Quark Masses:
An Environmental Impact Statement

by

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Abstract

We investigate how the requirement that organic chemistry be possible constrains the values of the quark masses. Specifically, we choose a slice through the parameter space of the Standard Model in which quark masses vary so that as many as three quarks play a role in the formation of nuclei, while keeping fixed the average mass of the two lightest baryons (in units of the electron mass) and the strength of the low-energy nuclear interaction. We classify universes on that slice as *congenial* if they contain stable nuclei with electric charge 1 and 6 (thus making organic chemistry possible in principle). Universes that lack one or both such stable nuclei are classified as *uncongenial.*

We reassess the relationship between baryon masses and quark masses, using information in baryon mass differences in our world and the pion-nucleon sigma term $\sigma_{\Pi N}$. We generalize the Weizsacker semi-empirical mass formula through a degenerate Fermi gas model that handles the kinetic energy of new baryonic species as they begin to participate in the nucleus, and derive an expression for the asymmetry energy equivalent in the $SU(3)$ limit through a minimization procedure on the quadratic Casimir operator. We spell out the conditions for decay by weak nucleon emission. Finally, we study the congeniality of various regions in the quark mass space, primarily by direct comparison to analog nuclei in our universe. Considering only two light quarks u and d, we find a band of congeniality roughly 29 MeV wide in $m_u - m_d$, with our universe living comfortably away from the edge. We find multiple congeniality regions in the three quark mass space. For an important region around the $SU(3)$ limit, we have not determined conclusive results but we have constructed the machinery to aid in its analysis and formulated the relevant problems. We have succeeded in formulating a well defined question about congeniality, and have made concrete progress toward answering it.

Thesis Supervisor: Robert L. Jaffe
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Acknowledgments

I would first like to thank my thesis supervisor, Prof. Robert Jaffe, for the time and effort he has patiently invested in training me as a physicist since I first walked into his office a year and a half ago. He taught me my first lessons both on what it means to do theoretical physics and on how to have fun while doing it. I can't imagine a senior thesis supervisor better than Bob; his wonderfully inspiring mentorship has largely defined the latter part of my undergraduate experience.

This research project was done as a collaboration among Prof. Robert Jaffe, Dr. Alejandro Jenkins and myself. I would like to heartily thank both Prof. Jaffe and Dr. Jenkins for making our collaboration exciting, for taking the time to teach me physics (even when it took a whole lot of time!), and for being generously helpful when I was wrong or confused.

I also wish to recognize here the other important people who led me to the completion of this thesis, though indirectly.

I've been lucky to know Prof. Barton Zwiebach both as my academic advisor and as a teacher. Always welcoming and radiant, he has been my first stop when I needed advice or wanted to share exciting news. His amazing teaching, famous throughout MIT, needs no further words here.

Towards the end of high school, I had the pleasure of spending two summers in Prof. V. Adrian Parsegian's Laboratory of Physical and Structural Biology, at the NIH. Prof. Parsegian's excitement was contagious, and it was he who first taught me that science can be both simple and wondrous.

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Finally, I wish to thank my parents, Chava and Avi. They both drove me to work hard at excelling in what I like, fully supporting me throughout. My mother first

introduced me to the world of scientific research through visits to her lab, and later in life enabled and encouraged me to pursue science. She has taught me how being a researcher connects with being a human being, and continues to provide me with critical advice in general, always magically managing to be right. My father did much to nurture my analytic curiosity, from reading classic popular physics texts together through continuing to ask me the simple but brilliant questions that challenge and inspire me, keeping my perspective grounded. To whatever extent I am a physicist, I owe that to him. Toda, Ima veAbba!

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[1]Note that the first two sections in this document were generated collaboratively, but written primarily by Prof. Jaffe and Dr. Jenkins.

1 Overview

1.1 Introduction

It is logically possible that certain physical quantities might be environmentally selected, rather than uniquely determined by fundamental principles. For instance, early scientists such as Johannes Kepler sought an explanation for the sizes of the orbits of the planets in terms of fundamental mathematical laws, but we now recognize that these are the product of complicated dynamics that involve significant historical contingencies. Many solar systems are possible (indeed, many are now known to exist) with very different configurations. Today we understand that we naturally happen to live in one of the few solar systems whose features allow intelligent observers to evolve. In other words, we believe that some features of our solar system, such as the size of the earth's orbit, have an environmental explanation.

From time to time modern scientists have suggested that the values of certain physical constants, and perhaps even the form of some of the laws of nature, might similarly be environmentally selected [6,9]. This idea, known loosely as the *anthropic principle*, has received support recently on both the theoretical and phenomenological fronts. Inflationary cosmology suggests that our universe may be one of a vast, perhaps infinite, number of causally disconnected universes, and studies of string compactification suggest that at least in that version of quantum gravity, the laws of nature and physical constants could be different in each of those universes. Then, the story goes, each universe would evolve according to its own laws, and only some would lead to the evolution of physicists who would try to make sense of what they observe. In those universes the laws of nature and physical constants may be explained not by some deep principle, but rather by the simple fact that they be consistent with the evolution of intelligent observers.

Whatever one may think of these theoretical arguments, Weinberg's *prediction* of the value of the cosmological constant on essentially these grounds has to be taken seriously [11]. At a time when the experimentally measured value of the cosmological constant was consistent with zero, Weinberg pointed out that values outside a very small range (on the scale of fundamental physics) were inconsistent with structure formation, and that general considerations of probability theory suggested a value close to the bound imposed by the structure we see around us. Soon after, a value of the cosmological constant was measured close to Weinberg's prediction [13]. Weinberg's environmental explanation is all the more persuasive in the absence of any attractive alternative derivation of Λ from a dynamical principle.

Anthropic reasoning raises some deep, even metaphysical, questions that will be difficult to answer: Is there a multiverse to begin with? Are string theory and eternal inflation the correct tools with which to study the space of possible universes? What parameters of the Standard Model are environmentally determined, and what is the *a priori* distribution over which environmentally determined parameters range? These questions would have to be addressed before one could claim that some parameter, like the up-down quark mass difference, is *determined.* However, there is another interesting, but much less daunting question to be asked about the parameters of the Standard Model: To what extent is a particular parameter environmentally *constrained*? Over what range of a parameter do the laws of physics allow for the existence of an observer? This is the question we attempt to formulate and, at least in part, answer in this paper. The answer to this question might be of little significance outside of an anthropic framework; nevertheless, the question is well formulated and can, at least in principle, be answered if we understand physics well enough.

In order to determine the environmental constraints on the interactions and physical constants in our universe, we must be able to extract the physical consequences of *varying the laws and parameters* that are familiar to us. If we do not understand the theoretical landscape well enough to declare certain regions environmentally barren of observers or others fruitful, it will not be possible to place environmental constraints on the various ingredients of the Standard Model. The common environmental explanation for the radius of the earth's orbit provides a good example of how this process might work. It is widely accepted that the orbital radius of any planet occupied by carbon-based life forms would have to lie in a relatively narrow "habitable zone" where temperatures are neither too hot nor too cold for biochemical processes.

That statement relies on our ability to extrapolate biochemistry from our world to others where an important parameter, in this case temperature, is different. It also assumes that we have correctly identified biochemistry as we know it as the essential feature that allows carbon based life to evolve. These questions are within the purview of science and their answers put environmental *constraints* on the orbital radius of a habitable planet. Whether our emergence on earth was environmentally *determined* depends on whether the universe contains many planetary systems and on the distributions of orbital radii, compositions, and so forth.

Any environmental analysis of the Standard Model requires three basic ingredients: first, a criterion for an acceptable universe; next, careful attention to what is being held fixed and what is varying; and finally, enough knowledge of the Standard Model to make predictions of the environmental consequences of varying its structure or parameters.

The aim of this paper is to see what it entails to perform an environmental analysis of some limited aspect of the Standard Model. We choose to study quark masses because the quark mass dependence of the strong interactions is non-trivial and yet is understood well enough to offer some hope that we can make definitive statements about the potential for evolution of observers.

We propose to vary the masses of quarks *holding as much as possible of the rest of Standard Model phenomenology constant.* In particular we leave the mass and charge of the lightest lepton, the electron, unchanged. (In pursuit of our objective we are forced to increase the masses of the muon and tauon relative to the QCD mass scale, but this has no effect on the questions of interest.) We also leave the weak interactions of quarks and leptons unchanged. It is *not* sufficient to simply hold all the other parameters of the Standard Model fixed and vary only the quark masses. If one insisted on varying quark masses keeping all the other parameters of the Standard Model fixed, for example, at the scale of weak symmetry breaking, then the resulting changes in low energy physics would be too extreme for us to analyze. In Section II we describe in detail what else must vary along with the quark masses in order to keep the resulting worlds within our capacity to analyze them.

The worlds we study will look a lot like our own. They will have some stable hadrons. Some will be charged. The charged stable hadrons, with either positive or negative charge, will capture electrons or positrons to form neutral atoms[2] with *chemistry that will be essentially identical to ours.* This leads us to adopt the existence of stable nuclei with charge one (some isotope of hydrogen) and charge six (some isotope of carbon) to be the criterion for an acceptable universe. Note that "carbon" might have very different *nuclear physics* in worlds with different quark masses, but the *chemistry* of the element with charge six would be the nearly the same as the chemistry of carbon in our world. In worlds where the d and s quarks are the lightest, the stable baryons are all negatively charged, and "carbon" has baryon number six. In worlds where u, d, and s quarks are all very light compared to the mass scale of nuclear physics, nuclei have a strong tendency to be electrically neutral, and "carbon", if it exists at all, will have very large baryon number.

For simplicity we call a universe *congenial* if it seems to support the evolution of an observer. An *uncongenial* universe is the opposite. Our criterion for congeniality is, as already stated, the existence of stable nuclei with $Z = 1$ and $Z = 6$. We realize that these may not be necessary if some exotic form of life were possible. Likewise they may not be sufficient if there were some other obstruction to the development of carbon based life. The former is interesting but hard to investigate. The latter is very important and requires further, often difficult work. For example, Hogan has argued that the existence of a stable, neutral baryon would short circuit big-bang nucleosynthesis and/or leave behind deadly clouds of neutral baryons [14]. Some of the worlds we study have neutral baryons but may have different early histories than Hogan considers (for example, in some of our worlds deuterons are not stable but ^{3}H is, a possibility not considered by Hogan). This highlights a distinction between two types of criteria for congeniality. The first, which we adopt, is that the *laws of nature* should be suitable for the evolution of life (as we know it). The second, which we put aside for later work, is that the history of a universe evolving according to

[2] If the stable hadrons have negative charge, neutrinos could offset the negative lepton number of the positrons.

those laws should lead to intelligent life. Clearly the latter is a much more difficult problem, involving aspects of astrophysics, celestial dynamics, planetary physics, *etc.*, each of which has its own complex rules. Returning to the planetary analogy, it is akin to showing that planets in the habitable zone would have any number of other attributes, like atmospheres, water, plate tectonics, *etc.*, that might be necessary for life. Such considerations are clearly important — Weinberg's prediction of the cosmological constant is based on a study of the history of the universe, not the form of the Standard Model — but they are beyond the scope of this paper. We content ourselves with a variation on an old proverb: with $Z = 1$ and $Z = 6$ we expect that "life will find a way."

1.2 Variation of quark masses

Quark masses in the Standard Model, m_a, are determined by Yukawa couplings, g_a, by $m_a = g_a v$, where v is the Higgs vacuum expectation value (vev). In our world, the quark Yukawa couplings range from $\sim 10^{-5}$ for the u-quark up to ~ 1 for the t-quark. There is no known connection between quark masses and the scale of the strong interactions, Λ_{QCD}. We would like to consider the widest possible range of variation of quark masses, limited only by our ability to study their consequences with the tools of the Standard Model. Quarks with masses above Λ_{QCD} interact very differently from the lighter quarks in our universe, so we choose to ignore them henceforth. Quarks with masses below Λ_{QCD} we call light quarks. If Yukawa couplings are varied widely, the number of light quarks can range from zero to six, and their charges can be $+2/3$ or $-1/3$, constrained only by the fact that there are at most three of either charge.

In our world with light u and d quarks and a somewhat heavier s quark, the methods of $SU(3)_{\text{flavor}}$ perturbation theory have been moderately successful in baryon spectroscopy. Using $SU(3)_{\text{flavor}}$ perturbation theory we can extract enough information from our world to predict the masses of the lightest baryons in worlds with *up to three light quarks*, provided their mass differences are small enough to justify the use of first order perturbation theory. In practice the use of first order perturbation theory is not an important limitation because we find that a quark species does not participate in nuclear physics unless its mass is considerably less than the mass of the s-quark in our world. So first order perturbation theory in quark masses should be even better in the other worlds we explore than it is in our world. In fact, the principal limitation of perturbation theory in quark masses is the possible importance of higher order corrections to the extraction of the reduced matrix elements of important operators from the baryon masses in our world. The limitation to at most three species of light quarks is forced on us by the fact that we cannot extract $SU(4)_{\text{flavor}}$ reduced matrix elements from light baryon masses in our world.

To summarize, we consider worlds where baryons made of up to three species of light quarks participate in nuclear physics. The sum of the three light quark masses is limited to be less than their sum in our world ($m_T \equiv m_u + m_d + m_s \approx 100$ MeV [15]). (We circumvent ambiguities associated with the extraction and renormalization scale dependence of quark masses by presenting results as functions of the ratio of quark masses to m_T in our world.)

1.3 Our slice through the Standard Model

If one's object is to vary the light quark masses, it would, at first sight, seem that the natural choice would be to vary them while keeping all other parameters of the Standard Model fixed. This, however, turns out to be a difficult slice of parameter space to consider, because if $\alpha_s(M_Z^2)$ is held fixed then the effective value of Λ_{QCD} changes as quark masses change. Worse still, the masses of light baryons seem to be unexpectedly sensitive to sum of the quark masses, m_T. A well known argument correlates the m_T dependence of the nucleon mass with the value of the σ-term extracted from low energy pion-nucleon scattering ($\sigma_{\pi N}$). The rather large value of $\sigma_{\pi N}$ suggests that the baryon masses decrease by hundreds of MeV as m_T decreases to zero. Changing the mass of the baryons relative to Λ_{QCD} would make for unpredictable changes in nuclear forces. This makes environmental analysis quite difficult in these worlds.

In order to proceed, we will use other parameters of the Standard Model to restore the resemblance between these worlds and our own (while still changing the values of light quark masses). In particular, we adjust Λ_{QCD} *and the electron Yukawa coupling* y_e to keep the average mass of the two lightest octet baryons and some typical measure of strength of the low-energy nuclear interaction (to be discussed in Sec. 2.3) fixed as quark masses are varied. An example will illustrate the issue: imagine that the s-quark mass were reduced so that $m_s \approx 5m_d$ (as opposed to $m_s/m_d \approx 17 - 22$ as it is in our world [15]). The mass of the proton and neutron might decrease by several hundred MeV, while the mass of the pion and other features of the nuclear force would be little changed. Hyperons remain too heavy to participate in nuclear physics. We then choose to adjust $\alpha_s(M_Z)$ such that some measure of the strength of nuclear binding (to be discussed further in Sec. 2.3) decreases from its value in our world by the same factor as the mass of the nucleons was decreased. Now all the relevant mass scales of QCD have decreased by the same factor, so the result is *equivalent to increasing the mass of the electron* (and the other leptons) by the same factor. To make this world isomorphic to ours, we scale the electron Yukawa coupling so the ratio m_e/M_N returns to the value it takes in our world. With a trivial redefinition of the MeV we can regard this world as one where the mass of the nucleon, the mass of the electron, and the features of nuclear interactions have been held fixed as the mass of the strange quark has been decreased by a factor of ~ 5. One important virtue of this approach is that it makes our analysis independent of the still controversial σ-term influence on baryon masses.

The reader might object that by following this procedure we are no longer holding the other parameters of the Standard Model fixed as we vary the quark masses. However, there is nothing sacred about keeping the other parameters of the Standard Model fixed as the light quark masses are varied. That is one slice among many through the parameter space of the Standard Model, but it is one on which we do not know how to calculate. So we choose instead a nearby slice on which we do. In the absence of any information about the landscape of possible universes, there is no *a priori* reason to believe that it is any more (or less) interesting to vary the quark masses with Λ_{QCD} and the electron's Yukawa coupling fixed than to vary quark masses with those parameters varying. Both scenarios should be explored. We choose our approach because *the resulting universes can be studied with some certainty.* So doing, we accomplish two things: first, we show that it is possible to analyze the environmental impact of non-trivial variations in Standard Model parameters, and second, we find some of the limits of the domain of congeniality in which we live, and also discover some different (disconnected) domains of congeniality elsewhere in light quark mass parameter space. As our knowledge of QCD improves, in particular as lattice QCD matures, it may become possible to explore much larger domains of parameter space by simulating hadrons and nuclei on the lattice.

2 The Rules of our Game

When approaching such a speculative subject as the possibility that the masses of the light quarks might be environmentally constrained, we must be clear about the assumptions that we have made in formulating a well-posed problem. For our analysis to have any value, we must state clearly what the rules are for the anthropic game we will be playing, and we need to justify why that choice of rules is reasonable.

2.1 *A priori* distribution of quark masses

Our first step is to clarify the relation of our work to more general anthropic arguments. Although, as described in the Introduction, we focus on the more pragmatic problem of anthropic *constraint* rather than anthropic *selection*, the usefulness of our constraints is to some extent influenced by metaphysical questions like ones prejudiced by the *a priori* distribution of quark masses.

Anthropic reasoning can be presented in the language of contingent probabilities [27]. According to Bayes's theorem, the probability distribution for measuring some set of values for the quark masses can be expressed as

$$P(\{m_i\}|\text{observer}) = \frac{P(\text{observer}|\{m_i\}) \times P(\{m_i\})}{P(\text{observer})}\ , \tag{2.1}$$

where $P(\{m_i\})$ and $P(\text{observer})$ are the *a priori* distributions of quark masses and observers over the landscape. We presume that $P(\{m_i\})$ is determined by some fundamental theory such as string theory in an eternally-inflating scenario. The quantity $P(\{m_i\}|\text{observer})$ is the contingent probability distribution for the quark masses over the subset of the landscape that contains intelligent beings likes us, capable of measuring quark masses. In simple terms, it is the probability that we observe some set of quark masses. The quantity $P(\text{observer}|\{m_i\})$, the contingent probability for an observer existing, given a choice of quark masses, is the primary subject of this paper.

Sometimes, for example in Weinberg's analysis of the cosmological constant, one can argue that the *a priori* probability for the parameter of interest — in our case $P(\{m_i\})$ — does not vary significantly over the range where $P(\text{observer}|\{m_i\})$ is non-zero. If so, then the probability that we observe some set of quark masses, which is the object of fundamental interest in an anthropic context, is proportional to $P(\text{observer}|\{m_i\})$, which we analyze here.

We believe it is reasonable to assume that the *logarithms* of the quark masses are smoothly distributed over a range of masses small compared to the Planck scale. First, we note that an environmental analysis is predicated on absence of any other underlying explanation for quark masses. Therefore we assume that the probability of a pattern of masses is the product of independent probabilities,

$$P(\{m_i\}) = \prod_{i=1}^{6} P(m_i)\,. \tag{2.2}$$

Evidence that $P(m_i)$ varies smoothly in the $\ln m_i$ comes from the observed distribution of quark masses in our world, which is shown in Fig. 1. As far as we can tell, the values of "heavy" quark masses, m_c, m_b, and m_t, are not anthropically constrained. These quarks are so massive (compared to Λ_{QCD}) and so short lived that they seem to play no role in nuclear or atomic physics. Therefore, referring to eq. (2.1), it is reasonable to take $P(\text{observer}|\{m_i\})$ to be essentially independent of m_c, m_b, m_t. If we make this assumption, then the pattern of heavy quark masses that we see is a

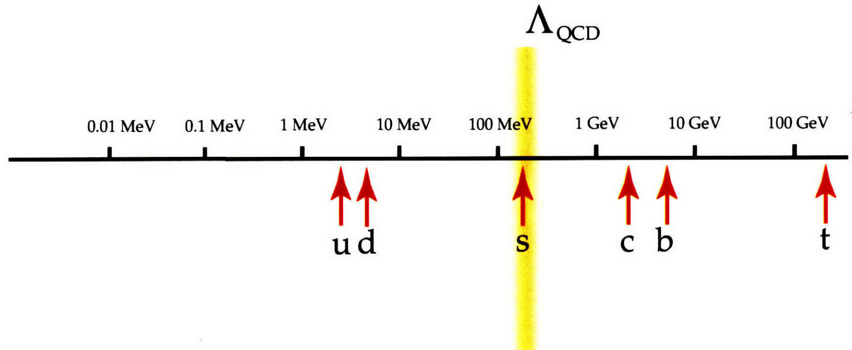


Figure 1: The quark masses and Λ_{QCD}, shown on a logarithmic scale.

direct measure of the *a priori* probability distribution $P(m_i)$ for $i = c, b, t$. Although we have only three data points, the measured values of heavy quark masses range over more than two orders of magnitude, resembling an *a priori* distribution that is smooth in the *logarithm* of the quark mass. It is not unreasonable to assume that *all* quark masses follow the same *a priori* probability distribution, *i.e.* they vary smoothly with the logarithm of the mass.

The *a priori* distribution of quark masses is determined by the landscape, which entails Planck scale physics. So the scale for variation in the quark mass probability distribution should be M_{Planck}. Since the range of quark masses that allows for life seems to be fairly narrow (as we will quantify below) in units of M_{Planck}, it is reasonable to expand $P(m_i)$ over the range of interest as a power series in m_i/M_{Planck}. Only the zeroeth order term in that expansion is then relevant in Eq. (2.1) in which case $P(m_i)$ is simply proportional to the logarithm of m_i,

$$\Delta P(\{m_i\}) = \prod_{i=1}^{6} \Delta P(m_i) \propto \prod_{i=1}^{6} \Delta(\ln m_i) \propto \prod_{i=1}^{6} \Delta m_i/m_i \,, \tag{2.3}$$

for quark masses in the range of interest, which will be $m_i \lesssim \Lambda_{\text{QCD}}$. This assumption would fail if the quark masses are related by some as yet unknown symmetry or dynamical principle. It would also fail if they are somehow correlated with the number of e-folds undergone by our pre-inflationary patch during inflation and if the *a priori* probability were weighed by the corresponding inflationary volume factor. In that case there might be an enormous preference for values of the quark masses that lie on the edge of what is anthropically allowed [16]. The fact that this does not appear to be the case (i.e., that it does not appear to be the case that life would be impossible if the quark masses were varied very slightly in some direction, as we well quantify later on in this paper) argues against such an inflationary "probability pressure". For a proposed construction of a landscape of Yukawa couplings, see [17].

Note that an *a priori* logarithmic distribution between zero and an upper limit of Λ_{QCD} focuses much more attention on small quark masses than a linear distribution would. Therefore it is very interesting to explore the congeniality of worlds where *all light quark masses are very small compared to* Λ_{QCD}. We have able to make some progress in formulating the tools needed to explore this region of parameter space, and we can make some qualitative statements about its congeniality. Nevertheless we have not been able to explore this region as fully as we would like. Finally, we emphasize again that this excursion into a discussion of *a priori* probabilities is independent of the analysis of environmental constraints on quark masses that follows.

2.2 Parameterizing the masses of light quarks

As mentioned in Subsection 1.3, we will consider the variation of three light quark masses over a slice through the space of parameters of the Standard Model in which the average mass of the two lightest octet baryons is fixed with respect to the mass

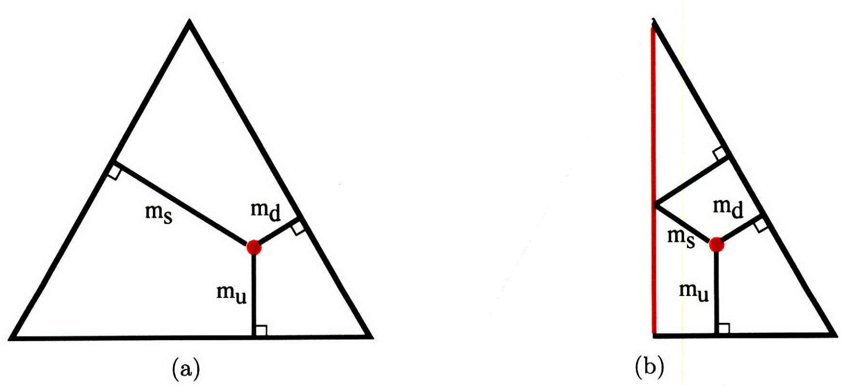


Figure 2: (a). Graphical representation of the landscape of light quark masses for fixed $m_T = m_u + m_d + m_s$; (b). Reduced landscape, assuming $m_s > m_d$.

of the electron. We shall also hold the strength of the low-energy strong nuclear interaction fixed, as far as possible.

We shall primarily deal with the charge assignments $(2/3, -1/3, -1/3)$ because the other possibility, $(2/3, 2/3, -1/3)$, has more limited domains of congeniality, and they can be characterized by analogy with domains in the more familiar case. We denote the quarks (m_u, m_d, m_s) in the former case and (m_u, m_c, m_d) in the latter case. We describe our parameterization of the space of light quark masses for the *uds* case; the *ucd* case is a trivial extension.

For a fixed value of $m_T = m_u + m_d + m_s$, we may represent the landscape of light quark masses by the points in the interior of an equilateral triangle with sides of m_T, as shown in Fig. 2(a). The value of each quark mass is given by the perpendicular distance from the point to the corresponding side of the triangle. So far we have not needed to make any assumptions about the structure of the CKM matrix. It suffices to assume that no elements vanish identically, so that all quarks decay to the lightest over time scales of order 10^{-8} seconds. For our purposes this removes any distinction between the worlds described by points in the right and left halves (related by $d \leftrightarrow s$) of the triangle of Fig. 2(a), so we will *define* the s quark to be the more massive of the charge -1/3 quarks, and sometimes restrict our attention to the reduced landscape shown in Fig. 2(b). The triangular landscapes relevant for quark charges $(2/3, 2/3, -1/3)$ can be obtained by the replacements: $m_s \rightarrow m_c, m_d \rightarrow m_u, m_u \rightarrow m_d$ with the convention that $m_c > m_u$.

The full space of light quarks is, of course, three-dimensional, and may be represented as a triangular prism, as in Fig. 3. The sides of any triangular slice are m_T, the sum of the three light quark masses. The variation of baryon masses and nuclear forces over the prism is influenced by the adjustments of $\Lambda_{\rm QCD}$ and y_e. Those adjustments are described in the following subsection, and the resulting variation of baryon masses is discussed in Section 5.6.

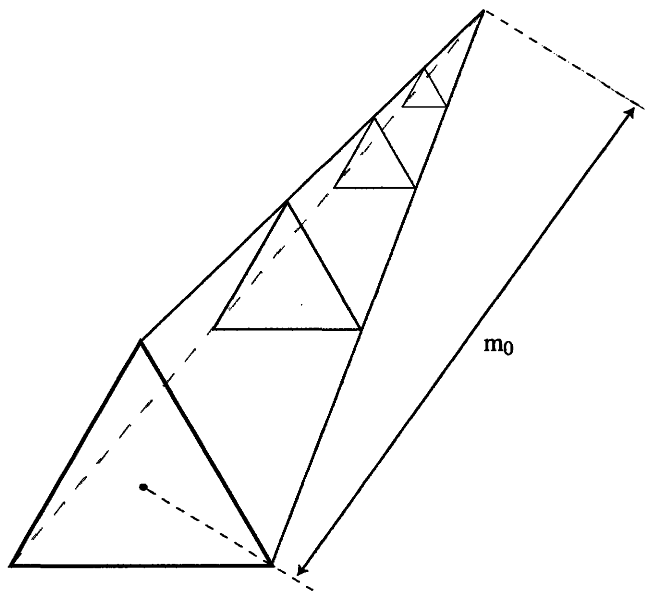


Figure 3: Representation of the full three-dimensional space of light quark masses as a triangular prism, with four different slices of constant $m_T = m_u + m_d + m_s \equiv \sqrt{6} m_0$ shown.

2.3 Strength of the nuclear interaction

The strength of the nucleon-nucleon interaction will change if the light quark masses are changed while keeping other Standard Model parameters fixed. In particular, the masses of pseudoscalar mesons vary dramatically for small quark masses, as dictated by the Gell-Mann-Oakes-Renner relation [18],

$$m_\pi = \frac{\sqrt{m_q \, |\langle \bar{q}q \rangle|}}{f_\pi} \,, \tag{2.4}$$

where f_π is the pion decay constant and $\langle \bar{q}q \rangle$ is the chiral-symmetry breaking vev of QCD, both of which are believed to be nearly independent of light quark masses.

If nuclear binding were dominated by single-pion exchange, eq. (2.4) would imply a very strong dependence of the nuclear interaction on the masses of the light quarks. In particular, increasing the masses of the up and the down quarks in our universe would be expected to significantly decrease and rapidly turn off nuclear binding. It is believed, however, that the attractive forces responsible for nuclear binding receive significant contributions from correlated two pion exchange in the region of 400 –

600 MeV invariant mass, an effect that is usually associated with the $f_0(600)$ (also known as the σ) meson [29–31]. The mass of the σ is not singular in the chiral limit, so there is reason to believe that nuclear binding depends less dramatically on light quark masses than the one pion exchange mechanism would lead one to believe.

The exact dependence of nuclear binding on quark masses is, of course, unknown, and has been the subject of considerable recent work [32–36]. To pursue our analysis we must avoid such uncertainties. As explained in the Introduction, as we vary quark masses, we use Λ_{QCD} to keep the strength of nuclear binding as close as possible to its value in our world. This approach has its limitations. If the light quark masses were all reduced to zero, then all QCD phenomena would controlled by a single parameter, Λ_{QCD} (or equivalently α_s at M_Z). We could change the y_e and the definition of the MeV to keep the masses of the lightest baryons and the electron at 940 and 0.511 MeV respectively, but there would be *no further freedom to readjust the strength of nuclear forces to mimic our world.*

However, when at least one of the light quark masses is non-zero, the range of the nuclear interaction will scale differently from the baryon masses as we vary Λ_{QCD}. We will exploit this different scaling in order to define a slice through the parameter space of the Standard Model in which $\alpha_s(M_Z)$ is tuned to keep the strength of the nuclear interaction fixed, while also tuning the Yukawa coupling of the electron so as to keep fixed the ratio of the electron mass to the average of the masses of the two lightest baryons. By then choosing units in which the mass of the electron is the same as in our world, we obtain a subspace of the landscape in which nuclear physics is the same as in our world, except for the masses and quantum numbers of the participating baryons. To the extent to which this procedure works, this is the subspace which we will explore in the rest of this paper.

Our approach will break down when changes in quark masses change the nature of nuclear forces qualitatively. For example, if one quark mass were very small and the other two roughly equal — a very interesting case — then the lightest pseudoscalar mesons would form an $SU(2)_{\text{flavor}}$ doublet, not a triplet like the pions. We could choose to adjust Λ_{QCD} to restore some single measure of nuclear binding, like the

Fermi momentum, to its value in our world. But there would be no guarantee that other important aspects of nuclear physics would also be restored, and so nuclear binding - specifically, the binding energy function of nuclei made of Z protons and N neutrons, $B(Z, N)$ - may not be close to our world's.

2.4 Baryon masses and nuclear stability

The primary issue concerning us is finding the points in the landscape of quark masses, as we have defined it, that correspond to worlds in which charged nuclei would have lifetimes comparable to the age of our own universe. As discussed in the Introduction we set aside "historical" questions like whether those elements would be efficiently produced either in primordial nucleosynthesis or in subsequent stellar processes, or whether astrophysics would be likely to produce pockets of the universe in which those elements would be likely to remain undisturbed long enough for intelligent life to evolve. All such considerations could be cleanly superimposed upon this work later.

Our analysis proceeds in three steps. Having selected values of the light quark masses, we first determine the masses of the lightest baryons. Next, we establish which of them are light enough to participate in nuclear physics. Finally we study the stability of nuclei composed of these baryons.

Only members of the familiar $SU(3)_{\text{flavor}}$ octet of spin-1/2 baryons (except for a brief consideration of the spin-3/2 decuplet when only one quark is light) can play a role in nuclear physics. Most of the information necessary to relate octet baryon masses to quark masses can be obtained from a study of octet baryon mass differences in our world using first order perturbation theory in quark masses. First order perturbation theory is justified by the fact that the mass differences among quarks that can participate in nuclear physics is always small. Our world is a good example: Although $SU(3)_{\text{flavor}}$ symmetry breaking is significant, the strange quark does not participate in nuclear physics, so $SU(2)_{\text{flavor}}$ symmetry is sufficient to explore the dependence of the masses of protons and neutrons on the masses of the u and d quarks. Baryon mass differences cannot tell us the dependence of baryon masses on the *average quark mass*. We take this information from the σ-term in πN scattering. Finally, to get as accurate an estimate as possible of the connection between baryon masses and quark masses, we include estimates of the electromagnetic contributions to baryon masses. This program is described in the following section of the paper.

Except in the case where all three light quark masses are nearly equal, only the two lightest baryons participate in nuclear dynamics. In our world they are the proton and neutron. In a world where the s and d quarks are lightest, for example, the Σ^- and Ξ^- are the players. When all three quark masses are similar, then all eight members of the baryon octet may become engaged in nuclear dynamics. As already mentioned, this is the region where we have had the least success in exploring the landscape.

Finally, given the masses of the participating baryons, we need to determine the spectrum of nuclear states and establish which ones are stable against strong and weak decays. For large baryon number we develop an effective Hamiltonian based on the well-known Weizsäcker semi-empirical mass formula. Weizsäcker's formula was designed for our world, with two nearly degenerate nucleons with charges zero and one. We must generalize the formula to allow for larger mass differences between the participating baryons, to allow for different baryon charges, and in the case of roughly equal m_u, m_d, and m_s, we must generalize from $SU(2)_{\text{flavor}}$ to $SU(3)_{\text{flavor}}$. The last of these is a particularly interesting exercise.

The Weizsäcker formula becomes unreliable for very light nuclei — remember we are particularly interested in isotopes of hydrogen and carbon. For such light elements, it is usually possible to estimate nuclear masses from *analog nuclei* in our world. Take a nucleus composed of two baryon species in a world corresponding to some point in the landscape of light quark masses. It is analogous to one in our world if the two have the same nuclear structure (N_1 of the first baryon species and N_2 of the second) and the same binding energy $B(N_1, N_2)$, up to a Coulomb energy correction arising from different assignments of electric charge to its component baryon species. For example, in the world we have already mentioned where the s and d quarks have the same masses that the u and d quarks have in our world, the nucleus composed of one Σ^- and one Ξ^- (which is chemically the element *helium* in this world) should have the same energy as the deuteron in our world, except for the correction due to the Coulomb repulsion of its two charged constituents.

3 Octet baryon masses in $SU(3)$ perturbation theory

3.1 The flavor $SU(3)$ symmetry

The congeniality criterion of nuclear stability is determined in part by the values of baryon masses, whose distribution is not close to uniform but instead is set by the underlying distribution of the light quark masses, through the functional dependence of the baryon mass spectrum on quark masses. In this section we derive and discuss the effect of light quark masses on the baryon spectrum. The mass of a baryon is determined by Hamiltonians for the interactions of the quarks composing it

$$H = H_0^{\text{color}} + H^{\text{flavor}} + H^{\text{electromagnetism}}. \tag{3.1}$$

Baryons composed of the three light up, down and strange quarks are classified into irreducible representations (irreps) of the $SU(3)_{\text{flavor}}$ algebra, families of roughly similar masses, within the decomposition of the $SU(3)_{\text{flavor}}$ product of three quarks $3 \otimes 3 \otimes 3$. The lightest baryons belong to a representation known as the octet.

Mass differences among octet baryons are well approximated by first order $SU(3)$ breaking of non-zero quark masses. Electromagnetic mass corrections break $SU(3)$ together with quark masses, so a straightforward $SU(3)$ perturbation theory analysis would lump H^{em} together with H^{flavor}. However, electromagnetic mass corrections are experimentally known through the Cottingham formula; we use the values given in table 2, column 1 of [24]. With the small H^{em} corrections out of the way, carrying $SU(3)$ breaking to first order is equivalent to taking the part of H^{flavor} linear in the quark flavor operators $\Theta_u \equiv \bar{u}u$, $\Theta_d \equiv \bar{d}d$, $\Theta_s \equiv \bar{s}s$, or equivalently linear in the transformed basis of the three $SU(3)_{\text{flavor}}$ tensor operators

$$\Theta_0^1 = \sqrt{\frac{2}{3}}\,(\Theta_u + \Theta_d + \Theta_s)\,, \quad \Theta_3^8 = \Theta_u - \Theta_d, \quad \Theta_8^8 = \sqrt{\frac{1}{3}}\,(\Theta_u + \Theta_d - 2\Theta_s)\ . \quad (3.2)$$

These are normalized to obey the condition $\mathrm{tr}\,(\Theta_i \Theta_j) = 2\delta_{ij}$ (so that the octet operators are the two diagonal Gell-Mann matrices, generators of $SU(3)$). In this notation the superscript is the dimension of the irrep that the operator belongs to, and the subscript is the standard label within the irrep (with 0 as a reminder that the singlet is a constant).

Thus to first order in $SU(3)$ we can write H in the spherical basis,

$$H = H_0^{\text{color}} + m_0\Theta_0^1 + m_3\Theta_3^8 + m_8\Theta_8^8 + H^{\text{em}}\ , \quad (3.3)$$

where the first two terms transform like a singlet within the octet (in other words, they're the same for all eight octet baryons). The m constants are quark mass sums and differences:

$$m_0 = \frac{1}{\sqrt{6}}(m_u + m_d + m_s), \quad m_3 = \frac{1}{2}(m_u - m_d), \quad m_8 = \frac{1}{2\sqrt{3}}(m_u + m_d - 2m_s)\ . \quad (3.4)$$

These form a transformed set of orthogonal axes in the m_u, m_d, m_s space of quark masses we investigate. [3]

The eight octet baryons are labeled by their total isospin I, third component of isospin I_3, and hypercharge Y. We use the combined label within the octet μ to stand for these three labels, so $\mu \in \{1...8\}$.

[3] Note that because of the normalization condition for the spherical operators, these axes are scaled by $1/\sqrt{2}$ relative to the m_u, m_d, m_s directions, so that an interval of 1 on m_u, m_d, m_s is an interval of $1/\sqrt{2}$ on m_0, m_3, m_8.

3.2 Master formula for octet baryon masses

The mass of an octet baryon μ is $\langle 8;\mu|\, H \,|8;\mu\rangle$. Using the Wigner-Eckart Theorem we evaluate the third and fourth terms in H in (3.3) that transform in the octet:

$$\langle 8;\mu|\, \Theta^8_\alpha \,|8;\mu\rangle = \sum_{\gamma=1}^{2} \begin{pmatrix} 8 & 8 & 8_{(\gamma)} \\ \mu & \alpha & \mu \end{pmatrix} \langle 8||\Theta^8||8\rangle_\gamma \ . \tag{3.5}$$

The double bar term is a symbol standing for the Wigner reduced invariant matrix element (WRME) which depends only on the representation. The symbol in large parenthesis is an SU(3) Clebsch-Gordan coefficient, following J.J. de Swart's notation [1]. Since H includes both Θ^3_8 and Θ^8_8 we apply this formula with $\alpha = 3$ and $\alpha = 8$, but the WRME is the same for both. The sum over two values of γ is introduced because there are two 8-dimensional representations in the irreducible decomposition of $8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8_1 \oplus 8_2 \oplus 1$. [4]

The $SU(3)$ Clebsch-Gordan coefficient can be written as the product of an $SU(2)$ Clebsch-Gordan coefficient and an isoscalar coefficient:

$$\begin{pmatrix} 8 & 8 & 8_{(\gamma)} \\ \mu & \alpha & \mu \end{pmatrix} = C^{I_\mu I_\alpha I_\mu}_{I_{\mu 3} I_{\alpha 3} I_{\mu 3}} \begin{pmatrix} 8 & 8 & 8_{(\gamma)} \\ I_\mu Y_\mu & I_\alpha Y_\alpha & I_\mu Y_\mu \end{pmatrix} \ . \tag{3.6}$$

C is the $SU(2)$ coefficient connecting isospin I states through magnetic quantum number I_3, and the last term is the isoscalar coefficient connecting $SU(3)$ states through the magnetic quantum numbers of total isospin I and hypercharge Y. Viewed as objects in the adjoint representation of $SU(3)$, the operators Θ^3_8 and Θ^8_8 are the two central elements in the octet, with $I_{3\alpha} = 0$ and $Y_\alpha = 0$ (the Hamiltonian conserves I_3 and Y). For $\alpha = 3$, $I_\alpha = 1$, while for $\alpha = 8$, $I_\alpha = 0$.

Using the tables of isoscalar coefficients in [1] and standard tables of $SU(2)$ Clebsch-Gordan coefficients, we compute the baryon masses as formulas with five unknown parameters: the overall constant term $A_0 \equiv H^{\text{color}}_0 + m_0 \langle \Theta^1_0 \rangle$, the two WRMEs $\langle 1 \rangle \equiv \langle 8||\Theta^8||8\rangle_1$ and $\langle 2 \rangle \equiv \langle 8||\Theta^8||8\rangle_2$, m_3 and m_8. For example, the

[4] See the appendix for more information.

formula for the mass of the proton is (where the $SU(2)$ C-G coefficients are outside the square brackets)

$$M(p) = A_0 + 1 \cdot \left[-\frac{\sqrt{5}}{10} \langle 1 \rangle \, m_8 + \frac{1}{2} \langle 2 \rangle \, m_8 \right] + \frac{1}{\sqrt{3}} \cdot \left[\frac{3\sqrt{5}}{10} \langle 1 \rangle \, m_3 + \frac{1}{2} \langle 2 \rangle \, m_3 \right] \ . \quad (3.7)$$

To simplify the formulas, following the notation of Coleman and Glashow in [2], we define two new parameters proportional to the WRMEs,

$$F \equiv \frac{\sqrt{3}}{6} \langle 8 || \Theta^8 || 8 \rangle_2 \text{ and } D \equiv \frac{\sqrt{15}}{10} \langle 8 || \Theta^8 || 8 \rangle_1 \ , \quad (3.8)$$

so that, for example, the formula for the mass of the proton becomes

$$M(p) = A_0 + \left(\frac{3F - D}{\sqrt{3}} \right) m_8 + (F + D) m_3 \ . \quad (3.9)$$

3.3 Λ^0, Σ^0 Mixing

However, this procedure yields a Hamiltonian which is not quite diagonal as an 8×8 matrix in the $\mu = 1 ... 8$ basis. It has off-diagonal mixing elements for the two central particles Λ^0, Σ^0:

$$\begin{pmatrix} \Sigma^0 \\ \Lambda^0 \end{pmatrix} \text{ basis: } \quad H = A_0 I + \frac{2D}{\sqrt{3}} \begin{pmatrix} m_8 & m_3 \\ m_3 & -m_8 \end{pmatrix} \ . \quad (3.10)$$

To diagonalize this Hamiltonian, we need to use a different basis. The particles in the octet center which are objects of definite mass (eigenstates of the Hamiltonian) are, with $\Delta \equiv \sqrt{m_8^2 + m_3^2}$:

$$C_{high} = \frac{1}{\sqrt{2\Delta}} \begin{pmatrix} \sqrt{\Delta + m_8} \\ \sqrt{\Delta - m_8} \end{pmatrix} \quad \text{and } C_{low} = \frac{1}{\sqrt{2\Delta}} \begin{pmatrix} -\sqrt{\Delta - m_8} \\ \sqrt{\Delta + m_8} \end{pmatrix} \ . \quad (3.11)$$

This mixing between $\mu = 3$ and $\mu = 8$ does not matter in our universe, where $m_3/m_8 = .02$, so $\Delta = 1.0002 m_8 \cong m_8$, $C_{low} \cong \Lambda^0$, and $C_{high} \cong \Sigma^0$. It will matter for configurations of quark masses that give m_3 and m_8 of the same order of magnitude; in these cases isospin no longer approximately commutes with the Hamiltonian, and we will not be able to assign the particles C_{high} or C_{low} definite total isospin $I = 1$ or $I = 0$. Except for the small electromagnetic mass corrections, in the (C_{high}, C_{low}) basis the Hamiltonian is diagonal,

$$H = \begin{pmatrix} A_0 + \frac{2D\Delta}{\sqrt{3}} & \\ & A_0 - \frac{2D\Delta}{\sqrt{3}} \end{pmatrix} . \tag{3.12}$$

The values of H^{em} for (C_{high} and C_{low}) may be different than they are for the Σ^0 and Λ^0, but as these are neutral particles the corrections would be negligible.

For reference, we present the baryon mass formulas resulting from the linear expansion in quark masses and diagonalization procedures described above. The values for H^{em} are given in MeV.

$$M(p) = A_0 + \left(\frac{3F - D}{\sqrt{3}}\right) m_8 + (F + D) m_3 + 0.63 \tag{3.13}$$

$$M(n) = A_0 + \left(\frac{3F - D}{\sqrt{3}}\right) m_8 - (F + D) m_3 - 0.13 \tag{3.14}$$

$$M(\Xi^0) = A_0 - \left(\frac{3F + D}{\sqrt{3}}\right) m_8 + (F - D) m_3 - 0.07 \tag{3.15}$$

$$M(\Xi^-) = A_0 - \left(\frac{3F + D}{\sqrt{3}}\right) m_8 - (F - D) m_3 + 0.79 \tag{3.16}$$

$$M(\Sigma^+) = A_0 + \left(\frac{2D}{\sqrt{3}}\right) m_8 + (2F) m_3 + 0.7 \tag{3.17}$$

$$M(\Sigma^-) = A_0 + \left(\frac{2D}{\sqrt{3}}\right) m_8 - (2F) m_3 + 0.87 \tag{3.18}$$

$$M(C_{high}) = A_0 + \left(\frac{2D}{\sqrt{3}}\right) \sqrt{m_8^2 + m_3^2} - 0.21 \tag{3.19}$$

$$M(C_{low}) = A_0 - \left(\frac{2D}{\sqrt{3}}\right) \sqrt{m_8^2 + m_3^2} \tag{3.20}$$

In our universe C_{low} is known as the Λ^0 and C_{high} is known as the Σ^0. The average mass is $A_0 = H_0^{color} + m_0 \langle \Theta_0 \rangle$. The F matrix element roughly counts the simple canonical quark composition of each baryon, while the D matrix element describes more complicated effects and interactions. The presence of two octets in the $8 \otimes 8$ decomposition is critical for the complex spectrum of the octet baryons.

3.4 Extracting numerical parameter values

In order to extract the values of the parameters in the baryon mass formulas, we fit the experimentally well known octet baryon masses, with the electromagnetic mass contribution removed during the fit procedure. In our universe $\sqrt{m_8^2 + m_3^2} = 1.0002\ m_8 \cong m_8$, so the quark masses m_3 and m_8 always appear in combination with the WRMEs F and D. The four appear in the formulas as three independent parameters: $F \cdot m_8$, $D \cdot m_8$ and m_3/m_8, with $A_0 = H_0^{color} + m_0 \langle \Theta_0 \rangle$ as an additional parameter.

A_0 is simply the average baryon mass. Note that averages across an isospin line for a given I depend only on m_8 and not on the much smaller m_3, allowing for a more robust fit. We fit $F \cdot m_8$ and $D \cdot m_8$ to the four such m_8 combinations. For quark mass ratios in our universe we use the PDG values, obtained through lowest order chiral perturbation theory: $m_u/m_d = 0.56 \pm 0.15$ and $m_s/m_d = 20.1 \pm 2.5$. Fitting m_3/m_8 to the four remaining differences across isospin yields $m_3/m_8 = 0.0181$ MeV, quite close to the PDG's much more sophisticatedly determined result of $m_3/m_8 = 0.0197 \pm 0.0071$ MeV. We used the PDG value in our calculations. Note that we do not quote precision figures on our fitted parameters because in this work we are not interested in precise numerical results; in general, our results are trustworthy to within a few MeV.

Parameter	Fitted value
A_0	1150.8
$F \cdot m_8$	-109.4
$D \cdot m_8$	35.7

Table 1: $SU(3)$ perturbation theory parameters. All numbers are in MeV.

Baryon Mass	Experimental	Fitted	Residuals
p	938.27	939.87	1.60
n	939.57	942.02	2.45
Ξ^0	1314.83	1316.81	1.98
Ξ^-	1321.31	1323.38	2.07
Σ^+	1189.37	1188.42	-0.96
Σ^-	1197.45	1197.21	-0.24
Σ^0	1192.64	1191.82	-0.82
Λ^0	1115.68	1109.61	-6.07

Table 2: $SU(3)$ perturbation theory fit to octet baryon masses. All numbers are in MeV.

3.5 Determining $m_0 \langle\Theta_0\rangle$ from the pion-nucleon σ term

The pion-nucleon sigma term $\sigma_{\Pi N}$ is a measure of chiral symmetry breaking due to non-zero light quark masses. It can be written as

$$\sigma_{\Pi N} = \frac{m_u + m_d}{2} \langle N| \Theta_u + \Theta_d |N\rangle \ , \tag{3.21}$$

where $N = \frac{n+p}{2}$ [19]. Note that $(\frac{m_u+m_d}{2})(\Theta_u + \Theta_d)$ differs from $m_u\Theta_u + m_d\Theta_d$ only by a term second order in isospin violation, $\frac{1}{2}(m_u - m_d)(\Theta_u - \Theta_d)$, which disappears when taking the expectation value with respect to $N = \frac{n+p}{2}$.

Switching coordinates to the spherical operators

$$\sigma_{\Pi N} = \frac{(\sqrt{2}m_0 + m_8)}{\sqrt{3}} \langle N| \frac{(\sqrt{2}\Theta_0 + \Theta_8)}{\sqrt{3}} |N\rangle \ . \tag{3.22}$$

From the baryon mass formulas presented earlier we read off $\langle N| \Theta_8 |N\rangle$ as the coefficient of the m_8 term in the nucleons, $\frac{3F-D}{\sqrt{3}}$. Regrouping,

$$\sigma_{\Pi N} = (\frac{1}{\sqrt{3}} + \sqrt{\frac{2}{3}\frac{m_0}{m_8}}) \cdot \left(\sqrt{\frac{2}{3}\frac{m_8}{m_0}}[\langle N| \Theta_0 |N\rangle m_0] + \left(Fm_8 - \frac{Dm_8}{3}\right)\right) \ . \tag{3.23}$$

Given a value for $\sigma_{\Pi N}$ with experimental uncertainties, we can use the Fm_8 and Dm_8 fit values and m_0/m_8 derived from the PDG quark mass ratios to solve for the singlet flavor mass term $\langle N| \Theta_0 |N\rangle m_0$. This term separates the average mass of the octet baryons A_0 into a QCD component that does not scale with quark masses and this flavor singlet component that scales with the sum of quark masses m_0.

There was considerable discrepancy in the literature on experimental values of the $\sigma_{\Pi N}$ term. We found two reasonable estimates for the Cheng-Dashen point extrapolation of the term: the canonical value of $\sigma_{\pi N}(2\mu^2) = 64 \pm 8$ MeV given by R. Koch [22], and a more recent value of $\sigma_{\pi N}(2\mu^2) = 79 \pm 7$ MeV obtained by a GWU/TRIUMF group analysis [21]. Neither of these papers appeared to have an estimate of the physical sigma term $\sigma_{\pi N}(0)$, but a later analysis [20] that uses Koch's standard value quotes the sigma term as $\sigma \approx 45$ MeV with errors of ± 5 MeV for a related parameter. We have used $\sigma_{\Pi N} = 45 \pm 5$ MeV in our calculations, yielding

$$m_0 \langle \Theta_0 \rangle = 507 \pm 116 \text{MeV} . \tag{3.24}$$

Evidently about half of the average baryon mass comes from the constant term in the flavor Hamiltonian, and about half comes from the QCD Hamiltonian. We avoid relying on this controversial result by choosing to investigate a slice through the standard model that holds the average mass of the lightest pair of baryons constant at 938.92 MeV, thus keeping our results insensitive to the value of $m_0 \langle \Theta_0 \rangle$.

3.6 Coordinates in the space of light quark masses

We aim to explore the spectrum of universes with light quark masses given by points in the positive octant of m_u, m_d, m_s space. For a number of reasons, we choose to present our results through a different set of coordinates on this space:

1. The coordinate m_0 only comes into play in determining the average mass of baryons; instead of m_u, m_d, m_s it is simpler to use the transformed orthogonal coordinates m_0, m_3, m_8. For a given value of m_0, the mass space is an equilateral triangle, with the m_3 axis parallel to one of the sides and the m_8 axis perpendicular to it, as in Fig.4. As we'll discuss later, except for small scaling effects, by choosing to hold the average mass of the lightest pair of baryons fixed we remove all dependence on m_0 and reduce the dimensionality of the problem from the 3D prism to a its largest 2D triangular slice.

2. Since quark masses are not accurately known and are anyway scale dependent, the quark mass combinations in our formulas should appear as a ratio over some quark mass combination in our world. We choose $m_T \equiv m_u + m_s + m_d = \sqrt{6}\, m_0$ for this role, so all quark masses appear in units of the quantity "m_T in our universe", which we denote as $m_T^{\odot}$. [5] Note that with the currently accepted PDG quark masses, $m_T^{\odot}$ is about 100 MeV. To improve the reader's intuition in viewing our results we take $m_T^{\odot}/100$ as the unit of quark mass. We could write quark mass combinations in units of MeV, letting the MeV be understood as actually standing in for the unit $m_T^{\odot}/100$. To make these units explicit, we introduce the axes $x_0 \equiv m_0/m_T^{\odot}$ 100 MeV, $x_3 \equiv m_3/m_T^{\odot}$ 100 MeV, $x_8 \equiv m_8/m_T^{\odot}$ 100 MeV. Thus, to summarize,

$$x_0 = \frac{1}{\sqrt{6}} \frac{m_T}{m_T^{\odot}}\, 100 \text{ MeV} = \frac{(m_u + m_d + m_s)}{\sqrt{6}\; m_T^{\odot}}\, 100 \text{ MeV}, \tag{3.25}$$

$$x_3 = \frac{(m_u - m_d)}{2\; m_T^{\odot}}\, 100 \text{ MeV}, \quad x_8 = \frac{(m_u + m_d - 2m_s)}{2\sqrt{3}\; m_T^{\odot}}\, 100 \text{ MeV} \ . \tag{3.26}$$

The perturbation theory derivation of baryon mass formulas is valid roughly for $x_0 \leq 1.5\; x_0^{\odot} \approx 60$ MeV. A given value of x_0 defines a triangular slice on which the maximal range of the x_3, x_8 axes is given by

$$-\frac{\sqrt{6}}{2} x_0 \leq x_3 \leq \frac{\sqrt{6}}{2} x_0; \quad -\sqrt{2} x_0 \leq x_8 \leq \frac{\sqrt{2}}{2} x_0 \ . \tag{3.27}$$

For $x_0 = x_0^{\odot} = 40.82$ MeV, the point coordinates range from -50 MeV to 50 MeV on the x_3 axis and from -57.74 MeV to 28.87 MeV on the x_8 axis. Our universe is close to the m_u, $m_d = 0$ corner, at the point $x_0 = 100/\sqrt{6}$ MeV $= 40.82$ MeV, $x_8 = -51.50$ MeV and $x_3 = -1.016$ MeV. The term $m_0 \langle \Theta_0 \rangle$ does not contribute to baryon mass differences, so we separate it in the baryon mass formulas given below.

$$\overline{M_B}(x_0) \equiv (1150.82 \text{ MeV} - [m_0 \langle \Theta_0 \rangle]) + [m_0 \langle \Theta_0 \rangle] \frac{\sqrt{6}\; x_0}{100 \text{ MeV}} \ . \tag{3.28}$$

[5] In analogy with $M_{\odot}$ being the mass of the star in our solar system.

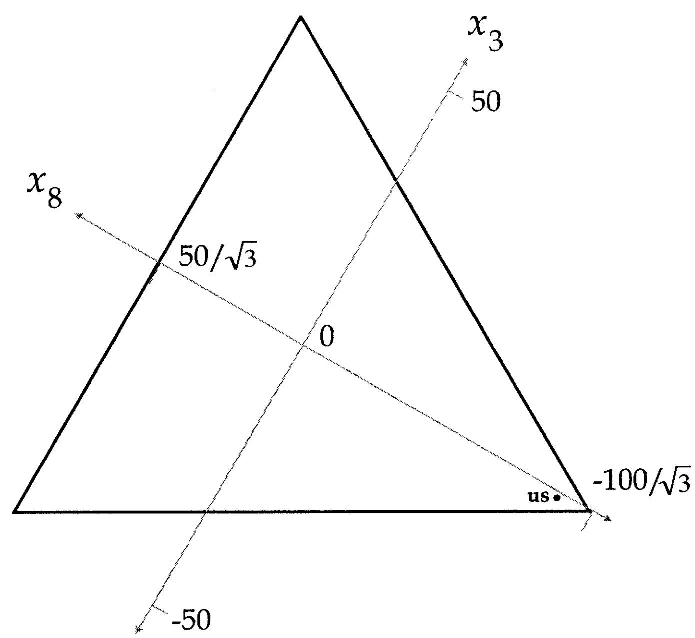


Figure 4: The quark mass space slice at $x_0 = x_0^{\odot} = 100/\sqrt{6}$ MeV, with the x_3 and x_8 axes marked, in units of MeV. Our universe is at the point by the lower right corner.

$$M(p) = \overline{M_B}(x_0) \; + 4.08(x_8) + 1.43(x_3) + 0.63 \text{ MeV} \tag{3.29}$$

$$M(n) = \overline{M_B}(x_0) \; + 4.08(x_8) - 1.43(x_3) - 0.13 \text{ MeV} \tag{3.30}$$

$$M(\Xi^0) = \overline{M_B}(x_0) \; - 3.28(x_8) + 2.82(x_3) - 0.07 \text{ MeV} \tag{3.31}$$

$$M(\Xi^-) = \overline{M_B}(x_0) \; - 3.28(x_8) - 2.82(x_3) + 0.79 \text{ MeV} \tag{3.32}$$

$$M(\Sigma^+) = \overline{M_B}(x_0) \; - 0.80(x_8) + 4.25(x_3) + 0.70 \text{ MeV} \tag{3.33}$$

$$M(\Sigma^-) = \overline{M_B}(x_0) \; - 0.80(x_8) - 4.25(x_3) + 0.87 \text{ MeV} \tag{3.34}$$

$$M(C_{high}) = \overline{M_B}(x_0) \; + 0.80(\sqrt{x_8^2 + x_3^2}) - 0.21 \text{ MeV} \tag{3.35}$$

$$M(C_{low}) = \overline{M_B}(x_0) \; - 0.80(\sqrt{x_8^2 + x_3^2}) \; . \tag{3.36}$$

Figures 6 and 7 graphically show the baryon masses from these formulas (without the electromagnetic corrections), moving along the perimeter of the triangle shown in Figure 5. The formulas are linear in x_3 and x_8 (or the radius $\sqrt{x_8^2 + x_3^2}$), so the baryon masses along the perimeter of a concentric similar triangle would produce the same plots, though with rescaled axes.

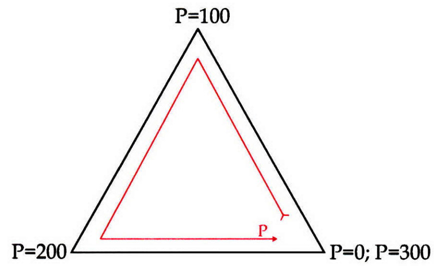


Figure 5: The triangle slice of $x_0 = x_0^{\odot} = 40.8$ MeV, with the perimeter P axis used in Figures 6 and 7 shown. The sides of the triangle are 100 MeV long in the x_3, x_8 coordinate system.

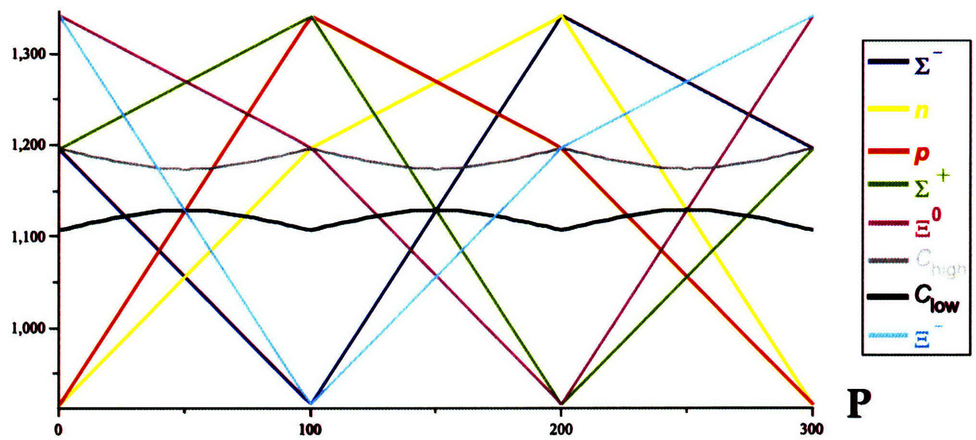


Figure 6: Baryon masses (without E&M) in MeV at mass points along the perimeter of the triangle slice with $x_0 = x_0^{\odot} = 40.8$ MeV. See Figure 5 for an explicit picture of the perimeter P axis.

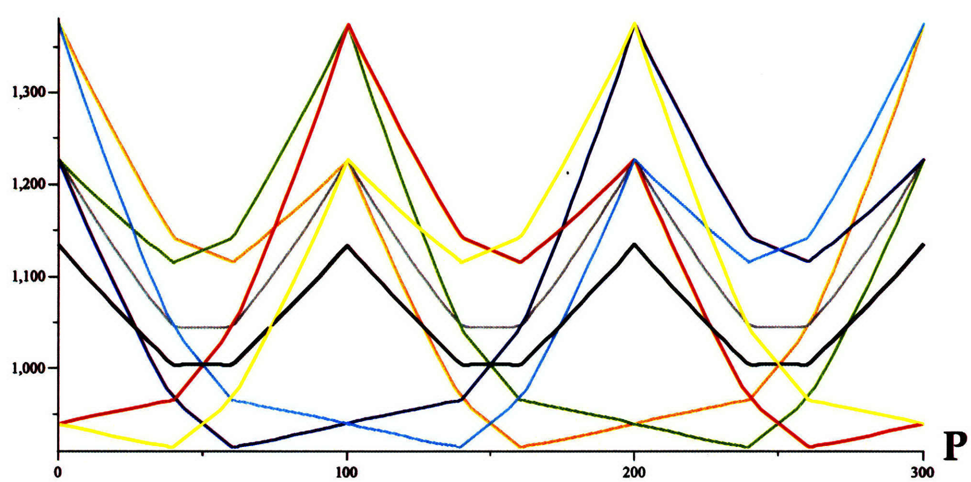


Figure 7: Baryon masses (without E&M) in MeV at mass points along the perimeter of the triangle slice with $x_0 = x_0^{\odot} = 40.8$ MeV, rescaled so that the average mass of the lightest pair of baryons is held fixed (potentially by varying the electron yukawa coupling). See Figure 5 for an explicit picture of the perimeter P axis.

4 The nuclear Hamiltonian and stability

4.1 Semi-empirical nuclear Hamiltonian

To understand the stability of nuclei with large A, we build a generalized semi-empirical nuclear Hamiltonian. We begin with the standard Weizsäcker semi-empirical nuclear mass formula,

$$H_W = \sum_i N_i M_i - \epsilon_v A + \epsilon_s A^{2/3} + \epsilon_c \frac{Z^2}{A^{1/3}} + \epsilon_{sym} \frac{I_3^2}{A} \mp \frac{\epsilon_{pair}}{A^{1/2}} . \tag{4.1}$$

As usual $A = \sum N_i = Z + N$ and $I_3 = (Z - N)/2$. The $\mp$ notation is for the pairing term: it is taken to be $+$ for nuclei with odd Z and odd N, $-$ for nuclei with even Z and even N, and zero for nuclei with odd A.

The first term in the Weizsäcker formula is the total mass of the constituent nucleons, followed by (negative) the binding energy, which in general makes nuclei stable (binding goes like volume, or A) but is hindered by surface energy corrections, the Coulomb repulsion energy, and a penultimate asymmetry term that forces the number of protons and neutrons to be similar. The final pairing term contributes to the binding of nuclei with even Z and even N and detracts from the binding of nuclei with odd Z and N. Aside from filled shell exceptions the formula fits measured nuclear masses for $A \geq 10$ quite well.

The coefficients in the formula arise from the nuclear forces that hold nuclei together, which we strive to hold constant by varying Λ_{QCD} as we explore quark masses. Their numerical value, which we take from [3], is the result of a fit to the binding energy per nucleon B/A. The volume coefficient ϵ_v is 15.56 MeV, the surface coefficient ϵ_s is 17.23 MeV, the Coulomb coefficient ϵ_c is 0.7 MeV, the asymmetry coefficient ϵ_{sym} is 93.12 MeV, and the pairing coefficient ϵ_{pair} is 12 MeV.

We shall generalize the Weizsäcker formula in two ways, aiding both computation and understanding of nuclear stability for various quark mass values. First, we will introduce the degenerate Fermi gas model in order to study when new baryon species begin participating in nuclear formation. The Fermi gas term exhibits non-analytic behavior corresponding to the "Fermi sea" spilling over when a new species becomes light enough. Second, we will take the current asymmetry term, which holds in the isospin $SU(2)$ limit, and generalize it to an alternative asymmetry term for the $SU(3)$ limit.

4.2 Fermi gas model for the nucleus

We model the nucleus as a noninteracting Fermi gas of baryons with masses M_i under constant pressure P_0. Our aim is not to reproduce the Weizsäcker formula, but to find $SU(3)$ breaking corrections that will generalize it to nuclei made of up to all eight octet baryons; we suppress Fermi gas surface corrections to the density of states dN (arising from wavefunction boundary conditions) under the assumption that their $SU(3)$ breaking will be negligible.

Letting N_i be the number of baryons of the ith species in the nucleus and k_i^F be their Fermi momentum, we begin with the defining equation

$$dN = 2_{(\text{spin})} \frac{d^3x \; d^3k}{(2\pi)^3} \text{ , yielding } \quad k_i^F = \left(\frac{3\pi^2 N_i}{V}\right)^{1/3} . \tag{4.2}$$

The internal energy is

$$U = \sum_i \left(\frac{(k_i^F)^5}{10M\pi^2} V + M_i N_i \right) , \tag{4.3}$$

and the chemical potential is defined by

$$\mu_i \equiv \left.\frac{\partial U}{\partial N_i}\right|_V = M_i + \frac{(k_i^F)^2}{2M} . \tag{4.4}$$

Note that we consistently ignore the $SU(3)$ symmetry breaking caused by what should be an M_i in the denominator, since its first order effects $\delta M_i / M^2$ turn out to be much smaller than all other terms. We define the symbol M to stand for the average mass of the pair of lightest octet baryons. This definition turns out to make sense as the zeroth order $SU(3)$ term, since in the quark mass regions investigated the baryons participating in nuclei formation have an average mass of or close to M. Thus we have chosen to hold M (rather than the average mass) fixed at 938.92 MeV as we explore quark masses, as described earlier.

We find it convenient to use the formalism of the grand canonical ensemble. The grand potential is the Legendre transform of the internal energy replacing N_i by μ_i as the independent variable, $\Omega_i(V, \mu_i) = U(N_i, V) - \mu_i N_i$. Its intensive counterpart is

$$\omega_i(\mu_i) \equiv \frac{\Omega_i(V, \mu_i)}{V} = -\frac{1}{15M\pi^2}\left[2M(\mu_i - M_i)\right]^{5/2} . \tag{4.5}$$

The total energy is the internal energy of the Fermi gas together with the work it does against the confining pressure P_0:

$$E(N_i, V) = U(N_i, V) + P_0 V = \sum_i (\omega_i(\mu_i)V + \mu_i N_i) + P_0 V . \tag{4.6}$$

Dynamical equilibrium requires that V be fixed, by minimizing E at fixed N_i:

$$\left.\frac{\partial E}{\partial V}\right|_{N_i} = 0 \quad \Rightarrow \quad \sum \omega_i(\mu_i) + P_0 = 0 . \tag{4.7}$$

P_0 can be written in terms of the Fermi momenta,

$$P_0 = -\left.\frac{\partial U}{\partial V}\right|_{N_i} = \frac{\sum_i (k_i^F)^5}{15M\pi^2} . \tag{4.8}$$

By the equilibrium condition of Eq.(4.7) and the expression for the grand potential in Eq.(4.5), we may express the total energy as

$$E(N_i) = \sum \mu_i N_i . \tag{4.9}$$

Expressing the Fermi momentum in Eq.(4.4) at equilibrium in terms of the confining pressure P_0, we obtain

$$E = \sum_i N_i M_i + \epsilon_0 \left(\sum_i N_i^{5/3}\right)^{3/5} , \tag{4.10}$$

with

$$\epsilon_0 = \left(\frac{15\pi^2}{2^{5/2}M^{3/2}}P_0\right)^{2/5} . \tag{4.11}$$

We find and fix the value of P_0 based on the Fermi momenta values of the nucleons in our universe, experimentally measured for typical heavy nuclei using quasi-elastic electron scattering[6] [23]. The bulk values $k_p^F = k_n^F = 245$ MeV yield a pressure $P_0 = 0.827$ MeV/fm^3 and energy $\epsilon_0 = 32.0$ MeV.

Expanding E in our universe as a power series in I_3,

$$\epsilon_0 \left(\sum N_i^{5/3}\right)^{3/5} = \epsilon_0 \frac{A}{2^{2/5}} \left(1 + \frac{4}{3A^2} I_3^2 + \ldots\right) , \tag{4.12}$$

we see that the Fermi gas energy contains terms that appear in the Weizsäcker formula, in addition to terms of higher order in I_3 that do not. In our universe nuclei never stray too far from $I_3 \approx 0$, so the higher order terms do not contribute and we can rewrite the Weizsäcker formula by adding the Fermi gas term $\epsilon_0 \left(\sum N_i^{5/3}\right)^{3/5}$ and subtracting its two lowest order corrections, $A\left(\epsilon_0/2^{2/5}\right) = A\ 24.2$ MeV and $\left(\epsilon_0 2^{8/5}/3\right) I_3^2/A = I_3^2/A\ 32.3$ MeV, from the corresponding terms in the original Weizsäcker formula.

The Fermi gas energy is a kinetic energy in the sense that it doesn't involve an interactions between nucleons. While the Fermi gas energy provides an asymmetry I_3^2/A term, it only accounts for a third of its empirically observed coefficient. The remaining $2/3$ of the Weizsäcker I_3^2/A term come from the strong interaction potential energy. The importance of the full term can be seen from the fact that the di-neutron and di-proton, which would have total angular momentum $j = 0$ but $I_3 = 1$, do not exist, while the deuteron with $j = 1$ but $I_3 = 0$ exists and is stable.

With a fixed nuclear pressure P_0, the density of nuclei varies with the Fermi momenta distribution. The coulomb term goes like $1/r$ and so depends on the Fermi

[6] Special thanks to Dr. T. William Donnelly, of the MIT Center for Theoretical Physics, who assisted us with finding the relevant experimental results.

momenta as well. In general this is a small effect, but we present it for consistency:

$$r = \left(\frac{9}{40MP_0} \left(\frac{3}{2\pi} \right)^{1/3} \sum N_i^{5/3} \right)^{1/5} = 1.55 \text{ fm} \left(\sum_i N_i^{5/3} \right)^{1/5} \approx 1.41 \; A^{1/3} \text{ fm} \; . \tag{4.13}$$

Note that the value of the nuclear radius r from the Fermi gas model above is not very accurate, as it doesn't include the effects of interactions. We do not use this r in our work, instead scaling it to reach the fitted value of the ϵ_c coefficient in the Weizsacker formula, resulting in the generalized Coulomb energy

$$E_c = 0.64 \left(\sum_i N_i^{5/3} \right)^{-1/5} (Q^2 - |Q|)^2 \; . \tag{4.14}$$

We use the Coulomb energy term when we analyze analog nuclei in universes where the two participating baryon species have different charges from the proton and neutron charges. In these cases the asymmetry in nuclear composition is never much more than 2/3 of one species and 1/3 of the other, yielding a difference for the Coulomb term of at most about 1%. Thus for the analog nuclei analysis, the unmodified Coulomb term $0.7 \; (Q^2 - |Q|)^2 / A^{1/3}$ is quite sufficient.

4.3 The asymmetry term in the $SU(3)$ limit

One third of the coefficient of the asymmetry term is accounted for by the Fermi gas term kinetic energy. The remainder of the term,

$$\left(\epsilon_{sym} - 2^{8/5} \epsilon_0 / 3 \right) I_3^2 / A = 60.82 \; I_3^2 / A \text{ MeV} \; , \tag{4.15}$$

must be accounted for by the potential energy interactions. It is reasonable to expect that the interactions will not exhibit the sort of non-analytic behavior seen in the Fermi gas term, so for a nucleus with only two baryon species we believe that the asymmetry term in Eq.(4.1) is correct even with a relatively large mass difference between the two species.

We wish to generalize the interaction dependent asymmetry term to apply to nuclei made of all eight octet baryons in the $SU(3)$ limit. While the asymmetry term is usually written as I_3^2, that cannot be physically correct - the nuclear Hamiltonian cannot break symmetry by depending on the third component of isospin. That term should actually be written as the total isospin squared, $I(I+1)$, since if nature was to minimize the energy term $I(I+1)$ for some nucleus constrained to have a given value of I_3 by its constituent up and down quarks, it would do it by putting (almost) all the isospin into the '3' axis, and so $I(I+1) \approx I_3^2$. Now it becomes clearer that $I(I+1)$ generalizes to the quadratic Casimir operator of SU(3) (one of the two operators which, by Schur's lemma, are proportional to the identity and so give a definite value for any irreducible representation). The strong interactions rearrange the quarks among baryons in a given nucleus to minimize its energy, changing N_i but preserving A, I_3 and Y. Given values of I_3 and Y that together with A define the quark makeup of a given nucleus, nature will choose a representation that will minimize the Casimir operator C, constrained by I_3 and Y.

We shall use the standard notation, where λ is the I_3 length of the top horizontal line for the given $SU(3)$ representation drawn on an eigenvalue weight diagram and μ is the length of its bottom horizontal line (see [1]). Equivalently, for a representation given by three rows of young tableaux of lengths (top to bottom) of a, b, c, we define $\lambda = a - b$ and $\mu = b - c$. The quadratic Casimir operator's value on a representation is given by

$$C = \left(\lambda^2 + \lambda\mu + \mu^2 + 3(\lambda + \mu)\right)/3 \tag{4.16}$$

up to constants that don't depend on $SU(3)$ representations.

Given the I_3 and Y values that define a given nucleus, look at candidate representations that include the point (I_3, Y) and check whether they have the minimal value of C. If (I_3, Y) is not on the boundary of the representation, one can decrease λ or μ or both to reach a representation that still contains (I_3, Y) but clearly has a smaller value for C. If (I_3, Y) is on the boundary but not in a corner, and instead is n steps away from the nearest corner, then either of the representations $(\lambda - 2n, \mu + n)$ or $(\lambda + n, \mu - 2n)$ will still contain (I_3, Y). A quick calculation shows that $C(\lambda-2n,\mu+n)=C(\lambda,\mu)-\lambda<C(\lambda,\mu)$ and equivalently for $(\lambda+n,\mu-2n)$, so the modified representation is favored. Thus the representation which minimizes C must have the point (I_3, Y) as a corner. Given this corner point, the representation is uniquely determined by reflecting the point over rays at angles $\theta = 30^o$, $\theta = 150^o$ and $\theta = 270^o$ successively, yielding its full set of six (or three) corner points. Thus the minimal representation and its associated minimized quadratic Casimir operator value, denoted by $\tilde{G}^2$ for now, are found as a function of I_3 and Y. The $\tilde{G}^2$ resulting from this minimization procedure can be explicitly computed:

$$\tilde{G}^2 = I_3^2 + \frac{3}{4}Y^2 + \frac{(|N_u - N_d| + |N_d - N_s| + |N_s - N_u|)}{2} \tag{4.17}$$

where N_u, N_d, N_s are coordinates along the axes counting the number of up, down and strange quarks (they can be written as complicated piecewise functions of I_3 and Y).

The quadratic Casimir operator can be freely modified by any function of A, since A is a constant under SU(3). We must modify the $\tilde{G}^2$ given above to reduce to $I_3^2 + |I_3| \approx I_3^2$ for our universe, where $Y = A$. This unique generalized asymmetry term quadratic Casimir operator is given by

$$G^2 = I_3^2 + \frac{3}{4}Y^2 - \frac{3}{4}A^2 - 3/2A + \frac{(|N_u - N_d| + |N_d - N_s| + |N_s - N_u|)}{2} \ . \tag{4.18}$$

The linear terms (A and differences in the number of u, d, s quarks) are always small compared to other terms in the Hamiltonian and will be ignored. Note that except for the mathematically unnatural normalization used in the definition of the hypercharge Y axis and for its linear terms, G^2 is simply the distance to the origin of the weight diagram squared. Like I_3^2 is the quadratic expansion around the $SU(2)$ limit, the quadratic Casimir operator is a quadratic expansion around the $SU(3)$ limit, and is only valid where $SU(3)$ symmetry is barely broken. In regions far from the $SU(3)$ limit where still more than two baryon species participate in nuclei, the kinetic Fermi gas model term accounts for some of the asymmetry term, but we do not know how to describe the effects of interactions.

We present the generalized semi-empirical nuclear Hamiltonian, with the Casimir operator symmetry term applicable near the $SU(3)$ limit. For the the isospin $SU(2)$ limit of our universe, plug in $Y = A$ in the Casimir operator to produce the $SU(2)$ limit asymmetry term.

$$H = \sum_i (N_i M_i) - \left(\epsilon_v + \frac{\epsilon_0}{2^{2/5}}\right) A + \epsilon_s A^{2/3} + \epsilon_0 \left(\sum_i \left(N_i^{5/3}\right)\right)^{3/5}$$
$$+ \left(\epsilon_{sym} - \frac{2^{8/5}}{3}\epsilon_0\right) \frac{1}{A} \left(I_3^2 + \frac{3}{4}Y^2 - \frac{3}{4}A^2\right)$$
$$+\epsilon_c\, 2^{-2/15} \left(\sum_i N_i^{5/3}\right)^{-1/5} \left(I_3 + \frac{Y}{2}\right)^2 \mp \frac{\epsilon_{pair}}{A^{1/2}} \quad . \qquad (4.19)$$

4.4 Nuclear stability

The strong interactions minimize H with respect to all the participating N_i while holding A, I_3 and Y fixed, converting H to a function $H(A, I_3, Y)$ that gives the energy of a given nucleus. For a given A, this energy function has a valley of stability in I_3 and Y. The weak interactions drive nuclei to the minimum of the valley through beta decays, which preserve A.

Other decay mechanisms change A, such as fission and specifically alpha decay. Alpha particle emission generalizes to the emission of the highly bound nucleus which has two baryons of each species, regardless of its charge.

The other decay process important for determining boundaries of stability is *weak nucleon emission*, which happens at large baryon mass differences. In weak nucleon emission, a heavier nucleon weakly decays into the lighter nucleon and is then immediately emitted. The nucleus gains the energy from the mass difference between the baryons. A similar process is responsible for the decay of hypernuclei in our universe.

For concreteness, we describe this process for a quark mass configuration similar to our universe's. When the proton is heavier than the neutron, weak neutron emission involves electron capture and neutron emission, and so occurs when

$$M(N,Z) + m_{e^-} > M(N,Z-1) + M_n \ . \tag{4.20}$$

Similarly, when the neutron is heavier than the proton, weak proton emission occurs when

$$M(N,Z) > M(N-1,Z) + M_p + m_{e^-} \ . \tag{4.21}$$

Let B be the binding energy function, and switch to the coordinates defined by

$$\begin{aligned} \delta M &= M_n - M_p \\ A = N + Z&;\ \Delta = (N-Z)/2 \\ N = \frac{A}{2} + \Delta&;\ Z = \frac{A}{2} - \Delta \ . \end{aligned} \tag{4.22}$$

The inequalities permitting weak nucleon emission in the heavy proton / heavy neutron cases become

$$B(A,\Delta) - B(A-1,\Delta \pm \frac{1}{2}) < \pm(m_{e^-} - \delta M) \ . \tag{4.23}$$

Approximating the discrete difference as a derivative,

$$\frac{\partial B}{\partial A} \pm \frac{1}{2}\frac{\partial B}{\partial \Delta} < \pm(m_{e^-} - \delta M) \ . \tag{4.24}$$

We are interested in nuclei already at the minimum of the valley of stability set by the weak interactions, with $\Delta = \Delta(A)$ set by

$$\frac{\partial H}{\partial \Delta} = 0. \tag{4.25}$$

Since $H = \frac{1}{2}A(M_n + M_p) + \delta M \Delta - B$, a nucleus at the weak interaction valley's minimum has

$$\frac{\partial B}{\partial \Delta} = \delta M \tag{4.26}$$

and so is unstable to weak neutron / proton emission when

$$\frac{\partial B}{\partial A} < \pm \left(m_{e^-} - \frac{\delta M}{2} \right) \; . \tag{4.27}$$

Note that at the minimum of the valley of stability,

$$\frac{dB(A, \Delta(A))}{dA} = \frac{\partial B}{\partial A} + \frac{\partial B}{\partial \Delta}\frac{\partial \Delta(A)}{\partial A} = \frac{\partial B}{\partial A} + \delta M \frac{\partial \Delta(A)}{\partial A}$$

so we can equivalently write the conditions for instability as

$$\frac{dB(A, \Delta(A))}{dA} < \pm \left(m_{e^-} - \delta M \left(\frac{1}{2} \mp \frac{\partial \Delta(A)}{\partial A} \right) \right) \; . \tag{4.28}$$

In our universe $\partial\Delta(A)/\partial A$ is zero for small nuclei and only gets as high as 0.1 for large nuclei, so the instability condition is roughly $dB/dA < \mp\delta M/2$. Note that previous descriptions ([25]) of this decay mode are off by roughly a factor of 1/2.

5 Regions in the quark mass space

We begin by considering various possibilities for quark mass configurations. At the end of this section we will present the regions that correspond to the various configurations within the quark mass space we investigate.

5.1 One light baryon

A universe where only one quark is light enough to participate in baryons would not be congenial. Only one baryon could participate in nuclear binding - for example, the Δ^{++} in the case of the up quark. Simple nuclei composed of that baryon would be akin to the di-proton, which is unbound due to its high symmetry energy penalty. The only atom would be hydrogen, or helium, depending on the single quark charge.

Because decuplet baryon masses are much higher than octet baryon masses, a decuplet baryon becomes the sole participant in nuclei only for very large quark mass differences. We do not have enough information to answer this question quantitatively, but a rough estimate indicates that it would require a value of x_0 of 2 to 3 times our universe's, and too large for our $SU(3)$ perturbation theory analysis to hold. No points in the mass space we investigate fall into the one light baryon category, though this region does exist in principle.

5.2 Two light quarks of the same charge

We first discuss the case of two light quarks with charges $-1/3, \; -1/3$. Within the primary mass space of the three light $u, \; d, \; s$ quarks we investigate, this occurs when the up quark is much heavier than the down and strange quarks. We will find below that even if the two light quarks have the same mass, nuclei with a charge greater than 4 electron charge units are unstable, so that universe is uncongenial. If the light down and strange quarks were to have a nonzero mass difference, nuclei at the bottom of the valley of stability set by $dH/dN = 0$ would still pay a high symmetry energy penalty and so would be even less stable than their $m_d = m_s$ counterparts.

It thus suffices to determine uncongeniality for the case of the down and strange quarks having an equal and small mass (with all other quarks too heavy to participate). The two participating baryons are the Σ^- and the Ξ^-.

Because both baryons are charged, the charge of a nucleus builds up as $Q = -A$. With no mass difference or Coulomb energy difference, the valley of stability lies on the line $N_{\Sigma^-} = N_{\Xi^-}$. Our universe does have a small up-down quark mass difference, but it is much smaller than the Fermi energy and thus is negligible in determining the valley of stability $\partial H / \partial N_i = 0$. Hence in our universe (before the Coulomb energy becomes important) we can find nuclei that lie on the corresponding line $Z = A/2$. Except for the different Coulomb energy, light nuclei in our universe have the same binding energy as the analog (same $N_1, \; N_2$) light nuclei in the universe with light down and strange quarks. The $Q = -6$ carbon equivalent has $A = 6$, which is too small for the semi-empirical nuclear Hamiltonian to be accurate - but the correspondence with analog nuclei in our universe allows us to directly equate the binding energy of light nuclei with the same $N_1, \; N_2$, once we adjust for the Coulomb energy.

Most nuclei are well approximated by a small sphere, so their Coulomb energy is given by summing pairwise Coulomb interactions between constituent nucleons, resulting in the standard term of the Weizsäcker formula,

$$E_C = 0.7A^{-1/3}(Q^2 - |Q|) \ . \tag{5.1}$$

However, the deuterium analog nucleus with $A = |Q| = 2$ is more complicated since deuterium is too weakly bound to be approximated as a sphere of charge. We calculate the Coulomb energy for the deuterium analog nucleus by modeling the deuteron nuclear potential as a finite spherical potential well. Its depth and radius are set by the conditions that the ground state wavefunction of the proton - neutron relative distance reduced system have a binding energy that agrees with the measured deuteron binding energy $B = 2.22$ MeV, and that it must have a root mean square (relative distance) radius that agrees with the measured deuteron diameter, so that $\sqrt{\langle r^2 \rangle} = 4.28 fm$. The potential giving the correct wavefunction has $V_0 = -19.54$ MeV and radius $b = 2.96 fm$. Treating the Coulomb interactions through first order perturbation theory, we calculate the Coulomb energy to be $\left\langle e^2 \frac{Q_1 Q_2}{r} \right\rangle = 0.57$ MeV.

We perform the analog nuclei analysis by first subtracting the Coulomb term from the energy for each analog nucleus in our world to find its binding energy without Coulomb repulsion, and then adding a Coulomb repulsion energy with $Q = -A$, to find the energy of the analog nucleus in the universe of light d and s quarks we investigate. For example, to analyze the $A = Q = 6$ chemical-carbon nucleus, we note that its composition of three of each baryon species make its nucleus analogous to that of the lithium isotope ^{6}Li in our universe. The resulting binding energies of chemical-helium $A = |Q| = 2$, chemical-beryllium $A = |Q| = 4$ and chemical-carbon $A = |Q| = 6$, derived from the analog nuclei of ^{2}H, ^{4}He and ^{6}Li, are 1.7 MeV, 23.9 MeV and 22.7 MeV respectively. Thus in the universe of light d and s quarks, the decay of carbon by fission into beryllium and helium is exothermic by 2.9 MeV.

Since the chemical-beryllium has the same shell structure as the alpha particle in our universe, this fission reaction is the equivalent of alpha decay, which occurs to nuclei as light as $A = |Q| = 6$ due to the increased Coulomb repulsion. Through the semi-empirical mass formula and some specific analog nuclei cases, heavier nuclei are found to also undergo alpha decay, with an even greater exothermic energy release of typically 8 MeV.. In our universe alpha decay is often impeded by the Coulomb barrier through which the alpha particle must tunnel on its way out; but the Coulomb barrier does not prevent fission in this case. Using simple formulas from the Gamow model for alpha decay [5], we find an extraordinarily short lifetime of the order of 10^{-18} seconds for the chemical-carbon fission process. Universes in which the two light quarks both have charge $-1/3$ do not have stable carbon or most atoms with charge greater than 4, and so are not congenial.

For universes with two light quarks of charges $2/3$, $2/3$, such as the up and charm quarks, even more instabilities occur. Both participating baryons have a charge of 2, so $Q = 2A$. A single baryon acts chemically like helium, and the stable alpha particle configuration is a chemical oxygen. But through the same analysis of analog nuclei described above, we find that all other nuclei in this universe are grossly unstable to fission; with only helium and oxygen and no hydrogen or carbon, these universes are clearly uncongenial.

5.3 Two light quarks of different charges

5.3.1 Analog nuclei calculations

Our universe has two light quarks u and d with different charges, $+2/3$ and $-1/3$. As shown in Fig.2, analyzing the regions with two light quarks of different charges is equivalent to analyzing the region around our universe with light u and d quarks. Thus we shall borrow the usual notation for N as the number of neutrons in a nucleus and Z as the number of protons, equal to its charge Q.

We first discuss the case of $M_p > M_n$. ^{14}C has a binding energy of 105.3 MeV. It is thus stable against alpha emission (endothermic by 8.35 MeV) and other fission processes. It undergoes electron capture to ^{14}B when $M_p - M_n > 19.344$ MeV, and decays by weak neutron emission to ^{13}B when $M_p - M_n > 20.318$ MeV. However, congeniality requires both $Z = 1$ and $Z = 6$, and the former sets the cutoff. When

$M_n < M_p + m_{e^-}$, free protons decay into neutrons (possibly by electron capture) and so cannot form hydrogen atoms. Deuterium could serve as a hydrogen replacement, but it is only weakly bound (by 2.22 MeV) and easily undergoes weak neutron emission already when $M_p - M_n > 1.71$ MeV.

In our universe, tritium beta decays into ^{3}He with a Q value of 0.019 MeV, even though tritium is bound more strongly than ^{3}He by 0.76 MeV, because the neutron is heavier than the proton by the larger 1.293 MeV. With a proton - neutron mass difference that lets free protons decay, tritium would not undergo beta decay, and could only decay through weak proton emission and subsequent dissociation into three free neutrons, a positron and a neutrino. The congeniality cutoff set by requiring some hydrogen isotope is therefore

$$M_p - M_n < B(^3\mathrm{H}) - m_{e^-} = 7.97 \text{ MeV} . \tag{5.2}$$

This is the main result for the limit of congeniality for $M_p > M_n$.

When $M_n > M_p$, congeniality boundaries are determined by the stability of carbon against all possible decay pathways. Alpha decay and fission processes depend only on the binding energies of the reactants and products, and so are independent of quark mass differences. The familiar ^{12}C has a binding energy of 92 MeV and is stable even against fission into three alpha particles. ^{10}C has a binding energy of 60.34 MeV, making it stable against fission into two alpha particles and two free protons by 3.7 MeV, and so even more stable against simple alpha decay (which would leave an unstable ^{6}Be behind).

Weak proton emission changes a given carbon nucleus into a lighter carbon isotope. ^{12}C decays by weak proton emission into ^{11}C when $M_n - M_p > 19.24$ MeV, ^{11}C decays when $M_n - M_p > 13.64$ MeV, ^{10}C decays when $M_n - M_p > 21.80$MeV, and for ^9C the cutoff is $M_n - M_p > 14.77$ MeV. Hence ^{10}C is stable against weak proton emission when $M_n - M_p < 21.80$ MeV.

Finally, beta decay occurs for a given Z, N nucleus in this region when $M(Z, N) > M(Z + 1, N - 1) + m_{e^-}$. The various carbon isotopes beta decay into nitrogen when

$M_n - M_p$ is greater than the following cutoffs: 18.64 MeV for ^{12}C, 15.25 MeV for ^{11}C and 24.40 MeV for ^{10}C. Thus, ^{12}C is stable against all decays when $M_n - M_p < 18.64$ MeV. ^{10}C becomes stable against β^+ decay already when $M_n - M_p$ is a few MeV, and remains stable until it weak-proton-emits into ^{9}C when $M_n - M_p \geq 21.80$ MeV. By this point ^{9}C also weakly-emits into ^{8}C which beta decays away. Thus the congeniality cutoff for the $M_n > M_p$ case is set by the stability of ^{10}C against weak proton emission:

$$M_n - M_p < 21.80 \text{ MeV} . \tag{5.3}$$

5.3.2 Semi-empirical Hamiltonian calculations of dB/dA

In analyzing the boundaries of congeniality for worlds with two light baryons, we have been able to use the analog nuclei method to determine the stability of carbon, without relying on the generalized semi-empirical Hamiltonian. However, it is still useful to see the predictions on stability boundaries against weak proton emissions for different nuclei generated by the semi-empirical Hamiltonian. These are shown in Figure 8, together with predictions directly using nuclear binding energy differences. It appears that the semi-empirical Hamiltonian's predictions of $dB/dA \approx \delta M/2$ are off by 0-2 MeV, a surprisingly large error; we can understand it through the following argument.

The values of parameters in the Weizsäcker semi-empirical mass formula are chosen so that the formula provides the best fit to the binding energy per nucleon B/A. We can write

$$\frac{B}{A} \equiv W(A) + W_{\text{error}}(A) \tag{5.4}$$

and note that the Weizsäcker formula provides an excellent fit for $A \geq 10$, so $W_{\text{error}}(A)$ is very small, $|W_{\text{error}}(A)| < 0.2$ MeV. However, observe that

$$\frac{dB}{dA} = W(A) + A\frac{dW(A)}{dA} + W_{\text{error}}(A) + A\frac{d}{dA}\left(W_{\text{error}}(A)\right) \tag{5.5}$$

so that the error on the Weizsäcker formula's prediction of dB/dA can still be quite large for large A, due to the $A\frac{d}{dA}\left(W_{\text{error}}(A)\right)$ term.

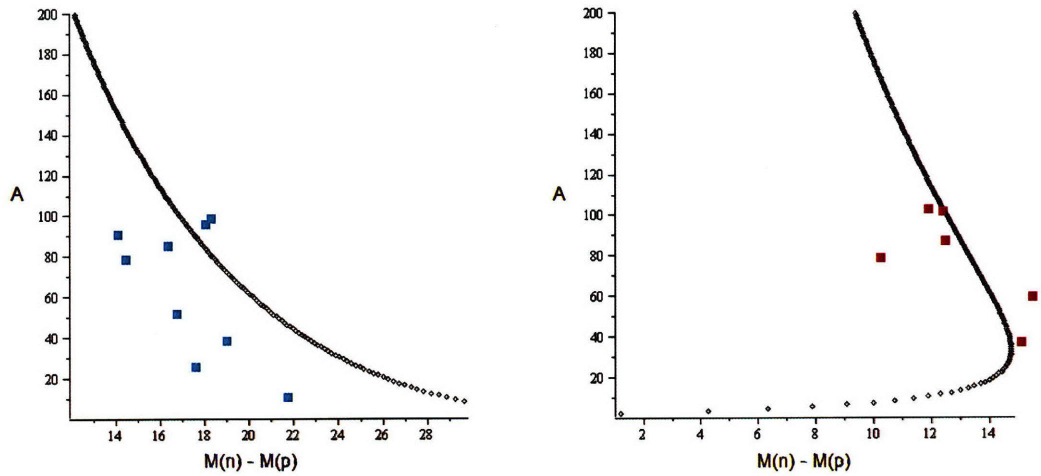


Figure 8: Boundaries in M(n)-M(p) for stability against weak proton emission of variously sized nuclei at the bottom of the valley of stability. On the left are decay transitions involving a pairing term penalty, while on the right are decay transitions in which the nucleus binding energy gains the pairing term. The black curve is the prediction of the generalized nuclear Hamiltonian, while the blue and red datapoints are from specific binding energies taken from nuclei in our world.

5.3.3 Congeniality in the two light quarks u, d mass space

The condition for congeniality we found above for universes with light u, d quarks, set by requiring that tritium (on one side) and carbon (on the other) be stable against weak nucleon emission, is

$$-7.97 \text{ MeV} \leq M_n - M_p \leq 21.80 \text{ MeV} . \tag{5.6}$$

To translate this condition into a condition on quark masses, we need formulas for baryon masses as functions of quark masses. Later on we shall use the $SU(3)$ octet model formulas derived above to find the region defined by this condition within the triangular quark mass space. However, note that the $SU(3)$ perturbation theory fit that gave us the baryon mass formulas actually gets the proton and neutron masses wrong by about 2 MeV. This is insignificant within the context of the entire 3 light quarks mass space, but if one is interested only in universes with light up and down quarks, we can do better. Looking only at $SU(2)$ isospin, we can immediately write that, subtracting electromagnetic corrections, the proton - neutron mass difference is simply proportional to the up - down quark mass difference:

$$M_p - M_n = (0.63 - (-0.13))\text{MeV} + 2.02x_3 \ . \tag{5.7}$$

Thus, considering only universes with two light quarks u and d, congenial universes form the region

$$-11.17 \text{ MeV} \leq x_3 \leq 3.57 \text{ MeV} \ . \tag{5.8}$$

Our universe with $M_p - M_n = -1.293$ MeV is at $x_3 = -1.02$ MeV, comfortably away from the edges of the congeniality band.

5.3.4 u, d domain congeniality boundary in x_8 and the three quark mass space

When the strange quark becomes light enough, a third baryon, the C_{low} (known as the Λ^0 particle in our world) will start to participate in nuclei. This marks the boundary of the congeniality region of the two light u, d quarks domain; while universes beyond this boundary may still be congenial, the analysis presented above for two light quarks and two participating baryons no longer applies.

Take $x_3 = 0$, so that $M_n = M_p$; our universe is almost indistinguishable from this point in terms of nuclear composition. In the following analysis we will refer to the C_{low} as a Λ^0, both for the sake of the reader's intuition and because when $x_3 = 0$, the Hamiltonian commutes with isospin and so $C_{low} = \Lambda^0$. Within a nucleus composed of Z protons and N neutrons, a regular nucleon (since $M_n = M_p$, for simplicity we'll take it to be a neutron) will turn into a Λ^0 particle when

$$M(Z, N, N_{\Lambda^0} = 0) \ > \ M(Z, N-1, N_{\Lambda^0} = 1) \ . \tag{5.9}$$

The third parameter in the nuclear binding energy $B(N_1, N_2, N_3)$ will continue to stand for the number of Λ^0 particles N_{Λ^0}.

Hypernuclei have been experimentally investigated in our world. For a hypernucleus that contains only one Λ^0 particle, define the binding energy of the Λ^0 particle in this (Z, N) nucleus as a difference in nuclear binding energies,

$$B_{\Lambda^0}(Z, N) \equiv B(Z, N, 1) - B(Z, N, 0) \ . \tag{5.10}$$

Thus the mass of a hypernucleus can be expressed as

$$M(Z, N-1, 1) = M_{\Lambda^0} + M(Z, N-1, 0) - B_{\Lambda^0}(Z, N-1) \ . \tag{5.11}$$

Fig. 2 of [26] provides experimental values of $B_{\Lambda^0}(Z, N)$ for various hypernuclei. For large nuclei around $A = 120$, the Λ^0 binding energy roughly saturates at $B_{\Lambda^0} =$ 23 MeV. For normal nuclei in our universe, B/A saturates to around 8 MeV, so $M(Z, N-1, 0) = M(Z, N, 0) - M_n + 8$ MeV. Combining this equation with Eq.(5.9) and Eq.(5.11) we find that the Λ^0 participates in nuclear physics when

$$M_{\Lambda^0} - M_n < 14.5\text{MeV} \ . \tag{5.12}$$

A theoretical calculation using only the kinetic Fermi gas model energy yields a cutoff of $M_{\Lambda^0} - M_n < 2^{-2/5}\epsilon_0 = 24$ MeV; evidently interaction effects discourage the Λ^0 from participating in nuclei for an additional 10 MeV. Also note that performing the previous analysis for carbon, we find that the Λ^0 is prohibited from participating in the carbon nucleus until even smaller mass differences of $M_{\Lambda^0} - M_n < 8$MeV, so we do expect universes to remain congenial a little bit beyond the boundary of the two participating baryons domain.

This analysis applies only around $x_3 = 0$; though we cannot provide quantitative results, we expect universes at larger $|x_3|$ to begin including Λ^0 particles earlier, at higher mass differences. Indeed, the width in x_3 of the congeniality band described in Eq.(5.6) goes up to $M_n - M_p = 21.8$ MeV; clearly such points are no longer in the two participating baryons domain substantially before the Λ^0 mass decreases to $M_p + 14.5$ MeV. The only point which we've determined through this analysis is the boundary of the two light baryons domain for $x_3 = 0$ Using the $SU(3)$ perturbation theory octet baryon mass formulas derived above, we can find this boundary:

$$x_8 \leq -2.97 \text{ MeV} . \tag{5.13}$$

Again using the $SU(3)$ octet baryon mass formulas, we can write the width of the congeniality band, in a manner analogous to Eq.(5.8) but more appropriate for inclusion within the full 3 quark triangle mass space:

$$-7.88 \text{ MeV} \leq x_3 \leq 2.52 \text{ MeV} . \tag{5.14}$$

5.4 One light quark with two participating baryons

When one quark is much lighter than the other two, but not light enough for a decuplet baryon to dominate nuclear formation, two octet baryons participate in nuclei. For example, if the down quark is very light, the n and Σ^- particles form nuclei. If the up and strange quarks have a similar or identical mass, these two baryons, one neutral and one charged, form nuclei roughly analogous to those in our universe. (Another case is of the up quark being very light; then the two light baryons p and Σ^+ both have charge 1, making this case equivalent to the scenario of two light quarks of the same 1/3 charge described above, which proved to be uncongenial.)

The preceding analysis for u, d universes thus carries over to the case of light down quark, but with an important exception. As explained at the end of Section 2.3, the meson spectrum in this case is qualitatively different from the meson spectrum in our world, and it is not clear at all whether our procedure for keeping the binding energy function $N(N_1, N_2)$ would work. To whatever extent it does, we can say that a congeniality band will exist in this case as well and find its width by converting the same baryon mass constraint into quark masses. Around the $m_u = m_s$ line, the mass difference between the two lightest baryons grows about twice as fast with quark masses as $M_p - M_n$ does around x_3 in our world, so the congeniality band here has a width around the $m_s = m_u$ axis of one half the width set in Eq.(5.14). We can also roughly guess how close to the $SU(3)$ center of the triangle mass space this congeniality band ends, by assuming that the 14.5 MeV mass difference to the third lightest baryon result of Eq.(5.12) stays the same, and extrapolating to quark masses from there. This assumption, though suspect, allows us to make a qualitative estimate. The rough boundaries of this congeniality band in Fig. 10 were drawn under these assumptions.

5.5 Three light quarks

When all three u, d, s quarks are light enough, all eight octet barons participate in nuclear formation. Kinetic energy Fermi seas are leveled across all eight baryons, and the full $SU(3)$ term asymmetry energy is reduced to close to its minimum. Without the asymmetry energy penalties, nuclei are in general much more strongly bound. However, because the average charge of the eight octet baryons is zero, nuclei with no net charge are easily created, and charge grows very slowly with A. Carbon can easily have $A > 10^5$ at quark mass differences of 1% or less of what they are in our universe. At exact $SU(3)$ symmetry everything is neutral on average, though away from this point baryon mass differences drive a charge assymetry.

The domain of three light quarks may be the most important, since if the a priori probability distribution for quark masses is logarithmic, much of the probability space will lie in this domain. Unfortunately, it is also extremely difficult to study. We have no firm conclusions for this domain yet, though we have been able to identify some of the relevant questions.

The first question is about the stability of the H Dihyperon [28]. When quark masses are all small enough, this six quark flavor singlet resonance may turn out to have a greater binding energy per quark than normal baryons. If it is absolutely stable in the $SU(3)$ limit domain, all other baryons would decay to it and the universe would be an uncongenial soup of neutral particles. More work, perhaps only with a lattice calculation, is needed to determine the binding energy of the Dihyperon and the answer to this question.

The second question arises from another stable configuration. Shell effects strongly favor nuclei constructed of two baryons of every species, analogous to the alpha particle in our universe. In the 3 light quarks $SU(3)$ limit, there is a nucleus made out of two baryons of every species; we've called it the Arkon, inspired by biblical object which contains every species two-by-two. The Arkon is likely to be highly stable, though because nuclear binding in the $SU(3)$ limit is qualitatively so different from our universe's, we know of no way to estimate its binding energy. It is neutral, so there is no Coulomb barrier to its emission. This second question is thus - are there any (or enough) large, charged nuclei that are stable against Arkon emission?

Finally, to study this domain one would need to determine how fast baryonic mass differences drive a nuclear charge asymmetry that creates a net charge. When mass differences are too close to zero, nuclei are created of both positive and negative charges, and would fuse together to form large neutral nuclei. If too many nuclei have no or small charge, especially if they already have large A, we may find that nuclei easily fuse together, generating large neutral clusters that prohibit chemistry.

While we can guess that exactly at the $SU(3)$ limit the universe is uncongenial, there is not much else we can say about this region. The domain bordering this region, in which more than two but fewer than eight baryons participate, is similarly beyond our reach. More work must be done to understand this critical region.

5.6 Quark mass space

Holding fixed the average mass of the lightest pair of baryons [roughly] stows away all dependence on the x_0 axis. Thus the full information of the prism is seen on the slice with largest x_0, since that slice gives the greatest range to the x_3 and x_8 axes. Each slice of smaller m_0 is roughly contained within the triangle slice of highest m_0, as a smaller concentric triangle cropped out within the larger slice; see Figure 9 for a full description.

Within this slice of largest x_0, a simple symmetry exists. As described in the Introduction, the down and strange quarks are essentially equivalent because they have the same charge, so the slice is symmetric about the axis $m_u = m_d$. Thus every point in the 3D prism is mapped by projection onto the largest 2D triangle slice, and every point on the triangle belongs to one of the regions we have considered above.

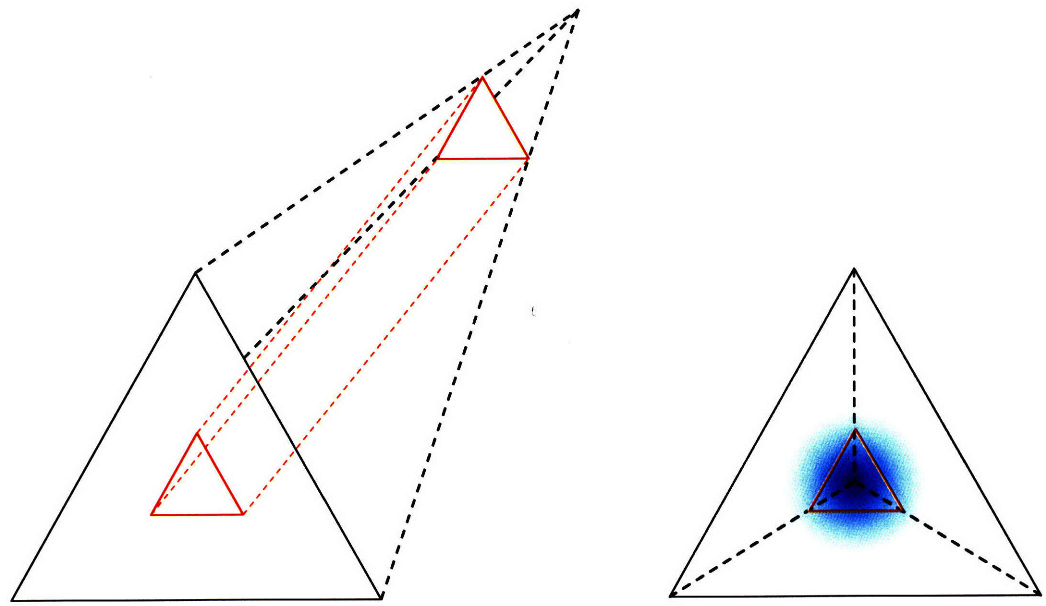


Figure 9: On the left, the mass space prism is shown with a uniform probability distribution pervading its space. Holding fixed the average mass of the lightest pair of baryons roughly eliminates all dependence on the x_0 axis. Thus the red triangle slice for some small x_0 value is identical to its projection across the x_0 axis into any other slice with a larger x_0. Specifically, all slices on the prism are equivalent to the 'shadow' they cast down the x_0 axis on the final, largest slice of the prism. Thus the full information of the prism is seen on the slice with largest x_0, on the right. Because the prism is effectively projected down the x_0 axis to its largest slice, the projected probability distribution achieves the qualitative pattern seen on the triangle on the right, with a higher probability weight given to the central $SU(3)$ region.

Before we present our final results on the triangle mass space, let us explain the 'roughly' caveats mentioned earlier. The rescaling procedure described at the beginning of this paper uses the electron yukawa coupling to scale the average mass of the lightest pair of baryons back so that its value, in units of the electron mass, is the same as it is in our universe. (We then redefine the MeV to return the electron mass to 0.511 MeV.) Explicitly for the region where the proton and neutron are the lightest baryons, we let y_e vary as a function of x_0, x_3, x_8 so that

$$\frac{\overline{M_B}(x_0) + 4.08\ x_8}{y_e(x_0, x_8)} \equiv \frac{938.92}{y_e^{\odot}}\ \text{MeV}\ . \tag{5.15}$$

All baryon masses are rescaled, and thus baryon mass differences get rescaled too.

For example,

$$M(p) - M(n) = (2.86\ x_3) \frac{938.92}{\overline{M_B}(x_0) + 4.08\ x_8}\ \text{MeV}\ . \tag{5.16}$$

This introduces nonlinear dependencies on the other axes, including x_0. For instance, the band of congeniality in x_3 we found earlier now changes its width as you move in x_8 or x_0. But for most variations in x_8 and x_0, the dependency is fairly small; it roughly comes as a fraction over 1000 MeV, and thus rarely gets over 15%. To a good approximation, the projection procedure described above still holds, and we can ignore the minor rescaling effects.

5.7 Final congeniality results and conclusions

We present our final congeniality results pictorially, on the triangle slice of $x_0 = x_0^{\odot} =$ 40.82 MeV. The picture extends naturally to triangles of slightly bigger x_0. Note that while most of the mass space is uncongenial, there are multiple quite substantial congeniality sections. The region that has the highest probability weight if the three quarks are logarithmically distributed, in the center, is still unknown.

The first conclusion of this manuscript is that it is indeed possible to ask and attempt to answer well defined physics questions that relate to the anthropic principle. We have chosen a slice through the standard model of particle physics and investigated the congeniality of points on this slice through methods in theoretical physics.

We have also reached some results in this investigation. Considering only two light quarks u and d, we derived the boundary on congeniality of Eq.(5.8), -11.17 MeV $\leq x_3 \leq 3.57$ MeV; expressed in $m_u - m_d$, it is roughly 29 MeV wide. We live at $x_3 = -1.02$ MeV, not near the edges of the congeniality band determined through our specific analysis. In the space of all three light quarks u, d, s, we derived the full congeniality picture of Figure 10. We found multiple domains of congeniality, and formulated the relevant problems for tackling the important central $SU(3)$ limit domain.

When choosing a standard model slice to investigate, it soon became clear to us

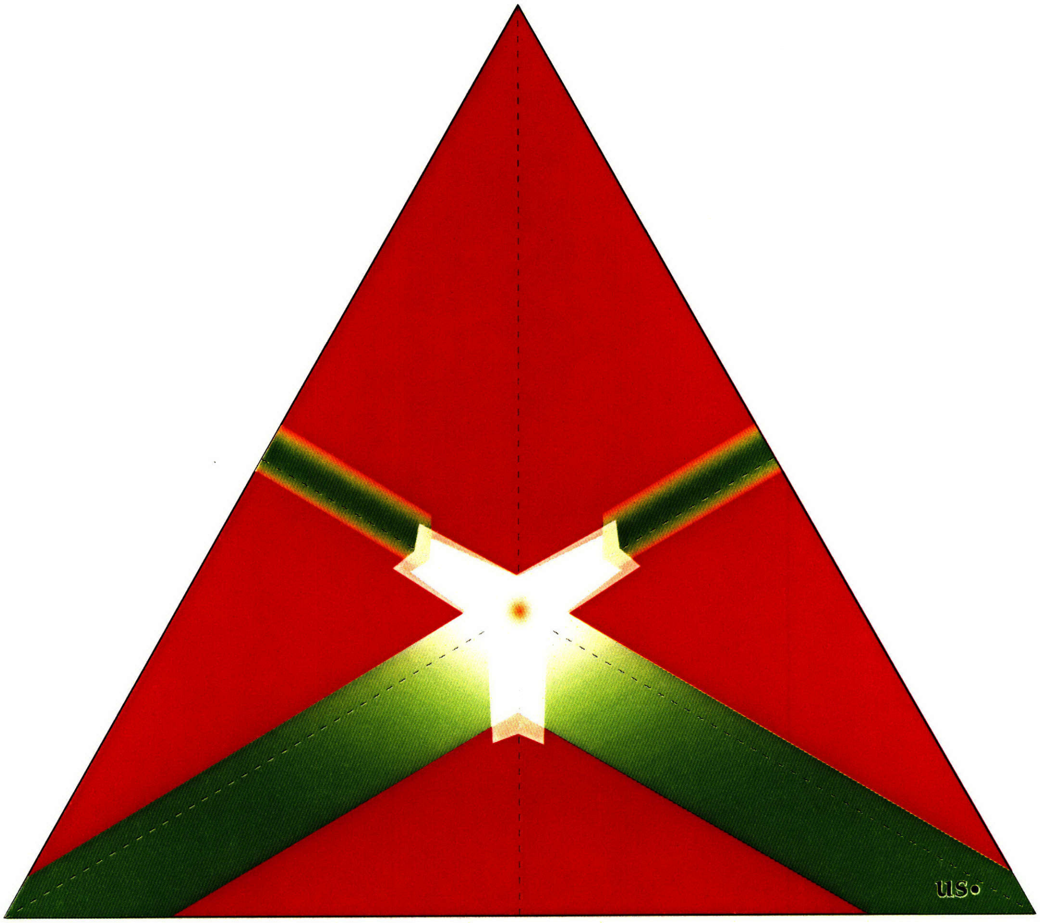


Figure 10: Final congeniality results on the triangle slice of $x_0 = x_0^{\odot} = 40.82$ MeV. Our universe is the point by the bottom right corner. The green bands are congenial, the red background is uncongenial and the central white region is the region with more than two participating baryons, about which we do not know much. Fuzzy borders imply a greater uncertainty in determining boundaries.

that we had to define the slice to keep most standard model phenomenology fixed if we wanted to make any progress. It is very easy to wander off of a familiar standard model slice and be on domains that are too complicated to investigate. Thus, while we have been able to carry out this investigation, defining the problem to investigate proved to be more subtle than we initially thought.

A Appendix: representation theory reminders

A.1 $SU(3)$ weight diagrams

Two matrices in the set of generators of $SU(3)$ are diagonal; aside from a constant factor, these are I_3 and Y, the third component of isospin and the hypercharge respectively. As matrices, they act on the fundamental representation of $SU(3)$, three dimensional vectors with the numbers of u, d, s quarks. Since quark states are eigenstates of I_3 and Y, they can be labeled by definite values of I_3 and Y as points in so called 'weight diagrams'. Weight diagrams are useful for understanding products of representations, such as how the baryon octet arises from the product of three quarks, as in Fig. 11. We begin by drawing a weight diagram for the fundamental quark representation, and couple it with itself twice by redrawing it multiple times, with the origin shifted to where each of the points in the diagram used to be. In Fig. 11 note the construction in three stages of the weight diagram for a system of three quarks, and then its decomposition into weight diagrams of irreducible representations. See [1] for an excellent explanation of weight diagrams and irreducible representations in this context.

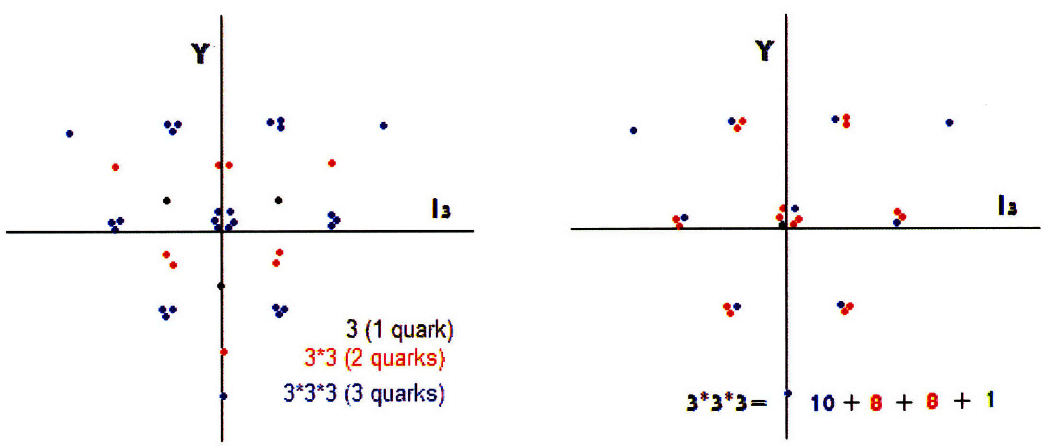


Figure 11: $SU(3)$ weight diagrams for the product of three quarks $3 \otimes 3 \otimes 3$. On the left, successive products are shown as they construct the full weight diagram in blue. On the right, this weight diagram is decomposed by color into its irreducible representations.

A.2 Combining Young tableaux

Another, perhaps more powerful way to visualize products of representations, involves Young tableaux. We give a brief introduction to their use in this context.

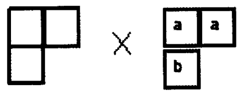
Representations, associated with particle multiplets, are related to sets of squares ('boxes') arranged in a grid, called Young tableaux or Young diagrams. A Young tableau belongs to $SU(N)$ for some dimension N. Its boxes must always be 'pushed' as far as possible to the upper left. Calculating the dimension of the representation associated with a diagram requires dividing a numerator by a denominator. Write a number in every box, starting with N in the top left one, increasing by 1 when you move right and decreasing by 1 when you move down; the product of all the numbers is the numerator. Now from the center of each box, draw a line all the way down and a line all the way to the right; the number of boxes that both lines together hit is the hook value associated with the box, and the product of the hook values is the denominator. For example, in $SU(2)$, the dimension of the following representation is $2 \cdot 1 \cdot 3 \cdot 4/(1 \cdot 2 \cdot 4 \cdot 1) = 3$: Note that a single box is the fundamental representation

2	3	4
1		

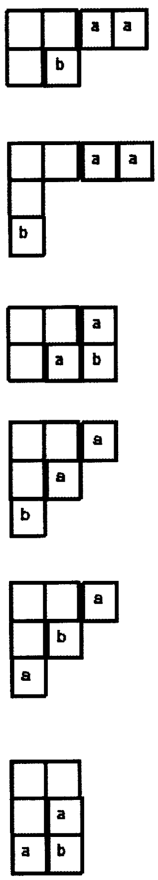
of dimension N, a full column of N boxes is the trivial singlet representation, and any longer column is rejected, as it has dimension zero.

Young tableaux combination rules allow one to find the irreducible decompositions of direct products of representations, or, in particle language, the multiplet coupling formulas. An example for coupling $8 \otimes 8$ in $SU(3)$ is shown in Figure A.2. In one of the two diagrams to be coupled, label the boxes in the first row by a, the boxes in the second row by b, etc. Using the unlabeled diagram as the upper left hand corner, attach the boxes from the labeled diagram in all permissible ways to find the decomposition of the product as a direct sum of diagrams. Begin by attaching all the a boxes, keeping only legitimate Young diagrams with at most one a per column; then attach the b boxes, etc. The labels serve to differentiate boxes from originally different rows and thus allow duplicate final diagrams with the same shape if their labeled boxes appear in different places. The labels also formulate the key requirement which must be satisfied at any point in the process: the sequence of letters in the diagram, ordered by reading from right to left and then top to bottom, must at any point in the sequence have at least as many a's as b's, at least as many b's as c's, etc. Thus *aabba* is allowed, but *abbaaaa* is rejected. To check for mistakes, note that the sum of the dimensions of the Young diagrams constructed in this way must be equal to the product of the dimensions of the two original diagrams. Thus we find that $8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8_1 \oplus 8_2 \oplus 1$.

Combining Young diagrams: SU(3) 8x8 example



Allowed letter sequences: **(Dimension)** **Rejected letter sequences:**



27

10

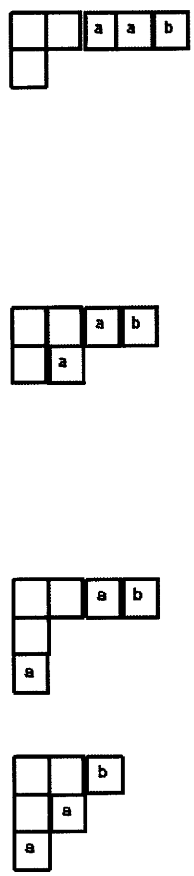
10*

8

8

1

8x8 = 64



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