

**Mitigating Airport Congestion: Market  
Mechanisms and Airline Response Models**

by

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## Abstract

Efficient allocation of scarce resources in networks is an important problem worldwide. In this thesis, we focus on resource allocation problems in a network of congested airports. The increasing demand for access to the world's major commercial airports combined with the limited operational capacity at many of these airports have led to growing air traffic congestion resulting in several billion dollars of delay cost every year. In this thesis, we study two demand-management techniques – strategic and operational approaches – to mitigate airport congestion.

As a strategic initiative, auctions have been proposed to allocate runway slot capacity. We focus on two elements in the design of such slot auctions – airline valuations and activity rules. An aspect of airport slot market environments, which we argue must be considered in auction design, is the fact that the participating airlines are *budget-constrained*.

- The problem of finding the best bundle of slots on which to bid in an iterative combinatorial auction, also called the preference elicitation problem, is a particularly hard problem, even more in the case of airlines in a slot auction. We propose a valuation model, called the *Aggregated Integrated Airline Scheduling and Fleet Assignment Model*, to help airlines understand the true value of the different bundles of slots in the auction. This model is efficient and was found to be robust to data uncertainty in our experimental simulations.
- Activity rules are checks made by the auctioneer at the end of every round to suppress strategic behavior by bidders and to promote consistent, continual preference elicitation. These rules find applications in several real world scenarios including slot auctions. We show that the commonly used activity rules are not applicable for slot auctions as they prevent straightforward behavior by budget-constrained bidders. We propose the notion of a *strong activity rule* which characterizes *straightforward bidding* strategies. We then show how a strong activity rule in the context of budget-constrained bidders (and quasi-

linear bidders) can be expressed as a linear feasibility problem. This work on activity rules also applies to more general iterative combinatorial auctions.

We also study operational (real-time) demand-management initiatives that are used when there are sudden drops in capacity at airports due to various uncertainties, such as bad-weather. We propose a system design that integrates the capacity allocation, airline recovery and inter-airline slot exchange procedures, and suggest metrics to evaluate the different approaches to fair allocations.

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## Credits

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# Chapter 1

## Introduction

The National Air Transportation system (NATS) is a core part of the national economy, generating revenue of about \$150 billion, transporting over 750 million people annually. In addition to being one of the largest US industries, it is a catalyst for economic growth and international competitiveness.

Air traffic is at its record-highs and is projected to grow at close to 3% annually. The Federal Aviation Administration (FAA) Aerospace Forecast for fiscal years 2008-2025 predicts that the US commercial aviation is on track to carry one billion passengers by 2016 with load factors well above 80% [AF07]. This sheer volume and sustained growth of the aviation sector is putting an enormous strain on the air transportation system. This is evident by extreme delays in the system. The *on-time performance*, defined by the US Department of Transportation (DOT) as the percentage of flights arriving no later than 15 minutes after scheduled arrival times, reached its record-lows in 2007 (see Table 1.1) – with 24.61% of all flights delayed (by more than 15 minutes), 2.52% of all flights cancelled, and almost 8,570 flights diverted in the first six months of the year [BTS08]. For 2007, the Air Transport Association (ATA) reported a total of 134M system delay minutes (up by 15% from 2006) that resulted in \$8.1B in direct operating cost and the estimated total cost to passengers was \$4.2B [ATA08].

All the components of an airline – passengers, crew and planes – operate as a network, interweaving into one another so closely that airline delays have strong net-

Year	On-time Arrivals (%)	Arrival Delays (%)	Flights Cancelled (%)	Flights Diverted	Total Flight Operations
1999	74.53	21.98	3.21	7,722	2,710,849
2000	73.58	22.72	3.44	7,372	2,811,192
2001	76.61	20.23	2.95	6,741	3,103,240
2002	81.32	17.22	1.31	3,963	2,620,287
2003	82.82	15.37	1.64	5,357	3,209,238
2004	77.88	20.26	1.66	6,650	3,505,494
2005	78.05	19.86	1.90	6,641	3,576,284
2006	76.63	21.66	1.50	7,806	3,504,336
2007	72.64	24.61	2.52	8,570	3,701,909
2008	73.38	24.11	2.26	8,784	3,604,175

Table 1.1: On-time Performance Statistics, January through June [BTS08].

work effects. A disruption in one operation can propagate into several other downstream operations creating havoc and propagating disproportionately through the entire network. For example, the summer of 2007 was one of the worst for flight delays with three-quarters of the flight delays nationwide originating from the New York area [NYC07]. As another example of network effects, it is often noticed that local weather delays in one geographical location affects other locations far removed from the inclement weather.

The primary cause of disruptions is the mismatch between the increasing demand for access to airports and the limited operational capacity restricting the number of landings and take-offs at airports. The limited capacity is due to the constraints on runway (spacing between the planes for safety), gate availability and air-traffic control. This imbalance between demand and capacity has led to significant air traffic congestion and flight delays, with delays starting at congested airports.

The demand-capacity mismatch occurs mainly due to limited capacity either because of a sudden capacity drop due to unforeseen circumstances, like bad-weather or because of excessive demand demand, i.e., over-scheduling of the flights, possibly at peak hours. As can be seen in Fig. 1-1, the Bureau of Transportation Statistics (BTS) attributes the largest percentage of National Aviation System (NAS) delays to weather (65.6%) and next to volume (18.91%) [BTS08]. A significant portion is attributed to volume even though only a small percentage of airports are congested

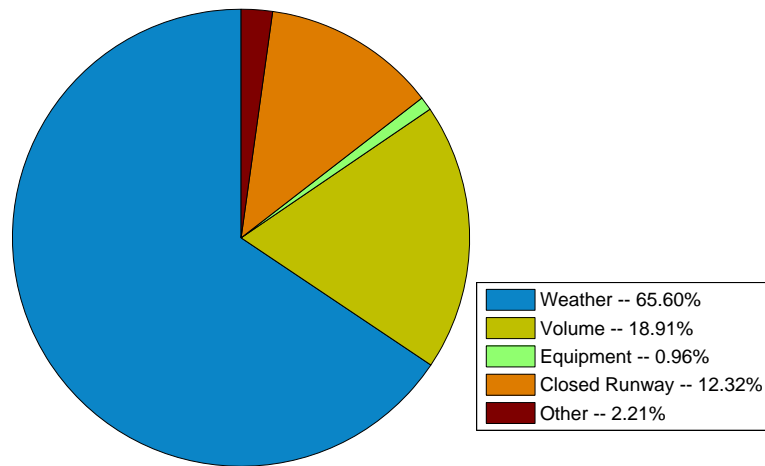


Figure 1-1: Causes of National Aviation System Delays (January – December, 2007) [BTS08]

(only 1% (35 of 3500) airports in the national airport system account for 75% of the commercial passenger activity [AF07]). Nonetheless due to the network structure of the operations, the delays at these airports propagate throughout the entire network, as in the summer of 2007, where three-quarters of the flight delays in the nation were attributed to the NY airports.

## 1.1 Procedures to Mitigate Congestion

Congestion mitigation procedures are critical to the nation’s air system and the economy to ensure that delays and congestion costs do not increase excessively with the projected increasing growth in air traffic. In fact, the role of Air Traffic Flow Management (ATFM) is to match the capacity of the air transportation system with the demand so as to mitigate congestion and ensure that aircraft can flow through the airspace safely and efficiently. We briefly describe the different methods to mitigate congestion which are also those adopted by ATFM.<sup>1</sup>

<sup>1</sup>In response to the record delays in summer of 2007, the DOT convened a special aviation rulemaking committee (NYARC) in the fall of 2007, to study the capacity enhancement and demand-management initiatives. [NYC07] summarizes their report.

- *Capacity enhancement measures* - These initiatives intend to increase the efficiency of existing capacities by reducing delays and maximizing the number of take-offs and landings at an airport. They include the addition of new runways and the deployment of modern technologies.
- *Demand-management techniques* - These policies influence the demand side with the FAA restricting the number of operations (i.e., capacity) to tolerable delay levels and allocating limited capacity to the airlines.

Capacity enhancement measures such as increasing runway capacity not only require a lot of time (in the order of decades for planning and implementation) but cost billions of dollars and are sometimes impossible to implement due to geographical factors.<sup>2</sup> For example, places like LaGuardia Airport (LGA) in New York cannot be expanded due to geographical constraints although it is a chronically congested airport in the US. On the other hand, the O’Hare International airport (ORD) in Chicago, also one of the very congested airports in the US, is undergoing a modernization plan to increase its capacity by the addition of new runways. This is expected to be completed by November 2008, costing \$6 billion and is expected to effectively increase the airport capacity by 60%.

Demand-management policies, unlike capacity enhancement measures, will not require long-term investments and will likely have more immediate effect on reducing delays and relieving congestion. These policies are essential because, the ATA reports, airport and airway infrastructure cannot be scaled to meet future demand as anticipated in the FAA aerospace forecasts [AF07]. Really, demand management should be an alternative, medium or short-term, measure in place only until capacity is enhanced, as capacity enhancement is the only long-run way of meeting the increasing demands.

In this thesis, we focus on demand-management policies to mitigate congestion.

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<sup>2</sup>In [NYC07], 17 short-term initiatives were proposed along with other long-term airspace redesign measures. 11 of these short-term measures have been completed and the rest are in progress. The US Government Accountability Office’s testimony suggests that the short-term initiatives will have fairly small impact on reducing the delays for the summer of 2008 [Uni08]

### 1.1.1 Demand-Management Policies

According to the US code, a *slot* is a reservation for an instrument flight rule takeoff or landing of an aircraft by an air carrier in air transportation [USC05]. ATFM estimates the available capacity at an airport based on the number of operations, i.e., number of landing and take-off slots, per hour for every 15 minutes based on constraints on the runway, gate availability and other safety constraints. Formally, an airport is congested when the landing capacity in a 15-minute period is less than the scheduled arrivals for that period. The demand-management technique adopted by ATFM at congested airports is simply to allocate these limited slots to the airlines. This can be interpreted as a resource allocation problem with the resource being the slots. Although demand-capacity mismatch can result from arrivals and/or departures, ATFM always controls only the arrivals and hence, just allocates landing slots to airlines. The rationale being that control of the arrivals will implicitly control the departures. Airports that are not congested do not need demand-management techniques because there is enough capacity for all scheduled operations at these airports.

ATFM's demand-management techniques are classified into two types based on the time-frame over which the landing slots are allocated to the airlines:

1. *Strategic (medium-term) initiatives*: With these initiatives, slots are allocated to the airlines over a medium-term horizon (usually on the order of a few years). The airlines are allowed to schedule planes only in the slots allocated to them. This approach is called a strategic demand-management technique as airlines can establish their priorities, anticipate issues and plan for the long-term. These initiatives are applied to airports that have prolonged periods of congestion everyday, even on a day with good weather. The major commercial airports facing these levels of congestion are LGA, ORD, DCA (Reagan National at Washington), JFK (Kennedy at New York) and EWR (Newark airport).
2. *Operational (real-time or short term) initiatives*: These initiatives are applied just for the day of operation when there is an unforeseen circumstance, like bad weather, causing a sudden drop in capacity on the day of operation. On these

days, 50% capacity drops over a period of 4-5 hours are not uncommon. Once ATFM receives an update about the new capacity estimate, it assumes that this value is deterministic and allocates the available arrival slots to all airlines scheduled to land. The times of the slots allocated often tend to be later than the scheduled arrival times (due to the capacity drop) and the planes are delayed in the departure cities accordingly. Because the capacity estimate typically is both stochastic and dynamic, not deterministic, due to the stochastic nature of weather and frequent updates, a new set of allocations are performed, each time a new capacity value is obtained. This approach is called operational or real-time demand-management technique because allocations are made in real-time and the airlines also have to respond to such allocations in real-time. Airlines in turn have to respond to such allocations in real-time.

In this thesis, we study both these approaches to demand-management but our primary focus will be on strategic approaches (Chapters 2, 3 and 4).

## **1.2 Thesis Contributions and Outline**

In the following, we briefly outline the structure and the contributions of this thesis.

### **1.2.1 Chapter 2: Strategic Approaches to Mitigate Airport Congestion**

Congestion at airports has led to the formation of slot controlled airports where there is a administrative limit on the number of landings, and hence take-offs, at an airport. In the US, JFK, EWR, LGA, ORD and DCA airports have been slot controlled in different ways since 1968. In Chapter 2, we provide the current procedures in the US of allocating landing slots, with their guiding principles of grand-fathering, lotteries and setting equal landing fees independent of demand levels. Such allocation schemes not only are inefficient methods of utilizing a scarce resource but also act as barriers for new entrants. Market mechanisms such as congestion pricing and slot auctions

have been proposed (in fact, since the 1980's) as an alternative method to current schemes to achieve efficient allocations. These schemes provide transparent demand-based pricing techniques to attain efficiency. Such techniques have been used in several other venues but have been vehemently opposed by the airlines. This is in part due to the fact that airlines will have to pay for something that is currently free. Recently, the DOT has announced plans to conduct slot auctions on an experimental basis at the three major airports in New York [DOT08] and the airlines and airports have reacted negatively to its implementation.

In this thesis, we focus on slot auctions as a strategic demand-management approach. We describe in detail in Chapter 2, the questions, concerns and unresolved issues that need to be addressed before the actual implementation of a slot auction. We lay emphasis on some important design aspects of the slot auction: we provide arguments that a slot auction should be designed as an *iterative combinatorial auction*, where the auction is conducted in rounds and airlines bid on packages of landing slots, and that airlines should be treated as *budget-constrained* bidders in this auction. We then propose our framework of a slot auction with its different components:

1. An airline response model with a *valuation model* that estimates the true value of any package of slots at the prices of the auction and a *strategic model* that decides the actual bids to be placed in the auction; and
2. an auctioneer module conducts the iterative combinatorial auction with *activity rules* to minimize gaming and promote consistent bidding, and a *winner determination* module that finds a provisional allocation and increments prices in each round of the auction.

## 1.2.2 Chapter 3: Airline Response to Airport Slot Auctions and Chapter 4: Activity Rules for Iterative Combinatorial Auctions

In Chapter 3 and 4, we will use the slot auction framework presented in Chapter 2 and focus on the valuation model and the activity rule in more depth.

In Chapter 3, we focus on the design of an airline valuation model to help airlines bid in a slot auction. Preference elicitation, the problem of finding the best bundle of goods in which to bid (landing slots in the case of airlines), is a hard problem in combinatorial auctions. It is even harder for airlines because they have to solve large scheduling, fleet and revenue management problems which often take several hours to solve to find the value of one bundle of slots. We propose the *Aggregated Integrated Airline Scheduling and Fleet Assignment Model* that can estimate the profitability of a bundle of slots by considering the network as a whole and performing both scheduling and fleet decisions. To the best of our knowledge, no previous work has studied this aspect of airline behavior. We refer to the model as an aggregate model because it uses different levels of discretizations of the time space network at different airports: a finer discretization in more congested airports with slot controls and a coarse one in less congested airports. This modeling approach allows us to dramatically improve the computational time needed to solve the model without losing information needed to build an accurate representation of the problem. This is particularly relevant in an iterative auction setting as these computations are performed between rounds of the auction.

We present experimental results using this model with increasing prices (as in an auction), on real data from a carrier. We observe the changing trends in the airline network as prices increase and observe that the model is: (a) computationally efficient; and (b) robust to uncertain data.

In Chapter 4, we focus on the design of activity rules for a slot auction. Activity rules are checks made by the auctioneer at the end of every round of an iterative auction to suppress strategic behavior by bidders and to promote consistent, contin-



ual preference elicitation. They are used in several real-world settings including the spectrum auction. We show that the commonly used activity rule, *Revealed Preference Activity Rule* (RPAR), prevents straightforward bidding strategies of a budget-constrained bidder. In fact, we also observe that they do not guarantee consistent bidding for bidders without budget constraints. The former is a critical issue in the context of slot auctions as airlines are in fact budget-constrained bidders. Good iterative auction design should promote straightforward, demand-revealing behavior on the part of bidders [AC04, Par06].

We characterize the set of *straightforward strategies* with the notion of a *strong activity rule*. We further design strong rules, which we refer to as Strong Revealed Preference Activity Rules (SRPAR), for a general iterative combinatorial auction for both budget-constrained bidders and quasi-linear bidders. We express them as simple linear feasibility problems. We observe an interesting dilemma: one cannot have a rule that is simultaneously strong for both budget-constrained bidders and quasi-linear bidders, and the choice of rule depends, to some extent, on the bid taker's beliefs about participants in the auction. We also propose simple relaxations to SRPAR that could be of interest in practical auction implementations to provide some leeway to the bidders in the bidding process. This work complements the current literature on iterative auction design, and slot auctions, in particular.

In our experimental simulations we compare SRPAR and RPAR for the *clock-proxy auction*, an iterative combinatorial auction proposed for practical settings like the spectrum auctions, when populated with straightforward bidders. SRPAR outperforms RPAR with respect to efficiency and revenue by 3.8% and 9.4% respectively (on average across the different distributions) at low budgets, with benefits falling off as budgets are increased.

### 1.2.3 Chapter 5: Real-Time Approaches to Mitigate Airport Congestion

In Chapter 5, our focus is on real-time demand-management techniques, where we propose a system design to evaluate different real-time allocations schemes based on different metrics. Due to unforeseen circumstances like bad weather, capacity of the runway in terms of the number of landings per unit time, can drop suddenly. We discuss the real-time procedures that the FAA adopts to assign airlines to the different landing slots. There are three different stages that are closely linked to obtain the final allocation of slots. The three stages are: (1) a primary allocation of slots; (2) airline recovery; and (3) inter-airline exchanges. As the names suggest, in the first stage the FAA allocates airlines to slots. In the second stage, airlines repair their schedules to accommodate the disruptions (that is, landing slots allocated are different from the slots in which the flights were scheduled to arrive). In the third stage, the airlines trade slots between each other based on their private objectives. Because it is impractical and impossible for the FAA to provide all the airlines with their most preferred slots, a trading scheme is essential for airlines to achieve their private objectives. Also, trading reduces the wastage of slots resulting from flight cancellations during recovery. After the first allocation of the slots, the three systems run in parallel for the duration of the bad weather as updates of the runway capacity enable new allocations and trades.

In this chapter, we provide a survey of the current allocation procedures that are considered ‘fair’ but might not be system optimal, and alternate procedures proposed in the literature, many of which are system optimal with regard to different metrics but are not considered ‘fair’. One can also divide the allocation procedures based on single-airport decentralized models and multi-airport simultaneous allocation models that ensure connectivity of aircrafts. Because the FAA is allocating the slots, these approaches do not consider flight cancellations, as airline recovery does. So, the question is that if connectivity really matter at the initial allocation stage when cancellations cannot be performed.

To answer this question, in our opinion, it is important to study the system as a whole, as the three stages are closely interrelated. In this chapter, we provide a review of airline recovery procedures and models. These procedures can be treated as airline preferences, like an airline response behavior to an allocation. We suggest using a network-based recovery model that estimates the true cost of a recovery considering the network operated by the airline. The literature in the context of real-time demand-management tend to use local recovery models, that do not consider network effects of the airline, over network-based ones for experimentation.

We then review inter-airline exchange mechanisms. We observe that in the mechanisms currently used, airlines cannot completely express their preferences and expose themselves to the risk of giving up something without the guarantee of getting something back in return (also known as the exposure problem in mechanism design). But on the other hand, a fully expressive exchange might be overly sophisticated leading to the problem of information overload, wherein airline controllers are unable to react due to the complexity. So, understanding airline preferences and designing an exchange that has the appropriate tradeoff between expressiveness and efficiency, is a problem open for future research.

To answer the questions on connectivity and fairness, we suggest a system design with all the three components - initial allocation, a network-based recovery model that captures recovery costs more accurately and for now, use of the exchange mechanism that is currently in place. For different allocation schemes, we can compute different metrics, including those that we have proposed in the chapter, to analyze if certain airline network structures (that can again be categorized with respect to some metrics that we suggest) tend to be more preferred over others. We also present data sources that we will need to run such experiments.

We conclude the thesis in Chapter 6.



## Chapter 2

# Strategic Approaches to Mitigate Airport Congestion

In this chapter we study medium-term strategic approaches to manage the capacity-demand mismatch in some major commercial airports where there is increasing demand to access the airport combined with the limited operational capacity because of constraints on runway, gate availability and air-traffic control. This mismatch leads to the growing air traffic congestion. Our focus is on managing the demand side by allocating the stagnant capacity at these airports to the airlines. This can be interpreted as a resource allocation problem with the resource being the slots.

We present current techniques and their drawbacks and suggest the use of market mechanisms, slot auctions, in particular, for efficiently allocating the limited resources. In the design of slot auctions, we lay emphasis that the auction should: (a) be designed as a price-based *iterative combinatorial auction* where in each round the auctioneer states the prices and the airlines respond with the bundles of slots in which they are interested; and (b) treat the participating airlines as *budget-constrained bidders* with values higher than their ability to pay for the slots. We propose a framework of the slot auctions, with airline response and auctioneer modules, that we adopt for the following chapters.

**Organization of the chapter.** The rest of the chapter is organized as follows. In [Section 2.1](#), we describe the current slot allocation mechanisms, the issues related to it and why it is challenging for the airlines to take measures to improve the situation. We then make a case for the use of market based approaches that provide economic incentives for efficient demand management and narrow our focus on slot auctions. In [Section 2.2](#), we discuss why are concerns, common questions and unresolved issues about slot auctions. We then dive into some important design characteristics and the different components of a slot auction in [Section 2.3](#). We focus on the details of two of the components of the slot auction in chapters [3](#) and [4](#) and use the framework presented in this chapter.

## 2.1 Airport Slot Allocation

### 2.1.1 Current Slot Allocation Procedures

In airports *with slot controls*, there is an administrative limit on the number of operations at an airport. The guiding principles of current slot allocation mechanisms at these airports include grand-fathering, lottery and use-it-or-lose-it rules. The details of each of these procedures are provided in the Code of the Federal Regulations [[CFR](#)]. According to grand-fathering, those operators who owned the slots on December 16, 1985 are allocated the slots subject to withdrawal. Any slot used on December 16, 1985 is distributed among the different operators based on the scheduling records of that day. If sufficient numbers of slots become available, they are given away by a random lottery held at most two times a year, with priority to the new entrants over the incumbents. According to the use-it-or-lose-it rules, if the slots are not used for 80% of the time over a two month period, they can be recalled by the FAA. These rules were established to ensure that the allocated slots are used. In airports *without slot controls*, the concept of slots does not exist. In theory, in these airports, airlines can schedule any number of flights they want.

Identifying a congested airport and deciding if an airport requires slot control

is, in theory, very simple. The process is to allow airports to operate without slot controls where airlines can schedule any number of flights. Observe whether the demand for travel measured by the number of flights scheduled exceeds the capacity of the airport and if delay is propagated to other airports. If this is the case, then the airport requires slot control. In the US, five congested airports, New York's JFK, EWR and LGA, Chicago's ORD and Washington's DCA, have been slot controlled, on and off, in one way or the other since 1968. Even a temporary lift in these slot controls have shown a drastic increase in demand that resulted in extreme congestion that rippled through the entire network. For example, the temporary enactment of the AIR 21 bill (semi-deregulation of the slot control) at LGA led to an immediate and extreme congestion of air traffic at LGA and was the cause of 25% of the national delays [Reg01]. And hence lifting controls should be done with great care.

Airlines pay the airports they operate in a *landing fee* for every landing, based on aircraft weight. The fee is the same irrespective of the congestion at an airport or the time of day.

**Issues with current allocation schemes.** Grand-fathering helps the airlines in long-term planning, investment decisions and making the slot allocation process less cumbersome for the FAA. However, it impedes competition, particularly, by not allowing new entrants to grow in these markets and not allowing changes over long periods [DoE01]. The runway resource at congested airports in peak periods is a valuable and a scarce resource and grand-fathering or a lottery are inefficient and random ways of allocating them.

At airports without slot controls, the current policies can encourage airlines to over-schedule, thus necessitating cancellations and delays of flights, leaving the system regularly in a crisis mode with re-scheduling the norm rather than the exception [BDH06]. A dilemma is that if a single airline cuts back its operations to reduce delays, its current market share is reduced and another airline, its competitor, might be incentivized to schedule more flights in order to increase its market share. The result is no reduction in total delays but a shift in market share, clearly not the desired outcome for the airline reducing its schedule. The airline reducing its schedule might

also be at the risk of permanently losing market share as the airline with more scheduled flights is likely to receive more slots in the event of an administrative procedure like grand-fathering in future [BDH06].

Weight-based prices can exacerbate congestion issues because the pricing schemes provides no incentive for airlines to use larger planes that can move more people during busy hours. In fact, airlines are motivated instead by customer pressures to provide high frequency service (translating into frequent operations with small planes) to match passenger demand. This, in a way, also helps airlines to satisfy the use-it-or-lose-it rules in slot controlled airports. Such policies by the FAA and competitive pressures have resulted in airline scheduling practices that can lead to extreme delays at popular US airports at peak times.

### **2.1.2 Case for Market Based Mechanisms: Auctions**

Alternate market mechanisms like congestion pricing, auctions or their hybrid have been proposed to rectify the shortcoming of the current system. There is an extensive literature that discusses the advantages of these market mechanisms [Dan92, FO01, GIP79, GIP89, DoE01, BDH06, BAB<sup>+</sup>07]. Well designed market based approaches allocate scarce resources efficiently (to the one who values it the most) and promote competition by allowing the participation of all airlines including new entrants.

Market mechanisms are based on a simple transparent demand-based pricing concept. Unlike politically determining who owns the scarce resources, market mechanisms use pricing to allocate them. Demand-based pricing has been widely used in other sectors, even in airline ticket pricing where ticket prices are adjusted based upon the supply and demand of certain routes: peak time and high demand tickets are more expensive than non-peak time and low demand tickets. In particular, using pricing at airports, market mechanisms would: (a) reveal the economic value of the slots; (b) develop a robust secondary market for slots that will lead to a greater efficiency in their usage; (c) increase the size of aircrafts used at the airports; (d) increase the number of passengers served; and (e) encourage airlines to tap into the un-utilized excess capacity elsewhere in the system, thus promoting a more uniform and efficient



utilization of airport resources. But because of the additional features that market mechanisms provide, unlike current mechanisms, they require considerable more research on their design.

**Congestion pricing vs. Auctions.** Congestion pricing charges airlines a flat fee based on the particular time of day. In turn, it allows the number of slots to be determined by the market. Unlike congestion pricing, in auctions, the number of slots to be auctioned and their leasing period are set first. The prices are set by the market. A drawback of congestion pricing is that the fee has to be iteratively set so that the demand does not exceed the capacity. On the other hand, in auctions, it is not necessary that the clearing price of the auction be the market price for the entire leasing period. Ausubel and Cramton [AC05] argue that auctions provide better stability of long-term leases and predictable (low) congestion levels, whereas congestion pricing gives airlines the flexibility to change schedules at the expense of less predictability in airport loads and delays.

In 2004, the FAA commissioned a study to better allocate landing slots at LGA, opening up the possibility of the use of market mechanisms. A proposed congestion pricing scheme and auction mechanism was laid out by a team of researchers and then presented to airline executives and the FAA through a series of conferences. A mock congestion pricing and auction mechanism was hosted by the National Center of Excellence for Aviation Operations Research (NEXTOR). Ball et al. summarizes the results of their findings [BAB<sup>+</sup>07]. More recently, the DOT has announced plans to conduct slot auctions on an experimental basis at the three major airports in New York [DOT08]. Indeed, market mechanisms, in particular auctions, was one of the demand-management initiatives in the New York Aviation Rulemaking Committee (NYARC) study [NYC07].

Our focus is on using auction mechanisms for allocating slots. But we first discuss some concerns and the many issues that need to be resolved for a successful implementation of a slot auction.

## 2.2 Airport Slot Auctions: Questions, Concerns and Unresolved Issues

Auctions for allocating landing slots was proposed for the first time by Grether et al. [GIP79, GIP89] in a report commissioned by the Civil Aviation Board and the FAA following the Airline Deregulation Act in 1978. Ever since, slot auctions have been proposed extensively by academics( [RSB82, Cho02, Fan03, LDC04, BDH06] to cite a few) as well as by industrial agencies such as Dot Econ Limited [DoE01] for the strong associated benefits of these auctions.

As one might imagine, there are several complex elements that need to be carefully designed and researched before implementing a new system of allocating slots. For a slot auction, these would include the number of slots to be auctioned, the design of the auction itself (the bidding language, choice and rules of the auction, and its complexity or simplicity leading to efficient outcomes), bidder aid tools, the ownership of the property rights, implementation issues including regulatory rules, compensating incumbent airlines who have spent heavily on the airport infrastructure, and possibly, where the government should invest the revenue raised from the auction.

The most common issue addressed in the literature deals with the actual design of the slot auction, the focus of [Section 2.3.1](#). There are several additional issues that need to be addressed for a successful implementation of a slot auction. We will first give a detailed description of: (a) why some stake holders are against the idea of an auction; (b) common questions that arise about the outcome of the auction; and (c) some unresolved questions and implementation details that require further research before implementing the auction. Many of these have been adapted from [BDH06]. Additional legal concerns are summarized in the report by the NYARC [NYC07].

**Why do some stake holders oppose the idea of a slot auction?** Several attempts towards auctioning slots, including the most recent announcement in 2008 about auctions [DOT08], have been vehemently opposed by some of the airlines and the New York Port Authority [PAN08]. The main reason being there is *transaction*

*of money* from the airlines to the government in an auction [BDH06]. Market mechanisms, in general, use pricing as the means of allocating scarce resources efficiently. The concern is that auctions are an easy way for the government to raise revenue. Airlines, on the other hand, feel threatened because they will have to pay for slots that they currently get for free. Furthermore, several incumbent airlines have spent heavily on the airport infrastructure (particularly, in gating and baggage facilities) and in many airports, they have long-term leases to have exclusive rights to the gates. So, airlines are indeed worried about the type of compensation they will receive for the investments they have already made.

The following observations mitigate these concerns:

- Auctions do raise revenue but they are no more than the market value associated with the scarce resource. This revenue could be used to pay for infrastructure necessary to safely administer and expand the airport and airspace. The auction should be carefully designed with efficiency objectives rather than maximizing the revenue from the auction so that it encourages new entrants and avoids monopoly at airports. In efficient auctions, the revenue generated is just the smallest amount to attain the efficient outcome.
- One option that can be considered in the design of slot auctions is to compensate incumbent airlines with vouchers for slots they currently own. The vouchers are in the form of money, allowing the airlines to use them to partially buy back slots in the auctions [BAB<sup>+</sup>07]. An alternative to using auctions and vouchers is the use of exchanges to allow airlines to trade slots between each other. Regarding investments, one approach is to allow the airlines to rent out their gates to other airlines at the “going” rate of a gate [BDH06]. This way, airlines that own gates can possibly get a return on their investment. The other way is that the government can compensate the airlines that currently own the gates by buying them and taking charge of the infrastructural requirements in the future using the revenue received from the auction. A gradual transition to an auction with only a small percentage of slots in the auction every year would

ease the pressure on the airlines.

Airline lobbyists and their supporters constantly argue that they need more capacity and technology, not pricing. Peters, the secretary of transportation, in an interesting article [Pet08] points out that this argument incorrectly assumes that these are competing concepts. According to her, the main reason that the government has failed to add capacity and modernize the air traffic control system is that the approach to paying for aviation infrastructure completely disconnects the price of using capacity from its true costs and thus promotes overuse at popular airports and during popular flying times. Indeed, as mentioned before, pricing resources for demand management has been used widely to manage efficient allocation of scarce resources, including the aviation sector (e.g., ticket prices).

**Common questions that arise about the outcome of the auction.** We address a few commonly posed questions that airline lobbyists debate about auctions.

- *Possible under-utilization of the airport?* At slot controlled airports where there is an administrative limit on the number of slots, airlines can schedule flights only up to the allowed capacity levels. Auctions would, in fact, use a similar administrative limit at these slot controlled airports, but would allocate the slots in a different manner. At airports without slot controls, it can very well be assumed that there is no problem of congestion (during good weather at least) and hence no strategic long-term allocation scheme is necessary. Thus, auctions would not result in under-utilization of the airport.
- *Do auctions reduce congestion?* Indeed, as explained above auctions only re-allocate a “fixed” set of resources in a more efficient manner. In both the administrative policies and auctions, a preset capacity limit decides the number of resources based on tolerable delay and cost values. So, the argument is that auctions do not deal with the limit and hence, do not reduce congestion on the runway as compared to that resulting from administrative controls. This is indeed true but as discussed in [Section 2.1.2](#), unlike administrative approaches, auctions help increase the size of the aircraft and the number of passengers

served, and achieve a more efficient use of the runway resources until there is a capacity expansion. Setting different slot limits at different airports and better managing the resources at congested airports, improves the whole system by reducing delay propagation.

- *Might auctions possibly reduce competition?* According to [PAN08], auctions would reduce competition, as several small airlines cannot pay for the slots. But theory says the opposite. Firstly, the auction process is transparent and open, allowing the airlines to participate, unlike the current system that barricades new entrants. Secondly, as long as an airline is able to manage its resources efficiently (e.g., to allocate large planes to bring in more passengers and more revenue), flying into these congested airports with slot controls should still be a profitable venture. In fact, one would potentially observe this kind of effect in the airlines post-auction. [PAN08] also suggests that auctions would lead to large players having a monopolistic hold in the market. In fact, auctions can be designed to guarantee that no airline can have a monopolistic hold of the market (by adding a side constraint about the allowable allocations in the auction).
- *Can auctions cause increased ticket prices?* Passengers incur several types of cost during travel – ticket cost, travel cost, delay cost. Currently, consumers pay higher delay costs in congested markets as measured by the associated wait times. In auctions, higher ticket prices may result due to pricing of slots. The increase in ticket prices depends on how much the airline can absorb in its operational costs and slot prices. For example, the recent increase in fuel prices did not immediately increase the ticket prices but some airlines eventually did increase the price on several itineraries. At least in the case of auctions, consumers have a choice of avoiding the cost increase [NYC07].

**Some unresolved issues about a slot auction.** We discuss a few issues that need to be resolved for a successful implementation of a slot auction. (Additional legal issues are discussed in [NYC07].)

- *Capacity and allocation of gates to slots.* A slot resource that is auctioned should be considered with gating and baggage facilities as it cannot be used otherwise. An auction assumes slots are interchangeable across different plane types and hence, across gates also. In practice, both these are not possible. The capacity of the airport changes with the sequence of flights, that is, with the mix of aircraft arriving, and the gates depend on the type of aircraft. The aviation director of the Port Authority [PAN08] vehemently disagrees to the concept of the “going rate” of a gate suggested in [BDH06]. The allocation of slots to different plane types is an interesting problem of determining slot capacities based on aircraft mix. And the problem of allocating appropriate gates to winners of the slots is a complex scheduling problem, one that might sometimes be infeasible. Moreover, the current system has gates grouped together by airline to facilitate passenger connections. The scheduling problem would also have to take this into account. Both these issues have not been addressed in any slot auction literature and are problems open for future research.
- *Property Rights.* Currently airlines enjoy a perpetual right to usage by the grand-fathering rights. Similarly, in an auction setting, it is important to define what ownership of a slot means. One approach that [BDH06] suggest is that the buyer of the slot would have exclusive rights to use the slot during a given time-window (e.g., within a 15-minute time window) every day. The right to ownership would be for a relatively long period, i.e., a leasing period (of say, five years). The slot should also include possible rights to other terminal area facilities, like gates and baggage facilities. Because secondary trading markets usually follow primary auction markets, allowing airlines to trade slots among themselves, it is important to define the property rights in the presence of such after markets. Property rights, then, should also define the tradable rights and rights to enforce them. (Refer to [BDH06] for more details on this secondary exchange mechanism of slot leases.)

- *Implementation.* There must be gradual deployment of the auction process in order for airlines to transition from the administrative process. One approach is to start with auctioning a small percentage of slots, possibly at the peak hours, at the most congested airports like LGA, to relieve the delay issues due to congestion at these airports. This way resources are redistributed resources and airlines can gradually tap into the un-utilized capacity elsewhere in the system. Based on redistribution and congestion levels, the auction can be expanded to include a percentage of the slots (even all of them) at an airport or at multiple congested airports simultaneously.
- *Small Communities.* Any market mechanism for slots where there is a transaction of money, might lead to the elimination of service to small communities, which are more likely to not be profitable. As a public policy, in case, this has to be avoided, incentives should be provided and the auction should be designed to also promote flights to small communities. But as highlighted in the NYARC report [NYC07], public policy regarding exceptions can undermine the benefits of a market-based approach.

## 2.3 Slot Auction Design

In this section, we will turn our attention from concerns and questions about a slot auction to its design characteristics, if implemented. Of course prior to the actual design of the auction, all the concerns discussed above have to be addressed. In particular, property rights, in terms of what the airline actually owns and its tradable rights, must be well defined and unambiguous. The overall goal of the slot auction should be that of efficiency in the allocation of slots to the airlines.

In the design of an auction, it is important to understand how the bidders, here, airlines, value slots. Airlines rely on passenger connections between different fleets and aircraft, and profitability is a function of the markets served and the frequency of service in these markets. Both these factors indicate that airlines value a group of

slots much more than the individual slots themselves. This means the slots are *complementary* goods for the airlines. In fact, slots are also *substitutable* goods because airlines are often times indifferent to arriving or departing a few minutes earlier or later as they can adjust to these small variations by minor speed changes in the air or by adjusting slightly their scheduled departure times. It is, thus, evident that airlines have a non-linear valuation over the slots at an airport.

In the auction literature, it is well-known that when there are complementary and substitutable goods, *combinatorial auctions*, where bidders bid on packages of goods, are best suited [CSS06]. This way, bidders are not at the risk of winning just a part of what they need with value that is disproportionately lower. The overall design of a slot auction should be that of a combinatorial auction where all the slots in the auction are auctioned simultaneously. In this auction, airlines submit their bids, i.e., preferences, in the form of packages or bundles of slots. In fact, the very first combinatorial auction was proposed for allocating slots at different airports [RSB82]. This auction by Rassenti, Smith and Bulfin is a *sealed-bid* auction with airlines specifying their value for the packages of slots directly to the auctioneer.

It has been well accepted, over the years, that among combinatorial auctions, *iterative*, especially *ascending price*, auctions are preferred over their sealed bid counterparts [Par06]. Such auctions, as the name suggests, are conducted in rounds. In each round, the auctioneer specifies the prices on the goods and the bidders respond with packages. The prices are updated each round based on a *price-updating rule* chosen by the auctioneer (e.g., incremented proportional to the excess demand). The auction stops when a *termination condition* is met (e.g., supply greater than demand). Parkes provides a detailed description of the design of combinatorial auctions in [Par06].

Iterative auctions are preferred because they help focus bidders attention on interesting bundles, especially when bidders have difficult preference elicitation problems, as in the case of airlines in our setting (see Chapter 3). Unlike sealed-bid auctions, these auctions have a transparent process and allow for price discovery and bidder feedback [Par06, ACM06, PU00a, Mil00]. Iterative auctions also tend to distribute (and, thus, decentralize) the computations associated with finding the winners of the



goods, amongst the different bidders.

Ball, Donohue and Hoffman provide a general framework for the design of iterative auctions in the context of auctioning slots at airports [BDH06]. We adopt a similar iterative auction framework that we describe in the following section.

In the design of an auction, we think airlines should be treated as *budget-constrained bidders*, i.e., have valuations that are greater than their ability to pay (for instance, due to liquidity or credit problems). The deregulated (post-1978) US airline industry has very low profit margins compared to an average US business. In more recent years, all the US airlines together have lost over 35 billion during the period 2001-2005 [ATA07]. Four of the six major U.S airlines declared Chapter 11 bankruptcy and have only recently emerged. The crisis began with the economic downturn in early 2001 and was fueled by the September 11 attacks [Bel05]. Airlines typically carry large amounts of debt and are especially vulnerable to fuel spikes, recession or economic shocks [ATA07]. Even for the wireless spectrum auctions, Cramton discusses that it is realistic to assume that *all* participating firms face budget constraints [Cra95] because bidders must raise funds before the auction starts. Fund raising which is a time-consuming and costly process, and arguably leads to budget constraints. The above reasoning justifies treating airlines as budget-constrained bidders in the slot auctions.

### 2.3.1 Components of a Slot Auction

The slot auction problem has two important components: the auctioneer and the airline response (See Fig. 2-1). The auctioneer is active in the auctioneer component while the bidders (airlines) are active in the airline response component.

The auctioneer accepts bids from all the airlines in each round of the auction. The first step is to ensure that the bidders satisfy the *activity rules*. Activity rules are the auction rules imposed by the auctioneer. An example of an activity rule is that quantity demanded decreases as price increases. Based on the bids, the auctioneer either terminates the auction or generates new ask prices for the next round. We call this the *winner determination* module because the auctioneer may also provide a

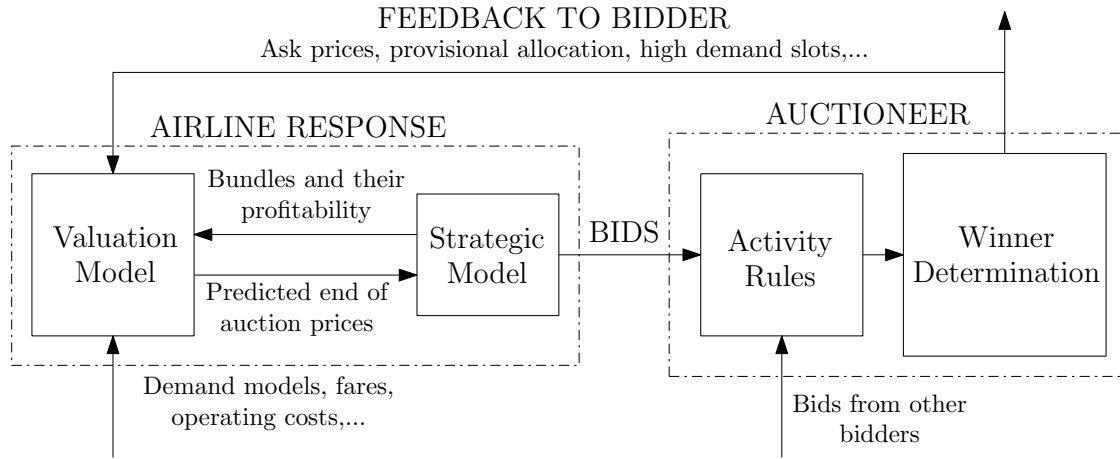


Figure 2-1: Slot auction model

provisional allocation to the airlines in each round.

The airline response model comprises of two components – the *valuation model* and the *strategic model*. The valuation model generates the profitability of the different bundles based on the forecasted demands, fares, ask prices and operational costs. The value of a bundle can have both a private and common value component (value that others place on it). Given the results of the valuation model, the strategic model generates the bids to be placed by the airline in each round of the auction. The airline’s goal is to win the best set of bundles at the end of the auction. The strategic model’s purpose is to help decide if the airline should bid aggressively, passively or truthfully at different stages of the auction, and to understand the trade-offs. These issues have been discussed in detail in Dong’s thesis [Don05]. Dong develops a strategic model for the airlines in a combinatorial clock auction that also returns predicted end of auction prices that can be used in the valuation model.

In the following two chapters, we focus on two components of the airport slot auctions. In [Chapter 3](#), we are interested in modeling airline behavior in auctions and propose a valuation model for the same. To the best of our knowledge, no previous work has studied this aspect of airline behavior. In [Chapter 4](#), based on our understanding of airline behavior as budget-constrained bidders, we provide some key design features, in the form of activity rules, to the slot auction mechanism. This

work complements the current literature on slot auction design and can also be used in other iterative auction settings as well.



# Chapter 3

## Airline Response to Airport Slot Auctions

In this chapter, we are interested in modeling airline behavior in a slot auction. We model slot auctions among the different airlines as iterative ascending price auctions among budget-constrained bidders. As discussed in [Chapter 2](#), one of the advantages of using an iterative ascending price auction is that it helps the bidders (the airlines in the slot auction) in price discovery. In price-based auctions, the auctioneers provide ask prices and the bidders need to respond with the bundle of slots in which they are interested. This preference elicitation is a particularly difficult problem, even more so in the case of airlines [[Par06](#)]. We build a response model for the airlines, in the form of a valuation model, to aid the airlines to find the slot bundles they prefer, in response to the auctioneer's prices. Though we present our model for price-based auctions in this paper, the same can be used for proxied auctions where direct-value information needs to be revealed.

We develop an airline response model, which we call the *Aggregated Integrated Airline Scheduling and Fleet Assignment Model (AIASFAM)*, that considers the airline network as a whole. Its solution returns the most profitable bundle of slots for the airline at the current auction prices, given the airline's budget, forecasted demands, fare estimates and operational costs. Our model is a network-based model that includes integrated decisions on both airline scheduling and fleet assignment. With our

aggregated-integrated model, our approach, even for these strategic long-term decisions, is highly tractable and efficient (i.e., the size of the model and the number of constraints are small) in contrast to traditional approaches. We present experimental results using this model with increasing prices (as in an auction), on real data from a carrier. We observe the changing trends in the airline network as prices increase and observe that the model is also robust to uncertain data.

**Organization of the chapter:** The rest of the chapter is organized as follows. We state our auction setting and assumptions in [Section 3.1](#). In [Section 3.2](#), we discuss the different challenges involved in developing a valuation model and propose an aggregated network-based approach. In [Section 3.3](#), we develop the airline response model AIASFAM and describe some of its variants. We then present our experimental findings in [Section 3.4](#).

## 3.1 Problem Setting

Let us suppose that the FAA conducts an auction to allocate slots over a finite specified period of time at an airport. A single airport is assumed for simplicity. The models described in this chapter can be easily extended to multiple airports as well. The number of slots to be auctioned depends on the runway, gating, baggage and terminal capacities and the deployment procedure (e.g., auction 20% of the landing slots every year). The leasing period of the slots should be long enough to allow airlines to plan their investments and short enough for the system to evaluate the market and the value of owning the slots.

For simplicity, we assume that only the landing slots are auctioned and the take-off slots are coupled with the landing slots. More generally, we can easily extend our models when both landing and take-off slots are auctioned together. A slot resource, as discussed earlier, should be considered with gating and baggage facilities as it cannot be used otherwise. Due to the physical limitations of aircraft movements and uncertainties in the system, a landing slot refers to a landing in a time-window. For this purpose, a day is divided into several such time-windows. Ball Donohue

and Hoffman suggest a 15-minute time window and state that larger windows would allow airlines to bunch their operations close to the peak, potentially increasing congestion [BDH06]. We assume that the time-windows are short enough that airlines are indifferent to landings within that time-window.

In price-based approaches, the auctioneer provides ask prices and the bidders respond with bundles i.e., the number of landings in the different time-windows. The ask prices are over landing slots in different time-windows. We assume linear prices, i.e., the price of a package is the sum of the prices of the items in the bundle. Thus, an ask price is a vector of prices where each element refers to the price of one landing slot in that time-window. Linear prices, because of their simplicity, have been used in other practical applications [AM06, Cra03, LJC<sup>+</sup>08]. Linear pricing is consistent with the procedure of distributing vouchers, to incumbent airlines, that are traded for individual slots at the end of the auction [BAB<sup>+</sup>07].

We model airlines as budget-constrained bidders, as discussed in Section 2.3. Models for firms with budget constraints allow for both hard and soft (i.e., flexible) budget constraints. Hard budgets are those that cannot be exceeded, while soft budget constraints are those that can be exceeded under certain circumstances. For simplicity, we restrict ourselves to bidders with hard budgets. We consider a model similar to that of Borgs et al. [BCI<sup>+</sup>05] where we assume a hard budget constraint and quasi-linear utility up to the budget. The utility function  $u$  then has the form:

$$u(x, p) = \begin{cases} v(x) - p(x) & \text{if } p(x) \leq B, \text{ and} \\ -\infty & \text{otherwise,} \end{cases} \quad (3.1)$$

where  $B$  is the budget,  $v$  is the value for the bundle  $x$  and  $p$  is the price. We call this the *budget-constrained utility function*. A special case of this utility function is the *quasi-linear utility function* which is obtained by setting  $B = \infty$ .

In the transition to an auction mechanism as a long-term allocation of slots, one way of compensating incumbents who have invested heavily on infrastructure and marketing is to provide them with vouchers, which are equivalent to cash. Vouchers can be provided to current holders of slots, for each time period, based on the current

allocation of slots during that time period. We assume for our models that vouchers at a particular time period can be exchanged for a winning slot in that time period and that unused vouchers can possibly be traded with other airlines. Vouchers are a mechanism to allow incumbents to make a gradual change to their current network without tremendous outlays of cash.

## **3.2 Valuation Model: Network-Based Aggregated Response Behavior**

Valuing even a single bundle can be difficult for the airlines, let alone several bundles. This is because the value of a bundle of slots in one airport depends on the operations of the airline in the entire network they are operating. To add to this, to access the value of a bundle, airlines have to solve scheduling, marketing and revenue-management problems that depend on uncertain future demand, fares and costs. These challenges translate to large-scale problems that sometimes require take several days to find a close-to-optimal solution. In an auction setting, we need a model that helps return profitable bundles both in an efficient and tractable manner between two rounds of an auction, and is robust to uncertain data, given the long leasing periods of a few years.

As a preliminary step, we ignore the effects of competition in designing a valuation model, despite the fact that airlines operate in a competition-driven industry. We very briefly touch upon plausible methods to capture competition in our model, but this is largely left as a topic for future research.

While developing a valuation model, one could adopt either a local or network wide profitability model. The local approach, as the name suggests, lets airlines evaluate its valuations under the assumption that the remaining network and operations are not affected. An example of the local approach would be to evaluate the profitability of the different bundles of slots based on the frequency of operations in and out of the airport



with the assumption that the remaining markets and operations are not affected. This approach is computationally very fast, but does not consider the network effects that changes in flights at an airport may create (e.g., if there are sufficient aircrafts for operations or if passengers can make the planned connections). The network wide approach, on the other hand, considers the network as a whole and thus, requires airlines to make both airline scheduling and fleet decisions over their network of operations (using models like the *Integrated Airline Schedule and Fleet Assignment Model (IASFAM)* of Lohatepanont and Barnhart. [LB04]). The IASFAM takes as input the demand and fare forecasts along with estimated operating costs, fleet sizes, the set of flight legs that the airline is potentially interested in operating and the passenger recapture levels between the different flight legs; and solves for both the airline scheduling and the fleet over their network of operations. The IASFAM is very detailed and hence, provides more accurate preferences than a local model, but it can require long solutions times.

In the mock slot auction [BAB<sup>+</sup>07], we observed that airlines adopted a local approach for speed and ease of computation. For our modeling, we instead adopt a network-based approach as it captures the network effects of the system and evaluates the preferences more accurately. To ensure tractability of the preference elicitation problem, we instead propose that an airline make decisions over an aggregated network. We propose a more aggregate time-line discretization than that used in traditional models, such as IASFAM, which typically employ a minute-by-minute discretization. Moreover, we suggest providing flexibility by allowing different discretizations for different airports, i.e., the granularity of discretization of an airport is a parameter of the problem. This allows modeling of congested airports with a fine degree of granularity and less-congested airports with a coarser one. This is because some congested airports have slot controls with finite capacity and an airline can potentially only use certain number of specified time intervals in those airports. So, by using a finer discretizations at these congested airports the model can respect such constraints. But if we divide all the airports uniformly and finely as the congested airports, it would be at the cost of increasing the size of the model and thus, its

running time. So, a coarser discretization would suffice at less-congested airports. Network sizes can be further decreased by coalescing cities that are geographically closely situated into a single node.

This aggregated network-behavior is a reasonable way of evaluating the different bundles for the following reasons:

1. The slot leases are for 3-5 years or more, hence, accurate predictions (of profitable markets, operational costs, fuel costs, fares etc) are difficult, if not impossible, to construct. Moreover, it was observed by Lohatepanont and Barnhart that IASFAM's solution can be very different even with small (5%) fluctuations in demand;
2. The airline does not know its competitors winning slots and their behavior; and
3. Both markets and participants could change over such long durations.

Such an aggregation as described above requires comparatively much less data than models like IASFAM and hence, the problem size is significantly smaller, making it far more tractable than IASFAM. Also, since the data is in a more aggregated fashion, it is less prone to data-uncertainty. IASFAM is a very good optimization model when all the data is estimated accurately. However this is not the case in the auction setting. So, decisions made at an aggregated level require less extensive data and will likely be more effective in evaluating different scenarios. However, such advantages come at the cost of counting of the planes in the network approximately. The following example illustrates the trade-off between plane count and aggregation.

**Example 3.2.1.** *Consider an airline operating between two airports A and B daily as shown in Fig. 3-1. The same set of operations (4 flight legs) between A and B are plotted for three different levels of time-line discretizations at the two airports. Based on the information from each (aggregated) network, we compute the minimum number of aircrafts required to perform the operations (i.e., cover the network) every day. Note that the maximum number is 4 aircrafts i.e., 1 aircraft per leg. The minimum can be computed by counting the number of planes that have to be on*

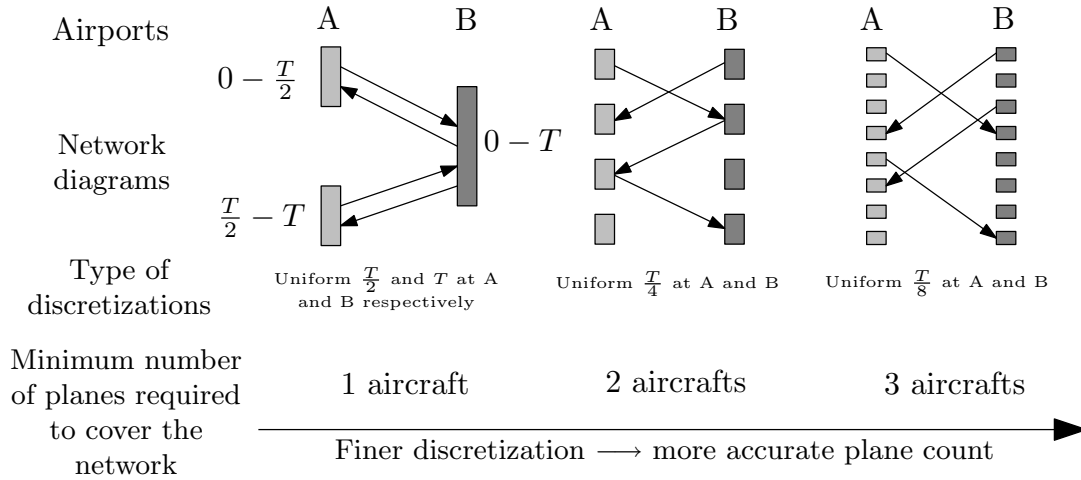


Figure 3-1: Tradeoff between time-line aggregation and plane count

the ground at the beginning of the day (time 0) at each airport to perform the sequence of operations. A departure before arrival implies that there exist at least one plane on the ground to perform that operation and vice-versa does not require any plane on the ground. We observe for the same network the number of aircrafts required increases with finer discretizations (from 1 to 3). In this example, any further discretization does not increase the number of aircrafts required over 3 aircrafts. The reason this is a concern is because for a sufficiently aggregated network of an airline with many more operations, the airline could potentially reassign its remaining planes elsewhere under the assumption that it can cover the network with fewer planes, when, in fact, it cannot.

This example illustrates the tradeoff between plane count and aggregate networks and that the networks need to be sufficiently disaggregate to model the plane count exactly. So, we can write an aggregate model that can be solved quickly but that can only approximately model limitations on the number of planes used or alternatively, we can write a finely discretized model that is computationally very challenging (or impossible to solve in reasonable time frame), which is capable of modeling plane count limitations exactly. The choice of the model depends on the size of the airline network, computational time needed for solution, accuracy of the data and time available between two rounds of an auction. In this thesis, we restrict to the aggregated

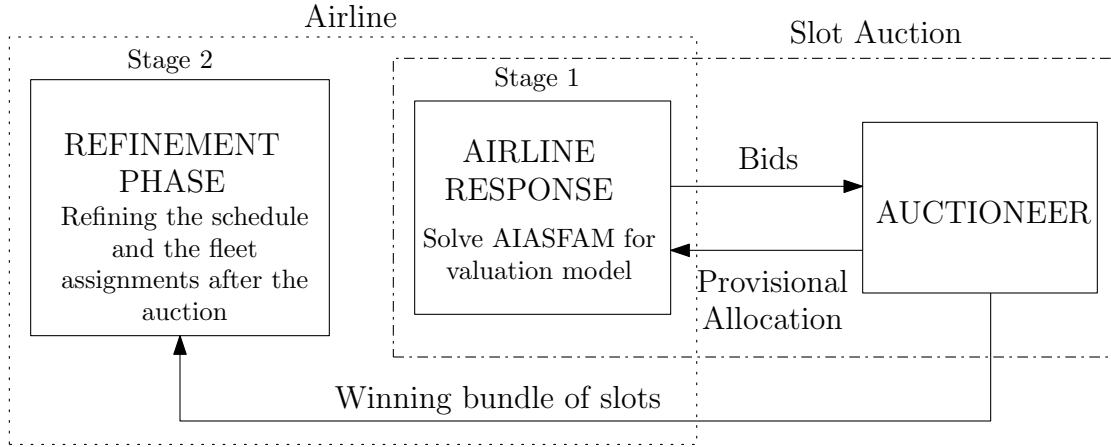


Figure 3-2: Proposed Airline Behavior

technique of valuing slots but propose alternative techniques of approximating and strengthening the plane count.

We call our model the *Aggregated Integrated Airline Scheduling and Fleet Assignment Model (AIASFAM)* because it is an aggregated decision version of the IASFAM. Our solution approach using AIASFAM is computationally fast and allows airlines to evaluate their valuations efficiently during the auction. We also propose different methods of strengthening the plane count constraint.

We propose an airline behavior in the auction as described in Fig. 3-2. In stage 1, in the auction, the airline can use the proposed AIASFAM model and then after the auction, when the airline implements an exact schedule for each season, they can adopt a detailed model similar to IASFAM (this way more accurate (short-term) data can be fed to IASFAM). In this chapter, our focus is on the valuation model in the airline response. But as discussed in Section 2.3.1 and Fig. 2-1, the airline response in stage 1, will include the interaction between the valuation model and the strategic model and the strategic model ultimately decides the bids to place in the auction.

In the next subsection, we formulate AIASFAM as an integer program to generate the most profitable bundle, and then describe some of our experimental results in Section 3.4.

### 3.3 Aggregated Integrated Airline Scheduling and Fleet Assignment Model (AIASFAM)

In this section, we describe a valuation model, AIASFAM, that can be used as a decision support tool by airlines to find their true preferences/valuations in a slot auction. In AIASFAM, in addition to scheduling and fleet decisions made at an aggregate level, the model incorporates a budget constraint (including vouchers). We do not explicitly add activity rules specified by the auction but they can be added as side constraints to the model. The model takes as input the set of flight legs the airline is potentially interested in operating, the forecasted fare and unconstrained demand data, the fleet data, the budget and the current ask prices of the slots. In the basic model we first present, we employ a leg-based model and hence, use prorated leg-based fares instead of itinerary-based fares and do not distinguish between the different fare classes. We have formulated the problem as a daily problem and assume a representative daily demand. This model assumes a free mobility between landing slots and airports. In general, this is more complex because of the investment of the airline in the terminal area facilities and the compensations they receive for when there is an auction.

#### Notation

##### Sets:

$A$  Set of all airports indexed by  $a$

$T_a$  Set of non-overlapping, but comprehensive, time-windows into which a day at an airport  $a$  is divided (either uniform or non-uniform)

$K$  Set of all fleet types indexed by  $k$

$F$  Set of all potential flight legs that might be operated indexed by  $f$ . A flight leg has four components  $(o_f, d_f, s_f, e_f)$ - origin, destination, time-window from which the flight leg departs the origin and the time-window at which the flight leg arrives at the destination, respectively. Note that this flight schedule is “aggregate” due to the time-windows at each of the airports.

$R(f)$  Set of flights  $f' \in F$  such that spilled passengers of  $f$  can be recaptured on  $f'$ ,  
 $\forall f \in F$ .

$O(a) = \{f \in F | o_f = a\}, \forall a \in A$

$I(a) = \{f \in F | d_f = a\}, \forall a \in A$

$O(a, m) = \{f \in F | o_f = a, s_f = m\}, \forall a \in A, \text{ and } \forall m \in \mathbf{T}_a$

$I(a, m) = \{f \in F | d_f = a, e_f = m\}, \forall a \in A, \text{ and } \forall m \in \mathbf{T}_a$

### Parameters/Data Needed

$a^*$  Airport where the landing slot auction is conducted

$p_m$  Ask prices for landing during time-window  $m$  at airport  $a^*$

$B$  Budget - Amount of money that an airline is willing to spend on the auction

$v_m$  Number of vouchers available to the airline in time-window  $m$  (can be non-integer because it is cash)

$r$  Discount at which unused vouchers are traded with another airline, that is, the cash obtained by an airline in return for an unused voucher is  $r$  times the market value of the corresponding slot ( $r \leq 1$ )

$L$  Maximum number of slots for which an airline can bid in a single bid

$O_m$  Maximum number of landings in time-window  $m$  allowed by airport  $a^*$

$Slots_m$  Additional slots in the airport  $a^*$  that are not auctioned and are owned by the airline. For example, if only 20% of the landing slots are auctioned,  $Slots_m$  refers to the slots that the airline owns in the 80% of the slots that are not auctioned.

$T_{f,k}$  Flying time in hours for flight leg  $f$  using fleet type  $k$

$N_k$  Number of airplanes that the airline has of fleet type  $k$

$H_k$  Average number of hours a fleet of type  $k$  flies

$S_k$  Number of seats in an aircraft of fleet type  $k$

$D_f$  Unconstrained demand for flight leg  $f$

$\lambda_{f,f'}$  The percentage of passengers spilled by  $f'$  that can be recaptured onto flight  $f$

$Fare_f$  Pro-rated fare for a flight leg  $f$

$C_{f,k}$  Operating cost of flying flight leg  $f$  using fleet type  $k$

## Variables

$b_m$  Number of landing slots in airport  $a^*$  in time-window  $m$  on which an airline bids

$w_m$  Maximum of the number of vouchers available in time period  $m$  and the number of slots bid in time period  $m$

$x_{f,k}$  Number of flights operating the flight leg  $f$  by fleet type  $k$

$P_f$  Number of passengers traveling on flight leg  $f$

$V_f$  Number of passengers spilled by flight leg  $f$

## Leg-based valuation model

$$\max \sum_{f \in F} Fare_f P_f - \sum_{f \in F, k \in K} C_{f,k} x_{f,k} - \sum_{m \in \mathbf{T}_{a^*}} p_m(w_m - v_m) + \sum_{m \in \mathbf{T}_{a^*}} r \cdot p_m(w_m - b_m)$$

$$\text{subject to } \sum_{m \in \mathbf{T}_{a^*}} p_m w_m \leq B + \sum_{m \in \mathbf{T}_{a^*}} p_m v_m \quad (3.2a)$$

$$w_m \geq w_m \quad \forall m \in \mathbf{T}_{a^*} \quad (3.2b)$$

$$b_m \leq w_m \quad \forall m \in \mathbf{T}_{a^*} \quad (3.2c)$$

$$\sum_{m \in \mathbf{T}_{a^*}} b_m \leq L \quad (3.2d)$$

$$b_m \leq O_m \quad \forall m \in \mathbf{T}_{a^*} \quad (3.2e)$$

$$\sum_{k \in K, f \in I(a^*, m)} x_{f,k} - b_m \leq Slots_m \quad \forall m \in \mathbf{T}_{a^*} \quad (3.2f)$$

$$\sum_{f \in F} T_{f,k} x_{f,k} \leq N_k H_k \quad \forall k \in K \quad (3.2g)$$

$$\sum_{f \in O(a)} x_{f,k} - \sum_{f \in I(a)} x_{f,k} = 0 \quad \forall a \in A, k \in K \quad (3.2h)$$

$$P_f - V_f - \sum_{f' \in R(f)} \lambda_{f,f'} V_{f'} \leq D_f \quad \forall f \in F \quad (3.2i)$$

$$P_f - \sum_k S_k x_{f,k} \leq 0 \quad \forall f \in F \quad (3.2j)$$

$$V_f \leq D_f \quad \forall f \in F \quad (3.2k)$$

$$x_{f,k}, P_f, V_f \in \mathbb{Z}^+ \quad \forall f \in F, k \in K \quad (3.2l)$$

$$w_m, b_m \in \mathbb{Z}^+ \quad \forall m \in \mathbf{T}_{a^*} \quad (3.2m)$$

The budget constraint (3.2a) which ensures that the payment is always less than the budget. Constraints (3.2b – 3.2c) along with the objective ensure that the variable  $w_m$  is set to the maximum of two quantities: (1) the number of vouchers available during the time-window  $m$ ; and (2) the number of landing slots in an airline’s bid in the time-window  $m$ . This is useful in writing the objective when the discount factor  $r < 1$ . Constraint (3.2d) limits the total number of landings on which an airline can bid in the auction. This is an external constraint imposed by the FAA on the airlines to promote competition so that the market power of an airport does not entirely lie in the hands of a single airline [BDH06]. Constraint (3.2e) limits the



landings by time-window to the maximum limit based on the optimal airport arrival rate for the auction. [Constraint \(3.2f\)](#) links the flight leg variable to the bidding variable. [Constraint \(3.2g\)](#) limits the total flying time by fleet type<sup>1</sup>. We will explain later in this section why we limit flying time of fleet rather than planes in a fleet. [Constraint \(3.2h\)](#) ensures balance by fleet type. [Constraints \(3.2i\)](#) and [\(3.2j\)](#) limit the number of passengers traveling on a leg to: the forecasted unconstrained demand for that leg less the passengers spilled by the leg including those that are recaptured; and the number of seats provided by the aircraft operating that leg respectively. And finally, [Constraints \(3.2k\)](#) limits the spill on a leg to its unconstrained demand.

The objective represents the profit (i.e., utility) to the airline. The first and second term are the total revenue and operational cost to the airline by operating a schedule. The third term is the payment to the auction and the fourth term is the return by trading the unused vouchers to other airlines.

### **Approximate techniques for counting planes in an aggregate network.**

[Example 3.2.1](#) illustrated that counting planes in an aggregate time-line network is an underestimate of the actual number of planes required to cover the operations. This, along with a count constraint that limits the number of planes in the network (aggregated in our case), results in overusing the planes i.e., more planes than the limit specified may be actually required to perform the same set of operations. So, in order to limit the actual number of planes used, a side constraint such as [Constraint \(3.2g\)](#), where a limit on the flying time of the fleet, can be imposed. This strengthens the explicit count constraint in an aggregated network. In fact, if the network is sufficiently aggregate, the time count would be a much tighter constraint than the explicit count constraint. For this reason, in the above formulation, we do not impose the explicit count constraint at all. Note that in our formulation, the network only has flight arcs (flow of planes in air between airports) and no ground arcs (flow of planes on the ground in an airport with time). Ground arcs are essential to perform an actual plane count. The addition of the ground arcs not only slowed the solution time considerably but did not improve the solution obtained in terms of the explicit

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<sup>1</sup>We evaluate the expected number of flying hours per plane by fleet type using historical data.

plane count.<sup>2</sup> In our experiments with the time count constraint we validate that the time count constraint is an effective constraint to cover all the operations with planes (See [Section 3.4](#)). The flying hours by plane by fleet can be tuned in order to obtain the appropriate plane count.

Other approximations can be added into the model to strengthen the count constraint. For example, we can add a constraint for each busy airport or hub to count the maximum number of planes on the ground in these airports and limit that to some values. Let  $z_{a,k}$  be the maximum number of planes of fleet type  $k$  on the ground at time 0 at airport  $a$ . Then,  $z_{a,k} \geq \sum_{t=0}^T \left( \sum_{f \in O(a)} x_{f,k} - \sum_{f \in I(a)} x_{f,k} \right) \forall T \in \mathbf{T}_a$ . These values of  $z_{a,k}$ 's can be bounded by different values.

### 3.3.1 Variations of the Valuation Model

In this section, we suggest various extensions of the leg-based model described in [Section 3.3](#).

**Itinerary-based model.** In the US, passengers typical travel on multiple leg itineraries and creating flight-leg interdependencies. We assumed in the leg-based valuation model proposed earlier that the legs are independent. This causes a revenue estimation error in the fleet assignment as it assumes leg-independence. Barnhart, Kniker, and Lohatepanont provide quantitative evidence about this and introduce itinerary-based fleet assignment models [[BKL02](#)]. Here, we extend the integrated leg-based models to an itinerary-based models to capture revenue more precisely.

#### Notation: Sets and parameters

$I$  Set of itineraries indexed by  $i$ . An itinerary consists of one or more, but typically less than three flight legs. Usually all possible itineraries are innumerable in number. We consider only itineraries with published fares.

$R(i)$  Set of itineraries  $i' \in I$  such that passengers spilled from  $i'$  are recaptured by  $i$ .

$D^i$  Unconstrained demand for itinerary  $i$

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<sup>2</sup>We describe the method to compute the actual plane count from an aggregate solution in the [Section 3.4](#)

$Fare_i$  Fare for the itinerary  $i$

$\delta_{f,i}$  1 if flight leg  $f$  is a part of itinerary  $i$ , 0 otherwise

$\lambda_{i,i'}$  The percentage of passengers spilled by itinerary  $i'$  that can be recaptured onto itinerary  $i$

## Variables

$P^i$  Number of passengers traveling on itinerary  $i$

$V^i$  Number of passengers spilled by itinerary  $i$

In the itinerary-based model we replace [constraints \(3.2i – 3.2k\)](#) in the leg-based valuation model by the following constraints:

$$\sum_{i \in I} \delta_{f,i} P^i \leq \sum_{k \in K} S_k x_{f,k} \quad \forall f \in F \quad (3.3a)$$

$$P^i - V^i - \sum_{i' \in R(i)} \lambda_{i,i'} V^{i'} \leq D^i \quad \forall i \in I \quad (3.3b)$$

$$V^i \leq D^i \quad \forall i \in I \quad (3.3c)$$

and replace the first term in the objective of the leg-based valuation model with the fare term  $\sum_{i \in I} Fare_i P^i$ .

The first constraint ensures that the total number of assigned seats on a flight leg does not exceed the capacity provided on that leg. The second constraint and third constraints are similar to [constraints \(3.2j – 3.2k\)](#). Observe that the number of variables and constraints increase by an order of magnitude as there are far more itineraries than the number of legs. This significantly increases the computational complexity of the model in addition to the detailed itinerary-based input that is required for the model. For example the Itinerary-Based Fleet Assignment Model (IFAM) takes several hours more to solve than the basic Fleet Assignment Model (FAM) because of the increase in the number of integer variables [\[BKL02\]](#). Even, IASFAM is an itinerary-based model and the solution time of this model is in the order of several hours [\[LB04\]](#).

**Non-linear dependency of demand and frequency.** In the above models we assumed that demand is independent of the frequency of the number of operations between a pair of airports. This is usually not the case. For example, some passengers prefer higher frequency of planes because they can either go on stand-by and take an earlier flight or could potentially take a later flight if they are delayed with a small extra fee. It provides passengers with a flexibility unlike the case with one single plane wherein passengers may have to wait for a whole day. For air-transportation, Teodorovic and KrmarNozic show that flight frequencies and departure times are among the most important factors that determine a passengers choice of an airline when there is high competition [TKN89]. The demand-frequency relationship is said to behave like an S-curve with high frequency markets having much higher market shares [Fru72]. Lohatepanont and Barnhart suggest one method of incorporating the changes in market demand with frequency by recapturing a certain percentage of spilled passengers based on the frequency in the market. We can extend similar techniques to AIASFAM also. A general method of capturing non-linear relationships is by first converting the non-linear curve to a piece-wise linear curve and use standard integer programming techniques to model the mapping (see chapter 10 in [BT97]).

The non-linear relationship between demand and frequency is also observed in the method in which airlines respond to their competitors wherein it is often times observed that airlines prefer to retain a certain percentage of market share in addition to goal of profit maximization. Maintaining a certain frequency in a market can be incorporated as a side-constraint to the model.

**Using predicted prices.** We mentioned in Section 2.3.1 that the strategic model could be designed to return predicted end of auction prices (Dong used machine learning techniques to return these prices [Don05]). The valuation model could use these prices to make more accurate valuations by using these predicted prices instead of ask prices for calculating the return from the unused vouchers.

**Mobility across airports.** The above model assumes that airlines can freely move operations between airports without incurring fixed costs. We know that this is not true. The fixed cost incurred by the airline can be included in the objective

of AIASFAM with a constraint that determines if there is operations in and out of an airport. But the cost of mobility can be depend on other factors also such as the return of investment by renting facilities to other airlines (or selling them) or the level of compensation from the auction and need not be always be a fixed cost.

**Enhanced balance constraints.** A additional set of constraints can be added to “enhance” the approximate count constraints. The following set of constraints can be added at some congested airports – ensuring that departures occur within acceptable departure windows, given the aircraft arrival time; and that arrivals occur within acceptable arrival windows, given the aircraft departure time. Let  $L_{a,k}^{\min}$  and  $L_{a,k}^{\max}$  refer to the earliest and latest time intervals (in the aggregated discretization) a flight of fleet type  $k$  can depart after its arrival into the airport  $a$ . So we have  $0 \leq L_{a,k}^{\min} \leq L_{a,k}^{\max} \leq |\mathbf{T}_a|$ .

*Type 1 Balance Constraints.* The total number of planes of fleet type  $k$  that arrive at an airport  $a$  at time interval  $m$  should be less than those that leave the airport within a specified departure time-window.

$$\sum_{i,s} x_{i,a,k}^{s,m} \leq \sum_{j,q} \left( \sum_{p=p_{1,a,k}^m}^{p_{2,a,k}^m} x_{a,j,k}^{p,q} + g_{a,k}^m \sum_{p=0}^{p_{3,a,k}^m} x_{a,j,k}^{p,q} \right) \quad \forall m \in \mathbf{T}_a, \forall a \in A, \forall k \in K \quad (3.4)$$

where

$$\begin{aligned} p_{1,a,k}^m &= (m + L_{a,k}^{\min}) \bmod |\mathbf{T}_a| \\ p_{2,a,k}^m &= \min\{p_{1,a,k}^m + L_{a,k}^{\max} - L_{a,k}^{\min}, |\mathbf{T}_a| - 1\} \\ p_{3,a,k}^m &= p_{1,a,k}^m + L_{a,k}^{\max} - L_{a,k}^{\min} - |\mathbf{T}_a| + 1 \\ g_{a,k}^m &= \begin{cases} 1 & \text{if } p_{1,a,k}^m + L_{a,k}^{\max} - L_{a,k}^{\min} > |\mathbf{T}_a| - 1 \\ 0 & \text{o/w} \end{cases} \end{aligned}$$

Here,  $p_{1,a,k}^m$  refers to the start time of the departure time-window.  $g_{a,k}^m$  is a binary parameter that is set to 0 if the time-window is completed the same day, in which case,  $p_{2,a,k}^m$  is set to the end of the time-window.  $g_{a,k}^m$  is set to 1 when the time-window rolls over to the next day, in which case,  $p_{2,a,k}^m$  is set to the end of the day and  $p_{3,a,k}^m$  is set to the end of the time window for the next day.

The first term in the right hand side of the constraint is time from the beginning of the departure interval to the end of the interval or the end of the day in case the interval spills over to the next day. And the second term is from the beginning of the day to the end of the time interval if the time interval spills over to the next day.

*Type 2 Balance Constraints.* The total number of planes of fleet type  $k$  that depart from an airport  $a$  at time interval  $m$  should be less than those that arrive at the airport within a specified arrival time window.

$$\sum_{j,t} x_{a,j,k}^{m,t} \leq \sum_{i,p} \left( \sum_{q=q_{2,a,k}^m}^{q_{1,a,k}^m} x_{i,a,k}^{p,q} + h_{a,k}^m \sum_{q=q_{3,a,k}^m}^{|\mathbf{T}_a|-1} x_{a,j,k}^{p,q} \right) \quad \forall m \in \mathbf{T}_a, \forall a \in A, \forall k \in K \quad (3.5)$$

where

$$\begin{aligned} q_{1,a,k}^m &= (m - L_{a,k}^{\min}) \bmod |\mathbf{T}_a| \\ q_{2,a,k}^m &= \max\{0, q_{1,a,k}^m - L_{a,k}^{\max} + L_{a,k}^{\min}\} \\ q_{3,a,k}^m &= q_{1,a,k}^m - L_{a,k}^{\max} + L_{a,k}^{\min} - |\mathbf{T}_a| \\ h_{a,k}^m &= \begin{cases} 1 & \text{if } q_{1,a,k}^m - L_{a,k}^{\max} + L_{a,k}^{\min} < 0 \\ 0 & \text{o/w} \end{cases} \end{aligned}$$

Here, the notation  $q_{1,a,k}^m, q_{2,a,k}^m, q_{3,a,k}^m, h_{a,k}^m$  have a meaning similar to  $p_{1,a,k}^m, p_{2,a,k}^m, p_{3,a,k}^m, g_{a,k}^m$  respectively but for the arrival time-window instead.

### 3.4 Experimental Results

We perform some experiments to understand: (a) the evolutions in the network of an airline that participates in a landing slot auction; (b) the tractability of AIASFAM; and (c) the effect of uncertain data on AIASFAM. We assume that the airline behaves straightforwardly (i.e., it is a utility maximizer) according to the valuation model described in [Section 3.3](#). We perform two types of experiments; we solve AIASFAM with: (1) increasing landing slot prices as in an auction and (2) varying levels of demand uncertainty.

We do not simulate an entire auction mechanism for various reasons. Firstly,

we would have to assume some behavioral model for all the other airlines. The results would be questionable if we assume that all airlines use the same valuation model. This would imply that: (a) all airlines behave straightforwardly; and more importantly, (b) that all airlines have independent private values i.e., the values of the airlines are independent of each other and that each airline's values depend on its own type which is a private information. Secondly, there are many other factors and design features of the auction that are yet to be addressed, including the method of compensation for airlines that have a heavy infrastructural investment in airports, the allocation of gates and baggage facilities along with slots; the property rights; the policies regarding the elimination of service to small communities; etc. There is no slot auction mechanism that has been designed that addresses all these factors associated with the airline industry. Thirdly, we do not have all the data from all the carriers that is needed to run an auction mechanism. Hence our experiments deal only with the straightforward behavior of a single airline that uses the valuation model.

Because we do not implement an auction mechanism, we resort to other ascending-price adjustment processes to feed the valuation model. We begin by constructing a target bundle defining the maximum number of landings for each time-window available to the airline and increase prices on all the time-windows proportional to the excess demand of the airline above the target landings for that time-window. We terminate when the price remains the same in two consecutive rounds i.e., the airline's bid does not exceed the target bundle in every time-window. We choose the target bundle in two ways. First, we select the same number of landings for all time-windows (10 for our experiments). This is representative of the fact that capacity at all times is the same. The other target bundle is chosen to lower the current (prior to the auction) operations of the carrier by 20% in each time-window. This type of bundle would be representative of a compression in the number of operations in the congested airport. We perform experiments for both types of target bundles and since the essence of the results are similar for both types of target bundles, we present the results only for the former type. We use this price-adjustment process as a representative way to observe

airline responses, although there may be several other ways of adjusting prices<sup>3</sup>. We also perform experiments on different price increment levels and we present results for a representative level. We assume the airline has no vouchers and a very high budget i.e., a quasi-linear utility for our experiments.

We use data provided by a US carrier, representing their operations for the quarter of 2004. The data consisted of operating schedule information, booking information and operating expenses for the quarter and the airline’s fleet information. Among the airports the airline was operating, we selected the airport whose slots were to be auctioned. We coded the leg-based model<sup>4</sup> in Java, used CPLEX as the optimization solver on a computer with 760MB RAM and solved all models to 0.5% relative gap.

We discretize time at different airports at different levels based on data on the levels of operations by the carrier and the congestion levels of the airports, as reported by from the Bureau of Transportation (BTS). In practice, airlines typically only engage in incremental schedule design; i.e., airlines do not build new schedules each planning period from scratch because building such a schedule requires data that the airlines might not have; requires changes to the current schedule that are likely to be impossible to implement; and is a computationally challenging (if not impossible) problem to solve. Hence, in our experiments, we construct the set of possible flight legs in the schedule generating only leg-based variables for all the legs in the operating schedule for that quarter and a few more for which we have demand information. We use pro-rated leg based fares that are obtained from average itinerary-based fares obtained from the booking data. We obtain the average demand for each itinerary from the itinerary-based booking information. We, first, unconstrain this data by multiplying the demand information for each itinerary by a constant, 1.2. We then obtain the unconstrained leg-based demand by summing all the itinerary-based demands for a leg. For the approximate count constraints by fleet type, we use the average number of flying hours per day by fleet for that quarter.

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<sup>3</sup>For example, we could increase prices based on historic congestion levels during the different time-windows.

<sup>4</sup>We attempted using an itinerary-based model instead. It had an order of magnitude more variables and constraints and did not converge in reasonable time in the computer we used. We did not implement the other extensions that we discussed in [Section 3.3.1](#).



For the passenger recapture constraints in our model, we divide the day into morning, afternoon, evening and night and allow 100% recapture of passengers within each part of the day. One could use a more sophisticated method of recapture but again this is at the risk of specifying a quantification for the percentage of recapture levels between different leg pairs. The accuracy of such predictions are questionable for long durations and are, in fact, hard even for short durations.

Finally, the model is driven purely by profits as described in [Section 3.3](#).

**Price-based experiments:** In each round in these experiments, we fed CPLEX the optimal schedule at previous prices as a starting solution for the current prices. The average running time of AIASFAM was 17.22 minutes. Relative to detailed models that can take several hours<sup>5</sup> to solve, this is fast and allows airlines to identify quickly the bundles on which they wish to bid.

In [figures \(3-3 – 3-5\)](#), we show the airline network obtained from solving the leg-based model AIASFAM at three different prices. In these networks, the nodes of the network represent the airports. An edge, along with its thickness, represents the number of planes that connect the cities in one direction. For the purpose of clarity, the arrows on the arcs have been removed. The color of an arc represents the fleet type. Note that the largest planes are colored in red. The auction is conducted at airport 48. It is the smaller second hub of the airline as seen in the figure.

In [Fig. 3-3](#), we depict the network at zero prices, round 0. With a small increase in price, round 5, the network has changed to that shown in [Fig. 3-4](#). Even with a small increase in prices, the frequency into 48 drops by 34% (from 150 landings to just 99 landings). The average number of seats per plane landing at 48 increased by approximately 17% (from 142 seats to 166 seats) indicating an up-gauging of planes into 48. [Fig. 3-5](#) shows the network at termination when the bid bundle is smaller than the target bundle. This is round 31. Observe that there continues to be a drop in frequency (to 45 landings i.e., down by 70%) and up-gauging at 48 (to 189 seats

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<sup>5</sup>IASFAM was reported to solve in 15-20 hours on a 2GB RAM machine running CPLEX 6.5 by Lohatepanont and Barnhart [[LB04](#)]. Although, IASFAM is an itinerary-based model with many more variables than a leg-based model (like AIASFAM in our experiments) which is an added reason why it takes much longer to solve compared to AIASFAM.

on average i.e., up by 33%). Even though the net revenue due to fares of passengers entering 48 drop by 10% and 20% in rounds 5 and 31 respectively, the fares per plane increase by 36% and 163% respectively. In all, the airline is eliminating markets that are no longer profitable and reducing frequency and up-gauging simultaneously in the profitable markets connecting 48. The reverse phenomena happens in some of the markets not connecting 48 (e.g., 71-27,71-22,49-13,49-96). The reason is that the airline is trying to capture other profitable markets with now, available the smaller planes.

Note that even though such high level of frequency changes are observed for a single airline, the auction by itself does not reduce frequency into 48 which maybe of concern for reasons such as reduced connections and tourism affecting the people in 48. This is because an auction has several participating airlines and it raises prices only till the point there is a unique buyer for all the capacity is auctioned. Unlike this, our experiments increased prices irrespective of other airline preferences for just a single airline till the bid is lower than the target bundle.

Notice that some new airports are now served by the airline (e.g., 14,16,70,72). We also observe that every airport in the network originally continues to have some level of service provided by the airline. This is probably because of the network structure of this airline; this need not be true in general. In fact, when unprofitable markets get eliminated, one would expect a loss in connectivity to some airports. In fact, the elimination of small communities is one of the main concerns of an auction mechanism unless the right incentives are provided. Here we observe that even though direct services in markets connecting 48 get reduced, connectivity is not lost – possibly because the prices are not high enough. Even smaller cities are still connected to 48 by at least a 2 leg connection either via 71 or via 49. This phenomenon can be attributed to the double hub network structure of this airline.

The last interesting phenomenon we observe is the development of a new hub, that is, 49. To manage demand, as you might expect, operations have shifted from the inexpensive hub to another airport. Congestion control at one airport can increase the number of operations at other airports. This enables a more uniform utilization

of resources, with the airline tapping into un-used capacity in the system. Care must be taken, however, not to increase operations to the extent that congestion is created at other airports that do not employ an auction or other congestion control schemes. It is to be noted that the assumption underlying this experiment is that airlines can freely move operations between airports without incurring fixed costs. We know that this is not true and is one issue in the slot auction design that needs to be resolved. This simplifying assumption, however, does provide insights regarding the evolutionary trends that one should expect if auctions are employed.

In Fig. 3-6, we show the increase in the number of passengers per arrival at 48 for subsequent rounds in the price adjustment process. The result is an improvement in runway utilization of 33%.

*Plane Time Count Constraint:* In the experiments, for the time count constraint, we use the average number of flying hours per day by fleet for the quarter. To show that this constraint is effective in covering the operations, we obtain the actual plane count of every aggregate solution to the AIASFAM and compare it against the size of the fleet. But in order to obtain the actual plane count, the aggregated solution has to be mapped onto the minute-by-minute schedule. We randomly assign a plane to any leg in minute-by-minute schedule that is encompassed with the aggregate variable. We then count the planes on the ground and in air, at the same time, to obtain the actual count. We average over 20 such random assignments and compare against the actual count. We observe that on average 0.8% fewer flights were used, with certain fleet types used more extensively than others. The former is good indicator that all the operations can be covered by planes. The latter result is because some of the fleet are not operationally as profitable as the others. To limit the use of profitable fleet, the flying hours can be used as a tuning parameter.

**Demand-based experiments:** We perform demand-based experiments to understand the effect of the accuracy of demand forecasts on the airline's bidding decisions. To understand this, we first fix the set of prices on the landings in each time-window. We present results for termination prices at an uncertainty level of 0% (round 31). We then randomly fluctuate the unconstrained itinerary-based demand by  $\pm x\%$ , where  $x$

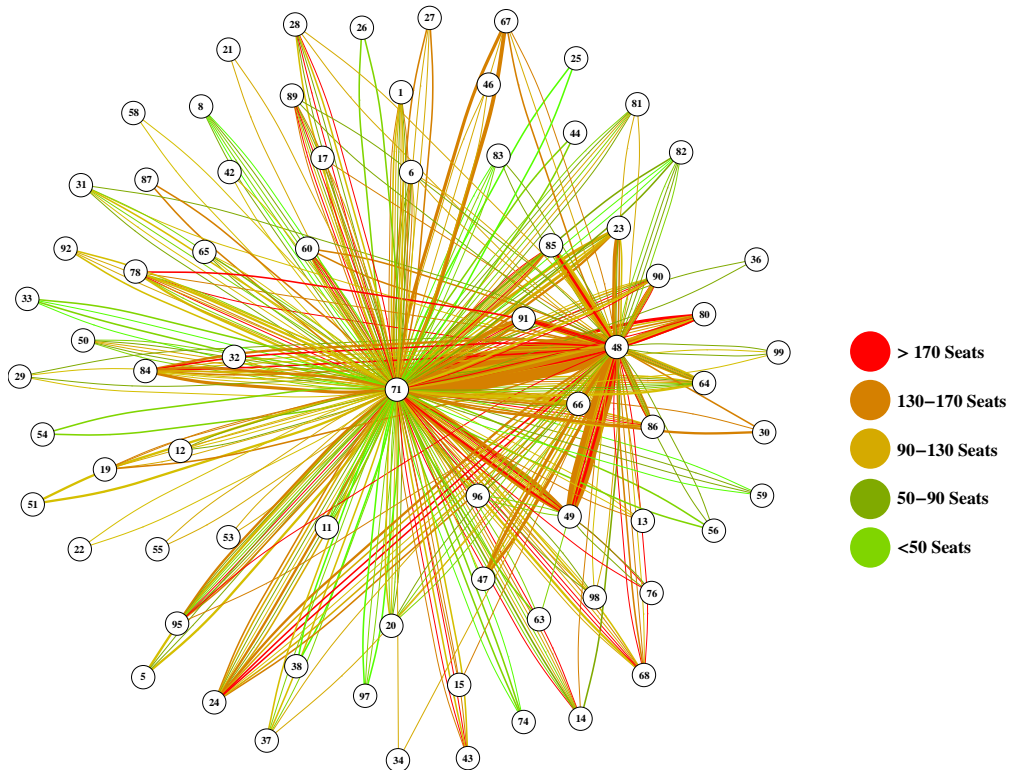


Figure 3-3: Auction conducted at node 48. Airline network at zero prices (round 0)

is the parameter determining the level of uncertainty. We present results for the average bundle and utility, i.e., profit or objective of the optimal solution to the model, taken over 20 instances. In Fig. 3-7, we show that the bids for different levels of demand uncertainty. They are less than 2 landings apart for any time interval and a 1 landing in total across all time intervals which is a very narrow range (less than 2% difference in total bundle size). We also observe that the utility for the different uncertainty levels is within 0.5% of the zero uncertainty level, even for up to 25% uncertainty in demand. Because the model is an aggregate model, the inaccuracies in the data did not affect dramatically the bids or the associated revenues and thus, the long-term decisions of the airline are somewhat insensitive to these fluctuations.

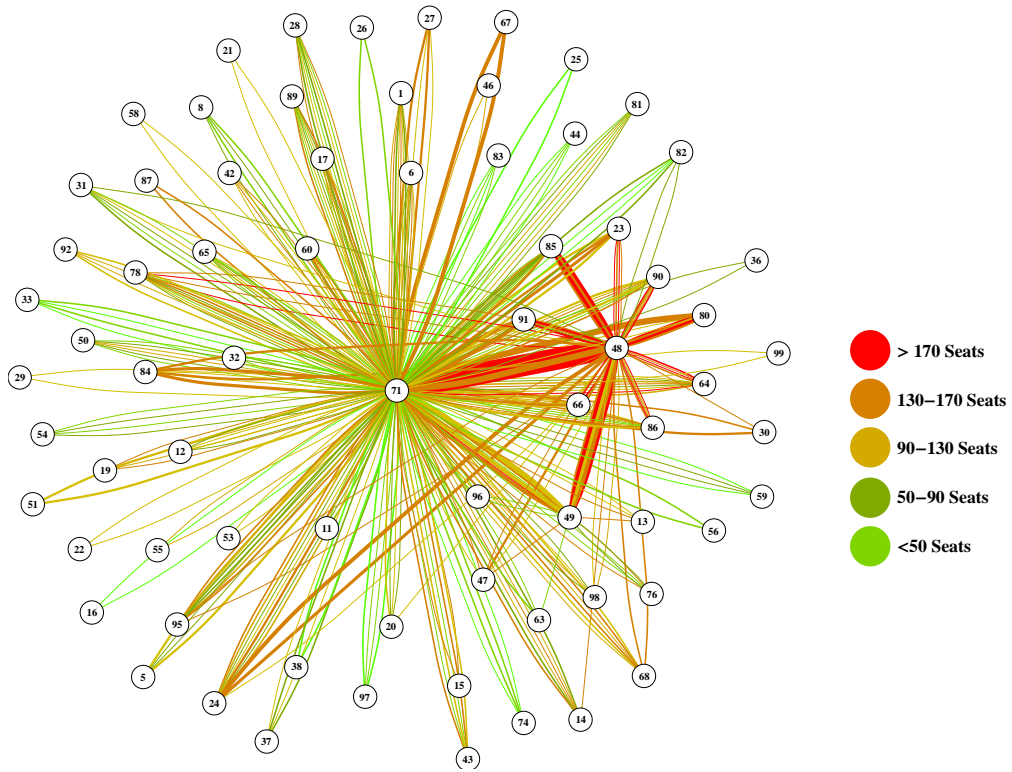


Figure 3-4: Auction conducted at node 48. Airline network at small prices (round 5)

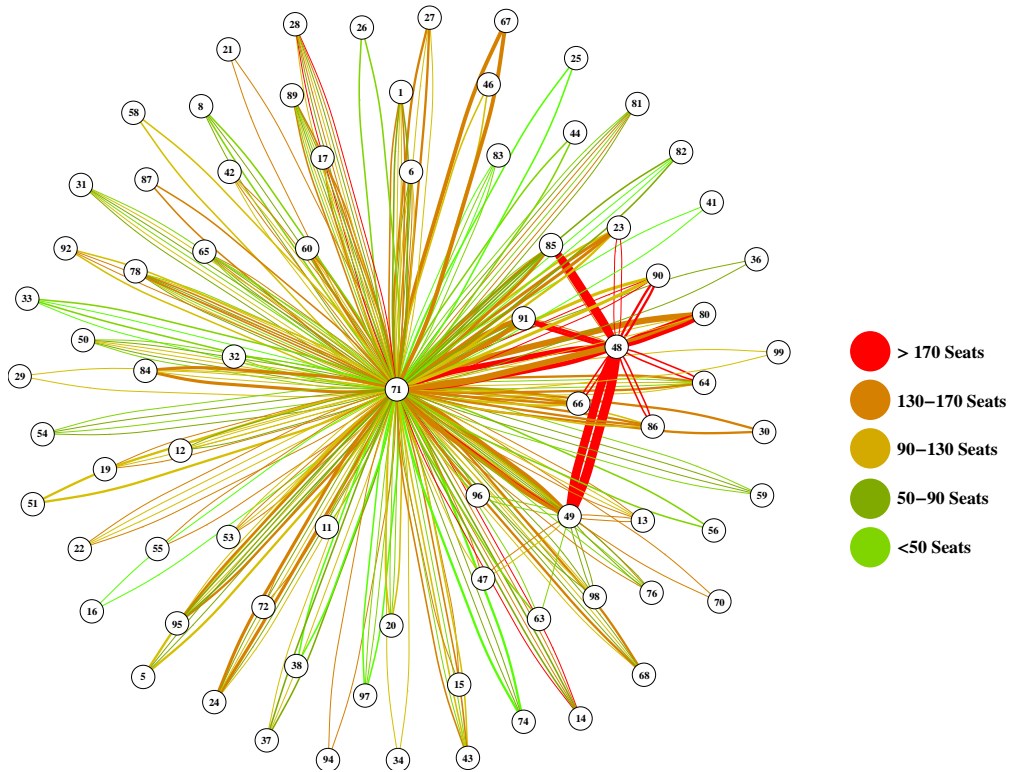


Figure 3-5: Auction conducted at node 48. Airline network at termination (round 31)

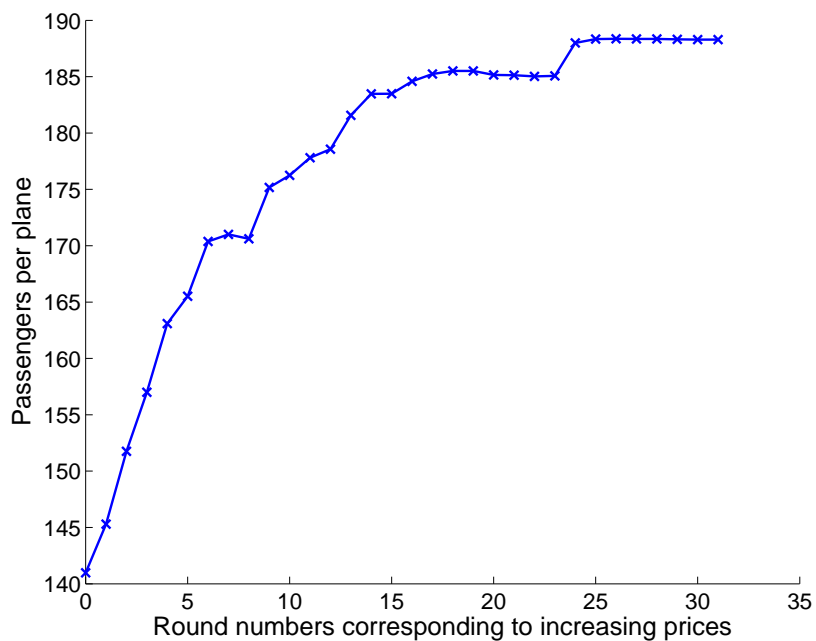


Figure 3-6: Number of passengers landing per plane with increasing price at airport 48.

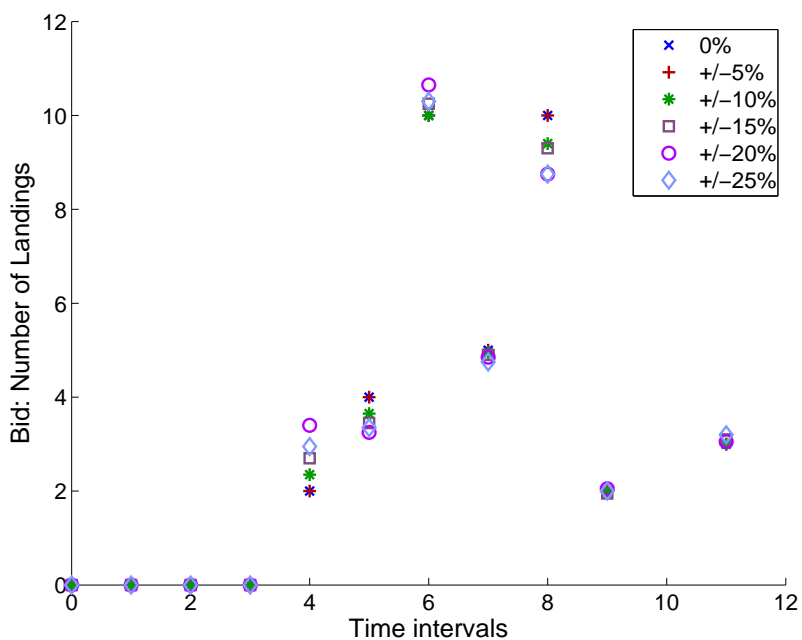


Figure 3-7: Number of landings per time-window for varying levels of demand uncertainty.

# Chapter 4

## Activity Rules for Iterative Combinatorial Auctions

Activity rules are used in auctions to suppress strategic behavior by bidders and promote consistent bidding. These rules find application in real world scenarios, for example in spectrum auctions, the airline landing slot auctions being considered by the FAA, and in procurement auctions. One aspect of practical market environments that we argue should be considered in activity rule design is the presence of *budget-constrained* bidders. The existence of such bidders is important, for example, in the FAA setting in which airlines are bidders. In this chapter, we are interested in the design of activity rules for iterative combinatorial auctions, in general, and the slot auctions with budget-constrained airlines, in particular.

### 4.1 Introduction

Combinatorial auctions provide a means of auctioning several related items, allowing bidders to place bids on packages of items rather than individual items. They are used in scenarios such as truckload transportation, bus routes, industrial procurement, and allocation of the radio spectrum, and have been proposed for airport arrival and departure slots [CSS06]. Among combinatorial auctions, iterative and especially *ascending price* auctions are more widely used than their sealed-bid counterparts, due

to the feedback and price discovery that they allow [Par06].

In high-stakes scenarios, such as auctions for the allocation of wireless spectrum [Cra95] or airport landing slots [BDH06], strategic behavior on part of the bidders can lead to large inefficiency. For example, bidders could underbid in the initial phase of the auction with a view to sniping at the end of the auction, which leads to poor price discovery and inefficient outcomes [ACM06, Mil00]. This necessitates the use of *activity rules*.

The importance of activity rules in suppressing insincere bidding and eliminating them in iterative auctions is well studied [Mil00, Wil01, ACM06]. Activity rules help in increasing the pace of the auction and the information available to the bidders during an auction. The idea is to reduce the strategic space as much as possible while still allowing for straightforward price and demand discovery. When coupled with careful design of pricing rules (e.g., via termination in the core [AM02, PU00a] or iterative Vickrey auctions [Aus04, MP07, VSV07]) activity rules help in achieving the efficient outcome with good revenue properties, and otherwise provide for straightforward demand revelation, wherein bidders must reveal demand in a way that can be rationalized by some single, fixed utility function.

One popular activity rule requires that the quantity demanded by a bidder be monotonically non-increasing as prices increase. However, it is known that when generalized to settings with heterogeneous items this rule – now called the *aggregate monotonicity rule* – permits some undesirable strategic behaviors. Ausubel, Cramton, and Milgrom [ACM06] proposed the *revealed preference activity rule (RPAR)* as an alternative. RPAR is the same as the monotonicity rule when the items are identical and with linear pricing across the items. But RPAR extends the monotonicity rule to the heterogeneous items scenario and has been suggested in several practical scenarios, including for use in the upcoming UK spectrum auction [Cra07]. RPAR is also advocated in the context of proposed auction designs for takeoff and landing slots at New York airports [BAB<sup>+</sup>07].

Many current day markets such as the cellular [Cra95] and airline [ATA07] industries involve budget-constrained bidders. Budget-constrained bidders have valuations



for resources that are greater than their ability to pay, for instance due to liquidity or credit problems. Budget-constrained bidders that bid straightforwardly may fail to satisfy RPAR and this forces budget-constrained bidders to behave strategically in order to satisfy the activity rule, running counter to the goals of activity rules. Surprisingly, we show that RPAR does not guarantee straightforward bidding even for bidders without budget constraints. We give an example of an auction where the bidders bid inconsistently, yet satisfy RPAR. This demonstrates the possibility of preventable manipulative behavior.

Both these drawbacks of RPAR illuminate why the design of activity rules needs to be revisited. We develop activity rules for a broadly applicable family of ascending price auctions, allowing for a variety of different prices including non-linear and non-anonymous prices. In the auctions that we consider, the auctioneer reports prices to bidders in each round and bidders respond with a demand set that defines a package of items. Our activity rules also extend to auctions in which (a) the bidder reports multiple packages of items, across which she is indifferent, in each round, and (b) in which the prices are not necessarily ascending. Given this, our results can find application to many auctions. Possible applications include the combinatorial clock auction [PRRS03], the clinching auction [Aus04, Aus06], RAD [KLPD05], *i*Bundle and ascending-proxy [PU00a, AM02], the clock-proxy auction [ACM06], *i*BEA [MP07], AkBA [WW00], and dVSV [VSV07].

The role of an activity rule in auctions, loosely speaking, is to preclude strategic behavior while allowing only straightforward behavior. A straightforward bidder reports in each round the package(s) of items that maximize her utility given the current prices. For a budget constrained bidder this requires not bidding more than her budget. We define a *strong activity rule* to be the “best possible” activity rule in the sense that it prevents all *preventable* strategic behavior without precluding straightforward behavior. We formulate the set of necessary and sufficient conditions for a rule to exactly characterize the set of straightforward bidding strategies, which depends in turn on the class of utility functions. Strong rules still allow for price discovery because a bidder is still guided in terms of the packages on which to bid

by the price trajectory in the auction. For this reason, iterative auctions with strong rules continue to have the advantage over sealed-bid auctions of facilitating preference elicitation in complex problem domains. Strong activity rules are particularly interesting because they minimize the strategic behaviors available to a bidder as much as possible while still allowing for pure demand revelation.

There are a variety of reasons for why a designer might in fact prefer to provide relaxed rules; e.g., for reasons of the complexity of the rules themselves, to allow for some mistakes for bidders, and to allow for some value interdependency and learning by bidders. For this purpose, it seems important to first understand the strong rule and then relax away from this rule. For example, relaxing away from RPAR in the context of budget-constrained bidders may still not provide for a set of constraints that are necessary (and thus not too strong) for budget-constrained bidders. Relaxing away from our strong activity rule will certainly allow for straightforward bidding behavior but will, in addition, permit some other behaviors. RPAR is in fact one such relaxation of a strong rule for quasi-linear bidders.

The requirements of a strong activity rule are formulated here as the feasibility problem of a linear program. Thus it suffices for the auctioneer to merely check feasibility of this optimization problem at the end of every round to verify that bidders satisfy the activity rule. We first derive a strong activity rule for budget-constrained bidders and suggest ways of relaxing the rule. As a special case, we then derive a strong activity rule for bidders without budget constraints that provides the appropriate strengthening of RPAR. We view these rules as a contribution to both the theory and practice of ascending auction design, wherein we also note that it is an easy matter to provide feedback to guide a bidder in meeting the rule, both in terms of the commitments that a bidder is (implicitly) making about her utility function through her bids, and also to guide a bidder in modifying bids in order to pass the rule.

In strengthening RPAR, we observe an interesting dilemma: one *cannot* have a rule that is *simultaneously* strong for both budget-constrained and quasi-linear bidders, and the choice of rule depends, to some extent, on the bid taker's beliefs about

the kind of utility functions of participants in the auction. Indeed, the aforementioned strong rule for budget-constrained bidders allows, but does not require, straightforward bidding for bidders without budget constraints. On the other hand, the strong rule for bidders without budget constraints does not allow straightforward bidding, and is too strong, for budget-constrained bidders.

In experimental simulations, we compare RPAR against our strong activity rule for budget-constrained bidders in the *clock-proxy auction* [ACM06]. This auction is advocated for practical settings such as the FCC wireless spectrum auctions, and consists of an ascending-price combinatorial clock auction phase that terminates with a “last-and-final” round in which bidders submit additional bids before the auction transitions to a sealed-bid (proxy) auction phase. Given our focus on issues related to the activity rule, we assume for the purpose of our simulations that bidders try to bid straightforwardly, and adopt optimization techniques to modify these bids as necessary when this behavior is blocked by the RPAR rule. This is what we refer to as *maximally straightforward* bidding.

In our experiments, the strong activity rule outperforms RPAR with respect to efficiency and revenue by 3.8% and 9.4% respectively (on average across the different distributions) at low budgets, with benefits falling off as budgets are increased. For certain distributions, we observe efficient and revenue improvements as high as 13.2% and 20.3% respectively, while for other distributions the improvements were not statistically significant.

**Organization of the chapter.** The rest of the chapter is organized as follows. In [Section 4.2](#), we describe the notation and give some preliminaries, and define the notion of a strong activity rule. In [Section 4.3](#), we discuss existing activity rules and RPAR in particular, describe some of their features and illustrate some key properties that they fail to achieve. We develop the strong activity rules in [Section 4.4](#) and compare them with other rules, providing also a discussion about extensions and modifications. We provide the details of our experimental set-up and results in [Section 4.5](#).

### 4.1.1 Related Work

In recent years more attention has been given to the possibility of budget-constrained bidders, largely because of the many opportunities to use auctions in practical settings. One such setting is that of airport slot allocation discussed in detail in [Chapter 2](#). As we had pointed earlier, the FAA, for the purpose of reducing delays and maximizing passenger throughput, is considering the use of the landing slot auctions as one of the options at congested airports like in the three major airports in New York [[Pet08](#), [DOT08](#), [FAA07](#)]. In [Section 2.3](#), we explain why treating airlines as budget-constrained bidders in landing slot auctions would be appropriate. Another example in which budget constraints play a major role is that of wireless spectrum auctions [[Sal97](#)]. Cramton states that it is realistic to assume that *all* firms participating in these auctions face budget constraints [[Cra95](#)]. Bidders must raise funds before the auction starts, a time-consuming and costly process that arguably leads to budget constraints.

In a more general context, Pitchik explains that capital market imperfections limit buyers' abilities to borrow against future income when investments are large [[Pit06](#)]. Che and Gale note that budget constraints also result from a problem of moral hazard; many organizations delegate acquisition decisions to purchasing units, while imposing budgets to constrain their spending [[CG98](#)]. This is observed even for low-valued goods, such as in the domain of sponsored search, in which advertisers can place caps on the amount they are willing to spend on Internet advertisements over a day.

Theoretical models for firms with budget constraints allow for both hard and soft (i.e., flexible) budget constraints [[KMR03](#)]. Hard budgets are those that cannot be exceeded while soft budget constraints are those that can be exceeded under certain circumstances. It is easy to model bidders without budget constraints as bidders with sufficiently high budgets. Auctions with budget constraints have been discussed in many works [[LR96](#), [CG98](#), [CG00](#), [Mas00](#), [BK01](#), [Pit06](#)]. However, none of this literature discusses activity rules, and every paper is restricted to domains with at most two items for sale. The focus is instead on equilibrium behavior. In the context

of combinatorial auctions, an impossibility result exists for truthful, deterministic and efficient multi-unit auctions in the presence of budget-constrained bidders [BCI<sup>+</sup>05]. Ausubel and Milgrom discuss a generalization of ascending-proxy auctions to allow for budget-constrained bidders, but remain in the context of proxied (i.e. sealed-bid) auctions [AM06].

The sufficiency conditions in the definition of a strong activity rule are a generalization of what is known as *rationalizability* (or *consistency*) in microeconomics. Afriat developed conditions for rationalizability for a concave utility function where the utility of a package of goods does not depend on the price as long as the package is affordable [Afr67].<sup>1</sup> Whereas agents in Afriat’s model are indifferent to the price as long as the total expenditure remains within their budget constraint, in our models bidders always prefer to spend less than more. Rochet [Roc87] has earlier provided sufficient conditions for rationalizability in the context of quasi-linear utility functions, and independently of our work Vohra [Voh07] obtained in the restricted setting of quasi-linear utility functions the same strengthening of these conditions as ours, making the conditions both necessary and sufficient. Along with a detailed study in the context of auction design, including an observation about the insufficiency of the often suggested RPAR rule, we develop strong rationalizability conditions (i.e., necessary and sufficient) in the context of budget-constrained bidders.

Activity rules have been discussed in a number of places in the auction literature. The activity rule used in the FCC auction, due to Milgrom and Wilson [Mil00], is a variation on an aggregate monotonicity rule in which quantities that are bid in the auction are required to weakly monotonically decrease across rounds. Similar rules have become standard in combinatorial auctions. For instance, in the combinatorial clock auction [PRRS03], a variation on the aggregate monotonicity rule is adopted. In the context of iterative, two-sided markets, Wilson [Wil01] describes detailed activity rules for an auction for electrical power generation, including a bid withdrawal and

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<sup>1</sup>Afriat’s Theorem can be viewed as the sufficiency condition for the specific utility function of the form  $\max_x \{v(x) | p \cdot x \leq B\}$  where  $v(x)$  represents the values of package  $x$  with linear price vector  $p$  and budget  $B$ . In particular, it can be shown that the Generalized Axiom of Revealed Preference (GARP) is equivalent to the sufficient condition for the above mentioned utility class [Var06]. Although presented for linear prices, it is immediate to generalize these methods to non-linear prices.

a revision rule. Mishra and Parkes study a special class of ascending price auctions with quasi-linear bidders and provide strong activity rules [MP07]. To the best of our knowledge, the only previous paper on the role of activity rules in the presence of budget constraints is by Day [Day07]. Day provides a rule that extends RPAR, in the sense that if there are no budget constraints then it is equivalent to RPAR. For this reason, the rule is not sufficient to prevent inconsistent bidding— and thus not strong—in the presence of non-budget constrained bidders.

An alternative to imposing constraints on bids placed across rounds is provided by *incremental, direct-revelation* mechanisms, in which a proxy agent is situated between each bidder and the auctioneer and receives explicit, partial information about a bidder’s utility function that is refined in response to price information, or other requests, from the auctioneer [PU00b]. Because the proxy agents have direct information, for instance constraints on a space of feasible utility functions, a strong activity rule is achieved by requiring tightening but not relaxing of constraints during the course of the auction; see also Lubin et al. [LJC+08].

## 4.2 Preliminaries

Let  $\mathcal{G}$  denote the set of items in an auction and let  $\mathcal{I}$  denote the set of bidders. We assume a private values model, with  $v_i(S) \geq 0$  denoting the *value* of bidder  $i$  in  $\mathcal{I}$  for bundle  $S \subseteq \mathcal{G}$ . We use package and bundle interchangeably in the chapter. We normalize  $v_i(\emptyset) = 0$  and assume that the auctioneer is indifferent across all allocations. We assume free-disposal, i.e.,  $v_i(T) \leq v_i(S)$  for all  $T \subseteq S$ , unless it is specifically mentioned.

Let  $p_i(S) \geq 0$  be the price the auctioneer sets for a bundle  $S$  for a bidder  $i$ . Prices may depend on the bidder if the prices are non-anonymous in nature. Prices are said to be *linear* if  $p_i(S) = \sum_{g \in S} p_i(\{g\})$  and *nonlinear* otherwise (i.e., if  $p_i(S) \neq p_i(S_1) + p_i(S_2)$  for some  $S = S_1 \cup S_2$  and  $S_1 \cap S_2 = \emptyset$ ). We will often drop the subscript  $i$  in our notation, because the context of the bidder is generally clear.

Bidders are modeled as utility-maximizing, and with a utility function that belongs

to a utility class  $\Theta$ . An instance  $\theta \in \Theta$  captures the set of all parameters that define the utility function  $u(S, p)$  for a bidder on bundle  $S$  at prices  $p$ . We consider two models of utility functions:

**Quasi-Linear** The utility of a bidder for bundle  $S$  at price  $p$  is given by  $u(S, p) = v(S) - p(S)$ , for some valuation function  $v : 2^{\mathcal{G}} \rightarrow \mathbb{R}$ . So, a quasi-linear bidder with valuation function  $v$  has type  $\theta = v$ .

**Budget-Constrained** We consider a model where a bidder has a hard budget constraint and a quasi-linear utility up to the budget. The utility function then has the form:

$$u(S, p) = \begin{cases} v(S) - p(S) & \text{if } p(S) \leq B \\ -\infty & \text{otherwise,} \end{cases} \quad (4.1)$$

where  $B$  is the bidder's budget. So, a budget-constrained bidder with valuation  $v$  and budget  $B$  has type  $\theta = (v, B)$ . We call this the *budget-constrained utility function*.

An *ascending price auction* is an auction with a single price path that is non-decreasing such that it ends with an allocation and payment for bidders [Cra98, Par06, MP07]. We focus in particular on the ascending price *demand-revealing* processes. The auctioneer specifies prices in each round and each bidder responds with a report about a package of items that maximizes her utility given the current prices. Prices are incremented from round to round by the auctioneer. The auction continues until a termination condition is met.

At the end of every round, we associate with each bidder a set of *price-bid pairs*,  $(p, S)$ , where  $p : 2^{\mathcal{G}} \rightarrow \mathbb{R}$  and  $S \subseteq \mathcal{G}$ , also known as the *history* of the bidder. History  $h$  of a bidder is the set  $h = \{(p^1, S^1), \dots, (p^t, S^t)\}$ , where  $S^i$  is the bid placed by the bidder when the prices are  $p^i$  and  $t$  is the cumulative number of bids placed by the bidder in all the rounds of the auction including the current round. Note that this may be more than the number of rounds, for instance in the case in which multiple bundles receive bids in a single round. Let  $H$  denote the set of histories.

An *activity rule* is any rule to check whether the history of each bidder satisfies some set of constraints. More formally, an activity rule,  $A$ , is specified by a function  $f_A : H \rightarrow \{0, 1\}$  where history  $h \in H$  satisfies the rule if  $f_A(h) = 1$  and violates the activity rule if  $f_A(h) = 0$ . If a bidder does not satisfy the activity rule in some round then some penalty is imposed, for instance, the bidder is precluded from bidding further or a default bid is submitted.

At a given price, a bidder *bids straightforwardly* if the bundle(s) she bids on are utility maximizing with respect to some utility function. Such a bidder need not be truthful and may bid straightforwardly with respect to some non-truthful utility function.

In the simple case, we consider auction processes in which bidders bid for a single package of items in each round. A simple generalization allows for auctions in which bidders respond with a set of packages across which they are indifferent and easily extends to non-ascending price processes also. We can also relax the assumption about bidder behavior and allow for approximate best-response of a bidder.

Let  $h_{\text{bundles}} \in SB(h_{\text{prices}}, \theta)$  denote a history  $h = (h_{\text{bundles}}, h_{\text{prices}})$  for a straightforward bidder that adopts utility function  $\theta$ , where  $h_{\text{prices}}$  and  $h_{\text{bundles}}$  refer to the price and bundle components of the history. A bidder is *consistent* given history  $h$  if there exists a utility function  $\theta \in \Theta$  such that the observed history  $h = (h_{\text{bundles}}, h_{\text{prices}})$  satisfies  $h_{\text{bundles}} \in SB(h_{\text{prices}}, \theta)$ . Consistency requires that there exists a single utility function that explains the bids of the bidder, under straightforward behavior. A bidder who is not consistent is called an *inconsistent bidder*. Note that a consistent bidder need not be truthful, but just follow a straightforward bidding strategy with respect to some utility function.

### 4.3 Existing Activity Rules

Perhaps the most commonly used activity rule requires *monotonicity in quantity*: as price increases the total quantity of items bid by each bidder has to decrease. We refer to this rule as the *Monotonicity Rule (MR)*. Defined in a setting with homogeneous



items, it is easy to see that a truthful bidder with quasi-linear utility will satisfy MR. Indeed, we will soon see that MR is a strong activity rule for homogeneous items and linear prices. On the other hand, when coupled with non-linear prices, a truthful bidder need not satisfy MR and thus the rule is too strong for such bidders. This is illustrated with the following simple example.

**Example 4.3.1.** *Consider a bidder, with values 12 and 15 for one unit and two units of the good. Suppose the prices for the two different bundles in round 1 are 5 and 9, and in round 2 are 8 and 10 respectively. Note that the prices are chosen to allow for volume discounts for the substitutable items. At these prices the bidder, when bidding straightforwardly with respect to her true values, demands one unit in round 1 and two units in round 2. Thus a truthful bidder would violate the Monotonic Rule and so be unable to express her true demand.*

This problem with MR is well understood and is one reason for the proposal to adopt RPAR. The problem continues even in a setting with heterogeneous goods, such as in spectrum auctions where the commodities are spectrum licenses, and the differences in the geographical location and bandwidth size make them a heterogeneous set of items. The appropriate form of monotonicity in this setting is *monotonicity of aggregate quantity*, with the rule referred to as *Aggregate Monotonicity Rule (AMR)*. The following simple example, this time with linear prices, shows that this AMR rule is too strong for bidders with quasi-linear utility functions.

**Example 4.3.2.** *Consider a bidder, who values a bundle with two units of item A at 10 and a bundle with one unit of item B at 15. Suppose the prices are linear and the price of one unit of A and one unit of B in round 1 are 2 and 8 respectively and in round 2 are 3 and 12 respectively. At these prices, the bidder, when bidding straightforwardly with respect to her true values, demands the bundle with one unit of item B in round 1 and two units of item A in round 2. Thus, a truthful bidder would violate the Aggregate Monotonicity Rule and be unable to express her truthful demand.*

But other pernicious problems exist with AMR, for example Ausubel, Cramton

and Milgrom [ACM06] observe that a “parking” strategy is popular in FCC auctions that use AMR. In this strategy, bidders bid on many units of cheap licenses before revealing their true demand at the end of the auction. In the FCC auction this is mitigated somewhat by defining the quantity in AMR in terms of MHzPOPs (i.e. bandwidth times population in the wireless region). This is a proxy for value, not just raw quantity, and prevents some extreme forms of strategic behavior. Thus, not only is AMR too strong in the sense of the earlier example but it is not strong enough in preventing some forms of preventable behavior.

RPAR was proposed by Ausubel, Cramton and Milgrom [ACM06] in response to these difficulties with AMR. But, we show that RPAR imposes constraints on bidder behavior that are too weak to ensure consistency with quasi-linear utility functions, while being too strong for budget-constrained bidders. Both observations appear to be novel to this thesis. A bundle,  $S$ , is represented by a vector  $s$  which specifies the quantity of each item and  $p$  is a price vector, defining a price on each of the items. Then the price of bundle  $s$  at price  $p$  is  $p \cdot s$ , where  $a \cdot b$  represents the inner product between two vectors. The activity rule RPAR works as follows. At every round  $t$  of an auction, RPAR checks that the bid  $s^t$  of a bidder at price vector  $p^t$  satisfies:

$$(p^t - p^r) \cdot (s^t - s^r) \leq 0, \tag{4.2}$$

for any bid  $s^r$  placed in an earlier round  $r$  by the same bidder at price  $p^r$  in the auction. In an auction with non-linear prices, RPAR can be easily extended to require:

$$p^t(s^t) - p^r(s^t) - p^t(s^r) + p^r(s^r) \leq 0, \tag{4.3}$$

which we refer to as the generalized form of RPAR.

**Claim 4.3.3.** [ACM06] *A truthful bidder with quasi-linear utility satisfies RPAR.*

In other words, RPAR does not preclude straightforward behavior of a quasi-linear bidder. This holds irrespective of whether the auction is defined for linear or non-linear prices and resolves the problems demonstrated with MR and AMR in the

earlier examples. Moreover, RPAR is in another way stronger than AMR, in that it precludes these parking-style manipulations [ACM06]. However, we observe two problems with RPAR.

**Claim 4.3.4.** *RPAR allows inconsistent bidders with quasi-linear utility functions and linear prices.*

*Proof.* To prove this claim, we present a counterexample in which a bidder satisfies RPAR but there exists no valuation function  $v$  consistent with her bids. Consider the case when multiple units of two distinct items are being auctioned. Consider a bidder who bids the following bundles,  $s^t$ , with respect to prices  $p^t$ : bundles (2,0),(1,2) and (0,2) at prices (0,0), (1,0) and (1,1) in rounds 1,2 and 3 respectively. Let us first check that the activity rules are satisfied:

$$\begin{aligned} (p^2 - p^1) \cdot (s^2 - s^1) &= (1, 0) \cdot (-1, 2) = -1 \leq 0 \\ (p^3 - p^1) \cdot (s^3 - s^1) &= (1, 1) \cdot (-2, 2) = 0 \leq 0 \\ (p^3 - p^2) \cdot (s^3 - s^2) &= (0, 1) \cdot (-1, 0) = 0 \leq 0. \end{aligned}$$

For the other direction, assume by way of contradiction that there exists a valuation function  $v$  that is consistent with the bids. Then the following is true:

$$v(s^t) - p^t \cdot s^t \geq v(s^r) - p^r \cdot s^r \quad \text{where } r \neq t, \quad r, t = 1, 2, 3$$

Substituting the values of  $p$ 's and  $s$ 's in the above constraints we get

$$\begin{aligned} -1 &\leq v(s^3) - v(s^2) \leq -1 \\ -1 &\leq v(s^2) - v(s^1) \leq 0 \\ 0 &\leq v(s^1) - v(s^3) \leq 0. \end{aligned}$$

If we add the first and the third constraint we get  $v(s^2) - v(s^1) = 1$  which violates the second constraint. Hence, there exists no consistent valuation function  $v$  for this bidder. This establishes our claim.  $\square$

Thus, RPAR is too weak and allows for some preventable bidding strategies. The problem is that RPAR only ensures pairwise consistency between the bids and not a global consistency across all the bids. Thus, even if the bids satisfy RPAR, there might be no underlying utility function corresponding to the bids.

**Claim 4.3.5.** *A truthful budget-constrained bidder need not satisfy RPAR, even in an auction with linear prices.*

*Proof.* Consider an auction with 2 types of items and a budget-constrained bidder with budget \$5,200 and values 7500 and 3000 for the bundles (1, 0) and (0, 3) respectively. Suppose the price vector in two successive rounds of the auction were (\$5,000, \$500) and (\$5, 500, \$900). Clearly, if the bidder bid straightforwardly, she would bid bundle (1, 0) in the first round and (0, 3) in the second round. However, these bids violate RPAR since  $(5500 - 5000)(0 - 1) + (900 - 500)(3 - 0) = 700 \not\leq 0$ .  $\square$

Thus, RPAR is too strong for budget-constrained bidders and precludes desirable bidding strategies. In the presence of budget constraints, a truthful bidder with budget  $B$  places a bid  $s^r$  in round  $r$  at price  $p^r$ , if

$$v(s^r) - p^r \cdot s^r \geq v(s) - p^r \cdot s, \quad \forall s \in R = \{s | p^r \cdot s \leq B\}. \quad (4.4)$$

In a subsequent round  $t$ , the player bids  $s^t$  at price  $p^t$ , if

$$v(s^t) - p^t \cdot s^t \geq v(s) - p^t \cdot s, \quad \forall s \in T = \{s | p^t \cdot s \leq B\}. \quad (4.5)$$

We say that sets  $R$  and  $T$  represent *budget feasible sets* at prices  $p^r$  and  $p^t$  respectively. Note that because  $p^t \geq p^r$ , we have  $R \supseteq T$ . RPAR is obtained by adding the above two inequalities, with  $s = s^t$  in the first inequality and  $s = s^r$  in the second inequality. This step requires the fact that  $s^t \in R$  and  $s^r \in T$ . The former is true since  $R \supseteq T$ . However, we can have  $s^r \notin T$  because of budget constraints. A situation such as in Fig. 4-1 can happen, in which  $S$  is a strict superset of  $T$  leading to the above claim.

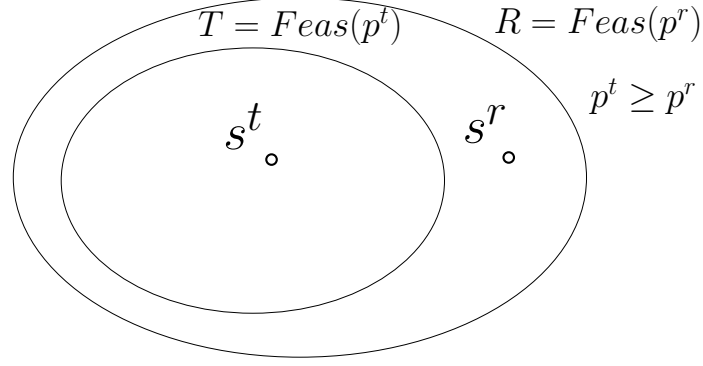


Figure 4-1: A scenario where RPAR might fail in the presence of budget constraints.

## 4.4 Strong Activity Rules

Having identified the problems with RPAR and motivated the need for activity rules we define *strong rules* that prevent all preventable strategic behaviors without precluding desirable behaviors.

**Definition 4.4.1. Strong Activity Rule:** *An activity rule  $A$  is said to be strong with respect to utility class  $\Theta$  if the following conditions are satisfied:*

**Necessary:**  $\forall \theta \in \Theta$  and  $\forall h = (h_{\text{bundles}}, h_{\text{prices}}) \in H$  such that  $h_{\text{bundles}} \in SB(h_{\text{prices}}, \theta)$ , then  $f_A(h) = 1$ ; and

**Sufficient:**  $\forall h = (h_{\text{bundles}}, h_{\text{prices}}) \in H$  such that  $f_A(h) = 1$ , there exists a  $\theta \in \Theta$  such that  $h_{\text{bundles}} \in SB(h_{\text{prices}}, \theta)$ .

Assuming a class of utility functions,  $\Theta$ , the *necessary* condition states that any consistent bidder will satisfy the constraints. The *sufficiency* condition states that if a bidder satisfies the constraints, then the bidder is consistent with respect to some utility function in that class. The constraints associated with RPAR are necessary for quasi-linear bidders but not budget-constrained bidders, while being insufficient for either. The necessary and sufficient constraints on behavior are the best we can achieve from an activity rule in the following sense: for any class of utility function  $\Theta$ , a strong activity rule allows all consistent straightforward strategies while removing all *preventable* strategic behavior.

For the most part, we limit attention to price-based demand-revealing processes that require only a single, best-response package of items to be reported in each round. We suggest a number of easy generalizations at the end of [Section 4.4.1](#). Note that we will not place any restrictions on the types of prices (linear or non-linear, including non-anonymous), or on the particular method by which prices are increased across rounds (ascending or non-ascending).

#### 4.4.1 Budget-Constrained Bidders

Consider a bidder with history  $h = \{(p^1, S^1), \dots, (p^k, S^k), \dots, (p^t, S^t)\}$ . The *Strong Revealed Preference Activity Rule* (SRPAR) defines constraints on the admissible utility function  $\theta = (v, B)$ , through constraints on variables  $\left(\{v(S^k)\}_{k=1}^t, B\right)$ :

$$\text{SRPAR: } v(S^k) - p^k(S^k) \geq v(S^l) - p^k(S^l) \quad \text{if } p^k(S^l) \leq B \quad \forall k, l \in \{1, \dots, t\} \quad (4.6a)$$

$$p^k(S^k) \leq B, \quad \forall k \in \{1, \dots, t\}. \quad (4.6b)$$

[Constraint \(4.6a\)](#) states that a bidder bids on a utility maximizing bundle  $S^k$  at price  $p^k$  among the other bids/bundles elicited by the bidder. The ‘if’ condition in [\(4.6a\)](#) is necessary to check if the bundle  $S^l$  is within budget so that a valid comparison is made. [Constraint \(4.6b\)](#) ensures that the bundle  $S^k$  is within budget.

Recall that we assumed the rules of the auction specify that only one of the maximizing bids need to be reported in each round. This is reflected in the “ $\geq$ ” inequality in [constraint \(4.6a\)](#) (as opposed to a strict “ $>$ ”) when comparing the utility of the two bundles  $S^k$  and  $S^l$ .

**Theorem 4.4.2.** *SRPAR is a strong activity rule for budget-constrained bidders, both with linear and non-linear prices.*

*Proof.* Consider a consistent, budget-constrained bidder and suppose she bids according to a utility function parameterized as  $\theta = (v_{\text{con}}, B_{\text{con}})$ . Thus, she behaves exactly

as a truthful bidder whose  $\theta = (v_{\text{con}}, B_{\text{con}})$ . That the rules will be necessarily satisfied by such a bidder is immediate, by the definition of SRPAR in [constraints \(4.6a – 4.6b\)](#). For the sufficiency direction, consider now a bidder who satisfies the activity rule SRPAR. Suppose  $(\{\tilde{v}(S^k)\}_{k=1}^t, \tilde{B})$  is a feasible solution for SRPAR. We observe that if  $(\{\tilde{v}(S^k)\}_{k=1}^t, \tilde{B})$  is a feasible solution then so is  $(\{\tilde{v}(S^k) + c\}_{k=1}^t, \tilde{B})$  for fixed number  $c$ . Hence without loss of generality, we assume that  $\tilde{v}(S^k) > 0$  for every  $k$ . Now consider a truthful bidder with budget  $\tilde{B}$  and a valuation function given as follows:

$$v(S) = \begin{cases} \tilde{v}(S^k) & \text{if } S = S^k \text{ for some } k \in \{1, \dots, t\} \\ 0 & \text{otherwise.} \end{cases} \quad (4.7)$$

This truthful bidder has the same history as the original bidder, thus proving sufficiency. Note that in this case, bidders might not satisfy free-disposal. However, we can modify the valuation function as follows so that the bidders satisfy free disposal.

$$v(S) = \max_{\{k \mid S \supseteq S^k; k=1, \dots, t\}} \{\tilde{v}(S^k), 0\}. \quad (4.8)$$

Clearly this valuation function satisfies free-disposal. Now if we show that  $v(S^k) = \tilde{v}(S^k)$  for all  $k$ , then the same proof as above proves sufficiency. Suppose this were not the case. Then it must be the case that for some  $l, k$  such that  $S^k \supset S^l$ , we have  $\tilde{v}(S^l) > \tilde{v}(S^k)$ . Due to free-disposal, the prices satisfy  $p^r(S^k) \geq p^r(S^l)$  for every round  $r$ . This means  $\tilde{v}(S^l) - p^r(S^l) > \tilde{v}(S^k) - p^r(S^k)$  for every round  $r$  and in particular when  $r = k$ , we have  $\tilde{v}(S^l) - p^k(S^l) > \tilde{v}(S^k) - p^k(S^k)$ . On the other hand, we have  $p^k(S^l) \leq p^k(S^k) \leq B$ . This contradicts that  $(\{\tilde{v}(S^k)\}_{k=1}^t, \tilde{B})$  is a feasible solution of SRPAR as it violates [constraint \(4.6a\)](#). This completes the proof.  $\square$

If a bidder is not budget-constrained and bids straightforwardly, this activity rule does not constrain the bidder. However, if such a bidder is not straightforward but instead consistent with some utility function,  $\theta = \{v, B\}$ , for some budget  $B$ , then SRPAR does not detect this behavior. This is necessarily so because SRPAR is

defined here to allow for utility functions in this larger class. For example, consider a quasi-linear bidder with values 3 and 5 for two items A and B. At prices (0,1) and (2,3), say the bidder specifies her demand as items B and A, respectively. Then the bidder passes the rule with budget 2 and values 3 and 4 for items A and B, respectively. But no straightforward bidder with quasi-linear utility could have this behavior because the prices have uniformly increased on both the items. In fact, because she is behaving consistently with respect to some utility function  $\{v, B\}$ , the activity rule cannot detect this behavior unless the auctioneer knows that the bidder is not budget-constrained.

**Implementing SRPAR.** SRPAR can be rewritten as a Mixed-Integer Program (MIP) with  $O(t^2)$  binary variables to simplify the 'if' condition in [constraint \(4.6a\)](#). But a rather simple observation about SRPAR simplifies its implementation. Consider an alternate definition in which a particular budget,  $\tilde{B}$  is imposed on the feasible space of utility functions. Given this additional constraint, SRPAR is an LP feasibility problem with at most  $O(t^2)$  constraints and  $O(t)$  variables. Let us call this LP corresponding to a specific  $\tilde{B}$  as LP- $\tilde{B}$ . In this case the strong activity rule would verify the existence of a feasible  $v(S^k) \forall k \in \{1, \dots, t\}$  by solving LP- $\tilde{B}$ . Define  $B_L = \max_{k \in \{1, \dots, t\}} \{p^k(S^k), 0\}$ .

**Observation 4.4.3.** *Consider some budget  $\tilde{B} \geq B_L$  and a corresponding feasible valuation function  $v$  for LP- $\tilde{B}$ . Then, for any  $B' \in [B_L, \tilde{B}]$ , the constraints of the LP- $B'$  are a subset of the constraints in LP- $\tilde{B}$ . So, any valuation  $v$  that is feasible for LP- $\tilde{B}$  is also feasible for LP- $B'$ .*

Therefore checking SRPAR at  $B_L$  (i.e., LP- $B_L$ ), which is a polynomial time check, is an activity rule that is sufficient (strong) for budget-constrained bidders but a relaxation of the basic SRPAR method. We make yet another simple observation about a case when SRPAR is equivalent to RPAR.

**Observation 4.4.4.** *In the limiting case, when a bidder just bids on a single bundle in all the rounds, LP- $B_L$  has no constraints to check for and SRPAR is always satisfied. In fact, the same is true about RPAR and hence they are equivalent in this*



*special case.*

**Requiring Every Best-Response Package.** Now consider the case when an auction requires that a bidder bids on every utility-maximizing bundle in each round. The additional bundles are recorded in the bidder's history with higher indices but at the same price. Then the “ $\geq$ ” inequality that compares the utilities in [constraint \(4.6a\)](#) is replaced by a strict inequality when  $p^k \neq p^l$ , and with an equality when  $p^k = p^l$  as shown below:

$$v(S^k) - p^k(S^k) > v(S^l) - p^k(S^l) \quad \text{if } p^k \neq p^l \text{ and } p^k(S^l) \leq B \quad \forall k, l \in \{1, \dots, t\}; \quad (4.9a)$$

$$v(S^k) - p^k(S^k) = v(S^l) - p^k(S^l) \quad \text{if } p^k = p^l \text{ and } p^k(S^l) \leq B \quad \forall k, l \in \{1, \dots, t\}. \quad (4.9b)$$

For computational purposes, we can relax the strict inequality with an addition of a small number  $\epsilon > 0$  to the right hand side as shown below:

$$v(S^k) - p^k(S^k) \geq v(S^l) - p^k(S^l) + \epsilon \quad \text{if } p^k \neq p^l \text{ and } p^k(S^l) \leq B \quad \forall k, l \in \{1, \dots, t\}. \quad (4.10)$$

This activity rule can be further strengthened to ensure that the bidder also respects free-disposal. Because all utility maximizing bundles are to be placed, then this also implies constraints on the utility of bundles that are not elicited. The modified activity rule is obtained by adding the following inequalities:

$$v(S^k) - p^k(S^k) \geq v(T) - p^k(T) + \epsilon, \quad \text{if } p^k(T) \leq B \quad \forall T \supset S^k, T \neq S^l, p^k(T) > 0 \quad \forall k, l \in \{1, \dots, t\} \quad (4.11a)$$

$$v(T) \geq v(S^k), \quad \forall T \supset S^k, T \neq S^l, p^k(T) > 0 \quad \forall k, l \in \{1, \dots, t\}. \quad (4.11b)$$

We introduce only as many constraints as bundles with *strictly positive price*, that is those bundles for which there is no subset with the same price. Thus, we

avoid an exponential number of constraints except when the prices, themselves, have an exponentially-sized representation.<sup>2</sup> A feasible solution to the linear feasibility problem gives a budget and possible values for all the elicited bundles and their supersets. To show the rule is sufficient, one can then set the value of all other bundles (i.e., the subsets) to zero. In [Section 4.4.3](#), we show an interesting case in which one can reduce these constraints to a simple local rule that just applies across consecutive rounds.

**Relaxed Rules.** In practice, relaxed activity rules can be important, for instance to allow for bidder mistakes and to allow bidders to adjust their values based on price feedback. On one hand, this latter value learning is already permitted within SRPAR because a strong activity rule certainly does not require that a bidder commits to a particular utility before participating. But on the other hand, additional relaxations may be useful. As a design principle, we advocate relaxing from strong rules whose properties are well understood. This will ensure that the rule that is achieved is not too strong, for example as could be the case when adopting an insufficient relaxation of RPAR in an environment with budget-constrained bidders. We suggest a rule that can be considered to be an appropriate relaxation of RPAR.

The SRPAR rule can be relaxed in different ways. One way is to allow for approximately straightforward bidding by a bidder, wherein the bidder is within some threshold  $U_{\text{error}} > 0$  of its utility-maximizing bundle in every round. In this case, we can modify SRPAR by adding an additional term  $U_{\text{error}}$  on the right hand side of the [constraint \(4.6a\)](#). Another example is to provide a bidder with an opportunity to skip the rule in one or more rounds. Another interesting relaxation can be obtained by fixing  $B$  in SRPAR to  $B_L$  and adding [constraints \(4.6a\)](#) for a pair of bundles  $l, k$  if they both exist (i.e., they both satisfy the 'if' condition at  $B_L$ ) or dropping them otherwise. The form of these constraints would appear similar to RPAR but they are imposed on a selected pair of bundles guided by  $B_L$  unlike RPAR that imposes it on all pair of bundles.

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<sup>2</sup>In technical terms we are working, here, in the so-called exclusive-or (XOR) bidding language in which the price on a bundle is the maximal price over the price quoted on all (weak) subsets of the bundle.

It would be interesting as a part of future work to study if there is a principled way of classifying strategic behavior and thus developing relaxed rules, where the auctioneer can specify the class of strategies that can be tolerated and then develop corresponding relaxed rules.

#### 4.4.2 Implicit Budget Bounds and Bidder Feedback

An important concern in regards to activity rules is that they not be viewed by participants in an auction as too complex. This is one reason to consider simplifications such as the last relaxation proposed above. One way to alleviate this concern is to generate information that can be used within a decision support or feedback tool provided for bidders to help with (a) focusing bidding on bundles that are within the bounds and observe the commitments that they are implicitly making with the bids, and (b) identifying violated constraints, if the activity rule is violated, and modifying bids to avoid violation. For instance, we present the implied bounds that can be derived on budget constraints and then discuss the relevance for bidder feedback. This kind of feedback is helpful to a bidder, not because a bidder need not know her own budget constraint, but because it allows a bidder to understand the commitments that she is making by following a particular bidding strategy in an auction.

**Bounds on Budget Constraints.** An auctioneer can generate implied constraints on budget from the bidder’s bidding history. With an empty history, the lower bound,  $B_L$ , is initialized to 0 and the upper bound,  $B_U$ , is initialized to infinity. As the stages proceed, the tightest possible lower bound for the budget is the revealed budget i.e.,  $B_L = \max_{k \in \{1, \dots, t\}} \{p^k(S^k), 0\}$ . To understand how to compute a tight upper-bound,  $B^* \in [B_L, B_U]$ , we appeal to [observation 4.4.3](#) and the general form of the SRPAR rules. By definition, this value  $B^*$  is the supremum over all  $\tilde{B} \in [B_L, B_U]$  such that LP- $\tilde{B}$  is feasible. Hence, clearly for all  $B_L \leq \tilde{B} < B^*$ , LP- $\tilde{B}$  is feasible and conversely for all  $\tilde{B} > B^*$ , LP- $\tilde{B}$  is infeasible. It turns out that  $B^*$  is in fact the smallest budget  $\tilde{B} \in [B_L, B_U]$  such that LP- $\tilde{B}$  is infeasible.<sup>3</sup>

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<sup>3</sup>To understand the behavior at  $B^*$ , suppose LP- $B^*$  is feasible. Then, consider LP- $(B^* + \delta)$  for a

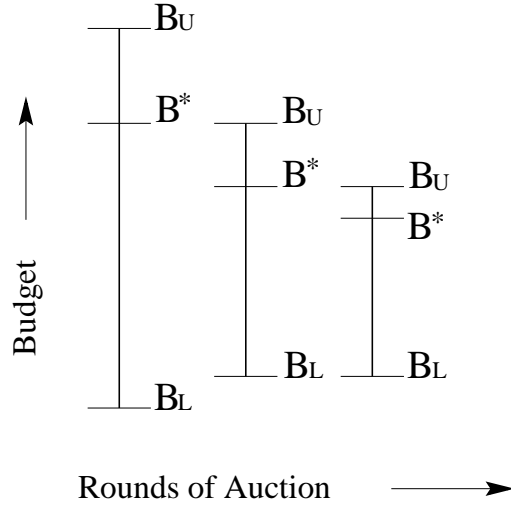


Figure 4-2: The upper bound  $B^*$  and lower-bound  $B_L$  of a bidder's budget that is implied by its bidding behavior, versus the number of rounds of the auction.

In each round of the auction, as new bids are placed, more constraints are added to the SRPAR. This can allow the upper-bound,  $B_U$ , to be tightened to the current  $B^*$  and the lower-bound,  $B_L$ , to be tightened to the maximum revealed budget (See Fig. 4-2). The following theorem helps to compute  $B^*$  to within  $\epsilon > 0$  accuracy, where  $\epsilon$  can be an arbitrary small number.

**Theorem 4.4.5.** *For ascending price auctions,  $B^* \in (\widetilde{B}_\epsilon, \widetilde{B}_\epsilon + \epsilon)$  where  $\widetilde{B}_\epsilon$  is the optimal objective value to the following integer program,  $P_\epsilon$ , with variables  $B, v(S^k)$ ,*

---

very small positive number  $\delta$ . LP- $B^*$  is the same as LP- $(B^* + \delta)$  because  $\delta$  is a very small number (more formally for small enough  $\delta$  the set of constraints of LP- $B^*$  and LP- $(B^* + \delta)$  are identical). Hence LP- $(B^* + \delta)$  is also feasible. But we know that for any  $\widetilde{B} > B^*$ , in particular when  $\widetilde{B} = B^* + \delta$ , LP- $\widetilde{B}$  does not have a feasible solution. Hence, by contradiction, LP- $B^*$  is infeasible.

$k = 1, \dots, t$  and indicator variable  $t_{kl} \in \{0, 1\}$ :

$$P_\epsilon : \quad \max B \quad (4.12a)$$

$$\text{subject to} \quad v(S^k) - p^k(S^k) \geq v(S^l) - p^k(S^l), \quad \forall k < l, \quad k, l \in \{1, \dots, t\} \quad (4.12b)$$

$$v(S^k) - p^k(S^k) \geq v(S^l) - p^k(S^l) - Mt_{kl}, \quad \forall k > l, \quad k, l \in \{1, \dots, t\} \quad (4.12c)$$

$$p^k(S^l) \leq B + Mt_{kl}, \quad \forall k > l, \quad k, l \in \{1, \dots, t\} \quad (4.12d)$$

$$p^k(S^l) + M(1 - t_{kl}) \geq B + \epsilon, \quad \forall k > l, \quad k, l \in \{1, \dots, t\} \quad (4.12e)$$

$$t_{kl} \in \{0, 1\} \quad \forall k > l, \quad k, l \in \{1, \dots, t\} \quad (4.12f)$$

$$B_L \leq B \leq B_U \quad (4.12g)$$

where  $M$  is a number larger than  $\max_{k,l \in \{1, \dots, t\}} p^k(S^l)$  and  $\epsilon > 0$  is any small positive number.

We provide some comments before proving the theorem. First, observe that the above theorem only gives an estimate of  $B^*$  to within  $\epsilon$  accuracy and not the exact value. However, this suffices for our purposes if we choose  $\epsilon$  to be smaller than the price increments between rounds. Second, we note that the MIP with  $O(t^2)$  variables and  $O(t^2)$  constraints is maximizing the budget,  $B$ , subject to SRPAR. The ‘if’ condition in [constraint \(4.6a\)](#) is rewritten using an indicator variable  $t_{kl} \in \{0, 1\}$ , which is 0 if bundle  $S^l$  at price  $p^k$  is below budget and 1 otherwise. Considering the cases  $k < l$ ,  $k > l$  and  $k = l$ , and recalling that  $p^k \geq p^l$  when  $k \geq l$  for ascending price auctions, [constraints \(4.12b – 4.12f\)](#) are obtained. Because of the ascending nature of prices, constraints similar to [\(4.12c – 4.12f\)](#) are redundant when  $k < l$ . But in a general demand revealing process where the ascending nature of prices is not present, we add these constraints, even in the case when  $k < l$ . Because of the  $B_U$  update from the

previous round, the MIP is infeasible for  $B \geq B_U$ . We retain the constraint  $B \leq B_U$  for clarity.

To provide some intuition, it is important to understand the role of  $\epsilon$  in SRPAR. In [constraint \(4.6a\)](#),  $\epsilon$  is the small addition made to the budget,  $B$ , to make constraint  $p^k(S^l) \leq B$  feasible, which in turn introduces constraint  $v(S^k) - p^k(S^k) \geq v(S^l) - p^k(S^l)$  in the LP- $(B + \epsilon)$ . Hence  $B^*$  is that budget at which the introduction of a new constraint makes LP- $(B + \epsilon)$  infeasible. With this intuition, we will now formally prove the result.

*Proof.* Note that  $B^* \leq B_U$  because LP- $B_U$  is infeasible.

- $B^* > \widetilde{B}_\epsilon$ : Suppose otherwise. Then, since LP- $\widetilde{B}_\epsilon$  is feasible, LP- $B^*$  must also be feasible, contradicting the definition of  $B^*$ . Hence,  $B^* > \widetilde{B}_\epsilon$ .
- $B^* < \widetilde{B}_\epsilon + \epsilon$ : Suppose otherwise, and that  $B^* \geq \widetilde{B}_\epsilon + \epsilon$ . Consider any  $B \in (\widetilde{B}_\epsilon, \widetilde{B}_\epsilon + \epsilon)$ . Observe that the MIP,  $P_\epsilon$ , is feasible at  $B$  as  $B < B^*$ . This contradicts that  $\widetilde{B}_\epsilon$  is the optimal solution of  $P_\epsilon$ . Hence,  $B^* < \widetilde{B}_\epsilon + \epsilon$ .

□

Thus, by solving the MIP,  $P_\epsilon$ , in each round, the auctioneer not only ensures the bidder satisfies the activity rule but can compute implied bounds on the budget of the bidder. Alternatively, the auctioneer can implement the following polynomial time algorithm instead of solving the MIP.

**Algorithm 4.4.6.** *Sort the prices  $p^k(S^l) \forall k, l \in \{1, \dots, t\}$ ,  $B_L$  and  $B_U$  in ascending order (without repetitions) and retain only elements above (and including) and below (and including)  $B_L$  and  $B_U$  respectively. Perform a binary search to find the largest element in this sorted array,  $\widetilde{B}_\epsilon$ , such that LP- $\widetilde{B}_\epsilon$  is feasible. If there is no such element then the MIP,  $P_\epsilon$ , is infeasible and the bidder does not satisfy the activity rules.*

**Example 4.4.7.** *It can be easily checked that the bidder in the example provided in the proof of [Claim 4.3.5](#) satisfies this new activity rule with  $\widetilde{B}_\epsilon = 5500 - \epsilon$  after the*

second round of the auction. Hence,  $B^* \in (5500 - \epsilon, 5500)$ . Thus, her budget is constrained to be between  $[5000, 5500)$  for all future rounds of the auction.

**Feedback.** Given this bound information, a decision-support tool for a bidder could report implied budget-constraint information by the bids that a bidder proposes in some round. This budget information can also be used in a decision support tool to guide bidding. For instance, a bidder could specify a (reported) valuation function to the tool in some round and a sub-interval of the range on feasible budgets, and the tool could suggest utility-maximizing bundles consistent with these current beliefs of the bidder. In this way the system can provide feedback to explain to a bidder what she can do in order to comply with the rule. A bidder can also *push* information to the auction rather than respond to demand queries via prices [SB06]. For instance, a bidder could report a range of possible budget constraints to the auctioneer, or a bidder-support tool, and the auctioneer (or tool) could then issue a warning to the bidder when the constraints on budget are implicitly violated by the bidder’s strategy.

### 4.4.3 Quasi-Linear Bidders

In this section, we consider bidders with quasi-linear utility functions,  $u(S, p) = v(S) - p(S)$  and no budget constraints. A bidder’s utility function is parameterized by  $\theta = v$ . In Section 4.3, we showed that RPAR is necessary but not sufficient in this context and is not a strong activity rule. Consider instead the following activity rule, SRPAR<sub>ql</sub>, obtained by substituting  $B$  with infinity in SRPAR and simplifying. In SRPAR<sub>ql</sub>, the auctioneer checks for feasibility of the following LP, where the variables are  $v(S^k)$ ,  $k = 1, \dots, t$ :

$$\text{SRPAR}_{\text{ql}} : v(S^k) - p^k(S^k) \geq v(S^l) - p^k(S^l), \quad \forall l, k \in \{1, \dots, t\}, l \neq k. \quad (4.13)$$

This constraint means the bundle  $S^k$  placed by the bidder should be one of the utility maximizing bundles at price  $p^k$  when compared to all other bundles,  $S^l$ , placed in the auction. The number of variables in the LP are  $t$  and the number of constraints are  $O(t^2)$ . We observe that SRPAR<sub>ql</sub> is a slightly enhanced (and stronger) form

of RPAR, which is itself obtained by only adding constraints for every pair  $l, k$  of rounds. The example in [Claim 4.3.4](#) which satisfied RPAR despite corresponding to an inconsistent bidder violates  $\text{SRPAR}_{\text{ql}}$ . The infeasible constraints in the claim exactly form the constraints of  $\text{SRPAR}_{\text{ql}}$  that the bidder would have to satisfy.<sup>4</sup> We have the following corollary to [Theorem 4.4.2](#) for this setting:

**Corollary 4.4.8.**  *$\text{SRPAR}_{\text{ql}}$  is a strong activity rule for quasi-linear bidders, both with linear and non-linear prices.*

Here again we assume that the rules of the auction specify that at least one (not all) of the utility maximizing bundles be placed in the auction. If the auction specifies otherwise, then we make modifications to  $\text{SRPAR}_{\text{ql}}$  that are directly analogous to those presented for SRPAR at the end of [Section 4.4.1](#).

**Simplifying: Homogeneous Items and Linear Prices.** In the case of homogeneous items and with linear prices, we show that  $\text{SRPAR}_{\text{ql}}$  is equivalent to the monotonic activity rule, MR. This proves that RPAR, which is also equivalent to MR with homogeneous items and linear prices, and MR are strong activity rules in this special scenario.

**Theorem 4.4.9.** *In an auction of homogeneous items with linear prices,  $\text{SRPAR}_{\text{ql}}$  is equivalent to RPAR and MR.*

*Proof.* RPAR is obtained from  $\text{SRPAR}_{\text{ql}}$  by adding constraints for pair  $l, k$ , thus proving  $\text{SRPAR}_{\text{ql}} \Rightarrow \text{RPAR}$ . Now consider a bidder that satisfies RPAR i.e.,  $S^k \subseteq S^{k+1} \forall k = 1, \dots, t$ . Consider a set of values  $v(S^k)$ ,  $k = 1, \dots, t$  as follows:

$$v(S^k) = \sum_{j=k}^{t-1} p^j [ |S^j| - |S^{j+1}| ] + p^t |S^t|,$$

---

<sup>4</sup>For linear prices, Vohra [\[Voh07\]](#) rewrites the conditions of Rochet [\[Roc87\]](#) in the same form as  $\text{SRPAR}_{\text{ql}}$ . Not only are these constraints an LP, but Vohra reinterprets them as a network flow problem. Network flow algorithms are computationally much faster than algorithms to solve general LP's as they exploit the network structure. This network flow re-interpretation works even for general prices. Consider a graph with one node for every bid  $k$ . For every pair of nodes  $(k, l)$ , the length of the arc from  $k$  to  $l$  is  $p^k(S^l) - p^k(S^k)$ . The activity rule  $\text{SRPAR}_{\text{ql}}$  is feasible if and only if this network has no cycle of negative length. Extending a similar approach to the case of a budget-constrained bidder, an arc of length  $p^k(S^l) - p^k(S^k)$  is placed between nodes  $k$  and  $l$  if the corresponding price  $p^k(S^l)$  is within the budget  $B$  ( $B = B_L$  is sufficient).



where  $|S|$  refers to the number of items in bundle  $S$ . It is easy to check that these values satisfy  $\text{SRPAR}_{\text{ql}}$ . This completes the proof.  $\square$

**Simplifying: Bundle Monotonicity in Ascending Auctions.** Mishra and Parkes [MP07], working in the context of ascending price auctions for heterogeneous items and with quasi-linear utility functions are able to propose simple, local rules (defined in terms of just the previous round of the auction) that are nevertheless strong. The auctions that they introduce require bidders to make a claim about *all* packages of items that maximize their utility in each round, and have a special price dynamics that provides for these best-response sets to monotonically increase across rounds when coupled with a straightforward bidder.

The auctions start with a zero price vector for every bidder, and the price increment at each round of the auction is performed as follows. The auction chooses a set of *critical bidders*,  $C$ , at the end of each round in some manner. The prices for the critical bidders are updated as follows:

$$p^r(S) = \begin{cases} p^{r-1}(S) + 1 & \forall S \in D(p^{r-1}) \\ p^{r-1}(S) & \text{otherwise,} \end{cases} \quad (4.14)$$

where  $D(p^r)$  is the *demand set* of the bidder when the prices are  $p^r$ . The price vector for non-critical bidders is unchanged. Bidders satisfy free-disposal and that bidder valuations are in integer amounts. Mishra and Parkes [MP07] show that for the class of ascending price auctions they study, local rules of *round monotonicity* and *bundle monotonicity* (RM & BM) are enough to ensure consistent bidding. We prove the equivalence between these local rules and strong activity rule,  $\text{SRPAR}_{\text{ql}}$ .

Consider the environment studied by Mishra and Parkes [MP07] with heterogeneous items and quasi-linear utility functions. The strong activity rule,  $\text{SRPAR}_{\text{ql}}$ , in round  $r$  of the auction is defined as follows, where we introduce the conditions required

for when bidders place *all* the utility maximizing rules (and to handle free-disposal),

$$A(r) : v(S) - p^k(S) = v(S') - p^k(S'), \quad \forall S, S' (\neq S) \in D(p^k),$$

$$\forall k \in R \quad (4.15a)$$

$$v(S) - p^k(S) \geq v(S') - p^k(S') + 1, \quad \forall S \in D(p^k), S' \in D(p^{k'}) \setminus D(p^k),$$

$$\forall k, k' \in R \quad (4.15b)$$

$$v(S) - p^k(S) \geq v(T) - p^k(T) + 1, \quad \forall S \in D(p^k), \forall T \supset S, T \notin D(p^{k'}),$$

$$p^k(T) > 0 \quad \forall k, k' \in R \quad (4.15c)$$

$$v(T) \geq v(S), \quad \forall S \in D(p^k), \forall T \supset S, T \notin D(p^{k'}),$$

$$p^k(T) > 0 \quad \forall k, k' \in R, \quad (4.15d)$$

where  $R = \{0, \dots, r\}$ . Note that since all values are integers, we have replaced  $\epsilon$  by 1 in the activity rules. Observe that this activity rule,  $A(r)$ , in round  $r$ , has exponentially many variables and constraints (as many as the number of positive prices). The local activity rules defined by Mishra and Parkes [MP07] are as follows:

**Definition 4.4.10.** (*Mishra and Parkes' activity rules 2007*)

- Round Monotonicity ( $RM_r$ ): For every bidder,  $D(p^{r-1}) \subseteq D(p^r)$  and in particular for non-critical bidders,  $D(p^{r-1}) = D(p^r)$ .
- Bundle Monotonicity ( $BM_r$ ): For every bidder, if  $S \subset T$  and  $S \in D(p^r)$  then  $T \in D(p^r)$ .

We now show that our activity rules (i.e.,  $A(r)$ ) are in fact equivalent to these two rules in the case when the price increments are as described above. The reason we present this theorem is to show that in some scenarios, simple local rules are available.

**Theorem 4.4.11.**  $SRPAR_{ql}$ , when instantiated in this environment with heterogeneous items and quasi-linear utility functions, is equivalent to RM and BM when the price increments are as described in Eq. (4.14). More formally, for each bidder and

each round  $r \in \{0, \dots, k\}$ , we have

$$A(r) \text{ is satisfied} \iff \forall j \in \{0, \dots, r\}, RM_j \text{ and } BM_j \text{ are satisfied.}$$

*Proof.* Mishra and Parkes [MP07] show that their conditions are strong in our sense. Still, it is instructive to show that  $A(r) \implies RM, BM$ . Consider any bidder. We need to show this bidder satisfies  $A(r)$ , then she satisfies  $RM_j$  and  $BM_j$  for each  $j \in \{0, \dots, r\}$ . We will prove this by induction on the round number  $r$ . To anchor the induction, consider the case when  $r = 0$ , i.e., at the beginning of the auction which starts with the zero price vector.  $RM_0$  is vacuously true. It is easy to observe that  $A(0)$  is not satisfied unless the demand set  $D(\mathbf{0})$  satisfies bundle monotonicity from [constraints \(4.15c – 4.15d\)](#) (because if  $T \notin D(\mathbf{0})$  for some  $T \supset S, S \in D(\mathbf{0})$ , then from [constraints \(4.15c – 4.15d\)](#), we have  $v(S) \geq v(T) + 1 \geq v(S) + 1$ , a contradiction).

Now by the induction hypothesis, suppose the statement is true for all rounds prior to  $r$ . We will then show that it is true for the current round  $r$  also. Suppose the bidder satisfies  $A(r)$ . Since the constraints of  $A(r-1)$  are a subset of the constraints of  $A(r)$ , we have by the induction assumption that the bidder satisfies  $RM_j$  and  $BM_j$  for all  $j < r$ . There are two cases to consider depending on whether the bidder is a critical bidder or not. We will consider both these cases in a unified fashion using the indicator variable  $\delta_C$  which is 1 if the bidder is a critical bidder and 0 otherwise.

- Round Monotonicity  $RM_r$ : Suppose  $RM_r$  is false. Then  $\exists Q \in D(p^{r-1})$  such that  $Q \notin D(p^r)$ . In round  $r-1$ , let the utility  $u^{r-1}(Q)$  of bundle  $Q$ , defined as  $v(S) - p^{r-1}(S)$ , be  $u$ . Then at round  $r$ ,  $u^r(Q) = u - \delta_C$  as  $p^r(Q) = p^{r-1}(Q) + \delta_C$ . We end at a contradiction if the utility of any bundle  $S \in D(p^r)$  is less than or equal to  $u - \delta_C$ , i.e.,  $u^r(S) \leq u - \delta_C$  because then [constraint \(4.15b\)](#) of  $A(r)$  is violated. There are two cases depending on whether  $S \in D(p^{r-1})$  or not.
  - Case 1:  $S \in D(p^{r-1})$ . Then  $u^{r-1}(S) = u^{r-1}(Q) = u$  (by [Constraint \(4.15a\)](#)). Since  $p^r(S) = p^{r-1}(S) + \delta_C$ ,  $u^r(S) = u - \delta_C$ .
  - Case 2:  $S \notin D(p^{r-1})$ . Then  $u^{r-1}(S) \leq u^{r-1}(Q) - 1 = u - 1$  (by [con-](#)

straint (4.15b)). Since  $p^r(S) = p^{r-1}(S)$ ,  $u^r(S) = u^{r-1}(S) \leq u - 1 \leq u - \delta_C$  (since  $\delta_C \leq 1$ ).

- Bundle Monotonicity  $BM_r$ : Suppose  $BM_r$  does not hold at round  $r$ . Then  $\exists T \supset S$ , such that  $T \notin D(p^r)$  and  $S \in D(p^r)$ . By round monotonicity,  $T \notin D(p^k)$  for any  $k \in \{0, \dots, r-1\}$ . We again consider two cases depending on whether or not  $S \in D(p^{r-1})$ .
  - Case 1:  $S \in D(p^{r-1})$ . Then by the induction assumption the bidder satisfies bundle monotonicity in round  $r-1$ . This means we have  $T \in D(p^{r-1})$ , which is a contradiction.
  - Case 2:  $S \notin D(p^{r-1})$ . This means  $S \notin D(p^k)$  for any  $k \in \{0, \dots, r-1\}$  by round monotonicity. This means that the price of bundle  $S$  has never changed from its initial price of zero (i.e.,  $p^r(S) = 0$ ) until the current round  $r$ . The same is the case with bundle  $T$  (i.e.,  $p^r(T) = 0$ ). Substituting this in  $A(r)$  in [constraints \(4.15c – 4.15d\)](#), we observe that  $A(r)$  is infeasible, which is a contradiction.

□

#### 4.4.4 Discussion: Comparing the Activity Rules

In [Table 4.1](#) we compare the different activity rules. We observe whether a rule that allows straightforward and truthful bidding (necessary condition), marked by T, and whether a rule ensures consistent bidding (necessary and sufficient condition), marked by C. Note that  $(C \implies T)$ . We mark an entry with ‘–’ when there exists a counterexample to show the failure of T (and thus also C). Subscript  $L$  or  $H$  refer to the case that the activity rule satisfies the property (T or C) *only* in the linear price setting or the homogeneous items setting respectively. Superscript \* refers to the case that the property requires the special price increment of [Eq. \(4.14\)](#) to hold.

We compare across the rules in the case of a general price structure and price path and denote by  $\succ$  if a rule dominates another rule, in the sense of a preference

Utility Class	Activity Rule							
	MR	AMR	RPAR	RM & BM	SRPAR <sub>ql</sub>	SRPAR		
Quasi-Linear	$\mathbf{C}_{L,H}$	$\rightarrow$	$\mathbf{C}_{L,H}$	$\rightarrow$	$\mathbf{C}_{L,H}$	$\mathbf{C}^*$	$\mathbf{C}$	$\mathbf{T}$
	-	$\leftarrow$	-		$\mathbf{T}$			
Budget-Constrained	-	$\leftarrow$	-	$\leftarrow$	-	-	-	$\mathbf{C}$

Table 4.1: Comparison of the strength of activity rules in different settings. Arrows indicate that one rule implies another rule. Entries in **bold** are associated with a proof or example.

ordering in which  $C$  is preferred by an auction designer to  $T$  and  $T$  is preferred in turn to having neither  $C$  nor  $T$ . We observe that, irrespective of whether the bidders are quasi-linear or budget-constrained, RPAR dominates AMR and MR and that SRPAR<sub>ql</sub> dominates RM & BM (for general price-increment processes). Furthermore, we observe that both SRPAR<sub>ql</sub> and SRPAR dominate RPAR because

	SRPAR		RPAR
Quasi-Linear	$\mathbf{T}$		$\mathbf{T}$
Budget-constrained	$\mathbf{C}$	$\gamma$	-

and

	SRPAR <sub>ql</sub>		RPAR
Quasi-Linear	$\mathbf{C}$		$\mathbf{T}$
Budget-constrained	-	$\gamma$	-

Thus, there is an unambiguous recommendation in favor of SRPAR over RPAR. On the other hand, it is difficult to compare rules SRPAR<sub>ql</sub> and SRPAR, because SRPAR<sub>ql</sub> dominates SRPAR for quasi-linear bidders but is dominated by SRPAR for budget-constrained bidders. It seems reasonable that allowing truthful bidding,  $T$ , should receive higher priority than ensuring consistent bidding, because at the very least we then avoid hurting straightforward bidders. On those lines, SRPAR has the  $T$  property independent of the utility class of the bidders, compared to SRPAR<sub>ql</sub>, and thus this rule is likely preferred to SRPAR<sub>ql</sub> unless there is good reason to believe that

there are no budget constraints. The choice of activity rule could also be personalized to individual bidders. Of course, one would prefer an activity rule that is always  $C$  in all scenarios, independent of the particular utility class of the bidders. Here we identify an interesting tension:

**Observation 4.4.12.** *If an activity rule is  $C$  for budget-constrained bidders, then it **cannot** be  $C$  for quasi-linear bidders; it can only be  $T$ . This is because as long as a quasi-linear bidder behaves consistently as a budget-constrained bidder, the activity rule cannot detect this behavior. See the example discussed in [Section 4.4.1](#)*

**Observation 4.4.13.** *If an activity rule is  $C$  for quasi-linear bidders, it can never be  $T$  for budget-constrained bidders. An easy counter-example in this scenario is as follows. Consider two bundles  $A$  and  $B$  that are being auctioned. Say in two rounds of the auction, the price of  $A$  and  $B$  increase by the same amount. Consider a bidder with values well above the prices. In the case that she is quasi-linear, the preferences should not change across the bundles. But in the case that she is budget-constrained, this might not be true depending on the actual prices of the bundles and the budget.*

These observations show that  $\text{SRPAR}$  and  $\text{SRPAR}_{\text{ql}}$  are theoretically optimal, in the sense of the necessary (T) and sufficient (C) aspects of strong activity rules that are introduced in this chapter. Thus, as an auction designer, it is useful to have some prior information about the types of utility functions to expect of bidders in choosing an appropriate activity rule. It might not always be clear if the bidders belong to one class or the other, in which case, the auctioneer should at the very least ensure that straightforward bidding is not precluded.

## 4.5 Experimental Simulations

In this section, we present the results of experimental simulations that are designed to validate the importance of strong activity rules in the context of budget-constrained bidders. We study activity rules in the context of the clock-proxy auction [[ACM06](#)]. This auction has been proposed for the landing slot auction at LaGuardia, for wireless

spectrum auctions and for auctions for power generation in the context of power generation capacity [CS07]. We describe two variations of the clock-proxy auctions, one that incorporates RPAR and one that incorporates SRPAR.

We study the immediate problems caused when rules prevent straightforward bidding. We implement a *maximally straightforward* bidding strategy, such that when a deviation from straightforward bidding is necessary in order to meet the rule, we seek a simple modification to the straightforward strategy by dropping packages during the clock phase and modifying bid values associated with bids in the transition to the proxy phase. We adopt problem distributions from the CATS test suite for combinatorial auctions [LBS06], including some of the problems originally described by Sandholm [San02]. They have been widely used in the literature on combinatorial auctions [SSGL05, PSB06]. A budget factor is used to assign a budget constraint to each bidder as a function of the largest value that bidder has across all packages.

#### 4.5.1 Instantiating the Clock-Proxy Auction

The clock-proxy auction starts with a clock phase and ends with a proxy phase. The purpose of the clock phase is to provide price discovery. In each round of the clock phase the auctioneer reports linear prices and bidders respond with a package of items. Bids are XOR in nature across rounds. Prices for items with excess demand are increased and the clock phase terminates when supply is weakly greater than demand on all items. In transitioning to the proxy phase, bidders can submit a final claim about their value on every clock bundle that they have mentioned together with values on a small number ( $E \geq 0$ ) of additional bundles. In our context of budget constraints, we also allow bidders to make a claim about their budget constraint at this transition point.

The proxy phase is a sealed-bid auction, but simulates an ascending-price auction with non-linear and non-anonymous prices. We refer to each simulated round in this proxy phase as a *proxy round*. Each bidder is represented here by a proxy that follows a straightforward bidding strategy with respect to the reported valuation and budget information. This strategy is not to be confused with the maximally straightforward

bidding strategy that we simulate for bidders in each actual round of the auction. This proxy bidding strategy simply defines the outcome of the final proxy phase. In our instantiation, the proxy agents bid on the set of packages in each round that maximize reported utility. A provisional allocation is computed in each proxy round to maximize revenue given bids and prices are increased to each losing bidder. The proxy phase, and thus also the entire clock-proxy auction, terminates as soon as supply is weakly greater than demand and no new bids are submitted.

We modify the behavior of the proxy phase from the standard clock-proxy auction to allow for the existence of budget-constrained bidders. Let  $p_{\text{ask}}(S)$  be the ask price the auctioneer specifies for bundle  $S$ . For bundle  $S$  where  $p_{\text{ask}}(S) \leq B$  and  $p_{\text{ask}}(S) \leq v(S)$ , the proxy bid price,  $p_{\text{bid}}(S)$ , is simply  $p_{\text{ask}}(S)$ . For bundles with  $p_{\text{ask}}(S) > B$  or  $p_{\text{ask}}(S) > v(S)$ , the bid price is  $p_{\text{bid}}(S) = p_{\text{ask}}(S) - \epsilon$ , where,  $\epsilon > 0$  is the bid increment in the proxy stage. By adopting this “ $\epsilon$ -discount” the effective ask prices on these bundles does not increase further.

A proxy agent reports packages in each round that (a) maximizes the quasi-linear utility (value minus price) to within  $\epsilon$  among all those bundles that are priced below value and below budget or priced just above the value (b) has quasi-linear utility at least as large as that in (a) but priced just above budget. This bidding strategy ensures that the proxy’s demand set is monotonically increasing across rounds. Because of the price dynamics, this has the effect of reporting packages,  $S \subseteq \mathcal{G}$ , in each round that satisfy:

$$v(S) - p_{\text{bid}}(S) + \epsilon \geq \max_{T \in \mathcal{G}_B} \{v(T) - p_{\text{bid}}(T)\}, \quad (4.16)$$

where the maximum is taken over a restricted set of bundles,

$$\mathcal{G}_B = \{T \subseteq \mathcal{G} \mid [v(T) \geq p_{\text{ask}}(T) \text{ and } p_{\text{ask}}(T) \leq B] \text{ or } [v(T) < p_{\text{ask}}(T)]\}. \quad (4.17)$$

We now provide details of the application of SRPAR and RPAR to the clock proxy auction, including a description of the impact of the activity rule on our construction for a maximally-straightforward bidding strategy.



**The Clock-Proxy Variation with SRPAR.** In the clock stage we implement SRPAR exactly as described in Section 4.4. At the transition to proxy we impose SRPAR directly for the clock bundles along with some additional constraints for the extra transition bundles. A bidder can report up to  $E \geq 0$  additional bundles that were not mentioned in the clock stage (the “clock bundles”), and for these bundles and the clock bundles, associate a valuation function  $\hat{v}$  that together with a claim about a budget constraint  $\hat{B}$  (perhaps infinite), provides a feasible solution to the SRPAR constraints in (4.6) along with the following constraints:

$$v(S^k) - p^k(S^k) \geq v(U) - p^k(U) \quad \text{if } p^k(U) \leq B \quad \forall \quad S^k \in \text{Bids}_{\text{clock}}, U \in \text{Bundles}_{\text{new}} \quad (4.18)$$

$$v(U) \geq v(S) \quad \forall \quad U \supset S, S \in \text{Bundles}_{\text{clock}}, \\ U \in \text{Bundles}_{\text{new}}, \quad (4.19)$$

where  $\text{Bids}_{\text{clock}}$  and  $\text{Bundles}_{\text{clock}}$  are the bids (bundles associated with a price) and bundles elicited in the clock stage respectively and  $\text{Bundles}_{\text{new}}$  are the additional transition bundles. Constraint (4.18) is the same as constraint (4.6a) in SRPAR. Constraint (4.19) just ensures free-disposal.

The **straightforward bidding strategy for budget-constrained bidders in clock-proxy with SRPAR** is defined as:

- (a) Clock stage: In each round, bid on the package that maximizes utility given a valuation function and budget constraint, and given current prices.
- (b) Transition stage: Select the additional  $E \geq 0$  bundles that maximize quasi-linear utility, given the final clock prices and ignoring the budget constraints. Report the true value for all clock bundles and all additional bundles together with the true budget constraint information.

**The Clock-Proxy Variation with RPAR.** In the clock stage we implement RPAR exactly as described in Sections 4.3. We also need an activity rule at the transition from from clock to proxy. For RPAR, we adopt a relaxed rule that is inspired in part

by some of the operational details in Hoffman [Hof04], while differing in substance in order to better allow for budget-constrained bidders during the clock stage<sup>5,6</sup>. The relaxed RPAR at the transition is parameterized by *relaxation parameter*,  $\alpha > 1$ , and defined as:

$$\alpha[v(S^k) - p^k(S^k)] \geq v(S^l) - p^k(S^l) \quad \forall S^k, S^l \in \text{Bids}_{\text{clock}} \quad (4.20)$$

$$\alpha[v(S^k) - p^k(S^k)] \geq v(U) - p^k(U) \quad \forall S^k \in \text{Bids}_{\text{clock}}, U \in \text{Bundles}_{\text{new}} \quad (4.21)$$

$$p^{\max}(S) \leq v(S) \quad \forall S \in \text{Bundles}_{\text{clock}}, \quad (4.22)$$

where  $p^{\max}(S)$  is the maximum price that the bidder has bid on bundle  $S$  during the clock stage. This activity rule ensures two things. Firstly it ensures that the clock bundles maximize the quasi-linear utility with an  $\alpha$  relaxation with respect to any other clock bundle and that they are valued higher than the maximum bid price on that bundle. Secondly it ensures that any transition bundle has lower quasi-linear utility than a clock bundle with a  $\alpha$  relaxation.

In defining a bidding strategy for budget-constrained bidders in clock-proxy with RPAR rather than SRPAR we adopt as close an approximation to a straightforward, truthful bidding strategy as is possible given the activity rule.

The following example shows that a bidder might be *unable* to meet RPAR at the transition to proxy, even without trying to submit additional bundles and for some  $\alpha > 1$ .

**Claim 4.5.1.** *A truthful budget-constrained bidder may be unable to meet the relaxed RPAR rule in the transition from clock to proxy by simply associating truthful values with each of the clock bundles, even when the relaxation parameter,  $\alpha$ , exceeds 1.*

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<sup>5</sup>Hoffman [Hof04] specified an upper bound on the values of clock as well as the new bundles based on their prices in the final rounds for a drop-out bidder. But we cannot upper bound the true value for budget-constrained bidders and hence do not impose these constraints. Furthermore, these authors also suggested to include a lower bound on the values of new bundles based on prices. We do not include this constraint because the prices at the end of clock phase need not be representative of the prices at the end of the proxy stage with budget-constrained bidders.

<sup>6</sup>Ausubel, Cramton and Milgrom [ACM06] also suggest the use of a relaxed activity rule, but for a different reason. Their concern is to address demand reduction in the clock phase, which can occur because of linear pricing. A relaxed activity rule allows bidders to reverse some of this demand reduction.

*Proof.* It suffices to provide an example of a scenario where this happens. We provide an example in the case when  $\alpha = 1.05$ . Consider an auction of two items A and B. Consider a bidder with a budget of 100 whose value for item A is 125 and whose value for item B is 110. Say the prices in the last two rounds of the auction are as shown in the following table. Also, shown are the packages,  $x^t$ , that form the best-response of the bidder:

Round	$p^t$	$x^t$
t-1	(80,75)	(1,0)
t	(110,100)	(0,1)

These two bundles satisfy RPAR because  $(110 - 80)(0 - 1) + (100 - 75)(1 - 0) = -5 < 0$ . However, RPAR constraint (4.20) for  $\alpha = 1.05$ , written in terms of the valuations reported at this transition from clock to proxy, requires  $1.05(v(A) - 80) \geq v(B) - 75$  and  $1.05(v(B) - 100) \geq v(A) - 110$ . Valuations,  $v(A) = 125$  and  $v(B) = 110$ , cause the second inequality to be violated. It is also interesting to observe that for  $\alpha = 1$ , the constraints are always infeasible for any valuations.  $\square$

So, bidders can be forced to report false values on clock bundles at the transition, and sometimes there can be *no* reports that will satisfy RPAR! To overcome this difficulty we further relax  $\alpha$ -RPAR at the transition, by always allowing every bundle in the clock phase to be submitted with a value no less than the maximum price bid on the bundle in the clock phase. This allows all bundles that receive a bid in the clock stage to be passed on to the proxy stage.

The **maximally straightforward bidding strategy for budget-constrained bidders in clock-proxy with RPAR** is defined as:

- (a) Clock stage: In each round, greedily pick a utility maximizing bundle (by definition, within budget) at the current prices satisfying RPAR.
- (b) Transition stage:
  - (b1) Select a subset of the clock bundles that will satisfy  $\alpha$ -RPAR at values discounted from the true values and to minimize the maximum difference

of the submitted values from the true values. We formulate this as a mathematical program described below. All other bundles are submitted at the maximal bid price from the clock phase.

- (b2) Sort the remaining (non-clock) bundles in order of decreasing quasi-linear utility at the final clock prices, and greedily pick the first  $E \geq 0$  packages (if any) that satisfy  $\alpha$ -RPAR when associated with the bidder's true value and with the values already assigned in step (b1) to clock bundles. The budget information is not considered in picking these extra bundles.

**Computing the Bidding Strategy in Clock-Proxy with RPAR.** The problem is to pick a subset of the clock bundles that will satisfy RPAR at values that are minimally discounted from true values. We break ties based on the total submitted value of the bundles. We formulate this problem as a MIP:

$$\min \quad M\delta - \epsilon \sum_{k \in D} \hat{v}(S^k) \quad (4.23a)$$

$$v(S^k) - \hat{v}(S^k) \leq \delta \quad \forall k \in T \quad (4.23b)$$

$$\hat{v}(S^k) = \hat{v}(S^l) \quad \text{if } S^k = S^l \quad \forall l, k (\neq l) \in T \quad (4.23c)$$

$$p^{\max}(S^k)z_k \leq \hat{v}(S^k) \leq v(S^k)z_k \quad \forall k \in T \quad (4.23d)$$

$$z_k + z_l = 2 - y_{kl} \quad \forall l, k (\neq l) \in T \quad (4.23e)$$

$$My_{kl} + \alpha [\hat{v}(S^k) - p^k(S^k)] \geq \hat{v}(S^l) - p^k(S^l) \quad \forall l, k (\neq l) \in T \quad (4.23f)$$

$$z_k \in \{0, 1\} \quad \forall k \in T \quad (4.23g)$$

$$y_{kl} \in \{0, 1, 2\} \quad \forall l, k (\neq l) \in T \quad (4.23h)$$

$$\delta \geq 0 \quad (4.23i)$$

where  $T = \{1, \dots, t\}$  and  $D = \{k | S^k \neq S^l, \forall k, l \in T\}$ , so that all bundles are accounted only once in the objective. This is necessary because a bundle could be elicited several times in the clock auction. Value  $\hat{v}(S^k)$  is the submitted value of bundle  $S^k$  whose true value is  $v(S^k)$ . Constant,  $M > 0$  is a large number and is set equal to  $10 * \max_{k \in T} v(S^k)$ . Constant,  $\epsilon > 0$  is a small positive number. Variable,  $\delta$  is

the maximum difference between the true value and the submitted value. Since  $\delta$  is constrained to be non-negative, the submitted value is always lower than the true value. This constraint is particularly critical when the value of a bundle is lower than the budget. The objective is *hybrid*, in the sense that it first minimizes  $\delta$  and then breaks ties based on the total submitted value. [Constraint \(4.23c\)](#) ensures that the values of two bundles elicited in different rounds of the auction are the same as long as the bundles are the same.  $z_k \in \{0, 1\}$  is a variable that is set to 1 if the bundle  $S^k$  is chosen by the MIP, and 0 otherwise. Variable  $y_{kl} \in \{0, 1\}$  is set to 0 only if both variables  $z_k$  and  $z_l$  are 1 and is used to enforce the activity rule between the pair of bundles  $S^k$  and  $S^l$ . [Constraint \(4.23f\)](#) checks that the activity rule is satisfied when both bundles  $S^k$  and  $S^l$  are chosen by the MIP.

## 4.5.2 Defining an Efficiency Metric for Budget-Constrained Bidders

For quasi-linear utilities, an *efficient allocation* is defined as the one that maximizes the total value of the allocation across the bidders. Although one can also adopt the same definition of efficiency for budget-constrained bidders, this benchmark is more problematic in this context because it is often unattainable when coupled with strategic bidders; e.g. the VCG mechanism fails (see [\[BCI<sup>+</sup>05\]](#)).

Consider a single item auction with two bidders, A and B. Say A and B have values of 10 and 8 for the item with budgets of 4 and 6 respectively. The maximum value allocation cannot be supported as a price equilibrium as B has sufficient budget to outbid A. Consider also limiting case of budget constraints  $B = 0$  for every bidder with multiple items. Now, because payments cannot be collected from bidders, this reduces to the setting of *voting theory*. The Gibbard-Satterthwaite theorem [\[Gib73, Sat75\]](#) states that any strategy-proof voting rule is dictatorial if there are at least three outcomes in the range of the rule. Thus, Pareto efficiency, but not allocative efficiency in the sense of maximizing total value, can be achieved.

An alternative metric, that is simple and somewhat intuitive, is to define the

efficiency of an allocation  $S^*$  in terms of the sum, over all bidders, of the *minimum of the value and the budget* ( $\min\{v(S_i^*), B\}$ ) for each bidder [BCI<sup>+</sup>05]. The best allocation is that which maximizes the sum of this “ $\min(v, B)$  value” over all bidders. The intuition behind the metric is that it will be hard to effect tradeoffs between the value of one bidder and another when these values are greater than the budgets of the bidders because prices cannot be used to effect the tradeoff. Indeed, this is achievable in the context of mechanism design for a single-item setting<sup>7</sup>. We choose to adopt both the total value and this total  $\min(v, B)$ -value in presenting our results. This latter metric is also used to provide a normalization when reporting the revenue achieved in the clock-proxy auction in the presence of budget constraints<sup>8</sup>.

### 4.5.3 Experimental Results

The distributions on bidder valuations that we adopt in our experiments are the *arbitrary*, *matching*, *paths* and *scheduling* distributions and the two legacy distributions ( $L2$  - uniform with linearly random and  $L4$  - decay with linear random) [San02]. For each distribution we generate 20 instances and present our results averaged over these instances. We choose distribution-specific parameters (such as *maxbid*) so that on average each bidder has a value for at least 10 bundles and adopt the *exclusive-or* (XOR) valuation logic, so that the bidder’s value for some bundle  $S$  is the maximal value over all bundles that are a subset of  $S$ . Having valuation functions with at least 10 bundles makes them sufficiently complex so that the distinction between RPAR and SRPAR matters. For the legacy distributions, we follow Parkes and Un-

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<sup>7</sup>It is unknown whether or not the target of the allocation that maximizes this  $\min(v, B)$  value is achievable in the combinatorial setting. Maskin [Mas00] elaborates issues with regard to the definition of efficiency for a single item and suggests a new definition called “constrained efficiency” in an all-pay single item auction setting. Its extension to multiple items, however appears to be an open problem.

<sup>8</sup>One can also consider a metric defined in terms of an approximate budget-constrained price equilibrium, where  $S^*$  and prices  $p^*$  are in an equilibrium when: (a) the allocation (approximately) maximizes each bidder’s utility at the prices and the bidder’s (given) budget constraint; and (b) the allocation (approximately) maximizes the seller’s revenue at the prices. Given this allocation, the efficient allocation would be defined to maximize the total value  $v(S)$  over all bidders, across all allocations  $S$  for which there exists an approximate, budget-constrained price equilibrium. While this can be formulated as an enormous MIP, we have been unable to find an operational methodology to compute this benchmark.

gar [PU00a] and generate valuations on bundles for each bidder independently, and join them together to form the input to our auction simulator.

We were unable to adjust the parameter in *paths* to achieve suitably large valuation functions (bidders valued on average only up to 3 bundles even with tuned parameters). Because of this the results for the *paths* distribution are the same for RPAR and SRPAR (as explained by [observation 4.4.4](#)) and are not presented here.

To analyze the efficiency and revenue results for various budgets, we introduce a notion called the *budget factor*: the budget factor  $BF \in (0, 1]$ , defines the budget  $B$  of a bidder as a fraction  $BF$  of the maximum value that the bidder has over all bundles. In our experiments, we make the simplifying assumption that all bidders have the same budget factor. We compute the following efficiency and revenue metrics:

VB/maxVB - Ratio of the total  $\min\{v(S), B\}$  of the allocation resulting from the auction to the maximum total  $\min\{v(S), B\}$  over all allocations.

CP/maxV - Ratio of the total value of the allocation resulting from the auction to the maximum total value over all allocations (the traditional definition of efficiency)

R/maxVB - The ratio of the revenue in the auction to the maximum of the total  $\min\{v(S), B\}$  over all allocations. We normalize the revenue by the maximum of the total  $\min\{v(S), B\}$  because this represents the maximum transferable monetary value from the bidders to the auctioneer at the specified budgets.

We vary budget factor  $BF$ , the number of *extra bundles*  $E \geq 0$  that can be submitted upon transition to the proxy phase, and the  $\alpha \geq 1$  relaxation to RPAR. We adopt the following notation in the figures:

$BF$  - Budget factor

$E$  - Number of extra bundles at transition

$A$  - The relaxation parameter  $\alpha$  for clock-proxy with RPAR

New - The performance of clock-proxy with SRPAR, our proposed new activity rules

Old - The performance of clock-proxy with RPAR, the standard revealed-preference activity rule and as relaxed by parameter  $\alpha \geq 1$ .

We illustrate in figures( 4-3– 4-7) the results for the L4 (decay) legacy, L2 (uniform) legacy, matching arbitrary and scheduling distributions. In each of these figures, subplots (a), (b), and (c) vary the budget factor  $BF$ , while subplots (d), (e), and (f) vary the number of extra bundles  $E$  at transition. Subplots (a) and (d) report the  $\min\{v, B\}$  efficiency metric (VB/maxVB), (b) and (e) report the traditional efficiency metric (CP/maxV), and (c) and (f) report the revenue metric (R/maxVB). The first set of subplots (a, b, c) fix  $E = 4$  and consider both  $\alpha = 1.05$  and  $\alpha = 1.0$  for RPAR. The second set of subplots (d, e, f) consider both  $BF = 0.3$  and  $BF = 0.7$ , and consider both  $\alpha = 1.05$  and  $\alpha = 1.0$  for RPAR. We have done experiments for several values of the parameters and observed a qualitatively similar trend across the parameters.

*Variation with BF.* One general observation based on the results in the figures is that SRPAR tends to dominate the performance of RPAR with respect to the  $\min\{v, B\}$  efficiency (subplots (a)) and revenue metrics (subplots (c)). The performance improvement provided by SRPAR over RPAR is most significant for low budget factors (3.8% and 7.8% on average) and qualitatively less at high budget factors. This can be seen in subplots (a, b, c) and by comparing the  $BF = 0.3$  and  $BF = 0.7$  results in subplots (d, e, f). The amount of improvement varies with distribution and is as high as 13% and 20% for the L4 legacy distribution (averaged over BF's 0.1 and 0.2) to an insignificant difference for the arbitrary distribution.

Even though we present the traditional efficiency metric, CP/maxV, in subplots (b), this metric is less meaningful when budgets are constrained because of the issues discussed in Section 4.5.2. As can be seen in the plots, although there is a marginal improvement for L4, L2 and arbitrary distributions, it marginally under performs for the matching and scheduling distributions.



While the performance of clock-proxy with respect to the efficiency metrics does not *strictly* improve with the budget factor, the performance generally improves with increasing  $BF$  for both RPAR and SRPAR. The revenue metric has a decreasing trend with budget factor in the matching distribution and an increasing trend in other distributions. The decreasing trend should not be a surprise because the goal of the clock-proxy auction is to maximize efficiency rather than revenue and it could very well be the case that they occur at different auction outcomes.

*Variation with  $E$ .* By varying the number of extra bundles,  $E$ , that can be submitted at the transition to proxy, we find that the  $\min\{v, B\}$  efficiency and revenue metrics improve with  $E$ , in return for allowing more bids to be submitted to the proxy. This is not the case for the  $\max V$  efficiency metric, possibly again because of the discrepancy in the choice of this metric for budget constrained bidders.

*Variation with  $\alpha$ .* Turning to the relaxation parameter  $\alpha$ , and clock-proxy with RPAR, we see from the subplots that the performance with  $\alpha = 1.05$ , a more relaxed rule, tends to dominate that for  $\alpha = 1.0$ , in general, but we do observe the reverse as well. The reason why such a behavior is possible is because we are working with a hybrid auction where (a) there is loss in information during the transition from clock to proxy and (b) the transitioning rules in the case of RPAR does not lead to submitting (i) true values for the clock bundles (but close to true values) and (ii) more bundles for the relaxed rule as compared to the tighter rule as one might expect. See the clock-proxy variation with RPAR in [Section 4.5.1](#).

Additional evidence that we observed in our experiments, and also suggestive of better performance with SRPAR than RPAR, is that the number of rounds in the clock auction with SRPAR tends to be greater than with RPAR. This is because not all straightforward bundles can pass RPAR. We also observe that the set of bundles submitted to the proxy in clock-proxy with SRPAR tends to be a *superset* of the bundles submitted by RPAR.

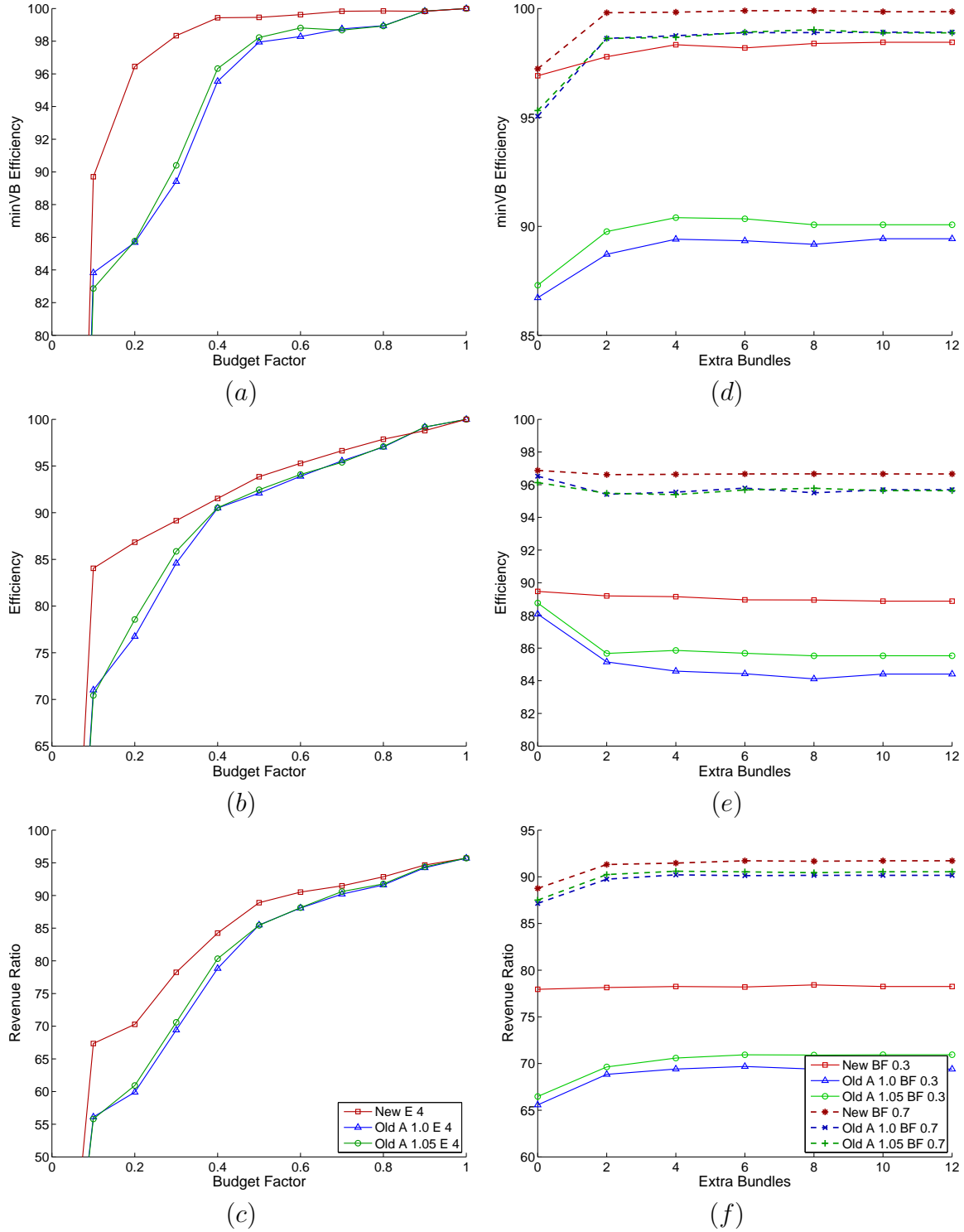


Figure 4-3: L4 Legacy – Decay and Linearly Random – 20 items and 20 bidders with 12.5 bundles per bidder.

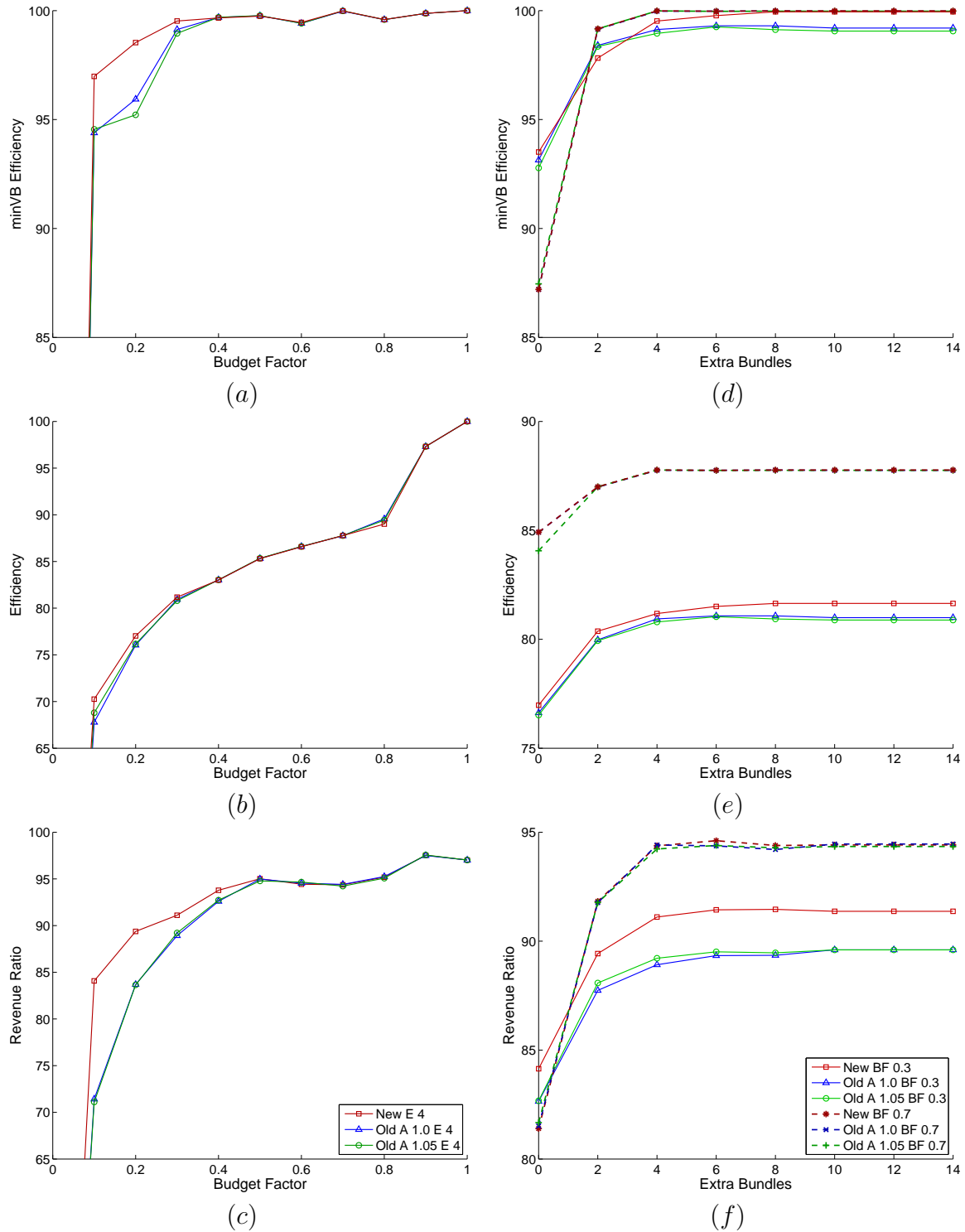


Figure 4-4: L2 Legacy – Uniform and Linearly Random – 20 items and 20 bidders with 15 bundles per bidder.

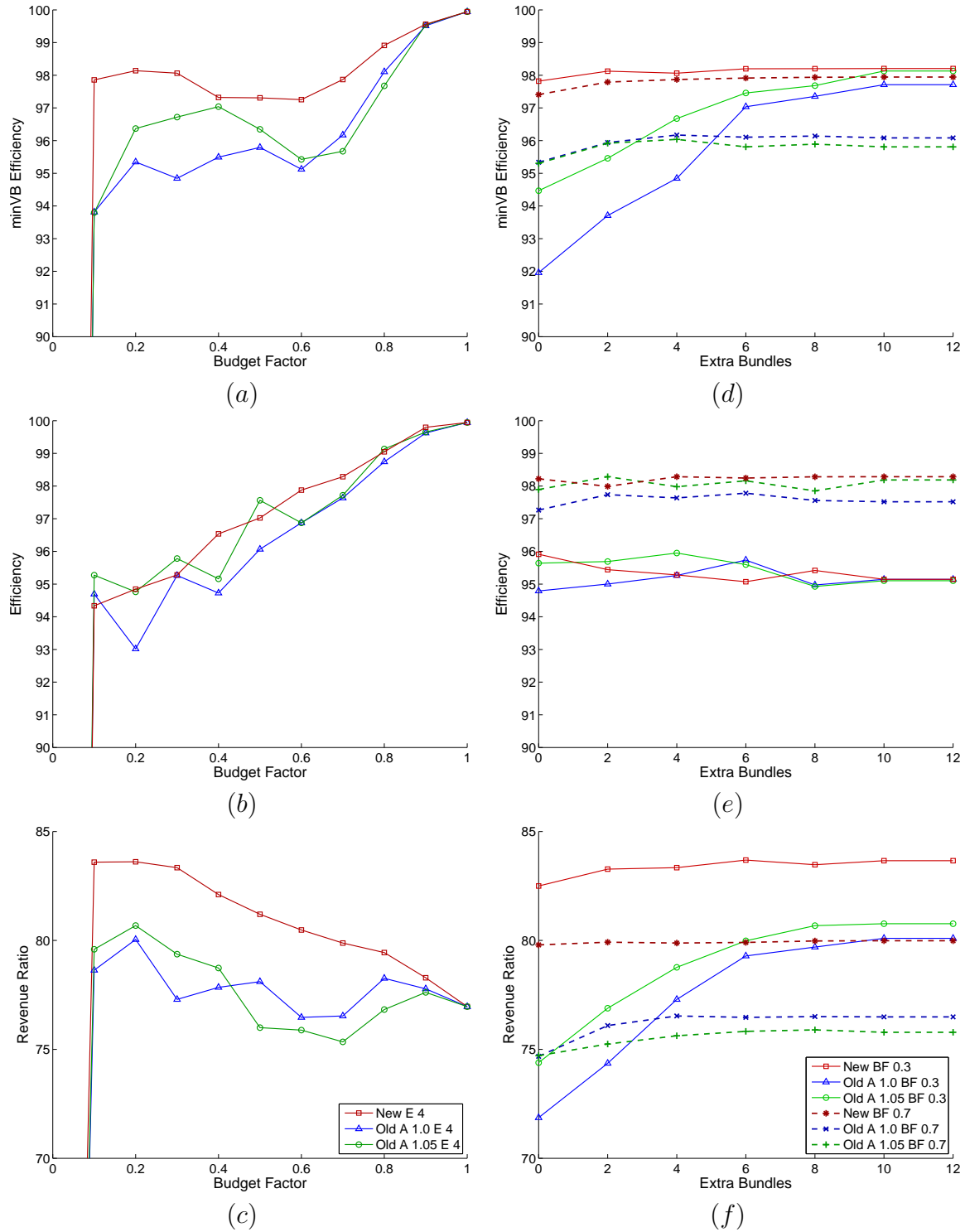


Figure 4-5: CATS – Matching – 32 items and total number of bundles randomly chosen between 400–500 (resulting in average of 13.3 bundles per bidder and 34 bidders).

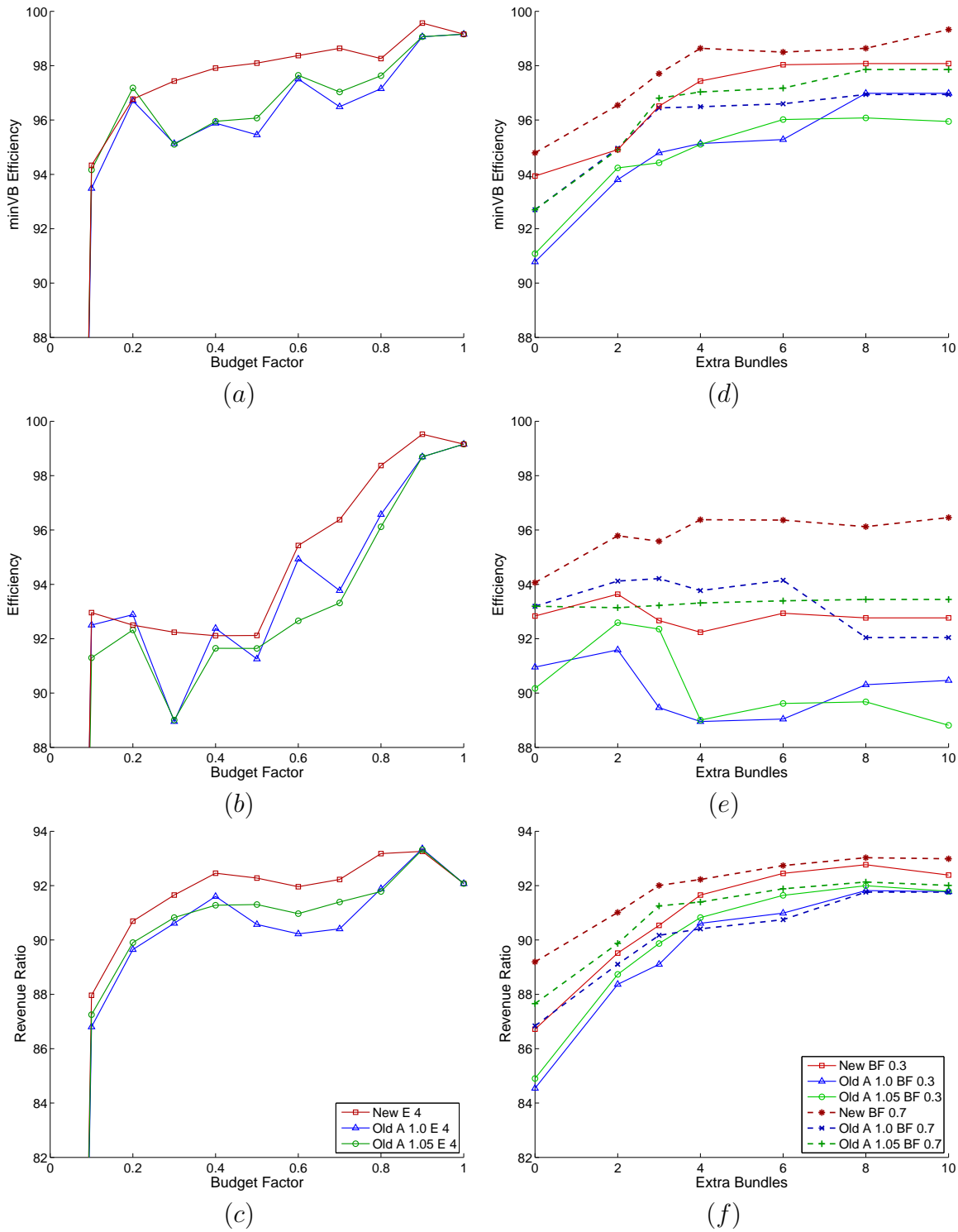


Figure 4-6: CATS – Arbitrary – 25 items and total number of bundles randomly chosen between 400–500 (resulting in an average of 8.4 bundles per bidder and 26 bidders).

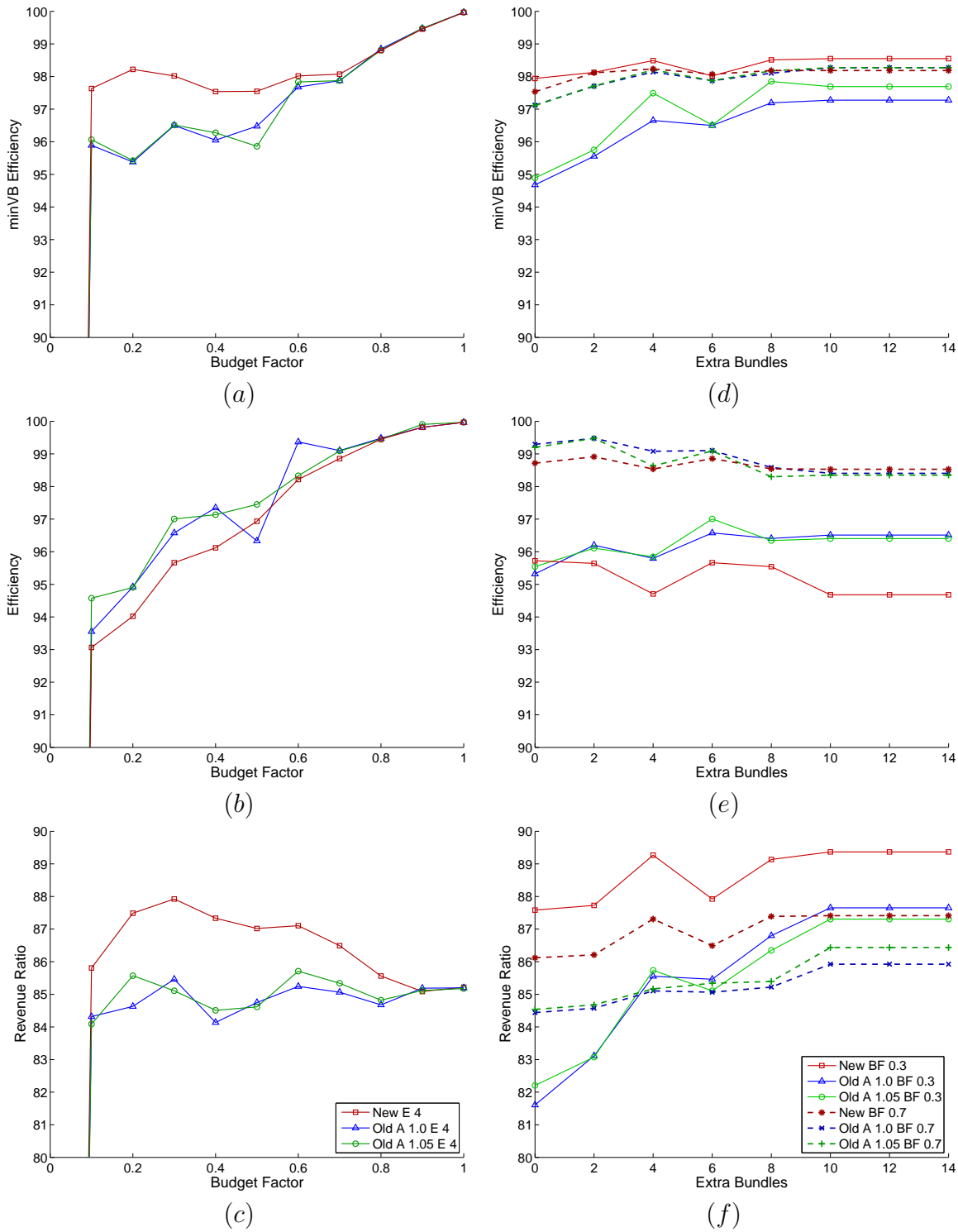


Figure 4-7: CATS – Scheduling – 25 items and total number of bundles randomly chosen between 200–400 (resulting in average of 18.6 bundles per bidder and 18 bidders.)

# Chapter 5

## Real-Time Approaches to Mitigate Airport Congestion

In this chapter, we study the current practices and future opportunities in managing real-time demand-capacity mismatches when there is a sudden drop in capacity in the terminal area (airports) due to various uncertainties (e.g., bad weather). We also briefly touch upon practices used during airspace capacity reductions also.

### 5.1 Introduction

Bad weather creates bottlenecks in the form of a sudden capacity drops in traffic flows within the air transportation system in airports as well as in airspace. Between April 2007 and March 2008, 94% of the days were affected by bad weather and on average there were about 4 airports with a capacity drop lasting about 6 hours every day.<sup>1</sup> It is not uncommon to observe extended periods with a 50% drop in capacity during bad weather. Sudden drops in capacity cause havoc in the entire system because airlines cannot operate as planned and this causes considerable delays to propagate in the system. For example, 65.6% of the delays in the NAS in 2007 were due to bad weather [BTS08]. So, efficient utilization of this capacity during bad weather is critical in order to contain the delays in the NAS.

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<sup>1</sup>Data from Metron Aviation.

The FAA should respond whenever the forecasted demand exceeds the forecasted capacity during any 15-minute interval, irrespective of the cause (which is usually weather, but could also be over-scheduling or aircraft congestion on the runway). The FAA manages minor or moderate drops in capacities by tactical airborne control procedures, such as re-routes and variations in speed. Major drops in capacities are resolved using ground holding tactics, where the aircraft are temporarily delayed at their respective departure airports.<sup>2</sup> The main idea is that as long as the delays are unavoidable, it is safer and less costly to delay the planes on the ground as airborne tactics are less safe and very expensive, especially in terms of fuel. (See [Bal07, H MV, VB06] for more information). This real-time response of the FAA is especially challenging due to the dynamic (with very little lead time) and uncertain nature of weather.

Ground holds issued by the FAA to manage airport capacity drops are called *Ground Delay Programs* (GDP) and airspace capacity drops are called *Airspace Flow Programs* (AFP). In a GDP, arrivals into a capacitated airport are controlled, and in an AFP, the flow of planes into a constrained airspace are controlled. GDPs have existed for more than 2 decades now, whereas AFPs are very recent (announced in 2006). Prior to the existence of AFPs, enroute congestion was also controlled by GDPs and were drafted under GDP for *severe weather avoidance procedures* (SWAP). But because several flights not in bad weather were controlled because of GDP-SWAPS and several flights in bad weather were not controlled by GDP-SWAP [Bre07], AFPs were introduced recently. In both, GDPs and AFPs, the delays are allocated to planes in order to match available capacities. This allocation of delays can be interpreted as a resource allocation problem with the resource being the arrival capacity, i.e., slots, assigned to the flights.

In the case of GDPs, airlines that are allocated new arrival slots, cannot operate as scheduled, and often there are associated delays resulting in passenger and crew misconnections. Airline recover their networks in real-time by swapping aircraft and

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<sup>2</sup>Like in the case of auctions, the notion behind controlling only arrivals for the demand-capacity imbalance is that controlling arrivals implicitly balances the departures also.



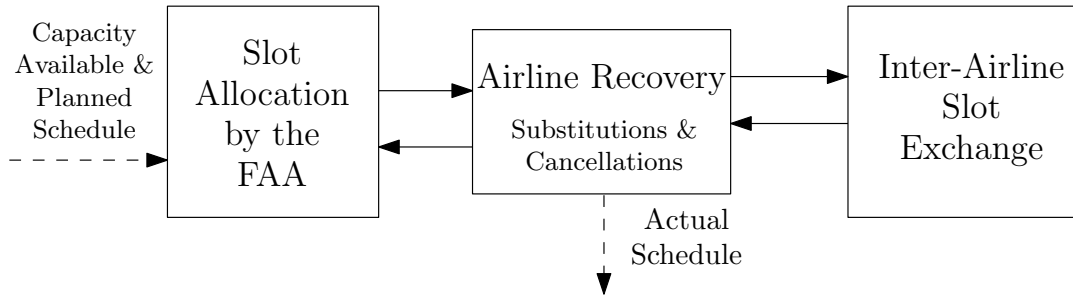


Figure 5-1: Slot allocation and exchange.

reassigning them to different flights, and by cancelling flights. These actions reallocate aircraft and crew resources to minimize both delay to passengers and the overall cost to the airline. Through recovery, airlines primarily aim to return to scheduled as quickly as possible. Slots assigned but not used by an airline are reallocated by the FAA to other airlines. This re-allocation can be thought of as an exchange of slots between airlines.

Fig. 5-1 provides a schematic diagram of this entire system. The iterative nature of the system is suggested by the pair of arrows between the different modules: capacity information in the system being updated with time; new allocation based on previous allocation and capacity update; exchange requests based on current allocation; and allocation of the exchange. The need for increased cooperation between the different stake holders, the FAA and the airlines, in this system has led to the emergence of the *Collaborative Decision Making* (CDM) paradigm. CDM is based on the recognition that data exchange and communication between the FAA and the airlines will lead to better decision making [Wam96]. In particular, CDM philosophy emphasizes that decisions with a potential economic impact on airlines should be decentralized and made in collaboration with the airlines whenever possible. CDM includes a wide-range of mechanisms for ATFM, but the primary tools include GDPs and AFPs.

In this chapter, we study the different components of this system and identify areas to direct future research. We pose some open questions and avenues for further improvement. We restrict our study primarily to GDPs because AFPs are still in the preliminary stages of development. Nevertheless, we point to some key differences

between GDPs and AFPs as we study the different aspects of a GDP.

**Organization of this chapter.** In [Section 5.2](#), we study the current capacity allocation procedures along with alternate procedures proposed in the literature. We then study airline recovery procedures in [Section 5.3](#). And in [Section 5.4](#), we discuss various design questions and challenges in designing an exchange. In [Section 5.5](#), we pose open questions, provide some preliminary results based on historical GDPs and provide a detailed experimental system design, with metrics and data sources, to address the open questions. Finally, we conclude with more open questions in [Section 5.6](#).

## 5.2 Capacity Allocation: Procedures During Ground Delay Programs

In this section, we describe the different elements of a GDP, the current procedures for slot allocation during a GDP, and finally compare these practices with it against alternate allocations schemes proposed in the literature. We adapt most of the information about GDPs, the current procedures, and literature from the chapter on ATFM by Hoffman, Mukherjee and Vossen [[HMV](#)].

During a GDP, there are always more flights vying for arrival slots than there are arrival slots. FAA gathers capacity information in the form of allowable landings (i.e., slots) in every 15-minute interval and assigns airborne and ground delay to the arriving flights to match the available capacity, while ensuring safe operations. For the purpose of planning GDPs, traffic managers assume a equispaced landing time for all aircraft even though different aircraft take different amounts of time to land. This is because the actual arrival order tends to shuffle anyway because of variable travel times and other minor uncertainties. The traffic managers specify a *program rate* that quantifies the number of landings allowed every 15 minutes (also called the airport arrival rate (AAR)). We refer to each landing interval as a landing slot. Traffic

managers treat airport capacity as a deterministic value. However, in reality, weather is highly stochastic and it is difficult to predict capacity exactly. Traffic managers address this issue by using airborne speed-control tactics and by dynamically revising the program, either by extending, shortening or cancelling the GDP.

Once capacity information is obtained, the set of flights affected by the GDP are identified as follows: flights are assigned to slots in the same sequence as the arrival schedule; all flights from and till the point at which an assigned slot is the same as the scheduled slot are identified as affected flights. The start time of a GDP is defined as the time when the first flight controlled by the GDP arrives and the end time of a GDP is when the last flight controlled by the GDP arrives.<sup>3</sup> Every arriving flight that is affected during a GDP is given a controlled time of arrival (CTA) corresponding to the time stamp of its allocated slot. We describe the details of the slot allocation procedure in the following section. Since en-route time can easily be estimated by the FAA and carriers, a controlled departure time (CTD) can also be computed by subtracting the en-route travel time from the CTA. The difference between the CTD of the flight and its desired departure time is the ground delay imposed by the FAA.

During a GDP, airlines have the same situational awareness as the FAA because of the Flight Schedule Monitor (FSM), a decision support system introduced by CDM. Airlines are informed about the control decisions of a GDP a few hours before the start of the GDP because flights have to be delayed on the ground before take-off. Although, in a special case of a GDP called *Ground Stops* controls are imposed with little or no notice. In a ground stop, the airport is closed for an emergency such as a sudden violent thunderstorm. However, in both GDPs and ground stops, flights that are airborne are exempt from the respective procedures. In a GDP, because it is difficult to make accurate forecasts well in advance, international flights and some long distance flights (that is, those exceeding a radius parameter chosen by traffic managers) are also exempt.

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<sup>3</sup>The only exception is the time when a GDP is cancelled ahead of time and all controlled flights are released. Here the end time of the GDP is the cancelled time even though the GDP already controlled some flights and the last controlled flight (that was held on the ground) could arrive well after the time the GDP was cancelled.

The boundaries of AFPs, unlike GDPs, correspond to lines in the airspace or polyhedral volumes of airspace. The rate of transition of planes across this line (either uni- or bi-directional) or through this affected region is controlled.

### 5.2.1 Current Slot Allocation Procedures During a GDP

The resource allocation problem to mitigate the demand-capacity imbalance by allocating ground delays, in other words slots, to the flights is well known in the literature as the *Ground Holding Problem* (GHP). Under CDM, the FAA initially allocates the available capacity during a GDP to the airlines with a procedure called the *Ration-By-Schedule* (RBS). Next, the FAA provides a medium for inter-airline exchange of slots using procedures like *Compression* and *Slot Credit Substitution*. The following description of these procedures are as described by Vossen and Ball [VB05].

The RBS algorithm is used to ration the arrival slots among the airlines on a first-scheduled-first-served basis by pushing back the arrival schedule to match the capacity. The strength of RBS is that it is very easy to implement, it is local and decentralized by airport and it is accepted by the airlines. The details of RBS are as follows:

**Algorithm 5.2.1. *Ration-By-Schedule.***

1. *Order the flights by their original scheduled time of arrival.*
2. *Select the first flight with an unassigned slot. If none exists, the algorithm terminates. Otherwise, assign the flight the first unassigned slot considering its scheduled time of arrival.*

The rationale behind slots being assigned based on scheduled arrival time as opposed to the most recent estimated arrival time (which could be different from scheduled arrival time because of various uncertainties) is that airlines are then not penalized for reporting a delay or cancellation (which is what happened prior to CDM in the *Grover Jack* approach [VB05]).

Vossen and Ball in [VB05] show that RBS, in fact, lexicographically minimizes the maximum ground delays allocated to the different flights. In practice, RBS also accounts for flights that are already airborne and those that are exempted from the GDP (for example, international flights, some long-haul flights) and respects the allocations of prior GDPs that were executed. The result of the RBS is an assignment of slots to airlines rather than to flights. Hence after this initial allocation, airlines are free to reschedule flights according to their private objectives through substitutions and cancellations. Substitutions and cancellations are very important from the standpoint of an airline. Vossen and Ball provide empirical results from actual GDPs showing that the number of substitutions and cancellations can be very large [VB06].

Substitutions and cancellations create gaps (open slots) in the arrival schedule. The utilization of the runway is improved using the *Compression* procedure that essentially moves flights up in the schedule to fill the open slots. The airlines have an incentive to provide information regarding open slots as they are in some sense “paid back” for the slots they release by getting priority for the next available slots that they are able to use. Compression is in fact a very simple inter-airline exchange mechanism premised on the principle that airlines like to schedule any flight as early as possible, with constraints restricting the new schedule no earlier than the original scheduled flight arrival times, that is, the earliest arrival time. The details of the compression algorithm (which is run periodically) is given below.

**Algorithm 5.2.2. *Compression.*** *The following steps are performed for each open slot  $s$  in the sequence of open slots,  $C$ , based on the current schedule. We refer to the owner of the slot  $s$  as  $A_s$ . We begin with the first open slot.*

1. *Check if any flight,  $f$ , of  $A_s$  can be moved **up** to slot  $s$  respecting its earliest arrival time. If there exists no such flight, go to step 2; otherwise swap the slot assignments of flight  $f$  to slot  $s$  and repeat step 2 for the newly vacated slot, denoted by  $s_f$ , whose owner continues to be  $A_s$ .*
2. *Determine the first flight  $f'$  in the schedule that belongs to another airline that can be moved **up** to use slot  $s$  respecting its earliest arrival time. If none exists,*

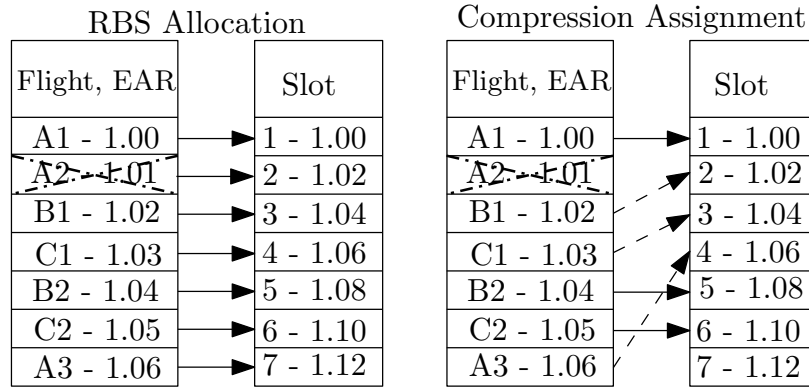


Figure 5-2: Illustration of the compression procedure

go to step 3; otherwise swap the slot assignment of flight  $f'$  to slot  $s$  and repeat step 2 with vacated slot  $s_{f'}$  whose new owner is  $A_s$ .

3. If no flight can be assigned to slot  $s$ , return to step 2 with the next open slot in  $C$ .

In Fig. 5-2, we illustrate an example of the compression algorithm implemented over an RBS allocation. In this example, A,B and C represent three different airlines with the earliest arrival time (EAR) as shown. We consider a scenario in a GDP where the time taken to land an aircraft doubles, from 1 to 2 minutes. RBS allocates the slots to the airlines by simply pushing the schedule down in time. We illustrate the compression algorithm when airline A cancels flight A2. The compression algorithm moves B1 and C1 up by one slot because of the following reasons: A3 does not arrive before 1.06; B1 arrives at 1.02; and C1 arrives at 1.04. A3 has a priority over every released slot and is allocated to the slot at 1.06 released by C1.

With compression, the slots that an airline cannot use are exchanged it in such a way that all airlines receive a reduction in delay. However, there are some key disadvantages of the compression procedure:

1. Compression requires a central authority like the FAA to run the procedure periodically (in discrete time-intervals not continuously) in real-time. Because it is not run continuously, traffic managers use the vacant slots of cancelled

flights during the time-interval to schedule general aviation flights that are not scheduled and do not compress often as the slots are occupied.

2. Compression lacks expressiveness being an inter-airline exchange. As a simple example, an airline cannot state explicitly what slot it needs in return for giving up a particular slot. In [Chapter 2](#), we discussed that airlines have a non-linear value for the slots over which they operate. This means they would be interested in a group of slots more than the individual pieces allocated separately. This is because airlines operate as a network and different elements of the network interact with each other and hence it might be essential to get all or none of the slots. See example in [Chapter 2](#). The same applied even to the case of the inter-airline exchange.
3. An airline has no guarantee when it cancels a slot that any of its other delayed flights will be moved up. Though there is a priority, there is no guarantee that this indeed will happen. Hence, airlines must cancel flights without the knowledge of the particular trade-offs being made. In the language of mechanism design, this is called the *exposure problem* i.e., the airline is exposed to the risk of cancelling a plane without receiving something in return.

To overcome some of these disadvantages, a more dynamic form of compression, called the *Slot Credit Substitution* (SCS)[\[How01\]](#), is currently in place. Under SCS, airlines can submit a “conditional cancellation” request of the form: “I am willing to release slot at time  $t_1$  by cancelling the flight  $f_1$ , if I can move flight  $f_2$  up into slots between time interval  $(t_2, t_3)$ ”. The FAA examines available flights in the time range  $t_1$  to  $t_3$  and tries to move a sequence of flights into earlier slots (including that at  $t_1$ ) to get a later slot (between  $t_2$  and  $t_3$ ) for the requesting airline. The FAA monitors such requests on a continuous basis, thereby implementing a continuous real-time compression procedure. SCS also provides increased trading opportunities over compression (i.e., more expressive than compression) because of the continuous response as well as the conditional nature of the requests. However, because SCS is limited to a 1-for-1 trading scheme, it has limited expressiveness and continues to

have the exposure problems when airlines prefer more complex schemes, for example, a 2-for-2 trade, which they must submit as two 1-for-1 trades.

Since March 2007, an *adaptive compression* procedure has been in effect [BE04]. This is essentially the compression algorithm running continuously in real-time, with airlines having the ability to control the flights that are a part of the adaptive procedure. The benefit of this procedure is that scheduled flights always get priority over flights that are not scheduled (general aviation) unlike the regular compression procedure that fitted them in the vacated slots before running the compression algorithm.

## 5.2.2 Comparing Current Procedures Against Alternate Procedures

In this section, we first compare RBS against other initial slot allocation procedures, then and then compare compression and SCS against other inter-airline exchange procedures proposed in the literature.

### Alternate Slot Assignment Procedures

Odoni was the first to formalize the Ground Holding Problem (GHP) of allocating ground delays to the flights during a GDP [Odo87]. There is extensive literature with models and algorithms that identify different strategies to assign ground delay. This has been summarized by Bertsimas and Odoni in [BO97] and by Hoffman, Mukherjee and Vossen in [HMV]. In this section, we highlight some important differences between RBS and the other models proposed in the literature.

Most past research has focused on a special case of the GHP - the single-airport ground holding problem (SAGHP). In this problem, it is assumed that only a single airport in the entire NAS is capacity constrained and all the remaining resources, including the other airports, are not constrained. The goal of the SAGHP is to assign ground delays only to the incoming flights at the capacity constrained airport. RBS, in fact, addresses a deterministic capacity version of the SAGHP. In deterministic



models, the capacity information for all future time intervals is assumed to be known in advance (see [Ter90]). As mentioned earlier, this is never the case. It is both a stochastic problem, because capacity estimates have a certain degree of uncertainty, as well as a dynamic problem, because forecasts must be updated frequently [Odo87]. There is a large body of literature in which the dynamic and stochastic versions of the SAGHPs are studied. This research highlights the trade-offs between airborne and ground delays, as detailed in [Odo87, ARJ87, RO93, AOR93, RO94, Rif98, Muk04, Han08].

A multi-airport ground holding problem (MAGHP), expands on the SAGHP by considering network effects, delay propagation and multi-airport capacity constraints. A MAGHP considers a network of airports and assign delays to flights, taking into account propagated delay of an aircraft downstream, i.e., flight connectivity. We illustrate the difference between MAGHP and SAGHP in the following example.

**Example 5.2.3.** *Consider an aircraft route from city  $U$  to city  $V$  and then from city  $V$  to city  $W$ . In the event where both cities  $V$  and  $W$  are affected by GDPs, MAGHP allocates slots for this aircraft in cities  $V$  and  $W$  so that the plane can actually make the connection at  $V$  and arrive at  $W$  at the allocated slot in  $W$ . SAGHP, on the other hand, might allocate slots in such a way that the aircraft is unable to arrive at the allocated slot at  $W$  if it lands at the allocated slot at  $V$ .*

Most of the literature on the MAGHP focuses on the deterministic capacity version of the problem. Vranas, Bertsimas and Odoni proposed the first model to solve MAGHP [BVO94b, BVO94a] and later, Bertsimas and Stock-Patterson proposed a formulation of MAGHP taking into account enroute capacities (airspace constraints) [BSP98]. Later, Bertsimas and Stock-Patterson further extended this to include re-routing [BSP00]. More recently, Bertsimas, Lulli and Odoni expanded the work of [BSP00] to achieve better computational performance [BLO08]. The Bertsimas, Lulli and Odoni model includes the en-route capacities with limited air sector capacities (although, airspace flow programs, introduced recently, does not constrain a sector but constrains the traffic across a line in space).

The advantage of a network-based MAGHP model is that it can simultaneously take into account several resource constraints, such as arrival and departure constraints at multiple airports, along with airspace constraints. The downside to network approaches is that they are computational much more difficult because of their large model sizes. The general MAGHP is known to be NP-Hard. Another important drawback of the network-based approach is that it fails to capture that *airlines have the ability to cancel flights*, which they often do as a part of their recovery (see [Section 5.3](#)). By cancelling flights, the airline has the ability to substitute its vacant slots with some of its other higher priority flights. Sometimes the airline is better off cancelling the flights than not cancelling them and dealing with the propagated delays and passenger and crew misconnections.

We make another observation of MAGHP allocations with different airline network structures. Consider a GDP at an airline's hub. Assume that the airline has enough resources to swap planes and crews so that it would rather not get an allocation from the network model that maintains connectivity because its response behavior could be very different. On the other hand, a regional carrier that might have *limited flexibility*, in terms of resources such as planes and crews that they can swap at the airport, could potentially be better off with a network model that ensures connectivity. We illustrate these with the following example.

**Example 5.2.4.** *In example 5.2.3, if airport  $V$  is the airline's hub then the airline might have enough resources (spare plane and extra crew) to depart leg  $V$  to  $W$  on time. In this case, an RBS allocation at  $W$  maybe be better than a MAGHP allocation because MAGHP allocation might provide the airline with a later slot than the RBS allocation. This is because MAGHP allocation assumes that the flight from  $V$  to  $W$  is going to use the same aircraft as that from  $U$  to  $V$  and since the latter is delayed the former cannot leave on time. But this is not the case. On the other hand if the airline is a regional carrier with just one flight into and out of  $V$ , then it would prefer the MAGHP allocation to the RBS allocation as in the latter case it would have to perform an additional step of trading with another airline and cannot be completely certain to get the slot it prefers.*

As we will see in [Section 5.3](#), airlines have several objectives in their recovery and the MAGHP network model does not capture the richness of the possible recovery strategies.

Apart from the modeling differences, the key difference between prior work in models for the GHP and RBS is in the notion of *fairness*. The GHP objective is to assign ground delays to individual flights in order to optimize a system-wide objective function, representing a central-planning paradigm [\[VB05\]](#). The CDM (and RBS) perspective, however, is that air traffic consists of a set of users (airlines) with diverse, often conflicting objectives and it is inappropriate to apply a common objective function across all airlines [\[BHOR03\]](#) and that airlines have some accountability to their published schedule. In essence, *fairness* as defined by CDM (implicitly) is achieved by awarding slots based on the planned schedule as published in the Official Airline Guide (OAG) which is created weeks in advance. With a slot allocation defined by this CDM approach, Ball et al. in [\[BHOR03\]](#) and later, Kotnyek and Richetta in [\[KR06\]](#) develop a static-stochastic version of the SAGHP. Fearing, Caramanis and Bertsimas also focus on fairness and formulate a version of the MAGHP that extends [\[BSP98\]](#) to include a network-based notion of fairness [\[FCB08\]](#). Their work is different from the usual MAGHP models in the sense that their model does not ensure flight connectivity but does take into account en-route capacities and hence is a network-based model. It extends the CDM notion of fairness at a single airport to the entire network. For the special case of no enroute capacities, the allocation of the Fearing, Caramanis and Bertsimas's model is exactly the same as independent RBS allocations at each of the capacity constrained airports.

There are fairness issues that result because of exemptions provided to long-haul flights. Airlines that disproportionately operate long-haul flights are often subjected to fewer GDP delays than those operating short-haul flights. For example, in RBS, all flights beyond a specific radius from the airport with the GDP are exempt from the GDP. The exemption of these flights, however, are an important mechanism for increasing overall system utilization by hedging against uncertainty in airport capacities.

The issue of what is *fair* is an underlying question for all the different alternate slot allocation schemes. All slot allocation mechanisms, including RBS, favor some airlines over the other based on the location of the GDP, type of carrier and the type of airline network structure. As discussed earlier this is either because of exemptions (as in the case of RBS) or because of partial assumptions on airline recovery (as in the case of the algorithms for SAGHP, that assume a single affected airport, or MAGHP, that do not assume cancellations of flights).

### **Alternate Inter-Airline Trading Schemes**

There is limited literature related to inter-airline trading schemes. The primary goal of this research has been to improve the expressiveness of the airlines by allowing airlines to state preferences over the slots in the trading mechanisms. Moreover, mechanisms detailed in the literature are quite restrictive, for example, most are restricted to operating without payments.

Vossen and Ball propose an alternative optimization-based method for compression in which airlines specify goal slots for flights among the slots they own [VB05]. The optimization model, an assignment problem assigning slots to flights, lexicographically minimizes the maximum deviation from the goal slots and minimizes total delay. Their objective and hence, definition of fairness, was motivated by RBS, as the authors showed that RBS lexicographically minimizes the maximum flight delays allocated to the airlines. The authors improve the slot exchange capability in [VB06] where airlines can submit to a central coordinator, like the FAA, general preferences of the form “ If I get these  $k$  slots, I am willing to give up these  $n$  slots”. The central coordinator then solves an optimization problem to determine the final allocation. They do not study any gaming issues that might result in such mechanisms.

Balakrishnan extends this approach to find a stable allocation where each airline declares the relative priorities of its flights and ranks the order of its preferences for all slots [Bal07]. This approach is well known in mechanism design as the *Top Trading Cycle* approach and has been used earlier for house allocation, school choice and kidney exchanges [AS99, AS03, RSU04]. The author shows that a stable allocation

need not exist when an airline owns more than one flight in a GDP, which could often be the case. Balakrishnan provides an algorithm to find the stable allocation, if it does exist, and proposes a sealed-bid mechanism with monetary payment schemes where airlines state directly to a central authority their value of trading one slot with another.

We will continue the discussion about exchanges in [Section 5.4](#) where we will discuss the objectives, questions and challenges in the design of an exchange.

### **5.3 Reconfiguration of Demand to Match Capacity: Airline Recovery**

In the context of operational demand-management (slot allocation and exchange between the airlines), the study of airline recovery goes hand-in-hand because of the following reason: an assignment of slots through operational demand-management is an assignment of slots to airlines and not flights because airlines have the ability to recover i.e., substitute and cancel planes. So, evaluation of the different demand-management schemes is based on the final distribution of slots to the flights based on airline recovery and not the initial allocation of slots to flights. Thus understanding the behavior of airlines to allocations, their options for recovery and their objectives is key in designing and developing an effective operational demand-management system and is the focus of this section.

A schedule disruption is often times caused when airlines cannot operate as planned because the resources they need, such as crew, planes, gates or landing slots become unavailable. Disruption to even one flight leg only can have widespread disruptive effects leading to irregular operations. Airport and airspace capacity shortages are one of the major causes of these disruptions. The other causes of schedule disruptions can be crew unavailability, mechanical failures, lack of ground resources, and longer than expected boarding or disembarking times.

To manage such disruptions, airlines operate control centers with real-time infor-

mation regarding the status of the network. During a disruption, airlines reconfigure their operations by replanning and recovering their aircraft, crew and passengers to match the available capacity. In this way, they *repair* the disrupted schedules in order to resume planned operations.

In this section, we provide an overview and a brief literature review of airline recovery processes used during irregular operations. We adapt most information in this chapter from Barnhart [Bar] and Bratu and Barnhart [BB06].

**Options for recovery.** We begin by briefly discussing the options that airlines have for schedule recovery.

- Swap resources such as planes and crews – Swapping resources can reduce considerable delay. For example, consider Fig. 5-3. We adapt this example from [Bar]. The schedule contains flight legs f1, f2, f3 and f4 with aircraft a1 operating f1 and f3; and aircraft a2 operating f2 and f4. Say f1 is delayed and f2 arrives as planned. If aircraft a1 and a2 are not swapped, the delay is propagated to passenger in f3 as they have to wait for the aircraft a1. Instead by swapping, the aircraft, no delay is propagated to downstream passengers as it is absorbed. Usually aircraft of the same type are swapped but sometimes aircraft of different type can also be swapped. In either case, several conditions must be met for a swap to occur: (a) the pilots must be able to operate the plane; (b) the crew schedules must be feasible; (c) the cost due to passenger disruptions resulting from different size planes must not outweigh the benefits of swapping; and (d) all maintenance requirements must be satisfied.
- Cancelling flight legs – Flight cancellations are usually invoked to decrease system wide delay propagation. For example, consider a situation where a late evening flight f1 is assigned to aircraft a1 and early morning flight f2 is assigned to an aircraft a2. If aircraft a1 is unavailable to operate f1 but aircraft a2 is, then either f1 can be cancelled or f1 can be operated with a2 and f2 can be cancelled, if a1 is still unavailable to operate f2. Cancelling a morning flight is preferred to cancelling a late-night flight to avoid passengers being disrupted

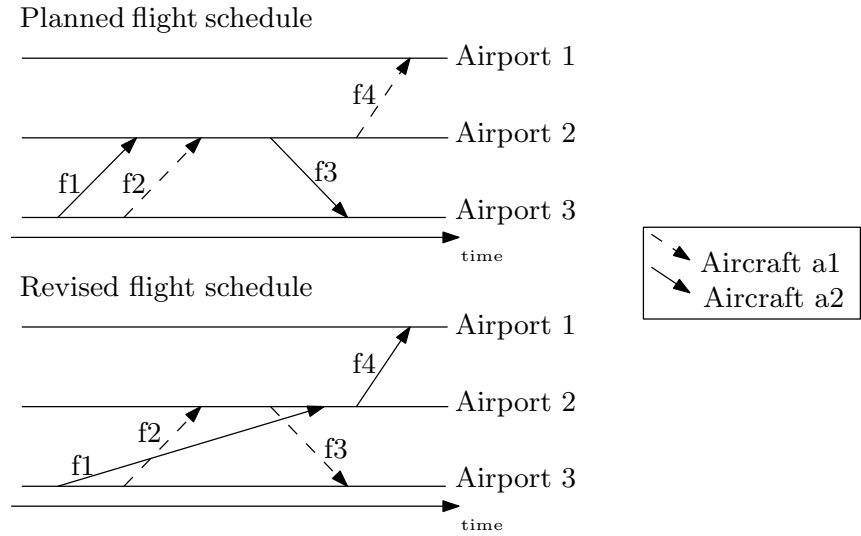


Figure 5-3: Example of an aircraft swap.

overnight, at least resulting in shorter delays. Cancellations must maintain aircraft flow conservation (i.e., balance) of the fleet type at each airport. Hence, cycles of two or more flight legs are typically cancelled, unless spare aircrafts are available to operate flights whose aircraft were assigned to cancelled flights.

- Delay or postpone departure times of flight legs – The purpose of delaying departure time is to prevent connecting passengers from missing their connections and/or ensure that necessary aircraft and crew are ready. Let us consider an example in the context of a GDP when two flights, f1 and f2 (arriving after f1), belong to an airline a. Based on the RBS allocation, the slot at t2 assigned to f2 is after the slot at t1 assigned to f1. If f2 is a high priority flight, compared to f1, with connecting passengers to an international flight and has an earliest arrival time before t1, then the airline is better off delaying f1's arrival to time t2 and allowing f2 to land at time t1. This example illustrates a scenario where departure is delayed to prevent passenger misconnections. Delaying departure is also useful when crew or aircraft are not ready and replacing them with other crew or aircraft is either not possible or expensive.
- Use *reserve crews* or spare planes – Airline typically have extra crew, known as reserve crew, and, possibly, aircraft resources to reduce the impact of disrup-

tions. Reserve crews are positioned strategically at hubs or other major airports and spare aircraft might be available at maintenance stations.

- Re-routing aircraft – Airlines are allowed to reroute planes. Examples include: (a) swapping aircraft flight assignments, or (b) re-routing an aircraft in air around bad weather. One of the differences between a GDP and an AFP is that for aircraft in a GDP, some amount of ground delay is inevitable, (unless the airline manipulates the delay but changing the arrival order of its flights) whereas in an AFP, the airlines can reroute around the constrained areas and remove themselves from the AFPs.
- Re-accommodation of disrupted passengers – Passengers whose itineraries are disrupted due to flight delays and cancellations must be reaccommodated on other available itineraries.

Even though capacity allocation is performed locally at each airport, airlines operate networks with aircraft, crew and passengers flowing over the network. Airline recovery, therefore, must take into account all network effects, with all schedule adjustments satisfying crew work rules, aircraft maintenance rules and safety regulations.

**Implementation.** There are three main resources in a recovery - aircraft, crew and passengers.

The *aircraft recovery problem* is to determine flight departure times, cancellations and revised routings based on maintenance requirements and aircraft balance constraints, including those at the start and end of recovery. At the end of recovery, aircraft must be positioned according to their original schedule locations to resume planned operations. Airline recovery can have several objectives such as minimizing the time to resume planned operations as scheduled originally; minimizing plane delay costs; minimizing passenger delay costs and loss of good will due to cancellations and delays; and minimizing the costs of using spare aircraft and reserve crews.

Comparing plane and passenger delay costs, it has become increasingly understood that delays to planes are different from delays to passengers [BB05]. Barnhart points out that if airlines do not capture the associated passenger disruption costs



in their recovery, the true cost of recovery is underestimated, potentially to a large extent [Bar]. For example, for the operations of a major U.S. airline, Bratu and Barnhart in [BB05] compare flight and passenger delays and found that

- Flight delays are not indicative of the magnitude of delay experienced by disrupted passengers. They point to an example on which the disrupted passengers experience an average of 419 minutes of delay where as the average flight delay was just 14 minutes.
- The cost of disrupted passengers are significant. In their study, only 4% of the passengers accounted for more than 50% of the passenger delay. These passengers are associated with significant recovery costs because of lodging, meals, re-booking (possibly on other airlines), and loss of passenger goodwill.

The *crew recovery problem* is to construct new schedules for disrupted and reserve crews to achieve schedule coverage. Objectives include minimizing cost, returning to plan as quickly as possible, and executing the least number of scheduled crew changes.

In *passenger recovery*, passengers who are disrupted (those who missed connections or whose flight was cancelled) must be accommodated on alternate itineraries.

Most airline recovery processes are sequential - aircraft recovery is followed by crew recovery and then, passenger recovery. This process simplifies and speeds up recovery considerably although it is sub-optimal as initial recovery decisions are made without regard to those that follow. Barnhart argues that even an estimate, although imprecise, of the downstream cost in aircraft recovery can lead to improved decisions [Bar]. Whatever the objective, the recovery model should be solved within a few minutes i.e., real-time, because otherwise the solution to the recovery can become infeasible.

### 5.3.1 Literature Review

Much of the literature in aircraft recovery deals with departure scheduling and cancellations, but ignores passenger disruptions and crew costs [JYKR93, CK97, TBY98].

Rosenberger, Johnson and Nemhauser present an optimization model for aircraft recovery that models passenger connections but does not fully capture passenger delays and costs [RJN03]. Bratu and Barnhart develop passenger-centric recovery models that make simultaneous decisions about aircraft, crew and passenger recovery [BB06].

Recovery models proposed in the event of a GDP include those of Vasquez-Marquez [VM91], Luo and Yu [LY97], Niznik [Niz01] and Vossen and Ball [VB06]. There is a large gap, however, between the airline recovery models proposed in a general context as discussed in the earlier paragraphs, and these recovery models. The GDP-based recovery models consider recovery primarily as a single airport problem, and formulate a simple assignment problem between flight legs and slots at that airport. These models do not consider resource swapping, such as swapping planes and crews, as a recovery option, and they do not capture passenger disruption costs, which can be quite considerable. When multiple airports in the airline's operations are subject to GDPs, it might be advantageous to expand on the single airport models and develop approaches for the entire system.

## 5.4 Inter-Airline Exchange

**Case for an exchange.** As mentioned earlier, airlines have the ability to substitute and cancel planes, irrespective of the type (local or network-based) or objective (fair or delay optimal) of the initial slot allocation procedure. So, in a GDP setting to reduce wastage of slots, it is essential to have an inter-airline exchange. Compression and SCS are examples, albeit quite simple, of such trading schemes. Well designed exchanges can considerably increase the efficiency of the system, as well as improve the utilization of slots by maximizing their usage. Vossen and Ball report a 24% increase in efficiency even with a simple 2-for-2 exchange [VB06].

An exchange is a decentralized approach that attempts to achieve a centralized efficient solution. Centralized slot allocation schemes are impractical, and sometimes impossible, to implement. This is, firstly, because different airlines can have very

different objectives, and secondly, airlines have complex recovery procedures that a centralized scheme cannot fully capture. Thirdly, such centralized models require proprietary information from the airlines, information that the airlines will not likely divulge. And fourthly, such models are large-scale models that need to be solved in minutes – an impractical feat. It is essential then to have a decentralized approach that provides autonomy of the different airlines to achieve an allocation that is preferred by all. It is this reasoning that led to decentralized schemes as the premise over which the CDM paradigm is built.

The initial slot allocation can be thought of as providing the airlines with initial ownership of slots that they can trade between other airlines, on a given day of operations. Airlines are both buyers and sellers of the slots in this exchange. We have motivated the problem for trading landing slots during reduced capacity periods, but this could very well be used for arrival and departure slots together, maybe also on other days with or without irregular operations. It is important to remember that any exchange that is designed for the airline should indeed be solved in real-time.

#### 5.4.1 Design Questions of an Exchange

In this section, we discuss some design questions that need to be addressed in developing an exchange. Ball, Donohue and Hoffman provide an interesting discussion of this topic in [\[BDH06\]](#).

**Exchanges at a single airport or multiple airports.** Current trading schemes, compression and SCS, occur at single airports. The design of a single airport efficient exchange is very much in its infancy, with multi-airport exchange far from implementation. In this chapter, we focus only on questions related to a single airport exchange, but we pose the question regarding multi-airport exchanges because there are situations where this could potentially be of interest. For example, say multiple airports are included in GDPs, along with AFPs in the airspace. With airlines operating as a network, an airline might value a set of slots in a sequence of airports affected by the GDPs, perhaps because these slots will service the path of one of its aircrafts.

The MAGHP is built on this notion (see example [5.2.3](#)), but one question to ask is whether such a market will be thick, meaning will there be enough bidders who buy and sell so that buyers get the packages they are looking for and the sellers can sell what they want? Theoretically it is interesting but one has to weigh its complexity with how often there are multiple airports that are connected by flights affected by GDP on the same day.

**No payments or payments in real or virtual currencies.** There are different approaches to designing an exchange. In the one without payments, airlines barter for slots among each other. Examples of such mechanisms include compression, SCS, or those proposed by Balakrishnan [[Bal07](#)] or Vossen and Ball [[VB05](#), [VB06](#)]. Airlines, however, might indeed be willing to make a monetary transaction to prevent excessive recovery costs. Although monetary exchanges can bring out the real economic value of the item, such exchanges have been vehemently opposed in the context of slot auctions. This is because airlines feel threatened to pay for slots that they currently get for free and that slot auctions are an easy way for the government to raise revenues. These issues are detailed in [Section 2.2](#). Another challenge associated with monetary exchanges is that the design of the initial allocation becomes crucial because it can create unwarranted transactions of money between the airlines. To avoid issues related to this, a mechanism can use virtual currency for trading. In this case, the initial allocation of currency to the airlines and its expiration time (so that a single airline does not hoard all the money) are important design elements.

**Package bidding.** As discussed in [Chapter 2](#), airlines have a non-linear value for the slots over which they operate. Because airlines operate as a network and different elements of the network interact with each other, an airline might be interested in getting a package of slots or none of them. See example in [Chapter 2](#). Package bidding can, therefore, potentially improve the performance of the exchange if airlines have an ability to state which slots they want to buy and which they want to sell. Package bidding avoids the exposure problem of getting only a part of the desired package – an issue in both the current schemes of compression and SCS. In the Vossen and Ball experiments, it was observed that package bidding with 2-for-2 slot

exchanges considerably increased the performance of the exchange [VB06]. Ball, Donohue and Hoffman suggest, however, that there is no overwhelming evidence that this complexity of package bidding is in fact necessary [BDH06]. They point out that a 2-for-2 bartering system can perform as well as a 1-for-1 system with side-payments. As with any design, it is important to study the improvement in the performance with different bidding schemes and different levels of expressiveness, and is especially important to understand how airlines value the slots.

**Batch vs. real-time exchange.** The adaptive compression and SCS mechanism currently employed are real-time continuous exchanges that respond as soon as a request (or bid) is placed by an airline in the system. An allocation of the continuous real-time exchange can depend on the order of the airline requests which can be treated, by some, as unfair. If an exchange is held at discrete time intervals, however, requests from all airlines can be batched until the end of the interval. This achieves a higher level of efficiency as more information is available to the system when making a decision about the allocation. This is a topic for further research as also pointed by [BDH06].

**Iterative price-based demand revealing exchanges or sealed-bid exchanges when there is a monetary transaction.** In price-based exchanges, prices are increased by a central clearing house and bidders respond to these prices with packages of slots they want at those prices (demand-revelation). In sealed-bid exchanges, the direct value information of the packages are actually revealed to the auctioneer. Price-based exchanges, in our opinion, are more suitable for the airline setting as: (a) airlines have to solve complex large-scale recovery models to obtain the true value of a single bundle; and (b) they will not likely reveal their value for a slot as it is proprietary, competition-sensitive information that they would not be willing to divulge.

## 5.4.2 Challenges of an Exchange

In this section, we discuss some challenges associated with the design of an exchange.

**Information overloading.** Even with simple exchange schemes like compression and SCS, industry experts believe that airlines have excessive pressures with the recovery process and the process of participating in such trading schemes [Wam04]. With inter-airline trading schemes that are more sophisticated, information overloading is a major concern. At the forefront of this issue is striking the correct tradeoff between building a sophisticated mechanism, with very good theoretical properties that requires a complex response process from the airlines, and implementing a simple and easy to respond mechanism with a reasonable set of theoretical properties.

**Design challenges.** (This discussion is adapted from Parkes, Kalagnanam and Eso [PKE01] and Balakrishnan Hamsa.) The objective of the inter-airline trading mechanism is to maximize allocative efficiency of the overall exchange based on the reported preferences of the airlines. In the design of an exchange, it is important to provide incentives to the airlines so that the following are satisfied:

- Airlines are not worse off by participating in the exchange. This is called *individual rationality* or voluntary participation, which means all agents, here airlines, have a positive expected utility to participate in the exchange.
- Airlines are encouraged to state their true preferences with minimal gaming opportunities. In mechanism design, this is more formally defined as *incentive compatibility* that states that truthful bidding is a Bayes Nash Equilibrium. In such equilibria agents maximize their expected utility by stating true values, given that every other agent also bids truthfully.

In the design of such ideal mechanisms we are faced with the following theoretical challenges.

- For mechanisms without payments, where airlines just state their preferences, Balakrishnan shows that a stable allocation need not exist. This means that airlines are sometimes better-off by forming smaller coalitions and hence, might break away, and no stable allocation can exist [Bal07].

- When monetary transactions are allowed, it is important that there is *budget-balance*. This means that the clearing house, the FAA in our context, may not run at a loss. We know, however, that individual rationality, budget-balance and exact efficiency is impossible because of the Myerson-Satterwaite impossibility result [MS83]. The implication is that even the well known Vickrey-Clarke-Groves (VCG) mechanism that is efficient, individually rational and strategy proof is not a budget-balanced mechanism.

To overcome this issue, for monetary exchanges, Parkes, Kalagnanam and Eso suggest an alternate sealed-bid combinatorial exchange which is individually rational, budget-balanced, fairly efficient and approximately incentive-compatible [PKE01]. They propose a *threshold* payment rule that minimizes manipulation and promotes efficiency. Balakrishnan suggests the use of the mechanism proposed by [PKE01] with threshold payments in the airline setting with airlines required to state their values for a 1-for-1 trade directly to the FAA [Bal07]. Lubin et al. develop the first fully expressive (k-for-n) iterative combinatorial exchange [LJC+08] extending the approach of [PKE01] for a general setting.

## 5.5 Experimental Setup

Based on the above discussion of the different components of the real-time capacity allocation problem, there are several questions that one can ask. We focus on one of the issues that we think is important in deciding the direction of future contributions to this area.

We investigate the importance of *connectivity* in the capacity allocation problem. Should the allocations be local or network-based? What are their trade-offs? Are multiple airports with GDP really an issue? In other words, are there markets that are connected that are affected by GDP at the same time (e.g., LGA, JFK and EWR are not connected by planes and top the list in the number of GDPs issued at these airports)? What percentage of the planes are affected by consecutive capacity

constraints? And how many consecutive capacity constraints does an aircraft meet on average, if affected? By consecutive capacity constraints, we mean consecutive GDPs. More generally, it can be GDPs and AFPs or just AFPs.

We think of RBS as a local approach and the different approaches to MAGHP discussed in [Section 5.2.2](#) as network-based allocation schemes. We want to understand if the different schemes favor airlines with different network structures. For example, if RBS approach is preferred at a hub of an airline, while a network-based approach is preferred for airlines with limited flexibility at the airport with GDP. If so, possibly, the preferred solution is a hybrid allocation model using a mix of local and network-based approaches, depending on the individual airline networks and preferences.

We perform preliminary analysis of past GDP data and propose an experimental design of the operational demand-management system that will enable us to answer questions on connectivity. This experimental design can also be used to answer other questions regarding the design of the initial allocation or exchange procedures. We present some metrics and the required sources of data to quantify and evaluate these metrics. We do not perform the actual experiments and leave them as a part of future work.

### 5.5.1 Preliminary Statistics about GDPs

Metron Aviation provided GDP data for all the GDPs from April 1, 2007 to March 31, 2008. There were 1359 GDPs during this period with an average length of about 395 minutes per GDP. Of the 366 days, 343 days experience a GDP, and there were approximately 4 GDPs per day on average.

Interestingly, 90% of the GDPs in the US are associated with 14 airports. Listing airports in order of decreasing numbers of GDPs we have - Newark (EWR), New York - LaGuardia Airport (LGA), San Francisco (SFO), Chicago O'Hare (ORD), New York - John F. Kennedy (JFK), Philadelphia (PHL), Boston - Logan(BOS), Atlanta (ATL), Minneapolis St.Paul (MSP), Teterboro (TEB), Houston (IAH), Las Vegas(LAS), Dallas Fort Worth (DFW) and Chicago - Midway (MDW). On at least



40% of the days, EWR, LGA, SFO, ORD and JFK had a GDP. Three of these airports belong to the New York – New Jersey area, and there are no flights between these airports.

The departure delay for each GDP is defined as the total over all the flights in a GDP of the controlled time of departure (CTD) at departure minus the estimated time of departure when the flight was first controlled (BETD). The average departure delay per aircraft associated with non-exempt aircraft was 67.4 minutes, with an average of about 184 non-exempt aircraft per GDP.

According to Metron Aviation, in the 2007 calendar year, the slot utilization was 97%. This means there were 3% of the slots allocated that were left open. A slot is considered open if the flight occupying the slot was cancelled from the program and was not replaced by another flight.

### 5.5.2 Slot Allocation, Recovery and Exchange system

To understand the importance of connectivity in the slot allocation stage, we envision a system which includes all the modules – slot allocation, exchange and recovery – as in [Fig. 5-1](#). Most experiments in the literature are performed either for a single airport GDP allocation (SAGHP) using only local recovery models (involving this the airport with GDP) or a network-based GDP allocation (MAGHP) without a recovery model for an airline. So, they either do not capture the true cost of recovery and network effects involved or do not capture the recovery itself. Through our proposed design, we aim to compare local and network allocation models with a more realistic system where airlines perform network based recovery in the presence of an exchange. We describe the details of the design, as well as some assumptions, about each component of the proposed system.

**Initial Allocation.** In comparing between local and network-based initial allocation schemes, we restrict to deterministic procedures at first. Among deterministic allocation schemes, we think of RBS as a local approach and network model by Bertsimas, Lulli and Odoni [[BLO08](#)] as a network-based scheme. If we also include

AFPs in the experiments, we can also compare RBS against Fearing, Caramanis and Bertsimas’s model [FCB08].

**Airline recovery.** Airline recovery is critical in understanding the role of connectivity in the initial allocation as slot are allocated to airlines and not to flights. The airline recovery module helps in identifying the airline’s preferred assignment of flights to slots. For airline recovery, we use recovery models like those by Bratu and Barnhart [BB06] (*Disrupted Passenger Model* (DPM)) because of the following reasons: they capture costs by optimizing the integrated response taking into consideration the interaction and interdependencies between crew and passengers.

**Inter-airline exchange.** It is important to include an exchange procedure in the system design as airlines often trade vacant unused slots of their cancelled flights to receive better slots in return for their remaining flights. Since the design of an exchange is still in the preliminary stages, we restrict to the simple compression procedure at first. We ignore unscheduled flights for the experiments. When comparing local and network-based allocations, it is important to note that to some extent a network-based scheme by itself accounts for an exchange as the allocation assumes flight connectivity which is one of the many goals in airline recovery.

**Implementation.** Initially, we restrict only to GDPs (and no AFPs). The program rate at the airports with capacity drops along with the planned schedule are the inputs to the slot allocation procedure. Our goal is to understand various metrics across the different airlines for different allocation schemes, so, we could either let all the airlines or a selected few airlines (based on different network structure, see Section 5.5.3) participate in the simulation. The issue with the latter is that we might have to artificially simulate compression for airlines that are not present in the simulations. In this sense, the former case with all airlines is preferable as long as we can obtain the necessary data. For simplicity, for the purpose of simulation, we assume that all participating airlines use the same recovery model although this is not the case in practice as different airlines may have different objectives.

Because we restrict to deterministic procedures, the drop in capacity is exactly known and there are no revisions to GDPs. Hence the initial allocation is performed

only once. After this, we solve the recovery and perform a few rounds of compression followed by recovery to obtain the final allocation and schedule of the airlines. Based on these schedules and the slots used we evaluate different metrics as described in [Section 5.5.3](#). We compare the different airlines based on these metrics for the different allocation schemes.

### 5.5.3 Metrics to Evaluate the Allocation

We propose a set of metrics classified by the different stakeholders involved – system, airlines and passengers – to evaluate the final allocation of the different initial allocation schemes. We also provide some metrics to characterize the different airlines that can help us in understanding the observations of the experiments. Our metrics relate only to deterministic schemes and primarily to GDPs. These metrics, however, can easily be extended to the case of AFPs and stochastic schemes.

#### **Metrics and procedures to evaluate the results.**

*System metrics.*

- Utilization of the slots allocated - This metric refers to the ratio of the number of slots utilized to the total available slots and is computed by airport and airline.

*Airline metrics* - The following metrics are computed by airline, market (O-D pairs, long-haul and short-haul flights) and aircraft fleet type. They are also evaluated for the overall system.

- Plane delay cost and minutes - This is the amount of plane delay and the cost associated with it. Based on the final schedule of the airlines we can compute the total delay. The delay cost can be based on ground and air minutes with non-linear cost functions.
- Average Ground delay associated with all the flights (and per flight controlled by a GDP) - This refers to the amount of ground delay associated with the planes.

- CDM's metric of fairness - This refers to the distance (in the number of slots) from the allocated position to the position in the initial scheduled ordering of the flights. This can be computed based on the initial schedule and the final schedule of the flights to slots.

*Passenger metrics* - The following metrics are computed by airline, market (O-D pairs, long-haul and short-haul flights), and aircraft fleet type. They are also evaluated for the overall system.

- Passenger delay minutes or disruption costs - This is the amount of delay associated with passengers and their corresponding disruption cost because of missed connections or cancelled flights.

**Characteristics of airlines.** The following list helps us compare metrics and network structures between different airlines. This can help a central planner identify possible fairness issues and an airline to redefine its network so that it is not penalized during the allocation process because of its type of operations.

1. Type of airline - legacy, regional or low-cost.
2. DOT on-time performance - cancellation rate and delay minutes.
3. Percentage of long-haul and short-haul flights.
4. Type of network structure for the airline.
5. Type of airports (hub or not) for the airline that get often affected with a GDP.
6. Percentage of flights affected by a GDP by airport and for the system.
7. Percentage of flights affected by consecutive GDPs among total affected by at least one GDP.
8. Slack in the airline network or schedule - Airlines provide slack in both the block times and the turn times. Based on estimated flying times between an origin and destination and the required turn times that can be computed from

an airline's schedule, the amount of extra buffer (slack) that is provided in the block times and the turn times can be evaluated.

9. Flexibility of recovery - Number of planes of the same type at the airports with GDP so that airlines have the flexibility of swapping their resources.
10. Percentage of connecting passengers at the airports with GDP.

#### 5.5.4 Data Needed and Possible Sources

*GDP information.*

- Real-time information of the GDP including the airports, the reason, the duration and the program rate is provided by the FAA.<sup>4</sup>
- Metron Aviation has information of all the GDPs including their start time, end time, control time, revisions, cause and number of exempt and non-exempt flights.

*Data required for allocation schemes and inter-airline exchange methods.*

- Schedule data - The Airline Service Quality Performance (ASQP) data source provides information about schedules.<sup>5</sup> Fields include aircraft tail number, carrier, gate departure delay, taxi-out time, airborne time, taxi-in time, and scheduled departure and arrival times.
- Exempt flights - The flight specific data from Enhanced Traffic Management System (ETMS) indicates the flights that are exempt. If this data set is unavailable, a simple heuristic can be used based on the number of flights that were exempt. Usually, the airborne flight, international flights and the long-haul flights (can be computed based on radius) are the flights that are exempt.

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<sup>4</sup>Real time GDP information <http://www.fly.faa.gov/ois/>

<sup>5</sup>Airline Service Quality Performance, <http://www.apo.data.faa.gov/asqp/entryASQP.asp>

*Data need for a recovery model.* For each participating airline we need the following information:

- Schedule data along with fleet information - same as above.
- Passenger disruption cost, plane delay cost (operational), passenger delay cost.
- Passenger flow data - This may be hard to obtain for all the airlines. Instead we can use the passenger delay calculator developed by Bratu and Barnhart [BB05] that computes passenger delay based on the booking data and flight schedule. Alternatively, we can use the technique proposed by Zhu [Zhu08]. She develops an optimization technique to estimate the booking data from aggregate passenger data based on 10% of random tickets collected by the FAA and the airline schedules that day. We can then use the passenger delay calculator to compute the delay or disruption cost to the airlines.
- Crew and maintenance information, if available.

## 5.6 Future Work and Open Questions

The problem of real-time congestion mitigation ties together three individually complex problems where one can raise several interesting open questions. As an operational (real-time) problem, the challenge is that all procedures should be quick to run, attain a certain level of efficiency and should necessitate a simple response behavior from an airline. As discussed in this chapter, information overloading, wherein the system is so complex that the controllers cannot respond, is a concern that has to be addressed while developing any sophisticated model. Here, we briefly describe some areas with open questions.

*Fairness:* The question of fairness in allocating the slots to the airlines is a wide open problem. Answering whether fairness means delay sharing or cost sharing or order preserving or system optimal in terms of delay to planes or passenger or fastest recovery, is quite complex and requires agreement of all the stake holders involved. Exemptions and uncertainty of weather adds another dimension to fairness.

*Enroute speeding.* Managing uncertainty in weather by controlling the enroute speed of planes during a GDP or AFP can reduce considerable amount of wastage in the slots and can smooth out arrival flows without holes in the landing sequence. Currently aircrafts travel in sequence along jet-routes where it can be difficult to pass ahead of another aircraft. Minor speed modifications are used by the FAA as an airborne tactic to queue up landings but the question is can they be used more extensively? Safety and deployment of parallel (either side-to-side or top and bottom) jet routes is an important question that has to be addressed first. Atilla is a commercially available tool for airlines to dynamically monitor their assets as well as make minor speed modifications<sup>6</sup>.

*Re-design airline networks.* Can airlines design their network in a way that potentially reduces the impact on their operations during uncertainties such as bad weather? Instead of building better and better recovery models, maybe the answer is re-designing their network so that it is robust to uncertainties. For example, if an airline splits its network, into two parts, so that each is self-contained and is such that the part most affected by the congestion is separated from the remaining pieces then the delay does not propagate into the system as much. In other words, can we divide airline networks into high and low priority networks so that the low priority network is the one that takes the hit during congestion?

*Inter-airline Exchange.* Inter-airline exchange is essential building block for achieving an efficient utilization of slots, in a decentralized fashion. A design of a scheme that understands and incorporates slot preferences of an airline and respects the proprietaries of their value/preference is a complex yet an interesting open problem.

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<sup>6</sup>The web link for Atilla is <http://www.freeflight.com/aerospace/products/attila/index.html>





# Chapter 6

## Conclusions

Airport congestion mitigation in the NAS is critical to the economy and nation's air system as congestion delays costs billions of dollars every year. The current airport and airway infrastructure cannot be scaled to meet future demand as anticipated by the FAA [ATA08]. So, in addition to the capacity enhancement measures, such as addition of new runways and the deployment of modern technologies, demand-management techniques, that control the level of demand level to match the available capacity (landing slots), are essential. In this thesis, we study two demand-management techniques – strategic (medium-term) and operational (real-time) approaches – to mitigate airport congestion and address different aspects of their design.

**Strategic demand-management approach.** Slot auctions have been proposed as an efficient means of allocating slot resources at congested airports. They have been considered as an alternative to grand-fathering and lottery that are currently used, so as to promote competition and efficiency. Several aspects of the design of slot auctions have to be resolved and addressed before its implementation. We focus on two aspects of the design – airline valuation model and activity rules. We argue that airlines must be treated as budget-constrained bidders in the slot auction.

*Airline Valuation Model.* We propose the Aggregated Integrated Airline Scheduling Fleet Assignment Model (AIASFAM) that identifies the most profitable

package of slots in return to the auction prices. In order to capture the profitability accurately, it is important that it is a network-based model that performs both scheduling and fleeting decisions. Models in the literature that make similar decisions on a network require detailed information and take several hours to solve [LB04]. We think it is important for a valuation model to be solved quickly between two rounds of an auction. We use different discretizations at different airports based on the level of congestion at an airport and improve the computational speed on the model. This thesis points the trade-off between the coarser levels of discretization resulting in faster computation time and plane count approximation and shows the need for developing approximate plane count constraints for faster solution times of large scale airline scheduling and fleet assignment models. We propose a time-count constraint and experimentally show its effectiveness in covering all the operations. The AIASFAM that uses the time count constraint takes less than 20 minutes on average for a real airline carrier. AIASFAM is computationally efficient and found to be also robust to data uncertainty in the experiments.

*Activity Rules.* We have introduced the concept of a strong activity rule. This is a property that we think activity rules should satisfy whenever possible because it disallows provably inconsistent bidding strategies but does not prevent a bidder from straightforward bidding. Good iterative auction design should promote straightforward, *demand-revealing* behaviors on the part of bidders [AC04, Par06].

But the oft-proposed revealed-preference activity rule (RPAR) fails this requirement on two accounts: it allows inconsistent bidding for quasi-linear bidders and rejects straightforward bids of budget-constrained bidders. Recognizing this shortcoming of RPAR, we developed two strong activity rules, one for budget-constrained bidders and one for bidders with quasi-linear utility functions. The strong activity rules (SRPARs) are formulated as linear feasibility problems and can easily scale to economically meaningful problems. We develop a simple

impossibility result to show that no rule can be simultaneously strong for both budget-constrained and non budget-constrained bidders, unless the bid-taker can identify whether or not a bidder is budget-constrained. On the other hand, while the SRPAR for budget-constrained bidders is not *strong* for bidders with quasi-linear utilities, it nevertheless allows truthful bidding in that setting and is likely to be useful in practice. We derive relaxations of SRPARs, that allow some strategic behavior without preventing straightforward bidding, and could be of interest in practical auction implementations.

The proposed strong activity rules provide a small, but significant improvement in the efficiency and revenue properties of the clock-proxy auction in experiments, when populated with maximally straightforward bidders. The relative performance from SRPAR over RPAR improves for tighter budget constraints, and when the RPAR-based clock-proxy auction provides bidders with less flexibility in the transition from clock to proxy.

**Real-time demand-management approach.** Ration-by-Schedule (RBS) is a first-scheduled-first-slot allocation policy that is currently used in a GDP when there is a sudden drop in capacity due to an unforeseen event. This is considered as a fair policy and is adopted by FAA and the airlines. Network-based policies, proposed in the literature, unlike RBS, strives to achieve a system optimal by partially capturing the airline recovery procedures, i.e., flight connectivity, in the initial allocation. We argue that the network-based policies do not, in fact, cannot capture the richness of the airline recovery procedures. We ask the question whether partial recovery hurts or helps the airlines. We study the trade-offs between local and network-based policies and present examples where some airlines based on their network structure and location of GDPs prefer different allocations. Since the initial allocation is followed by inter-airline exchanges, our hypothesis is that a simple decentralized local initial allocation may suffice, compared to a network-based allocation scheme, leaving it to the airlines to state their preferences and trade with other airlines. This calls for a well-designed exchange. Compression is a simplistic exchange that is currently in

place. In this thesis we propose a system design with all the three modules – initial slot allocation, network-based airline recovery and exchange like compression to evaluate the fairness levels and the need for flight connectivity with various metrics and different airline characteristics that we propose.

# Bibliography

- [AC04] LAWRENCE M. AUSUBEL and PETER CRAMTON. *Auctioning many divisible goods*. Journal of the European Economic Association, 2, 480-493, April-May 2004, 2:480–493, 2004.
- [AC05] LAWRENCE M. AUSUBEL and PETER CRAMTON. *Mock auction of LaGuardia arrival/departure slots*, 2005. (Presentation).
- [ACM06] LAWRENCE M. AUSUBEL, PETER CRAMTON, and PAUL MILGROM. *The clock-proxy auction: A practical combinatorial auction design*. In Cramton *et al.* [CSS06], chapter 5.
- [AF07] Federal Aviation Administration (FAA). *Aerospace Forecast for fiscal years 2008–2025*, 2007.
- [Afr67] SYDNEY N. AFRIAT. *The construction of utility functions from expenditure data*. International Economic Review, 8(1):67–77, 1967.
- [AM02] LAWRENCE M. AUSUBEL and PAUL MILGROM. *Ascending auctions with package bidding*. Frontiers of Theoretical Economics, 1(1):1–42, 2002.
- [AM06] ———. *Ascending proxy auctions*. In Cramton *et al.* [CSS06], chapter 3.
- [AOR93] GIOVANNI ANDREATTA, AMEDEO R. ODONI, and OCTAVIO RICETTA. *Models for the ground holding problem*. In GIOVANNI ANDREATTA and AMEDEO R. ODONI, eds., *Large-scale computation and information processing in air traffic control*, pages 125–168. Springer-Verlag, 1993.
- [ARJ87] GIOVANNI ANDREATTA and GIORGIO ROMANIN-JACUR. *Aircraft flow management under congestion*. Transportation Science, 21(4):249–253, 1987.
- [AS99] ATILA ABDULKADIROGLU and TAYFUN SONMEZ. *House allocation with existing tenants*. Journal of Economic Theory, 88:233–260, 1999.
- [AS03] ———. *School choice: A mechanism of design approach*. American Economic Review, 93(3):729–747, 2003.
- [ATA07] Air Transport Association. *Outlook: Reaching for the skies?*, 2007.
- [ATA08] ———. *Cost of Delays*, 2008.

- [Aus04] LAWRENCE M. AUSUBEL. *An efficient ascending-bid auction for multiple objects*. *American Economic Review*, 94(5):1452–1475, 2004.
- [Aus06] ———. *An efficient dynamic auction for heterogeneous*. *American Economic Review*, 96(3):602–629, 2006.
- [BAB<sup>+</sup>07] MICHAEL O. BALL, LAWRENCE M. AUSUBEL, FRANK BERARDINO, GEORGE DONOHUE, MARK HANSEN, and KARLA HOFFMAN. *Market-based alternatives for managing congestion at New Yorks LaGuardia airport*. In *Proc. of AirNeth Annual Conference*. 2007.
- [Bal07] HAMSA BALAKRISHNAN. *Techniques for reallocating airport resources during adverse weather*. In *Proc. of the IEEE Conference on Decision and Control*. 2007.
- [Bar] CYNTHIA BARNHART. *Irregular operations: Schedule recovery and robustness*. In CYNTHIA BARNHART, PETER BELOBABA, and AMEDEO R. ODONI, eds., *The Global Airline Industry*, chapter 9. Wiley. (Forthcoming).
- [BB05] STEPHANE BRATU and CYNTHIA BARNHART. *An analysis of passenger delays using flight operations and passenger booking data*. *Air Traffic Control Quarterly*, 13:1–27, 2005.
- [BB06] ———. *Flight operations recovery: new approaches considering passenger recovery*. *Journal of Scheduling*, 9:279–298, 2006.
- [BCI<sup>+</sup>05] CHRISTIAN BORGS, JENNIFER CHAYES, NICOLE IMMORLICA, MOHAMMAD MAHDIAN, and AMIN SABERI. *Multi-unit auctions with budget-constrained bidders*. In *Proc. of the ACM conference on Electronic commerce*, pages 44–51. 2005.
- [BDH06] MICHAEL O. BALL, GEORGE DONOHUE, and KARLA HOFFMAN. *Auctions for the safe, efficient and equitable allocation of airspace resources*. In Cramton *et al.* [CSS06], chapter 20.
- [BE04] MICHAEL BRENNAN and CHRISTOPHER ERMATINGER. *Principles of adaptive compression for ground delay programs*, 2004. Memorandum.
- [Bel05] PETER BELOBABA. *Impacts of 9/11 on us airline performance*. In *MIT Research and Development Conference*. November 2005.
- [BHOR03] MICHAEL O. BALL, ROBERT HOFFMAN, AMEDEO R. ODONI, and RYAN RIFKIN. *A stochastic integer program with dual network structure and its application to the ground-holding problem*. *Operations Research*, 51(1):167–171, 2003.

- [BK01] JEAN-PIERRE BENOIT and VIJAY KRISHNA. *Multiple-object auctions with budget-constrained bidders*. *Review of Economic Studies*, 68:155–179, 2001.
- [BKL02] CYNTHIA BARNHART, TIMOTHY S. KNIKER, and MANOJ LOHATEPANONT. *Itinerary-based airline fleet assignment*. *Transportation Science*, 36(2):199–217, 2002.
- [BLO08] DIMITRIS J. BERTSIMAS, GUGLIELMO LULLI, and AMEDEO R. ODONI. *The air traffic flow management problem: An integer optimization approach*. In *Integer programming and combinatorial optimization*, pages 34–46. 2008.
- [BO97] DIMITRIS J. BERTSIMAS and AMEDEO R. ODONI. *A critical survey of optimization models for tactical and strategic aspects of air traffic flow management*. Technical report, Massachusetts Institute of Technology, 1997.
- [Bre07] MICHAEL BRENNAN. *Airspace flow programs - a fast path to deployment*. *Journal of Air Traffic Control*, 49(1):51–55, 2007.
- [BSP98] DIMITRIS J. BERTSIMAS and SARAH STOCK-PATTERSON. *The air traffic flow management problem with enroute capacities*. *Operations Research*, 46(3):406–422, 1998.
- [BSP00] ———. *The traffic flow management rerouting problem in air traffic control: a dynamic network flow approach*. *Transportation Science*, 34(3):239–255, 2000.
- [BT97] DIMITRIS J. BERTSIMAS and JOHN N. TSITSILKLIS. *Introduction to Linear Optimization*. Athena Scientific, 1997.
- [BTS08] Bureau of Transportation Statistics. *Airline On-Time Statistics and Delay Causes*, 2008.
- [BVO94a] DIMITRIS J. BERTSIMAS, PETER B. VRANAS, and AMEDEO R. ODONI. *Dynamic ground-holding policies for a network of airports*. *Transportation Science*, 28(4):275–291, 1994.
- [BVO94b] ———. *The multi-airport ground-holding problem in air traffic control*. *Operations Research*, 42(2):249–261, 1994.
- [CFR] *Code of Federal Regulations, Title 14, Part 93*.
- [CG98] YEON-KOO CHE and IAN GALE. *Standard auctions with financially constrained bidders*. *Review of Economic Studies*, 65(1):1–21, 1998.
- [CG00] ———. *The optimal mechanism for selling to a budget-constrained buyer*. *Journal of Economic Theory*, 92:198–233, 2000.

- [Cho02] RIC J. CHOLANKERIL. *Evaluation of an auction mechanism for allocating airport arrival slots*. Master's thesis, Massachusetts Institute of Technology, 2002.
- [CK97] JIA-MING CAO and ADIB KANAFANI. *Real-time decision support for integration of airline flight cancellations and delays, part i: Mathematical formulation*. *Transportation Planning and Technology*, 20:183–199, 1997.
- [Cra95] PETER C. CRAMTON. *Money out of thin air: The nationwide narrowband pcs auction*. *Journal of Economics and Management Strategy*, 4:267–343, 1995.
- [Cra98] PETER CRAMTON. *Ascending auctions*. *European Economic Review*, 42(3-5):745–756, 1998.
- [Cra03] ———. *Electricity market design: The good, the bad, and the ugly*. In *Proc. of the Hawaii International Conference on System Sciences*. 2003.
- [Cra07] ———. *The revealed preference activity rule*, 2007. (Working Paper).
- [CS07] PETER CRAMTON and STEVEN STOFT. *Colombia firm energy market*. In *Proc. of the Hawaii International Conference on System Sciences*. 2007.
- [CSS06] PETER CRAMTON, YOAV SHOHAM, and RICHARD STEINBERG, eds. *Combinatorial Auctions*. MIT Press, 2006.
- [Dan92] JOSEPH DANIEL. *Peak-load-congestion pricing and optimal capacity of large hub airports: With application to the Minneapolis St.Paul airport*. Ph.D. thesis, MIT, 1992.
- [Day07] ROBERT W. DAY. *A revealed-preference activity rule for quasi-linear utilities with budget constraints*, 2007. (Working Paper).
- [DoE01] Dot Econ Ltd. *Auctioning Airport Slots*, 2001.
- [Don05] RUI DONG. *Combinatorial clock auction for airport time slots: An agent-based analysis*. Technical report, Harvard University, 2005. Senior thesis.
- [DOT08] Department of Transportation. *Aviation Congestion*, 2008.
- [FAA07] Federal Aviation Administration (FAA). *The Next Generation Air Transportation System Financing Reform Act of 2007*, 2007.
- [Fan03] TERENCE FAN. *Market Based Airport Demand Management - Theory, Model and Application*. Ph.D. thesis, Massachusetts Institute of Technology, 2003.
- [FCB08] DOUG FEARING, CONSTANTINE CARAMANIS, and DIMITRIS J. BERTSIMAS. *A practical approach to balancing fairness and delay in a network of ground-holding programs*, 2008. (Working Paper).



- [FO01] TERENCE P. FAN and AMEDEO R. ODonI. *The potential of demand management as a short-term means of relieving airport congestion*. In *Proc. of EUROCONTROL-FAA Air Traffic Management R&D Review Seminar*. 2001.
- [Fru72] WILLIAM E FRUHAN. *The Fight for Competitive Advantage: A Study of the United States Domestic Trunk Air Carriers*. Harvard University Press, 1972.
- [Gib73] ALAN GIBBARD. *Manipulation of voting schemes: A general result*. *Econometrica*, 41:587–602, 1973.
- [GIP79] DAVID M. GREThER, R. MARK ISAAC, and CHARLES R. PLOTT. *Alternative methods of allocating airport slots: Performance and evaluation*. Technical report, Polinomics Research Laboratories, 1979.
- [GIP89] ———. *The Allocation of Scarce Resources: Experimental Economics and the Problem of Allocating Airport Slots*. Westview Press, 1989.
- [Han08] MIKE HANOWSKY. *A Model to Design a Stochastic and Dynamic Ground Delay Program Subject to Non-Linear Cost Functions*. Ph.D. thesis, Massachusetts Institute of Technology, 2008.
- [HBPZ08] PAVITHRA HARSHA, CYNTHIA BARNHART, DAVID C. PARKES, and HAOQI ZHANG. *Strong activity rules for iterative combinatorial auctions*, 2008. (Under Submission).
- [HmV] ROBERT HOFFMAN, AVIJIT MUKHERJEE, and THOMAS W.M. VOSSEN. *Air traffic flow management*. (Working Paper).
- [Hof04] KARLA HOFFMAN. *Hybrid clock proxy auction - implementation details*, 2004. (Private Correspondence).
- [How01] KEN HOWARD. *Slot credit substitution*, 2001. Internal CDM Memorandum.
- [JYKR93] AUMAD I. Z. JARRAH, GANG YU, NIRUP KRISHNAMURTHY, and ANANDA RAKSHIT. *A decision support framework for airline flight cancellations and delays*. *Transportation Science*, 27:266–280, 1993.
- [KLPD05] ANTHONY M. KWASNICA, JOHN O. LEDYARD, DAVE PORTER, and CHRISTINE DEMARTINI. *A new and improved design for multiobject iterative auctions*. *Management Science*, 51(3):419–434, 2005.
- [KMR03] JANOS KORNAI, ERIC MASKIN, and GERARD ROLAND. *Understanding the soft budget constraint*. *Journal of Economic Literature*, 41(4):1095–1136, 2003.

- [KR06] BALZS KOTNYEK and OCTAVIO RICHETTA. *Equitable models for the stochastic ground holding under CDM*. *Transportation Science*, 40:133–146, 2006.
- [LB04] MANOJ LOHATEPANONT and CYNTHIA BARNHART. *Airline schedule planning: Integrated models and algorithms for schedule design and fleet assignment*. *Transportation Science*, 38(1):19–32, 2004.
- [LBS06] KEVIN LEYTON-BROWN and YOAV SHOHAM. *A test suite for combinatorial auctions*. In Cramton *et al.* [CSS06], chapter 18.
- [LDC04] LOAN LE, GEORGE DONOHUE, and CHUN-HUNG CHEN. *Using auction-based slot allocation for traffic demand management at Hartsfield Atlanta airport: A case study*. In *National Research Council Transportation Research Board Meeting*. January 2004.
- [LJC<sup>+</sup>08] BENJAMIN LUBIN, ADAM JUDA, RUGGIERO CAVALLO, SÉBASTIEN LAHAIE, JEFFREY SHNEIDMAN, and DAVID C. PARKES. *ICE: An Expressive Iterative Combinatorial Exchange*. *Journal of Artificial Intelligence Research*, 2008. (To appear).
- [LR96] JEAN-JACQUES LAFFONT and JACQUES ROBERT. *Optimal auctions with financially constrained buyers*. *Economic Letters*, 52:181–186, 1996.
- [LY97] SONGJUN LUO and GANG YU. *On the airline schedule perturbation problem caused by ground delay programs*. *Transportation Science*, 31(4):298–311, 1997.
- [Mas00] ERIC S. MASKIN. *Auctions, development and privatization: Efficient auctions with liquidity-constrained bidders*. *European Economic Review*, 44:667–681, 2000.
- [Mil00] PAUL MILGROM. *Putting auction theory to work: The simultaneous ascending auction*. *Journal of Political Economy*, 108:245–272, 2000.
- [MP07] DEBASIS MISHRA and DAVID C. PARKES. *Ascending price Vickrey auctions for general valuations*. *Journal of Economic Theory*, 127(1):335–366, 2007. Also, see technical report.
- [MS83] ROGER B. MYERSON and MARK A. SATTERTHWAITE. *Efficient mechanisms for bilateral trading*. *Journal of Economic Theory*, 29(2):265–281, 1983.
- [Muk04] AVIJIT MUKHERJEE. *Dynamic stochastic optimization models for air traffic flow management*. Ph.D. thesis, University of California at Berkeley, 2004.
- [Niz01] TIM NIZNIK. *Simplified substitutions*. In *Proc. of the AGIFORS Airline Operations Meeting*. 2001.

- [NYC07] New York Aviation Rule Making Committee. *Report*, December 2007.
- [Odo87] AMEDEO R. ODONI. *Flow control of congested networks*, chapter The flow management problem in air traffic control, pages 269–288. Springer-Verlag, 1987.
- [PAN08] The Port Authority of New York and New Jersey. *Hearing on congestion management in New York airspace*, 2008.
- [Par06] DAVID C. PARKES. *Iterative combinatorial auctions*. In Cramton *et al.* [CSS06], chapter 2.
- [Pet08] MARY E. PETERS. *End gridlock on runway*. The New York Times, July 2008.
- [Pit06] CAROLYN PITCHIK. *Budget-constrained sequential auctions with incomplete information*, 2006. (Working Paper).
- [PKE01] DAVID C. PARKES, JAYANT R. KALAGNANAM, and MARTA ESO. *Achieving budget-balance with Vickrey-based paymentschemes in exchanges*. In *Proc. 17th International Joint Conference on Artificial Intelligence (IJCAI-01)*, pages 1161–1168. 2001.
- [PRRS03] DAVID PORTER, STEPHEN RASSENTI, ANIT ROOPNARINE, and VERON SMITH. *Combinatorial auction design*. *Proc. of the National Academy of Sciences*, 100(10):11153–11157, 2003.
- [PSB06] ALEXANDER PIKOVSKY, PASHA SHABALIN, and MARTIN BICHLER. *Iterative combinatorial auctions with linear prices: Results of numerical experiments*. In *Proc. of the 8th IEEE international conference on E-Commerce Technology*, volume 0, pages 39–39. 2006.
- [PU00a] DAVID C. PARKES and LYLE H. UNGAR. *Iterative combinatorial auctions: Theory and practice*. In *Proc. of the National Conference on Artificial Intelligence*, pages 74–81. 2000.
- [PU00b] ———. *Preventing strategic manipulation in iterative auctions: Proxy agents and price-adjustment*. In *Proc. 17th National Conference on Artificial Intelligence*, pages 82–89. 2000.
- [Reg01] FEDERAL REGISTER. *High-density airports; take off and landing slots, slot exemption lottery, and slot allocation procedures: LaGuardia Airport, NY*. 66(113):31731–31748, June 2001.
- [Rif98] RYAN M. RIFKIN. *The static stochastic ground holding problem*. Master’s thesis, Massachusetts Institute of Technology, 1998.
- [RJN03] JAY M. ROSENBERGER, ELLIS L. JOHNSON, and GEORGE L. NEMHAUSER. *Rerouting aircraft for airline recovery*. *Transportation Science*, 37(4):408–421, 2003.

- [RO93] OCTAVIO RICHETTA and AMEDEO R. ODONI. *Solving optimally the static ground holding policy problem in air traffic control*. *Transportation Science*, 27(3):228–238, 1993.
- [RO94] ———. *Solving optimally the static ground holding policy problem in air traffic control*. *Transportation Research*, 28(Part A):167–185, 1994.
- [Roc87] JEAN-CHARLES ROCHET. *A necessary and sufficient condition for rationalizability in a quasi-linear context*. *Journal of Mathematical Economics*, 16(2):191–200, 1987.
- [RSB82] STEPHEN J. RASSENTI, VERON L. SMITH, and ROBERT L. BULFIN. *A combinatorial auction mechanism for airport time slot allocation*. *Bell Journal of Economics*, 13(2):402–417, 1982.
- [RSU04] ALVIN E. ROTH, TAYFUN SONMEZ, and M. UTKU UNVER. *Kidney exchange*. *Quarterly Journal of Economics*, 119(2):457–488, 2004.
- [Sal97] DAVID J. SALANT. *Up in the air:GTE’s experience in the MTA auction for personal communication services licenses*. *Journal of Economics and Management Strategy*, 6(3):549–572, 1997.
- [San02] TUOMAS SANDHOLM. *Algorithm for optimal winner determination in combinatorial auctions*. *Artificial Intelligence*, 135(1–2):1–54, 2002.
- [Sat75] MARK A SATTERTHWAITTE. *Strategy-proofness and Arrow’s conditions: Existence and correspondence theorems for voting procedures and social welfare functions*. *Journal of Economic Theory*, 10:187–217, 1975.
- [SB06] TUOMAS SANDHOLM and CRAIG BOUTILIER. *Preference elicitation in combinatorial auctions*. In Cramton *et al.* [CSS06], chapter 10.
- [SSGL05] TUOMAS SANDHOLM, SUBHASH SURI, ANDREW GILPIN, and DAVID LEVINE. *CABOB: a fast optimal algorithm for winner determination in combinatorial auctions*. *Management Science*, 51(3):374–391, 2005.
- [TBY98] BENJAMIN G. THENGVALL, JONATHAN F. BARD, and GANG YU. *Balancing user preferences for aircraft schedule recovery during irregular operations*. *IIE Transactions*, 32:181–193, 1998.
- [Ter90] MOSTAFA TERRAB. *Ground holding strategies for air traffic control*. Ph.D. thesis, Massachusetts Institute of Technology, 1990.
- [TKN89] DUSAN TEODOROVIC and EMINA KRCMAR-NOZIC. *Multicriteria model to determine flight frequencies on an airline network under competitive conditions*. *Transportation Science*, 23:14–25, 1989.
- [Uni08] United States Government Accountability Office. *DOT and FAA actions will likely have a limited effect on reducing delays during summer 2008 travel season*, July 2008.

- [USC05] *US Code Collection, Title 49, Sections 41714-41719*, 2005.
- [Var06] HAL R. VARIAN. *Revealed preference*. In MICHAEL SZENBERG, LALL RAMRATTAN, and ARON A. GOTTESMAN, eds., *Samuelsonian Economics and the Twenty-First Century*, chapter 6. Oxford University Press, 2006.
- [VB05] THOMAS W.M. VOSSEN and MICHAEL O. BALL. *Optimization and mediated bartering models for ground delay programs*. *Naval Research Logistics*, 53(1):75–90, 2005.
- [VB06] ———. *Slot trading opportunities in collaborative ground delay programs*. *Transportation Science*, 40(1):29–43, 2006.
- [VM91] ALBERTO VASQUEZ-MARQUEZ. *American airlines arrival slot allocation system (asas)*. *Interface*, 21:42–61, 1991.
- [Voh07] RAKESH VOHRA. *Paths, cycles and mechanism design*. Technical report, Kellogg School of Management, 2007.
- [VSV07] SVEN DE VRIES, JAMES SCHUMMER, and RAKESH V. VOHRA. *On ascending vickrey auctions for heterogeneous objects*. *Journal of Economic Theory*, 127(1):95–118, 2007.
- [Wam96] MICHAEL WAMBSGANSS. *Collaborative decision making through dynamic information*. *Air Traffic Control Quarterly*, 4:107–123, 1996.
- [Wam04] ———. *CDM initiative*, 2004. (Private Correspondence).
- [Wil01] ROBERT WILSON. *Activity rules for an iterated double auction*. In WILLIAM F SAMUELSON and KALYAN CHATTERJEE, eds., *Game Theory and Business Applications*, chapter 12. Springer, 2001.
- [WW00] PETER R. WURMAN and MICHAEL P. WELLMAN. *Aba: a progressive, anonymous-price combinatorial auction*. In *ACM Conference on Electronic Commerce*, pages 21–29. 2000.
- [Zhu08] YING ZHU. *Evaluating airline delays: the role of airline networks, schedules and passenger demands*. Master’s thesis, Massachusetts Institute of Technology, 2008.