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and ancillary services**

by

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Efficient bidding for hydro power plants in markets for energy and ancillary services

Dmitri Perekhodtsev* and Lester Lave**

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Abstract

In order to preserve stability of electricity supply generators must provide ancillary services in addition to energy production. Hydroelectric resources have significant ancillary service capability because of their dynamic flexibility. This paper suggests a solution for optimal bidding for hydro units operating in simultaneous markets for energy and ancillary services by estimating water shadow price from operating parameters of the hydro unit, expectations on prices of energy and ancillary services, and water availability. The model implications are illustrated on a numerical example of a hydro unit operating in markets of New York Independent System Operator.

Participation in ancillary services market increases or decreases water shadow price depending on water availability. As a result of participation in ancillary services markets, a unit with water availability given by a capacity factor of 0.6 increases the value of existing generating capacity by 25% and nearly doubles the value of incremental generating capacity.

JEL Codes: D21, D24, D41, D58, Q25, Q41

Key words: hydro optimization ancillary service energy market bidding water shadow price

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1. Introduction

Providing stable electricity supply requires ancillary services to counter the minute to minute rises and falls in electricity consumption and possible forced outages of generating or transmission facilities. Provision of ancillary services requires power plants to possess certain dynamic flexibility. Hydro generators are the most efficient source of ancillary services because of their good dynamic flexibility and may earn a substantial profit if ancillary services were purchased in a competitive market.

This paper studies the economics of provision of energy and ancillary services by hydroelectric resources and derives equations for optimal bidding for hydro units operating in simultaneous markets for energy and ancillary services.

We focus on two types of ancillary services that are provided on the market base in many areas of the United States: regulation or AGC (automatic generator control), and operating reserves. Regulation is needed to ensure the minute to minute balance between load and generation. It is provided by partially loaded generating units operating above the minimum operating limit able to respond to frequent signals from the system operator to increase or decrease the output. Operating reserve is needed to ensure that the load is met in case of a major outage of transmission or generating facility and is provided by partially loaded generators able to increase the output in case of such outage.

Since generating units provide energy, regulation, and operating reserves from the same generating capacity, the markets for energy and ancillary services are very interdependent.

After the most recent modifications the market of the New York Independent System Operator (NYISO) treats these interdependencies in the most consistent way. The NYISO collects bids that include the information on the generating cost and operation parameters of generating plants such as minimum load, maximum load, and the ramp rate. NYISO further minimizes the as-bid cost of meeting the energy load and requirements for regulation and operating reserve while satisfying the operation and system constraints. The prices of energy and ancillary services are set at the shadow price of the market clearing constraints for the corresponding product and as a result, generating units are dispatched to maximize their profits given the prices of energy and ancillary services.

Such market design is incentive compatible in that for a competitive generator bidding the actual generation cost and operation parameters is a weakly dominating strategy.

However, for hydroelectric resources bidding the generation cost is complicated. The direct cost of energy generation is virtually zero. But since the amount of water that can be used for generation is limited for pondage and pumped storages hydro plants, the generation bids must be based on the opportunity cost of water usage also known as the water shadow price. The water shadow price depends on parameters of the hydro project such as generation and ancillary services production capability, flow and reservoir constraints, and expectations of future energy and ancillary services prices and water inflow. This paper provides a model that estimates the water shadow price for hydro generators that operate in simultaneous markets for energy and ancillary services. Such model can help operators and owners of hydro facilities operating in such markets in New York, New England, California, and elsewhere to design efficient bidding strategies.

This model is further applied to calculate water shadow prices for a hypothetical hydro unit operating in New York markets for energy and ancillary services. We calculate the increase of the value of existing and incremental hydro generating capacity resulting from provision of ancillary services in the market.

The framework for calculating water shadow price can also be used as a step in the iterative process in modeling and forecasting energy and ancillary services prices. In particular, such modeling of ancillary services prices can be used by vertically integrated utilities with large hydro generating portfolio that currently do not administer markets for energy and/or ancillary services for internal valuation of generating assets. The paper provides an outline of such an iterative process.

The difficulty of optimization of the hydro-thermal generation mix is that the operation problem for fossil fuel plants is largely separable for each operation period (e.g. a day or an hour), while the hydro operation problem must be solved over a longer horizon. One of the fundamental works on the economics of such problem has been done by Koopmans [1]. It has been recognized that reservoir dynamics can be relaxed by using an additional set of Lagrange multipliers that can be viewed as water shadow prices. In a simple setup the water shadow price can be assumed to be constant over the cycle [2]. In [3, 4] a more realistic case is studied where water shadow price changes over the cycle any time the reservoir capacity constraints are reached. However, the water shadow price is still constant over the intervals when the reservoir capacity is not binding. The authors provide derivations of the rental valuation of the fixed inputs such as turbine and reservoir capacity for both river dams and pumped storage units based on the water shadow prices.

In this paper the general idea of variable water shadow price of [3, 4] is adopted. The estimates of water shadow prices are calculated as constants over pre-specified time intervals during which reservoir capacity constraints are unlikely to be binding. Although many river dams are optimized simultaneously as they share a common watershed, the water shadow prices are calculated individually for each hydro dam [5] since water is not perfectly transferable from one reservoir to another. The model of water shadow price estimation presented in this paper also assumes fixed coefficients of conversion of water into energy or fixed head for each dam. Cases of variable head are considered in [6-8].

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Studies of ancillary services appeared in energy economics literature long before the electricity markets started operating in several areas of the United States in 1997-1998. A survey done in 1996 across 12 US investor-owned utilities estimated the cost of these services to be up to 10% of the total cost of energy generation and transmission [9]. It was realized that in order to avoid the problem of missing markets there is a need for unbundling such services from the energy generation in the deregulated electricity markets [10].

Studies of the costs of ancillary service provision from fossil fuel plants include [11-14]. In [14] a simulation of the market for energy and ancillary services is presented for a fossil fuel mix and [15] studies the operation decisions and profits of a fossil fuel plant operating in markets for energy and ancillary services.

Currently, markets for ancillary services are administered in California, New York, New England, Pennsylvania-New Jersey-Maryland Interconnection (PJM), and Texas [16-21]. However, the first attempts to run ancillary services markets were not always successful. Some markets were noticed to provide wrong incentives and to be susceptible to market power exercise [22-25].

The model calculating the optimal bids for hydro plants presented here assumes perfect competition. Although there exists a large amount of literature on market power in energy markets there are not many studies of possible mechanisms of market power exercise by hydro plants [26].

The remainder of the report is structured as follows. Section 2 presents the model for estimating the water shadow price for hydro dams operating in markets for energy and ancillary services. Section 3 provides illustrative calculations of water shadow price based on NYISO price data. Section 4 shows how the value of existing and incremental hydro generating capacity changes as a result of optimal bidding in simultaneous markets for energy and ancillary services. Section 5 provides an outline of the iterative process that can be used for modeling and forecasting of energy and ancillary services in areas with large share of hydro generating capacity.

2. Estimation of water shadow price for river dams

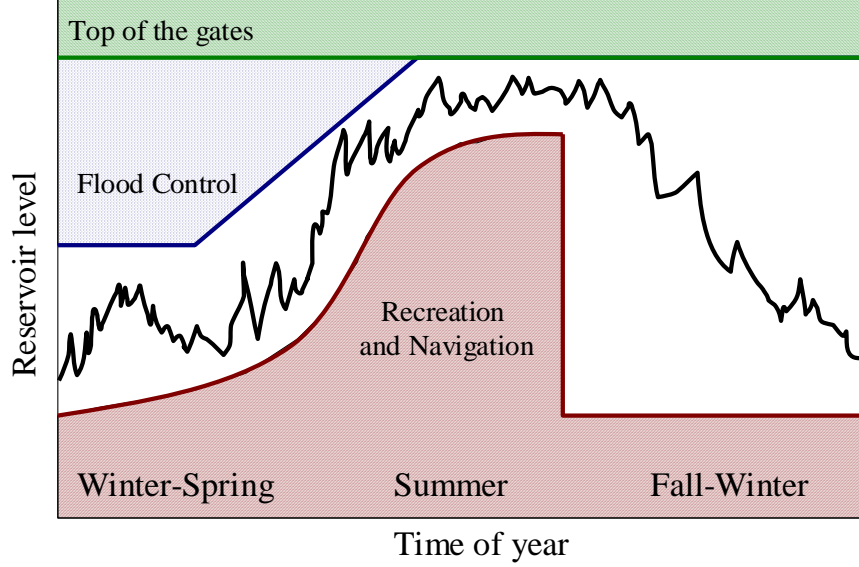
The direct cost of energy generation by hydro units is virtually zero compared to the fossil fuel plants. However, because of the scarcity of the water supplies the water shadow price can be viewed as a fuel cost for the purposes of designing bidding strategies for hydro plants. The idea behind the water shadow price is that generating one megawatt-hour in a given hour means not being able to generate one megawatt-hour in some future hour. Therefore, for a hydro unit the foregone earnings from future generation constitute the opportunity cost of current energy generation. Once the opportunity cost of energy generation or water shadow price is known, the optimal decision to provide generation or any of the ancillary services from a hydro plant is made the same way as for a combustion turbine with no startup costs [3, 4].

However, as opposed to the cost of fossil-fuel plants that is largely dependent on the fuel costs and is relatively stable over time, the shadow price of water in hydro plants varies a lot. It depends on the expected future prices of energy, as well as on parameters of a hydro project, such as storage capacity, maximum generating and pumping (in case of pumped storage) capability and efficiency, expected future natural water inflow and constraints on water use for navigation, flood control, recreation and other purposes.

In addition, as it will be shown in this section, the water shadow price depends on the ability of the hydro unit to participate in ancillary services markets and the expected future prices in these markets.

This paper only focuses on river dams. A similar analysis of pumped storage facilities is presented in [27]. River dams are characterized by a set of parameters and constraints. Maximum and minimum flow constraints determine the range of water flow that can be passed through the turbine in each hour. The level of the reservoir behind the dam also has to be within a certain range that can vary throughout the year. The upper bound of this range is determined by the top of the gates and the flood control constraints. The lower bound is determined by the navigation and recreation constraints that ensure that the reservoir level is high enough for these needs, the latter being particularly strict during summer. A typical “guide curve” for the reservoir level in the South East of the United States is shown on Figure 1.

Figure 1. A typical hydro guide curve in the South East



The supply of water in the dam is determined by the natural water inflow to the reservoir as well as the operation of the upstream dams. Some economic analysis of cascaded hydro systems can be found in [5].

The efficiency of converting water into energy varies with reservoir level or head elevation since energy is proportional to the height of the water drop. In addition, the conversion efficiency depends on the turbine load. However, in the analysis below the conversion efficiency will be assumed to be constant.

Consider the optimization problem of a river dam. The outflow from the reservoir is

$$f(t) = y(t) - e(t),$$

where $y(t)$ is hydro plant's output and $e(t)$ is reservoir water inflow. It is assumed that hydro plant's maximum output capacity y_{\max} is larger than $\sup|e(t)|$. This condition ensures that no spillage is ever necessary. With the initial reservoir level s_0 the reservoir level at t is:

$$s(t) = s_0 - \int_0^t f(t) dt$$

Hydro profit maximization problem given the deterministic continuous energy prices $p(t)$ over the time interval $[0, T]$ is therefore:

$$\begin{aligned} & \max_{y(t)} \int_0^T y(t)p(t)dt \\ & s.t. \\ & 0 \leq y(t) \leq y_{\max} \quad a(t), b(t) \quad , \\ & s_{\min}(t) \leq s(t) \leq s_{\max}(t) \quad g(t), d(t) \\ & s(T) = s_T, s_T \in [s_{\min}, s_{\max}] \quad l \end{aligned} \quad (1)$$

where s_{\min} and s_{\max} are the reservoir lower and upper constraints, which may vary over time as is shown in Figure 1. In (1) $a(t)$ and $b(t)$ are the Lagrange multipliers on the flow constraints; $g(t)$ and $d(t)$ are Lagrange multipliers on the upper and lower reservoir constraints. Finally, l is a Lagrange multiplier at the constraint on the terminal reservoir level s_T , which is fixed.

As shown in [4], the solution to this problem is given by:

$$y(t) = \begin{cases} y_{\max}, & p(t) > y(t) \\ [0, y_{\max}], & p(t) = y(t) \\ 0, & p(t) < y(t) \end{cases}, \quad (2)$$

where $y(t)$ is the water shadow price given by

$$y(t) = l - g(t) + d(t).$$

That is, “the hydro plant operates just like a thermal plant with a time-varying “fuel” price $y(t)$ ”. In a pool-based energy market a hydro unit would submit the energy bid equal to $y(t)$ and will be dispatched according to (2).

It is easy to see, that if on the interval $[t_1, t_2]$ reservoir constraints are not binding, then shadow price is constant on that interval and equal to $y_{t_1, t_2} = l_{t_1, t_2}$. On the other hand, [4] shows that $y(t) = p(t)$ if reservoir constraints are binding at time t .

Over each of the intervals $[t_1, t_2]$ where reservoir constraints are not binding the optimization problem (1) simplifies to:

$$\begin{aligned} & \max_{y(t)} \int_{t_1}^{t_2} y(t)p(t)dt \\ & s.t. \\ & 0 \leq y(t) \leq y_{\max} \quad a(t), b(t) \\ & \int_{t_1}^{t_2} y(t)dt \leq s_{t_1} - s_{t_2} + \int_{t_1}^{t_2} e(t)dt = S_{[t_1, t_2]} \quad l_{t_1, t_2} \end{aligned}, \quad (3)$$

where $S_{[t_1, t_2]}$ is the total amount of water available for the interval $[t_1, t_2]$. On such intervals the water price is constant $y(t) = y_{t_1, t_2} = l_{t_1, t_2}$.

To solve (3) for l_{t_1, t_2} it is convenient to introduce the notation of *price distribution function* over the interval $[t_1, t_2]$:

Definition:

Distribution function $F_p(a)$ of price $p(t)$ over the time interval $[t_1, t_2]$ is a measure of the subset of $[t_1, t_2]$ on which $p(t) < a$ relative to the total length of the interval:

$$F_p(a) = \frac{\text{meas} \{t : p(t) < a, t \in [t_1, t_2]\}}{t_2 - t_1}$$

A price distribution function is also called a *price duration curve*.

A price distribution function defined this way has properties similar to the cumulative probability distribution function:

$$F_p(-\infty) = 0$$

$$F_p(\infty) = 1$$

$$F_p' > 0$$

The strict inequality in the last condition implies that the price duration curve does not have flat regions, which is essential for many results of this paper. The actual prices may not have this property for different reasons. However, results of this paper can be easily generalized to deal with these situations by introducing appropriate tie breaking rules.

Proposition 1

The solution to (3) is given by (2), where $y(t) = y_{t_1, t_2}^* = \text{const}$ is obtained from equation

$$y_{\max} (1 - F_p(y_{t_1, t_2}^*)) = \frac{S_{t_1, t_2}}{(t_2 - t_1)}. \quad (4)$$

The solution of (4) exists and is unique if $F_p' > 0$.

Proof:

Suppose the solution to (3) is given by $y^0 < y_{t_1, t_2}^*$. Then from (2) the amount of water used will be

$$S^0 = (1 - F_p(y^0))(t_2 - t_1)y_{\max}.$$

Since F_p is nondecreasing, it follows that $S^0 > S_{t_1, t_2}$, which violates the constraint of (3).

Similarly, if $y^0 > y_{t_1, t_2}^*$, then $S^0 < S_{t_1, t_2}$, meaning that the objective in (3) is not maximized.

Since $\sup_{[t_1, t_2]} |e(t)| < y_{\max}$, it follows that $S_{t_1, t_2} < y_{\max}(t_2 - t_1)$. Therefore, at $y^0 = 0$ the sign of (4) is positive and at $y^0 = \sup_{[t_1, t_2]}(p(t))$ it is negative. Together with continuity of F_p this proves existence

of the solution. The uniqueness follows from the fact that $F_p' > 0$.

There is a simple intuition behind the solution for the water shadow price in (4). The right hand side of the equation gives proportion of the time that the hydro unit can generate at full capacity

given the amount of water available. For optimality the generation must be scheduled during the time with highest energy price. That is ensured by the left hand side of the equation.

The length of the time interval $[t_1, t_2]$ where the reservoir constraints are not binding must be on the order of $\frac{s_{\max} - s_{\min}}{y_{\max}}$, that is, the time needed to drive the reservoir from the upper constraint to the lower while operating at full turbine capacity. We have calculated this ratio for some river dams in the US to be about one month. This agrees with the fact that river dams are often optimized on a monthly cycle. Thus, throughout this paper the water shadow prices will be considered constant over each calendar month.

Ancillary services provision by river dams

If a river dam is capable of providing ancillary services such as regulation up to r_{\max} and spinning reserves¹ up to s_{\max} , and these services were priced at $p_r(t)$ and $p_s(t)$, its profit maximization problem over the period $[t_1, t_2]$ on which the reservoir capacity constraints are not binding modifies to:

$$\begin{aligned}
 & \max_{y,r,s} \int_{t_1}^{t_2} (y(t)p(t) + r(t)p_r(t) + s(t)p_s(t)) dt \\
 & s.t. \\
 & 0 \leq y(t) \leq y_{\max} \\
 & 0 \leq r(t) \leq r_{\max} \\
 & 0 \leq s(t) \leq s_{\max} \\
 & y(t) + r(t) + s(t) \leq y_{\max} \\
 & r(t) + s(t) \leq s_{\max} \\
 & y(t) \geq r(t) \\
 & \int_{t_1}^{t_2} y(t) dt \leq s_{t_1} - s_{t_2} + \int_{t_1}^{t_2} e(t) dt = S_{[t_1, t_2]} - I_{t_1, t_2}
 \end{aligned} \tag{5}$$

¹ Areas that administer markets for operating reserves usually distinguish several tiers of operating reserves different in the required response timeframe. Thus, NYISO distinguishes 10-minute reserves within the total operating reserves, and 10-minute spinning reserves within 10-minute operating reserves. Since higher tier reserves are always priced higher or the same as other reserve tiers in NYISO markets, and since fast ramping hydro resources are assumed to qualify for the highest reserve tier, the rest of paper only focuses on the 10-minute spinning reserves.

Here additional constraints require the sum of the energy and ancillary services output not to exceed the total generator capacity; the sum of ancillary services output not to exceed the spinning reserve capacity; and the energy output to exceed the regulation output to provide the room for regulation downwards movements². The water shadow price is still constant $y_{t_1,t_2} = l_{t_1,t_2}$ over the interval $[t_1, t_2]$, however, in general, is different from that in (3).

Throughout the rest of the paper it will be assumed that $s_{\max} = y_{\max}$ and $r_{\max} \leq 0.5y_{\max}$. The former is a reasonable assumption to make for hydro plants, whose dynamic characteristics allow them to ramp up to the full capacity within 10-15 minutes; the latter comes from the fact that the unit cannot provide more regulation than a half of the total output capacity since both regulation upwards and downwards by the amount of regulation capacity is required.

To simplify the proofs below it will also be assumed that $p_r \leq p_s \leq 0$. In the ancillary services markets of the New York Independent System Operator (NYISO) this was true over 95% of the hours in the studied period. Allowing for $p_s > p_r$ slightly modifies the decision rule in (6) below and calculation of water shadow price in (9) (see Appendix) but does not change the main findings of the paper.

Proposition 2

Given the water shadow price y_{t_1,t_2} a dispatch maximizing the profit of a hydro unit in (5) is:

$$(y(t), r(t), s(t)) = \begin{cases} (y_{\max}, 0, 0), & p(t) - y_{t_1,t_2} \geq p_r(t) \\ (y_{\max} - r_{\max}, r_{\max}, 0), & p_s(t) \leq p(t) - y_{t_1,t_2} < p_r(t) \\ (r_{\max}, r_{\max}, y_{\max} - 2r_{\max}), & 2p_s(t) - p_r(t) \leq p(t) - y_{t_1,t_2} < p_s(t) \\ (0, 0, y_{\max}), & p(t) - y_{t_1,t_2} < 2p_s(t) - p_r(t) \end{cases} \quad (6)$$

Proof:

For the reasons similar to those presented in [4] it follows that problem (5) is equivalent to the following linear programming problem solved at each $t \in [t_1, t_2]$:

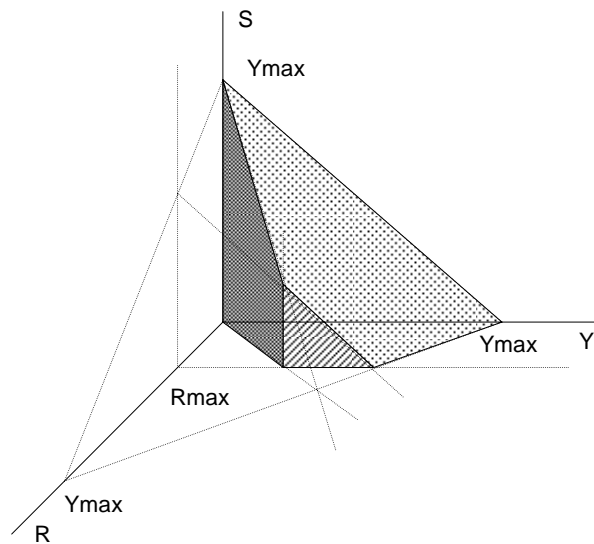
² Here regulation is assumed to be an energy-neutral service. A generator providing regulation has to be able to both ramp up or down by the amount of provided regulation. Movements in opposite directions offset each other on average and result in a net zero energy production.

$$\begin{aligned}
& \max_{y,r,s} y(p - y_{t_1,t_2}) + rp_r + sp_s \\
& \text{s.t.} \\
& 0 \leq y \leq y_{\max} \\
& 0 \leq r \leq r_{\max} \\
& 0 \leq s \leq s_{\max} \\
& y + r + s \leq y_{\max} \\
& r + s \leq s_{\max} \\
& y \geq r
\end{aligned} \tag{7}$$

that holds for every t . A solution to the linear programming problem (7) must lie on one of the corners of the polyhedron shown in Figure 2. This area is determined by the operation constraints in (7). The corners of this polyhedron are:

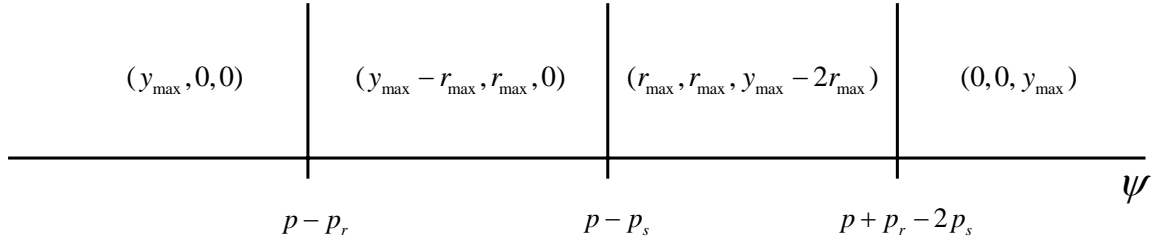
$$(0, 0, 0), (r_{\max}, r_{\max}, 0), (y_{\max} - r_{\max}, r_{\max}, 0), (y_{\max}, 0, 0), (0, 0, y_{\max}), (r_{\max}, r_{\max}, y_{\max} - 2r_{\max})$$

Figure 2 Simultaneous feasibility of energy and ancillary services output from a river dam



If the price of the spinning reserve is nonnegative, then the corners $(0,0,0)$ and $(r_{\max}, r_{\max}, 0)$ are dominated by $(0,0,y_{\max})$ and $(r_{\max}, r_{\max}, y_{\max} - 2r_{\max})$ leaving only four candidate solutions listed in (6). Having in mind that $p_r \geq p_s$, it is easy to check that each of the candidate solutions dominates the others on the intervals suggested in (6) illustrated on Figure 3.

Figure 3. Economic dispatch of hydro generators



In a simultaneous market for energy and ancillary services the hydro unit would submit the energy bid equal to $y(t)$ and its spinning and regulation capacity y_{\max} and r_{\max} and will be dispatched according to (6).

Proposition 3

If $s_{\max} = y_{\max}$ and $p_r \geq p_s \geq 0$, then the solution to (5) is given by (6), where $y(t) = y_{t_1, t_2}^* = \text{const}$ is obtained from:

$$G(y^*) = \frac{S_{t_1, t_2}}{(t_2 - t_1)}, \quad (8)$$

where

$$G(y^*) = y_{\max} (1 - F_{p - p_r}(y^*)) + (y_{\max} - r_{\max})(F_{p - p_r}(y^*) - F_{p - p_s}(y^*)) + r_{\max} (F_{p - p_s}(y^*) - F_{p - 2p_s + p_r}(y^*)) \quad (9)$$

(the t_1, t_2 subscript at y^* is dropped in (8) and (9)), and (8) has a unique solution.

Proof:

Similarly to Proposition 1 the form of (8) and (9) is dictated by the water balance constraint in (5) and the operation intervals in (6). The existence can be shown similarly to the proof in Proposition 1. For uniqueness consider $G_{\psi}(y)$:

$$G_{\psi}(y) = -f_{p - p_s}(y)(y_{\max} - 2r_{\max}) - r_{\max} (f_{p - p_r}(y) + f_{p - 2p_s + p_r}(y)) \quad (10)$$

$G_{\psi}(y)$ is non-increasing since $y_{\max} > 2r_{\max}$ from the property of regulation. If $p - p_s, p - p_r$, and $p - 2p_s + p_r$ do not have plateaus, that is, $F_{\psi}(y) > 0$ then $G_{\psi}(y)$ is strictly decreasing and (8) has a unique solution.

Change of water shadow price in presence of ancillary services markets

Unlike coal and gas generators whose cost determined by the cost of fuel is largely constant, for hydro units the shadow price of water depends on their operation capabilities. Formulas (4), (8), and (9) suggest that the water shadow price changes if a hydro unit is participating in the market for ancillary services.

To check the direction of the change in the water value with participation of hydro units in ancillary services markets examine

$$\frac{dy}{dp_s} \text{ and } \frac{dy}{dp_r},$$

assuming that $p_r, p_s = \text{const}$ and $p_r > p_s$.

Proposition 4

The shadow price of water goes down with the increase of the price of spinning reserves

Proof:

Consider $G(y)$ from (9). With $p_r, p_s = \text{const}$, it transforms to

$$\begin{aligned} G(y, p_r, p_s) &= \\ &= y_{\max} (1 - F_p(y + p_r)) + \\ &+ (y_{\max} - r_{\max})(F_p(y + p_r) - F_p(y + p_s)) + \\ &+ r_{\max} (F_p(y + p_s) - F_p(y + 2p_s - p_r)) \end{aligned}$$

The change in y in response to the change in p_s is

$$\frac{dy}{dp_s} = - \frac{G_{p_s}}{G_y}$$

From the proof of Proposition 3 it is known that $G_y < 0$, therefore, consider G_{p_s} :

$$G_{p_s} = - (y_{\max} - 2r_{\max})f(y + p_s) - 2r_{\max}f(y + 2p_s - p_r)$$

But since $y_{\max} > 2r_{\max}$ by definition of regulation capability, $G_{p_s} < 0$ and $\frac{dy}{dp_s} < 0$.

The intuition behind this result is that the ability to profitably provide spinning reserves allows hydro units to save water that otherwise could have been used for generation. Additional supply of the saved water drives the water price down.

Proposition 5

The change of the water shadow price in the presence of regulation markets is undetermined. If the energy price p has a unimodal distribution $f_p(\cdot)$ with the mode p_M then for river dams it can be stated that $\frac{dy}{dp_r} < 0$ if $y < p_M - p_r$ and $\frac{dy}{dp_r} > 0$ if $y > p_M - 2p_s + p_r$.

Proof:

Consider $G(y, p_r, p_s)$ as in Proposition 4.

$$G_{p_r} = -r_{\max} (f(y + p_r) - f(y + 2p_s - p_r))$$

The sign of the expression $f(y + p_r) - f(y + 2p_s - p_r)$ is not determined. However, if $f(\cdot)$ is unimodal with the mode p_M and $y < p_M - p_r$ then the expression is positive and $\frac{dy}{dp_r} < 0$ since in this case both $y + p_r$ and $y + 2p_s - p_r$ are on the increasing segment of $f(\cdot)$. Likewise, $\frac{dy}{dp_r} > 0$ if $y > p_M - 2p_s + p_r$.

The intuition behind this result is the following. Consider the case when a hydro unit provides regulation in a given hour but in the absence of the regulation market it would choose not to generate in this hour. Therefore, presence of regulation market requires the unit to use water that otherwise would not be used. That increases the water value for the subsequent hours. The other situation is that in a given hour the hydro unit provides regulation that offsets the energy production in the absence of regulation markets. The hydro unit is therefore saving water that otherwise would have been used for energy decreasing the water value for future use.

The total effect of regulation markets on water shadow price is determined by the ratio of hours of both types. In units with low water availability and therefore high water shadow price hours of the first type are more prevalent, therefore the water price tends to increase in presence of regulation markets. Likewise, units with high water availability and low water prices enjoy more hours of the second type over the cycle and their water price decreases in presence of regulation markets.

3. Bidding simulation

To apply formulas (8) and (9) or (15) for actual bidding the hydro operators must estimate the duration curves of prices of energy and ancillary services over the planning period. This can be done using methods from the option theory. First, parameters of the stochastic processes that govern prices of energy, regulation, and spinning reserve must be estimated. Second, Monte Carlo simulation of the paths of price differences: $p - p_s$, $p - p_r$, and $p + p_r - 2p_s$ should be

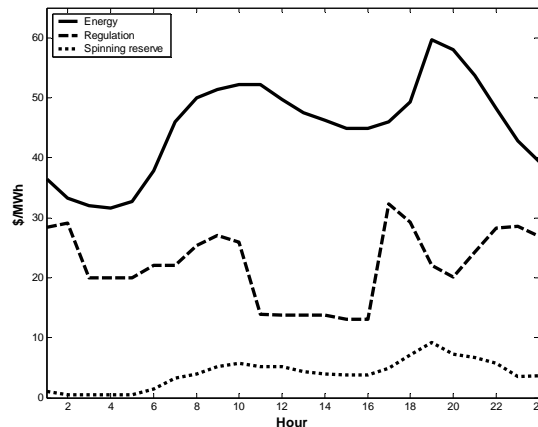
performed to construct the duration curves of these price differences. The detailed description of these techniques is beyond the scope of this paper and can be found elsewhere.

In the remaining of the paper we illustrate the efficient bidding of a hypothetical hydro plant operating in the integrated markets of the NYISO assuming that an expectation of the future price duration curves coincides with the actual NYISO prices. We simulate the bidding and the resulting profits for a hydro plant with a generating capacity of 100MW and regulation capacity of 40MW. We assume that the unit's guide curves and expected water inflow are such that the periods in which constraints are not binding are on the order of one month. We use publicly available hourly data on the day-ahead prices of energy at the Mohawk Valley zone of the NYISO which has the most concentration of NYISO hydro units, and prices of regulation and spinning reserves in the West ancillary services zone of the NYISO in March 2004. The month of March was chosen because the average prices of that month are representative of those over the whole year. The summary statistics of these prices and the average prices conditional on the hour of the day are presented in Table 1 and Figure 1.

Table 1. Descriptive statistics of NYISO price in March 2004 and in whole 2004 in Mohawk Valley energy zone and West ancillary services zone

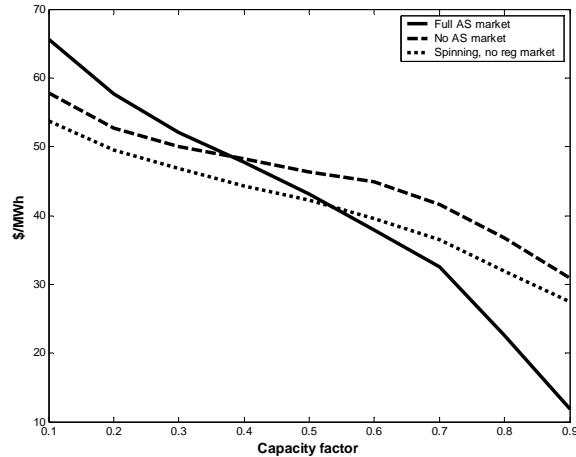
	March 2004			Calendar year 2004		
	Energy	Regulation	10-min Spin	Energy	Regulation	10-min Spin
Mean	45.65	22.23	4.05	49.53	22.55	2.36
Median	46.50	20.00	3.25	50.15	20.00	1.26
Stdev	9.72	13.18	4.19	13.69	16.21	3.54

Figure 4. NYISO energy and ancillary services prices in March 2004, conditional on time of day



The amount of available water is parameterized through unit's capacity factor defined as the ratio of the available monthly energy generation to the energy that the unit could have produced if operated at full capacity every hour of the month. Figure 5 shows the water shadow price calculated using formulas (8) and (15) for a unit participating in all three markets, for energy only markets, and for energy and spinning reserve markets for different values of capacity factor. The calculations used a linear interpolation of price duration curves from NYISO hourly price data. As was shown in Proposition 4, the shadow price of water goes down as a result of unit participation in spinning reserve market in addition to energy market. Participation in regulation market decreases the water shadow price if water availability is high and increases the water shadow price if water availability is low confirming the finding in Proposition 5.

Figure 5. Water shadow price vs. capacity factor and presence of AS markets



4. Hydro asset valuation

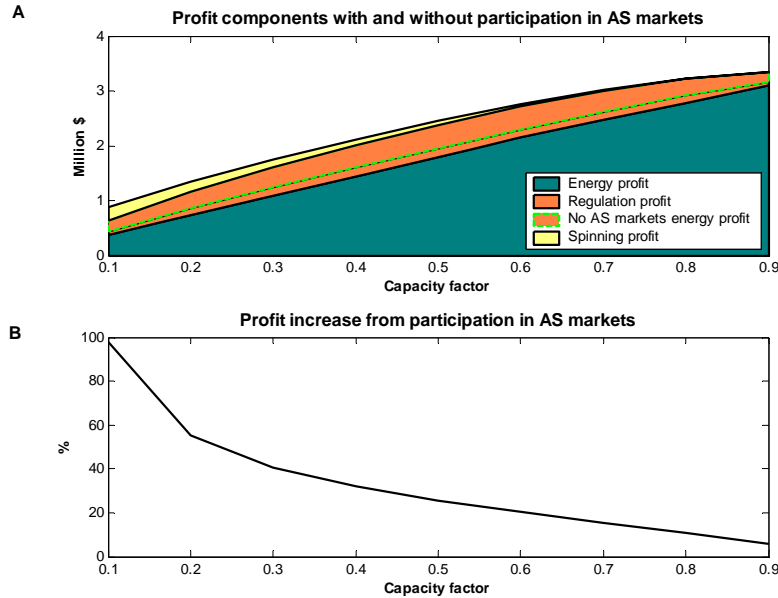
The water shadow price calculated above can be used to value hydro generating assets. This section focuses on two types of asset valuation: the incremental value of existing hydro capacity resulting from participation in AS markets and the value of additional generating or regulating capacity for hydro units participating in energy and ancillary services markets.

Incremental value from participation in AS markets

Clearly, a unit participating in markets for energy and ancillary services earns more profit than a unit that participates in energy markets only. Figure 6A shows how the profit per cycle and its components depend on the water availability (capacity factor) for a unit participating in ancillary

services markets. This profit is calculated as the value function of the optimization problem (5) given the decision rule (14) and the water shadow price from (8) and (15). As more water becomes available the relative benefit from participation in ancillary services markets falls as is shown in Figure 6B but remains fairly high at 20% at capacity factor of 0.6 and 10% at capacity factor 0.8.

Figure 6. Analysis of profit per cycle vs. capacity factor and AS market participation



Value of incremental generating and/or regulating capacity for hydro units providing AS

The framework presented here can be used to value the incremental capacity investments in existing hydro projects such as adding a turbine, increasing turbine efficiency, or adding regulation capability on the existing facility.

Proposition 6

For a hydro turbine operating in energy markets only, the value of incremental generating capacity $\frac{dp}{dy_{\max}}$ is greater than zero but less than the average value of the existing capacity

$$\frac{p(y_{\max})}{y_{\max}}.$$

Proof:

The profit per cycle of a hydro generator operating in energy market only is given as:

$$p(y_{\max}) = y_{\max} \int_y^{\infty} p dF(p), \quad (11)$$

Therefore,

$$\frac{dp}{dy_{\max}} = \frac{p}{y_{\max}} - y F(y) \frac{dy}{dy_{\max}} \quad (12)$$

From (4) it follows that $\frac{dy}{dy_{\max}} < 0$, which, together with the fact that $F(y) > 0$ suggests that

$$\frac{dp}{dy_{\max}} < \frac{p}{y_{\max}}$$

However, $p(y_{\max})$ is the value function of the constraint maximization problem (3). Therefore, relaxing the constraint by increasing y_{\max} has to increase $p(y_{\max})$. Therefore,

$$\frac{dp}{dy_{\max}} > 0$$

Proposition 6 suggests that for a unit that operates only in energy market the value of incremental generating capacity is less than the average value of the existing capacity since increasing the generating capacity increases the water shadow price and the unit generates less often. Values of

$\frac{p}{y_{\max}}$ and $\frac{dp}{dy_{\max}}$ simulated for NYISO energy prices in March 2004 are shown in Figure 7.

Figure 7. Profit per unit of existing capacity and incremental profit per incremental capacity for hydro turbine operating in energy market only

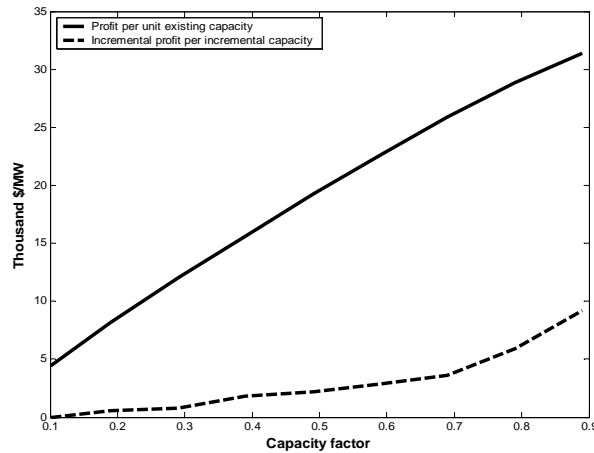


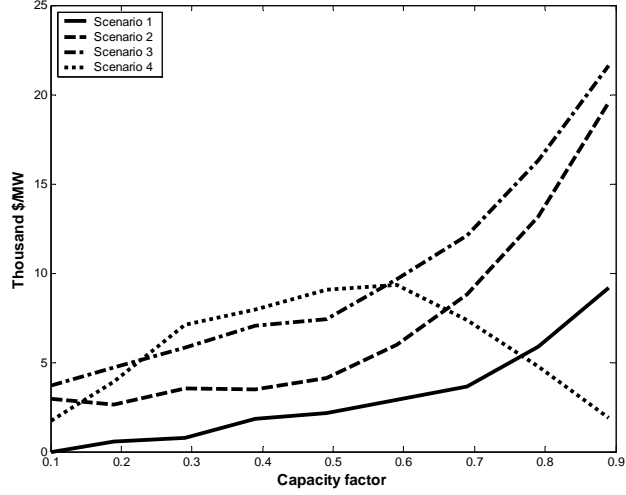
Figure 7 suggests that the value of incremental generating capacity is rather low for a unit operating only in energy markets. However, the value of the same incremental generating capacity increases dramatically for hydro plants that operate and optimally bid in markets for

energy and ancillary services. The main reason for that is that additional capacity that can provide both energy and ancillary services will not use water when providing ancillary services, and as a result, will not increase the water shadow price as much as the additional capacity that can only provide energy.

In Figure 8 we calculate values of incremental generating and/or regulating capacity under four different scenarios:

1. Unit only participates in energy markets $\left. \frac{dp}{dy_{\max}} \right|_{\substack{p_r=0 \\ p_s=0}}$,
2. Unit participates in energy and ancillary services markets but additional capacity cannot be used for regulation $\left. \frac{dp}{dy_{\max}} \right|_{r_{\max} = \text{const}}$,
3. Unit participates in energy and ancillary services markets and additional generating capacity increases the regulation capacity proportionally $\left. \frac{dp}{dy_{\max}} \right|_{r_{\max} = 0.4y_{\max}}$,
4. Unit participates in energy and ancillary services markets and the capacity increment can only be used for providing regulation $\left. \frac{dp}{dr_{\max}} \right|_{y_{\max} = \text{const}}$.

Figure 8. Value of a unit of incremental capacity under four scenarios



Profits and their derivatives presented in Figure 8 were calculated as the value functions of (5), using the decision rule (14), and water shadow price calculated from (8) and (15).

Figure 8 suggests that incremental generating and/or regulating capacity on a hydro facility operating in simultaneous markets for energy and ancillary services may be worth several times the value of incremental generating capacity on a hydro facility that only operates in energy markets.

5. Using the bidding strategy for modeling energy and ancillary services prices in hydro-thermal systems.

Formulas (8) and (9) or (15) for determining water shadow price for hydro units can be used in modeling and forecasting of energy and ancillary services prices for systems with hydro-thermal generation mix. We suggest the following iterative process.

0. On the initialization stage an arbitrary vector of water shadow prices \mathbf{wsp}^0 is picked for the fleet of hydro units for each hydro cycle (month).
1. Hourly energy and ancillary services prices \mathbf{p}^0 , \mathbf{p}_r^0 , \mathbf{p}_s^0 are obtained over the hydro cycle as a result of modeling the day-ahead security constraint unit commitment and economic dispatch in which hydro units are modeled with the generating cost equal to the water shadow prices \mathbf{wsp}^0 .

2. Water shadow prices \mathbf{wsp}^1 are updated using formulas (8) and (9), and current energy and ancillary services prices $\mathbf{p}^0, \mathbf{p}_r^0, \mathbf{p}_s^0$.

Steps 1. and 2. are repeated until convergence in water shadow prices for the hydro fleet over the hydro cycle.

With a single water shadow price per unit per cycle this method is expected to converge quickly due to a small number of variables over which the convergence is sought. This method was applied in [27].

6. Conclusion

The model presented in this paper shows that competitive hydro units must bid for the energy differently if, in addition to energy markets, they can provide their capacity in markets for ancillary services. They bid their water shadow price, which depends on the ancillary services capability, expected amount of water they can use over a cycle, and the expected duration curves of energy and ancillary services prices. The water shadow price decreases for units that also sell their capacity in reserves markets compared to hydro units operating only in energy markets. For hydro units that also sell their capacity in regulation markets the water shadow price may increase if the water availability over the cycle is low or decrease if the water availability over the cycle is high.

The change in water shadow price resulting from hydro participation in markets for energy and ancillary services may entail a long term adjustment in hydro constraints and re-allocation of water resources between energy generation and other water uses to increase the overall societal value of water resources.

This paper shows that the value of existing hydro generating plants operating in simultaneous markets for energy and ancillary services may be substantially higher than the value of hydro plants operating solely in energy markets. The increase is the most significant for units with low water availability since they are able to earn profits providing ancillary services without using their scarce water resources. However, even for a unit with moderate water availability parameterized by a capacity factor of 0.6 such increase has been calculated to be 25% based on the NYISO energy and ancillary services prices.

For hydro generators operating solely in energy markets expanding the generating capacity is not very profitable. If the water shadow price were constant, the additional capacity would earn the same profits per MW as the existing capacity. However, additional capacity increases the water shadow price making the whole facility generate less often. As a result, the net value per MW of

capacity upgrade is materially lower than the average value of existing capacity. Participation in simultaneous markets for energy and ancillary services may increase the value of incremental capacity several times. This happens because the capacity upgrade that can provide ancillary services in addition to energy has less effect on water shadow price. For a unit with the amount of water given by 0.6 capacity factor the value of a megawatt of additional capacity may be three times higher if this unit participates in simultaneous markets for energy and ancillary services.

These examples emphasize that allowing hydro units be paid the market price for the ancillary services that they provide is critical in giving right incentives for investment in upgrades, maintenance, and new capacity of hydro generating resources. Absence of explicit ancillary services markets results in a general misallocation of resources and an increased cost of energy procurement to customers. This is especially true for hydro generating resources because they have large ancillary services capacity and because of the scarcity of the water supply. Analysis presented in this paper can be applied with little changes to non-hydro generating units whose energy production is limited over a long horizon by either limited fuel access or environmental constraints.

The methodology of water shadow price evaluation presented here may be used in modeling energy and ancillary services prices in areas with large share of hydro generating capacity as described in Section 5. Such modeling may be useful in assessing investment opportunities for generating resources in areas that have explicit markets for energy and ancillary services. This modeling of energy and ancillary services prices may also be useful to negotiate compensations from system operators to generating plants for providing ancillary services in areas that do not have explicit markets for ancillary services.

7. References

1. T. C. Koopmans, Water Storage Policy in a Simplified Hydroelectric System, Cowles Foundation, Yale University, Cowles Foundation Discussion (1957).
2. J. Warford and M. Munasinghe, Electricity pricing: theory and case studies The John Hopkins University Press Baltimore-London (1982).
3. A. Horsley and A. J. Wrobel, Efficiency Rents of Storage Plants in Peak-Load Pricing, I: Pumped Storage, Suntory and Toyota International Centres for Economics and Related (1996).
4. A. Horsley and A. J. Wrobel, Efficiency Rents of Storage Plants in Peak-Load Pricing, II: Hydroelectricity, London School of Economics and Political Science, STICERD Theoretical Economics Discussion Paper: TE/99/372 (1999).
5. M. E. El-Hawary and G. S. Christensen, Optimal economic operation of electric power systems Academic Press New York (1979).
6. W. Bauer, H. Gferer and H. Wacker, Optimization strategies for hydro energy storage plants, *Zeitschrift fur Operation Research* (1984), **Series B**, 103-131.

7. H. Gferer, Optimization of hydro energy storage plants by variation methods, *Zeitschrift fur Operation Research* (1984), **Series B**, 87-101.
8. H. X. Phu, On the optimal control of a hydroelectric power plant, *System and Control Letters* (1987), **8**, 281-288.
9. E. Hirst and B. Kirby, Ancillary Services Cost for 12 US Utilities, Oak Ridge National Laboratory, Oak Ridge (1996).
10. E. Hirst and B. Kirby, Electric-Power Ancillary Services, Oak Ridge National Laboratory, Oak Ridge (1996).
11. D. Curtice, Cost of Providing Ancillary Services from Power Plants. Volume 1: A Primer, Electric Power Research Institute, Palo Alto, CA (1997).
12. A. A. El-Keib and X. Ma, Calculating Short-Run Marginal Costs of Active and Reactive Power Production, *IEEE Transactions on Power Systems* (1997), **12**, 559-565.
13. E. Hirst and B. Kirby, Ancillary-Service Details: Operating Reserves, Oak Ridge National Laboratory, Oak Ridge, TN (1997).
14. E. Hirst and B. Kirby, Creating Competitive Markets for Ancillary Services, Oak Ridge National Laboratory, Oak Ridge (1997).
15. E. Hirst, Maximizing Generator Profits Across energy and Ancillary Services Markets, Oak Ridge, TN (2000).
16. B. Kranz, R. Pike and E. Hirst, Integrated Electricity Markets in New York, *The Electricity Journal* (2003), **16**, 54-65.
17. K. W. Cheung, P. Shamsollahi, D. Sun, J. Milligan and M. Potishnak, Energy and Ancillary Service Dispatch for the Interim ISO New England Electricity Market, *IEEE Transactions on Power Systems* (2000), **15**, 968-974.
18. NYISO, NYISO Ancillary Services Manual, New York Independent System Operator, Schenectady, NY (1999).
19. PJM, Market-Based Regulation Business Rules, Pennsylvania-New Jersey-Maryland Interconnection (2000).
20. PJM, Spinning Reserves Market Business Rules, Pennsylvania-New Jersey-Maryland Interconnection (2002).
21. ISO-NE, Automatic Generation Control Market, New England Power Pool (2000).
22. L. Brien, Why the Ancillary Services Markets in California Don't Work and What to do About it, National Economic Research Associates, San Francisco, CA (1999).
23. H.-P. Chao and R. Wilson, Multi-dimensional Procurement Auctions for Power Reserves: Robust Incentive-Compatible Scoring and Settlement Rules, *Journal of Regulatory Economics* (2002), **22**, 161-183.
24. R. Wilson, Priority Pricing of Ancillary Services in Wholesale Electricity Markets (1998).
25. F. Wolak, R. Nordhaus and C. Shapiro, An Analysis of the June 2000 Price Spikes in the California ISO's Energy and Ancillary Services Markets (2000).
26. J. Bushnell, A mixed complimentary model of hydrothermal electricity competition in the western united states, *Operation research* (2003), **51**, 80-93.
27. D. Perekhodtsev, Two Essays on Problems of Deregulated Electricity Markets, in "Tepper School of Business", Carnegie Mellon University, Pittsburgh (2004).

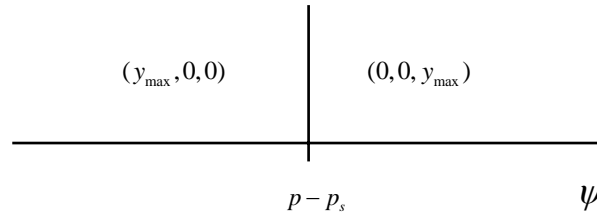
Appendix

In hours when $p_s > p_r$ the economic dispatch (6) does not maximize the profit of a hydro unit given by (5). In fact, in such cases the economic dispatch simplifies to:

$$(y(t), r(t), s(t)) = \begin{cases} (y_{\max}, 0, 0), & p(t) - y_{t_1, t_2} \geq p_s(t) \\ (0, 0, y_{\max}), & p(t) - y_{t_1, t_2} < p_s(t) \end{cases} \quad (13)$$

In such hours providing spinning reserves is always more profitable than providing regulation (Figure 9).

Figure 9. Economic dispatch of hydro units in hours when spinning reserve price exceeds regulation price



To calculate the water shadow price accounting for the possibility of such hours the economic dispatch must be generalized over (6) and (13), and the equations for the water shadow price (8) and (9) need to be modified accordingly.

The generalized dispatch is given by:

$$(y(t), r(t), s(t)) = \begin{cases} (y_{\max}, 0, 0), & p(t) - y_{t_1, t_2} \geq \max(p_r(t), p_s(t)) \\ (y_{\max} - r_{\max}, r_{\max}, 0), & p_s(t) \leq p(t) - y_{t_1, t_2} < \max(p_r(t), p_s(t)) \\ (r_{\max}, r_{\max}, y_{\max} - 2r_{\max}), & 2p_s(t) - \max(p_r(t), p_s(t)) \leq p(t) - y_{t_1, t_2} < p_s(t) \\ (0, 0, y_{\max}), & p(t) - y_{t_1, t_2} < 2p_s(t) - \max(p_r(t), p_s(t)) \end{cases} \quad (14)$$

The equation (9) will modify accordingly:

$$G(y^*) = y_{\max} (1 - F_{p - \beta}(y^*)) + (y_{\max} - r_{\max}) (F_{p - \beta}(y^*) - F_{p - p_s}(y^*)) + r_{\max} (F_{p - p_s}(y^*) - F_{p - 2p_s + \beta}(y^*)), \quad (15)$$

where $\beta = \max(p_r, p_s)$.