

Modality and Inquiry

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Abstract

The possibilities we consider or eliminate in inquiry are *epistemic possibilities*. This dissertation is mainly about what it is to say or believe that something is possible in this sense. Chapter 1 ('Epistemic Contradictions') describes a new puzzle about epistemic modals and uses it to explore their logic and semantics. Chapter 2 ('Nonfactualism about Epistemic Modality') situates the work of chapter 1 into a larger picture of content and communication, developing a broadly expressivist account of the language of epistemic modality. Chapter 3 ('Content and Modal Resolution') argues that states of belief should be understood as relativized to an inquiry, understood formally as a certain way of dividing up logical space.

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Contents

1	Epistemic Contradictions	9
1.1	Problem	9
1.2	Relational semantics for epistemic possibility	17
1.3	Domain semantics for epistemic possibility	20
1.4	Consequence	28
1.5	Content and communication	35
1.6	Epistemic necessity operators	43
1.7	Toward probability operators	44
1.8	Outstanding issues	49
2	Nonfactualism about Epistemic Modality	55
2.1	Introduction	55
2.2	Descriptivism about epistemic modals	57
2.3	Epistemic contradictions	60
2.4	Assertability and disagreement	63
2.5	Conflicting intuitions	65
2.6	States of mind	68
2.7	Expressing states of mind	84
2.8	Problems solved	91
2.9	On expressivism and nonfactualism	96
3	Content and Modal Resolution	99
3.1	The map picture	99
3.2	Deduction	103
3.3	Resolutions of logical space	106

3.4	Deduction redux	110
3.5	Availability	111
3.6	Directions for further development	115
	Bibliography	117

Chapter 1

Epistemic Contradictions

1.1 Problem

I want to make some observations about the language of epistemic modality and then draw some consequences.

The first observation is that these sentences sound terrible.

- (1) # It is raining and it might not be raining
- (2) # It is raining and possibly it is not raining
- (3) # It is not raining and it might be raining
- (4) # It is not raining and possibly it is raining

All of these sentences are odd, contradictory-sounding, and generally unassertable at a context. They all contain modal operators which, in these sentential contexts, are default interpreted epistemically. (Just what the epistemic reading of modal operators is remains to be made precise—getting clearer on that is the point of this paper—but the motivation for calling the reading ‘epistemic’ is the intuitive idea that epistemically modalized clauses convey information about some epistemic state or a state of evidence.) I will take it that at the relevant level of abstraction, the logical form of the first two sentences is this:

$$(\phi \wedge \Diamond \neg \phi)$$

and the logical form of the next two is this:

$$(\neg \phi \wedge \Diamond \phi)$$

using ‘ \Diamond ’ schematically for natural language epistemic possibility operators.¹ We will have a need to refer back to conjunctions of these forms often, so let me call an instance of one of these two schemata an *epistemic contradiction*.

Epistemic contradictions are defective. Why?

It is tempting to try to connect the defect to Moore’s paradox, as follows. As Moore and others have noted, sentences like these:

(5) It is raining and I do not know that it is raining

(6) It is not raining and for all I know, it is raining

are odd, contradictory-sounding, and unassertable, just like (1)-(4) above. Now plausibly, we have a grip on why Moore-paradoxical sentences are defective: they involve the speaker in some kind of pragmatic conflict. For instance, if it is conventionally understood that, in making an assertion in a normal discourse context, one usually represents oneself as knowing what one says, then in uttering (5) or (6), one will end up representing oneself as both knowing something and also as knowing that one does not know it. It is not coherent to intend to represent oneself in this way, and so one therefore expects (5) and (6) to strike us as defective. (The appeal to some pragmatic tension like this one is the usual response to Moore’s paradox, though the details vary across theorists.²) Note that this line

¹ I take it that in English these operators include, on the relevant readings, the pure modals ‘might’, ‘may’, and ‘could’, sentential operators constructible via expletives from these (‘it might be that’ etc.), and the sentential operators ‘possibly’ and ‘it is possible that’. I will abstract from any tense information contributed by the pure modals. Let me stress that by ‘ \Diamond ’ I do not have in mind complex operators containing overt attitude verbs, such as ‘for all I know it might be that’.

² See Hintikka [1962] and Unger [1975] for classic statements of the pragmatic approach, and Williamson [2000] and Stalnaker [2000] for more recent discussions.

of explanation does not appeal to any semantic defect in these sentences. In particular, it does not appeal to the idea that (5) or (6) are contradictory in the sense that their conjuncts have incompatible truth-conditions, or in the sense that they mutually entail each other's falsity.

Now we could take this sort of pragmatic account of Moore's paradox on board, and then try extending it to our epistemic contradictions. The simplest way to do that would be to conjecture that each epistemic contradiction entails, in a way obvious to any competent speaker, a Moore-paradoxical sentence. For instance, we could try saying that, holding context and speaker fixed, (1) and (2) each entail (5), and that (3) and (4) each entail (6). Since it is plausible that anything that obviously entails a Moore-paradoxical sentence will itself sound paradoxical, this would give us an explanation for why (1)-(4) sound defective. Note that this explanation would assume that epistemic possibility clauses licence the following entailments:

$\Diamond\neg\phi \models \text{I do not know } \phi$

$\Diamond\phi \models \text{For all I know, } \phi$

—relative, again, to a fixed context and speaker.

It is at least *prima facie* plausible that epistemic possibility sentences in context do licence these entailments, so perhaps something like this line of explanation for the infelicity of our epistemic contradictions will ultimately prove correct. But I am not actually interested in pursuing this issue now. Rather, my aim in this section to highlight a way in which epistemic modals give rise to their own sort of 'paradox', one that differs from Moore's paradox in significant respects. The puzzle I want to focus on emerges when we attempt to embed our epistemic contradictions. It turns out these conjunctions are much more difficult to felicitously embed than Moore-paradoxical sentences, and careful attention to this fact points to some interesting constraints on any theory of the meaning of epistemic modal operators.

Consider the following sentences.

(7) # Suppose it is raining and it might not be raining

(8) # Suppose it is not raining and it might be raining

Here we have (1) and (3) embedded under the attitude verb ‘suppose’. The resulting imperatival sentences are not acceptable. Indeed they are not even obviously intelligible. Substituting other natural language epistemic possibility operators yields equally defective sentences. Take ‘possibly’, for instance:

(9) # Suppose it is raining and possibly it is not raining

(10) # Suppose it is not raining and possibly it is raining

The fact is a general one about epistemic possibility modals. Intuitively, there is some element of inconsistency or self-defeat in what these sentences invite one to suppose.

We get similar results when we attempt to embed our epistemic contradictions in the antecedent position of an indicative conditional. For instance:

(11) # If it is raining and it might not be raining, then...

(12) # If it is not raining and it might be raining, then...

An indicative conditional that begins in one of these ways will strike any competent speaker as unintelligible, regardless of the consequent chosen to finish off the conditional. Even a conditional which merely repeats one of the conjuncts in the antecedent—say,

(13) # If it is raining and it might not be raining, then (still) it is raining

—strikes us as unintelligible rather than trivially true, the usual judgement for such conditionals. Again, as the reader may confirm for herself, this is a general fact about epistemic possibility modals, not an idiosyncratic feature of ‘might’. The intuitive judgements about these conditionals are not surprising, given the intuitive judgements about the ‘suppose’ sentences just described. For the interpretation of an indicative conditional plausibly involves something like temporary supposition of the antecedent, and again, we see there is some element of inconsistency or self-defeat in what these antecedents invite one to entertain.

Here are the facts in schematic form.

- #Suppose $(\phi \wedge \Diamond\neg\phi)$
- # Suppose $(\neg\phi \wedge \Diamond\phi)$
- # If $(\phi \wedge \Diamond\neg\phi)$, then ψ
- # If $(\neg\phi \wedge \Diamond\phi)$, then ψ

Our first observation was that epistemic contradictions are not acceptable as unembedded, stand-alone sentences. Our second observation is that epistemic contradictions are also not acceptable in the embedded contexts described above.³ We need an explanation for this second set of facts.

Finding an explanation proves not to be trivial. For starters, note that we will have no luck trying to explain this second set of facts by piggybacking somehow on a pragmatic explanation of Moore's paradox. Although our Moore-paradoxical sentences (5) and (6) are not felicitous unembedded, they are perfectly acceptable in the embedded contexts just described:

- (14) Suppose it is raining and I do not know that it is raining
- (15) Suppose it is not raining and for all I know, it is raining⁴
- (16) If it is raining and I do not know it, then there is something I do not know
- (17) If it is not raining but for all I know, it is, then there is something I do not know

(Indeed, a reason often cited in favour of the view that Moore-paradoxical sentences are not, semantically, contradictions is the very fact that sentences like (14) and (15) strike us as coherent requests.) Moore-paradoxical sentences serve to describe totally clear possibilities, possibilities we can readily imagine obtaining. The same apparently does not apply to epistemic contradictions. These sentences do not seem to describe coherent possibilities, as witness the fact that an invitation to suppose such a conjunction strikes us as unintelligible. The upshot here is that, unlike the unembedded case, there is no obvious way to explain the unacceptability of our epistemic contradictions in embedded contexts by appeal to Moore's

³ Plausibly they are not acceptable in *any* embedded context, but it will be useful to focus on the two contexts just described.

⁴ Feel free to replace the indexical 'I' in the imperatival sentences (14) and (15) with 'you', if you think that better makes the point.

paradox. Moore-paradoxical sentences are quite acceptable in these contexts. We might describe the situation roughly as follows. Like Moore-paradoxical sentences, epistemic contradictions are not assertable; but unlike Moore-paradoxical sentences, they are also not *supposable*, not *entertainable as true*.

How are we to explain this novel feature of our epistemic contradictions? Let me put the question in a somewhat more theoretically-loaded way. What truth-conditions for epistemic contradictions could suffice to explain why they do not embed intelligibly under ‘suppose’ and in indicative conditional antecedents? To answer this question, we need to know the truth-conditions of epistemic possibility clauses. But when we look closely at the facts, it turns out that we face a certain dilemma concerning the logical relationship between epistemic possibility clauses ($\Diamond\phi$) and their nonepistemic complements (ϕ), one which makes it hard to say what exactly the truth-conditions of epistemic possibility clauses, and hence our epistemic contradictions, could be. Let me explain.

To fix ideas, focus on epistemic contradictions of the form $(\neg\phi \wedge \Diamond\phi)$, and hold context fixed.⁵ Now either $\neg\phi$ is truth-conditionally compatible with $\Diamond\phi$, or it is not. Suppose first that the two are truth-conditionally compatible. Then their conjunction is, under some conditions or other, true; the truth-conditions of the conjunction $(\neg\phi \wedge \Diamond\phi)$ are non-empty. If the truth-conditions of the conjunction are non-empty, it seems there should be nothing at all preventing us from hypothetically entertaining the obtaining of these conditions. We ought to be able to do this simply as a matter of semantic competence. But we cannot. Evidently there is no coherent way to entertain the thought that it is not raining and it might be raining.

That suggests that we should drop the supposition that the two conjuncts actually are compatible. If we take it instead that $\neg\phi$ is truth-conditionally incompatible with $\Diamond\phi$, then we will have a ready explanation for our inability to entertain their conjunction. If there simply is no possible situation with respect to which $(\neg\phi \wedge \Diamond\phi)$ is true, then that explains why it is so hard to envisage such a situation. The conjunction is just semantically a contradiction. But although this line of explanation covers our intuitions about epistemic contradictions in embedded contexts, it comes at an unacceptably high price. If $\neg\phi$ and $\Diamond\phi$ are contradictory, then the truth of one entails the negation of the other. On ordinary

⁵ Where it creates no confusion, I will be loose about use and mention.

classical assumptions, this means that $\Diamond\phi$ entails the negation of $\neg\phi$ —that is, it means $\Diamond\phi$ entails ϕ . But that result is totally absurd. It would imply that the epistemic possibility operator \Diamond is a factive operator, something it very clearly is not. (It might be raining, and it might not be raining; from this we obviously cannot conclude that it both is and is not raining.)

So it appears we face a dilemma.

- $\neg\phi$ and $\Diamond\phi$ should be modelled as having incompatible truth-conditions, in order to explain why it is not coherent to entertain or embed their conjunction; but
- $\neg\phi$ and $\Diamond\phi$ should be modelled as having compatible truth-conditions, in order to block the entailment from $\Diamond\phi$ to ϕ .

A semantics for epistemic possibility modals should resolve this apparent tension. Note all of the preceding can be repeated *mutatis mutandis* for $(\phi \wedge \Diamond\neg\phi)$, our second kind of epistemic contradiction.

It will be helpful to give the problem an alternative formulation, in terms of consequence. This will let us state the problem at a somewhat higher level of generality. (It will also let us sidestep the intuitive, but at this point imprecise, notion of truth-conditions.) We can think of the problem as a tension between the following three constraints on the notion of consequence appropriate to the semantics of natural language.

Consequence is classical: \models respects classical entailment patterns.

Nonfactivity of epistemic possibility: $\Diamond\phi \not\models \phi$

Epistemic contradiction: $(\neg\phi \wedge \Diamond\phi) \models \perp$ ⁶

The principle of the nonfactivity of epistemic possibility is obvious. The principle of epistemic contradiction is much less obvious, but it is motivated by sentences like (8), (10), (12), and ordinary reflection on our inability to simultaneously coherently entertain instances of

⁶ We would also want the principle that $(\phi \wedge \Diamond\neg\phi) \models \perp$. If certain classical principles were assumed, we would get this second principle from the first for free. It will be convenient to just focus on the first principle for now.

$\neg\phi$ and $\Diamond\phi$. Despite motivation for both principles, however, it is clear that the principles are not jointly compatible, if the consequence relation is assumed to be classical.⁷

The nonfactivity of epistemic possibility is surely nonnegotiable. Given that we keep it, we seem to face a choice between the principle of epistemic contradiction and the thesis that the consequence relation is classical. If we reject epistemic contradiction, we need to explain what it is about our epistemic contradictions that makes them semantically defective in embedded contexts. This does not look easy to do. Again, if epistemic contradiction is false and $\neg\phi$ and $\Diamond\phi$ really are consistent in the sense appropriate to the correct semantics of the language, it is not clear why they should not be simultaneously entertainable as true, or why their conjunction does not embed intelligibly. On the other hand, if we keep epistemic contradiction, we need to clarify the nonclassical alternative notion of consequence in play.

That sets the stage. The task now is to spell out a logic and semantics for epistemic modals which makes sense of the facts, which resolves the tension just described. Here is the plan. I give a semantics which explains the phenomena in section 3. I consider the question of what notion of consequence is appropriate to that semantics in section 4. The discussion of consequence will set us up for a discussion, in section 5, of the pragmatics appropriate to the semantics. Equipped with a reasonable grip on the semantics and pragmatics of epistemic possibility operators, I turn in section 6 to the semantics of epistemic necessity operators. I then consider, in section 7, prospects for the extension of the semantics to probability operators. Probability operators, we will see, give rise to the same kind of problem epistemic possibility operators do, but also introduce their own challenges for analysis. In a closing discussion of outstanding issues, I attempt to catalogue some of the new questions raised by the semantics I give for these operators. Before introducing the positive proposal for the semantics of epistemic possibility modals, I want to begin by explaining why the problem I have set out in this section cannot be plausibly handled by a

⁷ If this is not obvious, remember that classically, $(\neg\phi \wedge \psi) \models \perp$ iff $\psi \models \phi$. Substituting $\Diamond\phi$ for ψ in this schema, we have the principle of epistemic contradiction on the left: $(\neg\phi \wedge \Diamond\phi) \models \perp$ iff $\Diamond\phi \models \phi$. Epistemic contradiction therefore classically entails factivity. (Note I use ‘factivity’ to describe an entailment property, not a presuppositional property.)

routine accessibility relation semantics for epistemic modals, since a semantics along those lines is perhaps the most familiar approach to the modals of natural language. This will help to clarify and motivate the need for the alternative semantics I describe.

1.2 Relational semantics for epistemic possibility

The idea for the semantics I want to consider and reject in this section is rooted in the classic work of Hintikka [1962], though to my knowledge Hintikka himself did not suggest it. The idea is to treat an epistemic modal clause effectively as a kind of covert attitude ascription, and to assume that attitude ascriptions are to be given the kind of semantics we find in epistemic logics of the sort inspired by Hintikka—logics conventionally interpreted on accessibility relation-based models (so-called *relational* or *Kripke* models). To make the semantics a little more realistic with respect to context-sensitivity, let me spell out the idea within a Kaplan-style two-dimensional semantics (see Kaplan [1989], Lewis [1980]).

Sentences in context are true (false) relative to possibilities. We may take possibilities to be possible worlds, or world-time pairs, or centered worlds, etc.; I will talk in terms of worlds, but nothing hangs on this. Natural language modals are treated as analogous to the modal operators of ordinary normal modal logic, with truth-conditions for modal clauses stated via quantification, in the metalanguage, over a domain of possibilities. Possibility modals—‘may’, ‘might’, ‘could’, ‘possibly’, etc.—require existential quantification. (Necessity modals—‘must’, ‘has to’, ‘necessarily’, etc.—require universal quantification.) The basic structure of the semantics of a possibility clause is this:

$$\llbracket \Diamond \phi \rrbracket^{c,w} \text{ is true iff } \exists w' (wRw' \wedge \llbracket \phi \rrbracket^{c,w'} \text{ is true})^8$$

We assume that the accessibility relation R is, in any given case, provided by context.⁹ On

⁸ ‘ $\llbracket \cdot \rrbracket$ ’ denotes the interpretation function of the model of the language, which maps well-formed expressions to their extensions relative to choice of context c and possible world w . By ‘is true’, I mean ‘= Truth’.

⁹ How is R provided by context? It could be the semantic value of a covert element in the underlying syntax of modal clauses; it could be specified as part of the definition of a model for the language; it could be an ‘unarticulated constituent’; or we could enrich the points of evaluation in our model, relativizing the truth of a sentence, not only to contexts and possibilities, but also to accessibility relations. Or perhaps something else. The choice does not really matter for our purposes. I am only

the approach to epistemic modals I now want to consider, what makes a modal epistemic is the kind of accessibility relation used in the truth-conditions for the clause. (Cf. Kratzer [1977, 1981], Lewis [1979b].) The accessibility relation R associated with an epistemic modal clause is one which relates the world w at which the clause is evaluated to a set of worlds not excluded by some body of knowledge or evidence in w . Let us think of a body of knowledge or evidence S in a possible world as determining a set of possibilities, the possibilities still left open by that knowledge or evidence in that world. Then the accessibility relation R associated with an epistemic modal is a relation of the form

$$wRw' \text{ iff } w' \text{ is compatible with evidential state } S \text{ in } w$$

where world w' is compatible with S just in case w' is left open by S in w . Think of ‘ S ’ as standing in for a description of an evidential state—‘what x knows’, ‘what x has evidence for’, and so on—for some contextually specified x . It determines a function from worlds to sets of worlds.

Put simply, then, the idea is that $\diamond\phi$ is a sort of description of an evidential state. Its truth turns on whether ϕ is left open by that evidential state in the world at which the clause is evaluated.

There has been much discussion of what exactly the rules are for determining S (and therefore the epistemic accessibility relation R) precisely—for determining the state of knowledge or evidence relevant to evaluating the truth of an epistemically modalized sentence in any given context. When we ask whether ‘It might be raining’ is true as tokened in a given context, whose state of knowledge do we look to in order to settle the question? Should S be understood as the epistemic state of the speaker of the context? Is it something broader—say, the group knowledge of the discourse participants? Does S include the knowledge possessed by nearby agents not party to the conversation? Does it include evidence readily available, but not yet known, to the interlocutors? And so on. (For relevant discussion, see Hacking [1967], DeRose [1991], Egan et al. [2005], MacFarlane [2006].) It is a striking fact that these questions do not have obvious answers.

interested in a general idea right now, namely that, relative to a fixed context, modals, in particular epistemic modals, express quantification over a domain of worlds which is determined as a function of the world at which the modal clause is evaluated.

Let us set aside these questions for now. For even bracketing the question of whether it is actually possible to sort out what the right S is in any given case, we can see that there is a more basic problem with this semantics. It is the problem this paper we began with. On a relational semantics of the sort just described, epistemic contradictions are mistakenly predicted to be entertainable as true, and mistakenly predicted to be felicitous in embedded contexts. Consider again (3):

(3) It is not raining and it might be raining

According the basic structure of the account on the table, this has nonempty truth-conditions. It is just the conjunction of a meteorological claim with (roughly) a claim about a contextually determined agent or group's ignorance of this meteorological claim. More precisely, the sentence in context is true at a world w just in case, first, it is not raining at w , and second, there is some world w' compatible with what some specific contextually determined agent or group in w knows (or has evidence for, etc.) in w such that it is raining in w' . Who exactly the agent or group is, and what exactly their epistemic or evidential relation is to the body of information said to be compatible with rain is, we assume, settled in some more detailed way by R . The point is just that however these details are cashed out, we will have a totally clear, entertainable possibility in (3). We have the sort of thing that is completely coherent to hypothetically suppose. The semantics of this clause will interact in a perfectly nice way with attitude contexts such as 'suppose' and with indicative conditional antecedents, at least on conventional assumptions about the semantics of these environments. (Indeed, the sentence should be exactly as embeddable as a Moore-paradoxical sentence, for the underlying idea of the semantics is that sentences like (3) just are Moore-paradoxical sentences.)

We can illustrate the point with an example. Consider the defective indicative conditional:

(18) # If it is not raining and it might be raining, then for all I know, it is raining

Now if the accessibility relation R for the epistemic modal in the antecedent is cashed out so that, whatever it is, it guarantees

$\Diamond\phi \models \text{For all I know, } \phi$

is valid given a fixed context—a weak assumption, and a standard one in the current literature—then we should expect (18) to strike us as sounding true. But clearly, the conditional is not true. It does not even make sense. The conditional is semantically defective, but this semantics does not capture the defect. This approach therefore misses the facts.

Why does it miss the facts? The problem, I suggest, is the idea, practically built into a relational semantics for modals, that the evidential state relevant to the truth of an epistemic modal clause is ultimately determined as a function of the evaluation world—the world coordinate of the point at which the modal clause is evaluated. If we model epistemic modals as if they behaved that way, epistemic modal clauses end up acting like (covert) descriptions of epistemic states. And as a result, sentences like (1)-(4) are incorrectly predicted to be as embeddable as the overtly epistemic-state-describing counterparts of these sentences—that is, Moore-paradoxical sentences.

1.3 Domain semantics for epistemic possibility

If we want to keep the intuitively reasonable idea that epistemic possibility clauses indicate, in some sense, that their complements are compatible with some evidential state or state of information, we need a better way of representing informational states in the semantics than via accessibility relations. Here is a fix.

Start again with a two-dimensional semantics in the style of Kaplan. Let me be a little more precise now about what the two dimensions are. The points of evaluation relative to which extensions are defined have two coordinates: a *context* coordinate and an *index* coordinate. Contexts are locations where speech acts take place. Following Lewis [1980], we may think of them as centered worlds, determining both a possible world and a spatiotemporal location within that world. Contexts have indefinitely many features—speakers, audiences, indicated objects, standing presuppositions, etc.—and these features may figure in the truth of sentences said in that context in indefinitely many ways. Indices are n -tuples of specific features of context, those features which are independently shiftable by operators in the language. Which features of the context are shiftable depends on what operators the

language contains. Our indices include at least a world parameter, since the fragment of English we consider has operators which shift the world at which a clause is evaluated.

Above our tacit assumption was that the index consisted *only* of a world parameter. Consequently there was no need to introduce the more general notion of an index. This notion only comes in handy when one posits an index with more than one parameter. That is what we do now. In addition to a world parameter, let our index include also an *information* parameter s . This coordinate will range over bodies of information, where a body of information is modelled as a set of worlds. Indices are therefore now pairs, $\langle s, w \rangle$; and the intension of a sentence relative to a fixed context is now a function from such pairs into truth values, rather than simply a function from worlds to truth-values. Our plan is to use this new s parameter to supply the domain of quantification for epistemic modal clauses. I will call this a *domain* semantics. Rather than quantifying over a set of worlds that stand in some R relation to the world of evaluation, as in a relational semantics, epistemic modals will be treated as quantifying over a domain of worlds provided directly by the index.¹⁰ Here are the truth-conditions:

$$\llbracket \Diamond \phi \rrbracket^{c,s,w} \text{ is true iff } \exists w' \in s : \llbracket \phi \rrbracket^{c,s,w'} \text{ is true}$$

Epistemic possibility modals simply effect existential quantification over the set of worlds provided by the information parameter. No covert material is assumed, and no accessibility relation is appealed to. We can observe immediately that iterating epistemic possibility operators adds no value on this semantics: $\Diamond \Diamond \phi$ is semantically equivalent to $\Diamond \phi$. The outer modal in $\Diamond \Diamond \phi$ serves only to introduce vacuous quantification over worlds. (This may explain why iterating epistemic possibility modals generally does not sound right, and why, when it does, the truth-conditions of the result typically seem equivalent to $\Diamond \phi$. I will generally ignore iterated epistemic modalities below.)

¹⁰ In adding a parameter to represent a set of worlds to the index and using it to give semantics for epistemic modals, I follow MacFarlane [2006]. MacFarlane's work helped me to see a cleaner formalization of the ideas in a previous draft of this paper. MacFarlane does not motivate (what I am calling) a domain semantics as over a relational semantics in the way I do here. He also does not enrich the information parameter probabilistically in the way described later (Sect. 7), and he has a quite different conception of the pragmatics of epistemic modal claims and of their informational content. I hope to discuss these differences elsewhere.

We can take it that the semantic role of s will be relatively minimal. Although denotations are now technically all relativized to a value for s , in most cases extensions will not be sensitive to it. Predicates will be assigned extensions relative only to worlds, as usual; logical connectives will be defined as usual;¹¹ and nothing new need be assumed about the semantics of names, generalized quantifiers, etc. Most clauses will continue to place conditions only on the world coordinate of the index, and will therefore retain their ordinary possible worlds truth-conditions. In such cases the information parameter s will be idle. We exploit s mainly in the definition of truth for epistemic modal talk (as above), and for certain constructions embedding such talk—in particular, attitude verbs and indicative conditionals. Let me now describe a domain semantics for these latter two constructions which will give us the desired predictions for our epistemic contradictions in embedded contexts.

Start with our troublemaking attitude verb ‘suppose’. For this verb, let us assume essentially an off-the-shelf possible worlds semantics, with one adjustment: the attitude verb will be taken to shift the value of s for its complement, replacing it with the set of worlds compatible with the agent’s suppositions. The truth-conditions of ‘ x supposes ϕ ’ are as follows:

$$\llbracket x \text{ supposes } \phi \rrbracket^{c,s,w} \text{ is true iff } \forall w' \in S_x^w : \llbracket \phi \rrbracket^{c,S_x^w,w'} \text{ is true}$$

where

$S_x^w =_{\text{def}}$ the set of worlds not excluded by what x supposes in w

Roughly: when you suppose what ϕ says, your state of supposition, abstractly represented by a set of worlds, includes the information that ϕ . What is supposed is what is true at every world compatible with what is supposed.

Semantically, the attitude verb does two things. First, it quantifies over the set of possibilities compatible with the attitude state. Second, it shifts the value of s to that set of possibilities. The second effect is what is unique to a domain semantics. This effect matters only when we come to evaluating the complement of the clause. Most complements of

¹¹ In particular, since negation and conjunction will occur often: $\llbracket \neg\phi \rrbracket^{c,s,w}$ is true iff $\llbracket \phi \rrbracket^{c,s,w}$ is false, and $\llbracket \neg\phi \wedge \psi \rrbracket^{c,s,w}$ is true iff $\llbracket \phi \rrbracket^{c,s,w}$ is true and $\llbracket \psi \rrbracket^{c,s,w}$ is true.

‘suppose’ ascriptions will not have truth-conditions which consult the s parameter in determining truth, and therefore this shiftiness will have no overall effect on truth-conditions. In such cases, the above semantics will yield the same predictions as a conventional accessibility relation semantics for attitude verbs. One type of complement which will consult the s parameter, however, is a complement containing an epistemic modal clause. As per the semantics just given above, epistemic possibility modals quantify over the set of worlds provided by the information parameter. Hence such a modal, when embedded under ‘suppose’, will quantify over supposition-worlds. We can see the interaction of the attitude verb and the modal by stating the truth-conditions for ‘ x supposes $\Diamond\phi$ ’ at the relevant level of abstraction:

$$\forall w' \in S_x^w : \exists w'' \in S_x^w : \llbracket \phi \rrbracket^{c, S_x^w, w''} \text{ is true}$$

We have two quantifiers here, one (universal) introduced by the attitude verb, the other (existential) introduced by the modal. They quantify over the same domain, since the quantificational domain of the epistemic modal is parasitic on that of the attitude verb. The modal picks up its domain from the information parameter, which has been shifted by ‘suppose’. Notice that the universal quantifier introduced by the attitude verb is vacuous. It has been ‘trumped’, as it were, by the epistemic modal. The attitude verb influences truth in this case only because it has provided the domain over which the embedded epistemic possibility modal quantifies. So the truth-conditions are really just this:

$$\exists w' \in S_x^w : \llbracket \phi \rrbracket^{c, S_x^w, w'} \text{ is true}$$

where here we have simply removed the vacuous universal quantifier.

The nonstandard way in which the modal and the attitude verb interact here is precisely what we want, for it lets us explain what is wrong with embedding epistemic contradictions. Take, for instance, a sentence of the form ‘ x supposes that $(\neg\phi \wedge \Diamond\phi)$ ’. It is straightforward to verify that, on the semantics just given, this sentence will be true just in case, first, in all the worlds compatible with what x supposes, $\neg\phi$ is true, and second, there is some world compatible with what x supposes where ϕ is true. That is, the truth-conditions are, at the relevant level of abstraction:

$$(\forall w' \in S_x^w : \llbracket \neg phi \rrbracket^{c, S_x^w, w'} \text{ is true}) \wedge (\exists w' \in S_x^w : \llbracket \phi \rrbracket^{c, S_x^w, w'} \text{ is true})$$

Obviously, there is no state of supposition S that could make this condition true, for the condition imposes contradictory demands on the state. (The same is true for ‘ x supposes $(\phi \wedge \Diamond\neg\phi)$ ’, since its truth-condition is the same, save for a switch in the location of the negation.) And this explains what is wrong with asking someone to suppose an epistemic contradiction. It is a request to enter into an impossible state of supposition, a request that cannot be satisfied. The empty set is not a state of information.

We can motivate a domain semantics of the sort I have been describing from a second direction, separate from the whole issue of epistemic contradictions. I have discussed only ‘suppose’ so far, but it is very natural to extend a domain semantics of this type to other attitude verbs, such as ‘believe’, ‘suspect’, ‘think’, and ‘know’. Take a sentence like:

(19) Vann believes that Bob might be in his office

On the natural reading of this sentence, it is intuitively plausible that the epistemic modal in the complement of this sentence is understood as directly quantifying over Vann’s belief worlds.¹² If we gave ‘believe’ a domain semantics structurally analogous to ‘suppose’ above, we could capture this easily. Again, the verb would shift the information parameter (this time to the set of worlds not excluded by Vann’s beliefs in the world of evaluation), and the modal would existentially quantify over that parameter. The sentence would be true just in case Bob’s being in his office is compatible with what Vann believes. That is the intuitively correct result.

By contrast, the story would have to be more complicated in a relational semantics. On the usual formulation of that semantics, (19) would be treated as a *second-order* attitude ascription. It would be understood as saying, roughly, that Vann believes that it is compatible with what Vann believes that Bob is in his office. This second-order ascription would entail the first-order ascription (i.e. that it is compatible with what Vann believes that Bob is in his office) in a relational semantics *only if* we made an assumption about the modal logic of belief—namely, the assumption that whatever you believe to be compatible with what you believe actually *is* compatible with what you believe. We can avoid the need to make such assumptions in a domain semantics.

¹² Here I am indebted to work by Tamina Stephenson; see Stephenson [2006].

Second, the second-order truth-conditions of relational semantics, whether or not they entail the truth-conditions supplied by the domain semantics, are plausibly just too strong to be right. Suppose my guard dog Fido hears a noise downstairs and goes to check it out. You ask me why Fido suddenly left the room. I say:

(20) Fido thinks there might be an intruder downstairs

That is good English. What does it mean? Does it mean, as a relational semantics requires, that Fido believes that it is compatible with what Fido believes that there is an intruder downstairs? That is not plausible. Surely the truth of (20) does not turn on recherche facts about canine self-awareness. Surely (20) may be true even if Fido is incapable of such second-order beliefs.

Let me close this digression on attitudes by stating a certain apparently true generalization about the logical relation between (some) attitude verbs and epistemic possibility modals. Following in the tradition of standard logics of knowledge and belief, we have treated attitude verbs as modal operators—specifically, as boxes, to be interpreted in terms of universal quantification over possibilities. What we have been observing is that, a least for many attitude verbs \blacksquare , it appears that \blacksquare interacts with epistemic possibility operators as follows:

$$\blacksquare\Diamond\phi \leftrightarrow \Diamond\phi$$

That is: attitude verb + epistemic possibility modal = dual of the attitude verb.¹³ What is nice about a domain semantics is that it underwrites this generalization easily, and without the need to make extra assumptions about the logics of the relevant attitude verbs.

Turn now to our other problematic embedded context, indicative conditional antecedents. Recall once more what needs to be explained:

If $(\phi \wedge \Diamond\neg\phi)$, then ψ

If $(\neg\phi \wedge \Diamond\phi)$, then ψ

¹³ Note that the principle admits of certain exceptions, some of which are discussed below (Sect. 5).

The explanation to be offered will mimic the explanation just given for attitude contexts. Again, we want to understand our epistemic contradictions as serving to place incompatible demands on the information parameter. We therefore need our semantics for indicative conditionals to interact in the right way with this parameter.

Here is what I suggest. Let us think of indicative conditionals as behaving semantically like epistemic modals. They place conditions, not on the world parameter of the index, but on the information parameter. The truth-conditions are as follows:

$$\llbracket \alpha \rightarrow \psi \rrbracket^{c,s,w} \text{ is true iff } \forall w' \in f_\alpha(s) : \llbracket \psi \rrbracket^{c,f_\alpha(s),w'} \text{ is true}$$

where f is an *information state selection function*, mapping an antecedent α and an information state s to another information state $f_\alpha(s)$. Intuitively, f selects an information state including the information α , and which otherwise differs minimally from s . This may be thought of as an adaptation of a Stalnaker semantics (Stalnaker [1968]) for conditionals to the setting of domain semantics.

We can think of the semantics as proceeding in two steps. First, the antecedent of a conditional shifts the information parameter, ‘updating’ it with the information the antecedent contains. Second, universal quantification occurs over that updated parameter. The whole conditional is true just in case the information in the consequent is ‘already included’ in the updated parameter.¹⁴

Just as Stalnaker’s semantics presupposes a similarity ordering among possible worlds, we presuppose a similarity ordering among states of information. And as in Stalnaker’s semantics, we can impose some formal constraints on the selection function. One obvious constraint is this one:

- **Antecedent acceptance.** For all antecedents α and information states s , α is accepted in $f_\alpha(s)$ in the sense that $\forall w \in s : \llbracket \alpha \rrbracket^{c,f_\alpha(s),w}$ is true

¹⁴ It may be that the two steps are the result of distinct compositional ingredients (Kratzer [1986]). Perhaps ‘if’-clauses serve to shift the information parameter only, with the universal quantification introduced separately by a (usually covert) epistemic necessity modal. We need not take a stand on the issue here.

This is the just-mentioned constraint that f selects an information state including the information α . If we want, we can impose other constraints that have their analogues in Stalnaker’s semantics—for instance, that the similarity ordering form a total order, or that if α is already included in s , then $f_\alpha(s) = s$. It would also be natural to assume that, for *indicative* conditionals, $f_\alpha(s)$ is generally understood to be a subset of s .¹⁵

Of course, it would take much more space than I have to defend a semantics of this form for indicatives adequately. I will just settle for pointing out that it gets the right result for our problem conditionals. The reason is that by the semantics, a conditional $\alpha \rightarrow \phi$ is true only if the selection function f_α involved in its composition can deliver an information state which accepts the antecedent. But if α is an epistemic contradiction, there will be no such set. This is for just the same reason as in the attitude case discussed above. An antecedent which is an epistemic contradiction will impose incompatible demands on the information parameter. This predicts that conditionals with epistemic contradiction antecedents are never true, hence that they should sound semantically defective. We have the desired result.

There is a clear sense in which our puzzle about epistemic possibility modals is now dissolved. Consider again our first formulation of the puzzle, as a dilemma about truth-conditions.

- $\neg\phi$ and $\Diamond\phi$ should be modelled as having incompatible truth-conditions, in order to explain why it is not coherent to entertain or embed their conjunction; but
- $\neg\phi$ and $\Diamond\phi$ should be modelled as having compatible truth-conditions, in order to block the entailment from $\Diamond\phi$ to ϕ .

We see that we have taken the second path, but avoided the associated horn, essentially by working with an enlarged conception of truth-conditions. Rather than modelling epistemic modal clauses as placing conditions on possible worlds relative to context (as would be typical on a relational semantics), we construed them as placing conditions on sets of worlds. $\neg\phi$ and $\Diamond\phi$ have compatible truth-conditions on our semantics because, relative to context, they place conditions on different index coordinates: $\neg\phi$ places a condition on the world parameter of the index, and $\Diamond\phi$ a condition on the information parameter.

¹⁵ Whether to treat this constraint as semantic or pragmatic is something we can leave open.

The incoherence of their conjunction in the various embedding environments discussed is explained, not by their joint truth at a point of evaluation being impossible, but by their failing to be jointly acceptable by a single state of information in the way that those environments require.

In the next section this notion of acceptance is more precisely defined, and its relevance to the appropriate definition of consequence for the semantics is considered.

1.4 Consequence

We were able to dissolve our puzzle without defining any notion of consequence. Our problem was solvable without any explicit commitment on that issue. Nevertheless, it is of interest to ask what notion of consequence is most appropriate to the semantics just provided—especially given our second setup of the puzzle, as a tension between the principle of epistemic contradiction and classical consequence. In this section, I will describe three notions of consequence, suggest that two are of primary interest, and ask where each of the two stand with respect to epistemic contradiction.

First, consequence might preserve *truth at a point of evaluation*, the notion recursively defined by our intensional semantics. We could call this *standard consequence*.

ϕ is a *standard consequence* of a set of sentences Γ , $\Gamma \models_s \phi$, just in case for every point of evaluation p , if every member of Γ is true at p , then ϕ is true at p .

I mention standard consequence only to set it aside. It is arguably not the notion we want if we are looking for a notion which tracks the intuitive notion of a conclusion following from a collections of premisses. The trouble is that the notion of truth that standard consequence preserves is, in an important sense, too general as applied to the *unembedded* sentences which constitute a set of premisses and a conclusion. To give a simple illustration, take the unembedded sentence ‘Jones has red hair’. Suppose we consider an occurrence of this sentence with respect to a context in which Jones has black hair (that is, a context which is such that in the world of the context, Jones has black hair). Is the sentence, as it occurs

in this context, true or false? False, intuitively. But given only our definition of truth at a point of evaluation, the question does not really make sense. According to that definition, sentences have truth values only with respect to a whole point of evaluation (a context *and an index*), and in stating the question, we have only specified the context coordinate of the point. But evidently we do have an intuitive notion of the truth or falsity of a sentence in context *simpliciter*. Given that we do, it would seem natural to define consequence so that it preserves this intuitive notion of truth.

Following Kaplan [1989], we can do that by first defining truth at a context in terms of truth at a point of evaluation. Let us write ‘ ϕ_c ’ for an occurrence of a sentence ϕ in a given context c . Then we can say that:

$$\phi_c \text{ is true iff } \llbracket \phi \rrbracket^{c, s_c, w_c} \text{ is true}$$

where w_c is the world of the context c , and s_c is the state of information determined by c . (More on s_c shortly.) A sentence in a context is true just in case it is true with respect to the point consisting of the context and the index determined by that context. Reflection on cases suggests that this definition does track the intuitive notion we intended to capture.¹⁶

With this notion of truth in hand, we can define our second notion of consequence. Call it *diagonal consequence*.

ϕ is a *diagonal consequence* of a set of sentences Γ , $\Gamma \vDash_d \phi$, just in case for any context c , if every member of Γ_c is true, then ϕ_c is true.

Diagonal consequence preserves truth at a context. It is perhaps the most intuitively natural definition of consequence available in a Kaplan-style two-dimensional semantics—given, at least, that consequence is to be understood in terms of some form of truth-preservation. Note that the only points of evaluation that matter in evaluating an argument for diagonal consequence are those points which are pairs of a context and the index determined by that context. We can call such points diagonal points, since these are the

¹⁶ e.g. ‘Jones has red hair’ is correctly predicted to be false with respect to the context described above, because it is false with respect to world coordinate of the index determined by the context. See Kaplan [1989] for further discussion.

points that would constitute the diagonal of the two-dimensional matrix associated with any given sentence. (Diagonal points are also sometimes called *proper* points.)¹⁷

Now let us raise the question of epistemic contradiction with respect to diagonal consequence. Is a contradiction a diagonal consequence of an epistemic contradiction such as $(\neg\phi \wedge \Diamond\phi)$? Or equivalently: is this sentence true at any diagonal points? Or equivalently again: are $\neg\phi$ and $\Diamond\phi$ diagonally consistent? To answer, we need to know when $\Diamond\phi$ is true at a context. To know that, we need a grip on what s_c , the state of information determined by a given context c , is.

But, as already alluded to above (Sect. 2), that last issue is a difficult one, and it is one I have avoided addressing. When is $\Diamond\phi$ true at a context? What body of information is relevant to determining whether a simple unembedded epistemic possibility claim is true or false? The answer is not clear. Obvious choices—such as the knowledge state of the speaker of the context, or the distributed knowledge of the discourse participants—appear to be subject to counterexamples, as noted already by Hacking [1967]; and recent work (Egan et al. [2005], MacFarlane [2006], Egan [2007]) suggests that the fix, if there is one, is not going to be straightforward.

Again, I want to sidestep this issue for now. Fortunately, we can answer our question about epistemic contradiction under diagonal consequence without a full theory of how the information parameter is ‘initialized’ by context. We need only capture some of the basic structural features the information parameter must have at diagonal points of evaluation. Two in particular are plausible. First:

Reflexivity: For every diagonal point of evaluation $\langle c, w, s \rangle$, $w \in s$

Roughly: what is true at a context is epistemically possible at that context. This is uncontroversial. Second,

Non-collapse: For some diagonal point of evaluation $\langle c, w, s \rangle$, $\{w\} \neq s$

¹⁷ Note that we could also define diagonal consequence in terms of truth at diagonal points of evaluation, as follows: $\Gamma \models_d \phi$ just in case for every diagonal point of evaluation p , if every member of Γ is true at p , then ϕ is true at p . This makes it obvious that diagonal consequence is a restricted version of standard consequence. (Standard consequence implies diagonal consequence, but not vice versa.)

Roughly: with respect to some contexts, what is possible is not, or not merely, what is actual. This, too, is uncontroversial. (And indeed presumably it is true for practically all diagonal points.) Given Reflexivity, it merely states that epistemic possibility does not collapse into truth. More than one world may be epistemically possible with respect to a context.¹⁸

Now it should be obvious, given these properties, that $\neg\phi$ and $\Diamond\phi$ are diagonally consistent, hence that $(\neg\phi \wedge \Diamond\phi) \not\equiv_d \perp$. For if more than one world may be epistemically possible with respect to a context, then for some ϕ false at that context, $\Diamond\phi$ is true. Hence $(\neg\phi \wedge \Diamond\phi)$ is true at the diagonal point determined by that context.¹⁹ So if the principle of epistemic contradiction is understood in terms of diagonal consequence, it is false.

What does this show? It shows that a prima facie natural, classical notion of consequence—diagonal consequence—is in fact compatible with our semantic explanation of the problematic embedding behaviour of our epistemic contradictions.²⁰ What it shows is that strictly speaking, diagonal consequence is not under direct threat by our puzzle about epistemic modals.

Diagonal consequence is under threat, however, from two other directions. First, as pointed out above, this notion of consequence requires the notion of truth at a context to be well-defined for epistemic modal claims. The current lack of consensus about how that definition is supposed to go—in our terms, about how the information parameter is to be ‘initialized’ by context—calls this assumption into question. Second, it may be argued that diagonal

¹⁸There will always be models in which non-collapse holds, and in this sense we do not really need to make special provision for it. But it will be helpful to single out this feature for discussion here.

¹⁹ Same goes, *mutatis mutandis*, for $(\phi \wedge \Diamond\neg\phi)$, as usual.

²⁰ A proponent of diagonal consequence still needs to explain what is wrong with epistemic contradictions in the unembedded case, given he cannot appeal to epistemic contradiction. But this could be done by piggybacking on Moore’s paradox. If we made the following popular assumption:

Speaker inclusion: For every diagonal point of evaluation $\langle c, s, w \rangle$, $s \subseteq S$, where S is the set of worlds not excluded by the knowledge of the speaker at c .

i.e. if we assumed that the information state determined by the context includes at least the knowledge of the speaker, then, unembedded, our epistemic contradictions would diagonally entail Moore-paradoxical sentences. Their badness could then be explained by whatever pragmatic explanation we give for the badness of Moore-paradoxical sentences generally.

consequence misses quite elementary patterns of inference. I have in mind especially the following line of objection:

Surely, any formal regimentation of the intuitive notion of consequence should substantially track our intuitions concerning what follows *on the supposition* of what. Now suppose that it is not raining. Given that supposition, might it be raining? Obviously not! Hence $\neg\phi$ and $\diamond\phi$ are incompatible. Diagonal consequence misses this.

(A line of thought rather like this one was voiced by Łukasiewicz, who proposed the following as an intuitive ‘general theorem’: ‘*If it is supposed that not- ϕ , then it is (on this supposition) not possible that ϕ .*’ Łukasiewicz [1920], p.156, his italics.)²¹

It would be of interest to find an intuitive notion of consequence for our semantics which did not face these two threats. In fact it is not difficult, given the semantics already in place, to define a such a notion of consequence. The notion of consequence I have in mind preserves, not truth, but a different property of sentences in context—one they have in relation to a state of information. We might call this property *acceptance*:

ϕ_c is *accepted* in information state s iff for all worlds w in s , $[[\phi]]^{c,s,w}$ is true.

The definition of acceptance mimics the domain semantics for attitudes given above. Intuitively, think of a sentence in context as determining a constraint on a state of information. A state of information accepts a sentence in context just when it satisfies the constraint determined by that sentence. If ϕ_c is nonepistemic, it places a condition on worlds, and the constraint it determines on a state of information is that each world compatible with the information satisfy that condition. If ϕ_c is epistemically modalized, then it places a global condition on a state of information (set of worlds), and the constraint on a state of information it determines is just that the state itself satisfy this global condition.

ϕ is an *informational consequence* of a set of sentences Γ , $\Gamma \vDash_i \phi$, just in case for every context c and body of information s , if every member of Γ_c is accepted in s , then ϕ is accepted in s .

²¹ Together with some other ‘general theorems’, Łukasiewicz used this principle to motivate his trivalent logic, which he interpreted as a modal logic.

If ϕ is an informational consequence of a set of sentences Γ , then any state of information which satisfies all the informational constraints imposed by the sentences of Γ (all evaluated with respect to a given context) already satisfies the informational constraint imposed by ϕ (at that context).

Informational consequence avoids the two threats described above. Unlike diagonal consequence, informational consequence does not require the idea of a diagonal point to be well-defined. It requires only the notion of truth at a point of evaluation, not the Kaplanian notion of truth at a context. Hence it avoids the assumption that this notion is in fact definable for epistemic modal claims. Second, informational consequence respects the intuitive pattern of inference from $\neg\phi$ to $\neg\Diamond\phi$. Indeed, it is worth noting that informational consequence validates the following three principles.

Lukasiewicz's principle: $\neg\phi \models \neg\Diamond\phi$

Epistemic contradiction: $(\neg\phi \wedge \Diamond\phi) \models \perp$

Nonfactivity of epistemic possibility: $\Diamond\phi \not\models \phi$

The first principle, which I have called *Lukasiewicz's principle*, expresses the intuition about consequence our objector had in mind above. The intuitive reason for its truth is that $\neg\phi$ and $\neg\Diamond\phi$ impose the same informational constraint. Given an arbitrary context, if a state of information accepts $\neg\phi$, then the state excludes all ϕ -possibilities; hence ϕ is not a possibility according to the state, hence $\neg\Diamond\phi$ is accepted with respect to that state. Epistemic contradiction is correct for a similar reason: $\neg\phi$ and $\Diamond\phi$ are associated with incompatible informational constraints. Given a fixed context, they cannot be both accepted by a single state of information.

Most important, these two principles are correct for the semantics under informational consequence *together* with the (non-negotiable) nonfactivity of epistemic possibility. The truth of nonfactivity is also easy to see. Relative to context, $\Diamond\phi$ merely asks for a state to contain at least one ϕ -world, whereas ϕ requires a state to be such that every world in the state is a ϕ -world. Hence $\Diamond\phi$ does not suffice for acceptance of ϕ .

Informational consequence is a nonclassical notion of consequence. This is because, as pointed out above, epistemic contradiction and nonfactivity are classically incompatible.

(Nonfactivity is also classically incompatible with Łukasiewicz’s principle.) A more detailed study of the logic that results from the combination of domain semantics for epistemic possibility modals plus informational consequence is better reserved for elsewhere, but let me just make an informal remark about the nature of the non-classicality. Informational consequence is built around the notion of acceptance. Acceptance is a gappy notion. Fixing context, there is a gap between (nonepistemic) ϕ being accepted with respect to some s and $\neg\phi$ being accepted (ϕ being *rejected*) with respect to that s . It may be that ϕ is neither accepted nor rejected. Nonepistemic sentences are therefore what we might call *acceptance-trivalent*.

The epistemic possibility operator \diamond exploits this trivalence: semantically it maps acceptance-trivalent sentences onto acceptance-bivalent ones. (Along with negation, it can be construed as an *acceptance-functional* operator.) It is the existence of a third acceptance value which introduces the nonclassical behaviour, and which lets us have both epistemic contradiction and nonfactivity.²²

Our semantics lets us define two notions of consequence, diagonal consequence and informational consequence. The former rejects epistemic contradiction, the latter accepts it. Informational consequence seems to have two theoretical advantages: it avoids the apparently troubled notion of truth at a context for epistemic modal claims, and it validates some natural forms of inference invalidated by diagonal consequence. In the next section

²² Informational consequence is similar to a notion of validity Frank Veltman defines over his update semantics (what he calls ‘validity 3’; see Veltman [1996], p.224). Though I lack the space to adequately discuss Veltman’s important work here, it should be noted that his semantics is, from an abstract point of view, very similar in its treatment of epistemic possibility modals to the domain semantics given above. On both approaches, the basic idea is to think of epistemic possibility clauses as expressing conditions on sets of worlds. Like domain semantics, Veltman’s semantics has no difficulty with epistemic contradictions. (Indeed, the facts discussed above concerning these conjunctions in embedded contexts provide strong evidence in favour of Veltman’s semantics as over a relational semantics for epistemic modals.) Whether a static domain semantics for epistemic modals is preferable to a dynamic semantics along Veltman’s lines is not a question I consider here. In focusing only on a static domain semantics, I have two simple motivations: first, to contribute to understanding what a static alternative to Veltman’s proposal might look like; and second, to make for an easier approach into the analysis of probability operators, by separating out questions of dynamics. The second motivation—only a methodological one—will become clearer later (Sects. 7-8).

I will discuss a further consideration relevant to the question of which of these notions of consequence has greater theoretical interest.

1.5 Content and communication

Distinguish two questions.

1. What is the compositional semantics of an epistemic modal clause?
2. What informational content do utterances of epistemic modal sentences communicate?²³

The questions are obviously related, but they should not be conflated. Very roughly, the first is a question of semantics, the second of pragmatics (or of the semantics-pragmatics interface). It is important to be clear that we have said a lot about the first question, and almost nothing about the second.

The second question has intrinsic interest. It might also be considered relevant to the question of what notion of consequence is of the most general theoretical relevance. For it might be held that a reasonable notion of consequence should be such as to preserve, in some relevant sense, the content communicated by sentences in context. (That is, it might be held that if $\Gamma \models \phi$, then for all c , the informational content communicated by ϕ_c is included already in the informational content communicated by the sentences in Γ_c .) That is not exactly a radical view, so it is worthwhile, in comparing diagonal and informational consequence, to ask whether one of them dovetails better with the actual informational or communicative content of epistemic possibility claims.

That requires asking what the communicative content of epistemic possibility claims is. In this section I will discuss just two of the myriad possible answers to this question. Then I will say how each answer connects to the issue of consequence. The two views about communicative content I want to describe both assume the same abstract picture

²³ Cf. Dummett's distinction between 'ingredient sense' and 'assertoric content' in Dummett [1973] and elsewhere.

of linguistic communication, so let me start by spending three paragraphs sketching that picture.²⁴

Think of linguistic communication as foremost a matter of coordination on a body of information. Participants in conversation begin with certain information presumed to be in common or mutually taken for granted, and the speech acts they perform in context are directed, and mutually understood to be directed, at variously influencing that common body of information. The attitude that communicating agents take towards the body of information they share is the attitude of *presupposition*. Presupposition is, in the intended sense, a public attitude: one presupposes propositional content *p* only if one presupposes that one's interlocutors also presuppose that *p*. When things are going as they should, the interlocutors of a discourse all make the same presuppositions, and we can say that everything that any participant presupposes is *common ground*, in the following sense:

It is *common ground* that *p* in a group just in case all members of the group presuppose that *p*, and all know that all presuppose that *p*, and all know that all know that all presuppose that *p*, etc.

What is common ground is what is common knowledge about what is presupposed. (When the agents in a discourse context are not all making the same presuppositions, something has gone wrong—the agents are misled about what is common ground—and the discourse context is defective, although the defect may never reveal itself.)

Given only what is common ground among a group of agents, one does not yet know how the agents of the context mutually regard the propositions in the common ground with respect to their other cognitive attitudes. To be given the common ground is only to be given a set of propositions mutually understood to be presupposed; it is not yet to be given that the agents also regard those presuppositions as knowledge, or as warranted belief, or conjecture, or fiction, or whatever. Using the notion of common ground, we can define a second notion which will let us articulate the status that the agents of a given context attach to the propositions they presuppose. Call this notion *conversational tone*:

²⁴ The picture is due in essentials to Stalnaker (see e.g. Stalnaker [1975]), but there are some nontrivial differences in formulation.

An attitude is the *conversational tone* of a group of interlocutors just in case it is common knowledge in the group that everyone is to strike this attitude towards the propositions which are common ground.

(It may be that a conversation is plausibly understood as having more than one conversational tone, but let me focus on the case where there is just one. And let me stipulatively exclude presupposition itself from the class of possible conversational tones.) When interlocutors coordinate on a conversational tone, they come into agreement about what counts as the correct non-public attitude to take towards what is common ground. This will be a reflection, *inter alia*, of the purpose of the discourse. If the conversational tone of our discourse is knowledge, then we regard our common ground as common knowledge, and we take our discourse to be trafficking, and aiming to traffic, in factual information. Similarly with belief. If the conversational tone is pretense, then we are not attempting to keep the common ground compatible with the truth, and we take ourselves to be trafficking in fiction. And so on, for all the various attitudes around and in between: the conversational tone may be belief, or suspicion, or supposition, or high-credence-that, or ironic non-belief, etc., depending on the interests and purposes of the interlocutors. It may also be a conditional attitude: the conversational tone may, for instance, be belief (in each q in the common ground) *conditional on* some specified p .

Now speech acts on this picture are understood as influencing, and intended to influence, the information that is common ground. (Their appropriateness is therefore partly dependent on the conversational tone.) Of central interest to us is assertion. We will take assertion to be a speech act whose conventionally understood effect is to update the common ground of the conversation by adding the informational content of the speech act to the common ground. To assert informational content p is just to propose to change the common ground in a certain way, viz., by adding to it p . The assertion is successful when the proposal is accepted.

That is the picture. The view of assertion it comes with carves out a certain theoretical role, viz., that of the *informational content* of an assertion. (We might also like to call it the *proposition asserted*, or what is *said* by the sentence in context.) What we need to do now is to say what occupies this role in the case of epistemic possibility claims. What content do such claims serve to assert?

According to the first of the two views I want to consider, the informational content of an unembedded epistemic modal claim is the *diagonal proposition* determined by the two-dimensional matrix provided by the semantics of the clause. We can write it as follows:

$$\lambda c. \llbracket \Diamond \phi \rrbracket^{c, s_c, w_c}$$

The diagonal is a function from centered worlds (contexts) to truth values, or equivalently, a set of centered worlds.²⁵ The diagonal of $\Diamond \phi$ is true with respect to a centered world just in case ϕ is compatible with the state of information determined by that centered world. There is a certain obvious theoretical attraction in taking the diagonal as the communicative content of (not just epistemic modal claims but) assertions in general. Since most ignorance can be understood as ignorance of features of context—ignorance of features of the world of the context, or of the location of the context within the world—we never know what context we occupy. So it is a natural idea to represent interlocutors as communicating information by uttering sentences which determine conditions on the context. Obviously, the idea of a diagonal proposition requires the notion of a diagonal point to be well-defined: it requires a position on how context supplies a value for the information parameter.

As you might predict, diagonal content goes naturally with diagonal consequence. Diagonal consequence preserves diagonal content in the sense that $\Gamma \vDash_d \phi$ iff every centered world c where the diagonals of the premisses in Γ are true at c is such that the diagonal of ϕ is true at c . Informational consequence does not preserve diagonal content in this sense, since (for example) there are centered worlds where the diagonals of $\Diamond \phi$ and $\neg \phi$ are both true.

²⁵ This proposal about the informational content of unembedded epistemic modal claims is akin to that of Egan [2007], though Egan does not arrive at his proposal via diagonalization.

I should stress that here I want to remain agnostic, in so far as I can, on the question of the metaphysical nature of the epistemic possibilities the diagonal carves up. My semantics makes it technically convenient to take the diagonal to divide the space of centered worlds, and that view of epistemic possibilities could be buttressed by Lewis [1979a]; nevertheless, it would be acceptable for my purposes to take diagonals to divide the coarser space of possible worlds (Lewis [1980] defines a diagonal along such lines), or perhaps something else. I abstract also from the pragmatic complexities introduced by the assumption of centered worlds; see Egan [2007] for discussion.

Now let me consider a second, very different response to the question of what content epistemic possibility claims serve to assert. According to this response, the question is actually confused. It just mistakes the speech act force of epistemic possibility claims. To say $\diamond\phi$ is not to propose to add some informational content, some proposition, to the common ground, as with assertions. Rather, it is to make explicit that ϕ -possibilities are compatible with the common ground—to make ‘explicit that the negation of ϕ is not presupposed in the context’ (to quote a passing suggestion of Stalnaker [1970a], p. 45). Suppose we followed Stalnaker in representing the information that is common ground by a *context set*, the set of possibilities where the propositions presupposed are all true. Then we could formalize this idea about the pragmatic effect of an epistemic possibility claim with the notion of acceptance defined above. To make an epistemic possibility claim in some context, on the present idea, is to propose to make it accepted with respect to the context set. What this speech act move exploits is not the diagonal of the epistemic modal sentence per se, but rather its horizontal at the context of utterance. The horizontal of an epistemic possibility claim determines a global condition on states of information (sets of possibilities), and the idea here is that in making such a claim, the speaker is proposing to make (or make explicit that) the context set satisfies this condition. (The horizontal associated with ϕ_c might be expressed as $\lambda s. [[\diamond\phi]]^{c,s,w_c}$.) To agree, in context, on $\diamond\phi$ is to explicitly coordinate on a body of presuppositions compatible with ϕ . A speaker who says $\diamond\phi$ is not expressing a proposition believed (known, etc.), but rather is expressing the compatibility of her state of mind with ϕ .

This account of the pragmatics of epistemic possibility claims goes naturally with informational consequence, because according to it, the communicative impact of such claims is understood in fundamentally in terms of acceptance, and informational consequence is what preserves acceptance. There is no ‘proposition expressed’ by an epistemic possibility claim on this picture, so there is no question of whether the proposition expressed is true or false. At most we can ask whether the claim is appropriate to accept or not, given the conversational tone(s) of the conversation.

Helping ourselves to the idea of a context set, we might summarize the two views just described about the pragmatics of epistemic possibility claims as follows.

- **Diagonal view:** To say $\diamond\phi$ in a context c is to propose to make $\dagger\diamond\phi_c$ accepted with

respect to the context set of c ²⁶

- **Informational view:** To say $\diamond\phi$ in a context c is to propose to make $\diamond\phi_c$ accepted with respect to the context set of c

How to choose?

The informational view has the same advantages over the diagonal view that informational consequence has over diagonal consequence: it avoids the need to define diagonal points of evaluation, and it gels better with intuition when it comes to inferences involving epistemic claims. The first point is obvious; let me give an illustration of the second.

Suppose the following. (1) Nobody—including ourselves—knows whether or not there is lead on Pluto, and indeed nobody is even close to having any evidence on the question of whether there is lead on Pluto. (2) As a matter of fact, there is no lead on Pluto. Now, on the basis of the information provided by these two premisses, is the following sentence true or false?

There might be lead on Pluto

There is strong pull to answer ‘false’. What that suggests is that the unembedded sentence ‘There might be lead on Pluto’ is not really understood as literally describing the condition of some agent’s evidential state, as on the diagonal view. (If it were, you would presumably be inclined to say ‘true’, since we have stipulated that, in the envisaged scenario, there is lead on Pluto for all anyone knows.) Rather, the behaviour of the sentence is akin to its behaviour in embedded contexts. The epistemic possibility operator is sensitive, not to the possession of some body of information by some agents, but rather only, as it were, to what is possessed: to the information itself. Its role is to place a condition on a possible body of information. In the sentence above, the modal is understood relative to the information conveyed by the premisses I asked you to suppose. When you evaluated ‘There might be lead on Pluto’ for truth, plausibly what you considered was whether lead’s being on Pluto would be compatible with the information you were asked to take for granted. We could say that you assessed whether the \diamond sentence was acceptable (in the technical sense) with

²⁶ The dagger ‘†’ is a two-dimensional modal operator which takes the diagonal of the sentence it embeds and projects it onto to the horizontal. See Lewis [1973], p. 63-4 for discussion.

respect to a certain temporary or ‘derived’ context set, one which included the information provided by premisses I asked you to take as given. Your judgement of falsity, on this interpretation of the facts, was really a (correct) judgement that the sentence could not be accepted with respect to that body of information.

I am about to conclude that intuition favours the informational view and informational consequence. Before that, let me consider a worry about that conclusion. The worry is that epistemic modal claims sometimes seem to communicate some kind of objective information, and it is not obvious how the informational view explains this. Take for instance:

(21) Cheerios may reduce the risk of heart disease

(22) Late Antarctic spring might be caused by ozone depletion²⁷

We tend to hear these sentences as (not just making certain possibilities explicit but) serving to communicate real information. Indeed, they strike us as the result of some actual research. This is intelligible on the diagonal view, according to which a proposition is expressed by epistemic modal claims. But how can we understand it on the informational view?

As follows. Sometimes when we converse we do so with the tacit aim of keeping our presuppositions compatible with (as it might be) the knowledge of the relevant experts. We try to get our presuppositions to relevantly overlap with expert knowledge. We try to obey a rule like:

Presuppose ϕ iff ϕ is known by the relevant experts

Our conversational tone is something like: treat as known by the relevant experts. In such cases epistemic modal claims, which on the informational view are pragmatically understood as imposing a condition on the information presupposed, will be assessed for correctness according to whether the informational condition they express is actually satisfied by the ‘target’ information—in this case, the relevant expert knowledge. They will therefore be ‘heard’ as communicating information about the knowledge of the relevant experts—concerning, as it might be, the health effects of Cheerios, or the causes of the

²⁷ The title of a 1987 article in *Nature*.

late Antarctic spring. In such contexts, epistemic possibility claims will be harder to make appropriately. These are contexts where it may be quite natural to say something like

(23) I do not know whether the late Antarctic spring might be caused by ozone depletion

On the natural reading of (23), what one grants is that one does not know whether something is an open possibility according to the target state of information that our presuppositional context aspires to. In these cases, we need to allow that interpretation may involve a tacit shift in the information parameter under the scope of ‘knows’, a shift to the target state of information for the context. Aside from Gricean considerations of charitable interpretation, it is not obvious whether general principles are involved in the interpretation of such tacit shifts.

(Of course, expert knowledge need not be the only sort of target information we attempt to keep our presuppositions in line with. We may have some specific body of evidence in mind, or we may be interested in what *could* be known about a topic if the investigative circumstances were ideal etc.)

I conclude that intuition favours the informational view and informational consequence. This conclusion suggests that we should not—or at least, we need not—actually think of the semantics proposed as a two-dimensional semantics. Were the semantics two-dimensional, the existence of diagonal points would be guaranteed. But if our conclusion is right, we need not assume the existence of diagonal points at all; hence we need not assume a purely two-dimensional semantics. The information parameter is perhaps better treated as semantically *sui generis*, not parasitic on Kaplanian contexts in the way that indices by definition are.

(If a purely two-dimensional semantics were found to be desirable on independent grounds, however, perhaps the best way to preserve the diagonal view in the face of the threats described above would be to effectively collapse it into the informational view, by letting the *s* parameter of a diagonal point be the context set of the context of that point. If diagonal points are defined this way, the two pragmatic moves technically come to the same thing. Note that this move would require abandoning Reflexivity, since the context

set of a conversation need not include the actual world. For this reason, the resulting definition of truth at a context would perhaps not be intuitive.)

1.6 Epistemic necessity operators

I have focused on the attractions of a domain semantics for epistemic possibility modals. A domain semantics for epistemic necessity modals has similar attractions. Here is the appropriate ‘dual’ semantics for epistemic necessity operators:

$$\llbracket \Box \phi \rrbracket^{c,s,w} \text{ is true iff } \forall w' \in s : \llbracket \phi \rrbracket^{c,s,w'} \text{ is true}$$

This semantics has three nice features. First, it explains what is wrong with

(24) # Suppose it is not raining and it must be raining

and its ilk. The explanation is along precisely the same lines as the domain semantics for epistemic possibility modals: $(\neg \phi \wedge \Box \phi)$ is an unacceptable sentence.

Second, it captures, to some degree, the sense in which epistemic necessity modals serve to indicate that a conclusion is being drawn from some (perhaps tacit) premisses. The reason is simple. On a domain semantics, $\Box \phi$ expresses a condition, not on possible worlds, but on bodies of information (sets of worlds). A body of information satisfies the condition expressed just in case ϕ follows from that information.

Third, in conjunction with our semantics for indicative conditionals, it explains the following observation. Observation: $(\alpha \rightarrow \psi)$ and $(\alpha \rightarrow \Box \psi)$ usually sound equivalent. Illustration:

Either the butler or the gardener did it. Therefore:

(C1) If the butler did not do it, the gardener did.

(C2) If the butler did not do it, the gardener must have.

(C1) and (C2) sound semantically equivalent. The explanation for why these two sentences sound equivalent on a domain semantics is that the sentences are equivalent. (C2) merely involves some additional vacuous quantification: the universal quantification introduced by the indicative conditional connective in (C2) is trumped by the quantification introduced by the embedded epistemic necessity modal. The conditional connective only influences interpretation in (C2) by shifting the information parameter over which the modal quantifies.²⁸

1.7 Toward probability operators

More trouble:

(25) # Suppose it is not raining and it is likely that it is raining

(26) # Suppose it is raining and it probably is not raining.

(27) # If it is not raining and it is probably raining, then...

(28) # If it is raining and it is likely that is not raining, then...

Certain probability operators—‘it is likely that’, ‘probably’, etc.—give rise to epistemic contradictions in the same way that epistemic possibility and necessity modals do. In the remaining pages I will sketch, in broad strokes, an approach to these operators, one developed in more detail elsewhere (Yalcin [In preparation]).

Abbreviate ‘it is probable that ϕ ’ and its kin (‘probably ϕ ’, ‘it is likely that ϕ ’) as $\Delta \phi$. The project is to state truth-conditions for $\Delta \phi$. The basic idea of the approach I want to recommend is simple: just upgrade the kind of object the information parameter can take as a value, from a set of worlds to a probability space. The intension of a sentence, relative to context, will be a function from world-probability space pairs to truth values.

²⁸ If the connective \rightarrow is decomposed into two semantic ingredients along the Kratzerian lines of n. 15 above, a second interpretation of the facts emerges. On the second interpretation, the only difference between (C1) and (C2) is in surface syntax: the epistemic modal explicit in (C2) is covert in (C1). Whether this interpretation is preferable depends on whether there is independent evidence for the presence of a covert modal in (C1)—not a question I will look into here. Suffice to say that both interpretations can be expressed in a domain semantics.

We will take it that a probability space \mathbf{P} determines a probability measure $Pr_{\mathbf{P}}$ over sets of possible worlds, and this measure is exploited in the semantics of Δ as follows:

$$\llbracket \Delta \phi \rrbracket^{c, \mathbf{P}, w} \text{ is true iff } Pr_{\mathbf{P}}(\{w : \llbracket \phi \rrbracket^{c, \mathbf{P}, w} \text{ is true } \}) > .5$$

Relative to context, $\Delta \phi$ determines a condition on probability spaces. The condition is satisfied just in case (roughly) ϕ is more likely than not according to the probability measure of the space.

There are some subtleties concerning what definition of ‘probability space’ is best suited to natural language probability operators. What follows is just one path through the decision tree; certainly, others are possible and worth exploring.

Think of a probability space (a state of information) as a certain triple $\langle \Pi, \pi, Pr \rangle$. Let me describe each member of the triple in turn. First, Π is a partition over the space of all possible worlds. The cells of this partition will represent the space of possible alternatives that are ‘recognized’ by the probability space (information state), in the sense that the grain of this partition will determine the possible worlds propositions that the probability measure of the space is defined over. A given Π may be said to recognize a possible worlds proposition p as an alternative just in case every cell in Π classifies with respect to p : just in case every cell ι in Π is such that, either every world in ι is a p -world, or every world in ι is $\neg p$ -world. To use a visual metaphor, Π provides a kind of ‘resolution’ over logical space: propositions not classified by Π are not ‘seen’ by the information state. (Cf. Lewis [1988] on subject matters.)

Second, π is a subset of Π . (It is therefore also a partition.) The cells of π are to be the *live* possible alternatives: they reflect what is *really* epistemically possible according to the probability space (information state). All of the probability mass of the probability measure will be located on the π -region of logical space.

Last, define Pr so that:

- (i) Pr assigns each cell ι in π a real value in the closed interval from zero to one, such that these values all sum to one
- (ii) For all propositions p that Π classifies, $Pr(p) =_{\text{def}} \sum_{\iota \subseteq p} Pr(\iota)$; otherwise $Pr(p)$ is undefined

My distinguishing π and Π may seem unnecessary. Why distinguish a special set π of epistemic possibilities? Why not leave π out of the formalism and let the epistemic possibilities just be those cells with nonzero probability? Because, at least not without further assumptions, it would be mistake to collapse epistemic possibility with nonzero probability. Continuous sample spaces in which probability zero events may nevertheless happen provide the usual counterexamples. See McGee [1994] and Hájek [2003] for further discussion of this issue.²⁹

Since we have changed the formal representation of information associated with the information parameter, and since epistemic possibility and necessity modals access this parameter, we need to update our semantics for these operators. The obvious idea would be to understand them as determining conditions on π , the epistemic possibilities associated with the relevant \mathbf{P} . To state the new semantics, a space-saving definition comes in handy: define truth and falsity with respect to a context, a probability space, and a *cell* ι as follows:

$$\llbracket \phi \rrbracket^{c, \mathbf{P}, \iota} \text{ is true (false) iff } \forall w \in \iota : \llbracket \phi \rrbracket^{c, \mathbf{P}, w} \text{ is true (false)}$$

Then the semantics for epistemic possibility and necessity modals is this:

$$\llbracket \Diamond \phi \rrbracket^{c, \mathbf{P}, \iota} \text{ is true (false) iff } \exists \iota \in \pi_{\mathbf{P}} : \llbracket \phi \rrbracket^{c, \mathbf{P}, w} \text{ is true}$$

$$\llbracket \Box \phi \rrbracket^{c, \mathbf{P}, \iota} \text{ is true (false) iff } \forall \iota \in \pi_{\mathbf{P}} : \llbracket \phi \rrbracket^{c, \mathbf{P}, w} \text{ is true}$$

The probabilistic semantics for $\Delta \phi$, $\Diamond \phi$, and $\Box \phi$ just given calls for one further assumption. Let us say that epistemic modal clauses carry a *classification presupposition*, to the effect that that the partition Π of the \mathbf{P} they are evaluated with respect to classifies the possible worlds proposition expressed by their complements ϕ . A probability space speaks to the question of whether a proposition is possible or probable only if the proposition is classified according to the space.

There are a lot of questions to be raised about this semantics, both technical and philosophical. Reserving extended discussion for elsewhere, let me devote the remainder of this section to a straight technical question: how do we use this semantics to explain what is

²⁹ Thanks here to Alan Hájek, Kenny Easwaran, and an anonymous reviewer. I have benefitted also from Aidan Lyons's unpublished work on this topic.

going on with our probabilistic epistemic contradictions in (25)-(28)? The obvious thing to do would be to follow the same strategy used earlier: first, define a notion of acceptance according to which the conjunctions are unacceptable, and second, give a semantics for the relevant embedding environments according to which these environments require acceptable complements.

The first step of this strategy is simple enough. We can update our definition of acceptance as follows:

ϕ_c is *accepted* with respect to \mathbf{P} iff $\forall \iota \in \pi_{\mathbf{P}} : \llbracket \phi^{c, \mathbf{P}, \iota} \rrbracket$ is true

On this definition of acceptance, it is trivial to verify that all the relevant epistemic contradictions ($(\neg\phi \wedge \Delta \phi)$, $(\phi \wedge \Diamond \neg\phi)$, etc.) are unacceptable.

The second step, however, is not as simple. Defining a semantics for attitude verbs and for indicative conditionals in the current probabilistic setting is a subtle matter, one raising considerations beyond the scope of this paper. I will have to settle for some sketchy and preliminary remarks on these constructions, the aim being only to give a sense of the prospects for probabilistic analyses and of the decision points that arise.

First, it is natural to conjecture that the semantics for acceptance attitude verbs ('believes', 'knows', 'accepts', 'supposes', etc.) can straightforwardly mirror our earlier domain semantics (Sect. 3). Let these verbs shift the value of the information parameter to the information state corresponding to the attitude state of the subject, and let the whole ascription require, for truth, that the complement of the verb be accepted with respect to that information state. The information parameter ranges over probability spaces, so the semantics assumes that these attitude states can be modelled by such spaces. The question arises how exactly to interpret the probabilities that go into modelling these attitude states. This issue is discussed, inconclusively, in the final section (Sect. 8).

Second, indicative conditionals. As with the attitudes, it would be natural to expect their analysis to be a probabilistic analogue of the domain semantics presented above. Here is a first pass. Conditionals express properties of probability spaces: an indicative conditional $(\phi \rightarrow \psi)$ in context is true with respect to a probability space \mathbf{P} just in case a certain other probability space (determined as a function of \mathbf{P}) which accepts the antecedent also accepts

the consequent. So the interpretation of an indicative will again involve an information parameter shift, a shift to a probability space accepting the antecedent. Which space do we shift to? The one that involves the ‘minimal change’ to \mathbf{P} needed to make the antecedent accepted with respect to that space. Here one can expect various theories of minimal change, which will need testing on specific examples. In the case where the antecedent is nonepistemic, a natural idea would be to shift to the probability space whose measure is just the conditionalization of the antecedent on the measure associated with \mathbf{P} . In the less common case where the antecedent is epistemic, it is less obvious what to say. (We could try shifting to the space \mathbf{P}' whose measure satisfies the condition and which is such that the relative entropy between the measures of \mathbf{P}' and \mathbf{P} is minimized; but care would need to be taken to avoid certain pitfalls for relative entropy minimization. See Grunwald and Halpern [2003] for a sense of the issues and references.)

It should be noted that a probabilistic semantics for indicative conditionals along the lines just described has a familiar independent motivation. Thanks to the triviality results of Lewis [1976] and others, it is well-known that if indicative conditionals express possible worlds propositions, the probabilities of the propositions they express could not in general be identical to the probabilities of their consequents conditional on the corresponding antecedents. It is also widely thought (thanks especially to Adams [1975]) that our tendency to accept an indicative conditional correlates closely with our intuitions about the corresponding conditional probability. Impressed by Adams’s thesis of acceptability and by the triviality results, many theorists have been tempted to conclude that indicative conditionals do not express possible worlds propositions; and from this they are tempted to effectively abandon semantics for indicatives altogether. I sympathize with the first temptation, but not the second. We can deny that indicative conditionals have possible worlds truth-conditions without denying that they have compositional semantic values. We can do it by saying that their compositional semantic values relative to context are effectively conditions on probability spaces—specifically, conditions on the relevant conditional probabilities. This would let us keep a tight semantic connection between indicative conditionals and the corresponding conditional probabilities without having to maintain, implausibly, that compositional semantics stops at ‘if’.

1.8 Outstanding issues

A lot of questions remain open. Here are some of them.

Outstanding semantic issues. There is still plenty of formal semantics left to do. The above semantics for epistemic modals should be connected in a natural way with the semantics of epistemic adjectives (as in ‘This a possible design for the new museum’). The work on probability operators should be connected with work on gradable adjectives generally, since these operators take all the same morphology and occur in comparative form (‘as probable as’, ‘more likely than’). The interaction of tense with this semantics for epistemic modals needs investigation. The attitude semantics given above should be shown to interact with plausible story about hyperintensionality. Finally, a detailed comparison of the static semantics I have sketched and a dynamic semantics for epistemic modals (along the lines of Veltman [1996] and Beaver [2001]) is in order.

The representation of uncertainty. In the previous section, I assumed without question that the representation of uncertainty appropriate to what I am calling probability operators is the probability space. But as it well known, there are numerous ways to represent uncertainty formally (see Halpern [2003]), and it may be questioned whether probability spaces really are appropriate to the semantics of (what superficially appears to be) natural language probability talk. Hamblin [1959], an impressive early investigation into this question, seems to favour a plausibility measure approach; and Kratzer [1991] gives a semantics for probability operators in terms of nonnumerical qualitative orderings of possibilities. It would be desirable to demonstrate, in so far as possible, that the resources of probability theory are in fact needed.

The interpretation of probabilities and probabilities in interpretation. Having assumed that probability spaces are in fact appropriate to the modelling of probability operators and related constructions, the question arises how best to understand the notion of probability at work in the semantics. There are a number of options here. Let me just mention two of interest, again reserving extended discussion for elsewhere.

First, we can try interpreting the probabilities along Bayesian lines, thinking of them as measuring degrees of confidence. On this interpretation, we can use the semantics to

formalize the idea that, in saying that it is probably raining, one thereby expresses one's credence in the proposition that it is raining—where this is not the same as saying that one's credence in the proposition is thus and so. What is the distinction between expressing one's credence and saying something about one's credence?

The contrast is the same here as it is with the expression of straightforwardly factual beliefs. Let Cleopatra say

Antony's fleet outnumbers the enemy's

She thereby expresses her belief that Antony's fleet outnumbers the enemy's, but she does not say that she has this belief. She is talking about the opposing fleets, not about her beliefs. (Gibbard [1990], p.84)

Exactly right. (I quote Gibbard out of context—the contrast he refers to is not my distinction between expressing one's credence and saying something about one's credence, but rather his distinction between expressing one's acceptance of a system of norms and saying that one accepts the system—but his analogy is perfect for the contrast I want to draw.) Now suppose Cleopatra says

Antony's fleet probably outnumbers the enemy's

On the Bayesianism-inspired interpretation of the semantics I want to consider, Cleopatra here expresses her state of high credence, or her confidence, in the proposition that Antony's fleet outnumbers the enemy's. She does not say that she is in this state of confidence. The only proposition in the vicinity is one about the opposing fleets.

How can our probabilistic semantics help to formalize this idea? Relative to context, the intension of 'Antony's fleet probably outnumbers the enemy's' determines a set of probability spaces—namely, the set of spaces in which the possible worlds proposition that Anthony's fleet outnumbers the enemy's receives a probability greater than one-half. If the probability spaces of the semantics are interpreted as idealized representations of credal states, then we can think of the sentence, relative to context, as expressing a property of credal states. Cleopatra expresses an aspect of her state of credence by uttering a sentence which, relative to context, expresses a property her credal state has, namely, the property of giving greater than one-half credence to the proposition that Anthony's fleet

outnumbers the enemy's. Thereby she 'gives voice' to that aspect of her credal state, but without uttering a possible worlds proposition about her credence. In a nod to the structurally similar view in metaethics, we could call this view about probability talk *credal expressivism*.

Credal expressivism is, I think, already tacit in the way that many Bayesians tend to informally describe epistemic modal beliefs. The usual way of modelling, within a Bayesian framework, someone describable as believing (for example) that it is probably raining would be to let the credence function characterizing their credal state map the proposition that it is raining to some highish value. Whether someone accepts what an epistemic modal clause says is thus generally taken to be a matter of their credence in the proposition expressed by the sentence embedded under the modal—not a matter of their credence in a proposition about their credence. In the attitude report, the modal tends to be treated, as it were, adverbially: the object of the agent's attitude is the proposition that it is raining, and the modal tells how strongly the proposition is believed. Attitude semantics for 'believes' along the probabilistic lines briefly sketched above can make semantic sense of this Bayesian tendency.

A second way of interpreting the probabilities is as measuring 'how far evidence supports or counts against various hypotheses about the world' [Mellor, 2005, p.79], or 'something like the intrinsic plausibility of hypotheses prior to investigation' ([Williamson, 2000, p.211]). This is sometimes called the *epistemic* or *evidential* interpretation of probabilities. While on the Bayesian interpretation probabilities measure the strength of an agent's confidence, on the evidential interpretation probabilities measure something like the objective degree of confirmation a body of propositions confers on a given proposition. We can still use the probability spaces of our semantics to characterize attitude states on this interpretation of the probabilities. But the import of the representation is quite different. The probabilities are now to be understood as a part of the informational content of the attitude state—not as measures of the strength of the attitude towards content. We could call this view *content probabilism*. It is the view that informational content itself is probabilistically articulated.

A challenge for the credal expressivist about epistemic modal talk is to make sense of the felicitous occurrence of epistemic modals in attitude contexts for which the corresponding

attitude does not, intuitively, come in degrees. A mixed strategy may therefore be in order as far as the interpretation of the probabilities goes: perhaps credal expressivism and content probabilism are each appropriate to different fragments of our folk probabilistic talk. The question of how the two might be integrated deserves investigation.

Probability in context. It is a short step from a probabilistic semantics and probabilistic representations of attitude states to a probabilistic pragmatics. Above (Sect. 5) I followed Stalnaker in treating the attitude of presupposition as central to characterizing the informational context (common ground) of a conversation, and in treating the informational context as central to the characterization of speech acts like assertion. Suppose now that, along content probabilist lines, we took it that the informational content of a state of presupposition could be characterized by a probability space, or by a set of such spaces. This would lead us naturally to a view we could call *context probabilism*.

Context probabilism: the common ground of a conversation is characterizable as a probability space, or as a set of such spaces.

Rather than representing the common ground by a context set, a set of possible worlds, we would represent it as a probability space—call it a *context probability space*—or as a set of probability spaces—what we might call a *context representor*. We could then think of sentences uttered in context as serving to determine constraints on probability spaces, and thereby on the common ground. To utter ϕ in c , we could try saying, is to propose to make it accepted by the context probability space (or by all the probability spaces in the context representor). This would let us model the communication of information, not only in terms of the elimination of possibilities, but also in terms of the elimination or evolution of the possible *probabilities* over possibilities. It would let us represent the transfer of purely probabilistic information.

What is at issue in the choice between representing the common ground as a context representor or as a single context probability space? If each thing presupposed in context determines a constraint on a probability space, then the context representor can be understood as just the set of probability spaces satisfying all those constraints. We can think of these as the probability spaces that are admissible given what is presupposed. This

representation of the common ground seems to have an advantage over the representation in terms of a single context probability space: it allows us to avoid the idealization that interlocutors in context coordinate on precise probabilities for the propositions they are concerned about. Nevertheless, we might try constructing a single context probability space from the context representor, by finding the probability space that satisfies all the constraints associated with representor and which otherwise maximizes entropy. Both representations deserve investigation.

The probabilistic representation of context we settle on—a context probability space, a context representor, or something else—will determine our options for modelling the dynamics of context change. In this paper I have mainly focused on static acceptability conditions: I have asked what properties an agent’s presuppositional state must satisfy in order to count as accepting the various epistemic modal claims I have discussed. What I have left out is an account of the dynamics of presupposition. If you are in some presuppositional state, and your interlocutor proposes that you move to a presuppositional state satisfying property F , exactly how should you shift your state in order to satisfy F ? Any complete version of context probabilism will have to address this question. Individual probability spaces can be understood to evolve by various forms of conditionalization, and by shifts in what counts as epistemically possible according to the space; context representors can be understood to evolve by changing their members, permitting or eliminating new probability spaces. If something like context probabilism is on track, the proper treatment of dynamics will be among the leading questions.

Chapter 2

Nonfactualism about Epistemic Modality

2.1 Introduction

When I tell you that it's raining, I describe a way the world is—viz., rainy. I say something whose truth turns on how things are with the weather in the world. Likewise when I tell you that *the weatherman thinks* that it's raining. Here the truth of what I say turns on how things are with the weatherman's state of mind in the world. Likewise when I tell you that *I think* that it's raining. Here the truth of what I say turns on how things are with my state of mind in the world.

Nothing like tedious platitudes to set the mood. Okay—what about when I tell you that it *might* be raining? Or that it is *probably* raining? Or that it *must* be raining? In these cases, am I again to be understood as describing a way the world is?

An affirmative answer would be nice. For it would mean less work. It would let us take the view that sentences like these—sentences with epistemic modal operators taking wide scope—are not special. It would let us apply to these sentences whatever semantic/pragmatic explanatory strategies we apply to other uncontroversially descriptive, fact-describing discourse.

And, from a distance at least, an affirmative answer seems anyway not hard to pull off. Epistemic modals are so-called, after all, because they seem to serve to communicate information about some epistemic state or state of evidence in the world. One could try, then, understanding epistemically modalized sentences—these sentences about what might or must be, or about what is probable—as telling how things are with some epistemic state or other, or with some body of evidence or other, in the world.

Since an affirmative answer means less work and looks not hard to pull off, little wonder that that answer is a very popular one. Indeed, it has some title to being called the *standard view* about epistemic modality in philosophy. (We will see evidence below.) We can spin the standard view either as a metaphysical thesis or a semantic thesis. The metaphysical thesis is *factualism* about epistemic modality. To a very rough first approximation, factualism is the idea that for it to be true that it might be (or must be, or probably is) raining is for the world to be configured in a certain way, for a certain state of affairs to obtain in the world. The semantic thesis is *descriptivism* about epistemic modal discourse. Descriptivism is the idea that epistemic modal talk serves fundamentally to describe reality, to say how the world, or some aspect of the world, is.

The thesis of this paper is that the standard view is mistaken. Though from a distance it may seem that epistemic modal discourse can be treated descriptively, observation at close range reveals serious in-principle obstacles to descriptive analysis. I will argue that there are elementary facts about the semantic behavior of epistemic modal operators that cannot be accommodated plausibly along descriptivist, factualist lines. Together with a number of pragmatic anomalies unembedded epistemic modal claims are now well-known to give rise to, the facts will motivate the development of a nonfactualist, nondescriptivist alternative. With caveats to be provided in due course, the positive account to be set out could plausibly be called a kind of *expressivism* about epistemic modal discourse.

I begin by setting out the standard factualist/descriptivist picture in more detail and by providing some specific examples of the descriptivist account of epistemic modal discourse. After that I make the case against the standard view. The positive account begins in section 6 ('States of mind'), and its development occupies the rest of the paper.

2.2 Descriptivism about epistemic modals

Loosely speaking, descriptivism about epistemic modal discourse is what you get when you begin with factualism about epistemic modality and semantically ascend. Semantic ascent affords a better view of the dialectical terrain, so much of my discussion will be keyed to the descriptivist reading of the standard view. Descriptivism, I already said, is the view that epistemic modal talk serves to describe reality. Let me clarify ‘epistemic modal talk’ and ‘serves to describe reality’.

By ‘epistemic modal talk’, I have in mind clauses that are modalized with natural language epistemic modal operators. For instance, ‘It is possible that it is raining’, ‘It might be raining’, ‘It could be raining’, ‘It is probably raining’, ‘It is likely that it is raining’, and ‘It must be raining’ all have readings on which the modals they contain are interpreted epistemically. (With ‘might’, ‘likely’, and ‘probably’, the epistemic reading is typically the preferred reading, if not the only reading; with ‘could’, ‘possible’, and ‘must’, other readings, such as a deontic reading, are often possible.) I don’t attempt an operational definition of this class of modals now; it is our eventual project to provide a theory which delimits the class more precisely. Only let me be explicit that by ‘epistemic modal operator’, I don’t have in mind complex operators such as ‘for all I know, it might be that’—operators with simple epistemic modals scoped under epistemic attitude verbs. The importance of excluding these complex operators will become clear later. I will also avoid interactions with tense, restricting myself to the case where these modals take apparently present-tensed complements.

By ‘serves to describe reality’, I mean that epistemic modalized clauses serve to represent the world, or one’s situation in the world, as being a certain way. More precisely: I mean that, relative to context, the semantic content of the clause determines, and is understood as determining, a condition on possible worlds or situations. The informational content of the clause has the effect of dividing the space of possible ways things might be into those which conform, and those which fail to conform, with how things are represented as being. Let me call a set of truth-conditions which serve to divide the space of possible worlds or situations *factualist truth-conditions*. A descriptivist provides factualist truth-conditions for epistemic modal talk.

I hope descriptivism sounds like the straightforward view it is. Some examples of descriptivist views will help to round out the picture. Start with descriptivism about epistemic possibility modals. G. E. Moore writes:

People *in philosophy* say: The propositions that I'm not sitting down now, that I'm not male, that I'm dead, that I died before the murder of Julius Caesar, that I shall die before 12 tonight are 'logically possible'. But it's not English to say, with this meaning: It's possible that I'm not sitting down now etc.—*this* only means 'It is not certain that I am' or 'I don't know that I am'. [Moore, 1962, p. 184]

Moore's view is descriptivist simply because according to it, epistemic possibility sentences in context are descriptions of the epistemic state of some agent in the world. Most descriptivists agree with Moore's basic idea, that these sentences serve to describe the epistemic situation of some agent or agents. The internal debate among descriptivists concerns the detailed nature of the description—for instance, which agents matter, or what aspect of the agents' evidential situation are relevant. For example, three or four epicycles of analysis down from Moore, DeRose proposes that

S's assertion "It is possible that P" is true if and only if (1) no member of the relevant community knows that P is false, and (2) there is no relevant way by which members of the relevant community can come to know that P is false [DeRose, 1991, p. 593-4]

while Stanley suggests that these sentences describe the epistemic state of some contextually given knower *A*:

It is possible_{*A*} that *p* is true if and only if what *A* knows does not, in a manner that is obvious to *A*, entail not-*p*. [Stanley, 2005, p. 128]

The particular motivations for these departures from Moore's position needn't detain us. The point is just that, although Moore, DeRose, and Stanley all differ on exactly what facts epistemic possibility sentences describe, they all agree that these sentences serve to describe some facts or other, some feature of the world.

Those are examples of descriptivism about epistemic possibility. What about epistemic necessity? If, as is widely assumed, epistemic necessity modals (\Box) are the logical duals of epistemic possibility modals (\Diamond) in the sense that

$$\Diamond\phi \leftrightarrow \neg\Box\neg\phi$$

then each of the above accounts of epistemic possibility straightforwardly generates an account of epistemic necessity. So, given duality, Moore's view would be that 'It must be raining', on the epistemic reading, is true just when 'I know it's raining' is; and so on for the other two views. It should be clear that the resulting positions on epistemic necessity are no less descriptivist than the positions on epistemic possibility they are constructed from. The duality of epistemic possibility and necessity is extremely plausible; I will assume it throughout.

Last, probability operators such as 'probably' and 'it is likely that', which I will abbreviate as ' Δ '. Here, a simplistic example of a descriptivist position can be abstracted from the Bayesian paradigm: for one to say 'It's probably raining' is for one to say that one's credence in rain is above one-half, or above some contextually-determined value.¹ In calling something 'probable', one describes one's credal state. A second position, closer in form to the descriptivist accounts of (non-probabilistic) epistemic modality just described, adverts to some tacit body of knowledge or evidence. Suppose a body of evidence induces, or is representable by, a probability measure over a domain of propositions. Then instances of $\Delta\phi$ can be understood to say that the proposition that ϕ has some highish value according to the measure induced by the body of evidence determined by the context in which the sentence is uttered. They would, in short, be factual claims about some contextually determined body of evidence in the world.

Descriptivism requires no fundamentally new semantic or pragmatic assumptions. Semantically, we can compositionally assign epistemic modal clauses possible worlds truth-conditions (or centered worlds truth-conditions) in perfectly ordinary fashion. (The stan-

¹ Jeffrey seems to have something like this in mind when he writes: 'If you say the probability of rain is 70% you are reporting that, all things considered, you would bet on rain at odds of 7:3' [Jeffrey, 2004, p. 3]. (For Jeffrey, one's credence just is a matter of one's disposition to bet.)

dard semantics is Kratzer's: see Kratzer [1977, 1981, 1991]; see also Lewis [1979b].) Pragmatically, too, we needn't make waves. We can retain a familiar picture of communication, a picture that gives no special place to epistemic modal talk. Whether I say that it is raining or I say that it is probably raining, the story about what is happening, at least in straightforward cases, can be the usual Gricean one: I believe myself to have some information, and wish to impart it to you; I say something whose truth turns on whether this information is true, presuming common knowledge of the language; in so doing I intend for you to come to accept that information, acting with the expectation that my intention to communicate that information is mutually recognized. More needs to be said to fill in the details, of course; but suffice to say that, for the descriptivist, the details will be filled in just the same ways for epistemic and non-epistemic modal talk alike.

Because descriptivism makes no semantic or pragmatic waves, there is a presumption in favor of it. In the next three sections I attempt to defeat this presumption, by describing some phenomena not amenable to descriptivist explanation.

2.3 Epistemic contradictions

Notice that the following sentences sound awful.

- (1) # It's raining and it might not be raining.
- (2) # It's raining and it probably isn't raining.

Let us call sentences like these—sentences of the schematic form $(\phi \wedge E\neg\phi)$, where E is an epistemic modal—*epistemic contradictions*.² Why do epistemic contradictions (1) and (2) sound awful?

At first glance, a descriptivist explanation seems easy enough. A descriptivist might try saying that these sentences sound terrible because, thanks to the semantics of the epistemic modals, these sentences both truth-conditionally entail

² A longer discussion would include sentences of the form $(\neg\phi \wedge E\phi)$, which are epistemic contradictions, too. Everything I say will carry over to sentences of this schematic form as well. See Yalcin [2007] for a much more detailed discussion of epistemic contradictions.

(3) # It's raining and I don't know its raining.

relative to context. Obviously, (3) is Moore-paradoxical. Therefore, says the descriptivist, (1) and (2) should be pragmatically defective in whatever way (3) is defective. The defect in (1) and (2) is parasitic, as it were, on (3). Epistemic contradictions are not contradictions in any semantic sense. They are just Moore-paradoxical sentences in new guise.

The situation is not so simple, however. (1) and (2) are more than merely pragmatically defective. The conjuncts in these sentences are incompatible in a more robust sense. We can see this when we attempt to embed these conjunctions into larger constructions. Consider, for instance, the imperatives:

(4) # Suppose it's raining and it might not be raining.

(5) # Suppose it's raining and it probably isn't raining.

These imperatives make no sense. The fact that they do not make sense is not explained by the assumption that the conjunctions they each embed both truth-conditionally entail (3), because (3) is perfectly easy to embed under 'suppose':

(6) Suppose it's raining and I don't know its raining.

(Indeed, the *intelligibility* of sentences like (6) is a popular motivation for denying that classically Moore-paradoxical sentences are contradictions in any semantically rich sense.) Or again, epistemic contradictions never sound acceptable in the antecedent position of a indicative conditional:

(7) # If it's raining and it might not be raining, then...

(8) # If it's raining and it probably isn't raining, then...

Conditionals that begin in this way seem beyond repair. But Moore-paradoxical sentences are acceptable in this environment:

(9) If it's raining and I don't know it, then I will get wet.

Compare that with the nonsensical

(10) # If it's raining and it might not be raining, then I will get wet.

The conditional (10) is particularly telling. If it really were the case that, relative to context, ‘It might not be raining’ entailed ‘I don’t know that it’s raining’, we would expect (10) to be about as acceptable as (9). But the difference in acceptability could hardly be greater.

The upshot is this. Epistemic contradictions ‘project their unacceptability’, as it were, in the embedded contexts described above. Moore-paradoxical sentences do not. The defect in sentences which embed epistemic contradictions is therefore not parasitic on Moore’s paradox. It must be explained in some other way.

And the problem is that is not at all clear how to explain it plausibly along descriptivist lines. Descriptivists want to tell us that epistemic contradictions such as (1) and (2) above have factualist truth-conditions. In particular, they want to tell us that these sentences have *non-empty* factualist truth conditions, truth-conditions that obtain in some possible situation.³ If the truth-conditions of (e.g.) ‘It isn’t raining and it might be raining’ are non-empty, however, it seems there should be nothing at all preventing us from hypothetically entertaining the obtaining of these conditions. We ought to be able to consider such a possibility simply as a matter of semantic competence. But we can’t. Evidently there is no coherent way to entertain the thought that it isn’t raining and it might be raining. Descriptivists fail to predict this.

It is not hard to see why. According to standard versions of descriptivism, the truth-conditions for (1) have the following schematic structure:

‘It’s raining and it might not be raining’, uttered at c , is true in w just in case

- (i) It’s raining in w ; and
- (ii) Some select epistemic state or body of evidence in w is thus and so in w

Descriptivists differ on how to precisify (ii), as we saw above. But on any plausible way of precisifying it, the result will be truth-conditions for (1) which obtain in some possible

³ Else the factualist would have to say that the conjuncts of an epistemic contradiction are truth-conditionally incompatible, hence that ‘It might be raining’ entails ‘It’s raining’, hence that epistemic possibility modals are factive operators—a totally unacceptable result.

(One might make the case that $\Box\phi$, unlike $\Diamond\phi$ or $\Delta\phi$, actually *is* truth-conditionally incompatible with $\neg\phi$. My discussion is therefore focused on the obviously nonfactive epistemic modal operators.)

situations, possible situations which should be straightforward to hypothetically entertain. That, again, is the wrong prediction, because ‘It’s raining and it might not be raining’ is not trivial to entertain-true. Indeed, plausibly it is impossible to entertain-true. That is why it sounds incoherent for me to ask you to entertain it, as in (4) above. The result is that descriptivism predicts coherence for constructions such as (4), whereas incoherence is what we find. All this may be repeated, *mutatis mutandis*, for (2), which trades the epistemic possibility modal for the probability operator ‘probably’.

Let me emphasize that the problem I have described is not due to particular features of what have called ‘standard’ descriptivism. That is, the problem is not just a problem for those versions of descriptivism which takes epistemic possibility clauses to be descriptions of epistemic states or states of evidence. It affects any descriptivism which allows that $\Diamond\phi$ and $\neg\phi$ (or $\Delta\phi$ and $\neg\phi$) are truth-conditionally compatible. (For as long as they are compatible, and as long as the truth-conditions are not implausibly complicated, their conjunction will describe a coherent and entertainable circumstance.) In other words, the problem affects any remotely plausible version of descriptivism.

There you have the elementary facts about the semantic behavior of epistemic modal operators that are difficult to handle under descriptivist assumptions. To this I want to add some further worries, these concerning the behavior of *unembedded* epistemic modal *claims*, rather than embedded epistemic modal clauses. These further worries occupy the next two sections.

2.4 Assertability and disagreement

We sometimes disagree, not merely about what is the case, but also about what might be the case, and about what is probably the case. The second concern about descriptivism is that it is hard to see how to deliver factualist truth-conditions for epistemic modal talk which make sense of this kind of disagreement.

The trouble was first noted by Huw Price. Price considers the idea of assigning $\Delta\phi$ factualist truth-conditions along the lines of ‘Given the existing evidence, it is probable that ϕ ’. He observes that the phrase ‘the existing evidence’ is ambiguous, admitting a

spectrum of readings from the more subjective to the more objective. He first attempts a subjective reading of the phrase, along the lines of ‘the evidence of which I [the speaker] am actually aware’. He objects that:

If I disagree with your claim that it is probably going to snow, I am not disagreeing that given *your* evidence it is likely that this is so... Indeed, I might *agree* that it is probably going to snow and yet think it false that this follows from your evidence. [Price, 1983, p. 404]

Here the problem is that the proposed truth-conditions for ‘It is probably going to snow’ are too weak to make sense of appropriate disagreement.

Next he tries an objective reading of ‘the existing evidence’, along the lines of ‘the evidence accessible in principle’. Such a reading would make disagreement intelligible: in the above example, for instance, you and Price would be differing over what is made probable by the evidence accessible in principle. But he objects that this more objective reading wouldn’t square with the facts about when $\Delta\phi$ is felicitous to assert:

...consider the surgeon who says, ‘Your operation has probably been successful. We could find out for sure, but since the tests are painful and expensive, it is best to avoid them.’ The accessibility, in principle, of evidence which would override that on which the [probability] judgment is based, is here explicitly acknowledged. [Price, 1983, p. 405]

Here the surgeon says $\Delta\phi$, but leaves open whether ϕ is probable given the evidence accessible in principle. No surprise he would leave *that* question open, after all. He simply doesn’t *have* the evidence accessible in principle. Hence his statement of $\Delta\phi$ is not well understood as speaking to a question about the evidence accessible in principle.

This now provokes the question: what or whose evidence is relevant to settling the truth of a given claim of $\Delta\phi$? We appear to need something *in between* the evidence of the speaker and the evidence available in principle. It is hard to see, however, how something in between could ever really *be* assertable for the speaker. Something in between, after all, is by definition beyond the scope of the speaker’s evidence. If we settled on something in between, our speaker would still be pictured as saying something whose truth turns on

a body of evidence that she doesn't have. It seems she would be pictured as speaking, and knowingly speaking, from a position of ignorance, making a stronger claim than is warranted by her evidence alone. Her speech act looks in danger of being irrational.⁴

The objection to descriptivism, then, is that it faces a tension. Either descriptivist truth-conditions systematically fail to capture the truth-value judgments that people actually make (by being too weak to capture the disagreement facts), or it captures these judgments but turns users of epistemic modal sentences into irrational asserters (by picturing them as making claims about/from a body of evidence they don't have). The difficulty here recurs exactly with epistemic possibility claims, as the reader may confirm by replacing 'probably' with 'possibly' in Price's examples.

2.5 Conflicting intuitions

The third problem with descriptivism is that it leads us to expect clear intuitions in cases where intuitions are not clear.

Consider the following case.

Fat Tony secretly plants highly compelling evidence of his murder at the docks. The evidence is discovered by the authorities, and word gets out about his apparent death. The next evening, from his safehouse, Fat Tony watches a panel of experts on the news discussing the situation.

Expert *A* has had a good look at the evidence found at the scene. "Fat Tony is dead," he says.

Expert *B* has also had a good look at the evidence, but his assessment is more cautious. "Fat Tony might be dead," *B* says.

⁴ Couldn't her evidence include information about the state of some other, not-yet-possessed body of evidence? And in that case, couldn't her evidence provide her with the warrant to make claims about what is made probable by this not-yet-possessed evidence? But it is unclear whether one can have evidence that some other, larger body of evidence makes *p* probable without one's own evidence itself making *p* probable. (See also section 3.1 of MacFarlane [2006] (this volume) for additional problems for this kind of approach.)

We can all agree that Expert *A*, however reasonable his speech act was in light of the information available to him, spoke falsely. Things are not as he says they are. Okay; what about Expert *B*? Is what he said true or false? Let me remind you that Fat Tony’s planted evidence was highly compelling. Let me remind you also that Fat Tony is *definitely not dead*. And, before you settle on an answer, let me ask you also to ponder whether Fat Tony himself should agree with your answer.

Now, what do you say about what *B* said—true or false?

It appears that, as a matter of empirical fact, intuitions are unclear about cases such as this—cases where an epistemic modal claim is assessed for truth from outside the discourse context.⁵ Some are inclined to say that *B* spoke truly; others are inclined to say that *B* spoke falsely; everyone else shrugs, or proposes to change the question to one with a clearer answer.

What needs explaining for eavesdropping cases such as this is not any univocal intuition we all have about the epistemic modal claim made in the case. There is no single intuition there to explain. Rather, what needs explaining is the absence of agreement, by competent speakers of English, on what the right answer is. What needs explaining are the conflicting intuitions. My point for now is just that conflicting intuitions are not expected on descriptivist assumptions. If *B*’s utterance is in the business of representing the world as being a certain way, as *A*’s presumably is, then either the world is that way, or it isn’t. Other things being equal, we’d expect intuition concerning the truth of *B*’s utterance to be about as clear as it is with *A*’s. Descriptivists have work to do, then, explaining why things are not equal.

A descriptivist might reply that this work is not really so hard. “Epistemic modals are, after all, highly context-sensitive on our view. Perhaps the lack of uniformity in judgments here

⁵ These are usually called *eavesdropping cases*. See MacFarlane [2003, 2006], Egan et al. [2005], Egan [2007] for discussion. These authors take the speaker judgments about eavesdropping cases to be less ambivalent than I do—on this matter we take different positions on what is an empirical question—and they use these cases to motivate different versions of *relativism* about epistemic modal talk. I lack the space to discuss relativism in adequate detail here, but see Yalcin [2006] for discussion of MacFarlane’s view. (Egan’s view, which could be interpreted as a factualist view, founders on epistemic contradictions.)

is simply due to the fact that subjects considering this case resolve this context sensitivity in different ways.”

But this reply is not satisfactory. If the interpretation of epistemic modals is as context-sensitive as suggested, presumably there is at least one reading of the modal according to which what *B* says above is both (1) true and (2) assertable for *B*. (Perhaps a reading along the lines of ‘The evidence in the reach of *B* leaves open the possibility that Fat Tony is dead.’) Now, where multiple interpretations of a speaker’s utterance are possible, subjects tend to gravitate towards true and assertable readings, for the simple reason that true and assertable readings tend to make most sense of what the speaker is doing—they tend to be easier to situate into a rational overall pattern of action. But this would lead us to expect a fairly robust judgment that what *B* says is true, the incorrect result.

It is striking, incidentally, that the body of evidence allegedly relevant to assessing the truth of an epistemic modal claim should be so obscure to speakers who actually use these sentences. If these sentences really do advert to some tacit body of evidence, as standard versions of factualism maintain, why are we competent speakers of the language not able to articulate what this body is? This opacity is puzzling. It is not a feature of context-sensitive language in general. For instance, when we use quantifiers in ordinary discourse, typically a restriction on the quantifier is provided tacitly by context. But with sufficient description of context, speakers can typically say what the restriction is; and where context is insufficient, speakers can typically indicate what further information is need to settle the question. Similarly for pronominal anaphora, and for demonstratives. But not so, it seems, for epistemic modals. Appeals to the context-sensitivity of epistemic modals seem to be of questionable explanatory power here, then.

Let me summarize. We have accumulated three desiderata for a theory of the meaning of epistemic modals. Such a theory should:

- I. Explain why **epistemic contradictions** are unembeddable.
- II. Explain the **assertability and disagreement** facts concerning epistemic modal claims in context.

- III. Explain the **conflicting intuitions** concerning epistemic modal claims in eavesdropping cases.

The first desideratum is plausibly understood as a constraint on the formal semantics of epistemic modals (together with the semantics of the relevant embedding environments). The second two desiderata are plausibly understood as constraining the pragmatics of epistemic modal claims—more precisely, their communicative content. The first of these pragmatic desiderata concerns the intra-contextual facts about how we assess epistemic modal claims *qua* participants in the discourse. The second concerns the extra-contextual facts about how we assess epistemic modal claims *qua* onlookers from outside the discourse.

Descriptivism, we have seen, is not well-equipped to satisfy these desiderata. It is time to take steps towards an alternative.

2.6 States of mind

If we want to understand what is going on with epistemic modal talk, we may be better served by taking a less direct approach. Let us take a step back from the linguistic facts and from direct questions about the truth-conditions of epistemic modal clauses. Let us ask instead:

What is it to be in a state of mind which accepts what an epistemic modal claim says?

I will suggest that descriptivism rests on a mistaken answer to this question, and that getting the answer right is the first step towards clarifying the meaning and role of epistemic modal discourse. The focus of this section will be on developing a model for what it is to believe that something might be so, or that something is probably so. Once we are clear on this, we will turn to the semantics and pragmatics of the language that is used to express these states of mind. It will be some time before we turn back to the desiderata recently described above; but when we finally do, we will be in position to accommodate them.

Epistemic Possibility

Begin with epistemic possibility. I believe that it is possible that Bob is in his office; Frank believes that it might be raining in Topeka. What kind of states of mind are we each in? Doxastic states of mind, trivially. How to model a doxastic state of mind? For our purposes, we may represent a doxastic state by its informational content, abstracting for now from its functional role in cognition and action. How, then, to represent the informational content of a doxastic state of mind?

Start with a familiar picture of informational content in general. Information is foremost that which eliminates possibilities. To gain information is to transition to a state of mind which leaves fewer possibilities open as candidates for actuality. As a first approximation, then, we may represent a body informational content as a set of possibilities, those possibilities left open by that informational content. So a state of belief is representable by a set of possibilities: intuitively, those not excluded by what is believed.⁶ We can think of this set as the set of possibilities at which each proposition believed by the agent is true. We may define *proposition* functionally, as whatever it is which is the potential object of belief. Of propositions we need only assume that they determine truth-conditions, again representable by a set of possibilities. (For convenience I will talk as if propositions just are sets of worlds, but strictly a determination relation is all that is needed.)

Equipped with this representation, we can provide an abstract picture of the descriptivist model of epistemic possibility beliefs—of what, according to the descriptivist, it is to believe that it is possible that Bob is in his office, or that it might be raining in Topeka. The picture is very simple. See figure 2.1.

The rectangle is logical space, the space of maximally specific metaphysical possibilities. A subset of those possibilities is the proposition that $\Diamond\phi$, here the set of possibilities contained within the dashed ellipse. *A* believes that $\Diamond\phi$ just when *A*'s belief worlds are a subset of the

⁶ As everyone knows, the classic possible worlds representation of belief faces acute problems, notably Frege's puzzle and an apparent commitment to logical omniscience. Do not be alarmed. Dialectically this classic picture will be serving as my point of departure, not arrival; and anyway, what problems it has crosscut the issues I want to discuss. Soon we will work this classic picture into something more realistic. Meanwhile it will let us provide a perspicuous representation of key features of descriptivism—features it has independent of the possible worlds representation.

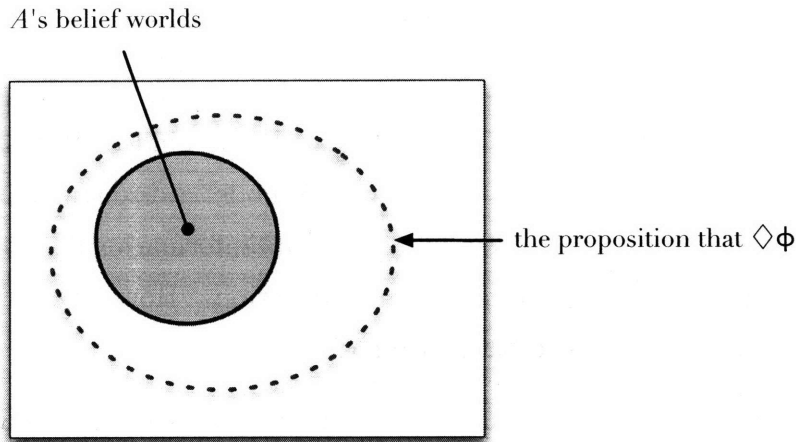


Figure 2.1: $B_A \diamond \phi$: The descriptivist model.

proposition that $\diamond\phi$. Thus for me to believe that Bob might be in his office is for a certain proposition—whatever proposition it is the descriptivist gives me—to be true throughout my belief worlds. Again, standardly the descriptivist’s truth-conditions are propositions about some body of evidence, where this body of evidence includes the knowledge of the agent doing the believing. As a result, the typical descriptivist picture is one according to which states of $\diamond\phi$ -belief are *second-order* states of mind, states of belief about (perhaps *inter alia*) one’s state of knowledge.

This way of thinking about the standard descriptivist picture provokes the question: when I believe Bob might be in his office, am I in a second-order state of mind?

We could try asking it like this. Is the question, “Why believe Bob might be in his office?” in part the question, “Why believe that I don’t know that Bob isn’t in his office?” Pretheoretically, the idea seems to have little motivation. Our initial question seems to be about Bob’s location, not about my views about Bob’s location. The question “Why believe Bob might be in his office?” seems instead equivalent to the question, “Why fail to believe that Bob isn’t in his office?” This latter question is clearly not a question about what to believe about one’s knowledge. It is just a question concerning what to believe about where Bob is.

These points are, I think, suggestive, but alone they are perhaps not decisive.⁷ Let us then consider the issue from another, rather different perspective. Suppose we are eating dinner, and my dog Fido comes into the room and heels by my chair. Occasionally I toss Fido a bone at dinner, but usually I don't. You ask why Fido is sitting there staring at me. I say:

(11) Fido thinks I might give him a bone.

An appropriate remark. What does it mean? Does it in part mean, as standard versions of descriptivism would require, that Fido believes that it is compatible with what he knows that I will give him a bone? That is not plausible. Surely the truth of (11) doesn't turn on recherche facts about canine self-awareness. Surely (11) may be true even if Fido is incapable of such second-order states of mind.

These considerations suggest that the question of whether $\Diamond\phi$ is 'transparent', as it were, to the question of whether ϕ . I think this is reflected in the kinds of reasons we understand to support epistemic possibility beliefs. Naively, correctly believing that $\Diamond\phi$ is a matter of there being an absence of conclusive reason to believe that $\neg\phi$. Correctly believing that ϕ is a matter of there being conclusive reason to believe that ϕ . Both kinds of reason concern how to settle one's doxastic state toward the proposition that ϕ . Believing that ϕ and believing that $\Diamond\phi$ are states of mind are supported by reasons of the same category.

It seems, then, that we have found another desideratum for a theory of epistemic modals. Such a theory should

- iv. Avoid the assumption that belief reports embedding epistemic modal clauses report **second-order states of mind** (i.e., beliefs *inter alia* about one's state of mind).

⁷ That is because it can be difficult to disentangle, from a first-person point of view, questions about what the world is like from questions about what one believes the world is like. As Evans famously observed, "If someone asks me 'Do you believe that there will be a third world war?', I must attend, in answering him, to precisely the same outward phenomena as I would attend to if I were answering the question 'Will there be a third world war?'" [Evans, 1983, p. 225]. (Though the point should not be overstated. The questions, "Why believe that ϕ ?" and "Why believe that you believe that ϕ ?" needn't always have the same answer.)

Again, this is another desideratum that descriptivism is not well placed to capture. If one expresses a proposition one believes when one says (e.g.) ‘It might be raining’, and that proposition has the epistemic-state-describing truth-conditions assigned to it by standard versions of descriptivism, it is a very short step to the thought that to believe it might be raining is to believe that very proposition. (Note that this point is no artifact of the possible worlds model I have used to illustrate descriptivism.)

Let us ask now: what minimal modification to the descriptivist model would be required to satisfy this new desideratum (IV)? I suggest that the modification is this one depicted below (figure 2.2).

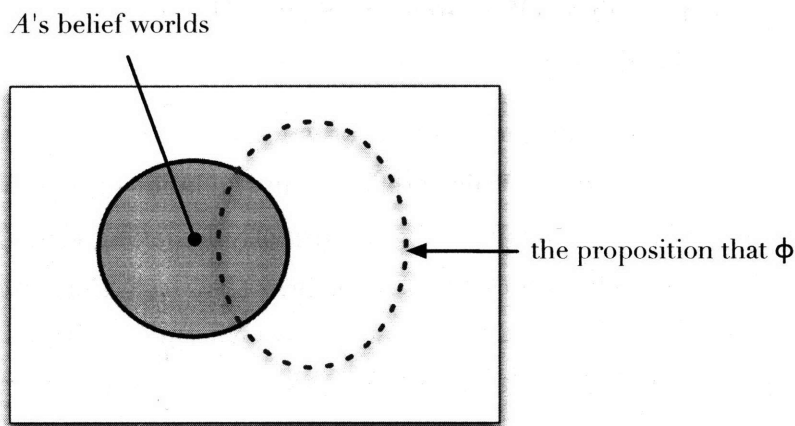


Figure 2.2: $B_A \diamond \phi$: Veltman's model.

On this revised model, due essentially to Frank Veltman⁸, there is no proposition that $\diamond \phi$ at work. The question of whether A believes that $\diamond \phi$ is just the question whether A 's belief worlds leave open possibilities wherein the proposition that ϕ is true. To believe Bob might be in his office is simply to be in a doxastic state which fails to rule out the possibility that Bob is in his office. It is a first-order state of mind.

Veltman's model is a considerable advance over the descriptivist model. It avoids the implausible idea that epistemic possibility beliefs are second-order states of mind, and in

⁸ See Veltman [1986, 1996] and (building on Veltman) Beaver [2001], where this model is tacit in the semantics developed in these works.

a way that lets us see why reasons that support belief that ϕ are *ipso facto* reasons that support belief that $\Diamond\phi$. A tempting thing to do now would be to craft a semantics and pragmatics for epistemic modals around Veltman's model, and see whether it does better than descriptivism on our earlier desiderata (I)-(III) above.

I will not do that now, however. (Of course, it has already been done by Veltman himself.) Although Veltman's model is surely on the right track, there is, I think, still room to improve on it in an important way. We have one more desideratum to uncover. Once we uncover it and upgrade Veltman's model accordingly, we can then raise the question of what semantics and pragmatics is suited to the (upgraded) model.

I turn then to a problem for Veltman's model.⁹ Recall Frank, who believes it might be raining in Topeka. Why does he believe this? We could imagine various accounts of how it happened. For instance: He left Topeka this morning and it looked cloudy then. Or the weatherman just now said the chance of rain was 30%. Alternatively, perhaps his evidential situation is more impoverished. Perhaps he has no noteworthy reasons in favor of believing that it's raining in Topeka; rather he merely notices his lack of sufficient reason to believe it isn't raining in Topeka. Perhaps on the way out the door, en route to Topeka, he glances by chance at his umbrella, and the question of rain in Topeka then occurs to him. He realizes he doesn't know whether to expect rain in Topeka. He then comes to think that it might be raining in Topeka.

This last kind of case raises a basic question. What is the difference between Frank's state of mind before the question of rain in Topeka occurs to him and his state of mind after? The question is a troubling one for Veltman's model. We know, on the model, that Frank's posterior state of belief must be one compatible with the proposition that it's raining in Topeka. But what, we ask, was his prior state of mind? The same: he had no prior beliefs one way or the other as concerns rain in Topeka, so what he believed was compatible with either circumstance. So he has transitioned from its being compatible with his doxastic state that it's raining in Topeka to... its being compatible with his doxastic state that

⁹ The problem was noted by Veltman himself at the University of Michigan Philosophy and Linguistics Workshop of 2006. (I do not know if he would agree with my statement of it.) Swanson [2006] also raises a version of this problem.

it's raining in Topeka. This is wrong: clearly some aspect of Frank's state of mind has changed, and our model ought to capture this change.

We could just as well make the point synchronically, by considering two states of mind at a single time rather than one across time. Compare Frank (in his posterior state) to Rem, a man living across the globe in Rotterdam. Rem has heard of Topeka, and he even knows roughly where it is on the map. But Topeka has no place in his life, and thoughts of Topeka simply have not crossed his mind all year. Like myriad other questions, the question of rain in Topeka today has just not occurred to Rem. Does Rem believe it might be raining in Topeka? It would be bizarre to answer affirmatively. It is true, we may stipulate, that for all Rem believes, it is raining in Topeka. For nothing he believes rules that possibility out. But this is merely to point out that 'Rem believes it might be raining in Topeka' and 'For all Rem believes, it is raining in Topeka' do not have the same truth-conditions. The states of mind of Frank and of Rem, we can say, are alike in as much as for all they each believe, it is raining in Topeka. But they differ in that Frank believes it might be raining in Topeka, whereas that is not so for Rem.

This gives us our last desideratum.

- (v.) Capture the difference between a proposition's merely being compatible with a state of mind and its being **considered possible** by that state of mind (or its being marked as an **open possibility** according to that state of mind).

To satisfy this desideratum, Veltman's model needs to be enriched. I propose to enrich it as follows.

Frank has considered the question of there being rain in Topeka. His is a state of mind that has taken note of a distinction: the distinction between there being rain in Topeka and there not being rain in in Topeka. Rem, in contrast, has not considered the question of rain in Topeka. His is a state of mind that has not taken note of that distinction. The respective states of mind of Frank and Rem differ, then, in the distinctions they have taken note of. What we therefore need is a representation of doxastic states of mind which tracks the distinctions that the agent being modeled takes note of.

A distinction—e.g., the distinction between rain and no rain in Topeka—may be represented by a line through logical space, one carving it into two regions, the rainy and the

rain-free. Suppose we collect all of the distinctions an agent takes note of, or counts as having taken note of, relative to some broad project of inquiry. That supplies us with an array of lines through logical space. Drawing them all at once, we then have a partition Π of logical space, a division of logical space into mutually exclusive and jointly exhaustive regions. We can then try saying this: the distinctions that an agent takes note of are the ones that ‘carve according to the lines’ of Π . The distinctions that an agent fails to take are those whose lines depart from the lines of Π . Pursuing this visual metaphor, call such a partition a *modal resolution*. It represents the agent’s ‘modal acuity’ as pertains to the project of inquiry in question, capturing only the level of specificity the agent may be said to be aware of in a broad sense. My plan is to index states of mind to a modal resolution. Some propositions will be *visible* to an agent in a state with resolution Π , namely, those whose boundaries respect the partition over logical space imposed by Π . All other propositions go unseen by the agent.

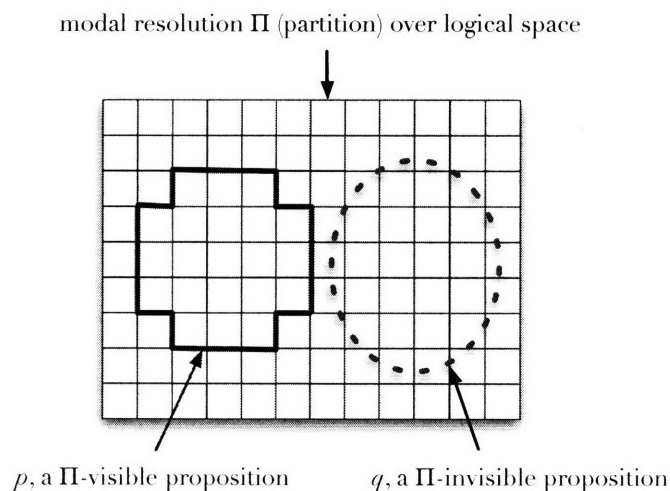


Figure 2.3: Visibility at a resolution.

A picture may help. See Figure 2.3. Here we see that p respects the grid imposed over logical space by our resolution Π . Hence it is visible with respect to Π . Not so for q , which cuts through the grid. Say a proposition p is Π -visible just in case each cell of Π either implies (is a subset of) p or contradicts (is disjoint from) p .

States of belief, I propose, are resolution-sensitive. Relative to a resolution, a doxastic state will select a set of cells at that resolution as candidates for actuality (in the sense that a cell is the *actual cell* just in case it contains the actual world). Formally, a doxastic state is now a (partial) function taking a resolution to a subpartition of that resolution. The cells of the subpartition may be thought of as the doxastically open possibilities for the agent at that resolution. We can call this partition the agent's *belief partition*.

To get a grip on this picture, it is helpful to think of a resolution as associated with, or even equivalent to, a question, in the following sense: the cells of the resolution give all the alternative complete answers to the question; the question asks which cell is the 'true' one, the one containing the actual world.¹⁰ On this way of approaching the model, the idea is that a doxastic state can be understood as a function from questions to answers. (The answers will usually only be partial, eliminating some but not all alternatives. And the question reflected by a resolution needn't be one particular easy to express in language: better to understand it as capturing a family of topically related questions on which the doxastic state takes a stance on—as capturing a relatively detailed project of inquiry.)

I said that a doxastic state is representable by a partial function on resolutions. Let me say how partial. Suppose we select, from the space of possible resolutions of logical space, the family of resolutions that have been active in the psychological life of the agent we are modeling, in the sense that these resolutions mark the various propositional distinctions (or questions, or subject matters) the agent can be said to have taken note of. These will reflect what we could call the agent's 'considered questions'—for short, her *inquiries*. Her doxastic state is a function defined on these inquiries. Elsewhere her doxastic state is undefined.

In saying her doxastic state is elsewhere undefined, let me be clear that the idea is not that human agents are incapable of considering more questions that they actually ever do consider. We are, of course, capable in principle of considering countless questions. The

¹⁰ A well-known theory of the semantics of interrogatives identifies their semantic values with partitions of logical space. See Groenendijk and Stokhof [1997] and references cited therein. Hamblin [1958] is the pioneering work. Other sources of inspiration for the resolution-sensitive model I am proposing include Lewis [1988], who models *subject matters* as partitions of logical space, and Schaffer [2004, 2005] who attempts to understand states of knowledge as relativized to questions. (Schaffer's formal development of this relativization differs from mine.)

idea instead is that when an agent considers a fresh question, she makes up her mind on it, and her doxastic state then becomes defined on a resolution which represents that question. What is ‘making up one’s mind’? It is natural to think that when an agent comes to a new question, what she attempts to do is to ‘increase’ the resolution of one of her existing inquiries, so as to make that resolution reveal the possible answers to the new question. Thereby she see what her positions on her other inquiries commit her to with respect to the new question. As rational agents we try, insofar as we can, to integrate all of our inquiries, so that information is pooled and so that answers are consistent.¹¹

The resolution-sensitive model of belief has various applications, full discussion of which is better reserved for elsewhere. Let me now turn to the solution this model presents to our problem for Veltman’s model. Recall the challenge was to say what about Frank’s doxastic state could have changed when he transitioned into believing that it might be raining in Topeka; or equivalently, to say what the difference was between Frank (after this transition) and Rem, whose mind is free of weather-in-Topeka thoughts. With a resolution-sensitive model, the thing to say is that Frank’s doxastic state came to be defined on a resolution making the proposition that it’s raining in Topeka (a) visible, and (b) compatible with his belief partition at that resolution.¹² This is the difference between him and Rem, who has not considered the question of rain in Topeka, and who (hence) has no inquiry making the relevant proposition visible.

So the new model of what it is for an agent to believe it might be that ϕ is as in figure 2.4. One can think of it as a sort of ‘pixelated’, low-res version of Veltman’s model.¹³

¹¹ We try, but apparently it is hard. There are only so many distinctions we can see at once, only so many we can bring together in a single state of mind. As a result there are severe limits on the extent to which we can unify our disparate inquiries into a single inquiry. And as a result it may be that we fail to believe the consequences of two propositions we believe. This can happen when the propositions are believed with respect to differing resolutions, resolutions not yet integrated. The issue of deductive omniscience in a resolution-sensitive setting is discussed in more detail in chapter 3.

¹² More precisely, compatible with the set of possibilities partitioned by his belief partition.

¹³ This diagram only partially represents A ’s doxastic state: it represents her state with respect to the inquiry reflected by II. We can assume that A ’s doxastic state is defined also on other ways of partitioning logical space. We can also assume that, unless A is deductively omniscient, the set partitioned by A ’s belief partition at each resolution is not always the same.

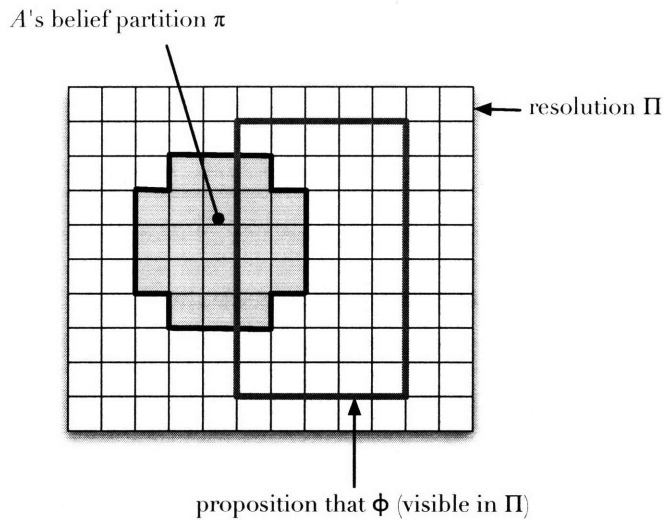


Figure 2.4: $B_A \diamond \phi$: Resolution-sensitive model.

Above I alleged that ‘A believes it might be that ϕ ’ and ‘For all A believes, ϕ ’ do not have the same truth-conditions. The difference can be reflected in a resolution-sensitive model. Figure 2.5 depicts a situation in which it would be true to say that for all A believes, ϕ , but false to say that A believes it might be that ϕ . The former is true for A with respect to the given resolution just in case there are ϕ -worlds within the union of A’s belief partition. This is just the classic possible worlds treatment of compatibility with belief. There are many more propositions compatible with what one believes than there are propositions one believes might be true.

I have gone on about what it is to believe something might be the case, and about what it is for something to be true for all one believes. But what about ordinary, vanilla belief in propositions—belief whose ascription does not involve epistemic modals? How is this represented in a resolution-sensitive model?

Here it should be clear that model enables a distinction between the propositions which are the *tacit commitments* of the belief state at a resolution and those propositions which are

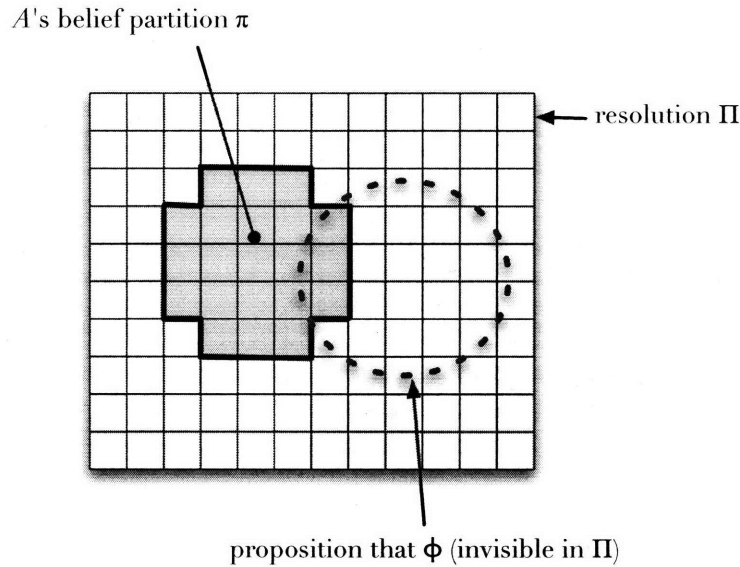


Figure 2.5: For all A believes, ϕ .

available or *accessible* to the agent. Some propositions are ‘accessibly believed’: they mark distinctions the agent can be said to recognize or have recognized; they reflect information available to the agent to guide thought and action. Formally, these are the propositions constructible entirely via unions of the cells of the resolution at which the doxastic state is being assessed. The resolution makes these propositions visible, and the proposition is true throughout the agent’s belief partition. Tacit commitments are like accessible beliefs in that they are also true throughout one’s belief partition, but they differ from them in that they are invisible at the resolution with respect to which the doxastic state is being evaluated.¹⁴

We can depict the difference between accessible belief and tacit commitment as in Figure 2.6. Here we see that while accessible beliefs and tacit commitments are both propositions true throughout the agent’s belief partition, accessible beliefs carve according to the resolution and tacit commitments do not.¹⁵ It is a natural thought that belief reports in natural

¹⁴ See chapter 3 for more on the idea of accessible belief. See also Swanson [2006] for a formally similar idea, though developed in a Bayesian setting.

¹⁵ As should be clear, accessible belief and tacit commitment are resolution-sensitive notions. A resolution-insensitive notion of tacit commitment might also be defined in the model: say that a proposition is a

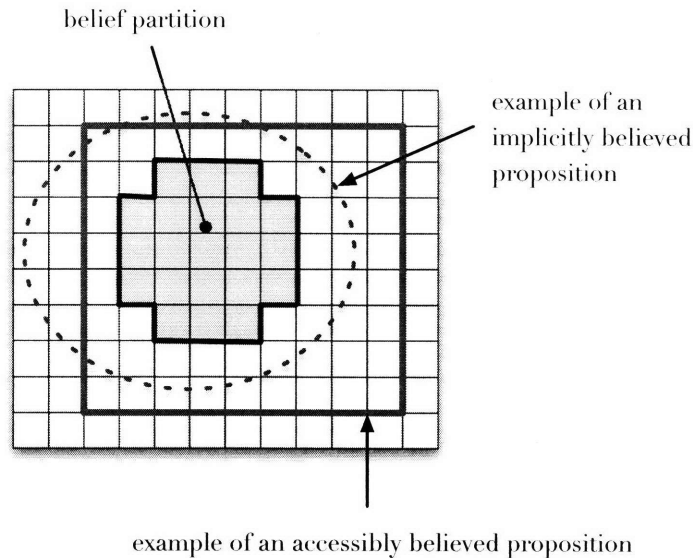


Figure 2.6: Accessible belief versus tacit commitment.

language sometimes presuppose that their complements are accessibly believed.¹⁶

Our resolution-sensitive upgrade to Veltman's model provides the resources to satisfy desiderata (IV)-(V). My plan is to craft a rough semantics and pragmatics for epistemic possibility and necessity talk around this model. The plan will be carried out in the next two sections (Sects. 7-8), where we will see that the semantics and pragmatics that results satisfies desiderata (I)-(III).

Before that, however, we will want to perform just one more important upgrade to the model. The upgrade is needed to handle probabilistic information and the associated probability talk.

global tacit commitment of an agent just in case the proposition is a tacit commitment with respect to all the agent's inquiries.

¹⁶ This, at any rate, would explain Fodor's intuition (in Fodor [1985]) that you don't quite count as believing that no grass grows on kangaroos unless you have actually considered the question.

Epistemic Probability

I said I would model a doxastic state as a partial function taking a resolution to a subpartition of that resolution, where the cells of the subpartition may be thought of as the doxastically open possibilities for the agent at that resolution. The cells are as specific as the possibilities get, as far as the agent's state of mind at that resolution is concerned.

Now it is time to recognize that these possibilities needn't all have the same status according to the agent. The agent may regard some of these open possibilities as more likely than others. More generally, she may regard it as more likely that actuality is contained within one region of her belief partition rather than another region. This is something we will need reflected in the model, if we want to capture what it is to believe something probable. *A priori*, there are a number of ways it might be done.¹⁷ The approach I will take is to define a simple probability measure over the propositions visible to the agent at a resolution. A doxastic state will now be a partial function from resolutions to pairs of a belief partition and a probability measure over (the propositions visible at) that resolution. We can call such a pair a *doxastic space*. Given resolution Π and the corresponding belief partition Π it determines, the measure is to be defined so that:

- (i) Pr assigns each cell ι in π a real value in the closed interval from zero to one, such that these values all sum to one.
- (ii) For all propositions p that are visible in Π , $Pr(p) =_{\text{def}} \sum_{\iota \subseteq p} Pr(\iota)$; otherwise $Pr(p)$ is undefined.

The probabilistic upgrade of the resolution-sensitive model is easy to visualize. It merely adds a quantitative, linear range of distinctions within the belief partition. We can represent this informally by the darkness of the shading of each cell in the partition, with the darker shades corresponding to greater probability.¹⁸ See Fig 2.7.

¹⁷ For example, one might exploit comparative possibility preorders or partial orders over the open possibilities; or ranking functions; or plausibility measures; or Dempster-Shafer belief functions; or sets of probability measures (representors). See Halpern [2003] for an overview of these formal tools.

¹⁸ More accurate, but perhaps less visually intuitive, would be to just write real numbers into each cell within the partition, such that their sum is one.

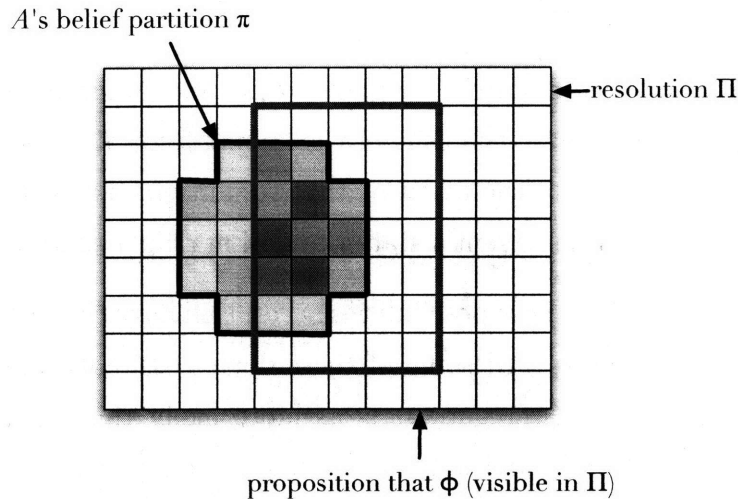


Figure 2.7: $B_A \Delta \phi$: Probabilistic resolution-sensitive model.

The probability of any visible proposition according to A 's doxastic space is just the sum of the probabilities of the cells in π where the proposition is true. Thus the probability of the proposition that ϕ above is given by the sum of the probabilities of the eleven ϕ -cells within π .

This model provides an easy way to say what, in abstract, it is for Frank to believe that it is probably raining in Topeka. He believes this, relative to some resolution Π on which his doxastic state is defined, just in case (a) the proposition that it's raining in Topeka is Π -visible, and (b) this proposition receives a probability greater than its negation, i.e., greater than $\frac{1}{2}$, at the corresponding doxastic space.¹⁹ (Letting A be Frank and ϕ be the proposition that it's raining in Topeka, the above diagram would be a fair representation of a situation in which Frank believes it's probably raining in Topeka, since it is clear that most of the probability mass is within the proposition that ϕ .) The model also provides for Frank's beliefs concerning the comparative probability of propositions: he believes rain is more likely than snow in Topeka (say) just in case his doxastic space assigns more probability to the former than to the latter proposition, with respect to the given resolution. Note that there is no proposition that $\Delta \phi$ in this picture.

¹⁹ In fact, the real value relevant to the evaluation of natural language 'probably'-clauses are shiftable by context, but I abstract from this complication here.

At one level of abstraction, this is the sort of model of what it is to believe something probable that Bayesians tacitly have in mind (*modulo* the resolution-sensitivity). To believe a proposition probable is just to be in a certain doxastic state of mind modelable by a probability space, where the measure of that space assigns the proposition a relevantly high value. On both the current picture and the usual Bayesian picture, believing propositions probable is *not* assumed to be a second-order state of mind—it is not a belief *about* one’s credence, say. And neither is it a state consisting in (full) belief in some proposition about one’s evidence, as the descriptivist proposals for $\Delta\phi$ discussed above would most naturally recommend.²⁰

This upgrade to the resolution-sensitive model extends its coverage to probabilistically articulated belief states, and it does so without compromising desideratum (IV).²¹

Earlier I adopted the view that information is that which eliminates possibilities. The probabilistic enrichment of doxastic states that I am recommending allows us to generalize this notion of informational content. Let us take it that information can do more than just eliminate possibilities. It may also shift the probabilities over the possibilities, without eliminating any possibilities altogether. Informational content itself, we can say, is probabilistically articulated. The informational content of a state of belief is the doxastic space of the state of mind at the resolution in question. We could call this view about informational content *content probabilism*. I want to endorse content probabilism for the whole family of what we could call *acceptance* attitudes: not only believing, which has been the focus of this section, but also states of presupposing, presuming, supposing, postulating, conjecturing, and knowing. Let us suppose that all of these states may have a probabilistically articulated informational content: all of them may be represented by diagram just like Figure 7. Doxastic spaces are just specific examples of what we could call *information spaces*.

Content probabilism differs from traditional subjective Bayesianism in that the probabilities of the model reflect, not the strength of one’s attitude towards content, but rather

²⁰ So the Bayesian picture itself needn’t be tied to anything like a descriptivist account of probability talk, as is sometimes assumed (e.g., by Jeffrey [2004]; see n. 2.2 above)—though it remains, of course, to articulate the non-descriptivist alternative.

²¹ Note that if we want a notion of *tacit* probabilistic beliefs, we will need to define a second probability measure over logical space, to settle the distribution of credence within the coarse cells of the resolution.

the content of the attitude itself. My aim in adopting this enriched conception of the informational content is, of course, to use it in an account of the semantics and pragmatics of epistemic modal talk. To this we now turn.

2.7 Expressing states of mind

The aim of the last section was to develop a picture of what it is to be in a state of mind that accepts what an epistemic possibility or probability claim says. It remains to say precisely how epistemic modal talk serves to express these states of mind. This is the question, both of the compositional semantics of epistemic modal clauses, and also of their pragmatic effect on the communicative contexts in which they are uttered unembedded. In outline, the plan here is straightforward. We have explained already what abstract property a doxastic state, or more generally a state of information, must have in order to count as accepting that $\Diamond\phi$. Now I will say that the compositional semantic value of an epistemic possibility clause, relative to context, just is that property. Pragmatically, I will say that one who makes an unembedded possibility claim is standardly understood as attempting to *engender coordination* on this feature of her state of mind—to get others to enter into that state. The same basic story will be told, *mutatis mutandis*, for $\Delta\phi$. And, treating $\Box\phi$ as the semantic dual of $\Diamond\phi$, we will automatically get a story about epistemic necessity.

Now to spell it out. First I state semantics for epistemic modals and for the relevant embedding environments. Then I situate the semantics in a broader pragmatics of communication. Once the semantic and pragmatic apparatus is in place, we will show that it satisfies desiderata (I)-(III).²²

Semantics

Start with a two-dimensional intensional semantics in the style of Kaplan [1989], built around a recursive definition of truth at a point of evaluation. (‘Truth at a point of

²² The semantic and pragmatic ideas to follow are discussed in more detail in Yalcin [2007].

evaluation' is a technical notion, at best indirectly related to any folk notion of truth.) The points of evaluation relative to which extensions are defined have two coordinates: a *context* coordinate and an *index* coordinate. Contexts are locations where speech acts take place. Following Lewis [1980], we may think of them as centered worlds, determining both a possible world and a spatiotemporal location within that world. Contexts have indefinitely many features—speakers, audiences, indicated objects, standing presuppositions, etc.—and these features may figure into the truth of sentences said in that context in indefinitely many ways. Indices are n -tuples of specific features of context, those features which are independently shiftable by operators in the language. Which features of the context are shiftable depends on what operators the language contains. We take it our indices include at least a world parameter, since the fragment of English we consider has operators which shift the world at which an embedded clause is evaluated.

Now in addition to context and index parameters, let our points of evaluation include also an *information parameter*, i , ranging over resolution-indexed information spaces. Formally we can take these to be triples of the form $\langle \Pi, \pi, Pr \rangle$ where Π is a resolution, $\pi \subseteq \Pi$, and Pr is defined over Π -visible propositions as above.

Although denotations are now technically all relativized to a value for the information parameter, in most cases extensions will not be sensitive to it. Predicates will be assigned extensions relative only to worlds, as usual; logical connectives will be defined as usual; and nothing new need be assumed about the semantics of names, generalized quantifiers, etc. Most clauses will continue to place conditions only on the world coordinate of the index relative to context, and will therefore retain their ordinary possible worlds truth-conditions. In such cases the information parameter i will be idle. We exploit i mainly in the definition of truth for epistemic modal talk and for certain related environments.

The semantics for epistemic possibility and necessity has two components. First, here is the definition of truth at a point of evaluation:²³

$$\llbracket \Diamond \phi \rrbracket^{c,i,w} \text{ is true iff } \exists w' \in \bigcup \pi_i : \llbracket \phi \rrbracket^{c,i,w'} \text{ is true}$$

²³ ' $\llbracket \cdot \rrbracket$ ' denotes the interpretation function of the model of the language, which maps well-formed terms to their extensions relative to points of evaluation. By 'is true', I mean '= Truth'.

$$\llbracket \Box \phi \rrbracket^{c,i,w} \text{ is true iff } \forall w' \in \bigcup \pi_i : \llbracket \phi \rrbracket^{c,i,w'} \text{ is true}$$

Given the partition π supplied by i , these modals just invoke quantification over the set of worlds π partitions.

Second, we add that these clauses carry a *visibility presupposition*, to the effect that the resolution Π supplied by i makes the embedded proposition that ϕ visible.²⁴ The visibility presupposition is where the resolution-sensitivity comes in. It makes the quantification over worlds invoked by epistemic modals equivalent, in effect, to quantification over partition cells—the coarser possibilities entertainable by mere mortals.²⁵ The general idea is that, relative to context, epistemic possibility and necessity clauses divide the space of information spaces, not the space of possible worlds. Informally, an epistemic possibility clause is true with respect to an information space just in case the partition of the space includes a ϕ -possibility (that is, a ϕ -cell, a cell for which ϕ is true throughout). Epistemic necessity clauses get the matching dual semantics: they are true just in case every cell in the partition of the space is a ϕ -possibility.

$\Delta\phi$ -clauses have a semantics which turns, as you might expect, on the probability measure of the information space it is evaluated with respect to. Here is the semantics:

$$\llbracket \Delta \phi \rrbracket^{c,i,w} \text{ is true iff } Pr_i(\{w : \llbracket \phi \rrbracket^{c,i,w} \text{ is true}\}) > \frac{1}{2}$$

Again the idea is that, relative to context, epistemic probability clauses divide the space of information spaces. $\Delta\phi$ is true at an information space just in case the measure of the space assigns the proposition that ϕ better-than-even odds.

We can think of these semantic values as picking out the relevant properties of states of mind we identified in the last section. (Indeed, that is exactly what they are crafted to do.) If you find the above truth-conditions for epistemic possibility clauses opaque, just look at Figure 4 above (p. 78): $\Diamond\phi$ is true at a point of evaluation $\langle c, i, w \rangle$ just in case the

²⁴ You will notice that, in assuming that the embedded ϕ expresses a proposition, I set aside iterated epistemic modalities. These are discussed briefly in chapter 1 above.

²⁵ Alternatively, we could have stated the truth-conditions by directly quantifying over partition cells, as done in Yalcin [2007]. (See chapter 1.)

information space of i stands to the proposition that ϕ as in Figure 4. Likewise, $\Delta \phi$ is true at a point of evaluation $\langle c, i, w \rangle$ just in case the information space of i stands to the proposition that ϕ as in Figure 7.

Now that we are doing semantics, we can be more precise about what we mean by ‘factualist truth-conditions’. Factualist truth-conditions are truth-conditions which are a function of the world coordinate of the points of evaluation in question. In this sense, the truth-conditions for epistemic modal clauses just provided are nonfactualist: they do not place conditions on the world coordinate of the points at which they are evaluated.

What does this technical property of the semantics ultimately amount to? That depends on how the semantics is integrated into a larger picture of the linguistic transfer of information. Let us turn now to situating this semantics with respect to pragmatics.

Pragmatics

Suppose that to be semantically competent with epistemic modal language is to know, at some relatively abstract level, that the meanings of epistemic modal clauses serve to divide the points of evaluation as I have described. How is the knowledge exploited in communication? We could put the question like this: what effect do unembedded epistemic modal claims have when there are expressed in a discourse context?

Proposal: when one says that it might be raining, or that it’s probably raining, or that it must be raining, one expresses one’s (probabilistic, resolution-sensitive) state of mind. One does so by saying something whose compositional semantic value, relative to context, just is that feature of one’s state of mind that one is aiming to express. Abstractly, to express such an aspect of one’s state of mind is to attempt to get one’s interlocutors to enter into that state of mind. Less abstractly, to make an epistemic modal claim is to propose to one’s interlocutors that their states of presupposition change so as to come satisfy the property the epistemic modal claim expresses as a matter of its semantics. One proposes to coordinate the presuppositional states of the interlocutors with respect to that property.

We can say what this means in detail by performing a probabilistic, resolution-sensitive upgrade of certain popular account of how assertions in general change the informational context of a discourse.²⁶ Begin by thinking of linguistic communication as foremost a matter of coordination on a body of information. Participants in conversation start with certain information presumed to be held in common or mutually taken for granted, and the speech acts they perform in context are directed, and mutually understood to be directed, at variously influencing that body of information. The attitude that communicating agents take towards the body of information they share is the attitude state I am calling *presupposition*. Presupposition is, in the intended sense, a public attitude: one is to presuppose only the information that one’s interlocutors do. Now states of presupposition, we allow, are resolution-sensitive. The set of possibilities left open by your state of presupposition is representable by the cells of a partition of possible worlds. When someone utters a sentence with factualist truth-conditions relative to context—that is, when someone asserts a proposition—and everyone understands what is said, the proposition becomes *visible* in the context, in the sense that the resolutions of the interlocutors’ presuppositional states make the proposition visible. The speech act of asserting that proposition is understood as a proposal to presuppose it. To presuppose it is just to eliminate those worlds (cells) incompatible with it from one’s presupposition partition.

Turning to epistemic modal claims, I want to say essentially the same thing: to say that $\Diamond\phi$ (or $\Box\phi$, or $\Delta\phi$) is to propose to the participants of the conversation that they come to presuppose that $\Diamond\phi$ (or $\Box\phi$, or $\Delta\phi$). The difference with factualist assertions lies only in what it is to presuppose these things. The semantics of these sentences makes their truth-conditions nonfactualist relative to context. One therefore cannot eliminate possible worlds from one’s presupposition partition according to whether they ‘satisfy’ these truth-conditions, since these clauses don’t determine a condition on worlds in the first place. Nevertheless, it is perfectly clear what it is to presuppose $\Diamond\phi$: it is for the proposition that ϕ to be visible at one’s state of presupposition, and for the informational content of one’s state to be compatible with that proposition, as in figure 4 above. (Similarly for $\Box\phi$: to presuppose this is for the proposition that ϕ to be visible for one’s state, and for the informational content of one’s state to entail that proposition.) This sentence in context

²⁶ The popular account is that of Stalnaker (Stalnaker [1978, 2002]).

semantically characterizes a property of a state of mind, and the speech act move of saying this sentence is understood to be a proposal, to one's interlocutors, that they make their presuppositional states satisfy this property.²⁷

The extension to probability talk is straightforward. We allow that states of presupposition may be, not only resolution-sensitive, but also probabilistically articulated. Then, just as with epistemic possibility and necessity sentences, $\Delta \phi$ uttered in context is understood to be a proposal to enter into a presuppositional state satisfying the property of states of mind the sentence expresses as a matter of its semantics.

That is the basic pragmatic tale I want to tell. It requires clarification on two points.

The first concerns the place of resolution in context. I said that it is in the nature of presupposition that "one is to presuppose only the information that one's interlocutor do". What does this mean? At minimum it means that each of the discourse participant's states of presupposition should have the same tacit commitments. But what about the resolutions of the discourse participants? Need these all be the same, in order for everyone to be presupposing correctly?

No, we do not need to assume this. One does not violate the presuppositions a discourse by recognizing more distinctions, among the possibilities compatible with what is presupposed, than one's interlocutors do. But while we needn't coordinate on all of the possible alternatives we might individually recognize, we surely do need to coordinate on *some* of them in order to communicate. To transfer information, we need to be able to jointly attend to ways of carving up logical space, so that we may coordinate on the region of logical space we want to consider or want to take ourselves to be in. It is plausible, therefore, that the resolution of any individual agent in a discourse must be a *refinement* of a single, relatively coarse modal resolution—where this single, relatively coarse resolution represents

²⁷ A natural conjecture is that often the point in uttering epistemically modalized sentences is to get listeners to adjust their resolutions—hence the possibilities they take note of—by getting them to *accommodate*, in the sense of Lewis [1979b], the visibility presuppositions of these sentences. Sometimes you just want to get your interlocutor to take note of the as-yet uneliminated possibility that ϕ . Saying $\Diamond\phi$ is a way to accomplish this via accommodation. Alternatively, sometimes you want to call attention to an as-yet unseen consequence (a tacit commitment) of what is being presupposed. Saying $\Box\phi$ is a way to accomplish this, again via accommodation.

the set of distinctions that are mutually recognized by all parties to the conversation.²⁸ (A resolution Π' is a *refinement* of resolution Π just in case every cell in Π is a union of cells in Π' , and some cell in Π' is not a union of cells in Π .) Let us call the resolution corresponding to this body of distinctions the *context resolution*. Think of it as a *minimal* resolution for the context, one each agent's state of presupposition should respect.

The second issue concerns the connection between the presuppositions of a discourse and the broader purposes of the interlocutors. My pragmatic story may feel somewhat insulated from reality. Factualist and non-factualist claims alike, I said, serve as proposals to change the presuppositions of the discourse. But what is the point of this game of updating presuppositions? How does it connect with the broader aims of the conversing agents?

Given only the information presupposed among a group of agents—representable, on my story, as an information space indexed to the context resolution—one does not yet know how the agents of the context mutually regard what is presupposed with respect to their other cognitive attitudes. One does not yet know whether the agents also regard those presuppositions as knowledge, or as warranted belief, or conjecture, or fiction, or whatever. If we want, we can define a second notion which will let us articulate the status that the agents of a given context attach to the information presupposed. Call this notion the *conversational tone*:

An attitude is the *conversational tone* of a group of interlocutors just in case it is common knowledge in the group that everyone is to strike this attitude towards what is presupposed.²⁹

When interlocutors coordinate on a conversational tone, they come into agreement about what counts as the correct non-public attitude to take towards what is common ground. This will be a reflection, *inter alia*, of the purpose of the discourse. If the conversational tone of our discourse is knowledge, then we regard the information presupposed as common

²⁸ This body of distinctions, we can take it, will typically include all those corresponding to any propositions expressed by clauses recently uttered in the discourse, since these propositions will be salient to all.

²⁹ A single discourse might have several conversational tones, but let us restrict attention to the case where there is just one. Conversational tone is meant to be an attitude over and above presupposition: presupposition itself, we can stipulate, is not a possible conversational tone.

knowledge, and we take our discourse to be trafficking, and aiming to traffic, in factual information. If the conversational tone is pretense, then we are not attempting to keep the information presupposed compatible with the truth, and we take ourselves to be trafficking in fiction. And so on, for all the various attitudes around and in between.

So what, ultimately, does it mean to propose to revise the standing presuppositions of a discourse—as I have claimed we do with declarative sentences of language (factualist and nonfactualist alike)? What it comes to, as far as the broader projects of the interlocutors are concerned, depends entirely on the conversational tone. Thus, for example, if the conversational tone is belief and I say that it might be raining, I am understood as expressing the compatibility of my doxastic state with rain, and recommending that feature of my doxastic state to you. Alternatively, if we are jointly reasoning under a counterfactual supposition and I say that it might be raining, I am understood as expressing the compatibility of my suppositional state with rain, and as recommending that feature of my suppositional state to you. The notion of conversational tone helps us to separate issues about the narrow pragmatic dynamics of presupposition change from issues about the broader role presupposition might take in any given discourse.

2.8 Problems solved

We have come a long way from the descriptivist picture. Our initial motivation for departing from that picture came from its failure to satisfy the desiderata (I)-(III) above. In the process of approaching the issues from the point of view of the states of mind epistemic modal claims express, we motivated two further desiderata ((IV)-(V)) and developed an abstract model satisfying them. It is time to show that this model of epistemic modal thought and talk gets the desired results for the three problems levied against the descriptivist picture at the outset.

Desideratum 1: explaining epistemic contradictions

We observed above the unembeddability of epistemic contradictions such as:

- (1) # It's raining and it might not be raining.
 (2) # It's raining and it probably isn't raining.

We were at a loss to explain this unembeddability along descriptivist lines. But given the apparatus in place, it is straightforward what to say. The defect in sentences embedding epistemic contradictions is parasitic on the fact that there is no state of mind that could be characterized as accepting an epistemic contradiction. On the model of the informational content of states of mind put forth in the last section, to accept the first conjunct of (1)—tacitly or not—is for the proposition that it's raining to be true throughout one's information space. But to accept the second conjunct is for the negation of that proposition to be *compatible* with one's information space. Obviously, there is no one state of mind satisfying both of these properties. Similarly for (2), whose second conjunct not only requires the proposition that it isn't raining be compatible with one's information space, but requires also that the proposition receive greater than .5 probability from the measure associated with the space.

Of course, it would be nice to have a formal semantics for attitude verbs that would vindicate this abstract explanation, in the sense of allowing us to prove mechanically that attitude verbs embedding epistemic contradictions describe incoherent states of mind. Here is the generic form of such a semantics:

$$\llbracket A_x \phi \rrbracket^{c,i,w} \text{ is true iff } \forall w' \in \bigcup \pi_{\mathbf{A}_x^w} : \llbracket \phi \rrbracket^{c,\mathbf{A}_x^w,w'} \text{ is true}$$

Where 'A' is a schematic letter over acceptance attitude verbs and 'A' is a schematic letter ranging over the resolution-indexed information spaces such verbs serve to express.³⁰ On this semantics, acceptance attitude verbs do two things. First, they invoke universal quantification over the coarse possibilities provided by the partition π of the information space of the associated attitude state (relative to a given resolution; in typical cases the 'given resolution', we can take it, will be the context resolution). Second, they shift the value of the information parameter to the information space of the attitude state. Again, if

³⁰ So a substitution instance of 'A_x' might be (something designating the words) 'Rem believes'; and a substitution instance of 'A_x^w' might be 'Rem's resolution-indexed doxastic space in w'—where again, the latter designates a triple $\langle \Pi, \pi, Pr \rangle$.

we have the intuition that attitude ascriptions are typically ascriptions of attitudes towards available or accessible information, we should want to add that attitude ascriptions carry a visibility presupposition with respect to the propositions which occur within the scope of their complements. (The presupposition is that these propositions are visible with respect to the resolution to which the state ascribed is indexed.)

Equipped with this semantics, it is trivial to prove that constructions like these (repeated):

- (4) # Suppose it's raining and it might not be raining.
- (5) # Suppose it's raining and it probably isn't raining.

implore one to enter into an incoherent state of mind, and thus are expected to be defective.³¹

Desideratum 2: explaining assertability and disagreement

We observed above that if epistemic modal claims are factual descriptions of evidence or evidential states, the question arises what evidence or evidential states is at issue in the

³¹ Proof: For notational simplicity, abbreviate \mathbf{A}_x^w as \mathbf{A} . Then from the definitions above, and from the standard definitions for conjunction and negation, we have:

$$\begin{aligned} & \llbracket \mathbf{A}_x(\phi \wedge \Diamond \neg \phi) \rrbracket^{c,i,w} \text{ is true iff} \\ & \forall w' \in \bigcup \pi_{\mathbf{A}} : \llbracket (\phi \wedge \Diamond \neg \phi) \rrbracket^{c,\mathbf{A},w'} \text{ is true iff} \\ & \forall w' \in \bigcup \pi_{\mathbf{A}} : \llbracket \phi \rrbracket^{c,\mathbf{A},w'} \text{ is true and } \llbracket \Diamond \neg \phi \rrbracket^{c,\mathbf{A},w'} \text{ is true iff} \\ & \forall w' \in \bigcup \pi_{\mathbf{A}} : \llbracket \phi \rrbracket^{c,\mathbf{A},w'} \text{ is true and } \exists w'' \in \bigcup \pi_{\mathbf{A}} : \llbracket \neg \phi \rrbracket^{c,\mathbf{A},w''} \text{ is true iff} \\ & (\forall w' \in \bigcup \pi_{\mathbf{A}} : \llbracket \phi \rrbracket^{c,\mathbf{A},w'} \text{ is true}) \text{ and } (\exists w' \in \bigcup \pi_{\mathbf{A}} : \llbracket \neg \phi \rrbracket^{c,\mathbf{A},w'} \text{ is true}) \end{aligned}$$

The two conjuncts of the last line impose incompatible demands on the attitude state \mathbf{A} in question: obviously, no \mathbf{A} is such that it both (1) includes only ϕ -possibilities and (2) includes some $\neg\phi$ -possibility. Thus ascriptions of epistemic contradictions are at best true only of absurd states of mind, states of minds that leave open no possibilities at all. One can perform formally analogous reasoning for $(\phi \wedge \Delta \neg \phi)$, $(\phi \wedge \Box \neg \phi)$, $(\neg \phi \wedge \Diamond \phi)$, etc.—constructions also plausibly called epistemic contradictions. The imperatives (4) and (5) are thus defective because they ask one to enter into a patently incoherent state of mind.

case of any given claim; and it is obscure how to settle that question consistent with the actual assertability and disagreement facts surrounding these claims.

Again, we get clear here by stepping back and understanding what it is to be in state of mind accepting an epistemic modal claim. Suppose I say

(12) It is probably raining in Topeka.

in a standard belief-communicating context, where the conversational tone is belief. You agree with my claim. In virtue of what do we agree? In virtue of our doxastic states mapping the context resolution to doxastic spaces which make rain in Topeka probable. Similarly, *mutatis mutandis*, for $\Diamond\phi$ and $\Box\phi$.

Notice that this feature of our states of mind, this feature we agree on, involves no proposition about evidence or evidential states. This is as it should be. It explains why we may agree that it is probably raining in Topeka, while simultaneously rejecting each other's grounds for believing that it is probably raining. When we agree on (12), we are not agreeing on what is probable *according to some body of evidence X*; indeed we might disagree about what is probable according to *X* for nearly all *X*. We are simply agreeing to coordinate our probabilistically articulated states of mind with respect to the probability our states confer on the proposition that it's raining in Topeka.

When is an epistemic modal claim assertable (or rejectable) for an agent in a context? Take the attitude that is the conversational tone of the discourse and find the agent's information space for this attitude with respect to the context resolution. Then the claim is assertable for the agent just in case this resolution-indexed information space satisfies the property semantically expressed by the epistemic modal clause in context. Conversely, the claim is rejectable for the agent just in case this resolution-indexed information space does not satisfy the property semantically expressed by the epistemic modal clause in context. This view about what makes for the assertability of epistemic modal claims allows us to see the disagreement facts about epistemic modal claims as rational. We needn't picture the speaker as claiming to speak for a body of evidence outside his own knowledge, in the way descriptivism is compelled to; and yet we can say how others may be in position to disagree with the speaker's claim.

Desideratum 3: explaining conflicting intuitions

We observed above that we tend to have conflicting intuitions about the truth value of an epistemic modal claim when we are outside the discourse context and in a better epistemic position (with the respect to the epistemically modalized proposition) than those within the discourse. When Expert *B* says ‘Fat Tony might be dead’ in the scenario envisaged earlier, it is not clear what truth value the claim deserves.

The reason it is not clear, I suggest, is that Expert *B*’s speech act does not serve to describe the world. There is no way the world could be, or could fail to be, which would settle the question of the truth of the sentence. For this sentence there is no answering the question,

- (T) Is the speech act **true** in the sense that its factualist truth-conditions characterize the actual world?

for it has a false presupposition. His utterance does not have factualist truth-conditions.

The point of the speech act on the story I recommend is, again, to engender coordination among one’s interlocutors with respect to the property of states of mind the sentence semantically expresses in context. When assessing this kind of speech act for correctness, we cannot ask (T). At best we ask one of two things:

- (R) Is the speech act **rational** in the sense that someone equipped with the evidence of the speaker would be responding appropriately to the evidence by accepting the content of the speech act?
- (A) Is the speech act **advisable** in the sense that a person equipped with full information about the relevant situation would be responding appropriately to that information by accepting the content of the speech act?³²

Given this distinction, it is easy to see that Expert *B*’s speech act was *rational* but *inadvisable*. His state of mind responded appropriately (in one sense) to the evidence, but it is not the state of mind we would recommend to him given our superior epistemic position.

³² This rational/advisable distinction is modeled on a distinction Gibbard makes for questions about what it ‘makes sense’ to do [Gibbard, 1990, pp. 18-9]. Gibbard’s discussion is influenced by Harman [1982a].

When we are asked about the truth value of claim in a given context, typically we understand the question to be (T). But where the claim is epistemically modalized, that question cannot arise. We therefore look for other criteria to assess the sentence for correctness; and the two kinds of features we check for instead, I suggest, are rationality and advisability. When ordinary speakers are asked, ‘Is what Expert B said true?’ some of them interpret the question as (R), and they answer ‘yes’. Others interpret the question as (A), and they answer ‘no’. Still others feel the intuitive pull of both interpretations. These enlightened subjects reject the question and say: ‘Look: Expert *B* was right to say what he did, given what he knows. But if he were to say that to me, I’d reject it, because I know the facts of the case.’ These speakers tacitly recognize that, as far as the correctness of the speech act goes, we can ask either of (R) or (A), but that no further question (T) arises.

The expressivist view of epistemic modal discourse I have put forward satisfies desiderata (I)-(V). The descriptivist view does not.

2.9 On expressivism and nonfactualism

In metaethics, the name ‘expressivism’ is sometimes attached to a cluster of theses to the effect that normative claims are fancy riffs on ‘Boo!’ and ‘Yay!’—that they are essentially yelps in linguistic dress, primarily ‘expressing’ some non-contentful attitudes pro and con and having no compositional semantics. That view—perhaps better called *emotivism*—seems to me to be implausible, and I hope it is clear that the theory I have defended has not much in common with it. Far from trying to avoid content, I have suggested that careful attention to epistemic modal thought and talk require us to enrich our representations of contentful states of mind, to see them as resolution-sensitive and probabilistically articulated. In this respect, my proposal resembles that of Gibbard [1986, 1990, 2003], whose approach—in some moods, at least—has been to understand normative discourse as calling for a richer kind of content. Semantically, both Gibbard and I propose that the informational content of a sentential clause relative to context may serve to place conditions, not merely on possible worlds, but on a further nonfactualist parameter, one not

corresponding to an objective feature of context.³³ And pragmatically, we both emphasize that the point in uttering sentences with the semantics we recommend is to engender coordination on the corresponding states of mind. (It should be obvious, however, that my view is not *noncognitivist*: I do not deny that epistemic modal claims can serve to express doxastic states.) In a sense, I go further than Gibbard from a semantic point of view, since he provides no detailed semantic analysis of deontic modals, no analysis which might parallel the semantics for epistemic modals given above.

The distinctive feature of expressivism about some fragment of language—or any rate, expressivism in what seems to me its most plausible form—is the denial that the sentences of the fragment semantically have factualist truth-conditions relative to context. It is not the denial of the view that the sentences can have truth values, where ‘truth’ is understood to refer to the technical notion deployed in the semantics. Epistemic modal clauses may perfectly well be true in the technical sense, i.e., as far as the recursive definition of truth at a point of evaluation in the semantics goes. What matters is that they are not ‘true’ in the sense of question (T) above: such clauses do not deliver factualist truth-conditions, and as a result they do not serve to describe the world.

I have presupposed in all this that the notion of describing the world is in reasonably good health, and moreover that I can technically approximate this notion with truth-conditions stated via metaphysically possible worlds. This, of course, may seem to beg the question against more ambitious varieties of nonfactualism about modality (e.g., Blackburn [1986]). As I see it, my conception of factualism definitionally excludes the possibility of nonfactualism (or anti-realism, or quasi-realism) about metaphysical modality in general, for I deploy a ‘robust’ notion of the metaphysical modality in saying what factualism is. Is this the right methodological course? It seems to me doubtful there is some more attractive, more metaphysically neutral way to state, with useful precision, the question of factualism about epistemic modality. But I can’t begin to justify this attitude here. My assumptions reflect decisions to theorize in a certain way, and the reader is free to judge for herself whether theorizing under these assumptions has yielded any insight.

³³ My parameter is the information parameter; Gibbard’s is a parameter for ‘systems of norms’ (Gibbard [1986, 1990]), or in more recent work, a parameter for ‘maximally complete plans for action’, what he calls *hyperplans* (Gibbard [2003]).

Chapter 3

Content and Modal Resolution

Above I put the idea of a modal resolution to largely semantic work, using it to mark distinctions we see in language. But the idea that states of mind are resolution-sensitive is not, or not directly, a claim about language at all; it is an idea about how to represent the content of attitude states. It would therefore be nice if we could motivate this idea *qua* representation of the content of attitude states, and independently of purely semantic questions. That is to say, it would be nice if we could motivate this idea by seeing how it could play a constructive, explanatory role in a plausible overall account of content.

I want to take a step in that direction in this chapter. Focusing on belief, I will begin by reviewing a version of a familiar story about belief and belief contents, what I will call the *map picture* of belief. I will suggest that the picture is incomplete in certain ways, and that the idea of resolution-sensitivity can help to complete the picture. My intention in this chapter is to ‘start fresh’ with the idea of a modal resolution, motivating it from problems in the philosophy of content.

3.1 The map picture

The motto of the map picture is: *belief is the map by which we steer*.¹ A number of versions of this picture have wide currency in philosophy. Let me review the main features of this view as I want to understand it here.

¹The motto is due in essence to Frank Ramsey, and was brought into currency by Armstrong [1973].

A belief state is a state of an agent that represents the world as being a certain way (the map aspect), and which plays a certain role in the explanation of action (the steering aspect). The representational dimension of belief we understand in terms of the idea of information. Say a state *carries information* about the environment when it systematically co-varies in the appropriate way with the condition of the environment, so that the condition of the state serves as an indicator how things are in the environment. We take belief to be a state which tends, under normal conditions, to carry information in this sense; it is a state that tends, normally, to indicate how things in the world are. The representational (informational) content of a state of belief we model as a set of possible worlds, the set of possibilities that are the way the state represents the world to be.

What makes some set of possibilities an agent's *belief* content is the role that these possibilities play in explaining how the agent 'steers'. To be in a belief state whose content is a set of worlds s is to be disposed to act in ways that would tend to satisfy one's desires, were one in an s -world. The belief state produces behavior which is such that, were the actual world in this set of worlds, the agent would act in a way which tends to satisfy her desires. Or again: given an agent's desires and behavior, we solve for the content of their belief by finding the class of possibilities where their behavior would lead to the satisfaction of their desires.²

Belief has a holistic character on this picture. One solves for the content of an entire belief state at once, not one belief at a time. The individual beliefs an agent has are understood derivatively, as the propositions true throughout their unstructured belief content. Thus to have a single nontrivial belief is to have a flood of other beliefs. We have nothing close to a one-one correlation between the individual believed propositions one believes and the complexity of the agent's internal state; there is, especially, no internal belief box containing a mentalese sentence for each proposition believed-true. Rather than the metaphor of an internal list of sentences, what we have instead is the map metaphor. States of belief represent richly, like maps. If a map represents things so that Oakland is east of San Francisco and south of Berkeley, it also represents everything entailed by Oakland's being

² For attempts to spell out 'systematically co-varies in the appropriate way' and 'tends, normally', see Dretske [1981a], Fodor [1990a,b], and references cited therein. For an attempt to clarify the interdependence of the contents of belief and of desire, see Dretske [1988].

east of San Francisco and south of Berkeley. For instance, it represents things so that San Francisco is west of Oakland, Berkeley is northeast of San Francisco, San Francisco and Berkeley are nonidentical and spatially apart, and so on. All these entailments are already included by the information given by the map. No extra ink needed. Consequences are free. So it is with belief on the map picture.³

A map is worth a thousand propositions, but it is indeterminate which thousand. A map represents a way the world might be, but it does not do so *via* representing a set of propositions, as, say, a list of sentences does. The information in a map might be distilled into a set of propositions, but this can be carried out in myriad ways, and the map itself does not privilege one way over any of the others. If there is any proposition which has a privileged status according to a map-like representation, it is the single proposition which states: *things in the world are arranged thusly*. That is, it is the proposition which is just the content of the map. Again, so it is with belief on the map picture.

Let me offer a sense of the leading motivations for this picture.

First, it is constitutive of this model that beliefs do not come and go one at a time, and that belief change is holistic in nature. When you form the belief that you left your keys in the car, you thereby learn that the keys are not in your pocket, and that they are not in this room, and that they are not in the building, and that to get your keys, you will have to go your car, and so on. When the propositions we believe change, they change as a whole system, and in a way that preserves their overall coherence. (Compare the way that the information conveyed by a map changes, when we shift the location of an item on it.) This is intuitive.

Second, we are often quite content attributing beliefs to agents whether or not the proposition said to be believed is one they have ever actively considered. Take Clyde, who swerves to avoid hitting a stray moose on the highway. It seems safe—i.e., true—to say that Clyde believes that the moose he just barely missed is larger than a golfball. (Pragmatically odd in some contexts, but safe.) This, even though no thoughts of golfballs have recently crossed Clyde's mind. Our comfort with the ascription seems related to the fact

³ To call consequences 'free' may be a bit misleading, insofar as this suggests a special class of propositions antecedently 'paid for'. The point is that, of themselves, entailments of what is believed do not impose any additional representational burden over their entailers.

that, had Clyde not believed this, he wouldn't have swerved so hard. The belief ascription seems appropriate because classifying the content of his state of belief with respect to this proposition does work explaining, *inter alia*, Clyde's failure to treat the moose as golfball-sized.

Once we let in one example like this, it is easy to see how to multiply them. The result is that if we want to vindicate the appearances of ordinary belief-desire explanations, it seems we have a need to allow for a whole body of default, or implicit, belief. A vast body of implicit belief is predicted, and made intelligible, by the map picture. It lets us see how ordinary belief-desire explanation needn't require the imputation of implausible level of explicit cogitation on the part of the agent being explained. An agent's belief state may be a reason for his action, whether or not the agent engages in any mental act of explicit reflection on his reasons.

Among other advantages, this enables explanations of communication and of coordinated behavior by appeal to the ideas of common belief and common knowledge, concepts whose application entails the truth of belief ascriptions of unbounded complexity. On the map picture we needn't impute mental representations of unbounded complexity to agents who are said to have common belief or common knowledge.

And then there are non-human agents. We ordinarily explain the behavior of non-human animals via the attribution of beliefs and desires, in a fashion entirely analogous to the explanation of human behavior. The map picture of belief can accept such explanations at face value. Attributions of belief on this picture do not involve the ascriber in any commitments about the specific structure of mental representation; *a fortiori*, there is no risk of representing ascribers as misconstruing this structure when they characterize the beliefs of agents of radically different cognitive design.

So the map picture has compelling features. Plausibly it is the holistic nature of belief on this story that accounts for the strengths just mentioned. But plausibly that same holism accounts for its central and best-known problem, a problem widely thought to vitiate the whole account.

3.2 Deduction

We are representing the content of a state of belief as a set of possibilities, the possibilities left open by what is believed. But if the propositions an agent believes are those which are true throughout her belief worlds, then if she believes p , and p entails q , then q is true throughout her belief worlds. Hence she believes q . Hence, more generally, she believes every consequence of what she believes. Above we focused carefully on the bright sides of this idea, but it is time to acknowledge that on the face of it, it simply looks like a severe distortion of the notion of belief as applies to agents like us. For the model to suggest that our logical powers are unlimited is off the rails. And it gets worse, as everybody knows. On this view of content, if an agent believes anything at all, she believes every necessary truth, since necessary truths are true at every possible world. Moreover there is exactly one necessary truth, because on this model propositions are individuated by their truth-conditions, and necessary truths do not differ in their truth-conditions.

Typically these two problems are lumped under the single heading *the problem of logical omniscience*, but let us separate them:

- **The problem of deduction.** How can we represent ordinary cases in which, as we would ordinarily like to put it, an agent believes p , but fails to believe a truth-conditionally weaker deductive consequence of p ?
- **The problem of equivalence.** If propositions are individuated by their possible worlds truth-conditions, how can we represent ordinary cases of agents who, as we would ordinarily like to put it, believe p but fail to believe q , even though p and q have the same truth-conditions?

Both of these problems are real problems, and they call for nontrivial adjustments to the standard possible worlds picture. I think the solutions they call for, however, are distinct. Here I focus on the problem of deduction. I hope to discuss the problem of equivalence elsewhere.⁴

⁴ In my view, part of the solution to the problem of equivalence calls for replacing possible worlds with a certain more fine-grained object, viz., (possible world, sequence of individuals)-pairs, or what I have elsewhere called *sequenced worlds*. (I discuss this adjustment in Yalcin, in connection with problems about attitudes *de se*. See Cumming [2006] for a similar idea, developed in the context of Frege's puzzle.

To get a feel for the problem of deduction, it will help to consider one attempt to overcome it. Some theorists sympathetic to the map picture have tried to treat the problem of deduction by distinguishing this principle:

(E) If S believes that p , and p entails q , S believes q .

which the map picture of belief as described seems unavoidably committed to, from this principle:

(D) If S believes that p , and S believes that q , then S believes that $(p \wedge q)$.

which, these theorists argue, they can reject. They reject (D) by, in effect, extending the story about belief. Rather than representing the beliefs of an agent by a single set of possibilities, as on the standard model, they represent the beliefs of an agent by a collection of sets of possibilities. On this enriched picture, agents do not have a single belief state; rather they have a set of belief states, or “separate systems of belief”, the contents of which are each represented by a set of worlds. These systems of belief may be “compartmentalized” from each other, so that the content of one system may include p but not q , and another q but not p . In such a case the agent may fail to believe some consequence r of p and q , because p and q are believed with respect to distinct belief states.⁵ Braddon-Mitchell and Jackson [1996] offer an illustration, in the context of advocating this kind of theory:

Jones may believe that Mary lives in New York, that Fred lives in Boston, and that Boston is north of New York, but fail to put all this together and form the belief that Mary will have to travel north to visit Fred. ... [However] Jones may, consistently with the theory, have a system of belief according to which P and a *different* system of belief according to which Q , and so fail to believe that $P \& Q$ by virtue of not having a system of belief according to which $P \& Q$. Indeed, it makes good sense that subjects should have different systems of belief, just as travellers often have a number of maps that they use on their travels. (p. 199)

Heim [1982], Cresswell and von Stechow [1982], and Ninan [2008] also develop this idea, though from different directions.)

⁵ Stalnaker [1984] was the first to suggest this kind of approach.

So adjust the guiding metaphor. Rather than a single map, an agent's beliefs are more like a set of maps. Each map in the set is internally consistent, but it may be that some of the maps conflict with each other about how things are. We still steer by a map, but not always by the same map. We could try putting the new motto like this: *belief is the possibly inconsistent atlas from which we select maps by which we steer*. Less catchy, true, but more accurate, or so it is suggested. Note that since agents are now allowed to have a number of states or systems of belief, believing is actually a *three*-place relation between an agent, one of the agent's belief states, and a proposition: one believes a proposition *with respect to* some state *s* of belief.⁶ Since we are multiplying the number of belief states one can have, it may help to introduce a term for the state of having the set of belief states one has. Let me use *doxastic state* in this way. So on this representation, an agent has in a single doxastic state (the atlas), and this consists in having a multiplicity of belief states (maps). The doxastic state, we could say, is fragmented in that the belief states it consists in are compartmentalized from each other. I will refer to this variously as the *fragmentation* or *compartmentalization* approach to belief, or simply as the *atlas* picture.

The atlas picture succeeds in evading commitment to (D), insofar as the belief ascriptions mentioned in (D) can be understood with respect to differing belief states. The idea of fragmentation does add complexity to the representation of belief, but this kind of additional complexity looks inevitable, at least if one wants to keep a representation of belief content broadly along these lines but also allow for the possibility of inconsistency. So the atlas picture seems a step in the right direction. Still, the question arises whether supplying a strategy for evading (D) is alone adequate to take the sting out of the problem of deduction.

I want to suggest that it is not. We still need to inquire into the source of fragmentation. Why does it happen at all? Why exactly is compartmentalization an inevitable fact of life for realistic agents, agents like ourselves? Understanding the nature of this limitation

⁶ There is alternative way of putting the proposal, depending on how we want to use 'agent'. If we adopt an idealized conception of agency according to which agents are definitionally understood to have only a single state of belief, then we can keep the familiar idea that believing is a two-place relation between an agent and a proposition. What we lose, on this use, is the assumption that we are agents. Rather, beings like us are represented as a bundle, or system, of agents. I will return to this way of thinking of things briefly in section 3.5.

is important, I think, if we are to really understand the atlas picture and the idea of compartmentalized belief. The usual line of explanation adverts in some way or other to the computational limitations of the agent. It is suggested that agents do not have anything close to the representational resources needed to register, or make *available* for cognition and action, all of the consequences of their beliefs. Without this capacity, it is inevitable that inconsistencies will go largely unregistered, and fragmentation unchecked.

This kind of reply is intuitive, and I think on the right track, but it should be observed that it employs distinctions that are not yet integrated into the abstract framework being assumed. What is for a proposition to be available in the relevant sense, and in virtue of what does this state of mind impose some nontrivial computational cost? It is not clear how these ideas are supposed to be taken on the picture I have so far described, because the relation between the belief content of an agent and the detailed nature of the agent's cognitive state is assumed on this picture to be highly opaque. To be in a belief state whose content is a set of worlds s is just to be disposed to act in ways that would tend to satisfy one's desires, were one in an s -world; this approach to settling the content of a state of belief makes and requires no distinction, among what is believed, between a special class of cognitively available propositions and the rest. The absence of such a distinction makes the version of the map picture of belief I have described incomplete, and I suggest that it is the real underlying problem in the problem of deduction.⁷

It would be a mistake to forsake the account of implicit belief that the atlas picture provides. But if want an account of the content of belief that goes beyond what is implicit, we need to extend the picture enough to include the idea of availability. Let me make a suggestion.

3.3 Resolutions of logical space

If we start, as the map picture does, with a picture of belief according to which it is primarily an attitude to possible states of the world and not to a class of propositions distinguishing those states, how can we make room for a special class of 'available propositions' without fundamentally distorting the picture? The solution is to recognize that, for

⁷ If I understand him, Stalnaker [1991] makes a similar point.

realistic believers, the possible states that belief is an attitude towards are not maximally specific. They are coarse possibilities, possibilities reflecting answers to only so many questions. This coarseness betrays a trace of our computational limitations, and of the notion of availability that we need. So a natural idea is that, insofar as we want a model of content that interfaces with these notions, information about this coarseness should somehow be included or reflected in the model. That is to say, the model should include information about the richness or coarseness of the possible alternatives that an agent's state of belief distinguishes, about what questions these alternatives speak to and fail to speak to.

What kind of formal element can play this role—can carry this kind of information in the model? As a first pass, it seems that a simple partition over logical space, a division of the space of possible worlds into mutually exclusive and jointly exhaustive regions, can play this role. We can think of a state of belief as relativized to such a partition, and think of the cells of the partition as the non-maximal possible alternative situations 'recognized' by the state. So the thought is to move from a representation of a belief state that looks like (A) to one that looks like (B) (see figure 3.1 below).

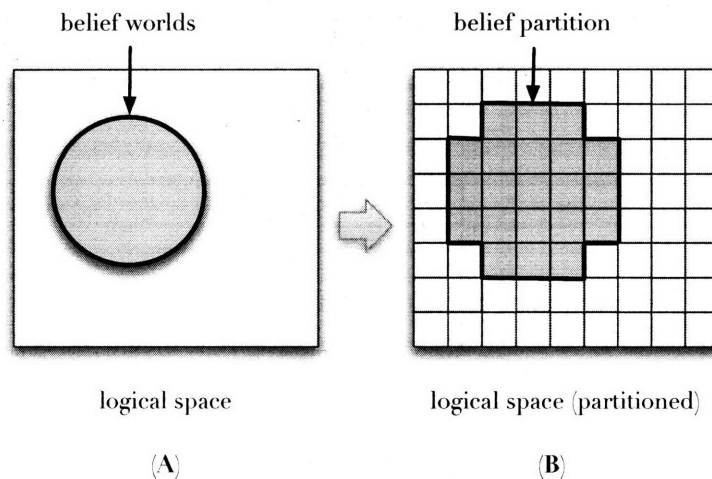


Figure 3.1: From possibilities as points to possibilities as partition cells.

A state of belief still determines a set of (maximally specific) possible worlds, but only insofar as it determines a coarser set of possibilities from the partition of logical space in question. We could call this latter set a *belief partition*. I will call a partition of logical

space used in this role a *modal resolution*. The cells of the modal resolution represent ways the world be, but they are not maximally specific in the way typically assumed of possible worlds; they settle some but not all questions. I will take it that realistic states of belief are relativized to finite resolutions.

Believed propositions are propositions which are true throughout the worlds contained within (partitioned by) a belief partition. With resolutions of logical space now in the picture, we can try using them to characterize belief content which is, in the desired sense, available. I will work with this idea: the propositions which constitute the available information of a body of belief content are those propositions constructible entirely from unions of cells of the resolution. I will sometimes call these propositions *accessibly* believed. Propositions believed, but not so constructible, are implicit in what is accessibly believed. Figure 3.2 gives examples depicting the distinction.⁸

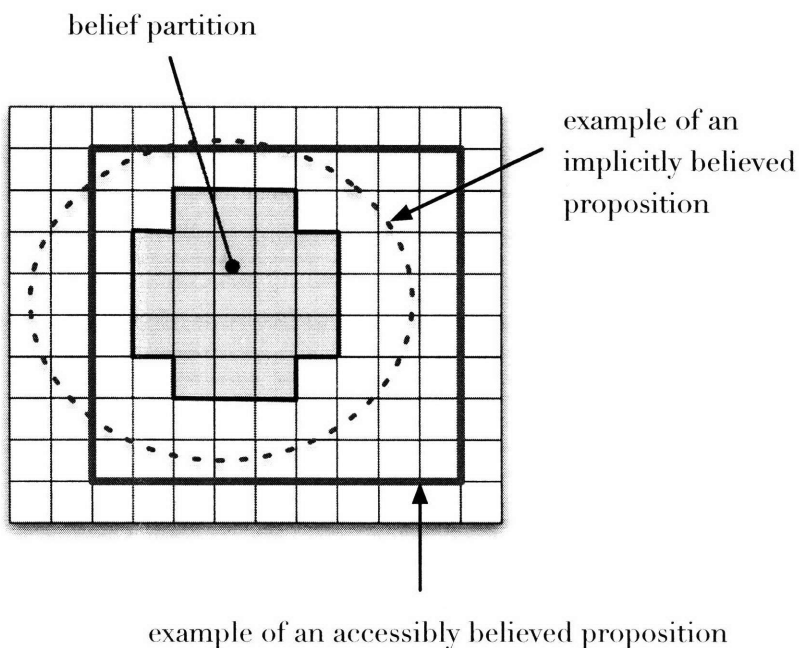


Figure 3.2: The distinction between available and implicit belief content.

(Let me hasten to add that this is not supposed to be some sort of conceptual analysis of the intuitive idea of available information. Rather, it is a formal representation of the

⁸Note that this figure does the same work as figure 2.6 above.

notion, a representation that is intended to limn some of its structural features. The hope is to use the representation to gain a better perspective on the intuitive notion.)

It is helpful to have a name for the distinctive property that accessible belief content has on this resolution-sensitive picture. Given a resolution Π over logical space, call a proposition p Π -*visible* just in case each cell of Π either implies (is a subset of) p or contradicts (is disjoint from) p . A proposition which is not visible is Π -*invisible*. Accessibly believed propositions are believed propositions which are visible at the resolution in question. Note that by definition, if a proposition is visible, so is its negation.

On the atlas picture, a doxastic state is represented as a set of sets of worlds. On the upgrade I now propose, a doxastic state is representable as a set of (partition-of-logical-space, subpartition)-pairs, or more intuitively, as a (partial) function from resolutions of logical space to a set of cells at that resolution. Think of it like this: given a menu of (coarse, non-maximal) alternatives for how things might be, as represented by a resolution, a doxastic state selects some set of these alternatives as its candidates for actuality.

Wherever you have a finite space of alternatives of the sort presented by a resolution, you can give a measure of how much information—in bits—would be needed to reduce those alternatives to one. This is just the logarithm, to the base 2, of the number of alternatives.⁹ The toy resolution in (B) above, for instance, distinguishes 81 possibilities; so given only this representation of the alternatives and no other information, it would require at least 6.33985 bits of information to cut those possibilities down to one. I will call this number in bits the *information potential* of a resolution. To get an intuitive grip on what this number means, look at a resolution from the perspective of this question: *what is the smallest number of propositions needed to construct it?* The answer to this question will be the information potential of a resolution, rounded up to the nearest integer. If a resolution consists of only four cells, its information potential is 2 bits, since one minimally needs two propositions to effect a four-way division of logical space. More generally, if the cardinality of a resolution is equal to 2^n for some n , then the number of propositions needed to draw the resolution will be n , and all of these propositions will be logically

⁹ Here I set aside complexities that arise when the alternatives are taken to vary in probability. Hartley [1928] was the first fix on a logarithmic measure, and the first to attempt to measure information in terms of the number of possibilities eliminated.

independent. If instead the size of a partition is m and $2^n < m < 2^{n+1}$, you will need n logically independent propositions *plus* one additional proposition entailed by one or more of those n propositions to construct the resolution.

The point of flagging the concept of the information potential of a resolution is to show that the model makes space for the idea of a clear, finite upper bound on how much information—at least in one very specific, technical sense of ‘information’—a given belief state might in principle carry. This is an important point, and it is a point at which our abstract story of content, of what is represented, makes contact with what does the representing. Although the idea of resolution-sensitivity does not come with any specific commitments about the detailed form by which the content of belief is represented, it does at least demand that the form have the complexity to *encode*, in bits, the information potential of the resolution in question. Resolutions impose what we could call an *encoding cost*. This is a way, albeit a highly abstract way, that this framework for representing belief imposes a constraint on a model of the mechanism of representation.

To flesh out this representation of belief content further, the thing to do is to put it to work. Let us see what the problem of deduction looks like from the perspective of this model.

3.4 Deduction redux

First, it is easy to see that when we take belief to be resolution-sensitive, principle (E), repeated here:

(E) If S believes that p , and p entails q , S believes q .

needs qualification. Taken as a principle governing *implicit* belief:

(E_i) If S implicitly believes that p , and p entails q , S implicitly believes q .

it is correct. Taken as a principle governing *available* belief, however:

(E_e) If S accessibly believes that p , and p entails q , S accessibly believes q .

it is incorrect. For q might be invisible. Any nontrivial item of available belief will have countless nontrivial implicit, invisible entailments.

This is a simple technical advance on the problem of deduction. More interesting, I think, than the technical advance is the intuitive representation of deductive inquiry that a resolution-sensitive picture allows. We can say that when we deduce, we enter into a state of mind making the premises visible, and we move to increase the resolution of that state of mind, so as to see what further propositions follow, or fail to follow, from the premises. Thought of in this way, deductive reasoning tends to require high-resolution states of mind, states of mind with high information potential. Resolutions with high information potential have a high encoding cost. Although this is obviously not yet anything like a cognitive model of deduction, it does give us a sense of the connection between our failure to be deductive omniscient and our limited representational capacities. The very high encoding cost of completely bringing together two compartmentalized states of belief—making the accessibly believed propositions of each visible at one resolution—may be a cost that an agent with limited representational capacities cannot afford.

3.5 Availability

I suggest that this makes an advance on the problem of deduction, and that it fills gaps in what I have called the atlas picture of belief. But one might object that resolution-sensitivity comes with a problem of deduction all its own. The problem is that, according to the model I have described, the class of accessibly believed propositions is closed under logical consequence, and that even this kind of closure is extremely unrealistic.

We can sharpen the problem. Let's say that S *believes* $_{\Pi}$ that p just in case S 's doxastic state maps Π to a belief partition throughout which p is true. Then the following principle:

(V) **Closure under visible consequence.** If S accessibly believes $_{\Pi}$ that p , and p entails q , and q is Π -visible, then S accessibly believes q .

is true on the resolution-sensitive model. The complaint is that this is implausible. The implausibility might be illustrated as follows. Consider a toy case where S accessibly believes,

with respect to a given resolution, four logically independent propositions, and that let us represent this state of belief simply as a cell of a sixteen-cell resolution. Question: how many propositions is S represented as accessibly believing with respect to this resolution? According to the definition of available belief I have given, what is accessibly believed are those propositions true throughout the belief partition and also constructible from the cells of the resolution. How many propositions satisfy these criteria in this case? A little reflection reveals that the answer is 2^{15} , or 32,768. But surely that is totally implausible; surely that is more information than is actually accessible for the agent. Moreover, were we to assume that S believes some fifth logically independent proposition, again representing S 's beliefs by a cell in the resulting resolution, S 's available beliefs would number in the *millions*. So there is a threat here that any resolution rich enough to characterize a nontrivial body of belief is also one which will make for *far too much* available belief.

This worry is less a reason to abandon the resolution-sensitive model—which, despite its commitment to (V), is an order of magnitude less abstract and idealized than the standard possible worlds model from which we started—than to look more closely at the intuitive notion of available information.

My notion of availability may seem expansive, but just how expansive it really is depends on what assumptions are made about the average information potential of the resolutions that an agent's doxastic state is defined on. On this I have not taken a stance; further investigation is needed before it makes sense to take a stance. Perhaps a realistic representation of a doxastic state on this model will be a function on *very low* resolutions of logical space—resolutions of, say, half a dozen cells or less, rather than the scores of cells that my diagrams have suggested. If so, the number of accessibly believed propositions at a given resolution will be reduced exponentially—rather than 32,000 available propositions, we will have 32 at most, assuming six cells. A doxastic state like this would presumably involve radical fragmentation. It would be natural to try to pair this kind of approach with attempts to represent intelligent thought and action as emergent from the interaction of a horde of primitive homunculi—see Dennett [1978], Fagin and Halpern [1987], and the introduction of Stalnaker [1999]—though note that the model I have described does not yet say anything about how these compartmentalized states of mind relate to one another.

On this last point, let me note an important possibility. I have been assuming that a

resolution of logical space either brings a proposition into focus—makes it visible—or leaves it out of focus—makes it invisible. But there is a middle ground. Just as photograph might have a high resolution foreground and a low resolution background, so a resolution of logical space might make a proposition visible with respect a region of logical space, leaving it invisible elsewhere. This would be to allow for *local* visibility, visibility with respect to a partition of some proper subset of logical space.

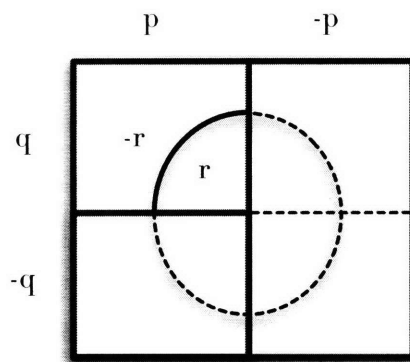


Figure 3.3: Figure and ground in logical space: a resolution with a region in focus.

Let me give an illustration. In figure 3.3 we have a partition of logical space into five cells (as reflected by the dark lines). The proposition p is visible with respect to all of logical space. The proposition q is not, but it is visible with respect to the space of p worlds. The proposition r is invisible with respect both logical space and the space of p worlds, but is visible with respect to the $p \& q$ worlds. One way of thinking of this is as a representation of an inquiry, or of a process of inquiry. An agent's doxastic state may map the partition $\{p, \neg p\}$ to p —that is, the agent may answer the question whether p affirmatively—and then ask, so to speak: ‘Given p , is q the case?’ in such a way that the agent may only be correctly represented as distinguishing q -visible alternatives *within* the p -space. Similarly for r . The representation of logical space here is ‘zoomed in’ on the p worlds, and zoomed in again on the $p \& q$ worlds, resolving this region more finely than any other area. A resolution with local visibility like this can be thought of as an ordered sequence of simple resolutions over an increasingly small region of logical space, with each successive resolution thought of as a stage in the inquiry. Perhaps states of belief should

be understood with respect to stages of more complex inquiries of this kind; if so, whether or not a proposition counts as available would depend on one's location in inquiry. I think this idea is worth exploring, but my aim is not to explore it here; this is a project that would shade, I think, into something like detailed cognitive modeling. The larger point is simply that the framework I have described helps us to state such questions precisely.

Let me note that the framework also enables other notions of availability, should we want them. We could distill from a resolution those smallest sets of propositions that are sufficient to determine the resolution, and treat the propositions in these sets as the most available propositions.¹⁰ Or we could just pick one of these sets of propositions as being the most accessible. In either case, a gradable notion of availability could then be defined in terms of the complexity of boolean formulae built out of propositional atoms expressing the most accessible propositions. Alternatively, we might treat the propositions which are the cells of the resolution as the most accessible, again recovering a gradable notion in terms of boolean complexity. No doubt other definitions are possible. Whether any of these notions are interesting depends, obviously, on the application. It seems plausible that the intuitive notion of availability is a gradable one; as Stalnaker [1991] notes,

There are questions I can answer quickly with a moment's thought or a minor calculation, and questions that I have the computational resources to answer eventually, but only after a lot of time and effort. For some questions of the latter kind, I may be able to say outright that I have the capacity to produce the answer eventually; for others, I may in fact be able to produce an answer, if I choose the right computational strategy, but may be unable to say whether I can actually produce the answer. How easy must the search or computation be in order for the answer to count as something the agent already knows or believes, and not something it has the capacity to come to know or believe? ... There is obviously a continuum here, and no very natural place to draw the line between information that is easily accessible and information that is not.¹¹

But whether a technical regimentation of that notion should also be gradable is an open

¹⁰ The cardinality of these sets will be the information potential of the resolution, rounded up to the nearest integer.

¹¹ Reprinted in Stalnaker [1999], p. 252.

question, one that depends on the explananda and the theoretical context. One might have a sharp line in one's model for some purposes, acknowledging that it is context-dependent and somewhat arbitrary exactly where it is drawn.

3.6 Directions for further development

Let me close by noting some directions in which the idea of resolution-sensitivity might be further developed.

First, I have developed the idea of resolution-sensitivity for belief, but inquiry should be extended to other attitudes. Similar considerations might drive resolution-sensitivity for knowledge, desire, and supposition. Supposed or entertained-true propositions are often paradigm cases of explicit or accessible content; similarly with the simple attitude of *thinking of* a proposition. And 'contrastivist' approaches to 'knows' in the style of Dretske and, more recently, Schaffer, seem *prima facie* to lend themselves to articulation in terms of resolution-sensitivity.¹²

Second, we should explore the extent to which an agent's capacity to enter into a state of mind making a proposition *p* visible can be *decomposed* into that agent's conceptual capacity to distinguish objects and properties. It is tempting to think that the notion of a resolution should be applied beyond—or below—the space of possible worlds, to the space of possible individuals or sets of individuals. The proper treatment of de re attitudes may call for an extension along these lines.

Above I began by pointing out that the idea that states of mind are resolution-sensitive is not a claim about language or about semantics. But it would be a mistake to conclude from this that semantics cannot help to constrain the theoretical application of that idea. Of course it can. Within semantics, two places in particular are obvious sources of direction. First, the literature on questions. The idea of using partitions to represent questions has been in wide currency in this literature since Hamblin [1958]. Work on the structure of questions can inform work on the structure of resolutions, and work on how they are composed compositionally can help to guide the project of decomposing the capacity to

¹²See Dretske [1981b], Schaffer [2004, 2005].

enter into a state of mind making a given proposition visible into other conceptual capacities. Second, the literature on so-called ‘hidden indexical’ analysis of attitudes.¹³ If belief is resolution-sensitive, it is a three-place relation between a person, a resolution, and a proposition. This raises of the question of how exactly reference to a resolution is achieved in belief reports, and one wants confidence that this question can be answered in a compositionally plausible way. ‘Hidden-indexical’ analyses also treat attitudes as three-place relations, so we should look to this literature to get a start in understanding how the idea of resolution-sensitivity can be made plausible from a compositional point of view.

¹³See e.g. Crimmins and Perry [1989], Schiffer [1992], Crimmins [1992]. Hidden-indexical analyses are usually motivated by versions of Frege’s puzzle.

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