Statistical Analysis of Illiquidity Risk and Premium in Financial Price Signals

by

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Abstract

Price is the most visible signal produced by competition and interaction among a complex ecology of entities in a system called financial markets. This thesis deals with statistical analysis and model identification based on such signals. We approach this problem at various levels of abstraction, with a particular emphasis on linking certain statistical anomalies identified to specific frictions that are only observable in a more microscopic view.

We first give a brief review of the framework for the analysis of financial prices. We highlight the important role of information by introducing the concept of informational efficiency. The main body consists of two parts. Part A consists of Chapters 3, 4 and 5. We first link unpredictability of financial returns, a direct consequence of the informational efficiency, to the expected covariance structure of resulting return signals. We discuss a particular algorithm designed to detect the existence of weak mean-reverting component in the observed returns. Applying this detection scheme to US stock returns between 1995 and 2007, we detect a statistically significant but continually decreasing mean-reverting component in the returns. To explain this observation, we link the mean-reverting component to the arrival structure of buyers and sellers and their interactions. We discuss a particular model for this interaction and apply various tests to establish the validity of the proposed model. Part A concludes with an application of these tools in analyzing the sequence of events in August 2007 which resulted in a breakdown of normal behavior of the system.

Part B, consisting of Chapters 6 and 7, also deals with the issue of predictability in financial returns, but at a different frequency and based on a different set of instruments. We first produce the evidence for an unusually high level of predictability among returns of certain classes of hedge funds. To explain this observation, we discuss a model built based on the notion of partially observed price signals. When prices are not observed, for example due to lack of trading, the most recent price is used to calculate the value of an investment, and this process results in perceived serial correlation in the calculated returns. We view this lack of trading as the second example of friction in this system, and set out to link this friction to the mean of the resulting returns signals. We find strong link between predictability and first moment in certain groups of returns used.

Thesis Supervisor: Andrew W. Lo
Title: Harris & Harris Group Professor

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Chapter 1

Introduction

A financial asset is a contract that gives its owner a claim on a future income. For example, the stock of a public company entitles the owner to future dividend payments of that firm. While the specific motives, information or objectives of individual buyers and sellers may be too complex and textured to be fully articulated, such a level of complexity is somehow reduced to a single number: the *price*. The price is the most visible result of competition and interaction among a complex ecology of entities with a wide range of skills, information levels, and objectives.

From one perspective, the *system of the world's financial markets* can be viewed as a vast *laboratory* in which various entities interact to produce a large volume of data ready to be used by scientists to achieve a more complete understanding of the interaction among the entities involved. In response to the complexity of this system and the scope of questions that arise in modeling and analyzing its behavior, recently researchers from a wider spectrum of social and natural sciences and even engineering have joined forces with economists to study and model the behavior of these systems. This thesis deals with statistical analysis, model identification and inference based on financial prices with a particular emphasis on linking certain statistical properties of prices to specific frictions in the *system*.

In Chapter 2, I will provide a short background of the conceptual framework and the issues involved in modeling the behavior of financial pricing systems. In particular, I will discuss the notion of *informational efficiency* of prices. The rest of the thesis will draw from this conceptual framework, but will take a very practical approach in analyzing a specific set of observations. It is my hope that the reader will find this thesis relevant in understanding the mechanism behind the very specific phenomena that I set out to analyze.

The main body of the thesis consists of five chapters that can be separated into two mostly independent parts. While both parts deal with the issue of predictability in financial time series, and link the source of the predictability to various frictions in the system, they deal with two relatively separate issues and work with time series from different sources and sampled at different frequencies. For this reason, I have separated all chapters into two parts each of which is self-contained and can be read independently.

All of the research presented here is either directly or closely related to the work I have done with my thesis advisor, Professor Andrew Lo. While I do take responsibility for the quality of the entire document, I would like to make a distinction between chapters that are directly the result of collaboration with Professor Lo, and those that are mostly based on my own extensions. Specifically, Chapters 5 and 7 are most directly related to work I have done with Professor Lo while the rest, and in particular, the models in Chapter 4 and 7 are based on my independent activity. But to keep the format consistent, I will use a first person plural narrative format for the entire document.

The first part of this thesis consists of Chapters 3, 4 and 5. These chapters deal with the issue of predictability of financial price signals at high frequency, for example daily or even intra-day level, and discuss the drivers that are behind price dynamics at these frequencies. In Chapter 3, we will focus on developing a test for the hypothesis that future prices are not predictable using a *linear predictor* based on the past prices; in other words, the changes in financial price signals should be *white noise*. We will review different methods for testing this form of predictability and highlight how a simple trading strategy, the *Contrarian Trading Strategy* first introduced in Lehmann (1990) and Lo and MacKinlay (1990b), is related to a measure of linear predictability in prices. We will then apply this strategy to the stock prices between 1995 and 2007 and detect a relatively large but continually decreasing deviation from unpredictability in prices. We provide an alternative view of the trading strategy as a simple filtering or *signal detection algorithm* designed to detect the existence of a *mean-reverting component* in financial return signals. Given this view, the empirical results prove that there is indeed a mean-reverting component in the return signals at these frequencies, and the strength of this component has been declining over time.

It will be argued that the deviation from unpredictability detected through these tests is a by-product of certain types of friction in the way that financial assets, stocks in this case, are traded. Therefore, while the perfect informational efficiency and unpredictability may be possible in an idealized *friction-less* world, there are many impediments in the real world that cause the actual behavior to deviate from what would be expected in the idealized setting. In Chapter 4, we develop a model for one particular type of *friction*. We argue that the mean reversion in price signals is a by-product of interactions between *buyers* and *sellers* and is driven by temporary imbalances caused by the asynchronous arrival of buyers and sellers. In order to alleviate these temporary imbalances, *dealers* are needed to sell the stock to buyers and buy it from sellers as they arrive. The dealers, however, need to be compensated. So on average, the dealers will buy the stock at a price slightly below what they would expect to be able to sell it for later (to the future arriving buyers) and sell it at slightly above the price that they expect to be able to buy the stock back for from future arriving sellers. We use this idea and develop a model based on Grossman and Miller (1988) to capture this phenomenon. Our model will link the time-series dynamics of observed prices to the time-series properties of the arrival processes for buyers and sellers. We will outline a specific hypothesis for the arrival processes of buyers and sellers and work out the implications of that hypothesis on the observed price dynamics. The model also produces testable implications for the link between price dynamics and other observables, such as volatility and transaction volumes, in the system. In the empirical testing, we extend our previous *signal detection algorithm* and apply that to test for various implications of the model under consideration. We find strong support for validity of the hypothesis proposed, in particular for the link between the strength of the mean-reverting component of returns and trading volumes or the volatility of returns.

Chapter 5 uses the results of earlier chapters to analyze changes in the system's dynamics based on various observables, such as prices and trading volumes, during the year 2007. Based on this analysis, we detect a period of unusual stress in the system starting in late July 2007. During this period, the pattern of trading volumes became more correlated with certain well-known quantitative valuation factors. Since these factors are commonly used by quantitative portfolio managers in making investment decisions, the higher correlation confirms that during this period there was a rush by these investors to reduce the size of their holdings. The pressure reached its highest level during the week of August 6, 2007 and resulted in a clear and distinct *regime shift* in the mean-reverting component of price changes. Starting on August 10, there seemed to be a sudden reversal towards normal system behavior. We conclude this chapter by showing some evidence on the increased linkage between different sectors of the hedge fund industry, and discuss the implications of these changes in the future stability of the system and price dynamics.

The second part of the thesis consists of Chapters 6 and 7. These two chapters concentrate on the sources of predictability in the return signals on a monthly basis based on signals generated by hedge funds and mutual funds. This analysis is primarily motivated by work in Getmansky, Lo, and Makarov (2004).

After documenting the unusually high level of predictability among certain hedge funds and validating the statistical significance of the observed patterns, we will discuss a model based on Lo and MacKinlay (1990a) that links the perceived predictability in returns to the issue of partially observed prices. The model is built based on the notion that prices are not observed if no trade takes place in a given time period. In the absence of an *observation* of the price signal, the most recent price is used to calculate the value of an investment and, hence, the return. This process, as we will make precise, would give rise to perceived predictability in observed returns. We will close this chapter by emphasizing an argument first made in Getmansky et al. (2004) that if serial correlation is indeed a proxy for lack of trading, assets that are more illiquid, i.e., trade less often, must exhibit higher serial correlation.

We start Chapter 7 by providing some additional evidence to support the view that serial correlation is a good proxy for illiquidity of assets. The rest of this chapter will focus on evaluating the link between illiquidity, as proxied by serial correlation of observed returns, and the expected return. We refer to the difference between the expected returns that can be contributed to higher illiquidity as the *illiquidity premium* and will discuss a *clustering* based approach to estimate this premium. We will discuss our methodology in detail and outline an approach meant to increase the precision of our analysis by adjusting returns for other common sources of co-movement. Overall, our analysis supports the existence of a positive illiquidity premium among certain categories of hedge funds and some categories of mutual funds. We also find that this premium has declined over the last four years of our sample and link that to changes in the overall behavior of this system.

Chapter 8 will summarize the main findings of the research and the original contributions.

Chapter 2

Conceptual Background

A financial asset is a contractual agreement that gives its owner a claim on a set of future cash flows. For example, the stock of a company gives the owner a claim on the future divided of that firm. Financial assets are different from physical assets, such as a house, since they don't have any physical usage. They derive their value purely and directly from the future set of cash flows. Hence, there must be a relationship between the prices of these assets and the promised future cash flows. Understanding how investors assign value to uncertain cash flows, based on their beliefs on the likelihood of different outcomes and their preferences for each possibility, is the subject studied under the general title of *Asset Pricing*.

Since setting up experiments with a meaningful incentive structure in a realistically competitive setting is almost never possible, the analysis of financial prices is almost entirely empirical. Models developed to explain certain behavior are calibrated to the observed realizations, and tools from statistical inference are applied to test if the calibrated results are consistent with the observed behavior. What makes the analysis of financial prices interesting is the central role that uncertainty plays in setting prices. This implies that a distinction, even if only conceptual, must be made between the first type of randomness that is due to the non-experimental nature of testing, and the second type of randomness that is the subject matter of the study. Finding ways to take into account this second type of randomness, which is often referred to as *uncertainty*, is what makes financial analysis interesting and challenging.

A distinction should also be made between the *positive* versus *normative*[2.1] nature of these models. On one hand, models can be aimed to provide a *description* of the actual behavior of the system. From this perspective, any deviation between the observed behavior and the behavior prescribed by the model points to a failure in the modeling approach. Alternatively, a model can be viewed as a *prescription* of what the behavior should be. In this latter view, any deviation from the prescribed behavior would represent an opportunity for an improvement in the system's design or, more shrewdly, an opportunity that an investor may be able take advantage of. In this sense, the analysis of financial pricing systems shares the same positive versus normative tension that exists in much of the social sciences, and in

[2.1] A positive statement is a statement about a fact and contains no notion of approval or disapproval. A normative statement is a statement about what is desirable and how the behavior should be. The same distinction can be extended to modeling the financial system.

particular economics, but is mostly absent in the natural sciences and engineering.

This conceptual background will be a brief overview and is only meant to provide context for the rest of this thesis for readers with no background in financial modeling and analysis. We will first review the general framework for developing asset pricing models in Section 2.1. There are many good references for the topics covered in Section 2.1 including the following two: Cochrane (2005) provides a thorough textbook treatment of Asset Pricing, while Campbell (2000) gives an overview of the same literature in a review-style paper. Section 2.2 is focused on the informational role of prices, and discusses in some detail the issue of informational efficiency. The exposition provided in this section is mainly based on Chapter 2 of Campbell, Lo, and MacKinlay (1997) and Chapter 3 of Ross (2005).[2.2] Brunnermeier (2001) and Grossman (1989) provide a more theoretical treatment of some of the issues.

2.1 Framework for Analyzing Prices

As mentioned earlier, financial assets are claims on a set of future cash flows. The prices of these assets are set as agents in the economy analyze various trade-offs available to them, and make their desired choice based on their preferences and available information. All pricing models, i.e., models to map the uncertain future cash flows to the current price of the asset, are built upon one or more of the following three principals: *Arbitrage*, *Optimality*, and *Equilibrium*.[2.3]

The basic intuition behind arbitrage-based pricing models is the absence of *arbitrage opportunities*, i.e., trades that cost nothing to set up[2.4] and yet can generate a positive payoff under some realizations of the uncertain future with no possibility of negative payoff in other states of the set of future possibilities. The most important special case of arbitrage-based pricing is the *law of one price*, which states that two assets with *identical payoffs* must have *identical prices*. For example, this is the law that enforces the price of shares of a particular stock trading on two different exchanges to be identical at *all* times. Any deviation would produce an arbitrage opportunity and such opportunities will be eliminated instantly due to the extremely competitive nature of financial markets. In fact, the activity of taking advantage of arbitrage opportunities by *arbitrageurs* is so essential to the behavior of much of modern finance that, by referring to the arbitrageurs as sharks, Ross (2002) notes: "Neoclassical finance is a theory of sharks and not a theory of rational *homo economicus*."

Arbitrage-based pricing models are an example of relative pricing which does not price any asset separately, but indicates if one asset is expensive or cheap relative to another asset. These models put the minimum set of requirements on the preferences of the agents in the system. Precisely for this reason, testing arbitrage-based pricing relationships is easier than

[2.2] Fama (1970), Fama (1991), and LeRoy (1989) are the classic references on this topic. Also see Malkiel (2003, 2005) for a more recent perspective.

[2.3] It should be emphasized that these principles are not mutually exclusive and there are some strong connections between them. For example, the existence of an arbitrage opportunity is not consistent with optimality or equilibrium.

[2.4] Arbitrage opportunities are constructed by financing the purchase of one group of assets by *short selling* another group of assets.

some of the other pricing models that we will review shortly. Even in this case, deviations can persist if the arbitrageurs are constrained in a way or there are other structural reasons that limit the arbitrage process. See, for example, Lamont and Thaler (2003), Froot and Dabora (1999) for empirical examples and De Long, Shleifer, Summers, and Waldmann (1990), Shleifer and Vishny (1997) or Abreu and Brunnermeier (2002) for examples of theoretical model dealing with such institutional or behavioral constraints. Perhaps a more important question is why such deviation may happen in the first place. Often behavioral reasons based on certain psychological pattern in decision making are offered as a potential reason (see for example, Hirshleifer, 2001 and Daniel, Hirshleifer, & Teoh, 2002), but there are competing explanations; see Ross (2002) for one such example.

The next level of pricing models are based on the *optimality condition* of investors. To develop this approach, one has to formalize the optimization problem of each investor by specifying the functional forms of their *utility function.*[2.5] To give the reader an idea, we start with the case that investors derive their utility from *consumption* at the current and next period, denoted by c_t and c_{t+1}, and they are *impatient*, i.e., prefer to consume earlier rather than later. A prototypical approach is to model investors as solving an optimization problem with an objective function of the following form

$$U(c_t, c_{t+1}) = u(c_t) + \delta u(c_{t+1}) \tag{2.1}$$

where c_t and c_{t+1} are the consumption in periods t and period $t+1$, respectively, and δ is a parameter that captures the impatience of investors.[2.6] In addition, assume that in making a decision when faced with uncertainty about the future outcomes, investors try to maximize their expected utility.[2.7] The first-order optimality condition for each investor implies that for each asset that the investor can purchase, the following relationship must hold at the optimal choices:[2.8]

$$p_t u(c_t) = \delta \mathrm{E}_t[u(c_{t+1}) p_{t+1}] \tag{2.2}$$

where p_t and p_{t+1} are the current and next period's price, respectively,[2.9] and E_t denotes the

[2.5]See Appendix G of Bertsekas (2000) for a review of utility functions.

[2.6]This is the standard model which is referred to as the *separable* class of utility functions since each part only depends on the current consumption. Since the form the utility function used in creating the model has profound implications on the behaviors of the system, researchers have modified all these assumption in order to expand the set of possible behaviors. For example, Campbell and Cochrane (1999) introduce the notion of *habit* into the utility and, hence, change the assumption that the utility only depends on the current consumption. Epstein and Zin (1991) is an example of an approach to produce *non-separable* utility functions.

[2.7]Similar to the previous assumptions, the assumption of expected utility maximization has been challenged and some alternatives have been proposed to incorporate the behavioral aspect of decision making, such as *loss aversion* or *ambiguity aversion.* See Starmer (2000) for a discussion on alternatives to expected utility theory. Chapter 8 of Campbell et al. (1997) provides a good overview including examples for loss- and ambiguity-aversion based models.

[2.8]Note that the agents also have a *budget constraint* that, in a sense, limits the alternative possible combinations of c_t and c_{t+1} they can have. The relationship given here holds when c_t and c_{t+1} are selected optimally subject to that constraint.

[2.9]Note that any payment from owning the asset at time $t+1$, for example a dividend, can be factored into the price without loss of generality.

expectation operator conditioned on the information at time t. The above expression can be restated as:

$$p_t = \mathrm{E}_t\left[\delta\frac{u(c_{t+1})}{u(c_t)}p_{t+1}\right] \tag{2.3}$$

The statement given in (2.3) is "the central asset pricing formula" and "most of the theory of asset pricing consistent of specialization and manipulation of this formula" (Cochrane, 2005, page 6). Yet the above expression stops short of a full description of what determines prices, i.e., relating the price to *exogenous* variables. (2.3) simply related one endogenous variable, the current price, to two other endogenous variables, the consumption and the next period's price. In a sense, this expression itself is a relative pricing model just as we discussed before. Yet this simple expression contains an enormous amount of information. In order to see the full power of this expression, let's restate it as:

$$1 = \mathrm{E}_t\left[\delta\frac{u(c_{t+1})}{u(c_t)}\frac{p_{t+1}}{p_t}\right] \tag{2.4a}$$

$$= \mathrm{E}_t\left[m_{t+1}r_{t+1}\right] \tag{2.4b}$$

where in the second expression $m_{t+1} = \delta u(c_{t+1})/u(c_t)$ and $r_{t+1} = p_{t+1}/p_t$. m_{t+1} is often referred to as the *stochastic discount factor* or the *pricing kernel.* [2.10] This restatement of the pricing equation is very useful in empirical analysis. For example, let's consider the case of a risk-free asset, i.e., an asset that promises a fixed and known payment at time $t+1$. Without loss of generality, assume this payment to be 1, i.e., $p_{t+1} = 1$. The formulation states that the current price should be $p_t = E_t[m_{t+1}]$. But we also observe the price of such asset through observing the risk-free interest rate in the system, i.e., $p_t = 1/(1+r_t^f)$, where r_t^f is the risk-free interest rate. Putting these two expressions together, we arrive at the following statement:

$$E_t[m_{t+1}] = \frac{1}{1+r_t^f} \tag{2.5}$$

Therefore, the risk-free rate puts a certain restriction on the behavior of the pricing kernel in terms of its conditional moment. Prices of other assets add additional constraints and, from there, one can start to build various moment conditions to test the desired model that characterizes the pricing kernel.[2.11] In spite of its elegance, testing the first-order condition given in (2.2) is mired with difficulty; issues such as setting the functional form of the utility function, specifying the information used in calculating the conditional expectation, and the consumption level to use in the model make the testing difficult and the results produced not

[2.10] The reason it is called a *discount factor* may be more clear once the expression is restated as $p_t = \mathrm{E}_t[m_{t+1}p_{t+1}]$. In this format, it is clear that m_{t+1} is a random variable used to discount various potential outcomes in the next period depending on the utility and consumption of the investor in each of these states. Since it is random based on the information at time t it is called the *stochastic.*

[2.11] This approach for building a test is called the *Generalized Method of Moments* approach. There are alternatives such as methods based on *Maximum Likelihood Estimation.* See Cochrane (2005) for an in-depth discussion of various methods for estimating and evaluating asset pricing models.

very precise. For this reason, most empirically testable models add additional assumptions to be able to aggregate agents' first-order condition to find certain properties that must be held in aggregate and, hence, in equilibrium. The Nobel prize-winning *Capital Asset Pricing Model* (CAPM) is an example of this approach. We will review CAPM later in this thesis.

Aggregation also has the benefit of allowing one to make general statements about the behavior of returns without dealing with the individual's conditional expectations. With respect to this later point, the conditional expectations are usually replaced with those conditioned on a subset of publicly available information that seem to be applicable to the model under consideration or simply replaced with the unconditional expectations (see Chapter 8 of Cochrane, 2005 for further discussion on this issue). Furthermore, using this approach gives us some hope to be able to finally link prices to truly exogenous variables. See Cox, Ingersoll, and Ross (1985) for an elegant example of an equilibrium model that links prices to exogenous variables.

For the purpose of this thesis, we are more interested in the behavior that is robust to such generalized, and potentially untestable, assumptions about the form of the utility function, the information structure and the way that conditional expectations are formed.[2.12] One behavior that turns out to hold almost perfectly with a relatively weak set of assumptions deals with predictability of financial prices that result from a competitive price-setting system. The issue of unpredictability is ultimately related to the role of prices as a tool for aggregating information across the system. We turn to this topic next.

2.2 Information and Prices

As discussed in the first section, prices are set by economic agents, the smallest unit of analysis in this system, as they decide how much of each financial asset to hold based on their objectives and risk preferences, as well as their *belief* about future outcomes. This last point, the fact that economic agents decide their holding of various financial assets based on their belief, implies that in some sense, the prices that prevail as a result of the individual's decision-making process already incorporate their respective information. This intuition is the basis for the notion of *informational efficiency* in financial markets. But "[e]xactly what is meant by this attractive phrase is not entirely clear" (Ross, 2005, page 42). Malkiel (1992) offers the following interpretation for this concept that gives this claim a more operational meaning:

> A capital market is said to be efficient if it fully and correctly reflects all relevant information in determining security prices. Formally, the market is said to be efficient with respect to some information set, Ω_t, if security prices would be unaffected by revealing that information to all participants. Moreover, efficiency with respect to an information set, Ω_t, implies that it is impossible to make economic profits by trading on the basis of Ω_t.[2.13]

[2.12] For a recent treatment of issues in this type of modeling and a discussion about the alternative approaches, see Farmer and Geanakoplos (2008).

[2.13] We have made minor modifications, such as adding Ω_t, to adapt the statement to current formulation.

The critical point from the above definition is the idea that markets are said to be efficient with respect to a particular piece of information if, once that information is revealed to all participants, prices do not change as a result. Depending on the information hypothesized to be reflected in the prices, one can distinguish between three forms of market efficiency. The *weak-form efficiency* is the situation that prices only reflect the *history of past prices* themselves. Prices are said to adhere to *semi-strong form efficiency* if they reflect *all publicly available* information. Finally, prices are said to be *strongly efficient*, i.e., adhere to the *strong form efficiency*, if they reflect *all privately available information* in addition to the past history of prices and public information.[2.14]

Informational efficiency is ultimately related to the success of the financial markets as a pricing system in aggregating distributed information. As expected, this would depended on many factors such as the structure of the market, the method of exchanging information, and the decision-making process of the agents. For example, it is possible to design structures in which *herding* is the more likely outcome and, in a sense, information cascades instead being aggregated even if all the individuals act rationally; see Acemoglu, Dahleh, Lobel, and Ozdaglar (2008), Brunnermeier (2001), Hirshleifer and Teoh (2002).[2.15]

In fact, by putting specific structural form on the way information is acquired or decisions are made, one can find very interesting paradoxes with the notion of informational efficiency. For example, if prices are informationally efficient then they are a sufficient statistics for all private signals and, hence, no one would have an incentive to collect private information. This point is made precise in Grossman and Stiglitz (1980) by putting proper structure around the information structure and the decision making of individual agents. This issue can be taken even one step further to claim that the new information is reflected in the prices without any trade taking place, see Tirole (1982) or reference to *No Trade Theorem* in Brunnermeier (2001). The key here is that the process by which information is acquired is common knowledge. So "while someone else doesn't know what you know, they do know that you may know something useful and that you know that they know it, and so on" (Ross, 2005, page 42). In conclusion, Ross (2005) summarizes various paradoxes of the efficient markets to say:

> As a matter of economic logic, though, markets cannot be perfectly efficient. If markets were perfectly efficient, then no one would have an incentive to act on their own information or to expend the effort to acquire and process information. It follows that there must be some friction in the market and some violation of market efficiency to induce individuals to acquire and process information.

This type of informational efficiency of financial markets, if true, implies that future returns, i.e., future changes in prices, are largely not due to the existing information, even if that information is only held by a group of participants, and, hence, must be largely

[2.14]Note that informational efficiency does not imply that everyone can infer the actual information content from the prices. See Brunnermeier (2001), page 24, for some discussion on the distinctions.

[2.15]One particularly interesting example is the *career risk*, i.e., the idea that individuals have a tendency to go with the crowed since being wrong when everyone else is also wrong has smaller career risk. See Devenow and Welch (1996) for some examples of this. Rajan (2006) discusses this as a potential risk for the overall stability of the system.

dependent on the new information, i.e., on the *news*.[2.16] Therefore, in an informationally efficient market, future returns are due to new information and hence must be *unpredictable.* The essence of this idea is succinctly summarized by the title of the Samuelson, 1965 paper: "Proof that Properly Anticipated Prices Fluctuate Randomly." This is the essential requirement on which we will be basing our analysis in the next chapter.

[2.16] Although we do not discuss this point here, the concept of information efficiency is very different from alternative notions of efficiency in economic structures. See Farmer and Geanakoplos (2008) for an overview of the distinction between informational and *Pareto* efficiency and Brunnermeier (2001) for a more rigorous discussion.

Chapter 3

Are Changes in Financial Prices White Noise?

This chapter is dedicated to the study of predictability of financial price signals. As we discussed in Section 2.2, a capital market is said to be efficient if it fully and correctly reflects all relevant information in determining security prices. One particular ramification of this type of efficiency, the so called *informational efficiency*, is that the changes in financial prices should be *unpredictable*. In this chapter, we will look at the issue of unpredictability is detail. In particular, we show that if the only information available for predicting future prices are the current and past prices, for the same security or other securities, and the predictor is only limited to using linear prediction approach, the test for unpredictability of changes in prices is *equivalent* to testing if the changes in prices are *white noise*. We will discuss a particular algorithm for testing this hypothesis. We also show that this algorithm is in fact designed to extract a *mean-reverting* component in the changes of financial prices. We will discuss the intuition behind why such mean-reverting component may exist and provide some interesting empirical results about the changes in the strength of this mean-reverting component over the last decade and also in 2007. This will set the stage for next chapter were we look at a particular model for the mean-reverting component of the price signals based on friction that govern the interaction of buyers and seller in real markets.

3.1 Some Notation and Definitions

First, we define some notation to avoid confusion moving forward. Let $p_{i,t}$ be the value of price signal for security i at time t. The price time series are not convenient for analysis since, among other reasons, they are neither scale independent[3.1] nor stationary.[3.2] For these reasons, the price time series are usually transformed into *returns* time series through one the following two methods:

[3.1] For example, consider bundling together 10 shares of IBM and calling it *Super IBM*. This later creation has statistical properties that are related to the original price signal. It is desirable to transform the price signals such that the resulting signal has statistical properties that are invariant to such trivial manipulations.

[3.2] Price signals are typically modeled as a stochastic process with a *unit-root*. See Hamilton (1994) for information on statistical tools that can be used in analyzing this type of stochastic processes.

Definition 1 *Let $p_{i,t}$ be the time series of prices for security i. This time series can be transformed into "returns" time series by one of the following two procedures:*

$$\textit{simple return:} \quad r^s_{i,t} = \ p_{i,t}/p_{i,t-1} - 1 \tag{3.1a}$$
$$\textit{compounded return:} \quad r^c_{i,t} = \ \log(p_{i,t}) - \log(p_{i,t-1}) \tag{3.1b}$$

Furthermore, we often work with the prices of many securities and through several time periods. Each of the above two transformations has an advantage in one type aggregation; the *simple return* is more convenient for cross-sectional (i.e., portfolio) aggregation and the *compounded return* is more convenient for aggregation through time. See Chapter 1 of Campbell et al. (1997) for further discussion.[3.3] To keep the discussion general, we will use notation $r_{i,t}$ to refer to security i's return in period t, which can be calculated using either of the above methods but we will specify in each case which of the methods is used. It should be noted that for small changes, which are of most interest in this analysis as we will be looking at changes over one-day intervals for example, the two quantities are very close to each other.[3.4]

Let N be the number of securities, and hence the number of price and return signals available for the analysis at each time t. Let $\mathbf{R}_t = [r_{1,t}, r_{2,t}, \cdots, r_{N,t}]^T$ be the $N \times 1$ vector of returns for time t. Assume $\mathbf{R}_t$ to be a jointly covariance stationary stochastic process with expectation $\mathrm{E}[\mathbf{R}_t] = \boldsymbol{\mu} = [\mu_1, \mu_2, \cdots, \mu_N]^T$. Denote by $\mathbf{\Gamma}_l$ the l-lag auto-covariance of $\mathbf{R}_t$; i.e., $\mathbf{\Gamma}_l = [\gamma_{i,j}(l)] = \mathrm{E}[(\mathbf{R}_t - \boldsymbol{\mu})(\mathbf{R}_{t+l} - \boldsymbol{\mu})^T]$.

We will also use the concept of *vector white noise* in the discussion below so it is worth defining it properly here. Let $\mathbf{W}_t$ be an $N \times 1$ vector stochastic process. $\mathbf{W}_t$ is a vector white noise if $\mathrm{E}[\mathbf{W}_t] = \mathbf{0}$, and $\mathrm{E}[\mathbf{W}_t \mathbf{W}^T_{t+l}] = \mathbf{0}$ for all $l > 1$. Note that the elements of $\mathbf{W}_t$ can be contemporaneously correlated but must be uncorrelated across time.

3.2 A Framework for Analyzing Predictability of Prices

We now formulate the problem of predicting future changes in financial prices. We have to decide on the information set to be used for prediction, the prediction approach (linear, non-linear of a particular form, etc.) and the prediction evaluation metric (such as mean squared error, mean absolute error, etc.).

With respect to the first issue, we will limit the information set *only* to the most recent set of price changes. In other words, we will limit the predictor to only use the $N \times 1$ vector of $\mathbf{R}_{t-1}$, as defined in that notation's section above, for predicting the period t's price changes. With respect to the second issue, we will limit the analysis only to the *linear* predictor class. Furthermore, we will use the *minimum mean squared* criteria for both selecting the best estimator and also for comparing estimator in this section.

[3.3] Some other complications not mentioned here are dividends and events such as stock splits. Most data sources used in the empirical analysis of this thesis use a specific approach in addressing these issues in turning prices into returns. In most cases, these data sources provide a time series of return, and not prices, as an input to the empirical analysis.

[3.4] This point is easy to see using Taylor expansion.

Future returns are said to be predictable given the estimation scheme, if the average minimized prediction error criteria, in this case the *minimum mean squared error*, conditioned on the available information is smaller that the same criteria unconditionally. This statement should be clear as we work out the detail of the estimator form and the expression for the *minimum mean squared error* in each case as given next. We consider two cases in turn.

- **Base Line**: As the base line, consider the case that the estimator completely ignores all available information and predicts the best *unconditional* prediction as the predicted value. We will denote this estimator by $\hat{r}_{BL}$ (Base Line). It is well-known that the estimator in this case is given by: $\hat{r}_{BL}(r_{i,t}) = \mu_i$.[3.5] The estimation error is simply $error_{BL}(r_{i,t}) = r_{i,t} - \hat{r}_{BL}(r_{i,t}) = r_{i,t} - \mu_i$. Denote by $\lambda_\phi(i)$ the mean squared error of this estimate. In this case, we have:

$$\begin{aligned}\lambda_{BL}^{i} &= E[error_{BL}(r_{i,t})^2] \\ &= E[(r_{i,t} - \mu_i)^2] \\ &= \sigma_i^2 \end{aligned} \tag{3.2}$$

  This will be the base line for comparing the performance of other estimators that will be considered next.

- **Linear Estimator**: Linear Least Squares (LLS) estimator is optimal in this case. The estimator is given by

$$\begin{aligned}\hat{r}_{LLS}(r_{i,t}; \mathbf{R}_{t-1}) &= \mu_i + \mathbf{\Gamma}_{r_i,\mathbf{R}_{t-1}}^T \mathbf{\Gamma}_{\mathbf{R}_{t-1}}^{-1} \widetilde{\mathbf{R}}_{t-1} \\ &= \mu_i + \mathbf{\Gamma}_{r_i,\mathbf{R}_{t-1}}^T \mathbf{\Gamma}_{\mathbf{R}_t}^{-1} \tilde{\mathbf{R}}_{t-1}\end{aligned}$$

  where, $\mathbf{\Gamma}_{\mathbf{R}_{t-1}}$ is the covariance matrix for the elements included in $\mathbf{R}_{t-1}$, $\mathbf{\Gamma}_{r_i,\mathbf{R}_{t-1}}$ is the covariance between r_i and elements in $\mathbf{R}_{t-1}$, and $\widetilde{\mathbf{R}}_{t-1}$ is the deviation between the elements in $\mathbf{R}_{t-1}$ and their means, i.e., $\widetilde{\mathbf{R}}_{t-1} = \mathbf{R}_{t-1} - \boldsymbol{\mu}$. Note that due to the assumption of covariance stationarity, we have $\mathbf{\Gamma}_{\mathbf{R}_{t-1}} = \mathbf{\Gamma}_{\mathbf{R}_t}$ and $\mathbf{\Gamma}_{r_i,\mathbf{R}_{t-1}} = \mathbf{\Gamma}_{r_i,\mathbf{R}_t}$ In this case, the error is given by $error_{LLS}(r_{i,t}; \mathbf{R}_{t-1}) = r_{i,t} - \mu_i - \mathbf{\Gamma}_{r_i,\mathbf{R}_t} \mathbf{\Gamma}_{\mathbf{R}_t}^{-1} \tilde{\mathbf{R}}_t$. It is easy to see that the mean squared error in this case is given by:

[3.5] Even in this case, one can claim that the estimator is using the knowledge about the unconditional average as the prediction. Throughout this discussion, it is assumed that all the distributional properties of returns is known in advance and does not need to be learned from the data. Hence, the information available, such as recent returns, is *ignored* in this case in the sense that they do not enter or modify the predicted value in any way.

$$\begin{aligned}\lambda^{i}_{LLS} &= E\left[error_{LLS}(r_{i,t};\mathbf{R}_{t-1})^2\right] \\ &= E\left[\left(r_{i,t}-\mu_i-\mathbf{\Gamma}^T_{r_i,\mathbf{R}_{t-1}}\mathbf{\Gamma}^{-1}_{\mathbf{R}_t}\widetilde{\mathbf{R}}_{t-1}\right)^2\right] \\ &= \sigma_i^2-\mathbf{\Gamma}^T_{r_i,\mathbf{R}_{t-1}}\mathbf{\Gamma}^{-1}_{\mathbf{R}_t}\mathbf{\Gamma}_{r_i,\mathbf{R}_{t-1}} \end{aligned} \tag{3.3}$$

Comparing (3.2) and (3.3), it is clear that as long as $\mathbf{\Gamma}_{r_i,R_{t-1}}$ is not identically equal to zero one can achieve some reduction in mean squared error using linear estimation technique. This is the basis for the *Random Walk Model*[3.6] for financial prices and will be the basis of the hypothesis that will be tested in this section.

The hypothesis that will be outlined in the next section is to test if prices are predictable using past prices alone, hence testing the Weak-form Efficiency based on the linear prediction approach described above.

3.3 Hypothesis Testing

A hypothesis is a statement about the population parameter, for example "mean of a random variable equal to zero," or a certain relationship that must hold between different parameters, for example "for normally distributed random variables, the second and forth central moments have a ratio of 3 to 1." In either case, the appropriateness of the model can be validated by testing if the hypothesized relationship holds in the data. Since the population parameters are unobservable (after all if that was not the case there would be no point in statistical hypothesis testing), the appropriate parameter(s) must be estimated from a given sample of observations. Typically, a hypothesis test is specified in terms of a test statistic, a function of an observed data sample, which can be compared against the value of that statistic if the data was truly following the supposed data generating process. Since the test statistic is simply a function of observed data, it itself is a random variable and, therefore, appropriate statistical distribution must be derived when comparing the observed value of the test statistic in a given sample against its population counterpart if the hypothesis was true.

In this section, we will focus on a testing approach designed to detect deviation from Weak-form Efficiency coupled with Linear Estimator approach. As outlined above, this hypothesis is equivalent to testing the null hypothesis that the covariance between future returns and all lagged returns is *zero*. This null hypothesis is equivalent to testing if

$$\text{Null Hypothesis: } \mathbf{\Gamma}_1 = E[(\mathbf{R}_t-\boldsymbol{\mu})(\mathbf{R}_{t+1}-\boldsymbol{\mu})^T] \equiv 0 \tag{3.4}$$

Recall that we have assumed the stochastic process for $\mathbf{R}_t$ is a covariance stationary process with mean $\boldsymbol{\mu}$ and auto-covariance matrix given by $\mathbf{\Gamma}_l$. Under this maintained hypothesis, the null hypothesis outlined in (3.4) is equivalent to testing if the $\mathbf{R}_t = \boldsymbol{\mu} + \mathbf{W}_t$

[3.6] This is in fact an example of an *uncorrelated increment random walk.*

where $\mathbf{W}_t$ is a white noise vector stochastic process. Therefore testing the null hypothesis of (3.4) is equivalent to testing the null hypothesis that the changes in prices above the long-term mean, $\boldsymbol{\mu}$, is a white noise vector stochastic process.

To set the stage for the analysis, it would be appropriate to discuss how such a test would be implemented in the case of a single return series. In this case, the hypothesis proposed in (3.4) comes down to testing whether the covariance at one time lag, i.e., $\gamma_i(1) = \text{Cov}(r_{i,t}, r_{i,t-1})$ is equal to 0. This suggests a straightforward solution: estimate $\hat{\gamma}_i(1)$ from the sample and compare the difference between the estimate and the expected value under the null hypothesis (0 in this case) using appropriate sampling distribution. It turns out that looking at the estimated correlation, defined as $\hat{\rho}_i(1) = \hat{\gamma}_i(1)/\hat{\gamma}_i(0)$, has easier statistical properties to work with.[3.7] For example, a common statistic used to measure the null hypothesis of the above form is based on the value of the following quantity estimated using the available T samples of the returns for security i:[3.8]

$$Q_i(1) = T \cdot \rho_i^2(1)$$

Under the above null hypothesis, it is easy to show that $Q_i(1)$, known as the Box-Pierce Q-statistic, would be asymptotically distributed as χ_1^2. By summing up the square of various auto-correlations, the Box-Pierce Q-statistic is designed to detect deviation from the zero auto-correlation in either direction. This statistic is by no means the only, or even the preferred, way of testing for the above null hypothesis. There are many alternatives with some having more precision to detect certain deviation better than others. For example, Lo and MacKinlay (1988) use "variance ratio" defines as:

$$VR_i(2) = \frac{\text{Var}(r_{i,t} + r_{i,t-1})}{2 \cdot \text{Var}(r_{i,t})}$$

to test a similar hypothesis. It is easy to show that the variance ratio is simply equal to $VR_{i,2} = 1 + \rho_i(1)$. A test based on this statistic can be generalized to use longer horizon returns.

In summary, both the $Q_{i,1}$ and the $VR_{i,2}$ are simple functions of the first order serial correlation. But which one is more appropriate? To answer this question, one has to define the measure of *appropriate*. In the context of Neyman-Pearson hypothesis testing, there is a fundamental tradeoff between size and power of a statistical test, more commonly referred to as probability of Type-I and Type-II error.[3.9] So one possible way to answer the above question is by looking at which test has more power, i.e., higher probability of rejecting the null hypothesis if the *alternative* hypothesis is true for a test with a given size (recall the size

[3.7] We work out the detail of the statistics of the first order serial covariance and the first order serial correlation in Appendix A.4.3. The derivation outlines why $\hat{\rho}_i(1)$ has more convenient statistical properties than $\hat{\gamma}_i(1)$.

[3.8] This statistic is called the Q-statistic and was first developed in Pierce and Box (1970). Ljung and Box (1978) provide a modified version with better small sample distribution. Derivation of the asymptotic distribution of this statistic is straightforward using Generalized Method of Moment (GMM). See the Appendix in Campbell et al. (1997) or Chapter 14 in Hamilton (1994) for a review of GMM. We will also deal with this issue in Section 6.1 of Chapter 6.

[3.9] In statistics, the terms Type-I error (also known as false positive) and Type-II error (or a false negative) are used to describe possible errors made in a statistical decision process.

is the probability of rejecting the null if it is correct and typically set to 5% or 2.5%). One challenge in doing this type of analysis is determining the appropriate *alternative* hypothesis. Lo and MacKinlay (1989) look at this type of question in great depth.

3.3.1 Test Statistic

The testing approach we will follow extends this basic intuition by estimating a particular function of elements of the auto-covariance matrix in sample and comparing that against the value under the above null hypothesis. As it turns out, the easiest way to outline this test statistic is as the outcome of a particular algorithm. For reasons that will be clear shortly, we will refer to this algorithm as the *Contrarian Trading Strategy*.[3.10] A trading strategy is a procedure that specifies the amount of investment that must be made in each security i at time t. The algorithm works as follows.

Consider the case of N securities with period t return given by $r_{i,t}$. The amount invested in security i, denoted by $w_{i,t}$, is then selected as:

$$w_{i,t} = -\frac{1}{N}(r_{i,t} - r_{m,t}) \quad \text{where} \quad r_{m,t} = \frac{1}{N}\sum_{i=1}^{N} r_{i,t} \tag{3.5}$$

Let's define the profit, π_t, as the change in value of this investment by time $t+1$. Given the investment of $w_{i,t}$, the initial value of the investment is $\sum_{i=1}^{N} w_{i,t}$. Notice that by the construction outlined in (3.5), we have $\sum_{i=1}^{N} w_{i,t} = 0$. The number of units of asset i that can be purchased based on the initial investment of $w_{i,t}$ is given by $w_{i,t}/p_{i,t}$. Hence, the change in the value of the investment by time $t+1$ is simply:

$$\begin{aligned}
\pi_t &= \sum_{i=1}^{N} \frac{w_{i,t}}{p_{i,t}} \cdot p_{i,t+1} - \underbrace{\sum_{i=1}^{N} w_{i,t}}_{=0} \\
&= \sum_{i=1}^{N} w_{i,t}(r_{i,t+1} + 1) \qquad \text{where } r_{i,t+1} \text{ is the "simple return" from Definition 1} \\
&= \sum_{i=1}^{N} w_{i,t} r_{i,t+1} + \underbrace{\sum_{i=1}^{N} w_{i,t}}_{=0} \\
&= \sum_{i=1}^{N} w_{i,t} r_{i,t+1} \qquad (3.6)
\end{aligned}$$

[3.10] This strategy was first studied in Lehmann (1990) and Lo and MacKinlay (1990b). But in this treatment, we are using the strategy as a simple way of extracting a particular test statistic for testing the null hypothesis outlined in (3.4). This usage is very different from the original analysis conducted in those papers. The treatment here, in particular some of the mathematical derivations, is closely related to Lo and MacKinlay (1990b).

The expected value of π_t will turn out, as will be shown shortly, to be related to the auto-covariance matrix. The key to this particular trading strategy, and the reason it can be used as a test for the above null hypothesis, is that the weights, i.e., the $w_{i,t}$ specified in (3.5), are linear in the past returns. Therefore, the expected profit of the strategy will be a function of various elements of the first order auto-covariance matrix. To see this, substitute the definition for $w_{i,t}$ given in (3.5) into (3.6) and simplify as follows:

$$\begin{aligned} \pi_t &= \sum_{i=1}^{N} \left[-\frac{1}{N}(r_{i,t} - \frac{1}{N}\sum_{j=1}^{N} r_{j,t}) \right] r_{i,t+1} \\ &= \frac{1}{N^2} \sum_{i=1,j=1}^{N} r_{j,t} r_{i,t+1} - \frac{1}{N} \sum_{i=1}^{N} r_{i,t} r_{i,t+1} \end{aligned} \tag{3.7}$$

This final expression for π_t makes it clear that the expected profit, $\mathrm{E}[\pi_t]$, will be a function of expectation of the inner products of the current and the one time lagged returns, which in turn can be written in terms of the first order auto-covariance function. The following proposition makes this precise:

Proposition 3.1 *Consider the collection of N securities and denote by $\mathbf{R}_t$ the $N \times 1$ vector of their period t returns, $[r_{1,t} \cdots r_{N,t}]'$. Assume that $\mathbf{R}_t$ is a jointly covariance-stationary stochastic process with expectation $E[\mathbf{R}_t] = \boldsymbol{\mu} = [\mu_1 \cdots \mu_N]'$ and auto-covariance matrices $E[(\mathbf{R}_{t-l} - \boldsymbol{\mu})(\mathbf{R}_t - \boldsymbol{\mu})'] = \mathbf{\Gamma}_l = [\gamma_{i,j}(l)]$. Consider a net-zero investment strategy that invests $w_{i,t}$ dollars given by*

$$w_{i,t} = -\frac{1}{N}(r_{i,t} - r_{m,t}) \quad \text{where} \quad r_{m,t} = \frac{1}{N}\sum_{i=1}^{N} r_{i,t} \tag{3.8}$$

in security i. The expected profit, $E[\pi_t]$, where π_t is given by

$$\pi_t = \sum_{i=1}^{N} w_{i,t} r_{i,t+1} \tag{3.9}$$

is:

$$E[\pi_t] = \frac{1}{N^2}\boldsymbol{\iota}'\mathbf{\Gamma}_1\boldsymbol{\iota} - \frac{1}{N}tr(\mathbf{\Gamma}_1) - \sigma^2(\boldsymbol{\mu}) \tag{3.10}$$

where $\boldsymbol{\iota}$ is an $N \times 1$ vector of ones and

$$\sigma^2(\boldsymbol{\mu}) \equiv \frac{1}{N}\sum_{i=1}^{N}(\mu_i - \mu_m)^2 \ , \ \mu_m = \frac{1}{N}\sum_{i=1}^{N}\mu_i. \tag{3.11}$$

Proof: This is a special case of Proposition 4.3 for $q = 1$.

Proposition 3.1 shows that the trading strategy described in (3.5) generates an expected profit that is a function of the $\boldsymbol{\Gamma}_1$ and $\sigma^2(\boldsymbol{\mu})$. Under the null hypothesis of interest, see (3.4), the first part of this expression is zero. So the expected profit should be negative. This is summarized in the following corollary.

Corollary 3.1 *Under the null hypothesis of*

$$\boldsymbol{\Gamma}_1 = E[(\mathbf{R}_t - \boldsymbol{\mu})(\mathbf{R}_{t+1} - \boldsymbol{\mu})^T] \equiv 0 \tag{3.12}$$

The expected profit of the Contrarian strategy described by (3.8) where profit is calculated by (3.9) is:

$$E[\pi_t] = -\sigma^2(\boldsymbol{\mu})$$

where,

$$\sigma^2(\boldsymbol{\mu}) \equiv \frac{1}{N}\sum_{i=1}^{N}(\mu_i - \mu_m)^2 \quad , \quad \mu_m = \frac{1}{N}\sum_{i=1}^{N}\mu_i. \tag{3.13}$$

Proof: Follows immediately from Proposition 3.1.

Proposition 3.1 shows that the expected profit from the trading strategy described in this section amounts to calculating a particular combination of the elements of the first auto-covariance matrix, $\boldsymbol{\Gamma}_1$, as given by (3.10). Under the assumptions of the null hypothesis of interest, this expression should be a negative value given by Corollary 3.1 and expression (3.13).

Similar to the discussion we presented in Section 3.3 in connection with testing a null hypothesis of a similar nature for the case of a single time series, there are other possible combinations that can be used in testing the null hypothesis of interest. For example, it would be entirely possible to use sum of the squared values of the elements of $\boldsymbol{\Gamma}_1$ to construct the test statistics. So a natural question would be to ask what makes this particular combination of the elements of $\boldsymbol{\Gamma}_1$ preferable?

The discussion regarding the difference between the Variance Ratio Test and the Q-statistic still applies. This means that without a proper alternative hypothesis, there is no basis upon which to judge which particular functional form of the elements of $\boldsymbol{\Gamma}_1$ would be preferable. One reason that the proposed statistic in this section is superior to other alternatives is that it has a clear economic intuition. On this basis, the deviations from the null detected by this statistic can be directly mapped into a particular type of friction in the market. This economic intuition and the implications captured by deviation detected by this test will be explained after we present the empirical results in the next section.

3.4 Empirical Analysis

The data for the empirical analysis is obtained from Center for Research in Security Prices (CRSP). Please see Section A.1.1 for an overview of the CRSP database and various filters applied and data cleaning steps taken prior to this analysis. The main data taken from this database is the daily "Holding Period Return" for 1995 to 2007.[3.11]

Before moving on to the main empirical analysis, it is worth taking a look at the statistical properties of return signals that will be the subject of subsequent empirical analysis and develop a general sense of the time series properties of these signals. Table 3.1 shows this information. Each row shows the data for a given year. For example in 1995, the data from 5,307 individual return signals were used to calculate the reported statistics. The average of the daily means of these 5,307 price signals was only 13.73 basis points (bps)[3.12] or 0.1373%. These types of statistics are usually referred to as the *cross-sectional* statistics of the data since they are averages across the available cross-section. The cross-sectional standard deviation of the means was 19.59 bps. So the variance of the means, i.e., the $\sigma^2(\boldsymbol{\mu})$ to be used in (3.10) was tiny 0.04 bps. The returns usually have a large daily volatility, for example the cross-sectional average of daily standard deviations in 1995 was 295.9 bps (cross-sectional standard deviation of this was 156.2 bps).

Financial price signals generally have a large noise when measured at high-frequency (for example, at daily frequency as we do here). To give the reader a sense of this, we have reported the cross-sectional average and standard deviation of the SNR defined as μ_i/σ_i in the table as well. As can be seen, the signal-to-noise ratio is typically in the order of 10^{-2}, making any statistical analysis on individual price signals difficult.

Table 3.1 also reports the estimates of the first order serial correlations, ρ_1, for these return signals. Even though the individual serial correlations are generally negative, the magnitudes are generally small. This is similar to the observation made in a number of earlier studies, for example Lo and MacKinlay (1988), that the stock returns seem to have a small negative serial correlation that is of little economic impact. Also notice that even though the magnitude of serial correlation seems to have increased (so it became less negative) in the first 3 or 4 years of the sample, see Table 3.1, there does not seem to be a clear trend. The empirical analysis presented in the next section provides a more direct test of the predictability, primarily by looking at all elements of $\boldsymbol{\Gamma}_1$ instead of simply the diagonal elements reported in Table 3.1.

We now apply the trading strategy described in the previous section in equation (3.8) to these return signals. We will then calculate the realized profit for each day based on (3.9). Before we do this, however, we need to address one other potentially complicating factor. The realized profit defined in (3.9) is a linear function of the $w_{i,t}$. Therefore, a larger trading position, i.e., larger values of $w_{i,t}$, would result in a larger profit without really capturing the *economic* value of deviation captured by this mathematical expression. In order to have a proper economic intuition, we need to find a way to normalize the profit by the initial investment. Recall that the $w_{i,t}$ are the dollar investment amounts in security i for day t defined by (3.8). So the immediate approach would be to normalize the profit by the total investment needed, i.e., by $\sum_{i=1}^{N} w_{i,t}$. However, the ws defined in (3.8) by definition add up

[3.11]CRSP takes into account issues such as dividend and stock splits in calculating the returns based on prices using an approach similar to the "simple return" from Definition (1). Please see the documentation from CRSP for more information.

[3.12]*"Basis Points"* (commonly denoted by *"bps"*) is a commonly used unit to report small numbers, 1 bps=10^{-4} or 100 bps=1%.

Table 3.1: This table reports a summary of cross-sectional statistics for the following basic statistical measures of price signals: mean, standard deviation, first order serial correlation, and the signal-to-noise ratio (defined as mean/standard deviation) based on the daily holding period returns for years 1995-2007. 1 bps=10^{-4}.

		Mean (bps)			Standard Dev (bps)		Rho 1		SNR	
Year	Count	Average	StDev	Variance	Average	StDev	Average	StDev	Average	StDev
1995	5,307	13.73	19.59	0.04	295.9	156.2	-0.12	0.19	0.056	0.061
1996	5,952	8.85	20.65	0.04	324.2	179.6	-0.10	0.18	0.036	0.056
1997	5,984	11.06	20.40	0.04	315.7	156.5	-0.09	0.16	0.049	0.062
1998	5,908	1.30	25.38	0.06	382.9	236.5	-0.04	0.16	0.005	0.056
1999	5,370	10.32	31.78	0.10	385.4	207.2	-0.06	0.15	0.018	0.065
2000	5,362	-0.70	36.59	0.13	495.8	262.0	-0.06	0.15	0.010	0.063
2001	4,292	10.62	24.09	0.06	390.4	210.0	-0.04	0.15	0.035	0.054
2002	3,987	-2.19	21.57	0.05	353.0	187.0	-0.07	0.13	0.005	0.052
2003	3,681	19.38	18.13	0.03	257.0	118.7	-0.05	0.12	0.077	0.050
2004	4,060	8.80	16.05	0.03	248.9	127.6	-0.05	0.12	0.041	0.054
2005	4,037	3.59	16.55	0.03	234.3	116.6	-0.05	0.12	0.018	0.057
2006	4,028	7.40	14.59	0.02	228.8	104.2	-0.03	0.12	0.037	0.054
2007	4,012	0.01	19.61	0.04	258.1	116.7	-0.06	0.12	0.003	0.066

to 0 and the strategy described in (3.8) represented a \$0 investment strategy.[3.13] To obtain a sensible normalization, we will normalize the profit by the following normalization factor:[3.14]

$$I_t = \frac{1}{2}\sum_{i=1}^{N} |w_{i,t}| \tag{3.14}$$

and calculate the normalized profit, which we will refer to as the *return* as:

$$r_t = \frac{\pi_t}{I_t} \quad \text{where, } \pi_t \text{ is defined in (3.9) and } I_t \text{ is defined in (3.14)}$$

The result of applying this strategy to daily stock returns since 1995 is presented in Figure 3.1. The average strategy profit or return for each year of this period is displayed. The general pattern of the expected profits and returns are similar. A clearly declining pattern in both the average profit and the average returns are observable.

The results for the statistical test of the null hypothesis of zero profit is reported in Table 3.2. The realized values of profit or return, π_t and r_t, may exhibit heteroskedasticity or serial-correlation since subsequent periods share some data (namely return for day t, $\mathbf{R}_t$, is used in calculating the profit and return for day t and also to establish w's for day $t+1$). Therefore, care must be taken in conducting the statistical test of these values. The generally accepted approach in statistics is to use an estimator for the standard error of the mean estimator that is robust to such dependency. The statistics reported in Table 3.2, and also later in

[3.13] Such strategies are referred to as *arbitrage strategy* in finance.

[3.14] This normalization factor is motivated by the regulation applied to entities, such as broker-dealers and hedge funds, that may be engaged in these types of trades. Please see Khandani and Lo (2007) for a discussion of this point.

Table 3.3, are based on the Newey-West estimator using 3 lags.[3.15]

A t-stat value of greater than 2, associated with a test with size of 5%, is the generally accepted level of significance for these types of tests. It can be seen that even in the most recent year, 2007, the null hypothesis of zero expected profit can be rejected at all conventional levels of significance.

Based on the reported return, we can get a sense of the actual economic significance of these returns. For example in year 1995, the return for this strategy was about 1.2%/day or about 350% per year (there are about 250 trading days in each year). Of course, in 1995 applying this type of strategy, i.e., trading across several thousand stocks each day, was not possible as we will discuss further in the next section. But even in the most recent year, the return was about 40-50%/year. So it is hard to argue that the economic significance of the deviations is too small to be of interest to the very competitive business of investment management. So what are the results telling us? This is the discussion that we will elaborate on in the next section.

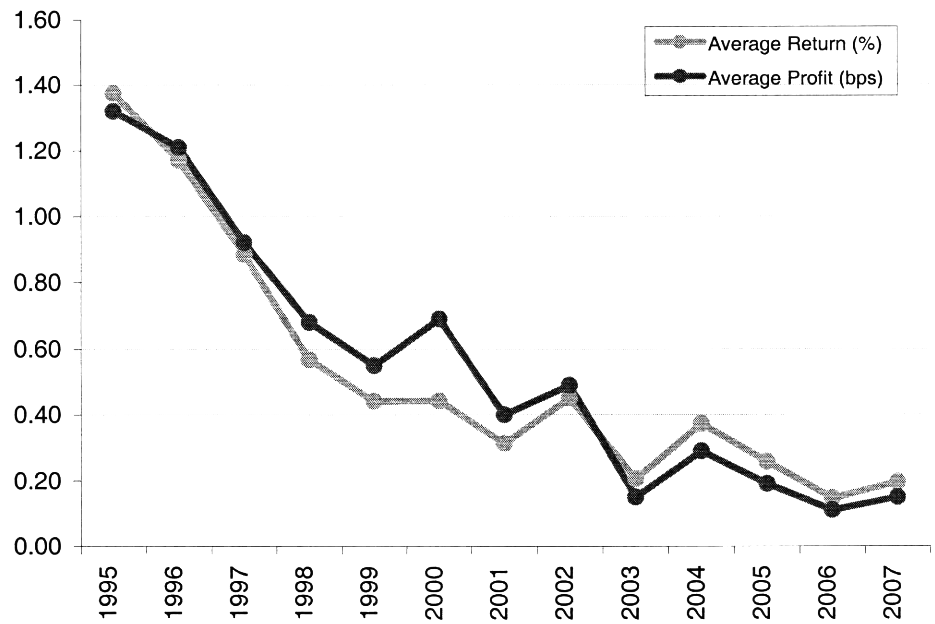

Figure 3.1: This Figure shows our measure of the average value of the contrarian signal in stock prices between 1995 and 2007. This value is measured using the average profit and the average for the contrarian trading strategy between 1995 and 2007. The trading strategy invests $w_{i,t}$ dollars in security i on day t where $w_{i,t} = -\frac{1}{N}(r_{i,t} - r_{m,t})$ and $r_{m,t} = \frac{1}{N}\sum_{i=1}^{N} r_{i,t}$. Profit ($\pi_t$) is calculated as $\pi_t = \sum_{i=1}^{N} w_{i,t} r_{i,t+1}$ and return (r_t) is calculated as $r_t = \frac{\pi_t}{I_t}$ where $I_t = \frac{1}{2}\sum_{i=1}^{N} |w_{i,t}|$. Average returns are reported in percentage points. The average profits are reported in basis points or "bps" where 1 bps$=10^{-4}$.

[3.15] See the Appendix A.1.2 for further discussion on this issue and an introduction to the Newey-West estimator.

Table 3.2: Formal statistical test of average profit and average return of a trading strategy that invest $w_{i,t}$ dollars in security i on day t where $w_{i,t} = -\frac{1}{N}(r_{i,t} - r_{m,t})$ and $r_{m,t} = \frac{1}{N}\sum_{i=1}^{N} r_{i,t}$. Profit ($\pi_t$) is calculated as $\pi_t = \sum_{i=1}^{N} w_{i,t} r_{i,t+1}$ and return (r_t) is calculated as $r_t = \frac{\pi_t}{I_t}$ where $I_t = \frac{1}{2}\sum_{i=1}^{N} |w_{i,t}|$. Under the null hypothesis of no-linear predictability, the average profit should be zero. A T-stat of greater than 2, associated with a test with size 5%, is the generally accepted level of significance. T-stats are calculated using the Newey-West approach using 3-lags. See Appendix A.1.2 for a discussion of the issues involved with developing appropriate statistical significance in this case. Average returns are reported in percentage points. The average profits are reported in basis points or "bps" where 1 bps=10^{-4}.

Panel A: Daily Profit Data

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Average Profit (bps)	1 32	1 21	0 92	0.68	0 55	0 69	0.4	0 49	0.15	0.29	0 19	0.11	0 15
T-stat of Average	46 44	32 66	12.90	8.70	7 06	3 68	2.98	7 04	5.36	9.82	7 63	4 61	3 78
Aveage Count	4,780	5,272	5,392	5,195	4,736	4,566	3,782	3,485	3,375	3,740	3,721	3,764	3,522

Panel B: Daily Return Data

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Average Return (%)	1.38	1.17	0.88	0.57	0.44	0 44	0 31	0 45	0 21	0 37	0 26	0.15	0 20
T-stat of Average	47 65	36.32	19 37	11 75	7 78	4 03	3 87	7 77	5 96	10 53	8.41	4 78	5.13
Aveage Count	4,780	5,272	5,392	5,195	4,736	4,566	3,782	3,485	3,375	3,740	3,721	3,764	3,522

3.5 A Signal Detection Scheme

Recall that the investment this strategy makes on day t in stock i, or $w_{i,t}$, is based on that stock's day t return and the average return of all stocks on that day as specified in (3.5) which is repeated here to facilitate the discussion:

$$w_{i,t} = -\frac{1}{N}(r_{i,t} - r_{m,t}) \quad \text{where} \quad r_{m,t} = \frac{1}{N}\sum_{i=1}^{N} r_{i,t} \tag{3.15}$$

So the investment in each stock is proportional to the amount by which it has under performed the average as calculated in $r_{m,t}$. Similarly, the investment is negative for stocks that have outperformed the average on that day. In summary, this strategy is betting on the fact that days with positive return will be, on average, followed by days with negative return and vice-versa. The fact that the profits reported in Figure 3.1 and Table 3.2 are positive show that this "bet" is a profitable bet on average; i.e., it has a more than fair chance of being correct.

From the engineering perspective, this trading algorithm is designed to detect the existence of a *mean-reverting component* in prices that are otherwise simply *white noise* signals. We will refer to this as the *contrarian signal component* of prices. Hence, our algorithm is simply a procedure for detecting this type of signal and the magnitude of the resulting average profits or returns are different transformation of the strength of the contrarian signal. The results suggest that prices do have a small contrarian component and the strength of this component of price signals has declined overtime. But why may this be the case? Here is one possible reason.

Recall that for every buyer there is a seller. But there is a difference between the times that buyers have initiated a transaction and times that sellers have done so. Perhaps, stocks that have outperformed the average on a given day tend to be more often traded in buyer-initiated trades. For these stocks, there is a supply-demand imbalance in the direction of excess buyers. The strategy discussed in this section simulates the behavior of an entity who is willing to sell such stocks. On the other hand, stocks that have under performed are the ones that have been more often initiated by sellers. Again, in this case, the simulated strategy is acting as an entity who is willing to buy such stock. In both cases, the simulated strategy is capturing the behavior of an entity who is willing to act as a balancing force in the supply-demand imbalances that exist. So what type of entity is being captured by this simulated strategy?

We argue that this is the behavior of *dealers* or what is more commonly referred to as *market-makers*. These are the entities that, similar to a car dealership, are willing to buy or sell stocks to interested sellers or buyers as they arrive in the marketplace. So the profit documented in the empirical analysis may simply show the amount of profit captured by such dealers acting in this market. Perhaps in the earlier parts of the sample this behavior was not really achievable as it would have required simultaneously trading across more than 5,000 stocks. One would expect the profit to decline as the technology for this type of activity becomes more widely available, and as more competitors enter the market. These expectations are consistent with the pattern of observed profits and returns documented above.

Table 3.3 gives some additional support for this argument. In this table, we have documented the profit of this strategy when applied to 10 size-sorted (when the total *Market Capitalization* is used as the size) Deciles of stocks.[3.16] Notice that the profits are always more substantial among smaller stocks. Smaller stocks tend to be less widely traded and, hence, providing the dealership service for these stocks is akin to being a dealership for exotic cars; such dealers are expected to collect a higher premium for providing their service. Motivated by this intuition, we will be providing a model for dealership's behavior in chapter 4.

In order to set the stage for Chapter 5, we have shown the behavior of this strategy in 2007 in Figure 3.2. It can be seen that there was a substantial breakdown in early part of August 2007. We will show in Chapter 5 that the model described in Chapter 4 can be used to explain the mechanism behind this apparent system breakdown in 2007.

[3.16]The Deciles are constructed on the first trading day of January and July of each year and kept intact for the subsequent 6 months.

Table 3.3: Formal statistical test of average profit and average return of the contrarian trading strategy applied to 10 size-sorted Deciles of stocks for each year between 1995 to 2007. Please see the caption of Table 3.2 for a description of the strategy and values reported. Average returns are reported in percentage points. The average profits are reported in *basis points* (bps) where 1 bps=10^{-4}. The t-statistic calculated based on the Newey-West estimator with 3-lags is reported in the parenthesis.

Panel A: Daily Profit Data (bps)

Year	Smallest	Dcecile 2	Dcecile 3	Dcecile 4	Dcecile 5	Dcecile 6	Dcecile 7	Dcecile 8	Dcecile 9	Largest	All
1995	4 24(50.5)	3.36(39.6)	2.24(32 6)	1.79(27 2)	1.12(18 3)	0 61(14.2)	0 19(4.5)	-0 01(-0.3)	-0 01(-0.6)	0.02(1.0)	1.32(46.4)
1996	4.31(40.4)	3 04(33 3)	2.17(31 3)	1.60(22 6)	0.98(11.8)	0.58(10.1)	0 20(3 6)	-0.10(-1.9)	-0.02(-0 5)	0.02(0 6)	1 21(32.7)
1997	3.26(34.1)	2 29(21.7)	1 61(15.4)	1.20(11.3)	0 74(6.4)	0.32(3 9)	0.07(1.0)	-0 08(-1.0)	0.08(1.3)	0 11(2.7)	0.92(12 9)
1998	3.01(15.1)	1.92(10.8)	1.49(11.0)	0.86(6.9)	0.43(2.9)	0 06(0 6)	-0.05(-0 6)	-0.14(-1.5)	0 01(0.1)	0.09(1.6)	0.68(8 7)
1999	3.05(23 4)	1.78(10.7)	1.03(9.5)	0 53(3.8)	0 05(0.3)	-0.08(-0.6)	-0.19(-1 3)	-0.38(-3.1)	0 01(0.1)	0.05(0.7)	0.55(7.1)
2000	3 20(22 8)	2.10(12.3)	1.26(5 9)	0 19(0.8)	0.11(0.5)	0 00(0 0)	-0.16(-0.6)	0 28(0 9)	0.00(0.0)	0.06(0 2)	0.69(3 7)
2001	2.04(19.9)	1.28(13.6)	0.69(5.0)	0 34(1.9)	0 09(0.5)	0 17(0.9)	0.23(1.2)	-0 14(-0.9)	-0 19(-1.2)	-0.16(-1.0)	0.40(3 0)
2002	1.43(17.0)	0 75(14.1)	0.56(6.4)	0 36(4 2)	0.33(3 6)	0.31(3.8)	0 31(3.5)	0.22(2.4)	0 13(1 3)	0 11(1 2)	0 49(7.0)
2003	0.76(14.0)	0.20(5.3)	-0.06(-0.9)	0 04(0.7)	0.09(1.7)	0 16(3 2)	0.14(3.2)	0.10(2.9)	0.02(0 7)	0.02(1 1)	0.15(5 4)
2004	1.05(12.4)	0.39(7 4)	0.27(5 3)	0.36(7.6)	0 22(4.7)	0 23(5 2)	0.15(3.6)	0.10(3.2)	0 02(0.8)	-0.01(-0 5)	0.29(9 8)
2005	0.97(10.3)	0 31(7.3)	0.11(2.6)	0 09(2.2)	0 08(2.4)	0.09(2.2)	0 04(1.4)	0.05(2.0)	0.01(0 4)	0.01(0 8)	0.19(7.6)
2006	0 68(10.5)	0.20(4.4)	0.10(2.1)	0 07(1.6)	0.04(1.0)	-0 01(-0.2)	-0.02(-0.7)	0 03(1 1)	0.03(1 5)	-0.00(-0 2)	0 11(4 6)
2007	0.65(7.9)	0 24(4.0)	0 21(2 8)	0.20(2 9)	0 17(2 7)	-0.06(-0.8)	0 01(0.1)	-0.04(-1.1)	-0 07(-1.6)	-0 03(-1.0)	0.15(3.8)

Panel B: Daily Return Data (%)

Year	Smallest	Dcecile 2	Dcecile 3	Dcecile 4	Dcecile 5	Dcecile 6	Dcecile 7	Dcecile 8	Dcecile 9	Largest	All
1995	3.57(54.8)	2.75(45 8)	1.94(32.7)	1.62(29.2)	1.07(18.8)	0.61(14.4)	0.21(4 6)	-0.01(-0 3)	-0.02(-0.6)	0.04(1 0)	1.38(47 6)
1996	3.58(44.7)	2.47(34 0)	1.82(32.0)	1 34(23.9)	0.84(13.1)	0.52(10.9)	0.19(3.5)	-0 11(-2.0)	-0 04(-0.8)	0 02(0.4)	1.17(36.3)
1997	2.83(36 6)	1.94(25.4)	1.34(19.2)	1.02(14 3)	0.62(8.1)	0 28(4.9)	0.04(0 8)	-0.12(-2 0)	0.06(1.1)	0.14(3 0)	0 88(19 4)
1998	2.38(18 8)	1.45(12 8)	1 11(13.8)	0 62(7.7)	0 29(3 0)	0 03(0.4)	-0 04(-0.7)	-0.12(-1.7)	0 03(0.4)	0 10(1.9)	0.57(11.8)
1999	2.56(26.2)	1.41(15 2)	0.82(10.1)	0 38(4 1)	-0.01(-0.1)	-0.11(-1 2)	-0 21(-2 3)	-0.35(-3 8)	-0 01(-0 1)	0 06(0.8)	0 44(7 8)
2000	2.58(26.1)	1.59(14.5)	0.92(7.3)	0.14(1.0)	0 03(0.3)	-0 02(-0.2)	-0.14(-1.0)	0.16(1.0)	0.00(0 0)	0.03(0.2)	0.44(4.0)
2001	2.15(23.8)	1 25(15.4)	0.57(5 8)	0.24(2.2)	-0 01(-0.1)	0 06(0 6)	0 13(1.3)	-0 10(-0.9)	-0.11(-1 0)	-0.11(-0.9)	0.31(3.9)
2002	1.67(19 5)	0.85(15.1)	0.53(6 5)	0.29(4 2)	0 28(3.9)	0 26(3.7)	0.28(3.6)	0.20(2 4)	0 11(1.2)	0 09(1.0)	0 45(7.8)
2003	1.00(14 2)	0.26(5.4)	-0.07(-1.0)	0 04(0 6)	0.11(1 9)	0 20(3.6)	0.18(3 4)	0 15(3.1)	0.04(0 9)	0.05(1.3)	0.21(6.0)
2004	1.17(13.2)	0.48(8.5)	0.31(5.5)	0.38(7.8)	0 25(4.8)	0.29(5 5)	0 22(4.1)	0.15(3 4)	0.05(1.1)	-0 01(-0.3)	0.37(10 5)
2005	1.05(12 0)	0.39(7.8)	0.13(2.5)	0 11(2.2)	0.09(2.3)	0 11(2 1)	0.05(1.4)	0 08(1.9)	0.01(0.4)	0.02(0 7)	0 26(8 4)
2006	0.86(11.9)	0 26(4.3)	0.11(2.0)	0.06(1 3)	0 05(1.1)	-0.02(-0.3)	-0.02(-0.6)	0.05(1.2)	0 06(1.7)	-0 00(-0.0)	0.15(4.8)
2007	0.74(9.2)	0.26(3 8)	0 25(3.4)	0 24(4 3)	0 21(3.9)	-0.08(-0.9)	0 03(0.7)	-0.04(-0.8)	-0 07(-1.2)	-0 04(-0.9)	0.20(5 1)

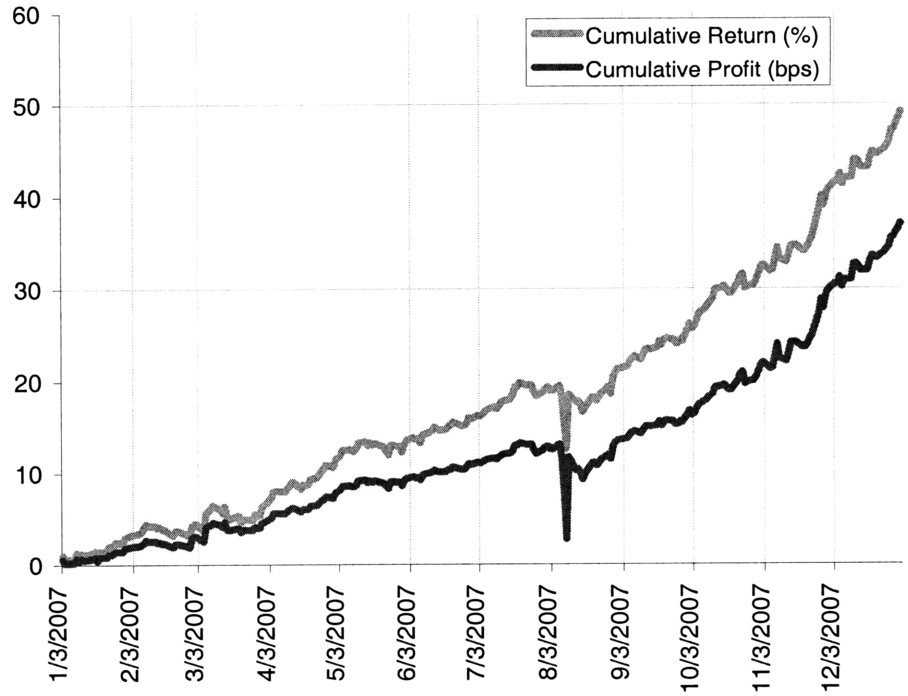

Figure 3.2: The cumulative profit and return of the contrarian trading strategy in 2007. Cumulative profit is simply the running sum of π_t for the days since January 1, 2007. The Cumulative return, r_t^c, is calculated as $r_t^c = \prod_t(1 + r_t) - 1$. Please see the caption of Figure 3.1 for a description of the strategy, as well as the method for calculating profit, π_t, and returns, r_t. Returns are reported in percentage points and profits are reported in basis points or "bps" where 1 bps$=10^{-4}$.

3.6 Chapter Conclusions

This chapter started with a relatively abstract notion of informational efficiency of price signals. We discussed how this notion is related to the predictability of prices. We looked at both linear and non-linear predictors and elaborated on the implications of each for the dynamics of price signals. We looked at methods for testing predictability of the next day's return based on the most recent set of returns and linear predictors. It was shown that the profit of the Contrarian Trading Strategy is one possible sample statistic for testing this type of predictability.

In the empirical analysis, we detect a relatively large but continually decreasing deviation from unpredictability in returns for the period of 1995 to 2007. We argued that the trading strategy used is a simple procedure for detecting a mean-reverting signal among otherwise white noise signals. Hence, our results suggest that there is indeed a weak mean-reverting component in the price signals and the strength of this component has declined in during this sample.

We argued that the deviation from unpredictability detected through these tests is a result of the arrival of buyers and sellers and the resulting supply-demand imbalance. As mentioned before, while the perfect informational efficiency and unpredictability may be possible in an idealized "friction-less" world, there are many impediments in the real world that cause the actual behavior to deviate from what would be expected in the idealized setting. Finally, we showed the performance of this test in two particular instances: among different subset of stocks based on the company size and also in year 2007, to highlight the kind of frictions that we believe play a major role in the behavior documented using these tests. This set the stage for the model we will discuss in Chapter 4 regarding the underlying frictions captured by the tests in this chapter.

Chapter 4

Short-Term Deviation from White Noise

In this chapter, we will take a closer look at predictability in price signals. As we argued before, the profit of the trading strategy discussed in the last section is a function of the elements of the auto-covariance matrix and can be used to detect small deviations from perfect unpredictability of the underlying price signals. The evidence provided before shows that the degree of deviation from efficient prices, i.e., the success of linear predictors using only prior prices in the framework discussed in the last chapter, has declined over time. Furthermore, the deviations have been consistently stronger among smaller stocks. This behavior suggests that the level of predictability may a by-product of certain "frictions" in the system that have been reduced over time. In other words, while under the idealized world outlined in Chapter 2, prices should fully reflect all available information, and hence be completely unpredictable, in the real world there are many impediments in the inner works of the market system and the resulting price signals may deviate from the idealized setting due to such frictions. In order to analyze the time-series properties of the resulting price signals, one has to explicitly model the important frictions that may give rise to deviation from perfect unpredictability.

We will explicitly model one such phenomenon and develop a hypothesis about the time-series dynamics of price changes. A nice feature of the model developed here is that it also produces testable hypotheses about the link between predictability of prices and other observables of the system. The empirical hypothesis testing results are presented at the end of this chapter.

4.1 Motivation

We will start by discussing a simple example to motivate the subsequent analysis. Consider analyzing the time series of prices at which a particular item, say a particular make and model of a car, is sold or purchased. Let's represent the fair price of the item at time t by p_t^*. What this means is that if you own this item and could instantly contact an interested buyer when you decided to sell, for example if there was an auction held at that moment to inform all interested buyers, you would be able to find someone to buy it from you for p_t^*. Of course, the assumption that you could find the other party instantly is not very reasonable and that is precisely the issue.

In reality, finding the buyer may be difficult and this difficulty gives rise to what is referred to as the *search friction.* One way to alleviate this issue is to have an organized market, such as the New York Stock Exchange, where all buyers and sellers come together. But even in such a setting, there is still a friction since buyers and sellers arrive asynchronously at the market; i.e., a buyer to offset an arriving seller may not arrive for a few seconds or minutes after the time that the sellers has arrived. In this section, we will create a model that links the *synchronicity friction* to predictability in the resulting price signals.

To better understand why this may be the case, consider a scenario in which buyers and sellers arrive randomly and try to buy or sell an item instantly. Since a seller may not be immediately available when a buyer arrives and vice versa, one can argue that *dealerships* are needed to alleviate this synchronicity issue. For example, a dealer (think of a car dealership, for example) can buy from the interested seller immediately and wait until an interested buyer has arrived. Assuming an interested buyer will eventually arrive, the dealership is not exposed to too much risk (they know a buyer will eventually come). But they still need an incentive and should charge a fee to provide this service. One way for the dealers to get this incentive would be to buy the item from the interested seller immediately but at a price slightly below the fair value, p_t^*, and then sell it to the interested buyer at a price slightly above the fair value. Let δ represent this spread, so the transaction prices are $p_t^* - \delta$ and $p_t^* + \delta$, respectively.[4.1] Let b_t be an indicator random variable which is 1 if the transaction at time t was initiated by a buyer and -1 otherwise. From the point of view of an outside observer looking at the prices at which the transactions occur, they will see a time series of prices governed by the following data-generating process:

$$p_t = p_t^* + \delta b_t \tag{4.1}$$

For now, we will assume that each b_t is $+1$ or -1 with probability $1/2$ independent of everything else. Let $\Delta p_t = p_t - p_{t-1}$. It is easy to calculate the following statistics for the observed price changes:

$$\begin{aligned}
\mathrm{Var}(\Delta p_t) &= \mathrm{Var}(\Delta p_t^*) + \frac{\delta^2}{2} && (4.2a)\\
\mathrm{Cov}(\Delta p_t, \Delta p_{t+1}) &= -\frac{\delta^2}{4} && (4.2b)\\
\mathrm{Cov}(\Delta p_t, \Delta p_{t+k}) &= 0, \quad k > 1 && (4.2c)
\end{aligned}$$

As can be seen in the above formulation, the subsequent price changes have a negative covariance. This negative covariance is the result of imperfection in the mechanism by which the arriving buyers and sellers engage in transactions. For example, if instead of randomly arriving buyers and sellers, we had a situation where each day *all* interested parties came together to do all their buying and selling, i.e., the auction scenario mentioned above, then perhaps there would be no negative covariance in the resulting prices. In other words,

[4.1] This approach was first proposed in Roll (1984). See also Campbell et al. (1997).

the observed negative covariance in this case is a result of the system's frictions and the actual mechanics of the trading process. For this reason, it is not hard to imagine that if a technological change, for example more easily available communication networks, made it easier for buyers and sellers to find each other, the resulting prices would move closer to the actual *fair* price, i.e., p_t^*. Furthermore, the variance of price changes is dominated by the variance of the underlying fair value, i.e., the $\text{Var}(\Delta p_t^*)$ term. This later issue makes it difficult to detect the small negative covariance term in the observed prices due to the underlying trading process. This simple intuition is the motivation behind the model we will discuss in the next section.

4.2 The Model

The model is described as the interaction between two types of entities: customers and dealerships. It builds upon the model developed in Grossman and Miller (1988). For simplicity, we assume that there is only one item that is being exchanged and customers are interested in holding this item (asset) as a part of their investment. Their motivation for holding this asset is assumed to be exogenous to the model being discussed. We also assume that some of these customers may decide due to exogenous reasons to reduce or increase their holding at arbitrary points in time. It is also assumed that in aggregate, all customers are still happy to hold the total amount of the asset outstanding. So their individual decision to buy more or sell some of their holding really comes down to locating an interested seller or buyer, which, due to the assumption about the aggregate behavior, always exists. One option is for each customer to do a search and wait until the offsetting customer has been located. Alternatively, we consider a situation where *Dealership* exists in order to alleviate the *synchronicity friction* such that the arriving customers don't have to wait until an offsetting customer has been located in order to engage in a transaction.

We start by modeling the arrival of buyers or sellers as a random processes. As it will turn out, this random processes drives the observed behavior of the system. Similar to the intuition behind (4.1), prices increase when extra buyers arrive and drop when extra sellers come to the market. Such increase or decrease is temporary, however, as offsetting buyers or sellers arrive over the next several time periods resulting in the reversal of any price increase or decrease induced by the imbalance. The exact behavior of the prices also depends on the characteristic of the dealer. In a sense that will be made precise soon, if the dealers are very aggressive then the temporary price change is smaller relative to the situations when the dealers are less aggressive and hence need a larger price drop or increase, after arrival of sellers or buyers, respectively, to accommodate the arriving customers. We next describe the two building blocks of this model: *Customer Arrival Process* and *Dealer's Maximization Objective.*

- **Customer Arrival Process**

  Let $\eta_t^b \sim N(\bar{v}, \sigma_\eta^2/2)$ and $\eta_t^s \sim N(\bar{v}, \sigma_\eta^2/2)$ be iid Gaussian random variables that represent the number of *new* arriving buyers and sellers at time t. *Offsetting* customers will arrive over the subsequent time intervals to offset each of these *new* customers completely. For example, due to arrival of η_t^s sellers at time t, a total of $\theta_1\eta_t^s$ buyers arrive by time $t+1$, another $\theta_2\eta_t^s$ buyers arrive by time $t+2$, etc. Given the assumption that customers' arrival will eventually fully offset each of η_t^b or η_t^s we need to have $\sum\theta_k = 1$. Please note the distinction between *new* and *offsetting* customers. The *cumulative* number of arriving buyers and sellers as time t, denoted by q_t^b and q_t^s respectively, are given by:

$$q_t^b = \eta_t^b + \theta_1\eta_{t-1}^s + \theta_2\eta_{t-2}^s \tag{4.3a}$$
$$q_t^s = \eta_t^s + \theta_1\eta_{t-1}^b + \theta_2\eta_{t-2}^b \tag{4.3b}$$

Let $\eta_t = \eta_t^b - \eta_t^s$. η_t represent the excess number of *new* arriving buyers at time t. Also denote by q_t the imbalance between the *cumulative* number of buyers and sellers at time t. Given the formulation above, q_t has the following Moving Average (MA) representations:

$$\begin{aligned} q_t &= q_t^b - q_t^s & (4.4a)\\ &= (\eta_t^b - \eta_t^s) + \theta_1(\eta_t^s - \eta_t^b) + \theta_2(\eta_t^s - \eta_t^b) - \cdots & (4.4b)\\ &= \eta_t - \theta_1\eta_{t-1} - \theta_2\eta_{t-2} - \cdots & (4.4c)\\ &= \theta(L)\eta_t & (4.4d) \end{aligned}$$

- **Dealer's Maximization Objective**

For the system to operate normally, the extra customers must be accommodated by the dealership. But the dealership needs to be compensated on average for holding the extra units. Intuitively, the price change $r_t = p_t - p_{t-1}$ must have a negative expected value after time intervals in which there was an excess buyer. The idea is that the dealers accommodate the extra buyers by selling them the item from their inventory but the price at which they will sell the item is higher than the price for which they expect to be able to buy the item back from the future arriving sellers. The same logic applies to cases when there is an excess arriving seller. In summary, we expect to have the following relationship between expected price changes, $\mathrm{E}[r_t]$, and the imbalance, q_t:

$$\mathrm{E}[r_{t+1}] = \mathrm{E}[p_{t+1} - p_t] < 0 \quad \text{if} \quad q_t > 0 \tag{4.5a}$$
$$\mathrm{E}[r_{t+1}] = \mathrm{E}[p_{t+1} - p_t] > 0 \quad \text{if} \quad q_t < 0 \tag{4.5b}$$

In order to quantify the link between price changes, $r_{t+1} = p_{t+1} - p_t$, that is needed to compensate the dealer, we will assume that each dealer is optimizing a simple two-period optimization process. Their objective, written in terms of the number of units of the asset they will purchase at this period, d_t, has the following form:

$$
\begin{aligned}
U(d_t) &= -\mathrm{E}_t[e^{-\alpha(d_t r_{t+1} - c)}] \\
&= -\mathrm{E}_t[e^{-\alpha d_t r_{t+1}} e^{\alpha c}]
\end{aligned} \tag{4.6}
$$

where $\mathrm{E}_t[\cdot]$ is the shorthand notation used to represent the expectation conditional on the information at time t, d_t is the number of units of the asset they will buy, r_{t+1} is the immediate price change over the next time interval, α is a parameter that controls the behavior of these dealers with lower values of α indicating a more risk-seeking (aggressive) behavior[4.2], and c is the cost of their operation. The utility function of the above form, i.e., exponential, is very common in this type of analysis.[4.3] Since the next period's price p_{t+1} is not known, the objective is to optimize the expected value of this utility function.

As discussed before, the model here is a simple representation of the impact of imbalance on prices. But prices also change due to reasons beyond this model and in order to capture those *exogenous drivers*, we model the period-to-period price changes, $p_{t+1} - p_t$, as the sum of a stochastic component that is not related to the arrival of buyers or sellers and a part that is driven by the imbalance. More precisely, we will conjecture that the return process has the following form:

$$
r_{t+1} = p_{t+1} - p_t \tag{4.7a}
$$

$$
= \text{exogenous price changes} + \underbrace{\text{imbalance driven price changes}}_{\text{part we try to model}} \tag{4.7b}
$$

The next proposition outlines the main theoretical result of this section. The proof is given in the Appendix A.2.1.

Proposition 4.1 *Let $\eta_t \sim N(0, \sigma_\eta^2)$ be a white-noise process representing the number of excess new buyers arriving at time t. Assuming that the total cumulative excess buyers is given by*

$$
q_t = \theta(L)\eta_t \tag{4.8}
$$

and assuming that dealers optimize an exponential utility function of the following form

$$
U(d_t) = -E_t[e^{-\alpha d_t r_{t+1}} e^{\alpha c}] \tag{4.9}
$$

[4.2]One way to link to form of the utility function is by looking at the coefficient of *absolute risk aversion* defined as $-\frac{U''(w)}{U'(w)}$. For $U(w) = -e^{-\alpha w}$ this coefficient is α

[4.3]Please see Bertsekas (2000), Appendix G, for an overview of decision theory under uncertainty, including a discussion on various commonly used utility function and the coefficient of *absolute risk aversion*.

where d_t is the number of units they will purchase, $r_{t+1} = p_{t+1} - p_t$, α is their parameter of risk aversion, and c is their cost. Under this structure, r_{t+1} will have the following form:

$$r_{t+1} = p_{t+1} - p_t = \nu_{t+1} - \alpha\sigma_\nu^2 q_t \tag{4.10}$$

where $\nu_{t+1} \sim N(0, \sigma_\nu^2)$ is the change in price due to exogenous events.

Proof: See Appendix A.2.1.

The above preposition creates a link between the imbalance quantity, q_t, and the changes in the observed price or r_t. We arrived at this result (please see the proof) by looking at the first-order conditions of the dealers according to the optimization problem formulated in (4.9). In the long-run, i.e., when the system reaches its steady-state, there should a link between the cost value, c in (4.9), and the dynamics of the observed prices. Intuitively, if price changes are too "volatile" compared to the cost of being a dealer, more dealers will enter the market. So when the system has reached its steady-state behavior, it should be the case that the sensitivity of price change to the imbalance quantity, $\alpha\sigma_\nu^2$ in (4.10), is somehow linked to the cost c in (4.9). The following proposition makes this link clear.

Proposition 4.2 *Assume the same data-generating process as outlined in Proposition 4.1. As outlined in Proposition 4.1, the sensitivity of the return, r_{t+1}, to imbalance, q_t, is $\alpha\sigma_\nu^2$. In steady-state, i.e., under the condition that dealers have entered the market such that there is no long-term gain from being a dealer, we should have:*

$$\alpha\sigma_\nu^2 \approx \frac{2c}{\sigma_q^2} \tag{4.11}$$

Proof: See Appendix A.2.2.

We now discuss some of the implications of this model to validate the intuition and develop further insight into the price dynamics.

4.2.1 Model Implications

- **No Imbalance:**

  If $q_t = 0$ then $r_{t+1} = \nu_{t+1}$. So if there is no imbalance, then the price change is driven by the exogenous source of uncertainty.

- **Return and Imbalance**

  The dealer's gain, r_{t+1}, is random as it depends on the exogenous source of uncertainty through ν_{t+1}. But their average gain is simply $E_t[r_{t+1}] = -\alpha\sigma_\nu^2 q_t$ (note that q_t is not random conditioned on the information available to the dealer at time t). The relation between customer arrival and the expected gain is consistent with the intuition we outlined in (4.5). For example, after the arrival of buyers (recall that q_t is positive for buyers) the prices are expected to depreciate (decrease) over the following time period. This means that in order to sell the extra number of units to excess buyers, the dealer will offer to sell the asset to them at a price, p_t, that is, on average above the price,p_{t+1}, they expect to be able to buy back the asset at the next time period. Similarly, if there are excess sellers (i.e., if q_t is negative), the price is expected to appreciate over the next time period, i.e., the dealers will buy the asset from seller at a price p_t slightly below the expected next period price of p_{t+1}.

- **Sensitivity to Volatility**

  As seen in (4.10), the sensitivity of the returns to imbalance quantity, q_t, is an increasing function of the volatility of the prices due to exogenous factors, σ_ν^2. The relationship is to be expected: if there is more uncertainty about the next period's price due to outside reasons, the dealers will need a larger price reduction (increase) in order to accommodate the excess sellers (buyers).

- **Sensitivity to Dealer's Aggressiveness**

  (4.10) shows that the relationship between return and imbalance quantity is an increasing function of the α parameter. Recall that α controls the risk aversion of the dealers, see (4.6), and lower values of α indicate a more aggressive, i.e. risk seeking, behavior by the dealers. For more aggressive dealers, the amount of price deviation needed to accommodate the extra customers is smaller, consistent with the expression given in (4.10).

- **Steady-State Behavior**

  In the long-run, however, the model needs a feedback loop where the price sensitivity, $\alpha\sigma_\nu^2$, is related to the cost that dealers have to pay, c in their optimization function given in (4.6), to be present in this market. (4.11) makes this link clear. As expected, as the cost of being a dealer becomes lower, more dealers will enter and the steady-state sensitivity of returns to the imbalance is reduced. (4.11) also indicates a connection between the form of the MA prices, $\theta(L)$, and the return sensitivity. Recall that by assumption we know what $\sum_{i=1}^{\infty}\theta_i = 1$. The lowest value for the sensitivity, $\alpha\sigma_\nu^2$, corresponding to the case that $\sum_{i=1}^{\infty}\theta_i^2$ is maximized, is achieved when one of θ_i is 1 and the rest are zero. The more widespread the value of θ_i are, the lower the $\sum_{i=1}^{\infty}\theta_i^2$ and the higher the price sensitivity will be. This is intuitively plausible because the more widespread value of θ_i correspond to a more slowly mean-reverting imbalance process, q_t. This corresponds to the case where the offsetting customers are slower to arrive at the market after the arrival of each new customer and it is intuitively plausible that the dealers would expect a higher return for providing immediacy to each new customer in this case.

In the next few pages, we will hypothesize a specific model for the evolution of imbalance and develop expectation for the price dynamics under that hypothesis. The empirical testing of those implications will be the subject of the empirical analysis that will follow.

4.3 Hypothesis

The model developed in the first part of this section gives us the theoretical underpinning that links time-series properties of price changes to the time-series characteristics of the imbalance quantity. The hypothesis that we will be dealing with in the rest of this section is based on testing a particular form for the evolution of the imbalance quantity. In other words, we will propose a particular form for $\theta(L)$ function given in (4.8) and then conduct some empirical validation to see if the proposed form is consistent with the observed behavior of price changes in the system. Our model also produces testable implications regarding the link between mean-reverting component of price changes and other observables of the system, such as trading volume or the volatility of price changes, that will give us additional degrees of freedom in our empirical validation.

To get started, let's consider the following data-generating process for the returns:

$$
\begin{aligned}
r_{i,t+1} &= p_{i,t+1} - p_{i,t} && (4.12a)\\
&= \mu_i + \nu_{i,t+1} + \lambda_{i,t} && (4.12b)
\end{aligned}
$$

where

$$
\begin{aligned}
\nu_{i,t+1} &= \beta_i f_{t+1} + \tilde{\nu}_{i,t+1} && (4.12c)\\
\lambda_{i,t} &= \epsilon_{i,t} - \left(\frac{1-\theta}{\theta}\right)\theta\epsilon_{i,t-1} - \left(\frac{1-\theta}{\theta}\right)\theta^2\epsilon_{i,t-2} - \cdots && (4.12d)
\end{aligned}
$$

According to (4.12b) in this formulation, $r_{i,t+1} = p_{i,t+1} - p_{i,t}$ consists of a mean, μ_i, and two random parts: $\lambda_{i,t}$ captures the prices changes due to customer imbalances while $\nu_{i,t+1}$ captures the prices changes due to exogenous reasons (see also the decomposition for price changes given in (4.7)). It is also assumed that f_t, $\tilde{\nu}_{i,t}$ and $\epsilon_{i,t}$ are zero mean white noise (i.e., uncorrelated both through time and in the cross-section) with variance of σ_f^2, $\sigma_{\tilde{\nu}_i}^2$ and $\sigma_{\epsilon_i}^2$, respectively. Note that sensitivity to f_t, the common factor, implies that $\nu_{i.t}$ are cross-sectionally correlated. Note that the total variance of the individual returns, i.e., the total variance of $\nu_{i.t}$, is partly due to the common factor of f_t. More specifically, the above formulation implies that $\sigma_{\nu_i}^2 = \beta_i^2\sigma_f^2 + \sigma_{\tilde{\nu}_i}^2$.

Comparing (4.12d) with (4.4b), it is clear that in the hypothesis under consideration here we have:

$$
\theta_k = \begin{cases} 1 & k = 0 \\ -\left(\frac{1-\theta}{\theta}\right)\theta^k & k \geq 1 \end{cases} \qquad (4.13)
$$

It is simple to see that

$$
\sum_{k=1}^{\infty} \theta_k = 1
$$

So this formulation satisfies one of the requirements for the customer arrival process, namely the requirement that for each arriving buyer or seller, the offsetting customer will arrive in subsequent time intervals to fully bring the system back to normal. Furthermore, comparing (4.12b) with (4.10) we can see that $\lambda_{i,t}$ is related to the imbalance quantity, q_t, by the following relationship:

$$
\lambda_{i,t} = \alpha\sigma_{\nu_i}^2 q_{i,t} \qquad (4.14)
$$

Note that the subscript i in (4.10) was left out since Proposition 4.1 was only dealing with the case of a single time series. Lastly, $\epsilon_{i,t}$ is related to the imbalance shocks, $\eta_{i,t}$, by the following relationship:

$$\epsilon_{i,t} = \alpha\sigma_{\nu_i}^2 \eta_{i,t} \tag{4.15}$$

In summary, $\lambda_{i,t}$ in (4.12b) captures both the imbalance quantity, $q_{i,t}$, and the sensitivity of the price changes to the imbalance, namely $\alpha\sigma_{\nu_i}^2$. In addition, $\epsilon_{i,t}$ in (4.12d) are the rescaled versions of the imbalance quantities $\eta_{i,t}$ in (4.4b) (note that the subscript i was left out to simplify the notation in the earlier discussion since we were dealing with the case of a single time series). These links will be important when we turn our attention to the empirical model validation in the next section.

4.4 Model Validation Strategy

The hypothesis proposed in (4.12) speculates a particular form for the mean-reverting part of price changes. Since the contrarian trading strategy was successful in Section 3.3.1 in detecting this type of signal out of a large cross-section of price changes, we will base our testing and validation approach on a similar algorithm.

The main difference is that instead of holding the constructed portfolio for one time period, as we did in Section 3.3.1, we will hold it for multiple periods. The pattern of the profit, and also the relationship between the profit and other observables of the system, such as the trading volume or volatility, can then be used to validate if the data-generating process outlined in (4.12d) is appropriate for describing the behavior of this system. We will first describe this algorithm.

Given a set N securities with period t return given by $r_{i,t}$, create a portfolio by investing $w_{i,t}$ dollars in security i where $w_{i,t}$ is given by:

$$w_{i,t} = -\frac{1}{N}(r_{i,t} - r_{m,t}) \quad \text{where} \quad r_{m,t} = \frac{1}{N}\sum_{i=1}^{N} r_{i,t} \tag{4.16}$$

Define the profit, $\pi_t(q)$, as the change in the value of this investment by time $t+q$. Given the investment of $w_{i,t}$, the initial value of the investment is $\sum_{i=1}^{N} w_{i,t}$ which is equal to 0 as discussed in Section 3.3.1. Furthermore, the number of units of asset i that can be purchased is given by $w_{i,t}/p_{i,t}$. Hence, the change in the value of the investment by time $t+q$ is simply:

$$
\begin{aligned}
\pi_t(q) &= \sum_{i=1}^{N} \frac{w_{i,t}}{p_{i,t}} \cdot p_{i,t+q} - \underbrace{\sum_{i=1}^{N} w_{i,t}}_{=0} \\
&= \sum_{i=1}^{N} \frac{w_{i,t}}{p_{i,t}} \cdot p_{i,t+q} \\
&= \sum_{i=1}^{N} \left(w_{i,t} \sum_{l=1}^{q} r_{i,t+l} \right)
\end{aligned} \tag{4.17}
$$

We will first work out the expected profit of the contrarian strategy as a function of the holding period, q, for a general covariance-stationary data-generating process. Proposition 4.3 summarizes that result. The result when applied specifically to the null hypothesis of interest in this section is given in Corollary 4.1. Proofs are give in Appendix A.2.3 and A.2.4, respectively.

Proposition 4.3 *Consider the collection of N securities and denote by $\mathbf{R}_t$ the $N \times 1$ vector of their period t returns, $[r_{1,t} \cdots r_{N,t}]'$. Assume that $\mathbf{R}_t$ is a jointly covariance-stationary stochastic process with expectation $E[\mathbf{R}_t] = \boldsymbol{\mu} = [\mu_1 \cdots \mu_N]'$ and auto-covariance matrices $E[(\mathbf{R}_{t-l} - \boldsymbol{\mu})(\mathbf{R}_t - \boldsymbol{\mu})'] = \boldsymbol{\Gamma}_l = [\gamma_{i,j}(l)]$. Consider a net-zero investment strategy that invests $w_{i,t}$ dollars given by (4.16) in security i. The expected profit, $E[\pi_t(p)]$, where $\pi_t(p)$ is given by (4.17), is given by:*

$$
E[\pi_t(q)] = \mathcal{M}(\boldsymbol{\Gamma}_1) + \cdots + \mathcal{M}(\boldsymbol{\Gamma}_q) - q\, \sigma^2(\boldsymbol{\mu}) \tag{4.18}
$$

where,

$$
\begin{aligned}
\mathcal{M}(A) &\equiv \frac{1}{N^2} \boldsymbol{\iota}' \mathbf{A} \boldsymbol{\iota} - \frac{1}{N} tr(\mathbf{A}) \\
\sigma^2(\boldsymbol{\mu}) &\equiv \frac{1}{N} \sum_{i=1}^{N} (\mu_i - \mu_m)^2 \quad \textit{and} \quad \mu_m = \frac{1}{N} \sum_{i=1}^{N} \mu_i .
\end{aligned} \tag{4.19}
$$

Proof: See Appendix A.2.3.

Corollary 4.1 *Under the data-generating process given in (4.12), the expected profit of the contrarian strategy is given by:*

$$
E[\pi_t(q)] \approx \frac{1 - \theta^q}{2} \left(\frac{1}{N} \sum_{i=1}^{N} \sigma^2_{\lambda_i} \right) \tag{4.20}
$$

Proof: See Appendix A.2.4.

(4.20) implies a concave relationship between the holding period, q, and the magnitude of the expected profit, $E[\pi_t(q)]$. This is one of the implications we will be testing for in the empirical analysis shortly

The contrarian strategy bets on reversion in the part of the price change that is due to imbalance quantity. Intuitively, one would expect to see a larger reversal among stocks that typically have larger imbalance, i.e., stocks that have larger average magnitude of $\lambda_{i,t}$. To see how this related to our testing approach, note that $\sigma^2_{\lambda_i} = \frac{\sigma^2_{\epsilon_i}}{1+\theta}$ (See (A.33) in the Appendix A.2.4). So we would expect a higher profit among securities with higher volatility of $\sigma^2_{\epsilon_i}$. From our earlier discussion, we know that $\epsilon_{i,t} = \alpha\sigma^2_{\nu_i}\eta_{i,t}$. This would point us to believe that the $\mathrm{E}[\pi_t(q)]$ would be higher if we apply the strategy to stocks with higher return volatility, i.e., higher $\sigma^2_{\nu_i}$, or higher imbalance volatility, i.e., stocks with larger swings in the magnitude of $\eta_{i,t}$. We will draw on these two intuitions in the empirical analysis.

In the same spirit, we expect the magnitude of the price reversal to be larger on days with larger imbalance quantities, for example on days that the $\epsilon_{i,t}$ are larger on average. To develop a test for this, we first work out the realized profit of the contrarian strategy in terms of the $\epsilon_{i,t}$. This result is given in Proposition 4.4.

Proposition 4.4 *Under the data-generating process given in (4.12), the realized profit of the contrarian strategy is given by:*

$$\begin{aligned}\pi_t(q) &= (1-\theta^q)\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon^2_{i,t}\right)\\ &-(1-\theta^q)(1-\theta)\theta\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon^2_{i,t-1}\right)-(1-\theta^q)(1-\theta)\theta^3\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon^2_{i,t-2}\right)-\cdots\\ &-\sigma^2(\boldsymbol{\beta})f_t(f_{t+1}+f_{t+2}+\cdots+f_{t+q})\end{aligned} \tag{4.21}$$

The above proposition decomposes the realized profit of the contrarian strategy into three components. Each of these components has a distinct interpretations and it worth looking at them in some detail.

- The first component, $(1-\theta^q)\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon^2_{i,t}\right)$, is undoubtedly positive. It is related to the arrival of offsetting customers for new customers that had arrived at the time the portfolio is established. The contribution is undoubtedly positive because by construction the imbalance is mean-reverting, i.e., $\theta_0 = 1$ in (4.13) and θ_k for all values of $k > 1$ is negative.

- The second group of terms, terms like $(1-\theta^q)(1-\theta)\theta\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon^2_{i,t-1}\right)$, are related to continual arrival of offsetting customers for customers that had arrived in prior time intervals. This is related to the fact that all θ_k for $k > 1$ in (4.13) are negative. Each such term has a negative contribution to the overall $\pi_t(q)$ magnitude.

- The final term, $\sigma^2(\boldsymbol{\beta})f_t(f_{t+1}+f_{t+2}+\cdots+f_{t+q})$, is due to realizations of the common factor. If the realization at the time of construction, f_t, has the same sign as the total of the realization of the subsequent q periods, the contribution is negative. On the other hand, the contribution is positive if these two random variables have opposite signs.

The essential insight from the above decomposition is that the term that contributes positively to the profit is the first term. Therefore, the realized profit is a linear and increasing function of $\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t}^2$ and, therefore, we expect the profit to be higher on days with higher $\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t}^2$. We also expect the profit to be lower on days when the lagged value of this measure, i.e., $\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t-1}^2$, is high since that term contributes negatively to the value of $\pi_t(q)$. We will test both of these in the empirical analysis. The main challenge is that the value of $\epsilon_{i,t}$ are unobservable and we need to create a proxy for their value based on observable quantities. How could we solve this problem?

One possible approach would be to use other observables that are correlated with the $\epsilon_{i,t}$. A candidate to be used to construct a proxy for the value of the $\epsilon_{i,t}$ is the observed volume. But how is volume related to the imbalance quantities? Recall that $\epsilon_{i,t}$ are rescaled versions of the imbalance quantities, $\eta_{i,t}$, where $\epsilon_{i,t} = \alpha\sigma_{\nu_i}^2\eta_{i,t}$. Also recall that $\eta_{i,t} = \eta_{i,t}^b - \eta_{i,t}^s$, where $\eta_{i,t}^b$ and $\eta_{i,t}^s$ are the number of arriving buyers and sellers respectively. But the number of transactions that take place in each time interval is a function of the total number of arriving buyers and sellers, which we denoted by $q_{i,t}^b$ and $q_{i,t}^s$ in our formulation. These quantities are in part due to new customers arriving on that day and partly due to the residual impact from previous days. The relationship we hypothesized was given by (4.3a) and (4.3b), which we repeat here for the $\theta(L)$ function of our null hypothesis:

$$q_{i,t}^b = \eta_{i,t}^b - \left(\frac{1-\theta}{\theta}\right)\theta\eta_{i,t-1}^s - \left(\frac{1-\theta}{\theta}\right)\theta^2\eta_{i,t-2}^s - \cdots \qquad (4.22a)$$

$$q_{i,t}^s = \eta_{i,t}^s - \left(\frac{1-\theta}{\theta}\right)\theta\eta_{i,t-1}^b - \left(\frac{1-\theta}{\theta}\right)\theta^2\eta_{i,t-2}^b - \cdots \qquad (4.22b)$$

In this framework, the observed volume on each day should be equal to the maximum of the above two quantities; for example, if there are 100 buyers, i.e., $q_{i,t}^b = 100$, and 123 sellers, i.e., $q_{i,t}^s = 123$ on a given day t, there will be 123 transactions on day t, 100 of those transactions will be between buyers and sellers and the other 23 will be between sellers (because there are extra sellers in this case) and the dealer. With this admittedly simplified framework in mind, we now measure the correlation between the volume$_{i,t} = \max(q_{i,t}^b, q_{i,t}^s)$, and $\epsilon_{i,t} = \eta_{i,t}^b - \eta_{i,t}^s$. Instead of trying to work out the correlation in closed form in this clearly simplistic setting, we only do a Monte Carlo simulation to ensure that the intuition holds; i.e., to ensure that the $\epsilon_{i,t}$ and volume$_{i,t}$ are correlated. Figure 4.1 shows the result of this Monte Carlo simulation for different values of the parameter θ. It seems there is consistent correlation between the (unobserved) imbalance quantity and the observed daily volume if our model linking the volume to imbalance is indeed correct. We will use this relationship to construct a proxy for the unobserved daily imbalance, i.e., $\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t}^2$, in the next section and use that proxy to test the relationship between the extracted signal strength, $\pi_t(q)$, and the imbalance given by (4.21) in Proposition 4.4.

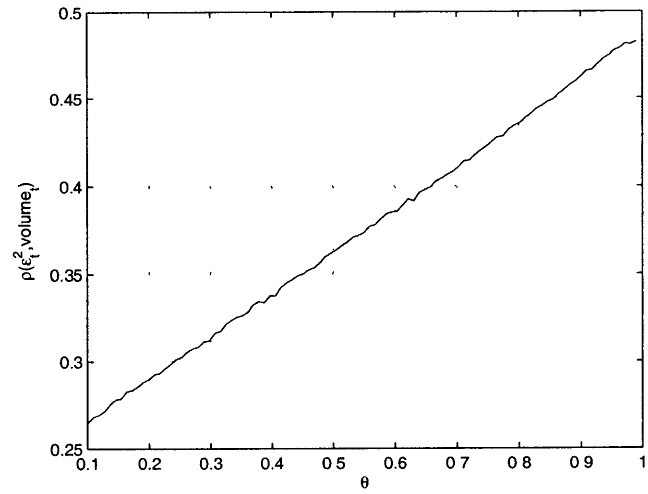

Figure 4.1: This figure shows the correlation between $\epsilon_t = \eta_t^b - \eta_t^s$ and $\text{volume}_t = \max(q_t^b, q_t^s)$ where:

$$
\begin{aligned}
q_t^b &= \eta_t^b - \left(\tfrac{1-\theta}{\theta}\right)\theta\eta_{t-1}^s - \left(\tfrac{1-\theta}{\theta}\right)\theta^2\eta_{t-2}^s - \cdots \\
q_t^s &= \eta_t^s - \left(\tfrac{1-\theta}{\theta}\right)\theta\eta_{t-1}^b - \left(\tfrac{1-\theta}{\theta}\right)\theta^2\eta_{t-2}^b - \cdots
\end{aligned}
$$

for a sample of 10,000 realization of ϵ_t, volume_t for each value of θ generated using realization of the standard normal random variables for η_t^b and η_t^s.

4.5 Empirical Analysis

This section contains the result of the empirical analysis of our model based on US stock returns between 1995 and 2007. Unlike the analysis in Chapter 3, we will now limit the study to using only stocks that are in the S&P 1500 index. We apply the contrarian strategy described by (4.16) to this universe of stocks and then calculate the profit for a 1- to 10-day holding period based on equation (4.17). We then calculate the average profit for each holding period over the entire sample. The result of this analysis is presented in Figure 4.5.

We showed in Corollary (4.1) that under the null of the data-generating process given in (4.12), the expected profit of the contrarian strategy will be of the following form:

$$
\mathrm{E}[\pi_t(q)] \approx \frac{1-\theta^q}{2}\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_{\lambda_i}^2\right) \tag{4.23}
$$

(4.23) implies a concave relationship between the holding period, q, and the magnitude of the expected profit, $\mathrm{E}[\pi_t(q)]$. The results shown in Figure 4.5 agree with this expectation surprisingly well.

We also mentioned before that the contrarian strategy is a tool to detect mean-reverting signal among otherwise white noise price signals. As discussed previously (see also (4.7)), this mean-reverting component in prices is driven by the reversion on the part of the price changes that is due to imbalance quantity. To see how this is related to the strength of the signal detected by our testing strategy, note that the expression for $\mathrm{E}[\pi_t(q)]$ given in (4.23) is an increasing function of the volatility of the imbalance part of the signal, namely $\sigma^2_{\lambda_i}$, also see (4.7) and compare with (4.12). In our formulation $\sigma^2_{\lambda_i} = \frac{\sigma^2_{\epsilon_i}}{1+\theta}$ and $\epsilon_{i,t} = \alpha\sigma^2_{\nu_i}\eta_{i,t}$. These relationships produce two testable hypotheses:

- We would expect the $\sigma^2_{\lambda_i}$ term and hence the value of $\mathrm{E}[\pi_t(q)]$ to be larger among stocks with higher volatility of returns, i.e., higher $\sigma^2_{\nu_i}$.
- We would expect the value of $\mathrm{E}[\pi_t(q)]$ to be larger among stocks with a higher volatility of the imbalance, i.e., higher swing in the magnitude of their $\eta_{i,t}$.

To test the first implication, we simply divide the stocks into two subsets based on their realized return over the twenty days prior to the day on which the portfolios are constructed.[4.4] twenty days is a somewhat arbitrary number and is selected to be short enough to capture changes in the volatility of stocks over time. The result of this test is shown in Figure 4.5. The data clearly supports the link between the magnitude of the reverting component and the volatility of the returns produced by our model.

The Monte Carlo analysis shown in Figure 4.1 points to a potential link between the daily volume and the magnitude of the imbalance. If this is the case, we would expect stocks with higher volatility of daily volume to have higher volatility of $\eta_{i,t}$. Similar to our earlier approach, we now divide the stocks into two subsets based on the volatility of their daily volume as measured by the ratio of the actual number of shares traded divided by the total number of shares outstanding[4.5] over the last 20 days. The result of this application is shown in Figure 4.4. Once again, the link between imbalance volatility and the strength of the mean reverting component is validated in the data.

Figure 4.5 shows a final set of tests where we have applied a two-step filter: first, we divided the universe of stocks into two subsets based on the volatility of their volume and then we further divided each subset into two groups based on the volatility of their realized returns. In all cases a window of 20 was used to measure the volatility of the daily turnovers (see footnote 4.5), and the volatility of the daily returns. The final outcome is the following 4 subsets of stocks based on the volatility of their returns and volatility of their trading volume: "Low Volume and Low Volatility," "Low Volume and High Volatility," "High Volume and Low Volatility," and "High Volume and High Volatility." According to this analysis, it seems that each filter achieves its goal of creating a separation in the strength of the mean-reverting signal. Neither of the filters consumes the other one and the filter based on the volatility of returns seems to create a larger separation in the strength of the mean-reverting signal.

So far the results presented in Figures 4.5, 4.4, and 4.5 have been based on dividing the universe into 2 or 4 subsets based on various measures and evaluating the strength of the

[4.4] [4]We have explicitly assumed that the realized volatility of the returns, $r_{i,t}$, is a good estimator of the volatility of the "exogenous" part of the return or $\nu_{i,t}$ in (4.12). This is clearly an upward biased estimator since the realized volatility is in part due to the imbalance part of the signal, i.e., $\lambda_{i,t}$ in (4.12). But as we argued before, this later component explains a small fraction of the daily volatilities so the bias should be relatively small.

[4.5] This measure called the "Turn Over" is a better measure of the volume as it is normalized for the trivial transformations such as stock splits. Also, see Lo and Wang (2000, 2006).

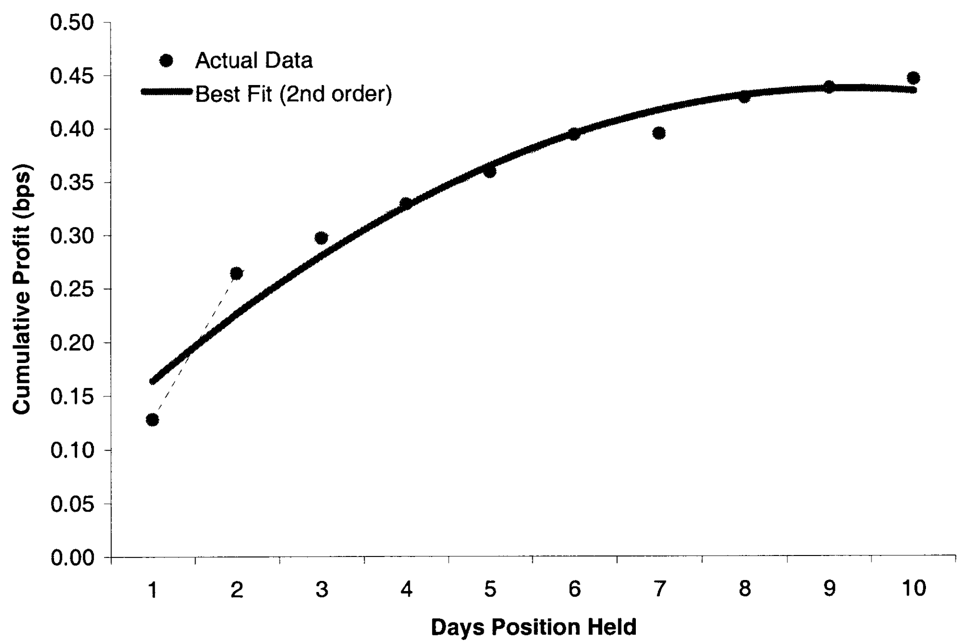

Figure 4.2: This figure shows the link between the strength of the mean-reverting component of price signals and the mean reversion horizon using a trading strategy that invests $w_{i,t}$ dollars in security i on day t where $w_{i,t} = -\frac{1}{N}(r_{i,t} - r_{m,t})$ and $r_{m,t} = \frac{1}{N}\sum_{i=1}^{N} r_{i,t}$ between 1995 and 2007 for 1- to 10-day holding periods. q-day holding period profit, $\pi_t(q)$, is calculated as $\pi_t = \sum_{i=1}^{N} w_{i,t}(r_{i,t+1} + \cdots + r_{i,t+q})$.

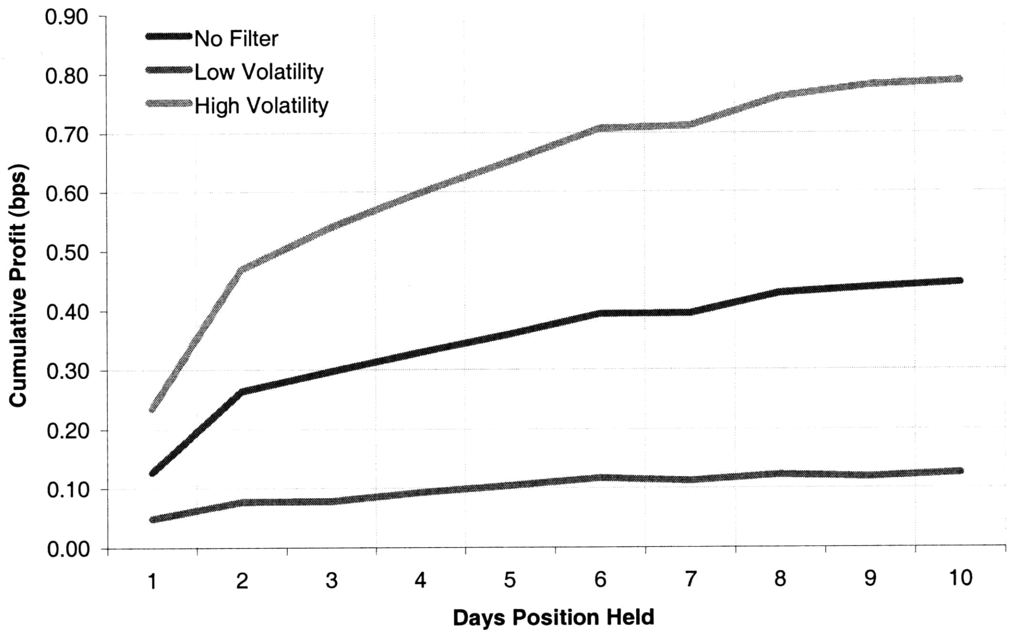

Figure 4.3: This figure shows the link between the strength of the mean-reverting component of price signals and the mean reversion horizon using our contrarian trading strategy for the unconditional case as well as the strength conditioned on past volatility. The unconditional case, labeled "No Filter" in the figure, is the same as the data shown in Figure 4.5. To extract the conditional signal strength, we simply divide the universe of stocks into two halves- "High Volatility" and "Low Volatility"- based on their realized volatility in the proceeding 20 days and then apply the standard contrarian trading strategy to each subset. Also see the caption of Figure 4.5 for a description of the strategy.

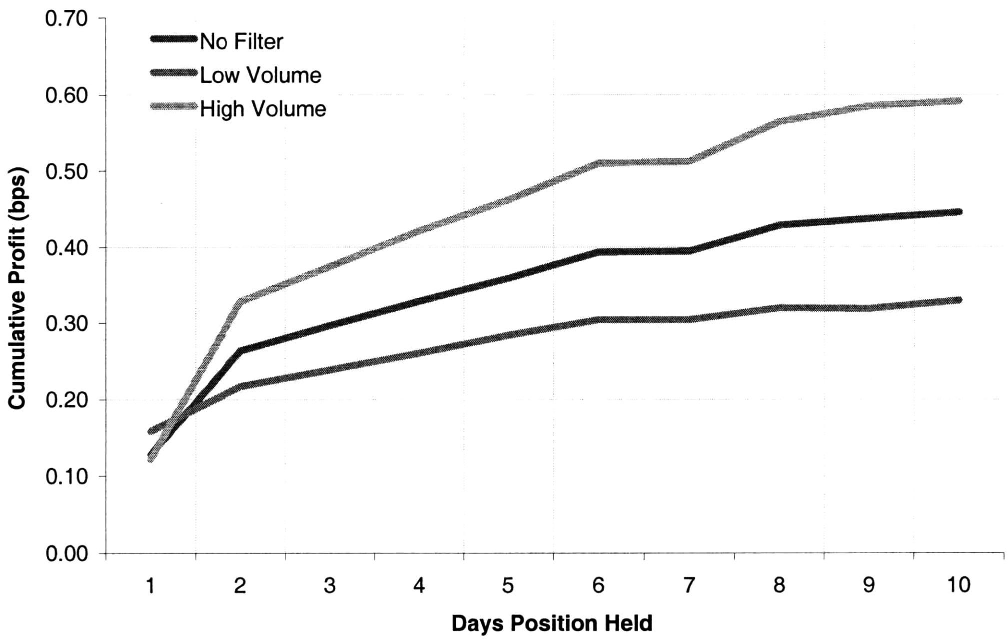

Figure 4.4: This figure shows the link between the strength of the mean-reverting component of price signals and the mean reversion horizon using our contrarian trading strategy for the unconditional case as well as the strength conditioned on past volume. The unconditional case, labeled "No Filter" in the figure, is the same as the data shown in Figure 4.5. To extract the conditional signal strength, we simply divide the universe of stocks into two halves-"High Volume" and "Low Volume"- based on their trading volume in the proceeding 20 days and then apply the standard contrarian trading strategy to each subset.
Also see the caption of Figure 4.5 for a description of the strategy.

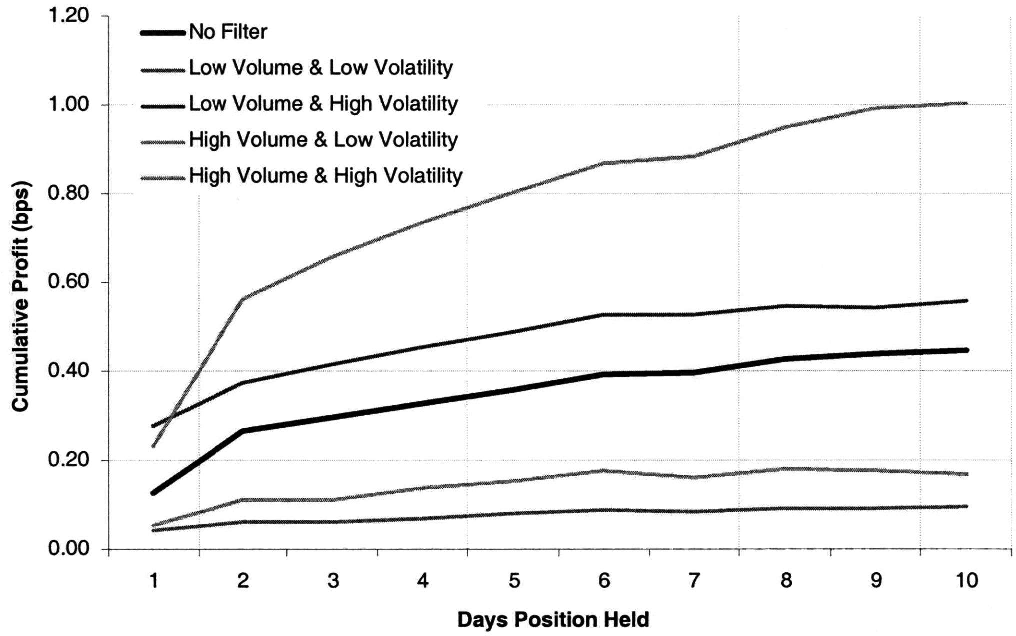

Figure 4.5: This figure shows the link between the strength of the mean-reverting component of price signals and the mean reversion horizon using our contrarian trading strategy for the unconditional case as well as the strength conditioned on past volume and volatility. The unconditional case, labeled "No Filter" in the figure, is the same as the data shown in Figure 4.5. To extract the conditional signal strength, we simply divide the universe of stocks into two halves- "High Volume" and "Low Volume"-based on their trading volume in the preceding 20 days. Each of these subset is divided into two halved based on the realized volatility in the proceeding 20 days. The standard contrarian trading strategy is then applied to each of the resulting four subsets: "Low Volume and Low Volatility," "Low Volume and High Volatility," "High Volume and Low Volatility," and "High Volume and High Volatility." Also see the caption of Figure 4.5 for a description of the strategy.

mean-reverting signal component in each subset. These tests were based on the fact that the mean-reverting part of the signals may be stronger among subsets with higher volatility or higher volume, two hypotheses motivated by our theoretical model.

We have yet another way of testing the validating of our model based on looking at the strength of the mean reverting component on days with high or low value of the imbalance quantity. Intuitively, we expect to see larger mean reversion after days with larger imbalance. Proposition 4.4 makes this link more clear and can be used as the basis to construct a test. We showed in Proposition 4.4 that:

$$\begin{aligned}\pi_t(q) &= (1-\theta^q)\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t}^2\right) \\ &\quad -(1-\theta^q)(1-\theta)\theta\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t-1}^2\right)-(1-\theta^q)(1-\theta)\theta^3\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t-2}^2\right)-\cdots \\ &\quad -\sigma^2(\boldsymbol{\beta})f_t(f_{t+1}+f_{t+2}+\cdots+f_{t+q})\end{aligned} \tag{4.24}$$

As discussed before, the essential insight from the above decomposition is that the realized profit is expected to be higher on days with higher $\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t}^2$. We also expect the profit to be lower on days when the lagged value of this measure, i.e., $\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t-1}^2$ is high.

To test this, we constructed a measure of $\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t}^2$ based on observed volumes. As the Monte Carlo simulation in Figure 4.1 argued, the magnitude of $\epsilon_{i,t}$ and volume$_{i,t}$ have a positive correlation. So one natural proxy for $\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t}^2$ is the following measure:

$$\frac{1}{N}\sum_{i=1}^{N}\text{E-Volume}_{i,t} \tag{4.25a}$$

$$\text{where}\qquad \text{E-Volume}_{i,t} = \text{Volume}_{i,t} - \overline{\text{Volume}}_{i,t} \tag{4.25b}$$

$$\overline{\text{Volume}}_{i,t} = \frac{1}{\tau}\sum_{j=1}^{\tau}\text{Volume}_{i,t-j} \tag{4.25c}$$

The results of the analysis based on the value of this proxy are presented in Table 4.1. We have used linear regression instead of dividing the days into two subsets based on the value of the above measure since the relation given in (4.24) is linear and enforcing this constraint may result in a clearer test. We have conducted a number of different alternative specifications and robustness checks and the result of all alternatives is presented in Table 4.1.

Panel A shows the result when only the current proxy for the value of $\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t}^2$ calculated through (4.25) is included on the right-hand side of the regression. Panel B shows the case that both current and lagged values of this proxy are included on the right-hand side. Specifically, we have calibrated linear models of the following form:

$$\text{Panel A:} \quad \pi_t(q) = \alpha + \lambda_{\text{Current}} \left(\frac{1}{N} \sum_{i=1}^{N} \text{E-Volume}_{i,t} \right) \tag{4.26a}$$

$$\text{Panel B:} \quad \pi_t(q) = \alpha + \lambda_{\text{Current}} \left(\frac{1}{N} \sum_{i=1}^{N} \text{E-Volume}_{i,t} \right) + \lambda_{\text{Lagged}} \left(\frac{1}{N} \sum_{i=1}^{N} \text{E-Volume}_{i,t-1} \right) \tag{4.26b}$$

where $\text{E-Volume}_{i,t}$ is defined by (4.25). As expected, the coefficient for λ_{Current} is positive, indicating a higher strength for the mean-reverting signal on days with larger volume, and hence larger imbalance. The coefficient for the lagged value of this measure, λ_{Lagged}, is negative, at values of q larger than 3, again consistent with the relationship given in (4.24). As a robustness check, we have estimated the same linear models in Panels C and D but included a different constant for each year to capture the non-stationary in the mean documented in Figure 3.1 in the last chapter. The analysis shows that our results are not driven by such non-stationary.[4.6]

While the measure defined in (4.25) is successful in capturing the link between the imbalance quantity and the strength of the mean-reverting signal, a simple improvement outlined next makes the link much stronger. This alternative measure is defined as:

$$\frac{1}{N} \sum_{i=1}^{N} (\text{N-Volume}_{i,t}) \tag{4.27a}$$

$$\text{where} \quad \text{N-Volume}_{i,t} = \frac{\text{Volume}_{i,t} - \overline{\text{Volume}}_{i,t}}{\sigma(\text{Volume}_{i,t})} \tag{4.27b}$$

$$\overline{\text{Volume}}_{i,t} = \frac{1}{\tau} \sum_{j=1}^{\tau} \text{Volume}_{i,t-j} \tag{4.27c}$$

$$\sigma(\text{Volume}_{i,t})^2 = \frac{1}{\tau} \sum_{j=1}^{\tau} (\text{Volume}_{i,t-j} - \overline{\text{Volume}}_{i,t})^2 \tag{4.27d}$$

Intuitively, this measure tries to incorporate the level of noise in the daily volumes. For example, the observation of high volume, $\text{Volume}_{i,t}$, for a stock that has had very volatile volumes, i.e., high $\sigma(\text{Volume}_{i,t})$, is assigned a lower importance than an observation of a high volume for a stock that has smaller volatility of daily volumes.[4.7] The same type of regression analysis based on current and lagged measure of this measure, with and without the time effect, is provided in Table 4.2. The positive link between the current values of this measure

[4.6] We suspected this to be a potential issue since the volume has increased in recent years just as the mean of the mean-reverting signal has declined (see Figure 3.1 for the evidence of the latter claim). So it is plausible that an interaction between these effects would be responsible for the pattern we observed in the effect of volume on the strength of the mean-reverting signal.

[4.7] The relationship to Whitening-Filter and Generalized Least Squares should be clear.

and the strength of the mean-reverting signal is statistically stronger now (compare the t-stat of Table 4.2 with those in Table 4.1). The negative relationship between the lagged value of this measure and the strength of the mean-reverting signal is also stronger and observed at all lags but it is not statistically significant at the conventional levels of significance based on the estimated t-stats.

Table 4.1: This table shows the sensitivity of the realized profit of the contrarian strategy to average volume on the construction day estimated through the following four regression models:

$$\text{Panel A: } \pi_t(q) = \alpha + \lambda_{\text{Current}} \left(\frac{1}{N} \sum_{i=1}^{N} \text{E-Volume}_{i,t} \right)$$
$$\text{Panel B: } \pi_t(q) = \alpha + \lambda_{\text{Current}} \left(\frac{1}{N} \sum_{i=1}^{N} \text{E-Volume}_{i,t} \right) + \lambda_{\text{Lagged}} \left(\frac{1}{N} \sum_{i=1}^{N} \text{E-Volume}_{i,t-1} \right)$$
$$\text{Panel C: } \pi_t(q) = \alpha_{1995} 1_{\{t \in 1995\}} + \cdots + \alpha_{2007} 1_{\{t \in 2007\}} + \lambda_{\text{Current}} \left(\frac{1}{N} \sum_{i=1}^{N} \text{E-Volume}_{i,t} \right)$$
$$\text{Panel D: } \pi_t(q) = \alpha_{1995} 1_{\{t \in 1995\}} + \cdots + \alpha_{2007} 1_{\{t \in 2007\}} + \lambda_{\text{Current}} \left(\frac{1}{N} \sum_{i=1}^{N} \text{E-Volume}_{i,t} \right) + \lambda_{\text{Lagged}} \left(\frac{1}{N} \sum_{i=1}^{N} \text{E-Volume}_{i,t-1} \right)$$

where $\pi_t(q)$ is the profit for the contrarian strategy established on day t and held for q days, $\text{E-Volume}_{i,t} = \text{E-Volume}_{i,t} - \overline{\text{E-Volume}}_{i,t}$ and $\overline{\text{E-Volume}}_{i,t}$ is the average of the daily volumes for security i calculated over the 20 days prior to day t.

	Holding Periods Length in Days									
	1	2	3	4	5	6	7	8	9	10
Panel A: Current Volume Only, No Time Effect										
Current Lambda(t-stat)	0.01(0.54)	0 07(1 81)	0.07(1.46)	0 07(1 38)	0.08(1 62)	0 09(1.63)	0 08(1 58)	0.09(1 61)	0.10(1.60)	0 10(1 59)
Panel B: Current & Lagged Volume, No Time Effect										
Current Lambda(t-stat)	0.01(0 34)	0 07(1 65)	0.07(1.32)	0 07(1 34)	0.10(1 73)	0 09(1.53)	0 09(1.49)	0 10(1 56)	0.11(1 56)	0 11(1 41)
Lagged Lambda(t-stat)	0.00(0.27)	0 00(0 06)	0.01(0 29)	-0.00(-0 09)	-0 02(-0 65)	-0.01(-0.32)	-0 02(-0.45)	-0 02(-0.52)	-0 03(-0 58)	-0 01(-0 25)
Panel C: Current Volume Only with Time Effect										
Current Lambda(t-stat)	0.01(0 58)	0 07(1 84)	0 07(1 49)	0.07(1.41)	0.09(1 65)	0 09(1 66)	0.09(1.62)	0 09(1.65)	0.10(1 64)	0 10(1 63)
Panel D: Current & Lagged Volume with Time Effect										
Current Lambda(t-stat)	0 01(0.35)	0 07(1.67)	0.07(1 33)	0 08(1 35)	0 10(1.74)	0 09(1.55)	0.10(1.51)	0.10(1 58)	0.11(1 58)	0.11(1.43)
Lagged Lambda(t-stat)	0.01(0 33)	0.00(0 13)	0.01(0.36)	-0 00(-0 03)	-0 02(-0 61)	-0 01(-0 28)	-0 02(-0.40)	-0 02(-0.47)	-0.02(-0.53)	-0 01(-0.19)

Table 4.2: This table shows the sensitivity of the realized profit of the contrarian strategy to average volume on the construction day estimated through the following four regression models:

$$\text{Panel A: } \pi_t(q) = \alpha + \lambda_{\text{Current}}\left(\frac{1}{N}\sum_{i=1}^{N}\text{N-Volume}_{i,t}\right)$$
$$\text{Panel B: } \pi_t(q) = \alpha + \lambda_{\text{Current}}\left(\frac{1}{N}\sum_{i=1}^{N}\text{N-Volume}_{i,t}\right) + \lambda_{\text{Lagged}}\left(\frac{1}{N}\sum_{i=1}^{N}\text{N-Volume}_{i,t-1}\right)$$
$$\text{Panel C: } \pi_t(q) = \alpha_{1995}1_{\{t\in 1995\}} + \cdots + \alpha_{2007}1_{\{t\in 2007\}} + \lambda_{\text{Current}}\left(\frac{1}{N}\sum_{i=1}^{N}\text{N-Volume}_{i,t}\right)$$
$$\text{Panel D: } \pi_t(q) = \alpha_{1995}1_{\{t\in 1995\}} + \cdots + \alpha_{2007}1_{\{t\in 2007\}} + \lambda_{\text{Current}}\left(\frac{1}{N}\sum_{i=1}^{N}\text{N-Volume}_{i,t}\right) + \lambda_{\text{Lagged}}\left(\frac{1}{N}\sum_{i=1}^{N}\text{N-Volume}_{i,t-1}\right)$$

where $\pi_t(q)$ is the profit for the contrarian strategy established on day t and held for q days, $\text{N-Volume}_{i,t} = \frac{\text{Volume}_{i,t} - \overline{\text{Volume}}_{i,t}}{\sigma(\text{Volume}_{i,t})}$ and $\overline{\text{Volume}}_{i,t}$ is the average of the daily volumes for security i calculated over the 20 days prior to day t and $\sigma(\text{Volume}_{i,t})$ the standard deviation of daily volume calculated over the same interval.

	Holding Periods Length in Days									
	1	2	3	4	5	6	7	8	9	10
Panel A: Current Volume Only, No Time Effect										
Current Lambda(t-stat)	0 14(1 69)	0.28(2.40)	0 24(1 61)	0 23(1.43)	0.28(1 84)	0 29(1.79)	0.30(1 79)	0 33(1.87)	0.36(1.95)	0.37(1.92)
Panel B: Current & Lagged Volume, No Time Effect										
Current Lambda(t-stat)	0 17(1 78)	0 30(2.40)	0.25(1 63)	0 26(1.54)	0 35(2.11)	0.34(1 92)	0.37(1.94)	0 38(1.93)	0 42(2.03)	0.39(1.80)
Lagged Lambda(t-stat)	-0 07(-1.20)	-0 04(-0.55)	-0 03(-0.36)	-0.07(-0.74)	-0 14(-1 29)	-0.12(-0 97)	-0.15(-1.10)	-0 11(-0.83)	-0.13(-0 92)	-0.05(-0.32)
Panel C: Current Volume Only with Time Effect										
Current Lambda(t-stat)	0 14(1 72)	0.28(2.43)	0.24(1.61)	0.23(1.43)	0 28(1 85)	0 29(1 81)	0.31(1 82)	0.34(1.90)	0.37(1.97)	0.38(1.95)
Panel D: Current & Lagged Volume with Time Effect										
Current Lambda(t-stat)	0.17(1 78)	0 30(2 42)	0 25(1 63)	0 26(1.53)	0 35(2.11)	0 35(1 94)	0.37(1 96)	0.39(1 95)	0.42(2.04)	0.40(1.82)
Lagged Lambda(t-stat)	-0 07(-1 14)	-0 04(-0 52)	-0.03(-0.34)	-0.07(-0 72)	-0 14(-1 28)	-0 12(-0 97)	-0 14(-1.09)	-0.11(-0 81)	-0.13(-0 90)	-0 04(-0 29)

4.6 Chapter Conclusions

In this chapter we dealt in greater depth with the issue of predictability in financial price signals. We proposed a model where predictability in prices was caused by predictability in the arrival of buyers and sellers. The model proposed produced testable hypotheses regarding the link between the strength of the predictable part of price signals and the holding horizon, the volatility of prices and even observed volumes. Our model validation strategy, based on an extension of the test statistic proposed in the last chapter, was able to validate all those hypotheses in the actual data.

The analysis in this chapter highlights the importance of understanding major frictions in financial markets and their implications on how different parts of this complex system interact and the effect of such friction on the eventual outcome of the interaction, such as the observed prices. Such understanding can shed light on the drivers behind the apparent breakdown in the normal course of behavior in August 2007 that was detected by our test in Figure 3.2. The next chapter is dedicated to studying that breakdown in some depth.

Chapter 5

Case Study: System Breakdown in August 2007

This chapter focuses on applying the tools developed in the last two chapters to study a specific example of price dynamics. We will argue that the major market disruption that happened in August of 2007 can be detected using tools and explained by the model discussed so far in this thesis.

We will start in Section 5.1 by giving the reader an overview of events leading to this period. We cite media coverage of this period in order to crystallize the magnitude and the spread of these events. Actual analysis starts in Section 5.2 where we will use our tools to analyze system dynamics during that period. Section 5.3 studies these dynamics in greater depth by bringing in information about which securities would have been of greatest interest to hedge funds that, based on the media reports given in Section 5.1, suffered the most severe losses in this period. It is not surprising that we will see the most dramatic change in the dynamics when we focus on the appropriate subset of securities. Section 5.4 brings in the data from intra-day transaction-by-transaction prices in order to give us more resolution into market dynamics during that time. Using these tools, in particular the intra-day prices, Section 5.5 focuses on detecting the epicenter of the crisis, to the nearest minute and even subset of stocks among which the crisis started. We will finally conclude in Section 5.7.

This chapter draws heavily from our papers on this topic, namely Khandani and Lo (2007, 2008). The interested reader should refer to our papers as this chapter is a short summary of a more detailed analysis presented there.

5.1 Background and Overview

The months leading up to August 2007 were a tumultuous period for global financial markets, with events in the U.S. sub-prime mortgage market casting long shadows over many parts of the financial industry. The blow up of two Bear Stearns credit strategies funds in June, the sale of Sowood Capital Management's portfolio to Citadel after losses exceeding 50% in July, and mounting problems at Countrywide Financial—the nation's largest home lender—throughout the second and third quarter of 2007 set the stage for further turmoil in fixed-income and credit markets during the month of August.

But during the week of August 6, something remarkable occurred. Several prominent hedge funds experienced unprecedented losses that week; however, unlike the Bear Stearns and Sowood funds, these hedge funds were invested primarily in exchange-traded equities, not in sub-prime mortgages or credit-related instruments. In fact, most of the hardest-hit funds were employing long/short equity market-neutral strategies—sometimes called "statistical arbitrage" strategies—which, by construction, did not have significant "beta" exposure and were supposed to be immune to most market gyrations. But the most remarkable aspect of these hedge-fund losses was that they were confined almost exclusively to funds using quantitative strategies. With laser-like precision, model-driven long/short equity funds were hit hard on Tuesday August 7, Wednesday August 8 and Thursday August 9.

In the following days, the financial press surveyed the casualties and reported month-to-date losses ranging from −5% to −30% for some of the most consistently profitable quant funds in the history of the industry. For example, the *Wall Street Journal* reported on August 10, 2007 that:

> After the close of trading, Renaissance Technologies Corp., a hedge-fund company with one of the best records in recent years, told investors that a key fund has lost 8.7% so far in August and is down 7.4% in 2007. Another big fund company, Highbridge Capital Management, told investors its Highbridge Statistical Opportunities Fund was down 18% as of the 8 of the month, and was down 16% for the year. The $1.8 billion publicly traded Highbridge Statistical Market Neutral Fund was down 5.2% for the month as of Wednesday... Tykhe Capital, LLC—a New York-based quantitative, or computer-driven, hedge-fund firm that manages about $1.8 billion—has suffered losses of about 20% in its largest hedge fund so far this month... (see Zuckerman, Hagerty, & Gauthier-Villars, 2007)

By Friday, August 10, the combination of movements in equity prices that caused the losses earlier in the week had reversed themselves, rebounding significantly but not completely. However, faced with mounting losses on August 7, 8, and 9 that exceeded all the standard statistical thresholds for extreme returns, many of the affected funds had cut their risk exposures along the way, which only served to exacerbate their losses while causing them to miss out on a portion of the reversals on the 10. For example, David Viniar, Chief Financial Officer of Goldman Sachs argued that:

> We were seeing things that were 25-standard deviation moves, several days in a row... There have been issues in some of the other quantitative spaces. But nothing like what we saw last week. (Thal Larsen, 2007)

The "perfect financial storm" was over, just as quickly as it descended upon the quants. But the impact was dramatic. For example, on August 14, the *Wall Street Journal* reported that the Goldman Sachs Global Equity Opportunities Fund had "lost more than 30% of its value last week..." (Sender, Kelly, & Zuckerman, 2007). The extraordinary fact that these losses seemed to be concentrated among quantitatively managed equity market-neutral or "statistical arbitrage" hedge funds caused this period to be referred to as the "Quant Meltdown" or "Quant Quake" of 2007. So what happened to the quants in August 2007?

Sudden changes in financial prices are typically accompanied by either changes in the underlying economic reality or sudden changes in the collective desire of market participants to hold a given financial asset. With respect to the first set of drivers, the impact of such broad-based revision in the underlying economic drivers should naturally be reflected in other prices that are representations of the same underlying reality. But, as discussed in Khandani and Lo (2007), the usual indicators such as various equity or bond indexes did not change much during that week. The goal of this study is to develop an understanding of the underlying drivers that caused these massive losses.

But because the relevant hedge-fund managers and investors are not able to disclose their views on what happened in August 2007, we proposed to construct a simple simulacrum of a quantitative equity market-neutral strategy and study its performance, as well as used other publicly available hedge-fund data to round out our understanding of the long/short equity sector during this challenging period. However, we recognize the difficulty for outsiders to truly understand such complex issues, and do not intend to be self-appointed spokesmen for the quants. Accordingly, we acknowledge in advance that we may be far off the mark given the limited data we have to work with, and caution readers to be appropriately skeptical of our analysis, as we are.

We argue that the losses reported in that week were due to a feedback loop started by the initial losses caused by the decision of some managers to sell their holdings. This initial impact, in turn, moved the prices in a way that cause a loss for other managers who held portfolios with similar holdings. In response, those managers decided (forced or otherwise) to reduce their holdings in the face of the prior losses. As with Long Term Capital Management (LTCM) and other fixed-income arbitrage funds in August 1998, the deadly feedback loop of coordinated forced liquidations leading to deterioration of collateral value took hold during the second week of August 2007, ultimately resulting in the collapse of a number of quantitative equity market-neutral funds, and double-digit losses for many others. But in this case, we argue that the feedback loop was made worse due to reduced activity of certain *market makers*. We will refer to this hypothesis about the sequence of "causes and effects" as the *Unwind Hypothesis*.

This hypothesis, if true, underscores the apparent commonality among quantitative equity market-neutral hedge funds, and the importance of liquidity in determining market dynamics. In the following few pages, we give a summary of our analysis. We conclude this chapter by showing that the pattern of greater commonality is not limited only to the "Quant" funds. This suggests that periods of this type of dislocation can happen due to greater commonality among investment strategies and underscores the importance of better understanding of this type of feedback dynamic.

5.2 Initial Inspection

Any change in the dynamic of customer arrival or the ability of the dealers to absorb the rate of arriving customers would be reflected by a corresponding change in the time-series properties of the price signals. For example, equation (4.10) in Proposition 4.1 in Section 4.2 showed that, under a very specific set of assumptions about the objective of the dealers and arrival process of the customers, the predictable part of the price signals essentially has the same structure as the predictable part of the customer arrival process.

We start the inspection by looking at price changes for the signature of price movement that would have been caused by the sudden arrival of a large buy or sell order for a specific group of stocks. We use the same contrarian strategy applied in the last two chapters. Recall that given a set N securities with period t return given by $r_{i,t}$, the contrarian strategy creates a portfolio by investing $w_{i,t}$ dollars in security i where $w_{i,t}$ is given by:

$$w_{i,t} = -\frac{1}{N}(r_{i,t} - r_{m,t}) \quad \text{where} \quad r_{m,t} = \frac{1}{N}\sum_{i=1}^{N} r_{i,t} \tag{5.1}$$

The profit, $\pi_t(q)$, measured as the change in value of this investment by time $t+q$, can be calculated as:

$$\pi_t(q) \;=\; \sum_{i=1}^{N}\left(w_{i,t}\sum_{l=1}^{q} r_{i,t+l}\right) \tag{5.2}$$

Figure 5.1 show the cumulative profit of this strategy when applied to the S&P 1500 Index during 2007.[5.1] The sudden drop and recovery of this strategy during the week of August 6, following several weeks of lower than expected performance, captures much of the dislocation during this period.

To develop some intuition for this dislocation, we need to recall the underlying economic motivation for the contrarian strategy that was first discussed in Section 3.5. By taking long positions in stocks that have declined and short positions in stocks that have advanced over the previous trading day, the strategy actively provides liquidity to the marketplace and acts as a balancing force for the constantly changing supply-demand imbalances. By definition, losers are stocks that have under performed relative to some market average, implying a supply/demand imbalance in the direction of excess supply that has caused the prices of those securities to drop, and vice-versa for the winners. By buying losers and selling winners, the contrarian are adding to the demand for losers and increasing the supply of winners, thereby stabilizing supply/demand imbalances.

By implicitly making a bet on daily mean reversion among a large universe of stocks, the strategy is exposed to any continuation or persistence in the daily returns, i.e., price trends or momentum.[5.2] Broad-based momentum across a group of stocks can arise from a large-scale liquidation of a portfolio that may take several days to complete, depending on the size of the portfolio and the urgency of the liquidation. In short, the contrarian strategy under performs when the usual mean reversion in stock prices is replaced by a momentum,

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[5.1] Components of the S&P 1500 as of January 3, 2007 are used. Strategy holdings are constructed and the daily returns are calculated based on the *Holding Period Return* from the CRSP dataset. See Appendix A.1.1 for a discussion of the CRSP dataset. Components of the S&P 1500 Index is obtained from the *Compustat*; see Appendix A.3.1 for an overview of the Compustat dataset.

[5.2] Note that positive profits for the contrarian strategy may arise from sources other than mean reversion. For example, positive lead-lag relations across stocks can yield contrarian profits (see Lo and MacKinlay, 1990 for details).

possibly due to a sizable and rapid liquidation.

Figure 5.1 also shows the total trading volume during this time. Serving as another observable of the internal dynamics of the system, the trading volume also shows some unusual patterns during this period. The above-normal volume observed in Figure 5.1 gives additional support to the idea that the disruption captured by the contrarian strategy in Figure 5.1 is caused by a sudden run for deleveraging, a process in which investors try to close out their investment positions to turn their capital into cash either to stay out of the market for some time or to obtain capital for other uses.[5.3]

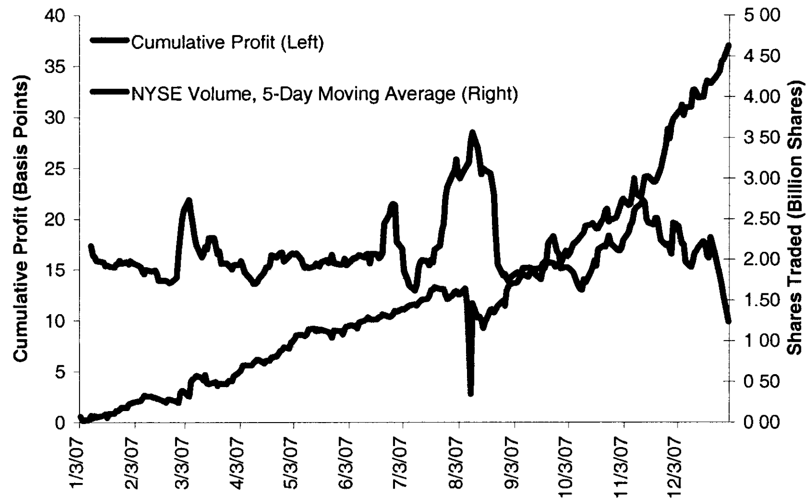


Figure 5.1: The cumulative profit of the 1-day contrarian strategy and the 5-day moving average of the NYSE volume during 2007. For the contrarian strategy, components of the S&P 1500 Index based on the membership as of January 3, 2007, are used.

But if the momentum was indeed caused by a sudden rush to deleverage, one would expect that in spite of immediate momentum in prices, perhaps lasting for a few days during that week, prices would eventually revert back to "normal" levels as the offsetting customers, in the sense discussed in the last chapter, would eventually arrive. To test the validity of this hypothesis, we apply the daily contrarian strategy to prices for the first two weeks of August 2007 and in each case keep the portfolio for 1 to 5 days after the initial construction. The result of this analysis is presented in Table 5.1. Each row of this table shows the cumulative profit from holding a portfolio constructed on the date shown under the "Construction Date" column based on the Contrarian Strategy shown in (5.1) where the profit is calculated using (5.2).

For example, for a portfolio constructed based on the price movement on Monday, August

[5.3]The first day with extremely high volume is June 22, 2007, which was the re-balancing day for all Russell indexes, and a spike in volume was expected on this day because of the amount of assets invested in funds tracking these indexes.

6, the 1-day holding profit is -1.64 bps - about 2 times the 1-day volatility of 0.88 bps for the first 7 months of year 2007. Prices continued to move unfavorably for this portfolio in the next two days, i.e., Tuesday and Wednesday of that week. By by the end of day 4, i.e., Friday August 10, the dynamics changed and previous losses turned into a profit. The change between the cumulative return by the end of day 3, Thursday August 9, and Friday is 1.39+1.96=3.35 bps. This may not seem like a large number but the usual shift between cumulative 3 days and cumulative 4 days return in the first 7 months of that year is 1.56-1.40=0.16 bps (see the last row in Table 5.1). So the 3.35 bps shows a move 20 times larger than the usual magnitude!

What is more interesting is that this pattern repeats itself for a portfolio constructed for each of the following 3 days, i.e., on Tuesday, Wednesday and Thursday of that week. We report some of the numbers reported in Table 5.1 just to give the reader a sense of the magnitude of the move. A portfolio constructed on Wednesday, August 8, had a cumulative profit (loss) of -6.11 bps by the end of Thursday. But the cumulative profit changed sign on Friday, turning into a positive cumulative profit of 5.79 bps. This again represents a jump of 5.79+6.11=11.9 bps or 33 times the usual movement of 1.24-0.88=0.36 bps (see the last row in Table 5.1) between day 1 and day 2 for the earlier times of that year. Note that, for portfolios constructed based on the price movements on each day of that week a loss was experienced before turning to a profit starting on August 10.

Table 5.1: Performance of the contrarian strategy applied to daily returns from August 1 to 15, 2007. Each entry shows the performance of the contrarian strategy based on daily returns, with positions established based on stock returns on the "Construction Date", and positions are held for 1 to 5 days afterward. Notice that the profit of the portfolio established on August 6 through 9 was negative before Friday; i.e., prices diverged more before finally starting to revert on Friday, August 10.

Construction Date	Total Holding Period Profit (bps)				
	1 Day	2 Days	3 Days	4 Days	5 Days
8/1/2007	0.12	-0.87	-2.28	-2.18	-0.29
8/2/2007	-0.53	-1.21	-1.95	-2.01	-2.15
8/3/2007	-0.27	-0.51	0.58	0.26	1.83
8/6/2007	-1.64	-2.00	-1.96	1.39	3.84
8/7/2007	-3.14	-4.90	1.50	4.36	4.93
8/8/2007	-6.11	5.79	11.94	12.71	12.54
8/9/2007	9.74	14.38	13.97	13.46	12.74
8/10/2007	-2.84	-3.34	-2.89	-3.59	-2.91
8/13/2007	0.22	0.96	4.45	5.41	4.42
8/14/2007	-0.70	-0.61	0.16	0.25	0.41
8/15/2007	-1.09	-0.47	0.25	0.62	1.37
2007 St. Dev	0.88	1.24	1.40	1.56	1.58

It seems that supply/demand imbalances returned to more normal levels as the daily contrarian strategies started to recover on August 10 (see Table 5.1), presumably as new capital flowed into the market to take advantage of opportunities created by the previous days' dislocation. The total amount of the strategy profit in five days is an indirect measure of the imbalance on the construction day.[5.4] As seen in Table 5.1, the imbalance was largest

[5.4]This can be seen based on the result in Proposition 4.4 in Section 4.4. As discussed in that section, the on August 8 and 9. This is consistent with the pattern observed in other measures presented in this chapter and also with the media reports of these events that we cited earlier.

Based on the reports that losses were most severe among quantitative hedge funds, one would expect to obtain a better view of the inner workings of this period by looking at features that make a security of particular interest to the "quant" funds. We will turn to this issue next.

5.3 Decomposing System's Behavior Based on the Quant Factors

As mentioned in the introduction to this chapter, much of the loss in those few volatile days in August 2007 was concentrated among the quantitatively management equity portfolios. The main construct of any quantitatively managed investment process is a forecasting framework for the future prices of financial assets. Given such a forecast, a manager can position himself to benefit by investing in assets that are undervalued based on the future forecasted price and short-sell assets that are over valued according to the forecast. In the most simplistic approach, securities can be ranked based on a combination of their characteristics that are known to be, or historically have been, related to the future returns. For example, the ratio of the price appreciation in the last month over the price appreciation over the proceeding 11 months is an example of a basic form of the *Price Momentum* metric. In order to better trace out the dynamics of the events in those days, we will use five specific quantitative valuation metrics to analyze price dynamics and trading volumes during 2007. The five metrics are as follows: *Price Momentum*, *Earnings Momentum*, *Book-to-Market*, *Earnings-to-Price*, and *Cash flow-to-Market*.[5.5]

In each month of the sample, each of these five quantitative valuation factors produces a numerical measure for each stock. These numerical measures can then be turned into decile rankings, with stocks in decile 10 being the ones that are relatively under-priced and hence expected to appreciate in price, and those in decile 1 being overpriced and expected to depreciate. Appendix A.3.2 gives the detail of the procedure for calculating these metrics.

To establish a potential link between the events in July and August 2007 and these "quant" factors, we perform two cross-sectional regressions each day from January to November 2007[5.6] using daily stock returns and turnover as the dependent variables:

first term of the profit is proportional to the amount of imbalance on the construction date.

[5.5] *Book-to-Market*, *Earnings-to-Price*, and *Cash flow-to-Market* are examples of the *Value* metric and are used extensively in the literature, see for example Lakonishok, Shleifer, and Vishny (1994). *Price Momentum* and *Earnings Momentum* are examples of the *Momentum* metric and have been studied extensively in connection with momentum strategies; see for example Chan, Jegadeesh, and Lakonishok (1996).

[5.6] At the time we obtained the Compustat data for this analysis, the Compustat database was still not fully populated with the 2007 quarterly data; in particular, the data for the quarter ending September 2007 (2007Q3) was very sparse. Given the 45-day lag we employ for quarterly data, the lack of data for 2007Q3 means that the deciles can be formed for only about 370 companies at the end of November 2007 (the comparable count was 1,381 in October 2007 and 1,405 at the end of September 2007). Since any analysis of factor models for December 2007 is impacted by this issue, we will limit our study to the first 11 months of 2007. See Appendix A.3.2 for more detail.

$$r_{i,t} = \alpha_t + \sum_{f=1}^{5} \beta_{f,t} D_{i,f} + \epsilon_{i,t} \quad (5.3a)$$

$$\mathrm{TO}_{i,t} = \gamma_t + \sum_{f=1}^{5} \delta_{f,t} |D_{i,f} - 5.5| + \eta_{i,t} \quad (5.3b)$$

where $r_{i,t}$ is the return for security i on day t, $D_{i,f}$ is the decile ranking of security i according to factor f,[5.7] and $\mathrm{TO}_{i,t}$, the turnover for security i on day t, is defined as:[5.8]

$$\mathrm{TO}_{i,t} \equiv \frac{\text{Number of Shares Traded for Security } i \text{ on Day } t}{\text{Number of Shares Outstanding for Security } i \text{ on Day } t} . \quad (5.4)$$

If, as we hypothesize, there was a significant unwinding of factor-based portfolios in July and August 2007, the explanatory power of these two cross-sectional regressions should spike up during those months because of the overwhelming price-impact and concentrated volume of the unwind. If, on the other hand, it was business as usual, then the factors should not have any more explanatory power during that period than any other period.

Figure 5.2 displays the R^2's for the regressions (5.3) each day during the sample period. To smooth the sampling variation of these R^2's, we also have displayed the 5-day moving average of these R^2's. These plots confirms the unusual trading volume documented in Figure 5.1 were more heavily concentrated among stocks that were over- or under-valued according to these quantitative valuation factors. Starting in late July, the turnover regression's R^2 increased significantly, exceeding 10% in early August. Moreover, the turnover regression's R^2 continued to exceed 5% for the last three months of our sample, a threshold that was not passed at any point prior to July 2007. As expected, the daily return regressions typically have lower R^2's, but at the same point in August 2007, the explanatory power of this regression also spiked above 10%, adding further support to the Unwind Hypothesis.

To obtain a more precise view of the trading volume during this period, we turn to the cross-sectional regression (5.3) of individual turnover data on exposures to decile rankings of the five factors of Appendix A.3.2. Figure 5.3 displays the estimated turnover impact $\hat{\delta}_{f,t}$ and R^2 of the daily cross-sectional regressions, which clearly shows the change in the trading activity and R^2 among stocks with extreme exposure to these five factors. The estimated impact is measured in basis points of turnover for a unit of difference in the decile ranking; for example, an estimated coefficient of 25 basis points for a given factor implies that ceteris paribus, stocks in the 10 decile of that factor had a 1% (4 × 25 bps) higher turnover than

[5.7]Note that the decile rankings change each month and they are time dependent, but we have suppressed the time subscript for notational simplicity.

[5.8]Turnover is the appropriate measure for trading activity in each security because it normalizes the number of shares traded by the number of shares outstanding (see Lo & Wang, 2000, 2006). The values of the decile rankings are reflected around the "neutral" level for the turnover regressions because stocks that belong to either of the extreme deciles—deciles 1 and 10—are "equally attractive" according to each of the five factors (but in opposite directions), and should exhibit "abnormal" trading during those days on which portfolios based on such factors were unwound.

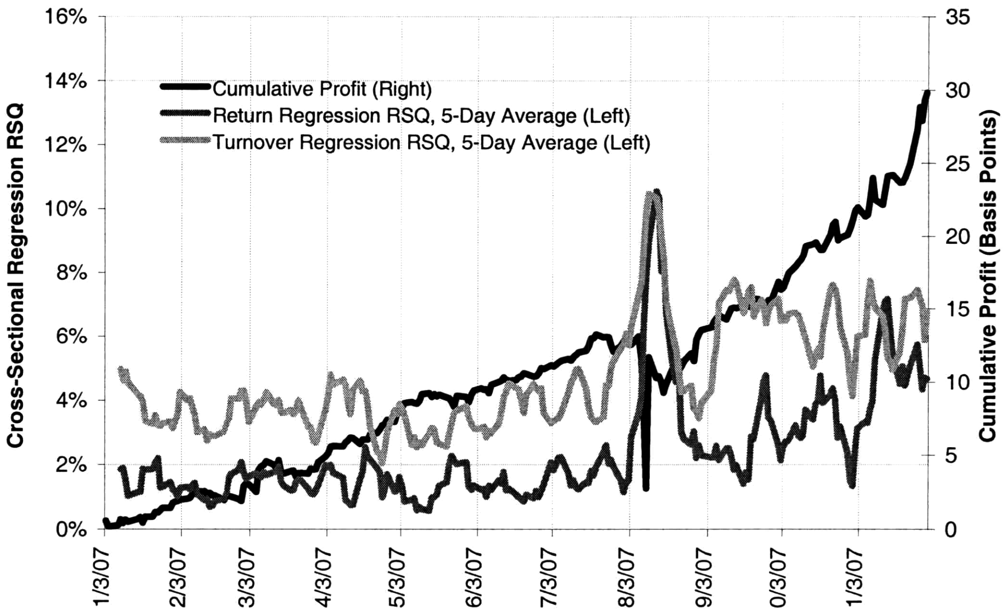


Figure 5.2: This Figure shows the 5-day moving average of the R^2's of the following cross-sectional regressions of returns and turnover: $r_{i,t} = \alpha_t + \sum_{f=1}^{5} \beta_{f,t} D_{i,f} + \epsilon_{i,t}$, and $\mathrm{TO}_{i,t} = \gamma_t + \sum_{f=1}^{5} \delta_{f,t} |D_{i,f} - 5.5| + \eta_{i,t}$, where $r_{i,t}$ is the return for security i on day t, $D_{i,f}$ is the decile ranking of security i according to factor f, and $\mathrm{TO}_{i,t}$, the turnover for security i on day t, is defined as the ratio of the number of shares traded to the shares outstanding. Five factors are as follows: *Price Momentum, Earnings Momentum, Book-to-Market, Earnings-to-Price, and Cash flow-to-Market.* See Appendix A.3.2 for more information on these factors.

stocks in the 6 decile. The estimated coefficients are always positive, implying that the securities ranked as "attractive" or "unattractive" according to each of these measures, i.e., Deciles 10 and 1, respectively, tend to have a higher turnover than the securities that are ranked "neutral" (Deciles 5 or 6).

The coefficients for most factors, particularly the Price Momentum factor, exhibited an increase during this period. Furthermore, the the R^2 of the cross-sectional regressions was substantially higher on those days in August. While these observations are all consistent with the Unwind Hypothesis, the explanatory power of these regressions and the estimated impact of the factors (other than Price Momentum) on August 8 and 9 were not markedly different than earlier in the same week. So what changed on August 8, 9, and 10 that yielded the volatility spike and change in the dynamics of the contrarian strategy returns and also in the reported losses among the most sophisticated quant hedge funds during this time?

We argue in the next section that a sudden withdrawal of liquidity by high-frequency dealers may be one explanation.

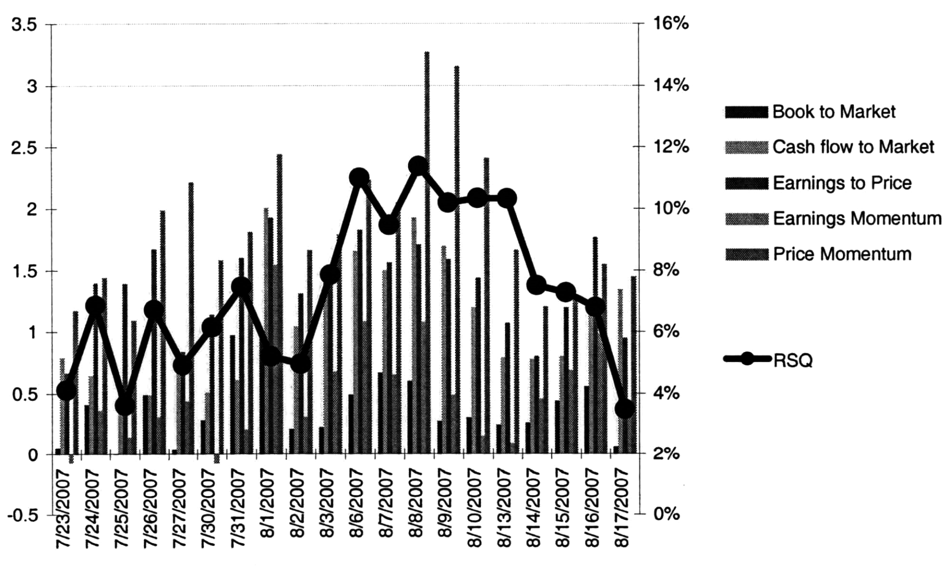


Figure 5.3: Estimated coefficients $\hat{\delta}_{f,t}$ and R^2 of the cross-sectional regression of daily individual-stock turnover on absolute excess decile rankings for five valuation factors from July 23, 2007 to August 17, 2007: $\text{TO}_{i,t} = \hat{\gamma}_t + \sum_{f=1}^{5} |D_{i,f} - 5.5|\hat{\delta}_{f,t} + \hat{\epsilon}_{i,t}$, where $\text{TO}_{i,t}$ is the turnover for stock i on day t and $D_{i,f}$ is the decile assignment for stock i based on factor f, where the five factors are: *Book-to-Market, Cash flow-to-Market, Earnings-to-Price, Price Momentum, and Earnings Momentum.*

5.4 High-Frequency Analysis

We now turn our attention to intra-day price dynamics by applying a similar strategy to transactions data from July to September 2007 for stocks in the S&P 1500 universe. We will use this data for two distinct purposes: to construct portfolios similar to the portfolios that were held by typical quant funds, and to more precisely study the changes in the price dynamics in the first half of August 2007.

Figure 5.4 displays the cumulative returns of long/short market-neutral portfolios based on the factors of Section 5.3 for the two weeks before and after August 6 (from 9:30 am on July 23 to 4:00 pm on August 17). Each portfolio consists of investing $1 long in the stocks in the 10 decile and investing $1 short in the stocks in the 1st decile of that month according to each of the five quant factors. Each $1 investment is distributed using equal weights among stocks in the respective decile. We then compute the value of long/short portfolios using the most recent transactions price in each 5-minute interval.[5.9] Portfolio are rebalanced on the first day of August.

The patterns observed in Figure 5.4 suggest that on August 2 and 3, long/short portfolios based on Book-to-Market, Cash flow-to-Market, and Earnings-to-Price were being unwound, while portfolios based on Price Momentum and Earnings Momentum were unaffected until August 8 and 9 when they also experienced sharp losses. But on Friday, August 10, sharp reversals in all five strategies erased nearly all of the losses of the previous four days, returning portfolio values back to their levels on the morning of August 6.

We now turn to applying the contrarian strategy to intra-day prices in order to study the changes in the price dynamics during this time. For computational simplicity, we use a simpler mean-reversion strategy than the contrarian strategy given in (5.1). This high-frequency mean-reversion strategy is based on buying losers and selling winners over lagged m-minute returns, where we vary m from 5 to 60 minutes. Specifically, each trading day is broken into non-overlapping m-minute intervals, and during each m-minute interval we construct a long/short dollar-neutral portfolio that is long those stocks in the lowest return-decile over the previous m-minute interval, and short those stocks in the highest return-decile over the previous m-minute interval.[5.10] The value of the portfolio is then calculated for the next m-minute holding period, and this procedure is repeated for each of the non-overlapping m-minute intervals during the day.[5.11]

Figure 5.5 plots the cumulative profit of this mean-reversion strategy from July 2 to September 28, 2007 for various values of m, and a clear pattern emerges. For $m = 60$ minutes, the cumulative return is fairly flat over the three-month period, but as the horizon shortens, the slope increases, implying increasingly larger expected returns. This reflects the fact

[5.9] We use the **Cumulative Factor to Adjust Price** (CFACPR) from the CRSP daily files to adjust for stock splits, but do not adjust for dividend payments. This may cause a small approximation error in the reported returns.

[5.10] Stocks are equal-weighted. In case there is a tie between returns for several securities that cross the decile threshold, we ignore all securities with equal returns to focus on the supply-demand imbalance, and also to enhance the reproducibility of our numerical results. No overnight positions are allowed.

[5.11] We always use the last traded price in each m-minute interval to calculate returns; hence, the first set of prices for each day are the prices based on trades just before 9:30 am plus m minutes, and the first set of positions are established at 9:30 am plus $2m$ minutes.

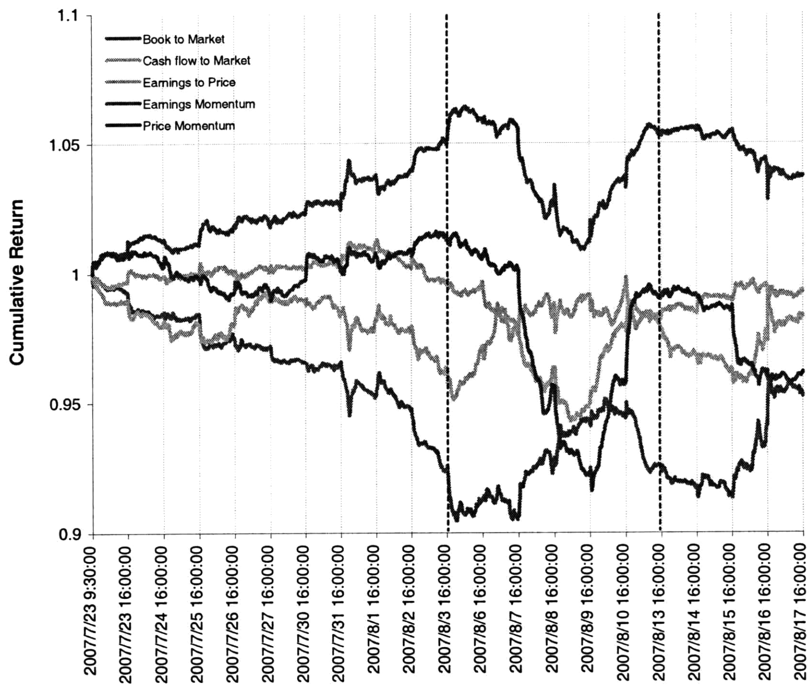


Figure 5.4: Cumulative returns for long/short portfolios based on five equity-valuation factors from 9:30 am July 23, 2007 to 4:00 pm August 17, 2007 computed from 5-minute returns using TAQ transactions data. Portfolios were rebalanced at the end of July 2007 to reflect the new factors rankings.

that shorter-horizon mean reversion strategies are closer approximations to marketmaking, with correspondingly more consistent profits. However, for all values of m, we observe the same dip in profits during the second week of August. Consider, in particular, the case where $m=5$ minutes—on August 6, the cumulative profit levels off, and then declines from August 7th through August 13th, after which it resumes its growth path at nearly the same rate. This inflection period suggests that marketmakers may have reduced their risk capital from August 7–13, returning to the market once the Quant Quake had passed. The logic is straightforward: the existence of marketmakers typically provides a counterbalancing force to attenuate such correlated liquidation-driven momentum among a large group of stocks. Therefore, a sudden withdrawal of marketmaking capital in the face of mounting price pressure would yield exactly the kind of price patterns observed over the week of August 6, 2007.

Although NYSE/AMEX specialists and NASDAQ dealers have an affirmative obligation to maintain orderly markets and stand ready to deal with the public, in recent years, a number of hedge funds and proprietary trading desks have become de facto marketmakers by engaging in high-frequency program-trading strategies that exploit mean reversion in intra-daily stock prices.[5.12] But in contrast to exchange-designated marketmakers, such traders are under no obligation to make markets, and can cease trading without notice. We conjecture that these traders may have left the market during the second week of August, either because of losses sustained during the start of the week, or because they were forced to reduce their exposures due to unrelated losses in credit portfolios and other investments within their organizations.

To explore the impact of varying holding periods on market making profits, we use lagged 5-minute returns to establish the positions of the mean-reversion strategy, and hold those positions for m minutes where m varies from 5 to 60 minutes, after which new positions are established based on the most recent lagged 5-minute returns. This procedure is applied for each day and average returns are computed for each week and each holding period. Figure 5.6 shows the average return for different holding periods for each week of the sample. With the notable exception of the week of August 6, the average return is generally increasing in the length of holding period, consistent with the patterns in the daily strategy in Section 4.5 and Figure 4.5.

To interpret the observed patterns in Figure 5.6, recall that this mean-reversion strategy provides immediacy by buying losers and selling winners every 5 minutes. As quantitative factor portfolios were being deleveraged and unwound during the last two weeks of July 2007, the price for immediacy presumably increased, implying higher profits for marketmaking strategies such as ours. This is confirmed in Figure 5.6. However, during the week of August 6, 2007, the average return to our simplified mean-reversion strategy turned sharply negative, with larger losses for longer holding periods that week. In particular, the pattern of losses in Figure 5.6 supports the Unwind Hypothesis in which sustained liquidation pressure for a sufficiently large subset of securities created enough price pressure to overcome

[5.12] The advent of decimalization in 2001 was a significant factor in the growth of marketmaking strategies by hedge funds because of the ability of such funds to "step in front of" designated marketmakers (achieve price priority in posted bids and offers) at much lower cost after decimalization, i.e., a penny versus 12.5 cents.

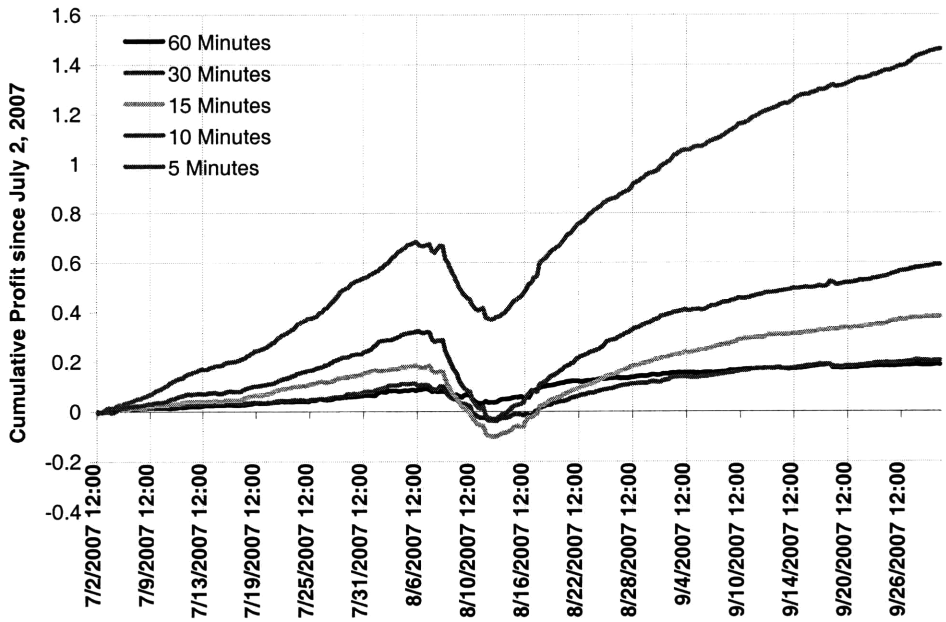


Figure 5.5: Cumulative profit of m-minute mean-reversion strategy applied to stocks in the S&P 1500 universe from July 2, 2007 to September 28, 2007 for $m = 5$, 10, 15, 30, and 60 minutes. No overnight positions are allowed, initial positions are established at 9:30 am plus m minutes each day and all positions are closed at 4:00 pm. Components of the S&P 1500 are based on memberships as of the last day of the previous month.

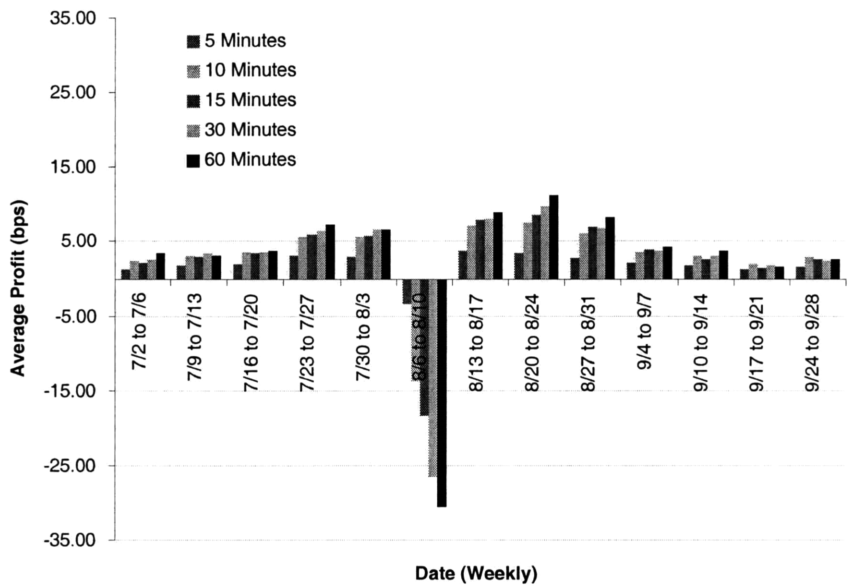


Figure 5.6: The average profit for the contrarian strategy applied to 5-minute returns from July 2, 2007 to September 28, 2007. Each day is divided into non-overlapping 5-minute intervals, and positions are established based on lagged 5-minute returns and then held for 5, 10, 15, 30, or 60 minutes. The average return for each holding period is calculated for each week during this sample. No overnight positions are allowed, initial positions are established at 9:40am each day and all positions are closed at 4:00 pm. Components of the S&P 1500 are based on memberships as of the last day of the previous month.

the profitability of our short-term mean-reversion strategy, resulting in negative returns for holding periods from 5 minutes to 60 minutes.

5.5 Detecting the Epicenter

The approach used in the last Section to detect the apparent *regime shift* during the week of August 6 2007 can be used to more precisely detect the exact time and even the group of stocks involved in this processes. In particular, we apply the contrarian strategy to the following subsets of stocks: three market-cap based subsets (Small-,Mid-, and Large-Cap subsets representing the bottom 30%, middle 40% and top 30% of stocks by market capitalization), five factor-based subsets (each subset consists of stocks in either decile 1 or decile 10 of each of the five quantitative factors of Section 5.3), and six industry based subsets, based on the twelve-industry classification codes available from Kenneth French's website.[5.13] To each of these subsets, we apply the simpler version of the contrarian strategy described in Section 5.4.

Table 5.2 and Figure 5.7 contain the returns of these portfolios from July 23 to August 17, the two weeks before and after August 6. Each entry in Table 5.2 is the average return of the 5-minute contrarian strategy applied to a particular subset of securities over the specified day. As discussed in Section 5.4, days with negative average returns in Table 5.2 correspond to those days when the trading activity started to exhibit momentum over the relevant holding period used for the strategy. This interpretation allows us to visually detect the intra-day emergence of price pressure and determine when the liquidation began and in which factor-based portfolios it was concentrated. Based on these results, we have developed the following set of hypotheses regarding the epicenter of the Quant Quake of August 2007:

1. The first wave of deleveraging began as early as August 1 around 10:45am, with the activity apparently concentrated among factor-based subsets of stocks. One can visually detect the sudden abnormal behavior of the long/short portfolios at the exact same time in Figure 5.4. Portfolios based on Book-to-Market and Earnings-to-Price dropped in value while portfolios based on the other three factors, Cash flow-to-Market, Earnings Momentum and Price Momentum, gained a little, suggesting that portfolios being deleveraged or unwound during this time were probably long Book-to-Market and Earnings-to-Price factors and short the other three. This wave of activity was short-lived and by approximately 11:30 am that day, markets returned to normal. By the end of the day, the contrarian strategy applied to all subsets except for Earnings Momentum and Book-to-Market yielded positive average returns for the day (see Table 5.2), implying that the liquidation may have been more heavily concentrated on portfolios formed according to these two factors.

[5.13] Industry sata was obtained from the data library section of Kenneth French's web site:

`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/`

Please refer to the documentation available from that site for further details. Note that industries with fewer than 100 stocks are included in the *Other Industries* subset.

2. The second wave started on August 6 at the market open, and lasted until approximately 1:00pm. Once again, the action was concentrated among factor-based subsets. This time, the price pressure due to the hypothesized forced liquidation was strong enough to overcome the idiosyncratic liquidity shocks and, as such, the contrarian strategy applied to all factor-based subsets of stocks yielded negative returns for the entire day. Earnings-Momentum and Book-to-Market portfolios within the financial sector suffered the largest losses, implying that the deleveraging was strongest among these groups of stocks. The patterns in Figure 5.4 suggest that the portfolios being deleveraged were probably long Book-to-Market, Price Momentum, and Cash flow-to-Market, and short Earnings Momentum and Earnings-to-Price. Appendix A.3.4 contains a more detailed analysis of the specific stocks that were affected. August 6 was remarkable in another respect—for the first time in our sample, the contrarian strategy applied to all stocks also registered a loss for the day (see Table 5.2), implying widespread and strong price pressure due to a forced liquidation on this day.

3. On August 7th, portfolios based on Price Momentum and Cash flow-to-Market continued to drift downward as Figure 5.4 shows, suggesting continued deleveraging among portfolios based on these two factors. Furthermore, the contrarian strategy applied to all stocks yielded a second day of negative returns, suggesting that the forced liquidation carried over to this day.

4. August 8 was the start of the so-called "Meltdown". On this day, the contrarian strategy suffered losses when applied to *any* subset of stocks (factor-based, industry, and market-cap). The sudden drop and subsequent reversal is clearly visible in Figure 5.4.

5. Starting on Friday August 10, the long/short factor-based portfolios sharply reversed their losing trend, and by the closing bell on Monday August 13th, all five long/short portfolios were within 2% of their values on the morning of August 8. We conjecture that this reversal was due to two possible causes: new capital that came into the market to take advantage of buying and selling opportunities created by the price impact of the previous days' deleveraging, and the absence of further deleveraging pressure because the unwind that caused the initial losses was completed.

6. In addition to the hypothesized forced liquidation, we conjecture that part of the losses from August 8 and 9 also stemmed from a reduction in liquidity, most likely from certain hedge funds engaged in high-frequency marketmaking activities. Unlike NYSE specialists and other designated marketmakers that are required to provide liquidity, even in the face of strong price trends, hedge funds have no such obligation. However, in recent years, such funds have injected considerable liquidity into U.S. equity markets by their high-frequency program-trading activities. The reason we believe that reduced marketmaking activity is partly responsible for the August 2007 Meltdown is as follows. As seen in Figure 5.3, the R^2's of the turnover regressions based on (5.3b) were also elevated during the week of August 6, with values of 10% or higher which were the highest R^2's for this regression during all of 2007. Also, the R^2 of the return-based regression (5.3a) was 13.9% on August 8, once again a

record-setting level for all of 2007. But these R^2's and impact estimates were—with the exception of the Price-Momentum factor—elevated throughout the week of August 6, hence they cannot explain the widespread losses that occurred on August 8 and 9. We suspect that at least part of the meltdown that began on August 8 was due to a specific reduction in marketmakers' capital, most likely by hedge funds engaged in high-frequency mean-reversion trading strategies.

7. We conjecture that the motivation for the reduction in marketmaking capital is the negative average returns for the all-stocks contrarian strategy during August 6 and 7th (see Table 5.2), which revealed a much larger pending unwind than marketmakers could handle, and those marketmakers who had the option of reducing their exposure, e.g., hedge funds, did so on August 8.

In Appendix A.3.4, we show how the contrarian strategy can be used to identify with even greater precision the specific stocks and sectors that were involved at the start of the Quant Meltdown of August 2007.

Table 5.2: The average returns for the contrarian strategy applied to various subsets of the S&P 1500 index using 5-minute returns for July 23, 2007 to August 17, 2007. Each day is divided into non-overlapping 5-minute intervals, and positions are established based on lagged 5-minute returns and held for the subsequent 5-minute interval. The average return for each subset of stocks over each day of this period is then calculated. No overnight positions are allowed, initial positions are established at 9:40am each day and all positions are closed at 4:00 pm. All entries are in units of basis points.

	By Size			By Factor				
Date	Small-Cap (Bottom 30%)	Mid-Cap (Middle 40%)	Large-Cap (Top 30%)	Book to Market (Decile 1&10)	Cash flow to Market (Decile 1&10)	Earnings to Price (Decile 1&10)	Price Momentum (Decile 1&10)	Earnings Momentum (Decile 1&10)
2007/7/23	6.19	3.45	1.94	5.43	5.97	5.89	6.38	3.09
2007/7/24	8.98	3.61	1.08	5.93	6.30	6.38	5.39	3.71
2007/7/25	7.98	1.51	0.63	5.98	5.19	7.22	6.53	1.38
2007/7/26	13.56	4.20	2.78	7.42	10.62	8.85	9.13	8.43
2007/7/27	9.63	4.83	2.04	6.81	10.49	8.52	7.58	4.72
2007/7/30	7.72	2.99	2.40	4.94	6.33	6.16	6.61	4.53
2007/7/31	7.40	2.52	0.53	2.77	3.71	1.43	2.74	3.19
2007/8/1	7.63	3.94	3.72	-0.83	0.56	0.44	0.42	-2.27
2007/8/2	9.28	7.69	1.64	4.80	6.63	7.34	7.84	6.30
2007/8/3	7.01	2.53	2.99	4.45	4.64	5.44	5.71	2.72
2007/8/6	-1.55	-1.02	-1.21	-6.52	-3.32	-3.33	-5.41	-8.11
2007/8/7	-1.02	-2.24	1.20	-0.68	0.10	-1.34	-1.04	-3.50
2007/8/8	-26.30	-18.07	-5.54	-16.16	-15.21	-18.81	-23.27	-20.08
2007/8/9	-7.93	-14.97	-2.57	-5.36	-7.93	-5.72	-9.31	-11.08
2007/8/10	-3.02	-8.89	2.54	-1.82	-0.25	2.02	-3.87	-1.58
2007/8/13	-8.10	-3.24	0.41	-7.22	-3.35	-5.05	-4.92	-4.49
2007/8/14	5.94	6.20	4.99	6.00	5.59	6.66	7.32	5.39
2007/8/15	9.31	6.42	4.62	9.25	11.12	10.98	11.09	5.57
2007/8/16	12.97	8.64	7.61	9.42	8.50	10.18	11.85	8.05
2007/8/17	18.17	14.11	6.22	16.86	16.82	17.86	17.71	15.61

	By Industry						
Date	Computer, Software & Electronics	Money & Finance	Wholesale & Retail	Manufacturing	Health Care, Medical Eq, Drugs	Other Industries	All Stocks
2007/7/23	7.58	4.38	3.18	4.07	3.98	4.09	4.09
2007/7/24	8.41	5.38	3.84	4.96	5.87	4.90	4.90
2007/7/25	7.07	1.93	2.41	2.03	4.63	3.56	3.56
2007/7/26	10.81	6.44	5.74	7.55	7.60	7.07	7.07
2007/7/27	9.53	3.64	6.53	7.29	5.43	5.88	5.88
2007/7/30	6.10	3.82	3.86	5.40	6.70	4.72	4.72
2007/7/31	8.02	2.30	3.60	3.18	7.94	3.74	3.74
2007/8/1	9.18	3.27	11.13	9.11	3.98	5.06	5.06
2007/8/2	9.53	5.30	9.42	6.00	10.09	6.74	6.74
2007/8/3	10.23	2.62	3.31	5.49	6.14	4.28	4.28
2007/8/6	3.01	-0.60	-1.57	-0.16	4.04	-1.30	-1.30
2007/8/7	0.48	-1.20	-2.20	2.40	5.58	-1.12	-1.12
2007/8/8	-20.89	-12.16	-24.57	-18.78	-16.36	-18.69	-18.69
2007/8/9	-4.37	-3.92	-11.69	-17.36	-7.01	-9.82	-9.82
2007/8/10	0.56	1.15	-10.56	-5.27	0.12	-4.38	-4.38
2007/8/13	-3.78	-1.88	-8.20	2.82	-1.07	-4.90	-4.90
2007/8/14	6.36	7.65	5.36	7.45	4.34	5.39	5.39
2007/8/15	8.32	6.78	5.59	11.63	5.30	6.79	6.79
2007/8/16	8.69	8.94	9.89	12.02	12.96	9.46	9.46
2007/8/17	14.67	15.66	11.58	17.94	13.71	13.03	13.03

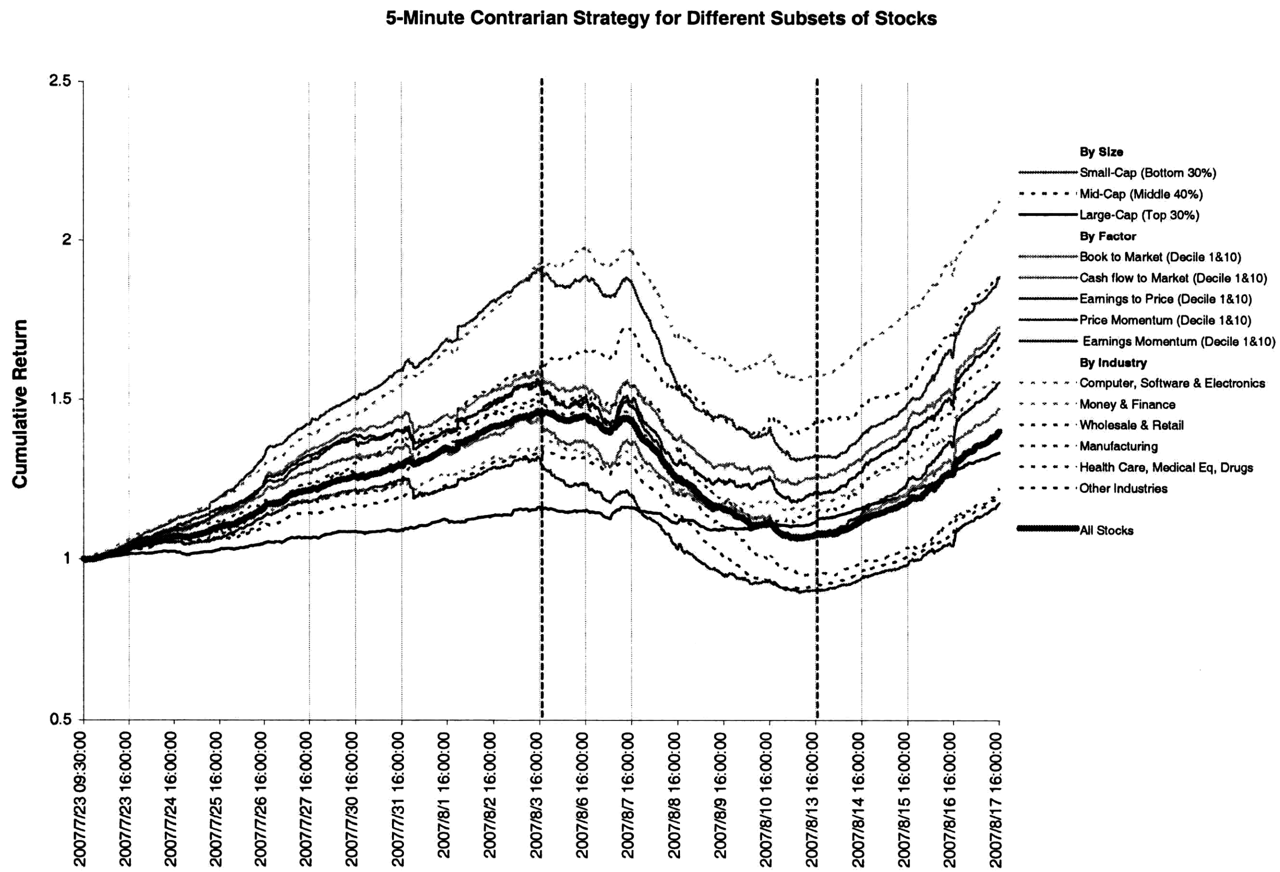


Figure 5.7: The cumulative returns for the contrarian strategy applied to various subset of the S&P 1500 index using 5-minute returns for July 23, 2007 to August 17, 2007. Each day is divided into non-overlapping 5-minute intervals, and positions are established based on lagged 5-minute returns and held for the subsequent 5-minute interval. The average return for each subset of stocks over each day of this period is then calculated. No overnight positions are allowed, initial positions are established at 9:40am each day and all positions are closed at 4:00 pm.

5.6 A Network View of the Hedge Fund Industry

One of the pillars of our Unwind Hypothesis that we have promoted in this chapter is the apparent large degree of overlap between holding of different hedge funds. We also hypothesized the sudden reduction in market-making activity, which in the current market conditions is provided by specialized hedge funds, was in part responsible for the massive losses in the week of August 6. The common theme of these observations is the large degree of interdependence and interconnectedness among different sectors. Although the focus of the study so far has been on a particular subset of hedge funds, namely quantitative equity hedge funds, the issue of interdependence is of great importance in the larger framework, specially when looking at the over stability of the system from a global perspective.

Perhaps some of the newly developed techniques in the mathematical theory of networks will allow us to construct systemic measures for robustness of the system. Given the lack of transparency in the hedge-fund industry, we have no direct way of gathering the data required to estimate the "network topology" that is the starting point of these techniques. One indirect and crude measure of the change in the "degree of connectedness" in the hedge-fund industry is to calculate the changes in the absolute values of correlations[5.14] between hedge-fund indexes over time. Using this measure, we will focus on the issue of "Connectedness" among different hedge fund sectors in this section.

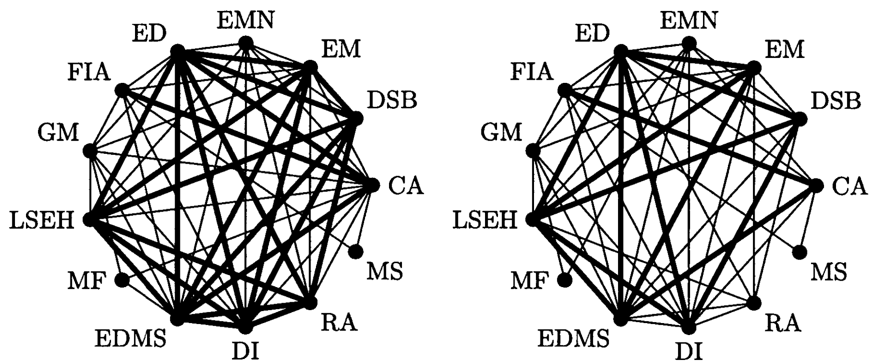
Using 13 indexes from April 1994 to June 2007 constructed by CS/Tremont,[5.15] we compare their estimated pairwise correlations between the first and second half of our total sample period: April 1994 to December 2000 versus January 2001 to June 2007. If, for example, the absolute correlation between Multi-Strategy and Emerging Markets was 7% over the first half of the sample and 52% over the second half, as it was, this might be a symptom of increased connectedness between those two categories.

Figure 5.8 provides a graphical depiction of this network for the two sub-samples, where we have used thick lines to represent absolute correlations greater than 50%, thinner lines to represent absolute correlations between 25% and 50%, and no lines for absolute correlations below 25%. For the earlier sub-sample, we estimate correlations with and without August 1998, and the difference is striking. Omitting August 1998 decreases the correlations noticeably, which is no surprise given the ubiquity and magnitude of the LTCM event. But a comparison of the two sub-periods shows a significant increase in the absolute correlations in the more recent sample. The hedge-fund industry has clearly become more closely connected.

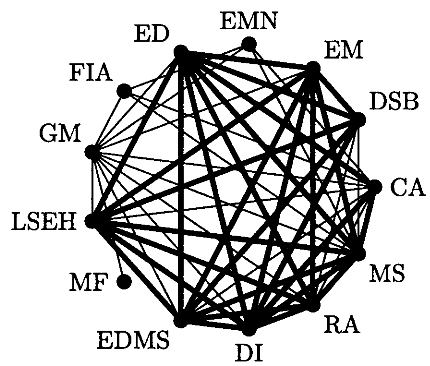
Perhaps the most significant indicator of increased connectedness is the fact that the Multi-Strategy category is now more highly correlated with almost every other index, a

[5 14]Because most hedge-fund strategies involve shortselling of one type or another, the correlations between the returns of various hedge funds can be positive or negative and are less constrained than, for example, those of long-only vehicles such as mutual funds. And because in our context, "connectedness" can mean either large positive or large negative correlation, we focus on the absolute values of correlations in this analysis.

[5 15]Specifically, we use CS/Tremont's Convertible Arbitrage, Dedicated Short Bias, Emerging Markets, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Global Macro, Long/Short Equity, Managed Futures, Event Driven Multi-Strategy, Distressed Index, Risk Arbitrage, and Multi-Strategy indexes. See Appendix A.4.1 for the definitions of these categories, and `www.hedgeindex.com` for more detailed information about their construction. All indexes start in January 1994 except Multi-Strategy, which starts in April 1994.



(a) April 1994 to December 2000, with (left) and without (right) August 1998



(b) January 2001 to June 2007

Figure 5.8: Network diagrams of correlations among 13 CS/Tremont hedge-fund indexes over two sub-periods, April 1994 to December 2000 (with and without August 1998) and January 2001 to June 2007. Thicker lines represent absolute correlations greater than 50%, thinner lines represent absolute correlations between 25% and 50%, and no connecting lines correspond to correlations less than 25%. CA: Convertible Arbitrage, DSB: Dedicated Short Bias, EM: Emerging Markets, EMN: Equity Market Neutral, ED: Event Driven, FIA: Fixed Income Arbitrage, GM: Global Macro, LSEH: Long/Short Equity Hedge, MF: Managed Futures, EDMS: Event Driven Multi-Strategy, DI: Distressed Index, RA: Risk Arbitrage, and MS: Multi-Strategy.

Table 5.3: The difference of the absolute correlation matrices of CS/Tremont Hedge-Fund Indexes using recent data (January 2001 to June 2007) and earlier data (April 1994 to December 2000), where the earlier correlation matrix is estimated with and without August 1998.

	Convertible Arbitrage	Dedicated Short Bias	Emerging Markets	Equity Market Neutral	Event Driven	Fixed Income Arbitrage	Global Macro	Long/Short Equity	Managed Futures	Event Driven Multi-Strategy	Distressed	Risk Arbitrage	Multi-Strategy
With August 1998 Included													
Convertible Arbitrage		4%	-6%	6%	-8%	-28%	17%	21%	-15%	-12%	-3%	-26%	21%
Dedicated Short Bias	4%		-17%	-40%	-2%	8%	26%	-7%	0%	-4%	-3%	8%	43%
Emerging Markets	-6%	-17%		-3%	-15%	-17%	20%	12%	3%	-23%	-14%	3%	58%
Equity Market Neutral	6%	-40%	-3%		-20%	29%	27%	8%	0%	-20%	-20%	-13%	37%
Event Driven	-8%	-2%	-15%	-20%		6%	28%	15%	-4%	1%	-6%	-21%	70%
Fixed Income Arbitrage	-28%	8%	-17%	29%	6%		-7%	15%	6%	-7%	19%	6%	-9%
Global Macro	17%	26%	20%	27%	28%	-7%		16%	31%	14%	34%	22%	36%
Long/Short Equity	21%	-7%	12%	8%	15%	15%	16%		35%	10%	8%	20%	56%
Managed Futures	-15%	0%	3%	0%	-4%	6%	31%	35%		-3%	-6%	-13%	25%
Event Driven Multi-Strategy	-12%	-4%	-23%	-20%	1%	-7%	14%	10%	-3%		-8%	-23%	62%
Distressed	-3%	-3%	-14%	-20%	-6%	19%	34%	8%	-6%	-8%		-25%	63%
Risk Arbitrage	-26%	8%	3%	-13%	-21%	6%	22%	20%	-13%	-23%	-25%		53%
Multi-Strategy	21%	43%	58%	37%	70%	-9%	36%	56%	25%	62%	63%	53%	
Excluding August 1998													
Convertible Arbitrage		25%	15%	17%	7%	-27%	24%	39%	-5%	1%	18%	-10%	13%
Dedicated Short Bias	25%		-6%	-32%	12%	3%	28%	-1%	13%	13%	13%	27%	39%
Emerging Markets	15%	-6%		9%	-2%	-11%	25%	20%	20%	-12%	8%	23%	54%
Equity Market Neutral	17%	-32%	9%		-9%	23%	20%	18%	-12%	-8%	-5%	-3%	36%
Event Driven	7%	12%	-2%	-9%		9%	35%	24%	11%	5%	1%	-9%	57%
Fixed Income Arbitrage	-27%	3%	-11%	23%	9%		-5%	22%	11%	-5%	28%	16%	-11%
Global Macro	24%	28%	25%	20%	35%	-5%		21%	22%	17%	48%	33%	34%
Long/Short Equity	39%	-1%	20%	18%	24%	22%	21%		49%	19%	23%	34%	52%
Managed Futures	-5%	13%	20%	-12%	11%	11%	22%	49%		11%	11%	2%	22%
Event Driven Multi-Strategy	1%	13%	-12%	-8%	5%	-5%	17%	19%	11%		12%	-10%	51%
Distressed	18%	13%	8%	-5%	1%	28%	48%	23%	11%	12%		-5%	54%
Risk Arbitrage	-10%	27%	23%	-3%	-9%	16%	33%	34%	2%	-10%	-5%		49%
Multi-Strategy	13%	39%	54%	36%	57%	-11%	34%	52%	22%	51%	54%	49%	

symptom of the large influx of assets into the hedge-fund industry. This increased correlation is also consistent with the hypothesis that forces outside the long/short equity sector may have caused an unwind of statistical arbitrage strategies in August 2007. In August 1998, multi-strategy funds were certainly impacted by their deteriorating fixed-income arbitrage positions, and no doubt many of them liquidated their statistical arbitrage portfolios to meet fixed-income margin calls. But because multi-strategy funds were not as significant a market force in 1998 as they evidently are now, their correlations to other strategies were not as large as they are today.

Table 5.3 contains a more detailed comparison of the two correlation matrices. The absolute correlation matrix from the earlier sample is subtracted from that of the more recent sample, hence a positive entry represents an increase in the absolute correlation in the more recent period, and is highlighted in red if it exceeds 20% (negative entries less than −20% are highlighted in blue). Table 5.3 confirms the patterns of Figure 5.8: absolute correlations among the various different hedge-fund categories have indeed increased in the more recent sample, with considerably more positive entries than negative ones.

To capture the dynamics of these changes in correlation structure among the CS/Tremont Indexes, in Figure 5.9 we plot the means and medians of the absolute values of 36-month rolling-window correlations between the indexes, with and without the month of August 1998.

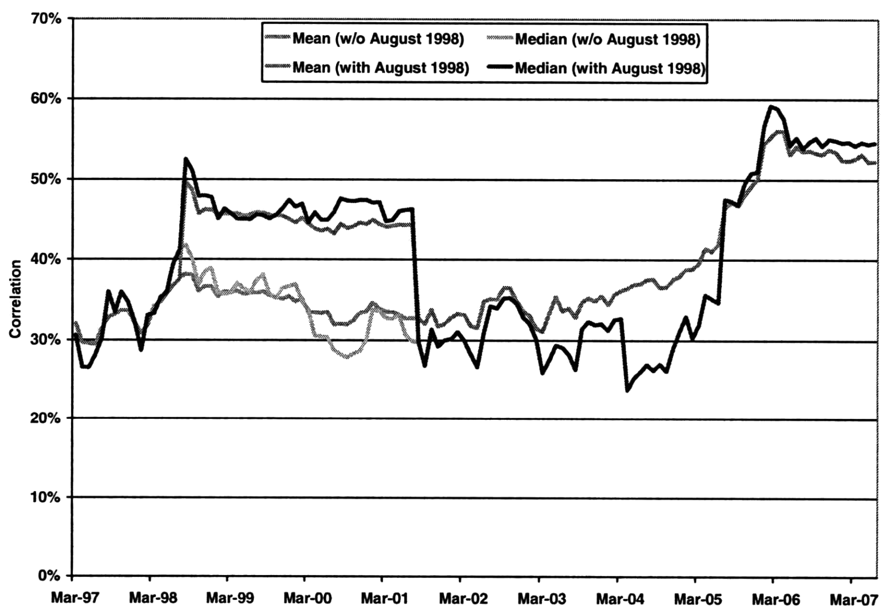


Figure 5.9: Mean and median absolute 36-month rolling-window correlations among CS/Tremont hedge-fund indexes from March 1997 to June 2007, with and without August 1998.

These graphs show that the mean and median absolute correlations among the indexes have been steadily increasing in recent years, especially after 2004. The inordinate amount of influence that August 1998 has on these correlations underscores the potential for system-wide shocks in the hedge-fund industry.

5.7 Chapter Conclusions

In this chapter we applied the tools developed in the last two chapters to analyze the events of August 2007 among equity hedge funds. We were able to link these events to a very distinct change in the price dynamics, where the usual mean reversion in returns was replaced by a momentum during a period of a few days. Given the model and the analysis presented in the last chapter, we were able to link the change in the price dynamics to a change in the arrival dynamics of customers. We complemented our analysis by presenting evidence about the explanatory power of typical quant factors in explaining the cross-section of trading activity and returns during the same time. Higher explanatory power during the volatile days supports our claim about higher commonality in customer arrival during that time.

We argued that certain aspects of risk in the hedge fund industry have increased due to a higher degree of similarity between assets held by various funds and a higher dependence of these hedge funds on other hedge funds and proprietary trading desks for providing liquidity to the market. In particular, we presented some evidence to support our claim that the volatility in August 2007 was in part due to a reduction in liquidity provision by certain high-frequency market makers. Finally, we presented some indirect evidence regarding the higher degree of correlation among a much larger and more diverse set of funds. This evidence suggests that transmission channels to direct the shocks between various hedge fund categories are stronger now.

Chapter 6

Long-Term Deviation from White Noise

This chapter builds upon the earlier chapters by looking at an alternative patterns of predictability in price signals. We will explore further the implications of the notion of informational efficiency.

In Section 3.2, we showed the link between the notion of informational efficiency and predictability in prices (or, to be more precise, price changes). In Section 3.3, we showed how testing for linear predictability is equivalent to testing the elements of the lagged covariance matrix. The contrarian trading strategy outlined in Section 3.3.1 was applied in Section 3.4 to detect major deviation from the hypothesis of unpredictability. Based on the results of that empirical analysis, we hypothesized that certain frictions may be the cause of this deviation. Chapter 4 was focused on studying one particular type of friction. These tools were used in Chapter 5 to study the sequence of events in August 2007 that, as we argued, was linked to the change in normal system dynamics caused by a shift in trade arrival and a reduction in the activity of market makers.

We continue the line of analysis in this chapter by looking at the evidence for predictability among a very different set of signals and at a substantially different trading horizon. The primary source of returns we use in this and the next chapter are the monthly returns of mutual funds and hedge funds. These returns are different from the returns we focused on in the earlier parts since they represent the returns generated by managers who are actively in the marketplace trying to *beat the market*. As we discussed in Chapter 3, the intuitive foundation of the informational efficiency hypothesis is that by the nature of competition in financial markets, market participants try to take advantage of all available information in predicting the fair price and take position in cases when their prediction of the price is different from the prevailing price. This competition causes the level of price to include information of all market participants. Hence any change in the price is due to *new* information not previously available to *any* market participant (see the discussion in Section 2.2). For this reason, serial correlation, as a simple measure of predictability, is often associated with market inefficiencies.[6.1] Using the same line of logic, the returns of *managed portfolios*, such

[6.1]This association is not totally correct as it has been shown that other sources, such as predictable changes in the investment universe (things such as periods of economic prosperity versus recession or periods of high as a hedge fund, should *definitely* be unpredictable.

After all, predictability in the returns, particularly among hedge fund returns, seems inconsistent with the popular belief that the hedge fund industry attracts the best and the brightest managers in industry. If a fund manager's returns are predictable, one implication is that the manager's investment policy is not optimal. To make this intuition more clear, consider the following thought experiment: if the manager's returns next month can be reliably forecasted as positive, the fund manager should increase positions this month to take advantage of this forecast, and vice versa for the opposite forecast. By taking advantage of such predictability, the fund manager will eventually eliminate it, along the lines of original proof in Samuelson (1965).

Several authors have documented significant deviation from unpredictability in the reported returns of hedge funds.[6.2] To bring this evidence to light, we have reported in Figure 6.1 the histogram of the estimate of the first order serial correlation of mutual funds and hedge fund in the universe of our data.[6.3] Three observations should be clear:

1. There is a large concentration of mutual funds with estimated serial correlation near 1.

2. With the exception noted in point 1, hedge funds tend to have higher estimated serial correlation as compared to the rest of mutual funds.

3. In both cases, the histograms are not centered around 0, i.e., negative serial correlation values are relatively rare and the average estimate seems to be reliably positive overall.

The information provided in Figure 6.1 lacks the rigors of an statistical test. We will turn to this issue in Section 6.1. But before doing that, let's try to build some intuition regarding the nature of mutual funds that exhibit very high serial correlation.

Consider a mutual fund that invests in 1-month US government debt, or what is called the Treasury Bills or the T-Bills.[6.4] The reported returns of such a mutual fund would be identical to the 1-month T-Bill rates. We plot the realizations of this rate and its mean for January 1986 to December 2006 in Figure 6.2. As can be seen, this rate has a significant variation around its sample mean, typically decreasing during recessions (such as early 1990s and early 2000s) and increasing during good times. This is an example of the long-term variations that we mentioned in footnote 6.1. Nevertheless, if we naively calculate the serial correlation for such a time series, we will find a very high value. For example, the estimated serial correlation for the time series shown in Figure 6.2 is 95.3%! This is consistent with the fact that the series is very predictable; for example, months that have an above mean

inflation versus low inflation), can also give rise to a small amount of predictability in the returns. See LeRoy (1989) and references therein for discussion of this topic.

[6.2]See Asness, Krail, and Liew (2001) for an early example.

[6.3]The hedge fund return data was obtained from the Lipper TASS Hedge Fund Database. Please see Appendix A.4.1 for an overview of this database. The mutual fund return data was obtained from CRSP Survivor-Bias-Free US Mutual Fund Database available from Center for Research in Security Prices. Please see Appendix A.4.2 for an overview of this database.

[6.4]Such fund would be similar to the *Money Market* funds offered by many financial institutions as an alternative to checking accounts.

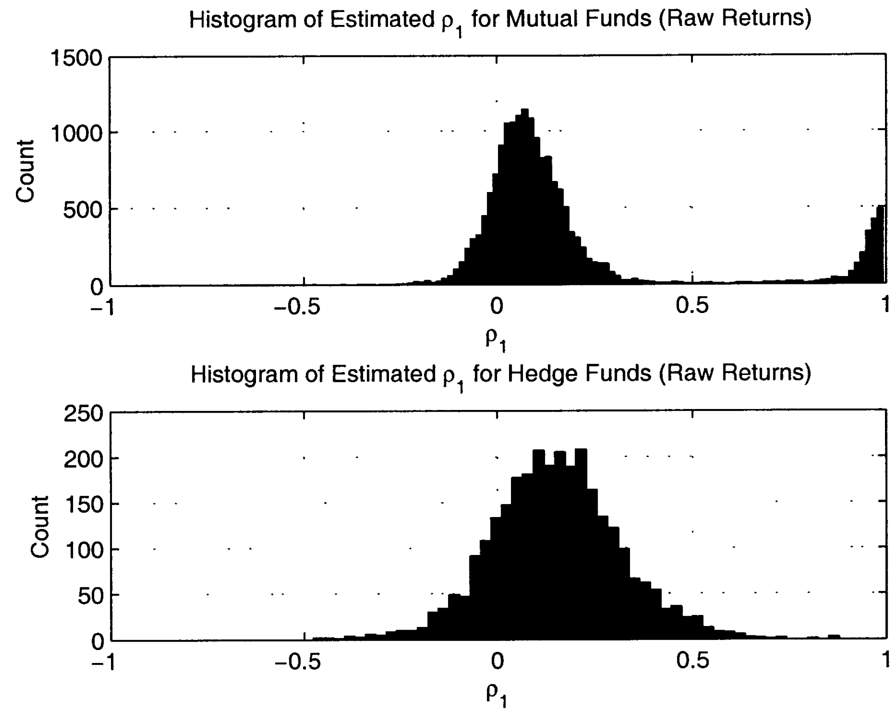


Figure 6.1: This figure shows the histogram of estimated first order serial correlation, $\rho_{i,1} = \text{Corr}(r_{i,t}, r_{i,t-1})$, in monthly returns for all hedge fund and mutual fund returns for the period of 1986 to 2006 where $r_{i,t}$ is the return for funds i in month t. See Appendix A.4.1 and A.4.2 for an overview of the data used for this analysis.

realization are typically followed by another month that has a realization above the sample mean and vice versa for a month with below sample mean realizations. But this is due to the fact that months with high T-Bill rates are months during economic expansion and months with low such rates are recession months, and since these periods tend to persist once started, we will find high persistence in the returns of our hypothetical mutual fund.

One simple way to remove this effect is to use fund returns above the T-bill rate to calculate the serial correlation values.[6.5] This procedure is repeated and the empirical histograms are reported in Figure 6.3. As can be seen, the exercise has removed the concentration of estimates around 1 (compare with Figure 6.1). But there still seems to be some mutual funds for which the level of estimated serial correlation is *too* large. In addition, even with this correction, hedge funds tend to have large serial correlation estimates. Understanding why this may be the case and its implications on the long-term behavior of the system are the objective of the rest of this chapter and the next chapter.

The rest of this chapter is structured as follows. In Section 6.1 we will discuss proper

[6.5]The T-bill rate, which is the rate for 1-month investment in the US government debt, is used since the reporting horizon of the mutual fund and hedge fund returns is also monthly.

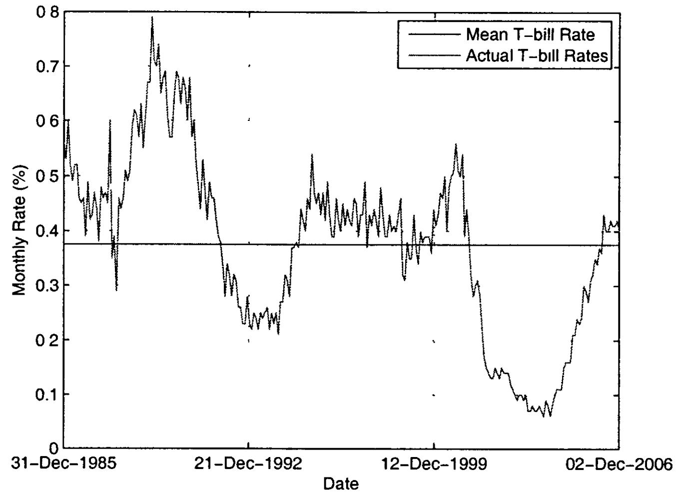


Figure 6.2: This figure shows why calculating serial correlation for a hypothetical mutual fund that invests in 1-month Treasury Bills (T-Bill) from 1986 to 2006 gives a very high level of serial correlation. Since, the T-bill rates vary widely around the sample mean during this time and the variation is persistent, the serial correlation calculated from this series will be very high; in fact, it will be near unity.

statistical distribution for the estimated serial correlation for the case of *usual* returns and the case of returns with serial correlation of almost 1 or what we, for now and for lack of a better word, refer to *unusual* returns. Section 6.1 will focus on the case of usual signals and Section 6.1.2 will address the case of unusual signals. Section 6.1.3 will apply the statistical tools developed in the two earlier sections to take a more precise view of the estimated serial correlation, i.e., the same data reported in histograms in Figures 6.1 and 6.3. Once the statistical significance of the estimated serial correlation values is established, we will, in Section 6.2.1, develop a model of a certain type friction that may be behind the observed unusually high serial correlation values. We will conclude in Section 6.3 by introducing one of the central concepts in finance, namely the link between *risk* and *return*. We will connect this idea with the framework familiar in statistical learning. This will set the stage for the analysis between the friction outlined in Section 6.2.1 and long-term return of the hedge funds and mutual funds in the next chapter.

6.1 Statistics of Serial Correlation of Signals Changes

In this section we develop proper sampling theory for the estimator of the first order serial correlation of return signals.[6.6] Let r_t denote the time t realization of the return signal and

[6] [6]As we discusses in Section 3.1, price signals are difficult to analyze directly due to scaling issue and also to a non-stationarity of the type we will discuss in this section. For this reason, prices are usually

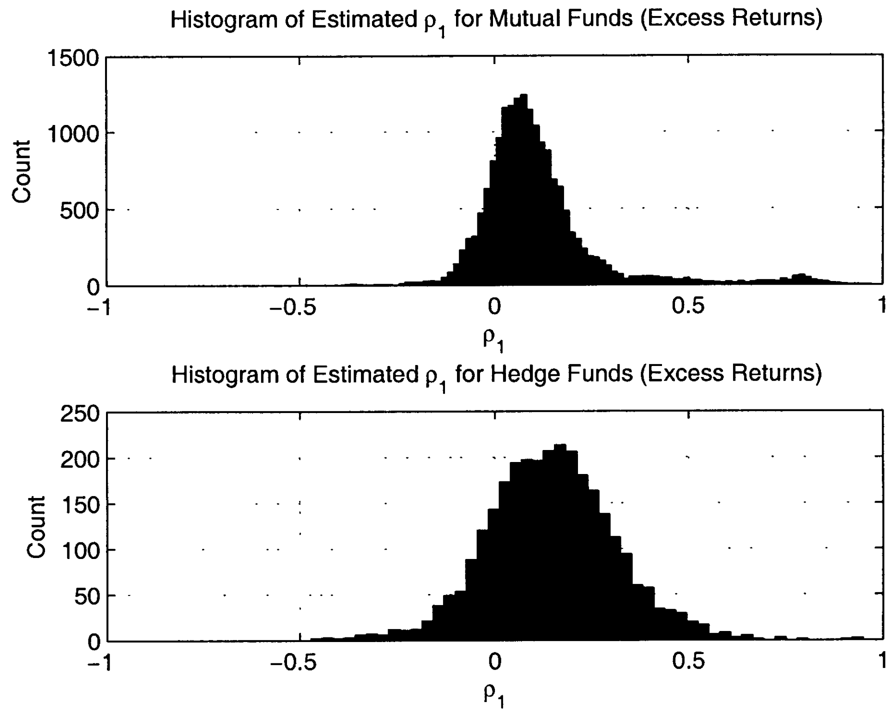


Figure 6.3: This figure show the histogram of estimated first order serial correlation, $\rho_{i,1} = \mathrm{Corr}(r_{i,t} - r_{f,t}, r_{i,t-1} - r_{f,t-1})$, in monthly excess returns for all hedge fund and mutual fund returns for the period of 1986 to 2006. Returns are measures in excess of the 1-month Treasury Bill (T-bill) rates. $r_{f,t}$ is the 1-month T-Bill prevailing at the beginning of month t and $r_{i,t}$ is the return for funds i in month t.

let T be the total number of realizations of this signal available for the analysis. The first order serial correlation is estimated by following expression:

$$\hat{\rho}_1 = \frac{(T-1)^{-1}\sum_{t=1}^{T-1} r_t r_{t+1} - \left[(T-1)^{-1}\sum_{t=1}^{T-1} r_t\right]\left[(T-1)^{-1}\sum_{t=2}^{T} r_t\right]}{(T-1)^{-1}\sum_{t=1}^{T-1} r_t^2 - \left[(T-1)^{-1}\sum_{t=1}^{T-1} r_t\right]^2} \tag{6.1}$$

The following two section provide the sampling distribution of this estimator under a null hypothesis of the following form:

$$r_t = \mu_t + \epsilon_t \tag{6.2}$$

transformed into returns by one of the two methods we mentioned in Definition 1. In fact, using returns as the starting point is so widespread that data sources usually provide the effective per-period return as the starting point of any analysis. The data sources used in this analysis (described in Appendix A.4.1 and A.4.2) directly provide the monthly return for various hedge funds and mutual funds.

where μ_t is the time-varying expected return and ϵ_t is the unpredictable part of the return. In particular, we will consider the following two cases:

- The case that μ_t is a constant equal to μ. The appropriate sampling distribution for $\hat{\rho}_1$ for this case is developed in Section 6.1.
- The case that μ_t is a random walk specified as $\mu_t = \sum_{i=0}^{t-1} \nu_i$ where ν_i are unpredictable but permanent shocks to the expected value. This type of null hypothesis is considered in Section 6.1.2.

The first case results in a stationary time series, while the second case results in a non-stationary time series. We will not consider the more general case of stationary time series in which $\mu_t = \theta(L)\nu_t$, where $\theta(L)$ is the usual polynomial in the lag operator L and constrained in a way that μ_t is a stationary process resulting in a stationary r_t process. Detecting this null-hypothesis without constraining the degree of persistence allowable in the μ_t process is not possible through the simple approach based on $\hat{\rho}_1$ that we are using. For testing this null hypothesis, one has to ultimately address the underlying drivers of variation in the mean along the lines of the drivers mentioned in footnote 6.1.[6.7] For the purpose of this study, we will not consider this more complicated version.

6.1.1 Stationary Case with Constant Mean

Recall that we are considering a null hypothesis where the data is generated through the following data-generating process:

$$H_0: \qquad r_t = \mu + \epsilon_t \tag{6.3}$$

where μ is the average return and ϵ_t is the unpredictable part of the return, so we have $E[\epsilon_t|\mathcal{H}_t]$ where $\mathcal{H}_t = \sigma(\epsilon_1, \cdots, \epsilon_t)$; i.e., $\mathcal{H}_t$ is the set of all available information at time t.[6.8] This null hypothesis corresponds to the weakest form of the random walk hypothesis, or the *Uncorrelated Increments* hypothesis presented in Chapter 2 of Campbell, Lo, and MacKinlay (1997). Under this hypothesis, one can find the following distribution for the estimated first order auto-correlation. Our proof, based on the Generalized Method of Moments is presented in the Appendix.

Proposition 6.1 *Under the null hypothesis of (6.3) where ϵ_t is a martingale difference sequence adopted to the filtration $\mathcal{H}_t = \sigma(\epsilon_1, \cdots, \epsilon_t)$ and under some additional technical regularity conditions listed in the proof, the first order sample auto-correlation has the following asymptotic distribution:*

[6.7] Getmansky et al. (2004) look at this issue using for example Hidden Markov Model (HMM) as the structure giving rise to the time variation in the mean.

[6.8] $\mathcal{H}_t = \sigma(\epsilon_1, \cdots, \epsilon_t)$ is the σ-algebra generated by $\{\epsilon_1, \cdots, \epsilon_t\}$. See Shreve (2004) for definition and relevant mathematical background.

$$\hat{\rho} \sim N(0, \theta^2) \tag{6.4a}$$

$$\theta = \frac{\left(\sum \epsilon_t^2 \epsilon_{t-1}^2\right)^{1/2}}{\sum \epsilon_t^2} \tag{6.4b}$$

Furthermore, θ can be replaced by its consistent estimator, $\hat{\theta}$, given by:

$$\hat{\theta} = \frac{\left(\sum (r_t - \hat{\mu})^2 (r_{t-1} - \hat{\mu})^2\right)^{1/2}}{\sum (r_t - \hat{\mu})^2} \tag{6.5a}$$

$$\hat{\mu} = T^{-1} \sum r_t \tag{6.5b}$$

Proof: See Appendix A.4.3.

The result given in Proposition 6.1 is robust to heteroskedasticity, i.e., conditional changes in the variance, in the returns signals. This is important as it is commonly known that financial return time series exhibit very strong clustering in their volatility. So it is important that the test statistic used for testing is based on the appropriate null that allows for conditional heteroskedasticity in the returns. Intuitively, accounting for this effect is equivalent to using the appropriate approach for estimating the standard errors of the estimator given in (6.1).

The test statistic presented in Proposition 6.1 is a subset of the more general Variance Ratio test developed in Lo and MacKinlay (1988). We have presented this here for completeness of the argument. Furthermore, the proof provided in Appendix A.4.3 is based on a different and much simpler approach.

6.1.2 Non-stationary Case

Consider the null hypothesis that the data is generated by a process of the following form:

$$\begin{aligned} H_0 \ : \ & r_t = \mu_t + \epsilon_t \\ & \mu_t = \mu + \sum_{i=0}^{t-1} \nu_i \end{aligned} \tag{6.6}$$

where both ϵ_t and ν_t are martingale difference sequences adopted to the filtration $\mathcal{H}_t = \sigma(\nu_1, \cdots, \nu_t, \epsilon_1, \cdots, \epsilon_t)$. Under this null hypothesis, it is assumed that $E[\epsilon_{t+1}|\mathcal{H}_t] = 0$, $E[\nu_{t+1}|\mathcal{H}_t] = 0$, and $E[\nu_{t+1}\epsilon_{t+1}|\mathcal{H}_t] = 0$. In non-mathematical terms, $\mathcal{H}_t$ represents all of the available information at time t. Our assumptions imply that shocks to expected returns and the unexpected part of the return are unforecastable and that these shocks are concurrently uncorrelated. These assumption are quite general and include many forms of leptokurtosis such as the stochastic volatility type of AutoRegressive Conditionally Heteroskedasticity (ARCH) or any of the generalizations of such models. As presented in the proof, all that is required for the main test of this null hypothesis to hold is the finiteness of moments up to the second order and some ergodicity conditions. Given all of these assumptions, it can be shown that the limiting distribution for the sample auto-correlation is given by the following proposition.

Proposition 6.2 *Under the null hypothesis that the data is generated by the process given in (6.6) where both ϵ_t and ν_t are martingale difference sequences adopted to the filtration $\mathcal{H}_t = \sigma(\nu_1, \cdots, \nu_t, \epsilon_1, \cdots, \epsilon_t)$ and under some additional requirements listed in the proof, we have the following limiting distribution for the sample auto-correlation:*

$$T(\hat{\rho} - 1) \xrightarrow{d} \frac{\frac{1}{2}(W(1)^2 - 1) - W(1)\int_0^1 W(u)du - \frac{\sigma_\epsilon^2}{\sigma_\nu^2}}{\int_0^1 W(u)^2 du - (\int_0^1 W(u)du)^2} \tag{6.7}$$

where $W(u)$ is the standard Brownian motion over interval $[0, 1]$.
Proof: See Appendix A.4.4.

Notice that the limiting distribution depends on the ratio of $\frac{\sigma_\epsilon}{\sigma_\nu}$. Any consistent estimator of this ratio can be used to conduct an asymptotic test. To conduct tests presented in this section, we have used the estimator outlined in the following corollary.

Corollary 6.1 *Under the null hypothesis outlined in (6.6), we have that*

$$\frac{\sigma_\epsilon^2}{\sigma_\nu^2} \xrightarrow{p} \frac{-Corr(\Delta r_t, \Delta r_{t-1})}{2Corr(\Delta r_t, \Delta r_{t-1}) + 1} \tag{6.8}$$

where $\Delta r_t = r_t - r_{t-1}$.
Proof: See Appendix A.4.5.

6.1.3 Empirical Application

We now apply the above sampling theory of the first order serial correlation to test the null hypothesis of unpredictability, i.e., $rho_1 = 0$, in the reported returns. Such a test would shed some light on the statistical significance of the values reported in Figures 6.1. To conserve space, we have only reported the results based on the actual returns, i.e., the statistical significance of the numbers shown in Figure 6.3 are not reported here. We have separated the results based on the category of the fund to highlight significant differences between categories that will be important in the next chapter. For definition of hedge fund categories see Appendix A.4.1.

As seen in Table 6.1, the estimated first order serial correlations are statistically significant among a large fraction of funds in certain categories of hedge funds. For example, the first order serial correlation is statistically significant among 79.4% of the Convertible Arbitrage hedge funds and about 52.4% of Event Driven funds. A similar metric is only 3.6% among Managed Futures or 8% among the Dedicated Short Biased hedge funds. Among mutual funds, Asset Allocation funds have the lowest level of serial correlation and the estimates are significant only among 0.4% of these funds. This is in contrast with Fixed Income mutual funds for which the null hypothesis can be rejected for 15.2% of all funds. Also note that the Money Market funds show a very high level of serial correlation and the null hypothesis can be rejected for 99.4% of all such funds. These funds, as we argued before, exhibit a unit-root in their expected return due to their exposure to the short-term interest rates. Also notice that negative and statistically significant first order serial correlation values are extremely rare.

The data in Table 6.1 shows that the null hypothesis of $rho_1 = 0$ can be rejected for 26.8% of mutual funds for which we don't have category information (row "Info. N/A"). As noted in the discussion prior to Table A.2 in Appendix A.4.2, these are typically the older funds which ceased to exist in the earlier part of the sample before the category information was made available. We suspect that some of these funds are in fact Money Market funds for which the returns are better represented by a unit-root process and can be separated using our unit-root test. Note that the ratio of $\frac{\sigma_\epsilon^2}{\sigma_\nu^2}$ is undefined under the null of $\rho_1 = 0$ and our test for unit-root is invalid for those funds. Table 6.2 reports two different counts; one is the percentage of funds for which the null of unit-root cannot be rejected and the second column reports the percentage of funds for which the null of unit-root cannot be rejected but the null hypothesis of $rho_1 = 0$ can be rejected. Given the point mentioned earlier about the $\frac{\sigma_\epsilon^2}{\sigma_\nu^2}$ in cases for which the rho_1 cannot be rejected, we will focus on attention only on funds for which that hypothesis is rejected. Based on this analysis, we suspect about 12.8% of funds with no category information to be in fact Money Market type funds. We will ignore these funds in our analysis in the next chapter.

Table 6.1: This table gives a summary of statistical tests for the null hypothesis of $\rho_1 = 0$ in hedge fund and mutual fund returns.

Fund Type	Category	Count	Average Rho_1	Average p-Value	Null of Rho_1=0 Rejected (5% Test)	Null of Rho_1=0 Rejected & Positive Rho_1	Null of Rho_1=0 Rejected & Negative Rho_1
Hedge Fund	Convertible Arbitrage	101	38.3%	0.06	79.2%	79.2%	0.0%
Hedge Fund	Dedicated Short Bias	25	9.2%	0.41	8.0%	8.0%	0.0%
Hedge Fund	Emerging Markets	182	17.4%	0.24	36.3%	36.3%	0.0%
Hedge Fund	Equity Market Neutral	153	11.4%	0.34	28.8%	24.2%	4.6%
Hedge Fund	Event Driven	254	22.7%	0.16	52.4%	51.6%	0.8%
Hedge Fund	Fixed Income Arbitrage	108	19.2%	0.27	25.9%	25.0%	0.9%
Hedge Fund	Fund of Funds	631	18.9%	0.21	42.0%	41.5%	0.5%
Hedge Fund	Global Macro	126	7.7%	0.41	8.7%	7.9%	0.8%
Hedge Fund	Long/Short Equity Hedge	906	12.6%	0.35	16.1%	15.5%	0.7%
Hedge Fund	Managed Futures	308	0.4%	0.51	3.6%	2.6%	1.0%
Hedge Fund	Multi-Strategy	133	17.8%	0.21	42.1%	41.4%	0.8%
Mutual Fund	Asset Allocation	1,133	5.3%	0.59	0.4%	0.4%	0.0%
Mutual Fund	Convertible	74	10.0%	0.37	6.8%	6.8%	0.0%
Mutual Fund	Equity	7,626	7.7%	0.48	5.5%	5.5%	0.0%
Mutual Fund	Fixed Income	4,088	8.2%	0.41	15.2%	15.2%	0.0%
Mutual Fund	Info. N/A	3,078	21.1%	0.35	26.8%	26.6%	0.2%
Mutual Fund	Money Market	1,560	94.2%	0.00	99.4%	99.4%	0.0%
Mutual Fund	Unclear (Multiple Categories)	50	10.8%	0.44	12.0%	12.0%	0.0%

Table 6.2: This table gives a summary of unit-root statistical test for different categories of mutual funds.

Category	**Count**	**Average Rho_1**	**Null of No-Unit Root Rejected (5% Test)**	**Null of Rho_1=0 Rejected and Null of No Unit-Root Not Rejected (5% Test)**
Asset Allocation	1,133	5.3%	49.5%	0.1%
Convertible	74	10.0%	64.9%	0.0%
Equity	7,626	7.7%	50.0%	1.2%
Fixed Income	4,088	8.2%	85.5%	1.3%
Info. N/A	3,078	21.1%	54.1%	12.8%
Money Market	1,560	94.2%	6.0%	93.6%
Unclear (Multiple Categories)	50	10.8%	54.0%	0.0%

6.2 Partially Observed Signals and Deviation from White Noise

We will now discuss a model for a particular type of friction that can give rise to observed serial correlation in prices. The model discussed here is a generalization of the model in Lo and MacKinlay (1990a). It works based on the idea that in the absence of a transaction, the price associated with an asset remains same as the price from the most recent transaction. For example, if the true price time series is $p_{i,t}$, the observed price time series, $p^o_{i,t}$, will be given by:

$$p^o_{i,t} = \begin{cases} p_{i,t} & \text{if there was a trade in interval } t \\ p_{i,t-1} & \text{if last trade was in interval } t-1 \\ p_{i,t-2} & \text{if last trade was in interval } t-2 \\ \vdots & \vdots \end{cases} \tag{6.9}$$

As we will show in this section, the effect of this stale pricing on the observed prices creates an illusion of predictability and serial correlation. We will first describe the idea through a simple simulation and then outline the model. To get started, we generate the sample path for 500 sets of prices for 180 time periods (that would be 15 years for monthly period length) based on a discrete version of a geometric Brownian motion data-generating process. More precisely, $p_{i,t}$, the price at time t for asset i, is given by:

$$p_{i,t} = \exp\left(\sum_{\tau=1}^{t}(\mu_i + m_\tau + \nu_{i,\tau})\right)$$

Under this data-generating process, the return for a security over a time interval is given by $\log(p_{i,t}) - \log(p_{i,t-1}) = \mu_i + m_t + \nu_{i,t}$. m_t represents the common random part of the security's return and $\nu_{i,t}$ represents the specific random part of the return for security i and μ_i is the average return. For the simulations, we set $\mu_i = 0.005$ and use independent draws for a zero-mean normal random variable to generate realizations of m_t and $\nu_{i,t}$ with $\sigma_m = 0.01$ and $\sigma_{\nu_i} = 0.05$.

Now consider the case when there is a 20% probability that each security is not traded in a given interval. In the absence of a trade, the observed price remains same as the price observed after the most recent transaction, i.e., the observed prices are determined through (6.9). Also consider a scenario where we have a portfolio with an equal amount invested in each of these 500 securities. The actual and observed value of the portfolio, $p_{p,t}$ and $p^o_{p,t}$, respectively, are simply the equal weighted average of the actual and the observed underlying prices, i.e., $p_{i,t}$ and $p^o_{i,t}$ respectively. Recall that, as discussed before, the observed prices are determined through (6.9) in the 20% of the time that there is no trade for a given security in a given time interval.

Figure (6.2) shows one realization of a sample path of prices generated through this process. We have calculated the actual and the observed value of the portfolio as the equal weighted average of the actual and observed set of prices. The portfolio values, both actual and observed, are then turned into returns by taking the first difference[6.9] of the value time series. The first order serial correlation of the returns time series is calculated for both actual and observed return and the $p-Value$ of the Q_3[6.10] test statistic based on the values of the first three serial correlations of the portfolio return are calculated in both cases. Both the magnitude of the first order serial correlation, 4.31% vs. 23.24%, as well as the $p-Value$ of the Q_3 indicate that the partially observed underlying prices can give rise to perceived serial correlation at the portfolio level. The model developed next will explicitly determine the expression for the portfolio level serial correlation under a more general data generating process.

6.2.1 The Model

Consider a collection of N securities and let $r_{i,t}$ represent the unobservable return at time t, for security i. We assume that contemporaneous returns have a common component captured by a linear structure given as follows:

$$r_{i,t} = \mu_i + \beta_i m_t + \epsilon_{i,t} \quad i = 1, \cdots, N \tag{6.10}$$

where m_t is a zero-mean common factor and $\epsilon_{i,t}$ is zero-mean noise specific to security i at time t. Since $\epsilon_{i,t}$'s represent the random returns specific to security i at time t, and we will model them as temporally and cross-sectionally uncorrelated at all leads and lags. We also assume that m_t for different values of t are independently and identically distributed and uncorrelated with $\epsilon_{i,t-k}$ for all i, t, and k. Finally, let $\text{Var}(m_t) = \sigma^2$.

For security i, let $\delta_{i,t}$ be the indicator random variable for the trade event at time t, i.e., $\delta_{i,t} = 1$ if a trade for security i takes place in time t and takes the value of 0 otherwise. If a security is not traded in a given time interval, the observed price will be unchanged during that time interval so the "reported" return is 0 even though its true or virtual return is given by (6.10). On the other hand, if a security is traded in time interval t, the observed return is the sum of the virtual return in all prior consecutive time periods in which the security was not traded. So the price "catches-up" to the true underlying price after each trade and remains constant at other times. Viewed in terms of the prices, the observed set of prices and underlying prices are related through (6.9) as described in the context of simulation in the last part. So far, the model is the same as that in Lo and MacKinlay (1990a).

[6.9]This is different from the return definition given in Definition 1 but this should be sufficient for the purpose of this illustrative simulation.

[6.10]Pierce and Box (1970) proposed the following statistic for testing the significance of the first k auto-correlation values:

$$Q_m = T\sum_{k=1}^{m} \rho^2(k)$$

Under the null hypothesis of no auto-correlation, this statistic is asymptotically distributed as χ^2_m. Ljung and Box (1978) proposed the following finite-sample correction which provides a better fit to the χ^2_m for small samples sizes:

$$Q_m = T(T+2)\sum_{k=1}^{m} \frac{\rho^2(k)}{T-k}$$

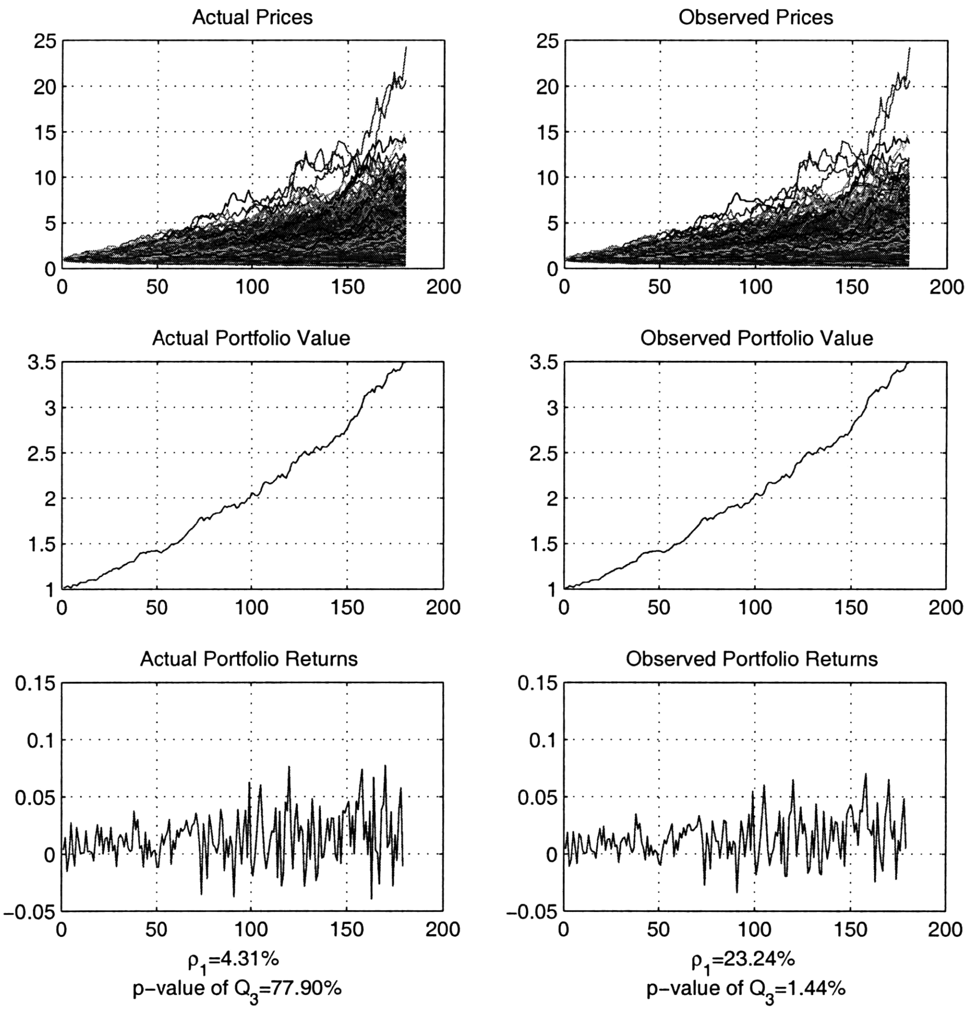


Figure 6.4: A simulation example of the partially observed price signals. Sample paths for prices of 500 securities for 180 time-periods are generated through a geometric Brownian motion data-generating process given by $p_{i,t} = \exp\left(\sum_{\tau=1}^{t}(\mu_i + m_\tau + \nu_{i,\tau})\right)$ where $\mu_i = 0.005$ and m_t and $\nu_{i,t}$ independent draws of normal random variable with zero man with $\sigma_m = 0.01$ and $\sigma_{\nu_i} = 0.05$. Each security has a 20% probability of not being traded in an interval in which case the "observed price", $p^o_{i,t}$, remains the same as the price after the most recent transaction, also see (6.9). Actual and observed values of portfolio are calculated as the equal weighted average of the actual and observed set of prices. The portfolio values, both actual and observed, are then turned into returns by taking the first difference of the time series. Finally, the first order serial correlation of the returns time series is calculated for both cases. The $p-Value$ of the Q_3 test statistic based on the values of the first three serial correlation of the portfolio return are reported for both cases. Both the magnitude of the first order serial correlation, 4.31% vs. 23.24%, and the $p-Value$ of the Q_3 indicate that the partially observed underlying prices can give rise to perceived serial correlation at the portfolio level.

In the model proposed in Lo and MacKinlay (1990a), it is assumed that $\delta_{i,t} = 1$ with a fixed probability p_i in each time period. Their model is capable of generating a negative auto-correlation for each security while generating a positive auto-correlation for a portfolio of securities. The intuition behind the model proposed here is to bridge this gap, i.e., by explicitly modeling the dependence between $\delta_{i,t}$ across i for a fixed t, we will propose a model that can generate both positive and negative correlation for portfolio returns.

For the case that $\delta_{i,t}$ are completely correlated, the model proposed here will generate negative auto-correlation for the portfolio, similar to the individual security return in the Lo and MacKinlay (1990a) model. In this case, creating a portfolio of securities does not achieve diversification across non-trading events and, hence, the portfolio is similar to a single security when it comes to the non-trading event or the non-trading induced auto-correlation. Therefore, the generated returns are negatively auto-correlated as was the case in the Lo and MacKinlay (1990a) model for a single security. On the opposite extreme, where $\delta_{i,t}$ are completely independent, the model will generate positive auto-correlation similar to the Lo and MacKinlay (1990a) model for a portfolio of securities. By adding a parameter that controls the level of dependence between $\delta_{i,t}$'s for a fixed t, we will be able to create a model that can generate both these extremes and values in between.

In order to model the cross-sectional dependence between $\delta_{i,t}$s across i, we define $\delta_{i,t}$ as the indicator random variable for the following event:

$$\delta_{i,t} = 1\{\rho_i \nu_t + \sqrt{1-\rho_i^2}\theta_{i,t} \leq \alpha_i\} \tag{6.11}$$

where ν_t and $\theta_{i,t}$ are both $N(0,1)$, ν_t are independent across t and $\theta_{i,t}$ are independent across both i and t. ρ_i is the parameter that captures the dependence among $\delta_{i,t}$'s across i. Finally, parameter α_i is set such that the unconditional trading probability is equal to p_i. Given that ν_t and $\theta_{i,t}$ are both $N(0,1)$ and independent, $\rho_i \nu_t + \sqrt{1-\rho^2}\theta_{i,t}$ also has $N(0,1)$ distribution. Hence α_i should be set according to:

$$\alpha_i = \Phi^{-1}(p_i) \tag{6.12}$$

where $\Phi(\cdot)$ is the CDF for $N(0,1)$ distribution. For our derivation, we will also need the conditional trade probability condition on the realization of the common factor given by ν. This conditional probability, denoted by $P_i^{\rho_i}(\nu)$, is given by the following expression:

$$\begin{aligned} P_i^{\rho_i}(\nu) &= P\left(\delta_{i,t} = 1 | \nu_t = \nu\right)) \\ &= \Phi\left(\frac{\Phi^{-1}(p_i) - \rho_i \nu}{\sqrt{1-\rho_i^2}}\right) \end{aligned} \tag{6.13}$$

By definition of α_i as given in (6.12), we have:

$$E_\nu[P_i^{\rho_i}(\nu)] = p_i \tag{6.14}$$

Also define:

$$D_i^{\rho_i} = E_\nu[P_i^{\rho_i}(\nu)^2] \tag{6.15}$$

and the following indicator random variable:

$$X_{i,t}(k) = \delta_{i,t}(1-\delta_{i,t-1})\cdots(1-\delta_{i,t-k}) \tag{6.16}$$

It is easy to see that the observed return is given by the following expression:

$$r_{i,t}^o = \sum_{k=0}^{\infty} X_{i,t}(k) r_{i,t-k} \tag{6.17}$$

Portfolio Construction

Let I_p denote the set of securities held in portfolio p. Let N_p be the size of I_p. We assume that each portfolio consists of securities with a common trading probability and correlation factor, i.e., that $p_i = p_p$ and $\rho_i = \rho_p$ for all securities in portfolio p. We also assume that all securities are equally weighted in the portfolio.[6.11] The observed return of the portfolio is approximately equal to the average of the individual return, i.e.:

$$r_{p,t}^o \approx \frac{1}{N_p} \sum_{i \in I_p} r_{i,t}^o \tag{6.18}$$

The approximation is due to the fact that the returns are continuously compounded and the logarithm of the sum is not the sum of logarithms.[6.12] If the individual returns are small and not too volatile, such an approximation is most likely valid in most cases. Using (6.17) and (6.18), the portfolio return can be written as:

[6.11] This assumption can be relaxed by replacing certain expressions in the results with a properly weighted average of the parameter that is assumed to be common across all securities. Since this model is only used for illustrative purposes, we will make these assumption to facilitate the discussion.

[6.12] Alternatively, if we assume returns are *simple returns*, see Definition 1, this expression would be exact but we would have approximation error in (6.16). As we mentioned in the discussion after Definition 1, each of the two methods for turning a price time series into a return time series has an advantage in one type of aggregation; simple returns are easier to aggregate cross-sectionally to form a portfolio but compounded returns are easier to aggregate across time.

$$
\begin{aligned}
r^o_{p,t} &= \frac{1}{N_p}\sum_{i\in I_p} r^o_{i,t} \qquad \text{using 6.18} \\
&= \frac{1}{N_p}\sum_{i\in I_p}\left(\sum_{k=0}^{\infty} r_{i,t-k}X_{i,t}(k)\right) \qquad \text{using 6.17} \\
&= \frac{1}{N_p}\sum_{i\in I_p}\left(\sum_{k=0}^{\infty} (\mu_i + \beta_i m_{t-k} + \epsilon_{i,t-k})\, X_{i,t}(k)\right) \qquad \text{using 6.10} \\
&= \sum_{k=0}^{\infty}\sum_{i\in I_p}\frac{1}{N_p}(\mu_i + \beta_i m_{t-k} + \epsilon_{i,t-k})\, X_{i,t}(k) \\
&= \sum_{k=0}^{\infty}\left(\frac{1}{N_p}\sum_{i\in I_p}\mu_i X_{i,t}(k) + m_{t-k}\frac{1}{N_p}\sum_{i\in I_p}\beta_i X_{i,t}(k) + \underbrace{\frac{1}{N_p}\sum_{i\in I_p}\epsilon_{i,t-k}X_{i,t}(k)}_{\stackrel{a.s}{\to}0}\right) \\
&\stackrel{a.s}{\longrightarrow} \sum_{k=0}^{\infty}\left(\frac{1}{N_p}\sum_{i\in I_p}\mu_i X_{i,t}(k) + m_{t-k}\frac{1}{N_p}\sum_{i\in I_p}\beta_i X_{i,t}(k)\right) \qquad (6.19)
\end{aligned}
$$

The last expression is *almost-sure* convergence which holds because $\epsilon_{i,t-k}$ are assumed to be cross-sectionally uncorrelated with zero common mean. The inner sums in expression (6.19) are the weighted cross-sectional averages of $X_{i,t}(k)$, which, as defined in (6.11), are dependent through the common factor of ν_t. The final expression enables us to find various time series properties of the observed returns we are interested in. But in order to arrive at the general result, we will take all expectations in two steps by first conditioning everything on the realization of ν_t's and then, in the second step, taking the expectation over values of ν_t's.

The first part of the above two-step approach involves taking the expectation over individual security $\theta_{i,t}$'s. The resulting expressions will be a function of the ν_t's as given here:

$$
\begin{aligned}
\frac{1}{N_p}\sum_{i\in I_p}\mu_i X_{i,t}(k) &\stackrel{a.s.}{\longrightarrow} \mathrm{E}_\theta\Big[\frac{1}{N_p}\sum_{i\in I_p}\mu_i X_{i,t}(k)\Big] \\
&\stackrel{a.s.}{\longrightarrow} \mu_p P_p^{\rho_p}(\nu_t)\left(1 - P_p^{\rho_p}(\nu_{t-1})\right)\cdots\left(1 - P_p^{\rho_p}(\nu_{t-k})\right) \\
&\stackrel{a.s.}{\longrightarrow} \mu_p Y_t^{\rho_p}(\vec{\nu}, k) \qquad (6.20a) \\
m_{t-k}\frac{1}{N_p}\sum_{i\in I_p}\beta_i X_{i,t}(k) &\stackrel{a.s.}{\longrightarrow} \mathrm{E}_\theta\Big[m_{t-k}\frac{1}{N_p}\sum_{i\in I_p}\beta_i X_{i,t}(k)\Big] \\
&\stackrel{a.s.}{\longrightarrow} \beta_p m_{t-k} Y_t^{\rho_p}(\vec{\nu}, k) \qquad (6.20b)
\end{aligned}
$$

where $\vec{\nu}$ is a vector that contains the ν_t's and

$$\mu_p = \frac{1}{N_p}\sum_{i\in I_p}\mu_i \quad (6.21a)$$

$$\beta_p = \frac{1}{N_p}\sum_{i\in I_p}\beta_i \quad (6.21b)$$

$$Y_t^{\rho_p}(\vec{\nu},k) = P_p^{\rho_p}(\nu_t)\left(1-P_p^{\rho_p}(\nu_{t-1})\right)\cdots\left(1-P_p^{\rho_p}(\nu_{t-k})\right) \quad (6.21c)$$

Using (6.20a) and (6.20b), expression (6.19) can be written as:

$$r_{p,t}^o = \sum_{k=0}^{\infty}(\mu_p+\beta_p m_{t-k})\,Y_t^{\rho_p}(\vec{\nu},k) \quad (6.22)$$

where, with a small abuse of notation, we have replaced the almost sure convergence with an "=" sign. Now, we can take the expectation over $\vec{\nu}$ to arrive at various time series properties of $r_{p,t}^o$. The following proposition summarizes the time series properties of this model:

Proposition 6.3 *Under the partially observed price signal model proposed in this section, the return of the equally weighted portfolio has the following time series statistics:*

$$E[r_{p,t}^o] = \mu_p \quad (6.23a)$$

$$Var(r_{p,t}^o) = \frac{2\mu_p^2(D_p^{\rho_p}-p_p)+\sigma^2\beta_p^2D_p^{\rho_p}p_p}{(2p_p-D_p^{\rho_p})p_p} \quad (6.23b)$$

$$Cov(r_{p,t}^o,r_{p,t+1}^o) = \frac{\mu_p^2(p_p^2-D_p^{\rho_p})+\sigma^2\beta_p^2(p_p^2-p_pD_p^{\rho_p})}{2p_p-D_p^{\rho_p}} \quad (6.23c)$$

$$Corr(r_{p,t}^o,r_{p,t+1}^o) = \frac{p_p\left((D_p^{\rho_p}-p_p^2)\zeta_a^2-(p_p^2-p_pD_p^{\rho_p})\right)}{(2p_p^2-2D_p^{\rho_p})\zeta_a^2-p_pD_p^{\rho_p}} \quad (6.23d)$$

$$Cov(r_{p,t}^o,r_{p,t+n}^o) = (1-p_p)^{n-1}Cov(r_{p,t}^o,r_{p,t+1}^o) \quad (6.23e)$$

$$Corr(r_{p,t}^o,r_{p,t+n}^o) = (1-p_p)^{n-1}Corr(r_{p,t}^o,r_{p,t+1}^o) \quad (6.23f)$$

where

$$\zeta_a = \frac{\mu_p}{\beta_p\sigma} \quad (6.23g)$$

Proof: See Appendix A.4.6.

The above proposition captures the two possibilities, for a single security and for a portfolio of securities, that was considered in Lo and MacKinlay (1990a) as special cases. The next two corollaries summarize these special cases.

Corollary 6.2 *If the non-trading events are completely correlated across the N securities, the entire portfolio behaves similar to a single security in terms of the non-trading events. The first order serial correlation is given by:*

$$Corr(r^o_{p,t}, r^o_{p,t+1}) = \frac{-\mu_p^2(1-p_p)}{\sigma^2\beta^2 + 2\frac{1-p_p}{p_p}\mu_p^2} \tag{6.24}$$

Proof: This special case can be captured by setting for $\rho_p = 1$. *In this case, we have* $D^1_a = p_p$. *Substituting this in (6.23d), the result follows immediately. This is the same as the result in Lo and MacKinlay (1990a) equation 2.23.*[6.13]

Corollary 6.3 *For the case that the non-trading events are completely uncorrelated across the N securities, the first order serial correlation is given by:*

$$Corr(r^o_{p,t}, r^o_{p,t+1}) = 1 - p_p \tag{6.25}$$

Proof: This special case can be captured by setting for $\rho_p = 0$. *In this case, we have* $D^1_a = p^2_p$. *Substituting this in (6.23d), the result follows immediately. This is the same as the result in Lo and MacKinlay (1990a) equation 2.26.* [6.14]

The above expression indicates that the observed value of the first order serial correlation is equal to 1 minus the trading probability. For example, if a portfolio contains securities that have a 20% chance of not trading in each time interval (so the trading probability is 80%), the theoretical value of the first order serial correlation under this model for the case where the trading events are completely uncorrelated will be 1 − 80% = 20%. Of course, this is simply one extreme of the range of possibilities encompassed in this model. Figure 6.2.1 more fully captures the magnitude of the first order serial correlation as a function of the trading probability for different values of the cross-sectional non-trading correlation parameter, ρ, for different values of $\zeta_a = \frac{\mu_p}{\beta_p \sigma}$.

Note that for a given level of ζ_a and non-trading probability, the observed first order serial correlation is a decreasing function of the cross-sectional non-trading correlation parameter, ρ. This is to be expected. For example, in Corollary 6.2 we showed that in the case where the non-trading events are fully dependent, this model reduces to the case of a single security in Lo and MacKinlay (1990a) and the serial correlation expression given in Corollary 6.2, which is negative for all parameter values.

6.3 Why Is This Important?

So far in this chapter we have documented the usually high level of serial correlation among hedge funds. The model analyzed in Section 6.2.1 linked this serial correlation to the issue of *observability* of the underlying signals. We showed that in cases where the underlying

[6.13]Note that Lo and MacKinlay (1990a) develop the expressions based on the *non-trading probability.* So p_i in their expression is equal to $1 - p_p$ in our expression here. Also, since they develop the expression for a single security, there is no concept of β; i.e., σ_i in their expression is equal to $\beta_p \sigma$ in our expression.

[6.14]Note that Lo and MacKinlay (1990a) develop the expressions based on the *non-trading probability.* So p_i in their expression is equal to $1 - p_p$ in our expression here.

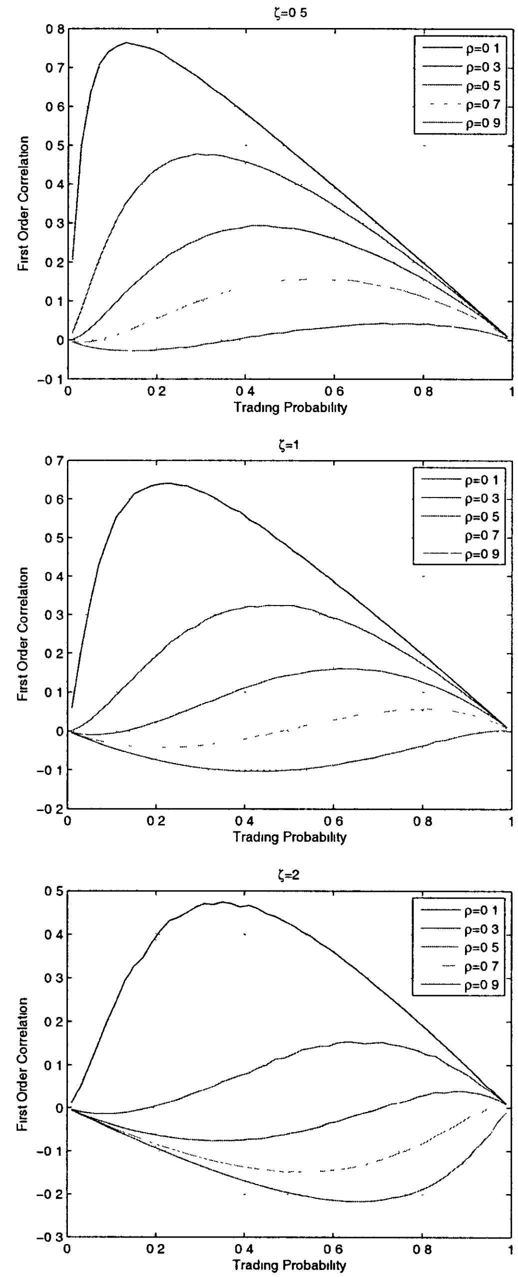


Figure 6.5: This figure shows the magnitude of the first order serial correlation of observed return on the non-trading model discussed in Section 6.2.1. The three plots are for different values of the $\zeta_a = \frac{\mu_p}{\beta_p \sigma}$. Each line shows the magnitude for a different value of the ρ parameter. Recall that this parameter controls the degree of correlation between non-trading events subject to a fixed unconditional trading probability. This probability is shown on the horizontal axis here in each plot.

assets don't trade frequently, the process of using *old* price signals in calculating the value of a portfolio of assets will result in a perceived predictability in the changes of the calculated portfolio value. The reader may wonder why the analysis presented here is important. Aside from the interest in analyzing the unexpected observation among hedge fund returns and understanding the frictions involved, the discussion of this section is also interesting as it applies to developing a better understanding of the drivers of the returns, i.e., appreciation in prices, in the long run. The next chapter is dedicated to analyzing the effect of this issue on the long-term price appreciation. But before we can do that, we need to provide some background.

In general, one of the important problems in finance is understanding the drivers of the expected returns. From a statistical perspective, this is done by linking the expected return to other observable characteristics of the security of interest. One of the first models proposed for this purpose is the Noble Prize winning Capital Asset Pricing Model (CAPM). This model creates a link between the expected return of security, i, and the covariance of the return of the security with a benchmark return with the following form:

$$\mathrm{E}[r_i] = \beta_i \lambda_m \tag{6.26a}$$

$$\text{where,} \qquad \beta_i = \frac{\mathrm{Cov}(r_i, r_m)}{\mathrm{Var}(r_m)} \tag{6.26b}$$

and r_m is the return of the market portfolio (usually taken as the portfolio of all stocks) and λ_m is a constant referred to as the market risk premium.[6.15] This model is remarkable because it indicates that the expected return should be linear in the covariance of the return of that security with the return of the market. But what is the market portfolio?

The CAPM is an equilibrium model which uses the fact that all risky assets, such as stocks, need to provide enough compensation for the investors so that in aggregate investors are happy holding those assets. When formulated properly, this idea guides us in selecting the market portfolio. According to this approach, the market portfolio should be the portfolio of *all* risky assets available to the investors when each of them is weighted by its market value. For example, the S&P 500 Index, which is a market weighted index of the 500 largest stocks in the US, is a commonly used proxy. The link between expected return and covariance with the market return can as be written in the form of a linear regression as follows:

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \epsilon_{i,t} \tag{6.27}$$

The CAPM model predicts that $\alpha_i = 0$. Hence, the part of the return that is perpendicular to the market return, i.e., $\alpha_i + \epsilon_{i,t}$, should have a zero-mean.[6.16] In a more abstract

[6.15]In reality, the model requires all returns to be measured in *excess* of the risk-free rate. While such considerations are essential in theoretical analysis and empirical testing, for the purpose of this discussion, and to keep the notation simple and clean, we will not write the return in the excess form.

[6.16]The CAPM is developed by making assumptions either about the utility function of the agents in the system or about the distribution of the returns. In the second method for deriving CAPM, it is needed to assume that returns are normally distributed. See any Financial Economics textbook for a treatment of and statistically friendly notation, the objective in this type of analysis is to find a set of characteristics as well as a functional form such that:

$$E[r_i] = f(\text{Characteristics of asset i}) \tag{6.28}$$

For example, although developed from rigorous theoretical characterization of optimization by agents and return characteristics, the CAPM implies that the only important characteristic is $\beta_i = \text{Cov}(r_i, r_m)/\text{Var}(r_m)$ and the $f(\cdot)$ is a linear function.

If serial correlation is in fact a by-product of lack of trading of the underlying security, then it would be reasonable to suspect that funds that tend to hold assets that are harder to trade, i.e., the more illiquid assets, tend to have higher average return relative to funds that hold easier to trade assets. The objective in the next chapter will be to test this hypothesis. We will do this by grouping funds used in this chapter into five clusters based on the estimated serial correlation and then conducting statistical tests about the difference in the long-term return of the portfolio with the highest serial correlation minus the portfolio with the lowest serial correlation.

6.4 Chapter Conclusions

We changed the attention in this chapter to the sources of predictability at monthly time scale. We started by documenting a peculiar observation that returns of certain investments, such as hedge funds, tend to display extremely high levels of predictability. We argued that this is in contrast with the general knowledge that hedge fund managers tend to be "smart" in using their informational advantage and, hence, that their returns should be unpredictable. We then proposed a simple model that generated predictability in observed returns of a portfolio based on the idea that if some of the securities held in the portfolio are not traded in every time interval, such as a month, the price of those securities used for calculating the value of the portfolio will be stale. The model proposed in this section for how this pricing process takes place is simple enough to be analytically tractable. As we discussed, this model can produce positive or negative serial correlation at the portfolio level depending on the structure of the cross-sectional dependence between how securities are traded.

We closed this chapter by discussing the general problem of analyzing the long term drivers of expected returns for financial prices. The final chapter of this thesis will elaborate the link between serial correlation of returns and the long-term expected returns.

CAPM and other equilibrium models.

Chapter 7

Linking Long-Term Deviation from White Noise to the Signal Mean

This chapter builds upon the issue of unusually high predictability among hedge funds that we disused in the last chapter. As we discussed in Section 6.3, if predictability is a by-product of unobservability of the underlying prices due to lack of transaction, one would expect that assets with higher serial correlation of returns are more *illiquid* in a sense. We are interested to see if there is any relationship between the serial correlation of returns, ρ_i, and the expected returns, $E[r_i]$, i.e.:

$$E[r_i] \stackrel{?}{=} f(\rho_i, \text{other characteristics of asset i}) \tag{7.1}$$

We will analyze this question using a variety of techniques based on the returns of a large sample of hedge funds, mutual funds, and US common stocks. For the case of hedge fund return data, we will use the TASS Hedge Fund Database. This is the same source of data that was used in the last chapter, details of which are discussed in Appendix A.4.1. We will also continue to use the CRSP Mutual Fund Database to obtain mutual fund return data. The details of this data source are discussed in Appendix A.4.2. We will not use Money Market mutual funds or mutual funds for which the hypothesis of unit-root was not rejected based on the statistical tests used in Section 6.1.3. We will also use the returns of 100 standard stock portfolios to extend the reach of our coverage. Stock portfolios, instead of individual stocks, are used to reduce the noise in our input data. Additionally, these 100 portfolios are one of the standard benchmarks used in this type of analysis. The details of this source are discussed in Appendix A.5.1. provides some additional analysis on the link between serial correlation of returns in various asset classes and their level of illiquidity.

This chapter is structured as follows. Section 7.1 In particular, in the case of hedge funds we have access to other measures of illiquidity, such as the Lockup Period or the redemption notice period,[7.1] which can be used to verify that serial correlation is indeed

[7.1] These are the restrictions commonly enforced on the investors of hedge funds by the fund managers which are commonly believed to associated with the level of illiquidity of their underlying asset. We will discuss this further in Section 7.1.

a proxy for the illiquidity of the underlying assets held by a fund. Section 7.2 discusses our methodology for analyzing the link between serial correlation and expected returns, i.e., our approach for calibrating a model for the generic relationship expressed in (7.1). We will outline an approach based on grouping individual funds or stock portfolios into clusters using their estimated serial correlation. The difference between the expected returns of different clusters can then be used to estimate the premium for holding the more illiquid assets. We also discuss two methods for taking into account *other characteristics* that can contribute to the return differential expressed in (7.1) by *adjusting* the returns for such characteristics. Looking at the *adjusted returns* will increase the ability of our approach to detect smaller return differentials by reducing the noise in our model. We will elaborate on these claims when we outline our approach in Section 7.2. Sections 7.3 and 7.4 apply this general method to our data set. Section 7.3 contains the results based on raw returns and we will report the results based on the adjusted returns in Section 7.4. In Section 7.5, we will develop this analysis further by looking at this measure of illiquidity among different categories of hedge funds. In Section 7.6, we will look at the evolution of this premium during the later part of our sample period, years 1998 to 2006. We will be able to draw some interesting insight on the effect of macro-economic variables on this premium. We will conclude in Section 7.7.

7.1 Motivation

We will start by looking at hedge funds and use this to promote our proposed measure of liquidity. In the case of hedge funds, we have access to some auxiliary variables that can be used to evaluate the liquidity of investment strategies followed by a given fund. For example, Liang (1999) uses the "Lockup Period," i.e., the number of days since the initial investment for which the investor's shares are "locked up" and cannot be redeemed, as a proxy for the liquidity of different hedge funds. In a more recent study, Aragon (2007) uses both the Lockup Period and the Redemption Notice Period, as the controlling variables to account for different liquidity characteristic of hedge funds. To give the reader a sense for the variability of these measures in our data, we have provided the average values and some distribution characteristics of these two measures for different categories of hedge funds in our data set in Table 7.1.[7.2] Each part of this table is sorted by the relevant measure in descending order. One can observe that categories such as Event Driven or Convertible Arbitrage appear near the top in both lists, while categories such as Managed Futures and Global Macro are at the bottom. Note that a substantial percentage of funds in each category have no Lockup Period, and for such funds, the redemption notice period is the only factor imposing liquidity constraint on the invertors. Lookup periods of more than 1 year are very rare, and hence, the redemption notice period is perhaps the primary liquidity constraint on the investors of most active hedge funds, although an investor would clearly consider both constraints in his or her decision to invest in a given fund. Although it possible to use these measures to proxy for the liquidity of a given hedge fund, our hope here is to create a liquidity measure that can be estimated directly based on observed returns and, hence, can be used to measure the liquidity of mutual funds and portfolios of stocks.

[7 2]See Appendix A.4.1 for details of our data set and the definition of hedge fund categories.

Table 7.1: This table shows the vast difference between the Redemption Notice Period and the Lockup Period among different categories of hedge funds. The data is sorted by the average of the relevant measure in each part. For the definition of these categories please see Appendix A.4.1.

Panel A: Redemption Notice Period

Category	***Count***	***Average Redemption Notice Period in Days***	***Redemption Distribution***		
			<10	***10-30***	***>30***
Event Driven	254	50	10%	33%	57%
Fund of Funds	631	40	16%	26%	58%
Convertible Arbitrage	101	37	15%	47%	39%
Fixed Income Arbitrage	108	34	29%	33%	38%
Multi-Strategy	133	34	20%	44%	35%
Equity Market Neutral	153	32	16%	50%	33%
Long/Short Equity Hedge	906	31	15%	59%	26%
Emerging Markets	182	27	32%	43%	24%
Dedicated Short Bias	25	25	28%	60%	12%
Global Macro	126	20	33%	55%	13%
Managed Futures	308	8	62%	34%	4%

Panel B: Lockup Period

Category	***Count***	***Average Lockup Period in Months***	***Lockup Distribution***		
			None	***Up to 1 Year***	***More***
Event Driven	254	5 4	60%	35%	6%
Long/Short Equity Hedge	906	4 4	65%	32%	3%
Convertible Arbitrage	101	3.1	74%	25%	1%
Multi-Strategy	133	2.8	74%	25%	2%
Equity Market Neutral	153	2 5	78%	20%	1%
Emerging Markets	182	2.1	84%	13%	3%
Fixed Income Arbitrage	108	1.9	82%	17%	1%
Dedicated Short Bias	25	1.9	80%	20%	0%
Fund of Funds	631	1.9	86%	12%	1%
Global Macro	126	1 0	93%	6%	1%
Managed Futures	308	0 5	96%	4%	0%

Following the argument of Getmansky et al. (2004), we take the view that the observed autocorrelation in returns is a by-product of difficulty to correctly price the different assets in a portfolio, and the "claimed" returns of any portfolio with this issue would tend to exhibit higher autocorrelation values. To make this more clear, and to provide the reader with some evidence on the universality of this issue, Table 7.2 shows the characteristics of the monthly returns for several representative equity and fixed income factors.[7.3] We have also shown the p-value for Ljung and Box Q-statistic[7.4] for the joint significance of the first three sample autocorrelation values. Looking at the returns of equity indexes, the hypothesis of no autocorrelation cannot be rejected for the indexes that include the largest and the most liquid set of assets, such the S&P 500 and the Wilshire 750 Large Cap indexes. The story is different among smaller stocks in the same market, captured by the S&P 600 Small Cap or the Wilshire 1750 Small Cap index, for which the null of no autocorrelation can be rejected at 5% significance level. In order to highlight the difference between the predictability of tradable and non-tradable assets, we have presented the return for two emerging market indexes. The first index, the S&P/IFC Emerging Markets Composite Index, is simply a "tracking" index, while the second index, the S&P/IFC Emerging Markets Investable Composite, is an "investable" index. Observe that the null hypothesis can be easily rejected for the "tracking" index while the null cannot be rejected in the case of the "investable" index. This observation argues that the process of pricing the portfolio involved in the investable index is more efficient, or perhaps that this index invests in a more liquid set of assets relative to the set of assets tracked by the first index. So even the simple Ljung and Box Q-statistic verifies the validity of the Samuelson (1965) argument regarding the lack of predictability in the returns for investable assets. The trend is similar among fixed income factors where the Q-statistic comes close to the critical value for rejecting the null, but this hypothesis cannot be rejected for the indexes tracking the more liquid US government bond securities. Again, lack of predictability can be rejected in the case of indexes tracking the corporate bond and mortgage-backed securities which are more illiquid. This observation suggests that the serial correlation is quite common, and not unique to hedge funds.

As mentioned before, the unique set of information available for hedge funds enables us to evaluate the degree to which serial correlation is a proxy for illiquidity of hedge funds.

[7.3] Data for all these factors is obtained from the Global Financial Database. All series are total returns and based on monthly frequency. Data from January 1986 to December 2006 was used when available to maximize the overlap with the data used in the rest of our study. Wilshire 1750 Small Cap was only available until March 2006, the Merrill Lynch Mortgages Index was available until February 2004, and the S&P/IFC Emerging Markets Investable Composite was available starting in January 1989.

[7.4] Pierce and Box (1970) proposed the following statistic to test the significance of the first k autocorrelation values

$$Q_m = T \sum_{k=1}^{m} \rho^2(k)$$

Under the null hypothesis of no autocorrelation, this statistic is asymptotically distributed as χ^2_m. Ljung and Box (1978) proposed the following finite-sample correction, which provides a better fit to the χ^2_m for small samples sizes:

$$Q_m = T(T+2) \sum_{k=1}^{m} \frac{\rho^2(k)}{T-k}$$

Table 7.2: Return statistics for several representative set of equity, fixed income and emerging market indexes. To maximize the overlap with the data used in the rest of this study, returns from January 1986 to December 2006 were used when available. See footnote 7.3 for the exact time periods. All values are based on monthly returns and not annualized.

Index Name	*Mean*	*StDev*	*Skewness*	*Kurosis*	*Rho_1*	*Rho_2*	*Rho_3*	*Q-Statistic (3 Lags) q-value*	*p-value*
S&P 500 Large Cap	1.1%	4.3%	-0.832	5.964	-1.3%	-3.8%	-1.0%	0.41	94%
S&P 400 Mid Cap	1.3%	4.8%	-0.862	6.210	6.4%	-8.5%	-9.1%	5.02	17%
S&P 600 Small Cap	1.0%	5.3%	-1.188	7.619	11.3%	-3.8%	-13.7%	8.33	4%
Wilshire 5000	1.0%	4.4%	-1.032	6.554	3.6%	-5.0%	-3.8%	1.31	73%
Wilshire 750 Large Cap	1.0%	4.4%	-0.840	5.716	0.5%	-5.1%	-1.4%	0.72	87%
Wilshire 1750 Small Cap	1.1%	5.4%	-1.060	6.842	13.3%	-6.2%	-12.0%	8.74	3%
S&P/IFC Emerging Markets Composite Global	1.0%	6.4%	-0.571	4.704	17.4%	9.0%	-4.2%	10.16	2%
S&P/IFC Emerging Markets Investable Composite	1.1%	6.5%	-0.570	4.731	13.9%	6.0%	-2.8%	5.18	16%
US Gov - 5 Year	0.6%	1.4%	-0.084	3.005	14.4%	-6.6%	1.3%	6.65	8%
US Gov - 10 Year	0.7%	2.2%	-0.019	3.425	9.3%	-11.1%	0.2%	6.17	10%
US Gov - 30 Year	0.8%	3.3%	0.144	3.943	6.9%	-11.4%	3.2%	5.92	12%
US AAA Corp Bond Index	0.8%	1.5%	-0.091	4.269	15.5%	-6.6%	-2.1%	8.00	5%
Merrill Lynch Mortgages Index	0.7%	1.1%	-0.159	4.074	15.1%	-11.7%	-2.2%	8.43	4%

Comparing the data provided in Table 7.1 with the data provided in Table 6.1 of Section 6.1.3 is supportive of the general notion that serial correlation in returns and illiquidity of the underlying assets are closely related. For example, Convertible Arbitrage and Event Driven, which appear near the top of both lists in Table 7.1, have the two highest average first order serial correlation values at 38.3% and 22.7%, respectively. Similarly, the Managed Futures and Global Macros funds have the two lowest values of average serial correlation in their returns, at 0.4% and 7.7%, respectively, and they also appear at the bottom of both lists in Table 7.1.

To make this claim more statistically rigorous, we have fitted a linear model to link the redemption notice period of a fund, which as argued above is the most relevant measure of the illiquidity for most hedge funds, to the first order serial correlation. Table 7.3 shows the estimates for the cross-sectional regression of

$$\text{redemption}_i = \hat{\alpha} + \rho_{1,i}\hat{\lambda} + \epsilon_i$$

for all hedge funds in our data set as well as for different categories separately. The point estimate of the linear regression coefficient, $\hat{\lambda}$, is positive and statistically significant when all funds are used. Looking at the details of the relationship in each of the 11 categories, we can see that in all but one category the point estimate is positive. It is also statistically significant at 5% level in 7 out of the 11 categories. The clear relationship observed in Table 7.3 gives us more confidence that the observed autocorrelation of returns as a valid proxy for illiquidity of the assets involved in the investment.

With the above argument in place, we now have more confidence to use serial correlation to evaluate the illiquidity of mutual funds and portfolios of US stocks for which other measure such as the Redemption Notice Period or Lock-Up Period are not relevant.

Table 7.3: Estimates of the cross-sectional regression of the form redemption$_i = \hat{\alpha} + \rho_{1,i}\hat{\lambda} + \epsilon_i$ for all hedge funds as well for each of the 11 categories. Redemption period is measured in days. The t-stat are reported in parenthesis.

Category	*Count*	*Alpha*	*T-stat*	*RSQ*
All	2927	25.8(40 6)	42 1(14 8)	7.0%
Convertible Arbitrage	101	26.0(4.64)	28.5(2.14)	4 4%
Dedicated Short Bias	25	21.9(4.42)	35.7(1.11)	5.2%
Emerging Markets	182	21.6(6 18)	31 8(1.88)	1.9%
Equity Market Neutral	153	27.9(12.9)	37 0(3.97)	9 5%
Event Driven	254	42.7(11.4)	31 6(2.31)	2.1%
Fixed Income Arbitrage	108	27 6(7 25)	34.1(2.51)	5 6%
Fund of Funds	631	32 9(18.5)	39.7(5.38)	4 4%
Global Macro	126	19 4(10.4)	12.4(1.03)	0 9%
Long/Short Equity Hedge	906	31 7(34 7)	-3 7(-0.7)	0 1%
Managed Futures	308	8.22(10.6)	16.6(2 53)	2.1%
Multi-Strategy	133	24 1(7.21)	55.4(4.11)	11.5%

7.2 Methodology

This section outlines our approach for testing the link between serial correlation and the expected returns. In mathematical terms, we are interested in identifying if there is a relationship such as:

$$\mathrm{E}[r_i] \stackrel{?}{=} \mathrm{f}(\rho_i, \text{other characteristics of asset i}) \tag{7.2}$$

Our approach is based on grouping funds into clusters based on their serial correlation. The average return of the funds in each of the resulting cluster can then be used to assess if there is any link between serial correlation, and hence the level of the illiquidity of the underlying assets, and average returns.

This approach has the advantage that it is non-parametric and does not enforce any particular form, such as linear, between the level of liquidity, as measured by serial correlation of returns, and the expected returns. In other words, by using a clustering-based approach we avoid defining any particular functional form for f($\cdot$) in (7.2). Furthermore, in the most basic approach that will be outlined shortly, we even avoid defining "other characteristics" in (7.2). The hope here is that the clusters we create are similar along all dimensions that can contribute to the return differential other than the level of serial correlation. Given this and if we have enough entries in each cluster such that the noise is "washed out," the average return of the entries in each cluster would give us a direct measure of the impact of serial correlation on the returns.

One shortcoming of this approach, like any other non-parametric technique, is that the approach may not be very precise and will be affected by the noise in the data; i.e., even if the entries in each cluster are uniform along all other dimensions that contribute to the expected returns other than the serial correlation, their return may be too noisy and the number of funds in each cluster may be too few to achieve enough diversification in each cluster such that the calculated average can be meaningful. Another shortcoming is that this approach may lack power if serial correlation is correlated with another "characteristic" that contributes to the expected returns. Under that circumstance, this approach will not be able to separate the effect of serial correlation from the effect of this other characteristic.

Addressing these shortcomings is difficult as it requires putting more structure on the form of f(·) in (7.2), which would require identifying the list of "other characteristics" that contribute to the expected returns and put structure on how those characteristics are related to expected returns. Below we will outline two alternative approaches for addressing these issues.

Detailed Approach

We will use non-overlapping periods to estimate the serial correlation and then to measure the average return to avoid any data mining or data snooping in our analysis. More specifically, our approach is as follows. We use the serial correlation estimated over the prior five years to rank available funds into five portfolios on January of each year. The first such ranking is constructed for January of 1991 since we use the data starting from January 1986 and the first time that five years of data is available is January of 1991. We then calculate the equal weighted *raw* or *adjusted*, where adjustment is done based on various methods that will be explained shortly, returns of funds in each of these portfolios for each month of the subsequent year. The final result is 192×5 monthly data points for January 1991 to December 2006 and each of the five portfolios. The time series of the returns of these five portfolios, which we will refer to as the "liquidity portfolios," will serve as the input to much of our analysis. To get a more direct measure of impact of liquidity, we also use the time series of the difference between the return of the most liquid portfolio, i.e., funds in the lowest serial correlation bin, and the most illiquid portfolio, i.e., funds in the highest serial correlation bin. We refer to this portfolio as the "liquidity spread portfolio" in the discussion that follows.

We now explain our approach for calculating "adjusted" returns. We use two different approaches based on using the historical correlation between returns of each fund or each of the five liquidity portfolios, which are constructed based on the procedure outlined above, and a set of pre-specified time series. We first describe our approach and then discuss the time series used.

- **Time Series Adjustment Approach:**

  The monthly returns for each of the five liquidity portfolios are linked throughout different years creating a time series with 192 monthly returns (January 1991 to December 2006). We then treat each of the resulting time series as the realization of the return associated with a particular level of asset liquidities. We then run a regression of the following form:

$$r_{p,t} \;=\; \hat{\alpha}_p + \sum_f \hat{\beta}_{p,f} f_t + \hat{\epsilon}_{p,t} \;\; \forall t \in \{\text{January}_{1991}, \cdots, \text{December}_{2006}\} \qquad (7.3)$$

where the set of factors, i.e., f_t, in the above regression, are time series selected from a set of 9 generic risk factors listed in Appendix A.5.2. We will elaborate more on this shortly. By using this regression we are trying to decompose the in-sample average return of each of the portfolios into components in the direction of each of the specified factors plus a residual average return. The resulting regression coefficients provide a measure of the sensitivity of the returns to each of the specified factors. Alpha from this regression provides a measure of adjusted returns. The same analysis is also repeated for the returns of the liquidity spread portfolio described above.

- **Residual Adjustment Approach:**

  While the above approach is ideal for addressing the most immediate question regarding the link between adjusted returns and liquidity, it suffers from two shortcomings. First, by conducting one time-series regression, we implicitly assume that the loading on various factors, i.e., $\hat{\beta}_{p,f}$ in (7.3), are constant through time. Qualitatively, this is similar to assuming that fund's characteristics are not changing through time. Such assumptions are common in the case of stock portfolios but may be less reasonable for hedge funds. Furthermore, obtaining a single adjusted return does not enable us to assess the change in the liquidity premium through time.

  We use a second adjustment approach to address these shortcomings. In this approach, we use the prior five years of returns to calibrate an optimal linear predictor for the return of each fund based on a pre-specified set of factors. The estimated betas and the realization of the factors in each month of the following year are used to calculate the prediction error or what we will be referring to as the "residual returns" for each fund. The following two questions specify this approach in mathematical terms:[7.5]

$$r_{i,t} = \hat{\alpha}_i + \sum_f \hat{\beta}^m_{i,f} f_t + \hat{\epsilon}_{i,t} \;\; \forall t \in \{\text{January}_{m-5}, \cdots, \text{December}_{m-1}\} \qquad (7.4a)$$

$$\tilde{r}_{i,t} = r_{i,t} - \sum_f \hat{\beta}^m_{i,f} f_t \;\; \forall t \in \{\text{January}_{m}, \cdots, \text{December}_{m}\} \qquad (7.4b)$$

  The equal weighted average residual returns for the funds in a given liquidity portfolio provide us with 192×5 monthly data points for January 1991 to December 2006 and each of the five portfolio. The average of each thess five time series provide a measure similar to the alpha calculated from the first approach. But the actual realization can also be used, as done in Section 7.6, to see the evolution of the liquidity premium.[7.6]

[7.5]Note that in (7.4b) we have enforced the assumption that the alpha from the first regression is zero and only used the estimated betas in calculating the residual returns.

[7.6]Of course the residual from (7.3) can also be used as a measure of liquidity premium for each month. Our approach is preferable as it uses two non-overlapping time periods for estimation of betas and the subsequent residual calculation.

Risk Factors

We use various subsets of 9 factors to control for the risk exposure of different funds.[7.7] The detailed description of these factors is provided in Appendix A.5.2, but we give a summary here to preserve the continuity of the exposition. The factors we used are as follows: *US Stock Market Index*, the *Lehman Brothers US Aggregate Government Bond Index*, the *Lehman Brothers Universal High-Yield Corporate Index*, the *Goldman Sachs Commodities Index*, *USD Trade Weighted Dollar Index*, the *CBOE Volatility Index Fama-French Small Minus Big (SMB)*, *Fama-French High Minus Low (HML)*, and the stock market *Momentum* factor.

The first five factors capture the broad sources of commonality due to equities, fixed income, credit, commodities and the currency markets. We also use three factors related to size, measured by Fama-French Small Minus Big (SMB) factor, value, measured by Fama-French High Minus Low (HML) factor, and the stock market momentum since these factors have been studied extensively in asset pricing literature and are known to contribute to the expected returns.

We also include the first difference in the CBOE Volatility Index to capture any exposure to changes of market volatility that a particular fund may be exposed to. Even though this factor does not translate immediately to returns using any investment strategy, it can still add value to our analysis by capturing communality due volatility exposure of different funds arising from non-linear instrument included in some trading strategies. This effect should be more significant for hedge funds but we have decided to keep this factor in the rest of our analysis in order to keep our results consistent across different asset classes. This factor in its initial format has a much higher level of volatility than all of the other factors. In order to avoid any numerical issues arising from this substantial difference, we decided to use a rescaled version of this factor by rescaling the monthly values to set their in-sample level of volatility to be the same as volatility of US Stock Market factor. This is purely a rescaling and won't change any of our analysis.

To given the reader a sense of the robustness of our analysis to different ways of controlling for risk, we use 4 different sets of factors in the risk-adjustment approaches outlined above. These four factor sets are as follows:

1. **Market Only**: Only the return of the US Stock Market index is used.
2. **4-Factor Set**: The US stock market factor plus size (SMB), value (HML), and the momentum factors are used.
3. **Broad Factor Set**: All 9 factors listed in above.
4. **Lagged Market**: Current and one lagged return of the US Stock Market Index are used.

In the last approach for risk adjustment we control for both current and lagged exposure to the stock market factor. This approach was first promoted by Scholes and Williams

[7.7]For reference on common factors among hedge funds, please see Fung and Hsieh (2001) and Hasanhodzic and Lo (2007). For mutual funds see Sharpe (1992).

(1977) in trying to estimate the "total" betas of a portfolio as the sum of both current and lagged market exposure. We should emphasize that the lagged exposure to a risk factor is a result of the illiquidity of the underlying assets. By including the lagged beta, we try to account for some of the illiquidity of the underlying assets. While this may be adequate for assets that draw most of their illiquidity exposure in form of lagged exposure to a given factor, for example different portfolios of US common stocks or equity mutual funds, it will probably be inadequate in capturing the illiquidity for hedge funds that most likely have a more complicated illiquidity exposure. Nonetheless, we have included this in our analysis to provide additional detail and to give more confidence in the robustness of our approach.

Exploratory Analysis

Before moving to our main analysis, we provide some exploratory data on the exposure of various fund categories to the risk factors used in this study. Table 7.4 give a summary of time-series regressions of the form (7.4a). As expected, hedge funds have much lower R^2 values and have positive alphas.

Table A.5 shows that some of the factors used have a positive serial correlation. For example, the Lehman US Universal High-Yield Corporate Index has a first order serial correlation value of 37.5%. In addition, high serial correlation is, in a sense, similar to having momentum in returns. In order to explore the link between serial correlation and exposure to other risk factors, we calculate the correlation between the beta for each factor and the estimated serial correlation by treating the estimate for each estimation window, k, and fund, i, as an observation of a pair of random variables $[\rho_i^k, \beta_{i,f}^k]$. All such observations are then pooled together and the cross-sectional correlation is calculated and reported in Table 7.5. Note that funds with higher serial correlation also tend to have higher exposure to the Lehman US Universal High-Yield Corporate Index (shown under column LH_HY) in Table 7.5, to the size (shown under column SMB in Table 7.5). The data does not show any connection between the momentum factor (column UMD in Table 7.5) and the serial correlation. The table shows that there is a very high correlation between exposure to the size factor and serial correlation of returns among stock portfolios. For mutual funds, the size factor seems to have the highest correlation among Equities funds and the high-yield index seems to have the highest correlation among Fixed Income funds. The results are a little less clear for hedge funds and there is a lot of heterogeneity among different categories. This analysis highlights the importance of taking into account and adjusting for the level of exposure to these factors in order to obtain results that can be statistically meaningful and also to avoid false positive, i.e., to avoid contributing return differential due to exposure to the high-yield factor, for example, with the differential due to illiquidity.

Table 7.4: Summary of the estimated exposures to various risk factors based on all five-year estimation windows from 1986 to 2006. For each factor, the median as well as the 25 and 75 percentile values, in parenthesis, is shown. We have also included the same statistics for the constant in regression, *alpha*, and the resulting time-series R^2 value. The reported alpha values are multiplied by 10,000 (i.e., translated into basis points) and the factors' exposures are multiplied by 100 in order to make the estimated values easier to display. The number of observations for each fund type is reported. The abbreviated names for factors are as follows: US Stock Market (MARKT), Lehman US Aggregate Government Bond Index (LH_GO), Lehman US Universal High-Yield Corporate Index (LH_HY), Goldman Sachs Commodities Index (GSCI), Trade Weighted USD Index (USD), Rescaled CBOE Volatility Index (VIX_S) as well as Small-minus-Big (SMB), High-minus-Low (HML), and the Momentum (UMD) factors.

	Observations	Alpha (bps)	MARKT	LH_GO	LH_HY	GSCI	USD	VIX_S	SMB	HML	UMD	RSQ (%)
All Funds Used in the Study	97,511	-2 25(-18 20,16 43)	57 48(2 91,97 27)	12 36(-8 31,52 92)	2 94(-3 37,10 49)	0 13(-1 39,3 03)	-0 47(-5 76,3 88)	0 65(-3 30,5 55)	2 00(-2 16,16 30)	4 44(-1 62,16 59)	0 40(-3 09,4 95)	84 58(67 55,92 49)
Hedge Funds												
All Hedge Funds	11,666	43 90(7 33,82 71)	21 16(2 99,52 83)	4 62(-16 46,27 82)	4 40(-7 49,17 60)	1 97(-1 10,6 61)	4 40(-7 44,19 19)	5 31(-3 49,16 88)	9 95(1 34,23 50)	6 53(-3 09,20 00)	2 71(-2 70,11 23)	41 99(27 39,58 59)
Convertible Arbitrage Hedge Funds	408	64 71(36 77,92 21)	-0 75(-8 01,6 53)	-2 38(-16 87,7 69)	13 00(6 14,24 50)	0 57(-0 63,2 64)	2 51(-5 67,12 62)	0 51(-4 43,5 65)	4 24(1 05,10 06)	0 79(-3 30,5 10)	-0 84(-2 92,1 84)	32 77(21 46,42 47)
Dedicated Short Bias	119	61 23(33 77,122 33)	-108 73(-140 52,-50 30)	4 73(-13 03,30 31)	-1 16(-10 64,9 96)	0 56(-4 54,6 12)	-4 12(-14 99,6 96)	-1 56(-21 20,6 92)	-17 67(-57 84,-5 70)	18 42(1 87,42 51)	-6 26(-15 66,8 02)	73 47(65 99,80 41)
Emerging Markets	749	51 10(-38 31,111 39)	62 15(23 90,101 75)	-15 03(-119 78,24 56)	21 40(4 44,50 71)	3 35(-1 20,12 34)	15 11(-4 65,48 58)	2 43(-14 42,19 97)	16 70(4 70,32 51)	15 69(2 28,36 73)	4 73(-4 69,15 83)	43 34(30 43,55 58)
Long/Short Equity	3,409	50 83(8 35,95 27)	51 68(24 07,81 39)	3 57(-20 00,25 05)	0 13(-11 75,14 92)	2 34(-2 09,7 68)	5 57(-9 10,22 03)	8 79(-4 59,23 92)	21 30(7 07,41 56)	9 53(-10 10,30 47)	3 61(-5 70,15 80)	51 32(35 95,67 29)
Equity Market Neutral	445	45 49(20 90,88 38)	2 65(-2 86,16 29)	3 87(-7 84,16 88)	0 08(-5 17,7 65)	0 67(-1 59,3 21)	1 08(-9 36,8 80)	0 33(-4 59,7 13)	1 24(-4 83,9 82)	0 75(-5 75,11 53)	1 90(-2 33,8 40)	27 02(19 87,39 10)
Events Driven	1,138	63 03(36 62,88 24)	10 30(2 05,25 86)	-6 17(-21 65,6 52)	14 51(2 77,29 76)	0 71(-1 72,3 43)	3 62(-3 63,11 26)	1 36(-4 74,7 61)	7 73(2 79,16 06)	8 28(2 76,16 51)	0 89(-1 87,4 11)	42 73(29 82,56 08)
Fixed Income Arbitrage	379	54 34(24 89,82 89)	0 78(-4 06,6 86)	4 75(-11 67,22 31)	5 31(-1 42,16 94)	0 52(-1 03,2 75)	2 38(-1 92,8 28)	1 66(-2 88,8 26)	1 07(-1 78,4 64)	1 89(-2 00,9 06)	0 43(-1 62,3 14)	26 26(18 16,39 41)
Fund of Funds	2,514	29 64(7 15,56 02)	20 28(7 87,37 60)	4 84(-10 64,19 31)	5 80(-0 73,12 40)	2 38(0 46,5 18)	6 20(-2 92,18 74)	6 81(1 06,14 51)	9 44(3 57,17 30)	6 24(0 57,14 10)	5 28(0 56,11 10)	48 53(34 12,63 48)
Global Macro	467	42 50(-9 80,87 84)	11 28(-3 74,45 74)	20 54(-3 33,55 54)	3 42(-13 15,20 94)	0 87(-3 51,6 90)	3 85(-18 48,28 09)	3 51(-8 24,16 91)	7 73(-1 96,20 95)	10 23(-5 56,24 40)	3 15(-3 75,13 15)	30 14(20 01,45 93)
Managed Futures	1,469	19 40(-25 23,75 59)	6 60(-13 00,27 39)	61 62(7 96,114 71)	-13 38(-34 91,7 95)	6 37(-2 76,16 46)	-1 39(-28 19,26 18)	9 09(-5 98,25 11)	4 09(-15 37,18 62)	4 25(-16 13,19 74)	1 67(-7 71,16 60)	26 90(19 34,36 61)
Multi-Strategy	569	50 35(15 09,81 50)	15 74(2 05,47 89)	0 44(-14 20,15 26)	4 85(-3 10,16 00)	1 59(-0 46,4 42)	3 78(-7 44,12 96)	5 75(-2 15,16 88)	8 14(2 00,18 68)	4 46(-1 57,13 61)	2 29(-1 15,6 91)	41 17(25 34,57 03)
Mutual Funds												
All Mutual Funds	85,845	-4 38(-19 31,10 59)	65 85(2 91,99 20)	14 08(-7 34,55 09)	2 83(-3 02,9 72)	0 00(-1 41,2 53)	-0 65(-5 59,2 84)	0 43(-3 29,4 38)	1 48(-2 41,14 04)	4 24(-1 49,15 88)	0 28(-3 14,3 91)	87 04(72 78,93 18)
Asset Allocation Mutual Funds	6,172	-5 63(-16 88,5 00)	59 93(49 32,68 30)	21 52(10 74,31 19)	1 60(-2 06,6 73)	0 31(-0 76,1 76)	-1 42(-5 44,2 46)	0 56(-2 34,3 95)	-1 08(-6 27,4 40)	6 51(0 87,14 95)	-0 80(-4 34,2 31)	93 78(88 90,96 63)
Equities Mutual Funds	40,038	-6 09(-28 91,15 55)	99 07(86 95,112 82)	-4 30(-19 70,8 57)	-1 76(-9 34,7 25)	1 59(-1 57,5 79)	-0 92(-13 35,8 11)	1 88(-4 28,10 55)	10 84(-6 22,37 94)	10 34(-13 16,35 53)	0 89(-6 35,10 86)	89 28(81 07,93 70)
Fixed Income Mutual Funds	30,121	-2 78(-12 50,8 13)	1 62(-0 71,4 65)	61 47(43 79,73 03)	5 24(2 22,11 63)	-0 57(-1 38,0 22)	-0 41(-2 11,1 24)	-0 21(-2 76,1 87)	0 55(-1 17,2 46)	2 99(0 14,6 31)	0 26(-1 06,1 31)	79 96(68 26,90 35)
Convertible Bond Mutual Funds	562	0 82(-13 52,18 06)	63 23(54 10,72 15)	-0 19(-14 23,11 00)	14 78(7 36,21 98)	2 46(0 30,4 71)	-1 60(-7 50,2 82)	6 15(2 57,10 84)	18 56(14 67,26 22)	2 85(-5 12,14 35)	4 93(-1 55,12 76)	87 43(83 10,90 99)
Stock Portfolios												
100 Value Weighted Portfolios	1,681	4 34(-22 62,31 62)	103 90(89 94,118 05)	-0 81(-18 66,18 37)	-2 59(-15 06,9 31)	-0 46(-5 61,4 51)	1 92(-8 56,12 49)	0 05(-8 54,7 93)	58 65(19 36,93 09)	35 66(1 16,66 09)	-4 96(-13 09,4 05)	84 02(77 55,88 30)

Table 7.5: This table provides an overview of the link between serial correlation and exposure to risk factors for different funds. The values are calculated as follows. Let $[\beta^k_{i,f}, \rho^k_i]$ be the estimated exposure to factor f and the serial correlation for fund i over the estimation interval k. The cross-sectional correlation between $\beta^k_{i,f}$'s and ρ^k_i's is then estimated across all estimates for the funds in the specified subgroup. The actual number of observations used to calculate the correlation is reported. See the caption of Table 7.4 for the abbreviated name of different risk factors used here.

	Observations	MARKT	LH_GO	LH_HY	GSCI	USD	VIX_S	SMB	HML	UMD
All Funds Used in the Study	97,511	-7.4	-11.23	10 32	-0 6	3.4	13.24	15.51	0 78	-1 75
Hedge Funds										
All Hedge Funds	11,666	-2.62	-16 67	11.65	-3.41	2.77	2.89	7.74	3.83	-7.04
Convertible Arbitrage Hedge Funds	408	-32.38	-1 83	-3.58	-20.29	-27.19	-13.62	-11.86	4.78	0.79
Dedicated Short Bias	119	14.81	-6.39	4 74	21.90	-6 10	-1.70	-6.63	-17.23	8 98
Emerging Markets	749	-0.84	-13.94	0.80	-5.99	7.86	5.77	12.83	7.71	-3.11
Long/Short Equity	3,409	-4 63	-10.21	4 28	-2.00	10 49	8 74	8.38	1.87	-6 22
Equity Market Neutral	445	4 05	6.81	-2.84	-6.80	-0.03	-6 07	4.88	7 63	-11.87
Events Driven	1,138	-5 80	-18 34	14 32	5 31	6 21	14.32	5.87	10 32	1.96
Fixed Income Arbitrage	379	-14.84	-14.62	-3 01	-10 82	-7 45	-12.35	8.12	0 09	3.54
Fund of Funds	2,514	8 36	-20.05	15.04	3.58	5 26	12.19	14.98	5 29	-2 59
Global Macro	467	-1.99	-8.64	-4 07	6.43	-9 43	-5 09	-2.10	14.40	-0.26
Managed Futures	1,469	-3.34	10 33	-4 78	6.72	-17.51	12 98	6.92	-8 22	-8 73
Multi-Strategy	569	-13.69	-31.39	14 91	0.81	6.49	13.13	3 78	8.87	-3.86
Mutual Funds										
All Mutual Funds	85,845	-5.16	-8.65	10.02	-1.48	0.17	15.13	16.64	-0.21	-1 96
Asset Allocation Mutual Funds	6,172	-0.59	-13 44	13 44	-4 77	-3.32	11.93	29 94	14 83	-4.91
Equities Mutual Funds	40,038	8.49	-11.85	2.11	-1 96	0.7	23 47	32 08	4.59	-1.4
Fixed Income Mutual Funds	30,121	-1.96	-28 37	21 26	2.84	-6.72	15.56	-11.45	-20 84	-2.62
Convertible Bond Mutual Funds	562	-40.61	12.8	16 71	-17.53	-18 67	-10.55	2 6	26 21	-38 64
Stock Portfolios										
100 Value Weighted Portfolios	1,681	-16.85	-1 91	9 75	-11 88	5 2	5.4	52.74	2.57	-4.98

7.3 Analysis Based on Raw Returns

Table 7.6 gives a summary of the average return for the five liquidity portfolios as well as the liquidity spread portfolio based on raw returns (i.e., returns unadjusted for the risk exposure) from 1986 to 2006.

Our analysis is limited by data availability issues. For example, as reported in Table A.2, our data set contains only 74 Convertible mutual funds. For this reason, we have not shown the results for this subgroup in Table 7.6 as in most years each of the five liquidity portfolios would have contained too few funds to diversify away the noise and produce statistically reliable numbers. This limitation is even more severe among hedge funds as seen in Table A.2. To address this issue, we have combined all hedge funds into three sub-groups based on the general knowledge regarding the liquidity of the instruments used in their investment strategies and the length of their redemption notice period reported in Table 7.1. These three sub-groups are as follows: the "Most Illiquid" subset contains Convertible Arbitrage, Fixed Income Arbitrage, Event Driven categories while the "Most Liquid" subset contains Managed Futures, Dedicated Short Bias, and Global Macro categories of hedge funds. The remaining five categories are placed in a the "Medium Liquidity" sub-group.[7.8]

The results shown in Table 7.6 provide the initial suggestion that there is a link between the average return and the serial correlation even before adjusting for other sources of risk. For example, the average return is almost monotonically increasing in the portfolio liquidity when all hedge funds are used in constructing the portfolios (see the second row in Table 7.6). Also, the difference between the highest and lowest portfolios, or what we have been referring to as the liquidity spread, is also positive although not statistically significant. A surprising observation is that the subset of hedge funds that contains the most liquid set of strategies shows the largest value for the liquidity spread, 4.24%/year, versus only 1.69%/year among the illiquid subset of funds. We will see later that this holds even after adjusting for the risk exposure.

The analysis based on all mutual fund data (see row 6 of Table 7.6) does not point to any interesting link between serial correlation and average returns. The link is much more clear among the Fixed Income mutual funds and to some extent among the Equities mutual funds, while there is no such effect observable for Asset Allocation funds. This should be expected as Asset Allocation funds tend to achieve their investment goal by investing in more liquid assets. Also as seen in Table 6.1 in Section 6.1.3, the null of zero serial correlation can only be rejected for 0.4% of Asset Allocation mutual funds while the similar metric is 5.5% and 15.2% for Equities and Fixed Income funds, respectively.

Lastly, the data in Table 7.6 points to a potential link between serial correlation and average return even among the 100 stock portfolios used in this study. Given the strong link between serial correlation and size reported in Table A.4 in Appendix A.5.1 and also in Table 7.5, one may suspect that the effect captured here is the well-known size effect. We will see in the next section that the addition of the size factor (Small-Minus-Big or SMB) does not fully eliminate the effect observed here.

[7.8] Fund of Funds are placed in this group even though they have a slightly longer notice period compared to Convertible Arbitrage and Fixed Income Arbitrage since we believe that the longer period is partially due to the delegated nature of fund management in these funds.

Table 7.6: Assets in the specified subset are grouped into five portfolios based on the first order serial correlation of returns estimated over prior five years. Equal weighted average return for each of the five groups is calculated for each month in the following year. This procedure is repeated for 1991 to 2006, giving a total of 192 data points between January 1991 and December 2006. Reported t-stats are based on the Newey-West estimator with 3-lags. "Difference" column reports the statistics for the return of High minus the return of Low portfolios. The 3 subset of hedge funds are as follows: *Illiquid Hedge Funds* subset contains Convertible Arbitrage, Fixed Income Arbitrage, and Event Driven, *Liquid Hedge Funds* subset contains Managed Futures, Global Macro, and Dedicated Short Bias. The remaining five categories are placed in the *Medium Liquidity Hedge Funds.* Stock portfolios are the standard 100 two-way sorted portfolio based on market capitalization and book-equity/market-equity ratio.

Funds Used	Average Return (% Annualized)						Count
	Low	2	3	4	High	Difference	
All Funds	9.18 (4.06)	9.42 (4.18)	8.94 (4.64)	8.33 (4 65)	9.25 (5.66)	0.07 (0.03)	192
All Hedge Funds	7 73 (4 17)	9 60 (5 02)	10 43 (4 50)	11 77 (6 00)	11 28 (6.25)	3.54 (1.70)	192
Illiquid Hedge Funds	9.09 (5.64)	11.20 (7 71)	10 95 (7.42)	11.70 (7.63)	10.78 (8.63)	1.69 (1.47)	192
Medium Liquidity Hedge Funds	11.12 (5 99)	11.80 (5.40)	13 51 (5.56)	11.91 (4.77)	11.97 (5 35)	0.86 (0.49)	192
Liquid Hedge Funds	3.31 (1.20)	6 75 (2.49)	7.72 (2 47)	6.92 (2.46)	7 55 (2.70)	4.24 (1.48)	192
All Mutual Funds	9 22 (3 84)	9 44 (3 89)	8 65 (4.48)	8 02 (4 39)	8 95 (4 94)	-0 27 (-0.12)	192
Asset Allocation Mutual Funds	8.87 (4.73)	9.27 (4 54)	8.90 (4.40)	8.67 (4 25)	8.73 (5.08)	-0.14 (-0.15)	192
Equities Mutual Funds	11 48 (3 68)	11 58 (3 45)	11 69 (3.39)	12 15 (3.31)	13.28 (3 53)	1 80 (0 96)	192
Fixed Income Mutual Funds	5 70 (5 77)	5 88 (5 74)	5.94 (6.06)	6 12 (6.33)	7.40 (7.61)	1.70 (2.56)	192
Stocks (100 Value Weighted)	15.18 (4 36)	14 91 (4 04)	15.58 (3.90)	16.97 (4.05)	18 88 (3.91)	3.70 (1.25)	192

7.4 Analysis Based on Adjusted Returns

Tables 7.7 and 7.8 give the summary of adjusted returns based on two alternative adjustment procedures outlined in the previous section. As mentioned before, we use four different sets of factors for adjustment to give the reader a sense for robustness of our approach. We have also repeated the relevant row from Table 7.6 to make it easier to compare the adjusted return with the returns prior to adjustment. This data is reported in rows labeled "raw" in Tables 7.7 and 7.8.

Results from the two approaches used for adjustment are qualitatively similar. The average adjusted return for the liquidity spread portfolios, i.e., the difference between the most liquid and the most illiquid set of funds, reported under the column labeled "Difference" is almost always positive and very often statistically different from zero. It seems that our adjustment approach has been successful is reducing the volatility of the portfolio returns and hence the estimated spreads are more precise and more often statistically significant.

Based on the data presented in Panel A of these two tables, the premium is even visible among all funds once the returns are adjusted for their risk exposure. But, as expected, the premium is more visible among hedge funds. For example, the liquidity spread for hedge funds estimated based on the first approach and using the Broad Factor Set is 3.96%/year. The second risk-adjustment approach based on the same set of funds and factors produces a premium of about 4.85%/year. The estimated premium among the most illiquid hedge funds is 3.90% and 3.87% based on the first and the second risk-adjustment approach in both cases using the Broad Factor Set. Similar to the trend seen in Table 7.6 for raw returns, the liquidity spread seems to be larger among the most liquid hedge funds. For example, Table 7.7 produces a risk-adjusted liquidity spread of 4.95% for this subset of funds based on the Broad Factor Set, while Table 7.8 shows that the second risk-adjustment approach produces even a larger spread 7.42%/year. Note that in all these cases the risk adjustment produced by the Lagged Market model is very similar to the values produced by the other models. So at least in this case it appears that simply including the beta with respect to lagged market return does not capture all of the illiquidity premium embedded in these returns.

Among different mutual funds categories, Asset Allocation mutual funds almost never show any statistically significant liquidity spread (see rows 6 through 10 in Panel C of Tables 7.7 and 7.8). The Fixed Income funds (see rows 16 through 20 in Panel C of Tables 7.7 and 7.8) show positive liquidity spreads which are in all but one case statistically significant. For example, the liquidity spread based on the Broad Factor Set using the first approach is 2.74%/year, while the second approach produces a slightly smaller but still statistically significant spread of 1.11%/year. The results among the Equities mutual funds (rows 11 through 15 in Panel C of Tables 7.7 and 7.8) are not as clear. For example, the first risk-adjustment approach produces illiquidity premiums that are not distinguishable from zero for this subset of funds, while the second approach produces a premium that come close to the critical level in only one case (see row 14 in Panel C of Table 7.8). Contrary to the case of hedge funds, the *Lagged Market Model* seems to capture most of the illiquidity premium among Equities mutual funds. This should not come as a surprise as the lagged exposure to the market factor is the result of the illiquidity of the underlying assets and since the equity factor is the most important risk factor among Equities mutual funds (see Table 7.4 for the supporting evidence), and controlling for illiquidity by allowing for lagged exposure to this factor captures much of the illiquidity premium among these funds. Hedge funds, on the other hand, draw their illiquidity from a much broader set of factors and hence the simple control for lagged market factors does not change the results in any significant way as seen in Tables 7.7 and 7.8.

The results from the 100 stock portfolios shown in Panel D of these two tables are also mixed. For the adjustment models other the *Lagged Market Model*, the estimates are always positive but not statistically significant in most cases. While controlling for the exposure to the size (SMB) factor seems to reduce the magnitude of this premium, for example reducing it from 4.37%/year to 2.14%/year in Table 7.8, it does not seem to make it economically irrelevant. In fact, the estimates after controlling for the size exposure seem to be more accurate as seen by the larger t-statistics (compare row 2 vs. row 3 of Panel D in Table 7.7 and Table 7.8). Lagged exposure to the US Stock Market Factor seems to capture most of the illiquidity premium among this group of assets.

Table 7.7:

Assets in the specified subset are grouped into five portfolios based on serial correlation estimated over proceeding five years. Equal weighted average return for each of the five groups is calculated for each month in the following year. The resulting 192 such monthly returns (January 1991 to December 2006) are linked across time and time-series regressions are ran to calculate the *risk-adjusted alpha* based on the following regression:

$$r_{p,t} = \hat{\alpha}_p + \sum_f \hat{\beta}_{p,f} f_t + \hat{\epsilon}_{p,t} \ \forall t \in \{\text{January}_{1991}, \cdots, \text{December}_{2006}\}$$

T-stat calculated based on the Newey-West estimator with 3-lags are reported in parenthesis. Factor sets are as follows:

Market Only contains the concurrent return of US stock market.
4-Factor Set contains the return for US stocks market plus size, value, and momentum factors.
Broad Factor Set is as described previously (see caption of Table 7.4).
Lagged Market contains the concurrent and lagged return of US stock market.

Funds Used	Factor Set	Alpha (Annualized in %)						Count
		Low	2	3	4	High	Difference	
Panel A: All Funds								
All	Raw	9.18 (4.07)	9.42 (4.20)	8.94 (4.66)	8 33 (4 66)	9.25 (5 67)	0.07 (0 03)	192
All	Market Only	1 61 (1 61)	1.63 (1.87)	2.69 (3 17)	3.61 (3 08)	5 47 (5 02)	3 86 (2 20)	192
All	4-Fcator Set	1.00 (0.95)	0.60 (0.71)	0.89 (1.26)	1 42 (1 16)	3 15 (2.81)	2 15 (1 13)	192
All	Broad Factors Set	-0 75 (-0.84)	-0.90 (-1.22)	-0.75 (-1.06)	-0.70 (-0 68)	0 94 (1.04)	1 69 (1 04)	192
All	Lagged Market	1.45 (1.42)	1 55 (1.69)	2.64 (2.83)	3.44 (2.73)	5 07 (4.62)	3 62 (2 08)	192
Panel B: Hedge Funds								
All Hedge Funds	Raw	7.73 (4.19)	9 60 (5 04)	10 43 (4.52)	11.77 (6 02)	11.28 (6.26)	3 54 (1.70)	192
All Hedge Funds	Market Only	4.93 (2.71)	7.00 (4.28)	7 03 (3 76)	7.59 (5.44)	7 82 (4.69)	2 89 (1 20)	192
All Hedge Funds	4-Fcator Set	3.30 (1 91)	5 45 (3 04)	5.25 (2 64)	5 24 (4.13)	6 04 (3.66)	2.74 (1 16)	192
All Hedge Funds	Broad Factors Set	0.99 (0.64)	2.95 (1 78)	3.05 (1.76)	3 42 (3 03)	4.95 (3.86)	3 96 (2.30)	192
All Hedge Funds	Lagged Market	5.52 (3 00)	6.70 (4 04)	6 88 (3 59)	6 78 (4.46)	6 61 (3.54)	1 09 (0.43)	192
Illiquid Hedge Funds	Raw	9 09 (5 65)	11 20 (7.72)	10 95 (7 43)	11.70 (7.65)	10 78 (8.64)	1 69 (1 47)	192
Illiquid Hedge Funds	Market Only	6.39 (4 80)	9.07 (7.52)	8 67 (6.16)	9 82 (6.94)	9 66 (7.66)	3 27 (3 24)	192
Illiquid Hedge Funds	4-Fcator Set	5.44 (3 61)	7.52 (7 15)	7 36 (5 48)	8.83 (5.90)	8 62 (7.43)	3 18 (3 08)	192
Illiquid Hedge Funds	Broad Factors Set	3.75 (3.17)	6.29 (5 38)	6 99 (6 06)	7 81 (6.24)	7 65 (7.12)	3 90 (4.10)	192
Illiquid Hedge Funds	Lagged Market	5 55 (3 81)	8 00 (6 93)	7.66 (5 39)	8 43 (5 65)	8.59 (6 69)	3.04 (2 87)	192
Medium Liq. Hedge Funds	Raw	11.12 (6 00)	11.80 (5.41)	13 51 (5 57)	11 91 (4.78)	11 97 (5 36)	0.86 (0.49)	192
Medium Liq Hedge Funds	Market Only	6 22 (5.10)	6 97 (4.54)	7 78 (4 74)	6 23 (3.44)	7.56 (3 77)	1.34 (0.66)	192
Medium Liq. Hedge Funds	4-Fcator Set	5.08 (4.64)	5.72 (3.78)	5.40 (3.74)	3 56 (2.18)	5.44 (2.85)	0.37 (0.17)	192
Medium Liq. Hedge Funds	Broad Factors Set	2.95 (2.61)	4 68 (3.07)	3 30 (2.45)	2.19 (1.68)	4 29 (2 88)	1 34 (0 78)	192
Medium Liq. Hedge Funds	Lagged Market	6.31 (5.33)	6 31 (4.13)	6.93 (4.04)	5.09 (2.59)	6.31 (2 86)	0 00 (0 00)	192
Liquid Hedge Funds	Raw	3.31 (1.20)	6 75 (2 49)	7.72 (2 47)	6.92 (2.47)	7.55 (2.70)	4.24 (1.48)	192
Liquid Hedge Funds	Market Only	2 61 (0.88)	6 75 (2 39)	9.72 (2 96)	8 02 (2.91)	7.76 (3.00)	5.15 (2 03)	192
Liquid Hedge Funds	4-Fcator Set	-0 58 (-0.19)	3.28 (1.09)	7.96 (2.29)	5 46 (1.82)	5.67 (2 06)	6.26 (2.30)	192
Liquid Hedge Funds	Broad Factors Set	-3 43 (-1.25)	-0 87 (-0 32)	2.44 (0.85)	1.11 (0 43)	1 52 (0.56)	4 95 (1 88)	192
Liquid Hedge Funds	Lagged Market	3 85 (1.29)	8 16 (2.72)	10.67 (3.19)	8.49 (3.07)	8 90 (3.29)	5 05 (1 98)	192

Table 7.7 (Continued)

Funds Used	Factor Set	Alpha (Annualized in %) Low	2	3	4	High	Difference	Count
Panel C: Mutual Funds								
All Mutual Funds	Raw	9.22 (3.85)	9 44 (3 90)	8 65 (4 49)	8.02 (4.40)	8 95 (4.95)	-0.27 (-0 12)	192
All Mutual Funds	Market Only	1.17 (1 17)	1 00 (0 94)	2 29 (2.70)	3 30 (2 60)	4 89 (3.73)	3.71 (1.82)	192
All Mutual Funds	4-Fcator Set	0.63 (0.58)	0.13 (0 12)	0 63 (0 87)	1 16 (0 84)	2.46 (1 68)	1.84 (0.79)	192
All Mutual Funds	Broad Factors Set	-1 05 (-1 12)	-1.03 (-1 13)	-1 09 (-1 57)	-1 00 (-0 90)	0 25 (0.22)	1 30 (0 68)	192
All Mutual Funds	Lagged Market	0.93 (0.90)	0.90 (0 81)	2 29 (2 47)	3.20 (2.35)	4 57 (3.39)	3.64 (1.75)	192
Asset Allocation Mut Funds	Raw	8.87 (4.75)	9 27 (4.56)	8 90 (4.41)	8 67 (4.26)	8.73 (5 09)	-0.14 (-0 15)	192
Asset Allocation Mut Funds	Market Only	2 34 (3 52)	2.12 (3 76)	1 89 (2.89)	1.54 (2.11)	3.46 (3 49)	1.12 (1 17)	192
Asset Allocation Mut Funds	4-Fcator Set	1 50 (2.20)	1 12 (2 51)	0 75 (1.50)	0 09 (0.19)	1 55 (2.39)	0.05 (0.07)	192
Asset Allocation Mut. Funds	Broad Factors Set	-0 37 (-0 92)	-0 65 (-2 40)	-0.73 (-2.17)	-1.38 (-3 27)	-0.46 (-0.76)	-0 09 (-0.13)	192
Asset Allocation Mut. Funds	Lagged Market	2 45 (3 85)	2 18 (3 79)	1.95 (2.93)	1.62 (2 04)	3 26 (3.01)	0 81 (0.81)	192
Equity Mutual Funds	Raw	11.48 (3 69)	11.58 (3.46)	11 69 (3.40)	12.15 (3 32)	13.28 (3.54)	1.80 (0.96)	192
Equity Mutual Funds	Market Only	0 67 (0 85)	-0 08 (-0 10)	-0 36 (-0.44)	-0 31 (-0 28)	1 26 (0 72)	0 60 (0.33)	192
Equity Mutual Funds	4-Fcator Set	-0.92 (-1 26)	-1.19 (-1.42)	-1.39 (-1.89)	-1.37 (-1.61)	-1.39 (-1.47)	-0 47 (-0.41)	192
Equity Mutual Funds	Broad Factors Set	-0.58 (-0.77)	-0 60 (-0.69)	-0 87 (-1.16)	-0 60 (-0.70)	-1 20 (-1 28)	-0 62 (-0.52)	192
Equity Mutual Funds	Lagged Market	0.37 (0 45)	-0.28 (-0 30)	-0 56 (-0 61)	-0 58 (-0 50)	0 51 (0 29)	0.14 (0 08)	192
Fixed Income Mutual Funds	Raw	5 70 (5 78)	5 88 (5.75)	5.94 (6 07)	6 12 (6 35)	7 40 (7 62)	1 70 (2 56)	192
Fixed Income Mutual Funds	Market Only	4.89 (4.59)	5.45 (4.94)	5 45 (5 29)	5 68 (5 58)	6 89 (7.16)	2 00 (2 89)	192
Fixed Income Mutual Funds	4-Fcator Set	3 69 (2.94)	3.96 (3.12)	3 94 (3 47)	4 24 (3.82)	5 65 (5.45)	1.96 (2 66)	192
Fixed Income Mutual Funds	Broad Factors Set	-1.12 (-1 74)	-0.86 (-1.44)	-0 64 (-1.31)	-0 35 (-0.95)	1 62 (3.35)	2.74 (3.56)	192
Fixed Income Mutual Funds	Lagged Market	4.92 (4 49)	5.55 (4 81)	5.52 (5.14)	5.83 (5.54)	6 80 (6.96)	1.87 (2.98)	192
Panel D: Stocks								
Stocks (100 Value Weighted)	Raw	15.18 (4 37)	14 91 (4 05)	15 58 (3.91)	16.97 (4.06)	18.88 (3 92)	3.70 (1.25)	192
Stocks (100 Value Weighted)	Market Only	3 84 (1 77)	3 39 (1 48)	2.71 (1.34)	4.66 (1.75)	6.88 (1.99)	3.04 (1 00)	192
Stocks (100 Value Weighted)	4-Fcator Set	0.08 (0 09)	-1 39 (-1 19)	-0 64 (-0 63)	-0 48 (-0.47)	2 19 (2.13)	2.10 (1 55)	192
Stocks (100 Value Weighted)	Broad Factors Set	0 94 (0.91)	-0 33 (-0 29)	-0.66 (-0.65)	-0 17 (-0.16)	1.69 (1.79)	0.75 (0 54)	192
Stocks (100 Value Weighted)	Lagged Market	2 89 (1 19)	2 42 (0.98)	1.78 (0 80)	3.15 (1.12)	3 72 (1.12)	0.83 (0 29)	192

Table 7.8:

Assets in the specified subset are grouped into five portfolios based on serial correlation estimated over preceding five years. The return for each asset attributable to a set of pre-specified factors are taken out using the *Residual Calculation* formula below where the β values are estimated based on the preceding five years of data using the *Sensitivity Estimation* formula given below. The equal weighted average of the residual return for each portfolio in each month of that year is calculated to give us one data point for that month. The statistics of the mean of these portfolios calculated across the 192 months between January 1991 and December 2006 are reported. T-stat calculated based on the Newey-West estimator with 3-lags are reported in parenthesis. The four different sets of factors used are described in the caption of Table 7.7.

$$\text{Sensitivity Estimation:} \quad r_{i,t} = \hat{\alpha}_i + \sum \hat{\beta}^m_{i,f} f_t + \hat{\epsilon}_{i,t} \quad \forall t \in \{\text{January}_{m-5}, \cdots, \text{December}_{m-1}\}$$
$$\text{Residual Calculation:} \quad \tilde{r}_{i,t} = r_{i,t} - \sum \hat{\beta}^m_{i,f} f_t \quad \forall t \in \{\text{January}_m, \cdots, \text{December}_m\}$$

Funds Used	Factor Set	Average Return (% Annualized)						Count
		Low	2	3	4	High	Difference	
Panel A: All Funds								
All	Raw	9 18 (4 06)	9 42 (4 18)	8 94 (4 64)	8 33 (4 65)	9 25 (5 66)	0 07 (0 03)	192
All	Market Model	1 69 (2 47)	1 43 (2 13)	1 70 (2 55)	2 31 (3 24)	4 73 (6 11)	3 04 (3 71)	192
All	4-Factor Set	0 56 (0 84)	0 85 (1 40)	0 97 (1 58)	1 40 (2 02)	2 95 (4 91)	2 39 (3 26)	192
All	Broad Factor Set	-0 52 (-1 09)	-0 26 (-0 51)	-0 12 (-0 25)	0 08 (0 21)	2 00 (4 33)	2 52 (4 62)	192
All	Lagged Market	2 08 (3 09)	1 37 (1 92)	1 39 (1 88)	1 80 (2 37)	3 69 (4 67)	1 61 (1 89)	192
Panel B: Hedge Funds								
All Hedge Funds	Raw	7 73 (4 17)	9 60 (5 02)	10 43 (4 50)	11 77 (6 00)	11 28 (6 25)	3 54 (1 70)	192
All Hedge Funds	Market Model	4 71 (2 82)	7 92 (4 39)	8 09 (3 88)	8 38 (5 36)	7 64 (5 53)	2 93 (1 51)	192
All Hedge Funds	4-Factor Set	3 56 (2 13)	6 01 (3 51)	6 60 (3 26)	6 34 (4 79)	5 21 (4 51)	1 64 (0 94)	192
All Hedge Funds	Broad Factor Set	1 19 (0 64)	4 38 (2 32)	5 41 (2 46)	5 71 (3 85)	6 04 (5 46)	4 85 (2 67)	192
All Hedge Funds	Lagged Market	5 52 (3 37)	8 71 (4 40)	7 92 (3 96)	7 76 (4 81)	5 93 (4 43)	0 41 (0 23)	192
Illiquide Hedge Funds	Raw	9 09 (5 64)	11 20 (7 71)	10 95 (7 42)	11 70 (7 63)	10 78 (8 63)	1 69 (1 47)	192
Illiquide Hedge Funds	Market Model	4 67 (3 97)	7 34 (6 09)	9 31 (7 16)	8 33 (5 94)	8 90 (8 08)	4 23 (4 14)	192
Illiquide Hedge Funds	4-Factor Set	2 66 (2 11)	5 44 (4 14)	5 73 (4 16)	4 94 (2 69)	5 59 (4 76)	2 93 (2 40)	192
Illiquide Hedge Funds	Broad Factor Set	2 58 (2 07)	7 27 (6 17)	7 39 (5 41)	5 27 (3 10)	6 45 (5 72)	3 87 (2 73)	192
Illiquide Hedge Funds	Lagged Market	4 26 (3 67)	5 81 (4 77)	7 86 (6 01)	6 50 (4 67)	7 22 (7 06)	2 97 (2 88)	192
Medium Liquidity Hedge Funds	Raw	11 12 (5 99)	11 80 (5 40)	13 51 (5 56)	11 91 (4 77)	11 97 (5 35)	0 86 (0 49)	192
Medium Liquidity Hedge Funds	Market Model	7 11 (6 05)	8 02 (5 28)	8 58 (5 21)	6 42 (3 46)	7 42 (4 43)	0 31 (0 18)	192
Medium Liquidity Hedge Funds	4-Factor Set	5 21 (4 27)	5 89 (4 55)	6 30 (4 63)	4 58 (2 95)	5 71 (4 24)	0 50 (0 32)	192
Medium Liquidity Hedge Funds	Broad Factor Set	3 04 (2 55)	4 79 (3 11)	5 98 (4 54)	3 28 (1 50)	6 62 (5 04)	3 58 (2 26)	192
Medium Liquidity Hedge Funds	Lagged Market	7 04 (6 12)	7 37 (4 81)	7 66 (4 57)	4 87 (2 60)	5 43 (3 33)	-1 61 (-0.96)	192
Liquide Hedge Funds	Raw	3 31 (1 20)	6 75 (2 49)	7 72 (2 47)	6 92 (2 46)	7 55 (2 70)	4 24 (1 48)	192
Liquide Hedge Funds	Market Model	0 84 (0 29)	8 13 (2 79)	7 79 (2 44)	9 25 (3 15)	8 42 (2 91)	7 58 (2 59)	192
Liquide Hedge Funds	4-Factor Set	-1 18 (-0 41)	7 35 (2 58)	6 33 (2.00)	9 37 (3 06)	6 74 (2 29)	7 92 (2.70)	192
Liquide Hedge Funds	Broad Factor Set	-3 44 (-1 23)	3 42 (1 13)	2 20 (0 62)	4 42 (1 45)	3 98 (1 31)	7 42 (2 62)	192
Liquide Hedge Funds	Lagged Market	2 60 (0 92)	9 70 (3 26)	9 50 (2 98)	10 79 (3 48)	9 76 (3 21)	7 17 (2 28)	192

Table 7.8 (Continued)

Funds Used	Factor Set	Average Return (% Annualized)						Count
		Low	2	3	4	High	Difference	
Panel C: Mutual Funds								
All Mutual Funds	Raw	9 22 (3 84)	9 44 (3 89)	8 65 (4 48)	8 02 (4 39)	8 95 (4 94)	-0 27 (-0 12)	192
All Mutual Funds	Market Model	1 37 (2 01)	0 82 (1 16)	1 09 (1 63)	1 77 (2 48)	4 01 (5 12)	2 64 (2 99)	192
All Mutual Funds	4-Factor Set	0 30 (0 47)	0 38 (0 63)	0 42 (0 65)	0 94 (1 30)	2 34 (3 72)	2 04 (2 47)	192
All Mutual Funds	Broad Factor Set	-0 68 (-1 43)	-0 71 (-1 39)	-0 59 (-1 21)	-0 38 (-0 96)	1 00 (2 56)	1 67 (3 15)	192
All Mutual Funds	Lagged Market	1 68 (2 49)	0 74 (0 97)	0 82 (1 09)	1 32 (1 75)	3 04 (3 86)	1 35 (1 48)	192
Asset Allocation Mutual Funds	Raw	8 87 (4 73)	9 27 (4 54)	8 90 (4 40)	8 67 (4 25)	8 73 (5 08)	-0 14 (-0 15)	192
Asset Allocation Mutual Funds	Market Model	1 67 (2 91)	1 35 (2 41)	1 23 (1 88)	1 21 (1 73)	2 61 (2 95)	0 94 (1 45)	192
Asset Allocation Mutual Funds	4-Factor Set	1 07 (2 03)	0 86 (1 84)	0 58 (1.17)	0 38 (0 62)	1 20 (1 78)	0 14 (0 29)	192
Asset Allocation Mutual Funds	Broad Factor Set	-0 55 (-1 53)	-0 54 (-1 65)	-0 62 (-1 79)	-0 93 (-1 86)	-0 24 (-0 49)	0 30 (0 68)	192
Asset Allocation Mutual Funds	Lagged Market	2 24 (3 93)	1 46 (2 57)	1 29 (1 94)	0 94 (1 32)	1 78 (1 91)	-0 47 (-0 66)	192
Equities Mutual Funds	Raw	11 48 (3 68)	11 58 (3 45)	11 69 (3 39)	12 15 (3 31)	13 28 (3 53)	1 80 (0 96)	192
Equities Mutual Funds	Market Model	0 17 (0 23)	-0 08 (-0 11)	-0 28 (-0 37)	-0 13 (-0 13)	1 40 (0 83)	1 24 (0 68)	192
Equities Mutual Funds	4-Factor Set	-1 20 (-1 51)	-0 89 (-1 30)	-0 74 (-1 07)	-0 68 (-0 92)	0 12 (0 13)	1 32 (1 15)	192
Equities Mutual Funds	Broad Factor Set	-1 15 (-1 71)	-0 47 (-0 65)	-0 31 (-0 45)	-0 18 (-0 24)	0 88 (0 91)	2 04 (1 98)	192
Equities Mutual Funds	Lagged Market	0 71 (0 94)	-0 00 (-0 00)	-0 61 (-0 82)	-1 13 (-1 04)	-0 76 (-0 43)	-1 47 (-0 75)	192
Fixed Income Mutual Funds	Raw	5 70 (5 77)	5 88 (5 74)	5 94 (6 06)	6 12 (6 33)	7 40 (7 61)	1 70 (2 56)	192
Fixed Income Mutual Funds	Market Model	4 05 (3 87)	4 04 (3 74)	4 05 (4 03)	4 40 (4 58)	5 49 (6 24)	1 45 (2 25)	192
Fixed Income Mutual Funds	4-Factor Set	3 04 (2 81)	3 14 (2 86)	3 24 (3 22)	3 72 (3 84)	4 58 (5 37)	1 55 (2 48)	192
Fixed Income Mutual Funds	Broad Factor Set	0 12 (0 24)	0 16 (0 32)	0 09 (0 24)	0 24 (1 06)	1 23 (4 08)	1 11 (2 05)	192
Fixed Income Mutual Funds	Lagged Market	4 18 (3 92)	4 20 (3 83)	4 28 (4 08)	4 47 (4 49)	4 73 (5 09)	0 55 (0 87)	192
Panel D: Stocks								
Stocks (100 Value Weighted)	Raw	15 18 (4 36)	14 91 (4 04)	15 58 (3 90)	16 97 (4 05)	18 88 (3 91)	3 70 (1 25)	192
Stocks (100 Value Weighted)	Market Model	2 64 (1 47)	2 19 (1 16)	2 29 (1 18)	3 87 (1 64)	7 02 (2 19)	4 37 (1 48)	192
Stocks (100 Value Weighted)	4-Factor Set	-0 36 (-0 36)	-0 96 (-0 97)	-1 02 (-1 02)	0 06 (0 05)	1 78 (1 98)	2.14 (1 94)	192
Stocks (100 Value Weighted)	Broad Factor Set	-0 40 (-0 37)	-0 68 (-0 69)	-0 63 (-0 63)	0 08 (0 07)	2 00 (2 08)	2 41 (2 09)	192
Stocks (100 Value Weighted)	Lagged Market	2 90 (1 64)	1 31 (0 68)	0 32 (0 16)	0 77 (0 31)	1 89 (0 57)	-1 01 (-0 34)	192

7.5 Illiquidity Premium by Hedge Fund Category

This section contains the analysis for each of the 11 hedge fund categories. Given the wide array of strategies followed by hedge funds and in particular the non-linearity and volatility exposure of some of those strategies, it would be reasonable to expect that risk adjustment based on the Broad Factor Set would produce the best set of results. Motivated by this intuition and in order to conserve space, we only report the risk-adjusted results based on the *Broad Factor Set*. Due to the data availability issues discussed previously, for some categories we can only construct the liquidity portfolios for the later part of the sample. The actual number of months involved for calculating each of the values presented in Table 7.9 are also reported. Since we can construct the liquidity portfolios for each of the 11 categories in the period after January 1998, the next section will use the period of 1998 to 2006 to discuss the evolution and the drivers of the illiquidity premium.

The most illiquid categories of funds, in particular Convertible Arbitrage and Fixed Income Arbitrage funds, exhibit large and in most cases statistically significant illiquidity premium. The analysis reported in Table 7.9 suggests that the high illiquidity premium among the most liquid hedge fund categories (recall that this subset include Global Macro, Dedicated Short Bias, and Managed Futures) found in Table 7.7 and 7.8 is primarily driven by the large premium among Managed Futures funds. One surprising result in Table 7.9 is the negative illiquidity premium among Global Macro hedge funds. Although the premium is only statistically significant in Panel A, the result does seem to be robust as it is negative in all three cases. This also seems to be robust to different set of factors used for risk adjustment, for example, using the second approach but using the Market factor as the only risk factor produces a premium of -6.5%/year while using the 4-Factor Set produces a premium of -8.2%/year (note that these results are not reported in Table 7.9).

Table 7.9 also reports the results for the average of the 11 individual time series, one for each of the 11 categories. This result is labeled as "All (Category Neutral)" since it includes all hedge fund returns available and it also includes the same 1/5 of the funds in each category. The liquidity spread constructed based on this return has the benefit that it is not biased to include funds with higher serial correlation (such as Convertible Arbitrage) on its plus side and funds with low serial correlation (such as Managed Futures) on its negative side. Next, we will use the time series of this liquidity spread to visualize the evolution of this premium in the last 9 years of our sample.

7.6 Evolution of the Illiquidity Premium: 1998-2006

In this section we focus on the evolution of the illiquidity premium and try to connect the realized return with various drivers that we expect to be related to the illiquidity premium. 1998 is the first year for which all 11 hedge fund categories have at least 5 funds with the required data, i.e., with five year of reported returns so the serial correlation and betas can be estimated. For this reason, we have focused our analysis here for the period after 1998. Even this short period of time is interesting as it starts shortly before the fall of Long-Term Capital Management in 1998 and ends with several years of great stability, low volatility and significant increase in risk taking in the period of 2004-2006. We would expect that funds holding the most illiquid assets were hit during the 1998 volatile period. Perhaps that volatility would force some of the players out of the market, resulting in a higher premium for holding the illiquid assets in the years following 1998. According to this logic, the tremendous growth of the assets under management by hedge funds, and also the reduction in volatility, which in turn allowed the illiquid assets to be held by leveraged investors at a higher leverage ratio, would have increased the competition for these illiquid assets. This process would have in turn increased the liquidity of these assets and reduced the premium for holding these assets in the latter part of our sample. Before discussing our results, we should remind the reader that given the limited amount of data available, our analysis lacks the required level of significance in some cases.

We start off by creating an overall measure of the illiquidity premium by taking the equal weighted average of the 11 liquidity spreads corresponding to each of the 11 hedge fund categories. This corresponds to the time series of the return used for the analysis reported in the row labeled as "All (Category Neutral)" in Table 7.9. Figure 7.1 shows the cumulative sum of this measure between January 1998 and December 2006. This figures shows that the first year of our sample, and in particular the second half of 1998, was a

Table 7.9: Similar to the analysis presented in Tables 7.7 and 7.8 but applied to each of the 11 hedge fund categories. The *All(Category Neutral)* time series is the equal weighted average of the above 11 time series. Panel A contains the raw returns, Panel B contains alphas with respect to the Broad Factor Set. Panel C contains the residuals with respect to the Broad Factor Set.

Panel A: Raw Returns

	Annualized Mean (%)						
	Lowest	2	3	4	Highest	Difference	Count
Convertible Arbitrage	7.34 (1.57)	10.80 (4.49)	10.49 (4.63)	6.01 (2.65)	9.81 (7.12)	2.47 (0.56)	144
Dedicated Short Bias	-3.00 (-0.44)	-8.04 (-1.08)	-4.97 (-0.51)	1.68 (0.25)	4.38 (0.90)	7.37 (1.22)	120
Emerging Markets	8.46 (1.47)	13.15 (1.91)	13.75 (2.13)	16.39 (2.64)	10.47 (1.72)	2.01 (0.65)	144
Long/Short Equity	14.14 (5.75)	14.69 (5.23)	13.43 (4.51)	14.82 (5.18)	15.46 (5.78)	1.32 (0.75)	192
Equity Market Neutral	9.48 (5.39)	-2.06 (-0.83)	6.99 (3.82)	8.59 (4.08)	8.10 (4.45)	-1.38 (-0.67)	120
Events Driven	11.28 (6.73)	12.14 (8.03)	12.64 (7.11)	11.77 (7.47)	11.77 (8.33)	0.49 (0.45)	192
Fixed Income Arbitrage	-0.81 (-0.41)	8.55 (4.24)	5.19 (2.56)	6.23 (3.51)	3.95 (1.17)	4.76 (1.30)	131
Fund of Funds	7.53 (3.91)	8.84 (5.05)	9.96 (4.92)	10.22 (4.12)	9.87 (5.64)	2.34 (1.02)	192
Global Macro	10.74 (3.57)	9.54 (2.39)	6.79 (1.71)	13.22 (2.91)	3.48 (1.32)	-7.26 (-2.11)	180
Managed Futures	3.63 (1.23)	6.08 (2.00)	8.50 (2.72)	6.94 (2.01)	7.53 (2.38)	3.91 (1.51)	192
Multi-Strategy	11.42 (3.00)	9.80 (1.99)	15.59 (3.31)	11.82 (4.03)	6.93 (1.85)	-4.49 (-0.82)	108
All (Category Neutral)	7.93 (5.62)	9.18 (6.02)	9.72 (6.15)	10.74 (6.15)	9.52 (6.67)	1.59 (1.30)	192

Panel B: Alpha After Adjusting for Broad Factor Set

	Alpha (Annualized in %)						Count
	Low	2	3	4	High	Difference	
Convertible Arbitrage	-0.23 (-0.07)	6.45 (2.89)	6.91 (4.19)	4.53 (2.24)	9.69 (6.70)	9.91 (2.98)	144
Dedicated Short Bias	2.92 (0.49)	2.36 (0.52)	1.46 (0.17)	12.24 (3.33)	7.49 (2.47)	4.57 (0.62)	120
Emerging Markets	-0.09 (-0.02)	-2.05 (-0.44)	0.54 (0.11)	2.29 (0.48)	-1.08 (-0.25)	-0.99 (-0.28)	144
Long/Short Equity	6.74 (5.15)	3.33 (2.73)	2.62 (1.64)	4.03 (2.33)	4.19 (3.76)	-2.55 (-1.82)	192
Equity Market Neutral	9.42 (5.25)	-3.59 (-1.55)	2.29 (1.22)	5.79 (3.03)	5.10 (3.14)	-4.32 (-2.05)	120
Events Driven	5.76 (4.21)	6.81 (5.28)	7.99 (6.54)	7.15 (4.90)	7.61 (7.01)	1.86 (1.90)	192
Fixed Income Arbitrage	-2.51 (-1.73)	3.01 (1.38)	3.87 (1.69)	2.23 (0.99)	4.57 (1.44)	7.08 (2.08)	131
Fund of Funds	0.28 (0.13)	2.39 (1.41)	3.01 (1.70)	3.92 (2.16)	3.73 (2.97)	3.45 (1.36)	192
Global Macro	4.43 (1.58)	3.46 (0.86)	-3.21 (-0.77)	5.07 (1.04)	-1.30 (-0.47)	-5.73 (-1.51)	180
Managed Futures	-3.90 (-1.23)	-1.24 (-0.43)	2.66 (0.82)	-1.56 (-0.51)	1.00 (0.29)	4.91 (1.73)	192
Multi-Strategy	3.43 (1.34)	5.96 (1.20)	9.77 (2.73)	7.73 (4.70)	3.64 (1.02)	0.21 (0.05)	108
All (Category Neutral)	2.30 (2.16)	2.99 (2.37)	3.34 (2.34)	4.44 (2.82)	3.92 (3.38)	1.62 (1.39)	192

Panel C: Residual After Adjusting for Broad Factor Set

	Annualized Mean (%)						
	Lowest	2	3	4	Highest	Difference	Count
Convertible Arbitrage	-1.93 (-0.52)	5.57 (3.11)	5.83 (2.95)	3.88 (2.03)	8.06 (5.51)	9.99 (2.72)	144
Dedicated Short Bias	5.49 (1.17)	4.18 (0.85)	3.37 (0.57)	5.53 (0.84)	8.63 (2.84)	3.15 (0.71)	120
Emerging Markets	1.43 (0.33)	6.23 (1.18)	11.90 (2.14)	12.01 (2.24)	8.03 (1.38)	6.59 (1.66)	144
Long/Short Equity	4.49 (3.70)	5.73 (4.40)	4.24 (3.12)	2.35 (1.16)	7.95 (5.42)	3.45 (2.11)	192
Equity Market Neutral	8.13 (4.95)	-4.86 (-1.64)	4.86 (1.90)	8.32 (3.13)	8.39 (3.34)	0.26 (0.10)	120
Events Driven	4.68 (3.41)	7.75 (5.85)	5.45 (3.18)	6.98 (5.23)	5.99 (4.02)	1.31 (1.03)	192
Fixed Income Arbitrage	-2.58 (-1.06)	2.00 (0.90)	5.59 (2.70)	3.33 (1.75)	4.08 (1.26)	6.66 (1.60)	131
Fund of Funds	1.51 (0.66)	4.03 (2.12)	3.26 (1.85)	3.79 (1.34)	5.26 (4.37)	3.75 (1.68)	192
Global Macro	5.90 (1.92)	6.71 (1.75)	-0.47 (-0.12)	3.65 (0.74)	-0.39 (-0.15)	-6.28 (-1.70)	180
Managed Futures	-2.94 (-0.95)	0.37 (0.10)	4.32 (1.26)	2.82 (0.73)	4.59 (1.40)	7.53 (2.78)	192
Multi-Strategy	1.85 (0.46)	1.40 (0.34)	7.83 (1.63)	9.20 (4.72)	7.39 (3.02)	5.54 (1.24)	108
All (Category Neutral)	1.51 (0.97)	4.14 (2.41)	3.63 (2.52)	3.99 (1.79)	6.18 (4.84)	4.67 (3.68)	192

difficult year for funds holding the illiquid assets. In the following 4 year, funds holding these assets performed well. In fact, by the end of year 2002 the combined return of the illiquidity portfolio had reached 30% based on the cumulative sum. The last four years of the sample show a substantial drop in this premium, perhaps driven by the issues we mentioned earlier such as increase in the assets under management of the funds competing for these illiquid assets and the general reduction in volatility and resulting higher leverage for holders of these assets.

Figure 7.6 shows the cumulative return of the liquidity spread portfolio for each of the 11 categories.[7.9] As reported in Table 7.9, Global Macro funds seem to have a negative liquidity spread, while categories such as Fixed Income and Convertible Arbitrage experienced two of the highest cumulative returns during this period. Without any exception, the slope for all 11 time series in Figure 7.6 shrunk towards zero in the second half of the sample, giving a very strong indication that the pattern observed in Figures 7.1 based on the average of these 11 time series was not strongly influenced by any one of them.

To get a better sense for the macro predictors of the liquidity spread, we regressed the monthly returns of the category neutral liquidity spread (i.e., the time series that was cumulated through time to produce Figure 7.1) against the lagged values of a few different factors that we expected to be related to the liquidity premium. In particular, we used the lagged value of the CBOE Volatility Index, the lagged value of 3-Month Treasury Yields, the 3-Month LIBOR Spread (3-Month LIBOR minus 3-Month Treasury) as well as the lagged values of the Term Spread (10 year minus 1 year US Treasury), Default Spread (yield of BAA minus AAA bonds). The result of these regressions is shown in Table 7.10. This analysis shows that the premium for holding the illiquid assets is higher in more volatile periods. This effect seems to be overshadowed when other factors are added to the regression. Overall, the premium seems to be higher in periods with higher volatility, higher Treasury rates as well as periods with higher Default and Term Spread. These results are consistent with the basic economic intuition. For example, during periods of higher volatility, some of the leveraged players are forced to sell and hence the premium for being able to hold these assets during these times should be higher. Similarly, periods with lower Treasury yield, Default or Term Spread would force the asset managers to shift their attention to holding more illiquid assets as they *search for yield* (see also Rajan (2006) for a very interesting discussion of this issue). This would, in turn, increase the competition and reduce the premium for holding these assets.

[7.9]Note that the data reported in this figure is based on the return since January 1998, while the results in Table 7.9 use all available data for each category which goes to earlier than 1998 in most cases.

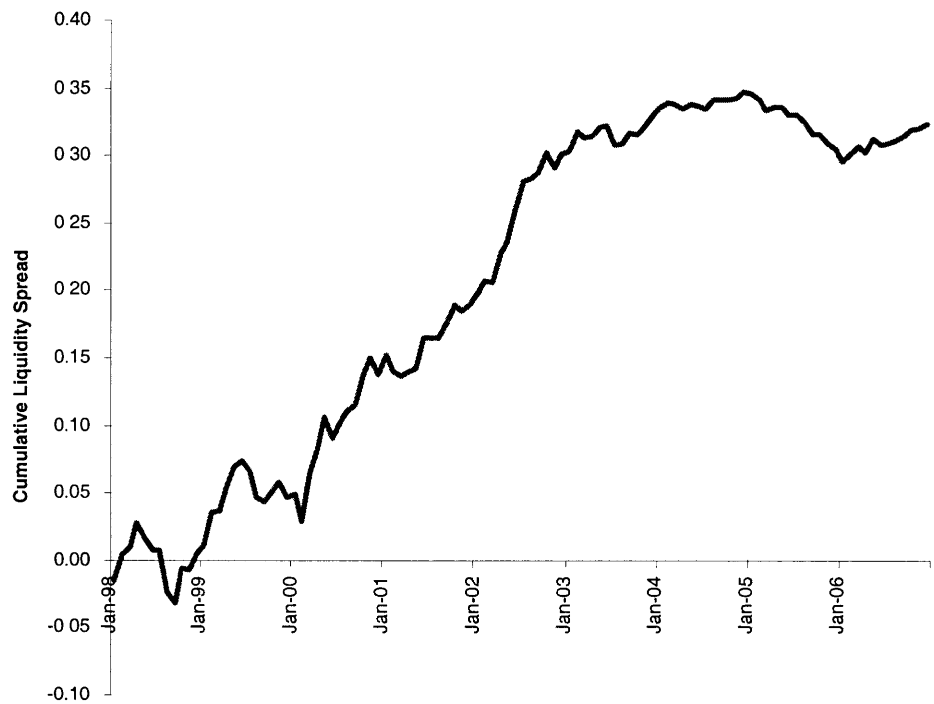


Figure 7.1: This figure shows the cumulative monthly returns of the *Category Neutral Liquidity Spread Portfolio* from January 1998 to December 2006. The *Category Neutral Liquidity Spread Portfolio* is the equal weighted average of the 11 *Liquidity Spread Portfolios.* Each *Liquidity Spread Portfolio* is the difference between the equal weighed average residual return for all funds in the high serial correlation quintile minus the low serial correlation quintile where the portfolios are constructed based on the prior five-year serial correlation and the *Broad Factors Set* based on the beta values estimated over the prior five years are used to calculate the residuals.

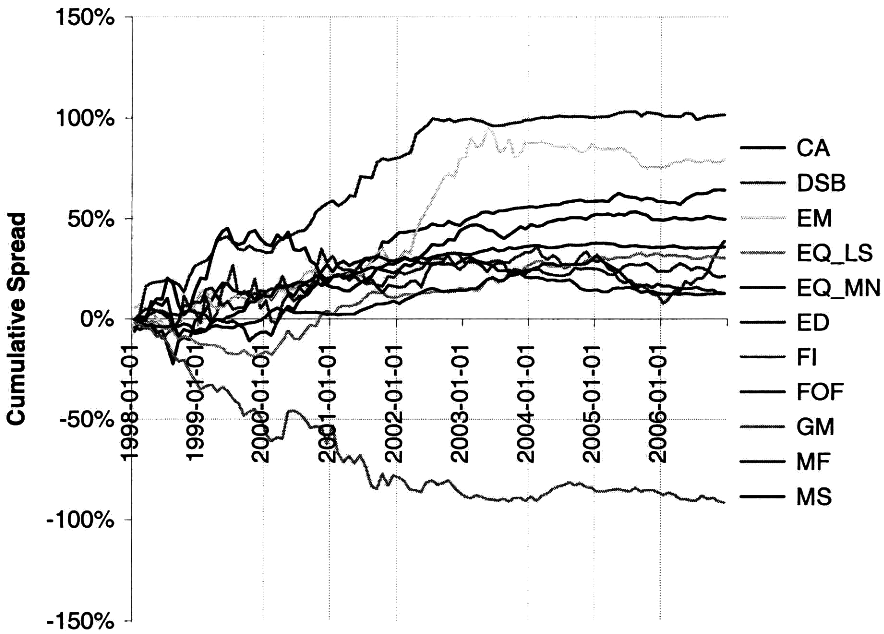


Figure 7.2: This figure shows the cumulative monthly returns of the *Liquidity Spread Portfolio* for different categories of hedge funds based on monthly returns from January 1998 to December 2006. Each *Liquidity Spread Portfolio* is the difference between the equal weighed average residual return for all funds in the high serial correlation quintile minus the low serial correlation quintile where the portfolios are constructed based on the prior five-year serial correlation and the *Broad Factors Set* based on the beta values estimated over the prior five years are used to calculate the residuals. See the caption of Table 7.8 for details of the residual calculation procedure and definition of *Broad Factors Set.* Hedge fund categories are as follows: Convertible Arbitrage (CA), Dedicated Short Bias (DSB), Emerging Markets (EM), Long/Short Equity (EQ_LS), Equity Market Neutral (EQ_MN), Events Driven (ED), Fixed Income Arbitrage (FI), Fund of Funds (FOF), Global Macro (GM), Managed Futures (MF), and Multi-Strategy (MS).

Table 7.10:

This table shows the basic statistics and the sensitivity of the *Category Neutral Liquidity Spread Portfolio* to different economic variables based on the data from January 1998 to December 2006. See caption of Table 7.1 for a description of the *Category Neutral Liquidity Spread Portfolio*. Data from January 1998 to December 2006 is used. All values are reported in the unites of basis points. T-stats are based on the Newey-West estimator with 3 lags. *VIX* in the COBE Volatility Index, *Term Spread* in the difference between 10 year and 1 year US government bond yields, *Defaults Spread* is the difference between yield on the BAA and AAA bonds, and *3M LIBOR Spread* is the difference between 3-month LIBOF rates and the 3-month T-Bill rates. Data is obtained from Federal Reserve monthly statistical release and are in units of percentage points.

Panel A: Basic Statistics

Mean	Standard Deviation	Skewness	Best	Worst
29.69 (2.53)	110	-1764	354(03-2000)	-385(08-1998)

Panel B: Regression Against Lagged Volatility and Interest Rate Variables

Constant	VIX	3M Treasury	3M LIBOR Spread	Default Spread	Term Spread	RSQ (%)	Count
29.69 (2.53)						0.00	108
-11.30 (-0.55)	1.84 (1.80)					1.84	108
-9.19 (-0.29)	1.85 (1.86)	-0.65 (-0.11)				1.85	108
-10.03 (-0.32)	1.92 (1.98)	5.96 (0.66)	-78.54 (-1.08)			3.01	108
-125.50 (-2.24)	1.21 (1.08)	13.12 (1.62)	-68.39 (-0.92)	116.18 (1.74)		6.12	108
-275.60 (-3.05)	-0.05 (-0.05)	44.62 (2.26)	-56.02 (-0.73)	121.59 (2.05)	51.28 (1.94)	8.60	108

7.7 Chapter Conclusions

We started this chapter by presenting some additional analysis to support the claim that serial correlation can be used as a proxy for the illiquidity of the underlying asset. We then turned to the analysis of the linkage between serial correlation and the mean of returns as a way to measure the impact of illiquidity on the returns, or what is usually called the illiquidity premium. We looked at the link between serial correlation of returns and their mean using a variety of statistical techniques.

In general, we found an economically and statistically significant link between average returns and serial correlation of returns. We found the link to be more important for certain categories of hedge funds and for Fixed Income mutual funds and more visible after we had applied our adjustment approach to reduce the noise in the data. We estimated the liquidity spread among hedge funds at about 3.96%/year. The similar measure among Fixed Income mutual funds was about 2.74%/year. We did not find much indication of the illiquidity premium among Equities and Asset Allocation mutual funds or the 100 portfolios of US common stocks. Looking at the difference among various hedge fund categories, we found that the categories traditionally known to involve the illiquid assets, such as Convertible Arbitrage and Fixed Income arbitrage exhibited large illiquidity premium (9.91%/year and 7.08%/year, respectively), but Managed Futures hedge funds that are not usually mentioned in connection with illiquidity exposure also exhibited a large illiquidity premium of about 4.91%/year. We also found that the Global Macro funds exhibited a negative illiquidity premium in our sample, which was somewhat surprising.

Lastly, we applied our methodology to analyze the evolution of the illiquidity premium in the period of 1998 to 2006. We found that while 1998 was a difficult year for funds with significant illiquidity exposure, the subsequent four years brought great return for these funds. We argued that the increased competition and higher leverage finally reduced the illiquidity premium towards zero in the last four years of our sample.

Chapter 8

Summary, Contributions, and Conclusions

This thesis dealt with statistical analysis and model identification based on financial prices. We took the view that prices are signals produced by the local financial markets, and, hence, they should be a reflection of the information and objectives of the entities acting in this system as well as frictions that constrain these interactions. We approached this problem at various levels of abstraction, with a particular emphasis on linking certain statistical anomalies identified at higher levels of abstraction to specific frictions that are only observable in a more microscopic view. With this perspective in mind, we looked at the issue of linear predictability in financial prices.

In this section, we first give a brief summary of the thesis, the main themes, and the finding of the work. We then review the original contributions of the research. Just as the thesis was separated into two parts, this summary is also organized into two parts.

Part A focused on the issue of linear predictability at daily or intra-day frequency. We discussed a signal-extraction algorithm for extracting the predictable part of the price movements. In the empirical application, we found that changes in financial prices have a weak mean-reverting component. We then linked the predictability to microscopic interactions among buyers, sellers, and dealers. The model proposed to make this link precise also produced a number of hypotheses about the behavior of the system and links among various observables, all of which stood well in empirical testing. We finally used this view and the tools developed here to look at a sequence of events in August 2007 that caused major losses for a certain class of hedge funds. We were able to document a distinct regime shift in the mean-reverting component of the prices during a few days in August 2007. We linked this period to the changes in the arrival behavior of buyers, sellers, and dealers.

While the trading algorithm used in Chapter 3 was originally used in Lehmann (1990) and Lo and MacKinlay (1990b), the perspective of looking at this trading strategy as a signal-extraction algorithm is new and original. It is precisely this alternative interpretation that results in the extensions of this algorithm that we used in Chapter 4. In particular, the analysis in Section 4.4 can be viewed as pre-filters applied to the signal prior to applying the signal-extraction procedure. The model proposed in Chapter 4 is motivated by the model in Grossman and Miller (1988), but it is a substantial extension of that model. Most importantly, the analysis presented in Chapter 5 regarding the intricate dynamic of trading mechanism, and its breakdown in August of 2007 is entirely new and has received a lot of attention from the academic, policy-making, and industry audiences alike. The evidence regarding higher level of linkage between various hedge fund sections, i.e., various parts of the world's financial system, which was presented in Chapter 5, is also new and should be of interest from a global stability and linkage perspective.

Part B focused on the issue of linear predictability at monthly frequency. We documented an unusually high level of predictability among hedge funds, which, as we discussed, should show the lowest level of predictability. We proposed a model to link this unpredictability to the unobservability of the underlying prices due to lack of trading. This suggests that assets with less liquidity, i.e., assets that trade less frequently, should exhibit a higher level of linear predictability. Using this concept, we set out to analyze the link between illiquidity and the expected values for returns produced by a large and heterogeneous, with respect to liquidity, group of assets. Overall, our analysis supported the existence of a positive illiquidity premium among certain categories of hedge funds and some categories of mutual funds. We also find that this premium has declined over the last four years of our sample and link that to changes in the overall behavior of this system.

The model proposed in Chapter 6 is an extension of Lo and MacKinlay (1990a). But it is a neat and important extension as it unifies the case of a single return signal and the case of a portfolio of assets, the two cases that were treated differently in Lo and MacKinlay (1990a), into one coherent treatment. The link between linear predictability and illiquidity was first promoted by Getmansky et al. (2004). But the analysis in Chapter 7 that links this hypothesized relationship with the difference in expected returns and, hence, the calculation of the illiquidity premium, is original. In particular, the finding that there has been a substantial change in the illiquidity premium in the last four years of our sample has important implications from the global system analysis perspective.

Appendix A

Appendix

The appendix provides explanations, proofs, or overview of the data. This information was not presented in the main body to enhance the continuity of the exposition. The appendix is separated into a number of sections according to the relevant chapter number in the main text.

A.1 Appendix for Chapter 3

This part of the appendix contains supplemental materials and relevant data descriptions for Chapter 3. Appendix A.1.1 describes the *The Center for Research in Security Prices* (*CRSP*) dataset, which is the main source of historical stock prices and returns used for the analysis presented in this thesis. Appendix A.1.2 provides a short background on the issues related to dealing with heteroskedasticity and autocorrelation in conducting basic statistical tests.

A.1.1 Overview of the CRSP Data

The *Center for Research in Security Prices* (*CRSP*) is a research center at the Graduate School of Business of the University of Chicago. It maintains the most comprehensive collection of price, return, and volume data for all equities listed on all US markets, including the NYSE, AMEX and NASDAQ markets.

The main data used from this database is the daily *Holding Period Return*. This entry is the change in the total value of an investment in a stock over some period of time per one dollar of initial investment. We will use this data primarily at daily frequency. It should be noted that the Holding Period Return incorporates the effect of distribution through dividend in the price. This database provides additional information such as the total number of shares and the closing prices, among other things, which are used in places where we need to rank stocks based on the total market capitalization (market capitalization equals the total number of shares multiplied by the price of each share). For the study in Chapter 3, we only use prices for US-based companies (share type "10" or "11"), and limit the study to stocks that were in the sample on the first day of January or July in the respective year, and had at least 100 days of reported prices for the given year. For the study in Chapter 4, we furthermore limit the set to the stocks that are part of the *S&P 1500 Composite Index*, roughly corresponding to the largest 1,500 companies traded on the US exchanges. The data for S& 1500 Index memberships is obtained from the *Compustat* database and re balanced once a month on the last trading day. Please see Appendix A.3.1 for an overview of the Compustat database.

A.1.2 Heteroskedasticity and Autocorrelation in Statistical Test

We often encounter situations in which we need to test a hypothesis about the mean of a random variable. In other cases, the test can be formulated in the form of testing the mean of a random variable after transforming the data. In the cases that we deal with most often in this thesis, the random variable is the realization of a stochastic process indexed by time. The profit of the Contrarian Trading Strategy in Section 3.4 is an example of such a setting.

The most direct approach would be to calculate the sample mean and use that for hypothesis testing. While under the independent and identically distributed (iid) assumption about the random variables, the standard central limit theorem applies, some adjustment must be made when random variables are autocorrelated and also the conditional variance changes in the sample. The following proposition gives this necessary adjustment to the variance of the sample mean. To keep the result in the most general form, we present it for the case of vector random processes.

Proposition A.1 *(See Hamilton (1994), Chapter 10) Let* $\mathbf{y}_t$ *be an n-dimensional covariance-stationary vector process with moments given by:*

$$E[\mathbf{y}_t] = \boldsymbol{\mu} \tag{A.1}$$
$$E[(\mathbf{y}_t - \boldsymbol{\mu})(\mathbf{y}_{t-j} - \boldsymbol{\mu})^T] = \boldsymbol{\Gamma}_j \tag{A.2}$$

Assume the autocovariances are absolutely summable. Let the sample mean be given by:

$$\overline{\mathbf{y}}_T = \frac{1}{T}\sum_{t=1}^{T}\mathbf{y}_t \tag{A.3}$$

Then:

1. $\overline{\mathbf{y}}_T \xrightarrow{p} \boldsymbol{\mu}$

2. $\lim_{T\to\infty}\left[T \cdot E[(\overline{\mathbf{y}}_T - \boldsymbol{\mu})(\overline{\mathbf{y}}_T - \boldsymbol{\mu})^T]\right] = \mathbf{S} = \sum_{v=-\infty}^{v=+\infty}\boldsymbol{\Gamma}_v$

3. $\sqrt{T}(\overline{\mathbf{y}}_T - \boldsymbol{\mu}) \xrightarrow{d} N(0, \mathbf{S})$

4. *(Newey-West Estimator) Define*

$$\tilde{\mathbf{S}}_T = \hat{\boldsymbol{\Gamma}}_0 + \sum_{v=1}^{q}\left[1 - \frac{v}{q+1}\right](\hat{\boldsymbol{\Gamma}}_v + \hat{\boldsymbol{\Gamma}}_v^T) \tag{A.4}$$

where $\hat{\mathbf{\Gamma}}_v$ is the sample estimator for $\mathbf{\Gamma}_v$ defined as

$$\hat{\mathbf{\Gamma}}_v = \frac{1}{T} \sum_{t=v+1}^{T} (\mathbf{y}_t - \overline{\mathbf{y}}_T)(\mathbf{y}_{t-v} - \overline{\mathbf{y}}_T)^T \tag{A.5}$$

then $\hat{\mathbf{\Gamma}}_v$ is positive semi-definite by construction and is a consistent estimator of $\mathbf{S}$ as long as both T and q go to infinity such that $q/T^{1/4} \to 0$.

We appeal to this result for most of our statistical tests in Section 3.4. We used the value of $q = 3$ for most our tests but the reported results are robust to different values of q.

A.2 Appendix for Chapter 4

This appendix contains background data and proofs related to Chapter 4.

A.2.1 Proof for Proposition 4.1

We need to prove a simple lemma first. The proof of Proposition 4.1 will be given after this.

Lemma A.1 *Let x be a normally distributed random variable with mean μ and variance of σ^2, i.e., $x \sim N(\mu, \sigma^2)$. Consider an objective function of the form*

$$U(d) = -E\left[e^{-\alpha d x}\right] \tag{A.6}$$

The value of d that maximizes the above objective function is

$$d = \frac{\mu}{\alpha \sigma^2} \tag{A.7}$$

and the value of the objective function at the optimal point is

$$U\left(\frac{\mu}{\alpha\sigma^2}\right) = -\exp\left(-\frac{1}{2}\frac{\mu^2}{\sigma^2}\right) \tag{A.8}$$

Proof: From the moment-generating function for the normal random variable, we know that $E[e^x] = \exp(\mu + \sigma^2/2)$. Noting that $-\alpha d \sim N(-\alpha d\mu, \alpha^2 d^2 \sigma^2)$, we know that

$$-E\left[e^{-\alpha d x}\right] = -\exp\left(-\alpha d\mu + \frac{\alpha^2 d^2 \sigma^2}{2}\right) \tag{A.9a}$$

so,

$$\frac{\partial - E\left[e^{-\alpha d x}\right]}{\partial d} = -\exp\left(-\alpha d\mu + \frac{\alpha^2 d^2 \sigma^2}{2}\right)\left(-\alpha\mu + \alpha^2 d\sigma^2\right) \tag{A.9b}$$

$$\frac{\partial^2 - E\left[e^{-\alpha d x}\right]}{\partial d^2} = -\exp\left(-\alpha d\mu + \frac{\alpha^2 d^2 \sigma^2}{2}\right)\left(\left(-\alpha\mu + \alpha^2 d\sigma^2\right)^2 + \alpha^2\sigma^2\right) \tag{A.9c}$$

Setting (A.9b) to zero, we get the desired optimality condition. The second derivative given by (A.9c) is always negative, which means the above optimal point is the global minimum. (A.8) will follow immediately by substituting the optimal value of d into (A.9a)

Proof of Proposition 4.1

First, notice that the objective function of the dealers is given by

$$U(d_t) = -E_t[e^{-\alpha d_t(p_{t+1}-p_t)}e^{\alpha c}] \tag{A.10}$$

where d_t is the number of units they will purchase, $p_{t+1} - p_t$ is the random price change, α is their parameter of risk aversion, and c is their cost. We will refer to d_t as their demand. The only part of the above objective that depends on their demand is the term $e^{-\alpha d_t(p_{t+1}-p_t)}$. So the optimal point for the objective given in (A.10) is the same as the optimal point for the *reduced* objective given by

$$\tilde{U}(d_t) = -E_t[e^{-\alpha d_t(p_{t+1}-p_t)}] \tag{A.11}$$

The hypothesized price dynamic is given by

$$r_{t+1} = p_{t+1} - p_t = \nu_{t+1} - \alpha\sigma_\nu^2 q_t \tag{A.12}$$

So,

$$p_{t+1} - p_t \sim N(-\alpha\sigma_\nu^2 q_t, \sigma_\nu^2) \tag{A.13}$$

Using this fact and based on the reduced utility function given in (A.11), lemma A.1 implies that the demand is given by

$$\begin{aligned} d_t &= \frac{E_t[p_{t+1} - p_t]}{\alpha \mathrm{Var}(p_{t+1} - p_t)} \\ &= \frac{-\alpha\sigma_\nu^2 q_t}{\alpha\sigma_\nu^2} \\ &= -q_t \end{aligned} \tag{A.14}$$

Therefore, under the price dynamics given in (A.12), the quantity demanded by the dealers is exactly equal but opposite to the imbalance quantity q_t. So this price dynamics represents one possible fixed point equilibrium for the system. We conjecture that this is the unique equilibrium but in general it is very difficult to prove uniqueness in this type of model.

A.2.2 Proof for Proposition 4.2

Recall that the utility function for the dealership is given by

$$U(d_t) = -E_t[e^{-\alpha d_t(p_{t+1}-p_t)}e^{\alpha c}] \tag{A.15}$$

Using (A.13) and (A.8) from Lemma A.1, we can calculate the value of the objective function at the maximum. We will denote this a $\tilde{U}(q_t)$ since it will only be a function of q_t. It is given by

$$\tilde{U}(q_t) = -\exp\left(\alpha c - \frac{1}{2}\alpha^2 {\sigma_\nu}^2 q_t^2\right) \tag{A.16}$$

This gives us the expected benefit the dealers extract from being in the market at time t. Note that it is increasing an a function of the imbalance q_t; i.e., the dealers expect to extract a higher benefit after larger imbalance periods. The idea is that in the long run, the benefit dealers extract from being a dealer should exactly offset their cost. So in the steady-state, the expectation of the above expression when the expectation is taken over q_t should be 1.

Recall that $q_t = \theta(L)\eta_t$. Hence, we have:

$$q_t \sim \mathcal{N}(0, \sigma_\eta^2 \sum_{i=0}^{\infty} \theta_i^2) \tag{A.17}$$

so,

$$\frac{q_t^2}{\sigma_\eta^2 \sum_{i=0}^{\infty} \theta_i^2} \sim \chi^2(1) \tag{A.18}$$

We need to use the moment-generating function for $\chi^2(1)$ random variable. If $x \sim \chi^2(1)$ then:

$$E\left[e^{tx}\right] = (1-2t)^{-1/2} \tag{A.19}$$

Using (A.18) and (A.19) to simplify the expectation of $\tilde{U}(q_t)$ we have:

$$\begin{aligned} E\left[\tilde{U}(q_t)\right] &= E\left[-\exp\left(\alpha c - \frac{1}{2}\alpha^2 {\sigma_\nu}^2 q_t^2\right)\right] \\ &= -\exp(\alpha c)\left(1 + \alpha^2 \sigma_\nu^2 \sigma_\eta^2 \sum_{i=0}^{\infty} \theta_i^2\right)^{-1/2} \end{aligned}$$

In the steady-state, we need

$$\exp(\alpha c)\left(1 + \alpha^2 \sigma_\nu^2 \sigma_\eta^2 \sum_{i=0}^{\infty} \theta_i^2\right)^{-1/2} = 1 \tag{A.20}$$

Using the approximation that $\exp(x) \approx 1 + x$ for small x and assuming that the cost, c, is small we find the following solution:

$$\alpha\sigma_\nu^2 \approx \frac{2c}{\sigma_\nu^2 \sum_{i=0}^{\infty} \theta_i^2} \tag{A.21a}$$

$$\approx \frac{2c}{\sigma_q^2} \tag{A.21b}$$

A.2.3 Proof for Proposition 4.3

Recall that we are considering the profit of a market-neutral strategy which invests a $w_{i,t}$ defined as:

$$w_{i,t} = -\frac{1}{N}(r_{i,t} - r_{m,t}) \quad \text{where} \quad r_{m,t} = \frac{1}{N}\sum_{i=1}^{N} r_{i,t}$$

in security i. Define $\pi_t(q)$ as the profit for this strategy for a position put together at time t and held for q periods. This quantity is given by:

$$\begin{aligned}
\pi_t(q) &= \sum_{i=1}^{N} w_{i,t}\frac{p_{i,t+q}}{p_{i,t}} \\
&= \sum_{i=1}^{N}\left(w_{i,t}\sum_{l=1}^{q} r_{i,t+l}\right) \\
&= \sum_{i=1}^{N}\left(-\frac{1}{N}(r_{i,t} - r_{m,t})\sum_{l=1}^{q} r_{i,t+l}\right) \\
&= -\frac{1}{N}\sum_{i=1}^{N}(r_{i,t}(r_{i,t+1} + \cdots + r_{i,t+q})) + r_{m,t}(r_{m,t+1} + \cdots + r_{m,t+p})
\end{aligned}$$

Now, take the expectation:

$$E[\pi_t(p)] = -\frac{1}{N}\sum_{i=1}^{N}\left(\gamma_{i,i}(1) + \cdots + \gamma_{i,i}(q) + q\mu_i^2\right) + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\gamma_{i,j}(1) + \cdots + \gamma_{i,j}(q) + q\mu_i\mu_j\right) \tag{A.22}$$

Now, look at the group of terms at each lag and try to simplify. For lag l, the group of terms has the structure:

$$-\frac{1}{N}\sum_{i=1}^{N}\left(\gamma_{i,i}(l)+\mu_i^2\right)+\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\gamma_{i,j}(l)+\mu_i\mu_j\right) = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_{i,j}(l)-\frac{1}{N}\sum_{i=1}^{N}\gamma_{i,i}(l)+ \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\mu_i\mu_j-\frac{1}{N}\sum_{i=1}^{N}\mu_i^2$$

$$= \frac{1}{N^2}\boldsymbol{\iota}'\boldsymbol{\Gamma}_l\boldsymbol{\iota}-\frac{1}{N}\mathrm{tr}(\boldsymbol{\Gamma}_l)+ \frac{1}{N^2}\boldsymbol{\iota}'(\boldsymbol{\mu}\boldsymbol{\mu}')\boldsymbol{\iota}-\frac{1}{N}\mathrm{tr}(\boldsymbol{\mu}\boldsymbol{\mu}')) \quad \text{(A.23)}$$

where $\boldsymbol{\iota}$ is an $N \times 1$ vector of ones. To simplify this, let's define the following operator

$$\mathcal{M}(\mathbf{A}) \equiv \frac{1}{N^2}\boldsymbol{\iota}'\mathbf{A}\boldsymbol{\iota}-\frac{1}{N}\mathrm{tr}(\mathbf{A})\cdot \quad \text{(A.24)}$$

Note that $\mathcal{M}(\cdot)$ is linear so

$$\mathcal{M}(\mathbf{A}+\mathbf{B}) = \mathcal{M}(\mathbf{A})+\mathcal{M}(\mathbf{B}) \quad \text{(A.25a)}$$

$$\mathcal{M}(\alpha\mathbf{A}) = \alpha\mathcal{M}(\mathbf{A}) \quad \text{(A.25b)}$$

In addition, for any column vector $\mathbf{c}$, $N \times 1$, define $\sigma^2(\mathbf{c})$ as

$$\sigma^2(c) \equiv \frac{1}{N}\sum_{i=1}^{N}(c_i-c_m)^2 \quad \text{where} \quad c_m=\frac{1}{N}\sum_{i=1}^{N}c_i\cdot \quad \text{(A.26)}$$

Also note that if $\mathbf{A}=\mathbf{c}\mathbf{c}'$ where $\mathbf{c}$ is an $N \times 1$ column vector it is easy to show that

$$\mathcal{M}(\mathbf{A}) = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}c_ic_j-\frac{1}{N}\sum_{i=1}^{N}c_i^2$$

$$= -\sigma^2(\mathbf{c}) \quad \text{(A.27)}$$

Putting all these together, and substituting (A.23) into (A.22), we get:

$$E[\pi_t(q)] = \mathcal{M}(\boldsymbol{\Gamma}_1)+\cdots+\mathcal{M}(\boldsymbol{\Gamma}_q)+q\mathcal{M}(\boldsymbol{\mu}\boldsymbol{\mu}')$$

$$= \mathcal{M}(\boldsymbol{\Gamma}_1)+\cdots+\mathcal{M}(\boldsymbol{\Gamma}_q)-q\sigma^2(\boldsymbol{\mu}) \quad \text{(A.28)}$$

A.2.4 Proof for Corollary 4.1

This corollary deals with the expected profit of the contrarian strategy under the following data generating process:

$$
\begin{aligned}
r_{i,t+1} &= p_{i,t+1} - p_{i,t} \\
&= \mu_i + \nu_{i,t+1} + \lambda_{i,t} \qquad \text{(A.29a)}
\end{aligned}
$$

where,

$$
\begin{aligned}
\nu_{i,t+1} &= \beta_i f_{t+1} + \tilde{\nu}_{i,t+1} \qquad \text{(A.29b)} \\
\lambda_{i,t} &= \epsilon_{i,t} - \left(\frac{1-\theta}{\theta}\right)\theta\epsilon_{i,t-1} - \left(\frac{1-\theta}{\theta}\right)\theta^2\epsilon_{i,t-2} - \cdots \qquad \text{(A.29c)} \\
f_t &= \rho f_{t-1} + z_t \qquad \text{(A.29d)}
\end{aligned}
$$

In (A.29a) of the above formulation, $r_{i,t+1} = p_{i,t+1} - p_{i,t}$ consists of a mean, μ_i, and two random parts: $\lambda_{i,t}$ captures the prices changes due to customer imbalances while $\nu_{i,t+1}$ captures the prices changes due to exogenous reasons. In the above formulation, z_t's, $\tilde{\nu}_{i,t}$'s and $\epsilon_{i,t}$'s are zero mean white noise (i.e., uncorrelated both through time and in the cross-section) with variance of σ_z^2, $\sigma_{\tilde{\nu}_i}^2$ and $\sigma_{\epsilon_i}^2$, respectively. Note that sensitivity to f_t, the common factor, implies that $\nu_{i,t}$'s are cross-sectionally correlated. Note that this date generating process is slightly more general than the one proposed in (4.12) in that we now allow the common factor to have a non-zero serial correlation denoted by ρ. Clearly setting $\rho = 0$ gets us back to the basic case.

The profit expression derived in Proposition 4.3 are a function of the variance-covariance matrix of $r_{i,t}$'s. Given the assumption that $\epsilon_{i,t}$ and z_t are uncorrelated at all leads and lags, the variance-covariance of $r_{i,t}$'s can be decomposed to a part due to customer imbalances and another part due to the common factor. We will refer to these parts are $\mathbf{\Gamma}_{l,\lambda}$ and $\mathbf{\Gamma}_{l,f}$, respectively, which are related to the overall variance-covariance matrix $\mathbf{\Gamma}_l$ by:

$$
\mathbf{\Gamma}_l = \mathbf{\Gamma}_{l,\lambda} + \mathbf{\Gamma}_{l,f} \qquad \text{(A.30)}
$$

Since the $\mathcal{M}(\cdot)$ operator used in the calculation of the contrarian profit is linear, the above decomposition will simplify our work in calculating the profits. We will now turn to calculating each component of $\mathbf{\Gamma}_l$.

Calculation of $\Gamma_{l,f}$

f_t is a simple AR(1) process so its variance and covariance are given by :

$$
\begin{aligned}
\sigma_f^2 &= \frac{1}{1-\rho^2}\sigma_z^2 \\
\gamma_{f_t,f_{t+l}} &= \frac{\rho^l}{1-\rho^2}\sigma_z^2 = \rho^l\sigma_f^2
\end{aligned}
$$

Also recognize that f_t is multiplied by β_i in the $r_{i,t}$, which will cause the covariance terms due to the common factor between different securities to scaled accordingly. Putting these together, we have:

$$\Gamma_{l,f} = \boldsymbol{\beta}\boldsymbol{\beta}^T \rho^l \sigma_f^2 \tag{A.31}$$

where $\boldsymbol{\beta}$ is a column vector of all β_i's. Appealing to the linearity and other properties of $\mathcal{M}(\cdot)$ discussed in the last section, we have:

$$\begin{aligned}\mathcal{M}(\boldsymbol{\Gamma}_{l,f}) &= \mathcal{M}(\boldsymbol{\beta}\boldsymbol{\beta}^T \rho^l \sigma_f^2) \\ &= -\rho^l \sigma_f^2 \sigma^2(\boldsymbol{\beta})\end{aligned} \tag{A.32}$$

Calculation of $\Gamma_{l,\lambda}$

Variance of $\lambda_{i,t}$ is given by:

$$\begin{aligned}\sigma_{\lambda_i}^2 &= \left(1 + \left(\frac{1-\theta}{\theta}\right)^2 \theta^2 + \left(\frac{1-\theta}{\theta}\right)^2 \theta^4 + \cdots \right) \sigma_{\epsilon_i}^2 \\ &= \left(1 + \left(\frac{1-\theta}{\theta}\right)^2 \left(\frac{\theta^2}{1-\theta^2}\right)\right) \sigma_{\epsilon_i}^2 \\ &= \left(1 + \frac{1-\theta}{1+\theta}\right) \sigma_{\epsilon_i}^2 \\ &= \frac{2}{1+\theta} \sigma_{\epsilon_i}^2\end{aligned} \tag{A.33}$$

To calculate the covariance between $\lambda_{i,t}$ and $\lambda_{i,t+l}$, we need to look at the cross-product of the MA coefficients for the (infinite) common $\epsilon_{i,t}$'s between $\lambda_{i,t}$ and $\lambda_{i,t+l}$. This expression is given by:

$$\begin{aligned}\gamma_{\lambda_{i,t},\lambda_{i,t+l}} &= \left(-\left(\frac{1-\theta}{\theta}\right)\theta^l + \left(\frac{1-\theta}{\theta}\right)^2 \theta^{l+2} + \left(\frac{1-\theta}{\theta}\right)^2 \theta^{l+4} + \cdots \right) \sigma_{\epsilon_i}^2 \\ &= \left(-\left(\frac{1-\theta}{\theta}\right)\theta^l + \left(\frac{1-\theta}{\theta}\right)^2 \theta^l \frac{\theta^2}{1-\theta^2}\right) \sigma_{\epsilon_i}^2 \\ &= \left(\frac{1-\theta}{\theta}\right)\theta^l \left(-1 + \frac{1-\theta}{\theta}\frac{\theta^2}{1-\theta^2}\right) \sigma_{\epsilon_i}^2 \\ &= \left(\frac{1-\theta}{\theta}\right)\theta^l \left(-1 + \frac{\theta}{1+\theta}\right) \sigma_{\epsilon_i}^2 \\ &= -\left(\frac{1-\theta}{\theta}\right)\theta^l \frac{1}{1+\theta} \sigma_{\epsilon_i}^2 \\ &= -\left(\frac{1-\theta}{2\theta}\right)\theta^l \sigma_{\lambda_i}^2\end{aligned} \tag{A.34}$$

where we use our expression for $\sigma^2_{\lambda_i}$ to simplify this. Putting these together, we have the following expression for $\mathbf{\Gamma}_{l,\lambda}$

$$\begin{aligned}
\mathbf{\Gamma}_{l,\lambda} &= \text{diag}(-\frac{1-\theta}{2\theta}\theta^l\sigma^2_{\lambda_1}, \cdots, -\frac{1-\theta}{2\theta}\theta^l\sigma^2_{\lambda_N}) \\
\text{and } \mathcal{M}(\mathbf{\Gamma}_{l,\lambda}) &= \frac{N-1}{N^2}\sum_{i=1}^{N}\frac{1-\theta}{2\theta}\theta^l\sigma^2_{\lambda_i} \qquad (A.35)
\end{aligned}$$

Profit Expressions

We can now combine the expression for $\mathbf{\Gamma}_{l,f}$ and $\mathbf{\Gamma}_{l,\lambda}$ to find the expression for the profit of the contrarian strategy. Recall that the profit is given by:

$$E[\pi_t(q)] = \mathcal{M}(\mathbf{\Gamma}_1) + \cdots + \mathcal{M}(\mathbf{\Gamma}_q) - q\sigma^2(\boldsymbol{\mu}) \qquad (A.36)$$

Due to linearity of $\mathcal{M}(\cdot)$ and using (A.30), we can rewrite this as:

$$E[\pi_t(q)] = \mathcal{M}(\mathbf{\Gamma}_{1,\lambda}) + \cdots + \mathcal{M}(\mathbf{\Gamma}_{q,\lambda}) + \mathcal{M}(\mathbf{\Gamma}_{1,f}) + \cdots + \mathcal{M}(\mathbf{\Gamma}_{q,f}) - q\sigma^2(\boldsymbol{\mu}) \qquad (A.37)$$

The two parts of this expression can be simplified as follows:

$$\begin{aligned}
\mathcal{M}(\mathbf{\Gamma}_{1,\lambda}) + \cdots + \mathcal{M}(\mathbf{\Gamma}_{q,\lambda}) &= \sum_{l=1}^{q}\left[\frac{N-1}{N^2}\sum_{i=1}^{N}\frac{1-\theta}{2\theta}\theta^l\sigma^2_{\lambda_i}\right] \\
&= \frac{N-1}{N^2}\sum_{l=1}^{q}\sum_{i=1}^{N}\frac{1-\theta}{2\theta}\theta^l\sigma^2_{\lambda_i} \\
&= \frac{N-1}{N^2}\sum_{i=1}^{N}\frac{1-\theta}{2\theta}\theta\frac{1-\theta^q}{1-\theta}\sigma^2_{\lambda_i} \\
&= \frac{N-1}{N^2}\sum_{i=1}^{N}\frac{1-\theta^q}{2}\sigma^2_{\lambda_i} \qquad (A.38) \\
\mathcal{M}(\mathbf{\Gamma}_{1,f}) + \cdots + \mathcal{M}(\mathbf{\Gamma}_{q,f}) &= -\sum_{l=1}^{q}\rho^l\sigma^2_f\sigma^2(\boldsymbol{\beta}) \\
&= -\rho\frac{1-\rho^q}{1-\rho}\sigma^2_f\sigma^2(\boldsymbol{\beta}) \qquad (A.39)
\end{aligned}$$

Substituting (A.38) and (A.39) into (A.37) we arrive at the final expression:

$$E[\pi_t(q)] = \frac{N-1}{N^2}\sum_{i=1}^{N}\frac{1-\theta^q}{2}\sigma^2_{\lambda_i} - \rho\frac{1-\rho^q}{1-\rho}\sigma^2_f\sigma^2(\boldsymbol{\beta}) - q\sigma^2(\boldsymbol{\mu}) \tag{A.40}$$

The second part of this expression is zero for the case that $\rho = 0$. Also, We showed in Table 3.1 that for practical signals the last part, i.e., $\sigma^2(\boldsymbol{\beta})$, is very small and can be ignored. Finally, for large values of N we can use the approximation that $\frac{N-1}{N^2} \approx \frac{1}{N}$. Putting these together we have:

$$E[\pi_t(q)] \approx \frac{1-\theta^q}{2}\left(\frac{1}{N}\sum_{i=1}^{N}\sigma^2_{\lambda_i}\right) \tag{A.41}$$

A.2.5 Proof for Proposition 4.4

This proposition deals with the form of the realized profit of the contrarian strategy under the following data generating process:

$$\begin{aligned} r_{i,t+1} &= p_{i,t+1} - p_{i,t} && \text{(A.42a)}\\ &= \mu_i + \nu_{i,t+1} + \lambda_{i,t} && \text{(A.42b)} \end{aligned}$$

where,

$$\begin{aligned} \nu_{i,t+1} &= \beta_i f_{t+1} + \tilde{\nu}_{i,t+1} && \text{(A.42c)}\\ \lambda_{i,t} &= \epsilon_{i,t} - \left(\frac{1-\theta}{\theta}\right)\theta\epsilon_{i,t-1} - \left(\frac{1-\theta}{\theta}\right)\theta^2\epsilon_{i,t-2} - \cdots && \text{(A.42d)} \end{aligned}$$

In the above formulation, f_t's, $\tilde{\nu}_{i,t}$'s and $\epsilon_{i,t}$'s are zero mean white noise (i.e., uncorrelated both through time and in the cross-section) with variance of σ^2_f, $\sigma^2_{\tilde{\nu}_i}$ and $\sigma^2_{\epsilon_i}$, respectively. Recall that the contrarian strategy invests $w_{i,t}$ defined as:

$$w_{i,t} = -\frac{1}{N}(r_{i,t} - r_{m,t}) \quad \text{where} \quad r_{m,t} = \frac{1}{N}\sum_{i=1}^{N} r_{i,t}$$

in security i. The cumulative q period profit, $\pi_t(q)$, is given by :

$$
\begin{aligned}
\pi_t(q) &= \sum_{i=1}^{N} w_{i,t} \frac{p_{i,t+q}}{p_{i,t}} \\
&= \sum_{i=1}^{N} \left(w_{i,t} \sum_{l=1}^{q} r_{i,t+l} \right) \\
&= \sum_{l=1}^{q} \left(\underbrace{\sum_{i=1}^{N} w_{i,t} r_{i,t+l}} \right)
\end{aligned} \tag{A.43}
$$

Let's simplify the expression in the parenthesis above:

$$
\begin{aligned}
\sum_{i=1}^{N} w_{i,t} r_{i,t+l} &= \sum_{i=1}^{N} \left(-\frac{1}{N} \left(r_{i,t} - \frac{1}{N} \sum_{j=1}^{N} r_{j,t} \right) \right) r_{i,t+l} \\
&= -\frac{1}{N} \sum_{i=1}^{N} r_{i,t} r_{i,t+l} + \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} r_{j,t} r_{i,t+l} \\
&= \left(-\frac{1}{N} + \frac{1}{N^2}\right) \sum_{i=1}^{N} r_{i,t} r_{i,t+l} + \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} r_{j,t} r_{i,t+l} \\
&= \frac{1-N}{N^2} \sum_{i=1}^{N} r_{i,t} r_{i,t+l} + \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} r_{j,t} r_{i,t+l}
\end{aligned} \tag{A.44}
$$

We can now simplify each part in turn. Expanding (A.42) from above we know $r_{i,t}$ and $r_{i,t+l}$ terms have the following form:

$$
\begin{aligned}
r_{i,t} &= \mu_i + \beta_i f_t + \tilde{\nu}_{i,t} + \epsilon_{i,t-1} - \left(\frac{1-\theta}{\theta}\right) \theta \epsilon_{i,t-2} - \left(\frac{1-\theta}{\theta}\right) \theta^2 \epsilon_{i,t-3} - \cdots \\
r_{i,t+l} &= \mu_i + \beta_i f_{t+l} + \tilde{\nu}_{i,t+l} + \epsilon_{i,t+l-1} - \left(\frac{1-\theta}{\theta}\right) \theta \epsilon_{i,t+l-2} - \left(\frac{1-\theta}{\theta}\right) \theta^2 \epsilon_{i,t+l-3} - \cdots
\end{aligned} \tag{A.45}
$$

The key is to use the fact the f_t's, $\tilde{\nu}_{i,t}$'s and $\epsilon_{i,t}$'s are zero mean and uncorrelated both through time. Hence, the law of large number can be applied in each of the summation terms above to drop many of the cross-terms and to obtain convergence in probability type of results. We look at each term in turns:

$$\frac{1-N}{N^2}\sum_{i=1}^{N} r_{i,t}r_{i,t+l} \xrightarrow{p} -\frac{1}{N}\sum_{i=1}^{N}\left(-\frac{1-\theta}{\theta}\theta^l\epsilon_{i,t-1}^2 + \left(\frac{1-\theta}{\theta}\right)^2 \theta^{l+2}\epsilon_{i,t-2}^2\right) - \cdots$$
$$-\frac{1}{N}\sum_{i=1}^{N}\beta_i^2 f_t f_{t+l} \qquad (A.46)$$
$$\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} r_{j,t}r_{i,t+l} \xrightarrow{p} \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\beta_i\beta_j f_t f_{t+l} \qquad (A.47)$$

Terms involving f_t's from the two expression can be collected and simplified using (A.24), (A.25b) and (A.27) as follows:

$$\begin{aligned}
-\frac{1}{N}\sum_{i=1}^{N}\beta_i^2 f_t f_{t+l} + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} r_{j,t}r_{i,t+l} &= \mathcal{M}(\boldsymbol{\beta} f_t \boldsymbol{\beta}' f_{t+l}) \\
&= -f_t f_{t+l}\mathcal{M}(\boldsymbol{\beta}\boldsymbol{\beta}') \\
&= -f_t f_{t+l}\sigma^2(\boldsymbol{\beta}) \qquad (A.48)
\end{aligned}$$

Analyzing the collection of terms involving the $\epsilon_{i,t}$'s is simpler when we look at the contribution of such term in (A.46) as it is aggregated over $l \in \{1, 2, \cdots, q\}$ as needed in (A.43). After some simplification we have:

terms involving $\epsilon_{i,t-1}$'s:
$$\frac{1-\theta}{\theta}\left(\theta + \theta^2 + \cdots + \theta^q\right)\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t-1}^2\right)$$
$$= (1-\theta^q)\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t-1}^2\right) \qquad (A.49)$$

terms involving $\epsilon_{i,t-k}$'s for $k > 1$:
$$\left(\frac{1-\theta}{\theta}\right)^2\left(\theta^{k+2} + \theta^{k+3} + \cdots + \theta^{k+1+q}\right)\left(-\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t-k}^2\right)$$
$$= (1-\theta^q)(1-\theta)\theta^{2k-1}\left(-\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t-k}^2\right) \qquad (A.50)$$

Using (A.48), (A.49), and (A.50) to simplify (A.43) we get:

$$\begin{aligned} \pi_t(q) &= (1-\theta^q)\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t-1}^2\right) \\ &\quad -(1-\theta^q)(1-\theta)\theta\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t-2}^2\right) - (1-\theta^q)(1-\theta)\theta^3\left(\frac{1}{N}\sum_{i=1}^{N}\epsilon_{i,t-3}^2\right) - \cdots \\ &\quad -\sigma^2(\beta)f_t(f_{t+1}+f_{t+2}+\cdots+f_{t+q}) \end{aligned} \tag{A.51}$$

A.3 Appendix for Chapter 5

This Appendix contains various supplemental data for Chapter 5. Appendix A.3.1 provides an overview of the Standard & Poor's Compustat database. Appendix A.3.2 discusses five specific quantitative valuation metrics used in Section 5.3. Appendix A.3.3 provides a short overview of the NYSE Trade and Quote (TAQ) data source used in the high-frequency analysis in Section 5.4. Finally, Appendix A.3.4 provide some detail on the specific stock involved in the un-wind starting on August 6th.

A.3.1 Overview of Compustat Data

Balance-sheet information is obtained from Standard & Poor's Compustat database via the Wharton Research Data Services (WRDS) platform. We use the "CRSP/Compustat Merged Database" to map the balance-sheet information to CRSP historical stock returns data. From the annual Compustat database, we use:

- **Book Value Per Share** (item code BKVLPS)
- **Basic Earnings Per Share Excluding Extraordinary Items** (item code EPSPX)
- **Net Cashflow of Operating Activities** (item code OANCF)
- **Fiscal Cumulative Adjustment Factor** (item code ADJEX_F)

We also use the following variables from the quarterly Compustat database:

- **Quarterly Basic Earnings Per Share Excluding Extraordinary Items** (item code EPSPXQ)
- **Cumulative Adjustment Factor by Ex-Date** (item code ADJEX)
- **Report Date of Quarterly Earnings** (item code RDQ)

There is usually a gap between the end of the fiscal year or quarter and the date that the information is available to the public. We implement the following rules to make sure any information used in creating the factors is, in fact, available on the date that the factor is calculated. For the annual data, a gap of at least 4 months is enforced (for example, an entry with date of December 2005 is first used starting in April 2006) and to avoid using old data, we exclude data that are more than 1 year and 4 months old, i.e., if a security does not have another annual data point after December 2005, that security is dropped from the sample in April 2007). For the quarterly data, we rely on the date given in Compustat for the actual reporting date (item code RDQ, *Report Date of Quarterly Earnings*) to ensure that the data is available on the portfolio construction date. For the handful of cases that RDQ is not available, we employ an approach similar to that taken for the annual data. In those cases, to ensure that the quarterly data is available on the construction date and not stale, the quarterly data is used with a 45-day gap and any data older than 135 days is not used (for example, to construct the portfolio in April 2007, we use data from December 2006, and January or February 2007, and do not use data from April or March 2007).

A.3.2 Construction of Typical Valuation Vectors

We focus our analysis on five of the most studied and most highly cited quantitative equity valuation factors: three value measures, Price Momentum, and Earnings Momentum. The three value measures, Book-to-Market, Earnings-to-Price, and Cashflow-to-Market, are similar to the factors discussed in Lakonishok et al. (1994). These factors are based on the most recent annual balance-sheet data from Compustat and constructed according to the procedure described below. The two remaining factors—Price Momentum and Earnings Momentum—have been studied extensively in connection with momentum strategies (see for example Chan et al. (1996)). The Earnings Momentum factor is based on quarterly earnings from Compustat, while the Price Momentum factor is based on the reported monthly returns from the CRSP database. At the end of each month, each of these five factors is computed for each stock in the S&P 1500 index using the following procedure:

1. The Book-to-Market factor is calculated as the ratio of the **Book Value Per Share** (item code BKVLPS in Compustat) reported in the most recent annual report (subject to the availability rules outlined in Appendix A.3.1) divided by the closing price on the last day of the month. Share adjustment factor from CRSP and Compustat are used to correctly reflect changes in the number of outstanding common shares.

2. The Earnings-to-Price factor is calculate based on the **Basic Earnings Per Share Excluding Extraordinary Items** (item code EPSPX in Compustat) reported in the most recent annual report (subject to the availability rules outlined in Appendix A.3.1) divided by the closing price on the last day of the month. Share adjustment factor available in CRSP and Compustat are used to correctly reflect stock splits and other changes in the number of outstanding common shares.

3. The Cashflow-to-Market factor is calculated based on the **Net Cashflow of Operating Activities** (item code OANCF in Compustat) reported in the most recent annual data (subject to the availability rules outlined in Appendix A.3.1) divided by the total market cap of common equity on the last day of the month. Number of shares outstanding and the closing price reported in CRSP files are used to calculate the total market value of common equity.

4. The Price Momentum factor is the stock's cumulative total return (calculated using holding period return from CRSP files which includes dividends) over the period span-

ning the previous 2 to 12 months.[A.1]

5. The Earnings-Momentum factor is constructed based **Quarterly Basic Earnings Per Share Excluding Extraordinary Item** (item code EPSPXQ in Compustat) using the standardized unexpected earnings, SUE. The SUE factor is calculated as the ratio of the earnings growth in the most recent quarter (subject the availability rules outlined in Appendix A.3.1) relative to the year earlier divided by the standard deviation of the same factor calculated over the prior 8 quarters (see Chan et al. (1996) for a more detailed discussion of this factor).

At the end of each month during our sample period, we divide the S&P 1500 universe into 10 deciles according to each factor. Decile 1 will contain the group of companies with the lowest value of the factor; for example, companies whose stocks have performed poorly in the last 2 to 12 month will be in the first decile of the Price Momentum factor. Deciles 1 through 9 will have the same number of stocks and decile 10 may have a few more if the original number of stocks was not divisible by 10. We do not require a company to have data for all five factors or to be a U.S. common stock to be used in each ranking. However, we use only those stocks that are listed as U.S. common shares (CRSP Share Code "10" or "11") to construct portfolios and analyze returns.[A.2] For example, if a company does not have 8 quarters of earnings data, it cannot be ranked according to the Earnings Momentum factor, but it will still be ranked according to other measures if the information required for calculating those measures is available.

This process yields decile rankings for each of these factors for each month of our sample. In most months, we have the data to construct deciles for more than 1,400 companies. However, at the time we obtained the Compustat data for this analysis, the Compustat database was still not fully populated with the 2007 quarterly data; in particular, the data for the quarter ending September 2007 (2007Q3) was very sparse. Given the 45-day lag we employ for quarterly data, the lack of data for 2007Q3 means that the deciles can be formed for only about 370 companies at the end of November 2007 (the comparable count was 1,381 in October 2007 and 1,405 at the end of September 2007). Since any analysis of factor models for December 2007 is impacted by this issue, we will limit all our study to the first 11 months of 2007.

A.3.3 Overview of TAQ Data

The NYSE Trade and Quote (TAQ) database contains intra-day transactions and quotes data for all securities listed on the NYSE, the American Stock Exchange (AMEX), the National Market System (NMS), and SmallCap issues. The dataset consists of the Daily National Best Bids and Offers (NBBO) File, the Daily Quotes File, the Daily TAQ Master File, and the Daily Trades File. For the purposes of this study, we only use actual trades as reported in

[A.1] The most recent month is not included, similar to the Price-Momentum factor available on Kenneth French's data library (see footnote 5.13).

[A.2] This procedure should not impact our analysis materially as there are only 50 to 60 stocks in the S&P indexes without these share codes, and these are typically securities with share code "12", indicating companies incorporated outside the U.S.

the Daily Trades File. This file includes information such as the security symbol, trade time, size, exchange on which the trade took place, as well as a few condition and correction flags. We only use trades that occur during normal trading hours (9:30am to 4:00pm). We also discarded all records that have a *Trade Correction Indicator* field entries other than "00"[A.3] and removed all trades that were reported late or reported out of sequence, according to the *Sale Condition* field.[A.4] During the 63 trading days of our sample of TAQ data from July 2, 2007 to September 28, 2007, the stocks within the universe of our study—the S&P 1500—yielded a total of approximately 805 million trades, ranging from a low of 4.9 million trades on July 3, 2007 to a high of 23.7 million trades on August 16, 2007. The cross-sectional variation of the number of trades was quite large; for example, there were approximately 11 million trades in Apple (AAPL) during our sample period while Lawson Products (LAWS) was only traded 6,830 times during the same period. On average, we analyzed approximately 11.3 million trades per day to develop our liquidity measures.

Using transactions prices in the Daily Trades File, we construct 5-minute returns within each trading day (no overnight returns are allowed) based on the most recent transactions price within each 5-minute interval, subject to the filters described above. These returns are the inputs to the various strategy simulations reported in Section 5.4.

A.3.4 Extreme Movers on August 6, 2007

Simulations of simple strategies such as the contrarian strategy can be used to pinpoint the beginning of market dislocations when applied to transactions data. Recall that the intra-day contrarian strategy of Section 5.4 invests \$1 long in the worst performing decile and \$1 short in the best performing decile of lagged 5-minute returns. Given the position $\omega_{i,t}$ of security i at time t, the security's contribution to the portfolio's profit or loss over the next period is simply $\omega_{i,t}r_{i,t}$. If this value is negative, it suggests that the security experienced either a negative return following a period of under-performance (recall that we invest \$1 long in the worst performing decile), or a positive return following a period of out-performance. While such an outcome may be purely random, a sufficiently high number of such occurrences over a given day indicates a price trend for that security and systematic losses for the contarian strategy. Therefor, the number of periods in which a security exhibited negative contributions to the portfolio:

$$\sum_t I_{\{\omega_{i,t}r_{i,t}<0\}} \tag{A.52}$$

can be used as a metric to detect the start of an unwind of mean-reversion strategies, as well as a possible decline in market liquidity due to losses accumulated by marketmaking strategies.

Under the scenario of pure randomness, i.e., independently and identically distributed

[A.3]According to the TAQ documentation, a Trade Correction Indicator value of "00" signifies a regular trade which was not corrected, changed or canceled. This field is used to indicate trades that were later signified as errors (code "07" or "08"), canceled records (code "10"), as well as several other possibilities. Please see the TAQ documentation for more details.

[A.4]These filters have been used in other studies based on TAQ data; see, for example, Christie, Harris and Schultz (1994) or Chordia, Roll and Subrahmanyam (2001). See the TAQ documentation for further details.

mean-zero returns, each security has a 1/5 chance of being included in the contrarian portfolio in each time period (recall that we long and short the bottom- and top-performing deciles). Once the portfolio is established, each position has a 1/2 chance of contributing a loss (negative returns following a period of under performance or position return following a period of outperformance).[A.5] Therefore, each security has a 1/10 chance of contributing a negative value to the return of the contrarian strategy over each interval so the expected value of (A.52) for each security on any given day is 7.6 (recall that the contrarian strategy takes position 76 times each day, starting at 9:40am and closing final positions at 4:00pm).

We have ranked securities according to this metric for August 6, 2007 and list the securities with the top 50 values in Table A.1. We have also reported the decile ranking of each security according to each of the five valuation factors as well as their market-capitalization decile. The *Open*, *High*, *Low* and the *Closing* price as well as the *High-Low* spread, as a measure of the intraday volatility, and the overall return for the day are also reported.

The stocks' factor rankings in Table A.1 do not look random, but clearly show that the extreme losers were concentrated in the financial sector, and had extreme factor rankings in at least three of our valuation factors and in size—high Book-to-Market, high Earnings-to-Price, low Earnings Momentum, and low market cap.

[A.5]Recall that we are using 5-minute returns, which is close to zero mean, hence the loss probability of 1/2 is a reasonable approximation.

Table A.1: Top 50 securities with highest loss rankings from the contrarian strategy applied to 5-minute returns of stocks in the S&P 1500 on August 6, 2007. Securities are ranked based on $\sum_t I_{\{\omega_{i,t} r_{i,t} < 0\}}$ where $\omega_{i,t}$ is the weight assigned to security i based on the returns over the preceding 5-minute interval and $r_{i,t}$ is the return over the subsequent 5-minute interval. The realized value for this metric is contained in the column "Periods with Loss".

				Factor and Size Deciles						Price and Return Data					
Ticker	Name	Industry	Periods with Loss	B/M	CF/M	E/P	ERN MOM	PRC MOM	SIZE	Open ($)	High ($)	Low ($)	Close ($)	High-Low Spread (% of Close)	Day Return
RDN	RADIAN GROUP INC	Money & Finance	35	10	10	10	1	1	6	23 27	24 50	17 44	23 23	30%	0%
SPF	STANDARD PACIFIC CORP NEW	Other Industries	34	10	1	10	1	1	3	12 25	12 28	7 51	10 56	45%	-14%
FFIV	F 5 NETWORKS INC	Computer, Software & Electronics	31	2	2	2	10	10	7	83 83	84 00	70 30	72 43	19%	-14%
IMB	INDYMAC BANCORP INC	Money & Finance	29	10	1	10	1	1	4	19 18	21 15	18 25	20 03	14%	4%
SMP	STANDARD MOTOR PRODUCTS INC	Computer, Software & Electronics	27	10	9	4	5	10	1	10 92	10 92	7 88	8 62	35%	-21%
MTG	M G I C INVESTMENT CORP WIS	Money & Finance	26	10	9	10	1	3	6	33 75	35 55	28 93	33 28	20%	-1%
BZH	BEAZER HOMES USA INC	Other Industries	25	10	1	10	1	1	2	11 32	11 60	10 12	10 96	14%	-3%
FRNT	FRONTIER AIRLINES HLDGS INC	Other Industries	25	10	8	1	2	1	1	5 25	5 27	4 51	4 89	16%	-7%
GFF	GRIFFON CORP	Manufacturing	25	9	2	10	1	2	2	15 70	15 92	12 00	12 98	30%	-17%
VCI	VALASSIS COMMUNICATIONS INC	Other Industries	24	4	6	9	1	1	2	9 52	9 66	7 67	7 88	25%	-17%
LAB	LABRANCHE & CO INC	Money & Finance	24	10	10	10	1	1	1	5 10	6 37	5 10	6 19	21%	21%
LFG	LANDAMERICA FINANCIAL GROUP INC	Money & Finance	24	10	9	9	1	9	4	57 10	58 43	54 32	57 01	7%	0%
MESA	MESA AIR GROUP INC NEV	Other Industries	24	10	1	10	1	1	1	6 45	6 45	5 42	6 11	17%	-5%
ROIAK	RADIO ONE INC	Other Industries	23	10	9	1	4	2	2	4 87	4 92	3 51	4 05	35%	-17%
CHUX	O CHARLEYS INC	Wholesale & Retail	22	10	10	5	5	7	1	16 76	16 76	15 47	16 39	8%	-2%
CBG	C B RICHARD ELLIS GROUP INC	Money & Finance	22	1	3	4	10	9	8	31 16	32 29	28 08	32 10	13%	3%
CFC	COUNTRYWIDE FINANCIAL CORP	Money & Finance	22	10	1	10	2	3	9	24 70	26 75	23 64	26 75	12%	8%
OMG	O M GROUP INC	Manufacturing	22	7	9	2	1	9	4	43 50	44 04	40 29	42 59	9%	-2%
CHP	C & D TECHNOLOGIES INC	Computer, Software & Electronics	22	N/A	1	1	6	1	1	4 85	4 85	4 13	4 32	17%	-11%
NDN	99 CENTS ONLY STORES	Wholesale & Retail	22	8	2	2	5	7	3	11 71	12 15	11 20	11 96	8%	2%
CELL	BRIGHTPOINT INC	Wholesale & Retail	22	4	1	7	1	2	3	12 70	12 73	11 85	12 34	7%	-3%
ABK	AMBAC FINANCIAL GROUP INC	Money & Finance	22	10	9	10	1	3	8	62 42	64 58	57 80	64 32	11%	3%
PNM	P N M RESOURCES INC	Other Industries	21	10	8	8	5	3	5	22 77	23 50	21 05	22 37	11%	-2%
ASTE	ASTEC INDUSTRIES INC	Manufacturing	21	3	2	3	9	10	4	50 23	52 87	47 61	52 50	10%	5%
SRDX	SURMODICS INC	Other Industries	21	2	3	2	4	8	3	44 92	48 85	44 52	48 57	9%	8%
ETFC	E TRADE FINANCIAL CORP	Money & Finance	21	7	8	9	4	2	8	15 98	16 29	14 73	16 19	10%	1%
CAS	CASTLE A M & CO	Wholesale & Retail	21	5	3	9	4	4	2	29 29	29 29	26 86	28 00	9%	-4%
UTI	UNIVERSAL TECHNICAL INSTITUTE IN	Other Industries	21	2	6	5	2	7	2	23 06	23 80	22 00	23 30	8%	1%
SPC	SPECTRUM BRANDS INC	Computer, Software & Electronics	21	10	10	1	1	2	1	4 50	4 50	3 77	4 15	18%	-8%
SRT	STARTEK INC	Other Industries	21	9	8	3	1	1	1	10 27	11 19	10 19	11 08	9%	8%
KBR	K B R INC	Other Industries	20	5	10	2	N/A	N/A	7	31 93	32 60	31 15	32 49	4%	2%
UNF	UNIFIRST CORP	Wholesale & Retail	20	8	8	6	8	9	2	37 51	39 11	35 25	38 76	10%	3%
MHO	M I HOMES INC	Other Industries	20	10	1	10	1	1	1	23 66	23 91	22 49	23 84	6%	1%
PRAA	PORTFOLIO RECOVERY ASSOCIATES IN	Money & Finance	20	4	5	6	10	9	3	46 52	51 64	44 26	51 52	14%	11%
CHB	CHAMPION ENTERPRISES INC	Other Industries	20	5	5	10	1	9	3	11 54	11 54	10 26	10 88	12%	-6%
BSC	BEAR STEARNS COMPANIES INC	Money & Finance	20	9	1	10	1	2	9	106 89	113 81	99 75	113 81	12%	6%
FED	FIRSTFED FINANCIAL CORP	Money & Finance	20	10	1	10	3	3	2	40 73	43 00	38 73	41 75	10%	3%
LEH	LEHMAN BROTHERS HOLDINGS INC	Money & Finance	20	8	1	10	10	5	10	56 50	58 50	52 63	58 27	10%	3%
NILE	BLUE NILE INC	Wholesale & Retail	20	1	2	2	9	10	4	82 00	84 81	78 20	82 00	8%	0%
MEE	MASSEY ENERGY CO	Other Industries	20	6	8	2	6	3	5	19 10	19 14	17 90	18 07	7%	-5%
BBX	BANKATLANTIC BANCORP INC	Money & Finance	19	10	1	6	N/A	1	1	7 75	8 48	7 53	8 39	11%	8%
MBI	M B I A INC	Money & Finance	19	10	7	10	4	4	8	50 81	56 20	48 95	56 20	13%	11%
OMN	OMNOVA SOLUTIONS INC	Other Industries	19	2	7	2	1	2	1	5 22	5 42	4 80	5 27	12%	1%
IFC	IRWIN FINANCIAL CORP	Money & Finance	19	10	10	10	3	1	1	10 18	10 37	9 32	10 00	11%	-2%
PMTC	PARAMETRIC TECHNOLOGY CORP	Computer, Software & Electronics	19	2	2	3	10	8	5	17 31	17 49	16 16	16 61	8%	-4%
MTEX	MANNATECH INC	Health Care, Medical Eq, Drugs	19	5	9	10	7	4	1	9 19	9 43	8 59	8 93	9%	-3%
CAR	AVIS BUDGET GROUP INC	Other Industries	19	10	10	1	N/A	6	6	24 05	24 97	21 22	23 28	16%	-3%
MRO	MARATHON OIL CORP	Other Industries	19	5	9	10	7	8	10	50 29	50 73	46 97	49 24	8%	-2%
RSCR	RES CARE INC	Health Care, Medical Eq, Drugs	19	8	5	8	5	4	2	18 71	19 13	17 62	18 60	8%	-1%
CAE	CASCADE CORP	Manufacturing	19	4	5	6	10	10	3	68 05	68 57	63 51	66 14	8%	-3%

A.4 Appendix for Chapter 6

This appendix contains the background and some supporting material for Chapter 6 of this thesis. Appendix A.4.1 provides an overview of the TASS Hedge Fund Database while Appendix A.4.2 provides a similar overview for the CRSP Mutual Fund Database. Appendix A.4.3, A.4.4, and A.4.5 give the proofs for the statistical tests used in Section 6.1. Finally, Appendix A.4.6 gives the proof for the time-series properties of the portfolio's observed return under the model of Section 6.2.1.

A.4.1 Overview of TASS Hedge Fund Database

Hedge fund data was obtained from the *Lipper TASS Database*.[A.6] This database contains the historical returns as well as legal structure, investment style, management fee style, contact information, fund flow, and self-claimed sources of risk exposure for different funds. The database is divided into two parts: "Live" and "Graveyard" funds. Hedge funds are recorded in the Live database if they are considered active as of the date of the snapshot. Once a hedge fund decides not to report its performance, liquidates, closes to new investment, restructures, or merges with other hedge funds the fund is transferred into the Graveyard database. A hedge fund can only be listed in the Graveyard database after having been listed in the Live database. Since the TASS database fully represents returns and asset information for live and dead funds, the effects of "survivorship bias" are minimized. However, the Graveyard database became active only in 1994, so funds that were dropped from the Live database prior to 1994 are not included in the Graveyard database, creating the possibility of a certain degree of survivorship bias. The database is also subject to "backfill bias." This issue arises because when a fund decides to be included in the database, TASS adds the fund to the Live database and includes all available prior performance of the fund. Since funds do not need to meet any specific requirements to be included in the TASS database, funds are more likely to decide to apply for TASS membership after having experienced several good months of returns. Given the voluntary nature of reporting, another potential issue sometimes associated with such a data set is the "self-selection bias" arising from the fact that funds with very good or very bad performance may decide to no longer report their returns. Please refer to Agarwal and Naik (2005) for a more comprehensive review of other data sources available for hedge fund returns and other potential biases associated with such data sets.

Hedge fund data was obtained from TASS in January 2008. However, there can be up to several months of delay in fund reporting. For this reason, we have decided only to use the data up to December 2006 in our study.[A.7] As of January 2008, this database consists of information on 8,729 funds. Each fund is assigned to one of the 11 investment category

[A.6]We thank the MIT Sloan School of Management for their support in obtaining access to this data set. In particular, Svetlana Sussman has be instrumental in making the necessary arrangements.

[A.7]This could bias our data set to slightly overestimate Graveyard funds, and underestimate Live funds as of December 2006 since TASS categorizes funds based on the information available until January 2008. This effect is not relevant in our analysis since we will not do a separate analysis based on this classification and Live and Graveyard information in Table A.2 is simply provided for better assessment of the diversity in the data set.

styles and on the information disclosed regarding the nature of their activities. A summary definition of these 11 styles is given below for reference. We have limited our study to only funds with at least 5 years of monthly reported history in our study period - January 1986 to December 2006. Funds that reported returns in frequencies other than monthly, such as quarterly, have been excluded from the study as well. Table A.2 gives a breakdown of the funds used in this study. Table A.3 gives important statistics of our data set.

TASS Primary Categories Definitions

The following is a list of descriptions of the categories for which CS/Tremont constructs indexes, taken directly from the CS/Tremont web site (www.hedgeindex.com).

Convertible Arbitrage. This strategy is identified by investment in the convertible securities of a company. A typical investment is to be long the convertible bond and short the common stock of the same company. Positions are designed to generate profits from the fixed income security as well as the short sale of stock, while protecting principal from market moves.

Dedicated Short Bias. This strategy is to maintain net short as opposed to pure short exposure. Short-biased managers take short positions in mostly equities and derivatives. The short bias of a manager's portfolio must be constantly greater than zero to be classified in this category.

Emerging Markets. This strategy involves equity or fixed income investing in emerging markets around the world. Because many emerging markets do not allow short selling, nor offer viable futures or other derivative products with which to hedge, emerging market investing often employs a long-only strategy.

EquityMarket Neutral. This investment strategy is designed to exploit equity market inefficiencies and usually involves being simultaneously long and short matched equity portfolios of the same size within a country. Market neutral portfolios are designed to be either beta or currency neutral, or both. Well designed portfolios typically control for industry, sector, market capitalization, and other exposures. Leverage is often applied to enhance returns.

Event Driven. This strategy is defined as "special-situations" investing designed to capture price movement generated by a significant pending corporate event such as a merger, corporate restructuring, liquidation, bankruptcy or reorganization. There are three popular sub-categories in event-driven strategies: risk arbitrage, distressed securities, and multi-strategy.

Risk Arbitrage. Specialists invest simultaneously in long and short positions in both companies involved in a merger or acquisition. Risk arbitrageurs are typically long the stock of the company being acquired and short the stock of the acquiring company. The principal risk is deal risk, should the deal fail to close.

Distressed. Hedge fund managers invest in the debt, equity or trade claims of companies in financial distress and general bankruptcy. The securities of companies in need of legal action or restructuring to revive financial stability typically trade at substantial discounts to par value and thereby attract investments when managers perceive a turn-around will materialize. Managers may also take arbitrage positions within a company's capital structure, typically by purchasing a senior debt tier and short selling common stock, in the hopes of realizing returns from shifts in the spread between the two tiers.

Multi-Strategy. This subset refers to hedge funds that draw upon multiple themes, including risk arbitrage, distressed securities, and occasionally others such as investments in micro- and small-capitalization public companies that are raising money in private capital markets. Hedge fund managers often shift assets between strategies in response to market opportunities.

Fixed Income Arbitrage. The fixed income arbitrageur aims to profit from price anomalies between related interest rate securities. Most managers trade globally with a goal of generating steady returns with low volatility. This category includes interest rate swap arbitrage, the United States and non-US government bond arbitrage, forward yield curve arbitrage, and mortgage-backed securities arbitrage. The mortgage-backed market is primarily US-based, over-the-counter and particularly complex.

Global Macro. Global macro managers carry long and short positions in any of the world's major capital or derivative markets. These positions reflect their views on overall market direction as influenced by major economic trends and or events. The portfolios of these hedge funds can include stocks, bonds, currencies, and commodities in the form of cash or derivatives instruments. Most hedge funds invest globally in both developed and emerging markets.

Long/Short Equity. This directional strategy involves equity-oriented investing on both the long and short sides of the market. The objective is not to be market neutral. Managers have the ability to shift from value to growth, from small- to medium- to large-capitalization stocks, and from a net long position to a net short position. Managers may use futures and options to hedge. The focus may be regional, such as long/short US or European equity, or sector specific, such as long and short technology or healthcare stocks. Long/short equity hedge funds tend to build and hold portfolios that are substantially more concentrated than those of traditional stock hedge funds.

Managed Futures. This strategy invests in listed financial and commodity futures markets and currency markets around the world. The managers are usually referred to as Commodity Trading Advisors, or CTAs. Trading disciplines are generally systematic or discretionary. Systematic traders tend to use price and market specific information (often technical) to make trading decisions, while discretionary managers use a judgmental approach.

Multi-Strategy. Multi-Strategy hedge funds are characterized by their ability to dynamically allocate capital among strategies falling within several traditional hedge fund disciplines. The use of many strategies, and the ability to reallocate capital between strategies in response to market opportunities, means that such hedge funds are not easily assigned to any traditional category. The multi-strategy category also includes hedge funds employing unique strategies that do not fall under any of the other descriptions.

A.4.2 Overview of CRSP Mutual Fund Database

The mutual fund data was obtained from "The CRSP Survivor-Bias-Free US Mutual Fund Database" based on the downloaded data in February 2008.[A.8] This data set also suffers from some biases noted in the accompanying documentation provided by CRSP. For example, there is a selection bias favoring the historical data for the best past performing private funds which became public. In addition, upon any split, the past history is inherited by any resulting fund which introduces some return averaging bias in the data source.

We will use funds with at least 5 years of monthly reported history in our sample period of January 1986 to December 2006. Funds that had missing months were excluded from the study. To make data comparable with the hedge fund data, we have assigned "Live" and "Graveyard" classification to mutual funds based on their reporting history. A mutual fund is counted as Live if it reported returns in December 2006, and counted as Graveyard otherwise. We use the "Main Category" field of CRSP data to categorize mutual funds into one of the following categories: Asset Allocation, Convertible, Equity, Fixed Income, and Money Market. This field has only been available since July 2003 and about 20% of the funds that meet the minimum reporting history requirement do not have any category information. These funds will be included in any analysis that does not require category information with the exception of any such fund for which we fail to reject the null hypothesis of unit-root at 5% significance level as outlined in Section 6.1.3. Furthermore, we have used historical end-of-year category information. As a result, there are 50 funds that have had more than one category classification over our sample study period. These funds will be included in the analysis that does not require category information, but will be left out from any analysis relevant to different categories of funds. We will exclude all Money Market funds from our study as the returns for these funds are better represented by a unit-root type non-stationary process; see Section 6.1.2 for the details. The remaining 15,654 funds will be used in any analysis that does not require category information. Table A.2 gives a breakdown of the funds used in this study. Table A.3 gives important statistics of our data set.

[A.8]This data was obtained through MIT Sloan access to Wharton Research Data Services (WRDS).

Table A.2: Breakdown for the composition of hedge fund and mutual fund data used in this study. Please see Section 6.1.3 for more detail about the unit-root test.

Panel A: Hedge Funds

Category	***Live***	***Graveyard***	***Combined***
Convertible Arbitrage	57	44	101
Dedicated Short Bias	12	13	25
Emerging Markets	102	80	182
Equity Market Neutral	106	47	153
Event Driven	157	97	254
Fixed Income Arbitrage	69	39	108
Fund of Funds	437	194	631
Global Macro	56	70	126
Long/Short Equity Hedge	562	344	906
Managed Futures	135	173	308
Multi-Strategy	110	23	133
	To be Used in the Study		2,927

Panel B: Mutual Funds

Category	***Live***	***Graveyard***	***Combined***	***Failed the Unit Root Test***
Asset Allocation	981	152	1,133	1
Convertible	59	15	74	0
Equity	6,580	1,046	7,626	88
Fixed Income	3,578	510	4,088	53
Info. N/A	10	3,068	3,078	395
Money Market	1,335	225	1,560	1,460
Unclear (Multiple Categories)	50	0	50	0
	To be Used in the Study		15,654	

Table A.3: Summary statistics for all funds with at least 5 years of reporting history after January 1986. Results are given for different categories of hedge funds and mutual funds, based on both raw and excess returns. All numbers are based on monthly returns and not annualized. The last column shows the p-value of Ljung and Box Q-statistic calculate based the first 3 lags autocorrelation values. We also provide summary statistics for all funds excluding funds with a unit-root in their reported returns; see Section 6.1.3 for details.

Fund Type	Category	Count	Mean		Standard Deviation		Skewness		Kurtosis		Sharpe Ratio		Rho 1		Q-Stat (3 Lags) p-Value	
			Average	SD	Average	SD	Average	SD	Average	SD	Average	SD	Average	SD	Average	SD
By Fund Type and Category																
Hedge Fund	Convertible Arbitrage	101	0 82%	0 40%	1 93%	1 52%	-0 27	1 55	7 63	11 77	0 77	11 77	38 3%	17 3%	0 05	0 13
Hedge Fund	Dedicated Short Bias	25	0 21%	0 47%	5 91%	3 34%	0 30	0 39	5 28	2 60	0 09	2 60	9 2%	12 5%	0 37	0 26
Hedge Fund	Emerging Markets	182	1 33%	1 12%	6 63%	4 10%	-0 33	1 50	9 17	7 89	0 27	7 89	17.4%	11 2%	0 24	0 27
Hedge Fund	Equity Market Neutral	153	0 72%	0 39%	2 16%	1 48%	0 34	0 93	5 57	3 99	0 46	3 99	11 4%	20 0%	0 27	0 29
Hedge Fund	Event Driven	254	0 98%	0 61%	2 45%	2 57%	-0 20	1 34	7 93	9 20	0 54	9 20	22 7%	15 2%	0 15	0 23
Hedge Fund	Fixed Income Arbitrage	108	0 75%	0 53%	2 13%	1 66%	-1 32	2 91	17 22	23 58	0 57	23 58	19 2%	20 4%	0 26	0 32
Hedge Fund	Fund of Funds	631	0 70%	0 36%	2 25%	1 68%	-0 13	1 27	7 27	7 36	0 43	7 36	18 9%	15 0%	0 23	0 28
Hedge Fund	Global Macro	126	0 86%	0 94%	4 70%	2 90%	0 38	0 97	6 16	3 83	0 23	3 83	7 7%	13 6%	0 35	0 28
Hedge Fund	Long/Short Equity Hedge	906	1 18%	0 64%	4 75%	2 79%	0 39	1 14	6 72	5 72	0 30	5 72	12 6%	14 1%	0 30	0 29
Hedge Fund	Managed Futures	308	0 91%	0 72%	6 05%	3 85%	0 31	0 81	5 41	3 91	0 18	3 91	0 4%	11 7%	0 40	0 30
Hedge Fund	Multi-Strategy	133	0 95%	0 54%	3 04%	2 78%	-0 05	1 97	9 97	13 19	0 48	13 19	17 8%	17 2%	0 19	0 25
Mutual Fund	Asset Allocation	1,133	0 52%	0 24%	2 63%	0 79%	-0 50	0 39	4 13	1 84	0 22	1 84	5 3%	6 7%	0 65	0 24
Mutual Fund	Convertible	74	0 72%	0 20%	3 24%	0 81%	-0 35	0 48	5 41	2 20	0 23	2 20	10 0%	6 1%	0 48	0 28
Mutual Fund	Equity	7,626	0 73%	0 49%	5 34%	1 91%	-0 35	0 52	4 34	3 23	0 15	3 23	7 7%	8 0%	0 55	0 28
Mutual Fund	Fixed Income	4,088	0 44%	0 13%	1 25%	0 62%	-0 49	0 60	4 94	4 90	0 41	4 90	8 2%	10 9%	0 26	0 22
Mutual Fund	Info N/A	3,078	0 53%	0 39%	2 92%	2 64%	-0 38	0 92	5 08	7 13	0 75	7 13	21 1%	30 0%	0 37	0 32
Mutual Fund	Money Market	1,560	0 28%	0 50%	0 41%	6 59%	-0 01	0 81	2 76	7 45	1 84	7 45	94 2%	10 0%	0 00	0 06
Mutual Fund	Unclear (Multiple Categories)	50	0 56%	0 28%	3 31%	1 87%	-0 39	0 60	4 00	1 34	0 41	1 34	10 8%	11 2%	0 62	0 29
By Fund Type Only																
	Hedge Funds	2,927	0 95%	0 67%	3 87%	3 13%	0 06	1 39	7 43	8 62	0 37	8 62	14 9%	16 7%	0 27	0 29
	Mutual Funds	15,654	0 61%	0 41%	3.71%	2 46%	-0 41	0 61	4 63	4 33	0 27	4 33	8 3%	10 8%	0 46	0 30

A.4.3 Proof of Proposition 6.1

We use the *Generalized Method of Moments* (*GMM*) approach to prove Proposition 6.1. The necessary result is given by the following Lemma. Please see Chapter 14 of Hamilton (1994) for a review of the GMM theory.

Lemma A.2 *(Generalized Method of Moments) Let w_t be an $(h \times 1)$ vector stochastic process. Let θ_0 denote an unknown $(a \times 1)$ vector of coefficients that characterizes the density of observed values. Finally, let $\varphi(w_t, \theta)$ be an $(r \times 1)$ vector valued function, $\varphi : (R^h \times R^a) \rightarrow R^r$. We assume the following technical conditions hold:*

- $\{w_t : t \in \{1, \infty\}\}$ *is stationary and ergodic*
- $\theta_0 \in \Theta$, *where Θ is an open subset of R^a*
- $\forall \theta \in \Theta$ $\varphi(\cdot, \theta)$ *and* $\frac{\partial \varphi}{\partial \theta}(\cdot, \theta)$ *are Borel measurable*
- $\varphi(w, \cdot)$ *is a continues on Θ for all w*
- $E[\frac{\partial \varphi}{\partial \theta}(w, \cdot)]$ *exits and it is full rank*
- *For true θ, θ_0, we have $E[\varphi(\theta_0, w_t)] = 0$.*

Let $\hat{\theta}_T$ solves $\frac{1}{T}\sum_t^T \varphi(w_t, \theta) = 0$. Hansen (1982) shows that the following holds:

$$\hat{\theta}_T \xrightarrow{p} \theta_0 \tag{A.53a}$$

$$\sqrt{T}(\hat{\theta}_T - \theta_0) \xrightarrow{d} N(0, V_\theta) \tag{A.53b}$$

where

$$V_\theta = H^{-1}\Sigma H^{-1^T} \tag{A.54a}$$

$$H = \lim_{T\rightarrow\infty} E\left[\frac{1}{T}\sum_{t=1}^{T}\frac{\partial \varphi}{\partial \theta}(w_t, \theta_0)\right] \tag{A.54b}$$

$$\Sigma = \lim_{T\rightarrow\infty} E\left[\frac{1}{T}\sum_{t=1}^{T}\sum_{s=1}^{T}\varphi(w_t, \theta_0)\varphi(w_s, \theta_0)\right] \tag{A.54c}$$

Proof of Proposition 6.1

The proof follows almost immediately by setting up the appropriate moment conditions and appealing to Lemma A.2. In this case, we need to estimate three parameters of the time series: mean, μ, variance, d_2, and autocovariance, γ_1. Hence, the moment conditions can be set up as follows:

$$\varphi(w_t, \theta) = \begin{bmatrix} r_t - \mu \\ (r_t - \mu)^2 - d_2 \\ (r_t - \mu)(r_{t-1} - \mu) - \gamma_1 \end{bmatrix} \tag{A.55}$$

where

$$w_t = \begin{bmatrix} r_t \\ r_{t-1} \end{bmatrix}$$

$$\theta = \begin{bmatrix} \mu \\ d_2 \\ \gamma_1 \end{bmatrix}$$

It is easy to show that in this case the H matrix in expression (A.54a), itself given by (A.54b), is equal to $-I$. So the variance of the estimator for θ, given by (A.54c), is equal to:

$$\lim_{T\to\infty} \mathrm{E}\left[\frac{1}{T}\sum_{t,s=1}^{T} \begin{bmatrix} r_t - \mu \\ (r_t - \mu)^2 - d_2 \\ (r_t - \mu)(r_{t-1} - \mu) - \gamma_1 \end{bmatrix} \begin{bmatrix} r_s - \mu, & (r_s - \mu)^2 - d_2, & (r_s - \mu)(r_{s-1} - \mu) - \gamma_1 \end{bmatrix}\right] \tag{A.56}$$

Since autocorrelation is a function of the three estimated parameters, the *Delta method* can be used to estimate the variance of the autocorrelation estimate once the variance of the θ estimate is known. Let $\rho = g(\theta) = \frac{\gamma_1}{d_2}$. Then

$$\frac{\partial g}{\partial \theta} = \begin{bmatrix} 0 \\ -\frac{\gamma_1}{d_2^2} \\ \frac{1}{d_2} \end{bmatrix} \tag{A.57}$$

Putting all these together, we have

$$\sqrt{T}(\hat{\rho} - \rho_0) \sim N\left(0, \begin{bmatrix} 0 & -\frac{\gamma_1}{d_2^2} & \frac{1}{d_2} \end{bmatrix} \Sigma \begin{bmatrix} 0 \\ -\frac{\gamma_1}{d_2^2} \\ \frac{1}{d_2} \end{bmatrix}\right) \tag{A.58}$$

where Σ is given by (A.56). Note that under this null hypothesis, $\gamma_1 = 0$ and, hence, the second entry in the partial derivation given on line (A.57) is zero. So all we need to calculate the variance of $\hat{\rho}$ is the $\Sigma_{3,3}$ where Σ is given by A.56. Furthermore, since there is no persistence in the returns, it is easy to see that all the terms $t \neq s$ in (A.56) are equal to zero. Noting that $\gamma_1 = 0$, this expression implies that:

$$\begin{aligned} \Sigma_{3,3} &= \lim_{T\to\infty} E\left[\frac{1}{T}\sum_{t=1}^{T}(r_t - \mu)^2(r_{t-1} - \mu)^2\right] \\ &= E\left[(r_t - \mu)^2(r_{t-1} - \mu)^2\right] \end{aligned} \tag{A.59}$$

Appealing to (A.58), we have:

$$\hat{\rho} \sim N\left(0, \frac{1}{T}\frac{\Sigma_{3,3}}{d_2^2}\right) \tag{A.60}$$

Finally, notice that each element of the final expression, $\Sigma_{3,3}$ and d_2^2 can be estimated by any consistent estimator. The most natural estimator is to replace each of these expression with their sample counterpart to get

$$\Sigma_{3,3} = T^{-1}\sum \epsilon_t^2\epsilon_{t-1}^2 \tag{A.61a}$$

$$d_2 = T^{-1}\sum \epsilon_t^2 \tag{A.61b}$$

Substituting (A.61a) and (A.61b) back into (A.62), we arrive at the desired result:

$$\hat{\rho} \sim N\left(0, \frac{\sum \epsilon_t^2\epsilon_{t-1}^2}{(\sum \epsilon_t^2)^2}\right) \tag{A.62}$$

A.4.4 Proof of Proposition 6.2

For the proof of Proposition 6.2, we will be using the following result from the *Functional Central Limit Theory.*

Lemma A.3 *(Functional Central Limit Theory) (Hamilton, 1994, Chapter 17 and Ibragimov & Phillips, 2004) Let ν_t and ϵ_t be two martingale difference sequences adopted to filtration $\mathcal{H}_t$, i.e., $E[\nu_t|\mathcal{H}_{t-1}] = 0$ and $E[\epsilon_t|\mathcal{H}_{t-1}] = 0$. Also assume that $E[\nu_t\epsilon_t|\mathcal{H}_{t-1}] = 0$. Furthermore assume that both ν_t and ϵ_t have finite second moments and are ergodic up to second order, i.e.:*

- $E(\nu_t^2) = \sigma_{\nu_t}^2 > 0$ *and* $E(\epsilon_t^2) = \sigma_{\epsilon_t}^2 > 0$
- $1/T\sum_{t=1}^T \sigma_{\nu_t}^2 \to \sigma_\nu^2$ *and* $1/T\sum_{t=1}^T \sigma_{\epsilon_t}^2 \to \sigma_{\epsilon_t}^2$
- $E|\nu_t|^{r_1} < \infty$ *and* $E|\epsilon_t|^{r_2} < \infty$ *for some* $r_1 > 2$ *and* $r_2 > 2$
- $1/T\sum_{t=1}^T \nu_t^2 \xrightarrow{p} \sigma_\nu^2$ *and* $1/T\sum_{t=1}^T \epsilon_t^2 \xrightarrow{p} \sigma_\epsilon^2$

Define μ_t as the partial sum of ν_t, i.e.:

$$\tilde{\mu}_t = \sum_{i=1}^{t-1} \nu_i \tag{A.63}$$

The following asymptotic results hold:

1. $T^{-1/2}\tilde{\mu}_t \xrightarrow{d} \sigma_\nu W(t/T)$
2. $T^{-3/2}\sum \tilde{\mu}_t \xrightarrow{d} \sigma_\nu \int_0^1 W(u)du$

3. $T^{-2}\sum \tilde{\mu}_t^2 \xrightarrow{d} \sigma_\nu^2 \int_0^1 W(u)^2 du$

4. $T^{-1}\sum \tilde{\mu}_t \nu_t \xrightarrow{d} 1/2\sigma_\nu^2 (W(1)^2 - 1)$

5. $T^{-1}\sum \tilde{\mu}_t \epsilon_t \xrightarrow{d} \sigma_\nu \sigma_\epsilon \int_0^1 W(u) dV$

6. $T^{-3/2}\sum \tilde{\mu}_t^2 \epsilon_t \xrightarrow{d} \sigma_\nu^2 \sigma_\epsilon \int_0^1 W(u)^2 dV$

where $W(u)$ and $V(u)$ are uncorrelated Brownian motion on $[0,1]$

Proof of Proposition 6.2

Recall that under the null hypothesis of the unit-root in the expected returns, the returns are assumed to be given by a data-generating process of the following form:

$$H_0 \;:\; r_t = \mu_t + \epsilon_t$$
$$\mu_t = \mu + \sum_{i=0}^{t-1} \nu_i$$

where both ϵ_t and ν_t are martingale difference sequences adopted to the filtration $\mathcal{H}_t = \sigma(\nu_1, \cdots, \nu_t, \epsilon_1, \cdots, \epsilon_t)$, i.e., we assume $E[\epsilon_{t+1}|\mathcal{H}_t] = 0$,$E[\nu_{t+1}|\mathcal{H}_t] = 0$, and $E[\nu_{t+1}\epsilon_{t+1}|\mathcal{H}_t] = 0$. To make direct application of Lemma A.3, write $\mu_t = \mu + \tilde{\mu}_t$, where $\tilde{\mu}_t$ has the same form as the expression given in A.63. Now assume we estimate the serial correlation by running the following regression:

$$r_{t+1} = \hat{\alpha} + \hat{\rho} r_t + \tilde{\eta}_t$$

We will then have:

$$\hat{\rho} = \frac{(T-1)^{-1}\sum_{t=1}^{T-1} r_t r_{t+1} - \left[(T-1)^{-1}\sum_{t=1}^{T-1} r_t\right]\left[(T-1)^{-1}\sum_{t=2}^{T} r_t\right]}{(T-1)^{-1}\sum_{t=1}^{T-1} r_t^2 - \left[(T-1)^{-1}\sum_{t=1}^{T-1} r_t\right]^2} \tag{A.64}$$

$\hat{\rho} - 1$ will be equal to:

$$\hat{\rho} - 1 = \frac{(T-1)^{-1}\sum_{t=1}^{T-1} r_t(r_{t+1} - r_t) - (T-1)^{-2}\sum_{t=1}^{T-1} r_t(r_T - r_1)}{(T-1)^{-1}\sum_{t=1}^{T-1} r_t^2 - ((T-1)^{-1}\sum_{t=1}^{T-1} r_t)^2} \tag{A.65}$$

We can replace $T-1$ with T as it will be inconsequential in large samples. Then rewriting this expression in terms of the quantity of interest, $T^{-1}(\hat{\rho}-1)$, we have:

$$T^{-1}(\hat{\rho} - 1) = \frac{T^{-1}\sum_{t=1}^{T-1} r_t(r_{t+1} - r_t) - \left[T^{-1/2}(r_T - r_1)\right]\left[T^{-3/2}\sum_{t=1}^{T} r_t\right]}{T^{-2}\sum_{t=1}^{T} r_t^2 - \left[T^{-3/2}\sum_{t=1}^{T} r_t\right]^2} \tag{A.66}$$

We find the asymptotic expression for various elements of the above expression and use the *Continuous Mapping Theorem* to find the desired asymptotic expression. Start by looking at the expression in the denominator.

$$\begin{aligned} T^{-2}\sum_{t=1}^{T} r_t^2 &= T^{-2}\sum_{t=1}^{T}(\mu + \tilde{\mu_t} + \epsilon_t)^2 \\ &= T^{-2}\sum_{t=1}^{T}\mu^2 + T^{-2}\sum_{t=1}^{T}\tilde{\mu_t}^2 + T^{-2}\sum_{t=1}^{T}\epsilon_t^2 \\ &+ T^{-2}\sum_{t=1}^{T}2\mu\tilde{\mu_t} + T^{-2}\sum_{t=1}^{T}2\mu\epsilon_t + T^{-2}\sum_{t=1}^{T}2\tilde{\mu_t}\epsilon_t \end{aligned}$$

Out of the six terms on the right-hand side of the final expression, only term $T^{-2}\sum_{t=1}^{T}\tilde{\mu_t}^2$ does not converge to zero in probability. The limiting distribution of this term is readily given by item 3 in Lemma A.3. $T^{-2}\sum_{t=1}^{T}\mu^2 \to 0$ since $T^{-1}\sum_{t=1}^{T}\mu^2 \to \mu^2$. $T^{-2}\sum_{t=1}^{T}\epsilon_t^2$ can be written as $T^{-1}(T^{-1}\sum_{t=1}^{T}\epsilon_t^2)$, which converges to zero by the second order ergodicity assumption presented in Lemma A.3. $T^{-2}\sum_{t=1}^{T}2\mu\tilde{\mu_t}$ can be written as $T^{-1/2}2\mu(T^{-3/2}\sum_{t=1}^{T}\tilde{\mu_t})$ which again converges to zero in probability due to item 2 in Lemma A.3. $T^{-2}\sum_{t=1}^{T}2\mu\epsilon_t$ can be written as $T^{-1}2\mu(T^{-1}\sum_{t=1}^{T}\epsilon_t)$, which converges to zero in probability due to ϵ_t, is a martingale difference sequence, i.e., zero mean, and first order ergodicity. And finally, $T^{-2}\sum_{t=1}^{T}2\tilde{\mu_t}\epsilon_t = T^{-1}(T^{-1}\sum_{t=1}^{T}2\tilde{\mu_t}\epsilon_t) \xrightarrow{p} 0$ using item 5 in Lemma A.3. Putting all these together, we have:

$$T^{-2}\sum_{t=1}^{T} r_t^2 \xrightarrow{d} \sigma_\nu^2 \int_0^1 W(u)^2 du \tag{A.67}$$

Finding the limiting expression for the second term in the denominator, $(T^{-3/2}\sum_{t=1}^{T} r_t)^2$ is much simpler. First, note that $T^{-3/2}\sum_{t=1}^{T} r_t = T^{-3/2}\sum_{t=1}^{T}\mu + T^{-3/2}\sum_{t=1}^{T}\tilde{\mu_t} + T^{-3/2}\sum_{t=1}^{T}\epsilon_t$. The first and third part converge to zero in probability for reasons already presented in the above argument. From item 2 in Lemma A.3 we have: $T^{-3/2}\sum_{t=1}\tilde{\mu_t} \xrightarrow{p} \sigma_\nu \int_0^1 W(u)du$. Combining these limiting expressions and appealing to the Continuous Mapping Theorem, we get:

$$(T^{-3/2}\sum_{t=1}^{T} r_t)^2 \xrightarrow{d} (\sigma_\nu \int_0^1 W(u)du)^2 \tag{A.68}$$

We apply a similar approach to find the limiting expressions for the two terms given in the numerator of expression A.66 as follows:

$$
\begin{aligned}
T^{-1}\sum_{t=1}^{T-1} r_t(r_{t+1}-r_t) &= T^{-1}\sum_{t=1}^{T-1}(\mu+\tilde{\mu}_t+\epsilon_t)(\nu_t+\epsilon_{t+1}-\epsilon_t)\\
&= T^{-1}\sum_{t=1}^{T-1}(\mu+\epsilon_t)(\nu_t+\epsilon_{t+1}-\epsilon_t)+T^{-1}\sum_{t=1}^{T-1}\tilde{\mu}_t(\nu_t+\epsilon_{t+1}-\epsilon_t)
\end{aligned}
$$

Using an argument similar to what is already presented in the last few lines, it is easy to show that:

$$
T^{-1}\sum_{t=1}^{T-1}(\mu+\epsilon_t)(\nu_t+\epsilon_{t+1}-\epsilon_t) \xrightarrow{p} -\sigma_\epsilon^2 \tag{A.69}
$$

Rewrite the second part as $T^{-1}\sum_{t=1}^{T-1}\tilde{\mu}_t(\nu_t+\epsilon_{t+1}-\epsilon_t) = T^{-1}\sum_{t=1}^{T-1}\tilde{\mu}_t\nu_t + T^{-1}\sum_{t=1}^{T-1}\tilde{\mu}_t(\epsilon_{t+1}-\epsilon_t)$. The limiting expression for the first part can be found using results presented in Lemma A.3. Rewrite this as:

$$
T^{-1}\sum_{t=1}^{T-1}\tilde{\mu}_t\nu_t \xrightarrow{d} 1/2\sigma_\nu^2(W(1)^2-1) \tag{A.70}
$$

The second part is a telescoping sum which can be simplified to

$$
\begin{aligned}
T^{-1}\sum_{t=1}^{T-1}\tilde{\mu}_t(\epsilon_{t+1}-\epsilon_t) &= T^{-1}\sum_{t=1}^{T-1}\sum_{i=1}^{t-1}\nu_i(\epsilon_{t+1}-\epsilon_t)\\
&= T^{-1}\sum_{i=0}^{T-2}\nu_i\sum_{t=i+1}^{T-1}(\epsilon_{t+1}-\epsilon_t)\\
&= T^{-1}\sum_{i=1}^{T-1}\nu_i(\epsilon_T-\epsilon_{i+1})\\
&= \epsilon_T(T^{-1}\sum_{i=1}^{T-1}\nu_i)-T^{-1}\sum_{i=1}^{T-1}\nu_i\epsilon_{i+1}\\
&\xrightarrow{p} 0
\end{aligned} \tag{A.71}
$$

where we used the fact that $T^{-1}\sum_{i=1}^{T-1}\nu_i \xrightarrow{p} 0$ since ν_i is a martingale difference sequence and ergodic for the first moment and $T^{-1}\sum_{i=1}^{T-1}\nu_i\epsilon_{i+1} \xrightarrow{p} 0$ since ϵ_{i+1} is a martingale difference sequence with respect to $\mathcal{H}_t$. The final term we have to calculate is $T^{-1/2}(r_T-r_1)$. The second part of this $T^{-1/2}r_1 \xrightarrow{p} 0$ by the assumption regarding finiteness of the second moment for ν_t and ϵ_t. The first part can be rewritten as $T^{-1/2}r_T = T^{-1/2}(\tilde{\mu}_t+\epsilon_T) \xrightarrow{d} \sigma_\nu W(1)$. So we have

$$T^{-1/2}(r_T - r_1)\left[T^{-3/2}\sum_{t=1}^{T} r_t\right] \xrightarrow{d} \sigma_\nu^2 W(1)\int_0^1 W(u)du \tag{A.72}$$

Substituting (A.67), (A.68), (A.69), (A.70), (A.71), and (A.72) into (A.66) and appealing to the Continuous Mapping Theorem, we get the desired result:

$$\begin{aligned} T(\hat{\rho} - 1) &\xrightarrow{d} \frac{\frac{1}{2}\sigma_\nu^2(W(1)^2 - 1) - \sigma_\nu^2 W(1)\int_0^1 W(u)du - \sigma_\epsilon^2}{\sigma_\nu^2\int_0^1 W(u)^2 du - (\sigma_\nu \int_0^1 W(u)du)^2} \\ &\xrightarrow{d} \frac{\frac{1}{2}(W(1)^2 - 1) - W(1)\int_0^1 W(u)du - \frac{\sigma_\epsilon^2}{\sigma_\nu^2}}{\int_0^1 W(u)^2 du - (\int_0^1 W(u)du)^2} \end{aligned} \tag{A.73}$$

A.4.5 Proof of Corollary 6.1

Recall that $r_t = \mu_t + \epsilon_t = \mu + \tilde{\mu}_t + \epsilon_t$. Taking the first difference of this we have:

$$\begin{aligned} \Delta r_t &= r_t - r_{t-1} \\ &= \nu_{t-1} + \epsilon_t - \epsilon_{t-1} \end{aligned} \tag{A.74}$$

Using the fact that ν_t and ϵ_t are martingale difference sequences adopted to filtration $\mathcal{H}_t$ allows us to easily calculate the $Corr(\Delta r_t, \Delta r_{t-1})$ as:

$$\begin{aligned} Corr(\Delta r_t, \Delta r_{t-1}) &= \frac{Cov(\Delta r_t, \Delta r_{t-1})}{Var(\Delta r_t)} \\ &= \frac{-\sigma_\epsilon^2}{\sigma_\nu^2 + 2\sigma_\epsilon^2} \end{aligned}$$

The expression for $\frac{\sigma_\epsilon^2}{\sigma_\nu^2}$ given in Corollary 6.1 follows immediately.

A.4.6 Proof of Proposition 6.3

Recall that for each security, we assigned the indicator random variable, $\delta_{i,t}$, that shows if security i was traded in period t. $\delta_{i,t}$ was defined as:

$$\delta_{i,t} = 1\{\rho_i\nu_t + \sqrt{1 - \rho_i^2}\theta_{i,t} \leq \alpha_i\} \tag{A.75}$$

where ν_t and $\theta_{i,t}$ are both $N(0,1)$, ν_t are independent across t and $\theta_{i,t}$ are independent across both i and t. ρ_i is the parameter that captures the dependence among $\delta_{i,t}$s across i. Note that that the unconditional trading probability is equal to p_i. This was assured by setting $\alpha_i = \Phi^{-1}(p_i)$. Let I_p denote the set of securities held in portfolio p. We also assumed that all the securities in this portfolio have the unconditional trading probability of p_p, i.e.,

$\forall i \in I_p, p_i = p_p$ and $\rho_i = \rho_p$. For these securities, the conditional probability condition on the value of ν is given by:

$$P_p^{\rho_p}(\nu) = \Phi\left(\frac{\Phi^{-1}(p_p) - \rho_p \nu}{\sqrt{1-\rho_p^2}}\right) \tag{A.76}$$

$P_p^{\rho_p}(\nu)$ is a random variable. It will be useful for the derivation to define its second moment as follows:

$$D_p^{\rho_p} = \mathrm{E}_\nu[P_p^{\rho_p}(\nu)^2] \tag{A.77}$$

As outlined in the discussion prior (6.22), the observed return of the portfolio is given by:

$$r_{p,t}^o \overset{a.s.}{=} \sum_{k=0}^{\infty} (\mu_p + \beta_p m_{t-k}) Y_t^{\rho_p}(\vec{\nu}, k) \tag{A.78}$$

where the portfolio mean, μ_p, and beta, β_p are defined as:

$$\mu_p = \frac{1}{N_p} \sum_{i \in I_p} \mu_i \tag{A.79a}$$

$$\beta_p = \frac{1}{N_p} \sum_{i \in I_p} \beta_i \tag{A.79b}$$

and

$$Y_t^{\rho_p}(\vec{\nu}, k) = P_{p,}^{\rho_p}(\nu_t)\left(1 - P_{p,}^{\rho_p}(\nu_{t-1})\right) \cdots \left(1 - P_{p,}^{\rho_p}(\nu_{t-k})\right) \tag{A.79c}$$

is a random variable defined in terms of realization of the common random variable ν that controls the non-trading event. The expression given in (A.78) already takes into account the cross-sectional expectation, i.e., over individual security $\theta_{i,t}$. We only need to take the expectation over $\vec{\nu}$ to arrive at various time-series properties of $r_{p,t}^o$ as given here:

$$
\begin{aligned}
\mathrm{E}\left[r^o_{p,t}\right] &= \mathrm{E}\left[\sum_{k=0}^{\infty}(\mu_p+\beta_p m_{t-k})\,Y_t^{\rho_p}(\vec{\nu},k)\right] \\
&= \sum_{k=0}^{\infty}(\mu_p+\beta_p m_{t-k})\,p_p(1-p_p)^k \\
&= \mu_p
\end{aligned}
\tag{A.80}
$$

$$
\begin{aligned}
\mathrm{E}\left[r^{o^2}_{p,t}\right] &= \mathrm{E}\left[\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}(\mu_p+\beta_p m_{t-k})(\mu_p+\beta_p m_{t-l})\,Y_t^{\rho_p}(\vec{\nu},k)Y_t^{\rho_p}(\vec{\nu},l)\right] \\
&= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\mathrm{E}\left[(\mu_p+\beta_p m_{t-k})(\mu_p+\beta_p m_{t-l})\right]\mathrm{E}\left[Y_t^{\rho_p}(\vec{\nu},k)Y_t^{\rho_p}(\vec{\nu},l)\right]
\end{aligned}
\tag{A.81}
$$

(A.81) has three different cases:

- $l < k$

$$
\begin{aligned}
\mathrm{E}\left[Y_t^{\rho_p}(\vec{\nu},k)Y_t^{\rho_p}(\vec{\nu},l)\right] &= D_p^{\rho_p}(1-2p_p+D_p^{\rho_p})^l(1-p_p)^{k-l} \\
\mathrm{E}\left[(\mu+\beta m_{t-k})(\mu+\beta m_{t-l})\right] &= \mu_p^2
\end{aligned}
$$

- $l = k$

$$
\begin{aligned}
\mathrm{E}\left[Y_t^{\rho_p}(\vec{\nu},k)Y_t^{\rho_p}(\vec{\nu},l)\right] &= D_p^{\rho_p}(1-2p_p+D_p^{\rho_p})^k \\
\mathrm{E}\left[(\mu+\beta m_{t-k})(\mu+\beta m_{t-l})\right] &= \mu_p^2+\beta_p^2\sigma^2
\end{aligned}
$$

- $l > k$

$$
\begin{aligned}
\mathrm{E}\left[Y_t^{\rho_p}(\vec{\nu},k)Y_t^{\rho_p}(\vec{\nu},l)\right] &= D_p^{\rho_p}(1-2p_p+D_p^{\rho_p})^k(1-p_p)^{l-k} \\
\mathrm{E}\left[(\mu+\beta m_{t-k})(\mu+\beta m_{t-l})\right] &= \mu_p^2
\end{aligned}
$$

Substitute these back into (A.81) to get:

$$
\mathrm{E}\left[r^{o^2}_{p,t}\right] = \frac{\mu_p^2(2D_p^{\rho_p}-p_p)+\sigma^2\beta_p^2 D_p^{\rho_p}p_p}{(2p_p-D_p^{\rho_p})p_p}
$$

Finally, we can use this to find the variance of observed returns

$$
\mathrm{Var}\left(r^o_{p,t}\right) = \frac{2\mu_p^2(D_p^{\rho_p}-p_p^2)+\sigma^2\beta_p^2 D_p^{\rho_p}p_p}{(2p_p-D_p^{\rho_p})p_p}
\tag{A.82}
$$

We can use a similar approach to find the autocovariance of the return:

$$\begin{aligned}\mathrm{Cov}(r^o_{p,t}, r^o_{p,t+n}) &= \mathrm{E}\left[r^o_{p,t} r^o_{p,t+n}\right] - \mu_p^2 \\ &= \mathrm{E}\left[\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} (\mu_p + \beta_p m_{t-k})(\mu_p + \beta_p m_{t+n-l})\, Y_t^{\rho_p}(\vec{\nu}, k) Y_t^{\rho_p}(\vec{\nu}, l)\right] - \mu_p^2\end{aligned} \tag{A.83}$$

The final expression has five different cases:

- $l < n$

$$\begin{aligned}\mathrm{E}\left[Y_t^{\rho_p}(\vec{\nu}, k) Y_{t+n}^{\rho_p}(\vec{\nu}, l)\right] &= p_p^2 (1 - p_p)^{l+k} \\ \mathrm{E}\left[(\mu + \beta m_{t-k})(\mu + \beta m_{t+n-l})\right] &= \mu_p^2\end{aligned} \tag{A.84}$$

- $l = n$

$$\begin{aligned}\mathrm{E}\left[Y_t^{\rho_p}(\vec{\nu}, k) Y_{t+n}^{\rho_p}(\vec{\nu}, l)\right] &= p_p^2 (p_p - D_p^{\rho_p})(1 - p_p)^{n+k-1} \\ \mathrm{E}\left[(\mu + \beta m_{t-k})(\mu + \beta m_{t+n-l})\right] &= \mu_p^2\end{aligned}$$

- $n < l < n + k$

$$\begin{aligned}\mathrm{E}\left[Y_t^{\rho_p}(\vec{\nu}, k) Y_{t+n}^{\rho_p}(\vec{\nu}, l)\right] &= p_p (p_p - D_p^{\rho_p})(1 - p_p)^{2n+k-l-1}(1 - 2p_p + D_p^{\rho_p})^{l-n} \\ \mathrm{E}\left[(\mu + \beta m_{t-k})(\mu + \beta m_{t+n-l})\right] &= \mu_p^2\end{aligned}$$

- $n + k = l$

$$\begin{aligned}\mathrm{E}\left[Y_t^{\rho_p}(\vec{\nu}, k) Y_{t+n}^{\rho_p}(\vec{\nu}, l)\right] &= p_p (p_p - D_p^{\rho_p})(1 - p_p)^{n-1}(1 - 2p_p + D_p^{\rho_p})^{k} \\ \mathrm{E}\left[(\mu + \beta m_{t-k})(\mu + \beta m_{t+n-l})\right] &= \mu_p^2 + \sigma^2 \beta_p^2\end{aligned}$$

- $n + k < l$

$$\begin{aligned}\mathrm{E}\left[Y_t^{\rho_p}(\vec{\nu}, k) Y_{t+n}^{\rho_p}(\vec{\nu}, l)\right] &= p_p (p_p - D_p^{\rho_p})(1 - p_p)^{l-k-1}(1 - 2p_p + D_p^{\rho_p})^{k} \\ \mathrm{E}\left[(\mu + \beta m_{t-k})(\mu + \beta m_{t+n-l})\right] &= \mu_p^2\end{aligned}$$

These can be substituted into (A.83) to find the autocovariance for general value of n. The final result is surprisingly simple:

$$\mathrm{Cov}(r^o_{p,t}, r^o_{p,t+1}) = \frac{\mu_p^2 (p_p^2 - D_p^{\rho_p}) + \sigma^2 \beta_p^2 (p_p^2 - p_p D_p^{\rho_p})}{2p_p - D_p^{\rho_p}} \tag{A.85}$$

$$\mathrm{Cov}(r^o_{p,t}, r^o_{p,t+n}) = (1 - p_p)^{n-1} \mathrm{Cov}(r^o_{p,t}, r^o_{p,t+1}) \tag{A.86}$$

Also using (A.82) to find the autocorrelation of the observed returns as given below:

$$\mathrm{Corr}(r^o_{p,t}, r^o_{p,t+1}) = \frac{p_p\left((D_p^{\rho_p} - p_p^2)\zeta_p^2 - (p_p^2 - p_p D_p^{\rho_p})\right)}{(2p_p^2 - 2D_p^{\rho_p})\zeta_p^2 - p_p D_p^{\rho_p}} \quad (A.87)$$

$$\mathrm{Corr}(r^o_{p,t}, r^o_{p,t+n}) = (1-p_p)^{n-1}\mathrm{Corr}(r^o_{p,t}, r^o_{p,t+1}) \quad (A.88)$$

where

$$\zeta_p = \frac{\mu_p}{\beta_p \sigma} \quad (A.89)$$

A.5 Appendix for Chapter 7

This appendix contains the background and some supporting material for Chapter 7 of this thesis. Appendix A.5.1 provides an overview of the 100 stock portfolios used in this chapter. Appendix A.5.2 contains some information on the various risk factors used to control for co-movement among funds and stock portfolios.

A.5.1 Overview of 100 Stock Portfolios

The stock return data set consists of the standard 100 Size and Book-to-Market sorted portfolios constructed at the end of each June by intersecting 10 portfolios based on size (market equity, ME) and 10 portfolios based on the ratio of book equity to market equity (BE/ME).[A.9] The historical data for these portfolios is available for a much longer period but we decided to use only the period 1986-2006 to put all our three data sources on a similar historical timeline in order to make comparisons between results easier. Out of the 100 portfolios, 3 portfolios[A.10] had missing data points due to lack of any security falling in the relevant intersection of size and BE/ME decile. We will only include these portfolio in our analysis if they have data for the relevant time period. Table A.4 presents the summary statistics of the monthly returns. To conserve space, we have aggregated the data two ways, either based on the size decile or the book-to-market equity decile to which a

[A.9] Data was obtained from the data library section of Kenneth French's web site:

`http : //mba.tuck.dartmouth.edu/pages/faculty/ken.french/`

According to the description supplied with the data, the size breakpoints for year t are the NYSE market equity decile at the end of June. Furthermore, the BE/ME for June of year t is the book equity for the last fiscal year end in t-1 divided by ME for December of t-1. The BE/ME breakpoints are also NYSE deciles. The portfolios for July of year t to June of t+1 include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for December of t-1 and June of t, and positive book equity data for t-1. Firms with negative book equity are not included in any portfolio. Please refer to the documentation available from that site for further details.

[A.10] Portfolio in the intersection of size decile 6-BE/ME decile 10 and the portfolio in the intersection of size decile 10-BE/ME decile 8 did not have data for July 2000 to June 2001. Similarly the portfolio in the intersection of size decile 10-BE/ME decile 10 did not have data for July 1999 to June 2000 and again from February 2001 to June 2001.

portfolio belongs. The data, when sorted by size decile, highlights in more detail the relation between autocorrelation and size as we previously elaborated on. This is consistent with our hypothesis that autocorrelation in returns is a proxy for illiquidity of an asset which in this case can be anecdotally related to the fact that smaller stocks are harder to trade and each trade will have a larger price impact.[A.11]

[A.11] See Mech (1993) and Lewellen (2002) for more extensive studies of sources of autocorrelation in portfolios of stocks.

Table A.4: This table contains relevant statistics for the 100 size and book-equity/market-equity (BE/ME) sorted portfolios. We have aggregated the results based on the relevant size or BE/ME decile and presented the statistics for both value and equal weighted returns. Note the apparent relation between size and autocorrelation. Values presented here are based on raw returns.

Decile	Mean			StDev			Skewness			Kurtosis			Sharpe Ratio			Rho_1			Q-Stastic (3 Lags) p-Value		
	Min	Average	Max	Min	Average	Max	Min	Average	Max	Min	Average	Max	Min	Average	Max	Min	Average	Max	Min	Average	Max
S1	0 15%	1 24%	1 65%	5 04%	6 41%	8 81%	-0 83	-0 12	1 04	6 05	8 38	15 54	0 02	0 21	15 54	13 2%	22 9%	33 1%	0 00	0 00	0 03
S2	0 19%	1 22%	1 80%	5 21%	6 56%	9 02%	-1 20	-0 34	0 66	5 48	7 73	11 92	0 02	0 20	11 92	6 6%	14 9%	25 6%	0 00	0 03	0 10
S3	0 56%	1 24%	1 60%	4 76%	6 04%	8 48%	-1 24	-0 82	-0 05	5 11	6 67	8 60	0 07	0 22	8 60	3 1%	14 3%	23 2%	0 00	0 05	0 27
S4	0 69%	1.15%	1 54%	4 96%	6 00%	8 34%	-1 33	-0 78	-0 11	4 79	6 74	8 67	0 08	0 20	8 67	6 1%	11 1%	20 8%	0 00	0 09	0 30
S5	0 75%	1 22%	1 43%	4 93%	5 85%	7 98%	-1 12	-0 78	-0 07	4 07	6 78	8 48	0 09	0 22	8 48	-0 2%	10 5%	16 2%	0 03	0 15	0 32
S6*	0 74%	1 20%	1 84%	4 63%	5 47%	7 86%	-0 93	-0 67	-0 27	4 62	6 07	6 93	0 09	0 22	6 93	4 1%	8 9%	12 6%	0 00	0 29	0 86
S7	1 05%	1 27%	1 44%	4 54%	5 36%	7 10%	-1 14	-0 69	0 33	4 40	7 01	9 00	0 18	0 24	9 00	-2 3%	6 0%	13 7%	0 03	0 36	0 97
S8	1 02%	1 22%	1 48%	4 45%	5 44%	7 57%	-0 92	-0 58	-0 02	3 90	5 81	8 52	0 15	0 23	8 52	-3 6%	4 3%	11 9%	0 03	0 45	0 89
S9	1 05%	1 21%	1 50%	4 18%	5 05%	6 24%	-1 08	-0 58	-0 32	3 76	5 18	6 72	0 20	0 24	6 72	-6 5%	2 8%	9 6%	0 14	0 52	0 95
S10*	0 58%	1 06%	1 25%	4 59%	5 30%	6 97%	-0 92	-0 39	0 43	4 21	5 68	10 16	0 11	0 20	10 16	-4 8%	-0 1%	3 9%	0 08	0 66	0 97
BE/ME1	0 15%	0 77%	1 26%	5 02%	7 64%	9 02%	-0 37	-0 09	0 42	4 07	5 50	7 85	0 02	0 11	7 85	1 0%	7 8%	22 9%	0 00	0 32	0 92
BE/ME2	0 60%	1 03%	1 22%	4 79%	6 33%	8 18%	-0 93	-0 37	0 53	4 61	6 15	8 25	0 07	0 17	8 25	-0 2%	7 5%	20 3%	0 00	0 16	0 79
BE/ME3	0 81%	1 12%	1 33%	4 98%	5 99%	7 71%	-0 95	-0 54	0 28	5 23	6 95	8 52	0 13	0 19	8 52	-1 1%	8 9%	20 9%	0 00	0 16	0 73
BE/ME4	1 03%	1 22%	1 50%	4 87%	5 70%	7 43%	-1 24	-0 64	0 66	4 97	7 43	11 92	0 19	0 22	11 92	-2 5%	8 3%	22 8%	0 00	0 21	0 97
BE/ME5	1 04%	1 22%	1 37%	4 59%	5 19%	5 78%	-1 19	-0 87	-0 31	5 59	6 90	9 00	0 20	0 24	9 00	0 7%	10 9%	20 3%	0 00	0 09	0 33
BE/ME6	1.05%	1 28%	1 60%	4 49%	5 15%	6 24%	-1 22	-0 60	1 04	5 00	7 35	15 54	0 21	0 25	15 54	-2 0%	9 4%	20 4%	0 00	0 28	0 95
BE/ME7	1 07%	1 36%	1 80%	4 76%	5 08%	5 74%	-1 33	-0 77	-0 39	4 21	6 60	9 18	0 22	0 27	9 18	-4 8%	10 4%	24 3%	0 00	0 37	0 89
BE/ME8*	0 58%	1 28%	1 55%	4 18%	4 92%	5 81%	-1 05	-0 65	-0 27	3 90	6 09	8 72	0 11	0 26	8 72	-6 5%	9 7%	24 5%	0 00	0 32	0 97
BE/ME9	1 13%	1 35%	1 65%	4 59%	5 32%	6 97%	-1 20	-0 66	0 43	4 55	7 25	10 16	0 16	0 26	10 16	2 1%	10 8%	27 1%	0 00	0 31	0 78
BE/ME10*	0 99%	1 38%	1 84%	5 50%	6 17%	6 89%	-0 95	-0 54	0 08	3 76	5 83	7 29	0 14	0 22	7 29	-3 6%	15 3%	33 1%	0 00	0 29	0 86

* Portfolio in the intersection of size decile 6-BE/ME decile 10 and the portfolio in the intersection of size decile 10-BE/ME decile 8 did not have data for July-2000 to June-2001 Similarly the portfolio in the intersection of size decile 10-BE/ME decile 10 did not hav data for July-1999 to June 2000 and again from February-2001 to June-2001 Please see the text for details We have excluded these portfolios in all calculation related to autocorrelation, and will only include these portfolio in the rest of our analysis if they have d for the relevant time periods

A.5.2 Overview of Generic Risk Factors

We use various subsets of 9 factors to control for the risk exposure of different funds. These factors are as follows. The first five factors capture the broad sources of common risk due to equities, fixed income, credit, commodities and the currency markets. The factors are as follows: Fama-French US Market Index, Lehman Brothers US Aggregate Government Bond Index, Lehman Brothers Universal High-Yield Corporate Index, Goldman Sachs Commodities Index, and USD Trade Weighted Dollar Index. We also use three factors related to size, measured by: Fama-French Small Minus Big (SMB) factor, the value, measured by Fama-French High Minus Low (HML), and the stock market momentum that have been studied extensively in asset-pricing literature.[A.12] We also include the first difference in the *CBOE Volatility Index* to capture any exposure to changes of market volatility that a particular fund may be exposed to. Even though this factor does not translate immediately to returns using any investment strategy, it can still add value to our analysis by capturing the volatility exposure of different funds arising from the nonlinear instruments included in some trading strategies. This effect should be more significant for hedge funds but we have decided to keep this factor in the rest of our analysis in order to keep all our results consistent across different asset classes. This factor in its initial format has a much higher level of volatility than all of the other factors. In order to avoid any numerical issues arising from this substantial difference, we decided to use a rescaled version of this factor by rescaling the monthly values to set their in-sample level of volatility to be the same as volatility of US Stock Market factor. This is a purely rescaling and mathematically won't change any of our analysis.

Table A.5 shows summary statistics of the factors used for sample of 1986-2006. It is worth noting that the only two factors for which the null of no autocorrelation can be rejected are the Lehman Brothers Universal High-Yield Corporate Index and the CBOE Volatility Index. The Lehman index most likely suffers more severely from non-trading mark-to-market inaccuracy in its pricing since it is the tracking index for high-yield corporate bonds that trade less frequently. The volatility factor can also have serial correlation, in this case negative, since it is well known that financial returns have stochastic volatility and can be fitted well using GARCH models. Of course the CBOE Volatility Index is based on option-implied volatility and the connection between this and the realized volatilities is not direct, but there should be a direct relation.

[A.12] Fama-French US Market Index, SMB, HML, and Momentum factors are obtained from WRDS. Goldman Sachs Commodities Index, and the USD Trade Weighted Dollar Index are obtained from the Global Financial Database. The total return of the Lehman Brothers US Aggregate Government Bond Index and Lehman Brothers Universal High-Yield Corporate Index are obtained from Data Stream. We use the monthly total return values for the US Market, Lehman US Government Bond, Lehman High-Yield and the Goldman Sachs Commodities Index to capture the effect of any dividend and/or coupon payments on the time series of returns.

Table A.5: Statistics for the factors that will be used to account for common sources of variations among returns of hedge funds, mutual funds, or portfolios of common stocks. Calculations are done based on monthly returns and values are not annualized. Data from January 1986 to December 2006 is used. The *Rescaled CBOE Volatility Index* has been rescaled to have the same level of in-sample volatility as the US Stock Market factor for 1986 to 2006 period. The *Broad Factor Set* that will be used in later parts of this paper for risk adjustment of return consists of the following factors: *US Stock Market*, *Lehman US Aggregate Government Bond Index*, *Lehman US Universal High-Yield Corporate Index*, *Goldman Sachs Commodities Index*, *Trade Weighted USD Index*, *Rescaled CBOE Volatility Index*, *Small-minus-Big (SMB)*, *High-minus-Low (HML)*, and the *Momentum (UMD)* factors.

Factor Name	*Mean*	*StDev*	*Skewness*	*Kurosis*	*Rho_1*	*Rho_2*	*Rho_2*	*Q-Statistic (3 Lags)* q-value	p-value
US Stock Market	1 03%	4 38%	-1 03	6 42	3 9%	-5 4%	-4 1%	1 52	68%
Lehman US Aggregate Government Bond Index	0 73%	1 63%	0 06	3 67	11 8%	-9 4%	-1 1%	6 42	9%
Lehman US Universal High-Yield Corporate Index	1 24%	3 67%	0 38	11 05	37 5%	6 3%	-2 5%	36 73	0%
Goldman Sachs Commodities Index	0 90%	5 42%	0 34	4 14	9 1%	-10 8%	2 6%	5 87	12%
Trade Weighted USD Index	-0 12%	2 54%	0 34	3 46	7 9%	0 6%	-1 4%	1 55	67%
Change in the CBOE Volatility Index	-2 58%	451%	2 78	26 82	-17 3%	-7 7%	-13 2%	13 50	0%
Rescaled CBOE Volatility Index*	-0 03%	4 38%	2 78	26 82	-17 3%	-7 7%	-13 2%	13 50	0%
Fama-French Small Minus Big Factor	0 06%	3 50%	0 83	10 98	-3 4%	2 7%	-13 4%	5 03	17%
Fama-French High Minus Low Factor	0 38%	3 18%	0 09	6 04	9 4%	5 7%	8 7%	4 96	18%
Momentum Factor	0 78%	4 44%	-0 68	9 25	-3 7%	-6 2%	4 7%	1 90	59%

Table A.6: Historical correlation between monthly returns for factor listed in Table A.5. Data from January 1986 to December 2006 is used. Factors are as follows: *US Stock Market (MARKT)*, *Lehman US Aggregate Government Bond Index (LH_GO)*, *Lehman US Universal High-Yield Corporate Index (LH_HY)*, *Goldman Sachs Commodities Index (GSCI)*, *Trade Weighted USD Index (USD)*, *Rescaled CBOE Volatility Index (VIX_S)*, *Small-minus-Big (SMB)*, *High-minus-Low (HML)*, and the *Momentum (UMD)* factors.

	MARKT	**LH_GO**	**LH_HY**	**GSCI**	**USD**	**VIX_S**	**SMB**	**HML**	**UMD**
MARKT	100%	7%	52%	-3%	8%	-61%	19%	-49%	-8%
LH_GO		100%	23%	-1%	-20%	15%	-20%	6%	14%
LH_HY			100%	-12%	9%	-33%	25%	-14%	-18%
GSCI				100%	-11%	0%	8%	3%	11%
USD					100%	-11%	5%	4%	-7%
VIX_S						100%	-22%	24%	10%
SMB							100%	-44%	12%
HML								100%	-9%
UMD									100%

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