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**Transmission Pricing of Distributed Multilateral Energy  
Transactions to Ensure System Security and Guide to  
Economic Dispatch**

by

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# Transmission Pricing of Distributed Multilateral Energy Transactions to Ensure System Security and Guide Economic Dispatch

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**Abstract**—In this paper we provide a simulations-based demonstration of a hybrid electricity market that combines the distributed competitive advantages of decentralized markets with the system security guarantees of centralized markets. In this market, the transmission service provider (TSP) guides an electricity market towards the optimal power flow (OPF) solution, even when maximizing its own revenue. End users negotiate with each other to determine an energy price and then submit separate bids for transmission to the TSP. The TSP returns with prices for transmission, allowing end users to respond. In simulations, this hybrid-decentralized market approaches the near-optimal results of fully coordinated and constrained markets. Additionally, this market exhibits properties that remove incentives for the TSP to withhold capacity. This hybrid market leads a market towards the optimum while allowing the TSP and the end users to act out of self-interest.

**Index Terms**—Electricity markets, transmission, optimum power flow.

## I. INTRODUCTION AND BACKGROUND

Electricity market structures can be designed along two extremes: entirely centralized or entirely distributed. Distributed markets require no central authority, removing a possible source of market power. However, in order to observe line constraints, decentralized markets often use post-trading alterations such as curtailment or adjustment bids, which usually result in suboptimal dispatch and create opportunity for the exercise of market power[1]. Centralized markets achieve OPF at the expense of the end users' autonomy. A central authority finds the optimum quantities for all end users given their bids and then imposes a transmission charge. Locational Marginal Cost Pricing (LMP) is one market structure based on this methodology [2]. Since transmission charges are imposed ex-post, risk adverse agents must use Transmission Congestion Contracts (TCCs) in order to insulate themselves from risk associated with locational price differentials [3].

We extend the hybrid market structure proposed in [4] to combine the distributed trading of decentralized markets with the system security and OPF assurances of centralized markets. In this model end users, generators and loads, first trade with each other for energy only. They then communicate the amount of power and the point of intended power injection to the TSP requesting its delivery. The TSP, in turn, communicates a transmission price for such delivery. In [5] a basic demo showing equivalence between maximizing benefits of the end users, and minimizing social welfare cost was presented. While in [4], a two-level iterative method was proposed in which the end users communicate power and a TSP communicates transmission

price, in this paper the bids by the end users are specified as transmission demand functions (not points). The end users then communicate transmission demand bids to the TSP without revealing their price for energy. This paper offers conceptually new idea of a transmission demand function separable from the energy bid function, making it possible for a TSP to become an active decision maker. These transmission demand bids express the end users' willingness to pay a fee, what we call a Transmission Service Charge (TSC), for injecting or withdrawing energy at a particular network location. The TSP then sets TSCs in order to maximize its revenue, subject to its own technical constraints. We distinguish the transmission-only TSCs from the LMPs, which reflect a bundled energy price and transmission charge. End users then respond to the TSP's charges by implementing the agreement that maximizes their profit. Since end users have the option to decline a TSC, they no longer need TCCs to insulate themselves from unexpectedly high transmission costs.

This paper proposes a basic mathematical framework for the hybrid market and illustrates a simulation of its feasibility. In section II-A, we describe a basic outline of our method and its underlying assumptions. Section II-B describes one iteration of the trading process. Section III-A graphically demonstrates the general properties of the trading process on an example network topology with five out of the ten lines constrained. In section III-C, we describe the overall trends that we observed after simulating our method on 102 different network topologies. In section IV, we conclude and outline considerations for future work.

## II. NODAL TRANSMISSION PRICING MODEL

### A. Method Outline

This section provides an overview of the separate trading for energy and transmission in the hybrid method. First, end users negotiate  $k$ -lateral agreements for energy, anticipating a TSC for their net injection. The end users then derive transmission demand bids and submit them to the TSP. The TSP then sets TSCs for each transmission demand bid. Finally, end users decide on an agreement to implement. A mathematical derivation appears in the appendix.

1) *k-lateral Agreements*: End users negotiate a  $k$ -lateral agreement without the assistance of a central coordinator. We model end users with net supply and demand functions, where positive quantities indicate generation and negative quantities indicate consumption. For one agreement,  $k$  end users detail

their bids to one another in order to formulate an agreement. The price of electricity in a  $k$ -lateral agreement is the same for all end users. Each agreement specifies quantities as a function of the TSC. Additionally, the sum of all quantities in each agreement is zero, i.e. the quantity generated is equal to the quantity consumed.

The intuition behind using  $k$ -lateral agreements to achieve OPF falls directly from nodal prices. Nodal prices facilitate the optimum quantities for all end users, but optimal nodal prices must be calculated centrally. By iterating through a series of  $k$ -lateral agreements, we seek to avoid this centralized calculation while achieving the same final energy allocation as a centralized OPF calculation.<sup>1</sup>

2) *Transmission Service Charges*: The TSP uses TSCs as a pricing signal to lead the market towards OPF without the centralized calculation required in LMP. In LMP, a transmission charge  $t_{ij}$  is defined for a bilateral flow from node  $i$  to node  $j$ . In our method, each agent  $i$  in a  $k$ -lateral agreement pays a transmission service charge  $t_i$  per quantity traded, where  $t_i$  may be positive or negative. The charge in our method is still locational, but it is defined for a net injection and not for a flow. The benefit of TSCs is that a  $k$ -lateral transaction need not be decomposed into a set of bilateral transactions in order to price transmission. TSCs are charged to net injections at each node, and can therefore be applied in a straightforward way to  $k$ -lateral agreements.<sup>2</sup>

3) *Transmission Demand Bids*: In our method, the TSP sets TSCs in order to maximize its own profit. In order to implement a strategy of profit maximization subject to flow constraints, the TSP needs to know the nodal quantities injected by each end user as a function of the set of TSCs  $\bar{t}$ . We introduce the notion of a *transmission demand bid* of each agent in a  $k$ -lateral agreement, the net injection or withdrawal of each agent as a function of  $\bar{t}$ . Given the transmission demand bids of each end user in a  $k$ -lateral agreement, the TSP can calculate the set of TSCs that will maximize its revenue subject to transmission constraints.

End users in a  $k$ -lateral agreement create their transmission demand bids by including a tax in their marginal cost or marginal utility equations. The transmission demands can then be submitted to the TSP, which uses them to calculate the set of transmission service charges that will maximize its revenue subject to flow constraints.<sup>3</sup>

4) *Optimizing Nodal (Transmission) Service Charges*: The TSP calculates transmission service charges in order to maximize profit, subject to transmission constraints. This method

works well in networks with many constrained lines. However, the maximizing revenue objective function increasingly fails to find a solution as the number of constrained lines falls to zero. Intuitively, zero constrained lines results in no TSP revenue since nodal transmission service charges are zero<sup>4</sup>.

Therefore, the TSP needs a different objective function for networks with fewer constrained lines. An alternative objective function, which works well empirically, minimizes the absolute value of the transmission service charges imposed. This function finds acceptable results because as the number of constrained lines goes down, the optimal nodal prices become increasingly uniform, thus requiring smaller locational price variation in order to achieve OPF. The two different objective functions have different characteristics. Maximizing the TSP revenue guarantees that the total social welfare of all of the participants increases, and that the TSP revenue is positive<sup>5</sup>. Minimizing the absolute value of the transmission service charges does not have the same properties; it can result in lower total social welfare, and may even result in negative TSP revenue. On average however, the min-absolute-value objective function facilitated the achievement of OPF in many of the simulated networks that were relatively unconstrained.

5) *Distributed Decision-Making by the End-Users*: After the TSP imposes TSCs on a proposed  $k$ -lateral agreement, the end users are not obligated to implement the deal. Agents are given the freedom to shop around for deals that are most profitable to them. Agents communicate to find a desirable agreement given the TSCs that the TSP imposes. Agents then ask the TSP to implement their most desirable agreement. The TSP is then obligated to implement this deal honoring the TSCs previously communicated to the agents. All other proposed bids and their corresponding service charges are discarded. If an agent still wants to produce or consume energy, it must resubmit a bid in the next round. In this simulation, we implement an evaluator to quickly find the most profitable deal. We propose that other decentralized mechanisms (i.e. voting) can also determine the most commonly desirable deal.

## B. An Example Iteration

In this section, we outline one iteration of the trading process. Initially, end users negotiate amongst themselves and agree on a bid for transmission capacity. Two possible combinations of three end users (1,2,3 and 1,2,4), shown below, have agreed on a bid for transmission capacity. (In our simulation, all possible combinations of end users create bids for transmission.)

<sup>1</sup>Optimal nodal prices are the set of locational marginal costs/utilities at OPF.

<sup>2</sup>By defining end users with net generation and load functions, this model does not deal with issues of assigning TSCs when a load and generation combo from a single node generates counterflow.

<sup>3</sup>An important caveat of this method is that the characterization of each agent's transmission demand bid cannot be done unilaterally; transmission demand bids as defined in our method require  $k$ -lateral coordination, as they require the solving of  $k$  linear equations for  $k$  unknowns, the agents' injections. It is valid to argue that this is a type of centralized coordination, especially as  $k$  approaches  $n$ , the number of agents in the network. However, in our simulations, we have set  $k$  to be 2 or 3, and this suffices to allow the trading process to converge to OPF. We consider coordination between 2 or 3 agents to be relatively decentralized, especially if  $n$  is large.

<sup>4</sup>We have not been able to determine a specific cut-off point, in terms of the number of constrained lines, below which the max-revenue objective function fails. However, whenever the quadratic optimization program that implements the max-revenue objective function increases beyond a predetermined number of iterations, we consider it to have failed. We use this cutoff point so that the time taken for simulation is reasonable.

<sup>5</sup>Mathematically, the max-revenue objective function ensures that the increase in TSP revenue between iteration  $i$  and iteration  $i + 1$  is exactly equal to the merchandising surplus of the network calculated at the nodal prices present at iteration  $i + 1$ , using the difference between the nodal quantities at iterations  $i$  and  $i + 1$ .

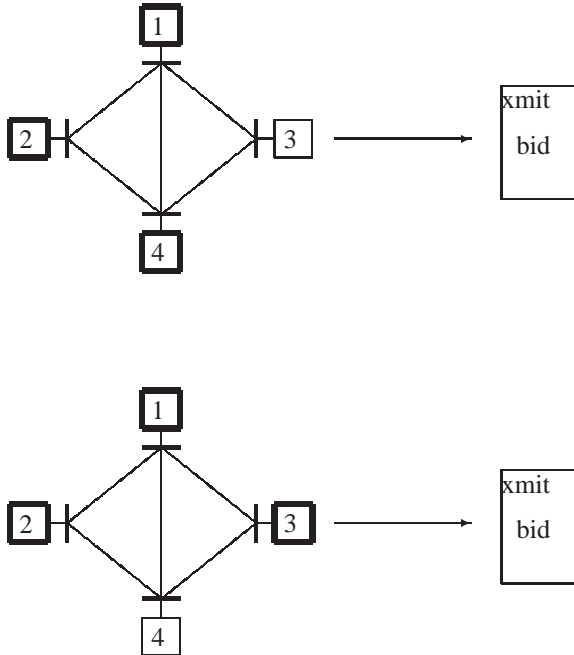
### III. RESULTS

#### A. Graphical Summary of Iterative Trading Process

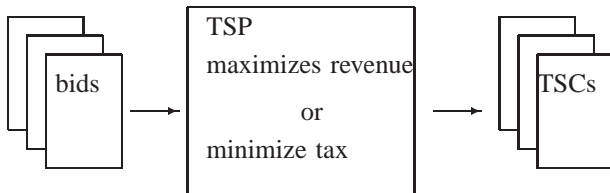
While the previous sections described the process by which the quantities traded and TSCs are set at a particular iteration, the actual trading process occurs over many iterations. This iterative trading process can be viewed as an alternative to using the spot market for arranging short term contracts. For example, the TSP can fix a time period during which agents can submit bids for transmission for the day-ahead power market. During this time, multiple iterations of the trading process described in II-B may occur, allowing each agent to buy or sell as much power as they require by seeking out their own trading partners.

The following figures describe the basic characteristics of the trading process in one particular network in which five out of the ten lines are constrained. In this network, the total social welfare converges to its maximal value, which is the value at optimal power flow, after 20 iterations. The revenue collected by the TSP at each iteration decreases as the trading process proceeds (figure 1).

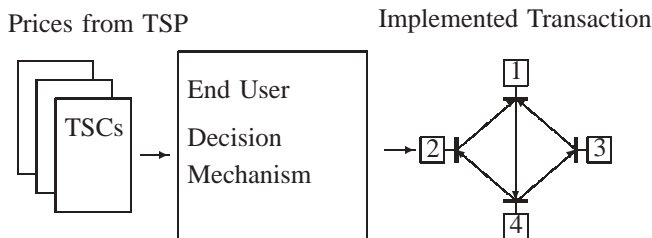
Figure 2 plots the flows and quantities traded at each iteration. Positive quantities indicate generation and negative quantities indicate consumption. The quantities traded in the transactions that define each iteration decrease as the trading process proceeds, since less profit can be made as the system approaches optimal social welfare. The flows also converge towards their values at optimal power flow. In this example, lines 1,2,3,5, and 9 have been constrained to one MW, whereas the other lines have a very large capacity. The figure shows that after 15 iterations, all of the constrained lines are at their full capacity.



The TSP considers each bid for transmission and assigns a transmission tax  $t_i$  to each end user for each bid. The TSP first tries to maximize its own revenues. However, if maximizing revenues results in a technically infeasible solution, the TSP will minimize the transmission tax in order to meet capacity constraints. Such a situation could arise, for example, if a TSP must subsidize an end user to induce sufficient counterflow.



Each set of end users that submitted a bid receive the TSP's proposed TSCs. One set of end users selects a deal to implement (the set that responds first to the TSP's reply).



The TSP is obligated to implement the deal selected by the end users. It updates the network state and then notifies each end user. Upon notification, end users may submit new bids or resubmit their old bids. A new set of TSCs is then calculated (based on the new network state) and communicated to the end users. This process iterates until all end users are satisfied.

#### B. Numerical Results

Out of 102 combinations of constrained lines in a 7 bus, 10 line network, 85 converge to within ten percent of the optimal power flow within 50 trading iterations, and 95 converge to within twenty percent within 50 iterations.

#### C. TSP Revenue and Transmission Capacity

While the revenue-maximizing TSP does appear to guide the market towards OPF, a market structure with a single grid operator may create the negative incentives for the TSP to relieve congestion. Under LMP, for example, the merchandising surplus, which is the profit collected by the TSP, increases as the number of congested lines increases. Therefore, additional mechanisms have to be created in order to prevent the grid operator from profiting by withholding transmission capacity [6]. Figure 3 compares the merchandising surplus collected by the TSP under LMP to the revenue collected by the TSP in the hybrid method, as a function of the number of constrained lines. In the hybrid method, the TSP revenue increases as the number of constrained lines decreases. In other words, TSP has no incentive to withhold transmission capacity.

We find that the increase in TSP revenue for networks with fewer constrained lines is associated with an increase in TSP revenue variation caused by using the min-absolute-value TSP

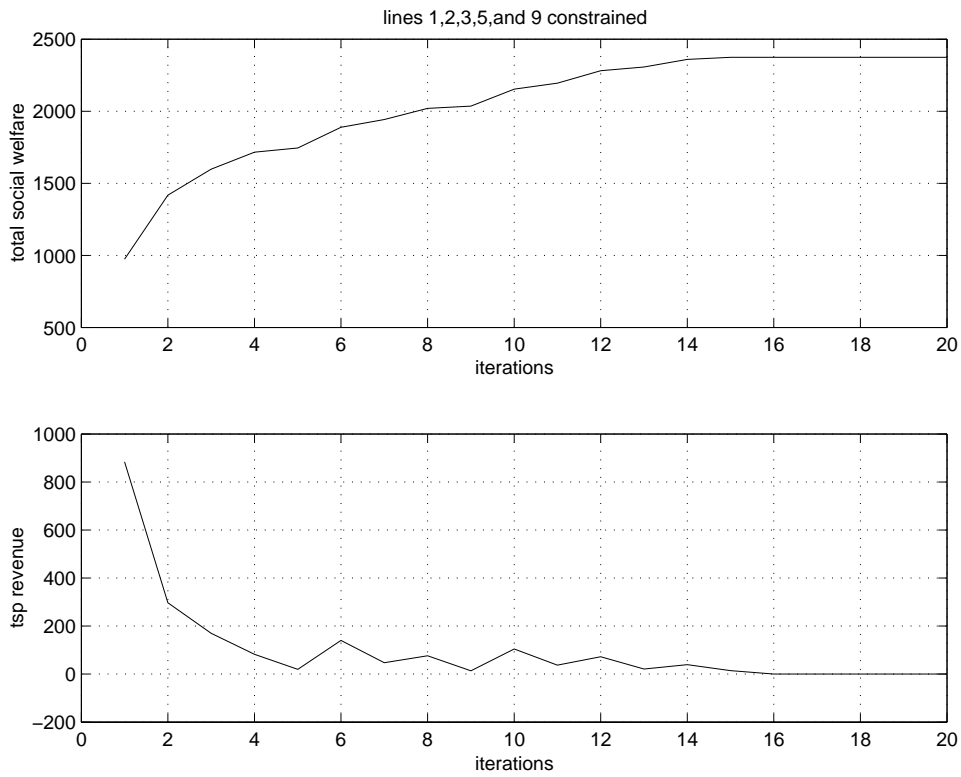


Fig. 1. Total Social Welfare and TSP revenue

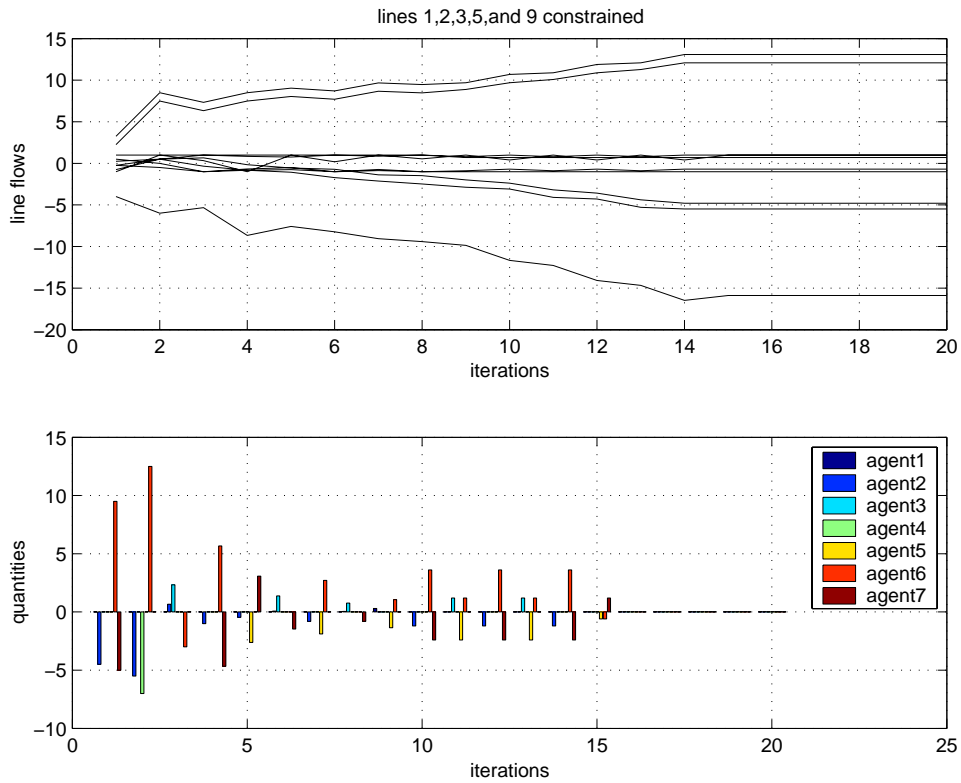


Fig. 2. Flows and Quantities

objective function in order to determine nodal transmission service charges. (As explained in II-A, the max-revenue objective function increasingly fails on such networks, so the

TSP is forced to switch to the min-absolute-value objective function) This objective function causes a greater variation in the TSP revenue, but in general increases the magnitude

of the TSP revenue. We have replicated the same behavior on smaller networks with 4 buses and 5 lines. Therefore, we find that TSCs that facilitate feasible flows on networks with fewer congested lines generally increases the TSP revenue as compared to networks with more congested lines.

Whereas we feel as though our market structure removes the incentive for the TSP to take transmission capacity offline unnecessarily, we do not claim that our market structure obviates the need for regulation of the TSP. Regulation is still necessary under our proposed market; however instead of investigating transmission outages, the regulatory body has the relatively less complicated task of monitoring the TSP for overcharges. A regulator still needs to oversee TSP activity as long as it remains a regulated monopoly.

#### IV. CONCLUSIONS

We have presented a method for nodal transmission pricing of decentralized  $k$ -lateral agreements as a means of facilitating the optimal power flow in a network. A qualitative difference between the method described here and the OPF-based approach comes from the fact that both end users and a transmission service provider are given flexibility to meet their own objectives and coordinate through interactions only. This ability is important for transmission service providers who have not been active decision makers in the power industry under restructuring. Through simulations we have shown that our method converges to OPF under a variety of conditions. In addition, the revenue of the TSP increases as the transmission capacity increases, creating positive incentives for the TSP to relieve some congestion. We view this hybrid method as a preliminary design for a new type of electricity market structure that uses transmission pricing signals and distributed decision making in order to facilitate optimal network operation. We have demonstrated that our method leads to the equivalent optimum to the OPF-based method. However, the method provides for wide range of distributed decision making by both the end users and the TSP during the bidding process. This is particularly important under various market uncertainties. While the end users eventually internalize the value of transmission service while bidding their transmission demand function, they are given flexibilities in managing uncertainties about the energy price and overall market conditions in ways the OPF solution does not allow for. However, we recognize many open questions and the need for further development of the proposed transmission pricing concept in this paper.

Several tasks remain to be investigated. We need to characterize the network conditions under which the max-revenue objective function fails, and understand the behavior of the min-absolute-value objective function. Another issue concerns grid expansion. We know that our method facilitates grid expansion through capacity increases, since the TSP profits from increasing line capacity. However, we do not address the question of detrimental grid expansion that may result in increased congestion rents [6]. The subject of transmission expansion is left for future research. Of particular interest is the question concerning the degree of market power exerted by

the large end users in our method relative to the market power in a bundled centralized market, assuming all other conditions the same. This critical question is likely to determine how truly separable transmission service is from the energy service and it is left for future research.

Beyond this simulation, one needs to generalize this method beyond strictly short-term operations concept. The method can be used over different time horizons by including the notion of a 'transmission supply curve' for the TSP. A TSP confronting requests over different time horizons may want to set aside some transmission capacity for future forecasted load. Instead of pricing agreements with fixed transmission service charges, a TSP can include some elasticity in its supply of transmission; for a greater charge (or lower subsidy), the TSP might be willing to support a larger quantity on the network.

#### V. APPENDIX: FRAMEWORK FOR TRANSMISSION PRICING OF AGREEMENTS BETWEEN $n$ AGENTS

We use the same setup as [3], in which there is one 'net' agent  $agent_i$  at each bus  $i$ , with cost function  $C_i(q_i)$ .  $C_i(q_i)$  is a strictly increasing, convex function. That is,  $\frac{dC_i}{dq_i} > 0, \forall q_i$ . Also,  $C_i(q_i) > 0$  for  $q_i > 0$  and  $C_i(q_i) < 0$  for  $q_i < 0$ . Positive quantities indicate generation and negative quantities indicate consumption. For positive  $q_i$ , the cost to  $agent_i$  is  $C(q_i)$ , and for negative  $q_i$ , the benefit to  $agent_i$  is  $-C(q_i)$ .

In an  $n$ -lateral agreement, each agent tries to maximize its profits. Defining  $Q_i$  as the amount currently being consumed/generated by  $agent_i$ , and  $\delta_i$  as the amount bought/sold in the transaction, an agent's profits can be calculated as follows.

$$\pi_i = p\delta_i + C_i(Q_i) - C_i(Q_i + \delta_i) - t_i\delta_i \quad (1)$$

where  $\pi_i$  and  $t_i$  are the profits and the transmission service charge paid by  $agent_i$ , and  $p$  is the energy price for the transaction. The same equation works for positive and negative  $\delta_i$ ; If  $\delta_i$  is positive, then the above equation states that the profits made by selling  $\delta_i$  is equal to the revenue from the transaction minus the cost increase. If  $\delta_i$  is negative, then the profits made by buying an additional  $\delta_i$  is equal to the increase in utility minus the price paid. In both cases, the transmission service charge is also factored into the equation.

Maximizing profit as a function of  $\delta_i$ , we get

$$\frac{d\pi_i}{d\delta_i} = 0 = p - t_i - \frac{dC_i}{d\delta_i} \quad (2)$$

There is one price for the  $n$ -lateral agreement, so we get

$$p = \frac{dC_i}{d\delta_i} + t_i = \frac{dC_j}{d\delta_j} + t_j : \quad i, j = 1 \dots n \quad (3)$$

We also have the additional balance constraint

$$\sum_{k=1}^n \delta_k = 0 \quad (4)$$

If we assume that the marginal costs are linear in the region of interest, then equations 3 and 4 define a system of  $n$  linear equations in  $n$  unknowns, the  $\delta_i$ 's. We can solve for

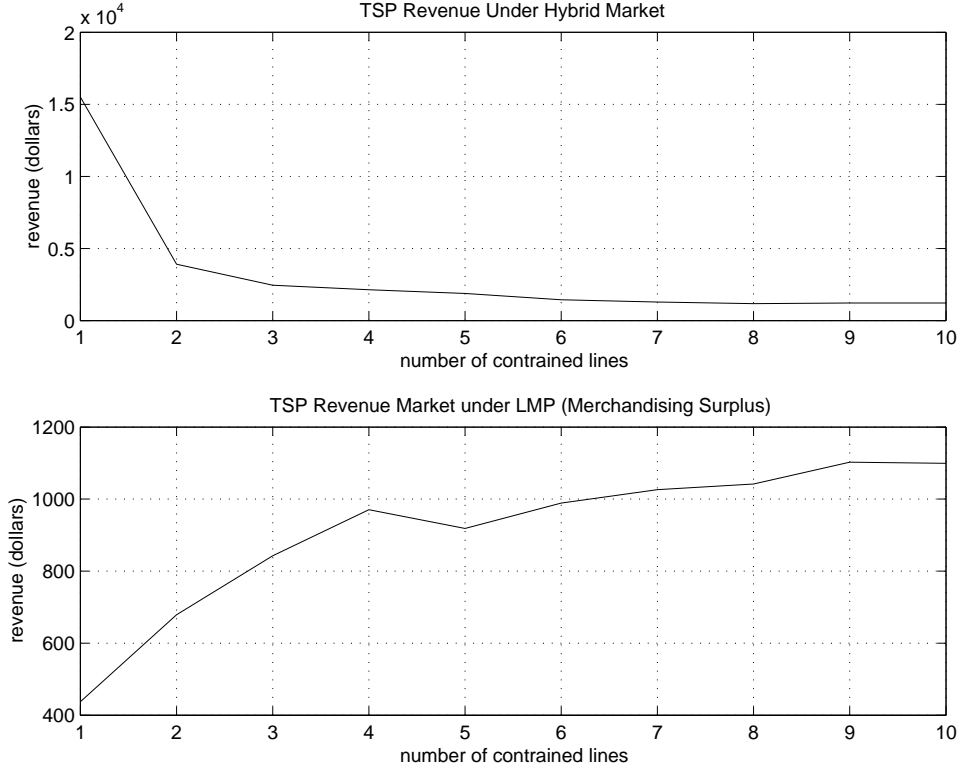


Fig. 3. TSP revenue and merchandising surplus as a function of the number of constrained lines

each  $\delta_i$  as a linear combination of the transmission service charges  $t_j$   $j = 1 \dots n$ .

Defining  $\bar{\delta} = (\delta_1 \delta_2 \dots \delta_n)$  and  $\bar{t} = (t_1 t_2 \dots t_n)$ , we have

$$\bar{\delta} = \tilde{A}\bar{t} \quad (5)$$

Equation 5 is the transmission demand relation. This is submitted to the TSP. The role of the TSP is to set the transmission service charges such that the agents and the TSP make a profit. We first examine how to calculate profit for the TSP. Using 1, we see that the TSP profit is a function of the pair  $(\delta_i, t_i)$ . Specifically,

- $(\delta_i > 0, t_i > 0)$  : TSP receives  $\delta_i t_i$
- $(\delta_i > 0, t_i < 0)$  : TSP pays  $\delta_i t_i$
- $(\delta_i < 0, t_i > 0)$  : TSP pays  $\delta_i t_i$
- $(\delta_i < 0, t_i < 0)$  : TSP receives  $\delta_i t_i$

The TSP profit can therefore be computed as a dot product.

$$\bar{\delta} \cdot \bar{t} = \sum_{k=1}^n \delta_k t_k \quad (6)$$

Using the objective of profit maximization results in a quadratic optimization problem for the TSP :

$$\text{Max } \bar{t} \cdot \bar{\delta} = \bar{t}' \tilde{A} \bar{t} \quad (7)$$

subject to

$$\left| \tilde{D} \tilde{A} \bar{t} \right| \leq \bar{c} \quad (8)$$

where transmission lines  $1 \dots m$  have capacities  $\bar{c} = (c_1 c_2 \dots c_m)$  and flows  $\bar{f} = (f_1 f_2 \dots f_m)$ , and given injections  $\bar{q} = (q_1 q_2 \dots q_n)$ , flows can be computed using the relationship

$$\bar{f} = \tilde{D} \bar{q} \quad (9)$$

where  $\tilde{D}$  is the matrix of distribution factors :

$$\tilde{D} = \begin{pmatrix} df_{11} & df_{12} & \dots & df_{1n} \\ df_{21} & df_{22} & \dots & df_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ df_{m1} & df_{m2} & \dots & df_{mn} \end{pmatrix}$$

In practice, when there are very few congested lines, the objective function defined by 7 fails to produce a feasible solution in a reasonable amount of time. In such cases, we use an objective function that minimizes the absolute value of the transmission service charges imposed .

$$\text{Min } \bar{t}' \tilde{I} \bar{t} \quad (10)$$

subject to 8, where  $\tilde{I}$  is the  $n * n$  identity matrix.

Simulation shows that maximizing profit for the TSP results in 'fair' profits for the agents, where 'fair' is measured in terms of total social welfare. Specifically, the previous formulation results in an optimal power flow when  $n$  is set to be the number of agents in the network.

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## VI. BIOGRAPHIES

**Marija Ilic** is a Professor at Carnegie Mellon University with a joint appointment in the Electrical Engineering and Engineering Public Policy Departments. She is currently on leave from MIT, where she is Senior Research Scientist in the Department of Electrical Engineering and Computer Science at MIT. At MIT she taught several graduate courses in the area of electric power systems and headed research in the same area. She has twenty years of experience in teaching and doing research in this area. Prior to coming to MIT in 1987, she was an Assistant Professor at Cornell University, and tenured Associate Professor at the University of Illinois at Urbana-Champaign. Her main interest is in the systems aspects of operations, planning and economics of electric power industry. She is an IEEE fellow.

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