Towards a Unified Theory of Procurement Contract Design: Production Flexibility, Spot Market Trading, and Contract Structure

by

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Abstract

We present in this work a unified approach and provide the optimal solution to the pricing problem of option contracts for a supplier of an industrial good in the presence of spot trading. Specifically, our approach fully and jointly endogenizes the determination of three major characteristics in contract design, namely (i) Sales contracts versus options contracts; (ii) Flat fee versus volume-dependent contracts; and (iii) Volume discounts versus volume premia; combining them together with spot market trading decisions and also the option of delaying production for the seller.

We build a model where a supplier of an industrial good transacts with a manufacturer who uses the supplier's product to produce an end good with an uncertain demand. We derive the general non-linear pricing solution for the contracts under information asymmetry of the buyer's production flexibility. We show that confirming industry observations, volume-dependent optimal sales contracts always demonstrate volume discounts (i.e., involve concave pricing). On the other hand the options contracts are more complex agreements, and optimal contracts for them can involve both volume discounts and volume premia.

Further, we find that in the optimal contracts, there are three major pricing regimes. First, if the seller has a higher discount rate than the buyer and the production costs are lower than a critical threshold value, the optimal contract is a flat fee sales contract. Second, when the seller is less patient than the buyer but production costs are higher than the critical threshold, the optimal contract is a sales contract with volume discounts. Third, if the buyer has a higher discount rate than the seller, then the optimal contract is a volume-dependent options contract and can involve both volume discounts and volume premia.

We further provide links between industry and spot market characteristics, contract characteristics and efficiency. Last, we look into an extension of our basic model, where we give an analysis for the case when the seller is given a last minute production option.

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Chapter 1

Introduction

1.1 Motivation

With the advances in technology and communications in this new era, as well as the effects of globalization of economies, companies from almost all industries find themselves increasingly under pressure to respond to the ever faster changes in demand and supply. This environment forces companies to employ more flexible forms of procurement strategies and tools, rather than the traditional, long-term, rigid, and often closed-relationship based delivery agreements which historically dominated industrial procurement. As a consequence, there is a rapid growth in the employment of two forms of flexible contracting strategies in recent years, namely: *options contracts* and the utilization of *spot trading*, which helps in making adjustments to existing contracts, according to the realization of uncertain demand.

The first one of these two strategies, the use of flexible option contracts in industrial procurement, is gaining prominence. Option contracts in this context refer to agreements between a supplier and a buyer of an industrial good, where the buyer purchases the right to receive a delivery of a certain industrial intermediate good from the supplier at a certain later date and a predetermined price. The buyer pays a reservation fee to purchase this right at the contract time. Upon the time of delivery, depending on the resolution of uncertainties (such as spot market price and/or consumer demand for the end product), the buyer decides the amount of the options contracts to execute. That is, according to the options agreement, the buyer chooses the amount (up to the number of options contracted) to be delivered by the supplier; and pays to the supplier an exercise price, for only the units executed but not the unexercised units reserved. This provides flexibility and increased efficiency in risk sharing between supply chain partners. Such contracts have now been used in a variety of industries and product categories, including: electricity, heavy equipment, tools and camera lens.

Advances in technology initiated electronic spot market trading, which complements many existing contracts. This gives many opportunities for companies to connect with new trading partners and to adjust to changing market conditions. In fact, today, in many industries in the U.S. and around the world, procurement is carried out as a combination of long term agreements between suppliers and buyers, and purchases in open spot markets (see, e.g., (9), (21)).

However, there are significant down-sides of trading industrial goods in spot markets compared to procuring through contracts with known partners. First, there is naturally a spread between the buy and sell prices in these venues. That is, the sell price of a product is typically lower than the buy price of the product.

Second, and more importantly, even very similar products might have different characteristics. For example, industrial goods from different suppliers have varying characteristics over a number of dimensions such as quality and compatibility. Thus, in the case of procuring from a new partner found on the spot market, not only is the buyer buying a product with a value potentially different from that of the specified supplier's product manufactured for his purposes; but he is also paying for adaptation costs, undertaking tests and spending extra efforts to ensure compatibility, reliability and match (see e.g., (7), (13)). This means cost procuring units from the spot market is higher than quoted due to the hidden transaction, adaptation and compatibility costs, which stems from a variety of reasons such as asset specificity, time specificity, complexity of product description, as well as product and transactional complexity ((1), (14), (24), (25)).

Inevitably, there is an inherent difference in the manufacturer's valuation of a good purchased from a regular trading partner versus a new outside supplier who is not his usual source. These inefficiencies create incentives to reduce reliance on spot markets for procurement and shift purchasing towards contracts with existing partners. Thus, the ability to adapt to trading with new suppliers becomes an important distinguishing characteristic between flexible and inflexible companies. If a manufacturer's production process is sufficiently flexible, its reliance on a given supplier is low; he can easily switch between long term partners and new suppliers found on a spot market. In such a case, the manufacturer will have a strong position compared to the supplier. On the other hand, if a manufacturer's production process is not very flexible, procuring from new suppliers imposes large compatibility costs, which lowers overall value; and hence he is highly reliant on his existing business partners.

As a result, the flexibility of a given manufacturer's production process becomes a major individual characteristic that determines his alternative outside production options, and hence an important factor in the pricing of a procurement contract. As such, it is natural that the prices for sales or option contracts rely on the supplier's information on the flexibility of the manufacturer's production process at the contracting stage. Typically, a manufacturer would like to represent himself as minimally reliant on the supplier as possible. Considering the sources of the relative flexibility of a manufacturer listed above, the "flexibility" of a manufacturer depends on many factors that do not vary with time, which include individual firm characteristics like internal organizational structure, relative freeness of the resources, temporary agility of the firm's processes, existing commitments, and the strength of the established relationships with other suppliers.

All these factors together create differences in the buyer's private valuation of the supplier's product, relative to that of buying from the spot market. Thus, when the supplier prices the option contracts, she is facing an informational disadvantage. Specifically, the supplier has to determine the price for the option contracts optimally without knowing the manufacturer's true intrinsic value of the contracts, as the value depends on his individual characteristics which are not fully known to the supplier.

The solution to this pricing problem is thus complex; it not only involves information asymmetry about the buyer's characteristics, but also a simultaneous consideration of decision making on several dimensions, including the quantity of contracts to be agreed upon, as well as the exercise regime of option contracts based on the realization of uncertainties in spot market prices and consumer demand. Further, the optimal exercise price for the contracts needs to be determined endogenously and simultaneously at the same time with the reservation price scheme. The structure of the optimal contracts under information asymmetry is wide-open and a variety of linear and non-linear pricing schemes involving patterns like fixed price-quantity contracts and contracts with volume-dependent incentives, namely volume discounts and/or volume premia, can emerge as optimal contracts.

These observations lead to important research questions: How do non-linear pricing schemes on option contracts and spot market trading interact? What is the optimal joint option pricing scheme, including the reservation price schedule and the exercise price, in the presence of spot market trading? When are volume discounts optimal? When is it optimal to employ flat pricing? When would a supplier offer options and when would she simply offer direct sales to the buyer? How do market and industry characteristics such as production costs, production flexibility, spot price distribution, bid-ask spread for the spot price, and demand distribution affect contract characteristics such as the exercise price, reservation price, and the contracted quantity?

The objective in this research problem is to answer man of these questions. Specifically, the goal for our research problem is four-fold. First, starting with a general class of options contracts, we present the general solution to the reservation pricing problem for the supplier. In particular, we characterize the optimal general non-linear pricing scheme for the options on delivery of the industrial good with the presence of spot trading as an outside option for both parties under information asymmetry on buyer's production flexibility, and uncertainty in demand and spot price. Second, we determine the conditions under which it is optimal to sell the capacity or offer options, and the conditions under which it is optimal to offer volume discounts instead of constant prices. Third, we demonstrate the effects of market and industry characteristics on the specification of optimal contracts. Finally, we extend our basic model by adding another dimension, looking at a delayed production opportunity for the seller.

The rest of this work is organized as follows. In this chapter, section 1.2 presents the literature review, and section 1.3 presents an overview of the whole problem. Chapter 2 presents the optimal contract design for our basic model, which includes the design and characteristics of the optimal contracting schemes. Chapter 3 presents the optimal determination of the exercise price and studies the effects of the industry and market characteristics on contract design. A numerical analysis section on uncertainty and efficiency is also included in the same chapter. Chapter 4 presents our model with delayed production opportunities, with results on its optimal contracting schemes and determination of optimal exercise price. Last but not least, Chapter 5 offers our concluding remarks and future possible research problems.

1.2 Literature Review

Supply chain contracting has received a considerable amount of attention in the literature in recent years. There is a large number of studies in the area that explore a variety of supply chain contracting schemes. (4) and (6) give comprehensive surveys of the literature in this area. Among many different contract structures studied in this literature are buy-back (7), "pay-to-delay" (3), quantity flexibility (22), and revenue sharing contracts (5).

Among these studies in the supply chain contracting literature, a number of papers explore option contracts for procurement. (8) studies option contracts (or "backup agreements") between catalog companies and manufacturers in the fashion industry. Examining data from the industry, they find that backup arrangements can have a substantial effect on expected profits and can increase quantity. (2) studies channel coordination with option contacts, showing that coordination can only be achieved through piece-wise linear exercise prices. However, they show that in order or coordinate the channel through linear prices, the supplier's individual rationality constraint has to be violated. (17) studies the bidding behavior in a market for supply option contracts with multiple suppliers and a single buyer. They show that in the Nash equilibrium of the bidding game, the suppliers show clustering behavior. They also show that, in general, the loss of supply chain profit due to competition is at most 25% of the centralized supply chain profits.

A second main branch of the supply chain management literature that is closely related to our paper deals with spot market trading in industrial goods. (11) provides a survey of the earlier literature in this area. (12) studies the effects of a secondary market for excess inventory on a supply chain with a large number of buyers and a monopolistic supplier. They demonstrate that a secondary market increases efficiency in allocation, but may decrease the supplier's profits. (20) considers a multi-period setting with both long term and spot purchases where unmet demand is carried to the next period. They identify the conditions under which each mode of procurement model is optimal. (23) studies the competition between informal relational contracts and auction markets in the presence of product quality differentiation among the suppliers and determine the conditions under which long term relational contracts will eliminate open auction markets and vice-versa. (18) studies a two sided business-to-business market, which the participants can sequentially use together with long term contracts. They identify the effects of the introduction of a two-sided spot market on the supply chain, and demonstrate that in most cases, the consumers are the main beneficiaries from an electronic industrial spot market. (19) studies sequential spot and long term trading in a two layer supply chain under asymmetric correlated information. They define a concept for liquidity (or market impact factor) for industrial spot markets and demonstrate the important role it plays in supply chain efficiency and the generation of value and surplus in the supply chain.

(27) examines the interaction between capacity option contracts and spot trading. They explore a model with a single seller and multiple buyers, where the seller and buyers first contract for capacity options, and can then trade in the open spot market if it is desirable. They show that the buyers' optimal reservation level follows an index that combines the seller's reservation and execution costs. (26) utilizes the same

framework to examine a setting with multiple sellers with heterogenous technologies and a single buyer of the product. They characterize the equilibrium and explore its efficiency properties. Using the same framework, (13) introduces the notion of codifiability of the product and study the role of adaptation costs that an industrial buyer incurs when purchasing from the spot market. They show that codifiability and spot price distribution have significant effect on options contract pricing. (16) studies a portfolio management approach to optimize supply option contracts in the presence of spot markets in a multi-period setting. They characterize the optimal replenishment policy for a portfolio of options, and show that this policy specifies that the use of each option contract and the spot market is dictated by a modified base-stock policy. (15) explores the mean-variance properties of supply option contract portfolios. They characterize the set of portfolios that a manufacturer must hold in order to achieve dominating mean-variance pairs.

Our model differs from the earlier work as we pursue an understanding of the endogenous determination of contract structures. To do this, and unlike the earlier literature, we explore the role of production flexibility in the presence of information asymmetry between the buyer and the seller. Starting from a very general class of contracts that encompasses common contracting structures, we jointly endogenize the determination of multiple dimensions of the nature of procurement contracts. Specifically, we aim to find the conditions under which the optimal contracts endogenously become sale contracts versus option contracts, and under which optimal contracts will be volume-dependent instead of flat-priced. We also seek to understand the role of market and industry variables, such as spot price distribution and the statistical properties of informational asymmetry on the determination of contract structure and pricing. We will explain how we put together all these elements in a unified model in detail in the section below where we present our problem description.

1.3 The Problem and Key Insights

Consider a single supplier, single manufacturer model. A supplier of an industrial good transacts with a manufacturer who uses the supplier's output as an input to produce an end good with an uncertain demand. There are two time periods. In the first period, the supplier offers an option contract that, if purchased, gives the right for the manufacturer to procure up to a certain agreed quantity of the supplier's product in the second period. The supplier is free to set a flexible reservation price schedule that is not necessarily linear, i.e., can involve volume-dependent incentives. In the second period, the demand for the manufacturer's product and the spot market price are realized. By observing these outcomes, the manufacturer chooses the amount of options to exercise (up to the quantity he purchased) and pays the pre-determined linear exercise price to the supplier. If either the supplier or the manufacturer has any unused intermediate good inventory in the end of the second period, they have the option to sell it to the spot market at its bid price, where there is often a bid-ask spread at the spot market.

If, in order to satisfy the demand, the manufacturer needs to produce more units than he contracted, he purchases the remaining balance at the spot market. However, purchasing from the spot market incurs a certain additional per unit adaptation cost burden for the manufacturer, which depends on his degree of production flexibility. Notice that this additional per unit adaptation cost burden is a strong indicator of the manufacturer's dependence on the supplier. Although the degree of this dependence may be partly known to the supplier, it has many components, such as the buyer's internal and external flexibility, finances, other commitments and technical idiosyncrasies that are private information to the manufacturer. This private information on his additional costs of adaptation when purchasing parts from other suppliers, consequently, is an important factor in the contract agreement and price determination process.

The work here first explores the optimal non-linear pricing schedule offered by the supplier for a fixed exercise price. We show that when: (i) the supplier is less patient (has a higher discount rate) than the buyer; (ii) the production costs and the bid-ask spread in the spot market are low relative to the expected spot price; and (iii) the exercise price for the options are sufficiently low, the optimal contracting scheme will involve flat pricing. On the other hand, if any one of these conditions is not satisfied, the optimal contracts will be volume-dependent, i.e. the total price will depend on the quantity and thus the price schedule can be non-linear. Futhermore, when the supplier is less patient (has a higher discount rate) than the buyer, the expected contracted quantity decreases with the exercise price of the options. Otherwise, when the supplier is more patient (has a lower discount rate) than the buyer is lower than the buyer, the expected quantity is non-monotonic in the exercise price. Specifically, contracted quantities increase for low exercise price levels, and decrease for high exercise price levels.

For the same model, we then study the optimization of the exercise price for the contracts. Endogenizing the determination of the exercise price, we can distinguish between option contracts (when the exercise price is strictly positive) and sales contracts (when the exercise price is zero). We show that when the seller is more patient (has a lower discount rate) than the buyer, it is optimal for the seller to offer option contracts by selecting a strictly positive exercise price. On the other hand, when the buyer is more patient (has a lower discount rate) than the seller, in the optimal contracts, the seller chooses to sell the intermediate good to the buyer by setting the exercise price to zero.

The analysis allows us to reach a general characterization of optimal procurement contract structures. Specifically, we show that there are three main modes of procurement contracts, and characterize the regions under which each will emerge: First, when the seller is less patient (has a higher discount rate) than the buyer, and the unit production costs are low relative to the buyer's discounted expected unit sales price in the spot market, the optimal contract is a flat-price sale scheme that offers the buyer to purchase up to a fixed quantity for a fixed price. Second, when the seller is less patient (has a higher discount rate) than the buyer, but the unit production costs are high relative to the buyer's discounted expected unit sales price in the spot market, the optimal contract is a sales contract with volume discounts. Finally, if the buyer is less patient (has a higher discount rate) than the seller, the optimal contract is an option contract with volume discounts and/or volume premia.

We also study the effects of spot market and industry characteristics on contract design. Our analysis provides previously unexplored links between variables such as spot price distribution and the bid-ask spread in the spot market and contract characteristics such as reservation and exercise prices and quantities contracted. We find that increased average spot prices, and buyer's production flexibility, or decreased uncertainty on the buyer's production flexibility, the ratio between seller's and buyer's discount rates and spot price variance, tend to increase the exercise price for the options. Increased average spot price, the ratio between seller's and buyer's discount rates and buyer's production flexibility, or decreased bid-ask spread, and production cost, tend to decrease expected contracting quantities. Further, increased expected spot prices, buyer's production flexibility, and the ratio between seller's and buyer's discount rates, or decreased production cost and spot market bid-ask spread, tend to increase the reservation price for the options.

We further consider an extension, where we add to the basic model another dimension of opportunities for the seller. This is a delayed production model, where the seller decides her production quantity in two separate occasions. In the first period, when seller offers the option contract to the buyer, she has the opportunity to decide on her production quantity after the buyer proposes his reservation quantity. Furthermore, there is a second production opportunity for the seller in the second period when the spot market and demand uncertainty states are realized, i.e. if the units produced by the seller at the first period are insufficient to meet the buyer's execution quantity, the seller can produce extra units in period two at a higher cost. On the other hand, if there are remaining units left after the buyer's execution, the seller will sell them all to the spot price at its bid-price.

Similar to the basic model, in this extension, we first explore the optimal non-linear reservation price schedule for a fixed execution price. We identify a complete set of conditions which determines the specific price schedule structure and its respective optimal contract offer. We show that there are exactly three possible combinations of optimal reservation price schedule structures: (i) *flat price schedule*, which is offered by the seller when either execution price is greater than her second stage production cost, or when we have the following three conditions satisfy: when the seller is less patient than the buyer, initial stage production cost is low enough and execution price is low enough; (ii) *volume-dependent price schedule*, which is offered when execution cost is in an intermediate range, and that the benefits to the buyer in engaging in options for the purpose of selling to spot market is higher than the cost incurred to the buyer; and (iii) *low volume dependent-high flat combination price schedule*, i.e. the seller offers a volume-dependent reservation price schedule to buyers with higher production flexibility, but a fixed price schedule to buyers with lower production flexibility. This type of price schedule is offered when neither the flat price condition nor the volume-dependent condition is satisfied. Apart from a full characterization of the optimal reservation price schedule, we also look into the delayed production option for the seller. Specifically, we determine the conditions under which it is favorable for the seller to produce up to reservation amount at the first period, versus to delay parts of her production till the second period.

For the delayed production model, we also study the optimization of the exercise price for the contracts, which further allows us to understand the general characteristics of optimal procurement contract structures. Similar to the basic model, we distinguish between option contracts and sales contracts. We show that when the seller is more patient than the buyer, it is optimal for the seller to offer option contracts by selecting a strictly positive exercise price with a flat reservation pricing schedule. On the other hand, when the buyer is significantly more patient than the seller, in the optimal contracts, the seller chooses to sell the intermediate good to the buyer by setting the exercise price to zero; if either the initial or later production cost is sufficiently low, a flat reservation pricing schedule is optimal, otherwise quantity discounts will be offered for an optimal reservation price schedule. Whereas, when the buyer is only slightly more patient than the seller, in the optimal contracts, the seller will select a strictly positive exercise price (but less than that of the seller more patient than buyer case) with an low volume dependent-high flat optimal reservation pricing schedule.

Chapter 2

Optimal Contract Design for Fixed Execution Price

2.1 The Basic Model

The problem considers a two layer supply chain. The supplier of an industrial good ("the seller" or "S") sells to a manufacturer ("the buyer" or "B"), who uses the intermediate good in his process to produce an end good.[1] The demand for the end good, D, is uncertain, with a continuous distribution function F_D, density function f_D and support $[\underline{D}, \bar{D}]$. The retail price for the end product is $p > 0$. There are two time periods in the model. At time $t = 1$, the buyer may reserve a production capacity, up to K units, from the seller by purchasing q capacity options, according to a price schedule $(R(q), w)$ the seller offers. The price schedule consists of a menu, $R(q)$, the reservation fee for the q options purchased, and a per-unit exercise fee, w. Each unit of capacity option purchased gives the buyer the right to buy one unit of the intermediate good from the seller at time $t = 2$ at the exercise price price $w \geq 0$. At time $t = 1$, the buyer decides on the amount of capacity to reserve, q, with the supplier and pays $R(q)$. The supplier, who has a production capacity K ($\geq \bar{D}$), then produces the goods at a unit production cost of $\beta_0 > 0$.

[1]As a language convention, we refer to the seller as "she" and the buyer as "he" throughout.

At time $t = 2$, the consumer demand for the buyer's final product, D, and the spot price, s, are realized. The spot price, s is uncertain at time $t = 1$, with support $[\underline{s}, \bar{s}]$, where $0 < \underline{s} < \bar{s}$. The distribution function F_s for s is continuous with density f_s. We assume that s has increasing hazard rate, i.e., $f_s(s)/\bar{F}_s(s)$ is increasing, this is a commonly used weak assumption satisfied by many common distributions such as normal, exponential and uniform. For notational convenience, define $g_s(s) \triangleq \bar{F}_s(s)/f_s(s)$, which is decreasing in s. As a regularity condition, we also assume that $\underline{s} > 1/f_s(\underline{s})$, which ensures a certain lower bound for the realization of the spot price. We also assume that $(d/ds)\log(f_s)$ is bounded on $[\underline{s}, \bar{s}]$, which ensures that the s distribution does not explode too fast over its support, this is also satisfied by almost every common distribution.

Observing D and s, the buyer has the choice of exercising his options at the strike price w (up to the purchased amount, q); or purchasing the intermediate good from the spot market. But if he decides to procure from the spot market instead of trading with his preferred supplier, his new product will either have a decreased market value, or incur additional adaptation costs to adjust for compatibility - to keep up to the standards of that of his specified supplier. These costs have been well established in the literature and stem from issues like asset specificity, time-specificity, hold-up costs and compatibility of the products ((10), (13), (14), (25)). Here, we denote the buyer's additional cost for each unit procured on the spot market to be $\theta > 0$. That is, each unit procured at the spot price s will have an effective total cost of $s + \theta$ for the buyer.

Notice that θ is a strong indicator of the buyer's dependence on his regular supplier, S. This indicator is only partly known to the supplier, as the buyer's dependence on his regular supplier has components which are private to the buyer, including his internal and external flexility, finances, other commitments and technical idiosyncracies. All these private information on this θ indicator is consequently an important factor in the contract agreement and price determination process. From the seller's perspective, θ is a random variable with support $[\underline{\theta}, \bar{\theta}]$, with continuous distribution function F_θ, and density function f_θ. We assume that θ also has an increasing hazard rate, which means that $g_\theta(\theta) \triangleq \bar{F}_\theta(\theta)/f_\theta(\theta)$, is decreasing. As regularity conditions for the s and θ distributions, we also assume $\underline{\theta} \geq \frac{1}{f_\theta(\underline{\theta})}$ and $f_s(\underline{s}) \in [\frac{1}{\phi \underline{s}+\underline{\theta}}, f_\theta(\underline{\theta})]$. The former maintains a balance in the low end of the θ distribution, and the latter ensures that the spot price distribution is more spread out in its lower end than the θ distribution.

The number of options the buyer decides to exercise naturally depends on his type, θ, the demand realization D, the spot price realization s, and the exercise price w. When the buyer places an order for exercising his options, the seller delivers the corresponding amount of the intermediate good. Furthermore, at the end of time $t = 2$, if either the seller or the buyer have any unused intermediate good inventory, they can sell it to the spot market at price $(1 - \phi)s$, where $0 < \phi < 1$ denotes the bid-ask spread in the spot market.

The supplier's discount rate between periods $t = 1$ and $t = 2$ is $r_S > 0$ and the buyer's discount rate is $r_B > 0$. For notational convenience, define $\rho = (1 + r_S)/(1 + r_B)$, and $\delta = \rho - 1$. Notice that $\delta > -1$. In order to prevent arbitrage, we assume $\beta_0 \geq E[s](1 - \phi)/(1 + r_S)$ and $\bar{s}(1-\phi) \leq \underline{s}+\underline{\theta}$. Finally, for simplicity, we also assume $p > \bar{s} + \bar{\theta}$, i.e., selling to the consumer market is profitable in any state realization.

2.2 The Optimal Contract Structure

In this chapter, we present the optimal design of the contract offered by the seller to the buyer for a given exercise price, w. For optimal options contract design, the seller considers the buyer's optimal actions throughout the time horizon, given any feasible contract that she offers. Thus, we start by analyzing the buyer's actions at time $t = 2$, for a given contracted quantity q. Building on this analysis, we then solve for the seller's optimal reservation price and quantity schedule problem.

2.2.1 The Buyer's Problem

The objective of the buyer is to maximize his expected profits. Given the contract's pricing schedule and exercise price, $(R(q), w)$, the buyer decides on the optimal capacity to reserve with the supplier, $q(R, w)$, at time $t = 1$. On the day of exercise, $t = 2$, the buyer either exercises his options or purchases from the spot market, whichever costs less. If there is a potential gain and the buyer has remaining options he does not use to satisfy consumer demand, he exercises all of his options to sell to the spot market. More specifically, at time $t = 2$, given the values $(R(q), w)$ and $q(R, w)$ (q in short), when the two uncertain states (s, D) are realized, the buyer optimally decides on three quantities:

(i) The number of options to exercise:

$$\begin{cases} q & \text{for } w < s(1-\phi); \\ \min(D, q) & \text{for } s(1-\phi) \leq w \leq s+\theta; \\ 0 & \text{otherwise.} \end{cases} \tag{2.1}$$

(ii) The quantity to purchase from the spot market:

$$\begin{cases} (D-q)^+ & \text{for } w \leq s+\theta; \\ D & \text{for } w > s+\theta. \end{cases} \tag{2.2}$$

(iii) The quantity to sell to the spot market:

$$\begin{cases} (q-D)^+ & \text{for } w < s(1-\phi); \\ 0 & \text{otherwise.} \end{cases} \tag{2.3}$$

We define the type θ buyer's expected discounted profit from purchasing q options as $\pi_B(q, w, \theta)$, where expectation is taken over demand and spot price; which is given by:

$$\begin{aligned} \pi_B(q, w, \theta) = -R(q) + \frac{1}{1+r_B}\{&(p - E_s[\min(w, s+\theta)])\, E_D[\min(D, q)] \\ &+ (p - E[s] - \theta)E_D[(D-q)^+] + E_s[(s(1-\phi) - w)^+]E_D[(q-D)^+]\}. \end{aligned} \tag{2.4}$$

In (2.4), the first expression accounts for the reservation cost of q units. The second accounts for the buyer's discounted profit from selling up to q units, where the buyer either exercises the options or buys from the spot market, whichever costs less. The third accounts for the buyer's discounted profit from selling above q units, where the only choice for the buyer is to purchase from the spot market. The last expression accounts for the buyer's discounted profit from selling to the spot market when there are extra units available and the exercise cost is less than the spot's bid-price.

2.2.2 Seller's Optimal Reservation Price Schedule

Given the optimal exercise and purchasing strategies of the buyer as discussed above, we define $\hat{\pi}_B(q, w, \theta)$ as the type θ buyer's expected discounted *net gains* from purchasing q options at exercise price w. By (2.4),

$$\hat{\pi}_B(q, w, \theta) = -R(q, w) + \varphi(q, w, \theta), \tag{2.5}$$

where

$$\varphi(q, w, \theta) = \frac{1}{1 + r_B}\bigg(\int_{w-\theta}^{\bar{s}} (s + \theta - w)\, dF_s(s) E_D[\min(D, q)] \\ + \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1 - \phi) - w)\, dF_s(s) E_D[(q - D)^+]\bigg). \tag{2.6}$$

More specifically, $\hat{\pi}_B(q, w, \theta)$ is the present value of the buyer's expected net benefit, acquired from purchasing q options, with the contract specifications posed by the seller, compared to purchasing no options. The first term in (2.5) is the reservation price $R(q)$ he pays to the seller at time $t = 1$. The second term accounts for: first, the buyer's discounted expected net gain from exercising a maximum of q units, when exercising options is less costly than procuring from the spot market. And secondly, as in (2.4), the term which accounts for the buyer's discounted expected profit from selling to the spot market when extra units of options are available after satisfying the consumer demand; and when the exercise price, w, is less than the bid price of the spot market $(s(1 - \phi))$.

For any given exercise price $w \geq 0$, the seller's problem is then to find the optimal reservation price schedule, $R(q(w,\theta),w)$ for maximizing her expected profits, $\pi_S(w)$, where $q(w,\theta)$ is the reservation quantity for a type θ buyer. We can write,

$$\pi_S(w) = E_\theta[R(q(w,\theta),w) + V(q(w,\theta),w,\theta)]\,, \tag{2.7}$$

where

$$V(q,w,\theta) = -\beta_0 q + \frac{1}{1+r_S}\Bigg\{\int_{\underline{s}}^{w-\theta} s(1-\phi)\,dF_s(s)q \\ + \int_{w-\theta}^{\frac{w}{1-\phi}} \left[wE_D[\min(D,q)] + s(1-\phi)E_D[(q-D)^+]\right]\,dF_s(s) + wq\bar{F}_s\left(\frac{w}{1-\phi}\right)\Bigg\}. \tag{2.8}$$

In (2.8), the first term represents the production costs. The second term represents the seller's discounted expected revenue when the spot price is sufficiently low that the buyer does not exercise any options, and the seller sells all she produces to the spot market. The third term in (2.8) is the seller's discounted expected revenue for the case where the buyer exercises some of the options, and the supplier sells the remaining to the spot market. Finally, the last term accounts for the seller's discounted expected revenues when the buyer exercises all options contracted.

The seller optimizes the contract schedule by taking into account the buyer's behavior in her own optimization problem. Specifically, the optimal contract has to be designed in such a way that the quantity purchased by a type θ buyer is indeed his optimal quantity given the contract terms. In addition, no buyer should have negative expected net gains upon contract agreement. Given these constriants, the seller's problem can be formulated as

$$\max_{R(\cdot,w)} \quad \int_{\underline{\theta}}^{\bar{\theta}} [R(q(w,\theta),w) + V(q(w,\theta),w,\theta)]\,dF_\theta(\theta) \tag{2.9}$$

$$\text{s.t.} \quad q(w,\theta) = \arg\max_{\xi\geq 0}\left[\hat{\pi}_B(\xi,w,\theta)\right], \forall\,\theta\,;$$
$$\hat{\pi}_B(q(w,\theta),w,\theta) \geq 0,\ \forall\,\theta\,.$$

In presenting the solution to (2.9), we start with identifying a set of conditions that determine whether the seller chooses a flat price schedule or a volume-dependent pricing scheme. The following lemma will be helpful in this characterization.

Lemma 2.1 *For* $w \in [0, \bar{s}(1-\phi)]$, *let* $G(w) \triangleq \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi) - w)\, dF_s(s)$. *If* $r_S > r_B$, *and* $\beta_0 \leq E[s](1-\phi)(1+r_B)^{-1}$, *then there exists a unique* $\tilde{w}_c \in [0, \bar{s}(1-\phi)]$ *such that* $G(\tilde{w}_c) = ((1+r_S)\beta_0 - E[s](1-\phi))\delta^{-1}$.

Proof of Lemma 2.1: First, notice that $\beta_0 \leq \frac{E[s](1-\phi)}{1+r_B}$ implies $(1+\delta)E[s](1-\phi) \geq (1+r_S)\beta_0$, which in turn implies $G(0) \geq ((1+r_S)\beta_0 - E[s](1-\phi))\delta^{-1}$, since $G(0) = E[s](1-\phi)$. Further, $G(w) = 0$ for all $w \geq \bar{s}(1-\phi)$. Combining this with the fact that G is strictly decreasing for $0 \leq w \leq \bar{s}(1-\phi)$ and $((1+r_S)\beta_0 - E[s](1-\phi))\delta^{-1} > 0$, the statement follows. ■

In certain cases, specifically when the number of options contracted exceeds consumer demand, and the option exercise price is lower than the bid price at the spot market, it is profitable for the buyer to exercise the remaining options to sell to the spot market. The expected per unit profit the buyer gets from such a transaction is $G(w)$ as defined in Lemma 2.1. Lemma 2.1 establishes certain mathematical properties of $G(w)$, and is critical for the conditions that yield to volume-dependent pricing in option contracts, which we give next.

Definition We say *the flat price conditions* are satisfied when *all* of the following three conditions are satisfied: (*i*) $r_S > r_B$; (ii) $\beta_0 \leq E[s](1-\phi)(1+r_B)^{-1}$; and (*iii*) $w \leq \tilde{w}_c$ where $\tilde{w}_c$ is as defined in Lemma 2.1.

The flat price conditions play a critical role in determining the nature of procurement contracts. They essentially require the supplier to be less patient than the buyer, and that the unit production cost and exercise price of the contract are sufficiently low. Specifically, note that for any $\tilde{w}_c \geq 0$, $G(\tilde{w}_c) = ((1+r_S)\beta_0 - E[s](1-\phi))\delta^{-1}$ is equivalent to the condition

$$\frac{G(\tilde{w}_c)}{1+r_B} = \beta - \frac{1}{1+r_S}\left(E[s](1-\phi) - G(\tilde{w}_c)\right). \tag{2.10}$$

The left hand side of (2.10) is the buyer's expected discounted benefit from having one remaining option, with an exercise price of $\tilde{w_c}$, in excess of the consumer demand, i.e., to sell to the spot market. The right hand side is the seller's *opportunity cost* of committing to one unit of option at exercise price $\tilde{w_c}$. Specifically, it is the cost of producing the unit, less the expected amount she can get from the spot market for that unit, adjusted for the expected exercise price, $\tilde{w_c}$, she might get from the buyer for that unit. Combining this intuition with the fact that G is decreasing in w, the flat price condition (iii) implies a positive gains from trade between the buyer and the seller for each remaining unit of option, in excess of demand, which is committed at exercise price w. Further, by Lemma 2.1, flat price conditions (i) and (ii) guarantee the existence of such $\tilde{w_c}$.

Given this intuition, we can now present the optimal contract offer by the seller for a fixed exercise price. The following proposition gives the result.

Proposition 2.1

(i) If the flat price conditions are satisfied, the optimal contracts are not volume-dependent. Rather, in the optimal offer, the reservation price is constant and given by $\varphi(K, w, \underline{\theta})$, and $q^*(w, \theta) = K$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

(ii) Suppose the flat price conditions are not satisfied. Then there exists $\bar{\delta} > 0$ such that, when $-1 < \delta < \bar{\delta}$, the optimal reservation price schedule for the seller is volume-dependent. Specifically the optimal quantity ordered for type θ buyer is

$$q^*(w, \theta) = \bar{F}_D^{-1}\left(\frac{(1 + r_S)\beta_0 - E[s](1 - \phi) - \delta G(w)}{\eta(w, \theta) - \delta G(w)}\right), \tag{2.11}$$

where

$$\eta(w, \theta) = \int_{w-\theta}^{\bar{s}} (\phi s + \theta - g_\theta(\theta))\, dF_s(s) + \delta \int_{w-\theta}^{\bar{s}} (s + \theta - g_\theta(\theta) - w)\, dF_s(s)\,, \tag{2.12}$$

and G is as defined in Lemma 2.1. The optimal total reservation fee paid by a *type θ buyer is*

$$R(q^*(w,\theta),w) = \left(\varphi(q^*(w,\theta),w,\theta) - \int_{\underline{\theta}}^{\theta} \left.\frac{\partial\varphi(q,w,a)}{\partial a}\right|_{q=q^*(w,a)} da\right), \quad (2.13)$$

where φ is as defined in (2.6). Further $q^*(w,\theta) < \bar{D}$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

Proof of Proposition 2.1: Let $q(w,\theta)$ be the type θ buyer's reservation quantity given exercise price w. Then, by (2.5), (2.6) and the envelope theorem, we have

$$\begin{aligned}
\left.\frac{d\hat{\pi}_B(q,w,\theta)}{d\theta}\right|_{q=q(w,\theta)} &= \left.\frac{\partial\hat{\pi}_B(q,w,\theta)}{\partial\theta}\right|_{q=q(w,\theta)} + \left.\frac{d\hat{\pi}_B(q,w,\theta)}{dq}\right|_{q=q(w,\theta)} \cdot \frac{dq(w,\theta)}{d\theta} \\
&= \left.\frac{\partial\hat{\pi}_B(q,w,\theta)}{\partial\theta}\right|_{q=q(w,\theta)} \\
&= \left.\frac{\partial\varphi(q,w,\theta)}{\partial\theta}\right|_{q=q(w,\theta)} \\
&= \frac{1}{1+r_B}\bar{F}_s(w-\theta)E_D[\min(D,q)] \geq 0\,. \qquad (2.14)
\end{aligned}$$

(2.14) implies that in any given feasible solution to (2.9), $\hat{\pi}_B(q(w,\theta),w,\theta)$ has to be increasing in θ. Therefore, the second set of conditions in (2.9) are satisfied if and only if $\hat{\pi}_B(q(w,\underline{\theta}),w,\underline{\theta}) \geq 0$. Further, in the optimal schedule, this constraint should be binding. Then by (2.5), (2.6), (2.14), and applying integration by parts, we have

$$\begin{aligned}
&\int_{\underline{\theta}}^{\bar{\theta}} R(q(w,\theta),w)\,dF_\theta(\theta) \\
=\ &\int_{\underline{\theta}}^{\bar{\theta}} \left(\varphi(q(w,\theta),w,\theta) - \hat{\pi}_B(q(w,\theta),w,\theta)\right)\,dF_\theta(\theta) \\
=\ &\int_{\underline{\theta}}^{\bar{\theta}} \varphi(q(w,\theta),w,\theta)\,dF_\theta(\theta) - \hat{\pi}_B(q(w,\underline{\theta}),w,\underline{\theta}) \\
&+\int_{\underline{\theta}}^{\bar{\theta}}\int_{\underline{\theta}}^{\theta} \left.\frac{\partial\varphi(q,w,a)}{\partial a}\right|_{q=q(w,a)} da\,dF_\theta(\theta) \\
=\ &\int_{\underline{\theta}}^{\bar{\theta}} \left(\varphi(q(w,\theta),w,\theta) - \frac{\bar{F}_\theta(\theta)}{f_\theta}\left.\frac{\partial\varphi(q,w,\theta)}{\partial\theta}\right|_{q=q(w,\theta)}\right) dF_\theta(\theta). \qquad (2.15)
\end{aligned}$$

Plugging (2.15) in the objective function in (2.9) we obtain

$$\pi_S(w) = \int_{\underline{\theta}}^{\bar{\theta}} \left(V(q(w,\theta),w,\theta) + \varphi(q(w,\theta),w,\theta) - g_\theta(\theta)\frac{\partial\varphi(q(w,\theta),w,\theta)}{\partial\theta} \right) dF_\theta(\theta), \tag{2.16}$$

where $g_\theta(\theta) = \bar{F}_\theta(\theta)/f_\theta(\theta)$. For notational convenience, define $H(q,w,\theta)$ as the integrand in (2.16). Notice that (2.16) can be optimized pointwise. By (2.6), for any constant θ, the first derivative is

$$\begin{aligned}\frac{dH(q(w,\theta),w,\theta)}{dq(w,\theta)} &= -(1+r_S)\beta_0 + E[s](1-\phi) + \delta \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi)-w)\,dF_s(s) \\ &\quad + \bar{F}_D(q(w,\theta))\Bigg\{ \int_{w-\theta}^{\bar{s}} (\phi s + \theta - g_\theta(\theta))\,dF_s(s) \\ &\quad + \delta\Big(\int_{w-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-w)\,dF_s(s) - \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi)-w)\,dF_s(s)\Big)\Bigg\}.\end{aligned} \tag{2.17}$$

Equating (2.17) to zero and solving for $q(w,\theta)$ yields

$$q^*(w,\theta) = \bar{F}_D^{-1}\left(\frac{(1+r_S)\beta_0 - E[s](1-\phi) - \delta G(w)}{\eta(w,\theta) - \delta G(w)}\right), \tag{2.18}$$

where

$$\eta(w,\theta) = \int_{w-\theta}^{\bar{s}} (\phi s + \theta - g_\theta(\theta))\,dF_s(s) + \delta \int_{w-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-w)\,dF_s(s), \tag{2.19}$$

and $G(w)$ is as defined in Lemma 2.1. From (2.17), the second derivative of H with respect to $q(w,\theta)$ is

$$\frac{d^2H(q(w,\theta),w,\theta)}{dq(w,\theta)^2} = -f_D(q(w,\theta))(\eta(w,\theta)-\delta G(w)). \tag{2.20}$$

Now, for notational purposes, we define

$$\begin{aligned}
\varrho(w,\theta) &\triangleq \eta(w,\theta) - \delta G(w) \\
&= \int_{w-\theta}^{\bar{s}} (\phi s + \theta - g_\theta(\theta))\, dF_s(s) \\
&\quad + \delta\left(\int_{w-\theta}^{\bar{s}} (s + \theta - g_\theta(\theta) - w)\, dF_s(s) - \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi) - w)\, dF_s(s) \right).
\end{aligned} \tag{2.21}$$

Taking the derivative of ϱ with respect to w,

$$\frac{d\varrho(w,\theta)}{dw} = \begin{cases} -\delta \bar{F}_s\left(\frac{w}{1-\phi}\right) & \text{for } w < \underline{s} + \theta, \\ -f_s(w-\theta)\nu(w,\theta) & \text{for } w \geq \underline{s} + \theta, \end{cases} \tag{2.22}$$

where

$$\nu(w,\theta) = \phi(w-\theta) + \theta - g_\theta(\theta) + \delta\left(g_s(w-\theta) - g_\theta(\theta)\right), \tag{2.23}$$

and $g_s(s) = \bar{F}_s(s)/f_s(s)$. Notice that

$$\begin{aligned}
\nu(\underline{s} + \underline{\theta}, \underline{\theta}) &= \phi\underline{s} + \underline{\theta} - \frac{1}{f_\theta(\underline{\theta})} + \delta\left(\frac{1}{f_s(\underline{s})} - \frac{1}{f_\theta(\underline{\theta})}\right) \\
&\geq \phi\underline{s} + \underline{\theta} - \frac{1}{f_s(\underline{s})} > 0,
\end{aligned} \tag{2.24}$$

since $\delta > -1$, and $(\phi\underline{s} + \underline{\theta})^{-1} < f_s(\underline{s}) \leq f_\theta(\underline{\theta})$. Now,

$$\frac{dg_s(s)}{ds} = \frac{d}{ds}\frac{\bar{F}_s(s)}{f_s(s)} = -1 - g_s(s)\frac{df_s(s)/ds}{f_s(s)}. \tag{2.25}$$

Since $g_s(s)$ is monotonically decreasing in s, finite at $\underline{s}$, and $d\log(f_s(s))/ds$ is bounded on $[\underline{s}, \bar{s}]$, we conclude that

$$\left|\frac{dg_s(s)}{ds}\right| < \infty, \text{ for all } \quad s \in [\underline{s}, \bar{s}]. \tag{2.26}$$

In addition, from (2.23), we also have

$$\frac{d\nu(w,\theta)}{dw} = \phi + \delta \left.\frac{dg_s(s)}{ds}\right|_{s=w-\theta}, \tag{2.27}$$

which is nonnegative for $-1 < \delta < \frac{\phi}{\sup_{s\in[\underline{s},\bar{s}]}\left|\frac{dg_s(s)}{ds}\right|} \triangleq \bar{\delta}$, since $g_s(s)$ is decreasing in s, and where $\bar{\delta} > 0$, by (2.26). Further

$$\frac{d\nu(w,\theta)}{d\theta} = (1-\phi) - (1+\delta)\frac{dg_\theta(\theta)}{d\theta} \geq 0\,, \tag{2.28}$$

since $g_\theta(\theta)$ is decreasing in θ. It follows that $\nu(w,\theta) \geq 0$ for all θ and $w \geq \underline{s} + \theta$. Combining this with (2.22), it follows that, for $\delta < \bar{\delta}$, $d\varrho(w,\theta)/dw < 0$ for $\underline{s} + \theta \leq w < \bar{s} + \theta$, and $d\varrho(w,\theta)/dw = 0$ for $w \geq \bar{s} + \theta$, for all $\theta \in [\underline{\theta}, \bar{\theta}]$. From (2.22), it also follows that for $0 \leq w < \underline{s} + \theta$, $d\varrho(w,\theta)/dw < 0$ if and only if $\delta < 0$. But

$$\varrho(0,\theta) = (1+\delta)\left(\phi E[s] + \theta - g_\theta(\theta)\right) > 0\,, \tag{2.29}$$

and

$$\varrho(w,\theta) = 0, \forall w \geq \bar{s} + \theta\,. \tag{2.30}$$

Therefore $\varrho(w,\theta) > 0$ for $0 \leq w < \bar{s} + \theta$. Combining this with (2.20) and (2.21), it follows that $H(q,w,\theta)$ is strictly concave for all q, which in turn implies that $q^*(w,\theta)$ as given in (2.11) is the pointwise optimum quantity for the seller to offer to type θ buyer. Now by (2.11), since $\varrho(w,\theta) \geq 0$, we have $q^*(w,\theta) = K$ if and only if $\delta G(w) \geq (1+r_S)\beta_0 - E[s](1-\phi)$. By Lemma 2.1, given $\delta > 0$ and $\beta_0 \leq \frac{E[s](1-\phi)}{1+r_B}$, there exists a w_c such that $\delta G(w_c) = (1+r_S)\beta_0 - E[s](1-\phi)$, and since G is strictly decreasing in w, for $0 \leq w \leq w_c$, $\delta G(w) \geq (1+r_S)\beta_0 - E[s](1-\phi)$. This means that, when $\delta > 0$, $\beta_0 \leq \frac{E[s](1-\phi)}{1+r_B}$, and $w \leq \tilde{w_c}$, $q^*(w,\theta) = K$ for all θ, i.e., the seller offers a constant contract. The price of the contract follows by plugging $q^*(w,\underline{\theta}) = K$ in $\hat{\pi}_B(q^*(w,\underline{\theta}), w, \underline{\theta}) = 0$, giving us $R(w) = \varphi(K, w, \underline{\theta})$ as stated. This proves part (i).

For part (ii), similar to above, $\varrho(w,\theta) \geq 0$ for any (w,θ). The optimal reservation price schedule is volume-dependent only when the seller is able to differentiate the buyer's type. By (2.11), $q^*(w,\theta)$ is dependent on θ if and only if $\eta(w) < (1+r_S)\beta_0 - E[s](1-\phi)$, which can only happen if any of the flat price conditions are violated. Further, $q^*(w,\theta) < \bar{D}$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. By (2.14), the optimal reservation price schedule is

$$\begin{aligned}
R(w,\theta) &\triangleq R(q^*(w,\theta),w) \\
&= \varphi(q^*(w,\theta),w,\theta) - \hat{\pi}_B(q^*(w,\theta),w,\theta) \\
&= \varphi(q^*(w,\theta),w,\theta) - \hat{\pi}_B(q^*(w,\underline{\theta}),w,\underline{\theta}) - \int_{\underline{\theta}}^{\theta} \left.\frac{d\hat{\pi}_B(q,w,a)}{da}\right|_{q=q^*(w,a)} da \\
&= \varphi(q^*(w,\theta),w,\theta) - \int_{\underline{\theta}}^{\theta} \left.\frac{\partial\varphi(q,w,a)}{\partial a}\right|_{q=q^*(w,a)} da\,. \qquad (2.31)
\end{aligned}$$

This completes the proof. ■

Proposition 2.1 presents the full solution for the optimal contract design for any given constant w. In addition, it separates the cases where it is optimal for the seller to offer a fixed bundle at a constant price, from where it is optimal for the seller to offer a volume incentive scheme, i.e. a different price for each different quantity contracted, where each different type buyer purchases a different quantity. As part (i) of the proposition states, when the seller has a higher discount rate, when the production costs are sufficiently low and when the option exercise price w is sufficiently low, the seller prefers to offer a constant price; the buyer, independent of his type, θ, chooses to purchase up to the seller's production capacity. However, when any of these three conditions are not satisfied, the seller finds it optimal to employ volume-dependent pricing.

Combining the definition of the flat-price conditions with the result of Proposition 2.1, we have the following corollary:

Corollary 2.1 *Higher r_S and $E[s]$ favor a flat reservation price contract. Higher r_B, w, β_0, and ϕ favor a volume-dependent reservation pricing scheme.*

The contract is an agreement between the buyer and the seller. As such, it reflects the incentives, preferences and alternative options of both parties. Although the seller determines the pricing of the goods she offers, she also considers the buyer's incentives. Corollary 2.1 reflects on this point. The corollary indicates that a higher seller discount rate or lower buyer discount rate moves the pricing regime from volume-dependent to flat-pricing. Given $r_S > r_B$, i.e., the seller has a higher discount rate than that of the buyer, she has a higher valuation of payments made in the first period than that of the second. Therefore, a higher r_S makes it easier for the seller to commit to a large bundle (of size K) up front. On the other hand, a higher r_B slows down the buyer's commitment; but the reduction in the number of options the buyer purchases also depends on his type, θ, and consequently, a volume incentive scheme becomes optimal for the supplier.

We can also see from Corollary 2.1 that a higher exercise price (w) can switch the pricing regime from flat-pricing to volume-dependent. This is because of the fundamental trade-off that the supplier faces when she prices the options contracts. Specifically, the seller strikes a balance between receiving payments today in the form of reservation fees ($R(q)$), and payments in the future in the form of exercise fees (w). These two factors (imperfectly) substitute for each other in determining the pricing schedule for the contracts. When w is large, the substantial part of the revenue for the supplier is in the second period. Given that the supplier is less patient than the buyer, she is less willing to commit to large quantities this period and hence switches to a sliding scale of commitment volume. A similar effect occurs when production cost (β_0) is large, a high production cost reduces the supplier's incentive to commit to large quantities and shifts the pricing regime to volume-dependent as stated. Finally, $E[s]$ and ϕ have a combined effect on the nature of the contract offer, as reflected in the expected resale price at the spot market, i.e., $E[s](1-\phi)$. When $E[s](1-\phi)$ is high, it is more desirable for both the buyer and the seller to commit to a larger amount. Thus, the supplier finds it optimal to offer a large number of contracts to all possible types at a constant reservation price. Thus, a high $E[s]$ or a low ϕ favor flat price contracts as stated in Corollary 2.1.

2.3 Volume Dependency

An important issue about the structure of a contract offer is the volume dependent nature of the reservation price schedule. A very common form of volume dependent pricing employed in practice in the industry is volume discounts, i.e., a reduction in average pricing with higher volume of purchases. Volume discounts imply a concave total price curve as a function of the quantity purchased. The opposite of volume discounts is volume premia, i.e., increasing average cost with quantity purchased. This is a reverse form of a volume incentive, which can be viewed as extra incentives given to the seller by the buyer to commit to high levels of production. Conversely, it can be the case that the seller takes advantage of the premium the buyer puts on the seller's product by charging more dependent types of buyers (higher θ types) higher average prices. Contrary to volume discounts, volume premia imply a convex total price curve.

Given the complexity of the transaction structures at $t = 2$, with options commitments and the resulting complexity of the two-sided determination of options contract terms, it is an open question to whether either or both types of volume dependent pricing structures can emerge in optimal contracts. The following proposition states the conditions under which the optimal contract offers volume discounts and/or volume premia to the buyer at a given part of the reservation price curve.

Proposition 2.2 *Suppose that the flat price conditions are not satisfied.*

(i) If $w \leq \underline{s}(1-\phi)$, the optimal contract offers quantity discounts for the entire reservation price curve, $R(q)$.

(ii) Suppose $w = \underline{s} + \tilde{\theta}$, where $\tilde{\theta} \in [\underline{\theta}, \bar{\theta}]$. Then for any $\theta \geq \tilde{\theta}$, $R(q)$ is locally concave at $q(\theta)$ if

$$(1+\delta) \geq \frac{\underline{s} + \tilde{\theta} - E[s](1-\phi)}{g_\theta(\theta) - \frac{dg_\theta(\theta)}{d\theta}(E[s] - \underline{s} + \theta - \tilde{\theta})}, \tag{2.32}$$

and locally convex at $q(\theta)$ otherwise.

Proof of Proposition 2.2: By Proposition 2.1, given that the flat price conditions are not satisfied, the optimal reservation fee, $R(w, \theta(q))$, will depend on θ. Using the implicit function theorem, we then have

$$\frac{dR(w,\theta(q))}{dq}=\frac{dR(w,\theta)}{d\theta}\left(\frac{dq^*(w,\theta)}{d\theta}\right)^{-1}, \tag{2.33}$$

and

$$\frac{d^2R(w,\theta(q))}{dq^2}=\left(\frac{d^2R(w,\theta)}{d\theta^2}\frac{dq^*(w,\theta)}{d\theta}-\frac{dR(w,\theta)}{d\theta}\frac{d^2q^*(w,\theta)}{d\theta^2}\right)\cdot\left(\frac{dq^*(w,\theta)}{d\theta}\right)^{-3}. \tag{2.34}$$

First, differentiating (2.11) with respect to θ, we have

$$-f_D(q^*(w,\theta))\frac{dq^*(w,\theta)}{d\theta}\varrho(w,\theta)+\bar{F}_D(q^*(w,\theta))\frac{d\varrho(w,\theta)}{d\theta}=0. \tag{2.35}$$

As we have shown in the proof of Proposition 2.1, $\varrho(w,\theta)\geq 0$, for all (w,θ). Now by (2.13),

$$\frac{dR(w,\theta)}{d\theta}=\left.\frac{d\varphi(q,w,\theta)}{dq}\right|_{q=q^*(w,\theta)}\frac{dq^*(w,\theta)}{d\theta}, \tag{2.36}$$

and

$$\begin{aligned}\frac{d^2R(w,\theta)}{d\theta^2}=&\left.\frac{d\varphi(q,w,\theta)}{dq}\right|_{q=q^*(w,\theta)}\cdot\frac{d^2q^*(w,\theta)}{d\theta^2}+\frac{\partial}{\partial\theta}\left.\frac{d\varphi(q,w,\theta)}{dq}\right|_{q=q^*(w,\theta)}\frac{dq^*(w,\theta)}{d\theta}\\&+\left.\frac{d^2\varphi(q,w,\theta)}{dq^2}\right|_{q=q^*(w,\theta)}\left(\frac{dq^*(w,\theta)}{d\theta}\right)^2.\end{aligned} \tag{2.37}$$

Using (2.35), and plugging (2.36) and (2.37) in (2.34), we obtain

$$\begin{aligned}\frac{d^2R(w,\theta(q))}{dq^2}&=\frac{d^2\varphi(q^*(w,\theta),w,\theta)}{dq^*(w,\theta)^2}+\frac{\partial}{\partial\theta}\frac{d\varphi(q^*(w,\theta),w,\theta)}{q^*(w,\theta)}\left[\frac{dq^*(w,\theta)}{d\theta}\right]^{-1}\\&=-\frac{f_D(q^*(w,\theta))}{(1+r_B)}\left(\frac{\varrho(w,\theta)}{d\varrho(w,\theta)/d\theta}\bar{F}_s(w-\theta)\right.\\&\qquad\left.-E_s[(s+\theta-w)^+]+E_s[(s(1-\phi)-w)^+]\right)\end{aligned} \tag{2.38}$$

Now suppose $w\leq \underline{s}(1-\phi)$. Then by (2.21),

$$\varrho(w,\theta)=\rho\left(\phi E[S]+\theta-g_\theta(\theta)\right), \tag{2.39}$$

and

$$\frac{d\varrho(w,\theta)}{d\theta} = (1+\delta)\left(1 - \frac{dg_\theta(\theta)}{d\theta}\right) > 0. \tag{2.40}$$

Plugging (2.39) and (2.40) in (2.38), it then follows that

$$\frac{d^2R(w,\theta(q))}{dq^2} = -\frac{f_D(q^*(w,\theta))\,(g_\theta(\theta) - g_\theta'(\theta)(\phi E[s]+\theta))}{(1+r_B)\,(1-g_\theta'(\theta))} \leq 0\,, \tag{2.41}$$

which proves part (i). Now, consider any $w = \underline{s} + \tilde{\theta}$, where $\tilde{\theta} \in [\underline{\theta}, \bar{\theta}]$, and for any $\theta \geq \tilde{\theta}$, using the condition $\bar{s}(1-\phi) \leq \underline{s} + \underline{\theta}$:

$$\varrho(w,\theta) = (1+\delta)\,(\phi E[s] + \theta - g_\theta(\theta)) - \delta\left(\underline{s} + \tilde{\theta} - E[s](1-\phi)\right) \geq 0, \tag{2.42}$$

and $d\varrho(w,\theta)/d\theta$ again satisfies (2.40). Plugging (2.42) and (2.40) in (2.38), we then have

$$\begin{aligned}\frac{d^2R(w,\theta(q))}{dq^2} &= \frac{f_D(q^*(w,\theta))\varrho(w,\theta)}{(1+r_B)\frac{d\varrho(w,\theta)}{d\theta}} \\ &\cdot \left(1 - \frac{(1+\delta)(1-\frac{dg_\theta(\theta)}{d\theta})[E[s]+\theta-\underline{s}-\tilde{\theta}]}{(1+\delta)(\phi E[s]+\theta-g_\theta(\theta)) - \delta(\underline{s}+\tilde{\theta}-E[s](1-\phi))}\right).\end{aligned} \tag{2.43}$$

Using (2.40) and (2.42), and simplifying (2.43), $d^2R(w,\theta(q))/dq^2 \leq 0$ if and only if

$$(1+\delta) \geq \frac{\underline{s} + \tilde{\theta} - E[s](1-\phi)}{g_\theta(\theta) - \frac{dg_\theta(\theta)}{d\theta}(E[s] - \underline{s} + \theta - \tilde{\theta})}. \tag{2.44}$$

This completes the proof. ■

Part (i) of Proposition 2.2 states that for low w values, the optimal contract *only* takes the shape of volume discounts. An important implication of this is for the case of $w = 0$, i.e., when the contract is a true "sales" contract rather than an "option" contract. The following corollary states the result.

Corollary 2.2 *When the contract terms are such that the supplier is selling the intermediate good to the buyer (i.e., $w = 0$), the volume-dependent price schedule, if employed by the seller, always involves volume discounts.*

This result is consistent with the wide-spread use of volume discount schemes in the industry, especially when the contract between the buyer and the seller is a sales contract. Compared to options contracts, sales contracts are relatively simpler since they do not offer flexibility to the buyer. The buyer commits to receiving all units at the time of the delivery and pays an upfront fee for it. Corollary 2.2 states that as the level of such fixed commitments increases with the quantity sold, the seller finds it more profitable to offer incentives to buyer to purchase more. These incentives strengthen the buyer's willingness to commit to larger quantities, and thus increasing supplier's profits.

On the other hand, option contracts are more complex agreements, especially in terms of the possibilities they imply on the behavior of the parties involved. As a consequence, their volume dependent pricing schedule is more complicated. As part (*ii*) of Proposition 2.2 indicates, volume discounts can exist in the optimal contract for at least part of the offer curve for large option exercise price w values as well. However, and remarkably, part (*ii*) of Proposition 2.2 also states that, unlike the sale reservation price curves, option reservation price curves can exhibit volume premia instead of volume discounts. This means that, the pricing of options shows significant differences in nature compared to the pricing of regular sales contracts. Further, it is possible that the same contract offer can involve volume discounts and volume premia at different parts of the pricing curve. Given this, when and at what ranges will the reservation price schedules in optimal options contracts show concave and convex pricing characteristics? The following corollary sheds some light on this question.

Corollary 2.3 *Suppose that the flat price conditions are not satisfied, and* $w = \underline{s}+\tilde{\theta}$*, where* $\tilde{\theta} \in [\underline{\theta}, \bar{\theta}]$*. Given (i)* $d^2 g_\theta(\theta)/d\theta^2 \geq 0$*, and (ii)* $\underline{s}+\tilde{\theta} \leq \left(E[s](1-\phi) + E[s] + \bar{\theta}\right)/2$*, there exists* $0 < \underline{\rho} \leq \bar{\rho} \leq 1$ *such that the optimal contract has quantity discounts for all* $q \geq q^*(w, \tilde{\theta})$ *if* $\rho \geq \bar{\rho}$*; and quantity premia for all* $q \geq q^*(w, \tilde{\theta})$ *if* $\rho < \underline{\rho}$.

Proof of Corollary 2.3: Looking at the denominator of the right hand side expression of (2.43), it is easy to show that if $\frac{d^2 g_\theta}{d\theta^2} \geq 0$, the whole right hand side expression is non-decreasing in θ. Furthermore, $\underline{s}+\tilde{\theta} \leq \left(E[s](1-\phi) + E[s] + \bar{\theta}\right)/2$ implies there exists $\bar{\rho}$ such that:

$$\bar{\rho} = \frac{\underline{s} + \tilde{\theta} - E[s](1-\phi)}{E[s] - \underline{s} + \bar{\theta} - \tilde{\theta}} \leq 1\,, \tag{2.45}$$

and also

$$\frac{\underline{s} + \tilde{\theta} - E[s](1-\phi)}{g_\theta(\theta) - \frac{dg_\theta(\theta)}{d\theta}(E[s] - \underline{s} + \theta - \tilde{\theta})} \leq \bar{\rho} \leq 1 \tag{2.46}$$

by using the fact that $g_\theta(\bar{\theta}) = 0$ and $dg_\theta(\bar{\theta})/d\theta = -1$. Also, there exists $\underline{\rho} > 0$ such that:

$$\underline{\rho} = \frac{\underline{s} + \tilde{\theta} - E[s](1-\phi)}{g_\theta(\tilde{\theta}) - \left.\frac{dg_\theta(\theta)}{d\theta}\right|_{\theta=\tilde{\theta}}(E[s] - \underline{s})} \leq \frac{\underline{s} + \tilde{\theta} - E[s](1-\phi)}{g_\theta(\theta) - \frac{dg_\theta(\theta)}{d\theta}(E[s] - \underline{s} + \theta - \tilde{\theta})} \leq \bar{\rho} \leq 1 \tag{2.47}$$

Using (2.35), (2.40), and (2.42), we also know that $dq^*(w,\theta)/d\theta \geq 0$ for all $\theta \geq \tilde{\theta}$. Hence, by (2.32), if $\rho \geq \bar{\rho}$ then $R(w, \theta(q))$ is concave in q for all $q \geq q^*(w, \tilde{\theta})$; and if $\rho < \underline{\rho}$ then $R(w, \theta(q))$ is convex in q for all $q \geq q^*(w, \tilde{\theta})$. ■

Corollary 2.3 states that both quantity discounts and quantity premia can easily be observed in large segments of the pricing curve. Condition (i) stated in the corollary is a relatively weak condition, which is satisfied by many common distributions such as exponential and uniform. Condition (ii) is also easily satisfied for many cases, in particular when the spot price distribution is sufficiently dispersed. Given these two conditions are satisfied, the high end of the pricing curve will demonstrate either volume discounts or volume penalties in its entirety. The former occurs for relatively high levels of supplier discount rates and the latter is optimal for relatively low levels of it.

2.4 Effects of Execution Price on Optimal Contracts

Another interesting issue of the problem is the effect of the exercise price, w, on the expected contracted quantity. As exercise price increases, one expects that options become less attractive to the buyer, and hence the demand for the contracts decreases. However, in the supplier's optimal design of contract offers, there is a counterbalancing factor to the effect of increasing exercise price. Given that the supplier adjusts her reservation price by taking the magnitude of w into account, reservation prices, $R(q)$, decrease with increasing exercise price, making the contracts better priced for the buyer. As a result, the buyer's demand can go up or down with increased w, and the final contract quantity depends on both the supplier and the buyer's relative valuations of the contracts. The following proposition presents the effect of the exercise price on expected contract quantity.

Proposition 2.3

(i) If $r_S \geq r_B$, the expected contracted quantity ($E[q^*(w,\theta)]$) decreases in the exercise price w.

(ii) If $r_S < r_B$, $E[q^*(w,\theta)]$ increases in w for $w < \underline{s} + \underline{\theta}$, and decreases in w for $w \geq \underline{s} + \bar{\theta}$. For $\underline{s} + \underline{\theta} < w \leq \underline{s} + \bar{\theta}$, $E[q^*(w,\theta)]$ can be increasing or decreasing in w.

Proof of Proposition 2.3: First, applying $\bar{F}_D$ to both sides of (2.11) and subsequently taking the total derivatives with respect to w, we have

$$-f_D(q^*(w,\theta))\frac{dq^*(w,\theta)}{dw}\varrho(w,\theta) + \bar{F}_D(q^*(w,\theta))\frac{d\varrho(w,\theta)}{dw} = \delta\bar{F}_s\left(\frac{w}{1-\phi}\right). \qquad (2.48)$$

As we have shown in the proof of Proposition 2.1, for all $w \geq \underline{s} + \theta$, $\frac{d\varrho(w,\theta)}{dw} \leq 0$. Further, when $w \geq \underline{s} + \theta$, we have $w \geq \bar{s}(1-\phi)$, which implies $\bar{F}_s\left(\frac{w}{1-\phi}\right) = 0$. As a consequence, by (2.48), $\frac{dq^*(w,\theta)}{dw} \leq 0, \forall w \geq \underline{s} + \theta$, for all θ.

Now, when $r_S \geq r_B$, we have $\delta > 0$, and thus by (2.22), for $0 \leq w \leq \underline{s} + \theta$, $\frac{d\varrho(w,\theta)}{dw} \leq 0$ for all θ. Further the right hand side of (2.48) is positive. It then follows from (2.48) that $\frac{dq^*(w,\theta)}{dw} \leq 0$. Therefore it follows that, when $r_S \geq r_B$, for all $w \geq 0$,

$$\frac{dE_\theta[q^*(w,\theta)]}{dw} = \int_{\underline{\theta}}^{\bar{\theta}} \frac{dq^*(w,\theta)}{dw}\, dF_\theta(\theta) \leq 0. \qquad (2.49)$$

This proves part (i). For part (ii), when $r_S < r_B$ and $w < \underline{s}+\theta$, by (2.22), $\frac{d\varrho(w,\theta)}{dw} \geq 0$. Plugging this in (2.48) and noticing that the right hand side is negative, it follows

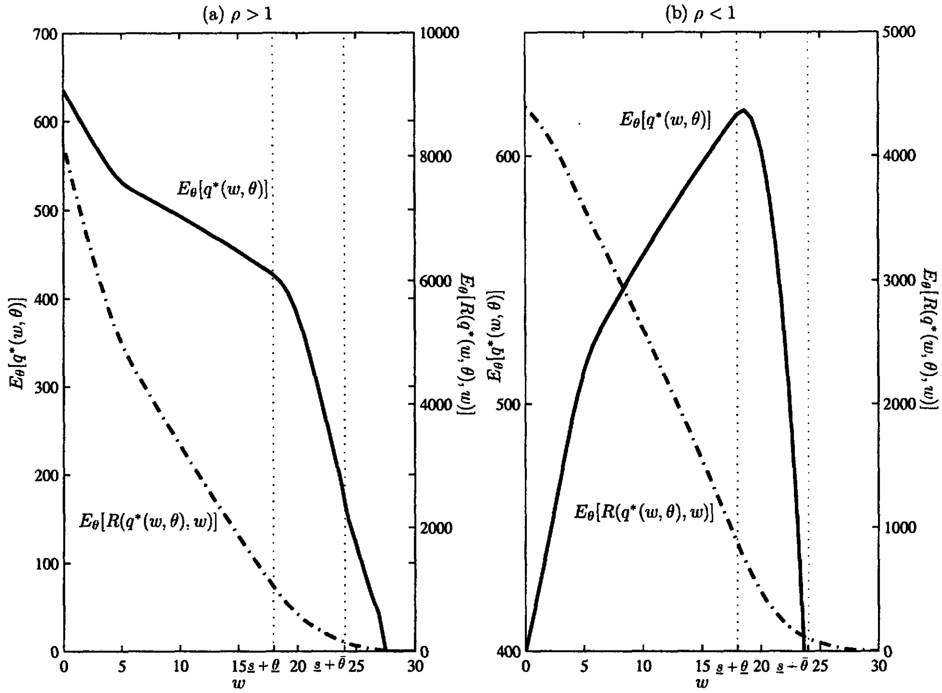

Figure 2-1: The effect of the exercise price (w) on the expected contracted quantity ($E[q^*(w,\theta)]$), and the expected reservation fee collected by the supplier ($R(q^*(w,\theta),w)$). For panel (a) $\rho = 15/11$, and for panel (b) $\rho = 11/15$. For both panels, s, θ and D have truncated normal distributions on $[10,18]$, $[8,14]$, and $[0,1200]$, means 14, 11 and 600, and standard deviations 5, 4 and 100, respectively. The remaining parameters for both panels are $p = 40$, $\beta = 8$, and $\phi = 0.65$.

that $\frac{dq^*(w,\theta)}{dw} \geq 0$ for $w < \underline{s}+\theta$ for all t. It then follows that for $w < \underline{s}+\underline{\theta}$, $\frac{dE_\theta[q^*(w,\theta)]}{dw} = \int_{\underline{\theta}}^{\bar{\theta}} \frac{dq^*(w,\theta)}{dw}\, dF_\theta(\theta) \geq 0$. On the other hand, as we established above that, for any given θ, when $w > \underline{s}+\theta$, $\frac{dq^*(w,\theta)}{dw} \leq 0$, it follows that when $w > \underline{s}+\bar{\theta}$, $\frac{dE_\theta[q^*(w,\theta)]}{dw} \leq 0$. This completes the proof. ■

Part (i) of Proposition 2.3 states that when the seller has a higher discount rate than the buyer and when the exercise price increases, in the optimal contracts, she does not decrease the reservation price substantially enough to generate an increased buyer's demand for the options. As a consequence, the expected total number of option contracts decreases with increased exercise price, as can also be seen in panel (a) of Figure 2-1. However, when the seller is more patient than the buyer, for low w values, she may find it optimal to decrease the reservation prices significantly in the optimal contracts to increase the quantity contracted. But even then, beyond a certain threshold w value, as w increases, the value of the options diminish sharply, making it not worthwhile for the seller to reduce the reservation price to keep the contracted quantity high, as also stated in part (ii) of Proposition 2.3. Consequently, the number of options sold can be maximized at an intermediate w level, as can also be seen in panel (b) of Figure 2-1. However, as can further be seen in the figure, the reservation price decreases with increased exercise price, even though the number of contracts signed increases sharply as demonstrated in panel (b). We will further use the results of Proposition 2.3 in Chapter 3.

Chapter 3

Optimal Contract Characteristics

Having found the optimal contract structure for a fixed exercise price w in Section 2.2, we now examine the determination of optimal exercise price by the seller. The answer to this question is important for understanding the full nature of the contracts offered by the seller to the buyer. A high exercise price contract shifts the supplier's collected fees to the future. Therefore, depending on the buyer's and the seller's specific characteristics, differences in the exercise price have important effects on contract design and the generation of supply chain surplus.

More specifically and importantly, the optimal exercise price further determines the structure of the contracts. If the supplier's optimal exercise price, w^*, is zero, then the contracts are traditional sales ones; as the buyer pays upfront for automatic future delivery. In contrast, when $w^* > 0$, the contracts are true options, since for that case, in certain state realizations (e.g., when the consumer demand is low or the spot price is low), the buyer chooses not to exercise the contracts. Thus, an important question to ask is: when are sales contracts are optimal and when are option contracts optimal? Or even further, given option contracts are optimal, how is the optimal exercise price, w^*, characterized?

Finally, it is important to characterize the effects of the industry and market characteristics on optimal contract design. The supplier's optimization of the exercise price yields endogenously determined answers to all these problems. In this chapter, we study the supplier's problem of exercise price optimization in detail, provide a full characterization of the solution, as well as a thorough analysis of optimal contract design characteristics.

3.1 Determination of Optimal Exercise Price

When to Sell and When to Offer Options?

By using the definition in (2.7), and the result of Proposition 2.1, the supplier's global optimization problem can now be written as

$$\max_{w \geq 0} \pi_S(w) \equiv \max_{w \geq 0} E_\theta[R(q^*(w,\theta),w) + V(q^*(w,\theta),w,\theta)]\,. \tag{3.1}$$

An important characteristic to notice from (3.1) is that, when optimizing w, the supplier needs to consider the trade-off between collecting revenues now (in the form of reservation fees, $R(q^*(w,\theta),w)$) and collecting revenues in the future (in the form of exercise fees as they affect $V(q^*(w,\theta),w,\theta)$, as defined in (2.8)). Therefore, one important factor that affects the supplier's decision will be her discount rate. However, the supplier is interacting with the buyer in signing the contracts, and hence she has to take into account the buyer's preferences when determining the optimal exercise price. The final outcome will reflect a combination of the preferences of both parties. The following proposition presents this outcome.

Proposition 3.1

(i) If $r_S > r_B$, $w^* = 0$. That is, selling the intermediate good is optimal for the supplier, rather than giving options to the buyer.

(ii) If $r_S = r_B$, the seller's profit is maximized by setting any exercise price w^*, where $0 \leq w^* \leq \underline{s} + \underline{\theta}$.

(iii) If $r_S < r_B$, there exists a strictly positive exercise price, w^*, where the seller's profit is optimized. That is, the seller strictly prefers giving options to selling the intermediate good. Further, $\underline{s} + \underline{\theta} \leq w^* \leq \underline{s} + \bar{\theta}$.

Proof of Proposition 3.1: First, substituting $q^*(w,\theta)$ in the seller's expected profit function, we have

$$\pi_S^*(w) = \int_{\underline{\theta}}^{\bar{\theta}} H(q^*(w,\theta), w, \theta)\, dF_\theta(\theta), \tag{3.2}$$

where $H(q,w,\theta)$ is the integrand in (2.16). Taking the total derivative with respect to w in (3.2), and applying the envelope theorem, we have

$$\begin{aligned}
\frac{d\pi_S^*(w)}{dw} &= \int_{\underline{\theta}}^{\bar{\theta}} \left[\frac{\partial H(q^*(w,\theta),w,\theta)}{\partial w} + \left.\frac{\partial H(q^*(w,\theta),w,\theta)}{\partial q}\right|_{q=q^*(w,\theta)} \cdot \frac{dq^*(w,\theta)}{dw} \right] dF_\theta(\theta) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial H(q^*(w,\theta),w,\theta)}{\partial w}\, dF_\theta(\theta) \\
&= \int_{\underline{\theta}}^{\bar{\theta}} \Bigg\{ -\delta \bar{F}_s\left(\frac{w}{1-\phi}\right) q^*(w,\theta) \\
&\qquad -\delta \left(F_s\left(\frac{w}{1-\phi}\right) - F_s(w-\theta) \right) E_D[\min(D, q^*(w,\theta))] \\
&\qquad +\delta g_\theta(\theta) f_s(w-\theta) E_D[\min(D, q^*(w,\theta))] \\
&\qquad -f_s(w-\theta)\left(\phi(w-\theta) + \theta - g_\theta(\theta)\right) E_D[\min(D, q^*(w,\theta))] \Bigg\}\, dF_\theta(\theta).
\end{aligned} \tag{3.3}$$

Since $\bar{s}(1-\phi) \le \underline{s} + \underline{\theta}$, and by (2.23), for any given $\theta \in [\underline{\theta}, \bar{\theta}]$,

$$\begin{aligned}
&\frac{\partial H(q^*(w,\theta),w,\theta)}{\partial w} \\
= &\begin{cases} -\delta \left[q^*(w,\theta) - F_s\left(\frac{w}{1-\phi}\right) E_D[(q^*(w,\theta)-D)^+] \right] & \text{for } w < \underline{s} + \theta; \\ -f_s(w-\theta)\nu(w,\theta) E_D[\min(D, q^*(w,\theta))], & \text{for } w \in [\underline{s}+\theta, \bar{s}+\theta]; \\ 0 & \text{for } w > \bar{s} + \theta\,. \end{cases}
\end{aligned} \tag{3.4}$$

where $\nu(w,\theta$ is given by (2.23). Plugging (3.4) in (3.3), we then have

$$\frac{d\pi_S^*(w)}{dw} = -\delta \int_{\underline{\theta}}^{\bar{\theta}} \left(q^*(w,\theta) - F_s\left(\frac{w}{1-\phi}\right) E_D[(q^*(w,\theta)-D)^+] \right) dF_\theta(\theta)\,, \tag{3.5}$$

for $w < \underline{s} + \underline{\theta}$, and

$$\frac{d\pi_S^*(w)}{dw} = -\int_{\underline{\theta}}^{\tilde{\theta}} f_s(w-\theta)\nu(w,\theta)E_D[\min(D, q^*(w,\theta))]\, dF_\theta(\theta) \\ - \delta \int_{\tilde{\theta}}^{\bar{\theta}} E_D[\min(D, q^*(w,\theta))]\, dF_\theta(\theta). \quad (3.6)$$

for any $w = \underline{s} + \tilde{\theta}$, where $\tilde{\theta} \in [\underline{\theta}, \bar{\theta}]$. Further, as shown in the proof of Proposition 2.1 that $\nu(w,\theta) \geq 0, \forall w \geq \underline{s} + \theta$; and by (3.4), $d\pi_S^*(w)/dw \leq 0$ for all $w \geq \underline{s} + \bar{\theta}$. Now, when $\delta > 0$, since $F_s(\frac{w}{1-\phi})E_D[(q^*(w,\theta) - D)^+] < q^*(w,\theta)$ for all θ, the integrand in (3.5) is positive for all θ. As a consequence, $d\pi_S^*(w)/dw < 0$ for $0 \leq w < \underline{s} + \underline{\theta}$. Similarly, since both terms in (3.6) are negative, $d\pi_S^*(w)/dw < 0$ also follows for $\underline{s} + \underline{\theta} \leq w \leq \bar{s} + \bar{\theta}$. Therefore, for $\delta > 0$, $\pi_S^*(w)$ is maximized at $w^* = 0$, which proves part (i). When $\delta = 0$, by (3.5), $d\pi_S^*(w)/dw = 0$ for $0 \leq w < \underline{s} + \underline{\theta}$, and by (3.6), $d\pi_S^*(w)/dw < 0$ for $\underline{s} + \underline{\theta} \leq w \leq \bar{s} + \bar{\theta}$. Hence $\pi_S^*(w)$ is flat on $w \in [0, \underline{s} + \underline{\theta}]$, strictly decreasing on $[\underline{s} + \underline{\theta}, \bar{s} + \bar{\theta}]$, and flat again for $w > \bar{s} + \bar{\theta}$, and thus is maximized for any $w \in [0, \underline{s} + \underline{\theta}]$, as stated in part (ii). Finally, when $\delta < 0$, by (3.5), $d\pi_S^*(w)/dw > 0$ for $0 \leq w < \underline{s} + \underline{\theta}$. Further, by (3.6), $d\pi_S^*(w)/dw > 0$ for $w = \underline{s} + \underline{\theta}$, $d\pi_S^*(w)/dw < 0$ for $w = \underline{s} + \bar{\theta}$, and $\pi_S^*(w)$ is non-increasing afterwards. Since $\pi_S^*(w)$ is continuous in w, it follows that for $\delta < 0$, $\pi_S^*(w)$ is maximized at a $w^* \in [\underline{s} + \underline{\theta}, \bar{s} + \bar{\theta}]$ as stated in part (iii). This completes the proof. ■

Proposition 3.1 states that the most important determinant for the nature of optimal contract structure, i.e. the sales versus options decision, is the relative magnitudes of the seller's and the buyer's discount rates. Part (i) of Proposition 3.1 states that when the seller's discount rate is higher than that of the buyer's, the supplier's optimal action is offering the buyer sales contracts. Given the seller is less patient than the buyer, it becomes an efficient revenue sharing arrangement to front-load the payments from the buyer to the seller by setting the exercise price to zero and maximizing the reservation fee. In this case, as can also be seen in panel (a) of Figure 3-1, the seller's expected profit is monotonically decreasing in w. That is, the discounted value of the revenue increase from increased exercise price cannot compensate for the

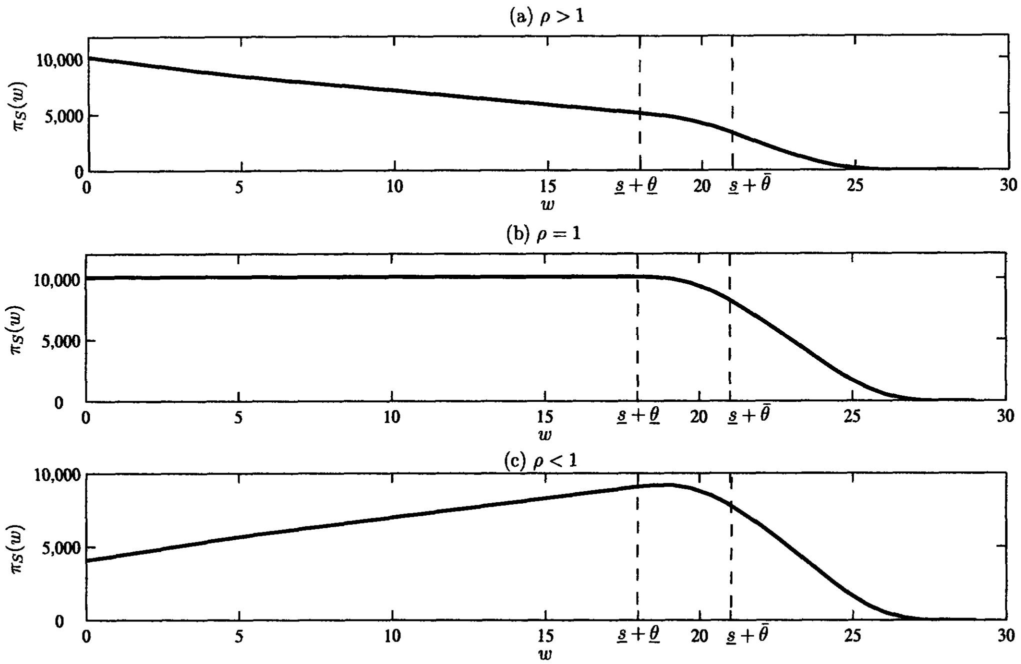

Figure 3-1: The supplier's expected profit (π_S) curves as a function of the exercise price, w for the three main regions stated in Proposition 3.1. In panel (a), $\rho = 1.5$, in panel (b) $\rho = 1$, and in panel (c) $\rho = 2/3$. In all panels, s and θ have truncated normal distributions on $[10, 18]$ and $[8, 11]$, means 14 and 9.5, and standard deviations 5 and 1.5, respectively. The demand is distributed as an increasing triangular distribution on $[0, 1200]$. The remaining parameters f or all panels are $p = 30$, $\beta = 8$, and $\phi = 0.65$.

loss the seller could originally get from the highest expected reservation price in the corresponding optimal contracts, as described in Proposition 2.1.

For the knife-edge case when the seller and the buyer have the same discount rate, there is a continuum of optimal exercise prices for the supplier as stated in part (*ii*) of Proposition 3.1. This is because the supplier now compensates exactly for the expected reduction in the optimal reservation fee by increasing the exercise price she gets from the buyer, i.e. the present value of the expected revenue increase in time $t = 2$ from execution, is exactly the expected minimum reduction in reservation fees in time $t = 1$, which is the amount the buyer requires from the seller to pay back in return for the reduction in option value for the buyer with increasing exercise price. Thus, in the optimal contracts, increasing w has the effect of transferring revenues from the present to the future present at the same rate, and hence the supplier's profit is constant in w over the range specified in Proposition 3.1, as can be seen from

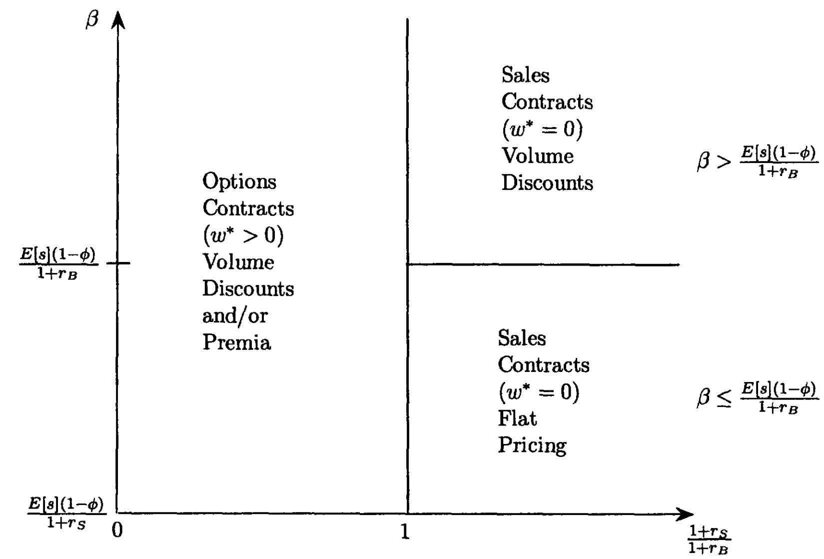

Figure 3-2: The partition of the parameter space according to full characterization of contract structures. The horizontal axis specifies ρ $((1 + r_S)/(1 + r_B))$. The vertical axis specifies β $(\geq E[s](1 - \phi)/(1 + r_S))$.

panel (b) of Figure 3-1.

However, when $r_B > r_S$, the seller's profit is non-monotonic in w. If the buyer is less patient than the seller, then in the optimal reservation price schedule, a relatively steep increase in the exercise price is accompanied by only a small decrease in the reservation price. Thus for low w values, the seller can keep the present value of the difference in total revenues with increased exercise price as net positive gains in her profit, i.e. the seller's profit levels increases with increased exercise price when w is low, as part (iii) of Proposition 3.1 indicates. However, as w increases, the value of the options start decreasing rapidly for the buyer. In this case, the supplier has to offer large discounts in reservation fees to the buyer in order to sell the options. Consequently, beyond a certain point, increased exercise price decreases the supplier's overall profit. The resulting profit curve is non-monotonic as depicted in panel (c) of Figure 3-1, and is maximized at a strictly positive w level in the interval $(\underline{s}+\underline{\theta},\ \underline{s}+\bar{\theta})$ as stated in part (iii) of Proposition 3.1.

Proposition 3.1 presents an important result by laying out the general structure of optimal procurement contracts. It states that when the seller has a higher discount rate, the optimal contracts are *sales contracts.* On the other hand, when the buyer has a higher discount rate the optimal procurement contracts are *options contracts.* Combining this with the result of Propositions 2.1 and 2.2, we obtain a full picture of contract design for our model, which is summarized in Figure 3-2. If the seller has a higher discount rate than the buyer and production costs are sufficiently low, the optimal contract is a flat fee sales contract; if the production costs are higher, the optimal contract is a sales contract with volume discounts. On the other hand, if the buyer has a higher discount rate, then the optimal contract is a volume-dependent options contract. Further, since by Proposition 3.1, the optimal exercise price is between $\underline{s} + \underline{\theta}$ and $\underline{s} + \bar{\theta}$, by part (*ii*) of Proposition 2.2, the reservation price curve can have concave and convex parts, i.e., the optimal contract offer can have volume discounts on the options as well as volume premia or even both.

3.2 Effects of Parameters on Contract Design

We next examine the effects of buyer, seller and market characteristics (such as production costs, spot price distribution, bid-ask spread in the spot market, and information asymmetry between the buyer and the seller about buyer's production flexibility) on the optimal contract design, including exercise price, reservation price schedule and expected contracted quantity.

We start with the effects on exercise price. The following proposition states the result.

Proposition 3.2 *If $r_B > r_S$, then there exists $\bar{\delta} > 0$ such that when $-\bar{\delta} < \delta < 0$, the optimal exercise price, w^*, increases with $E[s]$[1], and $E[\theta]$; and decreases with δ, $Var[s]$[2], and $Var[\theta]$.*

[1]Increase in mean of spot distribution here refers to a shift of the spot distribution to the right by the same amount

[2]For expositional purposes, for the rest of the paper, when we indicate an increase in variance of a distribution, we refer to an increase in variance that reduce the infimum of the support of that distribution.

Proof of Proposition 3.2: First, given that $r_S \leq r_B$, by Proposition 3.1, we know that $\underline{s} + \underline{\theta} \leq w^* \leq \underline{s} + \bar{\theta}$. Defining $\tilde{\theta}(w) \triangleq w - \underline{s}$, we then have $\tilde{\theta}(w^*) \in [\underline{\theta}, \bar{\theta}]$. Thus, at optimality and by (3.6)

$$\begin{aligned} \left.\frac{d\pi_S^*(w)}{dw}\right|_{w=w^*} &= -\int_{\underline{\theta}}^{\tilde{\theta}(w^*)} f_s(w-\theta)\nu(w,\theta)E_D[\min(D, q^*(w,\theta))]\, dF_\theta(\theta) \\ &\quad -\delta \int_{\tilde{\theta}(w^*)}^{\bar{\theta}} E_D[\min(D, q^*(w,\theta))]\, dF_\theta(\theta) \\ &= 0\,, \end{aligned} \tag{3.7}$$

where $\nu(w,\theta)$ is defined in (2.23), and $q^*(w,\theta)$ is defined in (2.11). Define $\hat{q}(\theta)$ as the optimal reservation level for a type θ buyer when exercise cost is $\underline{s} + \underline{\theta}$. By (2.11), we have

$$\hat{q}(\theta) = q^*(w,\theta)\big|_{w=\underline{s}+\underline{\theta}} = \bar{F}_D^{-1}\left(\frac{(1+r_S)\beta_0 - E[s](1-\phi)}{\phi E[s] + \theta - g_\theta(\theta)}\right). \tag{3.8}$$

By applying the implicit function theorem to (3.7), combining with (3.8) and simplifying, it then follows that, for $\delta < 0$,

$$w^* = \underline{s} + \underline{\theta} - \delta \frac{\int_{\underline{\theta}}^{\bar{\theta}} E_D[\min(D, \hat{q}(\theta))]\, dF_\theta(\theta)}{f_\theta(\underline{\theta}) f_s(\underline{s})[\phi\underline{s} + \underline{\theta} - g_\theta(\underline{\theta})] E_D[\min(D, \hat{q}(\theta))]} + O(\delta^2). \tag{3.9}$$

The effects of the parameters on w^* can then be derived by (3.8) and (3.9). As $Var[s]$ increases, we expect a lower $\underline{s}$ since distribution is extended more towards the extreme ends while keeping other values the same; which implies a lower w^*. The remaining comparative statics follow similarly. This completes the proof. ■

As the expected spot price or the buyer's production inflexibility increases, the buyer's alternative sources of procurement at $t = 2$ become less attractive. As a result, the seller charges a higher exercise price for the options as stated in Proposition 3.2. When the spot price variance increases, the probability that the buyer exercises his options decreases since it is more likely that the spot price will be lower than the option's exercise price. This decrease forces the supplier to reduce the exercise price to make the options more attractive to the buyer. Finally, when the uncertainty on the buyer's type increases, given the seller's information, the probability that the buyer exercises his options decreases, inducing the supplier to reduce the exercise price of the options in the optimal contracts as stated in Proposition 3.2.

The next proposition summarizes the effects of industry and market parameters on expected contracted quantity.

Proposition 3.3

(i) If $r_B < r_S$, expected contracted quantity, $E_\theta[q^*(w,\theta)]$, increases with δ, $E[s]$ and $E[\theta]$; and decreases with ϕ and β_0.

(ii) If $r_B \geq r_S$, then there exists $\bar{\delta} > 0$ such that when $-\bar{\delta} < \delta < 0$, expected contracted quantity, $E_\theta[q^*(w,\theta)]$, increases with δ, $E[s]$, and $E[\theta]$; and decreases with ϕ, β_0, and $Var[s]$.

Proof of Proposition 3.3: To see part (i), first we know that when $r_S > r_B$, by Proposition 3.1 that $w^* = 0$. Plugging in (2.11), and taking expectations over θ, we have

$$E_\theta\left[q^*(0,\theta)\right] = \int_{\underline{\theta}}^{\bar{\theta}} \bar{F}_D^{-1}\left(\frac{(1+r_S)\beta_0 - \rho E[s](1-\phi)}{\rho(\phi E[s] + \theta - g_\theta(\theta))}\right) dF_\theta(\theta)\,. \tag{3.10}$$

By (3.10), we can observe the effects of the parameters on $E_\theta\left[q^*(0,\theta)\right]$. As $E[s]$ increases, the numerator of the argument of $\bar{F}_D$ decreases and the denominator increases. Hence, the argument decreases and since $\bar{F}_D$ is monotonically decreasing, the integrand and consequently $E_\theta\left[q^*(0,\theta)\right]$ increases. The remaining comparative statics follow similarly. This completes the proof of part (i).

For part (ii), plugging (3.9) in (2.11) and (2.12), and simplifying,

$$E_\theta[q^*(w^*,\theta)] = \int_{\underline{\theta}}^{\bar{\theta}} \hat{q}(\theta)\, dF_\theta(\theta) + \delta \int_{\underline{\theta}}^{\bar{\theta}} \frac{\bar{F}_D(\hat{q}(\theta))}{f_D(\hat{q}(\theta))} \frac{E[s] - \underline{s} - \underline{\theta} + \theta - g_\theta(\theta)}{\phi E[s] + \theta - g_\theta(\theta)}\, dF_\theta(\theta) + O(\delta^2). \tag{3.11}$$

By (3.8) and (3.11), we can observe the effects of the parameters on $E_\theta\left[q^*(w^*,\theta)\right]$. Looking at the expression (3.8), as $E[s]$ increases, the numerator of the argument of $\bar{F}_D$ decreases and the denominator increases. Hence, the argument decreases and since $\bar{F}_D$ is monotonically decreasing, the integrand, $\hat{q}(\theta)$, and consequently $E_\theta\left[q^*(w^*,\theta)\right]$

increases. The remaining comparative statics follow similarly. This completes the proof of part (*ii*). ■

As mentioned above, increased spot price and increased buyer inflexibility increases the attractiveness of the options to the buyer. This increases the number of contracts purchased both when $w^* = 0$ and $w^* > 0$, as stated in parts (*i*) and (*ii*) of Proposition 3.3. An increased bid-ask spread reduces the amount the buyer can recoup for the intermediate goods he cannot use to satisfy consumer demand, making the options less attractive. As a result, the number of contracts he purchases decreases. An increase in spot price variance increases the probability that the buyer can supplement his procurement from the spot market at time $t = 2$. Consequently, when the seller is offering options, i.e., when $w^* > 0$, the options become less crucial for him and the expected contracted quantity decreases as stated in part (*ii*) of Proposition 3.3.

Finally, we examine how the expected reservation fee changes with industry and the market characteristics. The following proposition states the result.

Proposition 3.4

(*i*) *If $r_B < r_S$, expected reservation fee paid, $E_\theta[R(q^*(w^*,\theta),w^*)]$, increases with δ, $E[s]$ and $E[\theta]$; and decreases with ϕ and β_0.*

(*ii*) *If $r_B \geq r_S$, then there exists $\bar{\delta} > 0$ such that when $-\bar{\delta} < \delta < 0$, expected reservation fee paid, $E_\theta[R(q^*(w^*,\theta),w^*)]$, increases with δ, $E[s]$, $Var[s]$ and $E[\theta]$; and decreases with ϕ and β_0.*

Proof of Proposition 3.4: For part (*i*), again by Proposition 3.1, since $r_S > r_B$, $w^* = 0$. Plugging $w^* = 0$ in (2.11) and (2.31) and taking the expectation over θ,

$$
\begin{aligned}
& E_\theta[R(q^*(0,\theta),0)] \\
= & \frac{1}{1+r_B}\int_{\underline{\theta}}^{\bar{\theta}}\left\{E[s](1-\phi)E_D\left[\left(\bar{F}_D^{-1}\left(\frac{(1+r_S)\beta_0-\rho E[s](1-\phi)}{\rho(\phi E[s]+\theta-g_\theta(\theta))}\right)-D\right)^+\right]\right. \\
& +(E[s]+\theta-g_\theta(\theta)) \\
& \left.\cdot E_D\left[\min\left(D,\bar{F}_D^{-1}\left(\frac{(1+r_S)\beta_0-\rho E[s](1-\phi)}{\rho(\phi E[s]+\theta-g_\theta(\theta))}\right)\right)\right]\right\}dF_\theta(\theta). \quad (3.12)
\end{aligned}
$$

By (3.12), we can observe the effects of the parameters on $E_\theta\left[R(q^*(0,\theta),0)\right]$. As ρ increases, the numerator of the argument of $\bar{F}_D$ decreases and the denominator increases. Hence, the argument decreases and since $\bar{F}_D$ is monotonically decreasing, the integrand and consequently $E_\theta\left[R(q^*(0,\theta),0)\right]$ increases. The remaining comparative statics follow similarly. This completes the proof of part (i).

For part (ii), plugging (3.9) in (2.31), taking expectations and simplifying, we have

$$\begin{aligned}
&E_\theta[R(q^*(w^*,\theta),w^*)] \\
=\;& \int_{\underline{\theta}}^{\bar{\theta}} (E[s]+\theta-g_\theta(\theta)-\underline{s}-\underline{\theta})\,E_D[\min(D,\hat{q}(\theta))]\,dF_\theta(\theta) \\
&+\delta\left(\int_{\underline{\theta}}^{\bar{\theta}} [\kappa+E[s]+\theta-g_\theta(\theta)-\underline{s}-\underline{\theta}]\,E_D[\min(D,\hat{q}(\theta))]\,dF_\theta(\theta)\right. \\
&\qquad\left.+\int_{\underline{\theta}}^{\bar{\theta}} \frac{\bar{F}_D(\hat{q}(\theta))^2\,[E[s]+\theta-g_\theta(\theta)-\underline{s}-\underline{\theta}]^2}{f_D(\hat{q}(\theta))\,[\phi E[s]+\theta-g_\theta(\theta)]}\,dF_\theta(\theta)\right)+O(\delta^2)\,,
\end{aligned} \tag{3.13}$$

where $\hat{q}(\theta)$ is as defined in (3.8), and

$$\kappa=\frac{\int_{\underline{\theta}}^{\bar{\theta}} E_D[\min(D,\hat{q}(\theta))]\,dF_\theta(\theta)}{f_\theta(\underline{\theta})f_s(\underline{s})\,[\phi\underline{s}+\underline{\theta}-g_\theta(\underline{\theta})]\,E_D[\min(D,\hat{q}(\underline{\theta}))]}\,. \tag{3.14}$$

Note that $\kappa\geq 0$, given by the expression (3.14). Now,

$$\int_{\underline{\theta}}^{\bar{\theta}} (E[s]+\theta-g_\theta(\theta)-\underline{s}-\underline{\theta})\,dF_\theta(\theta)=E[s]-\underline{s}\geq 0\,. \tag{3.15}$$

Note that the integrand in (3.15) and also $E_D[\min(D,\hat{q}(\theta))]$ are both non-decreasing in θ. Further, $E_D[\min(D,\hat{q}(\theta))]\geq 0$ implies that

$$\int_{\underline{\theta}}^{\bar{\theta}} (E[s]+\theta-g_\theta(\theta)-\underline{s}-\underline{\theta})\,E_D[\min(D,\hat{q}(\theta))]\,dF_\theta(\theta)\geq 0\,. \tag{3.16}$$

Thus, the coefficient of δ in (3.13) is non-negative, which implies $E_\theta[R(q^*(w^*,\theta),w^*)]$ increases with δ. The remaining comparative statics follow similarly. This completes the proof of part (ii). ■

As before, an increase in the expected spot price or the buyer's expected inflexibility makes the options more valuable to the buyer. Consequently, the supplier increases the reservation price in the optimal contracts for both cases when $w^* = 0$ and $w^* > 0$ as stated in parts (i) and (ii) of Proposition 3.4. The effects of increase in discount rate differences and the spot price variance filter through the supplier's trade-off between the reservation and exercise prices of the options. As the supplier becomes relatively less patient compared to the buyer, she prefers to shift her revenues to earlier rather than later. Consequently, she decreases the exercise price as stated in Proposition 3.2, and increases the reservation price. Further, an increase in spot price variance decreases the probability of buyer exercising the options for any given fixed w. In response, the supplier finds it optimal to decrease the exercise price to increase the exercise probability. Part (ii) of Proposition 3.4 states that in the optimal contracts, the supplier is able to recuperate some of her losses from this decrease in exercise price by increasing the reservation fees. Lastly, an increase in the spot market bid-ask spread has a strong effect in reducing the value of the options. So much so that, even though the seller decreases the exercise price in the contract offer, she still has to reduce the reservation fees in order to maximize her profits as stated in part (ii) of the proposition.

The effect of production costs on contract characteristics deserves further attention. Proposition 3.4 states that as the supplier's unit production cost (β_0) increases, the expected total reservation price ($E_\theta[R(q^*(w,\theta),w)]$) decreases. That is, increased costs induce the seller to reduce the reservation revenue she is getting from the buyer. This is because, in the optimal contracts, the supplier actually chooses to increase the exercise price w^* as stated in Proposition 3.2. Further, as the unit production cost increases, the expected total quantity ($E_\theta[q^*(w,\theta)]$) also decreases, which causes an increase in the expected average reservation costs ($E_\theta[R(q^*(w,\theta),w)/q^*(w,\theta)]$), as can also be seen in panel (a) of Figure 3-3. Moreover, from the figure, as the production costs increase, the seller's optimal policy increasingly shifts towards collecting revenues later (through the exercise price w) rather than sooner (through average

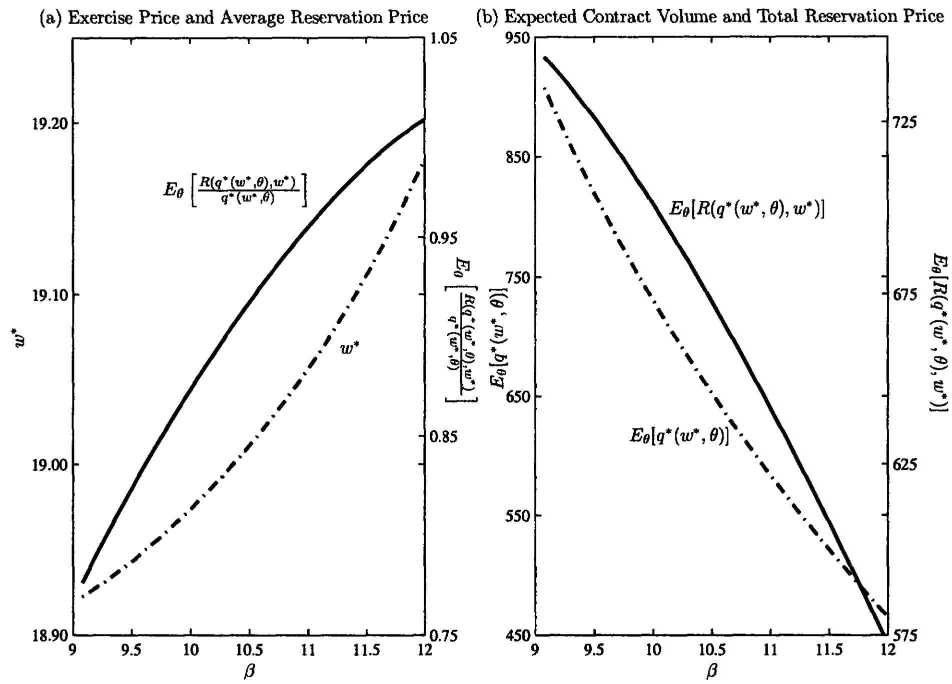

Figure 3-3: The effect of supplier's unit production costs (β) on the optimal contract characteristics. Panel (a) shows the effect of β on the optimal exercise price (w^*) and the expected average reservation price ($E[R(q^*(w,\theta),w)/q^*(w,\theta)]$). Panel (b) demonstrates the effect of β on the expected contracted quantity ($E[q^*(w,\theta)]$), and the expected reservation fee collected by the supplier ($R(q^*(w,\theta),w)$). s, θ and D have truncated exponential distributions on $[10,18]$, $[8,14]$, and $[0,1200]$, with means 14, 11 and 600, respectively. The other parameter values are $p=40$, $\phi=0.35$ and $\rho=0.73$.

unit reservation fees $R(q^*(w,\theta),w)/q^*(w,\theta)))$. Increased production costs, however, cause steep decreases in the quantity contracted; and as a result, increased average reservation fees cannot compensate for an increase in total expected reservation price, as noted in panel (b) of Figure 3-3.

3.3 Uncertainty and Efficiency

In this section, we provide a numerical analysis of the determinants of contract characteristics and supply chain efficiency. Specifically, we focus on the effects of distributions, i.e. mean and uncertainty, of the spot price and buyer flexibility, which are the two most important factors in determining contract characteristics.

In order to obtain a yardstick for measuring efficiency, we consider the supply chain's first-best profit. The supply chain's first-best profit is achieved under the assumption of an integrated channel with centralized decision making and no information asymmetry. Consider the integrated channel where the downstream unit has type θ, i.e., the additional cost when the system purchases from the spot market is θ, and the production level $y(\theta)$ at $t = 1$ is determined centrally. The corresponding profit level for the supply chain is thus given as

$$\pi_{SC}(y,\theta) = -\beta y + \frac{1}{1+r_{SC}}\Big(pE[D] + E[s](1-\phi)E_D[(y-D)^+] \\ - (E[s]+\theta)E_D[(D-y)^+]\Big), \quad (3.17)$$

where r_{SC} is the discount rate for the integrated supply chain. Given this set up, the following proposition presents the coordinated supply-chain's first-best solution.

Proposition 3.5 *The highest attainable profit level for the centralized system is*

$$\pi_{FB} = \frac{1}{1+r_{SC}}\left((p - E[s] - E[\theta])E[D] + \int_{\underline{\theta}}^{\bar{\theta}} (\phi E[s]+\theta)\int_0^{y^*(\theta)} x\, dF_D(x)\, dF_\theta(\theta)\right), \quad (3.18)$$

where $y^*(\theta)$ denotes the optimal production level of the centralized system with a type θ downstream unit and is given by

$$y^*(\theta) = \bar{F}_D^{-1}\left(\frac{(1+r_S)\beta - E[s](1-\phi)}{\phi E[s]+\theta}\right). \quad (3.19)$$

Proof of Proposition 3.5: First, notice that (3.17) is strictly concave in $y(\theta)$. Hence, by taking the first order condition in (3.17) for each θ-type, equating to zero and solving for y, the corresponding optimal production quantity for the supply chain is given as in (3.19). Substituting (3.19) into (3.17), and taking expectations over θ, the first-best profit level for the centralized system is

$$\begin{aligned}\pi_{SC} &= -\beta E_\theta[y^*(\theta)] + \frac{1}{1+r_{SC}} \int_{\underline{\theta}}^{\bar{\theta}} \Big(pE[D] + E[s](1-\phi)E_D[(y^*(\theta)-D)^+] \\ &\qquad\qquad -(E[s]+\theta)E_D[(D-y^*(\theta))^+] \Big)\, dF_\theta(\theta) \\ &= \frac{1}{1+r_{SC}} \Big((p - E[s] - E[\theta])\, E[D] \\ &\qquad\qquad + \int_{\underline{\theta}}^{\bar{\theta}} (\phi E[s] + \theta) \int_0^{y^*(\theta)} x\, dF_D(x)\, dF_\theta(\theta) \Big), \qquad (3.20)\end{aligned}$$

as stated. ■

Note that the integrated supply chain combines the supplier and the buyer, naturally, the discount rate for the integrated channel r_{SC} should be lower than $\min\{r_S, r_B\}$. That is, $0 \le r_{SC} \le \min\{r_S, r_B\}$. We consider this range in our numerical analysis for the supply chain's discount rate.

Utilizing the first-best solution described as in Proposition 3.5, we now calculate the percentage efficiency of the optimal procurement contracts and the division of the surplus between the supplier and the buyer. Table 3.1 provides the efficiency measures as the spot price and buyer flexibility distributions and the relative patience of the seller and the buyer (as represented by ρ) vary. Table 3.2 provides the corresponding contract characteristics. The tables present three production flexibility distribution cases to explore the effects of shifts in the expected value and variance: A base (or "standard") case ($\theta \sim U[20, 24]$, denoted by "ST"), a variance preserving case ($\theta \sim U[28, 32]$, denoted by "VP"), and a mean preserving case ($\theta \sim U[15, 29]$, denoted by "MP"). In addition to measuring supply chain efficiency with respect to the first best (π_{SC}/π_{FB}) and the supplier's share of the supply chain surplus (π_S/π_{SC}), we also measure the percentage gain from employing an options contract compared to the best sales contract for the supplier ($\Gamma_S = (\pi_S(w^*) - \pi_S(0))/\pi_S(0)$) and the supply chain ($\Gamma_{SC} = (\pi_{SC}(w^*) - \pi_{SC}(0))/\pi_{SC}(0)$) as a whole.

Table 3.1 demonstrates that decreased flexibility of buyer's production (increased $E[\theta]$), has inverse effects for the efficiency of the options and sales contracts. There are two underlying effects: First, a reduced buyer flexibility reduces the overall efficiency of the supply chain. Second, as can also be seen in Table 3.2, increased $E[\theta]$ increases the contracted quantity, making it closer to the first-best quantity. The former effect decreases the supply chain efficiency while the latter increases it. When $\rho < 1$, the supply chain operates with option contracts, which are more sensitive to the distribution of θ. As a result, the contracted quantity increase is more significant for these agreements with a shift in production flexibility, and the second effect overcomes the first one, increasing supply chain efficiency. A decrease in buyer's production flexibility also increases the supplier's share of the surplus, as it means an increased reliance of the buyer to the supplier. When the production flexibility decreases, the gains from employing the option contracts for the seller decreases, since the inflexibility of the buyer gives the seller the ability to better take advantage of the sales contracts. However, with increased buyer inflexibility, the supply chain gains from employing options instead of sales contracts increase.

As can also be seen from Table 3.1, because of the informational advantage the buyer has over the supplier, the buyer captures a significant portion of the generated supply chain surplus. As the uncertainty on the buyer type increases, the buyer's informational rent, and hence his surplus increases, while the supply chain's surplus decreases relative to the supply chain's first best solution. The supply chain also becomes less efficient with an increased variance of flexibility as the high informational asymmetry on the buyer's flexibility induces the contracted amount to be significantly lower than the efficient level. Interestingly, the supplier's relative gains from employing the optimal options contract versus the best sales contract increases with increased variance of flexibility, as the buyer's willingness-to-pay for the option contracts increases compared to that for the less flexible sales contracts. In contrast, the supply chain's benefit from employing option contracts versus sales contracts decreases. As the expected spot price increases, the supplier's share of the supply chain surplus increases as the spot market will be less attractive to the buyer, and hence the buyer will be more dependent on the supplier. The effect of expected spot price increases on relative efficiency of the supply chain is different for the cases with

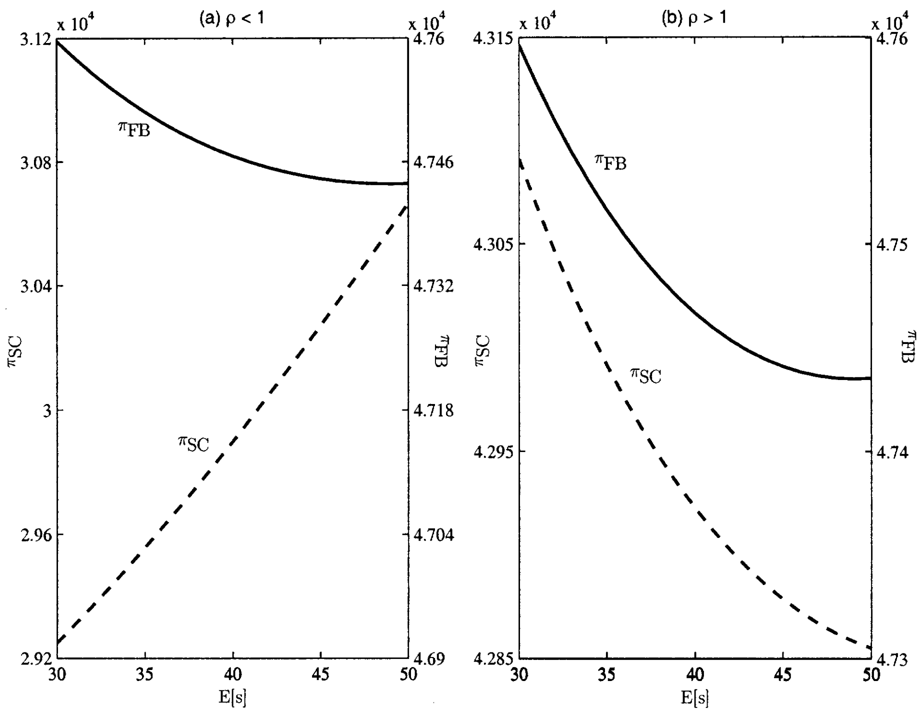

Figure 3-4: The contrast between the effect of increase in expected spot price on supply chain efficiency with option contracts (panel (a), $\rho = 0.8$) and sale contracts (panel (b), $\rho = 1.2$). For both panels, s has truncated normal distribution with standard deviation 3 and support $[E[s] - 4, E[s] + 4]$; D has truncated normal distribution with mean 600, standard deviation 400 with support $[0, 1200]$; and θ is $U[20, 24]$. The remaining parameters are $p = 100$, $\phi = 0.95$, and β=13.

$\rho < 1$ and $\rho > 1$ as presented in Table 3.1. When $\rho > 1$, the supply chain operates under sales contracts that are more rigid compared to the option contracts that emerge when $\rho < 1$. With option contracts $\rho < 1$, an increase in spot price can cause an increase in supply chain efficiency as can also be seen in panel (a) of Figure 3-4, while with sale contracts, there is a reduction in supply chain surplus faster than that of the first best as demonstrated in panel (b) of Figure 3-4. Thus, the observation that supply chain efficiency can in fact improve with increased spot price reveals the power of option contracts to serve as a shock absorbent for the whole supply chain against inclines in spot prices. This is also evident in the increase in the value of the options relative to the sales contracts for the supply chain (Γ_{SC}) with increased spot price as can be seen for cases where $\rho < 1$.

When spot price variance increases, given the expected spot price stays constant, the relative surplus distribution with the expected sales contracts which are optimal for the supplier for $\rho \geq 1$, do not change as the contracts are rigid and their use does not depend on the realization of the spot price. The latter effect can also be observed in Figure 3-5, where panel (a) of the figure shows that the optimal exercise price, w^* is monotonically increasing in $E[s]$. However, for cases with $\rho < 1$, the supply chain operates under the option contracts, which is sensitive to the realization of the spot price. When spot variance increases, the supplier's share of the supply chain surplus decreases, which can be explained as follows: increased spot variance allows the buyer to take advantage of the low end of the spot price distribution, thus decreasing the buyer's willingness to pay for the contracts. Further, the expected benefits of employing option contracts compared to sales contracts for the supplier, as well as for the supply chain, decreases. Supply chain efficiency also suffers from increased uncertainty in the spot market since, as can be seen from Table 3.2, such an increase in uncertainty substantially increases per unit reservation fees and reduces the contracted quantities, despite a significant cut in the exercise price.

Table 3.2 gives important insights for the optimal pricing strategies of the supplier. First, notice that there is a steep (discontinuous) decline in w and a jump increase in $E_\theta[R(Q^*(\theta))/Q^*(\theta)]$ as ρ increases around $\rho = 1$, since the supplier's pricing regime shifts from option contracts to sales contracts. For $\rho < 1$, an increase in the expected additional cost of purchasing from an outside supplier ($E[\theta]$) induces the supplier to shift her fees from reservation costs ($E_\theta[R(Q^*(\theta))/Q^*(\theta)]$) to exercise fees ($w^*$). This is because, for this range, the supplier is more patient than the buyer and takes advantage of the buyer's increased additional cost of outside procurement; she strategically prices the contracts to delay payments, thus maximizing her return. For $\rho \geq 1$ however, the effect on the unit reservation price is the opposite, as $w^* = 0$, and the supplier simply reflects the increased additional cost of the buyer's outside procurement on the sales price.

The effect of an increase in the uncertainty of the buyer's production flexibility on seller's pricing is significantly different. As can be seen from Table 3.2, such an increase forces the seller not only to decrease her average unit reservation price but also the exercise price for the options. There are two reasons for this observation. First, increasing the variance of θ increases the buyer's overall effective flexibility under option contracts since the option contracts cap the upside of the procurement costs. Thus, on average, the buyer will have more opportunities to procure form the spot market with an increased θ variance. Second, increased uncertainty on θ increases the information asymmetry between the buyer and the seller, increasing the information rent the seller gives to the buyer by reducing her prices. These two effects combined result in downward shifts in the optimal procurement fees from the contracts at both stages.

The effect of an increase in the spot price variance on the supplier's pricing is remarkable. Specifically, with an increased uncertainty in the spot market, the exercise price falls significantly, while the reservation price increases sharply. The reason for such a dramatic change in the supplier's pricing policy is that increased spot variance increases the buyer's opportunities in the lower end of the spot price distribution, making the option contracts less attractive for the buyer. This effect pushes the overall contract price down and forces the supplier to reassess her fee allocation structure. As can be seen from Table 3.2, in order to make the options more desirable for the buyer at the exercise time, she reduces the exercise price and balances it by increasing the average reservation cost.

Finally, and importantly, the effect of increased spot price on the contract structure reflects the nature of the volume dependency of the optimal option contracts. Notice that for the case with $\rho < 1$ and low θ variance, an increase in expected spot price results in a decrease in average reservation price. This is because increased contracted quantity and concavity of the reservation fee structure, together with the supplier's shifting of her fee collection towards the time of exercise for $\rho < 1$ put downward pressure on the average reservation fee. However, when the variance of θ increases, the volume dependency regime of the reservation fee structure can change and is sensitive to $E[s]$ as can be observed from Proposition 2.2. Specifically, by (2.32), an increase in $E[s]$ increases the concavity of the reservation schedule $R(q)$ at the higher end of the curve, and it can even shift the curve's shape from convex to concave in that region. Therefore, as can also be observed from Table 3.2, for the case "MP" and $\rho < 1$, the average reservation price is increasing with $E[s]$ at first. However, the effect is non-monotonic and the average reservation price decreases beyond a certain level of $E[s]$ as the contracted quantity increases and the reservation fee schedule becomes increasingly concave. Lastly, for the case with $\rho \geq 1$, since $w^* = 0$, the seller does not shift her fee collection towards the time of exercise; she thus increases the average reservation price as the buyer is more dependent on her. Therefore, as can also be observed from Table 3.2 and panel (b) of Figure 3-5, when the variance of θ is high and $\rho < 1$, the exercise average reservation price can increase initially with $E[s]$ but the effect can be non-monotonic. The average reservation price decreases beyond a certain level of $E[s]$ as the contracted quantity increases and the reservation fee schedule becomes increasingly concave. The pressure for decreasing average reservation price intensifies as the spot price variance increases and the sensitivity of the optimal reservation price curve's shape to the expected spot price increases, as can also be seen in panel (b) of Figure 3-5.

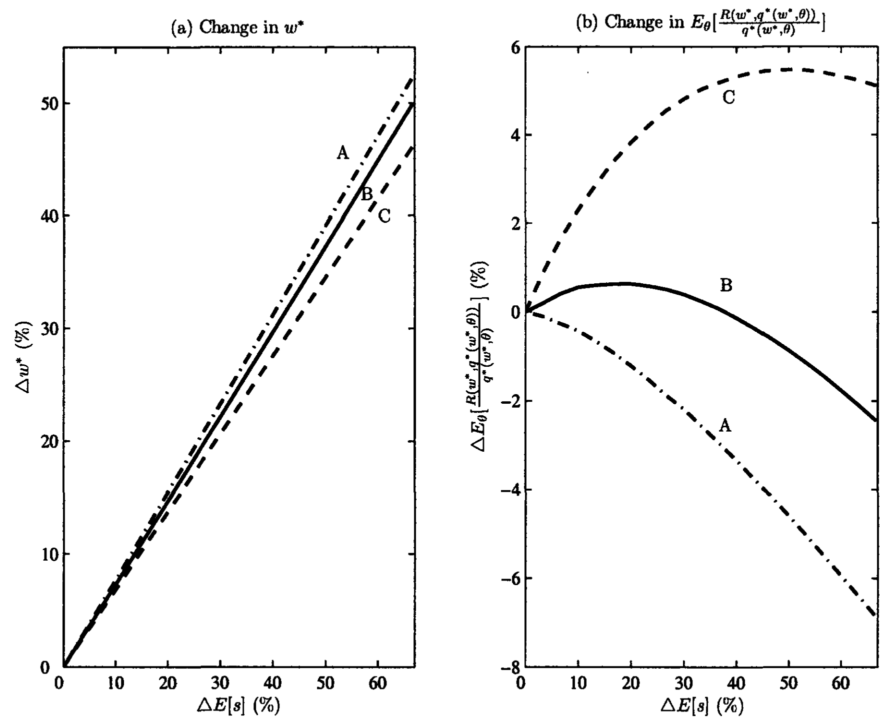

Figure 3-5: Percentage change in the optimal exercise price w^* (panel (a)), and average reservation price $E_\theta[R(q^*(w^*,\theta), w^*)/q^*(w^*,\theta)]$ (panel (b)) with increase in expected spot price. For case A, $\rho = 0.9$, for cases B and C, $\rho = 0.8$. The spot price s has a truncated normal distribution with standard deviation 40 and support $[E[s]-9.5, E[s]+9.5]$ for cases A and B, and standard deviation 3 and support $[E[s]-4, E[s]+4]$ for case C. For all cases, the demand has truncated normal distribution with mean 600, standard deviation 400 with support $[0, 1200]$; and θ is $U[15, 29]$. The remaining parameters are $p = 100$, $\phi = 0.75$, $\beta = 13$, and the initial mean spot price is 30.

			(a) $\rho = 0.8$				(b) $\rho = 0.9$				(c) $\rho = 1.0$		(d) $\rho = 1.1$		(e) $\rho = 1.2$	
σ_s	θ	$\Delta E[s]$	$\frac{\pi_{SC}}{\pi_{FB}}$	$\frac{\pi_S}{\pi_{SC}}$	Γ_S	Γ_{SC}	$\frac{\pi_{SC}}{\pi_{FB}}$	$\frac{\pi_S}{\pi_{SC}}$	Γ_S	Γ_{SC}	$\inf\left(\frac{\pi_{SC}}{\pi_{FB}}\right)$	$\frac{\pi_S}{\pi_{SC}}$	$\inf\left(\frac{\pi_{SC}}{\pi_{FB}}\right)$	$\frac{\pi_S}{\pi_{SC}}$	$\inf\left(\frac{\pi_{SC}}{\pi_{FB}}\right)$	$\frac{\pi_S}{\pi_{SC}}$
Low	ST	0	(61.38,86.02)	39.49	54.42	16.31	(66.36,93.00)	37.02	21.43	7.05	71.35	34.92	80.81	36.76	90.37	38.28
		2	(61.68,86.56)	42.38	52.63	17.25	(66.46,93.27)	39.83	20.87	7.44	71.25	37.64	80.75	39.44	90.34	40.93
		5	(62.13,87.38)	46.67	50.35	18.65	(66.61,93.68)	44.04	20.15	8.02	71.11	41.74	80.65	43.48	90.29	44.90
		10	(62.91,88.74)	53.67	47.38	20.95	(66.90,94.36)	50.98	19.19	8.98	70.89	48.60	80.51	50.19	90.21	51.49
		20	(64.49,91.46)	66.93	43.32	25.43	(67.50,95.72)	64.45	17.85	10.84	70.51	62.19	80.25	63.44	90.08	64.47
	VP	0	(62.64,88.03)	50.09	48.14	19.56	(66.89,94.01)	47.42	19.42	8.40	71.15	45.07	80.69	46.70	90.31	48.02
		2	(62.95,88.58)	52.88	47.05	20.47	(67.00,94.28)	50.19	19.06	8.78	71.07	47.82	80.64	49.39	90.28	50.67
		5	(63.43,89.40)	57.00	45.62	21.84	(67.19,94.69)	54.33	18.59	9.35	70.95	51.94	80.56	53.42	90.24	54.62
		10	(64.24,90.77)	63.68	43.65	24.08	(67.50,95.38)	61.11	17.94	10.28	70.77	58.79	80.44	60.10	90.18	61.16
		20	(65.86,93.47)	76.25	40.81	28.42	(68.15,96.73)	74.19	16.98	12.09	70.46	72.25	80.22	73.18	90.07	73.94
	MP	0	(60.48,84.75)	33.11	60.07	14.98	(65.84,92.26)	30.83	23.12	6.44	71.23	28.95	80.72	30.83	90.29	32.40
		2	(60.79,85.31)	36.00	57.65	15.86	(65.95,92.55)	33.60	22.40	6.81	71.16	31.60	80.67	33.46	90.28	35.01
		5	(61.26,86.14)	40.32	54.60	17.20	(66.13,92.99)	37.78	21.47	7.37	71.04	35.63	80.60	37.44	90.25	38.94
		10	(62.05,87.52)	47.42	50.67	19.43	(66.43,93.70)	44.73	20.24	8.31	70.85	42.40	80.48	44.11	90.19	45.52
		20	(63.65,90.26)	61.02	45.48	23.85	(67.06,95.10)	58.35	18.57	10.15	70.50	55.97	80.24	57.39	90.08	58.56
High	ST	0	(60.42,84.68)	38.54	48.36	14.50	(65.87,92.32)	36.57	19.06	6.27						
		2	(60.71,85.21)	41.47	47.04	15.42	(65.97,92.59)	39.39	18.67	6.65						
		5	(61.17,86.02)	45.83	45.37	16.80	(66.13,92.99)	43.63	18.17	7.23						
		10	(61.94,87.38)	52.95	43.16	19.08	(66.41,93.67)	50.62	17.49	8.18						
		20	(63.53,90.10)	66.44	40.13	23.56	(67.02,95.04)	64.20	16.55	10.05						
	VP	0	(61.66,86.66)	49.31	43.56	17.70	(66.40,93.32)	47.03	17.58	7.60						
		2	(61.98,87.21)	52.14	42.75	18.60	(66.51,93.59)	49.83	17.33	7.98						
		5	(62.46,88.02)	56.33	41.69	19.96	(66.70,94.00)	53.99	17.00	8.55						
		10	(63.27,89.39)	63.12	40.23	22.19	(67.01,94.68)	60.83	16.54	9.48						
		20	(64.89,92.10)	75.90	38.10	26.54	(67.67,96.04)	74.00	15.86	11.29						
	MP	0	(59.55,83.45)	32.20	53.30	13.21	(65.35,91.58)	30.37	20.41	5.66						
		2	(59.85,83.99)	35.12	51.44	14.08	(65.47,91.87)	33.16	19.90	6.03						
		5	(60.32,84.82)	39.49	49.09	15.40	(65.65,92.31)	37.35	19.23	6.59						
		10	(61.1,86.19)	46.68	46.06	17.62	(65.95,93.02)	44.34	18.35	7.52						
		20	(62.71,88.94)	60.47	42.05	22.02	(66.58,94.42)	58.06	17.15	9.37						

Table 3.1: The effect of distributions of spot price (s), buyer's flexibility (θ), and relative patience of seller to the buyer ($\rho = (1 + r_S)/(1 + r_B)$) on contract efficiency and gains from using options. Efficiency is measured by supply chain efficiency (π_{SC}/π_{FB}), and the supplier's share of supply chain profits (π_S/π_{SC}). For $\rho < 1$, the upper and lower bounds for the supply chain efficiency is given, while for $\rho \geq 1$, only the lower bound of efficiency is given as the upper bound for all cases is larger than 99%. The value of employing options instead of sales contracts are measured as percentage gain from employing an options contract compared to the best sales contract for the supplier ($\Gamma_S = (\pi_S(w^*) - \pi_S(0))/\pi_S(0)$) and the supply chain ($\Gamma_{SC} = (\pi_{SC}(w^*) - \pi_{SC}(0))/\pi_{SC}(0)$) for $\rho < 1$, since $w^* = 0$ for $\rho \geq 1$. The distribution parameters for the base case are: (a) spot market is truncated normal with mean 30, standard deviation 3, and bounds $[26, 34]$; (b) buyer's flexibility is uniform with bounds $[20, 24]$; and (c) demand is truncated normal with mean 600, standard deviation 400, and bounds $[0, 1200]$. Low spot variance (σ_s) is the base case, and high variance corresponds to $\sigma_s = 40$ and $E[s] \pm 9.5$ bounds of the spot price. For θ distribution, ST denotes the base case, VP denotes the variance preserving case with $\theta \sim$U$[28, 32]$, and MP denotes the mean preserving case with $\theta \sim$U$[15, 29]$. Expected spot price is varied by raising the base value by increments of $\Delta E[s] = 2, 5, 10, 20$. The remaining parameter values are $p = 100$, $r_S = 0.3$, $\phi = 0.75$, and $\beta = 13$. All figures in the table are in percentages.

			(a) $\rho = 0.8$			(b) $\rho = 0.9$			(c) $\rho = 1.0$		(d) $\rho = 1.1$		(e) $\rho = 1.2$	
σ_s	θ	$\triangle E[s]$	$E_\theta[Q^*(\theta)]$	w^*	$E_\theta\left[\frac{R(Q^*(\theta))}{Q^*(\theta)}\right]$	$E_\theta[Q^*(\theta)]$	w^*	$E_\theta\left[\frac{R(Q^*(\theta))}{Q^*(\theta)}\right]$	$E_\theta[Q^*(\theta)]$	$E_\theta\left[\frac{R(Q^*(\theta))}{Q^*(\theta)}\right]$	$E_\theta[Q^*(\theta)]$	$E_\theta\left[\frac{R(Q^*(\theta))}{Q^*(\theta)}\right]$	$E_\theta[Q^*(\theta)]$	$E_\theta\left[\frac{R(Q^*(\theta))}{Q^*(\theta)}\right]$
Low	ST	0	854.5	46.31	1.502	856.6	46.16	1.752	858.7	27.37	898.8	29.38	934.7	31.34
		2	875.5	48.30	1.483	877.4	48.15	1.727	879.4	28.16	919.6	30.22	955.8	32.24
		5	905.8	51.28	1.455	907.5	51.15	1.692	909.2	29.33	949.8	31.47	986.6	33.56
		10	953.5	56.26	1.410	954.9	56.13	1.635	956.2	31.22	997.9	33.48	1036.3	35.68
		20	1042.9	66.22	1.324	1043.7	66.11	1.530	1044.6	34.79	1090.6	37.23	1134.6	39.56
	VP	0	894.5	54.25	1.478	896.1	54.13	1.713	897.7	30.67	933.9	32.96	966.2	35.21
		2	912.3	56.25	1.461	913.7	56.13	1.692	915.2	31.44	951.6	33.79	984.3	36.09
		5	938.1	59.24	1.436	939.4	59.12	1.661	940.7	32.58	977.6	35.00	1011.0	37.38
		10	979.5	64.22	1.395	980.6	64.11	1.611	981.6	34.43	1019.7	36.97	1054.6	39.45
		20	1058.7	74.19	1.317	1059.3	74.10	1.517	1060.0	37.93	1102.0	40.65	1141.8	43.27
	MP	0	816.2	42.34	1.023	817.0	41.82	1.361	817.9	25.26	862.4	27.08	902.1	28.87
		2	841.1	44.27	1.039	841.9	43.77	1.371	842.8	26.07	887.1	27.95	926.8	29.79
		5	876.3	47.18	1.057	877.1	46.70	1.378	877.9	27.26	922.2	29.22	962.3	31.14
		10	930.6	52.06	1.074	931.3	51.62	1.377	932.0	29.19	977.1	31.27	1018.7	33.30
		20	1029.6	61.88	1.075	1030.0	61.50	1.348	1030.5	32.81	1080.0	35.08	1127.8	37.24
High	ST	0	848.1	40.98	3.697	853.4	40.74	4.246						
		2	869.7	42.95	3.645	874.6	42.73	4.183						
		5	900.7	45.93	3.569	904.9	45.72	4.092						
		10	949.5	50.89	3.448	952.9	50.70	3.948						
		20	1040.5	60.83	3.223	1042.5	60.66	3.683						
	VP	0	889.7	48.88	3.618	893.7	48.69	4.138						
		2	907.8	50.87	3.572	911.5	50.69	4.084						
		5	934.2	53.85	3.505	937.5	53.68	4.005						
		10	976.3	58.82	3.398	979.0	58.66	3.879						
		20	1056.7	68.78	3.197	1058.4	68.64	3.644						
	MP	0	807.9	38.01	2.893	812.8	36.95	3.681						
		2	833.7	39.85	2.905	838.1	38.84	3.671						
		5	869.9	42.65	2.912	873.8	41.71	3.648						
		10	925.7	47.38	2.900	928.8	46.53	3.587						
		20	1026.7	57.00	2.823	1028.6	56.30	3.428						

Table 3.2: The effect of distributions of spot price (s), buyer's flexibility (θ), and relative patience of seller to the buyer ($\rho = (1 + r_S)/(1 + r_B)$) on contract design. Specifically, the table displays the expected contracted quantity ($E_\theta[Q^*(\theta)]$), exercise price (w^*), and the average unit reservation price ($E_\theta[R(Q^*(\theta))/Q^*(\theta)]$). $w^* = 0$ for cases with $\rho \geq 1$. The parameter distributions and the table layout is the same as Table 3.1.

Chapter 4

The Delayed Production Model

In this chapter, we consider a separate model which adds another dimension to our previous problem. Here, the seller is offered a last minute production opportunity, at the time when the states for both demand and spot market are realized; on top of the initial production stage when the contracts are determined. With high levels of outsourcing in the procurement industry, we observe more and more such opportunities available. In particular, when execution prices are high, i.e. payments to the seller are mainly in execution form at time $t = 1$, there is high risk for the seller to commit to a production up to reservation amounts. Risk is even higher for the seller when demand and spot market variance is high, or when salvage value of the product is low (spot market has a high spread). Under such circumstances, it is natural for the seller to look into other last-minute production opportunities, where the seller might consider local production facilities instead of outsourcing her entire production, but of course last-minute production costs are higher. We include these aspects in this extension of the basic model, where we consider a delayed production option for the seller.

The seller here faces a revised problem. She can independently decide on her production quantity after the buyer decides on his reservation quantity. In specific, in addition to the the basic model which we considered earlier, the seller can decide on how much to produce, $y(w, \theta)$, at a per unit cost of β_0, after the buyer decides on his reservation quantity $q(w, \theta)$ in the first period. Furthermore, in the second period, if the buyer decides to execute an amount greater than the seller's initial production amount, she has another production opportunity in the second period, $t = 2$, but at a higher per unit production cost of β_1. For the remaining units, i.e. if the buyer's execution amount is less than initial production quantity, the seller sells them all to the spot market at its bid-price. Notice that a significant difference of this model from the basic model presented earlier is that the seller here has an option to delay parts of her production, if she finds this option favorable.

In order to prevent any arbitrage opportunity, we assume here that $\beta_0 \leq (1 + r_S)E[s](1-\phi)$ and $\beta_1 \geq \max((1+r_S)\beta_0, \bar{s}(1-\phi))$. The former condition on β_0 guarantees that the seller doesn't produce at an early stage for the purpose of selling to the spot market afterwards; while the latter condition on β_1 guarantees that production cost in the second period is higher than the first period, with the discounting factor included, and that there is no chance for the seller to produce for selling to the spot immediately after the spot price is realized. In addition, we assume that for the spot price's distribution, for any $\tilde{s} \in [\underline{s}, \bar{s}]$, we have $E_s[s - \tilde{s} | s \geq \tilde{s}] \geq \bar{F}_s(\tilde{s})/f_s(\tilde{s})$, which is a weak condition on the spot market satisfied by very general common distributions.

It is easy to understand that the buyer's problem here is exactly the same as that of our basic model; as for the buyer, his opportunities available, the benefits and the costs incurred in engaging the contract is still the same. On the other hand, the seller has new opportunities, and thus she has to make more decisions for optimizing her expected profits from the contract. Now, from the seller's perspective, using a similar analysis as before, we write the seller's discounted expected profits as follows:

$$\pi_S(w) = E_\theta[R(q(w,\theta), w) + V(y(w,\theta), q(w,\theta), w, \theta)], \tag{4.1}$$

where

$$V(y,q,w,\theta) = \begin{cases} V_1(y,q,w,\theta) & \text{for } y \geq q \\ V_2(y,q,w,\theta) & \text{for } y < q \end{cases}, \tag{4.2}$$

$$V_1(y,q,w,\theta) \triangleq -\beta_0 y + \frac{1}{1+r_S}\Bigg(E[s](1-\phi)y - E_s[(s(1-\phi)-w)^+]q$$
$$+ \int_{w-\theta}^{\frac{w}{1-\phi}} (w - s(1-\phi))\, dF_s(s) E_D[\min(D,q)]\Bigg), \quad (4.3)$$

$$V_2(y,q,w,\theta) \triangleq -\beta_0 y + \frac{1}{1+r_S}\Bigg(E[s](1-\phi)y$$
$$+ \int_{w-\theta}^{\frac{w}{1-\phi}} ((w-\beta_1)E_D[\min(D,q)] + (\beta_1 - s(1-\phi))\, E_D[\min(D,y)])\, dF_s(s)$$
$$+ \int_{\frac{w}{1-\phi}}^{\bar{s}} ((w-\beta_1)q + (\beta_1 - s(1-\phi))\, y)\, dF_s(s)\Bigg). \quad (4.4)$$

The notation here is consistent with the basic model. The seller's discounted expected profit has two major components, namely $R(q(w,\theta),w)$ which is the reservation price paid by the buyer to the seller; and all the remaining terms, represented as $V(y,q,w,\theta)$, which includes her discounted production costs and profits from the buyer's execution and/or selling to the spot market. Notice that we need two separate cases, as the first considers the case where production is beyond reservation amount, thus giving us $V_1(y,q,w,\theta)$, given by (4.3); while the second considers the case with a delayed production, giving us $V_2(y,q,w,\theta)$, given by (4.4).

In this model, the seller picks an optimal production quantity in her optimization problem; in addition to offering the optimal contract structure and the specifics of the optimal contract offer, including both the optimal reservation price schedule and execution price. Furthermore, the seller takes account of the buyer's buyer behavior in her own optimization problem. Specifically, the optimal contract has to make sure that the quantity purchased by a type θ buyer is indeed his optimal quantity given the contract terms. In addition, no buyer should have negative expected gains upon agreement. Given this, the seller's problem can be formulated as

$$\begin{aligned}
\max_{R(\cdot,w),y(w,\theta)} \quad & \int_{\underline{\theta}}^{\bar{\theta}} (R(q(w,\theta),w) + V(y(w,\theta),q(w,\theta),w,\theta))\, dF_\theta(\theta) \qquad (4.5)\\
\text{s.t.} \quad & q(w,\theta) = \arg\max_q \hat{\pi}_B(q,w,\theta), \forall\theta \in [\underline{\theta},\bar{\theta}]\\
& \hat{\pi}_B(q(w,\theta),w,\theta) \geq 0, \forall\theta \in [\underline{\theta},\bar{\theta}].
\end{aligned}$$

4.1 Choice of Delayed Production

We first look at the seller's optimal production quantity at period $t = 1$, given that the type θ buyer decides on reserving $q(w, \theta)$ units of option. We present the following lemma which tells that the seller does not produce more than reservation amount, i.e. she never produces extra units for the purpose of selling to the spot market; and further determines the optimal quantity the seller produces for the type θ buyer at period $t = 1$, defined as $\xi^*(w, \theta)$, if she considers the delayed production option.

Lemma 4.1 *$y^*(w, \theta) \leq q(w, \theta)$ for any given (w, θ). Specifically, at period $t = 1$, the seller either produces up to the reservation amount $q(w, \theta)$, or chooses a delayed production option, and produces only $\xi^*(w, \theta)$ units, which is strictly less than $q(w, \theta)$. The expression $\xi^*(w, \theta)$ is given by:*

$$\xi^*(w, \theta) = \bar{F}_D^{-1} \left[\frac{(1 + r_S)\beta_0 - E[s](1 - \phi) - \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s)}{\int_{w-\theta}^{\frac{w}{1-\phi}} (\beta_1 - s(1 - \phi))\, dF_s(s)} \right] \tag{4.6}$$

Proof of Lemma 4.1: Using the expressions (4.3) and (4.4), we have:

$$\left. \frac{dV(q, y, w, \theta)}{dy} \right|_{q \leq y} = \frac{dV_1(y, q, w, \theta)}{dy} = -\beta_0 + \frac{E[s](1 - \phi)}{1 + r_S} \leq 0 \tag{4.7}$$

which implies that $y^*(w, \theta) \leq q(w, \theta)$. Also, we have:

$$\begin{aligned} \left. \frac{dV(q, y, w, \theta)}{dy} \right|_{q > y} &= \frac{dV_2(q, y, w, \theta)}{dy} \\ &= -\beta_0 + \frac{1}{1 + r_S} \Bigg(E[s](1 - \phi) + \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s) \\ &\qquad + \int_{w-\theta}^{\frac{w}{1-\phi}} (\beta_1 - s(1 - \phi))\, dF_s(s) \cdot \bar{F}_D(y) \Bigg). \end{aligned} \tag{4.8}$$

Thus, for the case where the seller considers a delayed production option, i.e. $q(w, \theta) > y^*(w, \theta)$, the optimal production quantity for type θ buyer is given by the first order conditions of (4.8):

$$y^*(w,\theta) = \xi^*(w,\theta) = \bar{F}_D^{-1}\left[\frac{(1+r_S)\beta_0 - E[s](1-\phi) - \int_{\frac{w}{1-\phi}}^{\bar{s}}(\beta_1 - s(1-\phi))\,dF_s(s)}{\int_{w-\theta}^{\frac{w}{1-\phi}}(\beta_1 - s(1-\phi))\,dF_s(s)}\right]$$

This completes the proof. ■

Results from Lemma 4.1 tell us that the seller has two options for production. She can choose to produce up to reservation amount at the first period, or alternatively delay parts of her production to the second period; but if the buyer decides to execute an amount which exceeds the initial production amount, the seller has to produce the remaining required units at a higher cost of β_1 at the second period. For both options, if there are remaining units left after the buyer's execution at the second period, the seller sells them all to the spot market.

We now present a condition which is very critical for the analysis of the delayed production model, as the condition determines when it is optimal for the seller to choose the delayed production option for a type θ buyer. Here, we identify the *delayed production for type θ buyer condition* as describing the case when production is strictly less than reservation, i.e. $y^*(w,\theta) = \xi^*(w,\theta) < q(w,\theta)$.

Definition We say the *delayed production for θ buyer condition* is satisfied when

$$\rho > \hat{\rho}(w,\theta)$$

where $\hat{\rho}(w,\theta)$ is defined by

$$\frac{(\beta_1 - w)^+\left(\bar{F}_s\left(\frac{w}{1-\phi}\right)F_D(\xi^*(w,\theta)) + \bar{F}_s(w-\theta)\bar{F}_D(\xi^*(w,\theta))\right)}{\int_{\frac{w}{1-\phi}}^{\bar{s}}(s(1-\phi)-w)\;dF_s(s)F_D(\xi^*(w,\theta)) + \int_{w-\theta}^{\bar{s}}(s+\theta-g_\theta(\theta)-w)\;dF_s(s)\bar{F}_D(\xi^*(w,\theta))}, \tag{4.9}$$

and $\xi^*(w,\theta)$ is given by (4.6).

4.2 Optimal Contract Design

An important aspect of optimal contract design is the flexibility, and complexity involved in the different non-linear structures of reservation price schedules. As discussed in the basic model, there are volume-dependent price schedules versus non-volume-dependent ones. So, before we construct the optimal contract offered by the seller, we first identify the conditions which determine when the seller offers a flat pricing schedule versus a volume-dependent reservation price schedule. Interestingly, for this delayed production model, a combination of the two schedule types is also plausible, which we look in detail in this section. These definitions provide us the characteristics and also a set of complete conditions for determining the different possible contract structure types. The three possible sets of condition include: (i) flat price condition, if satisfied, implies a flat pricing schedule for buyers of all θ-types; (ii) volume dependency condition, if satisfied, implies a volume-dependent pricing schedule for buyers of all θ-types; (iii) neither conditions satisfy implies a *low volume dependent-high flat* pricing schedule. For such schedule types, the seller offers a volume-dependent reservation pricing schedule to buyers of lower θ-types, and a fixed pricing schedule to buyers of higher θ-types.

4.2.1 Flat versus Volume-Dependent Price Schedules

In order to identify the specific conditions for differentiating the different reservation price schedule structures, we need to consider two separate expressions which are critical for our definitions, namely $G(w)$, and $M(w)$). These two expressions are very important determinants for volume-dependency, i.e. which tells when the seller offers a flat pricing schedule, versus a volume-dependent pricing schedule. Notice that a flat pricing schedule is preferred only when the buyer's expected benefit of executing the contract, after satisfying demand, is no less than the seller's expected cost of engaging in the contract; in short, we can interpret this as a non-negative "expected net gain of execution after satisfying demand".

The first expression, $G(w)$, is the expected benefit for the buyer to execute one unit of contract for the purpose of selling to the spot market, when extra units are available after satisfying demand.

$$G(w) \triangleq \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi) - w)\, dF_s(s) \tag{4.10}$$

Notice that for the expression $G(w)$, we are considering the case when the seller considers no production delays. Specifically, in mathematical terms, we can interpret the condition for a flat pricing schedule with no delayed production as follows

$$\frac{1}{1+r_B} \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi) - w)\, dF_s(s) \geq \beta_0 - \frac{1}{1+r_S} \left(E[s](1-\phi) - \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi) - w)\, dF_s(s) \right). \tag{4.11}$$

This criteria is actually equivalent to the condition $w \leq \tilde{w}_c$ where we define $\tilde{w}_c$ as

$$\delta G(\tilde{w}_c) = (1 + r_S)\beta_0 - E[s](1-\phi) \tag{4.12}$$

and is actually consistent with part(ii) of the flat price condition presented below. The next expression we consider is $M(w)$, which is the expected per unit net gain of execution after satisfying demand, when there is delayed production. Specifically, when demand is satisfied, $M(w)$ is the buyer's expected benefit of executing one unit of contract for selling to the spot market, minus the seller's expected cost of engaging in this extra unit of contract by producing in period two.

$$M(w) \triangleq \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s) - \delta G(w) \tag{4.13}$$

For the delayed production case, the condition for flat pricing schedule can be represented as $M(w) \leq 0$, which can be equivalently represented in a more intuitive way as follows

$$\frac{1}{1+r_B} \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi) - w)\, dF_s(s) \geq \frac{1}{1+r_S} \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - w)\, dF_s(s). \tag{4.14}$$

The left hand expression of (4.14) is the buyer's per unit discounted expected net profit from executing the option after satisfying demand; while the right hand expression on (4.14) is the seller's discounted expected cost of engaging in this extra unit of contract by a delayed production in period two. It is clear that when $w \geq \beta_1$, the seller always benefits in engaging in the contract, even after the buyer satisfies demand; as execution cost, w, is higher than production cost β_1. Using the expressions defined, we now present a complete set of conditions for both flat-pricing and volume-dependent schedules.

(1) **Definition:** We say the *flat price condition* is satisfied for: (i) $w \geq \beta_1$; or (ii) when all of the following three conditions are satisfied: (a) $r_S > r_B$; (b) $\beta_0 \leq \frac{E[s](1-\phi)}{1+r_B}$; and (c) $w \leq \tilde{w}_c$, $\tilde{w}_c$ is defined in (4.12).

(2) **Definition:** We say the *volume-dependency condition* is satisfied when both of the following conditions are satisfied: (i) $w \in (\tilde{w}_c, \bar{s}(1-\phi))$; and (ii) $M(w) > 0$ where $\tilde{w}_c$ and $M(w)$ are defined in (4.12) and (4.13), respectively.

4.2.2 Optimization Problem Revised

As the definitions earlier suggest, there are conditions under which buyers of certain, or even all θ-types reserve quantities up to the seller's production capacity, i.e. $q^*(w,\theta) = K$. This means that the seller, in return, cannot differentiate the θ-type of the buyer through observing his behavior with regards to his reservation quantity. Differentiating the buyer's θ-type is important to the seller as we know that she picks her optimal production quantity to be $y^*(w,\theta) = \min(q^*(w,\theta), \xi^*(w,\theta))$, as presented earlier in Lemma 4.1. From (4.6), we know that $\xi^*(w,\theta)$ is dependent on θ, and under the "normal θ-dependent" case, the seller observes θ through the buyer's $q^*(w,\theta)$ decision, and makes her optimal production decisions accordingly. Now, a problem arises when $q^*(w,\theta)$ is no longer θ-dependent, i.e. the seller has no information on the buyer's θ type, thus she can no longer make her optimal production decisions from observing the buyer's $q^*(w,\theta)$ decision. As such, whenever there are buyers of certain θ-types reserving up to K units, we consider a separate optimization problem, which we present as follows

$$
\begin{aligned}
\max_{w,R^I(\cdot),R^D(\cdot),y^*(\cdot)} \quad & \int_{\theta\in\theta^D} \left[R^D(q(w,\theta),w) + V(y^*(w,\theta),q(w,\theta),w,\theta)\right] \, dF_\theta(\theta) \\
& + \int_{\theta\in\theta^I} \left[R^I(w) + V(y^*(w),q(w,\theta),w,\theta)\right] \, dF_\theta(\theta)
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \qquad & \theta^I = \{\theta : q(w,\theta) = K\} \\
& \theta^D = \left\{\theta : q(w,\theta) < \bar{D}\right\} \\
& \hat{\pi}_B(q(w,\theta),w,\theta) \geq 0, \ \forall\theta \\
& q(w,\theta) = \arg\max_q \hat{\pi}_B(q,w,\theta), \ \forall\theta
\end{aligned} \tag{4.15}
$$

where the expression $V(y,q,w,\theta)$ is given by (4.2), (4.3) and (4.4); and $\hat{\pi}_B(q(w,\theta),w,\theta)$ is given by (2.5).

Notice that for $\theta \in \theta^I$, we have the optimal reservation price schedule (given by $R^I(w)$) and also the optimal production quantities (given by $y^*(w)$) both being independent of θ. Taking all these into consideration, the optimal decision to be made by the seller, i.e. the optimal contract offer, is presented in the next section below.

4.2.3 Optimal Contract Offer for Fixed Execution Price

Before we present the optimal contract offer given by the seller, we first understand several independent results, i.e. "facts", which are very useful for the determination of an optimal contract offer later on. The proofs for these independent results are included at the appendix.

Fact 4.1 $\int_{w-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-w)\, dF_s(s) \geq 0$

Here, for notational purposes, we define an expression which is useful for many of our results presented afterwards:

$$
\eta(w,\theta) \triangleq \int_{w-\theta}^{\bar{s}} (\phi s + \theta - g_\theta(\theta)) \, dF_s(s) + \delta \int_{w-\theta}^{\bar{s}} (s + \theta - g_\theta(\theta) - w) \, dF_s(s). \tag{4.16}
$$

Fact 4.2 *$\eta(w,\theta) \geq \delta G(w)$, where $G(w)$ and $\eta(w,\theta))$ are given by (4.10) and (4.16), respectively.*

Fact 4.3 *For any given (w,θ), there exists $\hat{\beta}_1(w,\theta) \geq w$ such that for all $\beta_1 \leq \hat{\beta}_1(w,\theta)$*

$$\eta(w,\theta) - \delta G(w) \geq \int_{w-\theta}^{\frac{w}{1-\phi}} (\beta_1 - s(1-\phi))\, dF_s(s)$$

where $G(w)$ and $\eta(w,\theta))$ are given by (4.10) and (4.16), respectively.

Facts 4.2 and 4.3 are crucial for guaranteeing an optimal reservation quantity for any given type θ buyer; as the seller's expected discounted profits from offering the contract to a type θ buyer is concave in nature with the two results presented.

Fact 4.4 *For any given (w,θ), there exists $\hat{\beta}_1(w,\theta) \geq w$ such that for all $\beta_1 \leq \hat{\beta}_1(w,\theta)$:*

$$\rho > \hat{\rho}(w,\theta) \quad \Leftrightarrow \quad q^*(w,\theta) > y^*(w,\theta)$$

where $\hat{\rho}(w,\theta)$ is given by (4.9).

The previous result in Fact 4.4 presents a criteria which determines when it is optimal for the seller to delay her production for a θ-type buyer. Further, it allows us to segment the θ-types into two groups such that the seller produces up to reservation amount for one group, but delays her production for the other group. Here, we identify a buyer to be of *critical delayed type* $\hat{\theta}(w)$ if seller is indifferent between producing up to reservation amount for this buyer at the first period and delaying production till the second period. In mathematical terms, we represent this *critical delayed type* $\hat{\theta}(w)$ as

$$\hat{\theta}(w) = \{\theta : \hat{\rho}(w,\theta) = \rho\}, \tag{4.17}$$

and for notational purposes, we define $\hat{\theta}(w) = \underline{\theta}$ if $\rho > \hat{\rho}(w,\theta)$, $\forall\theta$; and $\hat{\theta}(w) = \bar{\theta}$ if $\rho \leq \hat{\rho}(w,\theta)$, $\forall\theta$. The next result suggests that for higher execution prices, the seller considers the delayed production option for all buyers. On the contrary, when execution price is low, the seller only considers the delayed production option for buyers of higher θ-types.

Fact 4.5 *There exists a unique $\hat{\theta}(w)$ in the range of $[\underline{\theta}, \bar{\theta}]$. Moreover,*

(i) for all $w \leq \underline{s} + \underline{\theta}$, $\theta \leq \hat{\theta}(w) \Leftrightarrow \rho \leq \hat{\rho}(w, \theta)$;

(ii) for all $w > \beta_1$, $\hat{\theta}(w) = \underline{\theta}$.

where $\hat{\rho}(w, \theta)$ and $\hat{\theta}(w)$ are defined in (4.9) and (4.17), respectively.

For a more complete analysis of optimal contract structures, we need to understand the relation between the seller's delayed production option and her choice of offering a flat versus volume-dependent reservation price schedules. We present the following two results which allows us to understand the linkage between the two seemingly independent choices for the seller. For notational purposes, we define δ^* as the critical relation between the seller's and buyer's discount rates such that:

$$\delta^* G(\tilde{w}_f) = (1 + r_S)\beta_0 - E[s](1 - \phi) \tag{4.18}$$

where we define $\tilde{w}_f$ by solving

$$(1 + r_S)\beta_0 - E[s](1 - \phi) = \int_{\frac{\tilde{w}_f}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s) \tag{4.19}$$

Note that equation (4.19), which defines the critical value, $\tilde{w}_f$, can be interpreted as:

$$\tilde{w}_f = \sup\left\{w : \xi^*(w, \theta) = K, \ \ \forall \theta \in [\underline{\theta}, \bar{\theta}]\right\}. \tag{4.20}$$

Equation (4.18) suggests two important result: (i) δ^* is positive; and (ii) the seller is being indifferent, in two-folds, at this critical relation between the seller's and buyer's discount rates. First, she is indifferent of whether the price schedule is flat or volume-dependent; and second, indifferent of whether production is delayed or its level is up to reservation amount. This critical relation, δ^*, is useful for the following result:

Fact 4.6 *(i) Suppose $\delta < \delta^*$. This implies that $\tilde{w}_c < \tilde{w}_f$ which is equivalent to $M(w) > 0$ for all $w \in [\tilde{w}_c, \tilde{w}_f]$;*

(ii) The reverse is also true. Suppose $\delta \geq \delta^*$, this implies that $\tilde{w}_f \leq \tilde{w}_c$ which is equivalent to $M(w) \leq 0$ for all $w \in [\tilde{w}_f, \tilde{w}_c]$,

where $\tilde{w}_c$, $M(w)$, δ^* and $\tilde{w}_f$ are given by (4.12), (4.13), (4.18) and (4.19), respectively.

According to the definitions of $\tilde{w}_c$ and $\tilde{w}_f$, presented in (4.12) and (4.19), respectively, we know that for any execution price lower than $\min(\tilde{w}_c, \tilde{w}_f)$, we have both optimal production and reservation quantities to reach the seller's capacity K. Interestingly, the result presented in Fact 4.6 suggests in part(i) that when the seller is relatively more patient than this critical δ^* relation, there is a range of execution prices where all the buyers' reservation quantities are θ-dependent and the seller does not consider the delayed production option. In mathematical terms, this can be represented by the relation

$$\delta G(w) < (1 + r_S)\beta_0 - E[s](1 - \phi) < \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s), \tag{4.21}$$

when we have $\tilde{w}_c < w < \tilde{w}_f$, where $G(w)$, $\tilde{w}_c$ and $\tilde{w}_f$ are given by (4.10), (4.12) and (4.19), respectively. An important note for this is that for any case where the seller is more patient than the buyer, i.e. $\delta \leq 0$, it falls into this category. On the contrary, in part(ii), when the seller is relatively less patient than this critical δ^* relation, there is a range of execution prices where all the buyers reserve up to K units while the seller considers the delayed production option; this can be mathematically represented by the relation

$$\int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s) < (1 + r_S)\beta_0 - E[s](1 - \phi) < \delta G(w), \tag{4.22}$$

when we have $\tilde{w}_f < w < \tilde{w}_c$, where $G(w)$, $\tilde{w}_c$ and $\tilde{w}_f$ are given by (4.10), (4.12) and (4.19), respectively. Notice that the seller's patience level, as compared to the critical relation, δ^*, shifts the regime from volume-dependent reservation price schedule with no delayed production to a flat price schedule with delayed production in optimal contracts. This can be explained by the fact that when the seller is less patient, she is more interested in earlier receiving payments, preferably in the form of a sales contract; and at the same time she is less willing to commit to high initial levels of production and thus delaying parts of her production to the second period.

We now present another important fact which states that for low levels of w, specifically for any $w \leq \tilde{w}_f$, the seller either considers a delayed production for all buyers; or produces up to level which meets each type θ buyer's optimal reservation quantity.

Fact 4.7 *For any $w \leq \tilde{w}_f$, $\hat{\rho}(w, \theta)$ is independent of θ. Moreover, $M(w) > 0$ if and only if $\rho \leq \hat{\rho}(w, \theta)$ for all θ; where $(\hat{\rho}(w, \theta), M(w), \tilde{w}_f)$ are defined in (4.9), (4.13) and (4.19), respectively.*

Given all the facts listed above, and also the definitions described in the previous section, we now present the seller's optimal contract offer for a fixed exercise price. The following proposition gives the result.

Proposition 4.1 *Suppose $\beta_1 \leq \underline{s} + \underline{\theta}$.*

(i) If the flat price condition is satisfied, the optimal contracts are not volume-dependent. In the optimal offer, the reservation price is constant and given by $\varphi(K, w, \underline{\theta})$; the optimal reservation and production amounts for buyers of any θ-type are given by $q^*(w, \theta) = K$, and

$$y^*(w,\theta) = \begin{cases} \bar{F}_D^{-1}\left[\dfrac{(1+r_S)\beta_0 - E[s](1-\phi) - \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s)}{\int_{\underline{s}}^{\frac{w}{1-\phi}} (\beta_1 - s(1-\phi))\, dF_s(s)}\right] & \textit{for } w \leq \underline{s} + \underline{\theta} \\ \\ \bar{F}_D^{-1}\left[\dfrac{((1+r_S)\beta_0 - E[s](1-\phi))}{\int_{\underline{\theta}}^{\bar{\theta}} \int_{w-\theta}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s)\, dF_\theta(\theta)}\right] & \textit{for } w > \underline{s} + \underline{\theta} \end{cases} . \tag{4.23}$$

(ii) If the volume-dependency condition is satisfied, then there exists $\hat{\beta}_1(w, \theta) \geq w$ such that for all $\beta_1 \leq \hat{\beta}_1(w, \theta)$, the optimal reservation price schedule for the seller is volume-dependent. Further, the seller delays parts of her production for all buyers of type θ greater that $\hat{\theta}(w)$. Specifically, given the optimal contract

offer, the optimal quantity ordered for type θ buyer is

$$q^*(w,\theta) = \begin{cases} \bar{F}_\theta^{-1}\left[\frac{(1+r_S)\beta_0 - E[s](1-\phi) - \delta G(w)}{\eta(w,\theta)}\right] & \text{for } \theta \le \hat{\theta}(w) \\ \bar{F}_D^{-1}\left[\frac{\int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s) - \delta G(w)}{\eta(w,\theta) - \int_{w-\theta}^{\frac{w}{1-\phi}} (\beta_1 - s(1-\phi))\, dF_s(s)}\right] & \text{for } \theta > \hat{\theta}(w) \end{cases}, \tag{4.24}$$

where $G(w), \eta(w,\theta)$ is defined in (4.10) and (4.16), respectively; and $\hat{\theta}(w)$ given by (4.17). Further $q^*(w,\theta) < \bar{D}$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. The optimal quantity produced for type θ buyer by the seller is

$$y^*(w,\theta) = \begin{cases} q^*(w,\theta) & \text{for } \theta \le \hat{\theta}(w) \\ \bar{F}_D^{-1}\left[\frac{(1+r_S)\beta_0 - E[s](1-\phi) - \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s)}{\int_{w-\theta}^{\frac{w}{1-\phi}} (\beta_1 - s(1-\phi))\, dF_s(s)}\right] & \text{for } \theta > \hat{\theta}(w) \end{cases}, \tag{4.25}$$

To the extreme, if $\hat{\theta}(w) = \bar{\theta}$, there is no delayed production. The total reservation fee paid by a type θ buyer is

$$R(q^*(w,\theta), w) = \varphi(q^*(w,\theta), w, \theta) - \int_{\underline{\theta}}^{\theta} \left.\frac{\partial \varphi(q,w,a)}{\partial a}\right|_{q=q^*(w,a)} da. \tag{4.26}$$

(iii) *Suppose both the flat price and volume-dependency conditions are not satisfied. The optimal contract offered by the seller is volume-dependent for the lower type θ buyers with no delayed production; and flat-priced for the higher type θ buyers with delayed production. Specifically, the optimal quantity ordered for type θ buyer is*

$$q^*(w,\theta) = \begin{cases} \bar{F}_\theta^{-1}\left[\frac{(1+r_S)\beta_0 - E[s](1-\phi) - \delta G(w)}{\eta(w,\theta)}\right] & \text{for } \theta \le \hat{\theta}(w) \\ K & \text{for } \theta > \hat{\theta}(w) \end{cases}, \tag{4.27}$$

where $q^*(w,\theta) < \bar{D}$ for all $\theta < \hat{\theta}(w)$. The optimal quantity produced for type θ buyer by the seller is

$$y^*(w,\theta) = \begin{cases} q^*(w,\theta) & \text{for } \theta \le \hat{\theta}(w) \\ \xi^*(w) & \text{for } \theta > \hat{\theta}(w) \end{cases}, \tag{4.28}$$

where

$$\xi^*(w) = \begin{cases} \bar{F}_D^{-1}\left[\frac{(1+r_S)\beta_0 - E[s](1-\phi) - \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s)}{\int_{\underline{s}}^{\frac{w}{1-\phi}} (\beta_1 - s(1-\phi))\, dF_s(s)}\right] & \text{for } w \leq \underline{s} + \underline{\theta} \\ \\ \bar{F}_D^{-1}\left[\frac{\bar{F}_\theta(\hat{\theta}(w))((1+r_S)\beta_0 - E[s](1-\phi))}{\int_{\hat{\theta}(w)}^{\theta} \int_{w-\theta}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s)\, dF_\theta(\theta)}\right] & \text{for } w > \underline{s} + \underline{\theta} \end{cases}, \tag{4.29}$$

and $\hat{\theta}(w)$ is given by (4.17). The total reservation fee paid by a type θ buyer is

$$R(q^*(w,\theta), w) = \begin{cases} \varphi(q^*(w,\theta), w, \theta) - \int_{\underline{\theta}}^{\theta} \frac{\partial \varphi(q,w,a)}{\partial a}\Big|_{q=q^*(w,a)} da & \text{for } q^*(w,\theta) < \bar{D} \\ \varphi(K, w, \hat{\theta}(w)) & \text{otherwise} \end{cases}. \tag{4.30}$$

Proof of Proposition 4.1: We use results from Fact 4.5, and the analysis given by the proof of Fact 4.4, which is included in the appendix of this Chapter; and by observing the expression (A.18) and (A.19), the optimal result reservation amount by the θ-type buyer equals K, i.e. up to the seller's production capacity, if and only if any one of the following cases is satisfied:

(F:I) $\hat{\theta}(w) = \bar{\theta}$ and $\delta G(w) \geq (1 + r_S)\beta_0 - E[s](1 - \phi)$;

(F:II) $\hat{\theta}(w) = \underline{\theta}$ and $\delta G(w) \geq \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s)$;

(F:III) $\delta G(w) \geq \max\left\{(1 + r_S)\beta_0 - E[s](1 - \phi), \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s)\right\}$.

Consider the first flat price condition, $w \geq \beta_1$. Fact 4.5 tells that $\hat{\theta}(w) = \underline{\theta}$ for all $w \geq \beta_1$, and from (4.14) we know that the right hand side expression is non-positive for $w \geq \beta_1$, which guarantees that $\delta G(w) \geq \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s)$, which is equivalent to (F:I).

Now, consider the second flat price condition, which is, we have all the following three conditions being satisfied: (i)$r_S > r_B$; (ii)$\beta_0 \leq E[s](1 - \phi)/(1 + r_B)$; and (iii)$w \leq \tilde{w}_c$ where $\tilde{w}_c$ is defined in (4.12). Note that the first two conditions guarantees the existence of $\tilde{w}_c$, as shown in (4.12). For any $w \leq \tilde{w}_c$, we have $\delta G(w) \geq (1 + r_S)\beta_0 - E[s](1 - \phi)$. We now use results from Fact 4.6. First, suppose $\delta < \delta^*$, we know $\tilde{w}_c < \tilde{w}_f$. If $M(w) > 0$, since $w < \tilde{w}_f$, we know $\hat{\theta}(w) = \bar{\theta}$ from Fact 4.7, hence equivalent to (F:II); for $M(w) \leq 0$, we have $\delta G(w) \geq \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s)$ as well, thus equivalent to (F:III). Now, suppose $\delta \geq \delta^*$, we know $\tilde{w}_f < \tilde{w}_c$. First consider $w \leq \tilde{w}_f$, from Fact 4.7, we know for $M(w) > 0$, thus $\hat{\theta} = \bar{\theta}$ which is equivalent to (F:II); and for $M(w) \leq 0$, we have $\delta G(w) \geq \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s)$, thus equivalent to (F:III). Now, for $w \in (\tilde{w}_f, \tilde{w}_c)$, Fact 4.6 tells us that $M(w) \leq 0$, i.e. $\delta G(w) \geq \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s)$, thus equivalent to (F:III). This completes the case for the second flat price condition. Considering all the possible cases, we conclude that the flat price condition implies $q^*(w, \theta) = K$ for all θ.

Given that $q^*(w, \theta) = K$ for all θ-type buyers under the flat price condition, the seller offers a constant contract. Thus, we consider a revised optimization problem presented in (4.15) where in this special case, we have $\theta^I = [\underline{\theta}, \bar{\theta}]$. For optimal reservation price, we look at the definition of $\hat{\pi}_B(q, w, \theta)$, where for $\theta \in \theta^I$, we have $\hat{\pi}_B(q, w, \theta) = -R^I(w) + \varphi(K, w, \theta)$. Thus, maximizing $R^I(w)$ subject to $\hat{\pi}_B(q, w, \theta) \geq 0$ for all θ is equivalent to setting $R^I(w)$ to the minimum of $\varphi(K, w, \theta)$ over all θ. Since $\varphi(q, w, \theta)$ is non-decreasing in θ from (2.14), we have our optimal reservation flat price to be $\varphi(K, w, \underline{\theta})$. For optimal production quantity, from (4.15):

$$y^*(w) = \arg\max_y \int_{\theta \in \theta^I} V(y, K, w, \theta)\, dF_\theta(\theta) \tag{4.31}$$

where we know $\theta^I = [\underline{\theta}, \bar{\theta}]$. Using results from (4.7) and (4.8), and differentiating the right hand expression of (4.31), we have:

$$\frac{d \int_{\underline{\theta}}^{\bar{\theta}} V(y, K, w, \theta)\, dF_\theta(\theta)}{dy}$$
$$= \begin{cases} -\beta_0 + \frac{E[s](1-\phi)}{1+r_S} & \text{for } y \geq K \\ -\beta_0 + \frac{1}{1+r_S}\Big(E[s](1-\phi) - \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s) & \\ \qquad + \int_{\underline{\theta}}^{\bar{\theta}} \int_{w-\theta}^{\frac{w}{1-\phi}} (\beta_1 - s(1-\phi))\, dF_s(s)\, dF_\theta(\theta) \Big) & \text{for } y < K \end{cases}$$

Thus the first order conditions give us the optimal production quantity to be:

$$y^*(w) = \bar{F}_D^{-1}\left[\frac{(1+r_S)\beta_0 - E[s](1-\phi) - \int_{\frac{w}{1-\phi}}^{\bar{s}}(\beta_1 - s(1-\phi))\, Fs(s)}{\int_{\underline{\theta}}^{\bar{\theta}}\int_{w-\theta}^{\frac{w}{1-\phi}}(\beta_1 - s(1-\phi))\, dF_s(s)\, dF_\theta(\theta)}\right] \tag{4.32}$$

Considering separate cases of $w \leq \bar{s}+\underline{\theta}$ and $w > \underline{s}+\underline{\theta}$ completes our proof for part(i).

Now, we consider the case for volume-dependent price schedules. Using a similar analysis given earlier in part(i), we know that the optimal reservation quantity is strictly less than $\bar{D}$ if and only if any one of the following cases satisfy:

(VD:I) $\hat{\theta}(w) = \bar{\theta}$ and $\delta G(w) < (1+r_S)\beta_0 - E[s](1-\phi)$;

(VD:II) $\hat{\theta}(w) = \underline{\theta}$ and $\delta G(w) < \int_{\frac{w}{1-\phi}}^{\bar{s}}(\beta_1 - s(1-\phi))\, dF_s(s)$;

(VD:III) $\delta G(w) < \min\left\{(1+r_S)\beta_0 - E[s](1-\phi), \int_{\frac{w}{1-\phi}}^{\bar{s}}(\beta_1 - s(1-\phi))\, dF_s(s)\right\}$.

Suppose the volume-dependency condition is satisfied, i.e. we have $M(w) > 0$ and $w \in (\tilde{w}_c, \bar{s}(1-\phi))$. For $M(w) > 0$, we know that $\delta G(w) < (1+r_S)\beta_0 - E[s](1-\phi)$ for all $\theta > \hat{\theta}(w)$. Consider three cases: (i) for $\hat{\theta}(w) = \bar{\theta}$, then this satisfy (VD:I); (ii) for $\hat{\theta}(w) = \underline{\theta}$, then $\delta G(w) < (1+r_S)\beta_0 - E[s](1-\phi)$ may not satisfy; but $w > \tilde{w}_c$ guarantees that $\delta G(w) < (1+r_S)\beta_0 - E[s](1-\phi)$, thus satisfying (VD:II); and (iii) for $\hat{\theta}(w) < \bar{\theta}$, for any $\theta \leq \hat{\theta}(w)$ $w > \tilde{w}_c$ guarantees us that $\delta G(w) < (1+r_S)\beta_0 - E[s](1-\phi)$, thus satisfying (VD:III). Since all the possible cases have been considered under the volume-dependency condition, we conclude that the volume-dependency condition implies $q^*(w,\theta) < \bar{D}$ for all θ.

Using the definition of (4.17), and results from Fact 4.2, Fact 4.3 and Fact 4.4, we know the type θ buyer's optimal reservation quantity is given by the first order conditions, i.e. by setting (A.18) and (A.19) to zero. Thus we have

$$q^*(w,\theta) = \begin{cases} q_1^*(w,\theta) & \text{for } \theta \leq \hat{\theta}(w) \\ q_2^*(w,\theta) & \text{for } \theta > \hat{\theta}(w) \end{cases}, \tag{4.33}$$

where

$$q_1^*(w,\theta) = \bar{F}_D^{-1}\left[\frac{(1+r_S)\beta_0 - E[s](1-\phi) - \delta G(w)}{\eta(w,\theta) - \delta G(w)}\right], \tag{4.34}$$

$$q_2^*(w,\theta) = \bar{F}_D^{-1}\left[\frac{\int_{\frac{w}{1-\phi}}^{\bar{s}}(\beta_1 - s(1-\phi))\,dF_s(s) - \delta G(w)}{\eta(w,\theta) - \delta G(w) - \int_{w-\theta}^{\frac{w}{1-\phi}}(\beta_1 - s(1-\phi))\,dF_s(s)}\right]. \tag{4.35}$$

Again, using the definition of (4.17), and results from Fact 4.4 and Lemma 4.1, the optimal production quantity for type θ buyer is given by:

$$y^*(w,\theta) = \begin{cases} q^*(w,\theta) & \text{for } \theta \le \hat{\theta}(w) \\ \xi^*(w,\theta) & \text{for } \theta > \hat{\theta}(w) \end{cases}$$

where $\xi^*(w,\theta)$ is given by (4.6). Using the exact same argument as in (2.15), the optimal reservation price paid by type θ buyer is

$$R(q^*(w,\theta),w) = \varphi(q^*(w,\theta),w,\theta) - \int_{\underline{\theta}}^{\theta} \left.\frac{\partial\varphi(q,w,a)}{\partial a}\right|_{q=q^*(w,a)} da,$$

this completes the proof for part(ii).

Now, we consider the case for combination pricing schedules. Using the result from Fact 4.5, there can only be two types of combination pricing schedules: first being a flat pricing schedule offered to buyers of lower θ types, and a volume-dependent pricing schedule for buyers of hight θ types; and vice versa for the second. We now show the first type cannot possibly happen.

Flat pricing schedule offered to buyers of type $\theta \le \hat{\theta}(w)$ implies $w \le \tilde{w}_c$ as discussed in (4.11) and (4.12). First suppose $\tilde{w}_f > \tilde{w}_c$, so for any $w \le \tilde{w}_c$, we also have $w < \tilde{w}_f$, if $M(w) > 0$, Fact 4.7 suggests that $\hat{\theta}(w) = \bar{\theta}$; on the contrary if $M(w) \le 0$, $q^*(w,\theta) = K$ for $\theta > \hat{\theta}(w)$, i.e. for both cases there is no volume-dependent part in the schedule. Now suppose $\tilde{w}_f \le \tilde{w}_c$. For any $w \le \tilde{w}_f$, the argument follows exactly the same as the earlier one for $\tilde{w}_f > \tilde{w}_c$; now for $w \in (\tilde{w}_f, \tilde{w}_c)$, from Fact 4.6, we know $M(w) \le 0$, which implies there is no volume-dependent part in the schedule. This proves that no such pricing schedule exist.

Now, we consider the case for "low volume dependent-high flat" pricing schedules. Using a similar analysis given earlier in parts(i) and (ii), we know that the optimal reservation quantity is strictly less than $\bar{D}$ for $\theta \le \hat{\theta}(w)$ only but equals K for $\theta > \hat{\theta}(w)$

if and only if all three of the following conditions satisfy:

(i) $\hat{\theta}(w) \in (\underline{\theta}, \bar{\theta})$;

(ii) $\delta G(w) < (1 + r_S)\beta_0 - E[s](1 - \phi)$;

(iii) $\delta G(w) \geq \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s)$

Notice that the intersection of the above three conditions is the exact complement of the set [(F:I),(F:II),(F:III),(VD:I),(VD:II),(VD:III)]. Thus, this implies that, all the three conditions satisfy if and only if neither the flat price conditions nor the volume-dependency conditions satisfy, which proves our first result for part(iii).

Given that buyers of type $\theta > \hat{\theta}(w)$ have $q^*(w,\theta) = K$ and that $\hat{\theta}(w)$ is strictly less than $\bar{\theta}$; we have to consider a separate optimization problem given by (4.15). Note that the determination of $R^I(w)$ is independent of $R^D(q^*(w,\theta), w)$, and thus the analysis for buyers for type $\theta > \hat{\theta}(w)$ is just the same as discussed earlier for the proof of part(i), taking $\underline{\theta}^I = \min\{\theta : \theta \in \theta^I\}$, where by Fact 4.5 and the definition of $\hat{\theta}(w)$ in (4.17), we have $\underline{\theta}^I = \hat{\theta}(w)$. Thus we have $R^I(w) = \varphi(K, w, \hat{\theta}(w))$, $q^*(w,\theta) = K$ and $y^*(w,\theta) = y^*(w)$ for all $\theta > \hat{\theta}(w)$ where $y^*(w)$ is given by (4.32).

For the volume-dependent portion of the price schedule, i.e. for $\theta \leq \hat{\theta}(w)$, we have to determine $R^D(q^*(w,\theta), w), q^*(w,\theta)$ and $y^*(w,\theta)$ for designing the optimal contract. Since this can be considered independently from the $\theta > \hat{\theta}(w)$ case, the analysis here is exactly the same as that of part(ii) earlier. Thus, for $\theta \leq \hat{\theta}(w)$, we have $q^*(w,\theta) = q_1^*(w,\theta)$, where $q_1^*(w,\theta)$ is given by (4.34), and $y^*(w,\theta) = q^*(w,\theta)$, and $R^D(q^*(w,\theta), w)$ is given by (4.26). Combining the results for the lower θ volume-dependent price schedule results with the higher θ flat price results, this completes our proof for part(iii). ■

4.2.4 Optimal Contract Offer Structures

The result presented in Proposition 4.1 is a strong one. Not only does it determine the specifics of an optimal contract offered by the seller given any fixed execution price, w, it also allows us to understand the nature of optimal contract structures in depth.

First, the result suggests that for any given fixed execution price, there are only three possible type of optimal contract structures, namely: (i) flat price schedule; (ii) volume-dependent reservation price schedule; and (iii) a mixed price schedule, in specific, volume-dependent for low quantities but flat for high quantities. The result further identifies a complete set of conditions which corresponds to each type of contract structure, thus establishing a framework which determines the specifics of the optimal contract offer for the whole parameter set.

Further, apart from understanding the volume-dependency nature of optimal contracts, a deeper understanding of the result allows us to illustrate the seller's utilization of the delayed production option. As Fact 4.5 suggests, there are three levels of utilization of this delayed production option:

(a) *Zero level*: production up to reservation amount for all buyer types;

(b) *High level*: delaying production for all buyer buyers;

(c) *Medium level*: production up to reservation amount for buyers of lower θ-types and delaying production for buyers of higher θ-types.

In fact we can distinctively identify the three utilization production levels in both the flat and volume-dependent price schedules; while only type (c) can be observed in low volume dependent-high flat price schedules. This implies that there are altogether 7 possible different variations of an optimal contract structure.

As presented clearly in the proof for Proposition 4.1, we managed to partite the whole parameter set such that each set corresponds to a specific set of conditions. Satisfying a specific set of condition is equivalent to saying that the optimal reservation quantities of the different θ buyers behave in a specific way; or interpreted differently, the optimal reservation price schedule which the seller offers is of a specific structure, which we describe in detail below. We partite the whole parameter set into three regions:

Flat Price Set:

First, we consider the set which corresponds to the *flat price condition.* As shown in the proof of Proposition 4.1, the condition satisfies if and only if buyer of all θ-types reserve up to the seller's production capacity, K, which can occur if any one of the three following conditions, identified as [(F:I),(F:II),(F:III)], is satisfied:

(F:I) $\hat{\theta}(w) = \bar{\theta}$ and $\delta G(w) \geq (1 + r_S)\beta_0 - E[s](1 - \phi)$;

(F:II) $\hat{\theta}(w) = \underline{\theta}$ and $\delta G(w) \geq \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s)$;

(F:III) $\delta G(w) \geq \max\left\{(1 + r_S)\beta_0 - E[s](1 - \phi), \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s)\right\}$.

Notice the relation $\delta G(w) \geq (1 + r_S)\beta_0 - E[s](1 - \phi)$ is equivalent to the seller offering a flat price schedule given there is *no delayed production*; and $\delta G(w) \geq \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s)$ is equivalent to the seller offering a flat price schedule given there is *delayed production.* Thus, if (F:I) is satisfied, it corresponds to: (i) the seller does not utilize the delayed production option; and (ii) a flat price schedule given there is no delayed production. This implies a flat price schedule of type (a), i.e. the sellers offers a flat price schedule with no delayed production for buyers of all θ-types. Using a similar analysis, (F:II) corresponds to a flat price schedule of type (b); whereas (F:III) corresponds to a flat price schedule of type (c).

Volume Dependent Set:

Second, we consider the set which corresponds to the *volume-dependency condition.* We know the condition satisfies if and only if buyer of all θ-types reserve a θ dependent quantity, which has to be strictly less than the seller's production capacity, K. This can occur if any one of the three following conditions, identified as [(VD:I),(VD:II),(VD:III)], is satisfied:

(VD:I) $\hat{\theta}(w) = \bar{\theta}$ and $\delta G(w) < (1 + r_S)\beta_0 - E[s](1 - \phi)$;

(VD:II) $\hat{\theta}(w) = \underline{\theta}$ and $\delta G(w) < \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s)$;

(VD:III) $\delta G(w) < \min\left\{(1 + r_S)\beta_0 - E[s](1 - \phi), \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s)\right\}$.

The relation $\delta G(w) < (1 + r_S)\beta_0 - E[s](1 - \phi)$ is equivalent to the seller offering a volume-dependent price schedule given there is *no delayed production*; and $\delta G(w) < \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1 - \phi))\, dF_s(s)$ is equivalent to the seller offering a volume-dependent price schedule given there is *delayed production.* Thus, (VD:III) corresponds to the seller offering volume-dependent price schedules for both the delayed production and no delayed production cases. Combining with results from Fact 4.5, this implies a volume-dependent price schedule of type (c), i.e. the seller offers a volume-dependent price schedule, with no delayed production for buyers of low θ-types, and delayed production for buyers of high θ-types. Using a similar analysis, (VD:I) corresponds to a volume-dependent price schedule of type (a); and (VD:II) corresponds to a volume-dependent price schedule of type (b).

Low Volume Dependent-High Flat Set:

Last but not least, we consider the set which corresponds to the complement of the union of the *flat price condition* and the *volume-dependency condition*, i.e. when neither the *flat price condition* nor the *volume-dependency condition* satisfy. As shown in the proof of Proposition 4.1, this happens if and only if all buyers of type $\theta \leq \hat{\theta}(w)$ reserve a θ-dependent quantity which is strictly less than K; and all buyers of type $\theta > \hat{\theta}(w)$ reserve up to K units. Note that this critical $\hat{\theta}(w)$ type is guaranteed to exist, which is expressed and analyzed in (4.17). Clearly, this can occur if and only if all of the conditions presented earlier, i.e. [(F:I),(F:II),(F:III),(VD:I),(VD:II),(VD:III)], do not satisfy. This is equivalent to having all of the three following conditions satisfy: (i)$\hat{\theta}(w) \in (\underline{\theta}, \bar{\theta})$; (ii) $\delta G(w) < (1 + r_S)\beta_0 - E[s](1 - \phi)$; and (iii) $\delta G(w) \geq \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s)$. Notice that condition(i) implies that there exist this critical $\hat{\theta}(w)$-buyer such that for buyers of type $\theta \leq \hat{\theta}(w)$, the seller produces up to their reservation amount; on the contrary, for buyers of type $\theta < \hat{\theta}(w)$ the seller delays parts of her production. Condition(ii) implies that given production equals reservation, the optimal price schedule the seller offers is volume-dependent; while condition(iii) implies that given there is delayed production, the optimal price schedule the seller offers is flat. Thus, having all the three conditions satisfy corresponds to a *low volume* *dependent-high flat* pricing schedule, and it is of type (c). Notice that for such pricing schedules, they cannot belong to type (a) or type (b), as imposed by condition(i).

4.2.5 Volume Dependency of Optimal Price Schedule

Another important aspect of the structure of a contract offer is the nature of volume dependency of a reservation price schedule. A very common form of volume dependent pricing employed in practice is volume discounts, i.e., a reduction in average pricing with higher purchases. Volume discounts imply a concave total price curve as a function of quantity purchased. The opposite of volume discounts is volume premia, i.e., increasing average cost with quantity purchased. This is a reverse form of a volume incentive, which can be viewed as extra incentives given to the seller by the buyer to commit to a high level of production. Conversely, it can be the case that the seller takes advantage of the premium the buyer puts on the seller's product by charging higher average prices to more dependent types of buyers (higher θ types). Contrary to volume discounts, volume premia imply convex total price curves.

Given the complexity of the transaction structure at $t = 2$ with options commitments and the resulting complexity of the two-sided determination of options contract terms, it is not surprising for either or both types of volume dependent pricing structures to emerge in the optimal contracts.

Here, we use a similar analysis as presented in Section 2.3 to understand the nature of volume dependency of an optimal price schedule. The following proposition states the conditions under which the optimal contract offers volume discounts.

Proposition 4.2 *Suppose that the flat price condition is not satisfied. If $w \leq \underline{s}(1 - \phi)$, the optimal contract offers quantity discounts for the entire reservation price curve, $R(q)$.*

Proof of Proposition 4.2: Using the exact same analysis for proof of Proposition 2.2, we have the volume-dependency of reservation price schedule to be determined by the sign of the expression below

$$\left.\frac{d^2R(w,\theta(q))}{dq^2}\right|_{q=q^*(w,\theta)} = \frac{d^2\varphi(q^*(w,\theta),w,\theta)}{dq^*(w,\theta)^2} + \frac{\partial}{\partial\theta}\frac{d\varphi(q^*(w,\theta),w,\theta)}{dq^*(w,\theta)}\left[\frac{dq^*(w,\theta)}{d\theta}\right]^{-1}, \tag{4.36}$$

where $q^*(w,\theta)$ is given by(4.33). We consider here both the production equals reservation and the delayed production cases, and that $w \leq \underline{s}(1-\phi)$. First for the production equals reservation case, optimal reservation is given by (4.34); differentiating the expression with respect to θ gives

$$\bar{F}_D(q_1^*(w,\theta))\frac{d\varrho(w,\theta)}{d\theta} - f_D(q_1^*(w,\theta))\frac{dq_1^*(w,\theta)}{d\theta}\varrho(w,\theta) = 0, \tag{4.37}$$

where $\varrho(w,\theta)$ is given by (A.3). By Fact 4.2, we know $\varrho(w,\theta) \geq 0$, and clearly, for $w \leq \underline{s}(1-\phi)$, we have

$$\varrho(w,\theta) = \rho(\phi E[s] + \theta - g_\theta(\theta)) \geq 0. \tag{4.38}$$

Further, differentiating $\varrho(w,\theta)$, given by (A.3), with respect to θ gives:

$$\frac{d\varrho(w,\theta)}{d\theta} = \begin{cases} \rho(1-g_\theta'(\theta)) & \text{for } w < \underline{s}+\theta \\ f_s(w-\theta)\big(\phi(w-\theta)+\theta-g_\theta(\theta) & \\ \quad +\delta(g_s(w-\theta)-g_\theta(\theta))\big) & \\ \quad\quad +(1-\rho g_\theta'(\theta))\bar{F}_s(w-\theta) & \text{for } w \geq \underline{s}+\underline{\theta} \end{cases}. \tag{4.39}$$

Second, we consider the delayed production case, where reservation is given by (4.35), differentiating the expression with respect to θ gives:

$$\bar{F}_D(q_2^*(w,\theta))\frac{d\varsigma(w,\theta)}{d\theta} - f_D(q_2^*(w,\theta))\frac{dq_2^*(w,\theta)}{d\theta}\varsigma(w,\theta) = 0 \tag{4.40}$$

where $\varsigma(w,\theta)$ is given by (A.8). For $w \leq \underline{s}(1-\phi)$, we have $\varsigma(w,\theta) = \varrho(w,\theta)$ thus given by the expression (4.38); and its derivative with respect to θ is given $\rho(1-g_\theta'(\theta))$, same as in (4.39). Thus there is no difference in the two expressions between the two cases. Thus, by plugging (4.38) and (4.39) into (4.36), we have for any $w \leq \underline{s} + \underline{\theta}$:

$$\frac{d^2R(w,\theta(q))}{dq^2} = -\frac{f_D(q^*(w,\theta))}{1+r_B} \cdot \frac{g_\theta(\theta) - g_\theta'(\theta)(\phi E[s] + \theta)}{1 - g_\theta'(\theta)} \leq 0, \forall\theta. \tag{4.41}$$

This completes our proof. ■

Proposition 4.2 states that for low w values, the optimal contract *only* takes the shape of volume discounts. An important implication of this is for the case of $w = 0$, i.e., when the contract is a true "sales" contract rather than an "option" contract. The following corollary states the result.

Corollary 4.1 *When the contract terms are such that the supplier is selling the intermediate good to the buyer (i.e., $w = 0$), the volume-dependent pricing, if employed by the seller, always involves volume discounts.*

This result is consistent with the wide-spread use of volume discount schemes in the industry, especially when the contract between the buyer and the seller is a sales contract. Compared to options contracts, sales contracts are relatively simpler in that they do not offer flexibility to the buyer. The buyer commits to receiving all units at the time of the delivery and pays an upfront fee for it. Corollary 4.1 states that as the level of such fixed commitments increases with the quantity sold, the seller finds it more profitable to offer incentives to buyer to purchase more. These incentives strengthen the buyer's willingness to commit to larger quantities, and thus increasing supplier's profits.

Proposition 4.2 states that the concavity nature of a reservation price schedule extends to any option with execution price up to $\underline{s}(1 - \phi)$. There is an important intuition behind this critical execution price. Note that when execution is no more than $\underline{s}(1 - \phi)$, buyers of all θ types execute all of his options, i.e. up to reservation amount, at the second period since $\underline{s}(1 - \phi)$ is the lowest possible bid-price of the spot market; i.e. under any realization of the spot market, it is profitable for the buyer to execute all of his options, for satisfying demand and selling the remaining units to the spot market. In retrospect, for any execution price is below $\underline{s}(1 - \phi)$, there is no difference in terms of flexibility and risk levels of the contract for both the buyer and seller; the implication of an execution price here is merely deferring a portion of the payment to the second period, but there is no uncertainty in terms of the level of payments paid by the buyer to the seller at any given time.

This explains why for execution price no greater than $\underline{s}(1-\phi)$, its corresponding optimal reservation price schedule can still easily preserve its simple concavity nature. On the other hand, "real" option contracts are more complex agreements in terms of the possibilities they imply on the behavior of the involved parties. As a consequence, their volume dependent pricing schedule is also more complicated.

4.3 Determination of Optimal Execution Price

In the previous sections, or specifically in Proposition 4.1, we manage to give a full characteristic for optimal contract structure for a fixed exercise price w for the delayed production model. For a complete characterization of optimal contract design, we now further examine the determination of optimal exercise price given by the seller. This is a very question to ask ourselves as it provides us a complete understanding of the nature of contracts offered by the seller to the buyer, in several dimensions.

A high exercise price contract shifts the supplier's collected fees to the future. Therefore, depending on the buyer's and the seller's specific characteristics, differences in the exercise price has important effects on contract design and the generation of supply chain surplus. More specifically and importantly, the optimal exercise price further determines the structure of the contracts. If the supplier's optimal exercise price, w^*, is zero, then the contracts are traditional sales ones; as the buyer pays upfront for automatic future delivery. In contrast, when $w^* > 0$ the contracts are true options, since for that case, in certain state realizations (e.g., when the consumer demand is low or the spot price is low), the buyer chooses not to exercise the contracts.

Thus, an important question to ask is: when are sales contracts are optimal and when are option contracts optimal? Further, given option contracts are optimal, how is the optimal exercise price, w^*, characterized? Clearly, the supplier's optimization of the exercise price yields endogenously-determined answers to these problems. We now study the supplier's problem of exercise price optimization in detail, and provide a full characterization of the solution as well as the analysis of contract design characteristics.

By using the same analysis as in Chapter 3, and also results from Proposition 4.1, the supplier's global optimization problem can now be written as

$$\max_{w \geq 0} \pi_S(w) \equiv \max_{w \geq 0} E_\theta[R(q^*(w,\theta),w) + V(y^*(w,\theta),q^*(w,\theta),w,\theta)]. \tag{4.42}$$

An important characteristic to notice from (4.42) is that, when optimizing w, the supplier needs to consider the trade-off between collecting revenues now (in the form of reservation fees, $R(q^*(w,\theta),w)$) and collecting revenues in the future (in the form of exercise fees as they affect $V(y^*(w,\theta),q^*(w,\theta),w,\theta)$, as defined in (4.2)). Therefore, one important factor that affects the supplier's decision will be her discount rate. However, the supplier is interacting with the buyer in signing the contracts, and hence she has to take the buyer's preferences into account when determining the optimal exercise price. The final outcome will reflect a combination of the preferences of both parties. The following proposition presents the outcome.

Proposition 4.3 *Suppose $\beta_1 \leq \underline{s} + \underline{\theta} - g_s(\underline{s})$.*

(i) If $r_S < r_B$, there exists a strictly positive exercise price, w^*, where $\underline{s}+\underline{\theta} \leq w^* \leq \underline{s} + \bar{\theta}$. Optimal reservation is flat pricing.

(ii) If $r_S = r_B$, the seller's profit is maximized by setting any exercise price w^*, where $\bar{s}(1-\phi) \leq w^* \leq \underline{s} + \underline{\theta}$. Optimal reservation schedule is low volume dependent-high flat.

(iii) If $r_S > r_B$, there exists critical relations between r_S and r_B, i.e. $\bar{\delta} \geq \underline{\delta} > 0$ such that for all $\delta \leq \underline{\delta}$, the optimal exercise price is $w^* = \bar{s}(1-\phi)$ and optimal reservation schedule is low volume dependent-high flat. For $\delta \geq \bar{\delta}$, $w^* = 0$. That is, selling the intermediate good is optimal for the seller, rather than giving options to the buyer. Furthermore, if $\beta_1 \leq \rho E[s](1-\phi)$ or $\beta_0 \leq E[s](1-\phi)/(1+$

r_B), optimal reservation is flat pricing; otherwise, we have a volume-dependent price schedule with quantity discounts and no delayed production.

Proof of Proposition 4.3: First, using the results from Fact 4.5 and substituting $q^*(w,\theta)$ in the seller's expected profit function, we have

$$\pi_S^*(w) = \int_{\underline{\theta}}^{\hat{\theta}(w)} H_1(q_1^*(w,\theta), w, \theta)\, dF_\theta(\theta) + \int_{\hat{\theta}(w)}^{\bar{\theta}} H_2(q_2^*(w,\theta), w, \theta)\, dF_\theta(\theta), \qquad (4.43)$$

where $H_1(q,w,t), H_2(q,w,\theta)$ are given by (A.16) and (A.17), respectively. Taking the total derivative with respect to w in (4.43), and applying the envelope theorem, we have

$$\begin{aligned}
&\frac{d\pi_S^*(w)}{dw} \\
= & \int_{\underline{\theta}}^{\hat{\theta}(w)} \left[\frac{\partial H_1(q_1^*(w,\theta), w, \theta)}{\partial w} + \left.\frac{\partial H_1(q_1^*(w,\theta), w, \theta)}{\partial q}\right|_{q=q_1^*(w,\theta)} \cdot \frac{dq_1^*(w,\theta)}{dw} \right] dF_\theta(\theta) \\
& + \int_{\hat{\theta}(w)}^{\bar{\theta}} \left[\frac{\partial H_2(q_2^*(w,\theta), w, \theta)}{\partial w} + \left.\frac{\partial H_2(q_2^*(w,\theta), w, \theta)}{\partial q}\right|_{q=q_2^*(w,\theta)} \cdot \frac{dq_2^*(w,\theta)}{dw} \right] dF_\theta(\theta) \\
= & \int_{\underline{\theta}}^{\hat{\theta}(w)} \frac{\partial H_1(q_1^*(w,\theta), w, \theta)}{\partial w}\, dF_\theta(\theta) + \int_{\hat{\theta}(w)}^{\bar{\theta}} \frac{\partial H_2(q_2^*(w,\theta), w, \theta)}{\partial w}\, dF_\theta(\theta)\,. \qquad (4.44)
\end{aligned}$$

Since $\bar{s}(1-\phi) \le \underline{s} + \underline{\theta}$:

$$\begin{aligned}
&(1+r_S)\frac{\partial H_1(q_1^*(w,\theta), w, \theta)}{\partial w} \\
= & \begin{cases} -\delta \left[q_1^*(w,\theta) - F_s\left(\frac{w}{1-\phi}\right) E_D[(q_1^*(w,\theta) - D)^+] \right] & \text{for } w < \underline{s} + \theta; \\ -f_s(w-\theta)\nu(w,\theta) E_D[\min(D, q_1^*(w,\theta))] & \text{for } w \in [\underline{s}+\theta, \bar{s}+\theta]; \\ 0 & \text{for } w > \bar{s}+\theta. \end{cases}
\end{aligned} \qquad (4.45)$$

$$
\begin{aligned}
&(1+r_S)\frac{\partial H_2(q_2^*(w,\theta),w,\theta)}{\partial w}\\
&= \begin{cases}
-\delta q_2^*(w,\theta) & \text{for } w < \underline{s}(1-\phi);\\
-\delta\Bigg(E_D[\min(D,q_2^*(w,\theta))] & \\
\quad +\bar{F}_s\left(\frac{w}{1-\phi}\right) E_D[(q_2^*(w,\theta)-D)^+]\Bigg) & \\
\quad +f_s\left(\frac{w}{1-\phi}\right)\frac{\beta_1-w}{1-\phi}\Bigg(q_2^*(w,\theta)F_D(q_2^*(w,\theta)) & \\
\qquad -\int_{\xi^*(w,\theta)}^{q_2^*(w,\theta)} x\,dF_D(x)\Bigg) & \text{for } w\in[\underline{s}(1-\phi),\bar{s}(1-\phi)];\\
-\delta E_D[\min(D,q_2^*(w,\theta))] & \text{for } w\in(\bar{s}(1-\phi),\underline{s}+\theta);\\
-f_s(w-\theta)\Bigg(\nu(w,\theta)E_D[\min(D,q_2^*(w,\theta))] & \\
-(\beta_1-(1-\phi)(w-\theta)) & \\
\cdot\left(\int_{\xi^*(w,\theta)}^{q_2^*(w,\theta)} x\,dF_D(x)+q_2^*(w,\theta)\bar{F}_D(q_2^*(w,\theta))\right)\Bigg) & \text{for } w\in[\underline{s}+\theta,\bar{s}+\theta];\\
0 & \text{for } w>\bar{s}+\theta.
\end{cases}
\end{aligned} \tag{4.46}
$$

where $\nu(w,\theta)$ is given by (2.23). Here, for notational purposes, we use $q^*(w,\theta)$ such that $q^*(w,\theta) = q_1^*(w,\theta)$ for $\theta \leq \hat{\theta}(w)$ and $q^*(w,\theta) = q_2^*(w,\theta)$ otherwise. Plugging (4.45) and (4.46) in (4.44), we then have for $w < \underline{s}(1-\phi)$:

$$
\frac{d\pi_S^*(w)}{dw} = -\frac{\delta}{1+r_S}\int_{\underline{\theta}}^{\bar{\theta}} q^*(w,\theta)\,dF_\theta(\theta); \tag{4.47}
$$

and for $w \in [\underline{s}(1-\phi), \bar{s}(1-\phi)]$:

$$
\begin{aligned}
\frac{d\pi_S^*(w)}{dw} = -\frac{\delta}{1+r_S}\int_{\underline{\theta}}^{\bar{\theta}} &\Bigg(E_D[\min(D,q^*(w,\theta))]\\
&+\bar{F}_s\left(\frac{w}{1-\phi}\right)E_D[(q^*(w,\theta)-D)^+]\Bigg)\,dF_\theta(\theta)\\
&\quad + f_s\left(\frac{w}{1-\phi}\right)\frac{\beta_1-w}{(1+r_S)(1-\phi)}\\
&\quad \cdot\int_{\hat{\theta}(w)}^{\bar{\theta}}\left(q_2^*(w,\theta)F_D(q_2^*(w,\theta))-\int_{\xi^*(w,\theta)}^{q_2^*(w,\theta)} x\,dF_D(x)\right);
\end{aligned} \tag{4.48}
$$

and for $w \in (\bar{s}(1-\phi), \underline{s}+\underline{\theta})$:

$$\frac{d\pi_S^*(w)}{dw} = -\frac{\delta}{1+r_S}\int_{\underline{\theta}}^{\bar{\theta}} E_D[\min(D, q^*(w,\theta))]\, dF_\theta(\theta); \tag{4.49}$$

and for $w \in [\underline{s}+\underline{\theta}, \underline{s}+\bar{\theta}]$, where we write $w = \underline{s}+\tilde{\theta}$, $\tilde{\theta} \in [\underline{\theta}, \bar{\theta}]$; we first know from Fact 4.5 that $\hat{\theta}(w) = \underline{\theta}$, and thus:

$$\begin{aligned}\frac{d\pi_S^*(w)}{dw} &= -\frac{\delta}{1+r_S}\int_{\tilde{\theta}}^{\bar{\theta}} E_D[\min(D, q^*(w,\theta))]\, dF_\theta(\theta) \\ &\quad - \frac{\delta}{1+r_S}\int_{\underline{\theta}}^{\bar{\theta}} f_s(w-\theta)\,(g_s(w-\theta)-g_\theta(\theta))\, E_D[\min(D, q_2^*(w,\theta))]\, dF_\theta(\theta) \\ &\quad - \frac{1}{1+r_S}\int_{\underline{\theta}}^{\bar{\theta}} \Bigg[(w-g_\theta(\theta)-\beta_1)\left(\int_{\xi^*(w,\theta)}^{q_2^*(w,\theta)} x\, dF_D(x) + q_2^*(w,\theta)\bar{F}_D(q_2^*(w,\theta))\right) \\ &\qquad + (\phi(w-\theta)+\theta-g_\theta(\theta))\int_{\underline{D}}^{\xi^*(w,\theta)} x\, dF_D(x)\Bigg] f_s(w-\theta)\, dF_\theta(\theta);\end{aligned} \tag{4.50}$$

and for $w \in [\underline{s}+\bar{\theta}, \bar{s}+\bar{\theta}]$:

$$\begin{aligned}\frac{d\pi_S^*(w)}{dw} &= -\frac{\delta}{1+r_S}\int_{\underline{\theta}}^{\bar{\theta}} f_s(w-\theta)\,(g_s(w-\theta)-g_\theta(\theta))\, E_D[\min(D, q_2^*(w,\theta))]\, dF_\theta(\theta) \\ &\quad - \frac{1}{1+r_S}\int_{\underline{\theta}}^{\bar{\theta}} \Bigg[(w-g_\theta(\theta)-\beta_1)\left(\int_{\xi^*(w,\theta)}^{q_2^*(w,\theta)} x\, dF_D(x) + q_2^*(w,\theta)\bar{F}_D(q_2^*(w,\theta))\right) \\ &\qquad + (\phi(w-\theta)+\theta-g_\theta(\theta))\int_{\underline{D}}^{\xi^*(w,\theta)} x\, dF_D(x)\Bigg] f_s(w-\theta)\, dF_\theta(\theta);\end{aligned} \tag{4.51}$$

and clearly, $\frac{d\pi_S^*(w)}{dw} = 0$ for all $w > \bar{s}+\bar{\theta}$. Now, when $\delta < 0$, i.e. $r_S < r_B$, expressions (4.47), (4.48) and (4.49) show us that $d\pi_S^*(w)/dw > 0$ for $w \in [0, \underline{s}+\underline{\theta}]$. Using the properties of $\nu(w,\theta)$ shown in section 2.2 and the fact that $\beta_1 \leq \underline{s}+\theta-g_s(\underline{s})$ and that $\delta > -1$, with some manipulation of algebra, we can easily show that (4.51) is non-positive, which is also the second big expression in (4.50). Hence, for $r_S < r_B$, we have $w^* \in [\underline{s}+\underline{\theta}, \underline{s}+\bar{\theta}]$. Using results from Proposition 4.1, we know optimal reservation schedule for $w \geq \underline{s}+\underline{\theta}$ is flat-pricing, thus giving our desired results for part(i).

Now, consider $\delta = 0$, i.e. $r_S = r_B$. We know the expressions (4.47) and (4.49) are zero, (4.48) is non-negative, also (4.50) and (4.51) are non-positive, which gives us $w^* = \{[\bar{s}(1-\phi), \underline{s} + \underline{\theta}]\}$; and using results from Proposition 4.1, we know optimal reservation schedule is low volume dependent-high flat, which completes our proof for part(ii).

Last, we consider the case for $\delta > 0$. Clearly, there exists some $\underline{\delta}$ such that for all $0 < \delta < \underline{\delta}$, the expressions (4.47) and (4.49) are non-positive but sufficiently small, the expression (4.48) stays positive, and both expressions (4.50) and (4.51) stays negative; thus giving us optimal $w^* = \bar{s}(1-\phi)$. Using results from Proposition 4.1, we know the corresponding optimal reservation schedule is low volume dependent-high flat.

A similar analysis of the expressions show us that if δ is large enough, i.e. there exists $\bar{\delta} > 0$ such that for all $\delta \geq \bar{\delta}$, we have $d\pi_S^*(w)/dw \leq 0$ for all w, thus $w^* = 0$. We now look at the optimal reservation schedule for sales contracts, i.e. $w^* = 0$. From results of Proposition 4.1, we know if $\beta_0 \leq E[s](1-\phi)/(1+r_B)$, the second set of the flat price condition is satisfied, i.e. flat price schedule is optimal. Also, if $\beta_1 \leq \rho E[s](1-\phi)$, we know $M(0) \leq 0$; and using results from Fact 4.7, we know $\hat{\theta}(w) = \underline{\theta}$ and thus we have an optimal flat pricing schedule. If either condition on β_0 or β_1 are not satisfied, it satisfies the volume-dependency condition, thus optimal reservation schedule is volume-dependent. From Fact 4.7, we know $\hat{\theta}(w) = \bar{\theta}$, i.e. there will be no delayed production; furthermore, using results from Proposition 4.2, we know optimal reservation schedule offers quantity discounts. This completes the proof. ■

The result from Proposition 4.3 is a strong one. This provides us a complete characterization for optimal contract design, showing how the nature of optimal contract structure, in many different dimensions, is being affected by parameters of the delayed production model. In specific, we manage to illustrate the nature of optimal contract structures in the following different areas:

The optimal execution price increases with the seller's relative patience level; when the seller is more patient, she is willing to commit first and wait for future payments rather than a higher reservation price, which can be noted by the fact that when the seller is more patient relative to the buyer, she offers a much higher execution price in optimal contracts than that of the case when seller is less patient. In particular, there is a major shift of regime from real options contract, i.e. execution price strictly positive, to a sales contract when seller is significantly less patient than the buyer, when $\delta \geq \bar{\delta}$. When the seller is too impatient, she prefers all payments to be paid at the first period, thus optimal contracts are traditional sales types as such.

Further, the seller's utilization of the delayed production option can also be illustrated here. There is a general trend in delaying production till the second period as the seller becomes more patient, keeping other factors equal. This can be explained by the fact that when the seller is more patient, optimal execution price increases, and since there are relatively more later payments for the seller, the seller finds it more risk to produce up to reservation amounts. Hence she is less willing to commit to higher levels of production in the first period, thus delaying production.

But of course, the cost of production in both periods will change the results significantly, as clearly illustrated by the sales contract case when $\delta \geq \bar{\delta}$. Results show that when both β_0 and β_1 are significantly high, the seller considers no delays in her production process. On the contrary, when either β_0 or β_1 is low enough, the seller considers a delayed production, while the actual amount is to be determined by the magnitudes of the corresponding production costs.

We also see a relation between the relative patience levels of the buyer and seller and the volume dependency in optimal reservation price schedules for options contracts. When seller is more patient, i.e. optimal execution price is higher, the corresponding reservation price schedules exhibit a flat price structure. As the seller's relative patience level decreases, we see a shift in the nature of reservation price schedules: first from flat schedules, then to low volume dependent-high flat schedules, last to volume dependent ones. For sales contracts, we see a difference between flat reservation price schedule and price schedules with volume discounts, which highly depends on the production costs at both periods. When both β_0 and β_1 are high, the seller is less willing to commit to high levels of production, thus a volume discount schedule is offered instead. Thus for both, we see how the seller's production cost and her relative patience level affect her optimal commitment levels, ultimately influencing the design of the optimal contracts she offers to the buyer.

4.4 Appendix: Proofs for Facts

Proof of Fact 4.1: For notation purposes, define

$$\zeta(w,\theta) \triangleq \int_{w-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-w)\,dF_s(s) \tag{A.1}$$

$$\frac{d\zeta(w,\theta)}{dw} = \begin{cases} -\bar{F}_s(w-\theta) & \text{for } w < \underline{s}+\underline{\theta} \\ -f_s(w-\theta)\,(g_s(w-\theta)-g_\theta(\theta)) & \text{for } w \leq \underline{s}+\underline{\theta} \end{cases} \tag{A.2}$$

Using the regularity conditions on $g_s(s)$ and $g_\theta(\theta)$, and from properties of $\zeta(w,\theta)$ given by (A.2), we know $\exists \hat{w}(\theta) \in [\underline{s}+\theta, \bar{s}+\theta]$ such that $g_s(\hat{w}(\theta)-\theta) = g_\theta(\theta)$ and thus giving us $\zeta(\hat{w}(\theta),\theta) = \min_w \zeta(w,\theta)$.

$$\begin{aligned} \therefore \zeta(w,\theta) &\geq \zeta(\hat{w}(\theta),\theta) \\ &= \int_{\hat{w}(\theta)-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-\hat{w}(\theta))\,dF_s(s) \\ &= \int_{\hat{w}(\theta)}^{\bar{s}} (s-(\hat{w}(\theta)-\theta)-g_s(\hat{w}(\theta)-\theta))\;dF_s(s) \\ &= \bar{F}_s(\hat{w}(\theta)-\theta)\left(E_s[s-\hat{w}(\theta)+\theta|s \geq \hat{w}(\theta)-\theta] - \frac{\bar{F}_s(\hat{w}(\theta)-\theta)}{f_s(\hat{w}(\theta)-\theta)}\right) \\ &\geq 0. \end{aligned}$$

where the last inequality holds due to the regularity conditions on $g_s(s)$. ■

Proof of Fact 4.2: For notational purposes, define

$$\varrho(w,\theta) \triangleq \eta(w,\theta) - \delta G(w) \tag{A.3}$$

where $(G(w), \eta(w,\theta))$ are given by (4.10) and (4.16), respectively. Note that another representation of $\varrho(w,\theta)$ is given by:

$$\varrho(w,\theta) = \rho \int_{w-\theta}^{\bar{s}} (\phi s + \theta - g_\theta(\theta))\, dF_s(s) + \delta \int_{w-\theta}^{\frac{w}{1-\phi}} (s(1-\phi) - w)\, dF_s(s) \tag{A.4}$$

To show that $\varrho(w,\theta) \geq 0$ we define and consider the expression:

$$\sigma(w,\theta) = \int_{w-\theta}^{\bar{s}} (\phi s + \theta - g_\theta(\theta))\, dF_s(s) - \int_{w-\theta}^{\frac{w}{1-\phi}} (w - s(1-\phi))\, dF_s(s) \tag{A.5}$$

Using the fact that $\rho > \delta$, and that the two integral expressions in (A.3) are both non-negative, we have $\sigma(w,\theta) \geq 0 \Rightarrow \varrho(w,\theta) \geq 0$. Taking the derivative of σ with respect to w:

$$\begin{aligned}
\frac{d\sigma(w,\theta)}{dw} &= -\left(F_s\left(\frac{w}{1-\phi}\right) - F_s(w-\theta)\right) - f_s(w-\theta)g_\theta(\theta) \\
&= \begin{cases} -F_s\left(\frac{w}{1-\phi}\right) & \text{for } w < \underline{s} + \theta; \\ f_s(w-\theta)\left(g_s(w-\theta) + g_\theta(\theta)\right) & \text{for } w \geq \underline{s} + \theta \end{cases}
\end{aligned} \tag{A.6}$$

Thus, $\sigma(\underline{s} + \theta, \theta) = \min_w \sigma(w,\theta)$. Using (A.6) and the regularity conditions on $(g_\theta(\theta), g_s(s))$ and the assumption $\underline{s} + \underline{\theta} \geq \bar{s}(1-\phi)$:

$$\begin{aligned}
& \sigma(w,\theta) \geq \sigma(\underline{s} + \theta, \theta) \\
= & \int_{\underline{s}}^{\bar{s}} (\phi s + \theta - g_\theta(\theta))\, dF_s(s) - \int_{\underline{s}}^{\frac{\underline{s}+\theta}{1-\phi}} (\underline{s} + \theta - s(1-\phi))\, dF_s(s) \\
= & \; E[s] - \underline{s} - g_\theta(\theta) \\
\geq & \; E[s] - \underline{s} - g_t(\underline{\theta}) \\
\geq & \; E[s] - \underline{s} - g_s(\underline{s}) \geq 0
\end{aligned} \tag{A.7}$$

thus giving us our desired result. ■

Proof of Fact 4.3: Define the following:

$$\begin{aligned}
\varsigma(w,\theta) &\triangleq \varrho(w,\theta) - \int_{w-\theta}^{\frac{w}{1-\phi}} (\beta_1 - s(1-\phi))\; dF_s(s) \\
&= \rho\left(\int_{w-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-w)\; dF_s(s) - \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi)-w)\; dF_s(s)\right) \\
&\quad -(\beta_1 - w)\left(F_s\left(\frac{w}{1-\phi}\right) - F_s(w-\theta)\right) \qquad \text{(A.8)}
\end{aligned}$$

where $\varrho(w,\theta)$ is given by (A.3). For notational purposes, we define

$$\alpha(w,\theta) \triangleq \int_{w-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-w)\; dF_s(s) - \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi)-w)\; dF_s(s) \qquad \text{(A.9)}$$

Differentiating (A.9) with respect to w gives:

$$\frac{d\alpha(w,\theta)}{dw} = \begin{cases} \bar{F}_s\left(\frac{w}{1-\phi}\right) - \bar{F}_s(w-\theta) & \text{for } w < \underline{s}+\underline{\theta} \\ -f_s(w-\theta)\left(g_s(s)(w-\theta) - g_\theta(\theta)(\theta)\right) & \text{for } w \geq \underline{s}+\underline{\theta} \end{cases} \qquad \text{(A.10)}$$

Using regularity conditions on $(g_s(s)(s), g_\theta(\theta)(\theta))$, and the properties of $\alpha(w,\theta)$ given by (A.10), we know $\exists\hat{w}(\theta) \in [\underline{s}+\theta, \bar{s}+\theta]$ such that $\alpha(\hat{w}(\theta),\theta) = \min_w \alpha(w,\theta)$.

$$\therefore \alpha(w,\theta) \geq \alpha(\hat{w}(\theta),\theta) = \int_{\hat{w}(\theta)-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-\hat{w}(\theta))\; dF_s(s) = \zeta(\hat{w}(\theta),\theta) \geq 0 \qquad \text{(A.11)}$$

where the last inequality follows from Fact 4.1.

$$\because \varsigma(w,\theta) = \alpha(w,\theta) - (\beta_1 - w)\left(F_s\left(\frac{w}{1-\phi}\right) - F_s(w-\theta)\right)$$

we have $\varsigma(w,\theta) \geq 0$, for all $\beta_1 \leq w$. We will now consider the case for $\beta_1 > w$, another representation for $\varsigma(w,\theta)$ is given as:

$$\begin{aligned}
\varsigma(w,\theta) = \rho&\left(\int_{w-\theta}^{\frac{w}{1-\phi}} (s+\theta-g_\theta(\theta)-\beta_1)\; dF_s(s) + \int_{\frac{w}{1-\phi}}^{\bar{s}} (\phi s+\theta-g_\theta(\theta))\; dF_s(s)\right) \\
&+ \delta(\beta_1 - w)\left(F_s\left(\frac{w}{1-\phi}\right) - F_s(w-\theta)\right) \qquad \text{(A.12)}
\end{aligned}$$

For notational purposes, we define:

$$\vartheta(w,\theta) \triangleq \left(\int_{w-\theta}^{\frac{w}{1-\phi}} (s+\theta-g_\theta(\theta)-\beta_1)\, dF_s(s) + \int_{\frac{w}{1-\phi}}^{\bar{s}} (\phi s+\theta-g_\theta(\theta))\, dF_s(s)\right) - (\beta_1-w)\left(F_s\left(\frac{w}{1-\phi}\right)-F_s(w-\theta)\right) \quad \text{(A.13)}$$

Since $\rho > \delta$ and $(\beta_1 - w)\left(\bar{F}_s\left(\frac{w}{1-\phi}\right) - \bar{F}_s(w-\theta)\right) \geq 0$, we have $\vartheta(w,\theta) \geq 0$ implies that $\varsigma(w,\theta) \geq 0$. With some manipulation of algebra, it is easy to show to the condition $\vartheta(w,\theta) \geq 0$ is equivalent to:

$$\beta_1 \leq w + \frac{\alpha(w,\theta)}{2\left(F_s\left(\frac{w}{1-\phi}\right) - F_s(w-\theta)\right)} \quad \text{(A.14)}$$

Using the result from (A.11), we can conclude that for any given (w,θ), there exists $\hat{\beta}_1(w,\theta) \geq w$ such that for all $\beta_1 \leq \hat{\beta}_1(w,\theta)$, we have $\varsigma(w,\theta) \geq 0$.■

Proof of Fact 4.4: Using the similar argument as in Proposition 2.1, the seller's expected profit, by taking into account the buyer's optimal behavior, can be represented as follows:

$$\pi_S(w) = \int_{\underline{\theta}}^{\bar{\theta}} H(q(w,\theta),w,\theta)\, dF_s(s)$$

where

$$H(q,w,\theta) = V(y^*(w,\theta),q,w,\theta) + \varphi(q,w,\theta) - \frac{\bar{F}_\theta(\theta)}{f_\theta(\theta)}\frac{\partial\varphi(q,w,\theta)}{\partial\theta} \quad \text{(A.15)}$$

For notational purposes, we write

$$H(q,w,\theta) = \begin{cases} H_1(q,w,\theta) & \text{for } q = y^*(w,\theta) \\ H_2(q,w,\theta) & \text{for } q > y^*(w,\theta) \end{cases}$$

where we define

$$H_1(q,w,\theta) = V_1(q,q,w,\theta) + \varphi(q,w,\theta) - \frac{\bar{F}_\theta(\theta)}{f_\theta(\theta)}\frac{\partial\varphi(q,w,\theta)}{\partial\theta} \quad \text{(A.16)}$$

$$H_2(q,w,\theta) = V_2(\xi^*(w,\theta),q,w,\theta) + \varphi(q,w,\theta) - \frac{\bar{F}_\theta(\theta)}{f_\theta(\theta)}\frac{\partial\varphi(q,w,\theta)}{\partial\theta} \tag{A.17}$$

and where $\xi^*(w,\theta)$ is given by (4.6). Next we look at the properties of $H_1(q,w,\theta)$, $H_2(q,w,\theta)$ and its relation to $\xi^*(w,\theta)$.

$$\begin{aligned}\frac{dH_1(q,w,\theta)}{dq} = -\beta_0 + \frac{1}{1+r_S}\Bigg(&E[s](1-\phi) + \int_{w-\theta}^{\bar{s}} (\phi s + \theta - g_\theta(\theta))\, dF_s(s)\cdot\bar{F}_D(q)\\ &+\delta\bigg(\int_{w-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-w)\, dF_s(s)\cdot\bar{F}_D(q)\\ &\quad+\int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi)-w)\, dF_s(s)\cdot F_D(q)\bigg)\Bigg);\end{aligned} \tag{A.18}$$

$$\begin{aligned}\frac{dH_2(q,w,\theta)}{dq} = \frac{1}{1+r_S}\Bigg(&\bigg(\int_{w-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-\beta_1)\, dF_s(s)\cdot\bar{F}_D(q)\\ &\quad+\int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi)-\beta_1)\, dF_s(s)\cdot F_D(q)\bigg)\\ &+\delta\bigg(\int_{w-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-w)\, dF_s(s)\cdot\bar{F}_D(q)\\ &\quad+\int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi)-w)\, dF_s(s)\cdot F_D(q)\bigg)\Bigg);\end{aligned} \tag{A.19}$$

$$\begin{aligned}&\frac{dH_1(q,w,\theta)}{dq} - \frac{dH_2(q,w,\theta)}{dq}\\ =\;& -\beta_0 + \frac{1}{1+r_S}\Bigg(E[s](1-\phi) + \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s)\\ &\qquad+\int_{w-\theta}^{\frac{w}{1-\phi}} (\beta_1 - s(1-\phi))\, dF_s(s)\cdot\bar{F}_D(q)\Bigg)\\ =\;& \frac{dV_2(q,y,w,\theta)}{dy}\bigg|_{y=q} \begin{cases} >0 & \text{for } q<\xi^*(w,\theta);\\ =0 & \text{for } q=\xi^*(w,\theta);\\ <0 & \text{for } q>\xi^*(w,\theta).\end{cases}\end{aligned} \tag{A.20}$$

$$\frac{d^2H_1(q,w,\theta)}{dq^2} = -\frac{f_D(q)}{1+r_S}\Bigg(\int_{w-\theta}^{\bar{s}} (\phi s+\theta-g_\theta(\theta))\;dF_s(s)$$
$$+\,\delta\Bigg(\int_{w-\theta}^{\frac{w}{1-\phi}} (s+\theta-g_\theta(\theta)-w)\;dF_s(s)+\int_{\frac{w}{1-\phi}}^{\bar{s}} (\phi s+\theta-g_\theta(\theta))\;dF_s(s)\Bigg)\Bigg); \quad \text{(A.21)}$$

$$\frac{d^2H_2(q,w,\theta)}{dq^2} = -\frac{f_D(q)}{1+r_S}\Bigg(\int_{w-\theta}^{\bar{s}} (\phi s+\theta-g_\theta(\theta))\;dF_s(s)$$
$$-\int_{w-\theta}^{\frac{w}{1-\phi}} (\beta_1-s(1-\phi))\;dF_s(s)$$
$$+\,\delta\Bigg(\int_{w-\theta}^{\frac{w}{1-\phi}} (s+\theta-g_\theta(\theta)-w)\;dF_s(s)$$
$$+\int_{\frac{w}{1-\phi}}^{\bar{s}} (\phi s+\theta-g_\theta(\theta))\;dF_s(s)\Bigg)\Bigg). \quad \text{(A.22)}$$

Using Fact 4.2 and Fact 4.3, we know both $d^2H_1(q,w,\theta)/dq^2$ and $d^2H_2(q,w,\theta)/dq^2$ are non-positive. Combining this with the property given by (A.20), we can conclude that

$$\left.\frac{dH_1(q,w,\theta)}{dq}\right|_{q=\xi^*(w,\theta)} > 0 \quad \Leftrightarrow \quad q^*(w,\theta) > \xi^*(w,\theta) \qquad \text{(A.23)}$$

Using (4.6) and (A.18), we can show that

$$\left.\frac{dH_1(q,w,\theta)}{dq}\right|_{q=\xi^*(w,\theta)} = \frac{w-\beta_1}{1+r_S}\Bigg(\bar{F}_s\left(\frac{w}{1-\phi}\right)F_D(\xi^*(w,\theta))$$
$$+\,\bar{F}_s(w-\theta)\bar{F}_D(\xi^*(w,\theta))\Bigg)$$
$$+\,\frac{1}{1+r_B}\Bigg(\int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi)-w)\;dF_s(s)\cdot F_D(\xi^*(w,\theta))$$
$$+\int_{w-\theta}^{\bar{s}} (s+\theta-g_\theta(\theta)-w)\;dF_s(s)\cdot\bar{F}_D(\xi^*(w,\theta))\Bigg)$$

Using Fact 4.1, $\left.\frac{dH_1(q,w,\theta)}{dq}\right|_{q=\xi^*(w,\theta)} > 0$ is equivalent to the condition

$$\frac{1+r_S}{1+r_B} > \hat{\rho}(w,\theta),$$

where $\hat{\rho}(w,\theta)$ is given by (4.9). Combining with (A.23) gives us the desired result. ■

Proof of Fact 4.5: The expression $\hat{\rho}(w,\theta)$ given by (4.9) suggests that for any $w \geq \beta_1$, $\hat{\rho}(w,\theta) = 0$. Since $\rho > 0$ by definition, we have $\rho > \hat{\rho}(w,\theta)$ for all θ, i.e. $\hat{\theta}(w) = \underline{\theta}$, for all $w > \beta_1$, which gives us the results of part(ii). Now we consider the behavior of $\hat{\rho}(w,\theta)$ and $\xi^*(w,\theta)$ for $w \leq \beta_1$. First, we differentiate (4.6) with respect to θ:

$$\begin{aligned} f_D(\xi^*(w,\theta))\frac{d\xi^*(w,\theta)}{d\theta}\int_{w-\theta}^{\frac{w}{1-\phi}}(\beta_1 - s(1-\phi))\,dF_s(s) \\ = \bar{F}_D(\xi^*(w,\theta))f_s(w-\theta)\,(\beta_1-(w-\theta)(1-\phi)) \quad \text{(A.24)} \end{aligned}$$

Thus giving us $d\xi^*(w,\theta)/d\theta = 0$ for $w < \underline{s} + \underline{\theta}$. Using this result and differentiating (4.9) with respect to θ:

$$\begin{aligned} \frac{d\hat{\rho}(w,\theta)}{d\theta}\left(G(w)F_D(\xi^*(w,\theta)) + \int_{w-\theta}^{\bar{s}}(s+\theta-g_\theta(\theta)-w)\,dF_s(s)\bar{F}_D(\xi^*(w,\theta))\right) \\ = -\hat{\rho}(w,\theta)(1-g'_\theta(\theta))\bar{F}_D(\xi^*(w,\theta)) \quad \text{(A.25)} \end{aligned}$$

Thus $d\hat{\rho}(w,\theta)/d\theta \leq 0$ for all $w \leq \underline{s}+\underline{\theta}$, which implies that there exists a unique $\hat{\theta}(w)$ such that $\theta \leq \hat{\theta}(w)$ if and only if $\rho \leq \hat{\rho}(w,\theta)$, which gives us the result of part(i). ■

Proof of Fact 4.6: We first look at the relation between $M(w)$, $\tilde{w}_c$ and $\tilde{w}_f$:

$$\delta G(\tilde{w}_c) = (1+r_S)\beta_0 - E[s](1-\phi) = \int_{\frac{\tilde{w}_f}{1-\phi}}^{\bar{s}}(\beta_1 - s(1-\phi))\,dF_s(s) \qquad \text{(A.26)}$$

$$M(w) = \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s) - \delta G(w) \tag{A.27}$$

Using δ^* defined in (4.18), and plugging it into (A.26), we have the relation:

$$\delta^* G(\tilde{w}_f) = \delta G(\tilde{w}_c) \tag{A.28}$$

Using the property that $G(w)$ is decreasing in w for all $w \leq \bar{s}(1-\phi)$, we know that $\delta < \delta^*$ implies $\tilde{w}_c < \tilde{w}_f$. Since both the leftmost and rightmost expression of (A.26) are decreasing in w for all $w \leq \bar{s}(1-\phi)$, we have for any $w \in (\tilde{w}_c, \tilde{w}_f)$:

$$\delta G(w) < (1 + r_S)\beta_0 - E[s](1-\phi) < \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s) \tag{A.29}$$

By plugging this result to (4.13), $\tilde{w}_c < \tilde{w}_f$ is equivalent to $M(w) > 0$, $\forall w \in [\tilde{w}_c, \tilde{w}_f]$. This completes the proof for part(i). A similar reverse argument gives the proof for part(ii), which we will not show here. ■

Proof of Fact 4.7: By the definition of $\tilde{w}_f$, given by (4.20), we know that $w \leq \tilde{w}_f$ implies $\xi^*(w, \theta) = K$ for all θ. Plugging this to (4.9) gives us:

$$\hat{\rho}(w, \theta) = \frac{(\beta_1 - w)\bar{F}_s\left(\frac{w}{1-\phi}\right)}{\int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi) - w)\, dF_s(s)}. \tag{A.30}$$

which is independent of θ. Also, we have for all $w \leq \tilde{w}_f$:

$$\begin{aligned}
M(w) > 0 \;\; &\Leftrightarrow \;\; \int_{\frac{w}{1-\phi}}^{\bar{s}} (\beta_1 - s(1-\phi))\, dF_s(s) > \delta \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi) - w)\, dF_s(s) \\
&\Leftrightarrow \;\; (\beta_1 - w)\bar{F}_s\left(\frac{w}{1-\phi}\right) - \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi) - w)\, dF_s(s) \\
&\qquad\qquad > \delta \int_{\frac{w}{1-\phi}}^{\bar{s}} (s(1-\phi) - w)\, dF_s(s) \\
&\Leftrightarrow \;\; \hat{\rho}(w, \theta) > \rho, \quad \forall \theta
\end{aligned}$$

Thus giving us our desired result. ■

Chapter 5

Concluding Remarks

In this thesis we presented a unified solution to the pricing problem of contracts for a supplier of an industrial good in the presence of spot trading and a buyer having varying degrees of production flexibility. In particular, we answered the following questions: (i) When is it optimal to offer sales contracts versus options contracts? (ii) When is it optimal to offer flat fee versus volume-dependent contracts? (iii) When does the optimal contract involve volume discounts and when does it involve volume premia? and (iv) When does the seller delay her production till after the realization of demand and spot price? We derived the optimal non-linear pricing of procurement options contracts for both the basic and delayed production models.

For the basic model, we showed that a higher discount rate for the supplier and a higher expected spot price favor a flat price schedule; while a higher discount rate for the buyer, higher production costs or wider spot market bid-ask spreads favor volume dependent contracts. Exploring the characteristics of the optimal contracts, we showed that while volume-dependent optimal sales contracts will always demonstrate volume discounts (i.e., will involve concave pricing), optimal options contracts can involve both volume discounts and and volume premia.

Further, we found that in the optimal contracts, there are three major pricing regimes. First, if the seller has a higher discount rate than the buyer and the production costs are lower than a critical threshold value, the optimal contract is a flat fee sales contract. Second, when the seller is less patient than the buyer but the production costs are higher than the critical threshold, the optimal contract is a sales contract with volume discounts. Third, if the buyer has a higher discount rate than the seller, then the optimal contract is an options contract with volume dependent pricing.

Finally, we also studied the effects of the industry and market characteristics on contract design. We found that increased expected spot prices, or decreased manufacturer's production flexibility, uncertainty of manufacturer's production flexibility, spot price variance, tend to increase the exercise price for the options. Increased average spot price, or decreased spot market bid-ask spread, production cost, spot price variance, manufacturer production flexibility, tend to increase expected contracting quantities. Increased expected spot prices, spot price variance, or decreased manufacturer production flexibility, spot market bid-ask spread, production cost, tend to increase the reservation price for the options.

For the delayed production model, we showed that the when the seller is significantly less patient than the buyer, optimal contracts are traditional sales ones; where its reservation price schedule exhibits volume discounts when the seller's production costs for both periods are sufficiently high. For options contract, we showed that optimal execution price increases with the seller's relative level of patience, and its corresponding reservation price schedule shifts from the regimes of being: (i) flat, (ii) low volume dependent-high flat; and (iii) volume-dependent as the seller gets more impatient, i.e. higher discount rates. With regards to the delayed production option for the seller, we showed that the seller finds the option favorable when she has a higher discount rate or when production costs are low enough.

In this study, we aimed to provide a novel way of looking at the determination of the structure of optimal procurement contracts. Our goal was to provide an approach to a unified design of procurement contracts with minimal pre-imposition of contract structures. As a consequence, we analyzed a very general class of option contracts that included a broad spectrum of commonly observed procurement contract structures. Our analysis not only provides insights for the endogenously determined nature of procurement contracts but also builds links between variables such as spot price distribution and bid-ask spread of the spot market and procurement contract characteristics, which weren't explored previously. All these provides a better understanding of the nature of complex procurement contracts, and further helps in laying the foundations for future studies that continue to build the unified theory of determination of procurement agreements. A few of the many possible future research problems include: (i) considering the risk-averse nature of both the manufacturer and the supplier ; (ii) a measurement of the benefits of the existence of the spot market; and (iii) an extension which allows renegotiation for the two parties.

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