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Multi-Factor Model of Correlated Commodity Forward Curves for Crude Oil and Shipping Markets

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Abstract

An arbitrage free multi-factor model is developed of the correlated forward curves of the crude oil, gasoline, heating oil and tanker shipping markets. Futures contracts trading on public exchanges are used as the primary underlying securities for the development of a multi-factor Gaussian Heath-Jarrow-Morton (HJM) model for the dynamic evolution of the correlated forward curves. An intra- and inter-commodity Principal Component Analysis (PCA) is carried out in order to isolate seasonality and identify a small number of independent factors driving each commodity market. The cross-commodity correlation of the factors is estimated by a two step PCA. The factor volatilities and cross-commodity factor correlations are studied in order to identify stable parametric models, heteroskedasticity and seasonality in the factor volatilities and correlations. The model leads to explicit stochastic differential equations governing the short term and long term factors driving the price of the spot commodity under the risk neutral measure. Risk premia are absent, consistently with HJM arbitrage free framework, as they are imbedded in the factor volatilities and correlations estimated by the PCA. The use of the model is described for the pricing of derivatives written on inter- and intra-commodity futures spreads, Asian options, the valuation and hedging of energy and shipping assets, the fuel efficient navigation of shipping fleets and use in corporate risk management.

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1. INTRODUCTION

The crude oil and tanker shipping markets are exposed to a variety of risks reflected in the high volatility of the prices of crude oil and its products – gasoline, heating oil, jet fuel – and tanker shipping freight rates. The mitigation of these risks has prompted the growth of the futures contracts of crude oil and its products that trade on public exchanges – the New York Mercantile Exchange (NYMEX) and the InterContinental Exchange (ICE) -- and of swaps and other customized derivatives that trade in cleared Over The Counter (OTC) markets aiming to mitigate counterparty risks.

The deep and liquid crude oil futures and forward paper markets have emerged as an important vehicle for price discovery, asset valuation, hedging and risk management. A robust model of the correlated dynamics of the forward curves of crude oil, its products and of the tanker shipping freight rates can be very valuable to market participants involved in the management of real assets – crude oil reservoirs, storage facilities, refineries, tanker shipping fleets – as well as investors who are primarily involved in the management of securities.

The forward curve of a commodity has embedded in it information about the economic factors that drive the short and long term evolution of the spot price. Therefore the futures contracts will be considered in the present study as the primary securities for the development of a multi-factor model of the underlying commodity markets – crude oil, gasoline, heating oil and tanker shipping freight rates. This approach reduces to standard spot price models of the crude oil price [Gibson and Schwatrz (1990), Ross (1997), Schwartz (1997), Schwartz and Smith (2000)], it allows for any number of factors and it accounts for cross-commodity correlation in their futures and hence their spot prices.

The arbitrage free evolution of the futures prices is modeled under the Heath-Jarrow-Morton framework developed for the modeling of the evolution of the term structure of interest rates [Heath, Jarrow and Morton (1992), Clewlow and Strickland (2000)]. Risk premia are not explicitly present in the HJM model, they are instead imbedded in the volatilities of the futures prices and imputed in the drifts of the spot price factor dynamics. In the present study the prices of futures and forward contracts are assumed to be identical, an assumption justified under deterministic interest rates or under stochastic interest rates with a term structure uncorrelated with the forward curve of the commodity under study [Cox, Ingersoll and Ross (1981)]. In practice the equality of the futures and forward prices is satisfactory assuming that the forward contract is free of credit risk. For the pricing of long-dated commitments it may be necessary to account for the futures-forward spread which is available in explicit form under a joint HJM model of the correlated term structures of interest rates and the commodity.

A statistical analysis of the log-returns of the futures prices of crude oil, gasoline and heating oil reveals that their probability distribution is approximately Gaussian, except perhaps for contracts with very short tenors. This reflects the efficiency of the deep and liquid crude oil futures markets where information flows are readily reflected in the prices of futures contracts which may be easily entered into and reversed. The drift of the log-returns of a futures contract depends on the slope of the forward curve which may be trading in contango, backwardation or in a composite formation. In the case of heating oil, shipping futures and other energy commodities (e.g. natural gas) a deterministic seasonality is often observed in the shape of the forward curve. Removing the deterministic drift associated with the slope of the forward curve and ensuring the stationarity of the remaining zero-mean price process, are essential for the statistical processing of the log-returns of the futures prices and the development of robust models under the HJM framework. This is accomplished by introducing futures processes with constant relative tenors, obtained by linear interpolation from the prices of futures contracts with fixed tenors. The deterministic drift of the constant relative tenor futures follows from the slope of the forward curve which may include seasonality. Moreover, the de-trended process has a stationary volatility, a property not enjoyed by the fixed tenor futures price process which has a volatility that increases as the contract approaches expiration by virtue of the Samuleson hypothesis which is strongly supported by market data.

The further statistical processing of the de-trended rolling tenor futures contracts is carried out by a Principal Component Analysis (PCA). The PCA is a powerful parametric free method for the derivation of a small number of independent statistical factors driving the fluctuations of the de-trended rolling tenor futures prices, and after interpolation, of the fixed tenor futures prices. This method is particularly effective for the explicit identification of factors from the fluctuations of the prices of a set of highly correlated securities. This is the case with the futures contracts of different tenors of a particular commodity and of the forward rates in the interest rate markets [Rebonato (2002)]. The PCA analysis of the individual forward curve of the commodity of interest – crude oil, gasoline, heating oil – enables the development of an arbitrage free model for the evolution of the futures price under the HJM framework. A small number of factors, their volatilities and their rate of decay with respect to the relative tenor of the underlying futures contract follow directly from the PCA which is an eigenvalue-eigenvector decomposition of the covariance matrix of the de-trended log-returns of the rolling tenor futures.

Demand for crude oil is largely driven by the demand for gasoline, aviation jet fuel, shipping bunker fuel, heating oil and other products produced by refineries. Therefore the statistical factors that drive the crude oil forward curve are likely to be correlated with the statistical factors driving the forward curves of gasoline or heating oil. Liquid futures also trade on ICE for gasoil which is used for the hedging of aviation jet fuel exposures. The statistical factors of crude oil, gasoline and heating oil follow in explicit form from the respective PCA analyses and their correlation follows by a simple matrix operation. The evaluation of the factor volatilities and cross-commodity factor correlations completes the derivation of the HJM model for the arbitrage free evolution of the correlated forward curves of crude oil, gasoline and heating oil which may be used for the pricing of derivatives, asset valuation and hedging.

Tests are conducted to determine the statistical properties of the factor volatilities and cross commodity factor correlations, aiming to determine if these parameters may be assumed to be constant and identify heteroscedasticity and seasonality, other than that present in the mean shape of the forward curve. This analysis is based on NYMEX crude oil, gasoline and heating oil futures data obtained from Datastream for the period 2003-2008.

A stochastic differential equation is derived driving the spot price process of the underlying commodity in the absence of arbitrage opportunities. This follows from the derivation of the stochastic differential equation governing the evolution of the futures prices under the Gaussian HJM model and the consistency condition that the spot and futures prices converge at the expiration of the futures contract. It is shown that the spot price evolution is driven by the same number of factors as the futures curve and the factor stochastic dynamics is mean reverting, with the factor rates of mean reversion being functions of the slope of the factor loadings with respect to the tenor. The short term dynamics is governed by a higher volatility while the long term dynamics is characterized by a lower volatility. This is consistent with the spot price model of Schwartz and Smith (2000). In the present HJM framework risk premia are not explicitly present, they are instead implicitly embedded in the factor volatilities estimated by the PCA which appear as parameters in the spot price stochastic dynamics.

As has been the case in the securities and crude oil markets, the development of robust marked-to-market models, derivative pricing and hedging methods for shipping derivatives is essential for the increase of their liquidity and their wide adoption by shipowners, charterers, banks and investors. Bulk shipping is a volatile industry providing ocean transportation services for the movement of commodities, crude oil and its products in the case of tanker shipping and iron ore, coal, grains, bauxite, alumina and phosphate rock in the case of dry bulk shipping. The commodity-like product produced by the shipping industry is ton-miles, Its price – the freight rate -- is determined by the supply of shipping tonnage and the derived demand for the transportation of liquid and dry bulk commodities in a perfectly competitive market. Two types of charter contracts prevail in the shipping industry. In a voyage charter the spot freight rate earned by the shipowner is expressed in dollars per ton of cargo ($/ton) while in a time charter the T/C rate earned is expressed in $/day. In the case of the tanker sector the freight rates are expressed as a percentage of the flat Worldscale (WS) spot rate expressed as $/ton and published yearly by the Worldscale

Association. The details of these and other charter contracts are presented in Stopford (1997). The prevailing freight rates in sub-sectors and routes of the bulk shipping industry are reflected in dry bulk and tanker indices published daily by the Baltic Exchange and Platts. They represent the most heavily traded routes within the dry bulk and tanker sectors and are discussed in Kavussanos and Visvikis (2006).

The spot or T/C freight rates of individual indices serve as the underlying assets for derivative securities that trade on public exchanges and over the counter (OTC). The public exchanges offering trading and clearing for shipping freight derivatives include the International Maritime Exchange (IMAREX) launched in 2000 and the New York Merchantile Exchange (NYMEX) since 2005. In 2006 the Singapore Exchange Limited (SGX) launched SGX AsiaClear for the OTC clearing of energy and shipping freight derivatives. The derivatives trading on IMAREX are dirty and clean oil & products tanker and dry bulk freight derivatives that settle against single route spot indices published by the Baltic Exchange and Platts. Basket dry bulk derivatives are also offered on IMAREX that settle against Baltic indices that represent the average T/C rates earned on the single route Capesize, Panamax and Supramax dry-bulk sub-sectors.

A large and growing market for shipping Forward Freight Agreements exits over the counter. As is the case with the vast crude oil OTC derivatives market, FFAs are bilateral agreements between two counterparties that settle against the arithmetic average of a spot freight rate index. The flexibility of OTC transactions allows the design and pricing of contracts tailored to the risk management needs of shipping companies, charterers, banks and investors. FFAs entail credit risk not present in the shipping futures contracts that clear on IMAREX. Clearing and settlement services for OTC FFAs are offered by the London Clearing House Clearnet (LCH.Clearnet), IMAREX and SGX. These services are essential for the growth of the shipping FFA and futures markets since they mitigate credit risk in an industry consisting of a large number of privately held shipping firms. A limitation of the OTC FFA market is that positions in derivatives are not easy to reverse at low cost prior to settlement. This flexibility is present in a liquid futures market which allows the implementation of dynamic hedging and other risk management strategies.

A multi-factor HJM model for tanker shipping futures and FFAs is developed along the same lines as in the crude oil, gasoline and heating oil markets. Most of the crude oil produced worldwide is transported by tankers and the value of the crude cargo is much larger than the freight rate cost. Therefore tanker shipping ton-miles may be viewed as an additional commodity driven by supply and demand dynamics of the crude oil its products over particular routes. A technical complexity present in the tanker shipping futures markets is that contracts settle against the arithmetic average of the underlying spot index. This requires an extension of the HJM model for the evolution of the shipping futures price process in the pre- and post-settlement periods. Otherwise, the modeling of the tanker shipping forward curve proceeds along the lines followed for the crude oil, gasoline and heating oil forward curves.

Tanker freight futures price series have been obtained for a major tanker shipping route for which liquid futures contracts trade on IMAREX. Constant relative tenor shipping futures prices have been obtained by interpolation from futures with fixed tenors, properly accounting for the length of the settlement period. The mean shape of a baseline tanker shipping futures curve is estimated and used to de-trend the log-returns of the traded futures contracts. Their evolution dynamics is then cast in the form of the HJM model and a small number of factors and their volatilities are estimated by a PCA. This leads to a model with lognormal evolution dynamics for the shipping futures. As in the crude oil market, the HJM model for the evolution of the tanker shipping futures leads to explicit dynamics for the evolution of the underlying spot index in the absence of arbitrage opportunities. This dynamics is driven by a number of factors which reveal the short term fluctuations around a long term trend of the spot shipping index under study along with the speed of their mean reversion.

The multi-factor correlated HJM models for the crude oil and tanker shipping futures markets lead to lognormal dynamics for the futures price processes with time dependent deterministic volatilities. This allows the explicit pricing of European derivatives written on the underlying spot commodity or index and a futures contract by using Black's formula. When liquid futures options are trading, e.g. in the crude oil market, the explicit formulae for calls and puts may be used to extract implied volatilities which may in turn be used to used to calibrate the factor volatilities of the particular forward curve under study. The pricing of options of intra- and inter-commodity futures spreads and baskets is also easy to carry out under the log-normal HJM framework using explicit formulae and efficient numerical methods. The accurate pricing of options on futures spreads and baskets depends critically on the correlations of the futures contracts in the spread. These in turn are functions of the factor volatilities and cross-commodity factor correlations the robust estimates of which is a focal point of the present study.

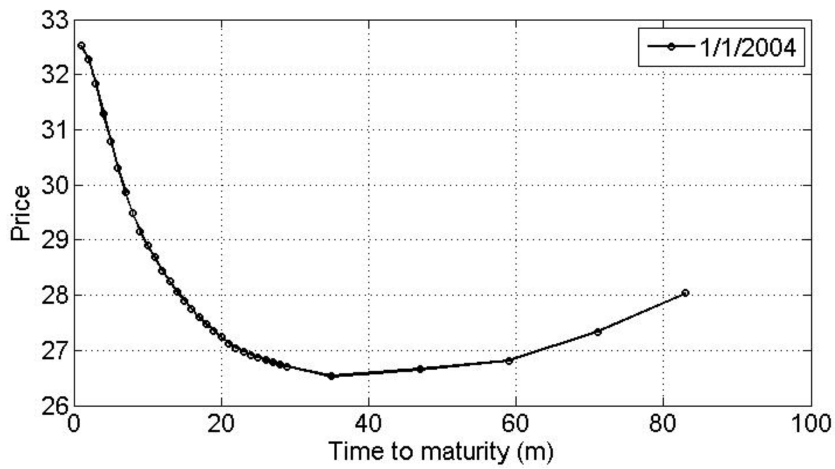
Options written on tanker freight rate futures are illiquid. Their pricing depends on the dynamics of the underlying futures price process which is lognormal under the present multi-factor HJM model. Therefore, European options on freight rate futures may be priced explicitly by using the Black formula. The option price in turn depends on the volatilities of the factors that drive the underlying futures process which are estimated by the PCA of the tanker shipping forward curve under study. The present HJM modeling framework leads to the explicit pricing of shipping futures options using Black's formula which in turn allows the estimation of implied volatilities where a liquid option market exists which may be used to better understand the dynamics of the shipping sector under study. Therefore, the present modeling framework strengthens the links between the modeling and pricing of derivatives in the crude oil and shipping markets and aims to enhance the understanding and eventually the liquidity and depth of the latter.

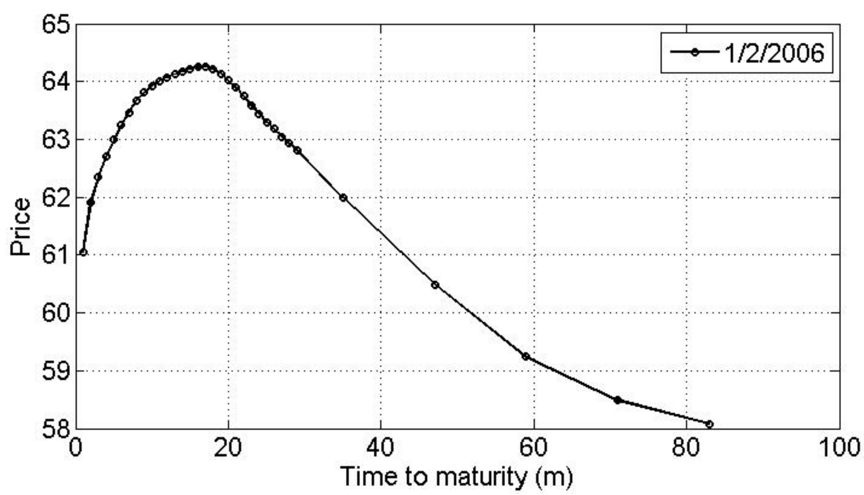
The derivative securities priced in the present study may be used as the fundamental building blocks for the valuation of a wide range of energy commodity, shipping assets and investment opportunities within the real options framework. The valuation is discussed of the option held by a refinery to convert oil into products over a specified time period. The value is also derived of the right to develop a hydrocarbon reservoir and of physical or synthetic storage of energy commodities. The valuation is discussed of a contract to transport a liquid energy commodity between two geographical locations where futures contracts written on the same physical commodity trade and when the optionality exists to control the vessel destination and speed. The valuation is discussed of a charter portfolio consisting of cargo vessels combined with a paper portfolio of shipping futures and futures options. The fuel efficient navigation of a shipping fleet is addressed by casting the seastate uncertainty in a lognormal diffusion framework which allows the explicit solution of the vessel fuel minimizing course and speed using methods of stochastic dynamic programming. Finally, the optimal dynamic management of futures and futures options portfolios is discussed when the underlying securities are governed by lognormal diffusions with time deterministic and stochastic coefficients.

The role of derivatives in corporate finance for the hedging of market risks faced by energy and shipping firms is addressed. The modeling of the default free interest rates and the pricing of credit risk using structural and reduced form models within the HJM framework is discussed. The common modeling framework of market risks that energy and shipping firms are exposed to enables its use for the evaluation of a wide range of integrated risk management strategies. They include the formulation and pricing of flexible long term contracts for the delivery of energy and shipping freight services, the minimization of firm cash flow variance, the selection of the optimal firm capital structure, and the design of value maximizing financial and investment policies via the proper mix of equity and debt.

2. CRUDE OIL FUTURES PRICE PROCESS

Assume that t=0 is an initial reference time hereafter assumed fixed. Denote by $S(t)=S_t$ the price of the underlying spot asset at the current time t -- crude oil or a shipping index -- by F(t,T) the price of a futures contract written on S_t with expiration date T. At expiration, the long futures position receives the difference S(T)-F(t,T) where S(T) is the price of the spot asset delivered by the short futures position. Evidently, the following consistency conditions must hold, F(t,t)=S(t) and F(T,T)=S(T). At time t futures contracts with fixed tenors T_j are assumed to trade with prices $F(t,T_j),\ j=1,...,N$.





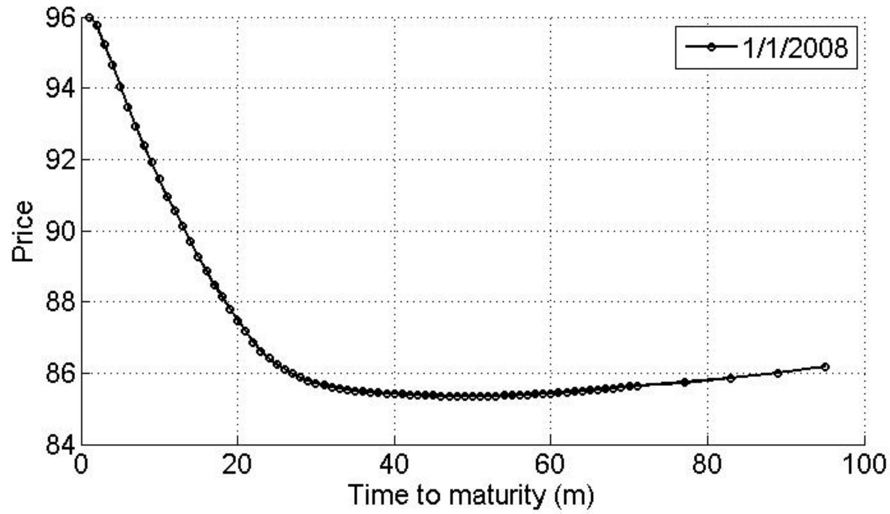


Figure 2.1: Crude oil Forward Curves at three dates

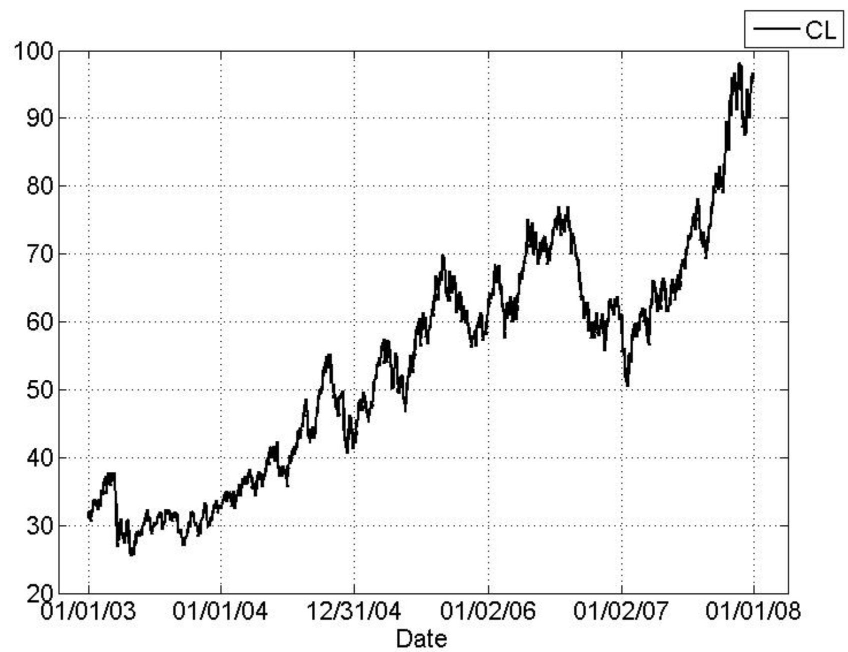


Figure 2.2: Crude oil Price from 1/1/2003-1/1/2008

Figures 2.1 plot the crude oil forward curve at three dates 1/1/2004, 1/2/2006 and 1/1/2008. On 1/1/2004 the forward curve was trading in backwardation, namely the futures contracts with tenors up to about 40 months were trading at a discount to the spot. Two years later on 1/2/2006 the forward curve was trading in contango for the front 20 months followed by a declining term structure from the 20^{th} to the 80^{th} month. On 1/1/2008 the crude oil forward curve was again trading in backwardation. The evolution of the crude oil spot price over this period is plotted in Figure 2.2. As time evolves the tenor of futures contracts shrinks as they approach expiration in a backwardation market, their price drifts upwards towards the spot. Moreover, as futures contracts approach expiration their volatility increases as positions are being offset or rolled over prior to expiration in order to prevent delivery. When the forward curve trades in contango the futures price drifts downwards as the contract approaches expiration again with an increasing volatility. The volatility increase and drifts towards expiration of the futures prices introduce a non-stationarity which complicates their statistical modeling.

It is therefore preferable to study the price evolution of futures prices with constant tenors rolling relative to the current time t. The prices of constant relative tenor contracts can be obtained by interpolation from the prices of traded futures contracts with fixed tenors. Their volatility is stationary and decreases with increasing relative tenor, by virtue of the Samuelson hypothesis. The drift to maturity associated with the slope of the forward curve is absent in the prices of the constant relative tenor futures. Their drifts instead depend on the drift of the spot price and vary as a function of the relative tenor. This variation controls the evolution of the shape of the forward curve, namely its transition from backwardation to contango and vice versa. The modeling of the prices of the constant relative tenor futures may be carried out robustly using the powerful statistical technique of Principal Components Analysis (PCA) which is particularly suited for the study of highly correlated securities. The PCA reveals a stable structure of the volatility term structure of the rolling tenor futures and produces a very small set of explicit statistical factors that dominate the evolution of the forward curve. The following stochastic dynamics is assumed to govern the evolution of the futures price processes with fixed tenors under the real world objective measure

$$\frac{dF(t,T_j)}{F(t,T_j)} = \mu(t,T_j)dt + \sum_{k=1}^{M} \sigma_k(t,T_j)\, dW_k(t), \ \ j = 1,\dots,N$$

(2.1)

$$d \ln F(t,T_j) = [\mu(t,T_j) - \tfrac{1}{2}\sum_{k=1}^{M} \sigma^2{}_k(t,T_j)]dt + \sum_{k=1}^{M} \sigma_k(t,T_j)\, dW_k(t)$$

$$dW_k(t)dW_l(t) = \delta_{kl}\, dt$$

The M-dimensional standard Brownian motions $(W_1,...,W_N)$ are assumed to be mutually independent and represent the M sources of uncertainty affecting all futures contracts trading on the forward curve of a given commodity. The factor volatilities $\sigma_k(t,T_j)$ are in the present study assumed to be deterministic time dependent quantities. The drift $\mu(t,T_j)$ is also time dependent and is assumed deterministic. Under these assumptions it follows that the de-trended futures prices follow a lognormal process an assertion which is supported by market prices as discussed in Section 3.

An implicit assumption in the model (2.1) is that M unobservable statistical factors affect the N futures contracts of the commodity forward curve under study. The assumption of their independence is not necessary, yet it turns out to be convenient and follows from the Principal Components Analysis (PCA) of the historical futures price series described below. The PCA analysis reveals a small number of factors d<M that dominate the fluctuations of the futures price process around their drift. It also produces estimates of the volatilities $\sigma_k(t,T_j)$ of the k-th factor affecting the j-th futures.

As the current time t approaches the fixed expiration date of the futures contract T_j, the volatility of the futures contract, and consequently the factor volatilities $\sigma_k(t,T_j)$, increase. This complicates the estimation of $\sigma_k(t,T_j)$. This complexity can be removed by introducing a set of rolling futures contracts $f(t,t+\tau_j)$ with constant relative tenors τ_j, j=1,…,N. The prices of this new set of securities may be obtained by linear interpolation from the market prices of traded futures contracts $F(t,T_j)$ using the relation

$$\ln f(t,t+\tau_j) \simeq \frac{(t+\tau_j-T_j)\ln F(t,T_{j+1})+(T_{j+1}-t-\tau_j)\ln F(t,T_j)}{T_{j+1}-T_j}, \; T_j < t+\tau_j < T_{j+1}$$

(2.2)

The relative tenors τ_j span the prices of liquid futures contracts with $t+\tau_1 > T_1$ and the rolling tenor futures contracts $f(t,t+\tau_j),\ j=1,...,N$ are expected to have stationary volatilities. The time t stochastic evolution of the process $f(t,t+\tau_j)$ follows from the evolution of the process $F(t,T_j)$ given by (2.1) and the use of (2.2) to define the drift and factor volatilities of $f(t,t+\tau_j)$

$$d\ln f(t,t+\tau_j) = [\mu(t,t+\tau_j) - \tfrac{1}{2}\sum_{k=1}^{M}\sigma^2{}_k(t,t+\tau_j)]dt + \frac{1}{f(t,t+\tau_j)}\left[\frac{\partial f(t,x)}{\partial x}\right]_{x=t+\tau_j} dt$$
$$+\sum_{k=1}^{M}\sigma_k(t,t+\tau_j)\,dW_k(t)$$

(2.3)

The drift of the constant relative tenor futures $f(t,t+\tau_j)$ is now seen to depend on the slope of the original futures curve with respect to the tenor. The factor volatility $\sigma_k(t,t+\tau_j)$ is assumed to be a stationary process. In the simplest setting it is assumed to be just a function of the relative tenor, hence $\sigma_k(t,t+\tau_j) \simeq \sigma_k(\tau_j)$. These constant volatilities will be estimated from the statistical processing of the de-trended prices of the price series $f(t,t+\tau_j)$. Upon estimation of the constant volatilities $\sigma_k(\tau_j)$ using the PCA analysis described below, the original volatilities $\sigma_k(t,T_j)$ follow by a reverse linear interpolation analogous to (2.2). Equation (2.3) may be recast in a more compact form which is amenable for the estimation of $\sigma_k(\tau_j)$ by the PCA analysis described below

$$d\ln p(t,\tau_j) = \nu(t,\tau_j)dt + \sum_{k=1}^{N}\sigma_k(\tau_j)\,dW_k(t),\ j=1,...,N$$
$$\ln p_j(t) \equiv \ln p(t,\tau_j) = \ln f(t,t+\tau_j)$$
$$\nu_j(t) \equiv \nu(t,\tau_j) = [\mu(t,t+\tau_j) - \tfrac{1}{2}\sum_{k=1}^{N}\sigma^2{}_k(t,t+\tau_j)] + \frac{1}{f(t,t+\tau_j)}\left[\frac{\partial f(t,x)}{\partial x}\right]_{x=t+\tau_j}$$

(2.4)

The PCA analysis proceeds as follows. Assume initially that the number of factors M is equal to N, the number of price series. The NxN covariance matrix Σ_{ij} of the price series $d \ln p_i(t)$ and $d \ln p_j(t)$ in the population is given by the relation

$$\begin{aligned}\Sigma_{ij} &= \frac{1}{dt} E\Big(d \ln p_i(t) - \nu_i(t),\, d \ln p_j(t) - \nu_j(t)\Big) \\ &= \frac{1}{dt} E\left(\sum_{k=1}^{N} \sigma_k(\tau_i)\, dW_k, \sum_{l=1}^{N} \sigma_l(\tau_j)\, dW_l \right) \\ &= \sum_{k=1}^{N} \sigma_k(\tau_i)\, \sigma_k(\tau_j)\end{aligned} \tag{2.5}$$

The left-hand side of (2.5) may be estimated from the price series of the rolling tenor futures prices $p_j(t_m),\ m = 1,...,N$ evaluated at times t_m assuming a constant interval $\Delta t = t_{m+1} - t_m$, say a day. The in sample estimate of the covariance matrix $[\Sigma]_{ij}$ is obtained by introducing the vector of the de-trended daily log-differences of the price series $p_i(t_m),\ m = 1,...,N$

$$\vec{x}_i = \begin{pmatrix} x_{i1} \\ x_{i2} \\ . \\ . \\ . \\ x_{iM} \end{pmatrix} = \begin{pmatrix} \ln \frac{p_i(t_2)}{p_i(t_1)} - \frac{1}{M} \ln \frac{p_i(t_{M+1})}{p_i(t_1)} \\ \ln \frac{p_i(t_3)}{p_i(t_2)} - \frac{1}{M} \ln \frac{p_i(t_{M+1})}{p_i(t_1)} \\ . \\ . \\ . \\ \ln \frac{p_i(t_{M+1})}{p_i(t_M)} - \frac{1}{M} \ln \frac{p_i(t_{M+1})}{p_i(t_1)} \end{pmatrix}, \quad i = 1,...,N \tag{2.6}$$

The in sample covariance matrix of the price series $p_i(t_m),\ m = 1,...,N$ follows from the definitions

(2.7) $$[X]=[\vec{x}_1,\vec{x}_2,...,\vec{x}_N] \\ [\Sigma]=[X]^T[X]$$

Comparing (2.4) and (2.6) we may assume the approximate equality of the in sample and population values of the covariance matrices

(2.8) $$[\Sigma]=[X]^T[X]\simeq\Sigma_{ij}=\sum_{k=1}^{N}\sigma_k(\tau_i)\sigma_k(\tau_j)$$

The form of (2.8) suggests the Singular Value Decomposition (SVD) of the symmetric positive definite matrix $[X]^T[X]$ which will relate the unknown volatilities $\sigma_k(\tau_i)$ to the positive eigenvalues and eigenvectors of the dispersion matrix $[X]^T[X]$. The SVD of $[\Sigma]$ takes the standard form

(2.9) $$\begin{aligned}&[U]^T[\Sigma][U]=[\Lambda]=diag(\lambda_i),\ i=1,...,N\\ &[U]^T[U]=[I]\\ &[\Sigma]=[U][\Lambda][U]^T=[U][\Lambda]^{1/2}[\Lambda]^{1/2}[U]^T=[V][V]^T\\ &[V]=[U][\Lambda]^{1/2}=[U]diag(\sqrt{\lambda_i})\end{aligned}$$

In (2.9) λ_i, i=1,…,N are the positive eigenvalues, $[U]$ is the orthogonal matrix containing the eigenvectors and the matrix $[V]$ has been defined as the product of $[U]=u_{ik}$ with the diagonal matrix containing the square root of the eigenvalues. Denoting by $[V]_{ik}=\upsilon_{ik}$ the typical element of the matrix $[V]$, we may write

(2.10) $$[\Sigma]_{ij}=\left([V][V]^T\right)_{ij}=\sum_{k=1}^{N}\upsilon_{ik}\upsilon_{jk}=\sum_{k=1}^{N}\sigma_k(\tau_i)\sigma_k(\tau_j)$$

The last equality of (2.10) yields the desired result, (2.11) $$\sigma_k(\tau_i) = \upsilon_{ik} = \sqrt{\lambda_k}\, u_{ik},\ i,k = 1,...,N$$

Equation (2.11) states that the volatility of the k-th factor as it affects the i-th price series is equal to the product of the square root of the k-th eigenvalue times the (i,k)-th element of the matrix of eigenvectors $[U]$. The eigenvalues are ordered so that $\lambda_1 > \lambda_2 > \ldots > 0$. The rate of decay of the eigenvalues may be quite rapid and the first few, say d<N, are often sufficient to describe most of the fluctuation of the price series. This value therefore defines the number of dominant factors affecting most of the variation of the forward curve under study. Examples illustrating this property of the PCA will be given in the next Section for the crude oil, gasoline and heating oil markets.

The constant volatilities of the rolling tenor futures contracts estimated by (2.11) may be used to determine the time dependent volatilities $\sigma_k(t,T_i)$ of the fixed tenor traded futures contracts using (2.2). This step along with the selection of the number d of dominant factors completes the estimation of the multi-factor model (2.1) for the traded futures of the commodity under study. The estimation of the drift $\mu(t,T_j)$ under the real world objective measure may be carried out independently using econometric techniques [Campbell, Lo and MacKinley (1997), Lo and MacKinley (1999)]. Yet, its value does not enter the estimation of derivative securities under the risk neutral measure when the drift $\mu(t,T_j)$ is zero and the futures price becomes a martingale. The risk neutral pricing of derivatives is discussed in Sections 5 and 6.

Correlated Commodity Principal Components Analysis

Consider now two commodity forward curves A and B and assume that a PCA analysis has been carried out of each forward curve individually using the method described above. Assume initially that the number of factors is equal to the number of traded futures contracts. It follows that the stochastic evolution of the futures of each commodity is given by the stochastic differential equations

$$
\begin{aligned}
&\frac{dF^A(t,T_j)}{F^A(t,T_j)} = \mu^A(t,T_j)dt + \sum_{k=1}^{N} \sigma^A{}_k(t,T_j)\, dW_k(t),\ j = 1,...,N \\
&dW_k(t)dW_l(t) = \delta_{kl}\ dt
\end{aligned} \tag{2.12}
$$

$$
\begin{aligned}
&\frac{dF^B(t,T_j)}{F^B(t,T_j)} = \mu^B(t,T_j)dt + \sum_{k=1}^{N} \sigma^B{}_k(t,T_j)\, dZ_k(t),\ j = 1,...,N \\
&dZ_k(t)dZ_l(t) = \delta_{kl}\ dt
\end{aligned}
$$

The primary output of each PCA analysis are the factor volatilities and the number of dominant factors d which is assumed to be the same for both commodities. The Brownian increments $dW_l(t)$ of commodity A are mutually independent and the same applies to the Brownian increments $dZ_k(t)$ of commodity B. This is the result of the individual PCAs carried out independently for commodities A and B. Yet, the cross-commodity Brownian increments may be correlated. It is therefore assumed that

$$
dW_k(t)\, dZ_l(t) = \rho_{kl}\ dt \tag{2.13}
$$

In (2.13) ρ_{kl} is assumed to be a constant NxN correlation matrix which is to be estimated from the prices of traded futures contracts of commodities A and B. It follows from (2.3)-(2.4) that the correlation coefficient ρ_{kl} between the Brownian shocks also applies to the rolling tenor futures contracts and can therefore be estimated from their price series. Define the de-trended log-return vectors for commodities A and B

$$
\vec{x}_i^A = \begin{pmatrix} \ln \frac{p^A{}_i(t_2)}{p^A{}_i(t_1)} - \frac{1}{M}\ln \frac{p^A{}_i(t_{M+1})}{p^A{}_i(t_1)} \\ \ln \frac{p^A{}_i(t_3)}{p^A{}_i(t_2)} - \frac{1}{M}\ln \frac{p^A{}_i(t_{M+1})}{p^A{}_i(t_1)} \\ . \\ . \\ . \\ \ln \frac{p^A{}_i(t_{M+1})}{p^A{}_i(t_M)} - \frac{1}{M}\ln \frac{p^A{}_i(t_{M+1})}{p^A{}_i(t_1)} \end{pmatrix}, \quad \vec{x}_i^B = \begin{pmatrix} \ln \frac{p^B{}_i(t_2)}{p^B{}_i(t_1)} - \frac{1}{M}\ln \frac{p^B{}_i(t_{M+1})}{p^A{}_i(t_1)} \\ \ln \frac{p^B{}_i(t_3)}{p^B{}_i(t_2)} - \frac{1}{M}\ln \frac{p^B{}_i(t_{M+1})}{p^B{}_i(t_1)} \\ . \\ . \\ . \\ \ln \frac{p^B{}_i(t_{M+1})}{p^B{}_i(t_M)} - \frac{1}{M}\ln \frac{p^B{}_i(t_{M+1})}{p^B{}_i(t_1)} \end{pmatrix}, \; i = 1,\dots,N \tag{2.14}
$$

Proceeding as in the case of a single commodity we define the dispersion matrices of commodities A and B and their cross-covariance matrices as follows

$$
\begin{aligned}
[X]_A &= [\vec{x}_1, \vec{x}_2, \dots, \vec{x}_N]_A \\
[X]_B &= [\vec{x}_1, \vec{x}_2, \dots, \vec{x}_N]_B \\
\\
[\Sigma]_{AB} &= [X]_A{}^T [X]_B
\end{aligned} \tag{2.15}
$$

The cross-covariance matrix may also be estimated from the stochastic differential equations governing the rolling futures prices of commodities A and B,

$$
\begin{aligned}
\Sigma_{iA,jB} &= \frac{1}{dt} E\left(d\ln p^A{}_i(t) - \nu^A{}_i(t), d\ln p^B{}_j(t) - \nu^B{}_j(t) \right) \\
&= \frac{1}{dt} E\left(\sum_{k=1}^{N} \sigma^A{}_k(\tau_i)\, dW_k, \sum_{l=1}^{N} \sigma^B{}_l(\tau_j)\, dZ_l \right) \\
&= \sum_{k=1}^{N} \sum_{l=1}^{N} \rho_{kl}\, \sigma^A{}_k(\tau_i)\, \sigma^B{}_l(\tau_j)
\end{aligned} \tag{2.16}
$$

Equating the sample cross-covariance matrix (2.15) estimated from the price series to its population counterpart derived from the model we obtain

(2.17) $$[\Sigma]_{AB} = [X]_A{}^T [X]_B \simeq \sum_{k=1}^{N}\sum_{l=1}^{N} \rho_{kl}\, \sigma^A{}_k(\tau_i)\sigma^B{}_l(\tau_j) \equiv \sigma^A{}_k(\tau_i)\,\rho_{kl}\,\sigma^B{}_l(\tau_j)$$

In the last equality of (2.17) the indicial summation notation was introduced for brevity. The factor volatilities that enter (2.17) have been estimated from the individual PCAs carried out for commodities A and B. Recalling (2.10) we may recast (2.17) in matrix form

(2.18) $$[\Sigma]_{AB} = [X]_A{}^T [X]_B = [V]_A [\rho][V]^T{}_B$$

The matrices $[V]_A, [V]_B$ have been obtained from the SVD of the covariance matrices of commodities A and B individually and satisfy the relations

(2.19) $$\begin{aligned}[\Sigma]_{AA} &= [X]_A{}^T [X]_A = [V]_A [V]^T{}_A \\ [\Sigma]_{BB} &= [X]_B{}^T [X]_B = [V]_B [V]^T{}_B\end{aligned}$$

The unknown correlation matrix $[\rho]$ follows from (2.18) explicitly in the form

(2.20) $$\begin{aligned}[\rho] = ([V]_A)^{-1}[\Sigma]_{AB}([V]^T{}_B)^{-1} &= ([V]_A)^{-1}[X]_A{}^T [X]_B([V]^T{}_B)^{-1} \\ &= [\Sigma_{AA}]^{-1}[V]^T{}_A [X]_A{}^T [X]_B [V]_B [\Sigma_{BB}]^{-1}\end{aligned}$$

The estimation of the factor volatilities of commodities A and B by independent PCAs and the factor correlation by (2.20) completes the statistical estimation of the cross-commodity multi-factor covariance structure using the price series of rolling tenor future contracts.

The modeling of the deterministic time dependent instantaneous volatilities $\sigma_k(t,T_i)$ and their calibration to market data lies at the core of the HJM model of the forward curve, extended here to N futures per commodity forward curve. The PCA analysis described above has relied on historical price data of liquid futures contracts for the direct estimation of the factor volatilities. Often it may be appropriate to define and model a single volatility per futures contract followed by the subsequent estimation of the factor loadings. This approach has certain advantages. The single volatility of each futures contract is related to the Black implied volatility which is forward looking and may be extracted from the prices of liquid futures options. Moreover, this volatility may be modeled as a stochastic process which may include jumps, a step that may be necessary for futures contracts with short tenors or for volatile forward markets like electricity and shipping with non-Gaussian log-returns.

Consider the stochastic evolution of a futures contract of a commodity with fixed tenor T_j. Factoring the instantaneous time dependent volatility from the factor volatilities we obtain

$$\begin{aligned}
\frac{dF(t,T_j)}{F(t,T_j)} &= \mu(t,T_j)dt + \sum_{k=1}^{N} \sigma_k(t,T_j)\, dW_k \\
&= \mu(t,T_j)dt + \sigma(t,T_j)\sum_{k=1}^{N} \lambda_k(t,T_j)\, dW_k \\
\sigma^2(t,T_j) &= \sum_{k=1}^{N} \sigma_k{}^2(t,T_j) \\
\lambda_k(t,T_j) &= \frac{\sigma_k(t,T_j)}{\sigma(t,T_j)} \\
\sum_{k=1}^{N} \lambda^2{}_k(t,T_j) &= 1
\end{aligned} \tag{2.21}$$

The quantity $\sigma(t,T_j)$ is hereafter referred to as the instantaneous volatility of the j-th futures contract of the commodity under study. The normalized intra-commodity factor loadings $\lambda_k(t,T_j)$ will be estimated using a PCA of the correlation matrix of the rolling tenor futures contracts, analogous to the one described above, but only after the instantaneous volatility $\sigma(t,T_j)$ has been estimated.

The instantaneous volatility may be calibrated against the Black implied volatilities of traded futures options. It is known that the Black implied volatilities are related to the time averages of the instantaneous variances over the tenor (t,T) of a futures options contract given by the expression

(2.22)
$$\sigma^2_{BLACK}(T_j - t) = \frac{1}{T_j - t}\int_t^{T_j} \sigma^2(s, T_j)\, ds$$

The availability of liquid futures options over a range of tenors T_j permit the estimation of a functional form of the instantaneous volatility $\sigma(t, T_j)$ by a nonlinear squares fit of the Black implied volatilities defined by (2.22). This approach has been adopted for the modeling and pricing of derivatives written on the term structure of interest rates [Rebonato (2002)].

Alternatively, the instantaneous volatility may be estimated from historical data and modeled prior to the estimation of the factor loadings by a PCA of the correlation matrix of the rolling tenor futures contracts. Recall the stochastic differential equation governing the price of the rolling tenor futures. Using the definition of the instantaneous volatility given by (2.21) we obtain

$$d \ln p(t, \tau_j) = \nu(t, \tau_j)dt + \sigma(t, t+\tau_j)\sum_{k=1}^{N} \lambda_k(t, t+\tau_j)\, dW_k(t),\ j = 1, \ldots, N$$
$$p_j(t) \equiv p(t, \tau_j) = f(t, t+\tau_j)$$
$$\nu_j(t) \equiv \nu(t, \tau_j) = [\mu(t, t+\tau_j) - \tfrac{1}{2}\sum_{k=1}^{N} \sigma^2{}_k(t, t+\tau_j)] + \frac{1}{f(t, t+\tau_j)}\left[\frac{\partial f(t, x)}{\partial x}\right]_{x=t+\tau_j}$$

(2.23)

It is reasonable to expect that the rolling tenor instantaneous volatility and correlations are stationary stochastic processes, unlike their fixed tenor counterparts which are clearly non-stationary as the life of a futures contract shortens towards expiration. The simplest approximation is to assume that $\sigma(t, t+\tau_j) \simeq \sigma(\tau_j)$, namely that the j-th rolling tenor volatility is constant. The same would apply to the factor loadings $\lambda_k(t, t+\tau_j) \simeq \lambda_k(\tau_j)$.

The volatility $\sigma(\tau_j)$ may be estimated from historical prices of the rolling tenor futures prices. Using (2.5)-(2.7) we obtain an estimate of the volatility of the j-th rolling tenor futures contract from the relation

(2.24) $$\sigma(\tau_j) = \vec{x}_j^{\,T} \vec{x}_j$$

The length M of the sample of rolling futures prices in the vector $\vec{x}_j$ will be selected along lines analogous to those used to estimate the volatility of other securities using historical price series. In volatile commodity and shipping markets, it is likely that the assumption that the volatility $\sigma(\tau_j)$ is constant may not be sufficient. A more accurate assumption is that it is a stationary process of the form

(2.25) $$\sigma(t, t+\tau_j) = \sigma(t, \tau_j)$$

The time dependence of $\sigma(t, \tau_j)$ may be deterministic or stochastic. Seasonality in the energy commodity and shipping markets may also be present in $\sigma(t, \tau_j)$. This process may again be estimated from historical data using (2.24), independently of the factor loadings, in light of their unit norm. This statistical estimation will reveal the degree to which it can be approximated by a deterministic or a stochastic process and if jumps are present. This step will permit the use of stochastic volatility models with jumps for the modeling of $\sigma(t, \tau_j)$. In the discrete case GARCH models may be used. Moreover, the model parameters are likely to depend on the magnitude of the rolling tenor τ_j. For small rolling tenors, the rolling futures price process may have fat tails and a stochastic volatility process may be appropriate. For large relative tenors the price process may be Gaussian and the assumption that the time dependence of the volatility $\sigma(t, \tau_j)$ is deterministic may be sufficient.

Following the estimation of $\sigma(t,\tau_j)$ from implied volatility or historical price data, the correlation matrix of the rolling futures price processes follows from the expression

(2.26) $$\rho_{ij} = \frac{\vec{x}_i^{\,T}\vec{x}_j}{\sigma(t,\tau_i)\sigma(t,\tau_j)} = \sum_{k=1}^{N} \lambda_k(\tau_i)\lambda_k(\tau_j)$$

Assuming that the time dependence in the covariance of the i-th and j-th price processes is mostly present in the respective volatilities $\sigma(t,\tau_i)$ and $\sigma(t,\tau_j)$, modeled as indicated above, the correlation matrix defined by (2.26) may be assumed to contain elements that are nearly constant. In such a case the factor correlations may be estimated by a direct implementation of the PCA described above. If significant time variability is detected in the correlation matrix estimated by (2.26), the factor loadings $\lambda_k(t,\tau_i)$ may be modeled using methods used in the securities markets discussed in Tsay (2005) and Engle (2009).

In the case of a pair of commodities A and B, the volatilities $\sigma^A(t,\tau_i)$, $\sigma^B(t,\tau_i)$ and factor correlations will be modeled independently from their respective forward curves, followed by the estimation of the cross-commodity factor correlation following the analysis described by equations (2.12)-(2.20).

Stochastic Volatility Models

In volatile energy commodity and shipping markets, or as futures approach expiration, the assumption that the logarithms of the futures prices are Gaussian distributed may need to be refined. When the energy commodity is non-storable, as is the case for electricity and shipping tonnage, sharp and asymmetric jumps in the spot and futures prices are known to occur. Therefore, extensions of the reduced form Gaussian price models developed above may be necessary by introducing jumps in the futures by allowing the volatility to follow a diffusion or a state-dependent process.

The same challenge has been dealt with in the equity markets where the modeling of the skew of the call and put prices observed in the market has led to the development of stochastic volatility models which may also involve jumps in the equity price and in the volatility. These models have been extensively studied and are widely used in practice. Most stochastic volatility models perform equally well in modeling the implied volatility skew and other departures from the Black-Scholes-Merton assumption of constant volatility. At the same time these models offer a reliable representation of the stochastic evolution of the underlying equity price. A popular model introduced by Heston (1993) has been studied extensively. Another choice is the GARCH model which has been mostly studied in a discrete setting. Its continuous time limit and relation to other stochastic volatility models, including Heston's, is studied by Lewis (2005). A distinct advantage of Heston's model is its analytical tractability. It leads to a closed form expression for the characteristic function of the underlying equity process. This property in turn leads to explicit expressions for equity derivatives defined as complex Fourier integrals which may be evaluated by contour integration, quadrature or by Fast Fourier Transforms. Similar closed form expressions of the characteristic function and derivative prices exist when jumps are allowed in the returns of the underlying process and its stochastic volatility.

In the context of the present multi-factor model of commodity and shipping futures, a non-Gaussian statistical structure designed to represent fat tails or to model skewness in the commodity futures options, is possible by allowing the factor vol+atilities to evolve according to the Heston model with jumps in the futures returns. Assuming for simplicity a one-factor model for the evolution of the futures price of a commodity or a shipping freight rate index and ignoring the effect of the tenor on the factor volatilities, a Heston stochastic volatility model with Merton-style jumps in the futures price takes the form under the risk neutral measure

$$
\begin{aligned}
&\frac{dF(t,T)}{F(t,T)} = \sigma_F \sqrt{\nu(t)}\, dW_F(t) + (e^{\alpha+\delta\varepsilon} - 1)[dJ_F(t) - \lambda_F dt] \\
&d\nu(t) = \kappa(\mu - \nu)dt + \sigma_\nu \sqrt{\nu} dW_\nu(t) \qquad (2.27) \\
&dW_F(t)\, dW_\nu(t) = \rho\, dt
\end{aligned}
$$

Jumps in the futures process (2.27) are represented by the Poisson process $dJ_F(t)$ which is assumed to have an intensity λ_F. The parameters (α,δ) controlling the jump size are constants with the random variable ε ~ N(0,1). The parameters of this futures model must be calibrated against market prices of futures and futures options. This model has been studied for equities and its characteristic function is available in closed form [Heston (1993), Gatheral (2006)]. Jumps may also be included in the volatility process in (2.27) as in the models considered by Bates (1996) and Pan (2002).

The joint characteristic function of the futures of two correlated commodities each modeled by (2.27) also exists in closed form and is discussed by Dempster and Hong (2000) and London (2007). This permits the valuation of derivatives either by complex contour integration, quadrature or FFT.

State Space Models

An alternative family of models for the treatment of price processes that exhibit nonlinearities are state space models where the drift and volatility of the underlying and the futures are nonlinear functions of the spot process itself, as opposed to simply functions of time.

The mathematical structure of these models is given by the pair of equations for the underlying spot process and its futures

$$\begin{aligned} \frac{dS_t}{S_t} &= \mu(S_t)dt + \sigma(S_t)dW_S(t) \\ \frac{dF(t,T)}{F(t,T)} &= \sigma_F(F)\,dW_F(t) \end{aligned} \tag{2.28}$$

Under the risk neutral measure the drift of the spot process needs to be adjusted by a market price of risk in order to ensure that its instantaneous drift is rdt, where r is the risk free interest rate. In (2.28) the dependence of the local volatilities $\sigma(S)$ and $\sigma_F(F)$ on the underlying state variables S or F may be assumed to have some analytical form to be determined upon calibration against price data from the energy and shipping spot and futures markets.

The nonlinear structure introduced by state-dependent models is consistent with the supply and demand fundamentals in the power and shipping markets. The latter produce a non-storable commodity – ton-miles -- where the supply of shipping tonnage may become inelastic in tight markets. This topic has been addressed by Adland and Cullinane (2006) for the tanker spot freight rates and the model (2.28) was found to represent well the underlying spot price process particularly away from equilibrium when the supply and demand fundamentals suggest tight markets, analogous to those encountered in the power sector [Joskow (2006)]. The model (2.28) is amenable to analytical treatment and has been studied by Albanese and Campolieti (2006). Explicit expressions are derived relating the underlying spot process and its futures process. The pricing is also presented of exotic derivatives as well as of the probability distribution of first passage time across one or two barriers.

The stochastic volatility and state space models outlined above may be extended to the multi-factor models of commodity forward curves developed above. The models (2.27)-(2.28) may be applied to the volatility $\sigma(t,\tau_j)$ of the stationary price process of the rolling futures contracts with relative tenors τ_j given by

$$
\begin{aligned}
& d\ln p(t,\tau_j) = \nu(t,\tau_j)dt + \sigma(t,\tau_j)\sum_{k=1}^{N}\lambda_k(\tau_j)\,dW_k(t),\; j=1,\ldots,N \\
& p_j(t) \equiv p(t,\tau_j) = f(t,t+\tau_j) \\
& \nu_j(t) \equiv \nu(t,\tau_j) = [\mu(t,t+\tau_j) - \tfrac{1}{2}\sum_{k=1}^{N}\sigma^2{}_k(t,\tau_j)] + \frac{1}{f(t,t+\tau_j)}\left[\frac{\partial f(t,x)}{\partial x}\right]_{x=t+\tau_j}
\end{aligned}
$$

(2.29)

The factor loadings $\lambda_k(\tau_j)$ may be assumed to be independent of time t and just functions of the rolling tenor. This enables the modeling of fat tails in the rolling futures returns while preserving the multi-factor structure of the forward curve of the energy commodity or shipping sector under study.

3. CRUDE OIL PRINCIPAL COMPONENTS ANALYSIS (PCA)

Prices of crude oil futures contracts trading on NYMEX have been obtained from Datastream and constant time-to-maturity prices $p(t,\tau_j)$ as observed at a date t were obtained using (2.2). These prices led to the construction of the static crude oil forward curve observed at three different dates with tenors up to 100 months, illustrated in Figure 2.1. Figure 2.2 illustrates the spot crude oil price over the period 1/1/2003-1/1/2008.

It may be seen from Figure 2.1 that the crude oil forward curve was trading in backwardation on January 1st 2004 and 2008. On January 1st 2006 it was trading in contango for the front 20 months followed and in backwardation from the 20th to the 80th month. The initial "mean" shape of the forward curve is assumed to be reasonably stable and to evolve slowly in time relative to the high frequency fluctuations of the futures prices around this mean shape. As discussed in Section 2 the slope of the mean forward curve contributes a significant component to the drift of the log-returns of the prices $p(t,\tau_j)$ given by (2.4) and used for the de-trending of their log-returns and estimation of their dynamic properties and volatility term structure discussed below.

The de-trended prices evolve through time as stationary random processes, yet their evolutions aren't independent because of the strong correlation between prices, for example, of oil futures with relative tenors 12 and 13 months. The consequence of the strong correlation of the prices of the rolling tenor futures prices is that the smoothness of the initial shape of the forward curve is preserved as prices along the forward curve fluctuate. The distribution of the de-trended log-returns is nearly Gaussian as illustrated in Figure 3.1 for the relative tenors 6 months, 3 years and 5 years. The co-evolution of the log-returns is described by their correlations. The correlation matrix of the constant relative-tenor crude oil futures is shown in Figure 3.2.

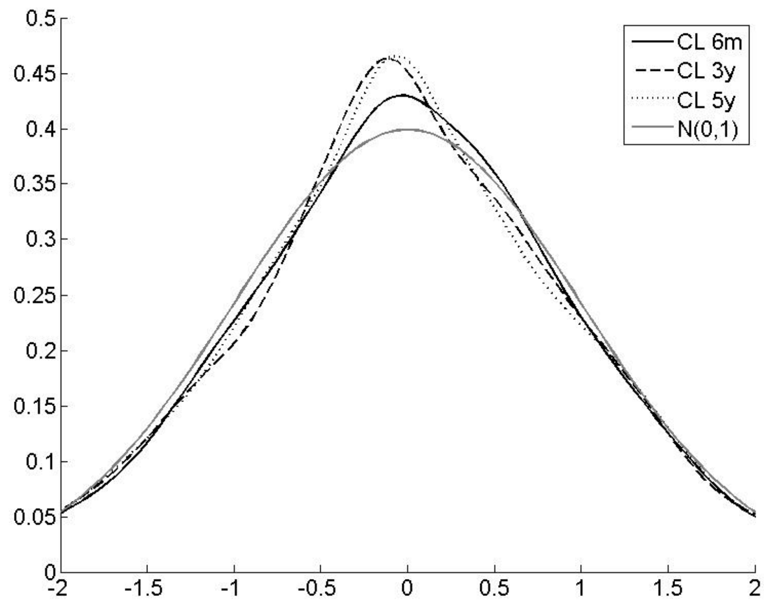


Figure 3.1: Distributions of crude oil 6m, 3y and 5y rolling tenor futures contracts, normalized to unit variance, obtained using a Gaussian kernel density estimator

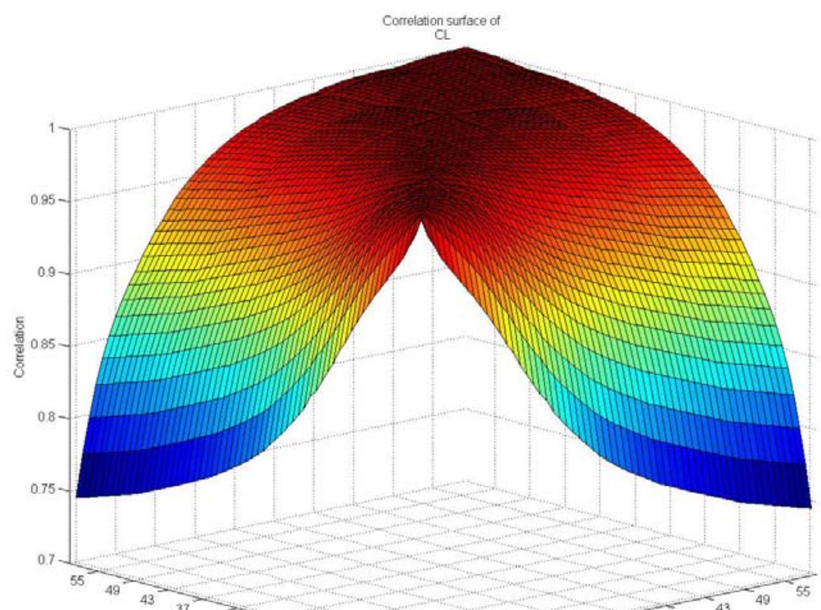


Figure 3.2: Correlation surface of crude oil futures, over the period 1/1/2003-1/1/2008

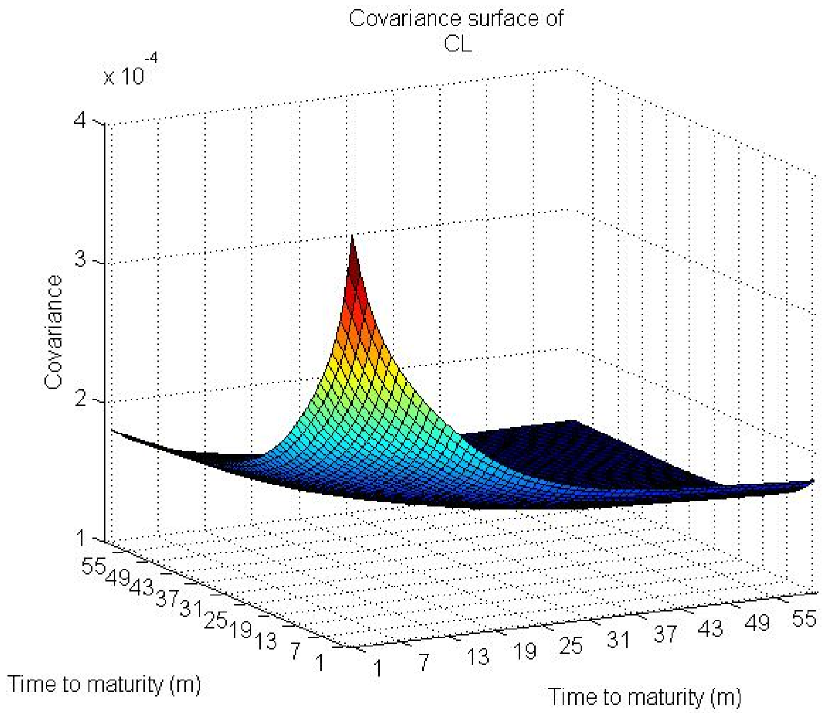


Figure 3.3: Covariance surface of crude oil futures, over the period 1/1/2003-1/1/2008

Principal Components Analysis of the Forward Curve

The joint distribution of the de-trended log-returns $d\ln p(t,\tau_j)$, assumed to be multivariate normal, is described by the NxN covariance matrix displayed in Figure 3.3. For the 1m-60m crude oil futures, this gives 1830 independent parameters. These would indeed be needed if the returns didn't have any structure. But when the returns are highly correlated as is seen in Figure 3.2, Principal Components Analysis (PCA) can be employed to reduce the dimension of the covariance matrix to a small set of significant factors.

Following its estimation the covariance matrix is diagonalized by a Principal Value Decomposition and the eigenvalues are listed in descending order. They are all positive, and generally the first few eigenvalues will explain the major part of the variance of the returns.

A PCA of the covariance matrix of the crude oil forward curve is performed with maturities 1 to 60 months, over the 5-year period 1/1/2003 to 1/1/2008.

Table 1. Eigenvalues and cumulative variance explained

	Eigenvalue λ_k	**Cumulative variance explained**
PC 1	1.0e-2	94 %
PC 2	5.36e-4	99.3%
PC 3	4.34e-5	99.7%
PC 4	1.31e-5	99.9%

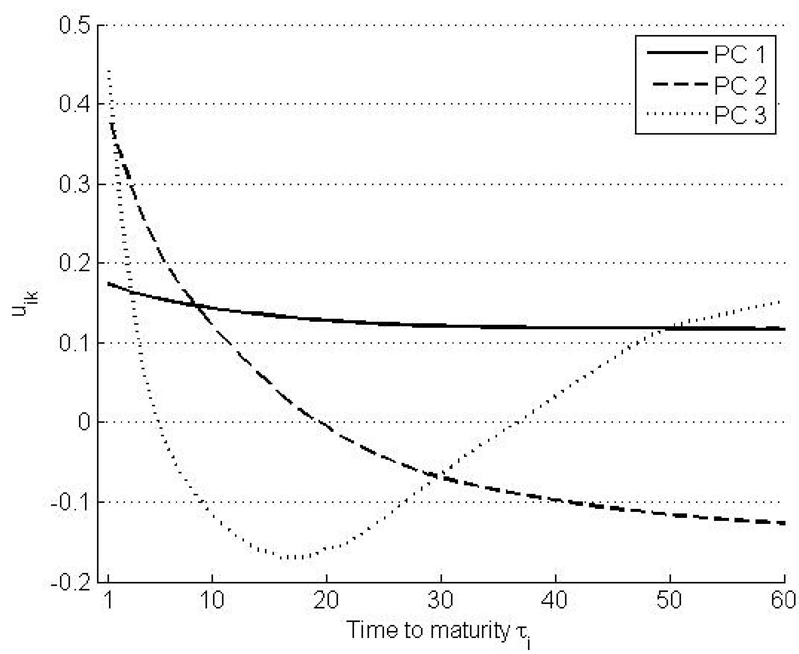


Figure 3.4: Principal component weights (eigenvectors), k=1,2,3

The high correlation between the futures contracts that was observed in Figure 3.2 means that only a few principal components are necessary to explain the variations of the forward curve. As has been found in earlier studies [Borovkova (2006), Geman (2008), Clewlow and Strickland (2000)], these factors correspond to:

- **A shift in the level of the curve**: the coefficients of the first principal component all have the same sign, and correspond to a movement in the same direction of all the prices. As they are highly correlated, this is the most significant effect. However, it is not a parallel shift: the closer maturity contracts, which are more volatile, will fluctuate more than the longer maturity contracts

- **A tilt of the curve:** the second principal component has positive weights for the short tenors and negative weights for the long tenors. This means that if the second factor shock (dW_2) is positive, the prompt contracts will shift up and the distant contracts will shift down.

- **A change in curvature:** the third principal component weights are positive for prompt contracts (1m-5m), negative for intermediate contracts (6m-36m), then positive again for distant contracts (37m-60m). This means that a positive dW_3 will send short and long-term contracts up, but middle-term contracts down.

Analysis of the Factor Returns

As has been seen in (2.9), the PCA is a decomposition of the covariance matrix as

(3.1) $$[\Sigma] = [U][\Lambda][U]^T$$

To relate the factors and the futures returns, let $[P] = [X][U]$ where *[X]* is the MxN data matrix containing the de-trended log-returns. Then

(3.2) $$[P]^T[P] = [U]^T[X]^T[X][U] = [U]^T[\Sigma][U] = [\Lambda]$$

such that the P_k's are uncorrelated, with variance λ_k. They are the factor log-returns and we can express the original price series X as a function of them:

$$
\begin{aligned}
&[X]=[P][U]^T \\
(3.3)\qquad &x_{tj}=u_{j1}P_{t1}+u_{j1}P_{t2}+\ldots+u_{j3}P_{t3} \\
&x_{tj}=\ln\left(\frac{f(t,\tau_j)}{f(t-1,\tau_j)}\right)=\sum_{k=1}^{N}u_{jk}\sqrt{\lambda_k}\left(W_k^{\,t}-W_k^{\,t-1}\right)
\end{aligned}
$$

This shows how to relate the principal components and the futures returns: *P* is the matrix of the *N* principal component log-returns. Their importance is decreasing, as the variance of the *k*-th column of *P* is λ_k. For this reason, we will only study P_1, P_2 and P_3 which are the log-returns of the independent stochastic processes $\sqrt{\lambda_1}dW_1$, $\sqrt{\lambda_2}dW_2$ and $\sqrt{\lambda_3}dW_3$. These are just a weighted time series of the futures log returns and can be studied as such, independently of the model where they are i.i.d $N(0,\lambda_k dt)$.

Table 2. Descriptive statistics of the log-returns of the principal components

	PC 1	PC 2	PC 3
Observations	1304	1304	1304
Mean	0.0077	0.0011	-1.7e-7
Median	0.0025	8.1e-4	1.1e-4
Minimum	-0.3374	-0.09	-0.04
Maximum	0.3612	0.13	0.04
Volatility (annualized)	162 %	37 %	10.6 %
Skewness	-0.056	0.17	0.16
Kurtosis	3.46	5.57	9.4
Jarque-Bera (p-value)	12.4 (0.0044)	364.3 (< 1e-3)	2250 (< 1e-3)
Jarque-Bera test	Rejected	Rejected	Rejected

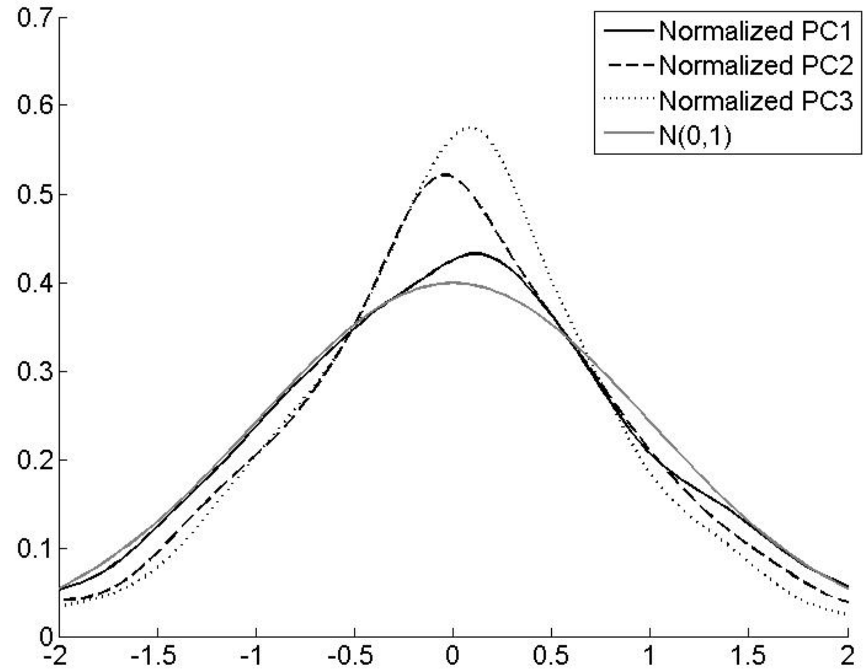
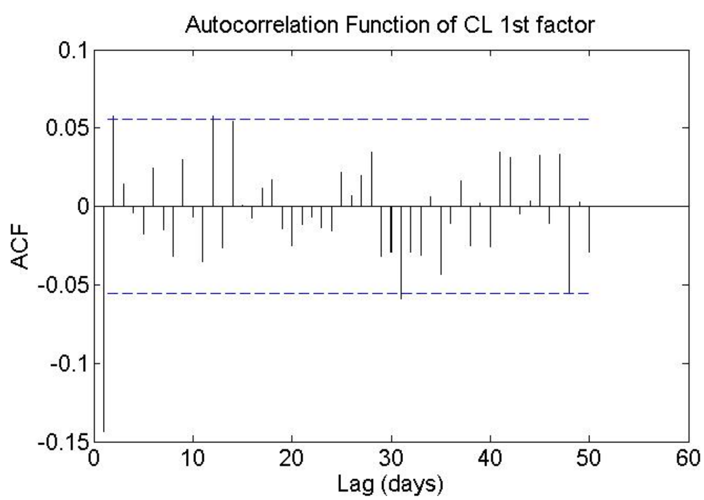
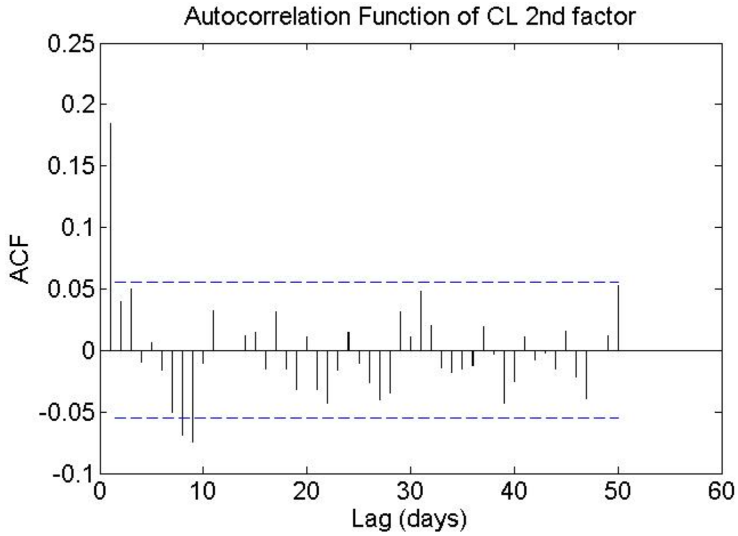


Figure 3.5: Distributions of the 3 PCs compared to the normal distribution. The distributions are estimated using a Gaussian kernel smoothing and normalized to unit variance





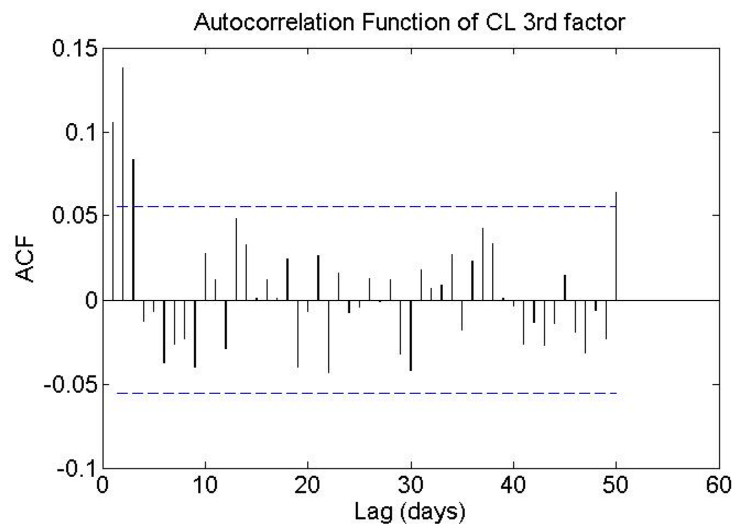


Figure 3.6: Autocorrelation function of PC 1,2,3. 95% confidence intervals in dashed line

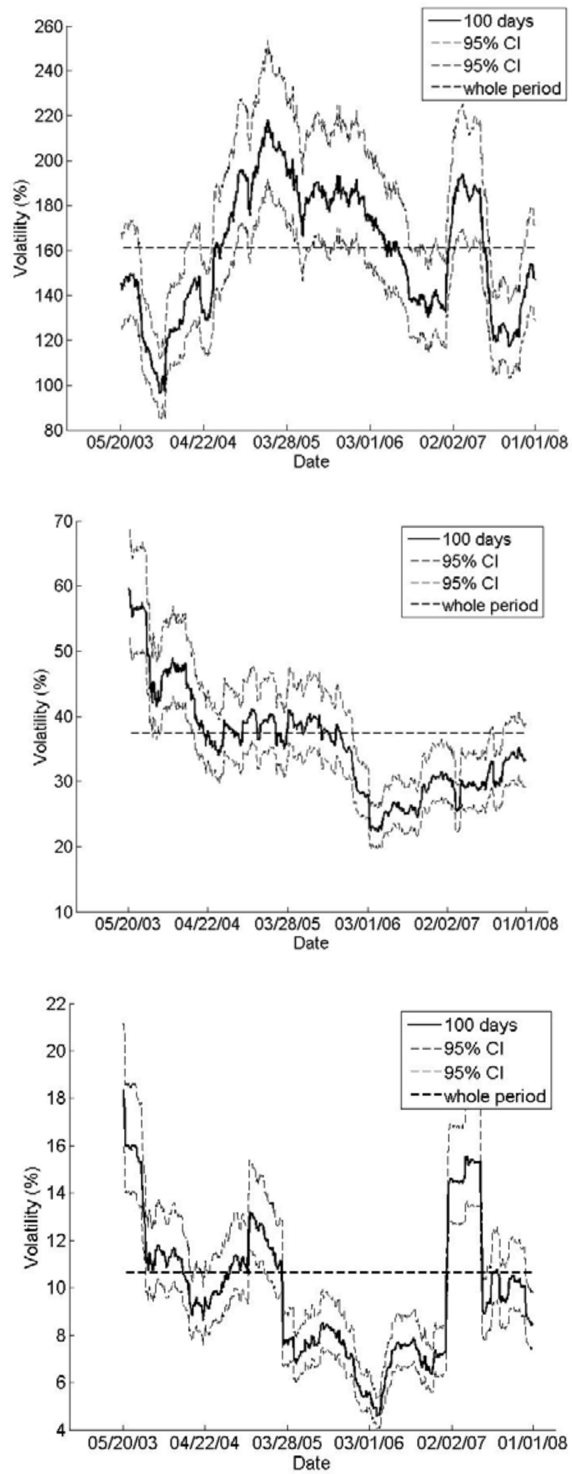


Figure 3.7: Stability of the volatility of Principal Components 1,2,3: Rolling 100-day volatility vs. volatility over entire period

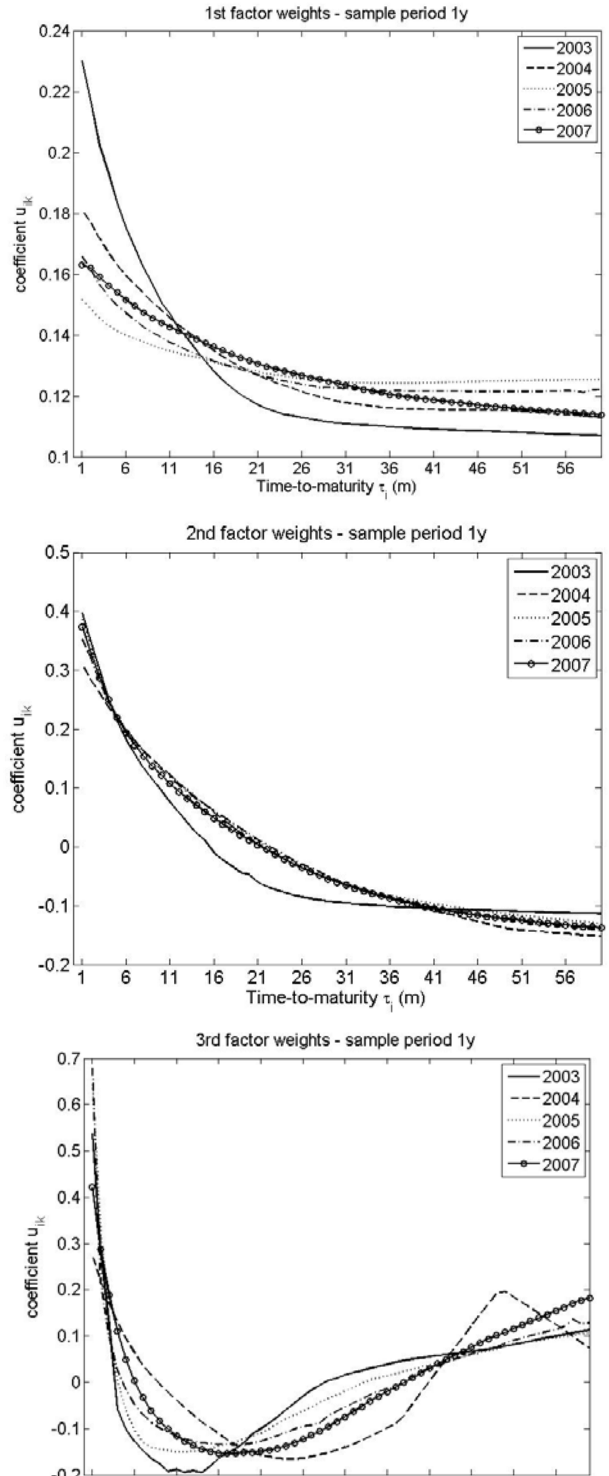


Figure 3.8: Stability of the PCA weights (U matrix): $U_{1,2,3}$ calculated over non-overlapping 1-year periods; Correspond to Principal components 1,2,3 in Figure 3.4

As can be seen in Table 2, the Jarque-Bera test rejects the null hypothesis of normality. But the returns are closer to normal than what has been exhibited by Geman and Kharoubi (2008) – which admittedly included the 1st Gulf war and other crises – or what is commonly observed in stock markets. According to Figure 3.6 they also exhibit some autocorrelation – around 0.15 – for the 1-day lag, but the autocorrelation function is known to be noisy and we will not give any importance to this finding.

In Figure 3.7 we compare the volatility of the principal components as measured over the whole period $\sqrt{\lambda_k}$ to the 100-day sliding window volatility. The assumption in Section 2 is that the constant time-to-maturity contracts are a stationary process, and this should entail that the principal components also follow a stationary process. While the sampled rolling 100-day volatility isn't constant, it doesn't move far from the long-term average, and in particular the 95% confidence interval stays very close to the 5-year volatility. It therefore seems reasonable to assume a constant volatility. If a more precise description (for short period risk forecasts for example) is needed, GARCH can be introduced.

The above results are presented keeping the weights u_{ik} constant, equal to the values calculated over 5 years. They depend on the shape of the covariance surface during the period. However, consistent with our assumption of stationary returns on the constant time-to-maturity contracts, this covariance surface is stable, and this is reflected in the stability of the weights u_{ik} sub-sampled in 1-year periods, as shown in Figure 3.8.

Principal Components as Indicators of Forward Curve Transitions

As seen from the shape and signs of the principal component weights in Figure 3.8, positive returns on the individual principal components will have different effects on the forward curve as a whole. The first principal component, giving the most variation, will push the whole curve up or down (as seen from 3.3). Figure 3.9 plots the log-price of the first principal component which is seen to drift upwards from early 2003 to mid-2006. Given the positive sign of the coefficient u_{1j}, plotted in Figure 3.8a as a function of the rolling tenor, the upwards drift of the first principal component corresponds to an upward shift of the entire forward curve, the shift being more pronounced for the prompt tenors and less pronounced for the distant ones.

The second principal component plotted in Figure 3.10, on the other hand, pushes near maturity prices up and long-maturity prices down. This is the result of the sign reversal of the coefficient u_{2j}, plotted in Figure 3.8b as function of the rolling tenor. So the second principal component has the potential to explain transitions of the forward curve from contango to backwardation. If a market is in contango and receives enough negative shocks from the second principal component it will go into backwardation.

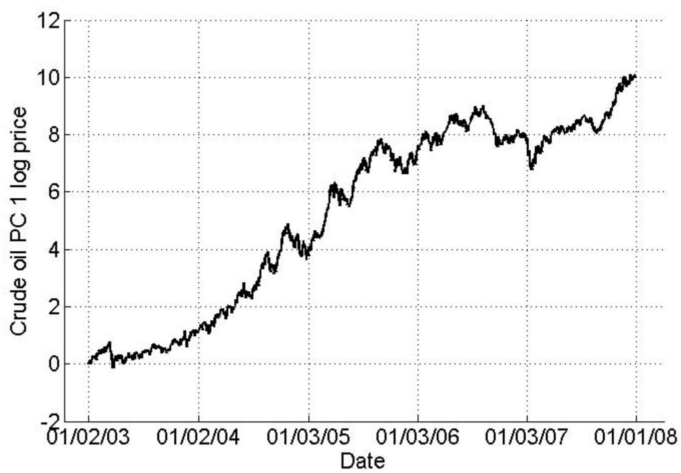


Figure 3.9: Log-price of Crude Oil Principal Component 1

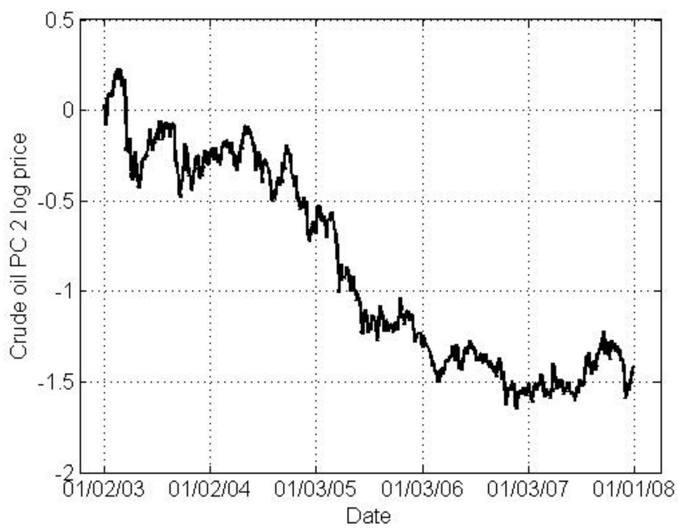


Figure 3.10: Log-price of Crude Oil Principal Component 2

There is a discernible downwards trend of the second principal component log-price plotted in Figure 3.10 between early 2004 and January 2007. Figure 3.11 shows the forward curves during that period, scaled by the front month price (such that we are only looking at the shape, not the level). There is a clear shape change during the period, but it is slow and the forward curve isn't in contango until early 2007. This is partly the result of the downwards drift of the second principal component during that period combined with the sign reversal of the coefficient u_{2j} for distant tenors. These results show that an indicator of the change of shape of the forward curve is the second principal component. This is consistent to what is suggested in Borovkova (2006), except that the present study carries a PCA on the log returns, not a PCA on the log prices.

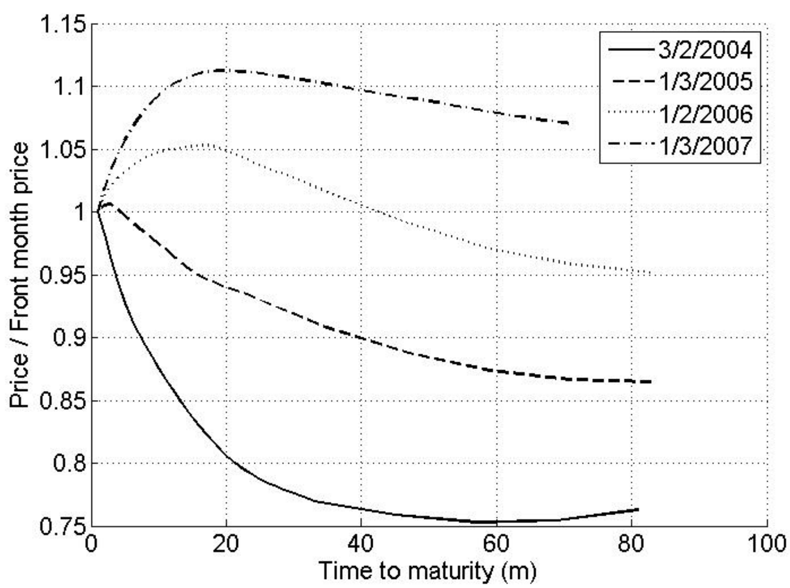


Figure 3.11: Forward curves of crude oil (scaled by the front month price) on different dates

Seasonality in the Heating Oil and Gasoline Markets

It is well known that the heating oil and gasoline markets are seasonal. This is linked to their different consumption during the winter and summer, and the associated building up of stocks. This pattern is apparent in the forward curves shown in Figure 3.12. For gasoline a pattern can also be spotted in the price series whereas for heating oil the seasonal pattern is almost impossible to spot (Figure 3.13).

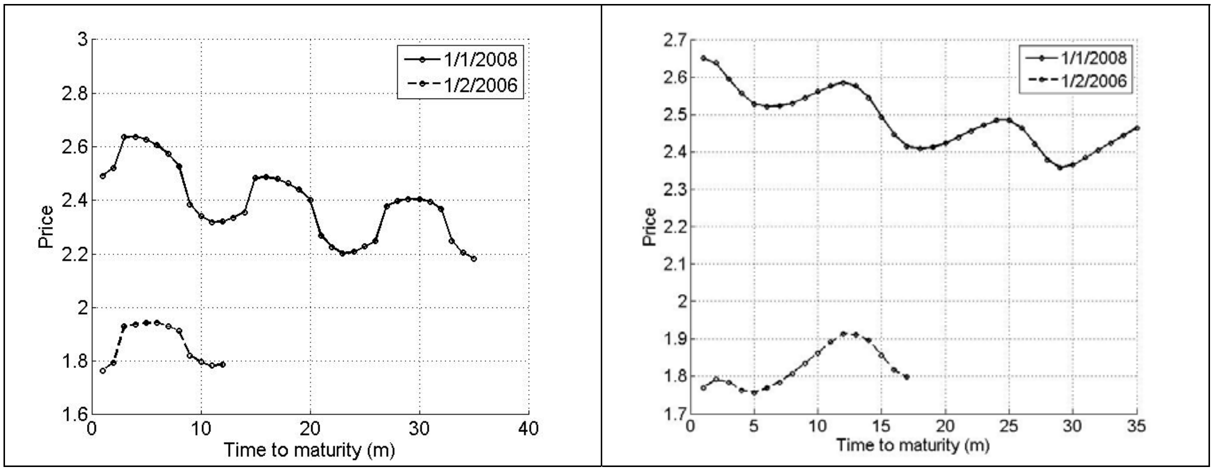


Figure 3.12: Forward curves of RBOB gasoline (left) and heating oil (right)

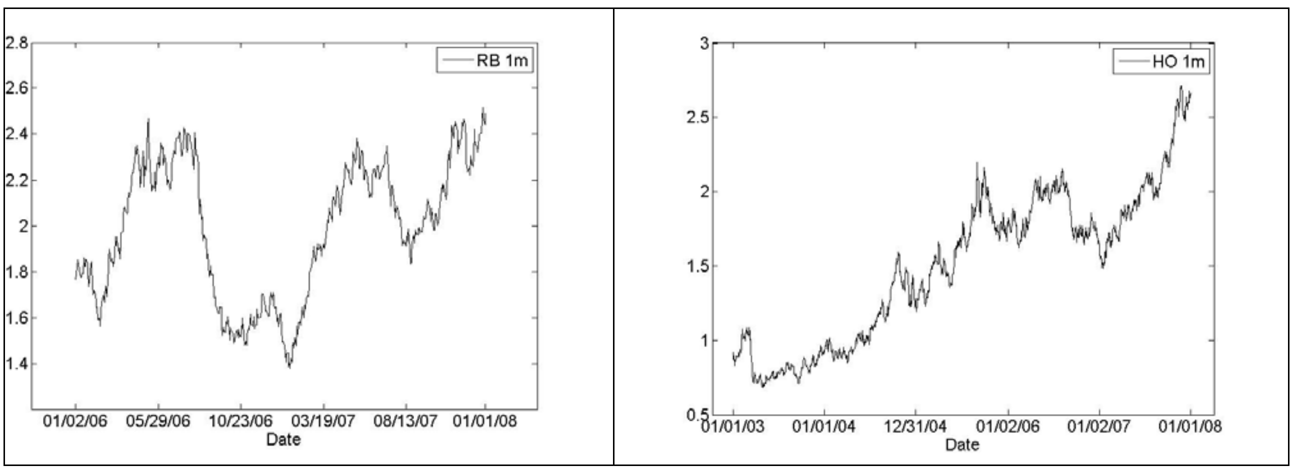


Figure 3.13: Price of RBOB gasoline (left) and Heating Oil (right)

When considering a futures contract with fixed expiration date, *F(t,T)*, its time evolution is not seasonal – the expiration date is constant, and is either a high-consumption or a low-consumption month. However, when considering constant *time-to-maturity* contracts $f(t,\tau)$ the actual delivery date of the contract is varying in time, therefore the time evolution will be seasonal. This is reflected in the stochastic differential equation derived in Section 2

$$\frac{df(t,\tau)}{f(t,\tau)} = \left(\frac{1}{f(t,\tau)} \frac{\partial f(t,\tau)}{\partial \tau} + \mu_t \right) dt + \sum_{k=1}^{d} \sigma_k(t,\tau) dW_k(t)$$

There exists a deterministic part in the futures price drift which arises from the up/down slope of the futures curve. To remove this expected evolution, we consider the de-trended log-returns:

$$\ln\left(\frac{f(t+\Delta t,\tau)}{f(t,\tau)}\right)-\frac{\partial \ln f(t,\tau)}{\partial \tau}\Delta t$$

Where $\frac{\partial \ln f(t,\tau)}{\partial \tau}$ is calculated at time t as $\frac{\partial \ln f(t,\tau_i)}{\partial \tau} \approx \frac{\ln f(t,\tau_{i+1}) - \ln f(t,\tau_i)}{\tau_{i+1}-\tau_i}$. We then perform the PCA on the de-trended log-returns.

Results

When performing the PCA on the original log-returns of heating oil, the 1st and 2nd principal components do not exhibit any seasonality. However the 3rd principal component picks up the seasonal variations as shown in Figure 3.14. After removing the expected returns the 3rd factor does not exhibit any seasonality, and the 1st and 2nd factor do not change.

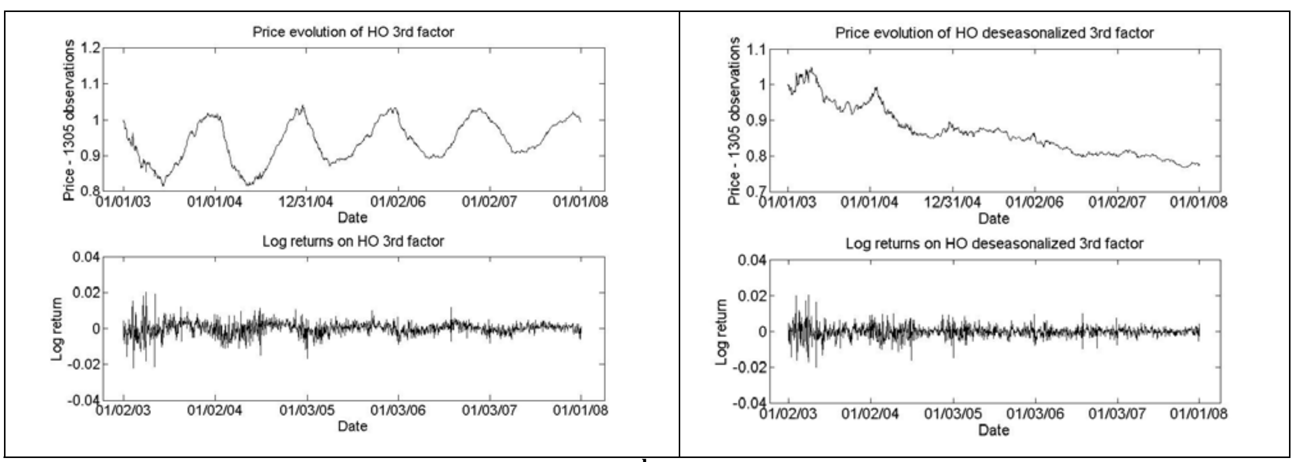


Figure 3.14: Time evolution of Heating Oil 3rd principal component – before and after de-seasonalizing

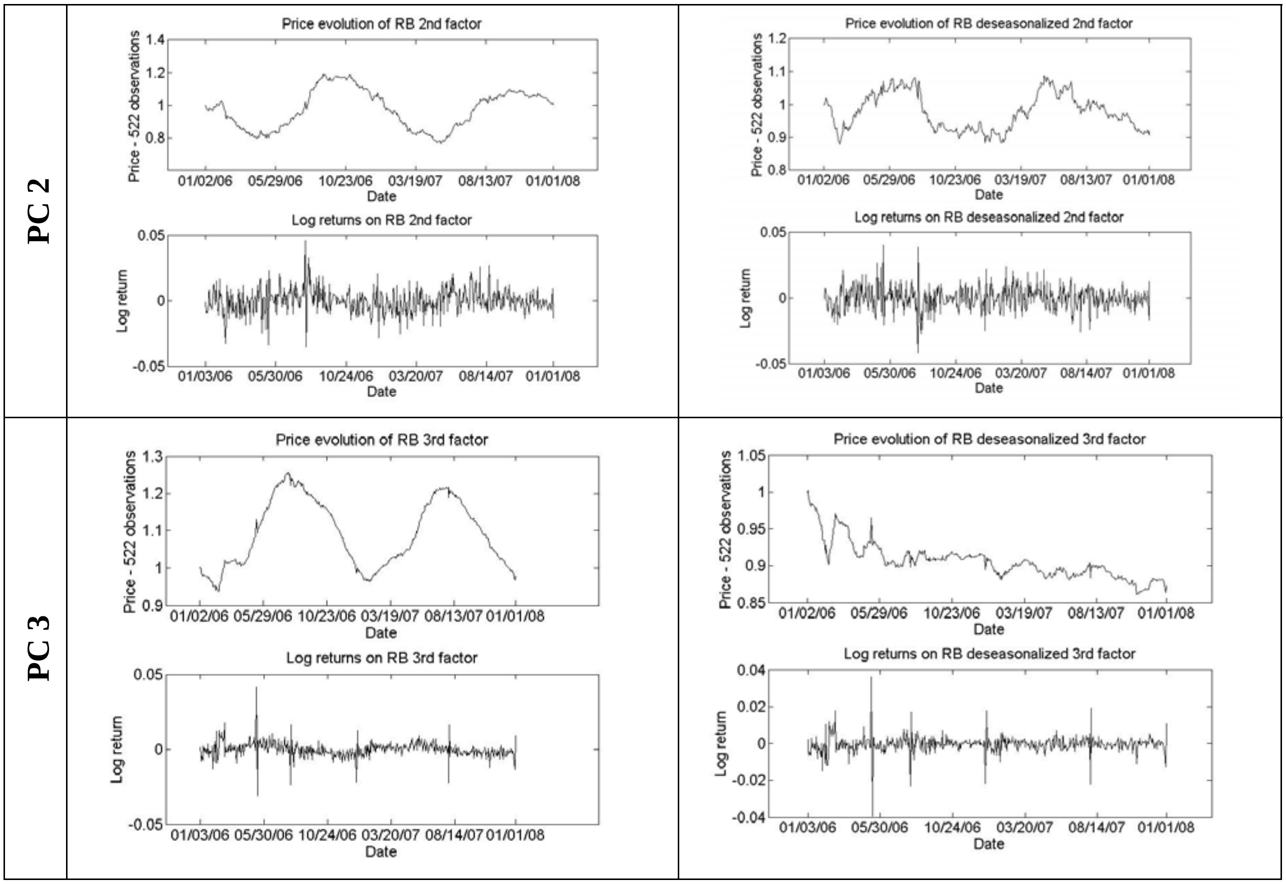


Figure 3.15: Time evolution of principal components 2 and 3 of RBOB gasoline before and after deseasonalizing

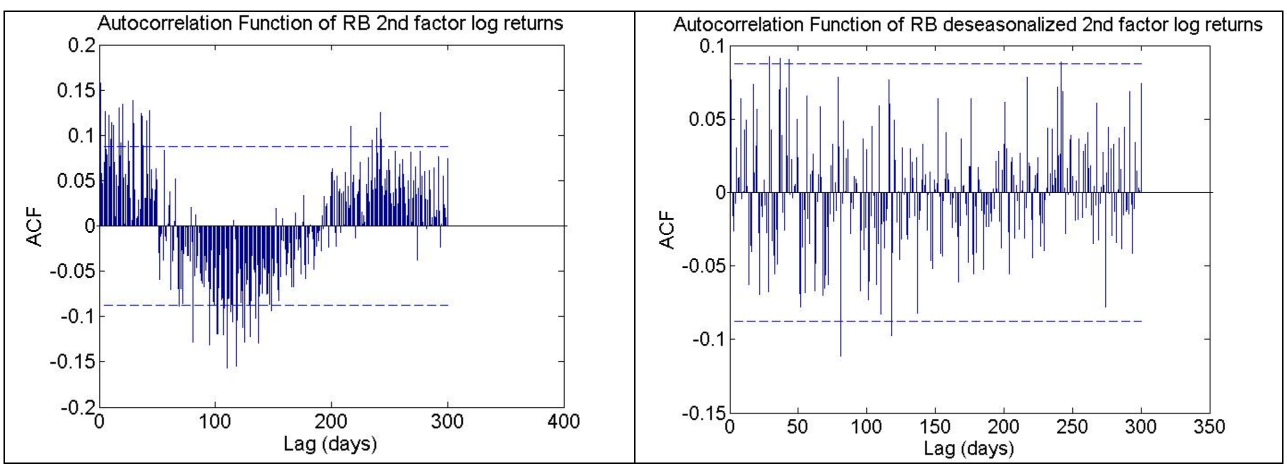


Figure 3.16: Autocorrelation function of PC2 of RBOB gasoline before and after deseasonalizing

The same procedure is applied to RBOB gasoline. In this case the 2nd and 3rd principal components exhibit seasonality – and do not after de-seasonalizing as seen in Figure 3.15. It is interesting to note that although the 2nd principal component still seems seasonal, it is not – this is apparent from the autocorrelation functions of the two as displayed in Figure 3.16.

Several texts [Clewlow and Strickland (2000)] consider the possibility of a seasonal variation of volatility. We calculate volatility quarterly for the different factors (after deseasonalizing, which doesn't change the volatility[1]) and find no evidence of any seasonal volatility, as seen in Figure 3.17.

Inter-Commodity Correlations

After having analyzed each market by itself, we turn to the case of several correlated markets. We will be concentrating on crude oil and heating oil. For each market, we have chosen three principal components explaining the major part of the variations of the forward curve. We will call these CL1, CL2, CL3 and HO1, HO2, HO3, respectively. The next step, as outlined in Section 2, consists in estimating the correlations between these principal components. Over the whole period we already know the intra-commodity correlations, so there are only 3x3 = 9 unknowns. We will also be looking at their stability during the 5-year period. The correlation matrix over the whole period and 95% confidence intervals are shown in Table 3. The correlations CL1-HO3, CL3-HO1 and CL3-HO3 are insignificant at the 95% level.

Table 3. Correlation matrix and 95% confidence intervals of the principal components of crude and heating oil

	CL 1	CL 2	CL 3	HO 1	HO 2	HO 3
CL 1	1	0	0	0.89 [0.88, 0.90]	-0.15 [-0.20, -0.09]	-0.02 [-0.07, 0.03]
CL 2		1	0	0.29 [0.24, 0.34]	0.47 [0.42, 0.51]	0.20 [0.15, 0.25]
CL 3			1	-0.03 [-0.09, 0.02]	-0.16 [-0.21, -0.10]	-0.02 [-0.07, 0.03]
HO 1				1	0	0
HO 2					1	0
HO 3						1

[1] This analysis has been carried out before and after deseasonalizing, and there is no significant difference: the drift is negligible compared to the standard deviation.

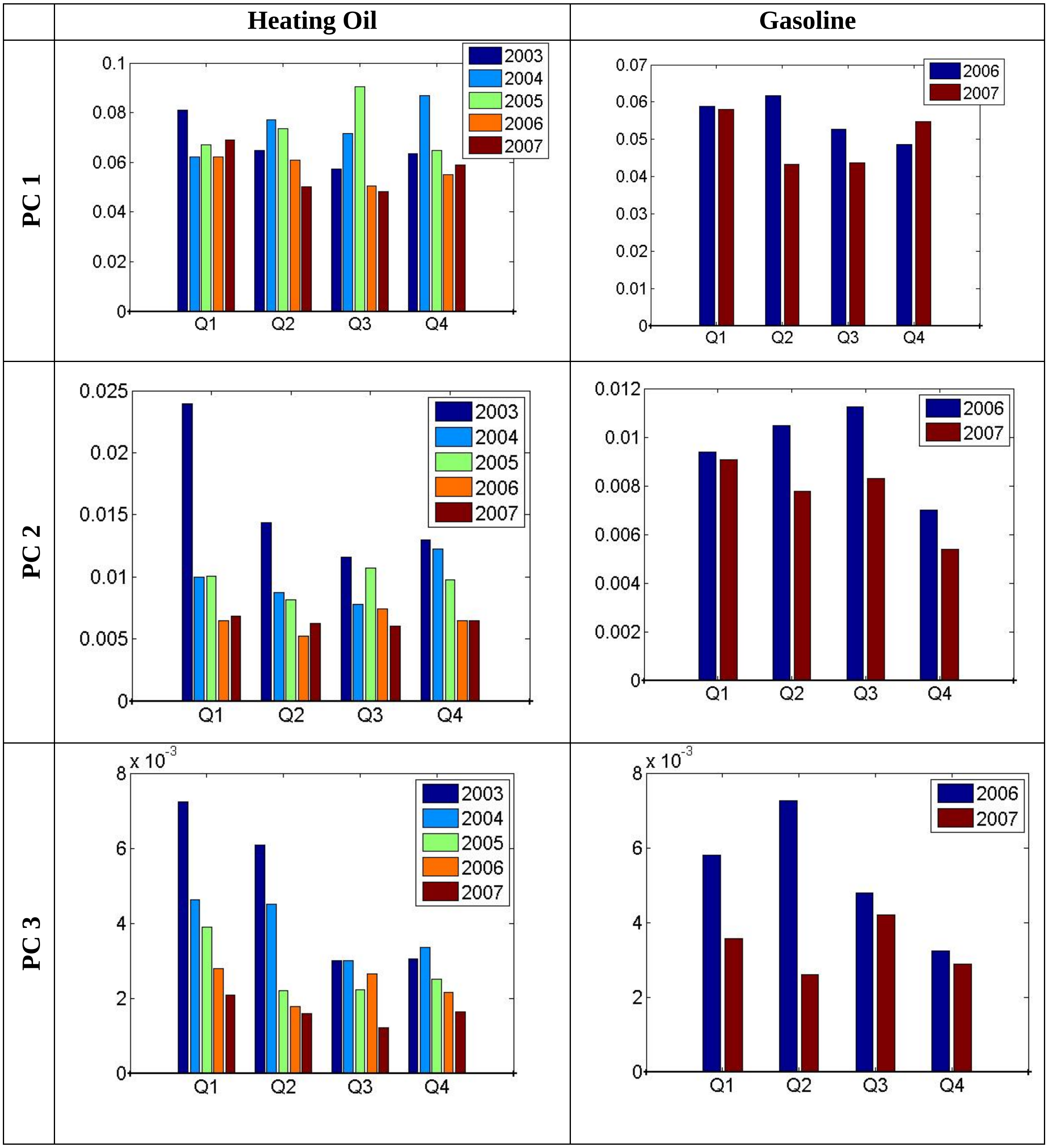
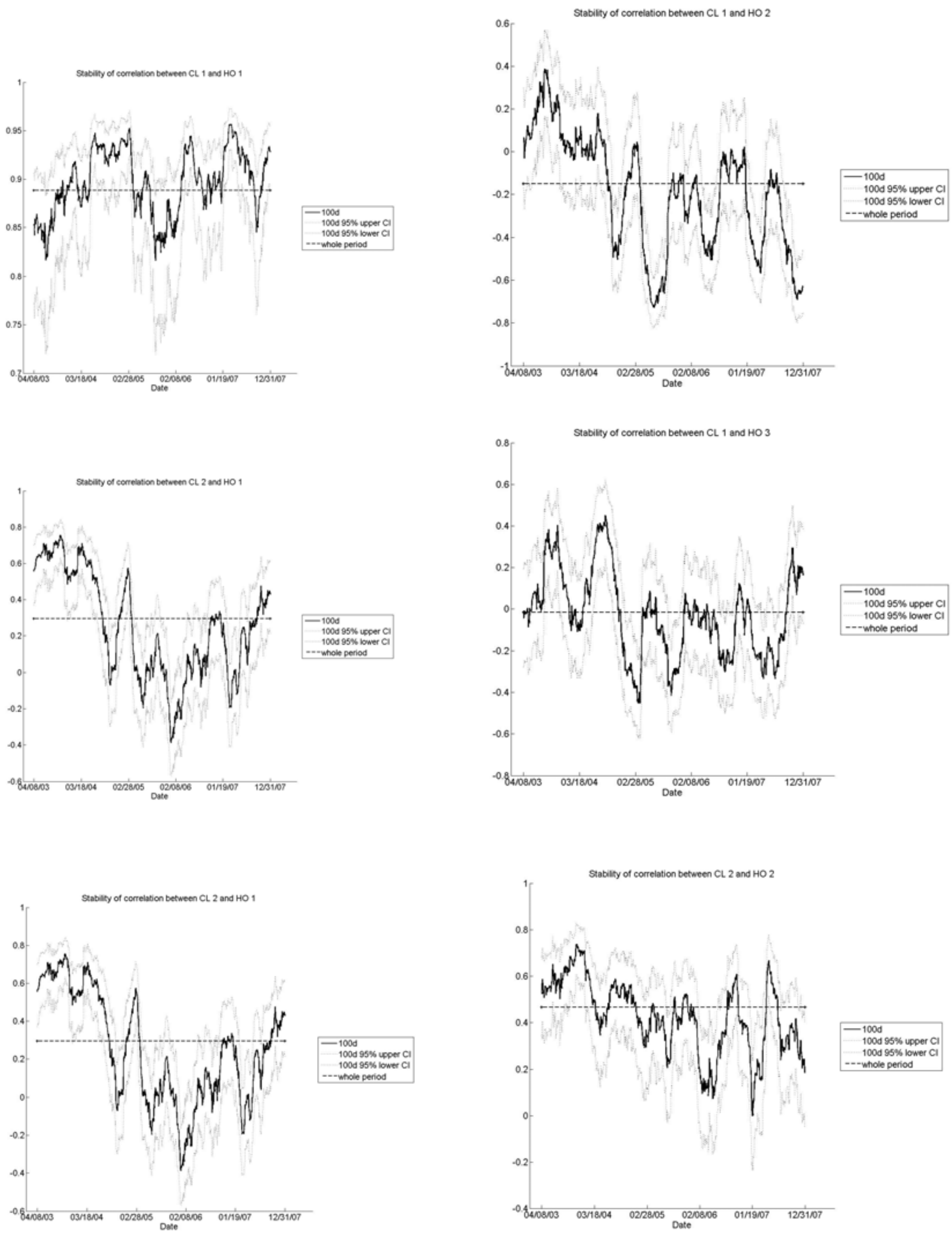
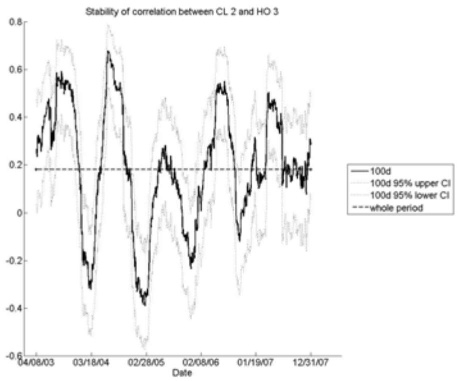
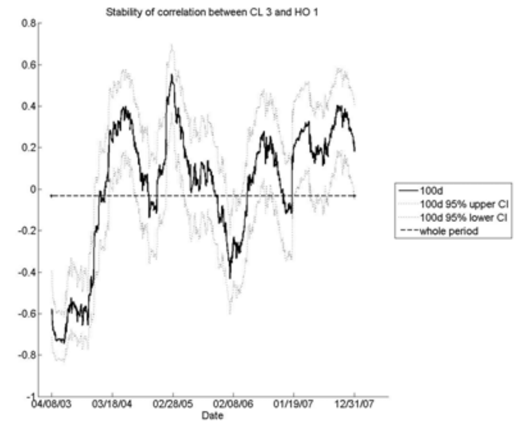


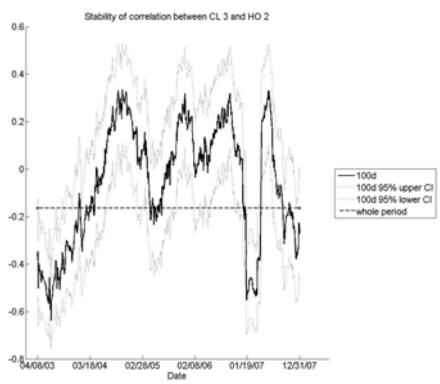
Figure 3.17: Quarterly standard deviations of heating oil and gasoline

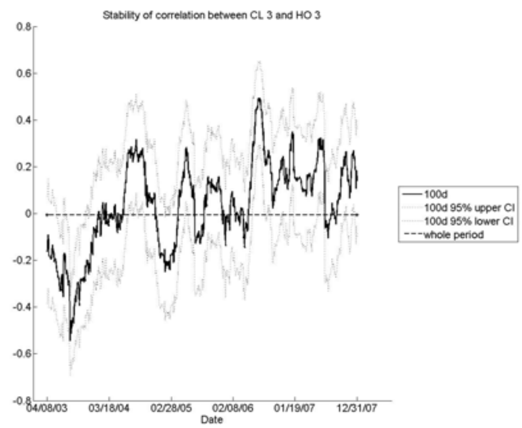
Figure 3.18: Correlations between the crude and heating oil principal components











The PCA of the crude oil market enables a fundamental analysis of the statistical factors responsible for the evolution of the forward curve. The correlation structure of the forward curve has been found to be stable over the period 2003-2008. This leads to the conclusion that the dynamics of the crude oil forward curve is governed by the statistical properties of a small set of factors which contain information on the economic drivers that control the transition of the forward curve from contango to backwardation. Moreover, the cross-commodity factor correlation allows this analysis to be extended to the study of the joint evolution of the forward curves of crude oil, gasoline, heating oil and other energy commodities in the crude oil complex. This analysis framework may be used for the design of a wide range of risk management and investment strategies in the crude oil and shipping markets discussed in Section 8.

4. CRUDE OIL SPOT PRICE PROCESS

Following the PCA described in Sections 2 and 3 assume that M dominant factors have been identified for a particular commodity forward curve. The price evolution of a futures contract associated with this forward curve is governed by the stochastic differential equation

$$
\begin{aligned}
&\frac{dF(t,T_j)}{F(t,T_j)} = \mu(t,T_j)\,dt + \sigma(t,T_j)\sum_{k=1}^{M}\lambda_k(t,T_j)\,dW_k;\ j = 1,\ldots,N \\
&\sigma_k(t,T_j) = \sigma(t,T_j)\lambda_k(t,T_j)
\end{aligned}
\tag{4.1}
$$

The solution of the stochastic differential equation (4.1) governing the futures price exists in closed form and is studied next. This solution suggests explicit evolution dynamics for the spot price under the risk neutral measure. In particular the mean reversion of the spot price around a long term stochastic trend is revealed in terms of the factor volatilities. The degree to which spot energy commodity prices mean revert has been researched extensively in the crude oil, natural gas and other commodity markers [Pindyck (1997)]. The same has been the case for the shipping markets discussed later in this article. The analysis presented below implies that the risk neutral dynamics of the spot commodity price exists in equilibrium with the forward markets and that price transmission mechanisms exist from futures to spot and vice versa.

The factor volatilities $\sigma_i(t,T) = \sigma(t,T)\lambda_i(t,T)$ have been estimated from the PCA described in Section 3. The solution of the stochastic differential equation (4.1) under the risk neutral measure follows explicitly upon integration from the initial time t=0 to the current time t<T

$$
F(t,T) = F(0,T)\exp\left[\sum_{k=1}^{M}\left\{-\frac{1}{2}\int_0^t \sigma_k^{\,2}(s,T)ds + \int_0^t \sigma_k(s,T)\,dW_k(s)\right\}\right]
\tag{4.2}
$$

It follows from (4.2) that the logarithm of the futures price at time t is Gaussian distributed, or

$$\text{(4.3)} \qquad \ln F(t,T) \sim N\left(\ln F(0,T) - \sum_{k=1}^{M} \frac{1}{2}\int_0^t \sigma_k{}^2(s,T)ds,\ \sum_{i=1}^{N}\int_0^t \sigma_k{}^2(s,T)\,ds \right)$$

In (4.2)-(4.3) t=0 F(0,T) is the known price at time t=0 of a futures contract that matures at T. The futures price at a future time t > 0, F(t,T), is a lognormal stochastic process with mean and variance defined by the first and second terms inside the parentheses of expression (4.3), respectively.

Invoking the equality of the futures and spot at the expiration, the spot price process $S^*(t)$ for the commodity under the risk neutral measure, when $\mu(t,T)=0$, follows from (4.2) in the form

$$\text{(4.4)} \qquad S^*(t) = F(t,t) = F(0,t)\exp\left[\sum_{k=1}^{M}\left\{-\frac{1}{2}\int_0^t \sigma_k{}^2(s,t)ds + \int_0^t \sigma_k(s,t)\,dW_k(s)\right\}\right]$$

and

$$\text{(4.5)} \qquad \ln S^*(t) \sim N\left(\ln F(0,t) - \sum_{k=1}^{M}\frac{1}{2}\int_0^t \sigma_k{}^2(s,t)ds,\ \sum_{k=1}^{N}\int_0^t \sigma^2{}_k(s,t)\,ds \right)$$

The implied spot price process at time t is also lognormally distributed under the risk neutral measure at time t with mean and variance given by (4.5).

Upon closer inspection of (4.4)-(4.5) a number of observations can be made about the structure of the stochastic process followed by the spot at time t. It depends upon the time t=0 price of a futures contract maturing at time t, hence it is a function of the initial shape of the futures curve. The price of the spot process at time t is a function of its path from the initial time t=0. In particular it depends upon the integral of the time history of the factor volatilities. Hence, the stochastic process (4.4) governing $S^*(t)$ is non-Markovian, since its price at time t depends upon its entire history over the interval (0,t). Markovian dynamics for $S^*(t)$ is however possible under restrictions on the time rate of decay of the factor volatilities σ_i(t,T) with respect to the tenor.

The evolution dynamics of the spot price process under the risk neutral measure may be derived by rewriting (4.4) in the form

(4.6)
$$\ln\left(\frac{S^*(t)}{F(0,t)}\right) = \sum_{k=1}^{M} s_k(t)$$
$$s_k(t) = -\frac{1}{2}\int_0^t \sigma_k^2(s,t)ds + \int_0^t \sigma_k(s,t)\,dW_k(s)$$

Therefore, the logarithm of the ratio of the spot price to the initial shape of the futures curve at the current time t=0 is the summation of N independent factor processes. Taking the differential of (4.6) the stochastic dynamics of the spot price process under the risk neutral measure yields the dynamics of the factors,

(4.7)
$$ds_k(t) = [-\frac{1}{2}\sigma_k^2(t,t) - \int_0^t \sigma_k(s,t)\frac{\partial\sigma_k(s,t)}{\partial t}ds]dt$$
$$+\sigma_k(t,t)\,dW_k(t) + \int_0^t \frac{\partial\sigma_k(s,t)}{\partial t}dW_k(s)$$

Following the analysis of Inui and Kijima (1998) in the interest rate markets, we may impose the following restriction on the rate of decay of the factor volatility with respect to its tenor

(4.8)
$$\frac{\partial\sigma_k(t,T)}{\partial T} = -\kappa_k(T)\,\sigma_k(t,T)$$
$$\kappa_k(T) = -\frac{\partial\ln\sigma_k(t,T)}{\partial T} = -\frac{\partial\ln\sigma_k(t,t+\tau)}{\partial\tau} \simeq -\frac{\partial\ln\sigma_k(\tau)}{\partial\tau}$$

where $\kappa_k(T)$ is a positive function of T. Upon substitution in (4.7) the factor dynamics takes the form

(4.9)
$$\begin{aligned} ds_k(t) &= [-\frac{1}{2}\sigma_k{}^2(t,t) + \kappa_k(t)\int_0^t \sigma^2{}_k(s,t)\,ds]dt \\ &\quad + \sigma_k(t,t)\,dW_k(t) - \kappa_k(t)\int_0^t \sigma_k(s,t)\,dW_k(s) \\ ds_k(t) &= -\frac{1}{2}\sigma_k{}^2(t,t)dt + \kappa_k(t)[\frac{1}{2}\int_0^t \sigma^2{}_k(s,t)\,ds - s_k(t)]dt \\ &\quad + \sigma_k(t,t)\,dW_k(t) \end{aligned}$$

It is seen from (4.9) that under the restriction (4.8) the process followed by the i-th factor is Markovian with mean reverting dynamics and a mean reversion coefficient $\kappa_k(t)$. Its drift includes an integral of the volatility over the interval (0,t) and its local volatility is the k-th factor volatility when t=T.

Taking the differential of the first equation in (4.6) the dynamics of the spot process under the risk neutral measure takes the form

(4.10)
$$\begin{aligned} d\ln S^*(t) &= d\ln F(0,t) + \sum_{k=1}^{M} ds_k(t) \\ \frac{dS^*(t)}{S^*(t)} &= \frac{\partial \ln F(0,t)}{\partial t}dt + \sum_{k=1}^{M} ds_k(t) \\ F(0,t) &= \overline{F}(0,t) + \sum_{j=1}^{L} \beta_j g_j(t) \end{aligned}$$

The lognormal evolution dynamics (4.10) of the spot process as implied by the entire futures curve under the risk neutral measure has a deterministic drift term a function of the time rate of change of the known log-futures at a tenor t, as observed at the initial time t=0. This drift is common to all factors and is known from the initial t=0 shape of the forward curve. The deterministic seasonality of the forward curve is contained in the function

$F(0,t)$, 0<t<T, and may be decomposed into the sum of a smooth term $\bar{F}(0,t)$ and the sum of L periodic functions $g_j(t)$ which represent the seasonality pattern of the energy or shipping market under study. The mathematical form of the periodic functions $g_j(t)$ may vary depending on the commodity market and may take a cosine, exponential or a power form as discussed by Pilipovic (2007). The smooth term and coefficients β_j multiplying the periodic functions may be determined by curve fitting the forward curve at time t=0.

Each independent factor has mean reverting stochastic evolution dynamics given by (4.9) with coefficients that are functions of the factor volatilities, including the coefficient of mean reversion $\kappa_k(t)$ defined by (4.8). As discussed above, a small number of dominant factors explain most of the fluctuations of the futures curves of energy commodities. The dominant factor corresponding to i=1 has the highest volatility and therefore represents the most volatile shocks to the futures curve. The volatility of the second factor is smaller by an amount equal to the ratio of the second to the first eigenvalue of the covariance matrix of the rolling futures prices. The rate of mean reversion of the k-th factor under the approximation (4.8) for the factor volatility term structure equals minus the derivative of the log-volatility of the rolling tenor futures with respect to the tenor. It follows that the factor volatilities which are found to posses a stable structure by the PCA contain all the necessary information for the derivation of the risk neutral evolution dynamics of the spot price over time horizons spanned by the forward curve. This information is useful for the derivation of risk management strategies in the energy and shipping sectors involving portfolios of the spot asset and its derivatives.

A popular parametric model for the factor volatilities that belongs to the class (4.8) is

$$\sigma_i(t,T) = \sigma_i \, e^{-\lambda_i (T-t)} \tag{4.11}$$

According to (4.11) two parameters need to be estimated per factor. A possible drawback of (4.11), discussed by Clewlow and Strickland (2000), is that the rate of decay of the factor volatilities may be too high for the crude oil and natural gas futures of distant tenors.

They are often seen to asymptote to a constant value albeit with decreasing futures liquidity. More general parametric models may be tested either by adding a factor specific constant in the right-hand side of (4.11) to which the volatility asymptotes for long tenors or by introducing more general analytical representations analogous to those used in the interest rate markets [Rebonato (2002)] . Alternatively, analytical representations of the function $\kappa_k(T)$ may be selected that lead to an accurate fit to the factor volatility dependence on the tenor derived from the PCA analysis.

The multi-factor spot price process (4.9) is consistent with the spot factor models of Gibson and Schwartz (1990), Ross (1997) and Schwartz (1997). The dynamics of the two factor spot price model of Schwartz and Smith (2000) is analogous to (4.9). In the present HJM framework risk premia do not appear explicitly as in the Schwartz and Smith model. They are instead imbedded into the futures prices and in particular the factor volatilities that drive the factor dynamics in (4.9).

Spot Price Process at a Distant Horizon

Participants in the commodity markets often face the obligation to deliver an amount of a certain commodity at a distant future date which exceeds the tenor of liquid futures contracts that trade on public exchanges and forward contracts that may trade over the counter. Examples include the purchase of fuel by transportation companies – crude oil by refineries, gasoline by transportation companies, jet fuel by airlines, heating oil by consumers, natural gas by utilities and bunker fuel by shipping companies. The tenor of such contracts and their risk management depends on the market participant's ability to price them and hedge them. Often more than one commodity is involved and the cross-commodity forward curve correlation must be modeled using the joint PCA developed in Sections 2 and 3.

Assume that liquid futures/forward curves are trading for the commodities of interest over tenors up to time T that can be used to calibrate the HJM model of the cross-commodity futures curves developed in the present study. A long dated commitment has been made to deliver a unit of a commodity at a distant time $T_D > T$. The price of the commodity needs to be estimated along with its sensitivity on the factors that impact the forward curve.

Under the risk neutral measure it follows from (4.4) that the stochastic price of the spot commodity at time T_D is given by the expression

$$S^*(T_D) = F(T_D, T_D) = F(0, T_D) \exp\left[\sum_{k=1}^{M} \left\{ -\frac{1}{2} \int_0^{T_D} \sigma_k^{\,2}(s, T_D) ds + \int_0^{T_D} \sigma_k(s, T_D)\, dW_k(s) \right\} \right]$$

(4.12)

The initial futures price $F(0,T_D)$ and factor volatilities $\sigma_k(s,T_D)$ at the distant horizon $T_D >$ T may be estimated by extrapolation from prices obtained from liquid futures contracts as described earlier.

Given these estimates, the following observations follow from (4.12). The expectation at the initial time t=0 of the spot price process at the distant horizon T_D under the risk neutral measure is $F(0,T_D)$. This is consistent with the definition of the futures price process as a martingale under the risk neutral measure. The uncertainty in this estimate is contained in the exponential term in (4.12) which as seen at the initial time t=0 is a random variable with mean equal to unity and a variance which is a function of the integrals of the factor volatilities estimated from the PCA of the commodity market under study. Therefore the uncertainly in $F(0,T_D)$ as the expected distant price of the spot commodity under the risk neutral measure may be extracted from the information contained in the entire forward curve studied by a PCA. Similar considerations apply to the expected spot price at a future time within the range of tenors of traded futures contracts by simply substituting T in (4.12).

5. OPTIONS ON SPOT, FUTURES, SPREADS AND BASKETS

It follows from the analysis of the previous section that the time-t price process followed by the logarithms of the futures price F(t,T) and the risk neutral spot price $S^*(t)$ are Gaussian. Assume here that t is the initial time, T is the tenor of the futures contract and introduce an intermediate time τ such that t< τ < T. This allows the futures price to be cast in the form

(5.1) $$F(\tau,T) = F(t,T)\exp\left[\sum_{i=1}^{M}\left\{-\frac{1}{2}\int_t^{\tau}\sigma_i^2(s,T)ds + \int_t^{\tau}\sigma_i(s,T)\,dW_i(s)\right\}\right]$$

Assume a deterministic term structure of interest rates and denote by B(t,T) the price at time t of a zero coupon bond that pays \$1 at its maturity T.

Options on Spot

The time t price of a call option with strike K that expires at time T written on the spot process $S^*(t)$ may be derived by first setting τ=T in (5.1)

(5.2) $$F(T,T) = S^*(T) = F(t,T)\exp\left[\sum_{i=1}^{M}\left\{-\frac{1}{2}\int_t^{T}\sigma_i^2(s,T)ds + \int_t^{T}\sigma_i(s,T)\,dW_i(s)\right\}\right]$$

It follows from (5.2) that under the risk neutral measure the spot is lognormally distributed at the expiration of the option. Therefore the Black-Scholes formula may be applied directly to price a European call option written on the spot

(5.3) $$\begin{aligned} C\left(t,S^*(t);K,T\right) &= B(t,T)E^*_t\left[\max(0,S^*(T)-K)\right] \\ &= B(t,T)\left[F(t,T)N(z) - KN(z-\sqrt{\Sigma})\right] \end{aligned}$$

$$z = \frac{\ln\left(F(t,T)/K\right)+\frac{1}{2}\Sigma^2}{\Sigma},$$

$$\Sigma^2 = \sum_{i=1}^{N}\left\{\int_t^T \sigma^2{}_i(s,T)\,ds\right\}$$

The price of a European put follows from put-call parity

(5.4)
$$P\left(t,S^*(t);K,T\right)=B(t,T)E^*_t\left[\max(0,K-S^*(T))\right]$$
$$=B(t,T)\left[-F(t,T)N(-z)+KN(-z+\sqrt{\Sigma})\right]$$

In (5.3)-(5.4) N(z) is the cumulative Gaussian probability distribution function and its derivative is the standard Gaussian density with unit variance.

Options on Futures

The price of a European call option written on a futures contract may be derived along similar lines. The time t price of the call option with strike K that expires at time τ written on a futures contract that matures at time T > τ follows by observing from (5.1) that the time τ price of the underlying futures contract is lognormally distributed.

The price of a call on this futures contract with strike K follows from (5.1) and the Black formula

(5.5)
$$C\left(t,F(t,T);K,\tau\right)=B(t,\tau)E^*_t\left[\max(0,F(\tau,T)-K)\right]$$
$$=B(t,\tau)\left[F(t,T)N(z)-KN(z-\sqrt{\Sigma})\right]$$
$$z=\frac{\ln\left(F(t,T)/K\right)+\frac{1}{2}\Sigma^2}{\Sigma},$$
$$\Sigma=\sum_{i=1}^{M}\left\{\int_t^{\tau}\sigma^2_i(s,T)\,ds\right\}$$

The put price follows from call-put parity

(5.6)
$$P\left(t,F(t,T);K,\tau\right)=B(t,\tau)E^*_t\left[\max(0,K-F(\tau,T)\right]$$
$$=B(t,\tau)\left[-F(t,T)N(-z)+KN(-z+\sqrt{\Sigma})\right]$$

Following a PCA and the estimation of the time dependence of the factor volatilities, the underlying futures and spot price processes and European options prices follow explicitly from expressions (5.1)-(5.6). They are all seen to be functions of the integrals of the factor volatilities over the time interval from the current time t to the expiration of the option contract.

Spread Derivatives of Correlated Commodity Futures

A number of cross-commodity transactions occur in the energy and shipping markets. Their value depends upon the differential of the spot or futures prices of two commodities. A typical example is the crack spread in the crude oil/refining industry which depends on the difference between the price of refined products and the price of crude oil. An example from the shipping industry involves the price differential between the "output commodity" the ton-miles and the input commodity the bunker fuel needed for the propulsion of cargo vessels.

In other energy transactions the spread between the prices of the same commodity at two distinct geographical locations are involved. Assuming that liquid futures or forward markets exist for the same commodity at each geographical location, they may be used for the valuation of energy transmission assets in terms of the futures spread option prices derived in the present Section. Similar considerations apply to the valuation of contracts involving the seaborne transportation of crude oil and products in tankers between two geographical locations where distinct futures or forward markets exist for the liquid energy cargo. Such seaborne transportation contracts have imbedded optionalities which result from the flexibility in the speed of the vessels, the choice of the destination port and the possible use of tanker fleets as substitute storage tanks at low or zero vessel speeds. These decisions may be taken in a continuous-time dynamic and value maximizing setting by using the pricing results of the present Section and stochastic dynamic programming. Related considerations apply to the transportation of Liquefied Natural Gas (LNG) a commodity of growing significance for which futures markets and the delivery infrastructure of the physical at expiration are not yet well developed.

For the pricing of options on spreads the joint evolution dynamics of the futures of commodities A and B must be considered. Using the results developed in Section 2 the futures prices of the two commodities follow upon integration of (2.12) from the current time t to time τ with $t<\tau<T_1<T_2$. Under the risk neutral measure the drifts in (2.12) vanish and we obtain

(5.7)
$$F^A(\tau,T_1) = F^A(t,T_1)\exp\left[\sum_{i=1}^{M}\left\{-\frac{1}{2}\int_t^\tau \sigma^{A2}{}_i(s,T_1)ds + \int_t^\tau \sigma^A{}_i\ (s,T_1)\,dW^A{}_i(s)\right\}\right]$$
$$F^B(\tau,T_2) = F^B(t,T_2)\exp\left[\sum_{i=1}^{M}\left\{-\frac{1}{2}\int_t^\tau \sigma^{B2}{}_i(s,T_2)ds + \int_t^\tau \sigma^B{}_i\,(s,T_2)\,dW^B{}_i(s)\right\}\right]$$

It follows from (5.7) that the marginal distributions at time τ of the futures prices of commodity A with tenor T_1 and commodity B with tenor T_2 are lognormal. Their logarithms are marginally normally distributed with means and variances given below

(5.8)
$$\ln F^A(\tau,T_1) \sim N[\ln F^A(t,T_1) - \frac{1}{2}\sum_{i=1}^{M}\ \int_t^\tau \sigma_i^{A2}(s,T_1)ds,\ \sum_{i=1}^{N}\ \int_t^\tau \sigma_i^{A2}(s,T_1)\,ds]$$
$$\ln F^B(\tau,T_2) \sim N[\ln F^B(t,T_2) - \frac{1}{2}\sum_{j=1}^{M}\ \int_t^\tau \sigma_i^{B2}\ (s,T_2)ds,\ \sum_{j=1}^{N}\ \int_t^\tau \sigma_j^{B2}(s,T_2)\,ds]$$

The covariance of two log-futures processes of the same commodity A with tenors T_1 and T_2, follows from (5.7) in the form

(5.9)
$$\begin{aligned}
&\operatorname{cov}\left(\ln F^A(\tau,T_1), \ln F^A(\tau,T_2)\right) = \\
&\qquad = E_t\left[\sum_{i=1}^{M}\int_t^\tau \sigma^A{}_i\ (s,T_1)\,dW^A{}_i(s)\ \times\ \sum_{j=1}^{M}\int_t^\tau \sigma^A{}_j\ (s,T_2)\,dW^A{}_j(s)\right] \\
&\qquad = \sum_{i=1}^{M}\int_t^\tau \sigma^A{}_i\,(s,T_1)\sigma^A{}_i\,(s,T_2)ds \\
&\qquad = \sum_{i=1}^{M}\int_t^\tau \sigma^A{}_i(s,T_1)\sigma^A{}_i(s,T_2)\,ds
\end{aligned}$$

Expression (5.9) offers an explicit connection between the factor volatilities estimated by the PCA of commodity A described in Section 2 with the covariance and hence the correlation of two futures contracts with distinct tenors trading on the forward curve of the same commodity. The variance of each futures contract is given by (5.8) and their covariance is given by (5.9) is all the information needed to price options written on the spread of the two futures contracts discussed below.

The covariance of the log-futures processes of commodities A and B with tenors T_1 and T_2 respectively, follows in the form

$$\begin{aligned} &\operatorname{cov}\left(\ln F^{A}(\tau,T_1),\ln F^{B}(\tau,T_2)\right)= \\ &\qquad = E_t\left[\sum_{i=1}^{M}\int_t^{\tau}\sigma^{A}{}_i\,(s,T_1)\,dW^{A}{}_i(s)\,\times\,\sum_{j=1}^{M}\int_t^{\tau}\sigma^{B}{}_j\,(s,T_2)\,dW^{B}{}_j(s)\right] \\ &\qquad = \sum_{i=1}^{M}\sum_{j=1}^{M}\rho_{ij}\int_t^{\tau}\sigma^{A}{}_i\,(s,T_1)\,\sigma^{B}{}_j\,(s,T_2)ds \end{aligned} \tag{5.10}$$

The derivation of (5.10) used (2.13) and the factor volatilities and cross-commodity factor correlations estimated by the two-step PCA of the correlated commodity markets under study. Equations (5.7)-(5.10) complete the derivation of the quantities necessary for the pricing of options on cross-commodity futures spreads.

The covariance between the futures contracts trading on the forward curve of a single, two or multiple correlated commodities may be used to price options written on baskets of the underlying spot commodities, portfolios of futures contracts and options on spreads involving spot commodities and their futures. Under the present joint log-normal framework explicit expressions for the price of options on spreads with finite strikes, and options on baskets may be carried out easily using efficient numerical integration as described below. These results are valuable to oil producers, refineries, tanker shipping companies, airline companies and other market participants exposed to the price differential of crude oil, its refined products and the freight rates of tankers transporting liquid energy cargoes.

In a general setting, consider a call option with strike K and expiration τ written on the spread of two futures contracts of commodities A and B with tenors T. Its price is given by the expression

$$
\begin{aligned}
(5.11)\qquad & C_S\left(t, F^A(t,T), F^B(t,T); K, \tau\right) = \\
& = B(t,\tau)E^*_t\left[\max(F^A(\tau,T) - H\,F^B(\tau,T) - K, 0)\right] \\
& = B(t,\tau)E^*_t\left[F^A(\tau,T) - H\,F^B(\tau,T) - K\right]^+
\end{aligned}
$$

The constant H is included to denote the relative weighting of the two futures contracts and may represent a volume or heat rate adjustment of an input/output commodity which may be crude oil/gasoline/heating oil in the case of a refinery or fuel oil/tanker freight rate in the case of shipping.

A closed form expression for the call option defined above exists under the present Gaussian statistical structure and has been derived by Pearson (1995). It follows from (5.7)-(5.10) that the log-futures of commodities A and B are jointly normally distributed at the expiration of the option at time τ. Assuming a common futures tenor, the covariance of the two log-futures is given by expression (5.10). It is known from Gaussian statistics that if the marginal distributions are Gaussian the joint distribution is joint Gaussian defined in terms of the marginal variances and the covariance defined by (5.10). Another property of joint Gaussian statistics is that the distribution of one variable conditional on the value of the other is also Gaussian.

Define

$$
(5.12)\qquad \begin{aligned} z_A &= \ln F^A(\tau,T) \\ z_B &= \ln F^B(\tau,T) \end{aligned}
$$

Let $f(z_A, z_B)$ be their joint Gaussian distribution at the time-τ expiration of the option. Under the risk neutral measure the price of the option follows from the definition of expectation from probability theory

(5.13) $$C_S = B(t,\tau)\iint dz_A dz_B \left[e^{z_A} - He^{z_B} - K\right]^+ f(z_A, z_B)$$

For all intents and purposes expression (5.13) is explicit since it may be evaluated by quadrature. However it may be reduced further by invoking the definition of the conditional probability distribution

(5.14) $$f(z_A, z_B) = f(z_A \mid z_B)\, f(z_B)$$

Upon substitution in (5.13) we obtain the nested integrals

(5.15) $$C_S(t) = B(t,\tau)\int dz_B \{\int dz_A \left[e^{z_A} - He^{z_B} - K\right]^+ f(z_A \mid z_B)\} f(z_B)$$

The inner integral with respect to the variable z_A treats z_B as a constant. Given that the conditional distribution of z_A is normal, by virtue of this unique property of Gaussian statistics, its exponential is lognormal and the inner integral is a call option written on z_A with a strike $He^{z_B} + K$ which can be evaluated by Black-Scholes. The outer integral can then be evaluated by quadrature over all values of z_B. The put option follows by put-call parity.

The prices for the call and put spread options written on the futures of two commodities may be readily applied to options written on spreads involving the spot commodity by invoking the relationship of futures and spot as the tenor tends to zero. They may be used to value crude oil, products and shipping assets involving both the physical and paper markets, develop dynamic hedging policies and identify arbitrage and investment opportunities.

Options on Commodity Futures Baskets

The analysis above extends easily to the pricing of options written on a basket of futures written on correlated commodities with known weights. The value of this option is of interest to refineries interested to price and hedge the differential of an optimal basket of their products -- fuels produced from the refining process – relative to the value of the input commodity -- crude oil. The explicit pricing of such a basket option will enable a refinery to determine the value maximizing composition of a basket of output fuels in various market conditions.

Assume that at the current time t, the commodity basket consists of fuels with weights a_i for which liquid futures $F_i(t,T)$ with common tenor T are assumed to trade. The value of a European option written on the basket with strike K and expiration time $\tau < T$ is defined as follows

$$C_B\left(t,K,\tau\right) = B(t,\tau)E^*_t\left[\max(\sum_{i=1}^{N} a_i F_i(\tau,T) - K, 0)\right] \tag{5.16}$$

Positive weights correspond to the output commodity products and negative weights correspond to the input commodity.

The futures price of each commodity is lognormally distributed and each commodity pair is jointly lognormally distributed. Invoking a standard result in Gaussian statistics, the log-futures prices z_i of the N commodities in the basket at time τ are jointly Gaussian distributed with probability distribution $f(z_1, z_2, ..., z_N)$. The covariance matrix of the random variables z_i is determined in terms of the variances and pairwise covariances using the results derived earlier in this Section. The value of the basket option follows from the definition of expectation in multivariate Gaussian statistics

$$C_B = P(t,\tau)\iint..\iint \, dz_1 dz_2 ... dz_N \left[\sum_{i=1}^{N} a_i e^{z_i} - K\right]^+ f(z_1, z_2, ..., z_N) \tag{5.17}$$

The evaluation of the multiple integral in (5.17) may be carried out directly by quadrature or by invoking the definition of conditional distributions in multivariate Gaussian statistics as in (5.14)-(5.15).

6. SHIPPING FREIGHT RATE FUTURES AND OPTIONS

The market for shipping derivatives, Forward Freight Agreements (FFAs) and freight futures, has experienced rapid growth over the past two decades. Forward Freight Agreements are bilateral forward contracts that trade over the counter. The flexibility of OTC transactions allows for the design and pricing of FFA contracts tailored to the risk exposures of shipping companies, charterers, banks and investors. The drawbacks of tailored FFAs are that they entail credit risk, they may not fully protect the identity of the counterparties and may not be easy to reverse at low cost prior to expiration. Consequently the popularity of shipping freight futures that trade on IMAREX has grown. As with commodity futures they offer protection from credit risk, protect the identities of counterparties and allow positions to be reversed prior to expiration hence enabling the implementation of dynamic hedging strategies. Concerns about the credit risk present in OTC FFAs has led to the emergence of Hybrid FFAs that are cleared through a clearing house, thus mitigating credit risk while maintaining some of the flexibility of OTC FFAs. Hybrid FFAs are currently clearing in the London Clearing House Clearnet (LCH.Clearnet) and in the Singapore Exchange AsiaClear (SGX Asia Clear).

In the absence of credit risk, under deterministic interest rates and for the same contract specification the price of an FFA and a freight futures are equal. Therefore the remainder of this section will consider the modeling and arbitrage pricing of freight futures, with the understanding that the same conclusions apply to FFAs. The forward curve model developed in the remainder of this section is based on tanker freight futures data obtained from IMAREX. An identical method may be used for the modeling of the dry bulk forward curve using data obtained form IMAREX or from the OTC market. The correlation between forward curves of individual routes within the tanker and dry bulk markets, may then be carried out by using the two step PCA method developed in Section 3 for the crude oil, gasoline and heating oil markets.

Tanker freight futures contracts trading on IMAREX settle against the arithmetic average of an underlying freight rate index for the tanker route under study compiled daily by the

Baltic Exchange or Platts. These are flow forward contracts which stipulate the cash delivery of the average of the underlying freight rate index over settlement periods that span the 6 front months, the 6 front quarters and the 2 front years. The last trading day of monthly futures contracts is the last day in the settlement period – a month -- when the long futures position receives from the short position the average of the underlying index over the month. The last trading day of the quarterly contracts is the last day of the first month of the quarter when the long position receives from the short the average of the index over the month and pays the futures price. The remainder of the settlement of the quarterly contract over its last two months is received by the long position in two monthly installments at the end of each of the two remaining months. Finally, the last trading of the yearly contract is the end of January when the long position receives from the short the average of the index over the month of January for the futures price. The remainder of the settlement of the yearly contract over the remaining eleven months of the year is received by the long position in eleven monthly installments at the end of each month equal to the monthly average of the underlying index. Dry bulk freight futures also trade on IMAREX with the same monthly, quarterly and yearly contract structure.

The design of the IMAREX freight futures contracts is such that the front monthly contracts with high liquidity overlap with the quarterly contracts with lower liquidity which in turn overlap with the yearly contracts. Another attribute of the contract design is that the settlement occurs over consecutive monthly sub-intervals for the quarterly and yearly contracts. In frictionless markets absence of arbitrage enforces restrictions on the relative pricing of the monthly quarterly and yearly contracts. These restrictions may be enforced by introducing a sequence of monthly futures contracts expiring at the end of each month and with the longest monthly expiration coinciding with the end of December expiration of the most distant yearly contract. The settlement price of each monthly contract paid by the short is the average of the underlying index over the index days of the respective month. Market prices for these generic futures are available from IMAREX for the front six months. The prices of the quarterly and yearly contracts may be expressed as weighted averages of the prices of these monthly securities over their tenor as discussed below. Consequently, the monthly futures contracts may be used as the baseline securities to construct the arbitrage free forward curve of the freight futures contract under study as described below.

Following the notation introduced in earlier Sections, assume that t=0 is the initial time, t is the current time, T_i, i=1,…,N are the expirations of the monthly futures contracts which coincide with the end of each month and c is the length of a month. Denote by S(t) the price of the underlying tnaker freight rate index and assume that the risk free interest rate is constant and equal to r.

Assuming a continuous time setting, the monthly futures settles continuously against the average of the spot freight rate index over the monthly settlement period (T-c, T), where T is the tenor of the shipping futures contract and c is the length of a month. The contract cashflow is assumed to occur at the end of the month. The arbitrage price of the monthly futures contract under the risk neutral measure in the pre-settlement period is given by the expression

$$\begin{aligned} F_S(t,T;c) &= \frac{1}{c} E_t^* \left\{ \int_{T-c}^{T} S(s)ds \right\}, \quad t < T-c \\ &= \frac{1}{c} \int_{T-c}^{T} F(t,s)ds \\ F(t,s) &= E_t^* \{S(s)\}, \ s > t \end{aligned} \tag{6.1}$$

In the settlement period of the monthly futures contract, T-c <t<T the arbitrage price of the futures contract is given by the expression

$$\begin{aligned} F_S(t,T;c) &= \frac{1}{c} \int_{T-c}^{t} S(u)du + \frac{1}{c} \int_{t}^{T} F(t,s)ds, \quad T-c < t < T \\ F(t,s) &= E_t^* \{S(s)\}, \ s > t \\ F_S(T,T;c) &= \frac{1}{c} \int_{T-c}^{T} S(u)du \end{aligned} \tag{6.2}$$

It follows from (6.2) that at its expiration t=T the monthly futures contract settles at the average of the underlying spot index over the monthly period T-c < t < T given by the last expression in (6.2). This average is the amount received by the long position at time t=T. Under constant interest rates and the contract cashflow occurring at the end of the settlement period, the prices of futures and forward contracts are equal.

Prices of the monthly tanker freight futures defined by (6.1) and (6.2) in the pre- and settlement periods are quoted on IMAREX for the front six months. It is assumed that prices in the settlement period are volatile as the futures contracts are being offset and only prices in the pre-settlement period given by (6.1) will be used for the statistical analysis and PCA of the forward curve. The following abbreviated definition of the monthly futures price will be used in the remainder of this section

(6.3) $$F_S(t,T) = \frac{1}{c}\int_{T-c}^{T} F(t,s)ds, \ \ t < T - c$$

The monthly futures price $F_S(t,T)$ at time t stipulates the following exchange of cashflows at the contract expiration at time T. The long position receives from the short position the difference between the average of the freight rate index over the month and $F_S(t,T)$, the futures price the long position locked at the contract inception at time t.

Consider the quarterly futures price $F^Q{}_S(t,T_1,T_2,T_3)$ at time t that expires at time T_3 the end of the third month in a futures quarter. Denote by $T_2 = T_3 - c,\ T_1 = T_2 - c,\ t < T_1 - c$. A long position in the quarterly futures contract established at time t for the price $F^Q{}_S(t,T_1,T_2,T_3)$, ensures that the long position will receive at the end of each month in the quarter the difference between the average of the freight rate index during that month and $F^Q{}_S(t,T_1,T_2,T_3)$. Cash flows under the IMAREX quarterly futures contract occur at the end of each month of the quarter, namely at times $T_1 = T_2 - c,\ T_2 = T_3 - c,\ T_3$, as opposed to just the end of the quarter at time T_3. Discounting these cashflows to the present time t under the risk neutral measure leads to the arbitrage futures price of the quarterly contract relative to the arbitrage price of the monthly futures price given by (6.1) and (6.3).

The present value of the fixed cashflows at the end of each month in the quarter, equal to the futures price $F^Q{}_S(t,T_1,T_2,T_3)$, is given by the expression

$$\left[e^{-r(T_1-t)}+e^{-r(T_2-t)}+e^{-r(T_3-t)}\right]F_S{}^Q(t,T_1,T_2,T_3)$$

The present value of the variable cashflows at the end of each month of the quarter, equal to the risk neutral expectation of the average of the freight rate index over the month discounted to the present time t, is given by the expression

$$\begin{aligned}
&\frac{e^{-r(T_1-t)}}{c}E_t^*\left\{\int_{T_1-c}^{T_1}S(s)ds\right\}+\frac{e^{-r(T_2-t)}}{c}E_t^*\left\{\int_{T_2-c}^{T_2}S(s)ds\right\}+\frac{e^{-r(T_3-t)}}{c}E_t^*\left\{\int_{T_3-c}^{T_3}S(s)ds\right\}\\
&=\frac{e^{-r(T_1-t)}}{c}\int_{T_1-c}^{T_1}F(t,s)ds+\frac{e^{-r(T_2-t)}}{c}\int_{T_2-c}^{T_2}F(t,s)ds+\frac{e^{-r(T_3-t)}}{c}\int_{T_3-c}^{T_3}F(t,s)ds\\
&=e^{-r(T_1-t)}F_S(t,T_1)+e^{-r(T_2-t)}F_S(t,T_2)+e^{-r(T_3-t)}F_S(t,T_3)
\end{aligned}$$

Since it is costless to enter a futures contract at time t, absence of arbitrage requires that the two present values derived above are equal. Therefore the risk neutral price of the quarterly futures contract follows in the form

$$\begin{aligned}
&F_S{}^Q(t,T_1,T_2,T_3)=w_1F_S(t,T_1)+w_2\,F_S(t,T_2)+w_3F_S(t,T_3)\\
(6.4)\qquad &w_i=\frac{e^{-rT_i}}{\sum_{j=1}^{N=3}e^{-rT_j}}
\end{aligned}$$

Therefore the arbitrage price of the quarterly futures contract is the weighted sum of the prices of the three consecutive monthly futures contracts. The two front quarterly contracts may overlap with the front six monthly futures contracts trading on IMAREX. Consequently absence of arbitrage requires that equation (6.4) holds if the quarterly and all monthly contracts in (6.4) trade.

In an analogous manner, the arbitrage price of the yearly futures contract is the weighted sum of twelve consecutive monthly futures contracts, or

$$F^{Y}{}_{S}(t,T_1,...,T_N) = \sum_{i=1}^{N=12} w_i\, F_S(t,T_i), \quad t < T_1 - c \tag{6.5}$$

The yearly futures contract may also be expressed as the sum of quarterly futures contracts by combining the definitions (6.4) and (6.5). It also follows from (6.4) and (6.5) that a long position in a quarterly or yearly futures contracts is equivalent in the absence of arbitrage to a weighted portfolio of futures positions in the monthly contracts. Therefore, the latter will hereafter be considered the generic contracts and their lognormal stochastic dynamics is modeled below. Options written on freight rate futures contracts also settle monthly, therefore their pricing will be shown to follow directly from the lognormal dynamics of the monthly futures prices using the Black formula.

The arbitrage pricing of shipping freight rate futures contracts given by (6.4) and (6.5) is analogous to similar flow futures contracts trading in the electricity markets, discussed by Benth and Koekebakker (2008). In general the arbitrage pricing relations derived in this Section for futures and options apply to other commodity markets where flow futures and options trade aiming to mitigate volatility risk via the averaging of the underlying over settlement periods of varying lengths.

Futures Price Process of Tanker Freight Rate Index

The first step towards the statistical analysis of the IMAREX tanker freight forward curve is the estimation of the initial monthly futures prices $F_S(t=0,T_i)$, $i=1,...,M$, $t=0<T_1-c$, where M is the tenor of the last month in the distant yearly contract. The steps involved in this task are described in Section 7.

Following the estimation of the initial shape of the forward curve constructed out of its monthly contracts, its risk neutral dynamics may be modeled as in the crude oil markets. It follows that under the risk neutral measure the monthly freight futures are assumed to satisfy the stochastic differential equation

$$\text{(6.6)} \qquad \begin{aligned} &\frac{dF_S(t,T_j)}{F_S(t,T_j)} = \sum_{k=1}^{N} \sigma_k(t,T_j)\,dW_k;\; j=1,...,M,\; t<T_j-c \\ &dW_i(t)dW_j(t) = \delta_{ij}dt \end{aligned}$$

The risk neutral log-normal dynamics given by (6.6) is directly analogous to the risk neutral dynamics of the futures contracts in the crude oil markets. The difference is that the time t is here assumed to be at least a month away from the tenor T_j of the monthly futures contract. The factor volatilities in (6.6) may again be estimated by implementing the PCA analysis developed in Sections 2-3 for the crude oil markets using the prices of the monthly futures contracts $F_S(t,T_j)$ estimated by the interpolation procedure outlined above and presented in more detail in Section 7.

The factor volatilities estimated from the PCA analysis presented in Section 7 are based on market prices of futures contracts trading up to the beginning of the settlement period. Market prices within the settlement period may be too volatile to be reliable for econometric fitting. Yet the validity of the stochastic differential equation (6.6) may be extended within the settlement period by observing that the volatility of the futures price $F_S(t,T_j)$ tends to zero as t approaches the contract maturity. This follows by taking the differential of the first equation in (6.2)

(6.7)
$$dF_S(t,T;c) = \frac{1}{c}\int_t^T dF(t,s)ds, \quad T-c<t<T$$

It follows from (6.7) that the volatility of the left-hand side of (6.7) tends to zero as the range of integration in the right-hand side tends to zero, as t approaches T and under certain regularity conditions for dF. Therefore a reasonable approximation of the factor volatility in the settlement period is

(6.8)
$$\sigma_k(t,T_j) \simeq \sigma_k(T_j-c,T_j)\frac{T_j-t}{c}, \; T_j-c<t<T_j$$

The use of approximation (6.8) in (6.6) completes the specification of the factor volatilities in the settlement period and until the contract expiration. This will enable the derivation of the stochastic differential equation governing the spot price process as implied by the traded futures prices. It will also be used to derive explicit expressions for the price of Asian options written on tanker freight futures. Both topics are discussed below.

Spot Price Process of Tanker Freight Rate Index

The form of the stochastic differential equation (6.6) is identical to the equation (4.1) governing the stochastic evolution of the futures contracts in the crude oil markets, with the drift set equal to zero in (4.1) under the risk neutral measure. Therefore the results derived in Section 4 apply directly to the shipping market.

Under the present multi-factor model of the tanker freight rate futures, the futures price and the implied price of the spot freight rate index averaged over a month follow in the form

$$F_S(t,T) = F_S(0,T)\exp\left[\sum_{k=1}^{M}\left\{-\frac{1}{2}\int_0^t \sigma_k^{\,2}(s,T)ds + \int_0^t \sigma_k(s,T)\,dW_k(s)\right\}\right]$$

$$\bar{S}(t) = \frac{1}{c}\int_{t-c}^{t} S(u)du = F_S(t,t) = F_S(0,t)\exp\left[\sum_{k=1}^{M}\left\{-\frac{1}{2}\int_0^t \sigma_k^{\,2}(s,t)ds + \int_0^t \sigma_k(s,t)\,dW_k(s)\right\}\right]$$

(6.9)

As expected the implied evolution dynamics of the average of the underlying spot index is lognormal and in analogy to the crude oil markets it is driven by a small number of factors with volatilities estimated from the PCA analysis described in Section 7 and their approximation (6.8) within the settlement period of the monthly freight futures contracts.

Using the results of Section 4 the stochastic evolution of the factors driving the spot freight rate index may be derived using (4.6)-(4.10). The rate of mean reversion of each factor may be determined by using the same reasoning as in the crude oil markets.

The freight rate index price at a distant horizon also follows directly from (6.9) under the risk neutral measure in the form

$$\bar{S}(T_D) = F_S(T_D,T_D) = F_S(0,T_D)\exp\left[\sum_{k=1}^{M}\left\{-\frac{1}{2}\int_0^{T_D} \sigma_k^{\,2}(s,T_D)ds + \int_0^{T_D} \sigma_k(s,T_D)\,dW_k(s)\right\}\right]$$

(6.10)

As in the crude oil markets the expected distant price of the spot freight rate is the futures price $F_S(0,T_D)$ which for $T_D > T$ may be extrapolated from the initial monthly futures curve estimated above. The variance of this estimate is supplied by the exponential term in (6.10) which is a function of the factor volatilities estimated by the PCA analysis described in Section 7 and extrapolated to the distant horizon T_D.

The spot tanker freight rate process is known to be volatile with log-returns that may exhibit fat tails. This non-Gaussian behavior may be modeled as in the crude oil market discussed in Section 2.

Correlated Route Shipping Forward Curves

Shipping FFAs and futures trade for a number of shipping sectors and routes. Each of these shipping market segments have their own forward curves which are correlated. Therefore their joint evolution needs to be modeled along the lines of the correlated forward curves in the crude oil market.

Consider routes A and B of the tanker or dry bulk shipping market with monthly futures evolution dynamics given under the risk neutral measure by the stochastic differential equations

$$\begin{aligned}
&\frac{dF_S{}^A(t,T)}{F_S{}^A(t,T)} = \sum_{k=1}^{M} \sigma_k{}^A(t,T)\, dW_k{}^A(t) \\
&\frac{dF_S{}^B(t,T)}{F_S{}^B(t,T)} = \sum_{l=1}^{M} \sigma_l{}^B(t,T)\, dW_l{}^B(t) \\
&dW_i^A(t) dW^A{}_j(t) = \delta_{ij} dt \\
&dW_i^B(t) dW^B{}_j(t) = \delta_{ij} dt \\
&dW_i^A(t) dW^B{}_j(t) = \rho_{ij} dt
\end{aligned} \tag{6.12}$$

The factor volatilities and cross-route factor correlations may be estimated by a two-step PCA analysis using the method described in Section 7, combined with the treatment of the cross-commodity correlation in the crude oil markets.

Assuming that t is the current time, the time τ price of the respective generic futures contracts is given by the familiar relations

(6.13)

$$F_S^{\ A}(\tau,T_1)=F_S^{\ A}(t,T_1)\exp\left[\sum_{k=1}^{M}\left\{-\frac{1}{2}\int_t^{\tau}\sigma_k^{\ A2}(s,T_1)\,ds+\int_t^{\tau}\sigma_k^{\ A}(s,T_1)\,dW_k^{\ A}(s)\right\}\right]$$

$$F_S^{\ B}(\tau,T_2)=F_S^{\ B}(t,T_2)\exp\left[\sum_{l=1}^{M}\left\{-\frac{1}{2}\int_t^{\tau}\sigma_l^{\ B2}(s,T_2)\,ds+\int_t^{\tau}\sigma_l^{\ B}(s,T_2)\,dW_l^{\ B}(s)\right\}\right]$$

The resulting joint lognormal evolution dynamics of the shipping futures $F_S^{\ A}(\tau,T_1),\ F_S^{\ B}(\tau,T_2)$ enables the pricing of options written on cross-sector shipping futures spreads using explicit formulae or efficient numerical techniques analogous to those in the crude oil markets. The availability of the prices of such spread options enables the valuation of a wide range of shipping assets discussed in Section 8.

Time Charter Rates (TC)

The time charter rate is the constant rate a shipowner receives from the owner of the cargo being transported over the charter period. The TC rate may be expressed in terms of the generic freight futures contract $F(t,T)$ defined in (6.1). Denote by TC(t,T) the TC rate at the current time t for a time charter that ends at a future time T with duration T-t. Assume that the TC rate is paid continuously to the shipowner and that the futures contracts $F(t,T)$ trade in a frictionless market. Absence of arbitrage requires that the revenue from entering at time t into a sequence of positions in the generic futures contracts $F(t,T)$ over the duration of the charter must be equal to the revenue from the charter contract.

(6.14)

$$\int_t^T e^{-r(s-t)} f(s,T)ds = TC(t,T)\int_t^T e^{-r(s-t)}ds$$

$$TC(t,T)=\int_t^T w(s,t,T)f(s,T)ds$$

$$w(s,t,T)=\frac{e^{-rs}}{\int_t^T e^{-rs}ds}$$

The TC rate may also be expressed in terms of the prices of traded weekly, monthly, quarterly or yearly futures contracts that settle at the end of the respective periods by sub-dividing the period of the time charter into the sub-periods of the corresponding contracts and discounting along the lines used to derive (6.4) and (6.5).

In (6.14) the time charter was assumed to start at the current time t. The same reasoning may be used to derive the arbitrage futures price at the current time t of a time charter to be entered into over a future period $(T_1, T_2) > t$. It follows that

(6.15) $$F_{TC}(t, T_1, T_2) = \int_{T_1}^{T_2} w(s, T_1, T_2) f(s, T) ds$$

Again the arbitrage price given by (6.15) may be expressed in terms of the prices of traded freight rate futures that settle weekly, monthly, quarterly of yearly.

Asian Rate Options on Monthly Freight Futures

The multi-factor model of the stochastic evolution of the monthly freight rate futures (6.6)-(6.8) forms the basis for the pricing of shipping options written on the futures and FFAs which are trading on IMAREX, Asia Clear SGX and OTC. These are average price Asian options with monthly expirations which settle at the average of the underlying index at the end of each month. They may be priced explicitly using the present multi-factor HJM model of the shipping futures using Black's formula.

Assuming that t is the current time, the monthly shipping futures at some future time τ<T is available explicitly in the familiar form

(6.16) $$F_S(\tau, T) = F_S(t, T) \exp\left[\sum_{k=1}^{M} \left\{ -\frac{1}{2} \int_t^{\tau} \sigma_k^{\,2}(s, T) ds + \int_t^{\tau} \sigma_k(s, T)\, dW_k(s) \right\} \right]$$

A European call option written on $F_S(\tau, T)$ with strike K and expiration at τ<T follows directly from Black's formula

$$C_S\left(t, F_S(t,T); K, \tau\right) = B(t,\tau) E^*_t \left[\max(0, F_S(\tau,T) - K)\right]$$
$$= B(t,\tau)\left[F_S(t,T) N(z) - K N(z - \sqrt{\Lambda})\right]$$

(6.17)
$$z = \frac{\ln\left(F_S(t,T)/K\right) + \frac{1}{2}\Lambda^2}{\Lambda},$$
$$\Lambda^2 = \sum_{k=1}^{M}\left\{\int_t^{\tau} \sigma_k{}^2(s,T)\, ds\right\}$$

The put price follows from put-call parity

(6.18)
$$P_S\left(t, F_S(t,T); K, \tau\right) = B(t,\tau) E^*_t \left[\max(0, K - F_S(\tau,T)\right]$$
$$= B(t,\tau)\left[-F_S(t,T) N(-z) + K N(-z + \sqrt{\Sigma})\right]$$

The formulae (6.16)-(6.18) are new explicit pricing formulae for Asian freight rate options based on a multi-factor log-normal model of the futures. A single factor Asian option pricing model was recently proposed by Koekebakker, Adland and Sodal (2007) which is based on a diffusion model for the freight rate spot price. Other Asian option pricing models based on a diffusion model for the underlying include Turnbull and Wakeman (1991) and Levy (1992).

In the special case $\tau = T$ expressions (6.17-(6.18) reduce to the price of Asian freight rate options trading on IMAREX. In this case it follows from the last equation in (6.2) and the definition of the call option (6.17) that $F_S(\tau = T, T)$ is the arithmetic average of the underlying spot index over the monthly settlement period.

Existing methods for the pricing of Asian options have relied on the development of explicit or approximate formulae based on a diffusion model for the evolution of the underlying index [Turnbull and Wakeman (1978), Levy (1990)]. The price of Asian calls and puts derived above for monthly futures are explicit and the result of the assumption that the futures contracts that settle against the arithmetic average of the underlying index follow a multi-factor lognormal diffusion which enables the direct application of Black's formula. A key input to the present price of Asian derivatives is the factor volatilities that appear in (6.16), estimated by the PCA of the commodity or shipping market under study and approximated in the settlement period using (6.8). Consistently with the experience gained from the pricing of equity options, the factor volatilities are the key inputs to the explicit Black-Scholes and Black formulae and effort must be devoted to their proper modeling and estimation.

Where liquid futures prices exist they can be modeled using the PCA developed above for the crude oil and tanker shipping markets. When the futures markets are not very liquid the price of the spot index which is quoted daily may also be used with (6.9) to estimate the factor volatilities. Another advantage of the Asian option pricing formulae (6.17)-(6.18) is that they may be hedged using the traded underlying futures contract. In the simplest case the hedge ratio of a delta hedge is the derivative of (6.17) or (6.18) with respect to $F_S(t,T)$.

Asian Options on Quarterly and Yearly Freight Futures

Options may also be written on the quarterly and yearly futures contracts with arbitrage free prices given by (6.4) and (6.5) in terms of the monthly futures contracts. The payoff of a call option written on a quarterly contract with strike K is given by the expression

(6.19)
$$\begin{aligned} C^Q{}_S\left(t, F_S{}^Q(t,T_1,T_2,T_3);K\right) &= e^{r(T_1-t)}E^*{}_t\left[\max(0,F_S(T_1,T_1)-K)\right] \\ &+ e^{r(T_2-t)}E^*{}_t\left[\max(0,F_S(T_2,T_2)-K)\right] \\ &+ e^{r(T_3-t)}E^*{}_t\left[\max(0,F_S(T_3,T_3)-K)\right],\ t<T_1-c \end{aligned}$$

$$F_S(T,T) = \frac{1}{c}\int_{T-c}^{T} S(u)du$$

As in the case of the monthly freight options contracts the last day of trading of the quarterly options contract is assumed to be the beginning of the quarter or the time

$t = T_1 - c$. If follows for the definition of the payoff of the call option written on a quarterly futures contract that its price is given by the sum of the options prices written on the sequential monthly contracts, or

$$C^Q{}_S\left(t, F_S{}^Q(t,T_1,T_2,T_3);K\right) = C_S\left(t, F_S(t,T_1);K\right) + C_S\left(t, F_S(t,T_2);K\right) + C_S\left(t, F_S(t,T_3);K\right)$$

(6.20)

The price of the yearly options contract follows in the form

$$C^Q{}_S\left(t, F_S{}^Q(t,T_1,\ldots,T_N);K\right) = \sum_{i=1}^{N} C_S\left(t, F_S(t,T_i);K\right) \tag{6.21}$$

The price of puts follows by put-call parity. If follows that the prices of the calls and puts written on monthly, quarterly and yearly freight rate contracts are available explicitly in terms of the Black price of the monthly options. This is the result of the assumption that the monthly futures price follows a lognormal diffusion and no need arises to assume a particular diffusion model for the quarterly or yearly futures process.

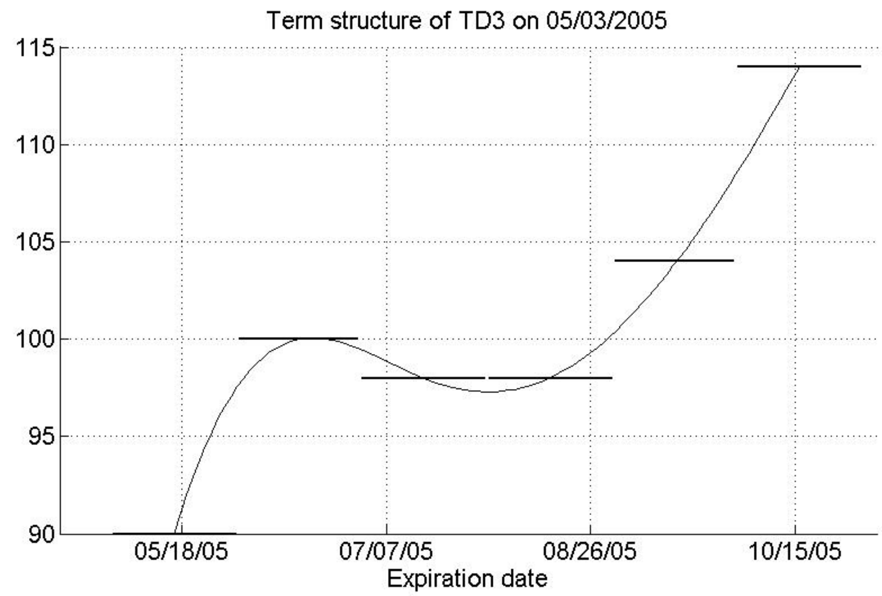
The modeling burden therefore falls upon the development of a robust multi-factor lognormal diffusion model governing the evolution of monthly futures prices. The parameters in that model that must be calibrated to the market data are the factor volatilities which are estimated in the next section for a major tanker shipping route as in the crude oil markets using a PCA. Where departures from normality are observed, as is the case in the shipping and electricity markets, the factor volatilities may be assumed to follow a stochastic process with jumps. The present modeling framework that relies on Black's formula also enables the estimation of the term structure of implied volatility from traded options contracts. Drawing upon extensive studies in the equities markets, the smile of the implied volatility surface suggests refinements of the diffusion process driving the underlying. More accurate multi-factor models for the process followed by the volatility may be derived using recent work on equity index options calibrated against variance swaps [Overhaus et. al. (2007), Bergomi (2009)].

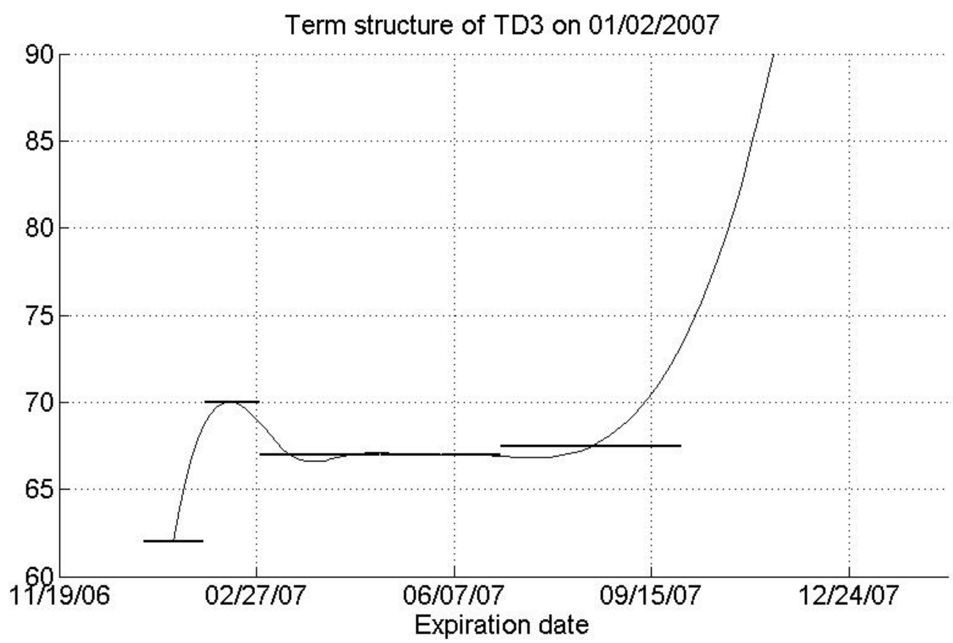
7. PRINCIPAL COMPONENTS ANALYSIS OF TANKER FREIGHT RATE FUTURES

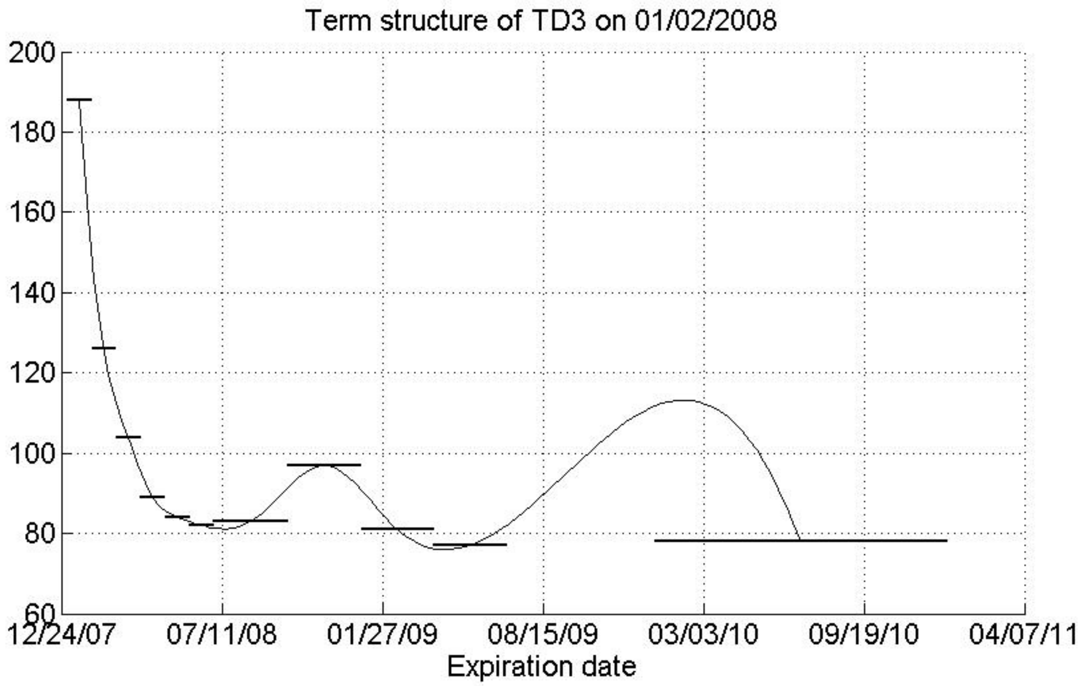
A principal components analysis was carried out for the tanker freight rate futures trading on IMAREX for the liquid contract TD3 corresponding to the Baltic Freight Index with the following specification: Very Large Crude Carriers (VLCC); Middle East to Japan; Cargo Size 260,000 metric tons. TD3 daily freight rate futures prices defined as in Section 6 have been obtained from IMAREX from April 2005 to February 2009 and used for the PCA described below.

The prices are quoted in terms of the Worldscale of the year of the contract. Worldscale is set yearly by the Worldscale association, and reflects the costs of transporting a ton of oil from one port to the other during the year before. For example, Worldscale 2009 for route TD3 is based on the costs in the period October 2007 to September 2008, and is effective from 1 January 2009 to 31 December 2009. This is WS100 and is in dollars per ton. WS120 means that the price of the contract is 120% of that cost.

Two problems arise from this scale when dealing with futures. First, the forward curve on a given date contains contracts that aren't quoted in the same unit (as seen on 2/6/2009, there are contracts for 2009 and contracts for 2010). This can be an issue when comparing contracts with each other. Second, the Worldscale for 2010 isn't known before late 2009, and thus the equivalent $/ton price can only be assessed based on forecasts of the flatrate. This is an issue in times of volatile markets: for example, the Worldscale for TD3 went up 37% from 2008 to 2009. This definition of Worldscale and its uncertain value in 2010 and beyond is perhaps a reason for reduced liquidity of the distant quarterly and yearly tanker futures contracts. This is not an issue with the dry bulk futures prices which are quoted in units equivalent to the spot rate and enjoy higher liquidity for distant tenors. Time charter (T/C) basket futures contracts also trade on IMAREX for the dry bulk market. Their definitions relative to the spot futures contracts is given by expressions (6.14)-(6.15).


Term structure of TD3 on 05/03/2005
115
110
105
100
95
90
05/18/05
07/07/05
08/26/05
10/15/05
Expiration date


Term structure of TD3 on 01/02/2007
90
85
80
75
70
65
60
11/19/06
02/27/07
06/07/07
09/15/07
12/24/07
Expiration date



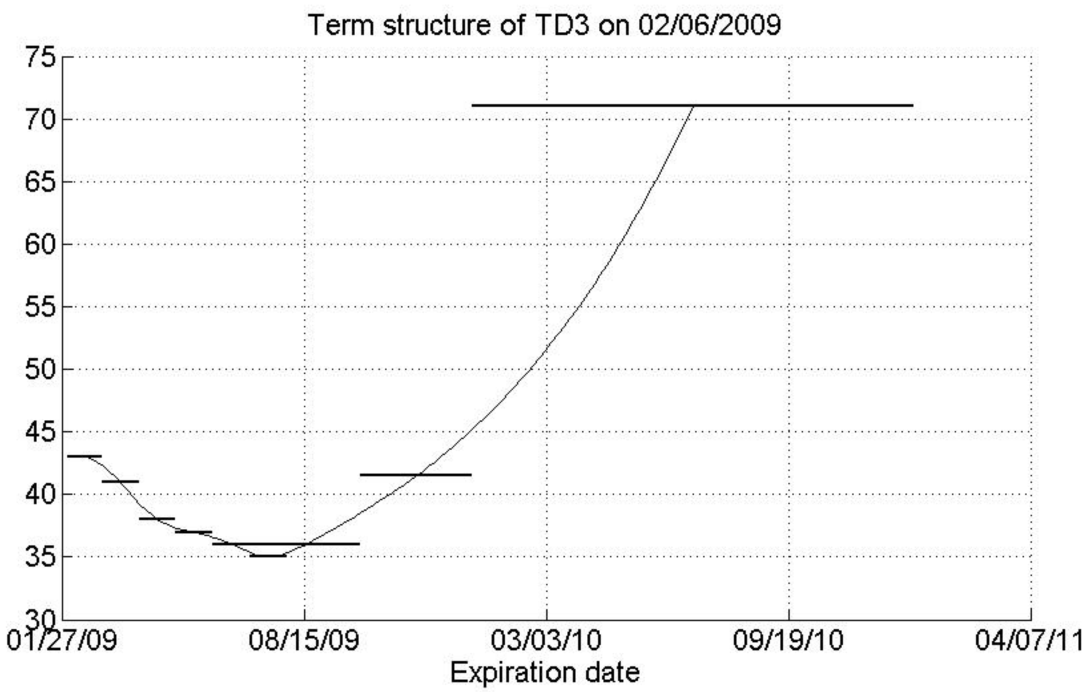


Figure 7.1:. Forward curves of TD3 on different dates

Figures 7 plot the TD3 forward curves on different dates. The length of the horizontal bars in the Figures illustrates the length of the settlement period (monthly, quarterly, yearly) and the height of the bars denotes the corresponding futures price. The mid-points of the bars are connected by a smoothed curve that illustrates the initial shape of the TD3 forward curve.

As expected the liquidity of the prompt monthly futures is higher that that of the quarterly and yearly contracts. The settlement of the monthly futures spans the month prior to the last settlement date and as discussed in Section 6 prices prior to the first day of the settlement period have been used in the present study. Figure 7.2 plots the futures price with a constant relative tenor of 2 months from April 2005 to February 2009. This futures price is the most prompt rolling tenor futures contract used in the present study and may be viewed as an approximate smoothed price of the underlying TD3 Baltic Index

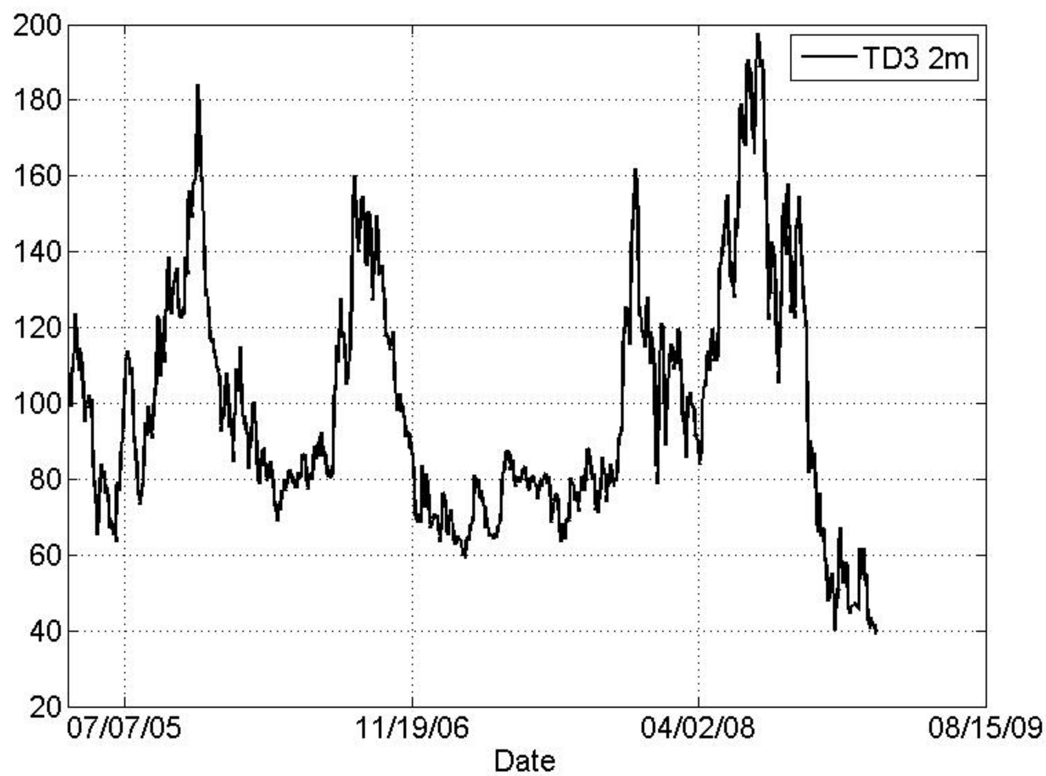


Figure 7.2: Price of the TD3 2m contract from April 2005 to February 2009

A qualitative inspection of Figure 7.2 indicates that the TD3 2 month futures is mean reverting with sharp upwards spikes possibly the result of a tight tanker market. The probability distribution of the log-returns of the 2 and 5 month rolling tenor TD3 futures is plotted in Figure 7.3 and compared to the standard normal distribution. Both distributions are leptokurtic indicating a departure from normality that needs to be accounted for in the modeling of the log-returns of the futures by introducing stochastic volatility models discussed in Section 2.

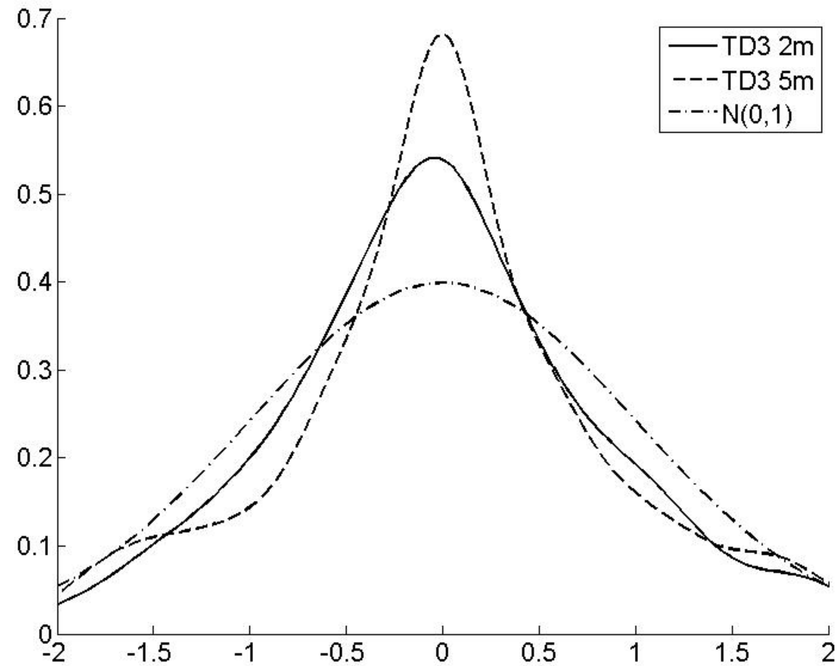


Figure 7.3: Distributions of the log returns on TD3 2m, 5m, normalized to unit variance, obtained using a Gaussian kernel density estimator

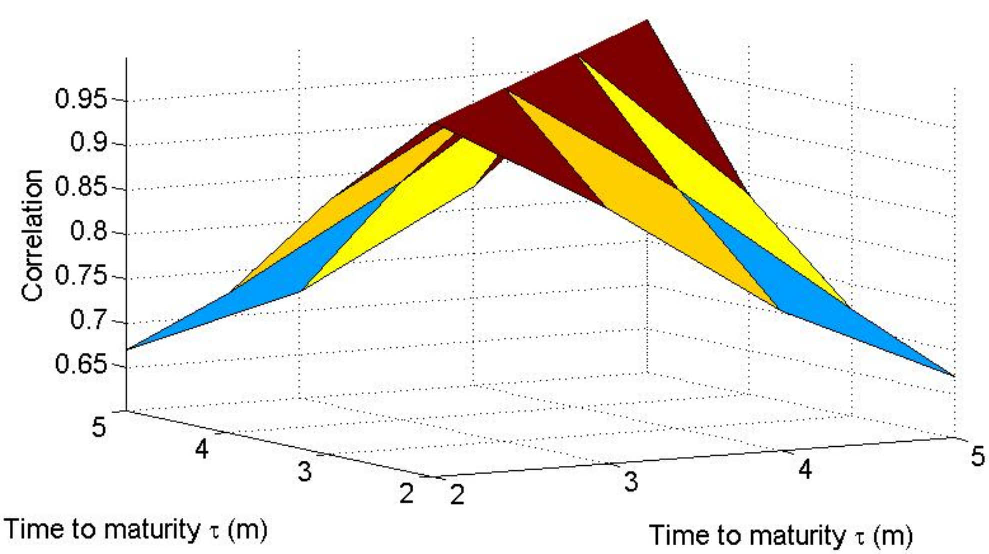


Figure 7.4. Correlation surface of TD3 futures, over the period 4/4/2005-2/6/2009

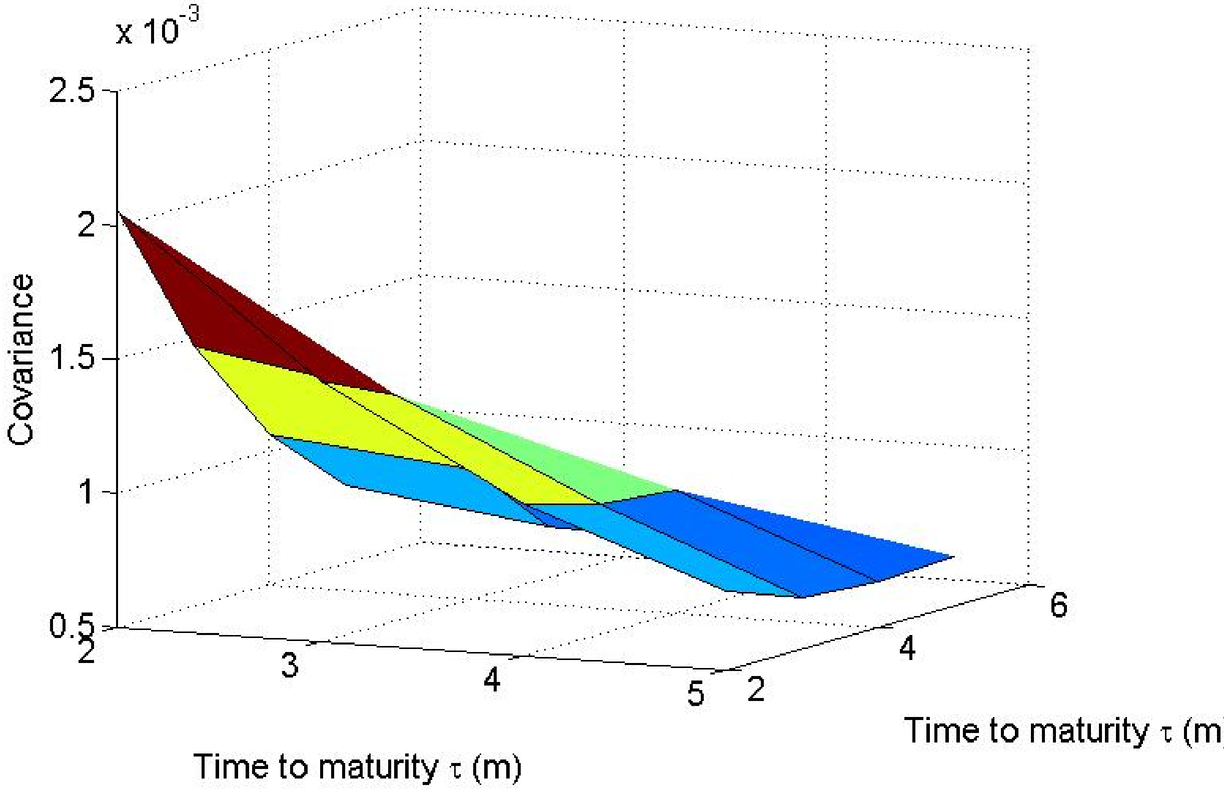


Figure 7.5. Covariance surface of TD3 futures, over the period 4/4/2005-2/6/2009

Figures 7.4 and 7.5 illustrate the correlation and covariance, respectively, of the futures contracts with rolling tenors ranging from 2 to 5 months. Both Figures indicate smooth declining surfaces which were used to carry out the PCA analysis described next.

Table 1 lists the first four eigenvalues obtained from the PCA analysis along with the percentage of the fluctuations explained by each principal component. As expected a small number of orthogonal factors is again sufficient for the description of the dynamics of the TD3 tanker freight rate forward curve up to a rolling tenor of 5 months. The less liquid quarterly and yearly futures contracts were not used in the PCA analysis.

Table 1. Eigenvalues and cumulative variance explained

	Eigenvalue λ_k	Cumulative variance explained
PC 1	4.3e-3	86 %
PC 2	4.3e-4	95 %
PC 3	1.8e-4	98 %
PC 4	8.3e-5	100 %

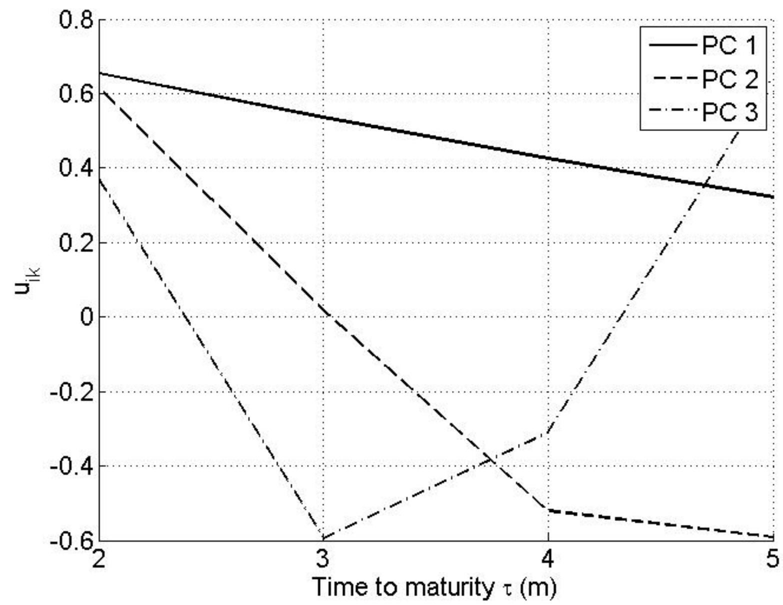


Figure 7.6: Principal component weights (eigenvectors), k=1,2,3

Figure 7.6 plots the shape of the first three principal component loadings as functions of the rolling tenor and their shape is seen to be qualitatively similar to the shape of the principal components for crude oil. The first principal component is relatively flat, the second has a steeper downward slope and becomes negative for a tenor of about 3 months and the third has a concave shape with a local minimum at the 3 month rolling tenor. Table 2 lists the descriptive statistics of the first three principal components.

Table 2. Descriptive statistics of the log-returns of the principal components

	PC 1	PC 2	PC 3
Observations	970	970	970
Mean	6.6e-19	4.7e-19	-2.18e-19
Median	-3.4e-4	-7.74e-4	-9.77e-5
Minimum	-0.25	-0.07	-0.08
Maximum	0.42	0.11	0.07
Volatility (annualized)	106 %	33 %	22 %
Skewness	0.36	0.48	-0.12
Kurtosis	6.61	5.28	6.87
Jarque-Bera (p-value)	548 (<1e-3)	247 (<1e-3)	609 (<1e-3)
Jarque-Bera test	Rejected	Rejected	Rejected

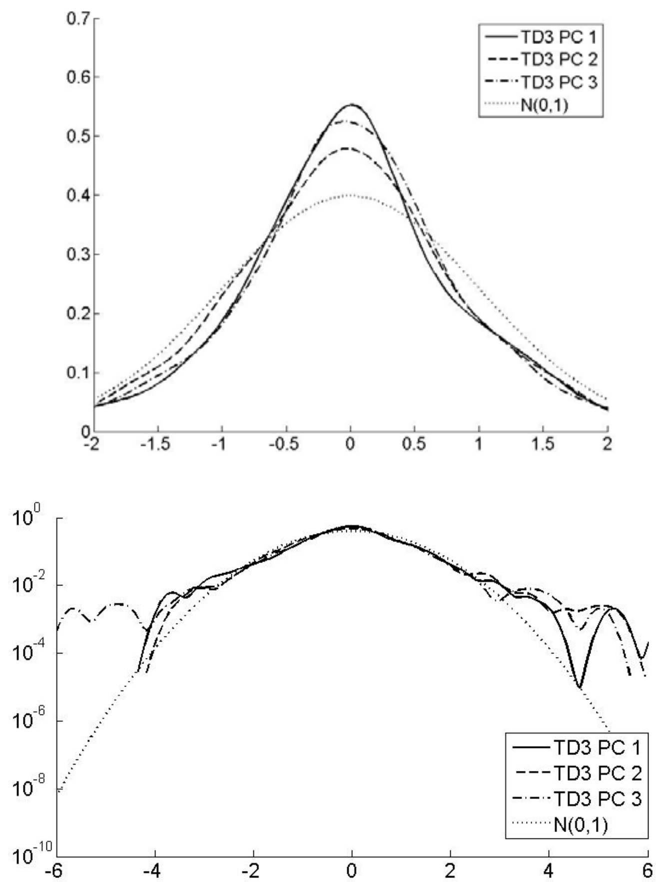


Figure 7.7: Distributions of the 3 PCs compared to the normal distribution. Linear scale (top) and log scale (bottom)

Figure 7.7 plots the probability distribution of the first three factors in linear and log-scale. As for the original futures prices they are also seen to be leptokurtic.

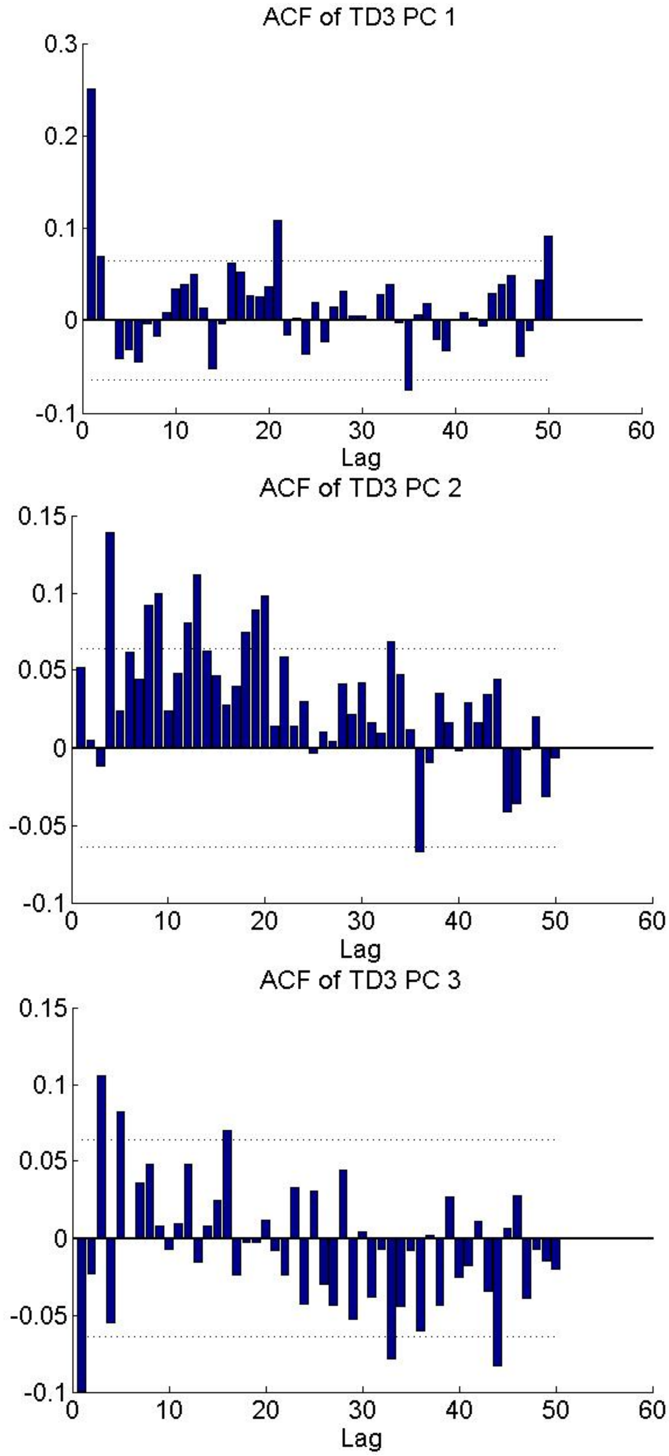
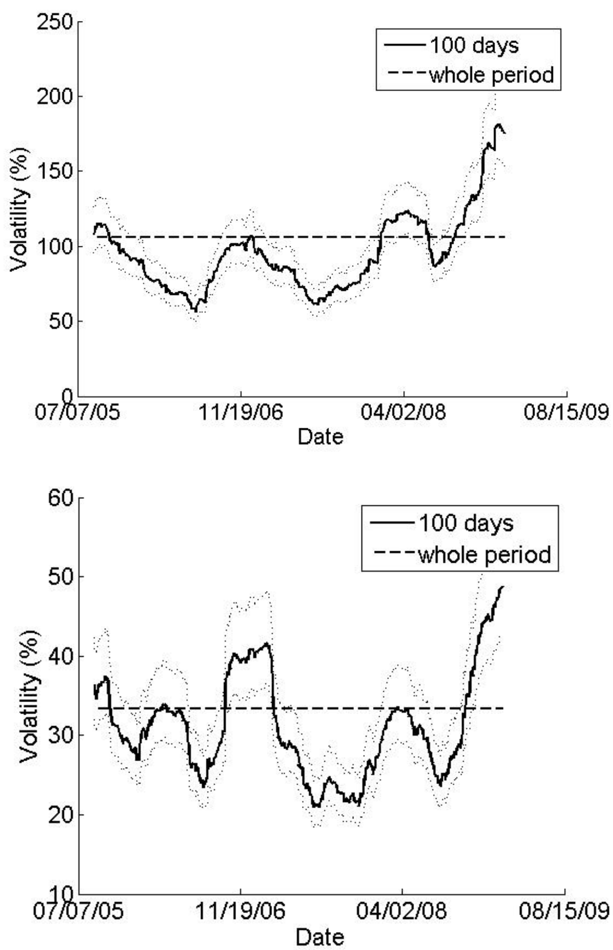


Figure 7.8. Autocorrelation of PC 1,2,3. 95% confidence intervals in dashed line

Figure 7.8. plots the autocorrelation functions of the first three factors. A significant autocorrelation is detected at the first lag of the first factor, and perhaps at the 10-20 day lag for the second factor. The possible significance of this autocorrelation will be investigated in a future study with the estimation of an auto-regressive model for the futures log-returns.

The rolling 100 day volatility of the first three factors, annualized, is presented in Figure 7.9 and compared with the volatility over the entire period. All three volatilities are seen to be stable indicating that the initial assumption that they are constant in the models presented in Section 6 is reasonable. A more detailed modeling of the volatility, possibly as a mean reverting jump diffusion including jumps in the futures returns as in the model (2.27), will be the subject of a future study.



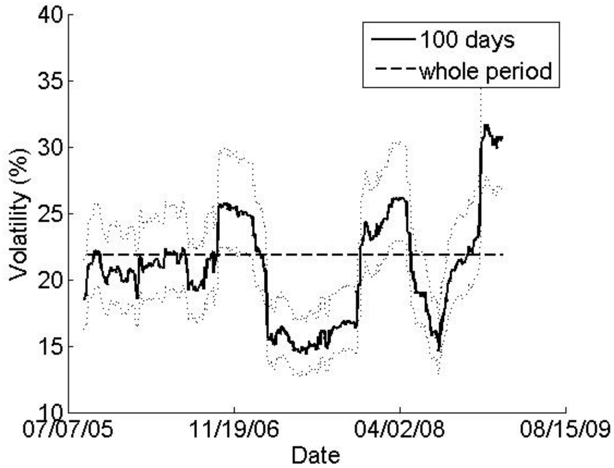
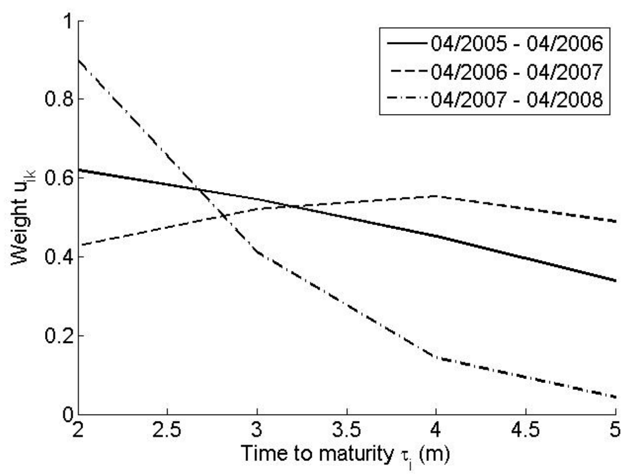
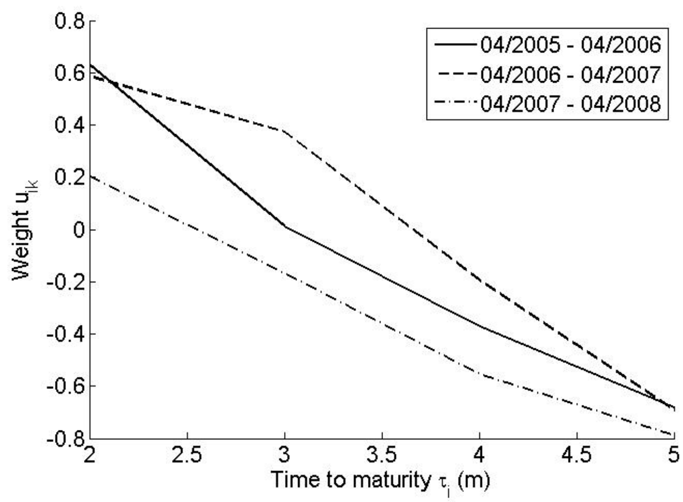


Figure 7.9: Stability of the volatility of Principal components 1,2,3: Rolling 100-day volatility vs. volatility of entire period





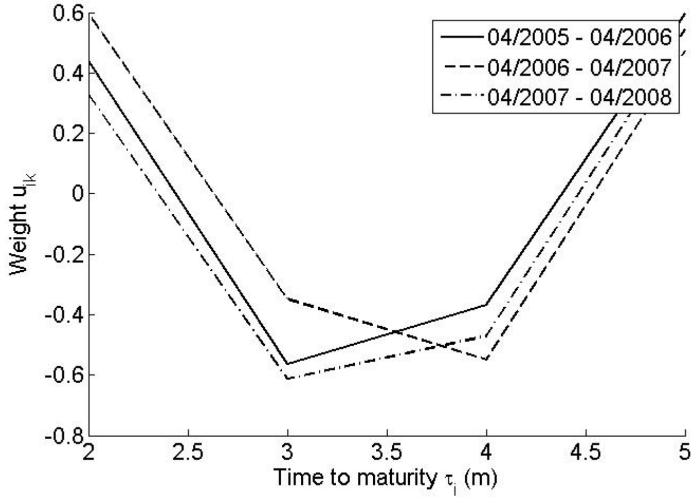
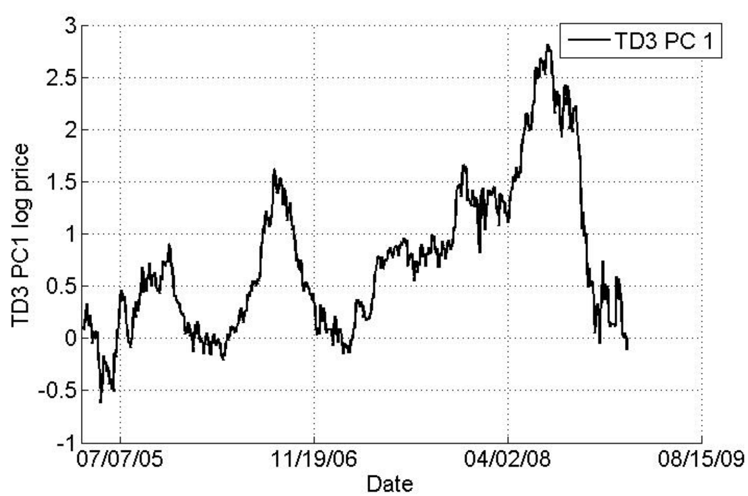


Figure 7.10: Stability of the PCA weights (U matrix): U1,2,3 calculated over non-overlapping 1-year periods. Corresponds to PC 1,2,3 in Fig 4

The stability of the principal components loadings is illustrated in Figure 7.10 evaluated over non-overlapping 1-year periods. The first principal component loading is more variable form year to year than the loadings of the second and third components. The stability increases with the index of the principal component, yet overall the stability is reduced relative to that observed in the crude oil market illustrated in Figure 3.8. It is noted that the loadings plotted in Figure 7.10 have been obtained from rolling futures log-returns up to a tenor of 5 months. As the tanker futures market deepens and liquid futures contracts with longer rolling tenors become available, as in the crude oil market, the stability of the factor loadings may increase.



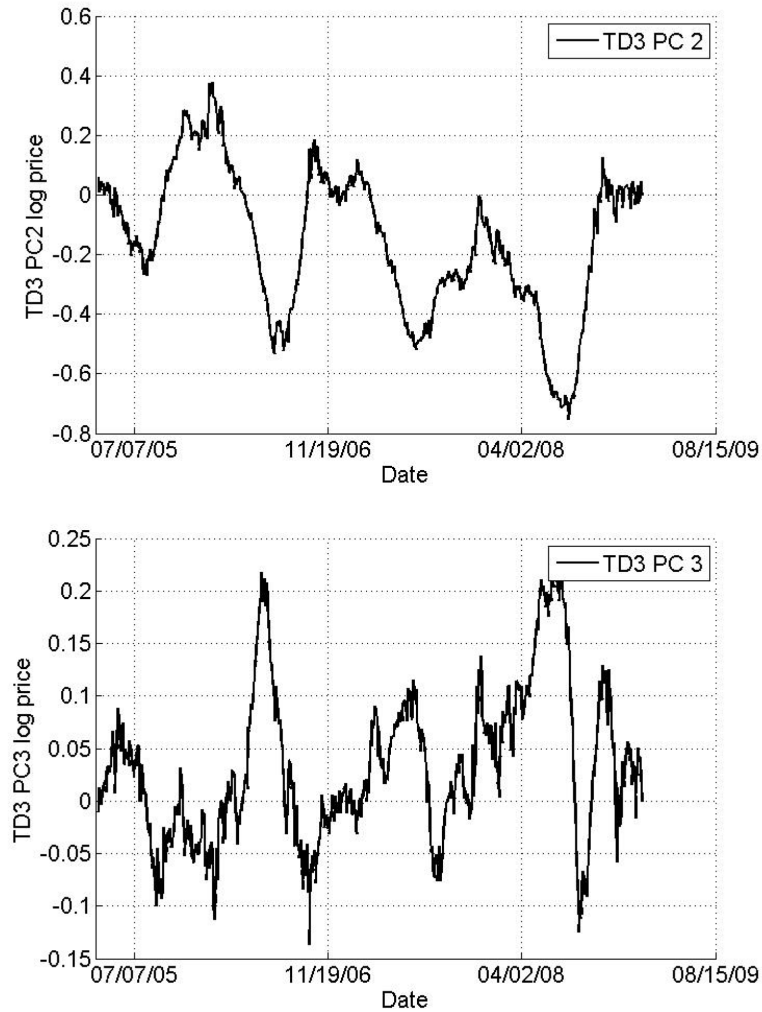


Figure 7.11: Log price of principal components 1,2,3 over the period

The time series of the first three factors are plotted in Figure 7.11 over the entire period of the data. As in the crude oil market, the first factor is responsible with up and down volatile shocks of the forward curve weighted by the slope of the first principal component loading plotted in Figure 7.6 and 7.10. The second factor drifts downwards and appears to be mean reverting or seasonal. This factor is again responsible for the rotation of the forward curve by virtue of the positive sign of the loading of the second principal component for short tenors and its negative sign for longer tenors, as seen from Figures 7.6 and 7.10. A downwards drift of the second factor tends to push the short tenors of the forward curve down relative to the long tenors therefore contributing to the transition of the forward curve from backwardation to contango. The third factor seems to be drifting sideways and no further conclusions can be drawn before a more detailed statistical analysis.

Correlated Route Futures in Tanker and Dry Bulk Shipping

The Principal Components Analysis presented above for the TD3 forward curve may be carried out for other routes and forward curves in the tanker and by bulk shipping markets, as in the PCA analysis of Koekebakker and Adland (2004) for the time charter freight rates for a Panamax dry bulk carrier. Since certain routes in the wet and dry bulk shipping may be highly correlated the two step PCA described in Section 3 for crude oil and heating oil extends naturally to the shipping markets. This analysis will enable the development of a wide range of hedging and investment strategies in shipping using the methods described in the next section. In particular, liquid futures trading for one route may be used to derive cross-route hedge ratios for risk management purposes. Moreover, arbitrage opportunities may be identified that will enable the design and optimal management of a shipping portfolio consisting of chartered cargo vessels and paper derivative positions.

8. ASSET VALUATION DYNAMIC HEDGING AND CORPORATE RISK MANAGEMENT

The use of futures for valuation, hedging and risk management offers advantages which flow from the ease of taking, offsetting and rolling futures positions, the liquidity of futures markets and the absence of counterparty risk when futures trade on an exchange. Moreover, futures and forward contracts are the fundamental building blocks for the pricing and hedging of a wide range of fixed-for-floating and floating-for-floating swaps involving one or two energy commodities and options written on swaps. When the use of the spot market is necessary, the use of the spot price models implied by the forward curve may be used. Participation in the paper futures, forwards and swaps markets may be the only option for market participants who are not in possession of the physical assets – the spot commodity, storage facilities, hydrocarbon reservoirs and shipping fleets. For firms in possession of real assets the present joint modeling framework of the spot and forward markets may be implemented for valuation, hedging, and to identify investment and arbitrage opportunities involving the physical and paper markets as discussed below.

Hedging of Energy Commodity and Freight Rate Risk

Participants in the energy and shipping markets may be categorized into producers, consumers and transformers. The first group includes crude oil, natural gas, coal producers and wind power generators. The second group includes commercial and industrial users of energy and the transportation industry – trucking, aviation, and shipping. The third group includes power generators, oil refiners and natural gas liquefiers. Producers face exposures to the price of a primary fuel – crude oil, natural gas, coal or wind. Consumers face exposures to the price of refined products – gasoline, jet fuel, diesel or shipping bunker fuel – as well as electricity produced from coal, natural gas or wind. Transformers face exposures to energy commodity price spreads – coal-natural gas-wind/electricity for power plants, crude oil/products for refineries, natural gas/liquefied natural gas (LNG) for energy companies, and bunker fuel/freight rates for shipping companies [Geman (2008), Leppard (2005), Schofield (2007), Kavussanos and Visvikis (2006)].

Inspired by the vast growth and success of the swap, futures and options markets for the risk management of financial securities, the energy commodity industry has witnessed the rapid growth of the crude oil OTC and futures paper markets where current volumes are ten times those in the underlying physical market. The futures markets have also experienced rapid growth for natural gas, crude oil products – gasoline, heating oil and gas oil – and more recently electricity. The growth of shipping futures and Forward Freight Agreements has also been rapid over the past ten years, yet still a tenth the size of the spot shipping charter market. It is an objective of the present article to increase the understanding of the pricing and use of freight rate derivatives in order to encourage their more widespread use as risk management instruments by the shipping industry.

There exist a number of fundamental differences between financial derivatives and energy commodity and shipping freight rate derivatives. Financial derivatives are settled in cash and conform more tightly to the no-arbitrage bounds while commodity derivatives often require the delivery of the underlying commodity and their no-arbitrage bounds may be wide. Therefore the pricing and use of energy commodity derivatives is influenced by the structural rigidities of the underlying spot commodity market. It has been argued by Davis (2002) that the term structure of crude oil can be divided into two segments, the first consisting of futures with tenors 0-18 months and the second consisting of tenors greater than 18 months. The first segment is linked to the physical market and is influenced by supply and demand, inventories, availability of storage and energy security. The distant tenors beyond 18 months are influenced by financial rather than physical economic factors including expectations, exchange rates, interest rates and inflation. It is interesting that crude oil PCA carried out in Section 3 indicates that the sign of the second principal component in Figure 3.4 reverses at a tenor of approximately 18 months and the negative peak of the third principal component plotted in the same figure occurs for the same tenor. Therefore, as alluded to in Section 3 the shape of the principal components and the statistical properties of the first few dominant factors are likely to contain useful information on the economic factors governing the evolution of the crude oil forward curve.

An attribute of commodity derivatives, not present in financial derivatives, is that the underlying physical commodity may exist in different grades. The key global marker crudes used as pricing benchmarks are; West Texas Intermediate (WTI), Brent, Dubai, Tapis and Urals. Liquid exchange traded futures markets exist for WTI and Brent trading on NYMEX and ICE, respectively. Therefore using WTI and Brent futures to hedge crude oil exposures in other grades, or exposures in products, will give rise to basis risk which must be properly managed. Basis risk also arises when an energy commodity producer, consumer or transformer is hedging an exposure in a refined product – e.g. aviation jet fuel or shipping bunker fuel – using liquid futures of a correlated commodity – e.g. crude oil, heating oil or gas oil. In such cases the robust modeling of the correlated forward curves of the respective commodities using the two-step PCA developed in the present article is essential for the derivation of accurate cross-commodity hedge ratios for the management of basis risk. The complexity in the design of intra- and cross-commodity hedging and risk management programs is considerable and is highlighted by the collapse of Metallgesellschaft discussed in Culp and Miller (1999).

The remainder of this Section discusses in more detail a few examples that highlight the use of futures and futures options for the valuation and dynamic hedging of assets and derivative portfolios in the energy and shipping markets. Futures, forwards and futures options are the fundamental building blocks for the pricing of swaps and swaptions that settle against a single price of the spot commodity or an index as well as their average over a settlement period. Fixed-for-floating swaps are a strip of forward contracts that settle at a set of regular dates, e.g. monthly or quarterly, that span the tenor of the swap. Their arbitrage pricing follows the same principles followed for the pricing of the individual futures positions. Intra- or cross-commodity floating-for-floating swaps may be priced by first pricing two fixed-for-floating swaps followed by the matching of the fixed leg payments. A discussion of the types of swaps encountered in the energy markets is presented by Leppard (2005). The pricing of interest rate and currency swaps when the interest rate term structures are stochastic is presented by Musiela and Rutkowski (2008).

Refineries, Power Plants and Transmission Assets

Refineries are energy assets exposed to the price differential of an input and an output commodity. The input commodity is crude oil and output commodities include heating oil, gasoline, jet fuel and other products. Analogous exposures are faced by power plants where the input commodity is coal, fuel, natural gas or wind and the output commodity is electricity.

A valuation of such cross commodity energy assets may be carried out by using derivative securities as building blocks and in particular options on cross commodity futures spreads priced in Section 5. Assume that the refinery will be in operation over the time interval (T_1,T_2) and that the ramp up/down times are negligible. At the current time t < T_1,T_2 the plant owns an option expiring at time τ which grants it the right to operate the facility over the time interval (τ, τ+dτ) if the price of the output commodity A is higher than the sum of the price of the input commodity B – after an adjustment for a heat rate H – and a fixed operating cost Kdτ. The differential value dv(t,τ) at time t of the right to operate the plant over the time interval (τ, τ+dτ) is equal to the price of a call option written on the cross commodity futures spread expiring at time τ with a strike equal to the operating cost Kdτ,

(8.1)
$$\begin{aligned} dv(t,\tau) &= C_S\left(t, F^A(t,\tau), F^B(t,\tau); K, \tau\right) d\tau = \\ &= B(t,\tau) E^*_t \left[F^A(\tau,\tau) - H\, F^B(\tau,\tau) - K \right]^+ d\tau \end{aligned}$$

Assume that the price of the call option in (8.1) has been obtained using the methods described in Section 5. Integrating over the time interval (T_1,T_2), the cumulative value derived from the operation of the refinery follows in the form

(8.2)
$$V(t) = \int_{T_1}^{T_2} C_S\left(t, F^A(t,\tau), F^B(t,\tau); K, \tau\right) d\tau$$

While the call option on the futures spread under the integral in (8.2) may not trade in the market, its explicit form reveals the dependence of the value V(t) on the underlying futures of commodities A and B with correlated dynamics. This permits the development of strategies to hedge the value V(t) discussed below.

The value V(t) is a stochastic process and its evolution dynamics may be derived from (8.2) and an application of Ito's theorem. This step reveals the Greeks of V(t), namely its sensitivities with respect to the time t prices of the futures of the underlying commodities A and B and their factor volatilities. Given the stochastic process followed by the value V(t), assuming that the current time is t and that an investment in this asset entails an irreversible sunk cost I, the value of an investment opportunity in this asset may be determined by implementing the real options framework [Myers and Majd (1984), McDonald and Siegel (1986), Dixit and Pindyck (1994)]. The value U(t) of the investment opportunity at time t is given by the expression

(8.3) $$U(t) = \sup_{s>t} E^{*}_{t}\left[e^{-r(s-t)} \left(V(s) - I \right)^{+} \right]$$

In a risk neutral setting, the interest rate r is assumed constant and time τ is the time when the investment will be made which is unknown at time t. The determination of the real option value U(t) may be carried out explicitly when the underlying value follows a geometric Brownian motion or a mean reverting process with constant coefficients. Alternatively, the method of stopping times for martingales may be used. Assume that V>I is the value of the underlying asset for which it is optimal to exercise the real option (8.3). The value of the investment opportunity may be expressed in the equivalent form

(8.4) $$U(t) = \sup_{V} \{ (V - I) E^{*}_{t}\left[e^{-r(s-t)} \right]_{V(s)=V} \}$$

In (8.4) the exercise value V appears as an intermediate variable with respect to which the right-hand side is to be maximized. The risk neutral expectation is taken with respect to the random stopping time s defined as the time at which the stochastic value process V(t) crosses the barrier V. This expectation is possible to evaluate explicitly for a number of stochastic processes for $V(t)$.

The same analysis applies when commodities A and B correspond to the same spot at two distinct and distant geographical locations where futures trade. Expressions (8.2)-(8.4) provide the value of the transmission asset carrying the commodity, a pipeline, an electricity transmission line or a tanker fleet used for the transportation of crude oil and products between two specific geographical locations.

Physical and Synthetic Storage

The valuation of physical storage facilities for crude oil, gasoline, heating oil and other energy commodities entails the dynamic optimization of injections and withdrawals of the energy commodity to/from the facility over a given time period. This dynamic injection/withdrawal process depends upon the shape and the volatility of the futures curve of the commodity of interest and is discussed below. As in other valuation problems discussed in the present section, the commodity futures and their options may be used as the fundamental securities for pricing and hedging. When liquid derivatives markets exist as in the crude oil and products markets, storage may be implemented synthetically by taking the appropriate positions in the futures markets.

Assume that at the current time t a firm has committed to deliver k units of a commodity currently in its possession at time T. Assume that liquid futures $F(t,s)$, $t<s<T$ and futures options trade for the commodity with evolution dynamics given by a multi-factor lognormal process. This commitment can be fully hedged by taking a short position in k futures contracts that expire at the horizon T, locking the delivery price $F(t,T)$.

In a more general setting assume that at time t positions are taken in N liquid futures and futures options contracts with tenors $T_i < T$, $i=1,\ldots,N$ and weights to be determined in an optimal manner. The value of the resulting futures and futures options portfolio becomes

$$V(t) = \sum_{i=1}^{N} v_i(t) F(t, T_i) + \sum_{i=1}^{N} Z_i(t) \begin{pmatrix} C \\ P \end{pmatrix} (t, T_i) \tag{8.5}$$
$$v_i(t) = e^{-r(T_i - t)} V_i(t)$$

The $V_i(t)$ positions in the underlying futures contracts and $Z_i(t)$ positions in futures calls and puts are assumed to have common expirations T_i. Ignoring for now the positions in the futures options, the futures portfolio may be selected at time t so as to meet obligations for the receipt/delivery of $V_i(t)$ units of the commodity into the storage facility at times T_i, by taking long/short positions in the respective futures contracts. If the futures contracts are allowed to expire, expression (8.5) provides the present value of the payments to be made at time T_i for the receipt of $V_i(t)$ units of the commodity.

The static value V(t) of this futures portfolio at time t may be maximized depending on the current shape of the forward curve over the tenor range (t,T). Such a static optimization is for example possible when the forward curve displays seasonality, as in the heating oil and natural gas markets. In this case the time t value V(t) provides the value of a storage facility used for the injection/withdrawals of $V_i(t)$ units of natural gas at times T_i, subject to physical constraints discussed in Eydeland and Wolyniec (2003).

The evolution dynamics of the futures contracts is given by the M factor model (2.1). Under the objective measure, a time dependent drift term $\mu_i(t)$ is added to the evolution of for the futures contract with expiration T_i. Assuming a zero interest rate for simplicity, the evolution dynamics of the futures portion of the portfolio (8.5) follows in the form

$$
\begin{aligned}
dV_F(t) &= \sum_{i=1}^{N} V_i(t) dF(t,T_i) \\
&= \sum_{i=1}^{N} V_i(t) F(t,T_i)[\mu_i(t)dt + \sum_{j=1}^{M} \sigma_j(t,T_i)\, dW_j(t)] \\
&= \sum_{i=1}^{N} V_i(t) F(t,T_i)\mu_i(t)dt + \sum_{j=1}^{M} f_j(t)\, dW_j(t)
\end{aligned} \tag{8.6}
$$

$$
f_j(t) = \sum_{i=1}^{N} V_i(t) F(t,T_i) \sigma_j(t,T_i)
$$

The evolution dynamics of the futures portfolio (8.6) is driven by the same M factors as the futures curve with time dependent deterministic drifts and volatilities $f_j(t)$ given by (8.6). It is also seen from (8.6) that the volatilities depend on the selection of the weights $V_i(t)$ in the futures portfolio. The same does not apply to the drift which must be estimated under the objective measure using the time series of the factors driving the commodity forward curve and econometric methods.

The dynamic trading of the combined portfolio (8.6) of futures and futures options positions may increase the value of synthetic storage discussed above. By adjusting the weights $V_i(t)$ and $Z_i(t)$ dynamically over the time period (t,T) the value V(T) may be maximized. In order to determine these optimal dynamic trading strategies, the evolution dynamics of the futures call and put prices must be derived. They follow from (8.5) and an application Ito's lemma

$$
\begin{aligned}
dC(t,F_i) &= C_t dt + \tfrac{1}{2} C_{FF} dF_i(t)\, dF_i(t) + C_F\, dF_i(t) \\
&= \left[C_t + \tfrac{1}{2} C_{FF}\, \Delta_i^2(t) + C_F\, F(t,T_i)\mu_i(t) \right] dt + C_F\, F(t,T_i) \sum_{j=1}^{M} \sigma_j(t,T_i)\, dW_j(t) \\
\Delta_i^2(t) &= \sum_{j=1}^{M} \sigma_j^2(t,T_i)
\end{aligned} \tag{8.7}
$$

For the purpose of developing dynamic hedging strategies of futures and futures options portfolios the stochastic differential equation (8.7) is cast in lognormal form

$$
\begin{aligned}
&\frac{dC(t,F_i)}{C(t,F_i)} = \mu_{Ci}(t,F_i)dt + \sum_{j=1}^{M} \sigma_{ij}(t,F_i)\,dW_j(t) \\
(8.8) \qquad &\mu_{Ci}(t,F_i) = \frac{C_t + \frac{1}{2} C_{FF}\,\Delta_i^{\,2}(t) + C_F\,F(t,T_i)\mu_i(t)}{C(t,F_i)} \\
&\sigma_{ij}(t,F_i) = \frac{C_F\,F(t,T_i)}{C(t,F_i)}\sigma_j(t,T_i)
\end{aligned}
$$

An analogous expression follows for puts from put call parity. In (8.8) the time dependent drift and volatility of the call option are known at time t in terms of the known values of the futures contracts $F(t,T_i)$.

Substituting (8.8) in the evolution dynamics of the combined portfolio of futures and futures options positions (8.6), we obtain the following stochastic differential equation

$$
\begin{aligned}
&dV(t) = \sum_{i=1}^{N} Z_i(t) \begin{pmatrix} C_t + \frac{1}{2} C_{FF}\Delta_i^{\,2}(t) + C_F\,F(t,T_i)\mu_i(t) \\ P_t + \frac{1}{2} P_{FF}\,\Delta_i^{\,2}(t) + P_F\,F(t,T_i)\mu_i(t) \end{pmatrix} dt + \sum_{j=1}^{M} G_j(t)\,dW_j(t) \\
(8.9) \qquad &G_j(t) = \sum_{i=1}^{N} [V_i(t) + \begin{pmatrix} C_F \\ P_F \end{pmatrix} Z_i(t)] F(t,T_i)\sigma_j(t,T_i)
\end{aligned}
$$

The time dependent coefficients in (8.9) are all available in explicit form from the closed form expressions of the options prices derived in Section 5.

An attractive property of the present multi-factor model of commodity futures is that the portfolio (8.5) is driven by as many sources of uncertainty as the number of dominant factors M<N which may be very small. This reduces the use of (8.9) for the valuation of synthetic storage and other applications into a simple computational task which may be carried out efficiently. Moreover, the existence of a small set of factors driving the forward curve indicates that the N futures and futures options positions are spanned by the M factors, therefore M ~ N. This suggests that the number of liquid futures and futures options positions needed for valuation and hedging may be equal to 2-3. The optimal

dynamic selection of the weights of the futures and futures options positions may be carried out using methods developed for the dynamic management of portfolios of securities. These methods rely on the use of stochastic dynamic programming methods which often lead to explicit expressions for the portfolio weights under a mean variance objectives, as discussed below.

Hydrocarbon Reservoirs

The valuation of oil and gas reservoirs may be carried out along similar lines to the valuation of synthetic storage, coupled with the real options framework. A typical investment opportunity by an oil company, discussed in Schwartz and Smith (2000), involves the development of an oil reservoir over the short or long term that would lead to the production of f(s) barrels of oil over the time interval (s,s+dt) with the reservoir productivity declining at an assumed exponential rate δ over a time period (τ,T). Investment in this reservoir entails an irreversible cost I which may occur over a short period or a longer time frame.

Assume that the current time is t and the cash flows resulting from the oil outflows from the reservoir will occur over the future time period (τ,T). Assuming that costs associated with the oil extraction process are small, the present value of this cash flow stream is given by the expression

(8.10) $$V_F(t) = \int_{\tau}^{T} ds\ B(t,s)\, f(s)\ e^{-\delta(s-\tau)} F(t,s), \quad t < \tau < T$$

In (8.10) B(t,s) the zero coupon bond assuming deterministic interest rates and the futures contracts F(t,s) are assumed to be quoted liquidly on public exchanges or OTC over the time period (τ,T).

The oil flow rate f(s) in (8.10) may be possible to select optimally in order to maximize the value $V_F(t)$, given the current and anticipated shape of the oil futures curve. The value

(8.10) may be increased further by the dynamic trading of futures options, assuming they are sufficiently liquid. This may be possible under the objective measure when a view exists on the evolution of the forward curve and its implied volatility.

The value of an investment opportunity in the hydrocarbon reservoir that entails a sunk cost of I, may be estimated by evaluating the real option value (8.3) or (8.4) with $V_F(t)$ given by (8.10). The optimal exercise of the American option imbedded in (8.3) or the evaluation of the expectation involving the stopped martingale in (8.4) may be carried out analytically when the evolution of $V_F(t)$ is approximated by a tractable stochastic process.

Seaborne Liquid Energy Cargoes

The valuation of seaborne liquid energy cargoes, e.g. crude oil, products, LNG, loaded in tankers or other commodities transported in bulk carriers share similarities to the valuation of storage discussed above. Assume that futures/forward markets exist for the commodity being transported at the port of loading and its destination. Transportation contracts for seaborne commodity cargoes may be valued as functions of futures spreads and futures options which may be traded dynamically over the duration of a voyage of optimal duration and destination.

Tankers engaged in the transportation of liquid energy cargoes may be viewed as crude oil transmission assets over particular sea-lanes or more generally as flexible storage facilities. Unlike land based storage, crude oil and products in tanker fleets contain surplus value associated with the optionality of the optimal time of delivery of the cargo. For certain term structures of the crude oil & products futures curves and their implied volatilities it may be advantageous to employ tankers as floating storage over a period of optimal duration controlled by the speed of a fleet which is instructed to low-steam or stay idle. An example is the profitable use of tankers as floating storage when the prompt crude oil forward curve is trading in extreme contango. This optionality combined with the dynamic trading of futures and futures options as in the case of synthetic storage discussed above leads to the maximization of the value of the cargo being transported. Such strategies may be derived in explicit form using the present Gaussian multi-factor cross-commodity model coupled stochastic dynamic programming methods discussed below

Dynamic Hedging and Optimal Portfolio Management

The use of liquid futures and futures options contracts to hedge energy assets requires the determination of the appropriate hedge ratios. This involves the evaluation of the sensitivity of the value of the energy asset on the risk factors that drive the forward curves of the pertinent commodity – the hedge ratio. The derivation of the hedge ratio entails the estimation of the differential of the asset value using the stochastic differential equation (2.1) governing the evolution of the futures price, the prices of calls and puts derived in Section 5, if present in the definition of the asset value, and Ito's lemma. An example of this process is offered above in connection with the valuation of storage. The hedge ratios are then obtained explicitly as the coefficients that multiply the sensitivities of the value of the asset to be hedged to the Brownian increments corresponding to each factor in (2.1). The number of factors is determined by the PCA of the commodity market under study and as seen from the results of Section 3 need not be more than 2-3. So typically a small number of hedge ratios is necessary in practice per commodity forward curve.

The evaluation of the hedge ratios under the current cross-commodity forward curve modeling framework enables the development of a wide range of dynamic hedging and risk management strategies involving physical or paper energy assets associated with one or multiple correlated commodities. When cross-commodity hedging is necessary, the same approach applies to the estimation of the hedge ratios. In such cases the two step PCA developed in Sections 2 and 3 must be used with the hedge ratios found to depend on the factor volatilities of each commodity as well as the cross-commodity factor correlations. Cross-commodity hedging may be necessary when the futures markets of the spot commodity to be hedged – e.g. aviation jet fuel – may not exist or be liquid enough. In such cases hedging may be implemented by using futures of correlated energy commodities with liquid energy markets – crude oil, gasoline, heating oil. The design of such cross-commodity hedging strategies may be carried out by using the two step PCA developed in

Sections 2 and 3. Emphasis must be placed on robust and parsimonious models that are market to market aiming to minimize basis risk.

The Gaussian evolution dynamics of the commodity and shipping futures derived above lead to stochastic differential equations with time dependent deterministic factor volatilities. Such evolution dynamics allows the derivation of explicit valuation and dynamic hedging strategies under quadratic mean-variance objectives outlined below. Consider the cash security $S_0(t)$, assume that the short interest rate r(t) is deterministic and time dependent and consider N risky securities, $S_i(t),\ i=1,...,N$, futures and assets that follow the multi-factor lognormal evolution dynamics of the form

(8.11)
$$\frac{dS_o(t)}{S_0(t)} = r(t)dt$$
$$\frac{dS_i(t)}{S_i(t)} = \mu_i(t)\,dt + \sum_{j=1}^{M} \sigma_{ij}(t)\,dW_j(t)$$

The evolution dynamics (8.11) is written under the objective measure and the drifts are assumed to be deterministic. The factor volatilities $\sigma_{ij}(t)$ are assumed to be time dependent deterministic quantities and are estimated using the methods described above. As deterministic quantities their values are assumed known at the current and future times t.

A portfolio consisting of the cash security and the N assets and securities governed by (8.11) has a value at time t given by the expression

(8.12)
$$V(t) = \sum_{j=0}^{N} \ w_i(t)\,S_i(t)$$

The portfolio weights $w_i(t)$ may take positive and negative values corresponding to long and short positions and are to be determined dynamically and in a continuous time setting subject to a mean variance objective to be met at a horizon T>t. The dynamic evolution of the self-financing portfolio is given by the relation

$$dV(t) = \sum_{j=0}^{N} w_i(t)\, dS_i(t) \tag{8.13}$$

A standard and general objective is to maximize the expectation of V(T) while penalizing its variance at the horizon T. This mean-variance objective may be cast in the form

$$\max \left\{ E[V(T)] - \mu Var[V(T)] \right\} \tag{8.14}$$

The parameter $\mu>0$ may be selected according to the application. This mean-variance dynamic portfolio optimization problem has been studied extensively in the securities markets since Merton (1971). In the present setup the coefficients of the stochastic differential equation (8.11) governing the securities prices are deterministic and time dependent.

The solution of the dynamic portfolio optimization problem defined by (8.11)-(8.14) may be obtained in closed form by solving the associated Hamilton-Jacobi-Bellman equations as described in Yong and Zhou (1999). The portfolio weights $w_i(t)$ over the time interval (t,T) follow explicitly in feedback form leading to closed form expressions for the expectation and variance of the portfolio value V(T) at the horizon. Extensions are also possible when the drifts and factor volatilities are stochastic processes. This case arises when the portfolio of securities (8.11) includes options. As can be seen from the stochastic differential equation (8.8) governing the evolution of the prices of calls and puts, the respective drifts and volatilities are functions of the futures prices which are stochastic processes. Therefore their future values are not known deterministically as is the case with the factor volatilities $\sigma_{ij}(t)$.

A number of the applications discussed above may be cast in the mean-variance optimization form described by (8.11)-(8.14). Essential in this process is the robust estimation of the factor volatilities which is possible in the commodities futures markets using the PCA described above. The estimation of the drifts is more challenging in principle. Yet, when a PCA model is available of a commodities futures curve, the drift of futures contracts of varying tenors may be decomposed into two components; the first is associated with a stable initial slope of the forward curve that may trade in contango, backwardation or in a composite formation; the second is associated with the slow transition of the forward curve form one formation to another associated with the drifts of the factors as discussed in Section 3.

Shipping Charter Portfolios

The valuation methods discussed above for energy assets may be applied to the optimal chartering of a fleet of cargo vessels by a shipping company. The risk management of a portfolio of real and paper shipping assets may be carried out in order to maximize value subject to financial constraints, for example the minimization of the volatility of the firm cash flows.

Assume that at time t $v_j(t)$ long/short positions are taken in shipping futures or FFAs for route j with tenors T_j and $z_i(t)$ positions in shipping freight rate futures options which have been priced in Section 6. By virtue of the arbitrage pricing of quarterly and yearly freight futures given by (6.4)-(6.5) and the pricing of freight rate options given by (6.17)-(6.21) the shipping derivatives portfolio would consist of monthly contracts with M tenors T_j spanning a time interval of interest.

Assuming zero interest rates for simplicity, the time t static value of this shipping futures and futures options portfolio is given by the relation

(8.15) $$V_S(t) = \sum_{j=1}^{M} v_j(t) F_S(t, T_j) + \sum_{j=1}^{M} z_j(t) \begin{pmatrix} C_S \\ P_S \end{pmatrix} (t, T_j)$$

At time t the static value of the portfolio (8.13) may be determined by selecting the magnitude and sign of the weights $v_j(t)$ and $z_i(t)$ in order to meet specific risk management objectives. For example if the firm is ready to commit M cargo vessels to M routes with charters initiating at times T_j the charter contract portfolio may be hedged by taking short positions in the corresponding shipping futures or FFAs at time t.

Dynamic Hedging of Shipping Assets

In a more general setting, futures positions in the shipping market may be taken by investors who do not own vessels and who have a view on the evolution of the freight rates over particular correlated routes. The futures options positions may enhance the value leading to the dynamic trading of a shipping derivatives portfolio analogous to that discussed earlier in connection with the valuation of storage. An investment that includes exposures in the real assets, the cargo vessels, or just the derivatives markets may therefore be designed to meet specific investment objectives or identify arbitrage opportunities. The hedging of shipping assets proceeds along the same lines as for energy assets discussed above. The freight rate futures follow log-normal diffusions and the Asian freight rate futures are priced by Black's formula. Therefore, hedge ratios may be derived explicitly. The same applies to the hedging of shipping assets associated with correlated shipping routes. The respective forward curves follow log-normal diffusions with factor volatilities estimated from individual PCAs for each route and the factor correlations follow from a two step PCA as in the crude oil and products markets. This enables the development and implementation of a wide range of dynamic hedging and risk management strategies in the tanker and dry bulk shipping sectors using (8.11)-(8.14) with the shipping portfolio (8.15).

Ship Routing and Fuel Efficient Navigation

The cost of bunker fuel represents a major expense in the shipping industry which currently consumes about 5% of the world oil production or about 4 million barrels of oil a day. The primary weather uncertainty faced by a cargo vessel is the severity of seastates to be encountered during a voyage which typically has a duration of weeks. The severity of the seastate along with the ship speed and heading determine the vessel fuel consumption which may be estimated using standard methods in naval architecture and marine hydrodynamics. A safe and optimal selection of the ship speed and course may lead to a significant reduction in fuel consumption. Generating 1 KW of propulsion power for one hour requires about 170g of fuel. A containership with a 60 MW main engine therefore burns about 245 tons of fuel daily or 3,430 tons during a 2 week trip across the Pacific. For a price of \$400/ton this represents a fuel cost of \$1,372,000 per crossing. Fuel savings during a typical trip were estimated by Avgouleas (2009) to be 10% or more when the vessel course and speed are optimally adjusted when sailing in a seastate, leading to \$137,200 of savings per trip for a containership.

The ship routing and fuel efficient ship navigation may be treated by invoking the log-normal diffusion models and optimal portfolio management methods presented above for the crude oil and shipping futures markets. The two state variables that govern the severity of a seastate are the significant wave height H and modal wave period T. The values of (H, T) may be assumed constant over the time scale of a stationary seastate which is assumed to be of the order of a few hours. Yet, they must be allowed to vary stochastically over a time scale of the order of a day to a week. The following joint-lognormal evolution process is assumed for the stochastic evolution of (H,T) pair over the long time scale

$$
\begin{aligned}
&\frac{dH(t)}{H(t)} = \mu_H(t)\,dt + \sigma_H(t)\,dW_H(t) \\
&\frac{dT(t)}{T(t)} = \mu_P(t)\,dt + \sigma_P(t)\,dW_P(t) \qquad (8.16) \\
&dW_H(t)dW_P(t) = \rho_{HP}(t)\,dt
\end{aligned}
$$

The stochastic dynamics (8.16) ensures that (H,T) remain positive at all times and that their drifts, volatilities and correlation as observed onboard the vessel are time dependent over the duration of the voyage. The coefficients of the joint log-normal evolution of (H,T) are subject to restrictions imposed by the physics of ocean waves, and are otherwise estimated from weather forecasts provided by a weather routing service over all likely trajectories of the vessel during the voyage. Assuming that a weather forecast is made available at the current time t for the rest of the voyage, the solution of the stochastic differential equation (8.16) up to a future time τ such that $t<\tau<T$ leads to the following expression for the random variables $(H(\tau),T(\tau))$

$$H(\tau) = H(t)\exp\left[\int_t^\tau \mu_H(s)ds - \frac{1}{2}\int_t^\tau \sigma^2{}_H(s)ds + \int_t^\tau \sigma_H\,(s)\,dW_H(s)\right]$$

(8.17)

$$T(\tau) = T(t)\exp\left[\int_t^\tau \mu_T(s)ds - \frac{1}{2}\int_t^\tau \sigma^2{}_T(s)ds + \int_t^\tau \sigma_T\,(s)\,dW_T(s)\right]$$

Let G the rate of consumption of bunker fuel in kg/sec for the propulsion of a cargo vessel. In calm weather, the value of G depends on the resistance and propulsion characteristics of the vessel, namely her hull shape, propeller design, engine characteristics, life of the vessel etc. In a seastate G also depends on the vessel added resistance in waves which in turn depends on the vessel seakeeping properties. All hydrodynamic quantities are assumed to have been computed a priori in the form of mean and RMS values in a seastate with known (H,T) values and stored in tabular form for use in the solution of the optimal control problem discussed below.

The fuel consumption in calm weather and in waves also depends on two "controls", u_1 the propeller revolutions per minute (RPM) and u_2 the vessel heading relative to an ambient seastate, current and wind. The two controls may be set in real time by the captain or the vessel navigation system. The effects of current and wind are ignored in the present treatment, yet they may be easily accounted for and treated along similar lines. When the vessel sails into a seastate the fuel consumption depends on the state variables (H, T) characterizing the seastate at time t. We may therefore cast the dependence of G on the weather state variables and controls as follows

(8.18) $$G \equiv G[u_1(t), u_2(t); H(t), T(t)]$$

Assume that t is the current time, T-t is the duration of the remainder of the voyage and treat T as a stochastic variable that depends on the speed and course of the ship which are not known with certainty at time t. The objective of a fuel efficient navigation policy is to minimize

(8.19) $$\min_{(u_1,u_2,T)} E_t\ \{\int_t^T G\left[u_1(\tau), u_2(\tau), H(\tau), T(\tau)\right] d\tau\}$$

This nonlinear stochastic optimal control problem is in principle difficult to solve. It can be simplified considerably by linearizing the fuel consumption rate G about the known fuel consumption if the vessel were to sail in calm weather during the entire trip. Assuming that the vessel speed and course deviations about their known calm weather values are not too large, a reasonable assumption, the optimal control problem (8.19) can be linearized by using Ito's theorem and cast into a form that may treated explicitly using (8.11)-(8.14). Closed form expressions follow for the optimal controls $u_1(\tau), u_2(\tau); t < \tau < T$ over the duration of the voyage that lead to the minimum possible fuel consumption over the trip, subject to constraints that ensure the vessel safety. The explicit form of these algorithms allows their easy and efficient implementation in real time onboard a vessel.

The optimal routing of cargo vessels and shipping fleets may be treated in a more general context by estimating the dependence of the net revenue of a vessel on a set of economic state variables, or factors, F_i, i=1,…,M, that follow the joint-lognormal evolution dynamics (2.1). In this setting a shipowner would be interested to determine the optimal vessel speed and course in order to maximize the net revenue R over a single trip or a number of consecutive trips. For example an optimal reduction of the speed of a shipping fleet would lead to the reduction of the supply of ton-miles and for fixed demand would lead to an increase of the freight rate revenue and a reduction in fuel consumption. In this setting the optimal control problem to be solved becomes

(8.20) $$\max_{(u_1,u_2,T)} E_t \{\int_t^T R\left[u_1(\tau), u_2(\tau); F_i(\tau), H(\tau), T(\tau)\right] d\tau\}$$

The solution of the optimal control problem (8.20) may be carried out explicitly assuming that the dependence of the net revenue on the economic factors is linear, upon linearization of the fuel consumption cost about the calm weather values and use of (8.11)-(8.14).

Derivatives in Corporate Finance

Concluding this section the role of derivatives on the capital structure, financial management and investment policies of firms in the energy and shipping industries is addressed. The topic of risk management within the field of corporate finance and the use of derivatives for hedging are discussed by Brealy and Myers (2000) and Stutz (2003). Accounting issues, taxation, the Modigliani-Miller invariance framework, firm valuation, financial distress costs, monitoring, agency costs and the use of derivatives are addressed. The present discussion focuses upon the role of risk management, derivative pricing and asset valuation and hedging by non-diversified firms in the energy and shipping industries with concentrated exposures to volatile energy prices, freight rates, bunker fuel costs and interest rate risks.

A first step towards the modeling and management of commodity market risk is the identification of a small set of factors that affect the forward curves and spot prices of crude oil, natural gas, shipping freight rates and other commodities. As discussed above the existence of liquid derivative markets enables the development of such factor models. Similar factor models have been widely used for the modeling of default free term structure of interest rates. As the derivative markets in the energy and shipping industry grow and deepen, price transmission mechanisms between the spot and futures develop and a dominant set of factors affecting each sector becomes easier to identify from the PCA of the forward curve.

Energy and shipping are capital intensive industries that rely on debt capital to finance their assets and operations. Consequently, these sectors are exposed to interest rate and credit risk. The modeling of the default free term structure of interest rates has been studied extensively and a number of robust models are widely used in practice. They include the Hull-White-Vasicek, the Cox-Ingersoll-Ross and the Black-Karasinski short rate models. These methods have been shown to be consistent with the Heath-Jarrow-Morton HJM model of the forward term structure of interest rates. Their properties and calibration to the market prices of traded interest rate securities and their derivatives is discussed by Shreve (2003). They may be used to model the default free interest rate risks energy and shipping firms are exposed to.

Credit risk may be valued by using the structural method of Merton (1974) or the reduced form intensity method of Duffie and Singleton (2003) and Lando (2004). The structural method of Merton's treats the firm equity and debt as claims contingent upon the value of the firm assets which may not be observable, and uses Black-Scholes for their pricing in a risk neutral setting. The reduced form method extends the HJM framework by introducing a hazard rate which enters as a yield premium added to the risk free rate. The hazard rate and the probability of survival of a firm may then be modeled as jump-diffusions and calibrated against the market prices of equity, debt and credit default swaps. The value of the firm assets in Merton's method is often not observed, an exception being the shipping industry where the cargo vessels trade in the second hand market. As pointed out by Schonbucher (2005), this unique property of firms with observable prices for their assets in a second-hand market enables the implementation of the Merton model for the pricing of equity and debt claims.

Energy and shipping firms often have publicly traded equity which may be used for the calibration of both structural and reduced form methods and the pricing of credit risk. This approach is adopted by Overhaus et. al. (2007) where the firm equity price, firm survival probability and the risk free short rate are modeled as jump-diffusion processes. Negative jumps in equity returns are correlated with negative jumps in the survival probability – positive jumps in the hazard rate – hence linking the equity price process with credit events. In the case of shipping this reduced form modeling framework may be coupled with a model for the prices of the cargo vessels in the second-hand market leading to a hybrid credit risk model that combines attributes of the structural and reduced form methods [Amman (2001). The factors affecting the yield premia of seasoned high yield bonds in shipping have been studied by Grammenos, Alizadeh and Papapostolou (2007). The data underlying this model form the basis for the calibration of a hazard rate process and the development of a reduced form model for the pricing of shipping credit risk. This reduced form model would permit the modeling both of interest rate and credit risk under the HJM framework and would enable the integrated management of interest rate, credit and commodity price risk.

Equity prices of tanker and dry bulk shipping firms may be strongly correlated with the price of the underlying freight rate index. Factors driving the equity prices of bulk shipping firms may therefore be revealed by the respective futures and FFA markets discussed above. As the liquidity of the shipping derivative markets grows, the factor volatilities of shipping futures and FFAs are likely to become more correlated with the equity volatility of shipping firms. This will enable the use of the shipping futures markets for the development of dynamic hedging strategies by firms aiming to address a wide range of financial management policies. They include the minimization of cash flow variance, selection of optimal firm leverage in order to take advantage of the tax shield on debt interest, fleet expansion via the proper mix of debt and equity, determination of dividend policies, structuring of equity hybrid derivatives and the enhancement of firm value.

9. CONCLUSIONS

The modeling, pricing, valuation and hedging has been presented of derivatives and assets in the crude oil and tanker shipping markets based on a Gaussian HJM multi-factor model of their forward curves based on a Principal Components Analysis (PCA). The approach draws upon the growing depth and liquidity of the commodity futures as the fundamental underlying securities used for price discovery and risk management in correlated energy commodity sectors like crude oil and its products as wet and dry bulk shipping sectors.

A number of exposures in the energy and dry bulk shipping sectors involve cross-commodity transactions. The modeling was carried out of correlated commodity forward curves using a two-step PCA. A cross-commodity HJM model was developed for correlated commodity futures curves which reveals a small number of 2-3 factors affecting each commodity market. An arbitrage free relation between the forward and spot markets was established and a multi-factor process for the spot price of the underlying commodity was derived revealing the mean reverting dynamics of short term transitory and long term persistent shocks, as implied by the forward curve. The factor volatility term structure was found to be stable for the crude oil, gasoline and heating oil markets. The time series of the dominant factors were derived and shown to govern the evolution of the forward curve, including its transition from backwardation to contango or into other composite formations. The explicit pricing was also considered of vanilla and spread options written on liquid underlying futures contracts that may be used as the fundamental securities for valuation and hedging.

Liquid commodity futures contracts and their derivatives are forward looking instruments that may be used for the valuation of a number of cross-commodity assets in the energy and shipping industries. They include refineries, power plants, oil and natural gas storage, energy transmission assets and seaborne liquid energy cargoes. When investments in these assets have not yet been made, the use of the real option framework was outlined for the valuation of the investment opportunity and the determination of the optimal exercise policy. A number of examples highlighting the use of the present multi-factor framework for the valuation and hedging of energy assets were presented.

The modeling framework was extended to the pricing of tanker shipping futures and Forward Freight Agreements in an arbitrage free setting. A HJM multi-factor model with time dependent volatilities was introduced for the shipping forward curve and was used for the modeling of shipping futures and Forward Freight Agreements. As the shipping derivatives markets grow in depth and liquidity the present modeling and pricing methods stand to foster a better understanding of the factors affecting the shipping markets and lead to the development of a wide range of risk management and investment strategies.

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