CURRENT ALGEBRA AND CHIRAL SYMMETRY

Chairman	R. Dalitz
Rapporteur	B. Zumino
Discussion leader	D. Volkov
Secretaries	M. Eliashvili
	R. Kallosh
	A. Slavnov

CURRENT ALGEBRA

B. Zumino

1. Introduction

In this report I shall describe what seem to me the most interesting developments in current algebra since the Vienna Conference. I shall emphasize those topics which are related to contributions to this conference. Please do not look for completeness, especially in the list of references, and forgive me if I failed to mention your work. My talk will be divided into two parts. The first part deals with the conventional vector and axial vector currents of the electromagnetic and weak interactions, the second with the energy momentum tensor and the closely related currents of the scale and conformal transformations. The study of the energy momentum tensor is a natural extension of the conventional current algebra, since it is the source of the hadronic gravitational field. The idea of current algebra is that one can, from the known properties of the electromagnetic, weak and gravitational interactions, infer certain symmetries of the strong interactions.

Some progress has been made recently in the study of the vector and axial vector currents. It seems to me that it is mostly in the question of unitarity and in forming a relatively consistent picture of the $SU(3) \times SU(3)$ symmetry breaking. On the other hand, in questions concerning scale invariance and the energy momentum tensor, many new ideas and new problems have arisen in recent time and a new exciting field of theoretical speculation has opened up. By the next conference, we shall probably have a better idea about the physical relevance of this approach.

2. Ward identities and effective lagrangians

Let us begin with the vector and axial vector currents. Traditionally [1], one writes down equal time commutators and (exact or partial) conservation laws for these currents. For instance, for $SU(2) \times SU(2)$ one has

$$[V_i^0(x, t), V_j^0(x', t)] = i\varepsilon_{ijk} V_k^0(x, t)\,\delta_3(x - x');$$

$$[V_i^0(x, t), A_j^0(x', t)] = i\varepsilon_{ijk} A_k^0(x, t)\,\delta_3(x - x');$$

$$[A_i^0(x, t), A_j^0(x', t)] = i\varepsilon_{ijk} V_k^0(x, t)\,\delta_3(x - x');$$

$$\frac{\partial V_i^\mu}{\partial x^\mu} = 0 \qquad \frac{\partial A_i^\mu}{\partial x^\mu} = \frac{1}{2} F_\pi m_\pi^2 \varphi_i.$$

In addition to the commutators written here, one can try to assume commutators between time and space components of currents and also commutators involving time derivatives [2]. These additional commutators contain Schwinger terms and may be quite complicated. In order to study the consequences of these relations, one can employ different techniques. A very convenient technique is the use of the Ward — Takahashi identities for the τ-functions. A τ-function involving a number of currents and of hadronic fields

$$\langle 0 | T^* (V_i^\lambda V_j^\mu \;\ldots\; A_k^\nu A_l^\rho \;\ldots\; \psi\psi' \;\ldots) | 0 \rangle = \tau_{ij\;\ldots\;kl}^{\lambda\mu\;\ldots\;\nu\rho}$$

provides the interesting amplitudes for electromagnetic and weak processes when the hadrons are put on the mass shell. Taking the divergence with respect to one index and using the conservation law for the current, one obtains the Ward identities. For instance

$$\frac{\partial}{\partial x^\nu} \tau_{ij\;\ldots\;kl}^{\lambda\mu\;\ldots\;\nu\rho} = \left\langle 0 \left| T^* \left(V_i^\lambda V_j^\mu \;\ldots\; \frac{\partial A_k^\nu}{\partial x^\nu} A_l^\rho \;\ldots\; \psi\psi' \;\ldots \right) \right| 0 \right\rangle$$

$+$ terms from equal time commutators.

As is well known, the additional terms on the right-hand side arise from the differentiation of the ϑ functions which give rise to equal time commutators. In the definition of the τ-functions, one must use a T^* product instead of a simple T product. When this is appropriately defined the τ-functions are covariant and satisfy Ward identities in which the Schwinger terms do not appear [3]. Sometimes, however, the cancellation between the Schwinger terms and the sea-gull terms of the T^* product is not complete and anomalous terms remain [4].

The Ward identities can be analyzed [5] by observing that the τ-functions (which correspond to all Feynman diagrams) can be put together from connected τ-functions W (which correspond to connected Feynman diagrams). These in turn can be analyzed in terms of one particle irreducible vertices A (which do not have one particle singularities; they correspond to Feynman diagrams which cannot be cut in two by cutting only one internal line). Clearly the W-functions are constructed in terms of irreducible A-functions by putting them together in tree-like structures. For instance see fig. 1 (p.498) where G is the exact propagator. In the low energy region, the one particle irreducible vertices can be approximated by polynomials in the momenta of the particles involved. This gives rise to the so-called hard-pion method [6].

In co-ordinate space the polynomial approximation corresponds to using a finite number of derivatives. Therefore the one particle irreducible vertices can be described in terms of a local Lagrangian [7]. These effective or phenomenological Lagrangians are to be used only in the tree approximation. The Ward identities for the τ-functions imply restrictions on the one particle irreducible verti-

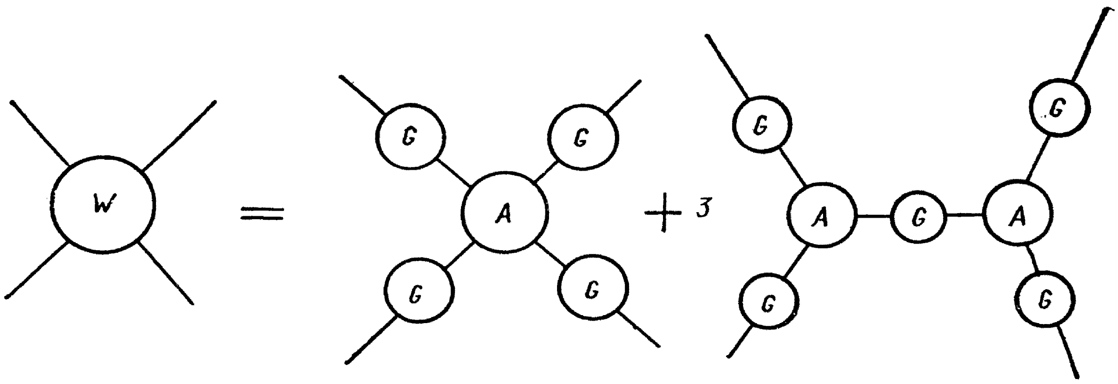

Fig. 1.

ces which can be stated as invariance properties of the phenomenological Lagrangian. These restrictions take the form of low energy theorems. In the case of a spontaneously broken symmetry, the invariance of the effective Lagrangian is with respect to a non-linear realization of the group [8].

In the hard pion calculations one applies the polynomial approximation in situations in which the momenta are no longer very small. Sometimes one even uses the effective Lagrangians for virtual particles and integrates the momenta to infinity. In these cases, of course, the results go far beyond the use of low energy theorems, but provide often useful indications of what one may expect in more realistic calculations. An interesting example is the fourth order calculation of the electromagnetic mass difference $m_{\pi_+} - m_{\pi_0}$ by Gerstein, Schnitzer, Wong and Guralnik [9]. They find that the mass difference is logarithmically divergent even if the pion mass is set equal to zero. Remember that, in second order, it is convergent in this limit and it agrees reasonably well with experiment [10]. It is not easy to trace the fourth order divergence to the non-vanishing of certain equal time commutators.

A lot of work has been done on the construction of effective Lagrangians invariant under a group. A particularly elegant technique uses geometrical ideas in the manifold spanned by the field variables [11]. For instance, the pion-pion interaction can be described by a Lagrangian of the form

$$-\frac{1}{2} g_{ij}(\vec{\varphi}) \frac{\partial \varphi_i}{\partial x^\mu} \frac{\partial \varphi_j}{\partial x_\mu} + f(\vec{\varphi})$$

where $g_{ij}(\vec{\varphi})$ gives the invariant metric associated with the group $SU(2) \times SU(2)$, the group manifold being parametrized by the pion field $\vec{\varphi}$. Using particular field co-ordinates, one would have, for instance

$$g_{ij}(\vec{\varphi}) = \frac{\delta_{ij}}{(1 + a^2 \vec{\varphi}^2)^2}; \quad a = \frac{1}{F_\pi}$$

while the symmetry breaking term could be assigned to a particular representation and could have the form

$$f(\vec{\varphi}) = -\frac{1}{2} m_\pi^2 \frac{\vec{\varphi}^2}{1 + a^2 \vec{\varphi}^2}.$$

Why should $g_{ij}(\vec{\varphi})$ be the metric of a group space? A more general point of view is taken in the interesting work by Volkov [12]. He also uses geometrical concepts in the space of the field variables but formulates requirements on this space which are more general than group invariance. For broken $SU(3) \times SU(3)$ he uses the natural hypothesis that the covariant derivative in the eight-dimensional Riemann space should exist for $SU(3)$ multiplets. This requirement gives strong limitations on the structure of the Riemann space and determines it up to one unknown function and possibly up to a constant. In the former case the corrections due to symmetry breaking (which means here deviation from the metric of a group space) begin to work in the quasikinetic terms of the Lagrangian corresponding to the metric tensor of such a space only for six or more external lines. This could be considered as an explanation of the fact that symmetry breaking is usually considered only as the mass term.

3. Algebraic form of sum rules

Assuming that the amplitudes have a certain high energy behaviour (convergence or superconvergence) inferred from experiment or from Regge theory, the low energy theorems can be transformed into sum rules and given an elegant algebraic form. This can be done by requiring that the tree diagrams of the effective Lagrangian behave not worse as $s \to \infty$ than Regge theory would require [13]. It can also be done, and was done this way earlier, by using directly the current commutators, for instance with the method of the infinite momentum limit [14]. For instance, for $SU(2) \times SU(2)$, one assumes that the forward isospin odd amplitude for scattering $\pi + \alpha \to \pi + \beta$ vanishes as $s \to \infty$. This gives rise to the commutation relations

$$[T_i, T_j] = i\varepsilon_{ijk}T_k;$$
$$[T_i, X_j(\lambda)] = i\varepsilon_{ijk}X_k(\lambda);$$
$$[X_i(\lambda), X_j(\lambda)] = i\varepsilon_{ijk}T_k.$$

Observe that T_i and $X_j(\lambda)$ are not operators but one-particle matrix elements directly related to observables. For instance, the rate for the collinear decay $\alpha \to \beta + \pi_i$ with helicity λ is given by

$$\Gamma(\alpha \to \beta + \pi_i) = \frac{(m_\alpha^2 - m_\beta^2)^3}{4\pi m_\alpha^3 F_\pi^2 (2J_\alpha + 1)} \, |[X_i(\lambda)]_{\alpha\beta}|^2.$$

Assuming that the isospin even amplitude behaves as δ_{ij} for $s \to \infty$ one obtains information on the mass matrix $m^2_{\beta\alpha} \equiv m^2_\alpha \delta_{\beta\alpha}$. It turns out to be the sum of a chiral invariant and of the fourth component of a chiral four vector

$$m^2 = m_0^2(\lambda) + m_4^2(\lambda);$$
$$[X_i(\lambda), m_0^2(\lambda)] = 0;$$
$$[X_i(\lambda), m_4^2(\lambda)] = i m_i^2(\lambda);$$
$$[X_i(\lambda), m_j^2(\lambda)] = i\delta_{ij} m_4^2(\lambda).$$

From these relations one finds mass formulae such as $m^2_{A1} = m^2_\rho + m^2_\varepsilon$. Other examples of such algebraic forms of sum rules are

$$[X_i, [X_j, mJ_\pm]] = \frac{1}{3}\delta_{ij}[X_l, [X_l, mJ_\pm]]$$

where J is the angular momentum matrix acting on helicity indices [15], and

$$\mu'(\lambda) X_i(\lambda) = X_i(\lambda + 1)\mu'(\lambda)$$

where $\mu'(\lambda)$ is the anomalous magnetic moment matrix, between states of helicity $\lambda + 1$ and λ.

The above examples show how the high energy requirements transform a «dynamical» symmetry of the non-linear or Goldstone variety into an algebraic symmetry operating on one particle states. Usually this method is applied to chiral groups [16]. In an interesting report to this conference [17], Ogievetsky applies it to SU (3), taking the hypercharge plus isospin group as the linearly realized subgroup. When the asymptotic requirements are imposed, one obtains mass formulae which are generalizations of the Gell — Mann — Okubo formula and which agree reasonably with experiments. He also studies the case of SU (3) $\times$ $\times$ SU (3) with hypercharge plus isospin as the linear subgroup. These generalizations are very appealing. Unfortunately, whenever one goes beyond SU (2) $\times$ $\times$ SU (2), at the level of exact symmetry one must assume vanishing masses for mesons which actually have a rather large mass.

4. Unitarity

One of the main theoretical problems of current algebra is that of unitarity. The Ward identities or the effective Lagrangians, treated in the tree approximation, satisfy the low energy theorems and crossing, but not unitarity. How should one improve on this, without loosing the low energy theorems and crossing? Essentially three approaches have been tried. The first, represented by the work of B. W. Lee and J. L. Basdevant [18] on π-π scattering consists in taking a renormalizable field theory model (the «linear» σ model) which satisfies the Ward identities, and calculating in perturbation theory. In order to improve the convergence of the series in a way which guarantees explicit unitarity they use the Padé approximation method. This approach is quite satisfactory, except perhaps for its being based on a specific field theory model. It has been extended to pion-nucleon scattering.

The second approach, exemplified by the work of Schnitzer, also on π-π scattering [19], consists in finding a crossing symmetric ansatz for the solution of the Ward identities and then imposing on it elastic unitarity. The resulting equations are then solved as well as one can. This approach is very satisfactory in principle but it is not clear how to go beyond elastic unitarity. In practice, the results obtained with these two approaches have been almost identical.

The third approach is to take the non-linear effective Lagrangian very seriously as a fundamental Lagrangian to be quantized. Since it corresponds to a non-renormalizable field theory, methods appropriate to such theories, such as the Efimov — Fradkin [20] method, have been used [21]. In particular several contributions in this direction have been presented to this conference. We consider it perfectly reasonable, in spite of the technical difficulties one encounters, to investigate how far one can go in widening the scope of quantum field theory and to study non-renormalizable and non-local field theories. However, we feel that these methods are out of place in the study of non-linear chiral Lagrangians, by their very nature phenomenological and not fundamental.

5. Outstanding questions

From the experimental point of view, the outstanding puzzles of current algebra have been for many years the following: $\pi^0 \to 2\gamma$, K_{l3}, $\eta \to 3\pi$. As we shall see, the first ($\pi^0 \to 2\gamma$) is probably solved by the Adler anomaly. The second (K_{l3}) is a puzzle only if the experiments confirm [22] a large negative value for the parameter ξ. The third ($\eta \to 3\pi$) is a puzzle because a point of view which works well in the analogous case of $K \to 3\pi$ gives here a vanishing amplitude [23]. A lot of theoretical effort has been put into finding theories of these effects which agree with experiment (e. g., the «weak» PCAC of Brandt and Preparata [24]). I believe, however, that the problems are still there. In order to solve the $\eta \to 3\pi$ puzzle, it has been suggested [25] that isospin invariance is perhaps broken not only by a minimal electromagnetic interaction but also by a term u_3 belonging to the $(3\bar{3}) + (\bar{3}3)$ representation of $SU(3) \times SU(3)$. The existence of such a term would be welcome also for the electromagnetic mass differences [26] and in some theories of the Cabibbo angle [27]. Is it really there? At the present time the evidence does not seem sufficiently strong to accept such a drastic departure from our usual picture of the fundamental interactions. On the other hand such a term could perhaps arise «spontaneously» and not require a change in our usual picture.

6. Anomalies

The puzzle of the $\pi^0 \to 2\gamma$ decay is the following. Sutherland and Veltman have shown [28], using gauge invariance and the PCAC equation for the neutral axial vector current, that the amplitude vanishes when extrapolated to zero pion mass. With the usual assumption on slow variation of form factors, the physical amplitude should then also be close to zero, contrary to experiment. This puzzle was solved when Adler and others [29] pointed out that the neutral PCAC equation acquires an additional term («anomalous» term) in presence of an electromagnetic field. This re-establishes a non-vanishing amplitude for $\pi^0 \to 2\gamma$ decay. The origin of the anomalous term can be understood as follows. Consider the τ-function of two electromagnetic currents and one axial vector current,

$$\langle 0 | T^* (V_\lambda V_\mu A_\nu) | 0 \rangle \sim R_{\lambda\mu\nu}(k_1, k_2).$$

In a theory with charged spinors the lowest order contribution is given by the triangle diagram (Fig. 2).

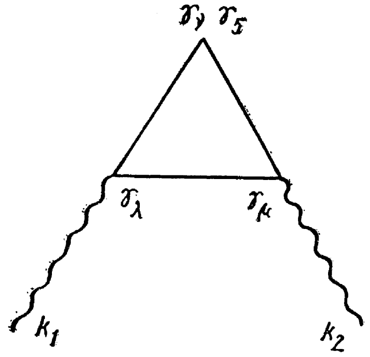

Fig. 2.

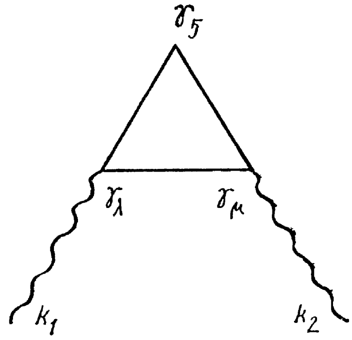

Fig. 3.

A formal calculation using the operator equations of motion shows that one should expect the Ward identity

$$-(k_1 + k_2)^\nu R_{\lambda\mu\nu} = 2mR_{\lambda\mu}$$

where $R_{\lambda\mu}(k_1, k_2)$ is given by the corresponding triangle diagram (Fig. 3). The same result follows if one works directly with the (linearly divergent) integral corresponding to the triangle diagram. However, it is well known in field theory that these formal manipulations are dangerous. Adler pointed out that if one defines the integrals by the Pauli — Villars regularization method, the above Ward identity is not satisfied, but rather one has

$$-(k_1+k_2)^\nu R^{\text{reg}}_{\lambda\mu\nu} = 2mR^{\text{reg}}_{\lambda\mu} + 8\pi^2 k_1^\alpha k_2^\beta \varepsilon_{\alpha\beta\lambda\mu}.$$

The anomalous term on the right-hand side originates from the last term in the correct equation

$$-(k_1+k_2)^\nu (R_{\lambda\mu\nu} - R^M_{\lambda\mu\nu}) = 2mR_{\lambda\mu} - 2MR^M_{\lambda\mu}$$

as the regulator mass $M \to \infty$. The PCAC equation must now be written

$$\frac{\partial A^\mu}{\partial x^\mu} = \frac{1}{2} F_\pi m_\pi^2 \varphi_\pi + \frac{\alpha S}{4\pi} \varepsilon_{\lambda\mu\nu\rho} F^{\lambda\mu} F^{\nu\rho}$$

where S is the square charge of the fundamental spinor fields which enter in the axial vector current averaged with weights proportional to their coupling to the axial vector current. For instance $S = \frac{1}{2}$ in a nucleon model (like the σ-model) while $S = \frac{1}{6}$ in the quark model. The magnitude and sign [30] of the amplitude is known; agreement with experiment requires $S = +0.44$, which seems to exclude the quark model and also certain integrally charged triplet models [31] which give $S = -\frac{1}{2}$. I find this success of the nucleon model rather puzzling.

Questions have been raised about the uniqueness of the result, in view of the possibility of using different regularization procedures. In a contribution to this conference, Dolgov and Zakharov [32] have argued that the result is ambiguous, because the anomalous term can be absorbed into a redefinition of the matrix elements of $\bar{\psi}\gamma_5\psi$ (to show this they use the ε regularization procedure in co-ordinate space). However, an unambiguous result can always be obtained by calculating first the imaginary part of an amplitude and then defining the amplitude by the requirement that the real part be well behaved at infinity. In this way the Adler anomalous term emerges uniquely. The only objection I can see to this point of view is that higher order electromagnetic corrections to the matrix elements of the axial current diverge and therefore asymptotic requirements are less convincing than in the case of the propagators of renormalizable theories where they ensure finiteness to all orders.

One may ask about corrections due to the strong interactions. It has been argued [33] that the above result for S is not affected, but there is disagreement about this [34]. We remark that the Adler anomalies do not explain the non-vanishing of the $\eta \to 3\pi$ amplitude. In a report to this conference, Abers, Dicus and Teplitz have described a careful search for anomalous graphs which could produce such an effect, with negative result.

Finally, let me mention that the anomaly in the Ward identity implies anomalies in equal time commutators [35].

7. A mathematical puzzle

The usually accepted form for the strong interaction Hamiltonian [36] is

$$H = H_{\text{inv}} + \varepsilon_0 u_0 + \varepsilon_8 u_8$$

where H_{inv} is invariant under the chiral $SU(3) \times SU(3)$ while u_0 and u_8 belong to the $(3\bar{3}) + (\bar{3}3)$ representation. For $\varepsilon_8 = -\varepsilon_0\sqrt{2}$ one would have $SU(2) \times SU(2)$ invariance and the pion would be massless. The observed pseudoscalar masses require $\varepsilon_8/\varepsilon_0 \approx -1.25$.

Kuo [37] has observed that the formally unitary transformation

$$W = e^{\frac{3}{2}\pi i Y_5}$$

transforms the Hamiltonian into

$$H' = WHW^{-1} = H_{\text{inv}} + \bar{\varepsilon}_0 u_0 + \bar{\varepsilon}_8 u_8$$

where

$$\bar{\varepsilon}_0 = -\frac{1}{3}\varepsilon_0 - \frac{1}{3} 2\sqrt{2}\varepsilon_8; \quad \bar{\varepsilon}_8 = -\frac{1}{3} 2\sqrt{2}\varepsilon_0 + \frac{1}{3}\varepsilon_8.$$

Since unitarily transformed operators should have the same spectrum of eigenvalues, the Hamiltonians H and H' should give the same mass spectrum (individual masses can of course exchange their role in the spectrum). However, the formulae for the pseudoscalar masses as functions of ε_0 and ε_8 do not appear to be explicitly invariant under the transformation. From this one has inferred that the approximations involved in deriving these mass formulae were unjustified. Alternatively, it has been argued that only explicitly invariant Hamiltonians are acceptable and that one must have $\varepsilon_8 = -\varepsilon_0\sqrt{2}$; other representations could occur in the symmetry breaking, but always in such a way that explicit invariance is preserved.

Clearly, a mathematical puzzle should be solved as such, and not by imposing restrictions on the physics. In this case the resolution is quite simple and is related to the Goldstone nature of the $SU(3) \times SU(3)$ symmetry and the non-invariance of the vacuum [38]. The correct mass formulae contain the parameters ε_0 and ε_8 as well as the vacuum expectation values of the scalar fields u_0 and u_8, and refer to a pseudoscalar octet plus a scalar correspondent of the kaon, the kappa. A Kuo transformation changes both sets of variables and actually leaves the mass spectrum invariant. The pion mass is invariant by itself while the kaon mass is exchanged with the kappa mass. Often the mass formulae are written by making the approximation $\langle u_8 \rangle \ll \langle u_0 \rangle$ and expressing the vacuum expectation value $\langle u_0 \rangle$ in terms of physical quantities such as F_π; the invariance is then lost. In conclusion there is no puzzle and the parameters ε_0 and ε_8 cannot be determined a priori but must be found by referring to experimental data.

8. The energy momentum tensor

The energy momentum tensor must be symmetric and conserved

$$\Theta_{\mu\nu} = \Theta_{\nu\mu} \quad \frac{\partial}{\partial x_\mu}\Theta_{\mu\nu} = 0.$$

Its matrix elements between physical particle states should be finite, just like those of the vector and axial vector currents. In a renormalizable field theory (finite S matrix) the finiteness of the matrix elements of local currents is an additional requirement, which in general must be separately verified. In theories which are exactly or partially invariant under a group, the resulting Ward identities connect τ functions involving currents with τ functions involving only field operators and can usually be used to show that the matrix elements of the currents are indeed finite. For that particular current which is the energy momentum tensor, one may expect that its finiteness will follow from the Ward identities (conservation identities) which follow from the above conservation equation, or more precisely from the general (Einstein) covariance satisfied by the

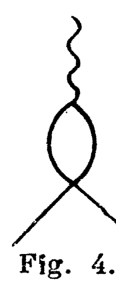

Fig. 4.

hadronic system in an external gravitational field. Actually this is not the case.

Coleman, Callan and Jackiw [39] have considered a scalar field with a self-interaction given by $\lambda\varphi^4$. In this very simple renormalizable theory the canonical (and symmetric) energy momentum tensor has divergent matrix elements. The lowest order divergent diagram is one on fig. 4 (see p. 503), where the wiggly line is an external graviton. In order to remedy this situation, the above authors have suggested the use of a modified or «improved» energy momentum tensor

$$\Theta_{\mu\nu} = \Theta_{\mu\nu}^{\text{can}} - \frac{1}{6}\left(\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} - g_{\mu\nu}\Box\right)\varphi^2.$$

They have shown that the additional term cancels the divergence, at least to lowest order [40] and if one performs the calculation by a particular regularization procedure. The modified $\Theta_{\mu\nu}$ gives rise to the same integrated generators of the Poincaré group and even satisfies formally the Dirac — Schwinger equal time commutation relations. Furthermore, its trace is given by

$$\Theta_\mu^\mu = -m_0^2\varphi^2$$

and therefore vanishes for $m_0 = 0$, while the trace of $\Theta_{\mu\nu}^{\text{can}}$ contains more singular derivative terms. In addition to the conservation identities, the modified $\Theta_{\mu\nu}$ satisfies certain «trace identities» which are smoother than those satisfied by $\Theta_{\mu\nu}^{\text{can}}$. Observe that a $\lambda\varphi^4$ interaction must often be introduced in order to make a theory renormalizable, as in the well-known case of the pseudoscalar pion-nucleon coupling.

The modified $\Theta_{\mu\nu}$ is simply related to the currents of the scale and conformal transformations. An infinitesimal scale transformation, of parameter ε, on a field ψ is given by [41]

$$\delta\psi = \varepsilon\left(x\cdot\frac{\partial}{\partial x} + d\right)\psi$$

where d is the dimension of the field (canonically $d = 1$ for scalars and vectors, $d = \frac{3}{2}$ for spinors, etc.). An infinitesimal proper conformal transformation of parameters a^μ is given by

$$\delta\psi = \left[\xi\frac{\partial}{\partial x} - 2dax - 2a^\mu x^\nu\Sigma_{\mu\nu}\right]\psi$$

$$\xi^\mu = a^\mu x^2 - 2x^\mu ax$$

where $\Sigma_{\mu\nu}$ are the spin matrices. If one extends the Poincaré group by the scale and conformal transformations, one obtains the 15-parameter conformal group. One can show that the currents associated with the scale and conformal transformations can be taken to be respectively

$$S_\mu = x^\nu\Theta_{\nu\mu}$$

$$C_{\mu\alpha} = 2x_\alpha x^\nu\Theta_{\nu\mu} - x^2\Theta_{\mu\alpha}$$

so that

$$\frac{\partial S_\mu}{\partial x_\mu} = \Theta_\mu^\mu, \qquad \frac{\partial C_{\mu\alpha}}{\partial x_\mu} = 2x_\alpha\Theta_\mu^\mu.$$

We see here that approximate scale and conformal invariance of the strong interactions, suggested first by Mack and Kastrup [42], is equivalent to approximate vanishing of the trace of the energy momentum tensor.

9. Scale invariance as a Goldstone symmetry

The trace of the energy momentum tensor does not vanish but is proportional to masses and dimensional coupling constants. If approximate scale invariance should mean that all these parameters are very small, it would clearly not be a very useful concept in the case of the strong interactions. On the other hand, one may investigate the idea (analogous to PCAC) that the trace of the energy momentum tensor is dominated by a particular scalar field (b is a universal constant)

$$\Theta^{\mu}_{\mu} = \frac{1}{b} m_{\sigma}^{2} \sigma$$

and that it would actually vanish if the mass m_σ were zero. The masses of all other particles can be arbitrarily large in this limit. This idea, which corresponds to a particular form of the trace identities, can be implemented by use of any of the techniques of current algebra, in particular by means of effective Lagrangians [43].

Interpreted in this way approximate scale invariance implies soft σ theorems analogous to the soft pion theorems; unfortunately the experimental verification of these ideas is still very remote. The simplest of these theorems says that the σ should couple to all other particles with strengths proportional to their masses (for spinors) or square masses (for scalars, vectors, etc.). If the σ is identified with the ε (700), which is strongly coupled to pions, its coupling to rho's or to nucleons should be enormous, a fact already empirically excluded. To avoid this difficulty, it has been suggested that the trace of the energy momentum tensor is dominated not by one but by more (according to some authors, three) scalar fields [44]. Actually the situation is not so bad. It was shown first by Ellis [45], using a conformal invariant effective Lagrangian, that the σ-pion coupling has the form

$$- b\sigma \left(\frac{\partial \vec{\varphi}}{\partial x^{\mu}} \right)^{2} - \frac{1}{2} d b m_{\pi}^{2} \sigma \vec{\varphi}^{2}$$

where d is the dimension of the $SU(2) \times SU(2)$ symmetry breaking term. On the mass shell this gives an effective coupling

$$- \frac{1}{2} b \left[m_{\sigma}^{2} + (d - 2) m_{\pi}^{2} \right] \sigma \vec{\varphi}^{2}.$$

Since the ε mass is much larger than the pion mass, this eliminates in part the above-mentioned difficulty. The resulting couplings between the ε and other particles are not inconsistent with what is known (for the ε-nucleon couplings from analysis of pion-nucleon and nucleon-nucleon scattering). The same can be said in relation to mass formulae. For instance, if one makes the hypothesis that scale invariance and $SU(3) \times SU(3)$ invariance are both broken by a single term which belongs to the $(3,\bar{3}) + (\bar{3},3)$ representation and has dimension d, one finds

$$m_{\sigma}^{2} = d(4 - d) \frac{b^{2} F_{\pi}^{2}}{8} (m_{\pi}^{2} + 2 m_{K}^{2}).$$

For $d = 1$, and identifying the σ with the ε (700), one finds for the universal constant $b \approx 1 \cdot 6/F_{\pi}$ ($F_{\pi} \approx 190\ MeV$). However, the question of the precise number and form of the terms which break scale invariance is still unresolved [46].

10. Scale invariance at small distances

We have seen how scale invariance gives rise to low energy theorems. We turn now to the study of some high energy questions. The problem is to study the behaviour of the product of two currents taken at nearby space time points when the separation goes to zero.

We begin with the vacuum expectation value

$$\langle 0 | J_\mu(x) J_\nu(0) | 0 \rangle.$$

If J_μ is the electromagnetic current this expectation value is related to the total cross-section for $e^+ + e^-$ annihilation into hadrons and the small x behaviour gives the high energy behaviour of the cross-section. In turn, the small x behaviour is related to the degree of divergence of the Schwinger term in the equal time commutator of J_0 and J_l. It has been pointed out some time ago by Gribov, Ioffe and Pomeranchuk [47] that, if the current is given by the ordinary bilinear expression for a charged spinor or scalar fields, the total cross-section goes like $1/E^2$ (E is the centre-of-mass energy of the electron pair). In the algebra of fields, where the electromagnetic current is proportional to the field of the neutral rho-meson, the total cross-section goes like $1/E^4$ as was first stated by Dooher [48]. This corresponds respectively to a Schwinger term diverging quadratically or convergent. In a contribution to this conference, Ioffe and Khoze have considered the case of charged vector mesons and obtained a Schwinger term diverging like $a\Lambda^2 + (b/\mu_0^2(\Lambda))\Lambda^4$ where $\mu_0^2(\Lambda)$ is the bare mass of the vector mesons and Λ an ultra-violet cut-off. This expression is valid for free vector mesons but their calculation gives strong indications that it remains valid in presence of strong interactions (unfortunately the corresponding theory would be unrenormalizable). If the bare mass is finite for $\Lambda \to \infty$, the cross-section will go like a constant, if the bare mass diverges with Λ, it will go like $1/E^\gamma$, with $0 < \gamma \leqslant 2$, depending on the degree of divergence of the bare mass. The case of the algebra of fields and that of charged vector mesons are instructive because they do not fall into the general pattern one would expect from purely dimensional considerations. It is easy to see, for instance, that in the algebra of fields, the time component of the current has dimension three (as must be) while the space components have dimension one. Under these circumstances one cannot expect the existence of a scale invariant «skeleton theory» which approximates the actual theory at small distances [49].

How does one study matrix elements other than the vacuum? Wilson and others have postulated the existence of an operator expansion in terms of singular functions with operator coefficients. For two currents J_1 and J_2

$$J_1\left(\frac{x}{2}\right) J_2\left(-\frac{x}{2}\right) \sim \frac{1}{x^2} \sum x^{\alpha_1} \ldots x^{\alpha_n} O^{(n)}_{\alpha_1 \ldots \alpha_n}(0).$$

The existence of this kind of expansions has been verified in perturbation theory to all orders for renormalizable field theory models by Zimmermann and others [50]. Actually in perturbation theory there occur also logarithms and in some cases ($\lambda\varphi^4$ theory) these logarithms seem to sum up to powers and can modify the main power $1/x^2$ with consequent loss of scaling in the form factors, as shown by Gatto and Menotti [51]. Nevertheless, one usually assumes that in real life the logarithms are not there or can be neglected. One can further make the hypothesis that only certain operators have low dimensions and occur in the first terms of the expansion. With this kind of assumptions, one can write precise expansions such as

$$J_\mu\left(\frac{x}{2}\right) J_\nu\left(-\frac{x}{2}\right) \sim \frac{1}{x^2}\left[\Theta_{\mu\nu} + g_{\mu\nu}\frac{x^\alpha x^\beta}{x^2}\Theta_{\alpha\beta} + 4\,\frac{x_\mu x_\nu x^\alpha x^\beta}{(x^2)^2}\Theta_{\alpha\beta}\right] + \cdots$$

Applications of this kind of expansions have been made in the interesting work of Gatto, Ciccariello, Sartori and Tonin and of Mack [52] reported at this conference. Their work is perhaps still based on too stringent assumptions [53]. It is of interest as an example of a kind of technique which generalizes that of the use of equal time commutators and which may be very fruitful. Observe, in particular, that relations like the above between the electromagnetic current and the energy momentum tensor can hardly be put on a group theoretic basis; here no symmetry connecting them is postulated, only an asymptotic connection [54].

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I would like to thank D. Amati, R. Brandt, S. Fubini, R. Gatto, J. Prentki and B. Renner for many conversations which have been very helpful in preparing this report. I am very grateful to R. Kallos and A. A. Slavnov for their help during the conference.

DISCUSSION

Kamal:

With what confidence can we take the value of ξ quoted by you? And if ξ is indeed close to —1, do you have any comments on the Brandt and Preparata's weak PCAC?

Zumino:

The value of ξ I quoted was taken from a recent review article by M. K. Gaillard and I. M. Chounet (CERN 70—14). I must say, however, that some people don't believe that value and I do not have a strong personal opinion on it. It is an experimental question which can only be resolved by more experiments. Concerning the weak PCAC of Brandt and Preparata, let me describe it briefly. In chiral dynamics we usually accept the idea that PCAC and the smallness of the pion mass are consequences of a particular form of symmetry breaking which makes $SU(2) \times SU(2)$ a good approximate symmetry. This gives a consistent picture. Brandt and Preparata assume instead that $SU(3)$ is a good approximate symmetry. For them the smallness of the pion mass is accidental and the use of PCAC depends on the convergence of certain dispersion integrals. I personally prefer the consistent picture based on approximate $SU(2) \times SU(2)$ symmetry.

Faissner:

Regarding the form factor ratio $\xi = f_-/f_+$ I should like to make the following comments: It is true that some recent measurements of the total polarization of the muon gave a value of ξ close to —1. This includes the result of the X — 2 experiment of Aachen et al., which in my opinion was done very carefully. However, some of the people involved in the experiment keep claiming that there might be some bias in the polarization measurement in the negative direction. As a matter of fact, all polarization measurements but one yield values of ξ lower than those obtained by the two other methods: Dalitz plot and $K_{\mu 3}/K_{e3}$ branching ratio. If one takes an average over all measurements, one obtains $\xi = -0.40 \pm 0.15$ (s. Particle Data Group, Phys. Letters, **33B**, August 1970, p. 27).

Thus, a small negative value of ξ, or even zero for that matter, is not disproven by the present data.

A. A. Migdal:

The problem of scale invariance in the field theory was investigated in several works by A. M. Polyakov and myself [1, 2, 3, 4]. These works were based on methods, developed by Gribov and myself [5]. The main result is that carefull summation of logarithmic terms in the perturbation theory leads to a convergent and scale invariant solution. The important point is that the dimensions of fields are anomalous and universal [4, 5] (they do not depend on bare parameters). It has been confirmed by an exact solution of a model [6]. In renormalizable theories it may be shown [4] that if a cut-off L tends to ∞ and a bare coupling constant remains finite and sufficiently large, then the renormalized constant tends to a universal limit, which obeys the eigenvalue condition of Gell — Mann — Low type. Scale invariance is valid in this case. The estimates of dimensions [4] show that the deviations from canonical values are small.

References

[1] A. M. Polyakov, JETP, 55, 1026 (1968)
[2] A. A. Migdal, JETP, 55, 1964 (1968).
[3] A. M. Polyakov, JETP, 57, 271 (1969)
[4] A. A. Migdal, JETP, 59, 1015 (1970)
[5] V. N. Gribov, A. A. Migdal, JETP, 55, 1498 (1968)
[6] V. N. Gribov, E. M. Levin, A. A. Migdal, JETP (to be published).

Zumino:

If I understand your comment, you are saying that it is possible to have scale invariance (with a noncanonical dimension for the fields) provided the renormalized coupling constant satisfies a certain eigenvalue equation. This fact was pointed out by a number of people. I first heard it some time ago from K. Johnson.

Nauenberg:

In your slide concerning Wilson's operator expansion I noticed you consider only $1/x^2$ singularities near the light cone, associated with the energy momentum tensor. Would you comment also on the anomalous dimension discussed by Wilson, and the relation of the light cone singularity to the inelastic electron scattering?

Zumino:

One knows that the $1/x^2$ singularity gives rise to scaling. However I would rather not go into inelastic electron scattering, which belongs to the subject of a different session. Concerning anomalous dimensions, I think that not very much is known. Wilson suggested that the interaction can change the dimensions of a field. His suggestion was based on an analogy with Johnson's exact solution of the two dimensional Thirring model. In relativistic renormalizable quantum field theory when the interaction is present one cannot in general ascribe a dimension to a field, not even an anomalous dimension.

J. G. Taylor:

I would like to comment on your remarks about the use of chiral Lagrangians as full-blown field theory.

Zumino:

I was expecting that.

Taylor:

Well, good, now you have it. The point is that field theory allows one to satisfy unitarity completely, so that the other methods you mentioned for incorporating it into chiral calculations pale into magnificence beside field theory results. And these results for the chiral case are that unitarization generates ultraviolet divergencies which can only be removed by destruction of the current algebra that one started from. I have no doubt that similar difficulties will be met by the other methods when they will go that far.

Let me finish by pointing out that you seem to be in the situation where one hand does not know what the other hand is doing. You denigrate the use of field theory for chiral Lagrangians, yet you also praise the use of the eikonal approximation to sum up soft meson contributions — a technique which clearly involves closed loops of particles though in a very distorted and approximated fashion.

Polyakov:

I should like to point out that things similar to the Wilson expansion, scale invariance, and anomalous dimensions in the field theory were investigated several years ago in connection with critical phenomena problems. These investigations have shown that logarithmics terms in the perturbation theory, which violate scale invariance, finally sum up to give power functions of distances and hence anomalous dimensions of fields. (See papers: A. M. Polyakov, JETP **55**, 1026 (1968), JETP **57**, 271 (1969); A. A. Migdal, JETP **55**, 1964, (1968); L. P. Kadanoff, Phys. Rev. Lett. **23**, 1430 (1969)).

My second comment is that though scale invariance is correct for interaction of virtual particles only and hence seems to be unobservable, it enables to obtain a number of predictions concerning deep inelastic lepton-hadron processes. Namely, for electron-positron annihilation into hadrons:

$$\sigma_N(s) = s^{-1-\delta}\chi(N/s^\delta) \quad \text{for } N \gg 1, \; s \gg m^2,$$

$$0 < \delta < \frac{1}{2} \; ; \; \chi(0) = 0, \quad \chi(z) \underset{z \to \infty}{\sim} \exp\left[-z^{\frac{1}{1-2\delta}}\right].$$

Here $\sigma_N(s)$ is the cross section for the production of N hadrons of any given sort, s is c. m. squared energy.

For deep inelastic scattering the following formula is available (with notations commonly used):

$$\int_1^\infty \frac{d\omega}{\omega^j}(\nu W_2) = F(j)(Q^2)^{-\gamma(j)}$$

$\gamma(2) = 0$, j is any number $\geqslant 2$.

This formula agrees with Bjorken's assumption only if $\gamma(j) = \text{const} = 0$. There are no reasons in the theory for this. It is also possible to show that the averaged multiplicity $\bar{N}$ is:

$\bar{N} = (Q^2)^\delta f(\omega, \ln Q^2)$, where δ is the same as for annihilation.

This comment is based partially on the paper: A. M. Polyakov, JETP 59, 542 (1970).

Y a n g:

In statistical mechanics in recent years there have been many discussions of scale invariance. These discussions were very stimulating. But I disagree with a previous comment by Dr. Polyakov. I do not believe that there are either mathematical reasons or physical insight that would conclusively lead to scale invariance.

L i p k i n:

Weinberg considers an algebra including four operators m_i^2, in addition to the generators of $SU(2) \times SU(2)$. Weinberg does not consider the commutators of these m_2^2, among themselves. But if you are crazy enough to consider them and simply assume that the algebra of these 10 operators closes, a miracle happens and several interesting results may be obtained automatically without any further assumptions. The algebra is $O(5)$ (or $Sp(4)$). The adjoint representation exactly accomodates the zero helicity states of the π, ρ, σ, A_1. The right mass formula comes out (The squared mass operator is in the algebra); namely equal spacing between π, ρ and A_1 with the σ degenerate with φ. The right mixing angle for the $\pi - A_1$ comes out — the factor $1/\sqrt{2}$ is a transformation coefficient connecting two subgroups of $O(5)$. Then the miracle stops and there is nothing further I have been able to obtain that is useful from this $O(5)$ algebra. Perhaps someone else can.

S h i r o k o v:

I should like to mention that I have derived the explicit form of the relativistic invariance conditions for the arbitrary set of the equal time commutators. These conditions provide the possibility to establish the invariance properties of the current algebra which includes both usual and Schwinger type terms. The energy stress tensor naturally enters the invariance conditions. This result is a part of the report presented to the field theory session.

T. T. W u:

I would like to go slightly further than Professor Yang. Not only are the so-called scaling laws in statistical mechanics not well-established, there are now experimental evidences and theoretical arguments against them. The situation was clearly presented almost a year ago by Barry McCoy in Physical Review Letters.

REFERENCES

In addition to the following references, I recommend the rapporteur talks by S. Weinberg at the 14th International Conference on High Energy Physics, Vienna 1968, and by B. Renner at the 1969 Lund International Conference on Elementary Particles.

1. M. G e l l - M a n n, Phys. Rev. 125, 1067 (1962).
2. For instance, in the algebra of fields:
   T. D. L e e, S. W e i n b e r g and B. Z u m i n o, Phys. Rev. Letters 18, 1029 (1967).

3. D. J. Gross and R. Jackiw, Nuclear Phys. **B14**, 269 (1969).
4. Wu-Ki Tung, Phys. Rev. **188**, 2404 (1969).
5. H. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1968); I. Gerstein and H. J. Schnitzer, Phys. Rev. **170**, 1638 (1968); Phys. Rev. **175**, 1876 (1968).
6. The hard pion method is sometimes formulated in a different but essentially equivalent way, see: R. Arnowitt, M. H. Friedman and P. Nath, Phys. Rev. Letters **19**, 1085 (1967); Phys. Rev. **174**, 1999 (1968); R. Arnowitt, M. H. Friedman, P. Nath and R. Suitor, Phys. Rev. **175**, 1802 (1968) and several subsequent preprints from North-eastern University; see also recent preprints by: K. J. Barnes and P. H. Dondi, Queen Mary College, where the question of symmetry breaking is especially considered.
7. See, B. Zumino, Proceedings of the Seminar on Vector Mesons and Electromagnetic Interactions, Dubna, September 1969; also Lectures at the Brandeis 1970 Summer Institute. The connection between the *W*-functions and the *A*-functions can be described elegantly in terms of a functional Legendre transformation. See also: B. W. Lee, Lectures at the 1970 Cargèse Summer School; An alternative approach to chiral Lagrangians is that of: R. Dashen and M. Weinstein, Phys. Rev. **183**, 1261 (1969).
8. See, e. g., S. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2239 (1969); C. G. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2247 (1969); a review on chiral effective Lagrangians is given by: S. Gasiorowicz and D. A. Geffen, Revs. Modern Phys. **41**, 531 (1969) where many references can be found; We quote here only some relatively recent papers which use chiral Lagrangians. For pion production in pion-nucleon collision: M. G. Olsson and L. Turner, Phys. Rev. Letters **20**, 1127 (1968); W. F. Long and J. S. Kovacs, Phys. Rev. **D1**, 1333 (1970); G. Murtaza and Harun-Ar-Rashid, Ann. Phys. (N. Y.) 59, 426 (1970); For pion production in nucleon-nucleon collisions: C. T. Grant, M. E. Schillaci and R. R. Silbar, Phys. Rev. **184**, 1737 (1969).
9. I. S. Gerstein, H. J. Schnitzer, T. Wong and G. S. Guralnik, Phys. Rev. **D1**, 3442 (1970).
10. T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low and J. E. Young, Phys. Rev. Letters **18**, 759 (1967).
11. See especially Ref. 12) and K. Meetz, J. Math. Phys. (N. Y.) **10**, 589 (1969); R. Finkelstein, Physica **44**, 260 (1969); C. J. Isham, Nuovo Cimento **59A**, 356 (1969); ibid **61A**, 188 (1969).
12. D. V. Volkov, several Kharkov preprints and report to this conference.
13. S. Weinberg, Phys. Rev. **177**, 2604 (1969); see also C. Cronström and M. Noga, Nuclear Phys. **B15**, 61 (1970).
14. R. Dashen and M. Gell — Mann, Proceedings of the 3rd Coral Gables Conference, 1966, and Phys. Rev. Letters **17**, 371 (1966); K. Bardakçi and G. Segrè, Phys. Rev. **159**, 1263 (1967); F. Gilman and H. Harari, Phys. Rev. **165**, 1803 (1968).
15. S. Weinberg, Phys. Rev. Letters **22**, 1023 (1969); S. S. Shei, Phys. Rev. **188**, 2275 (1969); S. Fubini and G. Furlan, Nuovo Cimento Letters **3**, 168 (1970); W. M. Kloet and A. H. Zimerman, Nuovo Cimento Letters **3**, 619 (1970).
16. A. Sudbery, Nuclear Phys. **B20**, 1 (1970).
17. V. I. Ogievetsky, to be published in Physics Letters.
18. J. L. Basdevant and B. W. Lee, Phys. Letters **29B**, 437 (1969).
19. H. J. Schnitzer, Phys. Rev. Letters **24**, 1384 (1970) and report to this conference. See also: A. M. S. Amatya, A. Pagnamenta and B. Renner, Phys. Rev. **172**, 1755 (1968); R. Arnowitt, M. H. Friedman, P. Nath and R. Suitor, Phys. Rev. **175**, 1820 (1968); Phys. Rev. Letters **20**, 475 (1968); L. S. Brown and R. L. Goble, Phys. Rev. Letters **20**, 346 (1968); E. P. Tyron, Phys. Rev. Letters **20**, 769 (1968); A. Neveu and J. Scherk, Nuovo Cimento Letters **1**, 414 (1969); R. Rockmore, Phys. Rev. Letters **24**, 541 (1970); R. Roskies, Phys. Letters **30B**, 42 (1969); Nuovo Cimento **65A**, 467 (1970);

J. J. Brehm, E. Golowich and S. C. Prasad, Phys. Rev. Letters 23, 666 (1969);
D. F. Greenberg, Phys. Rev. 184, 1934 (1969).
20. G. V. Efimov, Soviet Phys. JETP 17, 1417 (1963); Phys. Letters 4, 314 (1963); Nuovo Cimento 32, 1046 (1964); Nuclear Phys. 74, 657 (1965);
E. S. Fradkin, Nuclear Phys. 49, 624 (1963);
M. K. Volkov, Commun. Math. Phys. 7, 289 (1968).
21. See, e. g., R. Delbourgo, A. Salam and J. Strathdee, several Trieste preprints;
J. G. Taylor, University of Southampton preprint.
22. For a detailed discussion and a rather complete list of references on K_{l3} see a recent review paper by:
M. K. Gaillard and L. M. Chounet, CERN 70—14, May 1970.
23. A good review can be found in:
J. S. Bell and D. G. Sutherland, Nuclear Phys. B4, 315 (1968).
24. R. A. Brandt, M. H. Goldhaber, G. Preparata and C. Orzalesi, Phys. Rev. Letters 24, 1517 (1970) and several Rockefeller University preprints.
25. See, e. g., R. J. Oakes, Phys. Rev. Letters 30B, 262 (1969).
26. S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964);
R. Socolow, Phys. Rev. 148, B1221 (1965).
27. R. Gatto, G. Sartori and M. Tonin, Phys. Letters 28B, 128 (1968);
N. Cabibbo and L. Maiani, Phys. Rev. D1, 707 (1970);
R. J. Oakes, Phys. Letters 29B, 683 (1969).
28. D. G. Sutherland, Nuclear Phys. B2, 433 (1967); see also:
A. D. Dolgov, A. I. Vainshtein and V. I. Zakharov, Phys. Letters 24B, 425 (1967).
29. S. L. Adler, Phys. Rev. 177, 2426 (1969);
J. S. Bell and R. Jackiw; Nuovo Cimento 60A, 47 (1969);
C. R. Hagen, Phys. Rev. 177, 2622 (1969);
R. Jackiw and K. Johnson, Phys. Rev. 182, 1457 (1969);
B. Zumino, Proceedings of CERN Conference on Weak Interactions (1969);
Earlier relevant work is by J. Schwinger, Phys. Rev. 82, 664 (1951);
L. Rosenberg, Phys. Rev. 129, 2786 (1963).
The question of anomalies for external vector and axial vector fields has been studied by:
W. A. Bardeen, Phys. Rev. 184, 1848 (1969);
I. S. Gerstein and R. Jackiw, Phys. Rev. 181, 1955 (1969);
D. Amati, C. Bouchiat and J. L. Gervais, Nuovo Cimento 65A, 55 (1969);
The case of the $\eta \to 2\gamma$ decay was also studied in Adler's first paper above. See also:
R. Arnowitt, M. H. Friedman and P. Nath, Phys. Letters 27B, 657 (1968);
S. L. Glashow, R. Jackiw and S. S. Shei, Phys. Rev. 187, 1916 (1969);
M. G. Miller, Northeastern University preprint, July 1970, where phenomenological discussions are given of this and other processes.
30. S. Okubo, Phys. Rev. 179, 1629 (1969);
F. J. Gilman, Phys. Rev. 184, 1964 (1969).
31. For instance a version of the model described by M. Y. Hahn and Y. Nambu, Phys. Rev. 139, B1006 (1965).
32. A. D. Dolgov and V. I. Zakharov, contribution to this conference.
33. S. L. Adler and W. A. Bardeen, Phys. Rev. 182, 1517 (1969); they also consider higher order corrections due to insertions of photon lines in the triangle diagram.
34. See, K. Johnson and R. Jackiw, Ref. 29).
35. See, e. g., S. L. Adler and D. G. Boulware, Phys. Rev. 184, 1740 (1969);
D. G. Boulware and R. Jackiw, Phys. Rev. 186, 1442 (1969).
36. See, e. g., S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968);
M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. 175, 2195 (1968);
R. Dashen, Phys. Rev. 183, 1245 (1964).
37. T. K. Kuo, several Purdue University preprints.
38. S. Okubo and V. S. Mathur, in University of Rochester preprints, and in a contribution to this conference, have also emphasized the importance of the Goldstone nature of the $SU(3) \times SU(3)$ symmetry. However, it is not clear whether they have seen the full resolution of the Kuo paradox. In particular they mostly consider a spectrum consisting of only eight pseudoscalars and seem to attach undue importance to the possibility that the Kuo operator W may not exist as a unitary operator in Hilbert space, a point which seems to me irrelevant to the present question. The point is rather that the Goldstone bosons of $SU(3) \times SU(3)$, broken as described above, include in general the kappa meson and that the mass spectrum is invariant. The mass formulae needed to carry out the argument sketched above can be found in the Erice lectures by S. L. Glashow in «Hadrons and their Interactions», Edited by A. Zichichi, Academic Press, New York, 1968. For instance the

pion mass is given by

$$m_\pi^2 = \frac{\varepsilon_0 \sqrt{2} + \varepsilon_8}{\langle u_0 \sqrt{2} + u_8 \rangle}$$

which is explicitly invariant under the Kuo transformation.

39. C. G. Callan, S. Coleman and R. Jackiw, Ann. Phys. (N. Y.) **59**, 42 (1970).
40. Higher orders have been studied by K. Symanzik, DESY preprint;
C. G. Callan, Caltech preprints;
S. Coleman and R. Jackiw, MIT preprint.
The result seems to be that it is possible to define an energy momentum tensor finite to any given order in perturbation theory, but its form is not as simple as that given in the text. Furthermore the simple connection with scale invariance is not valid in higher orders. These complications are usually ignored in phenomenological applications.
41. See, e. g., G. Mack and A. Salam, Ann. Phys. (N. Y.) **53**, 174 (1969);
D. Gross and J. Wess, Phys. Rev. **D2**, 753 (1970).
42. G. Mack, Nuclear Phys. **B5**, 499 (1968); Phys. Letters **26B**, 575 (1968);
H. A. Kastrup, Nuclear Phys. **B15**, 179 (1969);
H. A. Kastrup and M. Dal Cin, Nuclear Phys. **B15**, 189 (1969).
43. P. G. O. Freund and Y. Nambu, Phys. Rev. **174**, 1741 (1968);
A. Salam and J. Strathdee, Phys. Rev. **184**, 1750 and 1760 (1969);
C. J. Isham, A. Salam and J. Strathdee, Phys. Letters **31B**, 300 (1970); and several Trieste preprints;
B. Zumino and J. Wess, work described in lectures given at the Brandeis 1970 summer Institute;
The study of the energy momentum tensor as a current was recommended on numerous occasions by M. Gell-Mann.
44. L. N. Chang and P. G. O. Freund, contribution to this conference and Caltech preprint;
P. Carruthers, Caltech preprint.
45. J. Ellis, Nuclear Phys. **B22**, 478 (1970).
46. See, e. g., M. Gell-Mann, lectures at the 1969 Hawaii Summer School for discussion on this point.
47. V. N. Gribov, B. L. Ioffe and I. Ya. Pomeranchuk, Phys. Letters **24B**, 554 (1967); see also
F. Hayot and H. Nieh, Phys. Letters **29B**, 689 (1969).
48. J. Dooher, Phys. Rev. Letters **19**, 600 (1967).
49. K. G. Wilson, Phys. Rev. **179**, 1499 (1969) and SLAC preprints.
50. W. Zimmermann, to be published;
R. A. Brandt and G. Preparata, CERN preprint;
Y. Frishman, Weizmann Institute preprint.
51. R. Gatto and P. Menotti, CERN preprint.
52. See also, S. Ciccariello, R. Gatto, G. Sartori and M. Tonin, Phys. Letters **30B**, 546 (1969); and to be published in Ann. Phys.;
G. Mack, University of Miami preprint and Trieste preprint. He emphasizes the universality (proportionality to the target mass) which follows if one assumes that the operator expansion is dominated by the energy momentum tensor.
These results are related to earlier work by:
C. Callan and D. Gross, Phys. Rev. Letters **22**, 156 (1969).
53. In particular it implies that the Callan — Gross integral for electroproduction is the same in the scaling limit for proton and neutron targets. This seems contradicted by the experiments.
54. The algebra of the vector and axial vector currents on the light cone has been discussed by a number of people. See in particular R. F. Dashen and M. Gell-Mann, Proceedings of the 3rd Coral Gables Conference, January 1966;
H. Leutwyler, Acta Physica Austriaca Suppl. V, 320 (1968);
J. Jersak and J. Stern, Nuovo Cimento **59A**, 315 (1969);
H. Leutwyler and J. Stern, several CERN preprints;
R. A. Brandt, Phys. Rev. Letters **23**, 1260 (1969) and Phys. Rev. **D1**, 2808 (1970); see also:
S. Okubo, Physics **3**, 165 (1967);
B. L. Ioffe, Phys. Letters **30B**, 123 (1969);
R. Jackiw, R. P. Van Royen and G. B. West, MIT preprint.