion and new investments will affect the decisions about the future investments and capacity expansions.

Another important challenge is to reach optimality for the overall system. The timing of investments, the inherent flexibility in technology choices and capacity decisions as well as the location of the infrastructure are examples of decisions that should be analyzed in a framework considering the overall system rather than a local subproblem to avoid suboptimization.

The third research challenge is to incorporate economical modeling into the production planning problem. In order to be able to capture both market driven production and operation, it is necessary to include the operational decision space of the infrastructure including using markets to resolve bottlenecks of the infrastructure and dynamic market driven operation of the infrastructure. In addition, to capture the operational flexibility and limitations of the system, it may be necessary to include detailed models of the technology.

In order to construct a mathematical model having all the properties of the value chain; it is necessary to incorporate nonlinear flow and reservoir models, gas quality in the fields, the contractual framework and LNG/LPG production models into the infrastructure development problem with all possible uncertainties. Then, this problem becomes a large-scale global optimization problem with stochasticity in it.

The first step chosen to start modeling the real value chain is to formulate a simple model for the production planning problem for a relatively small field and integrate the gas quality problem to this infrastructure development problem, since there are well known algorithms to solve large pooling problems and then the possible next step is to solve this problem at a larger-scale. Then, additional operational rules and uncertainties can be added to this model in each step.

5.2 Introduction to Stochastic Programming

Stochastic programming is a framework for modeling optimization problems that involve uncertainty. Whereas deterministic optimization problems are formulated with known parameters, real world problems almost invariably include some unknown parameters. Stochastic programming models take advantage of the fact that probability distributions governing the data are known or can be estimated. The goal is to find some policy that is feasible for all the possible data instances and maximizes the expectation of some function of the decisions and the random variables. More generally, such models are formulated, solved analytically or numerically, and analyzed in order to provide useful information to a decision-maker.

The most widely applied and studied stochastic programming models are two-stage linear programs. Here the decision maker takes some action in the first stage, after which a random event occurs affecting the outcome of the first-stage decision. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experienced as a result of the first-stage decision. The optimal policy from such a model is a single first-stage policy and a collection of recourse decisions (a decision rule) defining which second-stage action should be taken in response to each random outcome [29].

Solution approaches to stochastic programming models are driven by the type of probability distributions governing the random parameters. A common approach to handling uncertainty is to define a small number of scenarios to represent the future. In this case it is possible to compute a solution to the stochastic programming problem by solving a deterministic equivalent linear program. These problems are typically very large-scale problems, and so, much research effort has been devoted to developing decomposition algorithms that exploit the problem structure, which decompose large problems into smaller more tractable components [7].

An alternative solution methodology replaces the random variables by a finite random sample and solves the resulting deterministic mathematical programming problem. This is often called an external sampling method. External sampling methods typically take one sample before applying a mathematical programming method. A number of algorithmic procedures have been developed to take repeated samples during the course of the algorithm. This is often called the internal sampling method. However, both internal and external sampling methods are still immature, computationally expensive and can only solve relatively smaller problems [7]. Therefore, decomposition algorithms are preferred to solve large stochastic programs.

The basic idea behind decomposition algorithms is to decompose complex algorithms into smaller parts and try to use the fact that solving many simpler programs may be quicker than solving one large program. These algorithms are effective especially when the subproblems are easy to solve. Basically, decomposition algorithms works as shown in Figure 5-1. Each node in the figure represents a subproblem and the algorithm solves each subproblem separately and the solutions of parent nodes (represented as x_t in the figure and there is only one parent node in two-stage stochastic programs) are passed to child nodes and the solutions of child nodes (represented as Q_{t+1} in the figure and there is only one stage of child nodes in two-stage stochastic programs) are passed to parent nodes and both solutions are updated until they converge. One important point to mention is that the solution of child nodes have to be functions of the solutions of parent nodes since decisions in the previous stage always determine the outcome of the problem in the following stage. However, in most of the stochastic problem formulations, the solutions of parent nodes are also functions of the solutions of child nodes (For instance, second-stage operational variables effecting the planning problem in the first stage in infrastructure development problems.). Detailed information about the formulation of two-stage stochastic programs and solution techniques can be found in Birge & Louveaux (1994) [7] and Kall & Wallace (1994) [29].

5.3 Importance of Stochastic Pooling Problems in Natural Gas Infrastructure Development

Natural gas exploration and production is a highly capital-intensive industry. Facilities required for offshore exploration and production often remain in operation over the entire life-span of the project, typically 10-30 years and hence the operational use of infrastructure and the requirements for its design change over time. Therefore, decisions regarding investment in these facilities affect the profitability of the entire project. Given the large potential profits and high investments in each project, there is significant interest in de-

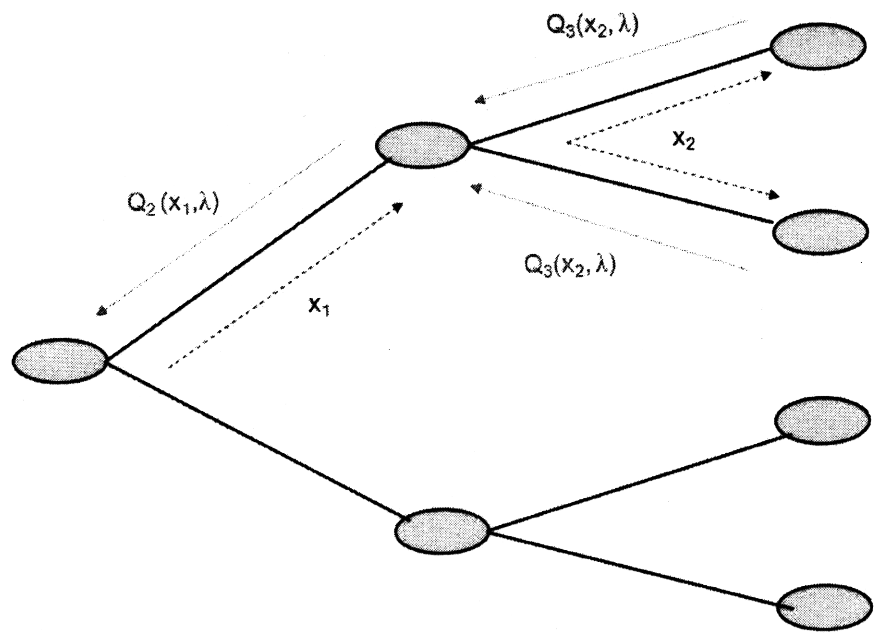


Figure 5-1: Basic illustration of decomposition algorithms in stochastic programming

veloping optimization models for planning in the natural gas exploration and production industry. A major challenge lies in the fact that decision-makers in this industry have to deal with a great deal of uncertainty. One of the most important sources of uncertainty is the quality of reserves. The existence of oil or gas at a site is indicated by seismic surveys and preliminary exploration tests. However, the actual amount of natural gas in these reserves, and the efficacy of extracting these remain largely uncertain until after the investments have been made. Both these factors directly affect the profitability of the project and hence it is important to consider the impact of these uncertainties when formulating the decision policy.

The opportunity for blending different sources of natural gas comes into the picture especially when the natural gas upstream infrastructure is being developed. When new wells or fields are being developed, it is possible to construct the pipeline system such that the gas from different wells are mixed together to satisfy the requirements for different qualities. However to construct the pipeline system optimally, a stochastic version of the pooling problem where the quality parameters in the wells are not known exactly has to be solved. Although advancing technology provides necessary tools to predict the content of the natural gas in different fields during the exploration stage, the content of the natural gas is still uncertain before drilling the well. Thus, stochastic programming principles has to be used to achieve an optimum solution in the infrastructure development problem.

5.4 Formulation of the Stochastic Pooling Problem

The problem to be considered as the stochastic pooling problem is to determine a minimum cost capacity expansion plan for the pooling network which meets demand and quality requirements and maximizes the operational profits for the natural gas production. Cost in this problem consists of two components: the initial capital cost of building the pools and the pipeline network, and the operational costs of the overall system to meet the demands of customers. Income comes from the sale of the natural gas, which meets the requirements, to customers. Because of the uncertainty in the quality variables of sources (the actual impurity levels of natural gas in the reserves are uncertain), the amount of gas in sources, prices and costs; these variables must be defined in terms of probabilistic measures and therefore this problem is a stochastic program.

The stochastic pooling problem naturally decomposes into two stages: determining the optimal investments in pooling capacity and necessary pipeline network, and determining the operating conditions to meet the customer requirements. The first stage is called the planning problem and the second stage is called the pooling problem. This natural decomposition can be exploited by decomposition algorithms. Using decomposition, the stochastic pooling problem can be divided into smaller problems, a master problem and a set of recourse subproblems. The master problem, which in this case is a mixed-integer linear program (MILP), is used to generate trial solutions for the optimal capacity expansion plan. The subproblems are used to determine the maximum profit operation and meeting the requirements. Basically, subproblems are deterministic pooling problems (as formulated in *Chapter 2*) which are solved to maximize the profit after the optimal pooling network is decided by the first stage capacity expansion problem. The planning problem has originated from the long-term analysis of the electricity transmission and distribution with price uncertainty. Basically, the planning problem is to make decisions about what to construct, where to construct and how many (much) to construct. The pooling problem forms the second stage. After the planning problem is solved at the first stage and the number of pools and connections to and from them (into the sources and end-product nodes) are decided by the solution of the planning problem; the pooling problem is solved as the second stage and profit is maximized.

The stochastic pooling problem can be solved iteratively, by decomposition algorithms, by alternately solving the master problem and the subproblems until an optimum is found. In this way, the complex nonlinear program for stochastic pooling problem is reduced to iterative solution of a MILP and a set of bilinear programs which reminds exactly the methodology of the proposed BD algorithm to find global optimum. Therefore, the BD algorithm can directly be used to solve stochastic pooling problems.

One of the most important issues in the stochastic version of the pooling problem is that in the literature, proof of convergence for two-stage stochastic programs is only provided for problems with a convex second stage. Unfortunately, the pooling problem is a nonconvex problem and hence the proofs from the literature cannot be applied directly to prove the convergence of the stochastic pooling problem. However, the BD algorithm guarantees to converge to global optimum in deterministic pooling problem and therefore, it is applied to the stochastic version without any proof of convergence, but as a future work global convergence for the BD algorithm in stochastic pooling problem has to be proved.

As explained, the first stage planning problem, which is an MILP, is to solve the optimal capacity expansion plan and can be represented mathematically as follows:

$$\min_{U,X,Y} \quad \sum_{j=1}^{p} U_j b_j + \sum_{i=1}^{n}\sum_{j=1}^{p} X_{ij} a_{ij} + \sum_{j=1}^{p}\sum_{k=1}^{r} Y_{jk} s_{jk} + \sum_{h=1}^{n_s} pr_h PP_h(U,X,Y) \tag{5.1}$$

$$\text{s.t.} \quad Y_{jk} - U_j \leq 0, \qquad j = 1,\ldots,p \tag{5.2}$$

$$X_{ij} - U_j \leq 0, \qquad j = 1,\ldots,p \tag{5.3}$$

$$U_j, X_{ij}, Y_{jk} \in \{0,1\}$$

In this representation, the problem consists of n sources, p pools, r end-products and n_s stands for the number of possible scenarios, where i is the index for sources, j is the index for pools, k is the index for products and h is the index for the scenarios. Moreover, U_j is the binary variable to indicate if the pool j is included in the network or not (1 if the pool is constructed and active, 0 if not constructed); X_{ij} is the binary variable to indicate if the pipeline from source i to pool j is included in the network or not (1 if the pipe is constructed and active, 0 if not constructed); Y_{jk} is the binary variable to indicate if the pipeline from pool j to end-product k is included in the network or not (1 if the pipe is constructed and active, 0 if not constructed). PP_h represents the operational cost function of the pooling network in scenario h (i.e. PP_h is the objective function of the second stage problem for the scenario number h.) and pr_h is the corresponding probability of the scenario h (i.e. pr_h represents the probability of scenario h to happen.). In addition, parameters in this representation are listed in Table 5.1.

Parameter	**Definition**
b_j	the investment cost of the pool j
a_{ij}	the investment cost of the pipeline from the source i to the pool j
s_{jk}	the investment cost of the pipeline from the pool j to the end-product k
PP_h	operational cost function of the pooling network in scenario h
n	total number of sources
p	total number of pools
r	total number of end-products
n_s	total number of possible scenarios
pr_h	probability of the scenario h

Table 5.1: Parameters and corresponding definitions for the first stage problem

As mentioned, the first stage problem determines the optimal investments in a pooling network that satisfies the given requirements and in the mathematical formulation of this problem, the objective function represents the total cost of the investments including the cost of constructing new pools and installing pipelines from sources to pools and from pools to demands. (5.2) and (5.3) ensure that if the pool j is not active (i.e. $U_j = 0$), the pipelines that connect the pool j to sources and demands cannot be active. As the planning problem formulation clearly shows, operational costs, profits and flow constraints are not included. In other words, the planning problem is only formulated to minimize the investment costs by considering the operational costs of the pooling network. On the other hand, the second stage problem determines the optimal operating conditions to meet the customer requirements. Formulation of the pooling problem in the second stage is basically same as the formulation in the deterministic case. The second stage problem is a general pooling problem with n sources, p pools, r products and l quality parameters. In this representation, i is the index for sources, j is the index for pools, k is the index for products and w is the index for qualities. In addition, f_{ij} is the variable for the total flow from the i^{th} source into pool j; q_{jw} is the variable for the w^{th} quality component of pool j and x_{jk} is the variable for the total flow from the j^{th} pool to product k. Again, U_j is the binary variable to indicate if the pool j is included in the network or not (1 if the pool is constructed and active, 0 if not constructed); X_{ij} is the binary variable to indicate if the pipeline from source i to pool j is included in the network or not (1 if the pipe is constructed and active, 0 if not constructed); Y_{jk} is the binary variable to indicate if the pipeline from pool j to end-product k is included in the network or not (1 if the pipe is constructed and active, 0 if not constructed). λ_{ijw} is the w^{th} quality component of the flow from the i^{th} source into pool j. Also, necessary parameters in this representation are listed in Table 5.2.

Parameter	**Definition**
c_{ij}	cost of the flow from the i^{th} source into pool j
d_k	unit price of product k
l	total number of component qualities
n	total number of sources
p	total number of pools
r	total number of end-products
S_k	demand requirement for product k
Z_{kw}	w^{th}quality requirement for product k
λ_{ijw}^{h}	w^{th} quality component of the flow from the i^{th} source into pool j in scenario h

Table 5.2: Parameters and corresponding definitions for the second stage problem

Then, for the scenario number h, the mathematical representation of the second stage pooling problem becomes:

$$PP_h(U,X,Y) = \min_{f^h,x^h,q^h} \quad \sum_{j=1}^{p}\sum_{i=1}^{n} X_{ij}c_{ij}^h f_{ij}^h - \sum_{k=1}^{r} d_k^h \sum_{j=1}^{p} Y_{jk}x_{jk}^h \tag{5.4}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} X_{ij}f_{ij}^h - \sum_{k=1}^{r} Y_{jk}x_{jk}^h = 0, \qquad j = 1,\ldots,p \tag{5.5}$$

$$q_{jw}^h \sum_{k=1}^{r} Y_{jk}x_{jk}^h - \sum_{i=1}^{n} \lambda_{ijw}^h X_{ij}f_{ij}^h = 0, \qquad j = 1,\ldots,p; \; w = 1,\ldots,l \tag{5.6}$$

$$\sum_{j=1}^{p} Y_{jk}x_{jk}^h - S_k \leq 0, \qquad k = 1,\ldots,r \tag{5.7}$$

$$\sum_{j=1}^{p} q_{jw}^h Y_{jk}x_{jk}^h - Z_{kw} \sum_{j=1}^{p} Y_{jk}x_{jk}^h \leq 0, \qquad k = 1,\ldots,r; \; w = 1,\ldots,l \tag{5.8}$$

$$X_{ij}f_{ij}^L \leq f_{ij}^h \leq X_{ij}f_{ij}^U, \qquad i = 1,\ldots,n_j; \; j = 1,\ldots,p$$

$$q_{jw}^L \leq q_{jw}^h \leq q_{jw}^U, \qquad j = 1,\ldots,p; \; w = 1,\ldots,l$$

$$Y_{jk}x_{jk}^L \leq x_{jk}^h \leq Y_{jk}x_{jk}^U, \qquad j = 1,\ldots,p; \; k = 1,\ldots,r$$

In this formulation, the objective function represents the difference between the cost of the flow from the source nodes and the returns from selling the end-products. (5.5) represents the mass balances for each active pool. (5.6) expresses the mass balance for each quality component. (5.7) ensures that the flows to each end-product node do not exceed the demands. (5.8) enforces that the quality requirements are satisfied at each end-product node. Moreover, binary variables indicating whether the pipelines are active or not (namely, X_{ij} and Y_{jk}) are added as multipliers to the lower and upper bounds of the flow variables (into the pools and from the pools respectively) in order to to set flow variables to zero in pipelines that do not exist.

In the stochastic pooling problem formulation, it is possible to set any combination of the network parameters as the uncertain parameters (e.g. demand requirements for products, quality requirement for product, prices, costs or quality parameters at sources) after excluding the known parameters which are provided or measured before formulating the problem. Actually, in reality, all of the parameters in the pooling problem are uncertain and hence this problem is a very difficult stochastic program. However, for convenience and better understanding of the performance of the proposed BD algorithm, in all example problems of this study, only the quality parameters of the flow from the sources into the pools are taken as uncertain parameters and the remaining parameters are held constant for all possible scenarios as explained in the next section. It is also important to mention here the fact that the second stage problem in the stochastic pooling problem formulation is a bilinear and hence nonconvex problem which makes this problem harder to solve.

5.5 Implementation of the BD Algorithm in Stochastic Pooling Problems

With a finite number of scenarios, two-stage stochastic programs can be modeled as large linear or nonlinear programming problems. This formulation is called the deterministic equivalent. Strictly speaking a deterministic equivalent is any mathematical program that can be used to compute the optimal first-stage decision, so these will exist for continuous probability distributions as well, when one can represent the second-stage in some closed form [29]. After formulating both stages, the stochastic pooling problem is reformulated to a single stage (i.e. deterministic equivalent) by using the basic conversion techniques which are discussed in Birge & Louveaux (1994) [7] and Kall & Wallace (1994) [29] and the resulting problem becomes a mixed-integer nonlinear program (MINLP). Since the second stage optimization variables (namely, f, q and x) are not functions of uncertain parameters

(λ in the example problems of this study) in the stochastic pooling program formulation, it is not necessary to introduce new second stage variables during reformulation [7] and therefore, the reformulation of the two stage stochastic pooling problem as a single stage MINLP (deterministic equivalent) can be written simply as (definitions of the variables and parameters in this formulation are given in the previous section):

$$\min_{U,X,Y,f,x,q} \quad \sum_{j=1}^{p} U_j b_j + \sum_{i=1}^{n}\sum_{j=1}^{p} X_{ij}a_{ij} + \sum_{j=1}^{p}\sum_{k=1}^{r} Y_{jk}s_{jk} + \sum_{h=1}^{n_s} pr_h\left(\sum_{j=1}^{p}\sum_{i=1}^{n} X_{ij}c_{ij}^h f_{ij}^h - \sum_{k=1}^{r} d_k^h \sum_{j=1}^{p} Y_{jk}x_{jk}^h\right) \tag{5.9}$$

$$\text{s.t.} \quad Y_{jk} - U_j \leq 0, \qquad j = 1,\ldots,p \tag{5.10}$$

$$X_{ij} - U_j \leq 0, \qquad j = 1,\ldots,p \tag{5.11}$$

$$\sum_{i=1}^{n} X_{ij}f_{ij}^h - \sum_{k=1}^{r} Y_{jk}x_{jk}^h = 0, \qquad j = 1,\ldots,p;\ h = 1,\ldots,n_s \tag{5.12}$$

$$q_{jw}^h \sum_{k=1}^{r} Y_{jk}x_{jk}^h - \sum_{i=1}^{n} \lambda_{ijw}^h X_{ij}f_{ij}^h = 0, \qquad j = 1,\ldots,p;\ w = 1,\ldots,l;\ h = 1,\ldots,n_s \tag{5.13}$$

$$\sum_{j=1}^{p} Y_{jk}x_{jk}^h - S_k \leq 0, \qquad k = 1,\ldots,r;\ h = 1,\ldots,n_s \tag{5.14}$$

$$\sum_{j=1}^{p} q_{jw}^h Y_{jk}x_{jk}^h - Z_{kw} \sum_{j=1}^{p} Y_{jk}x_{jk}^h \leq 0, \qquad k = 1,\ldots,r;\ w = 1,\ldots,l;\ h = 1,\ldots,n_s \tag{5.15}$$

$$U_j, X_{ij}, Y_{jk} \in \{0,1\}$$

$$X_{ij} f_{ij}^L \leq f_{ij}^h \leq X_{ij} f_{ij}^U, \qquad i = 1, \ldots, n_j;\ j = 1, \ldots, p$$

$$q_{jw}^L \leq q_{jw}^h \leq q_{jw}^U, \qquad j = 1, \ldots, p;\ w = 1, \ldots, l$$

$$Y_{jk} x_{jk}^L \leq x_{jk}^h \leq Y_{jk} x_{jk}^U, \qquad j = 1, \ldots, p;\ k = 1, \ldots, r$$

It is necessary to note here that the equations (5.12), (5.13), (5.14) and (5.15) have to be repeated for every scenario. In other words, for each scenario there are 4 constraints from the second stage problem. Therefore, the number of bilinear terms increases with the number of scenarios as well as the number of quality variables in each problem. Hence, the number of scenarios directly affects the solution times of the problem and as the total number of possible scenarios increase, the problem becomes harder to solve.

As the next step, the proposed BD algorithm is used to solve example stochastic pooling problems to verify the convergence of the algorithm and to check its performance. Although the essential methodology is same, the BD algorithm for stochastic pooling problems has to be slightly different than the one to solve deterministic pooling problems. In deterministic pooling problems, the BD algorithm decomposes problem into two smaller problems: primal problem (linear program) and relaxed master problem (nonlinear program in bilinear problems) and as in the deterministic case, also in the stochastic pooling problems the BD algorithm decomposes the problem into smaller problems for easy and quick solution process. However, in stochastic case, there are two sets of decisions that are made in consecutive stages and this structure naturally forms a mixed-integer first stage problem and a set of smaller bilinear second stage problems and since this natural structure of decomposition is different than the deterministic case, the proposed BD algorithm to solve stochastic pooling problems has some changes from the deterministic version. The main difference between the stochastic and the deterministic versions is the process of the formation of cuts in each iteration. In the deterministic version of the BD algorithm, the additional cuts are generated for the bilinear relaxed master problem in each iteration; however, in the stochastic version, the additional cuts are generated not for the bilinear subproblems but for the MILP first stage problem.

Before explaining the BD algorithm for stochastic pooling problems in detail, first of all, in order to be consistent with the stochastic programming literature, the first stage optimal planning problem is called the master problem and the second stage optimal operational problems are called the subproblems. The BD algorithm to solve the stochastic version of this problem can be formulated as follows: A typical iteration starts with the master problem without the additional cost coming from the subproblems (last summation in 5.1). In other words, in the intial master problem, only first stage decision variables are considered and the effect of the second stage variables to the first stage is neglected. The binary vectors from solution of the master problem (U^*, X^*, Y^*) are fixed as the first stage decision variables and subproblems for all scenarios are solved with these fixed decision vectors. The objective function value of the master problem with U^*, X^* and Y^* plus the summation of the objective functions of each subproblem times its probability (as explained in the problem formulation, a probability value is assigned to each scenario (subproblem).) updates the upper bound (UBD) value, if it is lower than the previous UBD. Then additional cuts are formed in the master problem by using the solutions of the subproblems and the master problem are solved again with these additional cuts. The new solution of the master problem provides new binary vectors (i.e. U^*, X^* and Y^* are updated after master problem is solved with additional cuts.). The objective function value of this updated master problem updates the lower bound (LBD) value, if it is higher than the previous LBD. Then the algorithm iterates and all of the subproblems are solved again with these new first stage decision values and a new UBD (if necessary), plus additional cuts for the master problem are determined. The algorithm iterates until the lower bound value becomes higher than the upper bound value. In other words, the solutions of the subproblems are used to find the upper bound (UBD); the solution of the relaxed master problem is used to find the lower bound (LBD). When LBD$\geq$UBD, algorithm terminates. A detailed description of the BD algorithm for stochastic pooling problems is provided in Algorithm 2.

A crucial point to note is that the subproblems in stochastic pooling problems are simply deterministic pooling problems (which are bilinear problems) since all binary first stage decision variables are already fixed. Therefore, it is necessary to use a global solver to solve the subproblems and since the proposed BD algorithm for the solution of the deterministic pooling problems is shown to converge to global optimum, it is used to solve the subproblems in the stochastic pooling problems. In other words, in the implementation, the proposed BD algorithm calls itself to solve the subproblems for every scenario. The reason behind using the BD algorithm is the size of the stochastic pooling problems. Even the largest deterministic pooling problem example solved in this study can be accepted as small compared to the stochastic pooling problems since the stochastic ones have large number of scenarios and each scenario itself is a large pooling problem and the BD algorithm has advantages over any B+B based global optimization algorithm in large problems since it decomposes very large problems into a number of smaller, more manageable problems instead of solving it as a whole. Hence, the BD algorithm proposed for the deterministic pooling problems is used as the solver for subproblems and it provides both a background for developing the BD algorithm for stochastic pooling problems and a tool to solve the subproblems in the stochastic version of the BD algorithm.

Another important change in the implementation of the stochastic version of the algorithm is the MILP solver since the LP solver (CPLEX [21]) in the BD implementation can also be used as MILP solver with appropriate parameters. It is important to note that to solve the subproblems, the BD implementation in C++ with the callable BARON library for the relaxed master problem is used as the BD solver in the stochastic implementation of the BD algorithm since it provides the best performance in the deterministic case.

To validate that the proposed algorithm works for stochastic pooling problems, 4 example pooling problems (which were created by the author) are solved. Example 1 has 1 pool, 3 sources, 2 end-products; Example 2 has 2 pools, 5 sources, 3 end-products; Example 3 has 8 sources, 4 pools, 5 end-products whereas Example 4 has 12 sources, 10 pools and 8 end-products. In all examples, only the quality parameters at source nodes are assumed as uncertain variables for convenience and all problems are solved with one, two and three quality variables. More information for both of these example problems including quality specs, demand requirements and cost coefficients are given in *Appendix C.* Again, for comparison purposes BARON Version 7.8 [42] is used as the other solver. To solve the example problems with BARON, their deterministic equivalent formulations are used, since BARON is not based on a decomposition algorithm. But, historically, B+B solvers are not very successful in solving stochastic programs, hence it is expected that the proposed BD algorithm may provide better results in stochastic pooling problem.

One of the principal practical difficulties with stochastic programming is that the number of possible scenarios is often large, leading to a large number of subproblems. A number of remedies have been proposed, including the use of random sampling to generate only a representative set of scenarios. However, still the solution of large stochastic problems is extremely difficult. Thus, easy examples with limited number of scenarios and uncertain parameters are selected in this study, since the initial goal is to show the proposed BD algorithm is suitable for stochastic pooling problems.

5.6 Results

All 4 examples are solved with 1, 2 and 3 uncertain quality variables. The solution times are given in Table 5.3, 5.4 and 5.5 for 1, 2 and 3 uncertain quality variables respectively.

Algorithm 2 Benders decomposition algorithm for stochastic pooling problems

```
{INITIALIZATION}
i (iteration) := 1, UBD := INF, LBD := -INF; C := Total Number of Scenarios;
{STEP 1: INITIAL MILP MASTER PROBLEM}
Solve Master Problem
    $\min_{U,X,Y} \quad b^{\mathrm{T}}U + a^{\mathrm{T}}X + s^{\mathrm{T}}Y$
    $\text{s.t} \quad BU + AX + SY \leq 0$
Let the solution be $U^*(i), X^*(i), Y^*(i)$
{STEP 2: NLP SUBPROBLEMS}
for $h = 1 to C$
Solve Subproblem $h$ (obj. function is $PP_h(U^*(i), X^*(i), Y^*)(i))$)
Let the solution be $f_h^*(i)$, $q_h^*(i)$ and $x_h^*(i)$ Let the objective function be $PP_h^*(i)$
endfor
if $b^{\mathrm{T}}U^*(i) + a^{\mathrm{T}}X^*(i) + s^{\mathrm{T}}Y^*(i) + \sum_{h=1}^{C} pr_h PP_h^*(i) \leq UBD$ then,
{RECORD BETTER SOLUTION}
$UBD := b^{\mathrm{T}}U^*(i) + a^{\mathrm{T}}X^*(i) + s^{\mathrm{T}}Y^*(i) + \sum_{h=1}^{C} pr_h PP_h^*(i)$. end if ($pr(h)$ is the probability of the scenario $h$)
{STEP 3: MILP MASTER PROBLEM} Solve Master Problem
    $\min_{U,X,Y} \quad b^{\mathrm{T}}U + a^{\mathrm{T}}X + s^{\mathrm{T}}Y + \theta$
    $\text{s.t} \quad BU + AX + SY \leq 0$
    $\theta \geq \sum_{h=1}^{C} pr_h PP_h^*$
Let the solution be $\hat{U}(i), \hat{X}(i), \hat{Y}(i)$, then $U^*(i) = \hat{U}(i), X^*(i) = \hat{X}(i), Y^*(i) = \hat{Y}(i)$
emphif $b^{\mathrm{T}}U^*(i) + a^{\mathrm{T}}X^*(i) + s^{\mathrm{T}}Y^*(i) + \theta^*(i) \geq LBD$ then,
{RECORD BETTER SOLUTION}
$LBD := b^{\mathrm{T}}U^*(i) + a^{\mathrm{T}}X^*(i) + s^{\mathrm{T}}Y^*(i) + \theta^*(i)$. end if
if $LBD \geq UBD$ then, STOP.
else, i := i+1, Go to STEP 2. end if
{END OF ALGORITHM}
```

Problem	BD	BARON
Example 1	7	1
Example 2	17	4
Example 3	23	11
Example 4	36	20

Table 5.3: Solution times of stochastic pooling problems with one quality variable (in minutes)

Problem	BD	BARON
Example 1	20	4
Example 2	55	13
Example 3	66	30
Example 4	No. Sol.	INF

Table 5.4: Solution times of stochastic pooling problems with two quality variables (in minutes)

All solutions are done in a computer with Intel 3.20 GHz Xeon processor.

The results clearly show that BARON provides better solution times than the BD algorithm. However, as the problems get complicated and number of variables increases the difference between the BD and BARON in terms of solution times decreases. This looks promising since the real planning problems that we are interested in are much larger than these examples. But, another observation is as problems get complicated, the BD algorithm gives no solution especially for more than one quality cases. Especially Example 4 with 12 sources, 10 pools and 8 end-products is a very complex problem and as shown in the Tables with more than one quality cases both algorithms struggle to solve Example 4. There are tighter bounding techniques available for stochastic programs, two most important of all are Edmundson-Madansky Bounds and Jensen Bounds. These techniques could help to

Problem	BD	BARON
Example 1	58	10
Example 2	92	25
Example 3	152	78
Example 4	No. Sol.	INF

Table 5.5: Solution times of stochastic pooling problems with three quality variables (in minutes)

reduce the solution times and solve larger problems with the BD algorithm, but, both of them proved to provide tighter relaxations only for convex stochastic programs. Therefore, in the next phase of the project, development of tighter relaxations for nonconvex stochastic programs will be the objective. Author believes that a decomposition algorithm with tighter bounds and an optimized code has still a better chance to solve stochastic problems than a B+B algorithm.

Moreover, since all of these examples are not real cases and created by the author as examples, it becomes difficult to create feasible examples as the problems get complicated. Complex examples such as Example 4 has many parameters to be adjusted in order to get a feasible problem and no one can guarantee the correctness of these parameters and the feasibility of the problem. Infeasibilities occur during the analysis process and one cannot determine whether these infeasibilities are results of incorrect parameters given by the author or the formulation itself. Therefore, it is crucial to look for the methods to generate feasible problems before proceeding further into the bounding techniques. Hence, as explained in the next chapter, the next step in this study will be to develop techniques to generate feasible problems automatically.

Chapter 6

Conclusion

The prime objective of this project is to solve the long term infrastructure development problem in the natural gas production industry with considering most of the possible uncertainties and to develop new optimization methods, decision support tools for the infrastructure development problem. In addition, it is necessary to demonstrate the methodology for analysis in important industrial cases. The complexity of the problem requires to develop new methodology and mathematical model for the design, development and operation of infrastructure in natural gas production under uncertainty and the main subsystems involved in the model will be the gas field, the surface processing facilities, the transportation facilities and the markets.

This is a very difficult goal to achieve when considering the requirement of huge mathematical models to be as close as possible to reality. Therefore, it is decided to start from relatively smaller problems by assuming most of the variables are known parameters and try to deal with only one aspect of the whole value chain. Because of the reasons explained above, the first stage in this project is chosen to be the planning problem only considering the pooling and blending of the natural gas from different fields with uncertain quality variables. To solve even this relatively small model a new BD algorithm has to be proposed because of the nonconvexity of the pooling problem.

In conclusion, it can be stated that for the pooling problems, the proposed BD algorithm which assumes the bilinear terms as complicating variables as a whole, is proven to converge to global optimal solution for all tested starting points regardless of the size of the problem. The BD algorithm is shown to be working for all example pooling problems without any problem and converges to a global optimum in all examples. But, the results illustrate that BARON gives better solution times than the BD implemented both in GAMS and in C++ with both custom B+B code (with range reduction) and BARON library. However, decomposition algorithms work better than the B+B based algorithms in stochastic programming and the goal of this study is to model the infrastructure development problem in terms of gas quality variables and therefore the results in deterministic pooling problems are not that important for this study.

The main goal of this study is to develop and implement a working BD algorithm in order to use it to solve stochastic pooling problems. Therefore, the proposed BD algorithm is used to solve simple planning problem examples in order to check the convergence and compare the solution times with BARON. The results clearly show that both BARON and the BD algorithm converges to same global optimum in most of the problems (in couple of complicated problems the BD algorithm cannot converge). However, BARON provides better solution times than the BD algorithm. However, as the problems get complicated and number of variables increases the difference between the BD and BARON in terms of solution times decreases. This looks promising since the real planning problems that we are interested in are much larger than these examples.

This project is still in progress and will continue as a PhD project and as the future work, the next step will be, as mentioned, to look for the possibility of tighter relaxations for non-convex stochastic programs and methods to generate feasible problems before proceeding further into the bounding techniques. After having an efficiently working BD algorithm for stochastic pooling problem, the long term objective is to have a more realistic model of the infrastructure development problem by adding more features to the basic stochastic pooling problem step by step and develop the BD algorithm so it can handle these large scale stochastic programs.

Appendix A

Example Pooling Problems

To validate the algorithm and check its performance, 7 example pooling problems are solved with both the proposed BD algorithm and BARON [42]. 3 of these example problems are taken directly from the literature (Haverly's, Adhya's and Fould's pooling problems), the remaining 4 problems are created by the author to check the performance of the algorithm in more complex pooling problems. In this chapter, detailed information is presented about these example problems including quality specs, demand requirements, cost coefficients and GAMS implementations. Since Haverly's pooling problem is formulated in detail in *Chapter 2*, this chapter excludes it and contains remaining 6 problems.

A.1 Adhya's Pooling Problem

One of the example problems used is taken from Adhya et. al. (1999) [1]. In this problem, the number of pools is 7, the number of sources is 8, the number of qualities is 4 and the number of end-products is 4. Necessary parameters (quality parameters, costs, prices and demand requirements) to construct this problem is given in Tables A.1, A.2, A.3, A.4 and A.5. GAMS implementation of Adhya's problem is also provided at the end of this Chapter.

Source quality parameters				
Sources	Qualities			
	1	2	3	4
1	0.5	1.9	1.3	1
2	1.4	1.8	1.7	1.6
3	1.2	1.9	1.4	1.4
4	1.5	1.2	1.7	1.3
5	1.6	1.8	1.6	2
6	1.2	1.1	1.4	2
7	1.5	1.5	1.5	1.5
8	1.4	1.6	1.2	3

Table A.1: Quality parameters in source nodes for Adhya's problem

Source costs	
Sources	Costs
1	15
2	7
3	4
4	5
5	6
6	3
7	5
8	7

Table A.2: Cost parameters in source nodes for Adhya's problem

Demand quality requirements				
Products	Qualities			
	1	2	3	4
1	2	2.2	2.25	1.1
2	3	1.4	2.5	0.6
3	1.5	1	2.9	1.9
4	2	3	0.75	0.5

Table A.3: Quality requirements in demand nodes for Adhya's problem

Demand flow requirements	
Products	Max. flow
1	30
2	25
3	75
4	50

Table A.4: Flow requirements in demand nodes for Adhya's problem

Prices	
Products	**Prices**
1	16
2	15
3	10
4	25

Table A.5: Prices in demand nodes for Adhya's problem

Source quality parameters	
Sources	**Qualities**
1	1
2	1.1
3	1.2
4	1.3
5	1.1
6	1.2
7	1.3
8	1.4
9	1.2
10	1.3
11	1.4
12	1.5
13	1.6
14	1.3

Table A.6: Quality parameters in source nodes for Foulds' problem

A.2 Foulds' Pooling Problem

Another example problem is taken from Foulds et. al. (1992) [12]. In Foulds' problem, the number of pools is 8; the number of sources is 14, the number of qualities is 1 and the number of end-products is 6. Necessary parameters (quality parameters, costs, prices and demand requirements) to construct this problem is given in Tables A.6, A.7, A.8, A.9 and A.10. GAMS implementation of this problem is also provided at the end of this Chapter.

Source costs	
Sources	**Costs**
1	20
2	19
3	18
4	17
5	19
6	18
7	17
8	16
9	18
10	17
11	16
12	15
13	17
14	16

Table A.7: Cost parameters in source nodes for Foulds' problem

Demand quality requirements	
Products	**Qualities**
1	1.05
2	1.1
3	1.15
4	1.2
5	1.25
6	1.3

Table A.8: Quality requirements in demand nodes for Foulds' problem

Demand flow requirements	
Products	**Max. flow**
1	30
2	29
3	28
4	27
5	26
6	25

Table A.9: Flow requirements in demand nodes for Foulds' problem

Prices	
Products	**Prices**
1	20
2	19.5
3	19
4	18.5
5	18
6	17.5

Table A.10: Prices in demand nodes for Foulds' problem

A.3 Example 1

The first example problem (Example 1) has 14 pools, 18 sources, 1 quality and 9 end-products. Necessary parameters (quality parameters, costs, prices and demand requirements) to construct this problem is given in Tables A.11, A.12, A.13, A.14 and A.15. GAMS implementation of this problem is also provided at the end of this Chapter.

A.4 Example 2

The second example problem (Example 2) has 14 pools, 18 sources, 6 qualities and 9 end-products. Necessary parameters (quality parameters, costs, prices and demand requirements) to construct this problem is given in Tables A.16, A.17, A.18, A.19 and A.20. GAMS implementation of this problem is also provided at the end of this Chapter.

A.5 Example 3

The third example problem (Example 3) has 16 sources, 10 pools, 6 end-products and 1 quality variable. Necessary parameters (quality parameters, costs, prices and demand requirements) to construct this problem is given in Tables A.21, A.22, A.23, A.24 and A.25. GAMS implementation of this problem is also provided at the end of this Chapter.

Source quality parameters	
Sources	**Qualities**
1	1.8
2	2
3	2.2
4	1.3
5	1.4
6	1
7	1.6
8	0.8
9	3
10	3.2
11	3.4
12	3.5
13	2.6
14	1.8
15	2.7
16	1.5
17	2.6
18	1.9

Table A.11: Quality parameters in source nodes for Example 1

A.6 Example 4

The fourth example problem (Example 4) Example 4 has 16 sources, 10 pools, 6 end-products and 8 qualities. Necessary parameters (quality parameters, costs, prices and demand requirements) to.construct this problem is given in Tables A.26, A.27, A.28, A.29 and A.30. GAMS implementation of this problem is also provided at the end of this Chapter.

Source costs	
Sources	**Costs**
1	10
2	5
3	6
4	8
5	13
6	25
7	16
8	18
9	35
10	5
11	20
12	15
13	11
14	24
15	20
16	25
17	10
18	14

Table A.12: Cost parameters in source nodes for Example 1

Demand quality requirements	
Products	**Qualities**
1	3
2	2.1
3	1.5
4	1.2
5	2.6
6	2.5
7	1
8	1.75
9	3.2

Table A.13: Quality requirements in demand nodes for Example 1

Demand flow requirements	
Products	**Max. flow**
1	75
2	85
3	80
4	50
5	130
6	120
7	100
8	90
9	95

Table A.14: Flow requirements in demand nodes for Example 1

Prices	
Products	**Prices**
1	30
2	15
3	25
4	40
5	30
6	35
7	22
8	10
9	15

Table A.15: Prices in demand nodes for Example 1

Source quality parameters						
Sources	**Qualities**					
	1	2	3	4	5	6
1	1.8	2.9	1.5	3	0.8	1.4
2	2.2	4.8	3.8	4.6	2.7	3.6
3	2	5	3	2.4	4	2
4	1.5	3.2	2.7	2.5	1.7	0.9
5	3.6	2.8	0.6	2	3.1	2
6	3.2	4.1	1.4	2.8	0.8	4.8
7	4	5	1.5	3.5	4.2	2.1
8	4.5	1.6	2.2	3.8	1.2	3
9	0.8	1.9	1.3	4	1.3	1.6
10	1.4	0.8	1.7	2.6	3.7	1.9
11	2.2	1.9	1.4	1	3.4	5
12	1.5	1	3.7	4.3	3.7	0.8
13	2.6	2.8	1.6	2.4	3.6	2
14	1.2	3.1	1.4	2.8	1	2.6
15	1.9	1.5	3.2	0.8	1.8	3.5
16	2.4	1.8	5	4	2.2	3
17	3.5	2.5	1.8	3.6	5	4.6
18	4.4	2.6	1.2	3	4.2	4

Table A.16: Quality parameters in source nodes for Example 2

Source costs	
Sources	**Costs**
1	10
2	5
3	6
4	8
5	13
6	25
7	16
8	18
9	35
10	5
11	20
12	15
13	11
14	24
15	20
16	25
17	10
18	14

Table A.17: Cost parameters in source nodes for Example 2

Demand quality requirements						
Products	**Qualities**					
	1	2	3	4	5	6
1	3.5	2.9	0.9	3.2	1.8	2.4
2	4.2	4	3.8	2.6	1.7	3
3	2.5	4.8	3.1	4.4	3.7	2.6
4	0.8	1.2	3.7	4.2	4.6	1.8
5	2.6	2	2.4	4	3	2.2
6	3.5	4	1	3.8	0.8	4
7	4	5	2.2	1.9	0.9	3
8	1.5	0.8	3.2	3.8	4.5	3.8
9	2.6	3.9	4.5	4.2	0.8	2.2

Table A.18: Quality requirements in demand nodes for Example 2

Demand flow requirements	
Products	**Max. flow**
1	75
2	85
3	80
4	50
5	130
6	120
7	100
8	90
9	95

Table A.19: Flow requirements in demand nodes for Example 2

Prices	
Products	**Prices**
1	30
2	15
3	25
4	40
5	30
6	35
7	22
8	10
9	15

Table A.20: Prices in demand nodes for Example 2

Source quality parameters	
Sources	**Qualities**
1	3
2	4
3	4.2
4	3.3
5	1
6	2.2
7	2.6
8	3.8
9	4
10	5
11	5.2
12	0.8
13	1.6
14	1
15	1.9
16	3.5

Table A.21: Quality parameters in source nodes for Example 3

Source costs	
Sources	**Costs**
1	30
2	40
3	45
4	38
5	18
6	30
7	32
8	45
9	55
10	50
11	20
12	19
13	20
14	28
15	30
16	45

Table A.22: Cost parameters in source nodes for Example 3

Demand quality requirements	
Products	**Qualities**
1	3
2	2.5
3	4.5
4	5
5	3.6
6	4

Table A.23: Quality requirements in demand nodes for Example 3

Demand flow requirements	
Products	**Max. flow**
1	50
2	20
3	25
4	40
5	30
6	60

Table A.24: Flow requirements in demand nodes for Example 3

Prices	
Products	**Prices**
1	80
2	90
3	25
4	30
5	40
6	75

Table A.25: Prices in demand nodes for Example 3

Source quality parameters								
Sources	**Qualities**							
	1	2	3	4	5	6	7	8
1	4	4.5	5	3.1	0.8	1.4	2.8	4.1
2	3.2	3.8	1	4	2.2	4.6	3	1.6
3	2.5	5.6	2.6	2.9	4	5	2	1
4	3.2	5.2	4.7	2	1.5	4	0.8	5
5	3	1.8	0.2	2	3.5	3	4.2	1.9
6	3.8	4	6	2.8	4.5	5.8	3.4	4.6
7	4	5.2	4.5	3.5	2	2	2.8	1
8	4.5	2.6	5.2	3.8	6.2	2.3	0.5	1.5
9	4.8	0.8	0.3	4	1.9	0.6	0.9	4
10	1	3.1	2	2.6	0.7	1	5	1.2
11	2.8	1	4.4	1	5.4	5.1	4	1
12	1.5	4.1	4.7	5.3	3.7	1	1.8	0.6
13	0.6	3	5.8	1.4	3.6	0.2	2	2.5
14	1.6	3.8	0.9	3.8	1	0.6	3	5
15	3.9	4.5	4.2	4	1.8	5.5	1.2	4.6
16	4.4	4.8	5.2	2.9	4.2	4.5	1.5	3.1

Table A.26: Quality parameters in source nodes for Example 4

Source costs	
Sources	**Costs**
1	30
2	40
3	45
4	38
5	18
6	30
7	32
8	45
9	55
10	50
11	20
12	19
13	20
14	28
15	30
16	45

Table A.27: Cost parameters in source nodes for Example 4

Demand quality requirements								
Products	**Qualities**							
	1	2	3	4	5	6	7	8
1	3.6	5	2.8	3.4	2.6	4.4	4	3
2	4	4.6	5.8	2.6	6	5	5	2.5
3	5.5	4.8	3.4	4.5	3.5	3.6	2.8	3.6
4	2.8	5.2	3.9	1.5	4	4.8	1.8	4.6
5	3.6	2.1	2.8	4	3	4.2	1	2
6	4	0.4	1	3	0.8	4.6	0.5	1.5

Table A.28: Quality requirements in demand nodes for Example 4

Demand flow requirements	
Products	**Max. flow**
1	50
2	20
3	25
4	40
5	30
6	60

Table A.29: Flow requirements in demand nodes for Example 4

Prices	
Products	**Prices**
1	80
2	90
3	25
4	30
5	40
6	75

Table A.30: Prices in demand nodes for Example 4

GAMS Implementation of Adhya's Pooling Problem

```
$ontext
    Gams Model of Adhya's Pooling Problem
    Author: Emre Armagan
    Date: June, 2007
$offtext

$eolcom #

# Set Declarations
    set comp /1*8/;
    set pro  /1*4/;
    set qual /1*4/;
    set pool /1*7/;

# components related parameters
table compparams(comp,*)
          1    2    3
   1      0   75   15
   2      0   75    7
   3      0   75    4
   4      0   75    5
   5      0   75    6
   6      0   75    3
   7      0   75    5
   8      0   75    7 ;

parameters cl(comp), cu(comp), cprice(comp);
cl(comp) = compparams(comp,'1');
cu(comp) = compparams(comp,'2');
cprice(comp) = compparams(comp,'3');

table cqual(comp,qual)
          1    2    3    4
   1    0.5  1.9  1.3    1
   2    1.4  1.8  1.7  1.6
   3    1.2  1.9  1.4  1.4
   4    1.5  1.2  1.7  1.3
   5    1.6  1.8  1.6    2
   6    1.2  1.1  1.4    2
   7    1.5  1.5  1.5  1.5
   8    1.4  1.6  1.2    3;
```

```
# pool related parameters
parameters psize(pool);
psize(pool) = 75;

# product related parameters
table prodparams(pro,*)
         1    2    3
   1     0   30   16
   2     0   25   25
   3     0   75   10
   4     0   50   25 ;

parameters prl(pro), pru(pro), pprice(pro);
prl(pro) = prodparams(pro,'1');
pru(pro) = prodparams(pro,'2');
pprice(pro) = prodparams(pro,'3');

parameter pqlbd(pro, qual);
pqlbd(pro, qual) = 0;

table pqubd(pro, qual)
         1    2    3    4
   1     2  2.2 2.25  1.1
   2     3  1.4  2.5  0.6
   3   1.5    1  2.9  1.9
   4     2    3 0.75  0.5;

# network related parameters
table ubq(comp, pool)
         1    2    3    4    5    6    7
   1     1    0    0    1    0    1    0
   2     1    0    0    1    1    0    1
   3     0    1    0    0    0    0    1
   4     0    1    0    0    1    1    0
   5     0    1    0    0    1    0    1
   6     0    0    1    0    0    0    1
   7     0    0    1    0    0    1    0
   8     0    0    1    1    0    1    0;

parameter ubz(comp, pro);
ubz(comp, pro) = 0;

$include pool.gms
```

GAMS Implementation of Foulds' Pooling Problem

```
$ontext
    Gams Model of Foulds' Pooling Problem
    Author: Emre Armagan
    Date: June, 2007
$offtext

$eolcom #

# Set Declarations
    set comp /1*14/;
    set pro  /1*6/;
    set qual /1*1/;
    set pool /1*8/;

# components related parameters
table compparams(comp,*)
         1    2    3
   1     0   50   20
   2     0   50   19
   3     0   50   18
   4     0   50   17
   5     0   50   19
   6     0   50   18
   7     0   50   17
   8     0   50   16
   9     0   50   18
  10     0   50   17
  11     0   50   16
  12     0   50   15
  13     0   50   17
  14     0   50   16 ;

parameters cl(comp), cu(comp), cprice(comp);
cl(comp) = compparams(comp,'1');
cu(comp) = compparams(comp,'2');
cprice(comp) = compparams(comp,'3');

table cqual(comp,qual)
         1
   1     1
   2   1.1
   3   1.2
   4   1.3
   5   1.4
   6   1.2
   7   1.3
   8   1.4
   9   1.2
  10   1.3
  11   1.4
  12   1.5
  13   1.6
  14   1.3 ;

# pool related parameters
parameters psize(pool);
psize(pool) = 75;

# product related parameters
table prodparams(pro,*)
         1    2    3
   1     0   30   20
   2     0   29 19.5
   3     0   28   19
   4     0   27 18.5
   5     0   26   18
   6     0   25 17.5 ;

parameters prl(pro), pru(pro), pprice(pro);
prl(pro) = prodparams(pro,'1');
pru(pro) = prodparams(pro,'2');
pprice(pro) = prodparams(pro,'3');

parameter pqlbd(pro, qual);
pqlbd(pro, qual) = 0;

table pqubd(pro, qual)
         1
   1  1.05
   2   1.1
   3  1.15
   4   1.2
   5  1.25
   6   1.3 ;
```

```
# network related parameters
table ubq(comp, pool)
        1    2    3    4    5    6    7    8
   1    0    0    1    0    0    0    1    0
   2    0    1    0    1    0    1    0    1
   3    0    0    0    1    0    0    1    1
   4    0    0    1    0    1    1    0    1
   5    0    1    0    1    1    0    0    1
   6    1    0    1    0    0    1    1    0
   7    1    0    0    0    1    0    0    0
   8    1    0    1    1    0    0    1    0;

parameter ubz(comp, pro);
ubz(comp, pro) = 0;

$include pool.gms
```

GAMS Implementation of Example 1

```
$ontext
    Gams Model of Example 1 Pooling Problem
    Author: Emre Armagan
    Date: July, 2007
$offtext

$eolcom #

# Set Declarations
    set comp /1*18/;
    set pro  /1*9/;
    set qual /1*1/;
    set pool /1*14/;

# components related parameters
table compparams(comp,*)
         1    2    3
   1     0  200   10
   2     0  200    5
   3     0  200    6
   4     0  200    8
   5     0  200   13
   6     0  200   25
   7     0  200   16
   8     0  200   18
   9     0  200   35
  10     0  200    5
  11     0  200   20
  12     0  200   15
  13     0  200   11
  14     0  200   24
  15     0  200   20
  16     0  200   25
  17     0  200   10
  18     0  200   14 ;

parameters cl(comp), cu(comp), cprice(comp);
cl(comp) = compparams(comp,'1');
cu(comp) = compparams(comp,'2');
cprice(comp) = compparams(comp,'3');
```

```
table cqual(comp,qual)
         1
   1   1.8
   2     2
   3   2.2
   4   1.3
   5   1.4
   6     1
   7   1.6
   8   0.8
   9     3
  10   3.2
  11   3.4
  12   3.5
  13   2.6
  14   1.8
  15   2.7
  16   1.5
  17   2.6
  18   1.9 ;

# pool related parameters
parameters psize(pool);
psize(pool) = 75;

# product related parameters
table prodparams(pro,*)
         1    2    3
   1     0   75   30
   2     0   85   15
   3     0   80   25
   4     0   50   40
   5     0  130   30
   6     0  120   35
   7     0  100   22
   8     0   90   10
   9     0   95   15 ;

parameters prl(pro), pru(pro), pprice(pro);
prl(pro) = prodparams(pro,'1');
pru(pro) = prodparams(pro,'2');
pprice(pro) = prodparams(pro,'3');

parameter pqlbd(pro, qual);
pqlbd(pro, qual) = 0;
```

```
table pqubd(pro, qual)
          1
   1      3
   2    2.1
   3    1.5
   4    1.2
   5    2.6
   6    2.5
   7      1
   8   1.75
   9    3.2 ;


# network related parameters
table ubq(comp, pool)
          1     2     3     4     5     6     7     8     9    10
   1      1     0     0     1     0     0     0     1     0     0
   2      1     0     0     1     0     0     0     1     0     0
   3      1     0     0     0     1     0     0     1     0     0
   4      1     0     0     0     1     0     0     1     0     0
   5      1     0     0     0     1     0     0     0     1     0
   6      0     1     0     0     1     0     0     0     1     0
   7      0     1     0     0     1     0     0     0     1     0
   8      0     1     0     0     0     1     0     0     1     0
   9      0     1     0     0     0     1     0     0     1     0
  10      0     1     0     0     0     1     0     0     0     1
  11      0     0     1     0     0     1     0     0     0     1
  12      0     0     1     0     0     1     0     0     0     1
  13      0     0     1     0     0     0     1     0     0     1
  14      0     0     1     0     0     0     1     0     0     1
  15      0     0     1     0     0     0     1     0     0     0
  16      0     0     0     1     0     0     1     0     0     0
  17      0     0     0     1     0     0     1     0     0     0
  18      0     0     0     1     0     0     0     1     0     0

         11    12    13    14
   1      1     0     0     0
   2      0     1     0     0
   3      0     1     0     0
   4      0     1     0     0
   5      0     1     0     0
   6      0     1     0     0
   7      0     0     1     0
   8      0     0     1     0
   9     0    0    1    0
  10     0    0    1    0
  11     0    0    1    0
  12     0    0    0    1
  13     0    0    0    1
  14     0    0    0    1
  15     1    0    0    1
  16     1    0    0    1
  17     1    0    0    0
  18     1    0    0    0 ;


parameter ubz(comp, pro);
ubz(comp, pro) = 0;

$include pool.gms
```

GAMS Implementation of Example 2

```
$ontext
    Gams Model of Example 2 Pooling Problem
    Author: Emre Armagan
    Date: July, 2007
$offtext

$eolcom #

# Set Declarations
    set comp /1*18/;
    set pro  /1*9/;
    set qual /1*6/;
    set pool /1*14/;

# components related parameters
table compparams(comp,*)
         1    2    3
   1     0  200   10
   2     0  200    5
   3     0  200    6
   4     0  200    8
   5     0  200   13
   6     0  200   25
   7     0  200   16
   8     0  200   18
   9     0  200   35
  10     0  200    5
  11     0  200   20
  12     0  200   15
  13     0  200   11
  14     0  200   24
  15     0  200   20
  16     0  200   25
  17     0  200   10
  18     0  200   14 ;

parameters cl(comp), cu(comp), cprice(comp);
cl(comp) = compparams(comp,'1');
cu(comp) = compparams(comp,'2');
cprice(comp) = compparams(comp,'3');
```

```
table cqual(comp,qual)
         1    2    3    4    5    6
   1   1.8  2.9  1.5    3  0.8  1.4
   2   2.2  4.8  3.8  4.6  2.7  3.6
   3     2    5    3  2.4    4    2
   4   1.5  3.2  2.7  2.5  1.7  0.9
   5   3.6  2.8  0.6    2  3.1    2
   6   3.2  4.1  1.4  2.8  0.8  4.8
   7     4    5  1.5  3.5  4.2  2.1
   8   4.5  1.6  2.2  3.8  1.2    3
   9   0.8  1.9  1.3    4  1.3  1.6
  10   1.4  0.8  1.7  2.6  3.7  1.9
  11   2.2  1.9  1.4    1  3.4    5
  12   1.5    1  3.7  4.3  3.7  0.8
  13   2.6  2.8  1.6  2.4  3.6    2
  14   1.2  3.1  1.4  2.8    1  2.6
  15   1.9  1.5  3.2  0.8  1.8  3.5
  16   2.4  1.8    5  3.8  1.2    3
  17   3.5  2.5  1.8  3.6    5  4.6
  18   4.4  2.6  1.2    3  4.2    4  ;

# pool related parameters
parameters psize(pool);
psize(pool) = 75;

# product related parameters
table prodparams(pro,*)
         1    2    3
   1     0   75   30
   2     0   85   15
   3     0   80   25
   4     0   50   40
   5     0  130   30
   6     0  120   35
   7     0  100   22
   8     0   90   10
   9     0   95   15 ;

parameters prl(pro), pru(pro), pprice(pro);
prl(pro) = prodparams(pro,'1');
pru(pro) = prodparams(pro,'2');
pprice(pro) = prodparams(pro,'3');

parameter pqlbd(pro, qual);
pqlbd(pro, qual) = 0;
```

```
table pqubd(pro, qual)
          1    2    3    4    5    6
    1   3.5  2.9  0.9  3.2  1.8  2.4
    2   4.2    4  3.8  2.6  1.7    3
    3   2.5  4.8  3.1  4.4  3.7  2.6
    4   0.8  1.2  3.7  4.2  4.6  1.8
    5   2.6    2  2.4    4    3  2.2
    6   3.5    4    1  3.8  0.8    4
    7     4    5  2.2  1.9  0.9    3
    8   1.5  0.8  3.2  3.8  4.5  3.8
    9   2.6  3.9  4.5  4.2  0.8  2.2 ;


# network related parameters
table ubq(comp, pool)
          1    2    3    4    5    6    7    8    9   10
    1     1    0    0    1    0    0    0    1    0    0
    2     1    0    0    1    0    0    0    1    0    0
    3     1    0    0    0    1    0    0    1    0    0
    4     1    0    0    0    1    0    0    1    0    0
    5     1    0    0    0    1    0    0    0    1    0
    6     0    1    0    0    1    0    0    0    1    0
    7     0    1    0    0    1    0    0    0    1    0
    8     0    1    0    0    0    1    0    0    1    0
    9     0    1    0    0    0    1    0    0    1    0
   10     0    1    0    0    0    1    0    0    0    1
   11     0    0    1    0    0    1    0    0    0    1
   12     0    0    1    0    0    1    0    0    0    1
   13     0    0    1    0    0    0    1    0    0    1
   14     0    0    1    0    0    0    1    0    0    1
   15     0    0    1    0    0    0    1    0    0    0
   16     0    0    0    1    0    0    1    0    0    0
   17     0    0    0    1    0    0    1    0    0    0
   18     0    0    0    1    0    0    0    1    0    0

         11   12   13   14
    1     1    0    0    0
    2     0    1    0    0
    3     0    1    0    0
    4     0    1    0    0
    5     0    1    0    0
    6     0    1    0    0
    7     0    0    1    0
    8     0    0    1    0
   9      0     0     1     0
  10      0     0     1     0
  11      0     0     1     0
  12      0     0     0     1
  13      0     0     0     1
  14      0     0     0     1
  15      1     0     0     1
  16      1     0     0     1
  17      1     0     0     0
  18      1     0     0     0 ;


parameter ubz(comp, pro);
ubz(comp, pro) = 0;

$include pool.gms
```

GAMS Implementation of Example 3

```
$ontext
    Gams Model of Example 3 Pooling Problem
    Author: Emre Armagan
    Date: July, 2007
$offtext

$eolcom #

# Set Declarations
    set comp /1*16/;
    set pro  /1*6/;
    set qual /1*1/;
    set pool /1*10/;

# components related parameters
table compparams(comp,*)
         1    2    3
   1     0  100   30
   2     0  100   40
   3     0  100   45
   4     0  100   38
   5     0  100   18
   6     0  100   30
   7     0  100   32
   8     0  100   45
   9     0  100   55
  10     0  100   50
  11     0  100   20
  12     0  100   19
  13     0  100   20
  14     0  100   28
  15     0  100   30
  16     0  100   45 ;

parameters cl(comp), cu(comp), cprice(comp);
cl(comp) = compparams(comp,'1');
cu(comp) = compparams(comp,'2');
cprice(comp) = compparams(comp,'3');

table cqual(comp,qual)
         1
   1     3
   2     4
   3   4.2
   4   3.3
   5     1
   6   2.2
   7   2.6
   8   3.8
   9     4
  10     5
  11   5.2
  12   0.8
  13   1.6
  14     1
  15   1.9
  16   3.5 ;

# pool related parameters
parameters psize(pool);
psize(pool) = 75;

# product related parameters
table prodparams(pro,*)
           1    2    3
   1       0   50   80
   2       0   20   90
   3       0   25   25
   4       0   40   30
   5       0   30   40
   6       0   60   75 ;

parameters prl(pro), pru(pro), pprice(pro);
prl(pro) = prodparams(pro,'1');
pru(pro) = prodparams(pro,'2');
pprice(pro) = prodparams(pro,'3');

parameter pqlbd(pro, qual);
pqlbd(pro, qual) = 0;

table pqubd(pro, qual)
           1
   1       3
   2     2.5
   3     4.5
   4       5
   5   3.6
   6     4 ;


# network related parameters
table ubq(comp, pool)
         1    2    3    4    5    6    7    8    9   10
   1     0    0    1    0    0    1    1    0    0    1
   2     0    0    1    0    0    1    0    0    1    1
   3     0    0    1    0    0    1    0    0    1    0
   4     0    0    1    0    0    1    0    0    1    0
   5     0    1    1    0    0    1    0    0    1    0
   6     0    1    0    0    1    1    0    0    1    0
   7     0    1    0    0    1    0    0    1    1    0
   8     0    1    0    0    1    0    0    1    0    0
   9     0    1    0    0    1    0    0    1    0    0
  10     1    1    0    0    1    0    0    1    0    0
  11     1    0    0    1    1    0    0    1    0    0
  12     1    0    0    1    0    0    1    1    0    0
  13     1    0    0    1    0    0    1    0    0    1
  14     1    0    0    1    0    0    1    0    0    1
  15     1    0    0    1    0    0    1    0    0    1
  16     1    0    1    1    0    0    1    0    0    1 ;


parameter ubz(comp, pro);
ubz(comp, pro) = 0;

$include pool.gms
```

GAMS Implementation of Example 4

```
$ontext
    Gams Model of Example 4 Pooling Problem
    Author: Emre Armagan
    Date: July, 2007
$offtext

$eolcom #

# Set Declarations
    set comp /1*16/;
    set pro  /1*6/;
    set qual /1*8/;
    set pool /1*10/;

# components related parameters
table compparams(comp,*)
         1    2    3
   1     0  100   30
   2     0  100   40
   3     0  100   45
   4     0  100   38
   5     0  100   18
   6     0  100   30
   7     0  100   32
   8     0  100   45
   9     0  100   55
  10     0  100   50
  11     0  100   20
  12     0  100   19
  13     0  100   20
  14     0  100   28
  15     0  100   30
  16     0  100   45 ;

parameters cl(comp), cu(comp), cprice(comp);
cl(comp) = compparams(comp,'1');
cu(comp) = compparams(comp,'2');
cprice(comp) = compparams(comp,'3');

table cqual(comp,qual)
         1    2    3    4    5    6    7    8
```

```
     1     4  4.5    5  3.1  0.8  1.4  2.8  4.1
     2   3.2  3.8    1    4  2.2  4.6    3  1.6
     3   2.5  5.6  2.6  2.9    4    5    2    1
     4   3.2  5.2  4.7    2  1.5    4  0.8    5
     5     3  1.8  0.2    2  3.5    3  4.2  1.9
     6   3.8    4    6  2.8  4.5  5.8  3.4  4.6
     7     4  5.2  4.5  3.5    2    2  2.8    1
     8   4.5  2.6  5.2  3.8  6.2  2.3  0.5  1.5
     9   4.8  0.8  0.3    4  1.9  0.6  0.9    4
    10     1  3.1    2  2.6  0.7    1    5  1.2
    11   2.8    1  4.4    1  5.4  5.1    4    1
    12   1.5  4.1  4.7  5.3  3.7    1  1.8  0.6
    13   0.6    3  5.8  1.4  3.6  0.2    2  2.5
    14   1.6  3.8  0.9  3.8    1  0.6    3    5
    15   3.9  4.5  4.2    4  1.8  5.5  1.2  4.6
    16   4.4  4.8  5.2  2.9  4.2  4.5  1.5  3.1 ;

# pool related parameters
parameters psize(pool);
psize(pool) = 75;

# product related parameters
table prodparams(pro,*)
         1    2    3
   1     0   50   80
   2     0   20   90
   3     0   25   25
   4     0   40   30
   5     0   30   40
   6     0   60   75 ;

parameters prl(pro), pru(pro), pprice(pro);
prl(pro) = prodparams(pro,'1');
pru(pro) = prodparams(pro,'2');
pprice(pro) = prodparams(pro,'3');

parameter pqlbd(pro, qual);
pqlbd(pro, qual) = 0;

table pqubd(pro, qual)
         1    2    3    4    5    6    7    8
   1   3.6    5  2.8  3.4  2.6  4.4    4    3
   2     4  4.6  5.8  2.6    6    5    5  2.5
   3   5.5  4.8  3.4  4.5  3.5  3.6  2.8  3.6
   4   2.8  5.2  3.9  1.5    4  4.8  1.8  4.6
   5   3.6  2.1  2.8    4    3  4.2    1    2
   6     4  0.4    1    3  0.8  4.6  0.5  1.5 ;

# network related parameters
table ubq(comp, pool)
         1    2    3    4    5    6    7    8    9   10
   1     0    0    1    0    0    1    1    0    0    1
   2     0    0    1    0    0    1    0    0    1    1
   3     0    0    1    0    0    1    0    0    1    0
   4     0    0    1    0    0    1    0    0    1    0
   5     0    1    1    0    0    1    0    0    1    0
   6     0    1    0    0    1    1    0    0    1    0
   7     0    1    0    0    1    0    0    1    1    0
   8     0    1    0    0    1    0    0    1    0    0
   9     0    1    0    0    1    0    0    1    0    0
  10     1    1    0    0    1    0    0    1    0    0
  11     1    0    0    1    1    0    0    1    0    0
  12     1    0    0    1    0    0    1    1    0    0
  13     1    0    0    1    0    0    1    0    0    1
  14     1    0    0    1    0    0    1    0    0    1
  15     1    0    0    1    0    0    1    0    0    1
  16     1    0    1    1    0    0    1    0    0    1 ;


parameter ubz(comp, pro);
ubz(comp, pro) = 0;

$include pool.gms
```

Appendix B

Gas Network Example

Gas network problems are a special kind of pooling problems where pools can be modeled as mixers and splitters. Modeling pools as mixers and splitters gives the opportunity to write mass balances for each quality separately.

For mixers, for a selected quality, mass balance can be written as the output volume flow rate equals to the sums of input volume flow rates and it is a linear equation. In other words, for the mixer shown in Figure B-1 (a), for a selected quality, mass balance can be written as $f_3 = f_1 + f_2$ where f_i are flow variables and it is a linear equation. However, for splitters, writing mass balances separately still introduces bilinear terms. In other words, for the splitter shown in Figure B-1 (b), for a selected quality, mass balance can be written as $q_1 f_3 = q_2 f_1 + (1 - q_2) f_2$ where f_i are flow variables; q_i are quality variables and and obviously, this equation is a bilinear equation. But, now since bilinear terms are only coming from the splitters instead of all of the pools, the number of bilinear terms reduces and therefore the complexity of the problem reduces greatly.

In order to test the performance of the proposed BD algorithm in a gas network problem, an example problem shown in Figure B-2 is studied. As shown in the figure this problem has 10 pools, 8 sources, 3 qualities and 4 end-products. Necessary parameters (quality parameters, costs, prices and demand requirements) to construct this problem is given in

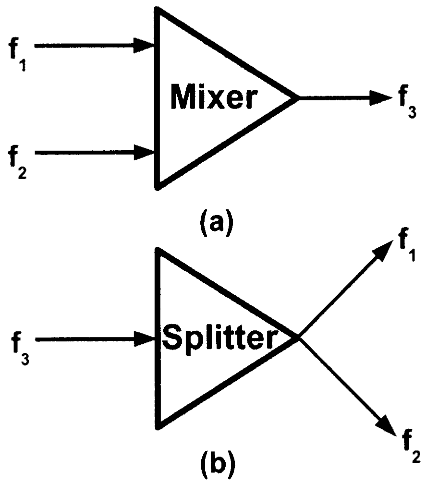


Figure B-1: Representation of a mixer (a) and splitter (b)

Tables B.1, B.2, B.3, B.4 and B.5. GAMS implementation of this problem is also provided at the end of this Chapter.

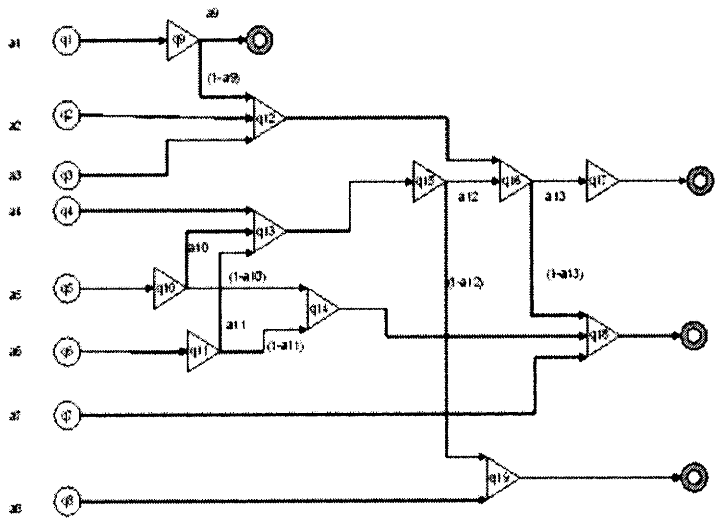


Figure B-2: The gas network example

Source quality parameters			
Sources	**Qualities**		
	1	2	3
1	2.5	2.9	0.8
2	2.4	1.8	2
3	1	3	2.4
4	1.5	2	1.8
5	1.8	1.9	0.6
6	0.9	1.4	2.4
7	1.2	1.5	3.5
8	2.4	1.9	1

Table B.1: Quality parameters in source nodes for the gas network example

Source costs	
Sources	**Costs**
1	15
2	10
3	20
4	5
5	10
6	15
7	25
8	20

Table B.2: Cost parameters in source nodes for the gas network example

Demand quality requirements			
Products	**Qualities**		
	1	2	3
1	2	2	3
2	3	1.5	2
3	1.5	3	1.5
4	2	2.5	0.75

Table B.3: Quality requirements in demand nodes for the gas network example

Demand flow requirements	
Products	**Max. flow**
1	50
2	100
3	75
4	80

Table B.4: Flow requirements in demand nodes for the gas network example

Prices	
Products	**Prices**
1	30
2	45
3	10
4	25

Table B.5: Prices in demand nodes for the gas network example

GAMS Implementation of Gas Network Example

```
$ontext
    Gams Model of Gas network example
    Author: Emre Armagan
    Date: July, 2007
$offtext

$TITLE Natural Gas Network Optimization Model

SETS

    NodeSet "Superset for Nodes"

    ArcSet   "Superset  for Arcs"

    Junctions(NodeSet) "Set of junctions where production should be set to zero"

    Wells(NodeSet) "Set of Wells"

    Splitters(NodeSet) "Set of Splitter"

    SplitOut(Splitters, ArcSet)

*  Mixers(NodeSet) "Set of Mixer"

    Demands "Set of demand"

    dNodes(NodeSet) "Demand Nodes"

    ddN(Demands, NodeSet) "correlation set between the demands and node"

    Components "Set of all components"

    Spec(components) "components on which specification is forced"
    ArcOrigin(ArcSet, NodeSet) "Arc origin to Node mapping"
    ArcEnd(ArcSet, NodeSet) "Arc end to Node mapping";


PARAMETERS


    fA(ArcSet)  "friction factor constant for arcs"
    A(Wells)  "Reservoir flow equation constants"
    B(Wells)  "Reservoir flow equation constants"
    C(Wells) "Reservoir flow equation constants"
    F(Wells) "Reservoir flow equation constants"
    E(Wells) "sqrt(B)"
    Pres(Wells) "Reservoir Pressure"
* IMPERIAL UNITS
    fAi(ArcSet) "friction factor constant for arcs"
    Ai(Wells) "Reservoir flow equation constants"
    Bi(Wells) "Reservoir flow equation constants"
    Ci(Wells) "Reservoir flow equation constants"
    Fi(Wells) "Reservoir flow equation constants"

    D(Demands) "Demands"

    Pdemand(Demands) "Requested Pressure at a demand"

    MW(Components) "Molecular Weight of components"
    yspec(Demands, spec) "Specification compositions on a component set
    (Mole fraction)";


SCALARS
    Ti "Duration in days for a time interval"
    rho "Density at standard temperature and pressure"
    convfactorP "Pressure conversion factor"
    convfactorV "Volumetric flow rate conversion factor"
    convfactor1 "Intermediate factors"
    convfactor2 "Intermediate factors"
    convfactor3 "Intermediate factors"
    convfactor4 "Intermediate factors"
    PdropScalefactor1 "Scale factor for Arc pressure drop"
    PdropScalefactor2 "Scale factor for Well pressure drop"
    MoleScalefactor "Flow scale factor";

*P A R A M E T E R     F I L E

$include gasplan-parameters


$ontext

Variable and Parameter Naming Conventions

fe = field exit (Gas collection Network exit, this is raw gas with liquids in)
```

```
ce = compressor exit
l = liquids
g = gases
sr = sour field
sw = sweet field
b = blending
fp= final field production
i = component flows
ds = desulfurization facility in sour field
dh = dehydration by adsorption in the sweet field
c = compression
st = storage
in = in
out = out
cf = component flows
$offtext

VARIABLES
* Objective value
    z
* Quantities at Each Node
    Mpcf(NodeSet, Components) "Component flow at each node"

* Quantities at Arcs
    PAin(ArcSet)  "Pressure at the origin of an edge (equal to the node before it)
    PAout(ArcSet) "Pressure at end of the arc (x10 bar)"

    MA(ArcSet) "Cumulative Flow in Arc (10^6 kg)"
    MAcf(ArcSet, Components) "Cumulative Component Flow in Arc (10^6 kg)"
    QA(ArcSet) "Volumetric flow in Arc in cu.m/day"

* Quantities at Wells
    Pfbhp(Wells) "Flowing Bottom Hole Pressure for well"
    Pfthp(Wells) "Flowing Tubing Head Pressure for well"
    Qp(Wells) "Volumetric flow rates at the wells"

* Spliting Ratio
    alpha(Splitters) "Split Fraction"

* Demand variables
    Fmolar(Demands, Components) "Component molar flow rate at demands"
    FTmolar(Demands) "Total molar flow rate";
*   x(Demands, Components) "Molar Composition at demands";
```

```
EQUATIONS

    ArcPressureFlowRelation(ArcSet)
    ArcMassVolumeRelation(ArcSet)
    TotalArcFlow(ArcSet)


    PositiveFlowConstraint(ArcSet)

    NodeArcMassBalance(NodeSet, Components)


    ArcPressureRelationsN9A
    ArcPressureRelationsN9B
    ArcPressureRelationsN9C

    ArcPressureRelationsN10A
    ArcPressureRelationsN10B
    ArcPressureRelationsN10C


    ArcPressureRelationsN11A
    ArcPressureRelationsN11B
    ArcPressureRelationsN11C

    ArcPressureRelationsN12A
    ArcPressureRelationsN12B
    ArcPressureRelationsN12C

    ArcPressureRelationsN13A
    ArcPressureRelationsN13B
    ArcPressureRelationsN13C

    ArcPressureRelationsN14A

    ArcPressureRelationsN15A
    ArcPressureRelationsN15B
    ArcPressureRelationsN15C

    ArcPressureRelationsN16A
    ArcPressureRelationsN16B
    ArcPressureRelationsN16C
    ArcPressureRelationsN16D
    ArcPressureRelationsN16E
```

```
ArcPressureRelationsN17A

ArcPressureRelationsN18A
ArcPressureRelationsN18B
ArcPressureRelationsN18C

ArcPressureRelationsN19A
ArcPressureRelationsN19B


JunctionNodes(Junctions, Components)

SplitterConstraintN9(Components)
SplitterConstraintN10(Components)
SplitterConstraintN11(Components)
SplitterConstraintN15(Components)
SplitterConstraintN16(Components)


BottomHolePressure(Wells)

TubingHeadPressure(Wells)

BHResRelation(Wells)

BHPTHPRelation1(Wells)
BHPTHPRelation2(Wells)

TubingHeadFlowConditionN1
TubingHeadFlowConditionN2
TubingHeadFlowConditionN3
TubingHeadFlowConditionN4
TubingHeadFlowConditionN5
TubingHeadFlowConditionN6
TubingHeadFlowConditionN7
TubingHeadFlowConditionN8


WellComponentFlows(Wells, Components)


DemandPressureConstraintN20
DemandPressureConstraintN21
DemandPressureConstraintN22
DemandPressureConstraintN23
```

```
    DemandConstraint(Demands)
    DemandMolarFlows(Demands, Components)
    DemandMolarSpecification(Demands, Spec)
*   DemandMolarComposition(Demands, Components)
    DemandTotalMoleFlow(Demands)
    Objective;




ArcPressureFlowRelation(ArcSet).. fA(ArcSet)*QA(ArcSet)*QA(ArcSet)
- PAin(ArcSet)*PAin(ArcSet) + PAout(ArcSet)*PAout(ArcSet) =E= 0;
ArcMassVolumeRelation(ArcSet).. MA(ArcSet) - QA(ArcSet)*rho*Ti =E= 0;

TotalArcFlow(ArcSet).. MA(ArcSet) - SUM(Components, MAcf(Arcset, Components)) =E=


PositiveFlowConstraint(ArcSet).. PAout(ArcSet) - PAin(ArcSet) =L= 0;


* RELATIONSHIP BETWEEN NODE and ARC VARIABLES

NodeArcMassBalance(NodeSet, Components)..
SUM(ArcSet$ArcOrigin(ArcSet, NodeSet), MAcf(ArcSet, Components))
- SUM(ArcSet$ArcEnd(ArcSet, NodeSet), MAcf(ArcSet, Components))
- Mpcf(NodeSet, Components) =E= 0;

NodeArcMassBalance(NodeSet)..
SUM(ArcSet, IN(NodeSet, ArcSet)*MAcf(ArcSet))
=E=  Mpcf(NodeSet, Components) ;


ArcPressureRelationsN9A.. PAin('A9') - PAout('A1') =L= 0;
ArcPressureRelationsN9B.. PAin('A10') - PAout('A1') =L= 0;
ArcPressureRelationsN9C.. PAin('A9') - PAin('A10') =E= 0;

ArcPressureRelationsN10A.. PAin('A11') - PAout('A5') =L= 0;
ArcPressureRelationsN10B.. PAin('A12') - PAout('A5') =L= 0;
ArcPressureRelationsN10C.. PAin('A11') - PAin('A12') =E= 0;


ArcPressureRelationsN11A.. PAin('A13') - PAout('A6') =L= 0;
```

```
ArcPressureRelationsN11B.. PAin('A14') - PAout('A6') =L= 0;
ArcPressureRelationsN11C.. PAin('A13') - PAin('A14') =E= 0;

ArcPressureRelationsN12A.. PAin('A15') - PAout('A10') =L= 0;
ArcPressureRelationsN12B.. PAin('A15') - PAout('A2') =L= 0;
ArcPressureRelationsN12C.. PAin('A15') - PAout('A3') =L= 0;

ArcPressureRelationsN13A.. PAin('A16') - PAout('A4') =L= 0;
ArcPressureRelationsN13B.. PAin('A16') - PAout('A11') =L= 0;
ArcPressureRelationsN13C.. PAin('A16') - PAout('A13') =L= 0;

ArcPressureRelationsN14A.. PAin('A17') - PAout('A14') =L= 0;

ArcPressureRelationsN15A.. PAin('A18') - PAout('A16') =L= 0;
ArcPressureRelationsN15B.. PAin('A19') - PAout('A16') =L= 0;
ArcPressureRelationsN15C.. PAin('A18') - PAin('A19') =E= 0;

ArcPressureRelationsN16A.. PAin('A20') - PAout('A15') =L= 0;
ArcPressureRelationsN16B.. PAin('A21') - PAout('A15') =L= 0;
ArcPressureRelationsN16C.. PAin('A20') - PAout('A18') =L= 0;
ArcPressureRelationsN16D.. PAin('A21') - PAout('A18') =L= 0;
ArcPressureRelationsN16E.. PAin('A20') - PAin('A21') =E= 0;

ArcPressureRelationsN17A.. PAin('A22') - PAout('A20') =L= 0;

ArcPressureRelationsN18A.. PAin('A23') - PAout('A21') =L= 0;
ArcPressureRelationsN18B.. PAin('A23') - PAout('A17') =L= 0;
ArcPressureRelationsN18C.. PAin('A23') - PAout('A7') =L= 0;

ArcPressureRelationsN19A.. PAin('A24') - PAout('A19') =L= 0;
ArcPressureRelationsN19B.. PAin('A24') - PAout('A8') =L= 0;



* JUNCTION NODES
JunctionNodes(Junctions, Components)..  Mpcf(Junctions, Components) =E= 0;


* SPLITTER CONSTRAINTS


SplitterConstraintN9(Components)..
MAcf('A9', Components - alpha('N9')*MAcf('A1',Components) =E= 0;
SplitterConstraintN10(Components)..
MAcf('A11', Components) - alpha('N10')*MAcf('A5',Components) =E= 0;

SplitterConstraintN11(Components)..
MAcf('A13', Components) - alpha('N11')*MAcf('A6',Components) =E= 0;

SplitterConstraintN15(Components)..
MAcf('A18', Components) - alpha('N15')*MAcf('A16',Components) =E= 0;

SplitterConstraintN16(Components)..
MAcf('A20', Components) - alpha('N16')*(MAcf('A15',Components)
+ MAcf('A18',Components)) =E= 0;

* Well Constraints


BottomHolePressure(Wells)..
Pres(Wells)*Pres(Wells) - Pfbhp(Wells)*Pfbhp(Wells)
- A(Wells)*Qp(Wells) - F(Wells)*Qp(Wells)*Qp(Wells) =E= 0;


TubingHeadPressure(Wells)..
B(Wells)*Pfthp(Wells)*Pfthp(Wells) - Pfbhp(Wells)*Pfbhp(Wells)
- C(Wells)*Qp(Wells)*Qp(Wells) =E= 0;



BHResRelation(Wells).. Pfbhp(Wells)-Pres(Wells) =L= 0;

BHPTHPRelation1(Wells).. Pfthp(Wells)-Pfbhp(Wells) =L= 0;
BHPTHPRelation2(Wells).. Pfbhp(Wells) - E(Wells)*Pfthp(Wells) =L= 0;

TubingHeadFlowConditionN1..   PAin('A1') - Pfthp('N1') =L= 0   ;
TubingHeadFlowConditionN2..   PAin('A2') - Pfthp('N2')=L= 0   ;
TubingHeadFlowConditionN3..   PAin('A3') - Pfthp('N3')=L= 0   ;
TubingHeadFlowConditionN4..   PAin('A4') - Pfthp('N4') =L= 0   ;
TubingHeadFlowConditionN5..   PAin('A5') - Pfthp('N5')=L= 0   ;
TubingHeadFlowConditionN6..   PAin('A6') - Pfthp('N6')=L= 0   ;
TubingHeadFlowConditionN7..   PAin('A7') - Pfthp('N7') =L= 0   ;
TubingHeadFlowConditionN8..   PAin('A8') - Pfthp('N8')=L= 0   ;


WellComponentFlows(Wells, Components)..
Mpcf(Wells, Components) - y(Wells,Components)*rho*Ti*Qp(Wells) =E= 0;
```

```
DemandConstraint(Demands)..
Ti*rho*D(Demands) + SUM(Components, SUM(dNodes$ddN(Demands, dNodes),
Mpcf(dNodes, Components))) =L= 0;

DemandPressureConstraintN20..  Pdemand('d1') - PAout('A9')  =L= 0;
DemandPressureConstraintN21..  Pdemand('d2') - PAout('A22')  =L= 0;
DemandPressureConstraintN22..  Pdemand('d3') - PAout('A23')  =L= 0;
DemandPressureConstraintN23..  Pdemand('d4') - PAout('A24')  =L= 0;

DemandMolarFlows(Demands, Components)..
Fmolar(Demands, Components) + (SUM(dNodes$ddN(Demands, dNodes),
Mpcf(dNodes, Components))*MoleScalefactor)/MW(Components) =E= 0;

DemandMolarComposition(Demands, Components)..
x(Demands, Components)*FTmolar(Demands) =E= Fmolar(Demands, Components);


DemandMolarSpecification(Demands, Spec)..
Fmolar(Demands, Spec) - yspec(Demands, Spec)*FTmolar(Demands) =L= 0;


DemandTotalMoleFlow(Demands)..
FTmolar(Demands) - SUM(Components, Fmolar(Demands, Components)) =E= 0;

Objective.. SUM((Components, dNodes), Mpcf(dNodes, Components)) - z =E= 0;


MODEL GasProductionPlanning /all/;

$include gasplan-bounds
*$include local-solution
OPTION NLP=BARON;
OPTION Limrow = 20;
OPTION Limcol = 20;
OPTION sysout=on;
GasProductionPlanning.optfile = 0;


SOLVE GasProductionPlanning USING NLP MINIMIZING z;


*file levels /local-solution.gms/;
*$include write-levels
```

Appendix C

The Stochastic Pooling Problem

To validate that the proposed algorithm works for stochastic pooling problems, 4 example pooling problems (which were created by the author) are solved. In all examples, problems are solved with 1,2 and 3 different quality variables and as an initial test of the algorithm, only the quality parameters at source nodes are assumed as uncertain variables for convenient analysis of the results. For convenience, only 7 possible scenarios are selected and in all of the example problems same scenarios are used. Moreover, in every possible scenario, all 3 source quality parameters are considered as having same values for simplicity. In other words, possible scenarios in all of the examples are determined as the following: Scenario 1 has 1 as the value of all 3 quality parameters at sources with the probability of 0.1; in Scenario 2, the value of the quality parameters at sources is 1.5 and the probability is 0.1; Scenario 3 has 2 as the value of the quality parameters with the probability of 0.2; Scenario 4 has 2.5 as the value of the quality parameters and its probability is 0.2; in Scenario 5 the value of the quality parameters is 3 and its probability is 0.25; Scenario 6 has 4 as the value of the quality parameters with the probability of 0.05, and Scenario 7 has 5 as the value of all 3 quality parameters at sources and its probability is 0.1. Table C.1 presents the quality parameters in the sources in all scenarios and their respective probability values in detail. This probability distribution is taken as same for all 3 qualities and parameters are

Probabilities of Scenarios				
Scenarios	**Probabilities**	**Source Qualities**		
		1	2	3
1	0.1	1	1	1
2	0.1	1.5	1.5	1.5
3	0.2	2	2	2
4	0.2	2.5	2.5	2.5
5	0.25	3	3	3
6	0.05	4	4	4
7	0.1	5	5	5

Table C.1: Source quality parameters in scenarios and the respective probability values

Investment costs of pools	
Pools	**Costs**
1	200

Table C.2: First stage investment costs of pools for Stochastic Example 1

used for all of them. When less than 3 quality variables is used, the remaining ones are neglected (i.e. when 1 quality variable is considered, the parameters for the second and third are neglected; when 2 quality variables are considered, the parameters for the third one are neglected.). More information for both of these example problems including quality specs, demand requirements, cost coefficients is given in following sections. In addition, GAMS implementation of the BD algorithm for stochastic programs is given in the end of this chapter.

C.1 Stochastic Example 1

Example 1 has 1 pool, 3 sources, 2 end-products. Necessary parameters to construct this problem is given in Tables C.2, C.3, C.4, C.5, C.6, C.7 and C.8.

Investment costs of pipes	
Sources	**Pools**
	1
1	100
2	50
3	100

Table C.3: First stage investment costs of pipelines (sources to pools) for Stochastic Example 1

Investment costs of pipes		
Pools	**End-products**	
	1	2
1	150	100

Table C.4: First stage investment costs of pipelines (pools to demands) for Stochastic Example 1

Source costs	
Sources	**Costs**
1	15
2	10
3	20

Table C.5: Second stage cost parameters in source nodes for Stochastic Example 1

Demand quality requirements			
Products	**Qualities**		
	1	2	3
1	3	1	2
2	4	2	4

Table C.6: Second stage quality requirements in demand nodes for Stochastic Example 1

Demand flow requirements	
Products	**Max. flow**
1	100
2	100

Table C.7: Second stage flow requirements in demand nodes for Stochastic Example 1

Prices	
Products	**Prices**
1	40
2	50

Table C.8: Second stage prices in demand nodes for Stochastic Example 1

Investment costs of pools	
Pools	**Costs**
1	400
2	400

Table C.9: First stage investment costs of pools for Stochastic Example 2

Investment costs of pipes		
Sources	**Pools**	
	1	2
1	100	25
2	50	150
3	100	200
4	150	50
5	100	100

Table C.10: First stage investment costs of pipelines (sources to pools) for Stochastic Example 2

C.2 Stochastic Example 2

Example 2 has 2 pools, 5 sources, 3 end-products. Necessary parameters to construct this problem is given in Tables C.9, C.10, C.11, C.12, C.13, C.14 and C.15.

Investment costs of pipes			
Pools	**End-products**		
	1	2	3
1	100	100	200
2	50	30	75

Table C.11: First stage investment costs of pipelines (pools to demands) for Stochastic Example 2

Source costs	
Sources	**Costs**
1	10
2	25
3	30
4	40
5	40

Table C.12: Second stage cost parameters in source nodes for Stochastic Example 2

Demand quality requirements			
Products	**Qualities**		
	1	2	3
1	1.5	1	2.5
2	3	2.8	3.5
3	1.7	2.6	1.9

Table C.13: Second stage quality requirements in demand nodes for Stochastic Example 2

Demand flow requirements	
Products	**Max. flow**
1	50
2	200
3	80

Table C.14: Second stage flow requirements in demand nodes for Stochastic Example 2

Prices	
Products	**Prices**
1	20
2	60
3	40

Table C.15: Second stage prices in demand nodes for Stochastic Example 2

Investment costs of pools	
Pools	**Costs**
1	100
2	200
3	300
4	400

Table C.16: First stage investment costs of pools for Stochastic Example 3

Investment costs of pipes				
Sources	**Pools**			
	1	2	3	4
1	100	25	150	75
2	50	150	50	25
3	100	200	100	100
4	150	50	200	75
5	100	100	30	60
6	100	200	50	125
7	150	50	70	175
8	100	100	75	100

Table C.17: First stage investment costs of pipelines (sources to pools) for Stochastic Example 3

C.3 Stochastic Example 3

Example 3 has 8 sources, 4 pools, 5 end-products. Necessary parameters to construct this problem is given in Tables C.16, C.17, C.18, C.19, C.20, C.21 and C.22.

Investment costs of pipes					
Pools	**End-products**				
	1	2	3	4	5
1	100	100	200	50	75
2	50	30	75	100	200
3	100	100	200	50	80
4	50	30	75	200	150

Table C.18: First stage investment costs of pipelines (pools to demands) for Stochastic Example 3

Source costs	
Sources	**Costs**
1	30
2	10
3	10
4	35
5	50
6	25
7	30
8	20

Table C.19: Second stage cost parameters in source nodes for Stochastic Example 3

Demand quality requirements			
Products	**Qualities**		
	1	2	3
1	2	1.2	2.5
2	3	2	2
3	1.5	2.4	1.9
4	3	3	3.5
5	1.8	4	3.8

Table C.20: Second stage quality requirements in demand nodes for Stochastic Example 3

Demand flow requirements	
Products	**Max. flow**
1	200
2	200
3	100
4	200
5	100

Table C.21: Second stage flow requirements in demand nodes for Stochastic Example 3

Prices	
Products	**Prices**
1	30
2	10
3	50
4	75
5	40

Table C.22: Second stage prices in demand nodes for Stochastic Example 3

Investment costs of pools	
Pools	**Costs**
1	500
2	400
3	300
4	200
5	300
6	400
7	400
8	100
9	100
10	50

Table C.23: First stage investment costs of pools for Stochastic Example 4

C.4 Stochastic Example 4

Example 4 has 12 sources, 10 pools and 8 end-products. Necessary parameters to construct this problem is given in Tables C.23, C.24, C.25, C.26, C.27, C.28 and C.29.

Investment costs of pipes										
Sources	**Pools**									
	1	2	3	4	5	6	7	8	9	10
1	100	25	150	75	100	125	250	175	100	50
2	50	150	50	125	150	200	50	30	50	75
3	100	200	100	100	100	100	150	125	40	25
4	150	50	200	75	60	40	240	175	160	275
5	100	100	30	60	120	150	50	100	200	50
6	100	200	50	125	150	60	250	50	150	40
7	150	50	70	175	100	80	100	30	100	175
8	100	100	75	100	50	75	50	25	30	200
9	100	100	30	60	75	50	30	150	50	60
10	100	200	50	125	30	200	25	50	200	180
11	150	50	70	175	50	250	100	100	250	50
12	100	100	75	100	125	125	140	175	100	225

Table C.24: First stage investment costs of pipelines (sources to pools) for Stochastic Example 4

Investment costs of pipes								
Pools	**End-products**							
	1	2	3	4	5	6	7	8
1	100	100	200	50	75	100	150	175
2	50	30	75	100	200	50	60	80
3	100	100	200	50	80	100	100	50
4	50	30	75	200	150	220	150	80
5	100	100	200	50	75	120	80	125
6	50	30	75	100	200	70	60	140
7	100	100	200	50	80	100	75	210
8	50	30	75	200	150	130	150	225
9	100	100	200	50	80	180	50	60
10	50	30	75	200	150	230	250	50

Table C.25: First stage investment costs of pipelines (pools to demands) for Stochastic Example 4

Source costs	
Sources	**Costs**
1	10
2	15
3	20
4	30
5	25
6	10
7	40
8	20
9	20
10	25
11	10
12	20

Table C.26: Second stage cost parameters in source nodes for Stochastic Example 4

Demand quality requirements			
Products	**Qualities**		
	1	2	3
1	1	1.5	2
2	2	3	1
3	1.5	2.5	2
4	2	3	1.5
5	1.8	0.9	4
6	2	2.5	3
7	4	3	3
8	2	1.2	3

Table C.27: Second stage quality requirements in demand nodes for Stochastic Example 4

Demand flow requirements	
Products	**Max. flow**
1	100
2	90
3	80
4	100
5	110
6	120
7	140
8	150

Table C.28: Second stage flow requirements in demand nodes for Stochastic Example 4

Prices	
Products	**Prices**
1	30
2	40
3	50
4	60
5	20
6	30
7	15
8	20

Table C.29: Second stage prices in demand nodes for Stochastic Example 4

GAMS Implementation of the BD Algorithm for Stochastic Pooling Problems

```
$ontext
    BD Algorithm for Stochastic Pooling Problems
    Simple Seven Scenario Problem (Example 1)
    Author: Emre Armagan
    Date: April, 2008
$offtext

$TITLE Stochastic Pooling Problem Example

# Set Declarations
    set comp /1*3/;
    set pro  /1*2/;
    set qual /1*1/;
    set pool /1*1/;
    set sce /1*7/;

# investment related parameters
parameters investpool(pool) ;

investpool(pool) = 200 ;

table investpipe1(comp, pool)
         1
   1   100
   2    50
   3   100 ;

table investpipe2(pool, pro)
         1    2
   1   150  100 ;

# source related parameters
table compparams(comp,*)
         1    2     3
   1     0  200    15
   2     0  200    10
   3     0  200    20 ;

parameters cl(comp), cu(comp), cprice(comp);
```

```
cl(comp) = compparams(comp,'1');
cu(comp) = compparams(comp,'2');
cprice(comp) = compparams(comp,'3');

# pool related parameters
parameters psize(pool);
psize(pool) = 200;

# product related parameters
table prodparams(pro,*)
         1    2    3
   1     0   100   40
   2     0   100   50 ;

parameters prl(pro), pru(pro), pprice(pro);
prl(pro) = prodparams(pro,'1');
pru(pro) = prodparams(pro,'2');
pprice(pro) = prodparams(pro,'3');

parameter pqlbd(pro, qual);
pqlbd(pro, qual) = 0;

table pqubd(pro, qual)
         1
   1     3
   2     4 ;

# scenario related parameters
table Sce1(comp,qual)
         1
   1     1
   2     1
   3     1 ;

table Sce2(comp,qual)
         1
   1   1.5
   2   1.5
   3   1.5 ;

table Sce3(comp,qual)
         1
   1     2
   2     2
   3     2 ;
```

```
table Sce4(comp,qual)
         1
   1   2.5
   2   2.5
   3   2.5 ;

table Sce5(comp,qual)
         1
   1     3
   2     3
   3     3 ;

table Sce6(comp,qual)
         1
   1     4
   2     4
   3     4 ;

table Sce7(comp,qual)
         1
   1     5
   2     5
   3     5 ;

cqual1(comp,qual) = Sce1(comp,qual) ;
cqual2(comp,qual) = Sce2(comp,qual) ;
cqual3(comp,qual) = Sce3(comp,qual) ;
cqual4(comp,qual) = Sce4(comp,qual) ;
cqual5(comp,qual) = Sce5(comp,qual) ;
cqual6(comp,qual) = Sce6(comp,qual) ;
cqual7(comp,qual) = Sce7(comp,qual) ;

# probability distribution
parameters prob(sce);
prob('1') = 0.1 ;
prob('2') = 0.1 ;
prob('3') = 0.2 ;
prob('4') = 0.2 ;
prob('5') = 0.25 ;
prob('6') = 0.05 ;
prob('7') = 0.1 ;

# network related parameters
table ubq(comp, pool)
```

```
        1
  1     1
  2     1
  3     1 ;

table uby(pool, pro)
        1    2    3
  1   100  100  100 ;

parameter ubz(comp, pro);
ubz(comp, pro) = 0;

*------------------------------------------
* Form the Benders master problem
*------------------------------------------
set
    iter 'max Benders iterations' /iter1*iter100/
    dyniter(iter) 'dynamic subset' ;

free variables
    zmaster           'objective variable of master problem'
    theta             'extra term in master obj' ;

equations
    masterobj         'master objective function'
    constr1(pool)     'constraint 1'
    constr2(pool)     'constraint 2'
    optcut(dyniter)   'Benders optimality cuts' ;
parameter
    cutconst(iter)    'constants in optimality cuts'
    cutcoeff(iter,j) 'coefficients in optimality cuts';

masterobj..
zmaster =e=   sum(pool, investpool(pool)*buildpool(pool))
+ sum((comp, pool),investpipe1(comp, pool)*buildpipe1(comp, pool))
+ sum((pool, pro),investpipe2(pool, pro)*buildpipe2(pool, pro))
+ theta ;

constr1(pool)..   buildpipe1(comp, pool) - buildpool(pool) =e= 0 ;

constr2(pool)..   buildpipe2(pool, pro) - buildpool(pool) =e= 0 ;

optcut(dyniter)..    theta =g= cutconst(dyniter) +
                                sum(pool, cutcoeff(dyniter,pool)*();
model masterproblem /masterobj, constr1, constr2, optcut/;

*------------------------------------------
* Form the Benders subproblem
*------------------------------------------
free variables
    zsub              'objective variable of sub problem'
;
equations obj                     'subproblem objective function'
          clower(comp)            'lower bound component availability'
          cupper(comp)            'upper bound component availability'
          plower(pro)             'minimum product production'
          pupper(pro)             'maximum product demand'
          pqlower(pro,qual)       'minimum product quality requirement'
          pqupper(pro,qual)       'maximum product quality'
          fraction(pool)          'fractions sum to one' ;

obj.. zsub =e= sum(pro, sum(pool$(uby(pool,pro) > 0),
              sum(comp$(ubq(comp, pool) > 0),
                cprice(comp)*y(pool,pro)*q(comp,pool)))
              - pprice(pro)*sum(pool$(uby(pool,pro) > 0),
              y(pool, pro))
              + sum(comp$(ubz(comp,pro)>0),
              (cprice(comp)-pprice(pro))*z(comp, pro)));

clower(comp).. sum(pool$(ubq(comp,pool)>0),
                  sum(pro$(uby(pool,pro)>0),
                    q(comp,pool)*y(pool, pro)))
               + sum(pro$(ubz(comp,pro)>0), z(comp, pro))
               =g= cl(comp);

cupper(comp).. sum(pool$(ubq(comp,pool)>0),
                  sum(pro$(uby(pool,pro)>0),
                    q(comp,pool)*y(pool, pro)))
               + sum(pro$(ubz(comp,pro)>0), z(comp, pro))
               =l= cu(comp);

plower(pro).. sum(pool$(uby(pool,pro)>0), y(pool,pro))
               + sum(comp$(ubz(comp, pro)>0), z(comp, pro))
       =g= prl(pro);

pupper(pro).. sum(pool$(uby(pool,pro)>0), y(pool,pro))
               + sum(comp$(ubz(comp, pro)>0), z(comp, pro))
               =l= pru(pro);
```

```
pqlower(pro, qual).. sum(pool$(uby(pool,pro)>0),
                        sum(comp$(ubq(comp,pool)>0),
                          cqual(comp, qual)*q(comp,pool)*y(pool,pro)))
                      + sum(comp$(ubz(comp, pro)>0),
                          cqual(comp, qual)*z(comp, pro)) =g=
                      sum(pool$(uby(pool,pro)>0),
                          pqlbd(pro, qual)*y(pool,pro))
                      + sum(comp$(ubz(comp, pro)>0),
                          pqlbd(pro, qual)*z(comp, pro));

pqupper(pro, qual).. sum(pool$(uby(pool,pro)>0),
                        sum(comp$(ubq(comp,pool)>0),
                          cqual(comp, qual)*q(comp,pool)*y(pool,pro)))
                      + sum(comp$(ubz(comp, pro)>0),
                          cqual(comp, qual)*z(comp, pro)) =l=
                      sum(pool$(uby(pool,pro)>0),
                          pqubd(pro, qual)*y(pool,pro))
                      + sum(comp$(ubz(comp, pro)>0),
                          pqubd(pro, qual)*z(comp, pro));

fraction(pool).. sum(comp$(ubq(comp,pool)>0), q(comp, pool)) =e= 1;

model subproblem
/obj, clower, cupper, plower, pupper, pqlower, pqupper, fraction/;

*-----------------------------------------
* solver options
*-----------------------------------------
option milp=cplex;
option nlp=baron;
option limrow = 0;
option limcol = 0;
subproblem.solprint = 2;
masterproblem.solprint = 2;
subproblem.solvelink = 2;
masterproblem.solvelink = 2;
*-----------------------------------------
* Benders algorithm
*-----------------------------------------
*
* step 1: solve master without cuts
*

dyniter(iter) = NO;
cutconst(iter) = 0;
```

```
cutcoeff(iter,pool) = 0;
theta.fx = 0;
solve masterproblem minimizing zmaster using milp;
display zmaster.l;
*
* repair bounds
*
theta.lo = -INF;
theta.up = INF;
scalar lowerbound /-INF/;
scalar upperbound /INF/;
parameter objsub(sce);
scalar objmaster;
objmaster = zmaster.l;
scalar iteration;
scalar done /0/;
loop(iter$(not done),
   iteration = ord(iter);

*
* solve subproblems
*
dyniter(iter) = yes;
loop(sce,
demnd(pool) = demand(pool,sce);
solve subproblem minimizing zsub using nlp;
objsub(sce) = zsub.l;
cutconst(iter) = cutconst(iter)-prob(sce)*(-plower.m(pro)-
clower.m(comp)-cupper.m(comp)-pupper.m(pro)-pqlower.m(pro, qual)
-pqupper.m(pro, qual));
cutcoeff(iter,pool) = cutcoeff(iter,pool)-prob(sce)*(-plower.m(pro)-
clower.m(comp)-cupper.m(comp)-pupper.m(pro)-pqlower.m(pro, qual)-
pqupper.m(pro, qual));
   );
upperbound =
min(upperbound, objmaster + sum(sce, prob(sce)*objsub(sce)));
*
* convergence test
*
   display lowerbound,upperbound;
   if( (upperbound-lowerbound) < 0.001*(1+abs(lowerbound)),
      display "Converged";
      done = 1;
   else
*
* solve masterproblem
*
solve masterproblem minimizing zmaster using milp;
lowerbound = zmaster.l;
objmaster = zmaster.l-theta.l;
   );
);
abort$(not done) "Too many iterations";
display bd.log;
display zmaster, zsub;
```

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