Airline Revenue Management Based on Dynamic Programming Incorporating Passenger Sell-Up Behavior

by

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ABSTRACT

Low-fare carriers with simplified and unrestricted fare structures have rapidly grown and captured an important share of demand in the markets they enter, forcing legacy carriers to inevitably simplify their fare structures to avoid distraction of their competitiveness. Consequently, traditional Revenue Management (RM) systems, which assume independent demand of fare classes, have become less effective for legacy carriers in dealing with passengers who tend to purchase the lowest fare available in the absence of distinctions among fare products.

This thesis studies two RM optimization algorithms based on dynamic programming, Lautenbacher DP (DPL) and Gallego-Van Ryzin DP (DP-GVR), that aim to control fare class closure using maximum expected revenue. The underlying principle of both DP methods considers the actual arrival pattern of passengers as a Markov decision process. DPL assumes independence of fare classes as do traditional RM methods, and determines which classes should be open for a given time frame. DP-GVR considers the fact that passengers may sell-up or buy down between fare classes, and determines which fare class should be the lowest class open for a given time frame.

The goal of this thesis is to evaluate the effectiveness of DPL and DP-GVR when they account for sell-up, using not only arbitrary sell-up assumptions but also estimated sell-up rates. Based on results obtained with the Passenger Origin-Destination Simulator (PODS), we compare the performance of both methods to traditional methods under various competitive settings.

Simulation results in a single origin-destination market demonstrate the potential of DPL over traditional methods when high passenger sell-up rates are assumed or estimated. The use of DPL achieves as much as 7.3% revenue improvement over EMSRb with Q-Forecasting at high demand. In contrast, the performance of DP-GVR is weaker especially against an advanced RM method, regardless of sell-up input or estimator used. On the other hand, results from a bigger network illustrate that an airline that practices DP-GVR performs much better against both simple and advanced competing RM methods. We conclude that the performance of the theoretically appealing DPL and DP-GVR depends on the environment in which they are used, the types of passenger sell-up estimator employed, as well as the Revenue Management method applied by the competitor.

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List of Abbreviations

ALF	Average Load Factor
AP	Advance Purchase
ASM	Available Seat Mile
AT90	Adaptive Threshold of 90% Load Factor
BTC	Bookings-to-Come
CAB	Civil Aviation Board
CFP	Conditional Forecast Prediction estimator
DAVN	Displacement Adjusted Virtual Nesting
DAVN/DPL	Displacement Adjusted Virtual Nesting with DPL control
DF	Demand Factor
DFW	Dallas-Fort Worth International Airport
DP	Dynamic Programming
DP-GVR	Gallego-Van Ryzin Dynamic Programming
DPL	Lautenbacher Dynamic Programming
DWM	Decision Window Model
EMSRb	Expected Marginal Seat Revenue
FA	Fare Adjustment
FC	Fare Class
FCFS	First Come First Serve
FCYM	Fare Class Yield Management
FP	Forecast Prediction estimator
FRAT5	Fare Ratio at which 50% of passengers will sell up
FT	Fixed Threshold
HBP	Heuristic Bid Price
HF	Hybrid-Forecasting
IC	Inverse Cumulative estimator
KI	Karl Isler's discrete Marginal Revenue Fare Adjustment
KS Fix	Ken Sejling's modification to Fare Adjustment
LCC	Low Cost Carrier
Loco	Lowest Competitor Open Class
LP	Linear Programming
MDP	Markov Decision Process
MR	Thomas Fiig's continuous MR Fare Adjustment
MSP	Minneapolis-St. Paul International Airport
NetBP	Network Bid Price
NLC	Network Legacy Carrier

OD	Origin-Destination
PE	Price Elasticity
PODS	Passenger Origin-Destination Simulator
ProBP	Prorated Bid Price
QF	Q-Forecasting
R1, R2, R3	Minimum-Stay, Change-fees, and Non-Refund. Restrictions
RM	Revenue Management
RMS	Revenue Management System
RPM	Revenue Passenger Mile
SAS	Scandinavian Airlines
SF	Scaling Factor of Fare Adjustment
TF	Time Frame
WTP	Willingness-to-Pay
Z-Factor	Assumed demand variance-to-mean ratio by DP optimizers

Chapter 1

Introduction

The goal of this thesis is to evaluate the revenue benefits of using dynamic programming based models in airlines' revenue management systems. Airlines have been looking for ways to improve models of passenger behavior in their systems, which is a challenge to airlines since the industry has evolved to the point where the assumptions made in the original systems are no longer valid. Despite recent developments to improve traditional models, revenues generated may still be non-optimal, and new optimizers that eliminate those assumptions may be required to reach optimality.

We will employ a simulation approach using the Passenger Origin-Destination Simulator (PODS), originally developed by Hopperstad, Berge, and Filipowski at the Boeing Company, to model the airline booking process with competing carriers trying to maximize passenger revenues over different competitive network configurations. Further development has been conducted by the PODS Consortium, a partnership between Massachusetts Institute of Technology and eight major international airlines.

1.1 Overview of Airline Revenue Management

Revenue management, or yield management, serves to design and manage service products to maximize revenue (Weatherford, 1991). It is an effective scheme to allocate a service provider's relatively fixed capacity and to achieve increased earnings from segmented markets. In the context of the airline industry, by thoroughly understanding customers' "willingness-to-pay" (WTP), airlines implement revenue management to maximize revenue by attracting as many high fare passengers as possible and filling up the airplanes at the same time. Ticket pricing, seat allocations, and overbooking are some important elements of a revenue management system. In the rest of this thesis, the term revenue management (RM) will be used as a synonym for seat allocation aspect of the system.

The concept of revenue management can be traced back to four decades ago, when American Airlines implemented a computer reservation system (SABRE) in 1968, which had the capability of controlling reservations inventory (Smith et al., 1992). During that period when the airline industry was regulated and the Civil Aeronautics Board (CAB) controlled all fares, reservation controls practiced by many airlines emphasized primarily controlled overbooking to reduce revenue loss due to no shows. The lack of flexibility to decide fare structures and fares consequently led to competition on service provision, trying to capture passengers through better quality of service and higher frequency. Simple inventory control was first practiced in early 1970s when British Oversea Airways Corporation (known as British Airways nowadays) began to deviate from single fare product and introduce discounted fares for reservations that were made twenty-one days before flight departure (McGill and van Ryzin, 1999).

The widespread development of revenue management came after the Airline Deregulation Act of 1978. This act loosened governmental control of airlines prices and schedules, and has led to more complicated fare structure offerings with sets of restrictions that included (1) advance purchase, (2) Saturday night minimum stay, (3) change fee, and (4) non- or limited refundability of cancelled bookings. Airlines have also differentiated fare classes by offering classes with different cabins and level of services, such as the First Class, the Business Class, and the Economy Class. Table 1 shows an example of a restricted fare structure offered by American Airlines for its BOS-SEA market in 2001.

Round Fare ($)	Class	Advance Purchase	Minimum Stay	Change Fee	Comments
458	N	21 days	Sat. Night	Yes	Tue/Wed/Sat
707	M	21 days	Sat. Night	Yes	Tue/Wed
760	M	21 days	Sat. Night	Yes	Thu-Mon
927	H	14 days	Sat. Night	Yes	Tue/Wed
1001	H	14 days	Sat. Night	Yes	Thus-Mon
2083	B	3 days	None	No	2X OW
2262	Y	None	None	No	2X OW
2783	F	None	None	No	First Class

Table 1: Example of Restricted Fare Product (AA, BOS-SEA, 10/1/2001)[1]

Using RM systems, airlines can determine how many seats to allocate initially to each fare class and how to dynamically adjust this allocation as bookings arrive and the departure time of the flight approaches (General Accounting Office, 1999). One key to maximize an airline's revenue is to keep the right number seats available for the full-fare business passengers who make reservations relatively close to the departure date, and prevent them from buying lower fare class available even if they meet all restrictions. Another key is to capture leisure passengers who have flexible schedules and are only willing to buy lower fare tickets. The seat allocation problem developed from the challenge of selling seats within the same cabin of a flight at different prices to the

[1] Belobaba (2007a)

customers of different fare classes. Airlines protect some seats away from the lower revenue fare classes in order to be able to satisfy demands from the higher revenue classes (Belobaba, 1998).

In addition, deregulation of the airline industry triggered a major restructuring of networks that further complicated seat allocation. Under deregulation the airlines moved to develop a hub and spoke system that often had passengers flying into a hub on their way to their final destination. The revenue management system must take into account that some seats on flights between cities must be reserved for connecting flights. Since then, airline revenue management systems have developed significantly from single-leg control to origin-destination, or network, control. New information technologies have played a critical role in the development of revenue management, and have led to more sophisticated revenue management capabilities. Since airline deregulation, revenue management techniques have had a significant impact on the development of in the industry, providing up to 4% to 10% increase in company revenues (Fuchs, 1987). For example, in 1997, American Airlines collected one billion dollars by implementing revenue management, representing most of the company's profit (Cook, 1998).

As mentioned above, seat allocation is one of the three major aspects of typical revenue management practices. RM is developed to address the problem that prices are usually substantially affected by external factors such as prices set by the competitors. To avoid losing market share, few legacy carriers nowadays are actually willing to set their fare structures substantially different, even if optimal, from what are practiced by their legacy counterparts or low-cost carriers. An effective seat capacity control results in revenue gains that may compensate for the limitation of price options and is thus necessary.

1.2 Evolution of the Airline Industry

Network legacy carriers (NLCs) traditionally apply RM to fully-restricted fare structures assuming demands for each fare product are independent and segmented. However, over the past few years, low-cost carriers (LCCs) with simplified or unrestricted fare structures have managed to capture an important part of the demand in the markets they enter. For example, they may set their fare scheme as simple as charging one-way tickets half that of round-trips and far less differentiated than that of legacy carriers. They also have lower operating costs and can offer low-fare products and frequent service on popular routes. An example of a less restricted fare product offered by Southwest Airlines is shown in Table 2.

Round Fare ($)	Class	Advance Purchase	Minimum Stay	Non-Refund	Comments
178	M	3 days	1 Night	Yes	Special Sale
402	H	7 days	1 Night	Yes	
438	Q	14 days	None	Yes	2X OW
592	B	7 days	1 Night	Yes	Off-Peak
592	B	7 days	1 Night	Yes	Peak
634	Y	None	None	No	Unrestricted

Table 2: Example of Less restricted Fare Product offered by LCC (SWA, PVD-SEA, 10/1/2001)[1]

Furthermore, the air traffic demand has changed in recent years – there have been huge losses of business traffic, and passenger willingness to pay has been decreasing. This trend can be accounted for by a number of factors, most notably the economic downturn since 2000. The loss of business bookings has lead to a serious decline in the average fare. In addition, both fuel prices and labor cost surges have placed most major airlines in financial jeopardy. Most of the largest US airlines have undergone bankruptcy protection, and the industry has reached a point where changes have to occur in the firm of consolidation, bankruptcy, and liquidation for U.S. major airlines. The surviving airlines have been trying to reduce their costs and at the same time increase their revenues in order to end the downturn of the current industry cycle.

To make the situation worse for legacy carriers, passengers are now getting an increased amount of information through the Internet. Online travel consolidating sites, such as Travelocity, Expedia, and Orbitz, enable LCCs to offer their discount fare much easier than before; higher level of transparency is provided to customers who can easily compare fare products of different airlines before making their booking decisions.

Full-fare passengers prefer to buy down to the lowest available class open of its competing airline, a low-cost carrier or a matched legacy airline, that offer fare structures with few restrictions and relatively cheaper prices. In response to these developments, legacy carriers have departed from their traditional differentiated, fully restricted products and matched the simpler fare structures set by LCCs. As a result, for markets where LCCs are present, legacy carriers would likely match some if not all fare structures of their competitors to avoid losing too much market share and revenue. In other words, fare structures of legacy carriers have become less-restricted by allowing certain fare classes to have same types of restrictions (or no restrictions) but differ by price only. Means of matching include compression of fare ratios, and total or partial removal of restrictions and advance purchase requirements. Table 3 shows an example of less restricted fare product by Delta Air Lines in 2005.

One-Way Fare ($)	Booking Class	Advance Purchase	Minimum Stay	Change Fee ($)	Comments
124	T	21 days	None	50	Non-Refund
139	U	14 days	None	50	Non-Refund
184	L	7 days	None	50	Non-Refund
209	K	3 days	None	50	Non-Refund
354	B	3 days	None	50	Non-Refund
404	Y	None	None	No	Full Fare
254	A	None	None	No	First Class
499	F	None	None	No	First Class

Table 3: Example of Less restricted Fare Product offered by a legacy carrier (DAL, BOS-ATL, 4/2005)[1]

Between 2000 and 2004, fares decreased on average by 31% in U.S. markets where LCCs reached 10% market share (Geslin, 2006). Based on the analysis of the fare data of the largest six legacy carriers of the United States, ECLAT Consulting found that the collapse in business revenue accounts for virtually all of the revenue loss the carriers have suffered (Aviation Daily, 05/2004). The need exists to improve the RM currently practiced to adapt to evolution of fare products and passenger behavior.

1.3 Forecasting and the Concept of Sell-Up

The objective of forecasting is to estimate bookings-to-come by fare class and by flight using historical unconstrained data obtained from previous booking records of the same flight. Forecasts need to be as accurate as possible in order for revenue management systems (RMS) to work optimally. In the absence of restrictions among fare products, the general trend is that passengers purchase in the lowest fare available, making it difficult for a conventional RMS to segment demand for different fare classes in its optimizer.

One way to address this problem is to apply the concept of sell-up probability, which is defined as the probability that a passenger is willing to buy a ticket at a higher fare for the same flight if the fare product of the initial booking request is closed. By taking this probability into consideration, the RM optimizer will close down lower fare classes whenever necessary and protect more seats for higher fare passengers, and passengers will higher willingness to pay higher fare will sell-up.

1.4 The Need for a Dynamic Programming Approach

Most traditional airline revenue management methods consider two types of booking passengers: (1) business passengers who are price-inelastic but may buy down to fare class at price lower than their WTP under simplified fare structure, as we discussed in the previous section, and (2) leisure passengers who are price-oriented but may be flexible to sell-up to the next lower fare classes open.

Traditional RM models assume that the arrival of fare class bookings is sequential in increasing order. That is, the closer the booking process approaches departure date, the more the bookings that come from business passengers and flexible, discount-fare passengers who are willing to sell-up. However, if the assumption of sequential bookings arrival is incorrect, or if passengers' flexibility to sell-up does not necessarily increase over time, then those methods may not generate optimal results.

Furthermore, conventional methods consider that passengers buy in all open fare classes no matter what the lowest open fare class is, but in unrestricted fare structures they will only buy in the lowest available fare class open. Consequently traditional RM methods are unable to distinguish between business and leisure demand, making airlines difficult to reach revenue optimality under less differentiated fare structures. Clearly, besides modifying traditional RM models to incorporate the concept of sell-up, the need exists to develop a new optimization method to determine what the lowest open class should be at each time of the booking process by considering demand that may potentially purchase the lowest fare class open at any particular time.

Much research effort has focused on deriving booking limit algorithms using dynamic programming that eliminate the assumptions of segmented fare class demand and sequential bookings. Methods based on dynamic programming consider the actual demand arrival pattern of passengers as a Markov decision process (Stidham et al., 1999). They divide the reservation processes into multiple decision periods, each of them small enough for one booking request, and decide whether or not to accept the request using dynamic programming optimization algorithms, the output of which can translate into an optimal protection of fare classes. Formulations of the two DP methods examined in this thesis are presented in more depth in §3.1 and §3.2.

1.5 Objectives of the Thesis

The objective of this thesis is to study the performance of two revenue management methods based on dynamic programming in unrestricted fare environments, namely the Standard Lautenbacher DP method[2] (DPL) and the Gallego-Van Ryzin DP method[3] (DP-GVR). Both the two DP methods and several traditional RM will be implemented in our simulations. Experimental results obtained from the simulator will be used to evaluate the performance of DP-based RM under various competitive settings.

As mentioned in §1.3, considering the concept of sell-up in the forecast and in the optimization process will allow more seats to be protected for higher fare passengers and force them to sell-up when they will be denied booking for this initial request. The key is to figure out how to use these sell-up probabilities accurately, and how to estimate them dynamically throughout the entire forecasting period as inputs in RM. Several advanced forecasting methods are included in the simulations. We also explore the efficacy of adapting these forecasting methods to competition by dynamically estimating the passengers' willingness to sell-up when competition comes into play.

All simulations and quantitative evaluations will be performed by the Passenger Origin-Destination Simulator (PODS), first developed by Hopperstad at the Boeing Company. Various levels of control over the passenger choice model, the environment of interest, and the airline RM methods settings are the main components that make the simulated booking process of an airline as accurate as possible.

1.6 Structure of the Thesis

This thesis will be divided into three major parts: (1) the literature review, (2) a discussion of revenue management method based on dynamic programming and the adaptive approaches to demand and sell-up forecasts, (3) and an analysis of the results of DP-based simulations with those modified forecasters and Fare Adjustment.

Chapter 2 presents a discussion of previous work done on airline revenue management with an emphasis on the problem of unrestricted fare environment examined in this thesis. Topics covered in this chapter include forecasting techniques, traditional revenue management models, and a discussion of recent work on developing dynamic models in seat allocation problems.

[2] Lautenbacher and Stidham (1999)

[3] Gallego and van Ryzin (1997)

Detailed methodologies for the dynamic programming based models incorporating recent development of forecasting techniques are presented in Chapter 3. Chapter 4 describes an overview of the Passenger Origin-Destination Simulator (PODS), and the simulation environments related to the new DP methods that are used to obtain statistical reports on traffic and revenue generated by the carriers.

In Chapter 5, we evaluate the performance of two dynamic programming methods by comparing outputs obtained from those simulation tests with results of previous studies. Such outputs include total revenue from bookings, load factors, fare class mixes, fare class closure patterns, and booking patterns by time before departure.

Finally, Chapter 6 serves to summarize the findings of this thesis and the potential of the use of dynamic programming in airline revenue management. Directions for future research are proposed as well.

Chapter 2

Literature Review

Airlines offer various fares to capture demand coming from different market segments and different time of seasons. Because of the keen competition in the airline industry since deregulation, competitors' fares often limit the ability of airlines to effectively segment demand. Revenue management has since become as indispensable tool to generate economic gain for airlines. Solely under the control of airlines, an effective seat inventory control determines optimal seat allocation strategies among fare classes. Major carriers worldwide have committed tremendous effort to design and study optimal seat allocation policy over the past two decades.

This chapter starts by reviewing the evolution of Revenue Management in traditional environments. We then present the advent of new fare environment associated with the emergence of low-cost carriers and the need of changes to both conventional optimization and forecasting methods. Finally, we will specifically focus on two dynamic programming based models to be used for simulations, Lautenbacher DP method (DPL) and Gallego-Van Ryzin DP method (DP-GVR). They are widely theoretically promising optimization tools to improve airline revenues.

2.1 Evolution of Revenue Management Methods

The first generation of Revenue Management systems appeared in the early 80s in the form of databases that airlines used to collect, store, and keep track of bookings. The second generation of RMS arrived in the mid-80s when airlines were able to follow bookings prior to a flight departure and compare to expected booking patterns. Thanks to the advances of operations research during the late 80's and early 90's, the third generation of RMS was born with three important revenue management components: (1) Overbooking, (2) Forecaster, and (3) Seat Allocation Optimizer (Refer to Figure 1). Using database of historical bookings, overbookings, cancellations, no-shows, and fare structures, the RM optimizer generates the optimal booking limits for each flight and fare class. More information about the evolution of RMS in the airline industry can be found in McGill and van Ryzin (1999), Barnhart et al. (2003), and Talluri and van Ryzin (2004).

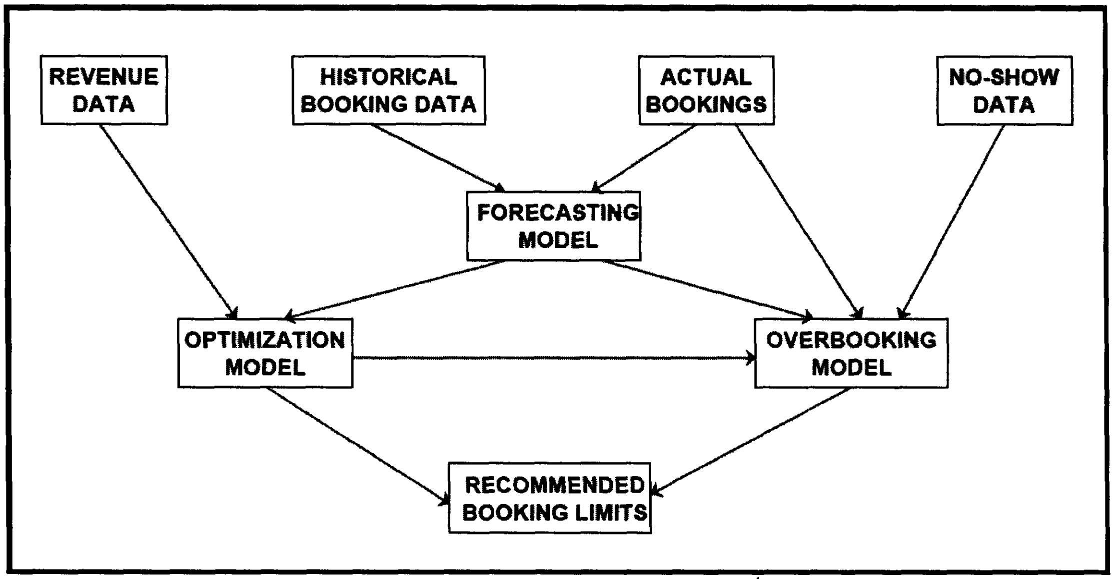

Figure 1: Third Generation RMS[4]

2.1.1 Seat Allocation Algorithms

The fundamental problem of the seat inventory control lies in managing the fixed and shared inventory of seats on a leg, so that a sufficient amount of seats are saved at full fare for passengers who are willing to pay higher fares, and seats that are not expected to be sold at low demand can be sold at discounted fares to passengers with lower WTP. There are two basic types of approaches for addressing the seat inventory control problem: leg-based models and network Origin-Destination models.

2.1.2.1 Leg-Based Fare Class Control

The application of fare class mix allocation to the seat inventory control problem was first developed for the case of a single-leg, two-fare class environment by Littlewood (1972), and subsequently extended by Buhr (1982), Richter (1982), and Wang (1983) to problems with multi-leg networks and multiple fare types. This commonly used approach is based on the concept of "serial nesting" of fare classes. Instead of allocating seats to partitioned classes, nesting protects seats for higher fare classes by limiting the number of seats sold in the lower fare classes according to demand forecast and expected seat revenue for each class.

Belobaba (1987) and Belobaba (1989) later developed a more generally applicable solution framework to the nested seat allocation problem that works with any number of fare classes using the concept of Expected Marginal Seat Revenue (EMSR). He

[4] Barnhart et al. (2003)

subsequently updated the framework that allows for joint upper classes to be protected from the next booking class right below, the algorithm known as EMSRb that has since become a prevalently used industry standard for establishing booking limits on a flight leg basis. Optimal formulations for the multi-nested class problem have been developed by Brumettle and McGill (1988), Curry (1990), Wollmer (1992), and Robinson (1995), whose results show that the nested booking limits produced by much simpler, computationally practical EMSRb techniques are indeed close to optimality.

The EMSRb method involves weighing expected seat marginal revenues, defined as "the expected fare of the booking class under consideration multiplied by the probability that demand will materialize for this incremental seat" (Belobaba, 1992), for each fare class and used them to derive leg-based protection levels for those fare classes. In other words, seats are protected for a booking fare class so long as the expected marginal of those seats is greater than or equal to the fare of the next lower class. Early simulation tests by Wilson (1995) show that implementing the leg-based EMSRb methods in a symmetric single market environment generated revenue benefits to the airline as well as the industry as a whole. Thorough description of the EMSRb heuristic can be found in Belobaba and Weatherford (1996).

The basic EMSRb algorithm is classified as a leg-based model because all legs are assumed to carry one itinerary. However, for most airlines that sell multiple-leg itineraries, their inventory is shared not only among fare classes but also between local and connecting passengers. The next advent in the development of RM from control by leg and fare class alone to joint control by both fare class and path is known as the Origin-Destination control. A path is defined as a set of single flight legs that comprise an itinerary between an origin and a destination within a network.

2.1.1.2 Network Origin-Destination Fare Class Control

For an airline that applies leg-based inventory control to accept a booking coming from a connecting itinerary, seats for all legs of the itinerary have to be available in the same fare class. Priority can thus be given to local passengers on a given flight leg at the expense of connecting passengers even though they may have higher contribution to the total revenue. O-D control models have been developed to account for network effects and produce booking limits of fare classes in an effort to maximize total revenue as opposed to yield. This approach is particularly important in the hub-and spoke networks that have flourished after deregulation of airline operations.

Optimal formulations mentioned in §2.1.1.1 for the multi-class problem have been extensively solved at the network level during the 80s. Glover et al. (1982) first framed the problem to be a large network flow problem that identifies flows on paths that maximize revenue using deterministic demand. Wollmer (1986) extended the model to implement with stochastic demand. Curry (1990) further extended Glover's formulation by formulating for multiple legs with multiple nested fare classes using continuous demand distribution, but assumed that capacities cannot be divided among nests for each path. Brumelle and McGill (1988) extended the models by Wollmer that assume discrete demand distribution, and found an optimal solution to a leg with multiple nested fare classes. There are both theoretical and practical difficulties in applying these models in real airline settings. Williamson (1992) and McGill and van Ryzin (1999) have performed critical reviews of these network optimization algorithms.

There are two approaches of Origin-Destination control models in common practice – virtual nesting methods and bid-price methods. The concept of "virtual bucketing" was initially developed by Williamson (1992), Vinod (1995), and Smith and Penn (1998). The initial idea was to nest all local and connecting fares of a given leg in various hidden buckets, known as virtual buckets, in the airline's own inventory system grouped by fare. Booking limits of each bucket is then determined by a leg-based inventory control model (EMSRb) using path forecasts aggregated into virtual buckets – demand forecasts for a given O-D routing. However, as these approaches gave higher priority to connecting passengers with higher fare, they did not address the situation that the connecting passenger may displace two local passengers whose total contribution to the overall revenue may actually be higher.

The Displacement Adjusted Virtual Nesting (DAVN) method, an adjustment to virtual bucketing made by Wysong (1988) and Smith and Penn (1998), addresses the problem based on the Network Revenue value, which is defined as the total itinerary fare minus the expected displacement cost that might occur on other legs when a request for a multiple-leg passenger is accepted on a given leg. These expected displacement costs are obtained by solving linear programming (LP) models, and the displacement-adjusted fares are put in virtual buckets. Williamson (1992) suggested a variety of methods to calculate those displacement costs. These displacement cost techniques were further refined by Tan (1994), and further developments can be found in Wei (1997) and Lee (1998). While incorporating network effects, inventory control algorithm is still implemented at the leg level. Further details about virtual bucketing and DAVN can be found in Lee (1998) and Belobaba (2002).

Another approach to O-D inventory control was the use of bid prices developed by Simpson (1989), Williamson (1992), and Smith and Penn (1998). The bid-price is calculated by summing up the displacement cost associated with each crossed leg in a given itinerary. In other words, the bid-price is the sum of the marginal values for an incremental seat on the all the legs of a given itinerary, local or connecting. Instead of generating protection levels for fare classes, the Bid-Price Control provides a standard as to the minimum amount an airline should accept a booking request – the request will be accepted if the bid-price is lower than the O-D fare, and will be rejected otherwise. Specific bid price algorithms consist of the Network Bid Price (NetBP) method, the Heuristic Bid Price[5] (HBP) method, and Prorated Bid Price[6] (ProBP) method.

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[5] Belobaba (1998)

2.1.2 Advent of the LCC Model and Fare Simplification

The success of low-cost carriers in recent years has had a significant impact on legacy carriers throughout the world. The major difference between the fare structure of low-cost carriers and that of a legacy carrier is the degree of demand segmentation. While legacy carriers typically offer multiple fare products ranging from high to low fare with various restrictions and capture higher revenue gains by inducing passengers to pay fares close to their WTP, LCCs have relatively much lower cost structures that allow them to offer unrestricted product structure and still be profitable. To avoid loss in market share for markets where a LCC is a competitor, legacy carriers are often forced not only to match fares but also simplify their own fare structure by removing certain restrictions and/or reducing the advance purchase requirements. Detailed analyses of comparison between traditional legacy carrier and LCC business models are provided in Gorin (2000), Weber Thiel (2004), and Dunleavy and Westermann (2005).

Ratliff and Vinod (2005) suggested that in this new competitive environment, the revenue management systems of legacy carriers based on segmented market structures have made it difficult for them to effectively segment demand, as legacy carriers are often compelled to match the fare structures and sometimes the lowest fare seat availability of their LCCs counterparts. More information about lowest open class matching can be found in Lua (2007).

As restrictions separating business and leisure passengers into their corresponding fare classes are removed, passengers naturally buy the lowest fare available regardless of their WTP. Boyd and Kallesen (2004) call these demand segments price-oriented and product-oriented passengers. Product-oriented demand refers to passengers who purchase a fare based on the product (i.e. restrictions and advance purchase requirements) it represents. Traditional RM methods assume independent fare class bookings in this behavior. Price-oriented demand, in contrast, corresponds to passengers who purchase at the lowest available fare regardless of product restrictions.

Hence, under less differentiated fare structures, the fundamental assumptions of traditional RM methods are violated because as business passengers are more willing to purchase at the lowest fare available when restrictions among fare classes become less differentiated, the traditional RM methods no longer can distinguish price-oriented from product-oriented passengers based on historical booking data. It becomes impossible to produce accurate independent demand forecasts for the higher fare classes. As a result, the forecaster produces a lower projection of higher fare demand, causing the optimizer to protect fewer seats in the higher fare classes and make more seats available for the lower fare classes. This "buy-down" phenomenon leads to an iterative effect known as "spiral-down", causing the airline that implement the traditional RM methods to generate

[6] Bratu (1998)

lower total revenue after each cycle (Ozdaryal and Saranathan, 2004) (Refer to Figure 2).

Cooper et al. (2003) and Kleywegt et al. (2004) develop mathematical models of the spiral down effect that occurs when traditional forecasting methods are used. Further detailed analyses about the buy-down phenomenon and spiral-down effect can also be found in Cusano (2003) and Cléaz-Savoyen (2005).

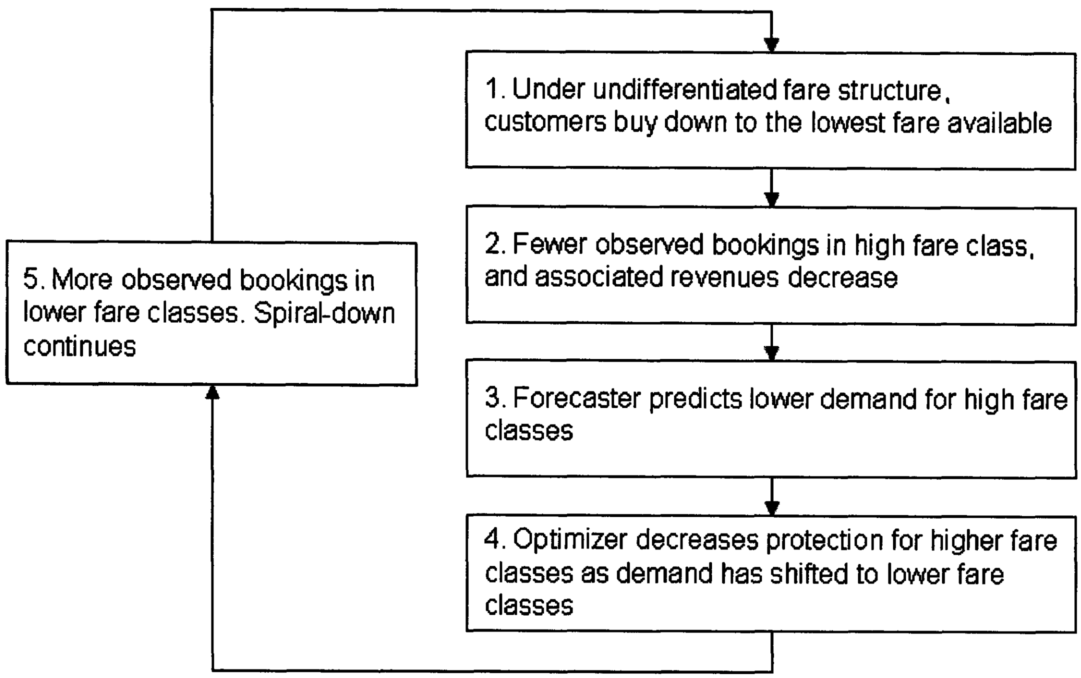


Figure 2: Spiral-Down Effect

2.2 Dynamic Programming Based RM methods

Most traditional airline revenue management methods assume that the arrival of fare class bookings is based on a predetermined, sequential order. That is, the closer the booking process approaches departure date, the more the bookings that come from business passengers and flexible, discount-fare passengers who are willing to sell-up. In addition, seat capacity control based on these models assumes that passengers' willingness to sell up is generally stable or increasing over time. Zhao and Zheng (1998) show that if this assumption is true, then traditional RM methods can lead to close to optimal results.

However, if the assumption of sequential bookings arrival is incorrect, or if passengers' flexibility to sell-up does not necessarily increase over time, then traditional methods likely lead to non-optimal solutions. This is particularly the case when a matching to LCC fare structures induces passengers with high WTP to buy down to lower fare classes. Dynamic programming models are theoretically the preferred optimization approach to seat allocation problems as they assume passengers to arrive in any order and consider explicitly the arrival process of passengers.

In developing RM methods based on dynamic programming, Stidham et al. (1999) consider the actual demand arrival pattern of passengers as a Markov decision process. They divide the reservation processes into multiple decision periods, each of them small enough for one booking request, and decide whether or not to accept the request using dynamic programming optimization algorithms, the output of which can translate into an optimal protection of fare classes.

Revenue management based on dynamic programming was initially developed by Mayer (1976), whose model divides the booking process into multiple periods and assumes that in each period, the discount-fare demand always arrives prior to the full-fare demand. Gerchak et al. (1985) study a two-class dynamic seat allocation problem using constant demand rates. Lee and Hersh (1993) consider a discrete-time, multiple-class dynamic seat allocation model with nonstationary demand, and further extend to allow group booking. Fare class demand is modeled as a Poisson process, while the entire booking process is modeled through a Markov Decision process. In other words, the state of the system is dependent only on the remaining time prior to departure as well as the remaining capacity at any given point in time.

Stidham et al. (1999) extend Lee and Hersh to allow cancellations and no-shows of passengers. The above models use discrete-time formulations, assuming that there is at most one arrival (or cancellation) in each period. Zhao and Zheng (1998) and Liang (1999) found a solution to Lee and Hersh multiple-class model framework but with continuous time. Bertsimas and Popescu (2003) design dynamic optimization techniques based on stochastic demand for multiple classes using bid-prices from a linear programming relaxation. Bertsimas and de Boer (2005) propose a stochastic gradient algorithm and approximate dynamic programming ideas to improve the existing nested seat allocation by EMSRb.

Vanhaverbeke (2006) examined two revenue management methods based on dynamic programming in PODS under fare structures with few or no restrictions: (1) the Standard Lautenbacher approach (DPL) developed by Lautenbacher (1999), and (2) the Gallego-Van Ryzin approach (DP-GVR) that was proposed by Gallego and van Ryzin (1997). We will focus on these two dynamic programming approaches in this thesis, as described in the following section.

2.2.1 Standard Lautenbacher DP method

The Standard Lautenbacher DP method (DPL) is a basic model developed at the single leg level and several papers mentioned above work as an extension to this model. However, this model incorporates important components common to all of the existing dynamic programming models and thus serves as a backbone when it comes to analyzing the performance of dynamic programming compared with non-dynamic traditional RM methods. Thus, DPL does not consider cancellations, overbooking, or no-shows.

Lautenbacher and Stidham (1999) develop a discrete-time, finite-horizon Markov Decision Process (MDP) to solve the single-leg revenue management problem by backward induction on the remaining time before departure. The booking period is divided into N decision periods in such a way that, during each decision period, the probability of two or more requests is negligible. These decision periods are numbered in reverse order, with period N corresponding to the start of the booking period, and period 0 corresponding to the scheduled departure time.

Each of the K fare classes, $1, 2, \ldots, K$, may arrive throughout the reservations horizon. At the moment a request arrives, the decision to accept or reject involves three factors: (1) remaining capacity, (2) remaining decision periods before scheduled departure, and (3) the fare class of the request. Fare class demand is modeled as a Poisson process, but request arrivals are generated based on independent MDP. Price for various fare classes are denoted as $p_1, p_2, \ldots, p_K$, with p_f being the price for the lowest fare class. Maximum expected revenue is denoted as $R_n(b)$ where n corresponds to the decision period $0, 1, \ldots, N$, and b denoting the number of booking requests that have been accepted. C is the capacity of the single flight leg. (Refer to Figure 3)

The general idea is that for each fare class f there is a probability $P_{f,n}$ that a request will arrive for this fare class during decision period n. $P_{0,n}$ is the probability that no booking request occurs for any fare classes during decision period n. If the request is accepted, the accepted fare p_f is contributed to the maximum expected revenue for the next decision period $n-1$. In other words, the maximum expected revenue for the next time frame is $R_{n-1}(b+1)$ if the request is accepted, and $R_{n-1}(b)$ otherwise. The maximum expected revenue for a given time frame with certain realized bookings is calculated by accounting for the probability that an accepted booking, a rejected booking, or no booking request may occur. The following algorithm summarizes the Standard DPL:

$$\sum_{i=0}^{K} P_{i,n} = 1$$

$$R_n(b)=\sum_{f=1}^{K} P_{f,n}\cdot \max\{R_{n-1}(b+1)+p_f\,,R_{n-1}(b)\}+P_{o,n}\cdot R_{n-1}(b).$$

with the boundary conditions :

$$R_0(b)=\begin{cases} 0 & \text{if } \; b\le C\,, \\ -R\,(b-C) & \text{if } \; b>C, \end{cases}$$

where

$$R \ge \max_f \{p_f\}$$

At each booking arrival, the potential revenue generated with accepting the request is weighed against the expected future revenue loss due to the removal of that seat from the available capacity. The expected marginal seat revenue of the $(b+1)^{th}$ seat in decision period $n-1$ when there are b realized bookings is defined as:

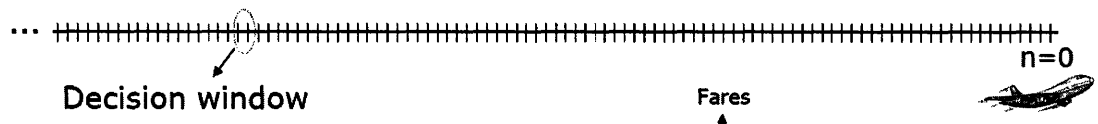
$$\Delta_{n-1}(b)=R_{n-1}(b)-R_{n-1}(b+1)$$

$$B_{f,n}=\min\{b\ge 0:\ \ \Delta_{n-1}(b)>p_f\}$$

where

$$B_{f,n}\le C$$

Optimal booking limits, $B_{f,n}$, for DPL are produced by backward induction. The policy is to accept a class *f* request in decision period n if and only if the condition $0\le b\le B_{f,n}$ holds. DPL is developed and applied more appropriately for fully or less-restricted environments because the model, like traditional RM methods, assumes independent demand for fare classes. Implementation of DPL in PODS simulations will be performed and analyzed in §3 and §4.



Fare Class		$P_{i,n}$: Prob. of a Bkg request in FC i, TF n	Expected revenue if accepted	Expected revenue if refused
1		$P_{1,n}$	$R_{n-1}(b+1) + p_1$	$R_{n-1}(b)$
2		$P_{2,n}$	$R_{n-1}(b+1) + p_2$	$R_{n-1}(b)$
f	:	$P_{f,n}$	$R_{n-1}(b+1) + p_f$	$R_{n-1}(b)$
K		$P_{K,n}$	$R_{n-1}(b+1) + p_K$	$R_{n-1}(b)$
Nobody shows up		$P_{0,n}$	$R_{n-1}(b)$	

Best expected revenue in Time-Frame (n-1) when b bookings occured

Bookings

$$R_n(b) = \sum_{f=1}^{K} P_{f,n} \cdot \text{Max}\{ R_{n-1}(b+1) + p_f,\ R_{n-1}(b)\} + P_{0,n}\, R_{n-1}(b)$$

Decision variable:
b: Number of bookings

Figure 3: The Standard Lautenbacher DP method (DPL)[7]

2.2.2 Gallego-Van Ryzin DP method

While the DPL model is applied more appropriately for fully restricted environments because of the assumption of demand independence among fare classes, the Gallego-Van Ryzin DP model (DP-GVR) is appropriate in fully unrestricted fare environments. RM methods that assume independence of segmented demand, including DPL, consider that passengers buy in all open fare classes irrespective of what the lowest open fare class is. In an unrestricted fare setting, however, passengers only buy in the lowest available fare class. Consequently these RM methods may not reach optimality in less-restricted fare structures.

Instead of allocating seats for each fare class, the objective of DP-GVR is to determine the lowest class that should be open at any given time. Optimal booking limits are determined only by the probability of sell-up by the passengers. Under an unrestricted fare structure in which there is no restrictions among fare classes which differ only in price, passengers will purchase in the lowest open fare classes. The DP-GVR model assumes that no passengers buy a ticket higher than the lowest open fare.

Gallego and van Ryzin (1997) first frame the problem and explain how to dynamically find an optimal pricing policy in a fare environment with only stochastic, price-oriented demand. Gallego and van Ryzin (2004) conclude that pricing policies

[7] Vanhaverbeke (2006)

derived from deterministic models produce close to optimal revenue. Although this implies RM methods based on dynamic programming may be relatively ineffective when pricing decisions are made correctly, in practice legacy carriers tend to match fares of LCC competitors to avoid losing market share, thus an optimal DP model for price-oriented demand is important to compensate for prices that are not optimally matched to demand.

The DP-GVR formulation first considers the arrival of random passengers through a Poisson process with rate $\lambda(p,s)$, defined as the number of booking requests s decision periods since the start of the booking period. The booking arrivals depend on the current (lowest) fare price, p. The booking period is divided into N subintervals, or decision periods, such that the probability of having more than one request during each decision period is negligible. The remaining capacity of the single flight leg is x.

In similar vein as DPL, DP-GVR allows that each of the K fare classes, $1, 2, \ldots, K$, may arrive throughout the reservation horizon. Price for various fare classes are denoted as $p_1, p_2, \ldots, p_K$, with p_f being the price for the lowest fare class. The maximum expected revenue for a given time frame with certain realized bookings is calculated by accounting for the probability that an accepted booking, a rejected booking, or no booking request may occur. The main difference here is that DP-GVR takes into consideration the possibilities of sell-up that depend on the set of open fare classes. $\Pr_{f,n}$ is the probability in decision period n that a passenger will sell-up to fare f. $\Pr_{0,n}$ is the probability that a passenger that arrives in decision period n decides not to choose any of the available fares. In other words, this is the probability that the passenger is spilled out. The following algorithm summarizes the DP-GVR model (Refer to Figure 4):

$$\sum_{i=0}^{K} \Pr_{i,n} = 1$$

$$J_n(x) = \max_f \{\lambda \cdot (\Pr_{f,n} \cdot (J_{n-1}(x-1) + p_f) + (1 - \Pr_{f,n}) \cdot (J_{n-1}(x))) + (1-\lambda) \cdot J_{n-1}(x)\}$$

The first term refers to when a passenger arrives and a booking is made in lowest class, whereas the second term corresponds to the case when either a passenger arrives and does not choose any of the available fare classes, or when a passenger does not arrive at all. If a passenger is willing to sell-up to fare class 1 with price p_1 which is the only class open, then the maximum expected revenue would be $p_1 + J_{n-1}(x-1)$, which refers to the maximum expected revenue for the next decision period $n-1$. The objective is to decide the lowest open fare class at each decision period n. The expected marginal seat revenue in decision period n when there are x remaining capacities is defined as:

$$\Delta J_n(x) = J_n(x) - J_n(x-1)$$

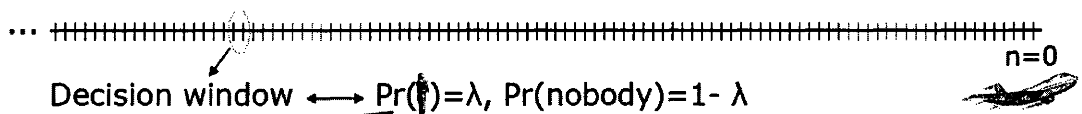
The sell-up probabilities are modeled through an exponential form $e^{-b_n \cdot (F_n - p_K)}$ where F_n is the price to charge and b_n is the constant in decision period n. The optimal price to charge is hence:

$$F_n^*(x) = \Delta J_n^*(x) + \frac{1}{b_n}$$

The idea is to close the lowest open fare class from p_f to p_{f+1} when the following condition holds:

$$\Pr_{f,n} \cdot \left(p_f - \Delta J_n(x)\right) < \Pr_{f+1,n} \cdot \left(p_{f+1} - \Delta J_n(x)\right)$$

$$\frac{\Pr_{f+1,n} \cdot p_{f+1} - \Pr_{f,n} \cdot p_f}{\Pr_{f+1,n} - \Pr_{f,n}} > \Delta J_n(x)$$



Fare Class		$Pr_{i,n}$: Prob. of sell-up to FC i, TF n	Expected revenue if accepts to sell-up	Expected revenue if refuses to sell-up
1		$Pr_{1,n}$	$J_{n-1}(x-1) + p_1$	$J_{n-1}(x)$
2		$Pr_{2,n}$	$J_{n-1}(x-1) + p_2$	$J_{n-1}(x)$
f	⋮	$Pr_{f,n}$	$J_{n-1}(x-1) + p_f$	$J_{n-1}(x)$
K		$Pr_{K,n}$	$J_{n-1}(x-1) + p_K$	$J_{n-1}(x)$
Wants nothing		$Pr_{0,n}$	$J_{n-1}(x)$	

Best expected revenue in Time-Frame (n-1) when there are x seats remaining

Remaining seats

$$J_n(x) = \mathrm{Max}_f \{\lambda.[Pr_{f,n}.(J_{n-1}(x-1)+p_f)+(1-Pr_{f,n})\, J_{n-1}(x)]+(1-\lambda).J_{n-1}(x)\}$$

Decision variable:
f: Lowest open class

Figure 4: The Gallego-Van Ryzin DP method (DP-GVR)[7]

2.3 Forecasting methods and Recent Developments

We mentioned in §1.2 and §2.1.2 that the legacy carriers have responded to the entrance of LCCs and implemented less differentiated fare classes, causing more passengers to exhibit price-oriented behavior and purchase at the lowest possible price. The traditional RM methods perform poorly under these conditions because a number of fundamental assumptions are violated. The combination of the inaccurate assumptions and the poor performance of traditional RM methods leads to what is known as the "spiral-down" effect, as described in §2.1.2. Optimal algorithms that do not make the traditional assumption of demand independence are however computationally difficult to formulate (Curry, 1990). To practically deal with price-oriented bookings in the optimal booking limit models, much research effort has been spent on modifying demand forecasts in recent years.

Weatherford (1999) and Zeni (2001) provide a summary of several traditional forecasting models commonly used in practice for RM. The pick-up forecasting model is a simple forecasting technique as forecasted incremental bookings based on historical trend is added to the number of current bookings to generate total forecasts. Other forecasting models include exponential smoothing, regression, multiplicative pick-up, and moving average. Descriptions of pick-up forecasting and other forecasting methods can be found in more depth in Gorin (2000) and Usman (2003).

Recent developments involve the notion of probability of sell-up, which is defined as the probability a passenger is willing to buy a higher fare ticket for the same flight when the passenger is denied booking for the requested fare class. Accounting for this probability in the forecast and in the optimization process will allow more seats to be protected for higher yield passengers and force them to sell-up when they will be denied booking for a lower class. The key is to figure out how to use these sell-up probabilities accurately, and how to estimate them dynamically throughout the entire forecasting period as inputs in RM.

"Q-Forecasting" (QF) and "Hybrid-Forecasting" (HF), developed by Belobaba and Hopperstad (2004), and "Fare Adjustment" (FA), developed by Fiig and Isler (2004) at Scandinavian Airlines (SAS) and Swissair, are recent developments in RM that incorporate the concept of sell-up probability in the seat allocation control models. The objective of Q-Forecasting and Hybrid-Forecasting is to forecast less segmented demand under fully unrestricted and less restricted fare environments, respectively, to be used as input in conventional RM. Fare Adjustment acts at the booking limit optimizer level. It incorporates potential of sell-up by adjusting the fares to feed the booking limit optimizer, resulting in greater protection for higher fare classes.

Recent research efforts have been made to incorporate these forecasting methods into traditional RM methods to adapt to less restricted fare structures. Cléaz-Savoyen (2005), Reyes (2006), and Soo (2007) present that Fare Adjustment is effective when implemented with Q-Forecasting or Hybrid Forecasting for airlines that use DAVN in a network setting. The use of Q-Forecasting and Hybrid Forecasting can prevent spiral-down and lead to revenues that are higher than those obtained with basic revenue management methods similar to those used by low-cost carriers.

Vanhaverbeke (2006) found that the performance of DPL in unrestricted and simplified fare structures is limited because its still assumes independent demand of fare classes, and generates results only slightly better than EMSRb but with much longer computation time. DP-GVR appears to be more promising but has generated even worse results than other RM methods under competitive scenarios in simulations. The challenge is that passengers' willingness to sell-up is difficult to estimate because it depends on competitor's fares and seat availability in future decision periods which vary with competition. Vanhaverbeke uses pre-determined estimates of sell-up that do not adapt to competition against more advanced RM methods and thus leads to poor performance. DP-GVR has significant sensitivity to forecasts of probabilities of sell-up that may need to be more accurately estimated in order to improve performance. A thorough description of how estimators are implemented in simulations will be presented in §3.

In this thesis, we will incorporate improved estimation of probabilities of sell-up in our simulation and investigate whether DPL and DP-GVR can prove its theoretical promises in simulations, and lead to robust improvement over traditional RM methods in unrestricted fare structures. Recent forecasting improvements incorporate adaptive estimation of passenger sell-up behavior based on historical bookings using (1) Forecast Prediction and (2) Inverse Cumulative approaches. A thorough explanation of the three approaches can be found in Guo (2007) and will be discussed in §4.2.1. They modify the forecaster by simultaneously improving the estimation of the demand based on the probability of sell-up. In addition, we will also examine the effect of incorporating Fare Adjustment, which proactively accounts for passenger sell-up in the seat optimizer, to DP methods within a fully unrestricted fare environment. The incorporation of these recent components in our simulations allows us to observe the performance of DPL and DP-GVR under various characteristics of the booking process.

In PODS, we will be simulating an airline which uses the two DP methods described within two network environments – a simple network consisting of only one OD market, as well as a more complicated, large airline network with close to 500 markets. Results allow us to look into the possibilities of using a dynamic programming approach in revenue management to aid airlines that are experiencing revenue deterioration in undifferentiated environments.

Chapter 3

Dynamic Programming RM and Forecasting Methodology

There are two major tasks in managing inventories in a traditional revenue management process: demand forecasting, either by flight legs or by origin-destination paths, and optimizing the closure of fare classes based on these forecasts. These two parts feed each other and are conducted separately, as shown in Figure 5. Forecasts are revised throughout the booking process based on the number of current bookings that have occurred at a given time. This can be done by dividing the period before departure into time frames and applying the RM process for each time frame. The data used by the forecaster are then unconstrained to estimate all potential demand that would have booked if no fare class was closed. The unconstrained demand forecasts are then inputted into the optimizer to determine which policy should be used throughout the booking period.

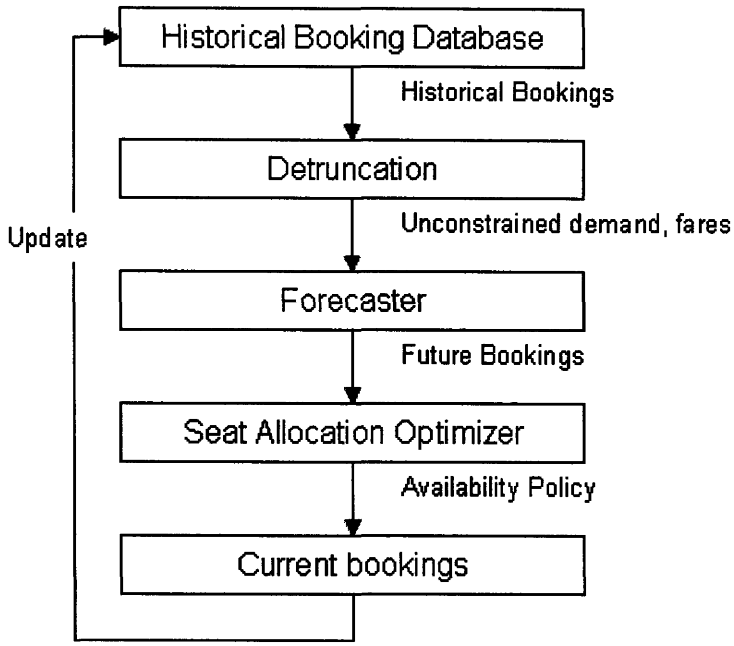


Figure 5: Traditional Revenue Management Process

The forecasts can be expressed as bookings occurring within a time frame or bookings-to-come, which is defined as forecasts of all bookings occurring between the current time frame and departure. Most traditional RM methods and DP methods use forecast of bookings-to-come. Furthermore, forecasts can be leg-based or path-based. Leg-based forecasts are class forecasts by flight legs. Path-based forecasts are class forecasts by origin-destination paths. Path-based forecasts can be converted into leg-based by summing up class forecasts of all paths associated with a given leg.

As described in the literature review, DP RM methods eliminate the assumption of sequential arrival order of bookings. Also, demand for each fare class is modeled as a Markov decision process. The booking period is divided into small decision periods. During each decision period, the probability to receive more than one booking request is negligible. Therefore, at a given time frame, a given flight leg with forecasts of 20 bookings-to-come should have longer decision periods than that with 60 bookings-to-come. When a booking request occurs in a decision period, the optimal policy produced by a dynamic programming algorithm decides whether to accept or refuse the booking.

Consider an example shown in Figure 6 when a request occurs for a given flight leg with c remaining seats at decision period $n+2$. When a request for booking occurs in a decision period, the decision to either accept or reject is made according to the optimal policy determined at the corresponding state. Each state is defined by decision period and remaining capacity. In this example, if a booking request arrives in decision period $n+2$ with c seats remaining, the request is refused.

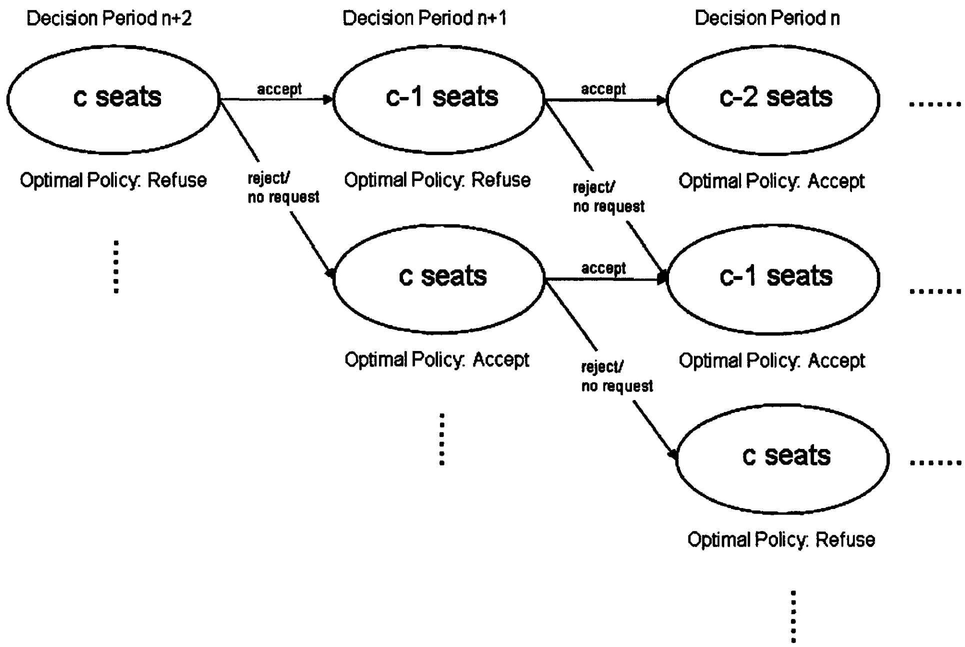


Figure 6: Example of an optimal policy based on a DP algorithm

Unlike traditional RM methods, the optimal policy produced by DP methods is not used to produce booking limits; it is directly applied to simultaneously control the closure of fare classes as bookings arrive within a time frame, and re-optimized before the start of the next time frame. While DPL determines which fare classes should be open for a given time frame, DP-GVR determines the lowest fare class to open.

3.1 Standard Lautenbacher DP

We have seen in the literature review that the algorithm that computes the expected maximum revenue in decision period n when b bookings have occurred is as follows:

$$R_n(b) = \sum_{f=1}^{K} P_{f,n} \cdot \max\{R_{n-1}(b+1) + p_f, R_{n-1}(b)\} + P_{o,n} \cdot R_{n-1}(b).$$

with the boundary conditions :

$$R_0(b) = \begin{cases} 0 & \text{if } \; b \le C, \\ -Z \cdot (b-C) & \text{if } \; b > C \end{cases}$$

where

$$Z \ge \max_f \{p_f\}$$

This algorithm accounts for the probability that an accepted booking, a rejected booking, or no booking request may occur, with the sum of these probabilities equal to one. That is,

$$\sum_{i=0}^{K} P_{i,n} = 1$$

DPL is performed on a leg level, so the probabilities of fare class booking requests have to be computed with leg/class forecasts of bookings-to-come for a given time frame. Depending on the fare environment, different forecasting methods can be used to obtain projected demand estimates by transforming the historical database in different ways using part or all of the available information. Pick-up forecasting is the traditional forecasting method commonly used under fully-restricted environments. The demand forecast at a given time is a function of the actual bookings plus the average demand that is expected to occur based on historical pick-up rates. With the introduction of unrestricted fare products, new forecasting methods are required to address the fundamental assumption, independence of demand by fare class, that is largely violated when using the Pick-up forecasting.

3.1.1 Q-Forecasting method

Q-Forecasting is designed to avoid spiral-down effect caused by interdependence among less differentiated fare classes. The general idea is to forecast only total demand at the lowest class and to account for the passengers' willingness to pay higher fares. Detruncated, or unconstrained, historical demand by time frame is obtained *only* for the class that was the lowest open fare class. The forecaster predicts an expected number of "Q-equivalent bookings" for each fare class, which is the equivalent demand for the Q class (lowest price) if Q is instead the lowest class open at that time. To do so, we use the probabilities of sell-up from Q class to higher fare classes. The number of Q-equivalent bookings for a time frame is thus equal to the number of bookings in a fare class divided by the probability of sell-up from Q class to this fare class. Table 4 illustrates an example of calculating Q-equivalent bookings for time frame tf .

$$hbk_{Q\to f,tf} = \frac{hbk_{f,tf}}{p\sup_{Q\to f,tf}}$$

where

- $hbk_{Q\to f,tf}$ denotes the estimated equivalent bookings for fare class Q in fare class f in time frame tf
- $hbk_{f,tf}$ denotes the mean unconstrained demand of fare class f in time

frame tf

- $p\sup_{Q\to f,tf}$ denotes the probability of sell-up from base fare class Q to a higher fare class f in time frame tf

Fare class f	$hbk_{f,tf}$	$p\sup_{Q\to f,tf}$	$hbk_{Q\to f,tf}$
1	0	2.5%	$0/0.025 = 0$
2	0	10%	$0/0.10 = 0$
3	1	25%	$1/0.25 = 4$
4	3	50%	$3/0.50 = 6$
5	7.5	80%	$7.5/0.80 = 9.4$
6	10	100%	$10/1.0 = 10$
Total Q-equivalent bookings for $tf = \sum_f hbk_{Q\to f,tf}$			**29.4**

Table 4: Example of calculating Q-equivalent bookings in time frame tf

Pick-up forecasting and detruncation are then used to estimate total unconstrained Q-equivalent bookings for tf. Using the same sell-up probabilities for fare classes, the total Q-equivalent bookings are then partitioned back into separate fare classes by estimating passengers that will sell-up to fare class f but not $f-1$. Class forecasts are therefore set equal to the potential demand of fare class f minus the potential demand of class $f-1$. These partitioned values represent the class forecasts that can be potentially realized when each fare class is the lowest class open. In other words, they will not materialize if the class is not the lowest open class. Table 5 illustrates such process using the same example as above, assuming the number of detruncated Q-bookings is 31.3.

$$fcst_{f,tf} = fcst_{tf} \cdot \left(p\sup_{Q\to f-1,tf} - p\sup_{Q\to f,tf}\right)$$

$$fcst\sigma_{f,tf} = fcst\sigma_{tf} \cdot \sqrt{p\sup_{Q\to f,tf} - p\sup_{Q\to f-1,tf}}$$

where

- $p\sup_{Q\to f,tf}$ denotes the probability of sell-up from base fare class Q to a higher fare class f (or $f-1$) in time frame tf
- $fcst_{f,tf}$ denotes the mean forecast for class f in time frame tf
- $fcst_{tf}$ denotes the total Q-equivalent bookings in time frame tf
- $fcst\sigma_{f,tf}$ denotes the standard deviation of forecast for class f in time frame tf
- $fcst\sigma_{tf}$ denotes the standard deviation of total Q-equivalent bookings in time frame tf

Fare class f	$p\sup_{Q \to f,tf}$	$fcst_{f,tf}$
1	5%	31.3*(0.025-0) = 0.78
2	10%	31.3*(0.10-0.05) = 1.57
3	25%	31.3*(0.25-0.10) = 4.70
4	50%	31.3*(0.50-0.25) = 7.83
5	80%	31.3*(0.80-0.50) = 9.39
6	100%	31.3*(1.00-0.80) = 6.26

Table 5: Example of calculating mean potential class forecasts in time frame *tf*

We should bear in mind that the probabilities of sell-up used to calculate potential demand are different among time frames. The reason is that bookings that occur at the end of the booking period tend to come from business passengers who have higher WTP than the early bookings that mostly comprise of leisure passengers with relatively lower WTP. The total forecasted bookings-to-come (BTC) for each fare class is finally determined by summing over all future time frames, as illustrated in Table 6.

f	Forecasted class bookings by time frame					Forecasted bookings-to-come
	tf^{*}	$tf+1$	$tf+2$	$tf+3$	$tf+4^{**}$	tf
1	0.78	1.27	1.85	4.61	7.12	16
2	1.57	2.54	3.91	10.90	3.46	22
3	4.70	6.40	7.83	6.30	4.60	30
4	7.83	9.12	15.65	10.13	3.89	47
5	9.39	12.13	15.65	4.21	1.97	43
6	6.26	16.34	13.24	3.67	0.56	40

* tf corresponds to the current time frame.

** $tf+4$ corresponds to the last time frame before departure.

Table 6: Example of calculating forecasted bookings-to-come in time frame *tf*

Q-Forecasting has recently been modified by Hopperstad to explicitly account for advance purchase (AP) requirements. This sets the partitioned class forecast to zero for all expired classes in each current or future time frame. In fact, since this thesis focuses on an unrestricted fare structure where there are no AP requirements, forecasted demand can potentially be realized in all fare classes in any time frames. Therefore, all class forecasts are included in the summation to generate bookings-to-come forecasts without the need to eliminate potential demand associated with closed fare classes due to AP requirements.

Q-Forecasting as illustrated by the example above partitions Q-forecasts by time frame into class forecasts using time frame sell-up probabilities, and produces forecasts of bookings-to-come by summing class forecasts of all time frames. An alternative process is to first compute the total Q-equivalent bookings-to-come by summing up Q forecasts for all time frames, and then partition into class bookings-to-come using the probabilities of sell-up-to-come computed as a weighted average of sell-up probabilities among all future time frames. A full description of the definitions, formulas, and derivations of Q-Forecasting can be found in Belobaba and Hopperstad (2004). Extensive simulation results analyzing the performance of this forecasting method can also be found in Cléaz-Savoyen (2005). Figure 7 is a flow chart that summarizes the Q-Forecasting process. Formulation of sell-up probabilities and their application in Q-Forecasting in PODS will be illustrated in §4.

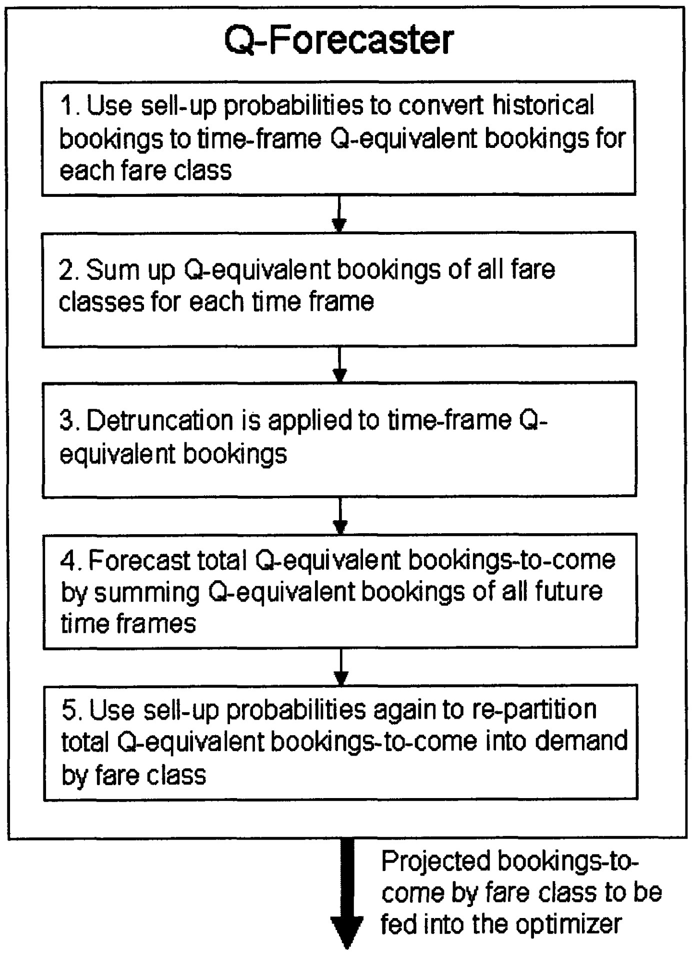


Figure 7: Q-Forecasting Process flow-chart

3.1.2 Hybrid-Forecasting method

While traditional Pick-up forecasting is used to estimate the product-oriented demand, Q-Forecasting can be applied to estimate the price-oriented demand. Hybrid-Forecasting method, developed by Belobaba and Hopperstad (2004), is a combination of the two concepts. In the simulations performed in this thesis, passengers are considered to be price-oriented if they are observed buying the lowest fare class available, that is, the next lower class is closed. If they purchase a fare product when the next lower class is still open, which means that the fare product is not the lowest price available, they are considered as product-oriented demand.

Hybrid-Forecasting is appropriate when an airline uses a semi-restricted fare structure, in which there are undifferentiated fare classes as well as higher fare classes that are differentiated by some restrictions from the lower classes. The idea is to classify all bookings into one of the two demand categories and apply a separate forecasting method for each. We mention in previous section that Q-Forecasting considers only observed bookings of classes that were the lowest open fare classes. The rest of the observed bookings are treated as historical data for the associated classes, and standard Pick-up forecasting is applied. The two sets of projected bookings-to-come (product-oriented and price-oriented) are then aggregated to feed the optimizer. Extensive simulation results of analyzing the performance of Hybrid-Forecasting can be found in Reyes (2006). Figure 8 summarizes the Hybrid-Forecasting process in a flow chart.

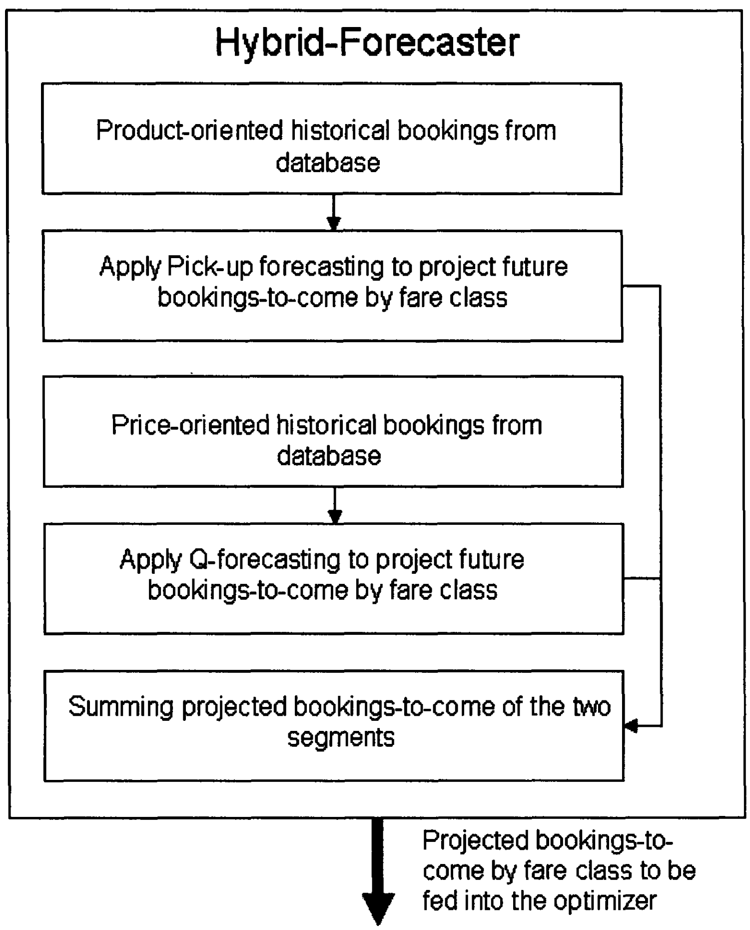


Figure 8: Hybrid-Forecasting process flow-chart

A shortcoming of classifying product and price-oriented demand this way is that passengers who buy in the lowest class open may not necessarily be price-oriented if that is the fare product they particularly seek to buy. On the other hand, passengers that book in the highest fare class may not necessarily be product-oriented if they are just enticed to sell-up to the highest fare class that is also the only class open.

3.1.3 Optimizer Based on DPL algorithm

We mention earlier in this chapter that the probabilities of fare class booking requests used in the DPL algorithm have to be computed with leg/class forecasts of bookings-to-come for a given time frame. Regardless of which forecasting method to be used in the forecaster, the output that we want to feed the optimizer is the projected bookings-to-come by fare class for each leg to determine the arrival rate of booking requests. In a network environment, fare class forecasts by path are essentially produced from observed bookings. They need to be converted to leg-based forecasts since the optimizer based on DPL uses leg/class forecasts.

Once the path forecasts of fare class bookings-to-come are received from the forecaster, they are first split out by time frame using estimated booking curves by class for each path. The obtained path/class forecasts for each time frame are then rolled up into leg/class forecasts by adding for each leg the forecasts of all paths that include that flight leg. Path booking curves can be estimated according to forecasted time-frame Q-equivalent bookings by path which is as an intermediate step of producing bookings-to-come forecasts in the forecaster. The number of decision periods divided for each time frame in the DPL algorithm is set equal to the sum of leg forecasts of all fare classes for that time frame. Average leg/class fares are computed as the average weighted fare by partitions across the associated paths and are thus different among time frames. DPL determines an optimal policy based on the maximum expected value computed for future time frames in reverse order. The policy is then applied in the current time frame and re-optimized again before the start of the next time frame (Refer to Figure 9).

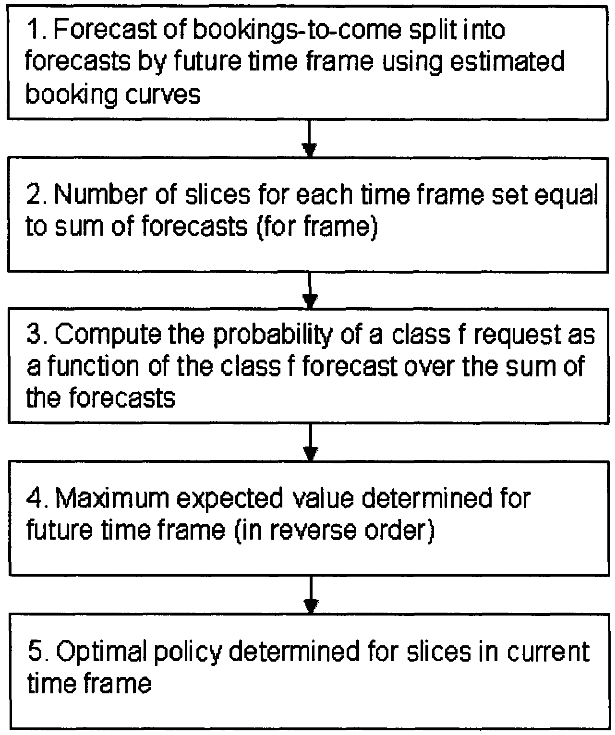


Figure 9: Bookings-to-come forecasts processed in DPL optimizer

As mentioned before, optimal policy by DPL is not processed into booking limits of fare classes in PODS. Using the policy to simultaneously close down or open fare class as bookings arrive is, in effect, equivalent to producing optimal booking limits, $B_{f,n}$, using backward induction. The policy is to accept a class *f* request in decision period n if and only if the condition $0 \le b \le B_{f,n}$ holds:

$$\Delta_{n-1}(b) = R_{n-1}(b) - R_{n-1}(b+1)$$

$$B_{f,n} = \min\{b \ge 0 : \quad \Delta_{n-1}(b) > p_f\}$$

where

$$B_{f,n} \le C$$

The following simple example of a single flight leg illustrates how DPL works under an unrestricted fare structure. For the simplicity of the problem, we assume that the total forecasted bookings-to-come produced by Q-Forecasting at the current and also the last time frame is 3 and consequently the number of decision periods is 3. We also assume that this is a 100-seat flight and 98 bookings have been observed. Table 7 presents the underlying 3-class average fares and the arrival rates of booking requests by class estimated from the leg/class forecasts for the current time frame.

Class f	p_f	$P_{f,n}$, $1 \le n \le 3^*$
0 (no request)	-	10%
1	\$500	45%
2	\$250	30%
3	\$125	15%

* $1 \le n \le 3$ corresponds to the last time frame before departure

Table 7: Example of Arrival rate of class bookings in current time frame

Table 8 shows how the values of $R_n(b)$ are computed in each decision period. The table is computed by starting on the top row, i.e. decision period 1 (closest to departure). The bold values represent the maximum expected revenue of a given decision period and observed bookings and consequently correspond to the optimal policy of DPL – the fare classes that should be open at the current time frame. In this example, we see that the airline should start the last time frame with class 1 and 2 open (n=3, b=98).

$$R_n(b) = \sum_{f=1}^{K} P_{f,n} \cdot \max\{R_{n-1}(b+1) + p_f, R_{n-1}(b)\} + P_{o,n} \cdot R_{n-1}(b)$$

n	Classes open	Observed bookings, b		
		98	99	100
1	1, 2, 3	**318.8**	**318.8**	< 0
	1, 2	300	300	< 0
	1	225	225	< 0
	None	0	0	**0**
2	1, 2, 3	**637.5**	350.6	< 0
	1, 2	618.8	379.7	< 0
	1	543.8	**400.3**	< 0
	None	318.8	318.8	**0**
3	1, 2, 3	742.8	358.8	< 0
	1, 2	**759.6**	400.1	< 0
	1	755.8	**445.2**	< 0
	None	637.5	400.3	**0**

Table 8: Example of computing the expected revenue for DPL

While the probability of class f request can be computed by the ratio of class f forecasts to the sum of all class forecasts, using such estimate does not satisfy the requirement that booking arrivals are not sequentially ordered by fare class. Although DPL sets optimal booking limit to control the partition the next booking request belongs to as it arrives, the optimizer does not really know which type of passenger will be arriving next. On the other hand, setting the probability of a class f request to simply the class f forecast over the sum of the forecasts assumes that the ratio of variance to mean is equal to 1, which is in most cases an overly certain estimation of demand distribution. To better model the arrival pattern of passenger types especially in the event when forecasts are more uncertain with higher variance, the first basic question we should ask is the following: Is the next (previously arriving) passenger who arrives different from the current passenger, i.e. book in a different fare class? Using the Q-equivalent mean demand and variance forecasts, we can express the arrival pattern as a Poisson process, and set a value on the probability that the previously arriving passenger is the same as that of the current one, i.e. book in at least the same fare class (Refer to Figure 10).

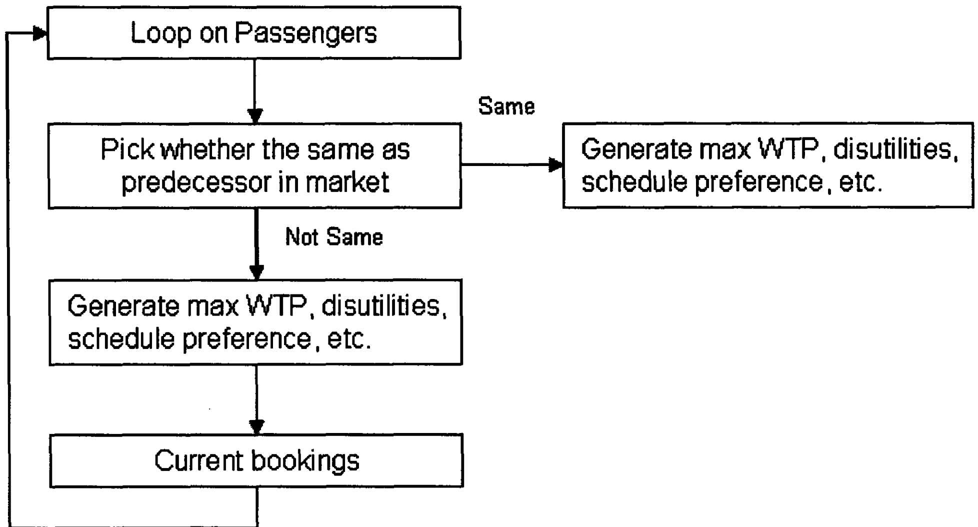


Figure 10: Modeling the arrival pattern of passenger types in DPL[8]

Assume that a class k request ($1 \le k \le K$) arrives in decision period n when the number of observed bookings is b. Then for slice $n+1$ (previous decision period) at booking level $b-1$, we define *psame* as the probability that the previously arriving passenger also had the same passenger choice characteristics, i.e. maximum willing-to-pay, disutilities, schedule preferences, etc., as the current passenger. That is, the passenger would book in one of the classes between 1 and k as his/her maximum WTP should be at least P_k. Then, the revised calculation of arrival probability for each of the fare classes is as follows:

$$P^*_{f,n} = \begin{cases} psame \cdot \dfrac{P_{f,n}}{\sum_{fl=1}^{k} P_{fl,n}} + (1 - psame) \cdot P_{f,n} & \text{for } 1 \le f \le k \\ (1 - psame) \cdot P_{f,n} & \text{for } k < f \le K \end{cases}$$

psame can be expressed by variance-to-mean ratio of the Poisson process known as the *Zfactor*. The higher the variance of forecasts is, the more likely the bookings arrive in sequential order by fare class, and the higher the *psame* will be. This *Zfactor*

[8] Guo (2006)

adjustment enables DPL to work with more uncertain forecasts by relaxing the assumption that variance and mean forecasts have to be equal. Let Z_{calc} be the value of *Zfactor* obtained from Q-equivalent demand mean and variance. Also let Z_{input} be a specified maximum value of *Zfactor* allowed in modeling the arrival pattern of passenger types. Then,

$$Zfactor = \min\left\{ Z_{calc}\ , Z_{input} \right\}$$

$$psame = \frac{Z-1}{Z+1}$$

When *Zfactor* has a value of 1, i.e. variance equals mean, the probability that the previously arriving passenger also came from the same type as the next arriving passenger is zero. Consequently, the probability of arrival of each fare class at the current decision period is unchanged from the previous one. On the other hand, for *Zfactor* of 2, the probability that the previously arriving passenger is the same as the next one is 1/3. In this thesis, we compare *Zfactors* 1, 2, 3, and 4 to test the sensitivity of *Zfactor* adjustment to the performance of DPL (See Table 9).

Zfactor	*psame*
1	(1-1)/(1+1) = 0
2	(2-1)/(2+1) = 1/3
3	(3-1)/(3+1) = 1/2
4	(4-1)/(4+1) = 3/5

Table 9: Computation of *psame* from *Zfactor* adjustment

3.1.4 Fare Adjustment

Fare Adjustment (FA) was originally developed by Fiig and Isler (2004) to improve the performance of DAVN in unrestricted and semi-restricted (a combination of differentiated and undifferentiated fare structures) fare environments. We mention earlier that Q-Forecasting is implemented in an unrestricted fare structure to improve forecasts of passengers that tend to book in the lowest open class. The objective of FA is to incorporate this sell-up behavior within the seat allocation optimizer, so that more high-class seats will be protected by closing down lower classes faster. Detailed description of the Fare Adjustment methodology can be found in Soo (2007) and Kayser (2008).

The general idea of adjusting OD fare as proposed by the FA method is that, instead of feeding the network LP all passengers' OD fare to calculate the displacement cost for a given leg (as described in §2.1.1.2), the method uses the "Marginal Revenue" (MR), which is calculated by subtracting from the OD fare the "Price-Elasticity Cost" (PE Cost) to account for the potential risk of buy-down under an unrestricted fare structure. In other words, while the "pseudo-fare" that is originally bucketed and optimized is calculated by OD fare minus the displacement cost, FA reduces this pseudo-fare by the PE Cost and sends the adjusted pseudo-fare to lower buckets. Under a semi-restricted fare structure, the PE cost is only applied to undifferentiated fare structure, consequently allowing the two different sets of fare structures to decouple and be managed independently. In a fully unrestricted fare structure, that is the main focus of this thesis, the subtraction of the PE Cost allows the undifferentiated fare to be mapped into a lower bucket. As the passengers' WTP increases, the PE Cost will also increase, reducing MR and closing down the lower fare classes more quickly.

$$Adjusted\ Fare_l = MR_{p \to l} - Displacement\ Cost_l$$

$$MR_p = OD\ Fare_p - PE\ Cost_p$$

We should note that, since DPL seat allocation optimization is performed on the leg level, OD Fare and MR that are originally computed by the path/class (p) are therefore needed to be rolled up into leg/class (l). Also, the average fares are not adjusted by displacement cost ($Adjusted\ Fare_l = OD\ Fare_{p \to l} - PE\ Cost_{p \to l}$). In DPL with QF and FA, the adjusted fares are different across future time frames, and are used in the backward recursions to solve through those time frames.

Ken Sejling proposed that class demand in the current or future time frames with zero or negative adjusted fares should be treated in the same fashion and should be set to be zero since they are certainly made unavailable to passengers (Belobaba, 2007c). Therefore, FA that is used in this thesis assumes that partitioned class forecasts are set to zero for all expired classes or all classes with negative adjusted fares in current or future time frame. This correction to the FA method is known as the "KS Fix".

3.2 Gallego-Van Ryzin DP

The objective of DP-GVR is to determine the lowest class that should be open at each decision period n. Optimal booking limits are determined only by the probability of sell-up by the passengers. The formula that is used to compute the expected maximum revenue in decision period n when b bookings have occurred is as follows:

$$\sum_{i=0}^{K} \mathrm{Pr}_{i,n} = 1$$

$$J_n(x) = \max_f \left\{ \lambda \cdot \left(\mathrm{Pr}_{f,n} \cdot \left(J_{n-1}(x-1) + p_f \right) + \left(1 - \mathrm{Pr}_{f,n}\right) \cdot \left(J_{n-1}(x) \right) \right) + (1-\lambda) \cdot J_{n-1}(x) \right\}$$

Under unrestricted fare structure in which there is no restrictions among fare classes which differ only in price, passengers will purchase in the lowest open fare classes. Q-Forecasting method is therefore appropriate to produce leg-based forecasts.

3.2.1 Optimizer Based on DP-GVR algorithm

Unlike DPL that receives path/class forecasts of bookings-to-come from the forecaster and rolls them up into leg-based forecasts by time frame, DP-GVR in PODS directly produces those forecasts on the leg level and use the associated sell-up probabilities by time frame in the recursion to generate optimal policy. It produces leg-based forecasts of Q-equivalent bookings-to-come by summing those forecasts for paths that include that flight leg. These forecasts are then split out by time frame according to the leg-based estimated booking curves:

$$fbtf = \frac{fbtc \cdot \left(pbook_{tf} - pbook_{tf-1} \right)}{1 - pbook_{tf-1}}$$

where

- *fbtf* denotes forecasts of Q-equivalent bookings by time frame
- *fbtc* denotes forecasts of Q-equivalent bookings-to-come
- $pbook_{tf}$ denotes the ratio of the number of historical bookings recorded until time frame *tf* to the total number of historical bookings until the day of departure

In DP-GVR, the number of decision periods divided for each time frame is set equal to the Q-forecasts for that time frame. Each leg/class fare is computed as the average historical mix of path/class fares. The forecasts are used to compute for the probabilities of sell-up for each fare class in the DP-GVR algorithms, as well as to model the arrival pattern of passenger types. Like DPL, DP-GVR determines an optimal policy based on the maximum expected value computed for future time frames in reverse order. The policy is then applied in the current time frame and re-optimized again before the start of the next time frame (Refer to Figure 11).

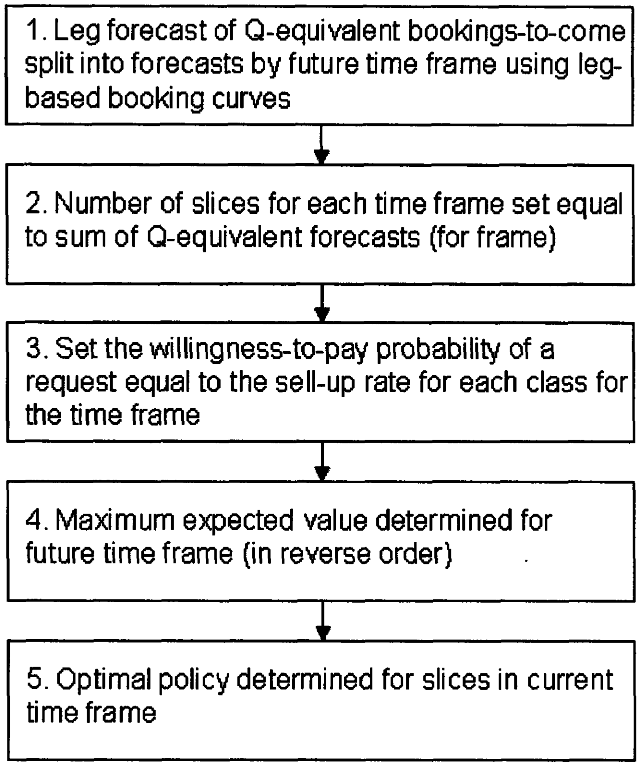


Figure 11: Bookings-to-come forecasts processed in DP-GVR optimizer in PODS

When a booking arrives during a decision period, there is a probability $\Pr_{f,n}$ that the passenger pays the current fare p_f that adds to the total revenue, but the inventory will then be reduced by one seat. If there is no arrival during the decision period, the capacity of the inventory still remains x. The policy is obtained by averaging the bid prices for the decision period associated with the current time frame. The bid price in decision period n when there are x remaining capacities is defined as:

$$\Delta J_n(x) = J_n(x) - J_n(x-1)$$

The sell-up probabilities are modeled through an exponential form $e^{-b_n \cdot (F_n - p_K)}$ where F_n is the price to charge and b_n is the constant in decision period n. The optimal price to charge is hence:

$$F_n^*(x) = \Delta J_n^*(x) + \frac{1}{b_n}$$

The idea is to close the lowest open fare class from p_f to p_{f+1} when the following condition holds.

$$\Pr_{f,n} \cdot \left(p_f - \Delta J_n(x)\right) < \Pr_{f+1,n} \cdot \left(p_{f+1} - \Delta J_n(x)\right)$$

$$\frac{\Pr_{f+1,n} \cdot p_{f+1} - \Pr_{f,n} \cdot p_f}{\Pr_{f+1,n} - \Pr_{f,n}} > \Delta J_n(x)$$

In the event that the WTP increases in decision periods of later time frames, the left side of the inequality will decrease, causing the control mechanism to close down lower fare classes. Note that the calculation of the optimal fare as a function of bid-price can be done over continuous fares, but for the study of this thesis this optimization method is performed over discrete fare by class to conform to the fare structure currently practiced by the industry.

The arrival pattern of passenger types in DP-GVR is modeled by the same approach as DPL. Again, if we assume that a class k request ($1 \le k \le K$) arrives in decision period n when the number of observed bookings is b, then for slice $n+1$ (previous decision period further from departure) at booking level $b-1$, the likelihood that this previously arriving passenger belongs to the same passenger type and would book in one of the classes between 1 and k is used to compute the maximum WTP. The revised algorithm of the maximum expected value in decision period n and b observed bookings for the simulations of this thesis is:

$$U_n(b) = U_{n-1}(b) + \max_f \left\{ pwtp_{f,n} \cdot \left(p_f + U_{n-1}(b+1) - U_{n-1}(b) \right) \right\}$$

where

- p_f denotes the leg-based decision fare for class f
- $pwtp_{f,n}$ denotes the probability passenger making a booking request in decision period n would be willing to pay at least the fare associated with class f

To illustrate how DP-GVR works under an unrestricted fare structure, the following single flight example will be used. We assume that the total forecasted bookings-to-come produced by Q-Forecasting at the current and also second-to-last time frame is 3 and consequently the number of decision periods is 3. Note that the optimizer needs to compute the forecasted class bookings by time frame on the leg level. It therefore uses leg bookings curves to split this bookings-to-come forecast across all future time frames. Let's assume 2 bookings in time frame tf and 1 in the last time frame $tf+1$. We also assume that this is a 100-seat flight that has currently has 98 observed bookings. Table 10 shows the underlying 3-class adjusted fares and the probabilities of willingness-to-pay in each decision period. It is important to understand that the adjusted fares associated with future time frames are used in the backward recursions solving through those time frames. Therefore, to make it simple, this example assumes that the last two time frames coincidently have the same adjusted fare, not that the current-time-frame adjusted fare is used throughout.

Class f	p_f	$pwtp_{f,n}$ $n = 2, 3^{*}$	$pwtp_{f,n}$ $n = 1^{**}$
1	\$500	20%	40%
2	\$250	45%	75%
3	\$125	100%	100%

* n=2, 3 represents the decision periods that share the same probabilities of WTP within same tf
** n=1 represents the decision period in the last time frame, $tf + 1$, before departure

Table 10: Example of Arrival rate of class bookings in current time frame

Table 11 shows how the values of $U_n(b)$ are computed in each decision period. The table is computed by starting on the top row, i.e. decision period 1 (closest to departure). The bold values represent the maximum expected revenue of a given decision period and observed bookings and consequently correspond to the optimal policy of DP-GVR, i.e. the lowest fare class open at the current time frame. In this example, we see that the airline should start the second-to-last time frame with class 1 being the lowest open class (n=3, b=98).

$$U_n(b) = U_{n-1}(b) + \max_f \left\{ pwtp_{f,n} \cdot \left(p_f + U_{n-1}(b+1) - U_{n-1}(b) \right) \right\}$$

n	Lowest Class Open	Observed bookings, b		
		98	99	100
1	1	**200**	**200**	0
	2	187.5	187.5	0
	3	125	125	0
2	1	300	**260**	0
	2	312.5	222.5	0
	3	**325**	125	0
3	1	**412**	**300**	0
	2	408.3	255.5	0
	3	385	125	0

Table 11: Example of computing the expected revenue for DP-GVR

Like DPL, the sell-up probabilities as output from the forecaster assumes that variance and mean forecasts are equal, which is in most cases an overly certain estimation of demand distribution. Then the revised calculation for the maximum willing-to-pay is as follows. *psame* can be expressed by *Zfactor* the same way as DPL, but using leg-based Q-equivalent mean forecasts and variance.

$$pwtp^{*}_{f,n,b-1} = \begin{cases} psame \cdot \dfrac{pwtp_{f,tf}}{pwtp_{k,tf}} + (1 - psame) \cdot pwtp_{f,tf} & for\ 1 \le f \le k \\ psame + (1 - psame) \cdot pwtp_{f,tf} & for\ k < f \le K \end{cases}$$

Depending on the number of seat remaining in each decision period, we will try to get from them the maximum expected value. We will then deduce in each decision period what the lowest open fare class should be. Figure 12 presents an example of the optimal policy for a flight leg when a request occurs for a given flight leg with c remaining seats at decision periods from n to $n+2$. For example, if there are $c-1$ seats remaining in period n, class 3 would be the lowest fare class open according to the policy.

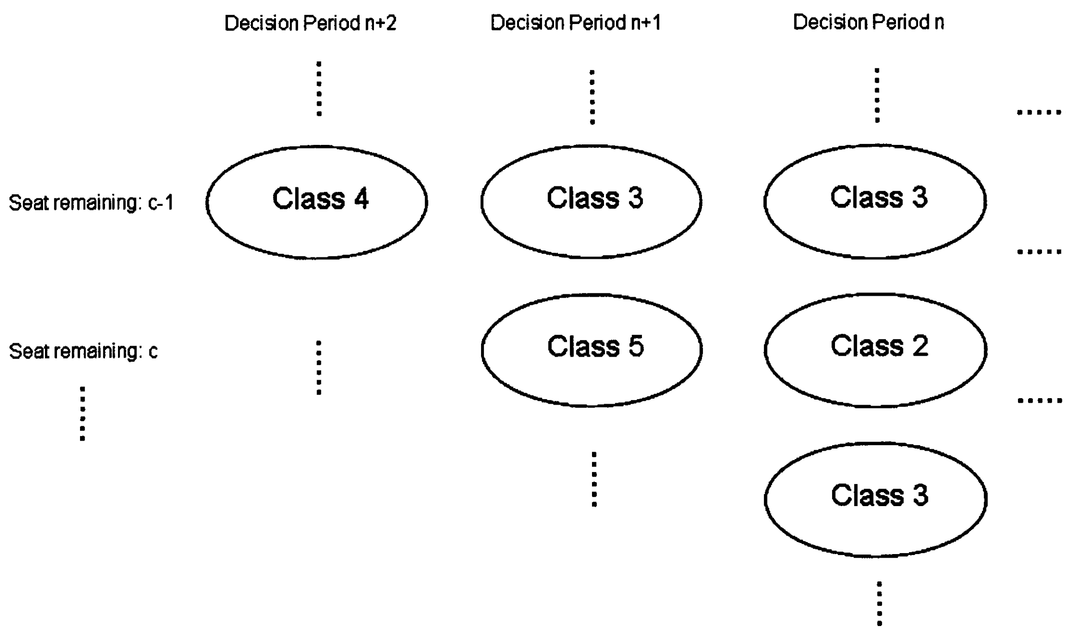


Figure 12: Example of lowest fare class to be open based on DP-GVR

3.3 Chapter Summary

In this chapter, we discuss two main RM methods that utilize dynamic programming. DPL determines which classes should be open for a given time frame, whereas DP-GVR determines the lowest fare class to open. We also discuss several techniques to deal with sell-up behavior of passengers when less restricted fare structures are applied. Q-Forecasting and Fare Adjustment are recent developments in RM that incorporate the concept of sell-up probability in the forecasting and seat allocation control models, respectively. The objective of Q-Forecasting is to forecast less segmented demand under fully unrestricted fare structure to be used as input to the conventional RM. On the other hand, Fare Adjustment acts at the booking limit optimizer level. It incorporates potential of sell-up by adjusting the fares to feed the booking limit optimizer, resulting in greater protection for higher fare classes.

Chapter 4

Simulation Environment

The Passenger Origin-Destination Simulator (PODS) was developed in 1997 at the Boeing Company by Hopperstad, Berge, and Filipowski as an evolution from its predecessor, the Decision Window Model (DWM)[9] for passenger choice. It is a software simulator of hypothetical airline networks used to test and analyze the performance of revenue management techniques, such as the optimization models and forecasting methods in particular, in different competitive and controlled simulation environments. For the purpose of this study, we first present an overview of the various component modules that comprise PODS and a few features of several important inputs that are required for our simulations. Then we describe how the forecasting and seat inventory control methods are modeled in PODS that relate to the theory introduced in §3. Detailed explanations of the operations of PODS can be found in Wilson (1995) and Lee (1998).

4.1 Overview of the PODS Structure

The basic idea of PODS is to simulate the interactions between passenger and airline decisions that occur in real-world air travel. On one end, the simulated passengers seek for air travel in their specific OD markets and decide among multiple airlines, paths, and fare classes available to them. On the other end, the airlines decide which air travel products to be made available to their customers based on their observations of booking behavior. Figure 13 describes how an airline booking process is simulated in the PODS structure of two separate but interactive components, namely, (1) the Passenger Choice Model and (2) the Revenue Management System.

[9] The Boeing Company (1994)

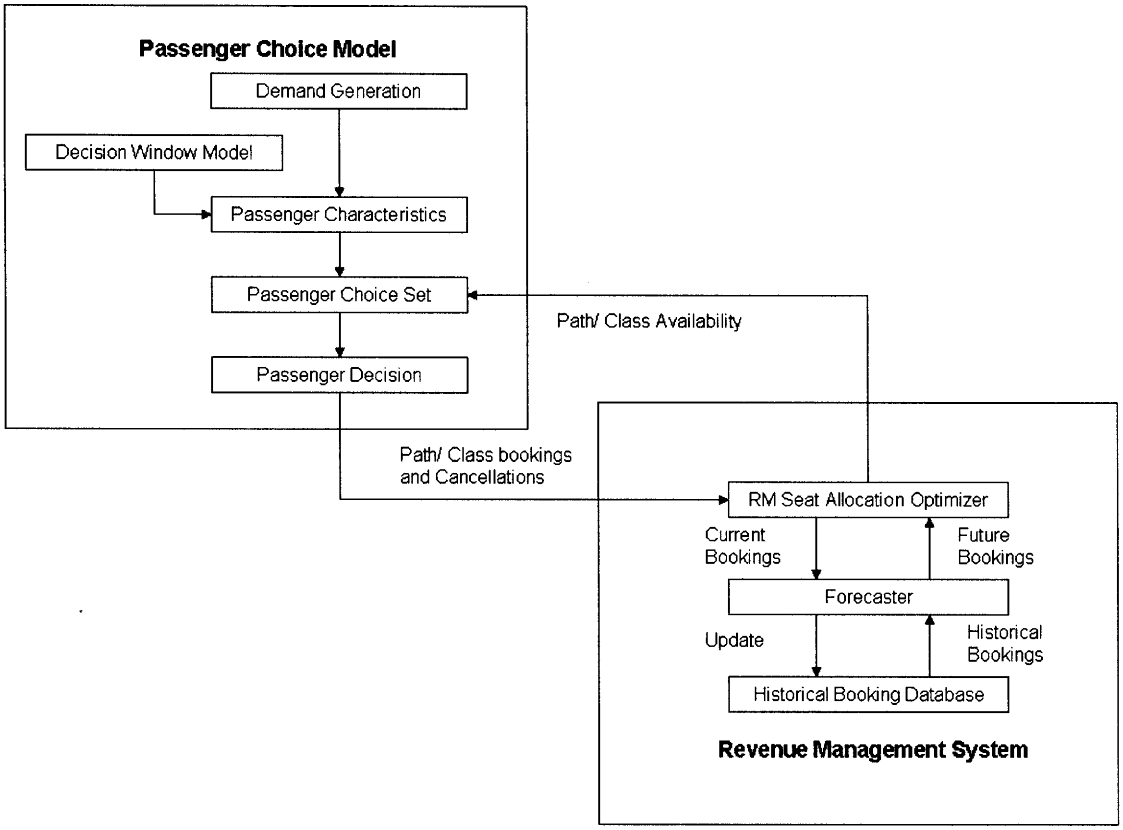


Figure 13: PODS Architecture

The Revenue Management System in PODS is similar to the third generation RM System described in §2.1. It consists of (1) a historical booking database, (2) a forecaster, and (3) a seat allocation optimizer. At the start of each simulation run, the forecaster takes in as inputs the historical bookings of a given flight from the database as well as the current booking levels to estimate future bookings, which are then fed into its optimizer to determine the seat protections and availability in terms of booking limits for each fare class for that time frame. Competing airlines within a network have a wide variety of choices for their own fare structures and forecasting and optimization methodologies. Each airline's seat availability policy is determined by the RM System at the start of each time frame.

The objective of the Passenger Choice Model is to generate passengers and simulate the decision processes according to the preferences of these passengers. Originally, the DWM model was used to determine passenger preferences based on the schedules, the image, and the aircraft types used by the airline (Belobaba, 2002). The Passenger Choice Model in PODS extends the schedule choice model of DWM by incorporating additional capabilities of simulating passenger choice by fare and restrictions. It first receives the seat protections and availability policy from the RM System, generates demand and individual characteristics of passengers, and assigns to them potential paths that fit their schedule decision windows. The model then determines whether each passenger is accepted to a path based on the availability of the path/fare product combination and his/her preferences. These preferences include the passenger's willingness-to-pay, and disutility costs associated with restrictions, a connecting path, and airline preference. Booking information is finally transferred back to the RM System, in which the historical database is updated for the next simulation run that repeats the process. More detailed descriptions of the passenger generation, characteristics, choice sets, and decisions in the Passenger Choice Model can be found in Carrier (2003). Typical components within a RM System in PODS are also fully discussed in Wilson (1995), Lee (1998), and Gorin (2000).

In PODS, a single simulation "run" consists of 5 independent "trials", each of which corresponding to an iterative result of 600 departure days, known as "samples", on each leg. The first sample is initiated by user-defined inputs, which are gradually updated with new computed data for the next sample. In each trial, the first 200 samples are discarded to eliminate the effects of initial conditions since each sample has some degree of correlation to the next sample. To ensure statistical significance of simulation results, the overall result for each simulated airline is thus obtained by averaging results of the last 400 samples for all of the 5 trials that add up to a total of 2000 daily simulations.

For each sample or departure, the booking process is divided into 16 time frames (TF). Booking limits are re-optimized at the start of each time frame until departure, whereas the interactions between passengers (bookings and cancellations) and airlines (close or reopen fare classes) are simulated within each of these time frames. It assumes that flights are open for bookings 63 days before departure, and the duration of each time frame becomes smaller as it approaches departure (Refer to Table 12).

Time Frame	Days until Departure	Time Frame Duration (days)
1	63	7
2	56	7
3	49	7
4	42	7
5	35	4
6	31	3
7	28	4
8	24	3
9	21	4
10	17	3
11	14	4
12	10	3
13	7	2
14	5	2
15	3	2
16	1	1

Table 12: Booking Process Time Frames

To control our simulation environment of this thesis, we perform tests based on a set of user-defined demand factors (DF). The demand factor is set to simulate periods of low and high demand. DF of 1.0, 0.9, and 0.8 reflect that experiments are performed at high, medium, and low demand intensity, respectively. Zickus (1998) provides a full summary of the major inputs as required by a PODS sample run.

What separates PODS from other transportation simulation models is its uniqueness of modeling passenger's preferences. Simulated passengers are first categorized to be one of the two passenger types – business or leisure, according to the arrival pattern of the pre-defined booking curves (Refer to Figure 14). It is important to note that the historical database in the RM System in PODS is updated and stored in the form of bookings by fare class. In other words, the seat allocation optimizer does not know the passenger type corresponding to each booking.

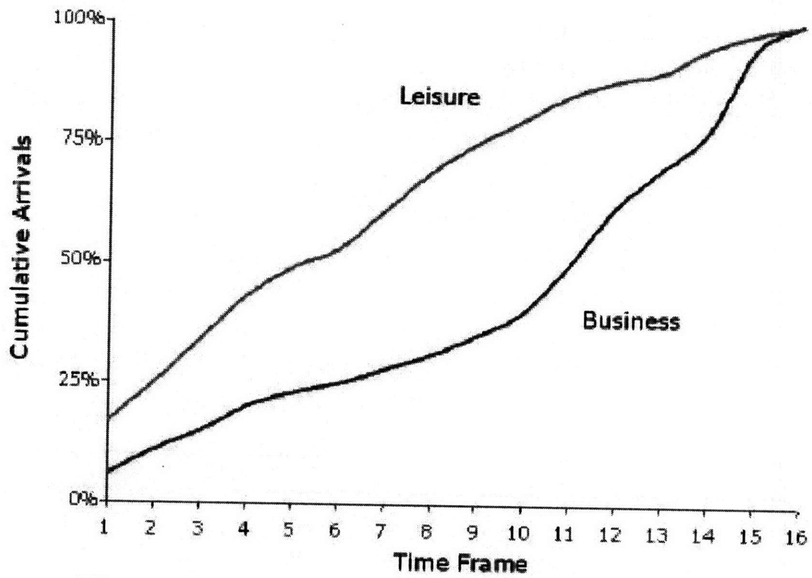


Figure 14: Booking Arrival Curves by Passenger Type[10]

Each passenger is then assigned a set of disutility costs associated with path characteristics and restrictions of each fare class based on the input Gaussian densities for that passenger type. Generalized costs are the sum of fare, disutility costs of restrictions, connecting costs, unfavorite airline costs, and replanning costs. Replanning cost is materialized if the path requires departure or arrival outside the decision window of the passenger. The current PODS simulations can apply up to three restrictions to each fare product: (1) Saturday night stay (R1), (2) cancellation or change penalty (R2), and (3) non-refundability (R3). The passenger is then simulated to determine the choice sequence among available fare products with the lowest equivalent fare value, computed by summing the product's nominal fare and generalized costs, always less than the maximum out-of-pocket fare they are willing to pay. This maximum WTP designated to

[10] Belobaba (2007b)

each passenger is modeled in PODS by a Gaussian distribution as the probability that a random passenger will pay a particular given fare (Refer to Figure 15).

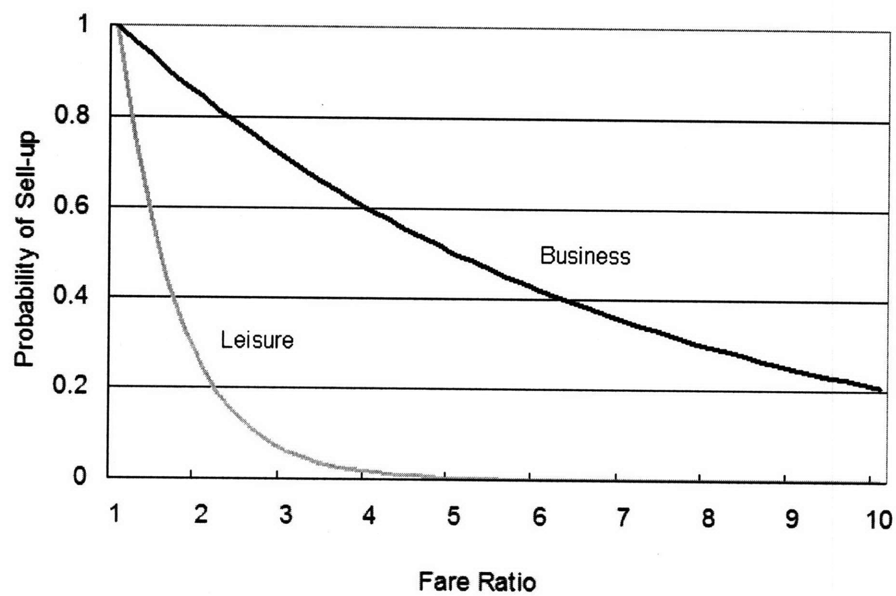


Figure 15: Willingness-To-Pay Curves by Passenger Type

4.2 Probabilities of Sell-up in PODS

As explained in §3.1, sell-up occurs under a less-restricted fare structure in which passengers tend to buy the lowest available fare. The distinction between business and leisure passengers having different disutility costs associated with restrictions has faded nowadays as there are no booking restrictions to segregate passengers. When a passenger is denied booking on a particular flight, he/she may be willing to pay more for the same flight and accepts the next higher fare available. In other words, a denied passenger may be either recaptured on the same flight in a higher fare class, or on another flight of the same airline in the same fare class. From the perspective of an airline, the goal of a seat allocation policy should be to get each passenger to pay his/her maximum WTP price. Therefore, accounting for sell-up in the seat inventory control process is important to increase airline profits.

In PODS, the probabilities of sell-up is governed by a value called "FRAT5", defined as the fare ratio of a higher fare to the lowest fare class at which 50% of the demand for the lowest fare class will sell-up to the higher class. In other words, the FRAT5 is a single parameter that stores information about passengers' sell-up behavior as it can be translated into probabilities that they will sell-up from Q-class to some higher-priced fare classes. Passengers with a high FRAT5 value are less price-sensitive than those with a lower FRAT5. The higher the FRAT5 value, the higher the probabilities of sell-up to fare classes, as shown in Figure 16.

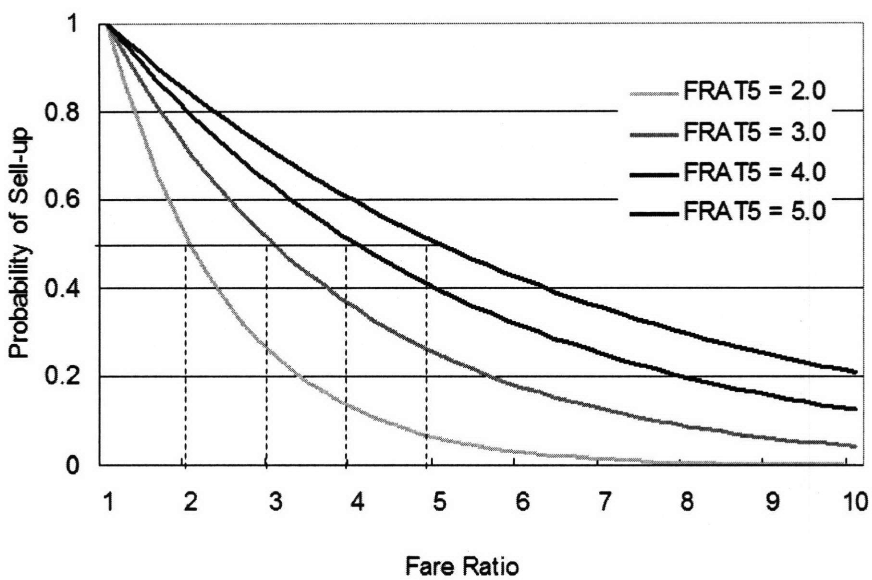


Figure 16: Relationship between FRAT5 and WTP Curves

For a given flight, it is expected that passengers' WTP and consequently the probabilities of sell-up increase as it approaches the date of departure, because business passengers, who are less price-sensitive are more willing to sell-up, tend to book later toward the end of the booking period. This also leads to the sell-up rates for higher fare classes to be higher. Thus, FRAT5 is expected to gradually increase from TF 1 to 16, resulting in a "FRAT5 Curve" of an S-shape that reflects the change in the business/leisure mix across time frames according to the arrival curves of both types of passengers (Refer to Figure 17).

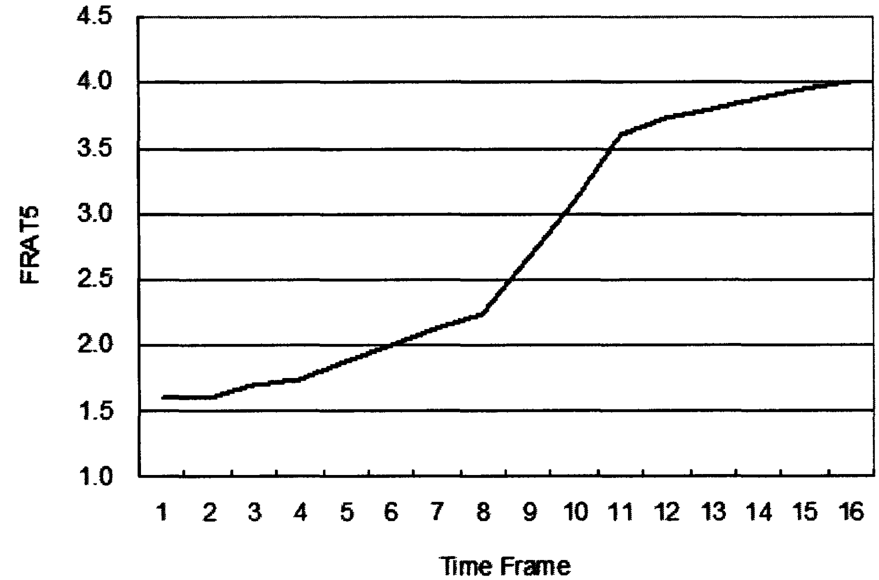


Figure 17: Typical FRAT5 Curve

4.2.1 Q-Forecasting in PODS

As explained in §3.1.1, Q-Forecasting manages passengers' sell-up by using historical booking data to estimate the number of potential future bookings in the lowest fare class, and then converting that value into an equivalent number of potential bookings in each of the higher fare classes. Each set of FRAT5 curve is denoted by a letter: "A", "C", and "E" are the sets mainly used for the study of this thesis. A FRAT5 curve can be perceived as the airline's estimates of passengers' sell up behavior. FRAT5 "A" assumes more aggressive sell-up rates than FRAT5 "C", and FRAT5 "E" being the least aggressive of the three (Refer to Figure 18). It is important not to confuse the airline's prediction with the underlying WTP of the simulated passengers that do not vary. Using a more aggressive FRAT5 curve would cause Q-Forecasting to predict higher probabilities of sell-up, and consequently lead to the optimizer protecting more seats for higher classes. Given that FRAT5 has been introduced, so how exactly is it used in PODS to implement Q-Forecasting?

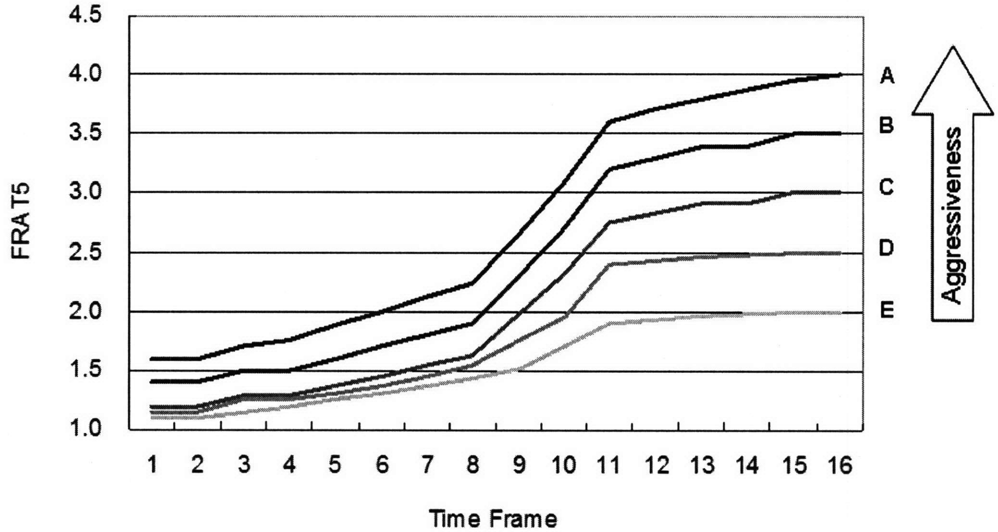


Figure 18: Different sets of FRAT5 Curves in PODS

We should remember that the first step in Q-Forecasting is to convert historical bookings into Q-equivalent bookings based on passengers' probabilities of sell-up. The number of samples from historical database to be used in the forecaster is 26. In PODS, the probabilities of sell-up are determined for each time frame and fare class depending entirely on FRAT5 values for the time frame and the fare associated with each class. Then, Q-equivalent bookings for each fare class are computed by dividing historical booking observations by the sell-up probabilities, as shown by the formulas below.

$$p\sup_{Q\to f,tf}\left(fare_f\right) = e^{-scon_{tf}\cdot\left(\frac{fare_f}{fare_Q}-1\right)}$$

$$scon_{tf} = -\frac{\ln(0.5)}{FRAT5_{tf}-1}$$

$$hbk_{Q\to f,tf} = \frac{hbk_{f,tf}}{p\sup_{Q\to f,tf}}$$

where

- $p\sup_{Q\to f,tf}$ denotes the probability of sell-up from base fare class Q to a higher fare class f in time frame tf
- $scon_{tf}$ denotes the sell-up constant in time frame tf
- $fare_f$ denotes the fare of the higher fare class f
- $fare_Q$ denotes the fare of the lowest-priced base fare class Q

- $FRAT5_{tf}$ denotes the fare ratio at which 50% of passengers will sell-up from $fare_Q$ in time frame tf
- $hbk_{Q \to f, tf}$ denotes the estimated equivalent bookings for fare class Q in fare class f in time frame tf
- $hbk_{f, tf}$ denotes the mean unconstrained demand of fare class f in time frame tf

The FRAT5 curves Vanhaverbeke (2006) used in his simulation tests on DP methods are sets of pre-determined input FRAT5 values. However, if such input FRAT5 does not reflect correctly the sell-up behavior of passengers, simulation results do not validate the real potential of a RM method since the optimizer may be sensitive to sell-up probabilities and require more accurate forecasts that reflect actual bookings. The challenge is that observed WTP of passengers tend to change with competition. An optimizer using a fixed input FRAT5 curve is not adaptive to situations where high-fare paying passengers may buy down to other airlines when at least one competitor has low or middle fare classes open later in the booking process. Using inaccurate probabilities of sell-up may lead to the optimizer generating non-optimal seat allocation policy and consequently lose out much potential demand.

Improving the forecasting of probabilities of sell-up is one of the major areas of research for airlines to test and evaluate the potentially better RM methods in less-restricted fare structures. The results Vanhaverbeke obtained with DP-GVR when competing against advanced RM methods were not as good as the theoretical advantages the model would suggest. Indeed, airlines would likely be hesitant to assume an arbitrary sell-up model in their RMS, and prefer to estimate sell-up probability using historical booking records.

In the simulations of this thesis, the forecaster manages to estimate the FRAT5 values dynamically from historical booking records at a particular time frame in order to make the Q-Forecasting process independent from the fixed input FRAT5 values. It would improve both the forecast of arrival rates as well as potential demand by fare class. These are the two forecasts on which DP optimizers use to generate optimal decisions. We hope this improvement would enable methods based on DP to achieve better results in unrestricted fare structures against advanced competitors. There are two types of FRAT5 estimator used in this study, namely (1) Forecast Prediction and (2) Inverse Cumulative methods. Detailed descriptions of the logistics behind these two estimators can be found in Hopperstad (2007) and Guo (2008).

4.2.1.1 Forecast Prediction Estimator

The goal of developing a FRAT5 estimator is to try to find a way to estimate probabilities of sell-up for forecasts of price-oriented passengers based on historical data without making too many empirical assumptions. Therefore, the historical observations used in our estimation of potential demand always involve only bookings that occur in the lowest open fare class. In addition, the estimators used in this thesis are not conditional on the competitor's availability, meaning that the estimated probabilities of sell-up only correspond to the open fare class of the incumbent airline without accounting for the lowest competitor open class (Loco).

The idea of Forecast Prediction (FP) is that historical bookings of a particular time frame are converted to the Q-equivalent forecasts using the previous estimates of probabilities of sell-up in previous samples. New estimates of sell-up probabilities are then calculated by the ratio of observed bookings to the average associated historical Q forecast. Each time a new sample comes in, the observed probabilities of sell-up are recalculated. The belief with this estimator is that after a sufficient number of recalculations, an accurate estimate of sell-up model can be achieved.

Table 13 illustrates an example of how observed probability of sell-up in a particular time frame is calculated using FP. Input sell-up probabilities are initially used to convert total class bookings into total Q bookings for each class. The next step is to compute the associated average Q bookings by summing those Q bookings of all classes and dividing by the number of samples. Finally, new estimate of sell-up probabilities can be obtained by dividing the average bookings for each fare class by the average Q bookings. Sell-up probabilities are recalculated when new samples are added.

Class/ Fare Ratio	Average bookings	Total bookings*	Previous $p\sup_{Q\to f}$ **	Total Q-bookings	$p\sup o_f$
1/ 4.0	5	10	15%	10/.15=67	5/33.4=15%
2/ 2.9	10	20	20%	20/.2=100	10/33.4=30%
3/ 1.8	10	30	35%	30/.35=86	10/33.4=30%
4/ 1.5	15	45	55%	45/.55=82	15/33.4=45%
5/ 1.3	20	80	75%	80/.75=107	20/33.4=60%
6/ 1.0	40	160	100%	160/1=160	100%
		Average Total Q-bookings		602/18 = 33.4	

* The total number of samples is 18.

** Input sell-up rates are used in the first sample initially.

Table 13: Example of Calculating Observed Sell-up Probabilities in FP estimator

Once the observed sell-up probabilities are obtained, the next step is to solve for the elasticity constant with a weighted least squares fit by time frame using fare ratios for which the forecasts and bookings occurred. The weight is set to be the total number of samples at that fare ratio. The following regression is performed for each time frame and select a and b that minimize the following:

$$\sum_{fare\ ratio} wt_{fare\ ratio} \cdot \left(p\sup o_{fare\ ratio,\ tf} - a \cdot e^{-b\cdot(fare\ ratio\ -1)}\right)^2$$

where

- $p\sup o_{fare\ ratio,\ tf}$ denotes the observed sell-up probability for the fare ratio in time frame tf

The following step involves applying a linear regression fit on the obtained sell-up constant, b_{tf}, across all time frames. Using the b_{tf} obtained from the previous regression, pick an intercept, int, and a gradient, $slope$, that minimize the following:

$$\sum_{tf} \left(\text{int} + slope \cdot tf - b_{tf}\right)^2$$

The estimated FRAT5 value can finally be calculated for each time frame using the int and $slope$ that give us the sell-up constant.

$$FRAT5_{tf} = \frac{-\ln(0.5)}{(\text{int} + slope \cdot tf)} + 1$$

Figure 19 summarizes the process of FP estimator for each time frame. We should note that this method assumes that passengers' WTP follows an exponential distribution, and regressions are performed within and across time frames to estimate FRAT5 values and sell-up probabilities.

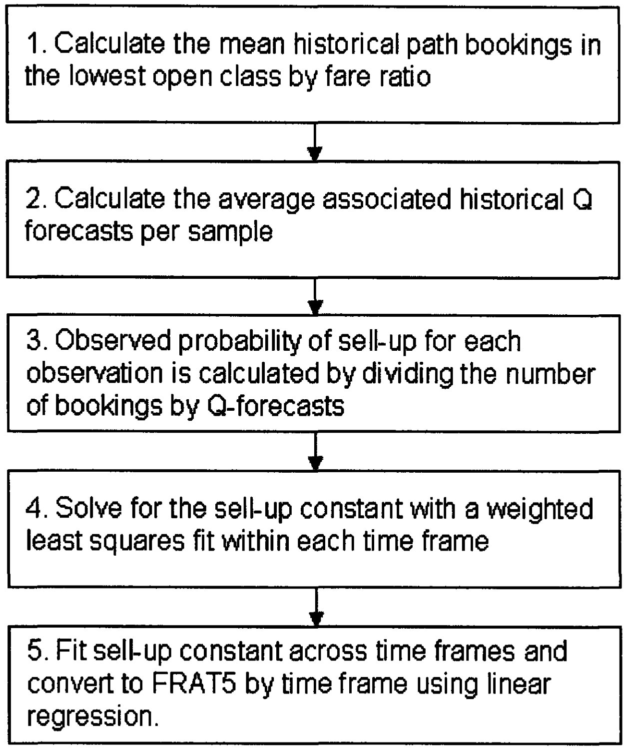


Figure 19: Forecast Prediction Estimator Flow Chart

4.2.1.2 Inverse Cumulative Estimator

The Inverse Cumulative Estimator implements a relatively simpler process that bases on the logic that a passenger booking in higher fare classes would certainly have booked at a lower fare class if this class had been open. Instead of producing estimates of sell-up probabilities from an average Q-equivalent bookings of a sample, the IC estimator computes for each class so-called "pseudo-bookings", which is defined as the sum of bookings reported in all higher classes, normalized by the number of observations. For example, the number of pseudo-bookings of class 4 is the sum of average historical bookings in classes 1, 2, 3, and 4. These pseudo-bookings are then normalized by the base-fare bookings, and finally used to estimate sell-up rates using two types of linear regression models very similar to the ones used by the FP estimator. Table 14 illustrates an example of how observed probability of sell-up in a particular time frame is calculated using IC.

Class/ Fare Ratio	Total bookings*	Potential Pseudo-Bookings	$p\sup o_f$
1/ 4.0	75	75	75/2155 = 3.5%
2/ 2.9	125	200	200/2155 = 9.3%
3/ 1.8	180	380	380/2155 = 17.6%
4/ 1.5	275	655	655/2155 = 30.4%
5/ 1.3	500	1155	1155/2155 = 53.6%
6/ 1.0	1000	2155	2155/2155 = 100%

* Total bookings are normalized by the number of observations.

Table 14: Example of Calculating Observed Sell-up Probabilities in IC estimator

As for the FP estimator, once the observed sell-up probabilities are obtained, the next step for IC is to solve the elasticity constant with a least squares fit by time frame using fare ratios for which there were forecasts and bookings. The following regression is performed for each time frame and select b that minimizes the following:

$$\sum_{fare\ ratio}\left(p\sup o_{fare\ ratio,\,tf} - e^{-b\cdot(fare\ ratio\ -\ 1)}\right)^2$$

where

- $p\sup o_{fare\ ratio,\,tf}$ denotes the observed sell-up probability for the fare ratio in time frame tf

The following steps are identical to FP. It performs a linear regression fit on the obtained sell-up constant, b_{tf}, across all time frames. Using the b_{tf} obtained from the previous regression, it then picks int and $slope$ that minimize the following:

$$\sum_{tf}\left(\text{int} + slope\cdot tf - b_{tf}\right)^2$$

The estimated FRAT5 value can finally be calculated for each time frame using the int and $slope$ that give us the sell-up constant.

$$FRAT5_{tf} = \frac{-\ln(0.5)}{(\text{int} + slope\cdot tf)} + 1$$

Figure 20 summarizes the process of IC estimator for each time frame. According to Zerbib (2006), an advantage of the IC estimator over the FP estimator is that the FRAT5 values estimated by IC seem to be more intuitive and more robust. A shortcoming of this method, however, is that the regression fit on sell-up constant by time frame is not weighted by the total number of observations. An observation having high-fare bookings should bare a lower weight if most observations contain low-fare bookings, and vice versa.

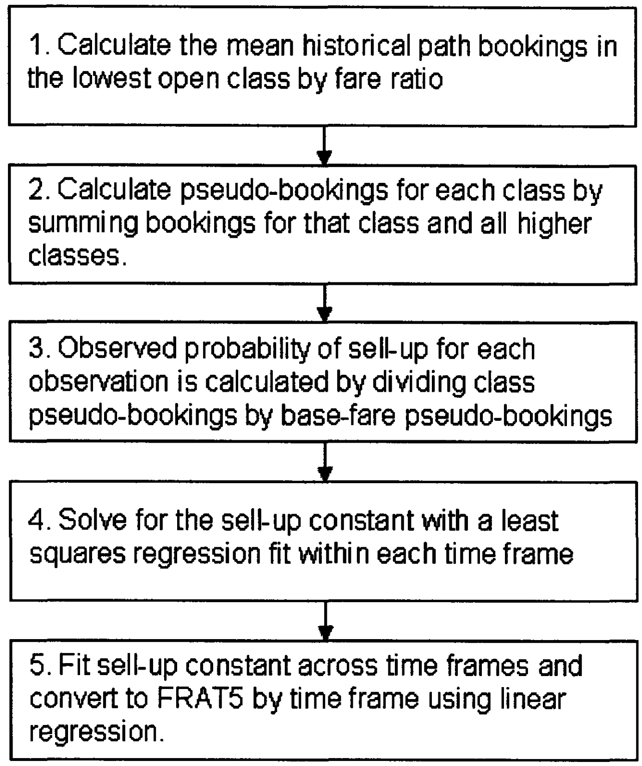


Figure 20: Inverse Cumulative Estimator Flow Chart

4.2.2 Hybrid-Forecasting in PODS

As explained in §3.1.2, we assume that under a semi-restricted fare structure environment, passenger demand is generally a combination of price-oriented and product-oriented demand. Hybrid-Forecasting first classifies historical bookings as either product-oriented or price-oriented. The price-oriented bookings are sent to the Q-Forecasting module in PODS which forecasts bookings in each undifferentiated fare class. In contrast, product-oriented bookings are sent to the traditional pick-up forecasting module which forecasts future product-oriented bookings in each fare class. The two sets of future bookings are then combined, and the aggregated booking forecasts are sent to the seat allocation optimizer.

As mentioned earlier in this chapter, that the historical database in the RM System in PODS contains data in the form of bookings by fare class. The seat allocation optimizer is limited in its knowledge of the travel market and the passenger type corresponding to each booking. In previous section, we discuss the importance of the optimizer being adaptive to competition. However, information about competing airlines is often limited to fare strictures; the airline has no access to the historical bookings and seat allocation policy of other competing airlines. Identification of passenger type within the Forecaster thus requires assumptions under certain situations to avoid blind classification.

Different rules of classifying product-oriented passengers can be chosen for Hybrid-Forecasting in PODS. In this study, we assume that a passenger booking in a fare class is classified as product-oriented if the next lower class is still available on the same path when the booking is made. However, if the next lower class has been closed due to advanced purchase requirements or seat allocation policy by RM, the booking is classified as price-oriented.

4.3 Seat Inventory Control in PODS

Often used as base case reference to measure the performance of revenue management methods, the First Come First Serve (FCFS) booking method basically allow passengers to book in the order of arrival during the booking process. There is no booking limit in this method, meaning passengers can book in a class that is not closed due to AP requirements until the plane is filled up. We will now describe several traditional RM methods that we will use to compare with DP methods in this thesis: Adaptive Threshold and EMSRb. DAVN is also a commonly used RM method to deal with large airline networks, but will not be used in this thesis.

4.3.1 Adaptive Threshold

Using a Threshold algorithm to manage booking limits is common for LCCs. A load factor threshold between 0% and 100% is associated with each fare class, and a class is closed down as soon as the load reaches the associated threshold level for that class. This method can be perceived as simple since it does not require forecasts based on historical observations in deciding seat allocation policy. In PODS, there are two different methods of applying the Threshold algorithm: Fixed Threshold (FT) and Adaptive Threshold (AT). FT does not allow the threshold values to be changed in the simulation for the entire booking period. AT specifies a load factor target at the beginning of the simulation, and in each time frame it uses the actual bookings recorded until that time frame to compute for an optimal threshold value that would achieve the target overall load factor. The type of Adaptive Threshold currently implemented in PODS is called Accordion Thresholds. Full description of the Threshold methods can be found in Gorin (2000). An Adaptive Threshold algorithm with a target overall load factor of 90% will be used in this thesis.

4.3.2 Fare Class Yield Management (FCYM)

Based on the concept of Expected Marginal Seat Revenue (EMSRb) developed by Belobaba (1987), the optimizer generally uses pick-up forecasting and probabilistic detruncation and deals with a nested booking inventory control on the leg level. The optimization assumes that the demand for each fare class is described by an independent Gaussian distribution. The mean and standard deviation are determined for each class based on detruncated historical data, and are used to produce mean and standard deviation for joint classes, which are defined as the combination of each fare class and its higher classes.

Figure 21 shows how nested limits work. Protected seats are seats that are saved particularly for fare classes which are higher than a given fare class. The booking limit for all bookings that occur in a fare class and its lower classes is determined by subtracting the number of seats to be saved for higher classes from the capacity.

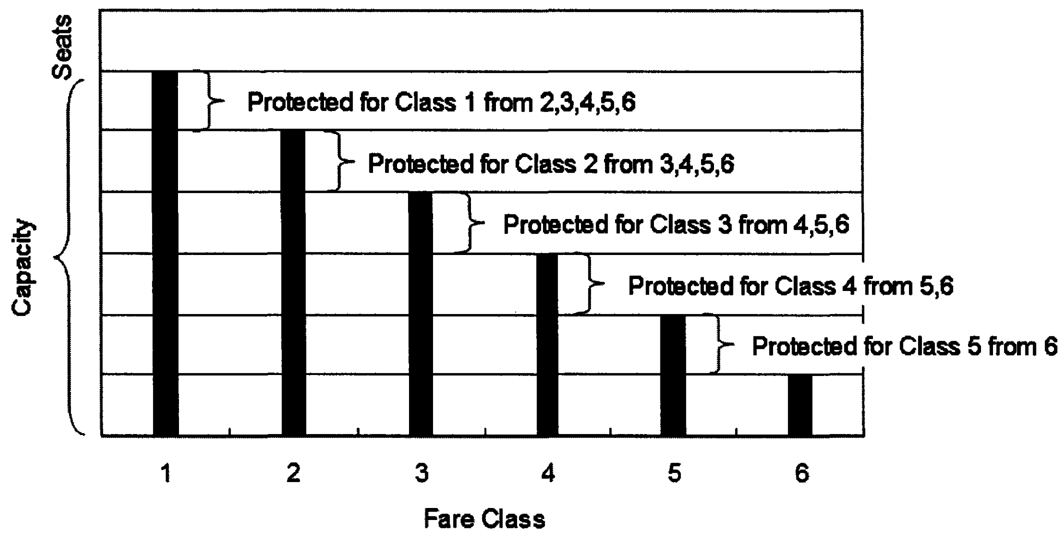


Figure 21: Nested Bookings Limits

4.3.3 Displacement Adjusted Virtual Nesting (DAVN)

Although DAVN is not used in the study of this thesis, this network O-D fare class control mechanism is prominently practiced by some airlines nowadays, and has been the basis of recent avenues of research in the PODS Consortium. The objective of DAVN is to apply a penalty to the connecting fares that accounts for the potential displacement of a local passenger. For a leg that is covered by the connecting itinerary, the passenger's total OD fare will be replaced in the bucketing by a so-called pseudo-fare computed by the actual fare minus the sum of the displacement costs associated with all other legs involved in the itinerary.

The bucketing is hence managed on the leg level. The objective is the same as EMSRb, that is to maximize total revenue subject to capacity and forecast constraints. The displacement costs as well as the size of the buckets are regularly re-optimized during the booking process.

The main focus of this thesis is to compare the performance of DP methods with traditional RM methods in unrestricted fare structure. Since the simulation environments under study involve simplified networks with little or no network effect, we will not utilize DAVN in our simulation tests.

4.4 DP Optimizers and Fare Adjustment in PODS

In PODS, FRAT5 values are used to capture the sell-up behavior in not only the forecaster but also in the DP-based seat allocation optimizer as well as Fare Adjustment. As discussed in §3.1.4, Fare Adjustment is a method employed by seat allocation optimizer to capture the risk of buy down by price-oriented passengers in an undifferentiated fare structure. As passengers' WTP increases, the PE Cost used in FA must increase in order to close lower classes faster. In PODS, there are two different ways to compute the PE Cost: (1) Thomas Fiig's continuous Marginal Revenue formulation (MR) and (2) Karl Isler's discrete Marginal Revenue formulation (KI). As shown in the formula below, the continuous MR assumes negative exponential sell-up model to compute for adjusted fare.

$$Adjusted\ fare^{MR}_{OD,f} = fare_{OD,f} - \frac{fare_{OD,f} \cdot (FA\ FRAT5 - 1)}{-\ln(0.5)}$$

where

- $Adjusted\ fare^{MR}_{OD,f}$ denotes MR adjusted Marginal Revenue for fare class f of path OD
- $fare_{OD,f}$ denotes the OD fare for class f of path OD

On the other hand, the discrete KI is generalized for all sell-up models and will be used for FA in this thesis.

$$Adjusted\ fare^{KI}_{OD,f} = \frac{p\sup_{Q\to f,tf} \cdot fare_{OD,f} - p\sup_{Q\to f-1,tf} \cdot fare_{OD,f-1}}{p\sup_{Q\to f,tf} - p\sup_{Q\to f-1,tf}}$$

where

- $Adjusted\ fare^{KI}_{OD,f}$ denotes KI adjusted Marginal Revenue for fare class f of path OD
- $p\sup_{Q\to f,tf}$ denotes the probability of sell-up from base fare class Q to a higher fare class f (or $f-1$) in time frame tf

- $fare_{OD,f}$ denotes the OD fare for class f (or $f-1$) of path OD

Cléaz-Savoyen (2005) explains that the FRAT5 values used for FA should be less than those used for Q-Forecasting, because unlike Q-Forecasting, the FRAT5 value for FA are "unconditional", meaning that the sell-up behavior is captured for all booking requests, including those that are booked in a class that is not the lowest available class. In other words, the sell-up rates we use when dealing with Q-Forecasting should be always greater than or equal to the rates we use for the FA method. To model this difference in PODS without the need of introducing a separate set of FRAT5, we apply an appropriate scaling factor, $f5scl$, that would best describe the passengers' WTP to be used in the FA method. We should note that the scaling factor has to be between 0 and 1.

$$FA\ FRAT5_{tf} = 1 + f5scl \cdot \left(FRAT5_{tf} - 1\right)$$

where

- $FA\ FRAT5_{tf}$ denotes the FRAT5 value used for Fare Adjustment in time frame tf
- $FRAT5_{tf}$ denotes the Forecasting FRAT5 value (used in Q-Forecasting) in time frame tf
- $f5scl$ denotes the scaling factor used to generate $FA\ FRAT5_{tf}$ from $FRAT5_{tf}$ in time frame tf, $0 \le f5scl \le 1$

Figure 22 compares a FRAT5-C curve and its corresponding FA FRAT5 curves with different levels of scaling factors from 0.1 to 0.5. Soo (2007) shows that FA should be more aggressive for more price-oriented passengers. This thesis focuses on unrestricted fare structure in which all passengers are classified to be price-oriented as they tend to buy in the lowest available class which differs from other classes by fare only. Therefore, in our simulation tests, we will mainly concentrate on FA using a scaling factor of 1.0. In some cases we will also run FA with less aggressive FA FRAT5 curves to seek for the optimal combination of FRAT5 and FA FRAT5 that might improve the performance of DPL.

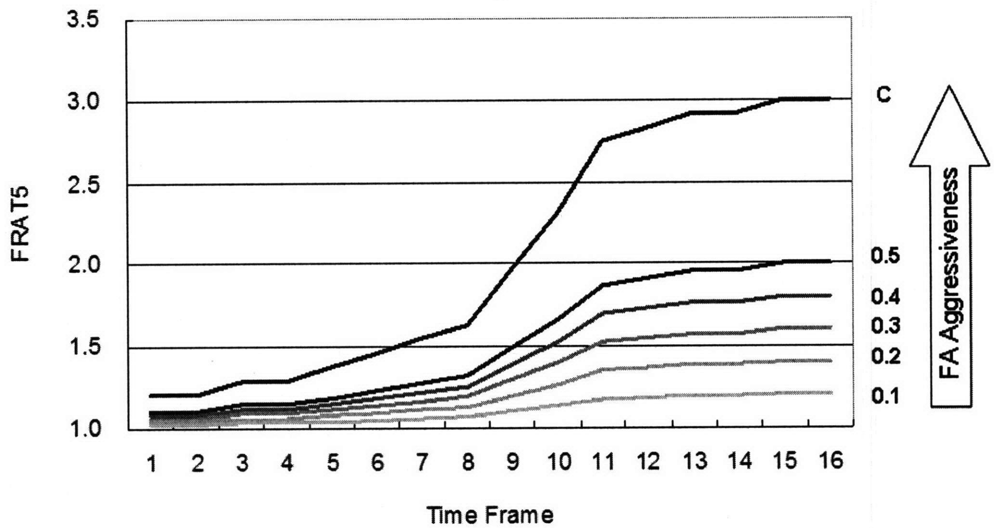


Figure 22: FA FRAT5s Values with Different Scaling Factors

Recall that in DP-GVR the sell-up probabilities by time frame are directly used by the optimizer to determine the lowest open fare class. The same algorithm used in Q-Forecasting, $p\sup_{Q\to f}$, is used to compute $pwtp_{f,tf}$ for each time frame tf. $pwtp_{f,tf}$ will then incorporate the concept of *psame* to determine the sell-up probabilities for all decision periods, $pwtp_{f,n}$ as they are iterated within that time frame, as explained in §3.2.1. The revised sell-up model by time frame used in DP-GVR with Fare Adjustment in PODS thus becomes:

$$pwtp_{f,tf}\left(Adjusted\ Pseudofare_{f,tf}\right) = e^{\left(\frac{\ln(0.5)}{FRAT5_{tf}-1}\right)\cdot\left(\frac{fare_f}{fare_Q}-1\right)}$$

where

- $pwtp_{f,tf}$ denotes the probability passenger making a booking request in time frame tf would be willing to pay at least the fare associated with class f
- $Adjusted\ Pseudofare_{f,tf}$ denotes KI adjusted pseudo fare computed by Marginal Revenue minus displacement cost for fare class f in time frame tf.

4.5 Chapter Summary

In this chapter, we present the Passenger Origin-Destination Simulator that is used for our experimentation in this thesis. In particular, we describe how an airline booking process is simulated in the PODS through interactions between passenger choices and airline decisions in revenue management. We also present how the forecasting and seat inventory control methodology as introduced in §3 are modeled in PODS to facilitate our experiments of this thesis.

The next chapter will be dedicated to the results of our simulations, specifically assessing the potential airline revenue gain that can be obtained with RM optimizers based on dynamic programming. We will also seek for additional gain that can potentially be achieved with DP methods by making modifications to the forecasting methods, specifically allowing for estimates of sell-up rates that are estimated as the closest match to the true sell-up rates of the simulated passengers.

Chapter 5

Simulation Results

In this chapter, we will present findings from our simulations in PODS regarding the effectiveness of two methods based on dynamic programming in Revenue Management, as well as the influence of sell-up models and forecasting on the revenues of airlines that implement these methods. This part of the thesis will be divided into two sections. The first section will present findings for a simple environment in which only two airlines compete for a single market. The second section will focus on a more complex setting of two airlines competing within a larger but symmetric network. We will compare these results to gains that can be obtained by using traditional methods that are still commonly practiced in the airline industry today. By the end of this chapter, we are able to further validate the experiments of Vanhaverbeke (2006) that DPL and DP-GVR perform well under certain situations in both simple and competitive frameworks. We will also have a good idea of the gains that can be achieved by accounting for a more accurate account of sell-up models.

As mentioned above, two main airline network environments have been developed in PODS for the experimentation of this thesis: (1) Single Market and (2) Network D6. We will be looking at results of implementing DPL and DP-GVR methods in these networks as compared to those obtained from using the traditional EMSRb method. Since the DP-GVR model takes into account the probabilities of sell-up and assumes that passengers may sell-up or buy-down between fare classes, we focus our study on testing under fully unrestricted fare structures to ensure passengers make their decision based on fare only.

In the Single Market case, simulation tests will be run against a simple RM method representing the type of RM approach used by low-cost carriers today, namely AT90, as well as against a more sophisticated method known as EMSRb, supplemented with forecasts estimated by Q-Forecasting (QF). In Network D6, we will inspect the performance of DPL and DP-GVR against three types of competing RM methods. Besides competing with AT90 and EMSRb with QF as for the case in Single Market, we would also like to test each of these DP methods against their respective same method, and compare results to those obtained when both airlines employ symmetric traditional methods.

We evaluate the performance of a RM method based on typical measures of general interest in the airline industry. The most common ones include the total revenue of each airline, average load factor, yield, and fare class mix. Average Load factor (ALF) is a measure of aircraft seat utilization, and is defined as the ratio of passengers to total capacity for a given flight. ALF is calculated by dividing the number of miles flown by a carrier-passenger, known as the Revenue Passenger Miles (RPM), by the number of miles flown by all the seats on each flight, known as the Available Seat Miles (ASM). On the other hand, yield is a measure of the average revenue each passenger pays for his or her ticket. It is calculated by dividing total revenue paid by passengers to the total RPMs.

Results obtained from our simulations are compared based on a reference point, or what we call the "base case" results. The base case for each test case is chosen to reflect what is perceived as the standard Revenue Management system traditionally used in the industry. Results from this baseline scenario will be used as benchmarks to allow for a systematic evaluation of our findings.

5.1 Single Market

Since both DPL and DP-GVR models are implemented on the leg level, performing our first series of simulation tests in the Single Market case with limited flight legs can provide a simpler and more controlled framework, and allow results to rely more on the pure performance of the RM methods themselves than the feedback effects that may linger in large airline network. As explained earlier, we will first consider a one-market scenario in which a competitor enters a non-stop market and competes with Airline 1 in the local market. The second half of the chapter will be dedicated to simulation results in a more complicated network.

5.1.1 Overview of Single Market

The Single Market case is the most basic airline environment in which there are 2 airlines serving an Origin-Destination (OD) market. Each airline operates 3 one-way, west-to-east flights daily with identical schedules (Refer to Figure 23).

Figure 23: Route Map of the Single Market Case

To control our simulation experiments, the simulations are set up to be perfectly symmetric for the two airlines. Both airlines offer identical 6-fare-class structure in which fare classes are differentiated by fare price only, as characterized in Table 15. All restrictions and advance purchase requirements are fully removed. For this test case, we assume both airlines offer an unrestricted fare structure with an average fare ratio (ratio of the highest fare to the lowest fare) of 4.

Fare Class	Average Fare	Restrictions			
		Advance Purchase	R1	R2	R3
1	\$500	0 days	No	No	No
2	\$400	0 days	No	No	No
3	\$315	0 days	No	No	No
4	\$175	0 days	No	No	No
5	\$145	0 days	No	No	No
6	\$125	0 days	No	No	No

Table 15: Fare Structure and Restrictions in Single Market

As mentioned in §4.1, the sets of restrictions that can potentially be included in the fare structure are Saturday-night stay (R1), cancellation or change penalty (R2), and non-refundability (R3). In the wake of losing market share to low-cost carriers that offer less-restricted, undifferentiated fare structure, fare structures of some legacy carriers have become less restricted by removing part or all restrictions, in the most extreme case leading to fare classes that differ by fare only.

5.1.2 Test Case 1: Against AT90

In the Single Market case, tests will be run against a basic RM method (AT90) and an advanced competitor (EMSRb with Q-Forecasting). In this section, we will be focusing on the impact of implementing DP methods for Airline 1 when it is competing against an airline that uses AT90. Recall from our introduction of AT90 (Refer to §4.1.5.1) that the method decides fare class closure based on optimal threshold that aim at an overall load factor of 90%. Hence, for this particular test case, the experimentation is set up to be that only one airline, Airline 1, accounts for sell-up in its Revenue Management (RM). We will see whether benefits can be achieved for Airline 1 using a DP method over the traditional EMSRb method, if the airline accounts for sell-up in both methods.

5.1.2.1 Specifications of Base Case and Simulations

The base case will be specified as that Airline 1 uses EMSRb with standard leg-based forecasting, which does not account for sell-up and is, in effect, equivalent to the first-come-first-serve (FCFS) under a fully unrestricted environment. The use of standard pick-up forecasting quickly leads to spiral-down effect: As passengers end up buying in the lowest fare (Fare Class 6), Airline 1 progressively makes Fare Class 6 the lowest available class open throughout the entire booking process before departure, and closes all fare classes at the same time when bookings reach the capacity.

On the other hand, the competing airline uses AT90. We will run simulations for different demand levels (0.8, 0.9, 1.0, and 1.1) and then analyze results. We will also evaluate the results of DP methods that assume different Z-factors (1, 2, 3, and 4). Table 16 describes the set-up of our simulations for this test case.

Test Case 1	Airline 1		Airline 2	
	Optimizer	Forecaster	Optimizer	Forecaster
Base Case	EMSRb	Standard Leg	AT90	-
1A	AT90	-	AT90	-
1B	EMSRb	Q-Forecasting	AT90	-
1C	DPL	Standard Leg	AT90	-
1D	DPL	Q-Forecasting	AT90	-
1E	DP-GVR	-	AT90	-

Table 16: Specifications of Test Case 1 (Against AT90, Single Market)

Once again, most of the results shown in the following sub-sections will be compared to the base case. The base case revenues, average load factors, and yields for different demand factors are shown in Table 17. Figure 24 illustrates the spiral-down effect as all bookings occur in Fare Class 6 only.

Demand Factor	Revenue ($)	Avg. Load Factor %	Yield ($/RPM)
0.8	31491	84.0%	0.0893
0.9	33014	88.0%	0.0893
1.0	33811	90.2%	0.0893
1.1	34700	93.5%	0.0893

Table 17: Base Case Results of Airline 1 against AT90 in Single Market

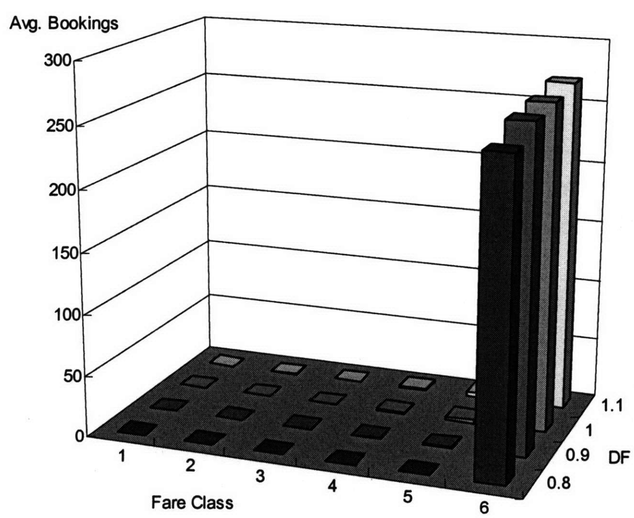


Figure 24: Base Case Fare Class Mix of Airline 1 against AT90 in Single Market

5.1.2.2 DPL with Standard Leg Forecasting vs. AT90

In our first simulation test set, we examine the results of Airline 1 that uses DPL with Standard pick-up forecasting (Test case 1C). Recall from §3.1.3 that Z-factor is an input in a DP-based optimizer that controls the variance-to-mean-demand ratio assumed in computing the arrival rates and sell-up probabilities. Table 18 and Figure 25 present the revenue, load factor, and yield of Airline 1 against AT90 across different Z-factors and demand factors.

Demand Factor	Z-factor	Revenue ($)	Load Factor %	Yield ($/RPM)
1.0	1	33811	90.2%	0.0893
	2	33811	90.2%	0.0893
	3	33811	90.2%	0.0893
	4	33811	90.2%	0.0893

Table 18: Results of DPL with Std. Leg Forecasting against AT90 in Single Market at Demand Factor 1.0

Our intuitive expectation is that the revenue of Airline 1 does not vary among Z-factors because spiral down should occur in Standard pick-up forecasting, which causes bookings to continuously and exclusively occur in Fare Class 6 (Refer to Figure 26). These results support the underlying principle of accounting for sell-up in forecasting when we evaluate the performance of different RM methods under fully unrestricted environments.

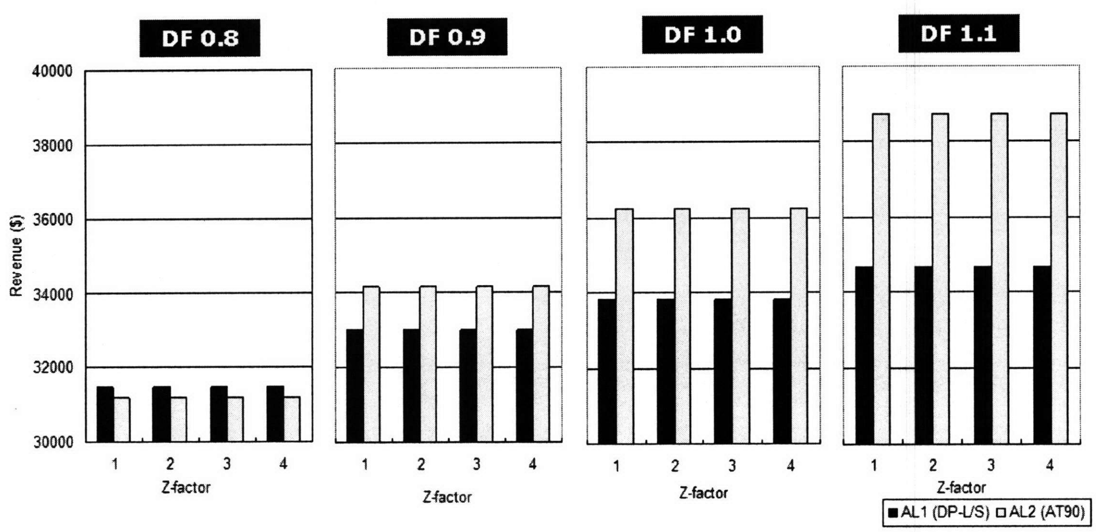


Figure 25: Revenues for DPL with Std. Leg Forecasting against AT90 in Single Market

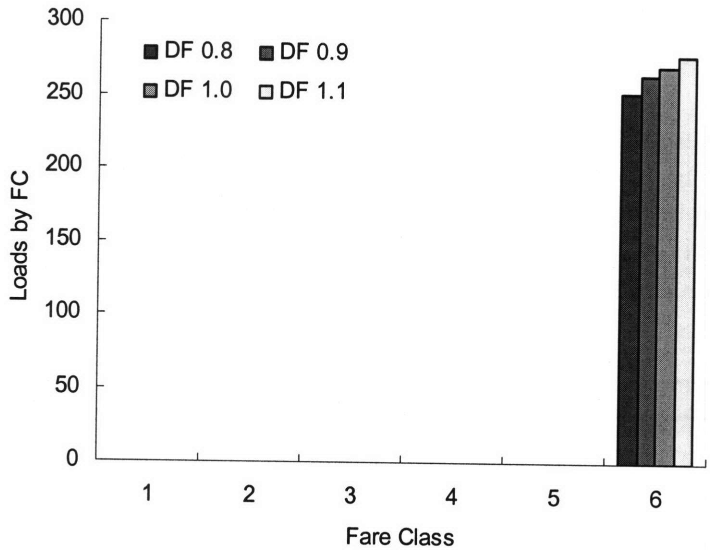


Figure 26: Fare Class Mix for DPL with Std. Leg Forecasting against AT90 in Single Market

5.1.2.3 DPL with Q-Forecasting vs. AT90

We will now turn our focus to Airline 1 that accounts for sell-up and employs DPL against AT90 (Test Case 1D). In the Single Market case, it is found that DPL with Q-Forecasting, assuming input FRAT5-C sell-up rates, helps the airline that implements the method. Table 19 shows the revenues, percentage revenue gains, average load factors, and yields obtained by Airline 1 at all four demand factors and all four Z-factors. We observe that at a Demand Factor of 1.0, Airline 1 sees a revenue gain that ranges from 36.7% to 41.0% depending on the Z-factor used. It appears that the higher the Z-factor, the higher the average load factor and yield, resulting in higher revenue improvement.

Demand Factor	Z-Factor	Revenue ($)	Load Factor %	Yield ($/RPM)
0.8	1	35164 (+11.7%)	83.5%	0.1002
	2	35087 (+11.4%)	83.6%	0.1000
	3	35341 (+12.2%)	83.5%	0.1007
	4	35410 (+12.4%)	83.5%	0.1009
0.9	1	40329 (+22.2%)	87.1%	0.1102
	2	40346 (+22.2%)	87.5%	0.1097
	3	41031 (+24.3%)	87.4%	0.1117
	4	41083 (+24.4%)	87.5%	0.1118
1.0	1	46535 (+36.7%)	88.6%	0.1251
	2	46739 (+37.3%)	89.6%	0.1242
	3	47954 (+40.9%)	89.4%	0.1277
	4	48001 (+41.0%)	89.5%	0.1277
1.1	1	49791 (+43.5%)	90.3%	0.1313
	2	49627 (+43.0%)	91.8%	0.1288
	3	50967 (+46.9%)	91.5%	0.1326
	4	51061 (+47.1%)	91.6%	0.1327

Table 19: Results of DPL with Q-Forecasting using FRAT5-C against AT90 in Single Market

The revenue plot in Figure 27 also shows that the revenue distinction among Z-factors is insignificantly small when the demand is low. This phenomenon can be explained by Figure 28, which presents the fare class mix of both airlines at different demand factors. When the demand is low, fewer high-fare bookings will be observed, causing the airlines to open up more low-fare classes and attract more low-fare bookings. Therefore, the probability that the previously arriving passengers belongs to the same passenger type as the next passengers (Recall from §3.1.3) will bring little influence on the arrival rates and consequently the protection of high-fare classes by the DPL algorithm.

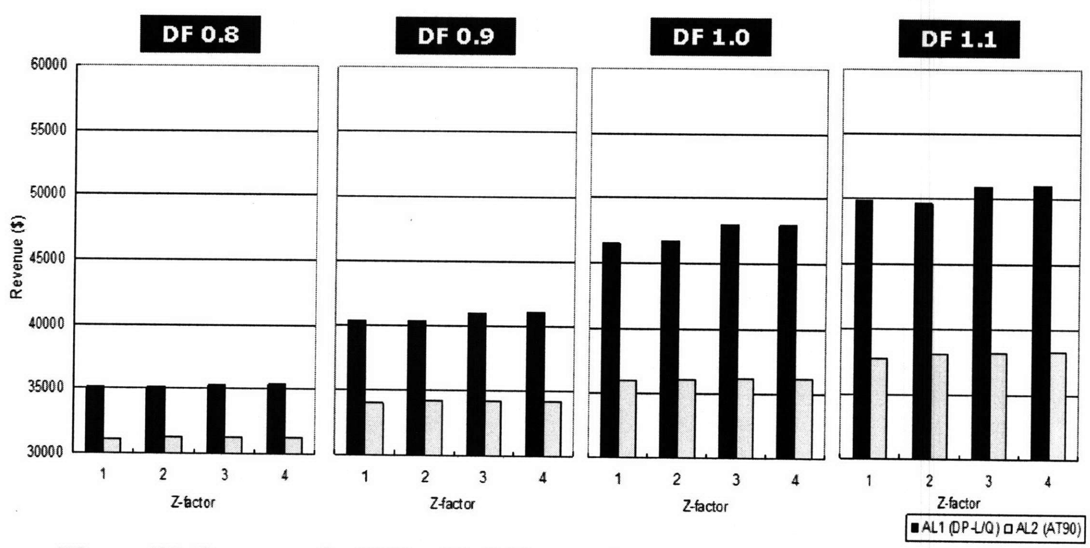


Figure 27: Revenues for DPL with Q-Forecasting against AT90 in Single Market

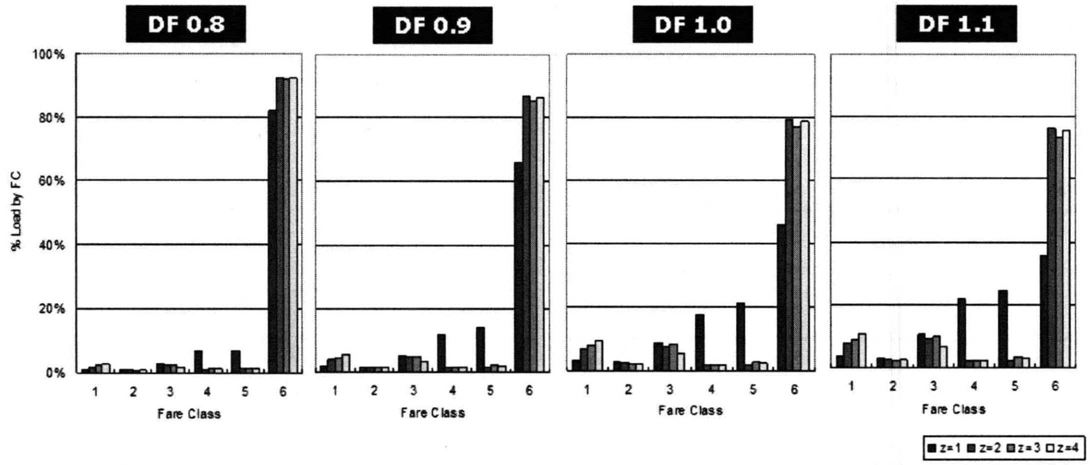


Figure 28: Fare Class Mix for DPL with Q-Forecasting against AT90 in Single Market

In summary, when Airline 1 implements DPL and is the only airline that accounts for medium sell-up, it achieves the highest revenue gains at a Z-factor of 4, regardless of demand factor. Moreover, as one would expect, the higher the demand factor, the larger the revenue gains, and the larger the percentage revenue gain as compared to the base case. A Z-factor of 4 will therefore be used for the best case comparison with traditional methods later in this section.

5.1.2.4 DP-GVR vs. AT90

The last simulation set for this test case is constructed for DP-GVR against AT90 (Test Case 1E), with Airline 1 assuming input FRAT5-C sell-up rates. Unlike DPL with Q-Forecasting, DP-GVR is found to be very sensitive to Z-factor values at high demand, and a small difference in this input does result in substantial differences in revenues. For example, at Demand Factor of 1.0, implementing DP-GVR causes Airline 1 to boost up its revenue by more than 10%, and as much as 20.2% when a Z-factor of 4 is used (Refer to Table 20).

Demand Factor	Z-Factor	Revenue ($)	Load Factor %	Yield ($/RPM)
0.8	1	28528 (-9.4%)	59.3%	0.1145
	2	28767 (-8.7%)	59.3%	0.1156
	3	28945 (-8.1%)	59.2%	0.1164
	4	29045 (-7.8%)	59.1%	0.1170
0.9	1	32254 (-2.3%)	67.6%	0.1137
	2	32805 (-0.6%)	67.4%	0.1160
	3	33269 (+0.8%)	67.1%	0.1181
	4	33523 (+1.5%)	66.9%	0.1194
1.0	1	37473 (+10.1%)	74.5%	0.1198
	2	39016 (+14.6%)	73.9%	0.1257
	3	40249 (+18.3%)	73.1%	0.1311
	4	40894 (+20.2%)	72.4%	0.1344
1.1	1	40441 (+16.5%)	78.6%	0.1225
	2	42325 (+22.0%)	77.6%	0.1298
	3	43783 (+26.2%)	76.4%	0.1364
	4	44552 (+28.4%)	75.5%	0.1405

Table 20: Results of DP-GVR using FRAT5-C against AT90 in Single Market

Looking at the performance measures, we see that Airline 1's load decreases as revenue increases, and its yield increases somewhat, which indicates that it is carrying "better" loads of traffic. By "better", we mean that Airline 1 forfeits opportunity costs due to flying more empty seats but manages to generate even more amount of revenue from accepting enough high-revenue passengers. However, DP-GVR is functioning poorly when demand is low, resulting in revenue loss of as much as 9.8% at Demand Factor of 0.8. Although it manages to improve revenue over base case by 28.4% at DF of 1.1, this revenue gain is much lower than that obtained by DPL with QF and EMSRb with QF. Revenues, loads, and yields all increase for Airline 2, indicating that Airline 1 is managing to improve its traffic without affecting its competitor.

As for the case with DPL with QF, the revenue distinction among Z-factors as shown in Figure 29 is smaller when the demand is low. We also see that Airline 2 is not sensitive to the Z-factor used in DP-GVR by Airline 1. Figure 30 supports the notion that more bookings are observed in higher-fare classes when the demand is high due to the fact that airlines can now be more selective in choosing passenger bookings without the fear of losing too much in foregoing loads when protecting for higher-fare classes.

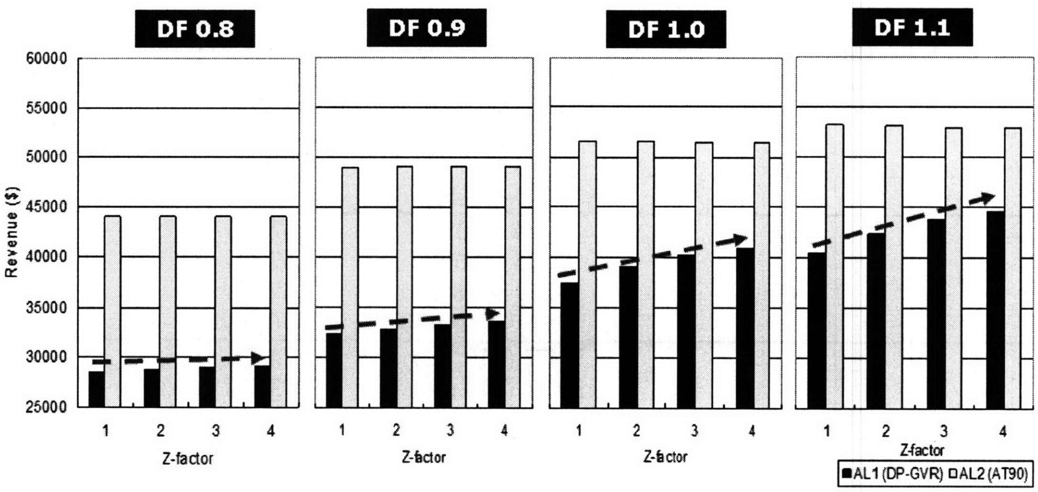


Figure 29: Revenues for DP-GVR against AT90 in Single Market

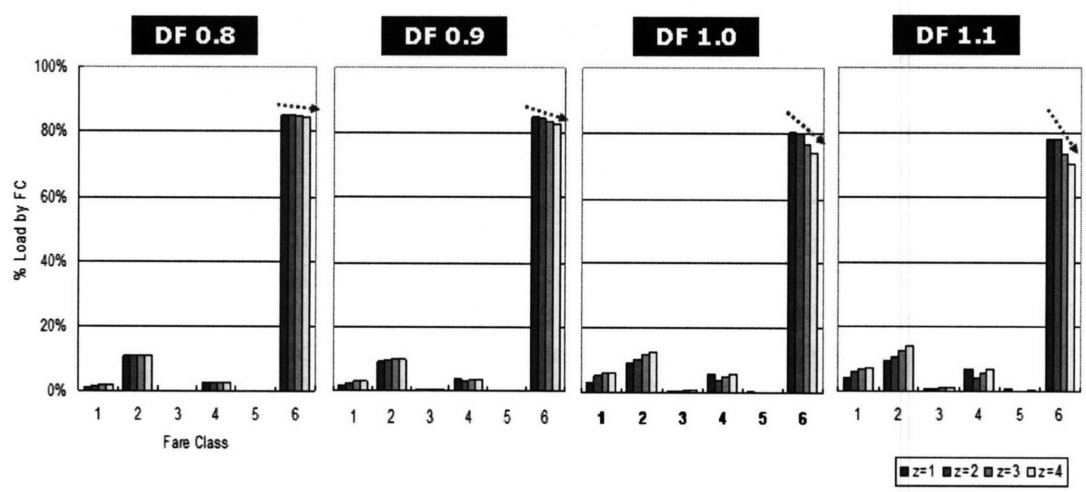


Figure 30: Fare Class Mix for DP-GVR against AT90 in Single Market

We highlight the load factor and yield graphs presented in Figure 31 here in order to stress that DP-GVR, which considers the probabilities of sell up to higher fare classes, is more sensitive to demand levels than other methods as a result. In a high demand situation where the number of passengers willing to sell up should expectedly increase, DP-GVR manages to increase its yield of Airline 1 by a higher percentage than what would be observed when traditional methods, which assumes independent bookings by fare class instead, are used. These results are consistent with Bohutinsky (1990) that the higher the demand for a flight, the greater the probability that sell-up will occur, and the more likely lower classes will be closed down by Revenue Management system to protect bookings for passengers that are willing to travel at a higher fare.

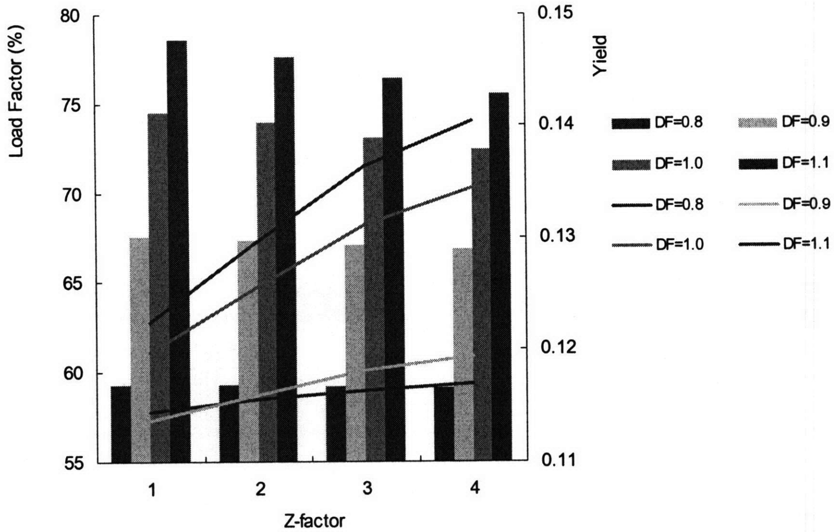


Figure 31: Load Factor and Yield for DP-GVR against AT90 in Single Market

In summary, DP-GVR appears to be less effective than DPL with QF against AT90 when FRAT5-C is used, regardless of the demand factor. As for the case in DPL with QF, the highest percentage revenue gain is achieved at a Z-factor of 4. As the mathematical formulations of DP-GVR show, the higher the Z-factor, the more likely the previously booking is assumed to belong to the same passenger type as the next arriving booking, causing the algorithm to protect more for higher fare classes that are predicted to possess higher arrival rates. A Z-factor of 4 will be used for the best case comparison with traditional methods in the next section.

5.1.2.5 Investigation of DP methods against AT90

Considering the performance of different RM methods with respect to the EMSRb versus AT90 base case scenario, we can see that DPL that incorporates sell-up behavior in its Q-Forecasting is the best performer in the single market case (Refer to Figure 32). As also shown in Table 21, the revenue gain of Airline 1 when using such RM technique (12.4%) has a slight edge over EMSRb with Q-Forecasting (12.0%) in the low demand environment, and the margin increases to as much as 7.5% when demand becomes higher. We also see that DP-GVR results in revenue loss (-7.8%) even against a simple AT90 when the demand is low. It indicates that Airline 2, which uses AT90 in aim at a load factor of 90%, does not lose revenues because it manages to keep filling its planes with the majority of low-fare passengers. There is not enough demand to allow Airline 1 to be successful in being selective in its own bookings, resulting in overprotection and consequently much lower load factor and revenue for Airline 1.

At first glance, the revenue summary plot seems to lean toward suggesting a competitive edge of DPL over other methods when Q-Forecasting is implemented alongside. Given that Q-Forecasting uses an arbitrary and fixed FRAT5C set in our analysis up to this point, it would interesting to investigate how sensitive DPL is on different input sets and if changing the airline's prediction of sell-up behavior would further improve the performance of DPL. Using different input FRAT5s on DP-GVR would also help explain whether the previously poor results of DP-GVR are attributable to an inaccurate prediction of passengers' WTP in forecasting rather than the optimization algorithm itself.

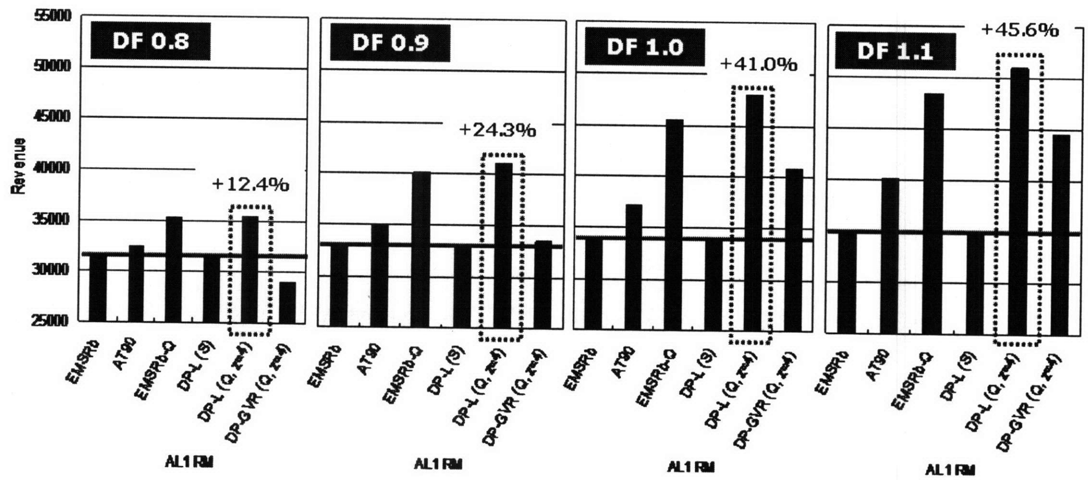


Figure 32: Revenue Comparison of Best Cases for different RM methods with FRAT5-C against AT90 in Single Market

DF	% Revenue gain over Base Case Scenario				
	AT90	EMSRbQ	DPL+Std.F.	DPL+QF	DP-GVR
0.8	+2.9%	+12.0%	0%	+12.4%	-7.8%
0.9	+5.8%	+21.4%	0%	+24.3%	+1.4%
1.0	+9.4%	+33.7%	0%	+41.0%	+20.2%
1.1	+14.3%	+38.1%	0%	+45.6%	+27.0%

Table 21: Revenue gains for different RM methods with FRAT5-C against AT90 in Single Market

Therefore, the next step in our analysis is to try various input sell-up rates in both the EMSRb heuristic and DP algorithms and see what happens to revenues for both airlines, but more importantly for Airline 1 that implements the modification. The reader will recall from §4.1.3 the three sets of FRAT5 curves to be used in Q-Forecasting for this thesis (Refer to Figure 18). Figure 33 summaries the sets of FRAT5 curves associated with FRAT5-A, FRAT5-C, and FRAT5-E inputs.

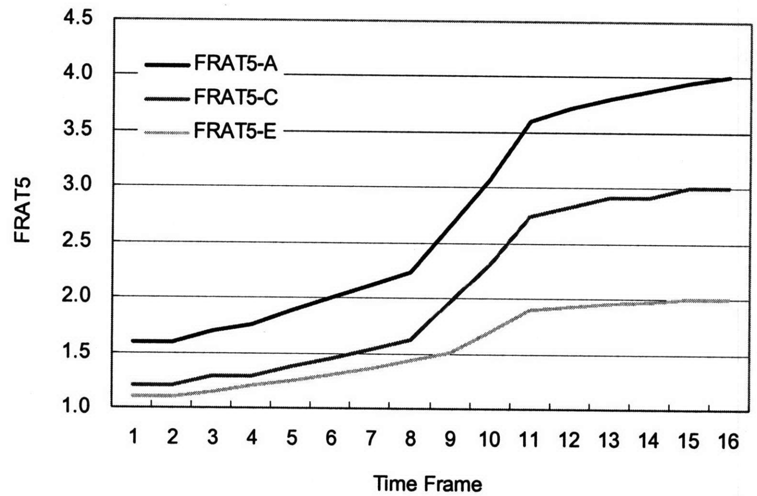


Figure 33: FRAT5 Curves for input FRAT5-A, FRAT5-C, and FRAT5-E

Figure 34 presents the revenue impact of different RM methods if the airline changes the input FRAT5 set in the forecaster (or the optimizer in the case of DP-GVR) at demand factor of 1.0. Other demand factors are also tested and their results are not shown here since similar patterns are resulted. It turns out that using a more aggressive FRAT5A in DPL with Q-Forecasting manages to increase the revenue by an additional 2.9% over the case when FRAT5-C is used, and still outperform EMSRb with Q-Forecasting although the revenue differential becomes smaller. The trend for DP-GVR is however the opposite. In a single market case, the load factors and the revenues of Airline 1 using DP-GVR against AT90 drop by a large margin when Q-Forecasting uses a high input FRAT5 set. These results indicate that DP-GVR is highly sensitive to how the airline predicts the passengers' WTP. If the airline assumes a fixed but less aggressive sell-up behavior similar to FRAT5-E, DP-GVR manages to achieve a revenue gain of 28.5% in comparison with the base case, a significant improvement although it still does not match well with other RM methods using high or aggressive input FRAT5s.

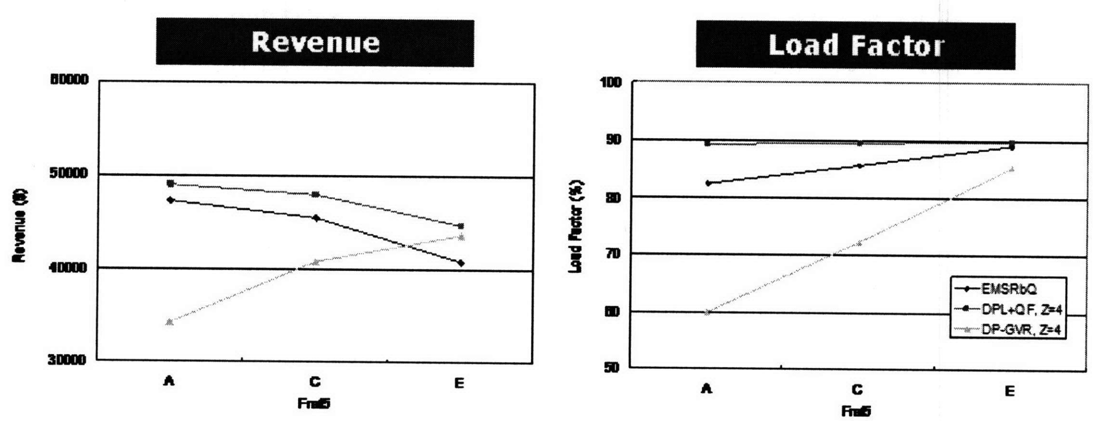


Figure 34: Sensitivity of different RMs to input FRAT5 sets against AT90 in Single Market

The low load factor of DP-GVR using an aggressive input FRAT5 indicates that it protects many seats for high-yield passengers but most of these passengers did not sell up. In other words, passengers' sell up behavior is not as great as what the airline estimates, causing DP-GVR to overprotect higher-fare seats that cannot be filled up, resulting in low load factor and thus low revenue. Although FRAT5E seems to match passengers' sell-up behavior the best among the three FRAT5s under study, it does not necessarily mean that passengers' maximum willingness-to-pay is low. It could be the case that a certain fare class is closed for Airline 1 but is still open by the competing airline, and under the unrestricted fare structure even a passenger with high WTP will buy down to the competing airline as it offers a relatively lower fare. FRAT5E is relatively better than more aggressive FRAT5s in capturing the buy down effect of passengers that may have lower tendency to sell-up.

It is interesting to see that using a more aggressive FRAT5 helps DPL and EMSRb with Q-Forecasting, a totally opposite trend to DP-GVR. Recall that both optimizers assume independent demands by fare class even though those demands are forecasted based on sell-up probabilities. It seems that these two RM methods are better off protecting more high-fare seats than the number of passengers willing to sell up. The increase in yield due to higher fare passengers manages to compensate the drop in load factor, causing these RM methods to achieve higher overall revenue.

Based on the poor results of a FRAT5-sensitive RM method in DP-GVR, we expect similar trend would occur DPL with Q-Forecasting incorporating Fare Adjustment, since sell-up probabilities are considered in the optimization algorithms as well. As shown in Figure 35, the introduction of FA to DPL with Q-Forecasting does not improve results. The revenue of Airline 1 deteriorates as the FRAT5 Scaling Factor becomes higher. However, the decrease in revenue is much smaller with a FRAT5 set similar to FRAT5-E that seems to match better the sell-up behavior of passengers. We note that this deterioration begins immediately, and displays a more rapid drop curve with a higher FRAT5. DPL/FA with QF using FRAT5-E, which has a much flatter revenue drop curve than FRAT5-A and FRAT5-C, finally outperforms those higher FRAT5s when the FA scaling factor exceeds 0.6.

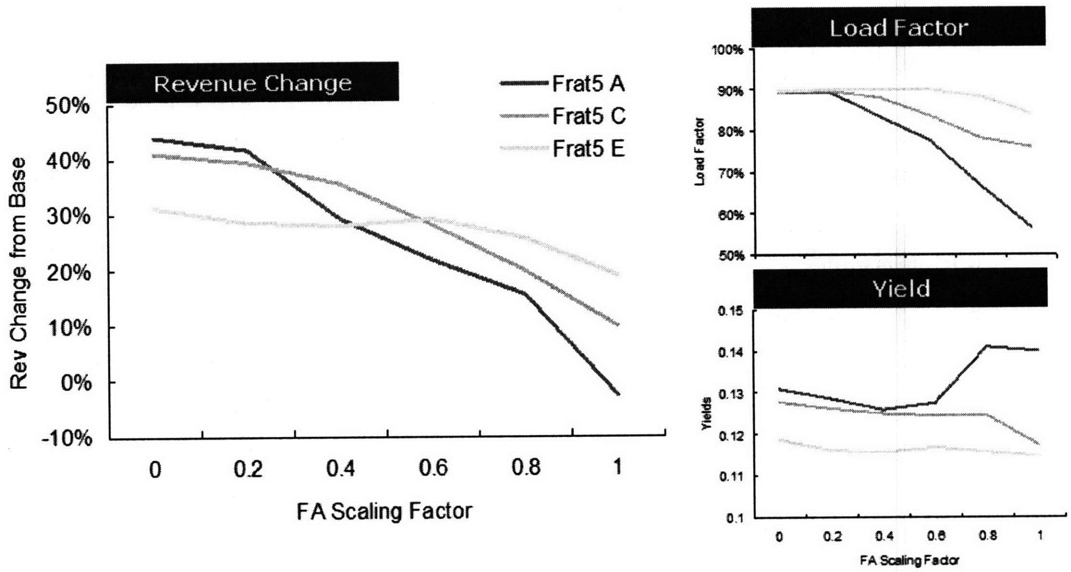


Figure 35: Impacts of Fare Adjustment on DPL with Q-Forecasting against AT90 in Single Market

The revenue drop can be explained by also taking a look at the load factor and yield of Airline 1 in Figure 35. The more aggressive the FA FRAT5s, the greater the sell-up probabilities and price elasticity costs, resulting in lower adjusted fares for low fare classes that are consequently closed more quickly in favor of the overprotected higher-fare classes. The increase in yield does not compensate for the fall in load factor, resulting in the revenue drop for Airline 1. Theoretically developed to encourage sell-up by closing lower fare classes earlier, Fare Adjustment seems to be too aggressive in doing so in this test case. FA appears to lead DPL optimizer to close lower fare classes too soon, decreasing load factor and increasing yield. We should not overlook the equivalency between DP-GVR and DPL/FA with Q-Forecasting as depicted in Figure 34 and Figure 35 – both RM methods produce better results using a less aggressive input FRAT5 set.

To proceed with our hypothesis that DP-GVR needs precise sell-up probabilities against AT90 to perform well, we would like to investigate whether using adaptive estimation of sell-up rates helps improve the performance of DP-GVR as compared to other RM methods. We would also seek for further improvement of DPL with QF over EMSRb with QF when the airline realistically estimates and adjusts FRAT5s at each time frame. Using the FP and IC estimators (described in §4.2.1 and §4.2.2, respectively) in our simulations, we compare potential revenue gains to those obtained with input FRAT5s and analyze the effectiveness of incorporating sell-up in the optimizers based on DP algorithms under the environment of this test case.

Figure 36 compares the revenue gains for different RM methods using input FRAT5s and the two estimators under study (FP and IC). Recall from Figure 34 that EMSRb with QF and DPL with QF perform better revenue-wise using aggressive FRAT5-A. FP and IC are found to improve both methods over using FRAT5-E except one case (EMSRb with

QF using IC), but are still outperformed by an input FRAT5-A. This is consistent with our previous hypothesis that EMSRb with QF and DPL with QF are better off using a more aggressive FRAT5s than the true sell-up behavior of passengers. From this figure, we see that the advantage of DPL with QF over EMSRb with QF becomes clearer with the use of estimated sell-up rates.

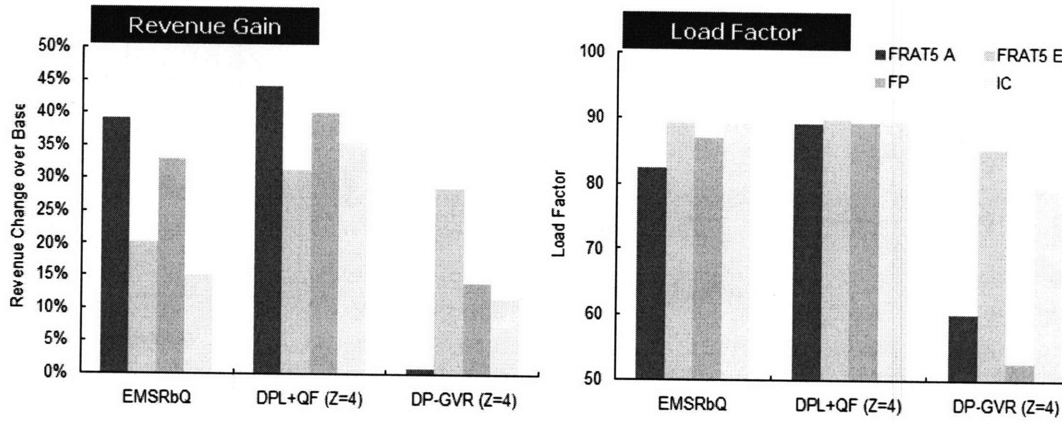


Figure 36: Revenue gains and Load Factors of RM methods against AT90 in Single Market using different methods of Sell-up Estimation

At first glance, FP appears to perform better than IC for any RM method in terms of revenue gain. However, both estimators do not seem to work effectively for DP-GVR against AT90. FP estimator manages to improve revenue over FRAT5-A by capturing less passengers (52.8% ALF) at higher yield, whereas IC estimator improves revenue by capturing higher load factor as compared to FRAT5-A (79.4% ALF) at lower yield. However, both estimators are unable to boost Airline 1's revenue at least to the level of FRAT5-E. If we look at the average estimated FRAT5 curves in Figure 37, we find that the IC FRAT5 curve matches well with FRAT5-E until time frame 10 when a sudden surge in sell-up rates is estimated. On the other hand, FRAT5s estimated by FP are even more aggressive than FRAT5-A in early time frames, and match quite closely with FRAT5-A curve later.

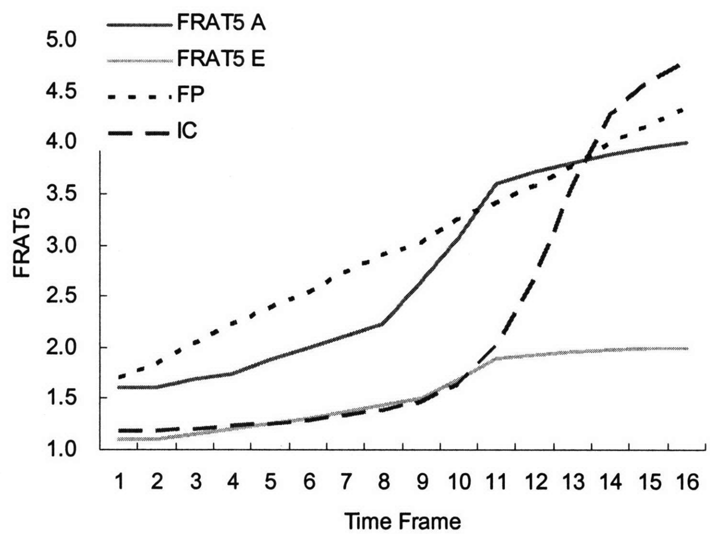


Figure 37: Comparison between Average Estimated FRAT5 Curves and input FRAT5 Curves for DP-GVR against AT90 in Single Market

Overall, this first analysis enables us to determine the "best" sell-up rates when one of the two airlines accounts for the possibility of sell-up in its RM system for our particular single market environment. We see that EMSRb and DPL with QF perform better by incorporating high sell-up models, and sacrificing spilling passengers towards the competing airline in favor of high-yield bookings despite the possibility of passenger loss due to buy-down. We also realize the limitation of DP-GVR in over emphasizing the low sell-up potential throughout the booking process caused by buy-down. It tries to be less protective throughout the booking process, but the results are not as glorious because of the expected head-to-head competition with AT90 for the low-fare, local passengers in a small OD market.

5.1.3 Test Case 2: Against EMSRb with Q-Forecasting

In this section, we focus on what happens when the competing airline upgrades its Revenue Management system and accounts for sell-up. Hence, for this particular test case, the simulations are set up to be that both airlines account for sell-up in their respective RM. We will investigate whether benefits can be achieved for Airline 1 using a DP method over the traditional EMSRb method, given that the airline accounts for sell-up when implementing both methods.

5.1.3.1 Specifications of Base Case and Simulations

Table 22 describes the set-up of our simulations for this test case. We will test the performance of EMSRb with Q-Forecasting, DPL with Q-Forecasting, and DP-GVR against a more advanced competitor in the Single Market Case. The base case will be that

Airline 1 still uses EMSRb with standard leg-based forecasting, which is equivalent to the first-come-first-serve (FCFS) under fully unrestricted environment – standard forecasts based on historical bookings-to-come leads to spiral-down and all bookings in the lowest fare class. On the other hand, the competing airline uses EMSRb with Q-Forecasting and input FRAT5-C. As previously, we will evaluate results across different demand levels (0.8, 0.9, 1.0, and 1.1). Various Z-factors (1, 2, 3, and 4) are again used in our DP simulations, and the one that leads to the highest revenue gains will be used for further investigation. Table 23 presents the base case results which results for this test case will be based on.

Test Case 2	Airline 1		Airline 2	
	Optimizer	Forecaster	Optimizer	Forecaster
Base Case	EMSRb	Standard Leg	EMSRb	QF, FRAT5-C
2A	AT90	-	EMSRb	QF, FRAT5-C
2B	EMSRb	Q-Forecasting	EMSRb	QF, FRAT5-C
2C	DPL	Standard Leg	EMSRb	QF, FRAT5-C
2D	DPL	Q-Forecasting	EMSRb	QF, FRAT5-C
2E	DP-GVR	-	EMSRb	QF, FRAT5-C

Table 22: Specifications of Test Case 2

Demand Factor	Revenue ($)	Load Factor %	Yield ($/RPM)
0.8	30760	82.0%	0.0893
0.9	33440	89.2%	0.0893
1.0	34478	91.9%	0.0893
1.1	35509	94.7%	0.0893

Table 23: Base Case Results of Airline 1 against EMSRb with Q-Forecasting in Single Market

5.1.3.2 DPL with Q-Forecasting vs. EMSRb with Q-Forecasting

We first consider the case when Airline 1 employs DPL with QF against EMSRb with QF (Test Case 2D), and both airlines assume medium sell-up rates using FRAT5-C. Recall that implementing DPL with QF against AT90 under high demand helps Airline 1 with a revenue gain of over 40% at the expense of Airline 2. Intuitively, the revenue improvement of Airline 1 should no longer be as much against a more advanced competitor. We establish in the results that when Airline 2 uses EMSRb and both airlines account for sell-up, Airline 1 undoubtedly does not benefit as much from the additional understanding and modeling of sell-up behavior. For example, Table 24 shows that at Demand Factor of 1.0, Airline 1 achieves revenue gain of more than 17%, and as much as 21.9% when Z-factor of 1 is used.

Demand Factor	Z-Factor	Revenue ($)	Load Factor %	Yield ($/RPM)
0.8	1	33535 (+9.0%)	80.6%	0.0991
	2	33283 (+8.2%)	80.9%	0.0980
	3	33339 (+8.4%)	80.8%	0.0982
	4	33264 (+8.1%)	80.9%	0.0980
0.9	1	37749 (+12.9%)	85.4%	0.1053
	2	37062 (+10.8%)	86.5%	0.1020
	3	37153 (+11.1%)	86.3%	0.1026
	4	36966 (+10.5%)	86.5%	0.1018
1.0	1	42044 (+21.9%)	87.9%	0.1139
	2	40616 (+17.8%)	89.8%	0.1077
	3	40930 (+18.7%)	89.5%	0.1089
	4	40475 (+17.4%)	89.6%	0.1075
1.1	1	44336 (+24.9%)	89.9%	0.1174
	2	42820 (+20.6%)	92.2%	0.1106
	3	43251 (+21.8%)	91.8%	0.1121
	4	42821 (+20.6%)	91.9%	0.1110

Table 24: Results of DPL with Q-Forecasting (FRAT5-C) against EMSRb with Q-Forecasting (FRAT5-C) in Single Market

In terms of the average leg load factors, Table 24 also shows that the average load factor for Airline 1 drops just a little from 91.9% in the base case to 87.9% when it uses DPL and accounts for sell-up in QF. However, it manages to achieve significant increase in yield from 0.0893 to 0.1139. Intuitively, this can be explained by the fact that both airlines are now protecting more seats for higher classes that more passengers are forced to sell up. We will investigate the results in more depth later in this chapter.

Looking at the performance measures, it appears that the efficacy of DPL with Q-Forecasting is greater when using a Z-factor of 1 against an advanced RM method. It allows Airline 1 to achieve comparable or even better revenue than its competitor when the demand is high. However, at higher Z-factors, DPL with QF is outperformed by the competing EMSRb with QF, and by a larger margin at a higher demand factor (Refer to Figure 38 and Figure 39). A Z-factor of 1 will therefore be used for the best case comparison with traditional methods later.

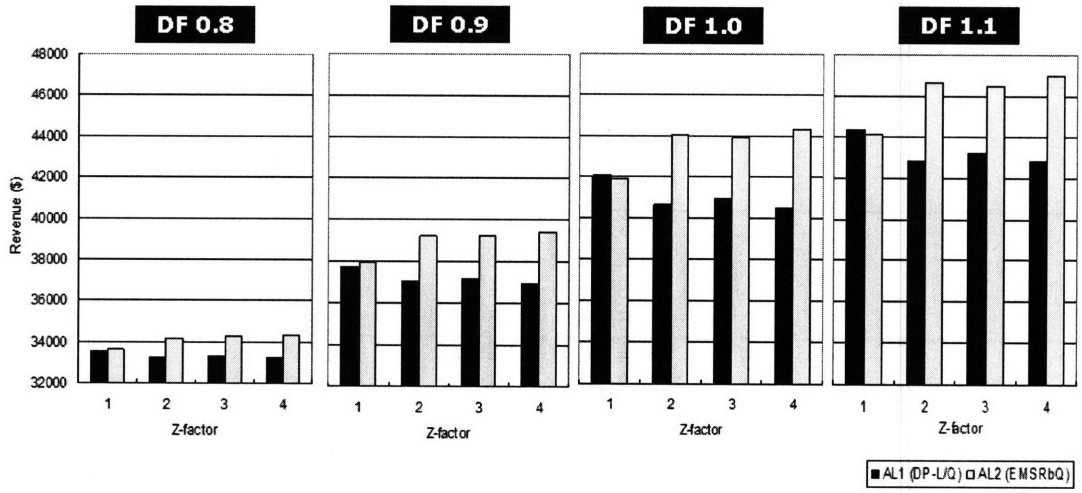


Figure 38: Revenues for DPL with Q-Forecasting against EMSRb with Q-Forecasting in Single Market

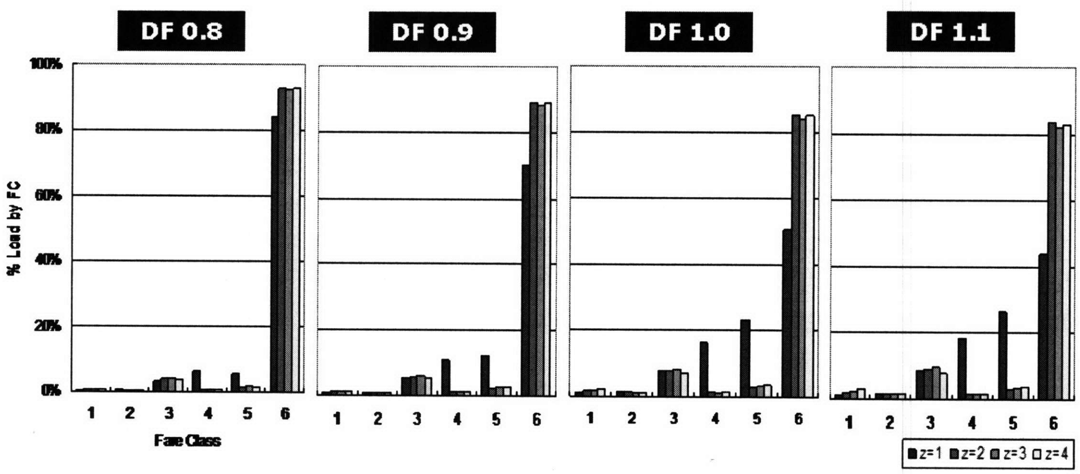


Figure 39: Fare Class Mix for DPL with Q-Forecasting against EMSRb with Q-Forecasting in Single Market

5.1.3.3 DP-GVR vs. EMSRb with Q-Forecasting

We now turn to look at the performance of DP-GVR against EMSRb with QF in the Single Market case (Test Case 2E), when both airlines assume medium sell-up inputs in FRAT-C. In contrast with the situation against AT90, DP-GVR performs poorly and even worse than the baseline revenue at low demand intensity. Highest revenue of Airline 1 is achieved at Z-factor of 4, but is still lower than the base case that employs EMSRb, a method that spirals down to be equivalent to First-Come-First-Serve (FCFS)! Intuitively, it appears that DP-GVR allows too many low-fare bookings and loses many potential high-yield passengers to Airline 2, resulting in a lower load factor that further deteriorates the revenue of Airline 1, as it has already had much lower yield (Refer to Table 25, Figure 40, and Figure 41). We will investigate the results in detail in the following section.

Demand Factor	Z-Factor	Revenue ($)	Load Factor %	Yield ($/RPM)
0.8	1	28207 (-8.3%)	70.2%	0.0957
	2	28267 (-8.1%)	70.2%	0.0959
	3	28343 (-7.9%)	70.0%	0.0964
	4	28371 (-7.8%)	70.0%	0.0965
0.9	1	31629 (-5.4%)	79.6%	0.0946
	2	31628 (-5.4%)	79.4%	0.0948
	3	31776 (-5.0%)	79.2%	0.0956
	4	31861 (-4.7%)	79.0%	0.0960
1.0	1	33824 (-1.9%)	84.2%	0.0957
	2	33739 (-2.1%)	83.9%	0.0957
	3	34139 (-1.0%)	83.6%	0.0972
	4	34327 (-0.4%)	83.5%	0.0979
1.1	1	35552 (+0.1%)	87.8%	0.0964
	2	35263 (-0.7%)	87.4%	0.0963
	3	35928 (+1.2%)	87.0%	0.0983
	4	36320 (+2.3%)	86.7%	0.0998

Table 25: Results of DP-GVR (FRAT5-C) against EMSRb with Q-Forecasting (FRAT5-C) in Single Market

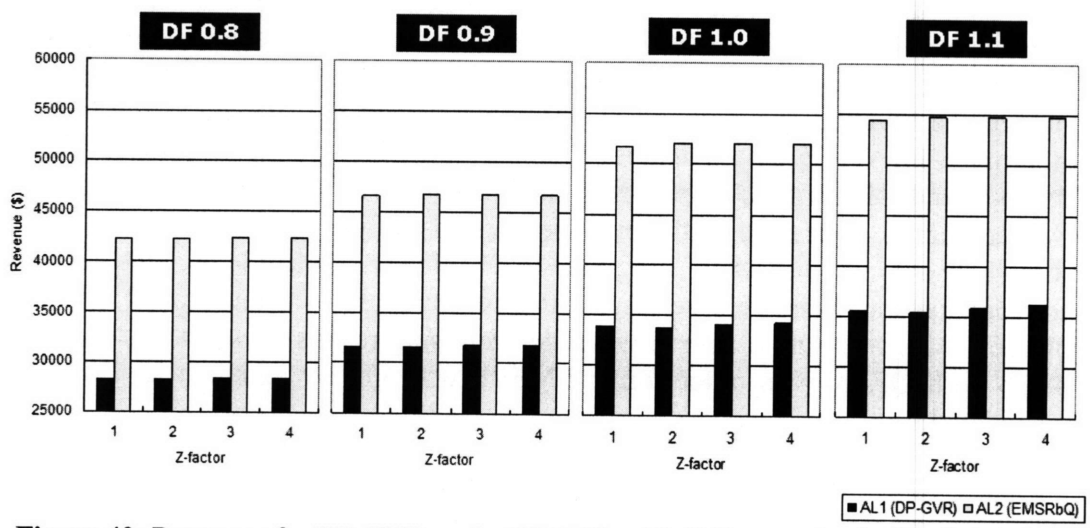


Figure 40: Revenues for DP-GVR against EMSRb with Q-Forecasting in Single Market

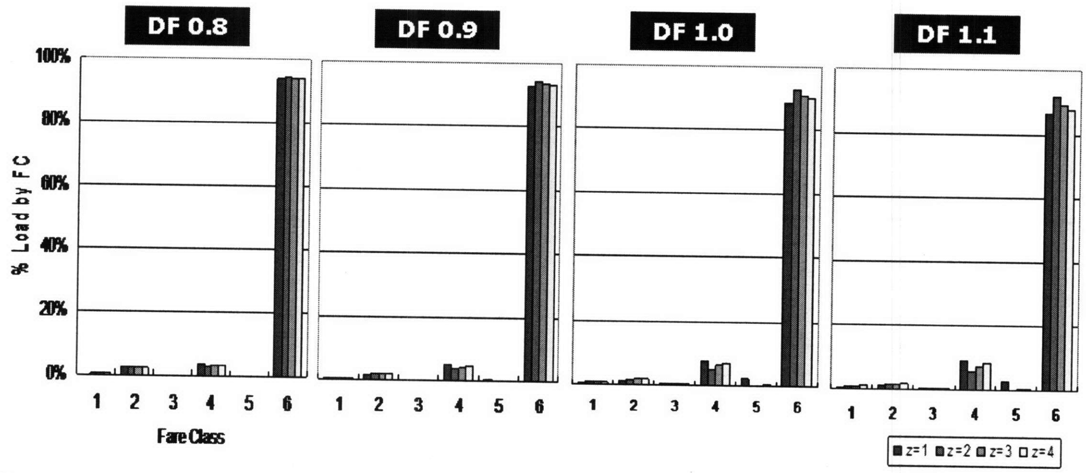


Figure 41: Fare Class Mix for DP-GVR against EMSRb with Q-Forecasting in Single Market

5.1.3.4 Investigation of DP methods against EMSRb with Q-Forecasting

The revenue graph in Figure 42 shows that when competing against EMSRb with Q-Forecasting, DP-GVR is unable to match the best results of other RM methods using Q-Forecasting. When using FRAT5-E, DP-GVR with a high Z-factor of 4 manages to minimize the revenue gap with other RM methods but still generates lower revenue. Furthermore, FRAT5-E is not the input leading to the best revenue of other RM methods. EMSRb-Q and DPL with Q-Forecasting obtain their best revenues with higher FRAT5. However, both revenues and load factors drop when applying higher FRAT5 to DP-GVR. Figure 43 also shows that when using FRAT5-C at lower demand level, DP-GVR obtains even worse revenue than the base case, with a revenue loss of as much as -7.8% (Refer to Table 26). DPL with QF is able to generate promising results as when Airline 1 applies EMSRb with QF. It even outperforms EMSRb with QF by as much as 1.4% at high demand level.

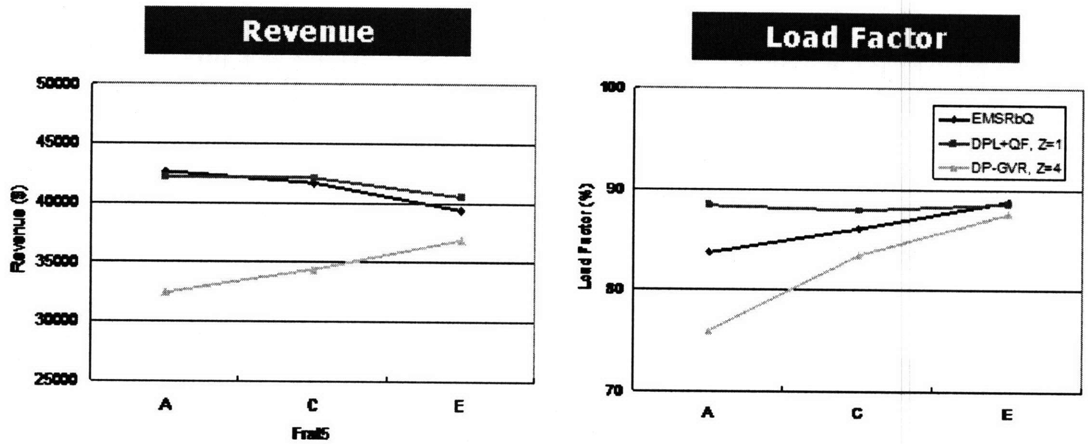


Figure 42: Sensitivity of FRAT5 values against EMSRb with Q-Forecasting in Single Market

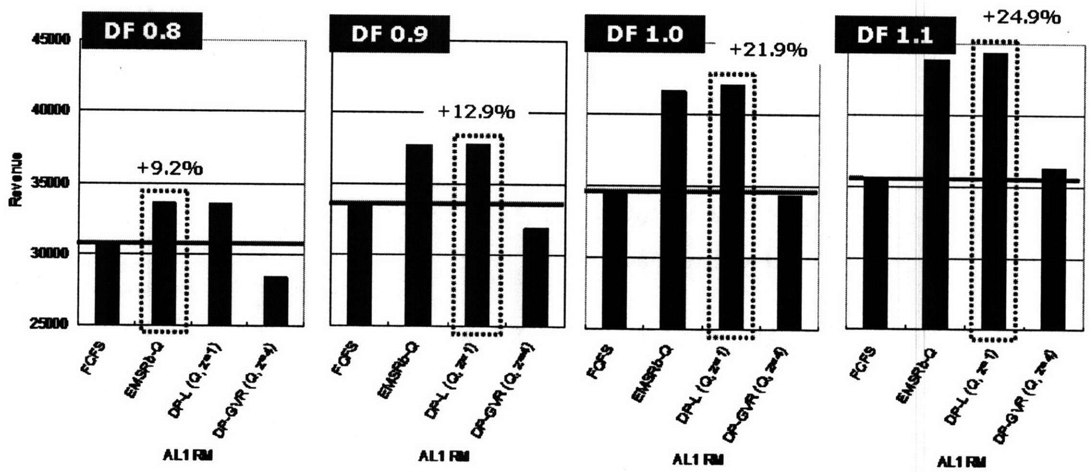


Figure 43: Revenue Comparison of Best Cases for different RM methods with FRAT5-C against EMSRb with Q-Forecasting in Single Market

DF	% Revenue gain over Base Case Scenario		
	EMSRbQ	DPL+QF	DP-GVR
0.8	+9.2%	+9.0%	-7.8%
0.9	+12.5%	+12.9%	-4.7%
1.0	+20.8%	+21.9%	-0.4%
1.1	+23.4%	+24.9%	+2.3%

Table 26: Revenue gains for RM methods against EMSRb with QF in Single Market

As for the case against AT90, we now examine the impact on Airline 1's revenue when various levels of Fare Adjustment are supplemented to DPL with Q-Forecasting. As shown in Figure 44, Fare Adjustment appears to bring potential revenue gain for each input FRAT5 series as it starts off with small FA scaling factors. However, the improvement does not continue indefinitely as the scaling factor grows. There exists a critical point where revenues begin to decline rapidly. In fact, as the input FRAT5 becomes more aggressive, the point at which Airline 1's revenue peaks occurs at a smaller scaling factor. For example, for input FRAT5-A, AL1's revenue slightly increases from +22.3% to a +23.0% peak at 0.2 scaling factor, after which it plummets due to overprotection. This phenomenon seems to suggest that little to moderate FA, depending on the aggressiveness of FRAT5 used for QF, can potentially generate additional revenue gains for Airline 1 over the case when no Fare Adjustment is applied at all. Such percentage benefits are perceived to be negligible for lower FRAT5 set.

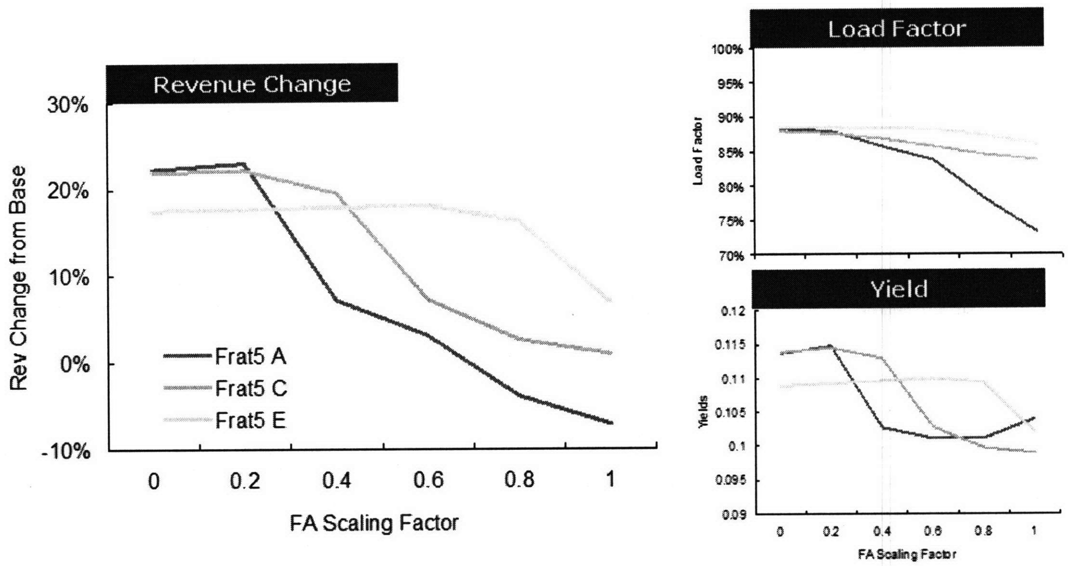
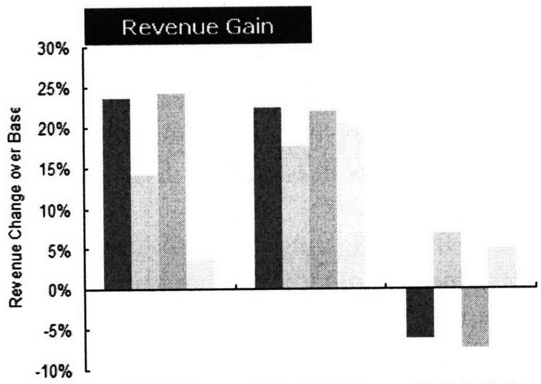


Figure 44: Impacts of FA aggressiveness on DPL with Q-Forecasting against EMSRb with Q-Forecasting in Single Market

Figure 45 compares the revenue gains for different RM methods using input FRAT5s and the two estimators under study (FP and IC). Recall from Figure 42 that EMSRb with QF and DPL with QF perform better revenue-wise using aggressive FRAT5-A. FP and IC are found to improve both methods over using FRAT5-E except one case (EMSRb with QF using IC). Using FP estimator in EMSRb with QF even gets slightly higher revenue than that obtained when FRAT5-A is used. On the other hand, using IC estimator in DP-GVR results in positive revenue gain, but is still worse than the case when FRAT5-E is used. The FP estimator does not help DP-GVR, as it produces a revenue loss that is even more than the case when FRAT5-A is used. It appears that EMSRb with QF is generally more adaptive to estimators since its decisions are mainly based on the forecasts of bookings-to-come, and booking class limits are frequently updated. In contrast, DP-GVR makes decisions on fare class closure based on FRAT5s and thus is sensitive to how they are estimated.



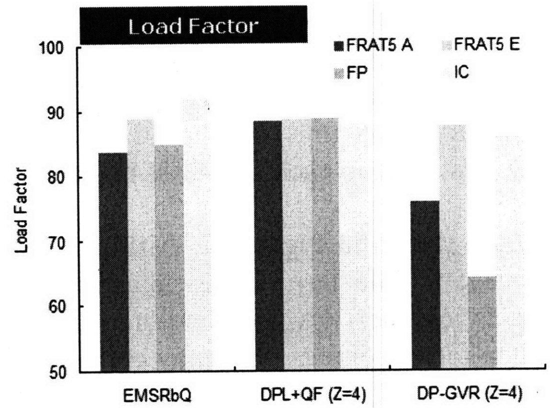


Figure 45: Revenue gains and Load Factors of RM methods against EMSRb with Q-Forecasting in Single Market using different methods of Sell-up Estimation

Looking at the average estimated FRAT5 curves in Figure 46, we find that the IC FRAT5 curve matches well with FRAT5-E until time frame 11 when a sudden surge in sell-up rates is estimated. On the other hand, FRAT5s estimated by FP are even more aggressive than FRAT5-A throughout the entire booking process except for the first 2 time frames. We realize that these estimated FRAT5 curves look very similar to the ones shown in Figure 37 for case against AT90.

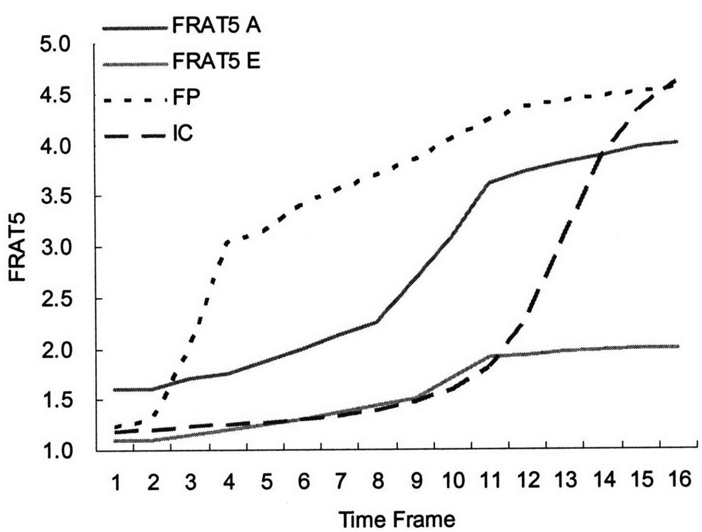


Figure 46: Comparison between Average Estimated FRAT5 Curves and input FRAT5 Curves for DP-GVR against EMSRb with Q-Forecasting in Single Market

An investigation of the fare class closures and cumulative bookings of the two airlines will explain why DP-GVR does not seem to perform well against EMSRb with Q-Forecasting. The fare class closure rates represent for each fare class the percentage of fare classes that are closed on paths over the network in each time frame. For example, if Fare Class 3 is closed in 40 paths out of the 100 paths that are served by Airline 1, the closure rate of Fare Class 3 for Airline 1 therefore equals 40%. In an unrestricted fare environment, in order to capture bookings in the highest fare class, an airline has to close all lower fare classes because passengers always buy in the lowest available fare class. Cumulative bookings, on the other hand, represent the cumulative average number of bookings made in each fare class on paths over the network in each time frame.

As shown in Figure 47, Airline 1 that uses DP-GVR and FRAT5-C does not close low and middle fare classes aggressively until the middle of the booking process. It accepts too many low fare bookings in the beginning and does not save enough seats for higher fare bookings that may arrive later. The competing airline, which implements EMSRb with QF and FRAT5-C, closes a significant portion of low fare classes early in the booking process. At the end of the reservation period, Airline 2 still has many middle fare classes open while Airline 1 has closed more of them. Figure 48 illustrates that Airline 1 accepts close to zero bookings once it closes down low and middle fare classes, whereas Airline 2 manages to accept many mid-fare bookings at the end. The reason is that passengers with high willingness-to-pay buy down and book seats in the middle fare classes of Airline 2. Its advanced RM method allows it to take advantage of this revenue opportunity by closing down the lowest fare class to boost mid-fare bookings. Why do DP-GVR and EMSRb with QF behave so differently in terms of closing down fare classes?

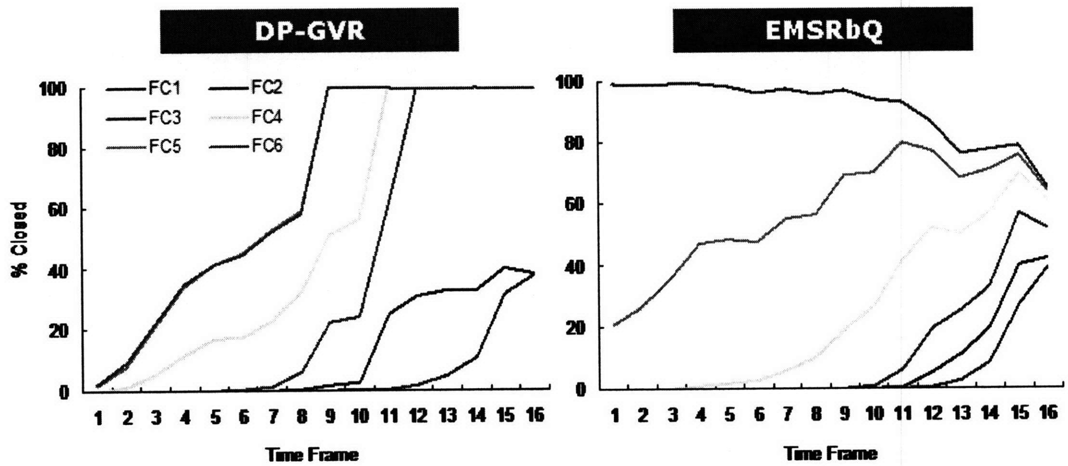


Figure 47: Fare Class Closure Rates for DP-GVR against EMSRb with Q-Forecasting in Single Market

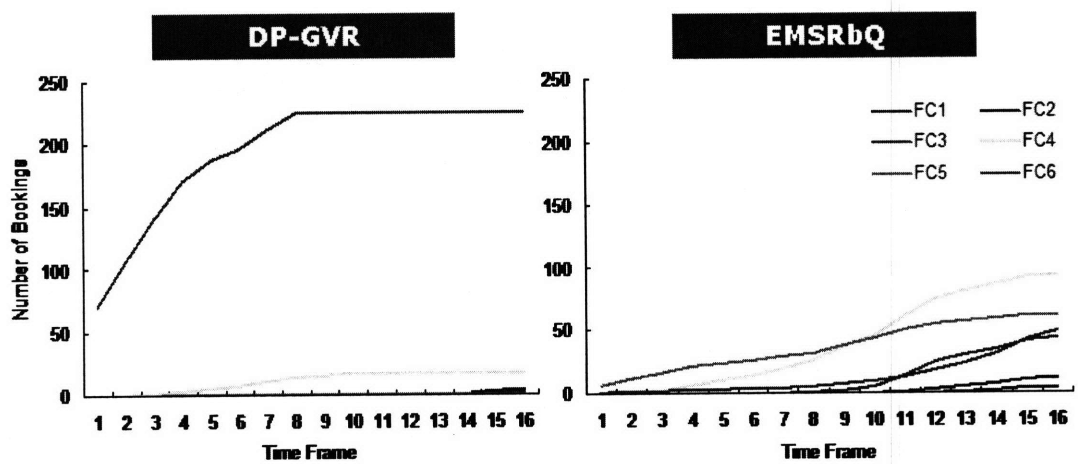


Figure 48: Cumulative Bookings by Fare Class for DP-GVR against EMSRb with Q-Forecasting in Single Market

Figure 49 compares the fare class mixes of the two airlines when Airline 1 uses DP-GVR with FRAT5-C and all four Z-factors at low and high demand level. DP-GVR only gets high loads in fare class 6 but very small loads in all higher fare classes. At low demand level when it cannot be as selective in its bookings, Airline 1 that implements EMSRb with QF does not accept as many mid-fare bookings as when the demand is high. This strengthens our analysis on the effect of fare class closure rates of both airlines. Both RM methods are using the same input FRAT5-C for QF and thus expecting high willingness-to-pay bookings to come later in the booking period. However, DP-GVR forecasts much smaller arrival rates at the end of the booking period as high willingness-to-pay passengers buy down to mid-fare classes of Airline 2. As a result, historical observations cause DP-GVR that assumes medium sell-up to capture more low-fare demand at the beginning and fill the plane when there *will not be* many high fare bookings later.

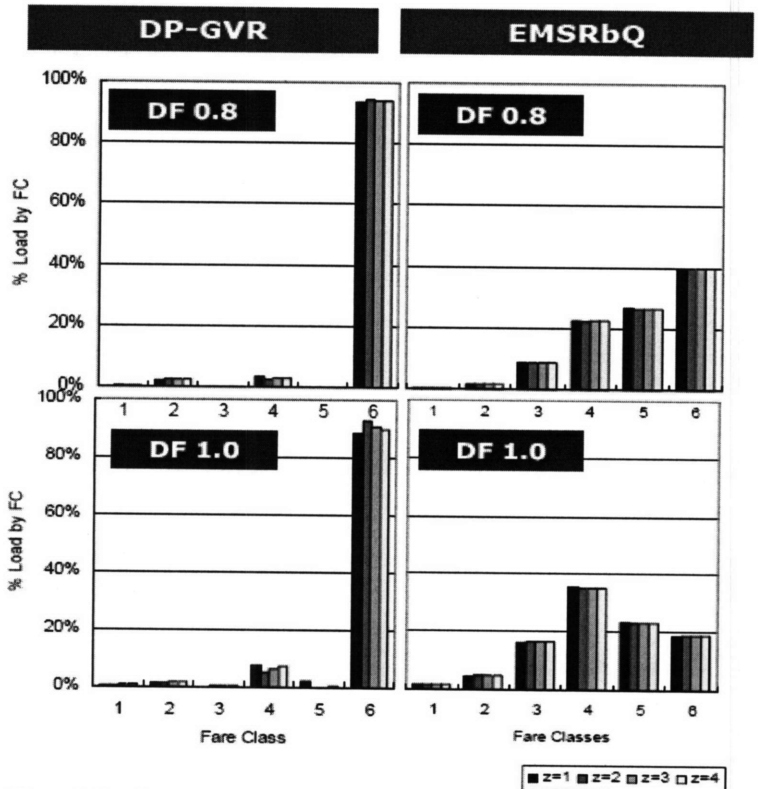


Figure 49: Fare Class Mix for DP-GVR against EMSRb with Q-Forecasting at Demand Factors 0.8 and 1.0

The promising results of DPL with QF can be accounted for by the similarity of the underlying assumptions of both DPL with QF and EMSRb with QF. Both RM methods assume independent fare classes with independent demands and utilize partitioned equivalent Q bookings to forecast bookings for each fare class. A comparison of fare class mixes of DPL with QF and EMSRb with QF as shown in Figure 50, along with the closure patterns of the 2 airlines as shown in Figure 51, illustrates the similarity of the two RM methods. Both airlines are closing their fare classes at similar rates throughout the reservation period. With roughly 60% closure rate of the lowest fare class, both RM methods manage to capture some demand in the middle fare classes 4 and 5 that arrive early in the period. At lower demand level, both RM methods do not protect as many seats for high fare seats with smaller arrival rates, causing both airlines to optimally slow down closure rates and fill more seats with low fare loads.

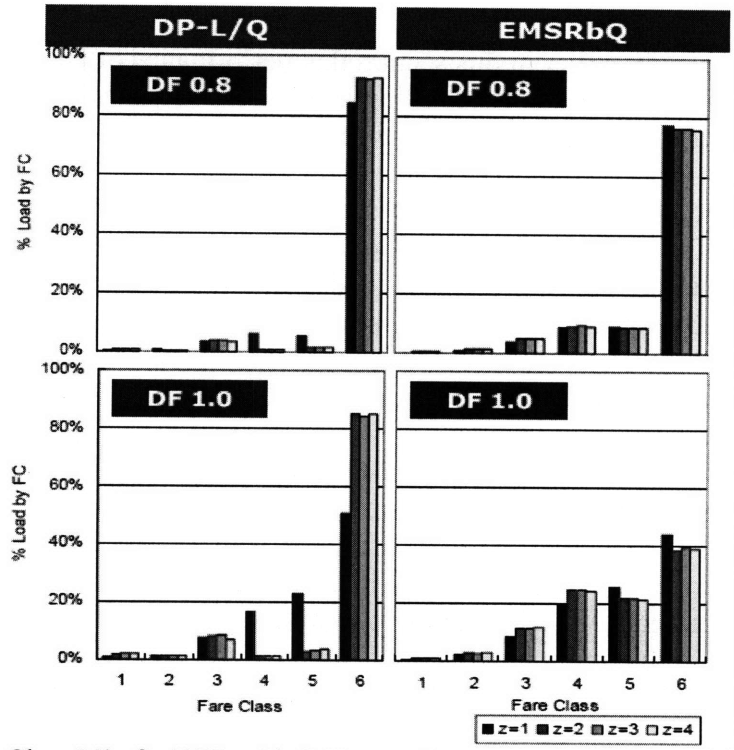


Figure 50: Fare Class Mix for DPL with Q-Forecasting against EMSRb with Q-Forecasting at Demand Factors 0.8 and 1.0

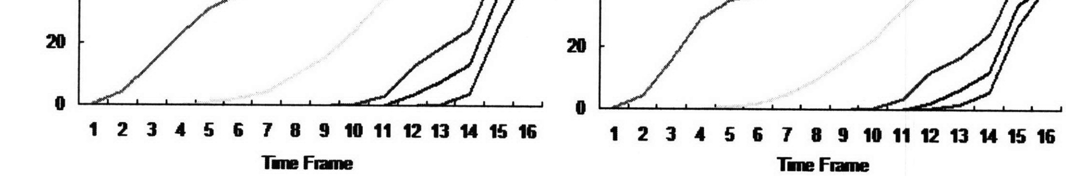


Figure 51: Fare Class Closure Rates for DPL with Q-Forecasting against EMSRb with Q-Forecasting in Single Market

Figure 50 also shows that performance of DPL with QF improves if we use a smaller assumed (input) Z-factor for the optimizer. At demand factor 1.0, fare mix is insensitive to Z-factors except at 1, when there is a significant shift to higher fare classes. With a lower input demand variance to the DP optimizer, DPL with QF becomes more willing to protect higher fare classes, thereby closes down low fare classes at a faster rate than higher Z-factors, resulting in slight improvement in revenue at Z-factor of 1.

We have discussed earlier in this section that performance of DP-GVR is sensitive to FRAT5 inputs. DP-GVR with Q-Forecasting and FRAT5-C generates poor results at both ends of the booking process because it loses revenue from two sources:

(1) Underprotection of high fare classes early in the booking period. Demand with high willingness-to-pay that would have been captured by Airline 1 but is rejected as the airline is already filled with low-fare demand;
(2) Overprotection of high fare classes late in the booking period. Demand with high willingness-to-pay that is willing to sell-up but is even more willing to buy down to middle fare classes made available by Airline 2.

Figure 52 evaluates how sensitive DP-GVR performs to input FRAT5. We note that as DP-GVR with Q-Forecasting uses an input set with high WTP such as FRAT5-A, low fare classes are closed down not long after the start of the booking process due to forecasts of higher sell-up probabilities. While FRAT5-A corresponds to more protection of high fare classes early in the booking period that seems to be an improvement over FRAT5-C, this closure pattern ignores the fact that at the end of the booking period most of the empty seats are not filled but still the airline keeps on rejecting mid-fare passengers. This results in low revenue and low load factor as observed in Figure 42. Using a lower WTP input set such as FRAT5-E, on the other hand, causes the airline to run out of seat earlier than when using higher FRAT5 and accept less high fare loads. Historical observations of low high fare arrivals used as input in the forecaster makes DP-GVR slow down fare class closure rates in favor of low fare demand that arrive more early in the booking period, causing DP-GVR to improve revenue to the level close to that by DPL with Q-Forecasting and EMSRb with Q-Forecasting.

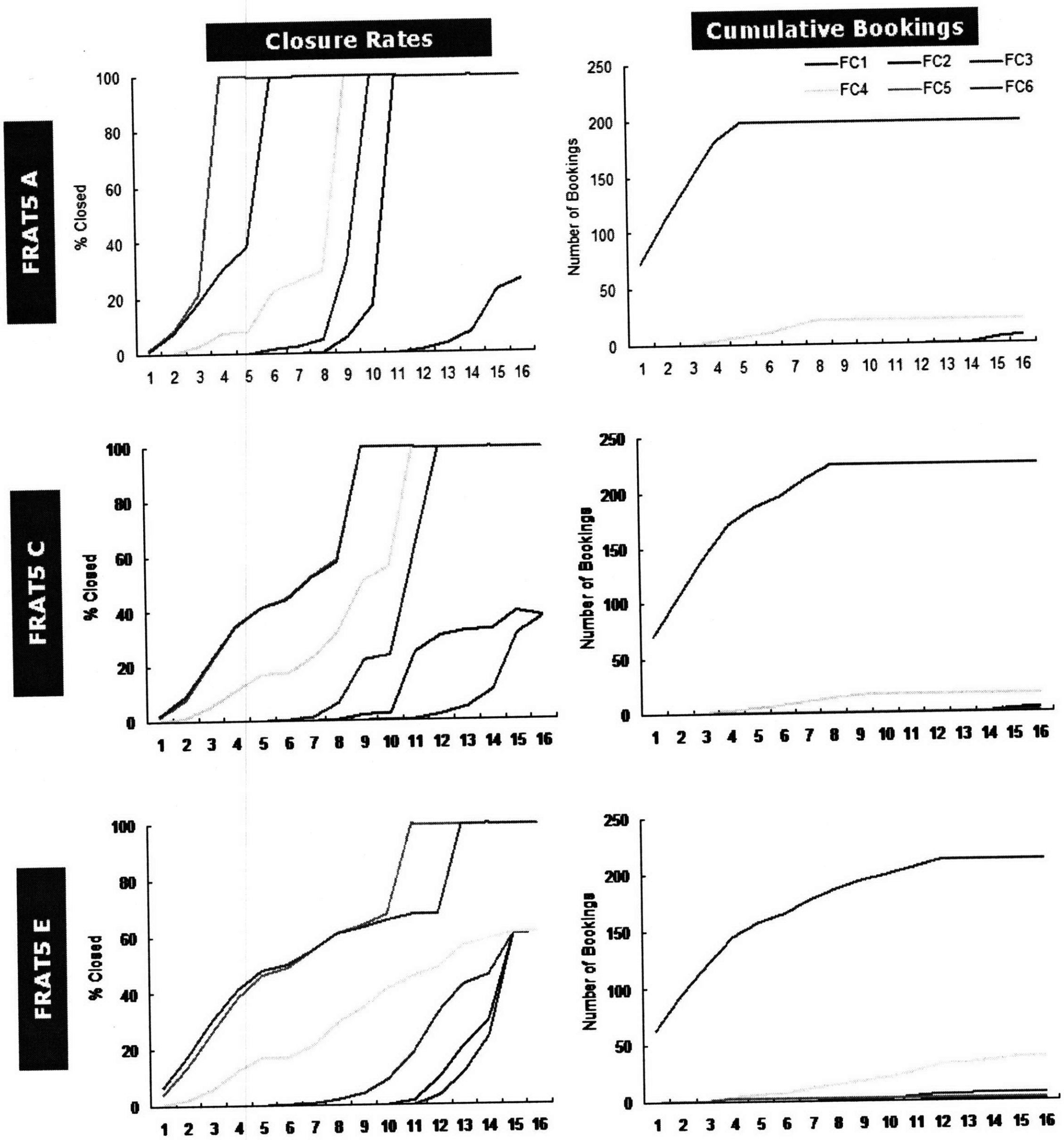


Figure 52: Fare Class Closures of DP-GVR against EMSRb with Q-Forecasting at various input FRAT5 sets and corresponding Cumulative Bookings by Fare Class

5.2 Network D6

After looking at the simulation results of these RM methods for the Single Market case, we can now extend our results obtained so far to a more complex airline network setting. Again, a relatively symmetric network in which airlines offer very similar fare structures and schedules becomes our main interest here. The reason is that we would like to ensure the RM systems to be the predominant distinction between airlines so that we are able to properly evaluate not only the RM methods themselves but also the adjustments made to their optimization or forecasting methods. As mentioned earlier, the implementation of each of the five methods – EMSRb with QF, EMSRb/FA with QF, DPL with QF, DPL/FA with QF, and DP-GVR, will be simulated in Network D6 against three competing RM methods: AT90, EMSRb/FA with QF-FP, and a symmetric RM method.

5.2.1 Overview of Network

Network D6 is a simplified representation of the US domestic airline network with two competing hub-and-spoke carriers. Each of the centrally located hubs has 20 Eastern spoke cities and 20 Western spoke cities. The incumbent airline, or Airline 1, is located at the Minneapolis-Saint Paul International Airport (MSP), whereas the competing airline, or Airline 2, is located at the Dallas-Fort Worth International Airline (DFW), as shown in Figure 53. To control our simulation experiments, both airlines experience symmetric head-to-head competition in all of their markets, and operate three one-way, west-to-east banks of connecting flight at its hub daily with the same schedules. Non-stop inter-hub flights are also offered by each airline.

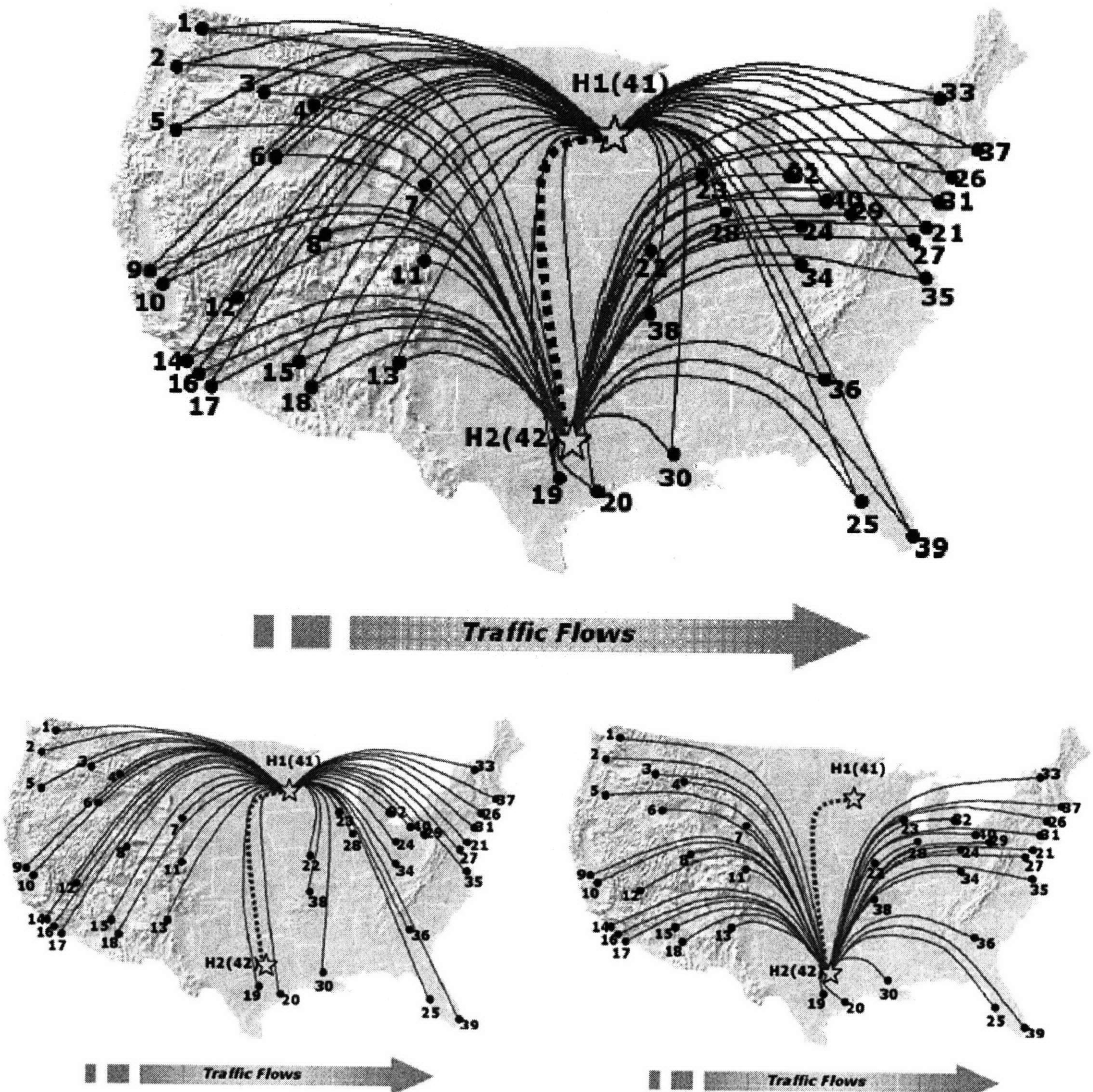


Figure 53: Route Map for Airline 1 (left) and Airline 2 (right) in Network D6

Therefore, each airline serves 482 Origin-Destination markets in each of its banks, and each market serves 42 local markets and 440 connecting markets. Detailed description about this network can be found in Lee**Error! Bookmark not defined.**. Besides symmetry in route map, both airlines are assumed to operate very similar schedules and offer identical 6-fare-class structure in which fare classes are differentiated by fare price only, as characterized in Table 27. For this test case, we assume both airlines offer an unrestricted fare structure with an average fare ratio of 4.1 across the network.

Fare Class	Average Fare	Restrictions			
		Advance Purchase	R1	R2	R3
1	$412.85	0 days	No	No	No
2	$293.34	0 days	No	No	No
3	$179.01	0 days	No	No	No
4	$153.03	0 days	No	No	No
5	$127.05	0 days	No	No	No
6	$101.06	0 days	No	No	No

Table 27: Fare Structure and Restrictions in Network D6

In the real world, no airlines would apply complete fare simplification across all of their markets. There should always be certain markets that are offered a combination of both restricted and unrestricted fare structures. A fully unrestricted fare structure is assumed in this thesis in order to control our simulation experiments and properly evaluate the potential revenue contribution of DPL and DP-GVR to an airline when passengers tend to buy in the cheapest fare available to them.

5.2.2 Test Case 3: Against AT90

Like in the case of single market, we first examine the performance of DPL and DP-GVR against a competitor using AT90. In his research, Vanhaverbeke (2006) tests DPL with Q-Forecasting alone against *AT80* in Network D6 and gets comparable revenue gain as other RM methods with Q-Forecasting but not as good as DP-GVR when a low FRAT5 set is used. We would like to extend his results to investigate if incorporating Fare Adjustment in the optimizer can improve the performance of DPL over other RM methods and close to DP-GVR. Here we only run tests with a medium FRAT5-C rather than various sets of fixed input FRAT5 curves. Sell-up FRAT5 estimators (Forecasting Prediction and Inverse Cumulative methods) are also used to ensure results to depend more on the impact of DP methods under competition than the accuracy of pre-determined sell-up probabilities.

5.2.2.1 Specifications of Base Case and Simulations

Table 28 describes the set up for simulations of this test case. For the baseline environment, both competitors employ AT90. Hence, for this particular test case, the experimentation is set up to be that only one airline, Airline 1, accounts for sell-up in its RM. Note that Fare Adjustment that is used in simulations of this test case assumes a Scaling Factor of 1.0. Table 29 and Figure 54 show the revenue and fare class mix of the two airlines in this base case scenario. Again, it is not surprising to see little difference in revenue and fare class mix between the two airlines that close fare classes by the same adaptive threshold levels in a symmetric network. For this network, we do not compare across various demand levels and only consider a demand factor of 1.0.

Test Case 3	Airline 1		Airline 2	
	Optimizer/Forecaster	FRAT5	Optimizer/Forecaster	FRAT5
Base Case	AT90	-	AT90	-
3A	EMSRb /QF	"C", FP, IC	AT90	-
3B	EMSRb, FA /QF	"C", FP, IC	AT90	-
3C	DPL /QF	"C", FP, IC	AT90	-
3D	DPL, FA /QF	"C", FP, IC	AT90	-
3E	DP-GVR	"C", FP, IC	AT90	-

Table 28: Specifications of Test Case 3

Airline	Revenue ($)	Load Factor %	Yield ($/RPM)
1	1118480	88.3	0.1033
2	1123303	88.2	0.1000

Table 29: Base Case Results of Airline 1 against AT90 in Network D6

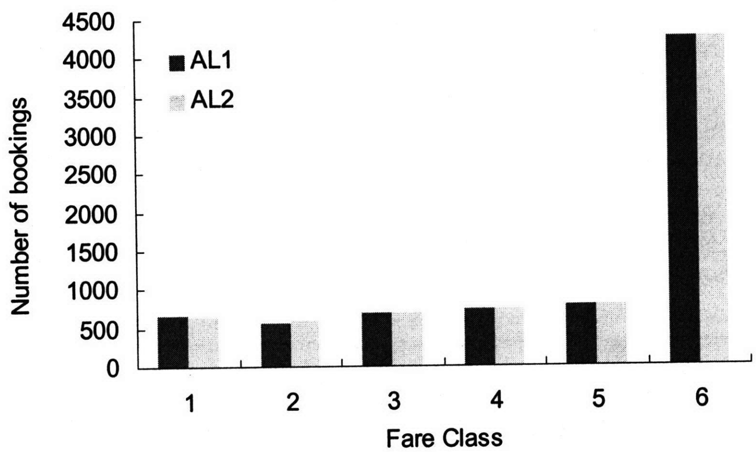


Figure 54: Fare Class Mix of Base Case in Network D6

5.2.2.2 Investigation of DP methods against AT90 in Network D6

Figure 55 presents the revenues of Airline 1 when using different RM methods: EMSRb with QF, DPL with QF, and DP-GVR. For the revenue graphs of Airline 1 and 2, the RM methods listed on both X-Axes correspond to the RM method used by Airline 1. For comparison purposes, each RM method is tested with both input (FRAT5-C) and estimated (FP and IC) sell-up models. The effect of incorporating Fare Adjustment for each method is also presented. We consider first the case when Airline 1 applies EMSRb with Q-Forecasting. The results for EMSRb with QF are consistent with Soo (2007) that Fare Adjustment generally helps the method regardless of the estimators used. FP estimator (+4.1%) seems to be a better choice than IC (+3.7%) when EMSRb with QF does not incorporate Fare Adjustment. However, FA, when applied with the IC estimator, leads to an increase in revenue by 1.5%, which is more than the gain provided by FA with FP. Nevertheless, EMSRb with QF and FA using FRAT5-C performs the best among all methods, having revenue gain of 12.1% (Refer to Table 30).

Unlike the Single Market case where it is the clearer winner, DPL with QF is outperformed by EMSRb with QF regardless of the estimator used. Again, FP has a slight edge over IC and FRAT5-C when used with DPL with QF. Both FP and IC are not helped by Fare Adjustment, which, however, improves the result with FRAT5-C by 4.3%. When adjusting fares using the IC estimates of sell-up probabilities, DPL with QF actually leads to Airline 1's revenue even lower than the benchmark revenue.

The highlight of Figure 55 belongs to DP-GVR, which leads to an increase in revenue by as much as 7.1% when IC estimator is used. IC performs slightly better than FRAT5-C in DP-GVR by 0.1%. Using FP estimator in DP-GVR does not result in as much revenue gain, but nonetheless significant. Although EMSRb with QF is able to generate more revenue with FRAT5-C, the reality is that airlines do not totally buy into an idea of risking their business with an arbitrary FRAT5. On the other hand, if we compare the load factor and yield graphs (Refer to Figure 56) between DP-GVR and EMSRb with QF and FA, we notice that the benefits provided by DP-GVR are achieved with even higher load factor and yield. This further strengthens the potential of DP-GVR over EMSRb with QF as it supports the common industry strategy of attracting as many high-fare passengers as possible and at the same time filling up the airplanes.

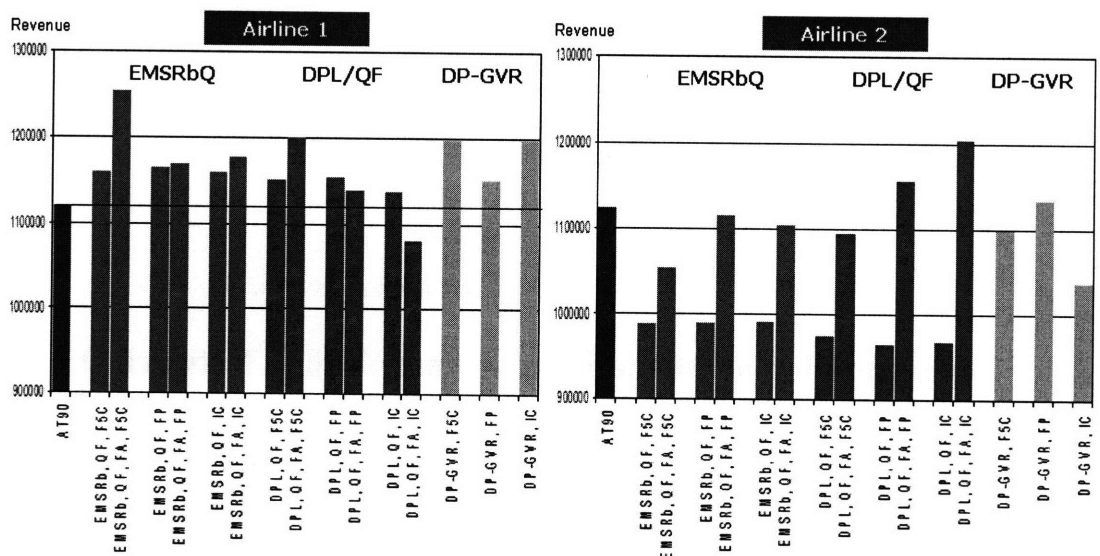
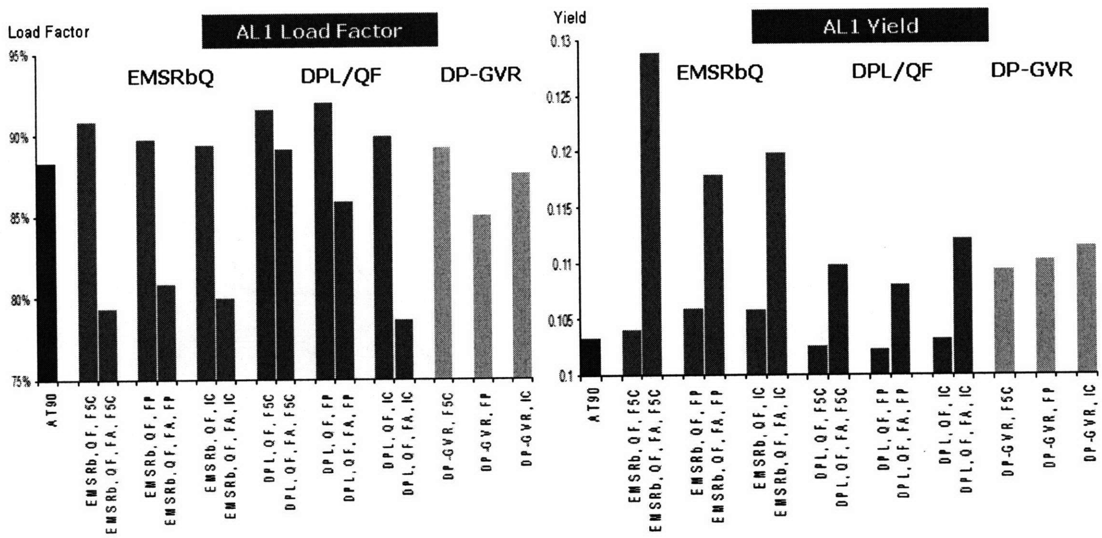


Figure 55: Revenue Summary of different RM methods against AT90 in Network D6

AL1 RM	FRAT5 Input or Estimator		
	FRAT5-C	FP	IC
EMSRb, QF	+3.7%	+4.1%	+3.7%
EMSRb, QF, FA	+12.1%	+4.5%	+5.2%
DPL, QF	+3.0%	+3.2%	+1.7%
DPL, QF, FA	+7.3%	+1.8%	-3.4%
DP-GVR	+7.0%	+2.9%	+7.1%

Table 30: Revenue gains of Airline 1 against AT90 in Network D6



* F5C stands for FRAT5-C.

Figure 56: Load Factors and Yields of Airline 1 using different RM methods against AT90 in Network D6

Depending on the RM method, there is not a unique estimator that performs the best for all the RM methods under study. Nevertheless, we observe that DP-GVR with a sell-up estimator generally performs well against AT90, and generates the highest revenue when IC estimator is used. We mention in the Single Market case about the equivalency between DP-GVR and DPL with QF and FA as both consider sell-up probabilities in their optimization algorithms. We notice that this is the case in Network D6 as well, since both DPL with QF and FA and DP-GVR produce comparable revenue gains (+7.3% and +7.0%, respectively) when FRAT5-C is used. However, this equivalency is not observed for FP and IC estimators.

Moreover, the comparison between average estimated FRAT5 curves and FRAT5-C curve, as shown in Figure 57, shows that the rankings of sell-up rates demonstrate 3 phases:

(1) From the beginning to Time Frame 8, IC FRAT5 curve starts off with slightly higher FRAT5 than FP, and both estimators predict higher sell-up rates than FRAT5-C inputs

(2) From Time Frame 8 to 11, FP FRAT5 curve matches closely and is slightly lower than FRAT5-C, but the sell-up estimates by IC remain low.

(3) From Time Frame 11 to the end, FRAT5 estimates by FP continue to surge and become much more aggressive than the others throughout the rest of the booking process. The IC FRAT5 curve gradually increases and finally surpasses FRAT5-C at Time Frame 14.

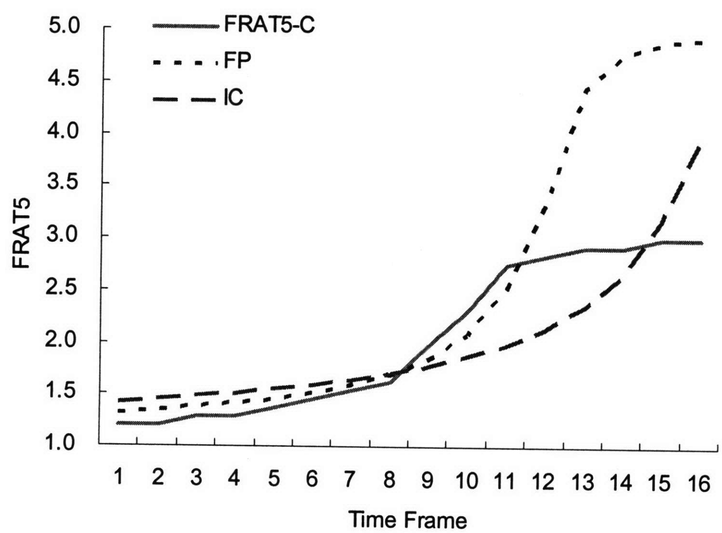


Figure 57: Comparison between Average Estimated FRAT5 curves and input FRAT5-C for DP-GVR against AT90 in Network D6

In order to highlight the success of DP-GVR using IC estimator against AT90, we investigate the fare class mix, closure patterns, and cumulative bookings of both airlines, and compare with the case when Airline 1 employs EMSRb with QF and FA using IC estimator. We examine the involved mechanisms behind how DP-GVR using IC estimator still manages to improve the revenue by 2% over a traditional EMSRb method that appears to have done "everything right" by accounting for sell-up behavior in Q-Forecasting and Fare Adjustment.

According to the closure patterns of fare classes as shown in Figure 58, we first notice that EMSRb with QF and FA closes most of their lower classes very earlier in the booking process and gradually closes higher fare classes until it runs out of seats on some paths after time frame 14 and consequently has to close some of fare classes 1. On the other hand, DP-GVR closes down low fare classes about 3 times less as much in early time frames, accounting for the buy-down effect of passengers to Airline 2 that employs a simple RM method if protecting higher-fare seats too aggressively that early. Thus, following the IC estimation of FRAT5, DP-GVR responds to the low protection of AT90 by Airline 2 by allowing more lower-fare class bookings in the beginning. However, we notice that DP-GVR manages to close down mid-fare classes faster than its competitor. This illustrates the power of estimator in DP-GVR, that while it prevents passengers in low-demand paths from buying down to a simple competitor earlier in the booking process, it still manages to realize the high sell-up potential for certain high-demand paths that do not observe much buy-down, and closes down mid-fare classes for those paths earlier in anticipation of future bookings in higher fare classes. Unlike DP-GVR, EMSRb with QF aims to target higher-yield passengers since time frame 1 with its very aggressive Fare Adjustment. This strategy proves to be effective against a simple competitor such as AT90, but the potential gain is not as high as DP-GVR using IC estimator.

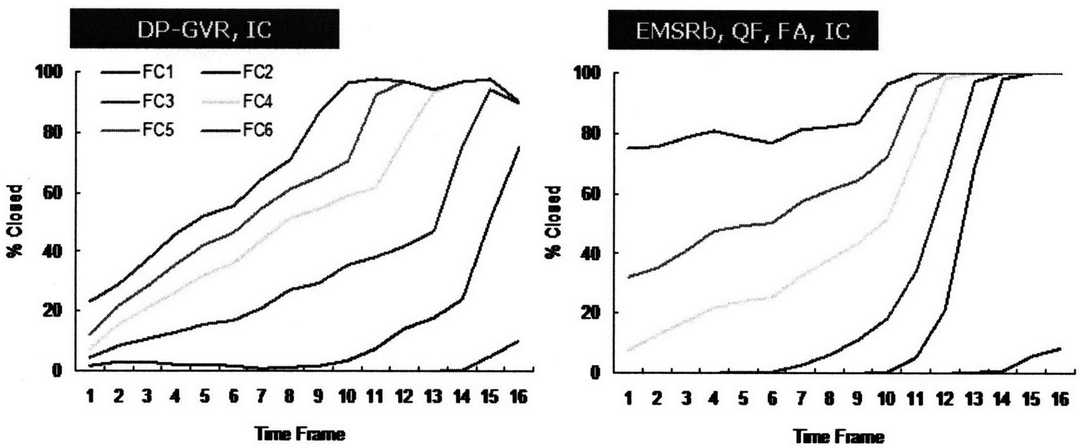


Figure 58: Comparison of Fare Class Closure Rates between DP-GVR with IC and EMSRb/FA with QF and IC against AT90 in Network D6

The cumulative booking curves presented in Figure 59 supports our analysis. We observe that DP-GVR estimates low sell-up rates in the beginning and competes with Airline 2 for these low-WTP passengers. EMSRb with QF and FA foregoes these low-yield passengers in anticipation of future bookings in higher-fare classes. Looking at the fare class mixes of both methods in Figure 60, we note that the potential benefits of capturing large loads of low-fare passengers earlier in DP-GVR manages to compensate the opportunity costs of future bookings in high-yield bookings in EMSRb/FA with QF, given that the competitor employs a RM method as simple as AT90.

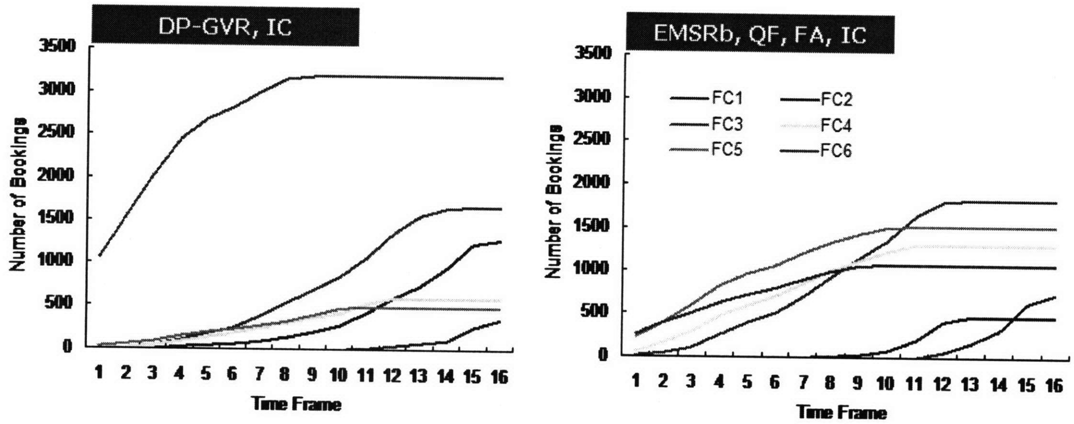


Figure 59: Comparison of Cumulative Bookings by Fare Class between DP-GVR with IC and EMSRb/FA with QF and IC against AT90 in Network D6

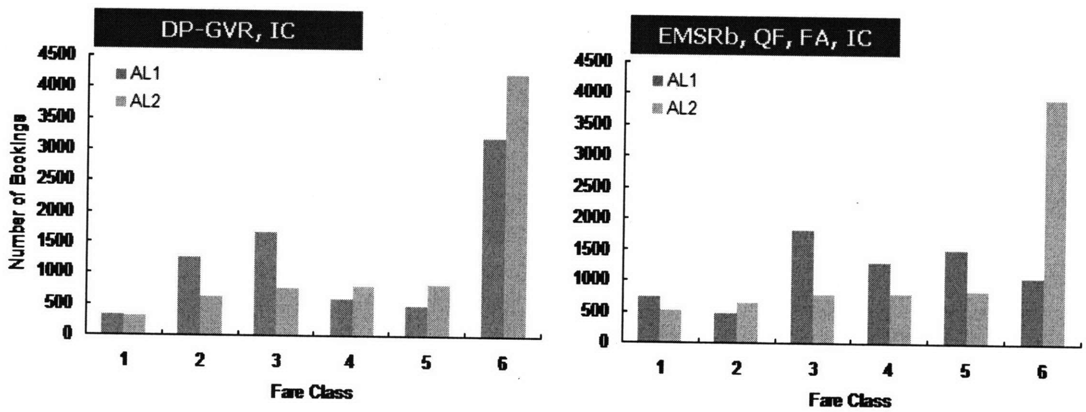


Figure 60: Comparison of Fare Class Mix between DP-GVR with IC and EMSRb/FA with QF and IC against AT90 in Network D6

We should not overlook potential improvement EMSRb with QF may achieve by using a smaller scaling factor in Fare Adjustment. However, searching for the peak revenue with the "right" scaling factor depends on the estimated FRAT5 and may not necessarily be uniform across all paths to generate the maximum revenue. The use of DP-GVR requires less effort in terms of the procedural steps to account for passengers' sell-up behavior, and, more importantly, still manages to improve revenue by 3.5% and 2% over EMSRb with QF and EMSRb/FA with QF, respectively, at a high average load factor.

5.2.3 Test Case 4: Against EMSRb with Q-Forecasting and Fare Adjustment

The experiments performed so far assumes that, if the competitor incorporates sell-up model in their forecaster, they implement FRAT5-C as the baseline estimate of passengers' maximum WTP in order to simplify analysis as well as to ease the simulation effort of PODS. For example, one of the test cases in the Single Market involves that the competing airline employs EMSRb with Q-Forecasting and FRAT5-C.

Given that in reality airlines are reluctant to assume an arbitrary FRAT5-C to model sell-up behavior of passengers, we would like to inspect the case in Network D6 against an advanced RM method using estimates of passengers' maximum WTP at each time frame rather than an arbitrary assumption. In doing so, we hope to extend our evaluation of DPL and DP-GVR to a more realistic and competitive setting. We would like to evaluate the impacts of estimating sell-up on the performance of DPL and DP-GVR when the competitor also estimates sell-up, and compare results to EMSRb when the competitor accounts for sell-up estimation. We observe in the Single Market case that FP over-predicts sell-up rates and estimates a FRAT5 curve that is even more aggressive than FRAT-A. We also recall from the Single Market case that RM methods that assume independent class demand tend to perform better with more aggressive fare class protection. Therefore, in this simulation test case, we assume that the competitor employs EMSRb with Q-Forecasting and Fare Adjustment using FP estimator. Again, this test case involves Airline 1 employing EMSRb with Q-Forecasting, DPL with Q-Forecasting, and DP-GVR.

5.2.3.1 Specifications of Base Case and Simulations

Table 31 describes the set up for simulations of this test case. The baseline scenario therefore involves an asymmetric RM structure – AT90 versus EMSRb with QF-FP and FA. For this particular test case, the experimentation is set up to be that both airlines account for sell-up in their respective RM except the base case. Fare Adjustment that is used in all simulations of this test case assumes Scaling Factor of 1.0. That is, both QF and FA assume the same set of FRAT5, whether it is an input or estimated set. Table 32 and Figure 61 present the revenue and fare class mix of the two airlines in this base case scenario. Consistent with our intuitive expectation, Figure 61 shows that Airline 2 which employs a more sophisticated RM method outperforms Airline 1 that uses a simple AT90 by capturing lower loads but higher-yield passengers. We do not compare across various demand levels and only consider a demand factor of 1.0.

Test Case 4	Airline 1		Airline 2	
	Optimizer/Forecaster	FRAT5	Optimizer/Forecaster	FRAT5
Base Case	AT90	-	EMSRb, FA /QF	FP
4A	EMSRb /QF	"C", FP, IC	EMSRb, FA /QF	FP
4B	EMSRb, FA /QF	"C", FP, IC	EMSRb, FA /QF	FP
4C	DPL /QF	"C", FP, IC	EMSRb, FA /QF	FP
4D	DPL, FA /QF	"C", FP, IC	EMSRb, FA /QF	FP
4E	DP-GVR	"C", FP, IC	EMSRb, FA /QF	FP

Table 31: Specifications of Test Case 4

Airline	Revenue ($)	Load Factor %	Yield ($/RPM)
1	1121313	89.0	0.1027
2	1166628	80.3	0.1140

Table 32: Base Case Results of Airline 1 against EMSRb with QF-FP and FA in Network D6

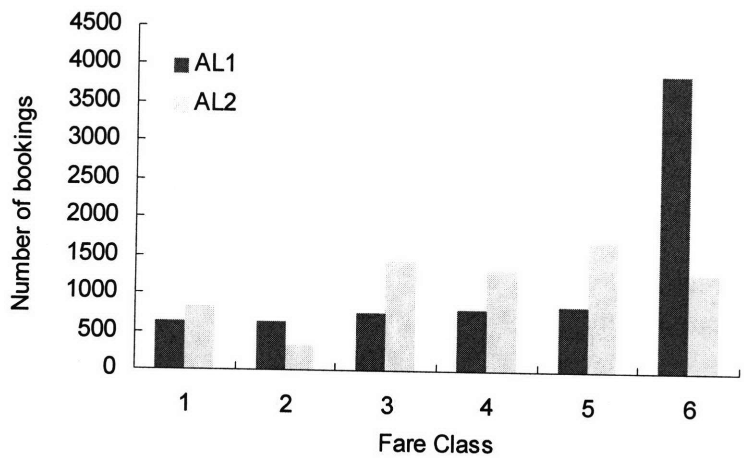


Figure 61: Fare Class Mix of Base Case against EMSRb with QF-FP and FA in Network D6

5.2.3.2 Investigation of DP methods against EMSRb with Q-Forecasting and Fare Adjustment in Network D6

We recall from §5.1.3.3 that DP-GVR appears to be an unappealing method against a competitive RM method in the Single Market case. This is however not the case with a large airline network. Figure 62 depicts that DP methods that incorporate sell-up rates in their algorithms are actually the only methods that manage to bring incremental revenue gain to Airline 1 with respect to the baseline case. We also note that the effectiveness of DPL/FA with QF depends on the estimator used, as there is substantial revenue discrepancy between FP and IC estimators when used in the method. Using IC estimator in DP-GVR helps Airline 1 at the expense of Airline 2, which revenue reduces significantly from its baseline revenue. However, when using FP estimator in DP-GVR, Airline 1's revenue is boosted by 1.5% while Airline 2 does not suffer as much, resulting in the total network revenue much higher than other methods (Refer to Table 33).

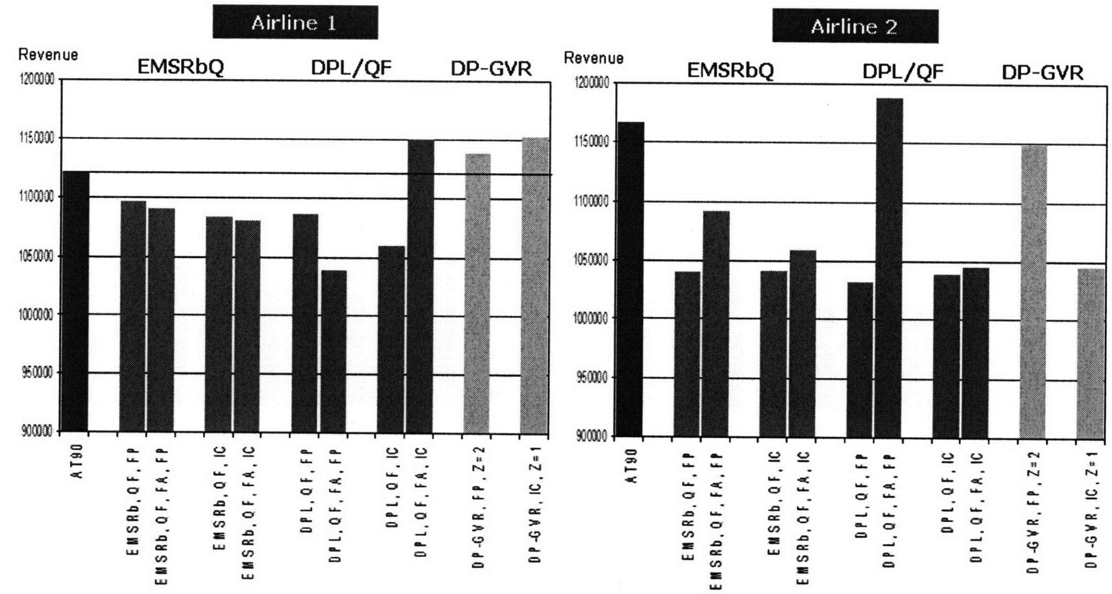


Figure 62: Revenue Summary of different RM methods against EMSRb with QF-FP and FA in Network D6

AL1 RM	FRAT5 Estimator	
	FP	IC
EMSRb, QF	-2.2%	-3.3%
EMSRb, QF, FA	-2.8%	-3.7%
DPL, QF	-3.1%	-5.5%
DPL, QF, FA	-7.4%	+2.6%
DP-GVR	+1.5%*	+2.7%

* Corresponds to the best case in DP-GVR that occurs at Z-factor of 2

Table 33: Revenue gains of Airline 1 against EMSRb with QF-FP and FA in Network D6

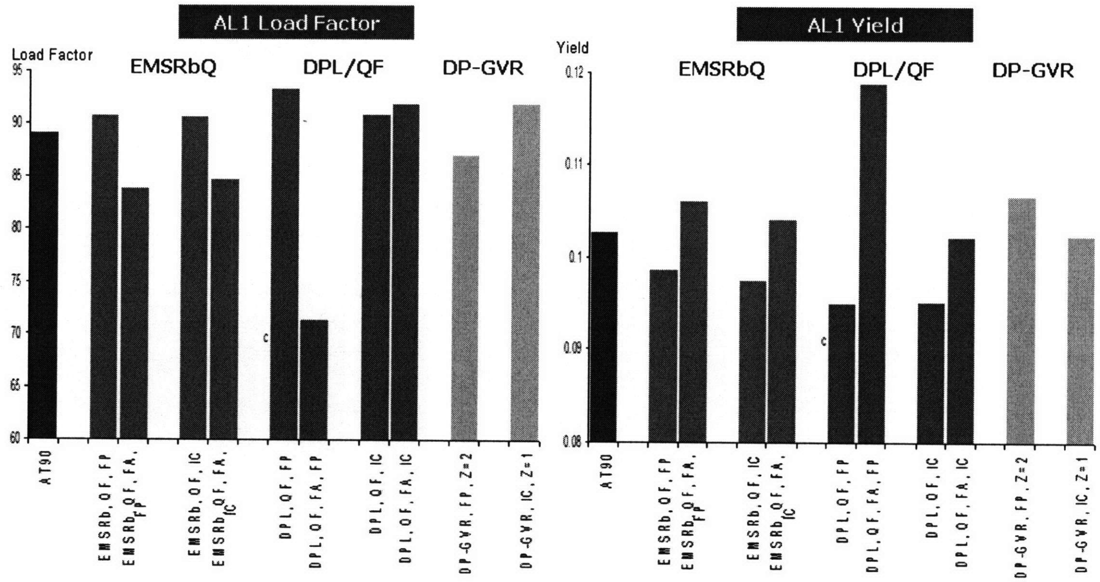


Figure 63: Load Factors and Yields of Airline 1 using different RM methods against EMSRb with QF-FP and FA in Network D6

Looking at the average estimated FRAT5 curves in Figure 64, we find that the IC method produces lower FRAT5 estimates than FP throughout the entire booking period. Also, unlike the case against AT90, FP FRAT5 curve does not experience exceptionally high surge at the end. Comparing this figure with Figure 33, we observe that the FP FRAT5 curve matches relatively closer to the FRAT5-A curve at the beginning but generates lower estimates than FRAT5-C later, whereas the IC estimates match quite closely to the assumed FRAT5-E inputs.

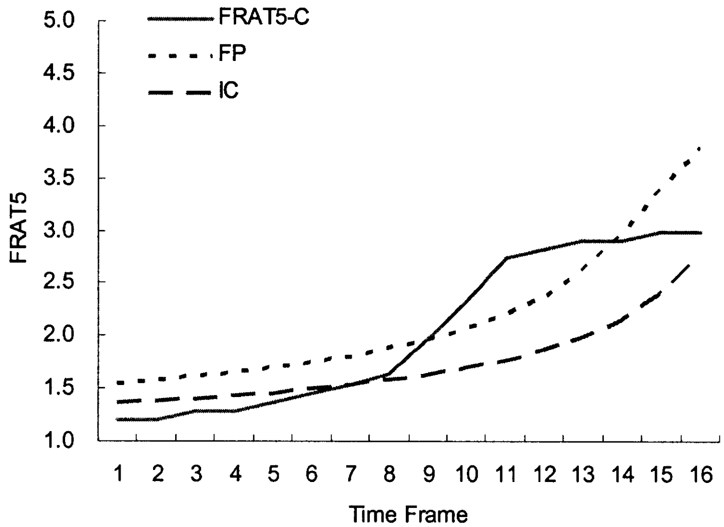


Figure 64: Comparison between Average Estimated FRAT5 curves and input FRAT5-C for DP-GVR against EMSRb/FA with Q-Forecasting in Network D6

Figure 63 suggests that using FP in DPL/FA with QF overprotects higher-fare seats at the expense of load factor, causing the revenue to drop below the baseline. DP-GVR, on the other hand, manages to capture very high load factor without sacrificing much yield, an achievement that none of the traditional methods are able to accomplish with an aggressive fare adjustment. Comparing the fare class closure rates as well as fare class mix of the 2 airlines in Figure 65, it appears that DP-GVR takes advantage of the aggressive fare adjustment and consequently overprotection of EMSRb/FA with QF by closing down classes at slightly slower rates, and captures enough spills at mid-fare to boost up its revenue. When the demand is high, Airline 2 is so aggressive and denies so many bookings that Airline 1 can become more selective with its own bookings using, simultaneously increasing both its yield and load factor, and eventually driving its revenue considerably upward.

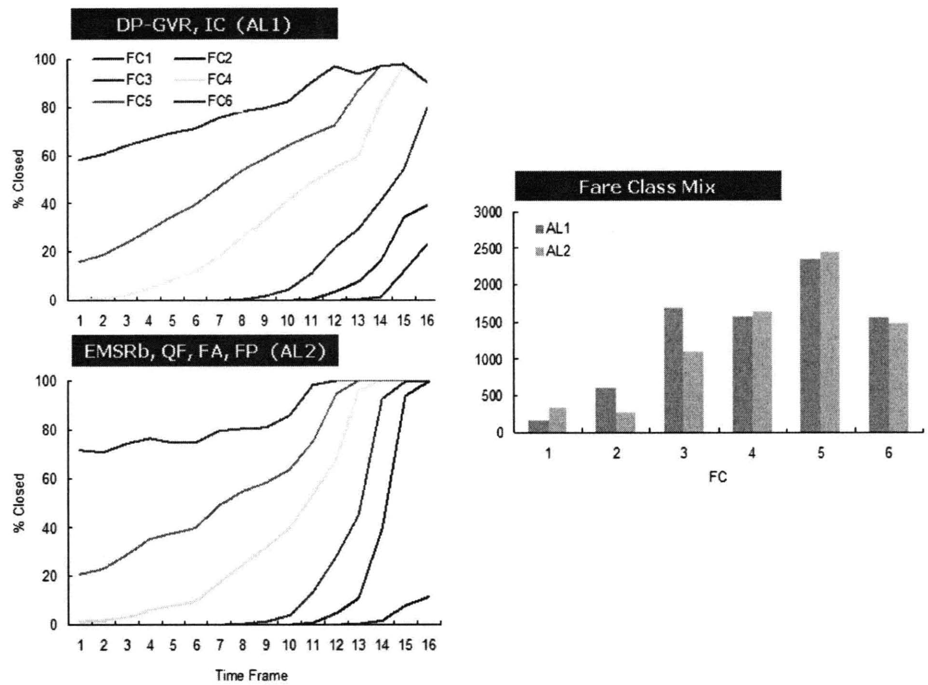


Figure 65: Fare Class Closures and Fare Class Mix for DP-GVR with IC estimator against EMSRb/FA with QF-FP in Network D6

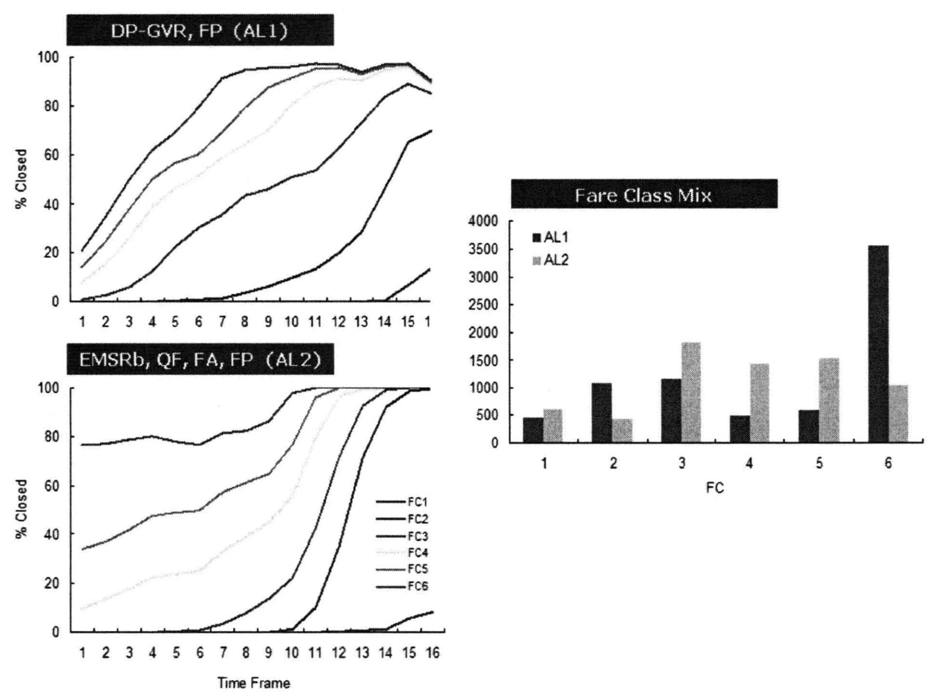


Figure 66: Fare Class Closures and Fare Class Mix for DP-GVR with FP estimator against EMSRb/FA with QF-FP in Network D6

Figure 65 and Figure 66 also illustrate the effect on the fare class closure patterns of the 2 airlines when both airlines use the same or different FRAT5 estimation methods. When switching from IC to FP estimator, DP-GVR does not close down as many low fare classes in the beginning to capture some spills, but closure rates are progressively more rapid later in the booking process. In wake of fewer seats filled in early time frames, Airline 2 tries to open mid-fare classes for a longer period to capture sell-up passengers, and ends up with a faster closure rate in the second half of the booking period when bookings begin to reach capacity. Airline 1 loses out many mid-fare passengers because as it captures more low-fare bookings in the beginning and leaves fewer seats the rest of the way, DP-GVR decides it would be best to protect remaining seats for high-yield passengers. Of course, buy-down phenomenon is expected from Airline 1 to its competitor, who opens more mid-fare classes during the rest of the process. This corresponds to the more low-fare bookings for Airline 1 and mid-fare bookings for Airline 2 when FP is used in DP-GVR. The higher closure rates also result in lower observed load factor for DP-GVR that uses FP estimator.

5.2.4 Test Case 5: Against Symmetric RM Method

Finally, we focus on the incremental benefit that each RM method can bring to an airline when the same optimizer and forecaster are also employed by its competitor. We observe in the previous test case that employing an advanced RM such as EMSRb/FA with QF by both airlines leads to deterioration in overall network revenue. Given the promising results of DP methods against both simplified and advanced competitor, we would like to examine the case in which both airlines use DP methods in conjunction with an aim of strengthening the validation of these methods.

5.2.4.1 Specifications of Base Case and Simulations

Table 34 describes the set up for simulations of this test case. Because these simulation tests are performed with different competing RM methods, we do not have a controlled setting in which the revenues produced by different RM methods are comparable with respect to a baseline scenario. Nevertheless, the main interest of this test case is to inspect not only the potential of a DP method by itself but also the impact it brings to the overall network when both airlines employ DP-based RM methods. As previously, we do not compare across various demand levels and only consider a demand factor of 1.0.

Test Case 5	Airline 1		Airline 2	
	Optimizer/Forecaster	FRAT5	Optimizer/Forecaster	FRAT5
Base Case	AT90	-	AT90	-
$5A_1$		FRAT-C		FRAT-C
$5A_2$	EMSRb /QF	FP	EMSRb /QF	FP
$5A_3$		IC		IC
$5B_1$		FRAT-C		FRAT-C
$5B_2$	EMSRb, FA /QF	FP	EMSRb, FA /QF	FP
$5B_3$		IC		IC
$5C_1$		FRAT-C		FRAT-C
$5C_2$	DPL /QF	FP	DPL /QF	FP
$5C_3$		IC		IC
$5D_1$		FRAT-C		FRAT-C
$5D_2$	DPL, FA /QF	FP	DPL, FA /QF	FP
$5D_2$		IC		IC
$5E_1$		FRAT-C		FRAT-C
$5E_2$	DP-GVR	FP	DP-GVR	FP
$5E_3$		IC		IC

Table 34: Specifications of Test Case 5

5.2.4.2 Investigation of DP methods against Symmetric RM methods in Network D6

A positively startling result of this test case, as shown in Figure 67 and Table 35, is that the employment of DP methods by both airlines, when accounting for sell-up behavior of passengers, actually helps both airlines. Although using FP estimator in DP-GVR hurts Airline 2, it manages to boost the revenue of Airline 1 slightly and consequently leads to overall network revenue much higher than traditional methods. We also see that, like the previous two test cases against AT90 and EMSRb/FA with QF, using IC estimator in the DP-GVR model has a tendency to generate lower estimates of sell-up probabilities, causing the method to achieve revenue gains at higher load factor and lower yield in general.

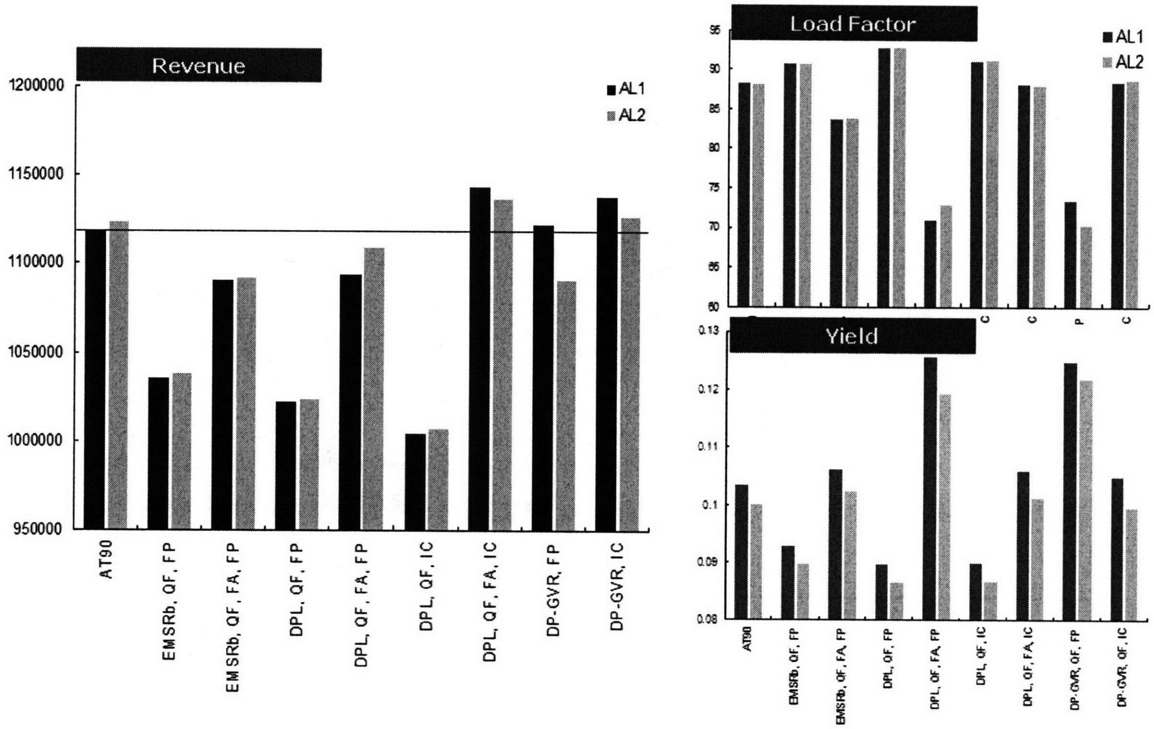


Figure 67: Revenue, Load Factor, and Yield for different RM methods against Symmetric RM method in Network D6

AL1 & AL2 RM	FRAT5 Estimator	
	FP	IC
EMSRb, QF	-7.4%	
EMSRb, QF, FA	-2.5%	
DPL, QF	-8.6%	-10.2%
DPL, QF, FA	-2.2%	+2.2%
DP-GVR	+0.3%	+1.8%

Table 35: Revenue gain for different RM methods against Symmetric RM method using Estimated FRAT5s as compared to AT90 vs. AT90 in Network D6

At this point, we should be aware that while sell up estimation is incorporated in the simulation tests in this thesis, these estimators that aim to truly match the sell-up behavior of passengers are by no means proven products. Another case we would like to consider here is to use an input FRAT5 series to compete against the same method. Figure 68 shows that when a medium FRAT5-C set is assumed, DP-GVR no longer performs better but indeed slightly worse than traditional methods using QF and FA. However, its improvements over AT90, EMSRb with QF, and DPL without fare adjustment are significant. These results shown in Figure 68 and Table 36 further validate our analysis that DPL and DP-GVR, when supplemented with sell-up behavior of passengers in their seat allocation policy, appears to indeed be a valuable tool for improving the performance of an airline in a fully unrestricted fare structure.

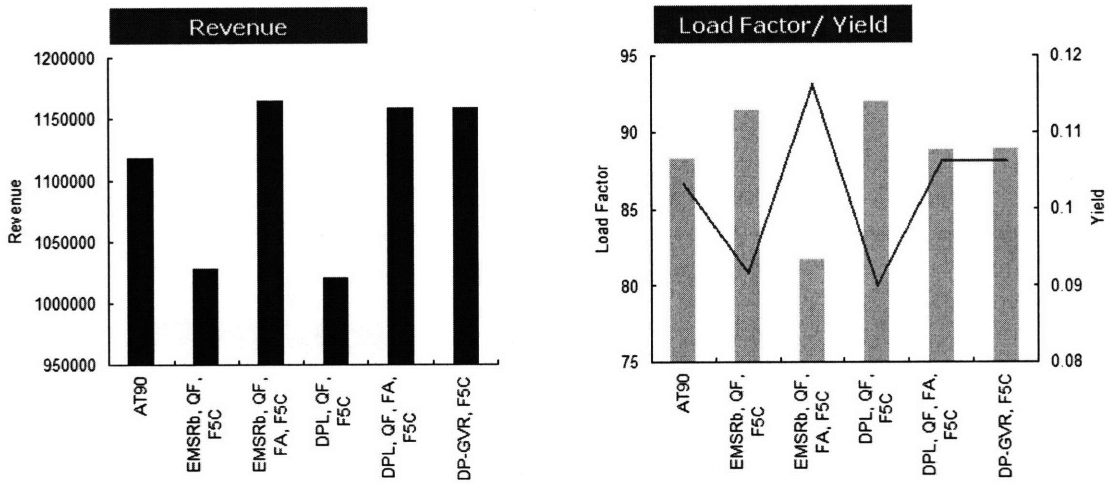


Figure 68: Results for different RM methods using input FRAT5-C against Symmetric RM method in Network D6

AL1 & AL2 RM	FRAT5-C
EMSRb, QF	-8.1%
EMSRb, QF, FA	+4.2%
DPL, QF	-8.7%
DPL, QF, FA	+3.6%
DP-GVR	+3.6%

Table 36: Revenue gain for different RM methods against Symmetric RM method using FRAT5-C as compared to AT90 vs. AT90 in Network D6

5.3 Summary of Findings

It is necessary to point out here that what our analysis focuses on is the possible benefits from DPL and DP-GVR for Airline 1 when assumed or estimated sell-up rates are used in the algorithms. While we do recognize the benefits are highly dependent on the choice of input sell-up rates or estimation method, our goal is not to critically review whether FP and IC methods produce accurate sell-up estimates, or prove which one is a better approach to use with DP methods than the other. Rather, we try to get a feel for the patterns of possible revenue gains that can be obtained by DPL and DP-GVR, not only when sell-up rates are arbitrarily chosen but also in cases when the airline is able to apply sell-up models based on their own estimates.

At this point of thesis, we have looked at the performance of DPL and DP-GVR in the both the Single Market and Network D6, as well as the effect of accounting for sell-up using various arbitrary and estimated rates. We would like to summarize our findings and draw preliminary conclusions for our simulations:

(1) In Single Market, we see the potential of DPL over traditional methods especially when aggressive sell-up behavior of passengers is assumed or estimated. In contrast, the performance of DP-GVR is poor especially against a competitor using an advanced RM method, regardless of sell-up input or estimator used.

(2) DP-GVR, when accurately accounting for sell-up, performs much better and even delivers results in the highest revenue gains under several scenarios tested in a bigger symmetric Network D6. DPL with Q-Forecasting performs worse than traditional methods if supplemented with Fare Adjustment that is overly aggressive.

(3) The performance of the theoretically appealing DPL and DP-GVR depends on the environment in which they are used, the type of sell-up estimator employed, and the Revenue Management method applied by the competitor.

Chapter 6

Conclusion

6.1 Summary of Thesis Objectives

The objective of this thesis is to study the performance of two revenue management methods based on dynamic programming in unrestricted fare environments, namely the Standard Lautenbacher DP method (DPL) and Gallego-Van Ryzin DP method (DP-GVR). We have discussed that the effectiveness of traditional Revenue Management methods has weakened due to the growth of simplified or unrestricted fare structures offered by low-cost carriers. These traditional methods consider that passengers buy in all open fare classes no matter what the lowest open fare class is, but as legacy carriers simplified their fare structures to avoid losing too much market share and revenue, passengers are expected to only buy in the lowest available fare class open. Consequently, these methods are unable to distinguish between business and leisure demand, making it difficult for airlines to reach revenue optimality under less differentiated fare structures. Clearly, besides modifying traditional RM models to incorporate the concept of sell-up, the need exists to develop a new optimization method to determine what the lowest open class should be at each time of the booking process by considering demand that may potentially purchase the lowest fare class open at any particular time.

The mechanisms of methods based on dynamic programming focus on controlling fare class closure using maximum expected marginal revenue, and eliminating the assumptions of segmented fare class demand and sequential bookings. The underlying principle of the dynamic programming methods considers the actual demand arrival pattern of passengers as a Markov decision process. They divide the reservation processes into multiple decision periods, each of them small enough for one booking request, and decide whether or not to accept the request using dynamic programming optimization algorithms, the output of which can translate into an optimal protection of fare classes. DPL assumes independence of fare classes as do traditional RM methods, and determines which classes should be open for a given time frame; DP-GVR considers the fact passengers may sell-up or buy down between fare classes, and determines which fare class should be the lowest class to open for a given time frame.

We have also discussed several techniques to deal with sell-up behavior of passengers when unrestricted fare structures are applied. Q-Forecasting and Fare Adjustment are recent developments in RM that incorporate the concept of sell-up probability in the forecasting and seat allocation control models, respectively. The objective of Q-Forecasting is to forecast demand under fully unrestricted fare structure to be used as input in the conventional RM. Fare Adjustment acts at the booking limit optimizer level. It incorporates potential of sell-up by adjusting the fares to feed the booking limit optimizer, resulting in greater protection for higher fare classes.

The main purpose of our simulations in PODS is to quantify the effectiveness of DPL and DP-GVR in different competitive settings, as well as the influence of sell-up models and forecasting on the airlines that implement these methods. We have looked at the simulated results of implementing DPL and DP-GVR methods in two unrestricted environments, a Single Market and a larger Network D6, and compared them to those obtained by the traditional EMSRb method. We have also emphasized that it is not the purpose of our study to quantify the sensitivity of the revenue gains to the choice of sell-up inputs or estimators, or prove which input or estimator is the better approach to use with DP methods than the other. Rather, we try to get a feel for the range of possible revenue gains that can be obtained by DPL and DP-GVR when they account for sell-up with not only arbitrary sell-up inputs but also sell-up models based on their own estimates.

6.2 Summary of Results

As previously mentioned, we have used PODS to simulate the performance of DPL and DP-GVR in a two-airline environment. The findings of our simulation tests illustrate the following set of overall conclusions under *fully unrestricted fare structures*:

(1) In a Single Market, we see the potential of DPL over traditional methods especially when aggressive sell-up behavior of passengers is assumed or estimated. In contrast, the performance of DP-GVR is weaker especially against a competitor using an advanced RM method, regardless of sell-up input or estimator used.

Table 37 and Table 38 present a summary of revenue gains obtained for Airline 1 as compared to the benchmark revenue corresponding to a First-Come-First-Serve (FCFS) against AT90 in the Single Market environment when the demand is low and high, respectively. When the demand is low, the RM method producing the best results against a simple RM method, assuming a medium level of sell-up probabilities, is DPL with QF with revenue improvement of 0.4% over EMSRb with QF. Against a more advanced RM method, EMSRb with QF is the best performer with 0.2% revenue improvement over DPL with QF. The comparable results between the two methods are accounted for by the similarity of the closure patterns of their respective fare classes. The poor results of DP-GVR against both simple and more advanced competitors are caused by the assumption of overly aggressive sell-up rates in FRAT5-C. The performance of DP-GVR demands on whether the employed estimator adapts to be more competitive and responsive to buy down of high-yield passengers. The over-reliance on estimates of passenger sell-up that do not respond to competition causes DP-GVR to continuously protect high-fare seats when those bookings never materialize.

DF = 0.8	FRAT5-C	RM method by Airline 2	
		AT90	EMSRb with QF
RM method by Airline 1	AT90	+2.9%	
	EMSRb with QF	+12.0%	+9.2%
	DPL with QF	+12.4%	+9.0%
	DP-GVR	-7.8%	-7.8%

Table 37: Summary of Revenue gains over FCFS using FRAT5-C at Low demand in Single Market

DF = 1.0	FRAT5-C	RM method by Airline 2	
		AT90	EMSRb with QF
RM method by Airline 1	AT90	+9.4%	
	EMSRb with QF	+33.7%	+20.8%
	DPL with QF	+41.0%	+21.9%
	DP-GVR	+20.2%	-0.4%

Table 38: Summary of Revenue gains over FCFS using FRAT5-C at High demand in Single Market

When the demand is high, implementing DPL with QF represents a clearer improvement over EMSRb with QF regardless of its competing RM method. DP-GVR performs better in high demand as well, although its results are still much worse than those obtained by DPL with QF and even EMSRb with QF. The comparative success of DP methods in high demand is explained by that there exists enough low-fare demand in the beginning for Airline 1 to capture and improve its revenue. Therefore, DPL with QF records higher arrival rates early in the booking process that it starts to focus more on capturing low-fare passengers at the start than EMSRb with QF. Despite some improvements, DP-GVR using FRAT5-C still does not manage to adapt to the lower buy-down behavior of passengers, resulting in low load factor and total revenue.

Incorporating various levels of fare adjustment in terms of scaling factors to DPL with QF results in a decline of revenue because the associated price-elasticity costs rely again on an input FRAT5 series that does not correspond to reality. Employing an adjusted fare in the DPL model that assumes independent bookings-to-come forecasts by fare class is in effect equivalent to implementing DP-GVR that incorporates sell-up rates directly in its decision algorithms. Against a competitive RM method, it is found that a small adjustment to the average fares can potentially generate additional revenue gains for Airline 1 over the case when no fare adjustment is supplemented at all. The "best" scaling factor is in the vicinity of 0.2, and allows revenue increases of at most 0.7% over the gains achieved by accounting for sell-up alone without fare adjustment.

The potential of DP-GVR is realized when a lower FRAT5 input is assumed. Against EMSRb with QF that assumes FRAT5-C, using the lower FRAT5-E in DP-GVR reflects more appropriately to the reality when passengers with high willingness-to-pay tend to buy down to the mid-fare classes that are re-opened by the competitor later in the booking process. Against AT90 which does not reopen fare classes once closed, DP-GVR manages to capture low to mid-fare passengers that buy down from Airline 2 as long as it assumes a low FRAT5 input, causing it to outperform EMSRb with QF and produce results comparable to DPL with QF.

(2) DP-GVR performs much better and even delivers results in the highest revenue gain under several scenarios tested in a bigger symmetric Network D6 when accurately accounting for sell-up. DPL with Q-Forecasting performs worse than traditional methods if supplemented with Fare Adjustment that is overly aggressive.

The results obtained in this bigger symmetric network strengthen the comparative success the DP methods have had in Single Market. Table 39, Table 40, and Table 41 summarize the revenue gain obtained for different RM methods over the AT90 baseline case when implementing with a medium FRAT5 input, FP estimator, and IC estimator, respectively. Unlike the Single Market case, we find that the use of fare adjustment leads to gains in revenue due to network effects beyond capturing sell-up in a large airline network. However, fare adjustment is found to be sensitive to estimators when practiced in DPL with QF and may produce extreme results not equivalent to what DP-GVR obtains when employing those estimators.

Although EMSRb with QF is able to generate significant revenue gain (+12.1%) when medium sell-up rates are assumed, the reality is that airlines do not totally buy into an idea of risking their business with an arbitrary FRAT5. Comparing the load factor and yield graphs between DP-GVR and EMSRb with QF and FA, we notice that the benefits provided by DP-GVR are achieved with higher load factor and yield. This further strengthens the potential of DP-GVR over EMSRb with QF as it supports the low-risk strategy favorable to most airlines, that is to attract as many high-yield bookings as possible and generate high load factor at the same time.

DF = 1.0	FRAT5-C	RM method by Airline 2	
		AT90	Symmetric RM
RM method by Airline 1	EMSRb with QF	+3.7%	-8.5%
	EMSRb/FA with QF	+12.1%	+4.2%
	DPL with QF	+3.0%	-8.7%
	DPL/FA with QF	+7.3%	+3.6%
	DP-GVR	+7.0%	+3.6%

Table 39: Summary of Revenue gains over AT90 base using FRAT5-C at High demand in Network D6

DF = 1.0	FP Estimator	RM method by Airline 2		
		AT90	EMSRb/FA with QF-FP	Symmetric RM
RM method by Airline 1	EMSRb with QF	+4.1%	-2.2%	-7.4%
	EMSRb/FA with QF	+4.5%	-2.8%	-2.5%
	DPL with QF	+3.2%	-3.1%	-8.6%
	DPL/FA with QF	+1.8%	-7.4%	-2.2%
	DP-GVR	+2.9%	+1.5%	+0.3%

Table 40: Summary of Revenue gains over AT90 base using FP Estimator at High demand in Network D6

DF = 1.0	IC Estimator	RM method by Airline 2		
		AT90	EMSRb/FA with QF-FP	Symmetric RM
RM method by Airline 1	EMSRb with QF	+3.7%	-3.3%	
	EMSRb/FA with QF	+5.2%	-3.7%	
	DPL with QF	+1.7%	-5.5%	-10.2%
	DPL/FA with QF	-3.4%	+2.6%	+2.2%
	DP-GVR	+7.1%	+2.7%	+1.8%

Table 41: Summary of Revenue gains over AT90 base using IC Estimator at High demand in Network D6

The success of DP-GVR against a simple competitor is based on the notion that the seats of Airline 2 are gradually filled up by AT90 and closed fare classes are not re-opened at the end of the booking process. Since demand with high WTP are not given a choice to buy down, DP-GVR manages to record high arrival rates in late time frames. Therefore, despite the relatively higher arrival rates of low-fare passengers at the beginning of the booking process, DP-GVR focuses more on the end of the booking period and closes down more low-fare classes earlier in anticipation of future high-yield bookings.

On the other hand, the success of DP-GVR against an advanced RM method is accounted for by its ability to target low-fare demand at the beginning of the booking process. Airline 2 overprotects high-fare classes by EMSRb/FA with QF-FP earlier and does not seem to slow down its closure rates late in the booking period due to the high sell-up estimates of its own. DP-GVR, who starts off with low closure rates to take advantage of high arrival rates of low-fare passengers, adapts to the overprotection of Airline 2 in later time frames by gradually closing down low-fare classes in anticipation of buy-down passengers to its mid-fare classes, and results in significant revenue improvement over base case (+2.7%) with high yield and load factor.

(3) The performance of the theoretically appealing DPL and DP-GVR depends on the environment in which they are used, the types of sell-up estimator employed, and the Revenue Management method used by the competitor.

We observe in Single Market the limitations in FP and IC estimators when applied to DP-GVR. The FP method starts off with low sell-up rates that stay for 2 time frames after which it estimates a huge surge in sell-up probabilities even higher than FRAT5-A the rest of the way. On the other hand, the IC method appears to make DP-GVR adapt well to the fare class closures by EMSRb with QF at first, as it produces sell-up estimates that closely match with the FRAT5-E curve for the first two-thirds of the booking process, after which the IC estimator predicts sell-up rates that may possibly be too high.

Results from Network D6 also illustrate that, an airline that practices DP-GVR can experience a significant positive impact on its total revenue against both simple and advanced competitor as long as it accurately adapts its closure of fare classes to be more competitive when high-yield passengers buy down. Compared to FP, the IC method appears to deliver results in the higher revenue with low sell-up estimates closer to the FRAT5-E inputs, and leads to revenue gain much higher than what traditional methods can provide when estimators are implemented.

Overall, the FP FRAT5 estimates tend to increase very progressively starting early in the booking period, whereas the IC FRAT5 estimates generally result in higher or similar revenues as compared to FP estimates. IC FRAT5 curves follow more of an exponential shape and match relatively closer to low FRAT5 inputs.

6.3 Directions for Future Studies

Two major directions for future research are suggested in this section, namely:

(1) The validation of our present research results in a mixed-fare environment of a larger airline network; and,
(2) The incorporation of competitor's response to improve the current implementation of estimating passenger sell-up behavior

6.3.1 Validation of DP methods in Mixed-fare Networks

Having demonstrated the potential benefits of employing DPL and DP-GVR in airline Revenue Management Systems (RMS) under fully unrestricted fare structures, we believe it is important to take the next step of our study to a higher competitive level. In this thesis, we have limited our simulations to two competing airlines with overlapping single market and hub-and-spoke networks. It would be interesting to see what would happen in a larger network where several more airlines compete for markets within asymmetric route structures.

On the other hand, in real world, no airlines would apply complete fare simplification across all of their markets. Neither do they have a single fare structure for all of their markets; most of their markets are generally offered a combination of both restricted and unrestricted fare structures. Hence, our simulations can perhaps be expanded to include cases of airlines competing with semi-restricted fare strictures. Network S1 and S4 are the recent networks developed for other researches in PODS. They consist of 4 airlines with different sizes and markets, in which one of the airlines represents a low-cost carrier that offers a fare structure with more compressed fares and fewer restrictions. Testing DPL and DP-GVR in these networks allow us to further validate the usefulness of these methods, as well as the impacts of specific revenue management enhancements, in more complex and asymmetrical environments.

Also an avenue of interest for future research would be a modification of DPL to be implemented in complex networks such as Network S1 and S4. The current implementation of DPL considers that the probabilities of fare class booking requests have to be computed with leg/class forecasts of bookings-to-come for a given time frame. The output that we want to feed the optimizer is the projected bookings-to-come by fare class for each leg to determine the arrival rate of booking requests. Based on previous researches, the benefits of network OD fare class control to account for network effects in hub-and-spoke networks are substantial as it produces booking limits of fare classes in an effort to maximize total revenue as opposed to yield. We therefore see the value of network OD adjustment in DPL, a methodology known as DAVN/DPL. This modified version still implements on the leg level; that is, like DPL, DAVN/DPL still receives path/class forecasts of bookings-to-come from the forecaster and rolls them up into leg-based forecasts by time frame. The tweak occurs in the average fares as they are now adjusted by displacement as optimized by linear programming (LP), and the resulting path/class adjusted fares are then mapped into virtual classes rolled up into leg/virtual-class. The adjusted fares associated with future time frame are then used in the backward recursions solving through those time frames (Refer to Figure 69).

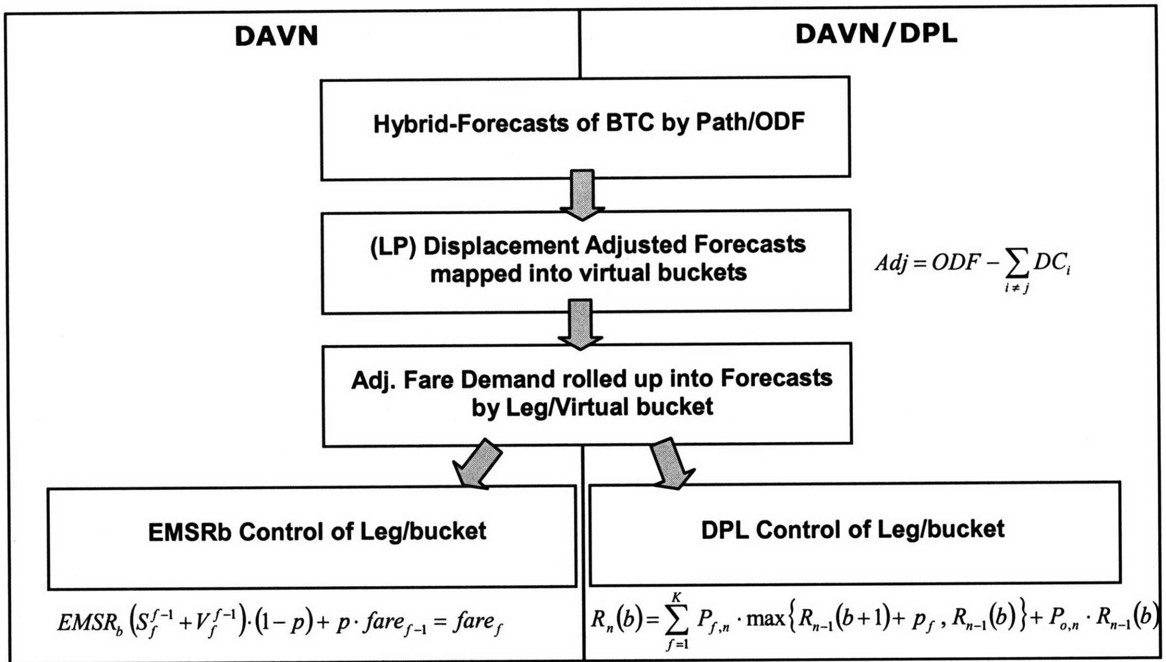


Figure 69: Flow Charts of DAVN and DAVN/DPL RM methods

6.3.2 Improvement of Sell-Up Estimators

Beyond extending the scope of simulations to be performed in a larger, semi-restricted network, the second suggested research direction is the development of a better method to estimate passenger's sell up behavior. For all the experiments in this thesis, we have applied two estimators, namely Forecast Prediction and Inverse Cumulative methods, to manage sell-up behavior. We have seen in our findings in both Single Market and Network D6 that neither method manages to stand out to produce the best results under all simulation experiments among FRAT5 inputs and estimators. One advantage the IC estimator has over the FP estimator may be that the FRAT5 values estimated by IC seem to be more intuitive and more robust. Consistent with the results from Cléaz-Savoyen (2005) and Vanhaverbeke (2006), however, these estimators are found to be far from proven products.

A suggestion regarding an improvement of the sell-up estimator involves allowing the Forecast Prediction method to be conditioned to the lowest competitor class open (Loco), known as the Conditional Forecast Prediction method (CFP). Inferred with the lowest open competitor's fare class in each time frame associated with each path examined, CFP makes use of this competitor availability information and computes for sell-up probabilities associated with each value of Loco for all future time frames. We believe that improving the sell-up estimation represents an important research agenda that could guide dynamic programming methods to the next breakthrough in Revenue Management.

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