APPLICATION OF THE OPTION VALUATION MODEL TO CONSTRUCTION PROJECTS

by

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ABSTRACT

The option valuation model is a financial model used in evaluating the price of put and call options in stock markets. The model may lend itself to more general applications. Its statistical representation of stock market mechanisms seems to be appropriate to describe risky assets whose development is influenced by an uncertain economic environment.

This thesis investigates its potential applications to construction projects. A model involving three sucessive options is proposed to describe the construction development process. This method may allow to valuate a piece of land, a project, or alternatives for the use or the renovation of a facility. This schema, along with situations where more specific options can be identified, is discussed in the case of buildings and industrial construction, and infrastructure planning.

Numerical applications are discussed on the example of a building which may be used for office space or apartments. The values of one or two development schemas are derived at different stages of the project, and sensitivity analyses with respect to the input parameters of the model are presented.

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INTRODUCTION

Among the many issues addressed before a construction project is undertaken, financial considerations are more and more perceived as critical. This sole factor is often able to counterbalance technical arguments in the final choice of the project, and a strong financial plan is becoming a prerequisite to any serious evaluation of such investments.

This fact probably shows that the construction environment is becoming increasingly complex and that decisions have to be made more carefully. An accounting for the time value of money is demanded by high interest rates. The uncertainty surrounding future economic conditions affects long run construction project plans. And methods of financing are becoming major issues for these capital intensive projects.

On the other hand, the demand for construction projects is changing, although it is still important. Infrastructure redevelopment, the construction of modern plants, and the development or renovation of certain urban areas should still provide substantial activity for the U.S. construction industry. However, new emphasis is put on well planned designs, flexibility, maintenance costs, and potential further developments.

The financial evaluation of these projects is becoming sensitive to this evolution in the environment and the demand. Evaluating a project as a large investment which will produce constant cash flows year after year is no longer appropriate. The sensitivity of the project to random future events, and the alternatives offered to the decision maker if the expectations are deceived, have to be taken into account. Traditional methods, however, do not adequately handle these uncertainties and opportunities, and more elaborate analysis methods are needed.

In an attempt to shed light on these questions, this thesis investigates the possible use of a new financial model. The option valuation model is presently used in finance to evaluate stock options and similar securities. The model may lend itself to more general applications, since it captures important criteria affecting the behavior of investors in an uncertain environment. Its statistical treatment of risky assets is an additional benefit.

This thesis therefore focuses on identifying and discussing construction alternatives which can be described in terms of options. The following points are addressed.:

- Does the financial model of option valuation fit to construction projects? This question will be discussed in Chapter 1.
- Can the construction process be modelled as a set of options, without reference to other valuation schemes? (Chapter 2)
- How do the specifics of building, industrial, and infrastructure construction influence the application of the option valuation methodology? (Chapters 3 to 5)
- Are applications of the model consistent with real world cases observations? (Numerical applications will be discussed in Chapter 6).

Focus will be placed on developing the option valuation methodology applied to construction projects, and on pointing out its capabilities, its weaknesses, and the validity of its assumptions. A final question would be to evaluate the impact of this approach on the design of facilities. That is, how can the value of these investments be optimized by developing real options, even if construction costs are increased?

CHAPTER I

OVERVIEW OF THE OPTION VALUATION MODEL

Although the financial concept of option was introduced early in many stock exchanges, it has not aroused much interest among researchers until the last two decades. The option valuation model appears as one of the most recent developments in finance. This chapter will review the basic model and the relevant subsequent developments, and compare this approach with other methods of valuation. Focus will then be placed on applications to real assets, and some recent results will be presented. (for a more detailed presentation of the option valuation model, see references [1] and [8])

1.1 Options as Financial Securities

Since the creation of the Chicago Board of Options Exchange in 1973, the word "option" means a very specific kind of security for investors. This security is related to a traded company's stock and its value depends on the variations of the stock's price. There are two basic types of options:

- a call option gives the option's owner the right to buy a stock at a specified exercise price on or before a specified exercise date.
- a put option gives its owner the right to sell the same stock at a specified exercise price on or before a specified exercise date.

Call and put options are not the only securities presenting these properties. Other securities such as warrants, callable bonds, convertible securities, or options on commodities or foreign exchange bonds are similar contracts between investors. All of these can however be defined in terms of the put and call options.

These definitions do not explain why these securities receive the attention they do and are more than a mere by-product of traditional stock markets. Why are they so successful and what determines their value? The unique set of parameters defining an option may explain this fact. The underlying stock's price (S) is variable overtime and can be assumed to follow gaussian movements, while the exercise price (E) is fixed. The time to maturity (τ) is meant to limit the changes in S to a specific period. This gamble between S and E can be described by Figure 1.1, where the case of a call option is represented.

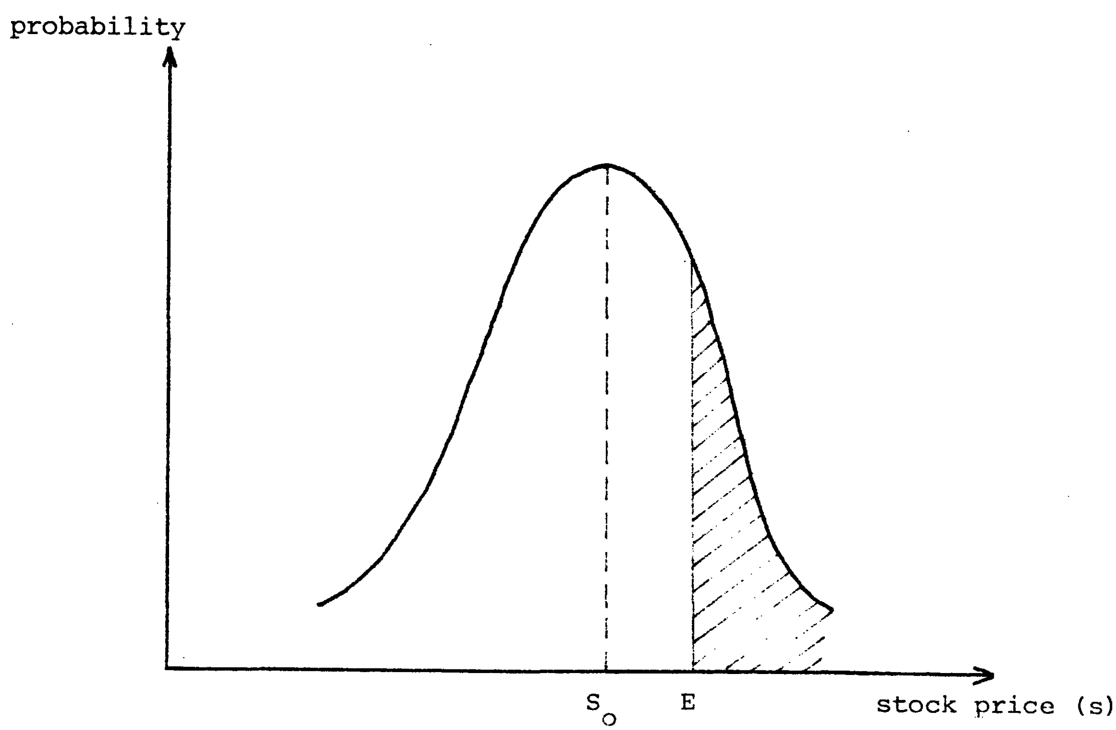


Figure 1.1: Distribution of a Stock's Value and the Exercise Price for a Call Option

In this figure, we have assumed that the exercise price, E, is higher than the stock price at the time of issue. Exercising the option, that is choosing to buy the stock at the agreed upon exercise price, is not profitable at this particular moment. However, the distribution curve shows that there is some probability that S will eventually exceed E. If this condition is met, it will be profitable to exercise the call option. This operation will yield S-E, which itself depends on the value of S. Or the investor may choose to sell his option if the time to maturity is not reached. The selling price will then be slightly higher than S-E since the option still offers an opportunity for future gains if the stock's price increases. These arguments show why an option is valuable, even if exercising it immediately is not profitable. It may also show why option valuation is not straightforward. The problem is to evaluate an uncertain payoff reflecting an unknown spot on the stock price's probability density function.

1.2 The Black Scholes Model

The search for an option valuation method has been a challenge to researchers in the '60s and early '70s. The first satisfactory solution was derived in 1973 by Fischer Black and Myron Scholes. In their model, the value of a simple call option is expressed as a function of the following parameters:

- $S(t)$: price of the underlying stock
- E: option exercise price
- τ: option time to maturity
- r: risk-free interest rate

σ^2: variance of the stock's rate of return $\left(\frac{\Delta S}{S}\right.$ for one unit of time$\left.\right)$.

The value of the call option, C, is derived as

$$C(S,E,\tau,r,\sigma) = S\ N(d_1) - E \cdot e^{-rt} \cdot N(d_2)$$

where $d_1 = \dfrac{\ln\ (S/E) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}$

$d_2 = d_1 - \sigma\sqrt{\tau}$

$N(\cdot)$ = cumulative standard normal distribution function

The theoretical study deriving this solution is based on a statistical model of the stock market. A brief outline of the procedure follows.

The first step is to describe the dynamics of the stock price, in accordance with the investors' expectations. Black and Scholes have assumed that the stock price is log-normally distributed (log(S) follows a normal distribution function) which can be mathematically represented by the differential expression:

$$\frac{dS}{S} = \alpha\ dt + \sigma\ dZ$$

where α = instantaneous expected rate of return of the stock

σ = variance on the movement of log S (hence σ is the variance on the rate of return $\frac{\Delta S}{S}$)

t = time variable

Z = stochastic varible for a normal distribution.

The second step is the derivation of a differential equation for the call option function $C(S,E,t,r,\sigma)$:

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + r S \frac{\partial C}{\partial S} - rC + \frac{\partial C}{\partial t} = 0$$

with three boundary conditions:

$C(0,E,\tau,r,\sigma) = 0$ (if S=0, the option is worthless)

$C(S,E,\tau,r,\sigma) \leq S$ (the option value is lower than the stock price)

$C(S,E,0,r,\sigma) = \text{Max}\ [0,S-E]$ (at maturity, the option is worth 0 or S-E)

Solving this equation is possible after noticing that it is a variation of the standard heat exchange equation. Hence the closed form solution described earlier.

The main assumptions which are made to derive this solution are the following:

(1) There are neither taxes nor transaction costs on the market. Trading takes place continuously, borrowing and short selling are allowed for all investors, and borrowing and lending rates are equal (The market is "frictionless")

(2) r and σ are known and constant

(3) the price is log normally distributed

(4) the stock pays no dividend

The most interesting feature of this approach is the small number of data necessary to use the formula. All the parameters are observable, except σ which results from the investors' expectations and can be approximated from past data. No risk-adjusted rate of return for the stock appears in the final expression. Nor does the probability distribution of future stock prices need be known.

Figure 1.2 shows sample calculations for different tradeoffs between S and E. Calculations for different values of the time to

Stock price (S)	τ = 3 months σ = .20	τ = 6 months σ = .20	τ = 3 months σ = .30
25	.067	.355	.307
30	1.589	2.483	2.166
35	5.787	6.612	6.013

Figure 1.2: Value of a Call Option Shown for Sample Calculations with E = \$30 and r = .10

Parameter	Effect of an Increase in the Parameter's Value
stock price (S)	increase
exercise price (E)	decrease
time to maturity (τ)	increase
interest rate (r)	increase
variance (σ^2)	increase

Figure 1.3: Effect of an Increase of the Basic Parameters

maturity and the variance show that an increase in one of these parameters increases the value of the option. Figure 1.3 shows the influence of the 5 parameters on the value of the option as given by the model. These results can be generalized to almost all options.

Finally, it can be noticed that the price of a put option can be easily expressed as a function of the same parameters, using the equation

$$P(S,E,\tau,r,\sigma) = C(S,E,\tau,r,\sigma) - S + Ee^{-r\tau}$$

1.3 Uses and Extensions of the Model

The Black-Scholes model is a useful tool to calculate the investment potential of an option or a similar security. The model can also be used to evaluate a security or a contract when part of the value comes from an option. It gives satisfactory results, specially when E and S are close numbers, and when the order of magnitude of the time to maturity τ is a few months or more. The tests are however difficult to interpret as the validity of the hypothesis, the measure of σ, and the formula itself cannot be addressed individually.

Another area of financial application is the valuation of equity and debt in a leveraged firm. Researchers have noted that shareholders hold a call option on the firm since they have to pay the value of the debt (exercise price) to get the value of the firm (see reference [1] for a discussion of this problem). This important application shows that the option methodology can be appropriate outside the field of the valuation of securities, and at a more general level.

Moreover, theoretical developments have considerably enlarged the potential applications of this approach. Robert C. Merton has studied the influence of the basic assumptions in the derivation of the Black-Scholes model. It results in particular that the no dividends assumption can be relaxed. This introduces a new term in the basic differential equation. Solving the equation becomes more difficult since no closed form solution can be derived. Finite elements techniques have to be used in order to approximate the option value. This framework can be appropriate for real assets since dividends can represent cash flows generated by an asset. It has been adopted to evaluate abndonment options (see reference [3]), in which an asset offers the alternative to switch from one stream of cash flows to another, or simply to be sold. This approach can be applied to salvage decisions, or a decision to change the use of an asset during its economical life. A simplified version of the abandonment option assuming an infinite time to maturity is presented in Appendix 2.

More sophisticated options have been studied recently. Stanley Fischer (reference [2]) studies the valuation of options when the exercise price is variable (variance σ_E, and expected rate of increase α_E). William Margrabe (reference [4]) derives the value of the option to exchange one risky asset to another. René Stulz (reference [9]) values a call option on the maximum or the minimum of two risky assets. In this latter case, the investor has the choice between two mutually exclusive alternatives, characterized by their values (V and H), variances (σ_V and σ_H) and their correlation (ρ). Exercising none of them is also possible. A closed form solution is derived with hypotheses similar to the simple call problem. The valuation formula is presented in Appendix 1, and will be used for numerical applications in part of this thesis.

1.4 The Evaluation of Risky Projects Using Options

1.4.1 Traditional Approaches in Dealing with Uncertainty

The Option valuation model recognizes that an option which is worthless now may offer a profit opportunity at a certain point in time depending on the value of an external parameter, the stock price. It takes into account the fact that the exercise decision depends on the variations of the stock price (therefore its variance) and can take place within a certain time span. Such circumstances can be encountered outside the stock markets, in particular in the uncertain environment of risky projects. These projects generally offer alternatives among which the decision maker must choose at different stages. Such decisions depend on unpredictable circumstances at the moment of the choice. The idea is to model these alternatives as options, so that the decision maker chooses to exercise the option or to ignore it.

Traditional methods based on the Net Present Value concept try to analyse these alternatives in terms of probabilities. When the exact revenues are not known, this method uses an evaluation of "expected cash flows", which are weighted averages of the possible cash flows with assumed probabilities. Uncertainty is also taken into account through the project risk adjusted discount rate. The valuation formula is the following

$$\text{Net Present Value} = \sum_{t=0}^{T} \frac{E(CF_t)}{(1+r)^t}$$

where $E(CF_t)$ is the expected cash flow at time t

$$E(CF_t) = \sum_{i=1}^{n} p(E_i) \times E_i$$

E_i = cash flow in alternative i

$p(E_i)$ = probability of the payoff i

When external variables have a significant effect on the project, or when managerial decision will allow to direct the project towards the most profitable alternatives after evaluation of the context at different points during the project, several techniques may complete this approach:

- sensitivity analysis, which evaluates the effect of changes in key parameters on the overall profitability
- decision tree analysis, which identifies the main alternatives through the project's life and evaluates the possible outcomes with their probabilities of occurence
- Monte Carlo techniques, which use the same framework as decision trees, but estimate the statistical distribution of the project's values through computerized simulation methods.

However, these approaches have several drawbacks:

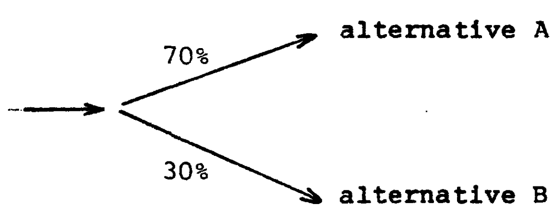
- they do not take into account explicit timing factors and choices. For example, the ability to delay a decision is not well-handled. The choice of the timing of the nodes in tree patterns is set in advance, often arbitrarily, and denies the operating and decision flexibility which is offered in most projects.
- they are based on discontinuous distributions. Statistical distribution function such as given by gaussian curves are approximated with a small number of discrete values

- they are subjective. Estimating the exact probabilities is difficult and may be affected by the lack of information as well as the attitude towards risk of the person doing the evaluation.
- they are very sensitive to the choice of the discount rate, which may also change at different stages of the project and is difficult to evaluate.
- their application is limited by the amount of calculations. Monte Carlo simulations, for example, require an intensive use of computers.

1.4.2 Advantages of the Option Methodology

The option approach may avoid many of these theoretical and practical difficulties. Given certain assumptions, a more straightforward process is offered, based on a totally different description of the decision process.

With this model, a key decision at any time during the project is evaluated as a two way problem: the choice is to exercise the option or not. Figure 1.4 shows how a choice between two alternatives encountered in a decision tree schema would be represented by an option approach.



- the base project is evaluated with alternative A
- the project contains an option to switch to alternative B

decision tree analysis — option approach

Figure 1.4: Comparison of the Option Approach with a Traditional Point of View Exhibited by Decision Tree Analysis

The option approach here is superior to the traditional approach as it incorporates two new fundamental dimensions. A time lag is allowed for the decision to go for alternative B through the time to maturity. This is particularly appropriate when, for example, choosing alternative B is possible at any time during several years, or even during the whole duration of the project. Second, the option model evaluates this alternative with a stochastic distribution of payoffs, instead of fixed payoffs and probabilities. This approach is more realistic in capturing the effect of uncertainty. Comparing the values of alternatives A and B may also be represented in this approach, since their variances, their correlation, and their tradeoff can be implemented in the model. They would be evaluated in the same way as one would intuitively consider these opportunities.

A third interest may also be mentioned: the simplicity of this approach. The study of decision tree analysis is limited since each additional node doubles the number of configurations to evaluate. In comparison, each additional option adds only one value to the main project, or is part of the evaluation if the project is linear. Options such as abandoning the project at any moment, waiting until undertaking a following development phase, or switching to another configuration may simply be added to the analysis without significantly increasing the number of calculations.

The option viewpoint can actually be considered as a higher level or more general tool compared to the methods described before. Correct statistical hypotheses are made to model the alternatives, but the complex mathematical development required by these hypotheses is handled by the model so that the user only needs to specify the relevant parameters. As a consequence, probabilities do not have to be evaluated since they are already taken into account by the model.

The option model finally seems to generalize the study of all the alternatives offered in the completion of a project, since it can represent explicit choices such as in decision tree analysis as well as more diffuse alternatives. It seems appropriate to study projects taking place in a risky environment, and how these projects can be adapted so as to maximize profit despite the hostile environment.

1.5 Applications to the Valuation of Real Assets

1.5.1 The Valuation of Oil Leases

The valuation of real assets is the first attempt to use the option methodology outside the field of finance. These assets clearly present option-like characteristics: they do not have value by themselves (an undeveloped piece of land, or mine does not produce revenue), and their value lies in the development opportunities that they offer. They can therefore be regarded as call options, in which the owner has to spend a certain investment to increase his chances of obtaining revenue.

Up to now, the most advanced development in this field is the valuation of oil leases by J. Paddock, D. Siegel and J. Smith (reference [7]). The development process of an oil tract from its purchase to its exploitation has been modelled by a series of three successive options: exploration, development, and extraction. Each stage requires that the company exercise the related option and gives rise to the following option. The exercise decision is subject to a favorable environment (e.g. the reserves appear to be substantial after the exploration phase, or oil prices are high enough to justify the oil tract development) and can be delayed until better conditions.

(lease purchase) ⟶ exploration option ⟶ development option ⟶ extraction option

This schema allows the derivation of the lease value by working recursively from the value of the extracted oil (as stock price) and using the option model at each of these three stages. The final lease values are greater than those obtained with traditional methods. They are closer to the observed winning bids for these tracts, although very different assessment of the oil reserves by oil companies do not permit efficient testing of this method.

This situation is also favorable to the application of the option model for practical reasons. Capital costs are very high for the development of these tracts, so that the financial decision process is here essential. These assets are however standardized, and market mechanisms are recognizable. Developed reserves are traded in secondary markets, and their value can also be derived through specific stocks, which are traded in the securities markets. Finally, uncertainty is an important element of the decision process. The final cash flows, for example, depend on the price of oil, which can be modeled as a stock with a significant variance.

These criteria appear to provide useful guidelines to the modelling of other circumstances where the option approach may apply. Part of this thesis has therefore been built upon the identification of similar situations and the application to the situations of the concepts developed in this study.

1.5.2 Theoretical Difficulties

Some reservations may be expressed concerning the validity of certain assumptions used in the financial models, in the context of real assets. One of these is related to the derivation of the Black Scholes formula and the following developments which rely upon arbitrage arguments. A comparison between two combinations of assets having the same payoff at any point in time allows valuation of these assets. This argument is correct in a world where assets are easily traded and duplicable. Real assets generally do not have these properties as:

- many real assets are unique and therefore not duplicable. A facility may be unique by its location, design, technology, etc. An oil field itself is unique in the exact composition of its reserves and cost of development and different tracts within this field also have different values.
- market mechanisms are generally limited. When they exist, their efficiency is very questionable, and the valuations may differ widely from one investor to another. The wide discrepancy of the traded assets and the low volume may add to the fuzziness of the valuation process.

These arguments show that at this point, a strict application of the model would probably be pointless given the assumptions necessary to apply essentially financial-market-based evaluation models to real assets. Moreover, the practical difficulties of acquiring objective data would also prevent an accurate evaluation. At this stage of development, these applications aim at exploring the capabilities of the option model and at judging the order of magnitude of the numerical results. They may also give some guidance on the main factors affecting decision processes which have not been well described until now.

CHAPTER II

A CONSTRUCTION PROJECT AS A SUCCESSION OF OPTIONS

2.1 The Model of the Process

From the moment a piece of land is purchased until the facility on this land is scrapped, a construction project passes through several stages. Each of these stages requires the developer and operator to make a decision involving a choice among several possible actions, the commitment of a certain budget for construction, and consequently an increase in the value of the asset in place based on the receipt of cash flows immediately or during subsequent phases. The importance and particularities of these options depend highly on the type of project under evaluation. Buildings, plants and infrastructure facilities do not offer the same alternatives, construction constraints and flexibility in the possible uses. Integrating all these specifics into a single schema requires therefore that we remain at a general level where large variations in the parameters are allowed.

The following process is a proposed model which tries to encompass as many kinds of construction as possible. Three options are defined: the design, the use, and the renovation options. This section will briefly outline these three options in the context of a real development process. The adaptation of the option model for a construction project will be studied in general terms in section 2, while each of these options, their parameters, and their results will be described in details in section 3.

2.1.1 The Design Option

Given a particular piece of land and the constraints due to its location (utilities, regulations, access, urban area...) the developer has a wide choice as to what general kind of construction will take place: factory, buildings of different shapes and sizes, parking, warehouse... The solutions differ by the general type of use, dimensions, structure, as well as cost. This option starts as soon as the land is purchased and has the particularity of an infinite time to maturity provided no regulation or practical problem imposes a time constraint. As a consequence, this point of view recognizes that a developer may wait a few years before undertaking a construction or simply selling the land. This is particularly favored if holding costs, e.g. taxes and interests, are low for an undeveloped land.

2.1.2 The Use Option

Exercising the design option means that the developer commits himself to a narrow range of uses. He has chosen an overall design, and as a result, will get a well defined shell construction after a certain duration allowing for the detailed design, bids, and gross construction phases. However, some flexibility may still be permitted: he may have a choice among different finishes determining the final use of a building, or different equipment changing the final product of a plant. The use option may also be viewed as a GO/WAIT decision before starting the operation phase. The cost of the finishes, or the start-up cost (including inventories and overhead) may be balanced against the immediate profitability of the operations so that the decision may be to wait for better economic conditions. In this case however, the cash flows which were planned during this period cannot be recovered.

design option--------------→use option--------------→renovation option--------------→(eventual other renovation option)

(design &

* (undeveloped land) * construction) * (operation phase) * (operation:other use)

land purchase	design option exercised	use option exercised	renovation operation	end of physical life

Figure 2.1: <u>The Option Model and Timing Pattern for a Construction Project</u>

2.1.3 The Renovation Option

This option can be defined as any possibility to change or enhance the use of the facility by means of a construction operation. For a building, a renovation operation leads to a change in the finishes in order to accomodate a new use or simply to make it more attractive for the same use, and therefore increases its value through the future cash flows. Other facilities present the same process: industrial units can be retrofitted to adapt to new market conditions, the equipment of a plant can be changed for the manufacture of a new product, infrastructure facilities can be reorganized if the volume or any other characteristic of their use has changed. On the whole, these operations involve a relatively small cost as well as a small loss of time, but their apparent effect may be important. They contribute to the profitability of the project by increasing the cash flows, in particular towards the end of the project's life. They constitute a certain hedge against the risk of depletion and should allow to take advantage of any opportunity during the operation phase.

2.2 Adaptation of the Option Valuation Model

Using the option model for a construction project requires one to define the five parameters used as inputs. The analogy with the stock model is straightforward and can be described by the following table:

Notation	Stock Option	Construction Project
S	Current stock price	Value of the construct project
E	Exercise price	Construction cost
τ	Time to maturity	Time to maturity (depends on option)
r	Risk-free interest rate	Risk free interest rate
σ	Variance of the rate of return	Variance of the rate of return

Figure 2.2: Comparison of the Variables for the Pricing Model of a Stock Option and a Construction Project

We can notice that the model stresses the difference between the value of the physical asset (S) and the construction cost (E).

In the case of a single independent option whose exercise leads to the receipt of cash flows, S represents the discounted present value of these cash flows. A project risk adjusted rate (r_1) is used, and with the same notations as in Chapter I, we obtain:

$$S = \sum_{t=0}^{T} \frac{CF_t}{(1+r_1)^t}$$

In the case of several successive options, an additional term is added. As exercising the option gives rise to the following option, the value of this second option, noted C, has to be added:

$$S = \sum_{t=0}^{T} \frac{CF_t}{(1+r_1)^t} + C$$

This scheme is encountered when the model is applied to an option on an option. In certain cases, when no cash flow is received in the next phase, S is exactly the value of the following option.

The example of the design option and the use option illustrate these considerations:

- in the design option, no cash flow is expected in the next phase (construction). S is therefore the value of the use option.
- in the use option, S is the net present value of the operating cash flows plus the value of one or more renovation options.

Furthermore, we will see in Chapter 6 that there exists an alternative method for valuing S through the stock market. S can be derived from specialized stock values if a company holding only the particular kind of asset under evaluation can be found. This solution has the advantage to suppress any reference to discounted cash flows methods, thus enhancing the specifics of the option approach.

S is sensitive to market conditions and its variability introduces the risk of the project. This risk is accounted for by the value of σ. However, $\sigma > 0$ implies that future payoffs can be greater than expected as well as less.

On the other hand, construction costs are easier to evaluate and less susceptible to variations. Specialized publications such as the "Means Guides" are commonly used by professionals for the evaluation of construction costs of buildings, and other publications are available for more specific calculations. The breakdowns of these costs are known in details, and their evolution is easy to forecast as construction cost generally do not show sudden variations. The approximation of a fixed construction cost can therefore be considered as satisfactory provided a mechanism corrects the effect of inflation (calculations in real terms, for instance).

In practice however, this fundamental difference between cost and value does not always appear clearly. In several cases, construction costs are currently used to evaluate a facility. A "replacement value" approach is used under which a building is evaluated by the cost of duplicating it, adjusted for a factor taking its age into account. This method ignores generally changes in technology or replacement of productive units as value.

In the case of infrastructure facilities, the values of the facilities are difficult to know since no cash flow is received. Comparing projects in terms of costs, or cost savings may be attractive but is not appropriate for an option approach. The model cannot be applied in this case unless a satisfactory method of evaluation is defined, for example in terms of social value or benefits to users.

2.3 Valuation of the Project at Each Stage

This section will describe in more details each of the options in the construction process model and the interpretation of their values. Figure 2.3 shows what are the imputs of the model (S, E, and τ) for each option and what are the results, in order to support as well as summarize the following discussion.

2.3.1 Value of a Piece of Land Through the Design Option

The value of a piece of land is generally defined as the greatest of its value under all the possible schemes of development. This approach is however based on a relatively rigid view of the ways the land may be used. In particular, it does not take into account two issues which account for a great part of the speculation on the market:

Design option

S = value of the use option

E = construction cost of the shell construction

τ = infinite

value of the option = value of the piece of land

Use option

S = Net present value of the expected cash flows + value of renovation option

E = Construction cost (finishes, start-up cost...)

τ = design, bids and construction phases duration

value of the option = value of the project as viewed during the construction phase (most of the construction cost is already spent)

Renovation option (put option)

S = net present value of remaining cash flows under the present use + future potential renovation options

E = net present value of the alternate use (net of conversion cost)

τ = economic life of the present use

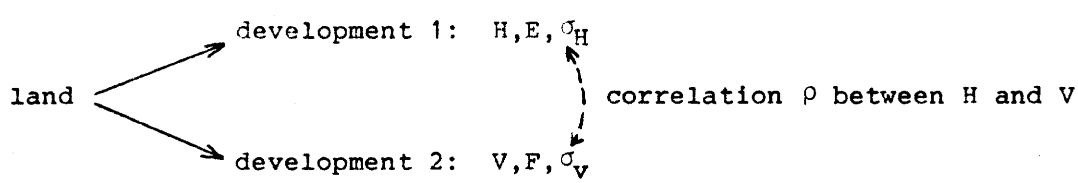
value of the option = added value due to the renovation operation opportunity

Figure 2.3: Summary of the Parameters for the 3 Options

- the choice offered to the owner to develop (or sell) his land at any time in the future, if the market condition is favorable. Owning undeveloped land is cheap, and this offers the opportunity to wait until the best moment for a construction project, or more often, to sell it when market prices are high, thus passing the option on the the buyer.
- the different possible types of development: the owner has the choice to undertake the most profitable one at any time. There is no unique best use through time, but a set of possibilities whose values depend on the location, size of the land as well as the development of the surrounding parcels and changing economic conditions. Each piece of land is also unique and may be appreciated differently by developers under uses related to their own business. At any time, the market reflects the highest of these values.

The timing issue can be captured by the design option under a model of simple call valuation. Assuming that the land offers a single bust opportunity for development, its value can be computed with the formula $C(S,E,\tau,r,\sigma)$ presented in 1.2, where S and E are the value and cost of the project and τ is a reasonably great number of years. One result of the option theory offers some insight into how the development process would take place: the model shows that it is always more profitable to sell the option rather than exercise it before its expiration date. In other words, the owner would have to give up some value to undertake the development, in comparison with a selling policy. With a great or infinite time to expiration, this would even mean that the development would never take place. This paradox was noted in the case of oil leases and is also true for land. Further research on this issue is needed since the basic crux of the paradox is that when a stock option is exercised, no new net supply of the underlying asset is created, unlike new oil reserves and new buildings when those options are exercised (see reference [7] for a discussion).

Calculating the additional value due to the opportunity to choose among several equivalent schemes of development require the use of a more sophisticated model of the call option on the maximum of several assets. In the case of two development alternatives whose characteristics are respectively H (value), E (construction cost), σ_H (variance) and V, F, σ_V, respectively, with a correlation ρ, the model for the maximum of two assets would provide this value as $f(V,H,E,F,\tau,r,\sigma_H,\sigma_V,\rho)$



Numerical applications of this valuation model are studied in Chapter 6. Although the context is not the valuation of land, the situation is similar (except τ), and the main results can be extended to this problem. In particular, increases in the variances (σ_V and σ_H) and a decrease in the correlation coefficient (ρ) increase generally the value of the option.

2.3.2 Alternatives During the Construction Phase Through the Use Option

The decision to start a project is a bet to the extent that nobody knows precisely what the market conditions will be when the facility is completed. Having to forecast a certain number of years in advance is a constraint creating major risks: the risk to end up with a facility which does not match the market demand, and the risk to miss a very profitable opportunity which could not be forecasted.

The premium given by the valuation of the use option over traditional approaches represents, in these conditions, the value of the ability to delay the major decisions until sufficient information is available. This point of view recognizes a value to the reduction of the construction time constraint through more flexibility. The developer can actually hedge against market uncertainties with this method.

The valuation of the use alternatives can be done at any moment during the design and construction phase. All other things being equal, the option value will depend on the time to maturity, τ. As τ decreases from the duration of the design and construction phase (2 to 5 years generally) to 0 (when the construction phase finishes), the value of the option will decrease, as a result of the decrease in the project's flexibility when the final decision becomes imminent. For $\tau=0$, the value of the option is exactly the present value of the most profitable alternative (as with the NPV method).

2.3.3 Value of Flexibility During the Operation Phase with the Renovation Option

The renovation option acts as an additive term to the total value of the present use of the facility. It can therefore be viewed directly as the value of the opportunity to change the operation's scheme at any time during the physical life. In the case where several renovation alternatives are possible, the value of the project can be even more increased as several option values are added, and each option may also involve other later renovation opportunities.

A renovation opportunity can be viewed as a put option to abandon the present use for another pattern of use. Such a point of view was developed by Stewart Myers and Saman Majd (reference [3]). The main differences with the simple put option are summarized here.

- this option offers the opportunity to switch from one uncertain stream of cash flows (value S) to another uncertain stream (value E). The model must therefore allow for an uncertain exercise price E with a variance σ_E. In order to simplify the calculations, E is defined as the net present value of the alternative use net of all cost of switching (E is therefore decreased by the cost of completing the renovation operation).
- S(t) and E(t) are the present values of the remaining cash flows at time t. S and E are decreasing functions of time since less and less cash flows are incorporated as time passes (Figure 2.4)

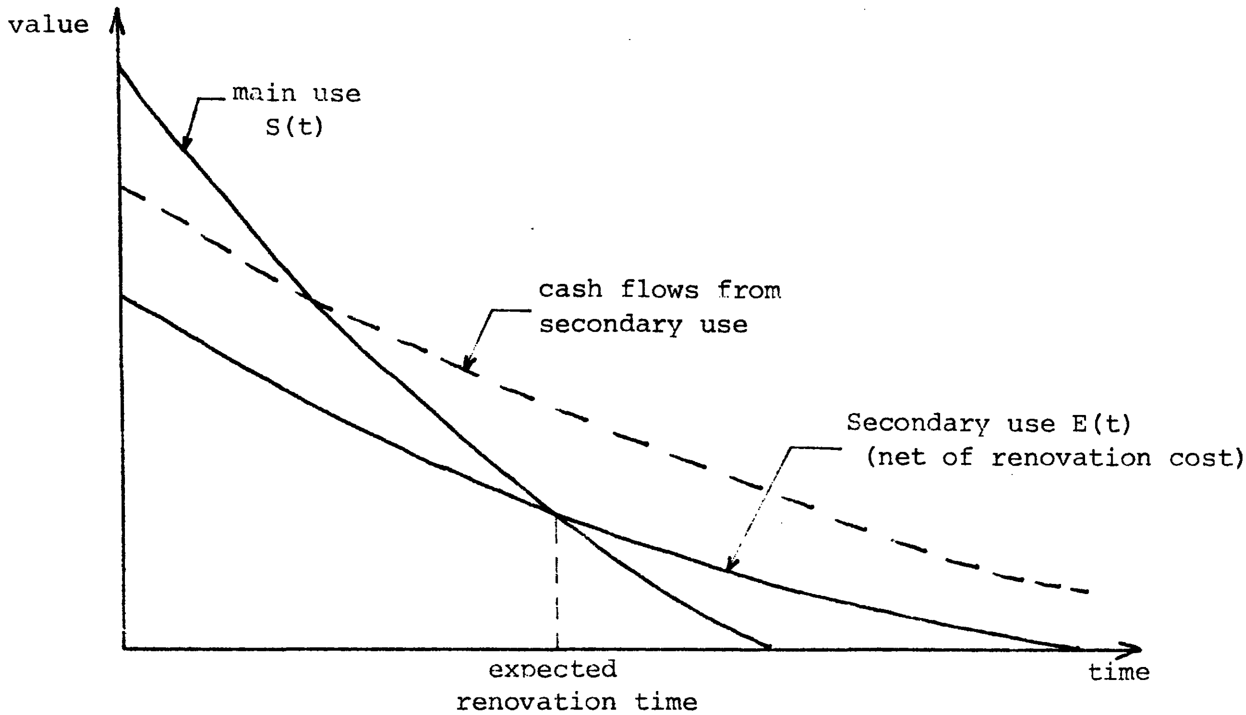


Figure 2.4: Value of the Main Use and the Renovation Alternative through Time

- the model includes dividend payments (cash flow C(t) received at time t). Solving the model in a general case would therefore require important calculations. The most simple cases are obtained when the "payout ratio" $\gamma_s = \frac{C(t)}{S(t)}$ is constant over time. A close form solution can be obtained if the time to maturity (usually the economic life of the use) is considered infinite.
- S(t) and E(t) are correlated. A correlation coefficient, ρ, is incorporated when the variances σ_E and σ_S are involved. The more uncorrelated the uses, the highest the value of the option.

These hypotheses make the resolution of this problem more complicated than for a simple option valuation. Using the exact curves of project value over time in the case of construction projects would increase significantly the amount of calculations. Orders of magnitude are however available in the case of simplified hypothesis. They depend, as usually, on the tradeoff between S and E, and the variances σ_E and σ_S and correlation ρ.

2.4 Other Options in the Construction Process

The process that has just been described is far from encompassing all the options relative to the construction phase or the project in general. The following chapters will describe how particular characteristics of different projects can also be described with the option methodology. Although these particular cases may not fit into a general description, some common features can be noticed concerning their occurence.

From a general point of view, each activity of the construction process can be viewed as an option. For example, the design phase is an option to the bidding phase. Bids are an option to start the construction of the foundations, completion of the foundations gives an option to start the structural work, and so on until the final completion. Such a detailed breakdown may probably go beyond the scope of the project financial evaluation. However, some of these steps may take a significant importance in the case of a project whose achievement would be questionable. Some typical concerns of the developer would then be:

- how to spend as little money as possible if the project is likely to be cancelled. The answer would probably be to delay the non-critical activities. Let us suppose, for example, that an independant unit has to be build on the site. This can be done at the beginning or at the end of the construction phase. If the project has some probability to be cancelled, it is advisable to wait until the end of the construction phase. This is actually viewing the completion of this unit as a put option allowing to recover its value if the project is cancelled before its construction has started.

- with the same perspective, some particular milestone in the project can define put options affected by an eventual change in the initial project. Before the milestone is reached, the completed part of the construction may have little effect on the final construction: this part may be used for different designs, or its cost may be recovered if the material in place can be sold or used in another project. After the milestone, the flexibility may be lost, or recovering the cost may become impossible. Therefore, an important decision may be required at this point in time. This decision is equivalent to exercising or not an option.
- more generally, each milestone between activities in the schedule offer the opportunity to stop the construction. Exercising this option gives immediately a complementary option, resuming the construction.

Finally, issues concerning the dimensionment and the long run development planning of important facilities seem to offer option-like characteristics. Incorporating demand forecasts and tradeoffs between the cost of different solutions are key issues in these problems. Particular cases where such issues are apparent will be described in Chapter 4.

CHAPTER III

APPLICATIONS TO BUILDING CONSTRUCTION PROJECTS

3.1 Specifics of Real Estate Investments

Buildings are probably the most immediate example of real assets. At the same time, real estate development is a sector of the U.S. economy where financial integration has always been important and where market characteristics are easily observable. This is therefore a favorable field of application of an option approacn, in particular since uncertainty affects the market value of these assets.

The valuation of a building generally depends on two general characteristics. Buildings can first be classified into general categories of uses, e.g., hotels, apartments, offices, shopping centers, and private houses. Each of these types define to a certain extent a market segment evolving independently on a supply-demand basis. Secondly, building values are affected by their location and relation to their immediate environment. Areas can be defined, and their evolution through time influences the value of different building types in different ways. Both factors are specific sources of risk for any construction project. Their influence over a year or a decade can be significant. Their conjunction however affects uniquely each building, and justifies the reservations expressed before regarding the treatment of real assets in the same manner as financial securities.

In this context, the approach developed in Chapter 2 may be of interest to a developer. Given a particular use, flexibility during the construction phase may allow him to choose the right configuration in order to satisfy the market demand. Being able to choose between two or three uses at the last moment, or later during the operational phase may be even more valuable. This would enable him to choose the more profitable of these alternatives, in a market where the uniqueness of the assets and the slow response of competition probably allow important differences to persist.

In addition, new investment schemes can be made possible by combining two uses. A risky investment can be undertaken if some possibility to switch to a safe use is available in case the project would fail. Conversely, a safe, but not very profitable investment can be worth undertaking if it allows later and instantaneously to capture a very attractive but risky opportunity. This latter configuration is described in figure 3.1: if the economic environment puts the outcome out in the right hand tail, the developer can go with the risky use, otherwise he will go with the safe use.

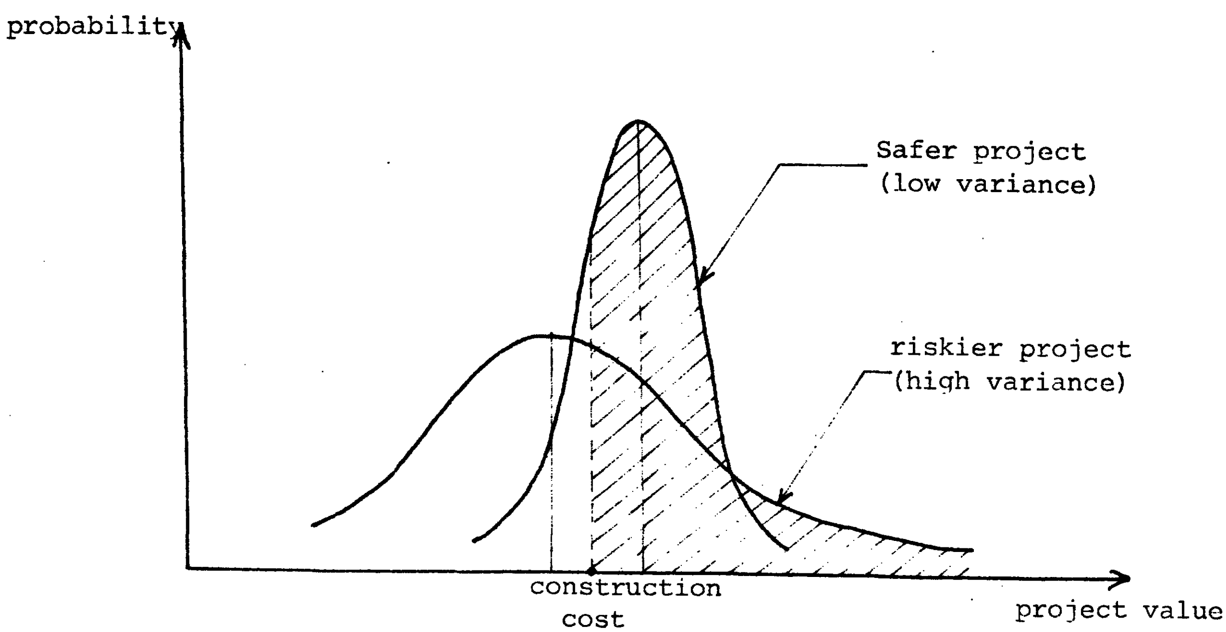


Figure 3.1: A Risky Opportunity Enhances the Value of a Safe Project

I will present in the next sections what the real options offered by a building are and how the general model fits to the well known building construction process. Examples and practical difficulties will then be analysed in the context of the present structure of this sector.

3.2 Options in the Building Development Process

Among the three options introduced before, the design option and the renovation option present little specifics in the case of buildings. The former deals with a larger approach of the construction process, while the latter does not need more elaborations over the presentation in section 2.3.3, although its formulation is particularly adapted to building projects. We will therefore focus on the use option, or in other words, the evolution of the project once a general shape and structure have been chosen.

When the design option is exercised, the overall design of the building defines the arrangement of the space within the enclosure. In addition, the organization of the piece of land is now fixed, and in particular, accesses, open spaces, and the relationship of the building with its immediate environment are defined. This process determines therefore the main parameters which will direct the development of the project through its physical life. Any further development will have to be accomodated within this simple framework.

What are in these conditions the possibilities of development? Several options are offered, which can be classified in two categories:

- mutually exclusive options: when two options apply to the same space, choosing one of them automatically excludes the other. Using the building for offices or as a hotel are examples of mutually exclusive options in the sense that both cannot be exercised at the same time.

- <u>independent options</u>: the building may offer additional options which do not depend on the exercise or not of any other option. Such options may be using the basement of the building as storage area, using the roof as a support, or using any available space on the land for a related construction.

The developer will probably be mostly concerned with the first type of options. The final use of the building requires obviously the most important decision. The value of having this type of choice would be given in our model by an option on the maximum of two or more streams of cash flow.

The second category represents additional features which can be implemented on the project at any time. They would also be accounted for under a use option model, as a simple call option: the option to extend the project before or during the operation phase if the total profitability can thus be increased. In certain cases, their value may be affected by the choice of the function of the building. For example, the use of the basement may have different values in the case of offices or apartments. Such options may thus also be considered as dependent on the choice of the main use.

All these options are represented in a typical case on figure 3.2. Alternatives A and B may be as different as a large flat building and a high rise building. Building A may be used as a shopping center, or for small, accessible offices, and B may be more suitable as an office building for larger companies or a hotel. Other options may be available for minor parts of these buildings. The

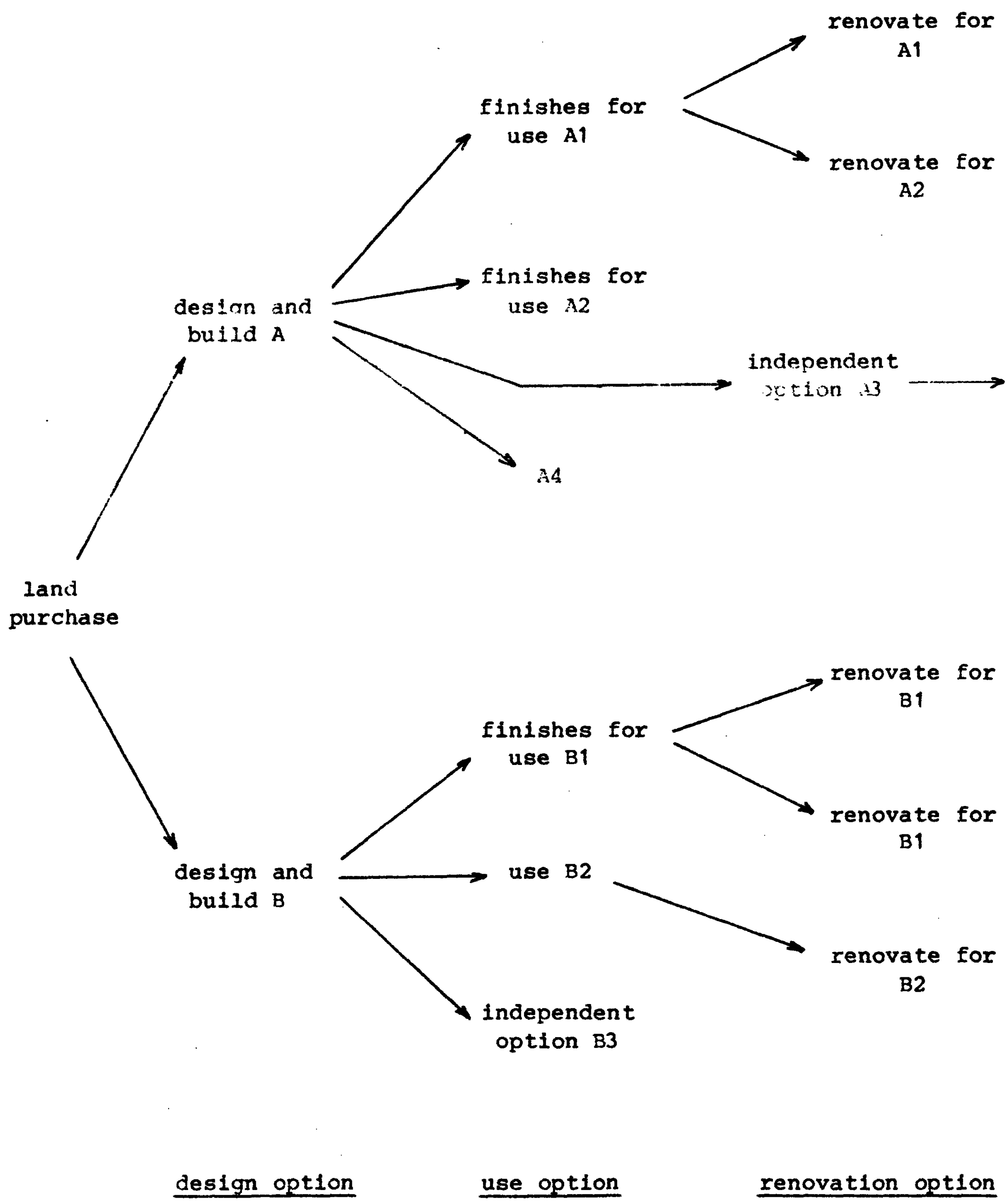


Figure 3.2: Succession of Options in the Building Development Process

alternative to sell the building and the land at any time may also be represented on this scheme. This is not an option in the sense of the financial model. It is simply a way to translate the value of the project at any stage into market terms. The modelling of the process with options should try to approximate as accurately as possible this market value, but does not include this alternative in the calculations.

3.3 A Parallel with the Real Construction Process

The use option becomes predominant at the very beginning of the construction operation: an architect is hired for the execution of the detailed design, and financial aspects of the operation have to be dealt with. The development process will then go on with the bids and construction phases, during which the developer may receive information on the market conditions relevant to his operation. Assuming that he wants the building to produce revenue as soon as possible, his decision on the final configuration will have to take place a few months before the end of the construction phase, in order to allow time for the completion of the finishes.

Figures 3.3 and 3.4 show how the real process is simplified under the assumptions of the proposed model. The comparison is made in terms of schedule (duration of the use option compared to a bar chart of the main activities) as well as construction spending.

In the comparison between the schedules, the maximum duration has been given to the use option. The design phase lasts at least one year as it includes many slow tasks such as organizing a design team with different professionals, having information flow between participants, the submission-correction-approval cycle between the design team and

```
                                  1-2 years
design                           |---------|
                                            2 months
bids                                       |--------|
                                                    9-24 months
shell construction                                 |-----------|
                                                          9-18 months
finishes                                                 |-----------|
operations                                                           |----------

                                        use option                    operations
                                 |-----------------------------------|----------
                                        2 to 5 years
```

Figure 3.3: Building Development Schedules: a Comparison Between the Actual Process and the Option Approach

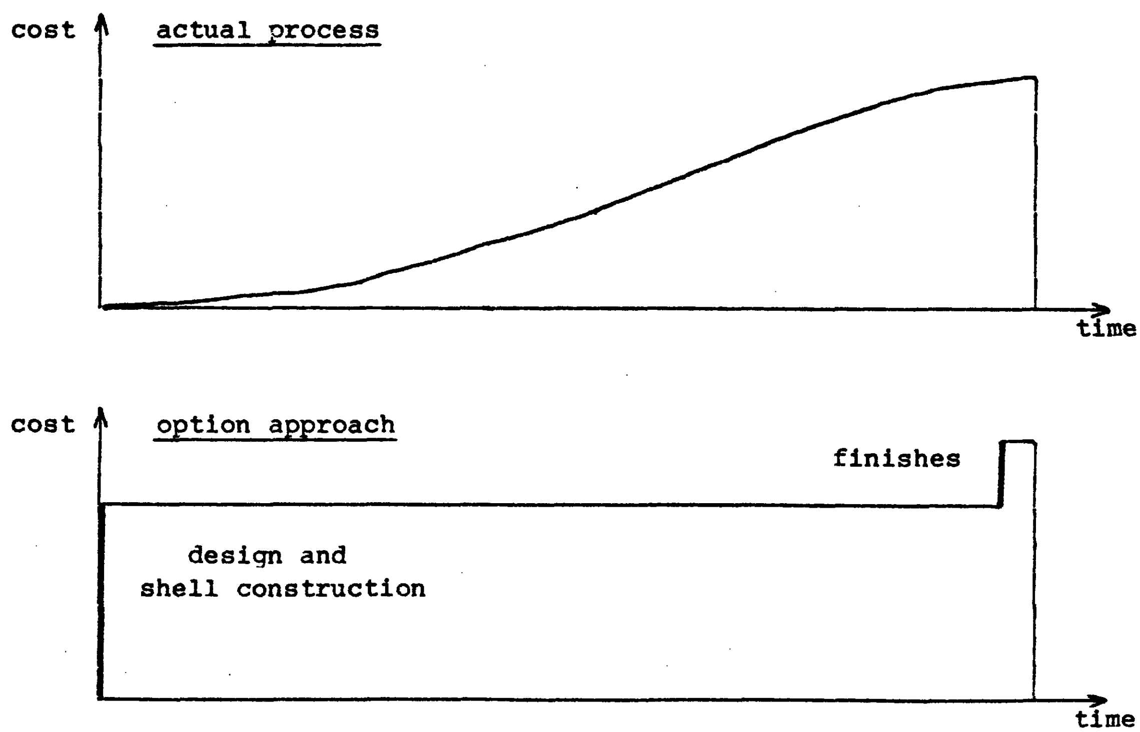


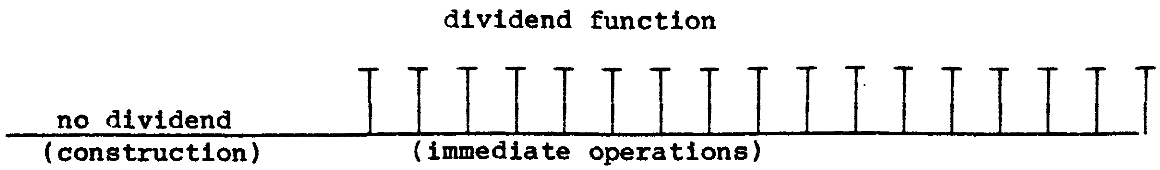
Figure 3.4: Cumulative Construction Cost Curves with the Actual Process and the Option Approach

the owner, obtaining permits from administrative authorities. The project can however still be modified or cancelled at this stage. On the other hand, the option extends until the building enters into the operation phase. The completion of the finishes is assumed to be instantaneous. These hypotheses are a very rough description of the actual process, since it supposes that the developer can wait until the end of the construction phase to make up his mind about the choice of the use of the building. This decision would take place, in the actual process, at least 9 to 18 months before the operation phase.

The comparison between the cumulative construction cost curves show that the actual S-shaped curve is approximated by two discrete expenditures. The construction cost of the building less the finishes is assumed to be known and spent at the beginning of the design phase. In the real process, this sum is spent later, mainly in the construction phase. This time lag may be accounted for by translating the real expenditures to the beginning of the design phase at a risk-free discount rate (i.e., this sum is put in the bank). The finishes work can be defined as partitions, wall, floor and ceiling finishes, and additional light electrical, mechanical, and other equipment. Mechanical and electrical rough-in may be considered as part of the shell construction provided a common configuration can be found for all the uses under consideration. The cost of the finishes generally amounts to 30% of the total construction cost (order of magnitude). In this model, completing the finishes yields immediately the whole value of the use, that is all the later revenues discounted to this moment.

These hypotheses will be used in the numerical example developed in Chapter 6. However, a criticism can be expressed concerning the timing assumption of this model. The assumption that the time to maturity of the option is limited to the duration of the construction phase takes into account the choices between 2 or 3 uses, but does not account for the possibility of delaying the operations phase after the building is completed. The interest of this viewpoint is that the owner may not be satisfied with the present level of rents for his type of building and may prefer to wait until a stronger demand allows him to charge more. Moreover, the importance of this choice appears in the fact that the first rent will determine the level of the rents during a large part of the operations phase through indexation formulas negotiated in the context or regulations. This decision is therefore essential for the profitability of the project, in particular if the owner has little choice over the use of his building.

A more sophisticated approach can take this aspect into account. A dividend payments model can be used with the hypothesis that the use option extends until the end of the physical life. The construction process and immediate operations phase would be modelled as a single phase where no dividends are perceived until the construction is completed, and a stream of dividends is available as soon as operations can begin. The developer may be allowed to delay his decision until after the construction is completed, but he would lose the dividends which would have been available if he had begun the operation phase immediately. As soon as he exercises the use option, the process will go on as modelled under the hypotheses described before. This is illustrated by Figure 3.5 presented below.



dividend function: $D(t)=0$ for $t<\tau_{construction}$

$D(t)=R$ for $t>\tau_{construction}$

maturity of the option: τ = physical life of the building

Figure 3.5: A More Elaborate Model for the Use Option

This model would simulate the developer's probable decision scheme in the sense that:

- as long as no dividend is paid, the option model suggests that the optimal policy is to wait until the maturity date. The developer whould therefore wait at least until the end of the construction phase before making a decision.
- as soon as dividends are paid, a penalization occurs. It is likely that the value of the option decrease with time, or that holding costs incur (financial costs), thus suggesting that the optimal exercise time is the end of the construction phase.

The complexity of the dividend function in these hypotheses does not allow for straightforward calculations. Deriving a numerical solution may not however present theoretical difficulties as solving the basic differential equation is theoretically possible with any kind of function. Moreover, finite element methods must accomodate easily such a step dividend function. These problems are not addressed in this thesis.

3.4 Examples and Practical Considerations

Figure 3.6 shows a listing of different types of buildings with their average cost per square foot and typical size. These data give an idea of the similarities between some uses in terms of complexity and size, although it gives little information about the design itself. Several types seem to offer enough similarities to be accomodated into relatively standarized shell constructions.

- Apartments, offices, hotels, and even colleges can probably be located in common types of buildings. In particular, mid-rise parallelepipedic buildings with roughly 50,000 sf of gross area would probably be appropriate for two or more of these uses provided their dimensions (width, floor to floor height) and structure allow for the partition into units whose geometry is adequate for each use.
- Schools and housing buildings for the elderly seem to offer this flexibility. Renovation operations conducted by municipalities have proved that converting buildings from the first use to the second one is not difficult and can provide substantial benefits for a low cost.
- Hospitals, research buildings, medical offices have similarities in the importance of HVAC, safety regulations, general organization as well as cost. Developing such a flexibility may be of particular interest for the evolution of a hospital. As technology and needs change very often, reorganizations are required after short periods of use. The best way to keep up with this very demanding path is to use new buildings. The solution may be to develop a lifecycle of uses

Square Foot Base Size							
Building Type	Median Cost Per S.F.	Typical Size Gross S.F.	Typical Range Gross S.F.	Building Type	Median Cost Per S.F.	Typical Size Gross S.F.	Typical Range Gross S.F.
Apartments, Low Rise	$ 38.80	21,000	9,700 - 37,200	Jails	$111.00	13,700	7,500 - 28,000
Apartments, Mid Rise	48.30	50,000	32,000 - 100,000	Libraries	70.10	12,000	7,000 - 31,000
Apartments, High Rise	53.20	310,000	100,000 - 650,000	Medical Clinics	65.40	7,200	4,200 - 15,700
Auditoriums	64.00	25,000	7,600 - 39,000	Medical Offices	61.90	6,000	4,000 - 15,000
Auto Sales	40.50	20,000	10,800 - 28,600	Motels	39.10	27,000	15,800 - 51,000
Banks	88.60	4,200	2,500 - 7,500	Nursing Homes	66.20	23,000	15,000 - 37,000
Churches	58.70	9,000	5,300 - 13,200	Offices, Low Rise	52.50	8,600	4,700 - 19,000
Clubs, Country	55.40	6,500	4,500 - 15,000	Offices, Mid Rise	57.60	52,000	31,300 - 83,100
Clubs, Social	56.90	10,000	6,000 - 13,500	Offices, High Rise	69.70	260,000	151,000 - 468,000
Clubs, YMCA	60.00	28,300	12,800 - 39,400	Police Stations	85.50	10,500	4,000 - 19,000
Colleges (Class)	77.60	50,000	23,500 - 98,500	Post Offices	65.60	12,400	6,800 - 30,000
Colleges (Science Lab)	90.50	45,600	16,600 - 80,000	Power Plants	432.00	7,500	1,000 - 20,000
College (Student Union)	83.40	33,400	16,000 - 85,000	Religious Education	48.60	9,000	6,000 - 12,000
Community Center	60.60	9,400	5,300 - 16,700	Research	83.70	19,000	6,300 - 45,000
Court Houses	81.10	32,400	17,800 - 106,000	Restaurants	77.40	4,400	2,800 - 6,000
Dept. Stores	35.70	90,000	44,000 - 122,000	Retail Stores	37.90	7,200	4,000 - 17,600
Dormitories, Low Rise	58.60	24,500	13,400 - 40,000	Schools, Elementary	57.30	41,000	24,500 - 55,000
Dormitories, Mid Rise	74.30	55,600	36,100 - 90,000	Schools, Jr. High	57.00	92,000	52,000 - 119,000
Factories	34.80	26,400	12,900 - 50,000	Schools, Sr. High	55.80	101,000	50,500 - 175,000
Fire Stations	63.30	5,800	4,000 - 8,700	Schools, Vocational	54.10	37,000	20,500 - 82,000
Fraternity Houses	55.10	12,500	8,200 - 14,800	Sports Arenas	43.50	15,000	5,000 - 40,000
Funeral Homes	55.30	7,800	4,500 - 11,000	Supermarkets	37.30	20,000	12,000 - 30,000
Garages, Commercial	40.10	9,300	5,000 - 13,600	Swimming Pools	70.20	13,000	7,800 - 22,000
Garages, Municipal	41.10	8,300	4,500 - 12,600	Telephone Exchange	97.60	4,500	1,200 - 10,600
Garages, Parking	19.10	163,000	76,400 - 225,300	Terminals, Bus	43.00	11,400	6,300 - 16,500
Gymnasiums	54.20	19,200	11,600 - 41,000	Theaters	52.80	10,500	8,800 - 17,500
Hospitals	112.00	55,000	27,200 - 125,000	Town Halls	62.20	10,800	4,800 - 23,400
Housing (Elderly)	55.00	37,000	21,000 - 66,000	Warehouses	25.10	25,000	8,000 - 72,000
Housing (Public)	42.10	36,000	14,400 - 74,400	Warehouse & Office	28.70	25,000	8,000 - 72,000
Ice Rinks	43.00	29,000	27,200 - 33,600				

Figure 3.6: Square Foot Costs and Typical Sizes for Several Categories of Buildings

SOURCE: Means Building System Cost Guide (reference 14) by which a hospital building would be turned into a research laboratory and later an office building so that medical activities would take place in up to date buildings while an economic operation would be also achieved through the entire life of the building.

- complex development projects where either one single high rise building accomodates several uses (for example shopping center at ground level, then parking, offices, even hotel in the subsequent floors) or a whole area is devoted to an administrative complex (offices, hotels, restaurants, parkings, etc...) give such options. The breakdown into different uses can be easily changed in this flexible framework and under the authority of a single developer.

Achieving this flexibility may have important consequences on the design: the spatial organization, structural design, dimensionment of the accesses, choice of glazing system may create additional costs. The profitability of the building would also be affected by its organization (notion of space efficiency) and general standard. The main use options as well as the additional independent options would induce choices in the initial design. A typical example is the option to use the basement of the building: this option depends on the initial choice on the type of moisture protection adopted for the foundations. The choice of a dry basement has to be made very early and cannot be reversed later. The trade off between the additional cost of a flexible design and the increase in value due to the use and renovation options has to be studied in order to make the decision at the beginning of the project.

The use of this valuation scheme is difficult in several cases: when buildings are owned by institutions, they are not considered as profit centers, which means that they do not produce revenue. Schools, hospitals, municipal buildings, and offices operated by large companies for their own needs are not operated on a profit basis, so that their value is not known. The parameter S cannot therefore be implemented in a straightforward fashion in the model. On the other hand, all income producing buildings such as offices, hotels, apartment buildings are easily valuable through the rents. Several solutions for this valuation will be presented in Chapter 6.

This situation is paradoxical: institutions have very favorable opportunities to take advantage of this flexibility, as owner and user coincide. On the other hand, income producing buildings are less flexible as contractual terms prevail between owner and users. For example, office space leases extend for 5 or 10 year periods, which means that no construction operation can take place during these periods. Renovation options can be exercised only at very specific moments.

Finally, we can notice the similarities between this financial approach and architectural concepts developed in the sixties. In reference [11], N.J. Habraken proposes the idea of building configurations based on a "support structure": "a support structure is a construction which allows the provision of dwellings which can be built, altered, or taken down, independently of the others." (the support structure is defined as the beams and floors, while dwellings apply to walls and all other equipment). Although this theory is developed primarily for housing projects, it shows that the idea of accomodating different configurations within a single skeleton construction and with flexibility through time is not new. As noted by this author, the interesting features of this approach are, besides flexibility, the ease of organizing renovation operations and maintenance, and of improving the building with new technology and renewals.

CHAPTER IV

POTENTIAL APPLICATIONS TO INDUSTRIAL PROJECTS

4.1 Specifics in the Application of the Three Option Model

The description of a construction project with the design, use, and renovation options is applicable to investments in manufacturing or process plants, as well as extractive facilities. These projects are generally larger than building construction projects, and the capital costs are more important. Many of them also involve a great part of risk: power plants, refineries, chemical plants are examples where a project can be cancelled during the construction phase due to a change in economic, politic, or technological conditions. Manufacturing plants involve a great deal of risk due to the competition in their market, regulations, the international environment, in particular when margins are tight. Our model takes into account some of these uncertainty factors and their effect on the decision process at different levels:

- the design option: an option to undertake a construction project is created by the conjunction of favorable factors such as the location of a site, availability of infrastructure, ease of organization, and availability of the appropriate technology. These factors are relatively known and they affect the cost of the project (exercise price). They should place the company in good position vis a vis the competition. Uncertainty appears in the cost of inputs, labor, raw materials and the demand of the market for the output. These factors affect the cash flows received from the operations. The option model takes them into account by the uncertain "underlying project's value." The tradeoff between a known cost of the project and an uncertain project's value is probably representative of the decision process at the overall evaluation stage.

- <u>the use option</u>: the decision to begin the operation phase may not be as simple as in the case of a building. The exercise price is here the cost of inventories, additional tools, organization, that is, the start-up cost of the factory. We may even include in this figure the loss which is likely to occur in the first months or years of operations. This investment may be large, and its magnitude may be close to the cost of the plant itself. As a consequence, the exercise of the use option may not be straightforward as in a building. It may appear more profitable to dismantle the plant and sell it at this stage. This is a difference with the building case, where the cost of the finishes usually cannot justify to cancel the project, and where selling at this point may not be easy.
- <u>the renovation option</u>: converting a facility to another production is feasible, at least in the manufacturing sector, and adapting the production process so as to include new technologies is possible as well. In this field, the profitability of a facility is affected by obsolescence (in the manufacturing process or the product itself), and not its age by itself, as in the case of a building. Lifecycle effects have to be taken into account, and their duration may be much shorter than the physical life of the facility. Therefore, it is important that the facility be designed to accomodate different production schemes, or different product lines. This gives a significant value to the renovation option.

As a conclusion, it can be noticed that significant differences exist between buildings and plants construction projects and their description under the options process. This general framework will however not be investigated in more details in this chapter. The theoretical description is not different from the preceding chapters, even if some options appear to be more valuable in this case. Furthermore, it is difficult to give general conclusions in the case of industrial projects. There is such a wide discrepancy in the sizes, breakdowns of costs, operation schemes, physical descriptions, that common points are difficult to observe, and a general description would be either imprecise or inaccurate for extreme cases. We will therefore focus on specific fields where an option approach can be developed. Examples will be studied, where the source of uncertainty is easy to identify, so as to point out typical situations.

4.2 Flexibility in the Production Mode

4.2.1 Choice of the Final Product

A first category of options can be defined as the options to change the final product of the facility. It can be valuable if there is a shift in the demand for this product, or if the current model of the product has to be changed for a new model. As these options appear in the operation phase, they can be classified as renovation options.

Such flexibility appears generally when the equipment is standardized. It can be enhanced by dividing the production process so that each workstation can be used for another type of production. These considerations have important consequences on the design of the facility: many professionals argue now in favor of simple, segmented, modular designs where each stage of the product's transformation is isolated in a specific unit, rather than a single sophisticated equipment making the whole transformation at once. Beside the possible advantages in cost or maintenance, the flexibility is increased by allowing a wider range of modifications. Each unit gives valuable options as it is standardized (therefore easy to reuse or sell) and independant (can be integrated into another production cycle). The advantage of a modular versus an integrated facility can be compared to holding a portfolio of options versus an option on a portfolio. In the first case, the most valuable options can be exercised independantly, while in the second case, there is only the choice between all or none of the options. Thus the value of a portfolio of options will be greater than an option on a portfolio with the same components.

Interesting examples where this flexibility has important consequences on the design can be found for manufacturing plants. For instance, car factories must be able to accomodate different models of cars, or even trucks or buses. The design of the production lines and the choice of the tools is influenced by the high probability that a line will have to be adapted to another product at some point in time. Therefore, a tradeoff between cost and flexibility has to be found. One extreme example is the case of the River Rouge plant where the Ford's model T was built in the 20's. This plant was designed to minimize cost, without any concession to flexibility. Ford would surely have been better off if he had realized how valuable is the option to change the manufacturing process at low cost. More recently, we may wonder where the optimum point is between an equipment designed specifically for a certain task on a particular model, and more flexible equipment which can accomodate changes in the basic model, but may be less efficient.

Other examples can be found in an international environment. Manufacturing in less developed countries is often attractive as labor costs are cheap and local markets offer some opportunities of development. However, uncertainty is high: exchange rates, political stability, taxation, and nationalization create very important risks. In case a sudden change happens in a particular country, flexibility in the production can allow two types of responses: the company can shift its production to another country and thus remedy a break in an internationally integrated production cycle; or it can change the production of its factory in order to adapt to new conditions on the local market or requirements dictated by the local government.

Finally, process plants offer identical features. For example, the final product can be modified by adding an additional treatment unit eliminating a minor component or achieving a higher degree of refinement. Using part of the plant for another production is also possible if a transitory stage of the product offers the opportunity to end up with a totally different final product. Changes concerning by-products or linked to environmental regulations can also be implemented: recycling steam or transforming minor products so as to make them marketable have appeared to be profitable in the seventies for certain types of facilities.

On the whole, the exact definition and the valuation of these options seem difficult. Valuing an alternative production scheme is probabily not easy as the organization has to be defined again. However, the examples of manufacturing plants show that a tradeoff between flexibility and efficiency is generally implicitely evaluated, even if no figures are derived. In other cases, even if different market scenarios are not examined, clever designs generally offer opportunities to improve the profitability of the production if an important change occurs. It would be interesting to evaluate these qualitative rules of good design in terms of options, and study their impact on different kinds of investments.

4.2.2 Choice of the Inputs

The price of the inputs can affect significantly the profitability of a production. Typical examples can be found in the chemical, petrochemical, and utilities industries, which depend to a high degree on the prices of hydrocarbon components, namely oil, natural gas, coal, and their derivatives. These industries have been facing a very uncertain environment since the 1974 and 1980 price increases. Furthermore, the availability of new technologies for substitution of the most critical products must be taken into account when new investments are studied. In this environment, it is not surprising that decision makers emphasize flexibility as a major feature of new plants. Here again, the option valuation model may be useful in valuing the opportunities to change the feedstock during the operation phase.

The example of ethylene production is striking. Until 1974, the producers used natural gas as the main feedstock. After the first energy crisis, natural gas became too expensive as well as somewhat scarce, so that most producers turned towards naphta and gas oil for their new investments. In 1980, the price of naphta tripled, so that natural gas became competitive again. As a result, many crackers have been retrofitted to process either pure natural gas, or natural gas as a certain percentage of the total feedstock (naphta). However, synthetic natural gas produced from coal will probably be available in the next decade when the technology is applied to large scale projects, and may become the most economical feedstock.

Similar evolutions have taken place for other productions. The production of electrical power from coal, fuel, or uranium raises similar problems to utilities. All these temporary market conditions are shorter than the usual physical life of a plant -- at least 10 years -- shifting market data must be considered when a company decides on an investment. A certain number of factors are then likely to affect the decision as well as the design:

Competitive feedstocks: the possible feedstocks may be as different as coal and its derivatives, natural gas, oil, uranium, or naphta, butane, ethylene, etc. But all these categories do not even represent a unique and homogeneous product. Coal, natural gas, and oil vary widely by their composition of sulfur and nitrogen components, as well as marginal components such as chloride (which affects corrosion rates). These components require specific treatment units. Flexibility among these products may not be easily achieved as illustrated by the problems of refining light, medium, or heavy oils imported from different producing countries in the world.

Market data: the possible fuels can be traced back to the original raw materials, which affect their prices, variances, correlations. These statistics are important, as discussed before, for applying the option model. However, more elaborate subproducts are also very sensible to the supply/demand equilibrium. While the supply of petroleum derivatives is a fixed proportion of the oil production (after cracking), the demand is determined by the competition between the different components for a very wide range of applications. The choice of a feedstock for a particular production depends on prices, the processes available, and technical considerations, but affects also the other productions.

As an illustration, figure 4.1 and 4.2 show the evolution of the prices of some feedstocks since 1973. In figure 4.1, a record of the prices of the main fossil fuels, i.e., fuel oil, natural gas, and coal is indicated on the basis of their heat content (in cents per MBtu delivered to utilities). The differences in their movements, and also their variances and correlations can be observed. Figure 4.2 describes the prices of intermediate chemical products obtained from natural gas and its derivatives, naphta and gas oil (both obtained from fuel oil cracking), or even coal, for which several processes are often in competition.

Technical solutions: the technical feasibility affects directly the cost of converting a plant to another feedstock. Some favorable factors are:

- a standardized design. For example, crackers have appeared to be able to accomodate several feedstocks, even if they were not designed for them
- a modular design, making part of the plant common for all feedstocks. For instance, all power plants are identical from the moment fuel has been burnt to produce steam (turbines and electrical equipment)

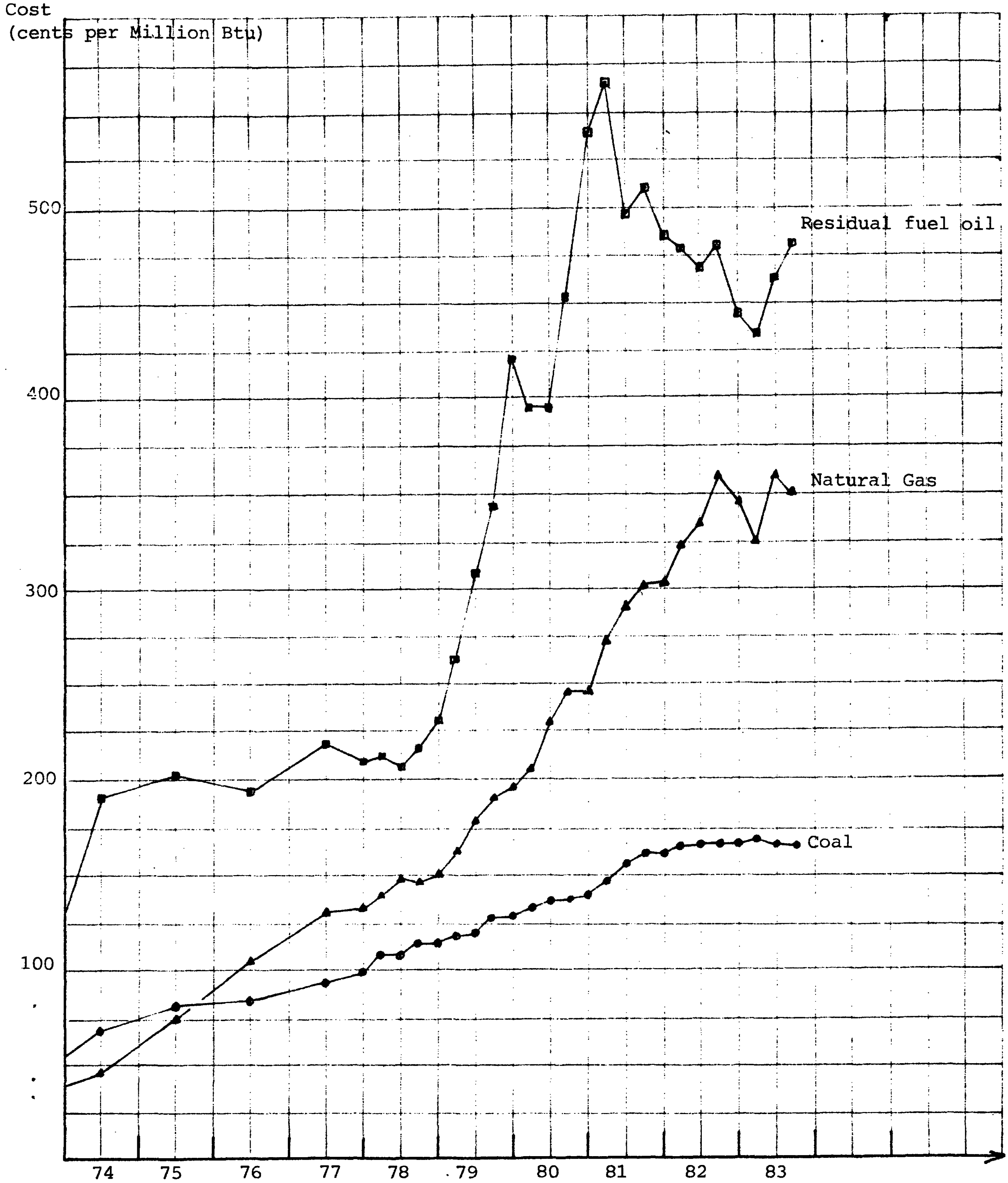


Figure 4.1: <u>Cost of Fossil Fuels Delivered to Steam Electric Utility Plants</u>

SOURCE: <u>Monthly Energy Review</u>, Energy Information Administration

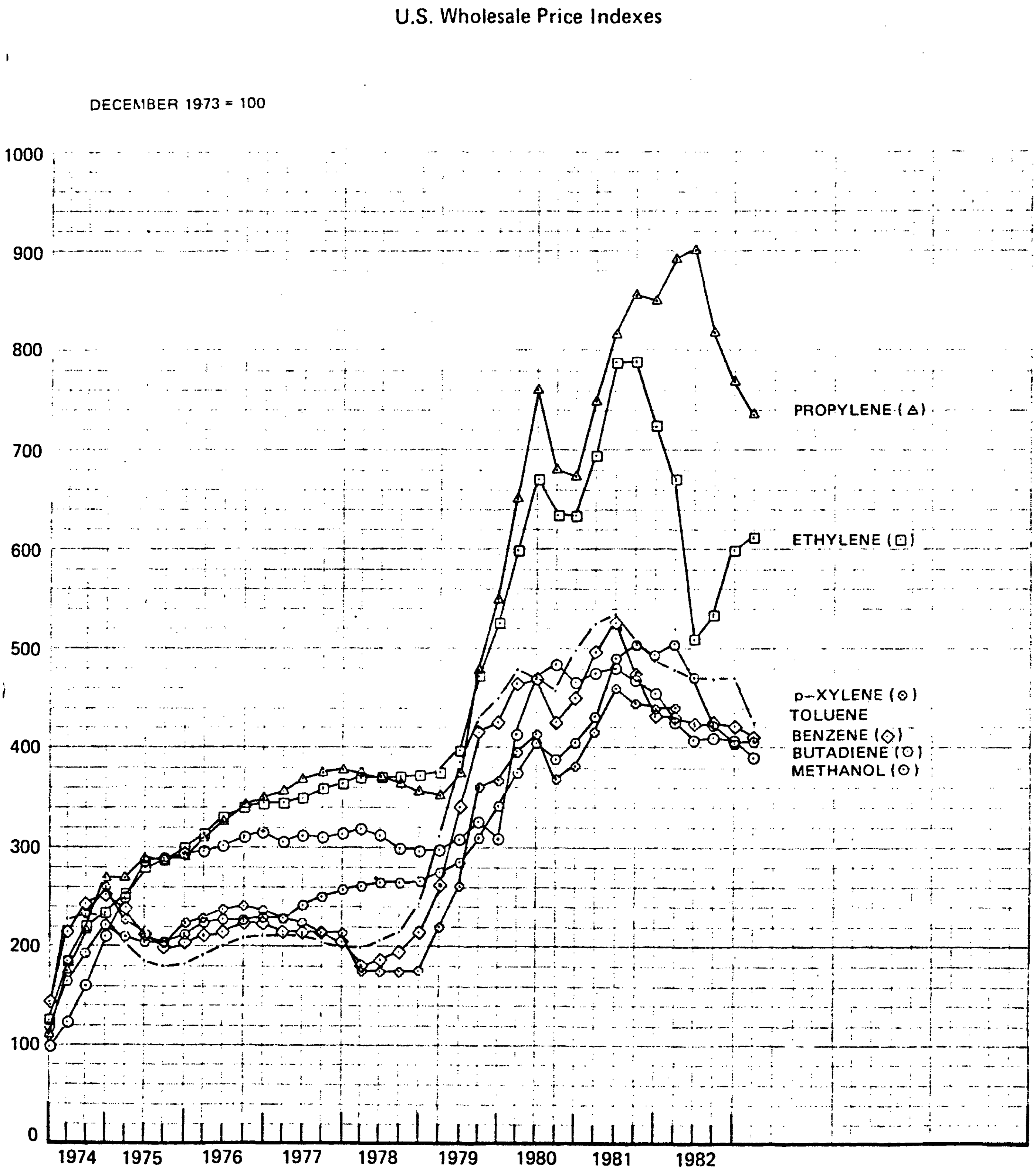


Figure 4.2: Evolution of the Price of Some Basic Chemical Products Since 1973

SOURCE: Reference [17]

- a feedstock requiring less treatments: most of the coal based power plants can be almost readily converted to fuel oil or natural gas as coal is the most demanding feedstock. The contrary is more difficult and requires more sophisticated treatment units. This is illustrated by the transformations at the Crystal River Plant in Florida, which was first operated with coal in 1966, converted to heavy fuel in 1970, and reconverted to coal firing in 1975 (see reference [20]).

Environmental requirements: this may create obstacles, as the subproducts of the plant may differ, thus requiring additional treatments. This may also lead the company to require a more refined feedstock, thus increasing the cost or decreasing the flexibility.

These options may be incorporated in the project's valuation as use options (change in the feedstock decided before the operation) renovation options, or both. In the case of a renovation option, (put option), the parameters of the model would be

S = present value of the cash flows using the present feedstock

E = present value of the cash flows with the alternative feedstock (net of all cost of reconversion)

r = riskless interest rate

σ^2 = variance of the cash flows

τ = useful life of the plant

In particular, the parameter σ would depend on the variance of the price of the feedstock, which can be easily approximated from a record of past prices.

In this case, the valuation with the option model may be straightforward provided data on costs and revenues can be found. The value of the options may also be significant as the variance is high, as well as easy to include in a project evaluation.

4.2.3 Case of Repeated Changes in Inputs or Outputs

In the last two sections, we have studied the value of modifying a plant if economic conditions change. Such an option is modelled as a put option, where the issue is to exchange the stream of cash flows resulting from the present production into a higher stream of cash flows under the new operations pattern. The basic model assumes that no later change is possible, that is the only choice is the abandonment value.

In practice, it is not unrealistic to allow for more than one retrofit operation. In this case, the value of one use at each point in time is the present value of the expected cash flows, plus any option created by this use

$$S = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t} + V$$

where S = value of the use under consideration

CF_t = expected net cash flow, year t

r = risk adjusted interest rate

V = value of another put option offered by this use

A more complicated pattern where a great number of nested options take place, can even be studied. Let us suppose, for example, that a process plant is able to operate with two different feedstocks. When feedstock 1 is used, the value of this use is the present value of the cash flows with this feedstock, plus the value of the option to switch to feedstock 2. When feedstock 2 is used, the value of use 2 includes the option to switch back to feedstock 1. Assume also that it does not cost anything to change the feedstock. The parameters of the options would then be:

$$\underset{\text{feedstock 1}}{\text{use 1}} \dashrightarrow \underset{\text{feedstock 2}}{\text{use 2}} \dashrightarrow \underset{\text{feedstock 1}}{\text{use 1}}$$

value of use 1 $$S_1 = \sum_{t=0}^{\tau} \frac{CF_{1t}}{(1+r)^{\tau}} + V_{12}$$

$$S_2 = \sum_{t=0}^{\tau} \frac{CF_{1t}}{(1+r)^{t}} + V_{21}$$

where V_{12} is the option to convert the operations to feedstock 2

V_{21} is the option to convert the operations to feedstock 1

τ is the physical life of the plant

With our model, V_{12} and V_{21} would be computed as

$$V_{12} = P(S_1, S_2, \tau, r, \sigma^*) \qquad \text{(put option } S_1 \rightarrow S_2\text{)}$$

$$V_{21} = P(S_2, S_1, \tau, r, \sigma^*)$$

$$\sigma^* = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$$

(σ_1 and σ_2 variances of uses 1 and 2, ρ correlation between 1 and 2)

We have just described here a plant which would be able to operate with two fuels with a total flexibility. The manager would only have to select the cheapest fuel at any time, thus making more profit than with any of the two fuels considered separately. The option model seems to provide a satisfactory description of this process, although the solution may not be easy to derive. It would be interesting to study how this model selects the appropriate use: the choice is not based on a long run evaluation of each use, but only on a short run tradeoff between the next cash flow under each use, and the option to switch uses each period.

Besides feedstocks, other examples of total flexibility can be found in the industrial environment. The manufacture of steel products may provide illustrations of such situations. In some cases, equipments can be adapted to cast several shapes of parts, provided the material is the same (the steel mill is actually the main part of the plant). For example, a new process has recently been developed, allowing to cast any kind of linear steel product from wire to structural steel beams. Changing the product is said to require only a change of the mold and takes half an hour.

Before leaving this topic, I would like to reconcile the effect of input and output prices, treated separately in this section. It can be noticed that the manufacturing process is actually a call option where the value of the asset comes from the revenues from the product sale, and the exercise price is the cost of the inputs. The option is exercised only if the revenue is superior to the cost of the inputs, or in other words if the unit price is larger than the variable cost. Manufacturing can therefore be viewed as a succession of independent options, for example, the options to decide to manufacture or not at the beginning of each year. Such a decision process may be encountered in the case of extractive industries, where the variable costs depend on the physical characteristics of the mines while revenues are governed by a world market.

4.3 Options in the Development Stage

4.3.1 Step-wise Development of a Project

Choosing the appropriate size for a project is a major task addressed during the preliminary stage of planning. If the market is uncertain, there is little chance that the chosen size matches exactly the most economical point of production. If the demand is also likely to grow during the project's life, there is no way to obtain a production at minimum unit cost throughout the production phase: a compromise has to be chosen between a large size plant operating at low capacity during the first years, and a smaller plant which is likely to be unsufficient if the demand actually increases.

In the case of industrial projects, flexible solutions can be found. For example, a two step process can be studied, in which the plant is initially dimensionned for a small, but sufficient capacity, and later extended to full capacity. Such a case has been examined by J.P. Asquith in "a Comparison of the Direct and Step-wise Completion of a Process Plant and the Capital Cost Involved" (Institution of Chemical Engineers, 1966) and I will try to interpret it here in terms of options.

A brief summary of this article is needed at this point. The issue is to dimension a process plant for full or two thirds of its final capacity. The final product is not specified in this article, and the size of the plant is expressed in terms of units (one unit represents a certain output requiring in particular one specific type of chemical reactor). The alternatives are therefore to build either a 6 unit plant, or a 4 unit plant which can be easily extended to 6 units.

This article deals with the technical issues involved in the design of the two alternatives.

- some parts of the plant can be divided into several identical units (2,3, or 6), thus making the step-wise completion alternative possible at competitive cost

- other parts such as utilities have to be built immediately for the full capacity due to scale economies and the difficulty to divide them into sub-units.

After selection of the most economical configuration for each alternative, cost data are given. The following table summarizes them:

4 units plant	4 units plant, designed to be extended to 6 units	6 units plant
this case is not evaluated	4 units: $6,426,000 2 additional units: $940,000	$7,000,000

Figure 4.3: Cost Data for Development Alternatives of a Process Plant

Unfortunately, data on the value of these possible configurations are not mentioned. The cash flows expected from this project are essential elements in the evaluation of the alternatives. The writer however assumes clearly that the demand is going to increase, making an extension from 4 units to 6 probable within 2 or 3 years. The forecasted output could therefore show a progressive increase, and its curve over time might look like Figure 4.4.

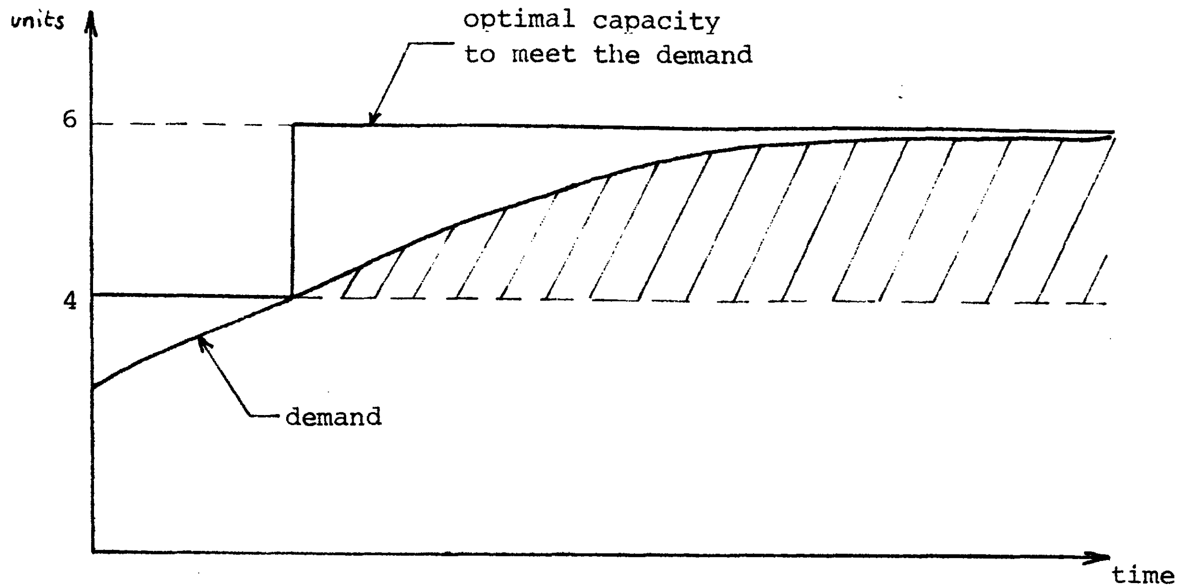


Figure 4.4: Assumed Economic Data for the Process Plant

In this figure are represented an increasing demand curve, and the curve of the capacity if the company plans the extension at the best moment. The shaded area represents the additional supply which will be available with the additional 2 units. It can be directly related to the additional cash flows produced by the extension of the plant.

The analysis may even be more complicated if the production costs are different for the studied alternatives. For example, operating a 6 unit plant at less then two thirds of its capacity may be more expensive than operating the 4 unit plant close to its optimal capacity. In a first approximation however, it will be assumed that the production unit costs are constant at any stage of the project. The value of the project would then be found by discounting the cash flows resulting from the sale of a certain output, produced at a constant unit cost.

In comparing these alternatives, there may be two ways to interpret the choices in terms of options.

Four Units and an Option for Two More Units Versus Four Units

The study by J.P. Asguith did not include the alternative to build only a 4 unit plant without an extension option. The reason must have been that, with a high probability of an extension after a few years, the company did not want to limit its capacity to this plant's output. However, the cost of this alternative would be useful to derive as it would allow to calculate the actual cost of the option to build two more units. If we suppose, for example, that a simple 4-unit plant costs $5,000,000, the actual cost of the extension option is:

$6,426,000 - $5,000,000 = $1,426,000

The second cost figure necessary for this evaluation is the exercise price. The article evaluates it as $940,000, representing the cost of the equipment to be added (2 more reactors, and other equipment).

The option valuation model would be useful in this analysis as it would derive the value of this option. This option can be modeled as a call option whose parameters are:

$S(t)$ = value of the additional 2 units (incremental cash flows allowed by the extension decision: this can be calculated using the shaded area on Figure 4.4)

E = exercise price = $940,000

τ = physical life of the project = 10 to 20 years

σ = variance of the rate of return on $S(t)$

r = risk free rate

The decision to buy or not this option (that is choosing to invest in 4 units or 4 units with the extension possibility) would simply be taken by comparing its value and cost:

if $C(S(t),E,\tau,\sigma,r) > \$1,426,000$ <u>buy the option</u>

if $C(S(t),E,\tau,\sigma,r) < \$1,426,000$ <u>do not buy it</u>

A few comments can be made about modelling this case with a call option.

This is an option on an asset which pays dividends. As long as a four unit plant supplies a sufficient volume to cover the demand, these dividends are equal to zero. As soon as the four units plant cannot meet this demand, the dividends become substantial as the 2-unit extension would produce revenue. Figure 4.5 shows these "dividends", that is the forgegone revenue lost by not exercising the 2-unit option.

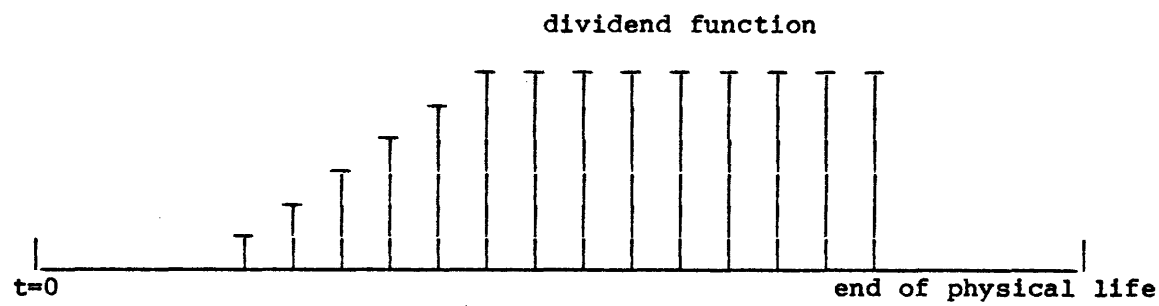


Figure 4.5: The Cash Flows Expected From the Additional 2 Units Modelled as a Dividend Function

The optimal timing to exercise the option may not be easy to identify. Two conditions are required:

(1) the value of the additional 2 units must be superior to the exercise price: $S(t) > \$940,000$

(2) even if (1) is realized, it is better to wait as long as no dividend is paid. It is not useful to build the additional two units if they are not going to be operative immediately. The optimal exercise decision should take place when the extension would begin to produce revenue, or more precisely when the revenue for the next period

exceeds the savings due to delaying the expenditure of the extensive operation from one period (evaluated at the risk free rate).

Finally, it is intuitive that the growth rate of the demand curve has an important influence on the value of the option. The option is more valuable if the demand is likely to grow rapidly, and we may wonder how this factor is taken into account in the model. The answer would be that this factor is implicitely included in the value of S(t). If the growth rate is expected to be high, S(t) will be higher as revenue will be produced early and each cash flow will be higher than with a lower growth rate.

These remarks show that this option would be similar to the model described in section 3.3 for the use option. This option offers the opportunity to wait when no dividend is paid, and to make a decision concerning the exercise when dividends are produced.

<u>Four Units and an Option for Two More Units Versus Six Units Immediately</u>

In the case of the comparison between these alternatives, another argument intervenes in the decision: building now a six unit plant allows to save on the total cost of the project. The design is simplified as it does not have to include an intermediary step for 4 units, and the construction of the whole plant at once may be cheaper (in particular non-standard equipments are cheaper if they are ordered in greater number at once). This issue illustrates the difference between financial and real assets: while 6 shares of stock can be purchased for 6 times the price of one share of stock, this rule does not apply for real assets if economies of scale intervene in the exercise price.

These two alternatives can be compared by using a put option model. Considering the final goal of obtaining a 6 unit plant, these two alternatives have different costs:

- the step-wise construction alternative costs $6,426,000 + $940,000 = $7,366,000
- the direct full-size construction alternative costs $7,000,000.

The first alternative is more expensive by $366,000, but we can consider that this additional cost is related to an additional feature offered by its process: in case the extension to six-units is cancelled, this alternative allows the company implicitely to recover $940,000 which will not be spent immediately.

This is a put option (abandonment option) whose exact parameters are the following:

- the initial project is a 6-unit plant costing $7,366,000.
- the put option offers the opportunity to convert this project into a 4-unit plant. In that case, $940,000 can be recovered from the initial budget.
- the time to maturity is the duration until the full size plant is expected to be completed. In the context of this particular project, it seems that the company is considering to operate a 6-unit plant no later than 3-4 years after the beginning of the operations. (This maturity date is actually uncertain.)

The comparison between the direct and step-wise construction alternatives will then have to evaluate these figures in the context of the real environment. Both projects will produce anyway the same revenue, therefore, the comparison is done in terms of costs, that is the ability to delay in higher expenditures or to save them if the demand does not grow. Figure 4.6 shows the curves of cost over time for these two cases. The value of the put option comes from the fact that the step-wise completion curve is constantly below the direct completion curve, in particular after the beginning of the operations.

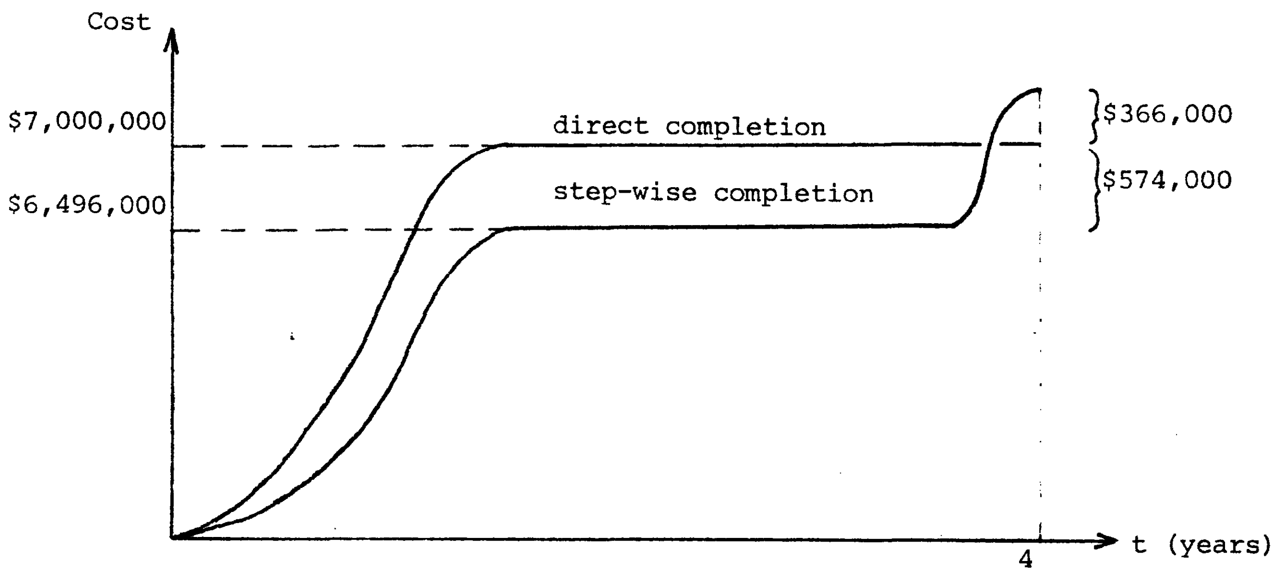


Figure 4.6: Cumulative Construction Cost Curves for the Direct and the Step-wise Completion Alternatives of a 6-unit Plant

In making a decision, the company has to judge whether it is worth spending $366,000 on the step-wise project in order to save $940,000 if the extension is not made at a later point in time. In other words, the plant is more expensive by $366,000 if the extension is done, but saves $574,000 if the extension is not done.

Note that no discount of the sums spent for the alternatives at different times has been done in this analysis. This argument should be taken into account in the evaluation of the extension operation three years after the 4-unit plant construction. If the direct and step-wise completion alternatives lead the same final cost, the step-wise alternative would be preferred for this reason.

4.3.2 Other Options in the Development of a Project

Other potential applications of the option approach are presented here. As no specific example was found during the documentation phase of this study, they are not discussed in details.

Option-like investments occur when a new site is developed for a plant, and when part of this investment can be used for other projects on the same site. This is the case for all expenses for utilities, infrastructure terminals such as roads, access to a river, or a linkage to a railroad, which allow the location of similar projects at a lower cost. In some cases, the expenses for utilities and infrastructure can amount up to one third of the total cost of a project. Allocating these expenses to different projects on the same site is possible, but an option approach of this problem may be even more appropriate. The present approach, would be rather to consider them as long run investments, so that their strategic character is recognized.

Other options may take place during the design and construction phases of a project. The completion of industrial projects is long: some projects such as power plants can take ten years or more from the beginning of the operations phase. These projects are sensitive to changes in technology, new environmental regulations, and also the prices of raw materials (see for example the evolution of copper prices in Figure 4.7). In this environment, some options may be interesting to develop in order to minimize the cost of the project in relation with a certain set of possible scenarios during the construction phase. The option to adopt a new technology, construct new treatment units, or change some materials may be valuable at certain moments of the construction phase if the environment changes.

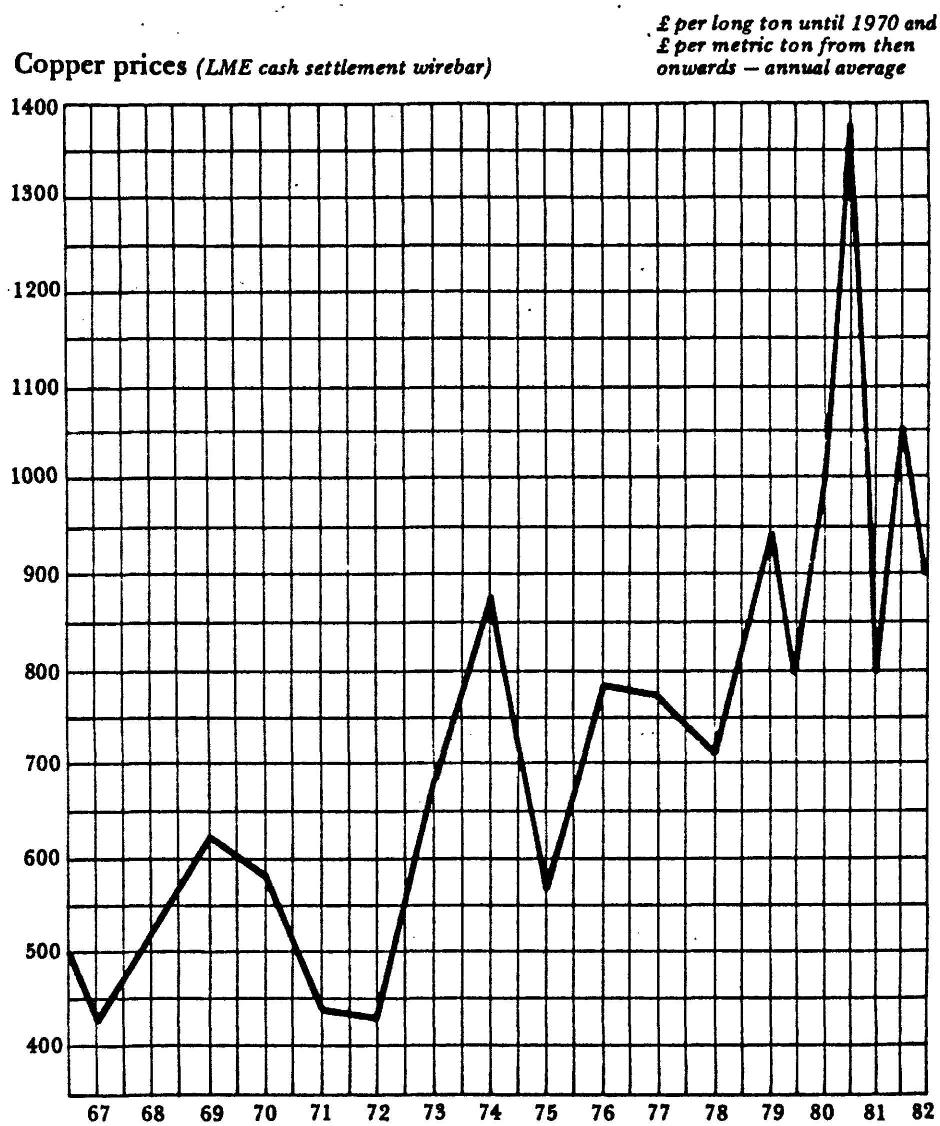


Figure 4.7: Evolution of Copper Prices from 1966 to 1982
(used in wires, beat exchangers, cooling systems)

SOURCE: Reference [21]

CHAPTER V

APPLICATIONS TO INFRASTRUCTURE PROJECTS

5.1 Option Approach in the Planning of Infrastructure Projects

The option valuation model may be appropriate to describe issues encountered in infrastructure development projects, related in particular to dimensionment decisions. The choice of a size for roads, pipes, and tracks is difficult and important since infrastructure facilities have to be studied in a long-run perspective, although the development of different areas may induce rapid changes in the needs. Furthermore, infrastructure elements are not independent from one another, and the insertion of a new project in an existing network, including the effect on any other branch, has to be studied. These decisions are crucial since errors may be very costly, or even impossible to correct.

Options may be identified at two levels. One project, considered independently may include options, for example to increase its capacity. Or, at the level of a whole network, a particular configuration on a part of the network may offer new development alternatives on other parts of the network.

The example of a highway project in an urban area may illustrate these points of view. The dimensioning choice is generally based on two estimates: an expected volume of traffic upon completion of the project, and a forecast of the traffic 20 years later. If the need is for a two lane highway immediately and a four lane highway in 20 years, two development alternatives can be considered:

- the agency may decide to build a 2-lane highway now and to allow an extension to 4 lanes later. This can be done by planning the project so that enough land will be purchased for the extension alternative. This investment involves therefore a call option (the extension possibility). This kind of investment scheme may be appropriate if the traffic will not increase in the immediate future or if funds are not available for a 4-lane highway.
- the decision may be to build immediately a 4-lane highway. This project in itself does not include an option since no further development will be made on the highway itself. But the full utilization of its capacity may depend on other investments such as debottlenecking an extremity of the highway, or developing other roads so that the highway be included in a new route. In that case, the value of the highway includes an option.

Both solutions lead to valuing the project as one asset plus one or more options. The asset relates to the immediate benefits provided by the project, while the option applies to potential benefits which will occur if future developments are undertaken. This point of view would account for the importance of intertemporal considerations in these projects. They take place in a growth environment, and they are very costly to duplicate, unlike most buildings or plants construction projects. Long run forecasts are therefore more important than elsewhere.

5.2 Infrastructure Facilities as Growth Opportunities

The point of view presented in section 1 can be related to the way some assets held by a firm can be valued through the stock exchange. In "determinants of corporate borrowing" (reference [6]), Stewart Myers notes that two kinds of assets can be defined:

"assets in place" are "assets whose ultimate value does not depend on further discretionary investments"

"growth opportunities" are "assets that can be regarded as call options in the sense that their ultimate values depend, at least in part, on further discretionary investments by the firm"

The development alternatives presented in section 5.1 can be described as growth opportunities. Furthermore, it seems that most bridges, tunnels, important crossroads or large facilities in an infrastructure network can be classified in this category. This is supported by the fact that they are often overdimensioned, even with regard to the forecast for their use.

Such a facility would be valued at its immediate benefits to the user plus an option to accomodate an additional volume.

Figure 5.1 shows the example of overdimensioned bridge: it is designed for 6 lanes (maximum volume 150,000 vehicles per day), while its access road is a 4-lane highway (maximum 100,000 vehicles per day).

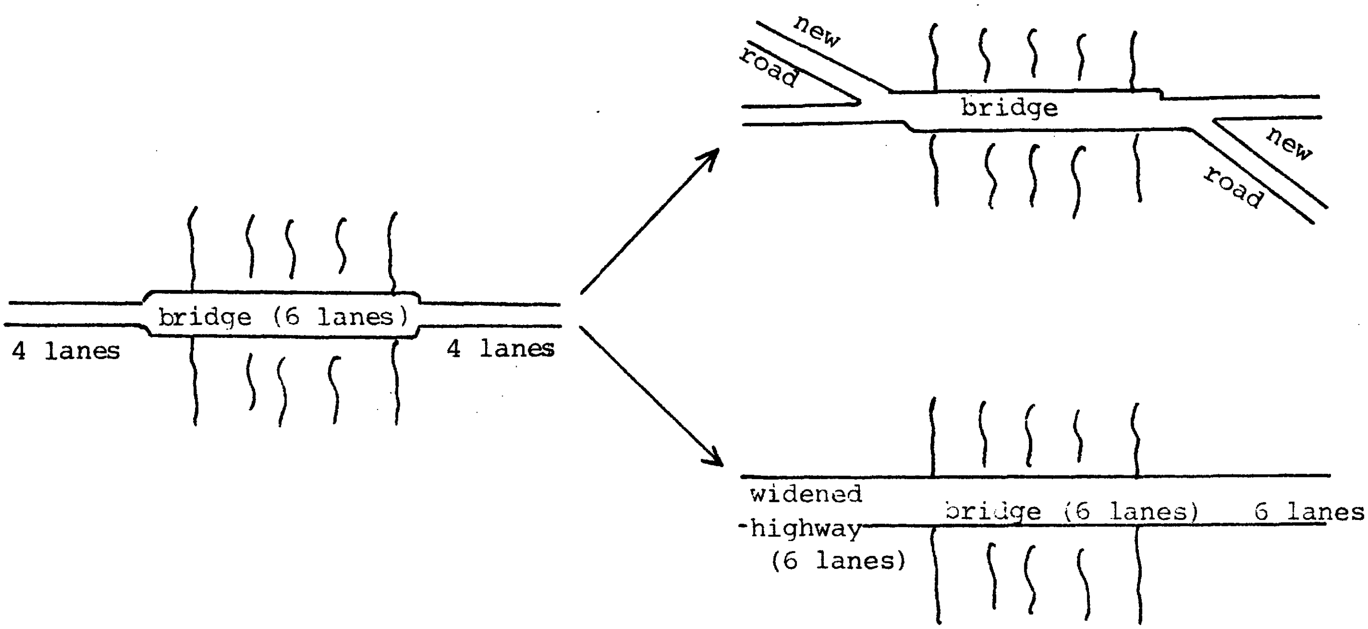


Figure 5.1: An Overdimensioned Bridge as a "Growth Opportunity"

In its initial configuration, the bridge cannot receive more than 100,000 vehicles as it is botlenecked on both extremities. Its value is the same as a 4-lane bridge.

However, the value of the bridge is higher if we consider the two options offered by this configuration: a new road can be connected to the bridge (daily volume 50,000 vehicles), or the access highway can be widened to 6 lanes. Both solutions provide the additional benefits of a new or a wider road (value S) without having to build a new bridge. The cost of the new road, or the widening of the old highway (exercise price E) is therefore lower than in a normal project.

5.3 Examples

The John F. Fitzgerald Expressway in Boston: in this project, the existing elevated highway is to be replaced by a 1-mile long tunnel. Its dimensioning is crucial as this tunnel will be a major exit of Boston for decades. The project is to be an 8-lane underground highway, with the elevated highway still in use even after completion (see reference [19]).

Figure 5.2 shows a description of the project and the curve of its cost versus capacity. The flat part of the curve around the 8 lanes proposed capacity shows that the size of this tunnel could be increased at a relatively low cost. According to an option approach, this increase in the cost may be worthwhile if it provides a more valuable option. The project's size would be actually determined by a tradeoff between the cost of overdimensioning and the value of the call option thus created.

High speed railway development projects

The construction of a new high speed track between two cities (named A and B) may create opportunities for developing other routes. In figure 5.3, it is assumed that a third city, named C is located halfway between A and B, but off the high speed track.

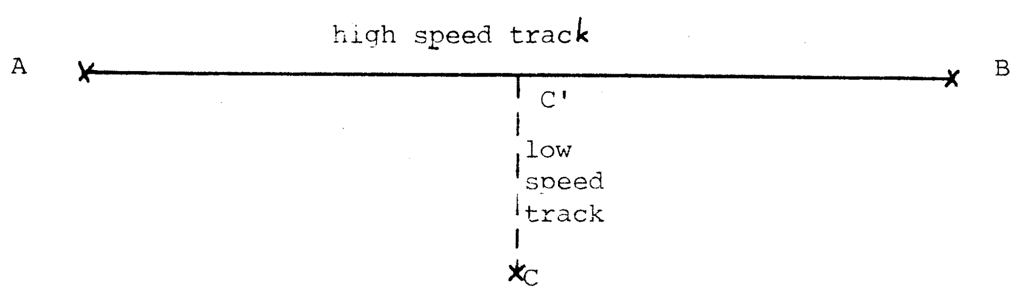
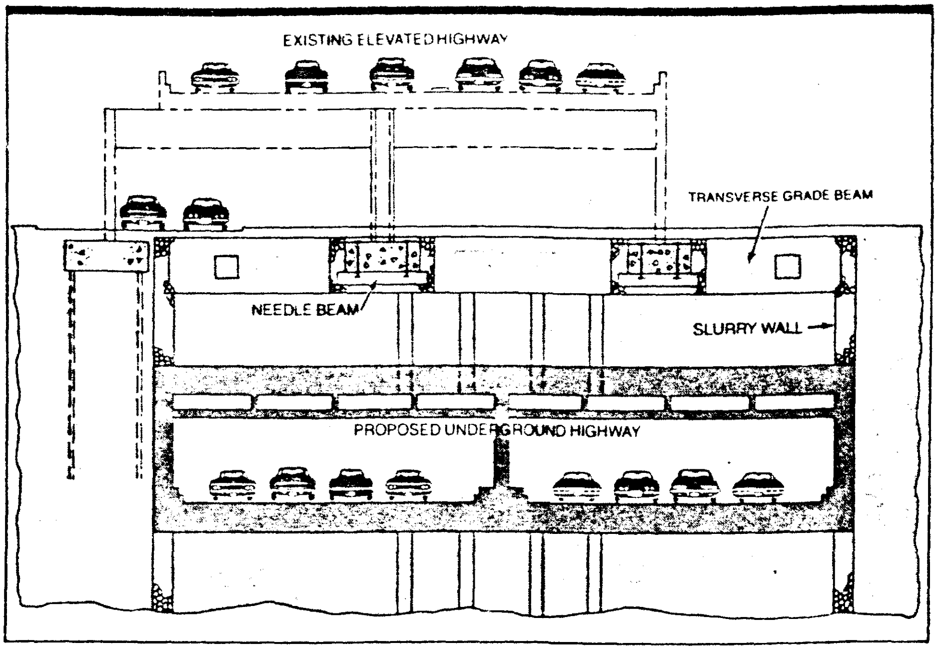


Figure 5.3: Options in a High Speed Railway Project



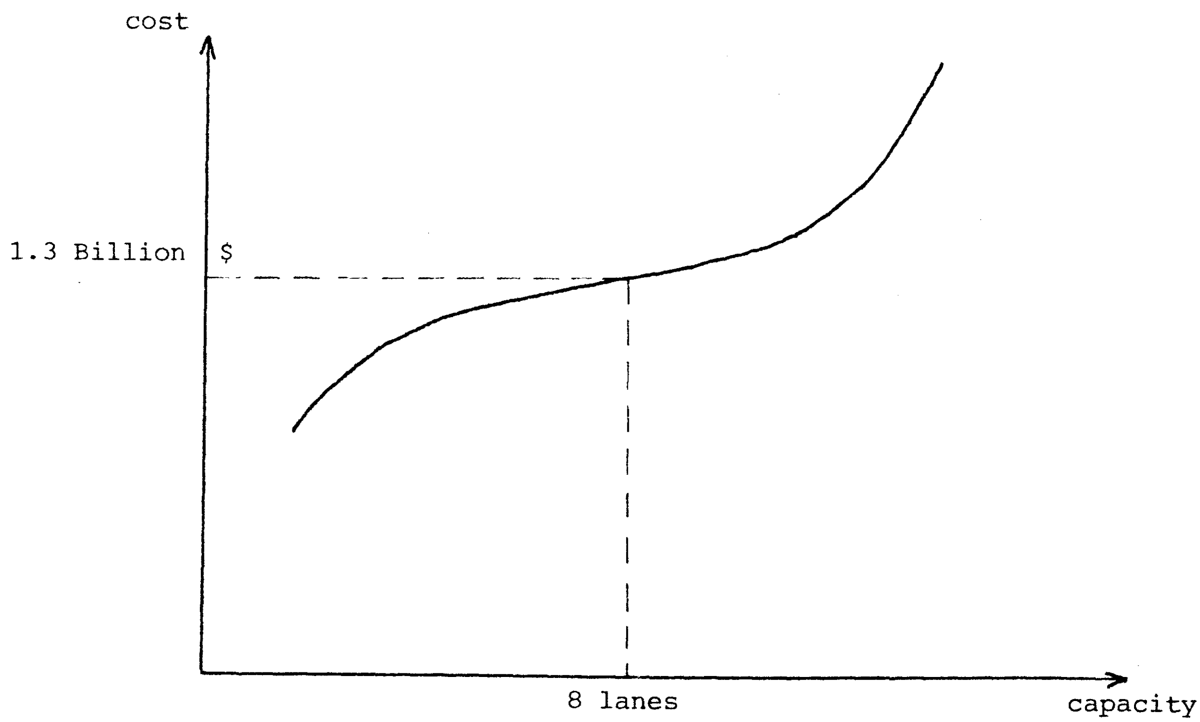


Figure 5.2: Description and Assumed Cost Data for the J.F. Fitzgerald Underground Highway Tunnel

This configuration gives opportunity for a better service to city C. Two options are allowed:

(1) Use part of the high speed track and the existing low speed track between C and C'. This investment requires only to buy high speed cars and engines which can ride at low speed between C and C'.

(2)Build a high speed track between C and C' in order to set a fully high speed service between A,B, and C. This investment is more expensive due to the new track construction.

The value of the initial project is increased by these two call options. It may be worthwhile undertaking this project even if line A-B is not profitable enough to justify the high capital investment. It can be noticed that in the case of the French TGV project, the first category of options has been exercised: part or the whole new Paris-Lyon track and existing low speed tracks are used by high speed trains serving several southeastern French cities.

Water resources or sewer systems networks

The design of these networks is difficult in growing population areas. The dimensioning of the piping systems and the treatment plants has to be carefully studied in order to satisfy the needs of several urban areas, each one having its own growth pattern. Several options may be studied in order to satisfy different population patterns, in 10 or 20 years. Such options may be complex due to the great number of parameters as well as their interdependancy.

5.4 Difficulties in the Valuation Process

The method described above may be useful in valuating facilities, and in particular when a project has to be chosen according to immediate as well as future potential benefits. When two alternatives are possible, and if their benefits and costs are similar, this method should allow us to identify the project presenting the best development potential in relation with other facilities and needs.

These considerations however do not solve the general problem of the valuation of the benefits provided by an infrastructure facility. There is for example no satisfactory method for the valuation of highway facilities. There is no market and these facilities do not produce revenue. The only available method is based on running costs: the evaluation of the benefits to the users is derived from such parameters as average speed, value of travel time, volume of traffic, maintenance and vehicle costs. (see reference [18]) This figure cannot be compared to financial data, even to the cost of the project. As a result, no positive NPV rule is used to evaluate a project. The rule is to rank alternatives according to their benefit/cost ratio and to choose the best projects within the available budget. This nonfinancial method cannot be accomodated in the option valuation model, where the value of the project (S) and its cost (E) cannot be dissociated and are compared to stocks in their movements.

This problem is the main difficulty in testing the model and interpreting its results. It may however be partly solved if the privatisation of this sector goes on. For instance, the recent sales of water treatment facilities to private companies may allow to get objective data through sale prices as well as the study of stock prices.

CHAPTER VI

NUMERICAL EXAMPLES

Even if the option pricing methodology seems to describe properly certain types of construction investments, some basic questions still have to be answered before any practical application can be considered. What are the results of the model, in comparison with other valuation methods? How can the input parameters be estimated? What is the sensitivity of the valuation to their values? Are the calculations reliable and can they be applied in the present form to real projects? This last chapter of the thesis will try to answer some of these questions by developing numerical examples and interpreting them.

For this purpose, an hypothetical building being able to accomodate two uses will be described in section 1 and used all along this chapter for different applications. The design, use, and renovation options discussed in Chapters 2 and 3 will then be illustrated. These calculations will involve the use of the models of simple call option, option on the maximum of two assets, and infinite put option introduced before.

6.1 Description of the Chosen Example

6.1.1 A Building for Office Space or Apartments

As noted in Chapter 3, applying the option methodology to building projects presents some analytical advantages. The existence of market mechanisms allows us to find more easily the data needed for the tests in a financial context, while cost figures and development schemes are relatively well known. Most of the data presented here can be found in references 10 and 12 to 15. An exposure to real estate practices also was very useful (in particular, to derive realistic rental values), and was provided by private conversations with Philip Trussel at the MIT Real Estate Office.

Most of the following applications will involve the valuation of a building project when two uses are possible. It was necessary to define a building type allowing to accomodate these uses within the same overall design. A mid-rise building project (3-4 stories) with roughly 50,000 square feet of gross area was chosen for uses as office or residential space.

Monthly rents for new apartments are about \$1 per square foot in Boston, which gives \$12 as annual rent. A gross income multiplier method can be used to derive the total value (present value of the income) from this use. This method is a convention used in the real estate industry as an approximation of the net present value of a real estate investment. We use it here due to the lack of more accurate market value data. This multiplier parameter varies generally between 5 and 8, and was chosen here as 7. We obtain therefore:

$$\text{value of the use as apartments} = \text{annual rent} \times \text{gross income multiplier}$$
$$= 12 \times 7 \approx \$85 \text{ p.s.f.}$$

This result is consistent with general information available.

The market for office space is less homogeneous than the housing market. In downtown Boston, annual rents amount up to \$35 per square foot for new space, while a significant lower amount can be observed in Cambridge and other suburban areas. We have assumed here that the annual rent for similar buildings in the same location is \$18 p.s.f., so that the total value for this use is $90 p.s.f. (this figure can best be derived using mortgage payments calculations. $18 generally covers an annual mortgage payment plus profit. $90 can be derived by comparison with the present value of the mortgage).

Figure 6.1 summarizes the assumed development budget. The Means Square Foot Cost Guide (reference 15) provides a good basis for this study. The cost of this type of building (including architect's fees) is close to $50 per square foot. As sitework, landscaping, and a possible basement are not included in this figure, the total construction cost was estimated as $60 p.s.f. In this figure, $20 can be considered as the cost of the finishes. Adding another $20 (30 to 35% of construction cost) for leasing, financing, contingency, taxes, and start-up cost, the total cost of the development operation was derived as $80 per square foot, while the remaining $5 to $10 account for the land value and profit.

6.1.2 An Evaluation Method Using Market Prices

Several methods are available to calculate precisely the value of the income produced by the building described above. A first method would be to study the rents of similar buildings in the same location (same range, same age) and to capitalize them at a required rate of return which is presently close to 17%. This is actually the basic method of evaluation of income producing buildings, but it requires accurate data about the rents and to study their evolution over the building's life. A second method would be to study the sale prices of similar buildings and to convert these data into square foot prices. This evaluation is easier, as it takes advantage of the evaluation by the market of the expected rents, maintenance costs, and

Basic construction cost:	shell construction	30	50	
	finishes	20		
Plus: sitework, basement, landscaping...			10	
Total construction cost				60
Other expenses (leasing, financing, contingency, etc...)				20
Development operation budget				80
Value as office building	(rent = $18 p.s.f.)			90
Value as apartments	(rent = $12 p.s.f.)			85

Figure 6.1: Development Cost and Expected Value of an Offices/Apartments Building Project

discount rates, into a single figure. It requires however to ascertain and to study a record of past sales of market prices in order to find a comparable building. These two methods are probably the most accurate ones. They have not been used in this study since no accurate data were accessible at this point.

A third method offers interesting insight on the value of these assets in a financial context. This method is based on a study of the stock prices of real estate investment trusts (REITs) and derives square foot values by relating them to the exact composition of their assets. These trusts appeared in great number in the '70s, and some are traded today on the New York Stock Exchange. The research should focus on finding a trust holding only the type of assets under evaluation (e.g. office buildings only), but such trusts are rare since many undiversified trusts disappeared during the particularly difficult 1976 year.

As an example, Figure 6.2 shows selected parts of the Standard and Poor report on the Hubbard Real Estate Investment Trust. Its assets, as given by the October 1983 company annual report, are the following:

distribution and service properties		1,503,000 sf
office space		745,000 sf
retail		981,000 sf
	total	3,229,000 sf

On the other hand, calculating its value is straightforward since the stock price and the value of the debt are known:

equity (5,945,681 shares at 22 3/4 $)	135,264,000
long term debt (mortgage, using book value)	16,501,000
total asset value	$151,764,000

Dividing this value by the total number of square feet gives $47.00 per square foot. In this calculation, we consider that all the value of the trust is invested in operating real estate assets, and we disregard in particular cash and other liquid assets. As a comparison, the study of three other REITs shows figures ranging from $34 to $50 per square foot.

The difference between this result and the values assumed in section 1.1 can be explained by the large discrepancy in this trust's assets:

- different buildings are included in this valuation. For example, shopping centers and warehouses are less valuable than office buildings on a square foot basis.
- the age of the buildings is an important factor since it affects rents and maintenance costs
- the valuation also depends on the location. Downtown and suburban properties do not show the same values.

This method however shows that the difference between real assets and financial security claims is not as large as one might expect at first. It justifies to some extent the application of the option valuation model to these assets. Another feature of this method is that it allows to study the value of the variance σ^2, which is an important parameter of the option model. This parameter might be calculated using a record of past stock prices (as for any evaluation of the price of an option on a stock using the option model). However,

Hubbard Real Estate Inv. 1168

NYSE Symbol HRE

Price	Range	P-E Ratio	Dividend	Yield	S&P Ranking
Jan. 27'84 22¾	1983-4 26-17⅝	11	³2.20	³9.7%	NR

Summary

This real estate investment trust invests in a diversified portfolio of income producing properties. In recent years the trust's investment program has emphasized the development and acquisition of properties with opportunities for growth in yield, supplanting its initial program which almost exclusively involved properties net leased to large corporations at fixed rates.

Current Outlook

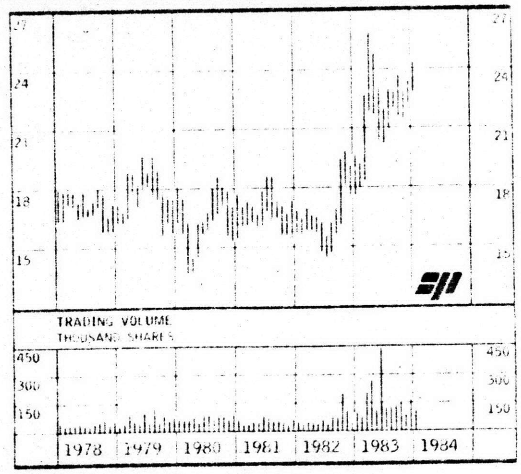
Net earnings for fiscal 1984 should compare favorably with the $1.99 a share of fiscal 1983.

Dividends are expected to approximate taxable ordinary income.

Operating earnings for the fiscal year ending October 31, 1984 are expected to continue to reflect HRE's redeployment of assets from high-yielding short-term investments into partnerships and joint ventures in income producing properties. Start-up of new investments are expected to restrict the yield on these assets, but more favor- ... the remaining net leased properties ... economy ...

1975 91 95 ... 9.3 10...

Data as orig. reptd. 1. Incl. prov. of losses & depr. 2. Incl. gain on sale ... ventures in 1982, 1981. d-Deficit.



Important Developments

Oct. '83—HRE sold an office/research facility to a subsidiary of Ashland Oil Inc., for $16 million. William F. Murdoch, Jr., president, said the trans- ...tion largely completed HRE's strategy of with- ...

Business Summary

Hubbard Real Estate Investments is a real estate investment trust that invests primarily in income-producing property such as stores, warehouses, office buildings, shopping centers, industrial buildings, apartment buildings, etc. In recent years the trust investment program has emphasized the redeployment of assets into the development and acquisition of properties with greater potential for future growth, from its initial program, which almost exclusively involved properties net leased to large corporations for long terms at fixed rates.

Investments have generally taken the form of direct ownership of properties and, more recently, have involved controlling interests in partnerships with local developers. Joint venture developments include a new office building in Portland, Oregon, another in Denver, and an existing office building in Charlotte, North Carolina.

At October 31, 1982 HRE's investments consisted of office buildings and research units (34%); warehouse, distribution and service centers (22%); general merchandising outlets (20%); supermarkets (18%); and shopping centers (6%).

The trust's adviser is Hubbard Advisory Corp., a subsidiary of Merrill Lynch, Hubbard Inc.

Div...

Divid... reinve...

Amt. of Divd. $	Decl.	Date	Record	Date
0.55	Mar. 8	Mar. 17	Mar. 23	Apr. 20'83
0.55	May 24	Jun. 17	Jun. 23	Jul. 20'83
0.55	Aug 25	Sep 19	Sep. 23	Oct. 20'83
0.55	Nov. 29	Dec. 19	Dec. 23	Jan. 20'84

Finances

In connection with the W.T. Grant bankruptcy, the trust regained control and possession of the properties which had been leased to Grant. These properties were rerented to new tenants on terms generally more favorable to the trust. HRE, from time to time, has received partial settlement towards its bankruptcy claim against Grant. While no assurance has been given as to the amount or timing of any further payment, any such recovery will be recognized as income in the period in which it becomes determinable.

Capitalization

Mortgage Payable: $16,501,000.

Shares of Beneficial Interest: 5,945,681 shs. (no par)

Institutions hold about 11%.

Shareholders of record: 8,472.

Office—125 High St., Boston, Mass 02110. Tel—(617) 426-6158. Pres—W. F. Murdoch Jr. VP-Secy-Treas & Investor Contact—B. M. Hall.
 Transfer Agent & Registrar—First National Bank of Boston. Organized in Massachusetts in 1969.

Figure 6.2: Standard and Poor Report on the Hubbard Real Estate Investment Trust (February 3, 1984)

differences may be observed between this value, calculated at an aggregate level, and local or category specific variances (This study has not been done here).

6.2 Design and Use Option: Case of One Use

We will discuss here the first two options of the proposed construction process model, when only the use as office building is considered. The results will also be useful as a reference for section 3, since this use is dominant in our example. Figure 6-3 shows the numerical results for the design and use option.

The procedure for the evaluation is to work backwards, starting from the value of the building in the operation phase. The value of the use option is computed with the indicated parameters. This figure (here $74.267 p.s.f.) is then plugged into the design option as value of the project. Hence the value of the piece of land for the developer is determined ($27.765 per square foot of gross area of the building).

Several approximations have been made in these calculations, and they must be considered in analysing the results. First, reservations can be expressed concerning modelling these two successive options as independent simple call options. The design option is an option on an option, and should be treated differently from a theoretical point of view. The option theory however does not provide specific numerical models for these types of options, although theoretical research has been devoted to their study. We have also considered that both options have the same standard deviation $\sigma=15\%$.

The use option has an overwhelming probability to be exercised, since it offers the opportunity to pay only $20 and get $90. This

Parameters

Value of the use (office space)	\$90 p.s.f.
Construction cost: shell construction	\$60 p.s.f.
finishes	\$20 p.s.f.

Variance $\sigma = .15$

Risk-free rate (real terms) $r = .08$

Time to maturity: piece of land hold for a maximum of 3 years

design and construction phases 3 years

Results

	design option →	use option
	S = \$74.267	V = \$90
Value of the land for this project	E = \$60	F = \$20
\$27.765 ←	σ = .15	σ = .15
	r = .08	r = .08
	τ = 3 years	τ = 3 years

Figure 6.3: Numerical Application of the Design and Use Option for a Single Use as Office Building

option is "way in the money," and its value is very close to the value of the project ($90) less its exercise price ($20). The value is actually $74.267 and not $70 since the exercise price is discounted at the risk free rate, 8% for three years. The model accomodates this particular situation by considering that all the terms due to the normal distribution function in the call valuation formula are almost equal to 1, so that the use option value is actually:

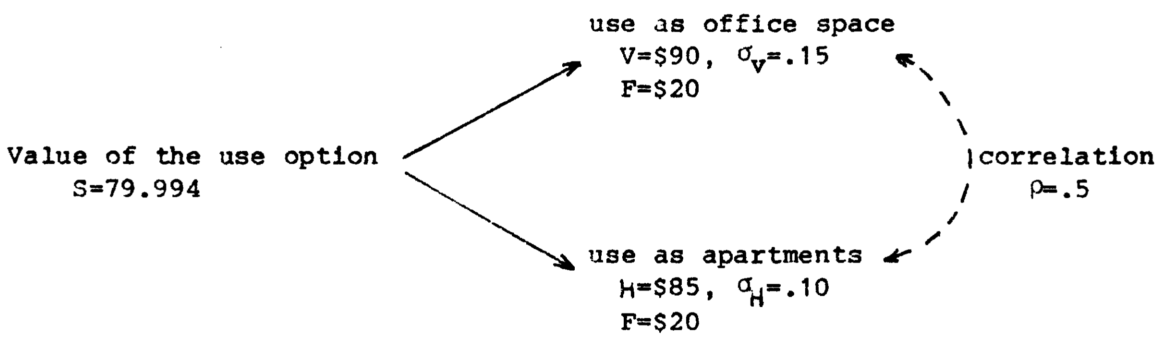
$$S = 90 - 20 \exp(-r\tau) = 74.267$$

In the design option, the time to maturity has been chosen as 3 years. This departure from the theoretical infinite time to maturity is intentional. Taking $\tau=+\infty$ would provide $74.267 as the value of the land, since an infinite call option has the same value as the project, whatever the exercise price. This unreallistic result is difficult to admit at this stage. At least, holding costs (e.g. taxes and interest) would reduce this value. We may also consider that the relationship between the project's value and the construction cost is not adequately described by the model of simple call option in a long run perspective (i.e. while E remains constant in real terms, S is expected to increase at an implicit project's rate superior to the risk free rate, 8%).

6.3 The Use Option: Case of Two Uses

In the case when using the building for offices or apartments is still possible until the end of the construction phase, the value of the use option can be found by using the theoretical model of option on the maximum of two assets, whose computer program was written for the purpose of these tests.

Figure 6.4 shows sample calculations for a "base case" where the project's values are, respectively 90 and 85 $p.s.f., and the standard deviations 15% and 10% for the office space or apartment uses respectively, and when the correlation between these uses is assumed to be .5. In this example, we do not mean that the developer will be uncertain about the final use until the end of the construction phase. This project would be probably viewed as an office building project since this use appears as more profitable. However, switching to the apartment use is still possible at no extra cost until the end of the construction phase, if this use appears to be more profitable at that time.



other parameters: τ = 3 years, = .08

call option on V: CV = 74.267 on H: CH = 69.267

additional value = 79.994 - 74.267 = $5.727 p.s.f.
(or 7.7% of the single use value)

Figure 6.4: The Use Option Valued by the "Maximum of Two Assets Model" when the Hypothetical Building can be used for Offices or Apartments

The value of this option is $79.994, and can be compared to the $74,267 value obtained for the use for office space only. The difference, $5.727 (or 7.7% of the initial value) can be interpreted as the value of the ability to pick up the most profitably use between the office space and apartment. This is an important result which will be characteristic of real estate development compared to stock options which have only "one use".

Figure 6.5 shows the sensitivity of this result to the parameters σ_H (variance of the use for apartments) and ρ (correlation between V and H).

σ_H \ ρ	-1	-.5	0	.5	.7	.9	1
.05	83.961	82.803	81.503	79.994	79.305	78.545	78.131
.10	86.894	85.015	82.803	79.994	78.545	76.684	75.424
.15	89.810	87.468	84.673	81.028	79.061	76.301	74.267
.20	92.699	90.050	86.894	82.803	80.629	77.688	75.424
.25	95.554	92.699	89.318	85.015	82.803	79.994	78.130

other parameters: V = 90 H = 85 F = 20

σ_V = .15 r = .08

call option on V only: CV = $74.267

Figure 6.5: Sensitivity Analysis with Respect to σ_H and ρ

Not surprisingly, the value of the option is minimal when ρ is equal to 1 (totally perfectly positively correlated projects), and increases significantly until ρ is equal to -1 (perfectly negatively correlated projects: V increases when H decreases and visa versa). The option value is also generally an increasing function of σ_H, except for highly correlated projects. When $\sigma_V = \sigma_H = .15$ and $\rho = 1$, we find the lowest value, which is exactly equal to the value of the option for a single use as offices, or $74.267. In this situation, the values V and H have exactly the same distribution. Given the initial estimates, V will always exceed H, so that this last alternative is not meaningful under this example assumptions. As a consequence, the option value for $\rho = 1$ (last column) drops from $\sigma_H = .05$ until $\sigma_H = .15$, and then increases again. When σ_H is low, asset H is valuable in case the value V drops in the future, and allows to "save" the project. When σ_H is high, H may exceed V if both asset values increase. This effect is also sensitive at a lower degree for other values of ρ, until $\rho = .5$ in the table.

In this analysis, we have limited the range of the values of σ_H between 5% and 25%. We assume therefore that the project's value H is less variable than most stocks, for which σ can easily be 40% or more. This is also true for the variance of V, for which 15% seemed to be a reasonable value although we have not done a specific study to support this assumption.

The effect of variations of σ_V has not been studied in details as it probably gives the same results as the study of σ_H. It appeared however interesting to study the effect of substituting σ_V for σ_H in the base case. The interpretation is the following. When $\sigma_V = .15$ and $\sigma_H = .10$, the developer is undertaking a risky office project (asset V) supported by a safer residential alternative (H). Thus, if the value of the office project goes down (lower rents and higher vacancies rates are expected at the moment of the choice), switching to the safer alternative allows to save the project. On the other hand, if the variances are reversed, that is if $\sigma_V = .10$ and $\sigma_H = .15$, the project will appear as a safe office project allowing to capture a riskier, but possibly more profitable residential opportunity. These two situations are illustrated by Figure 6.6.

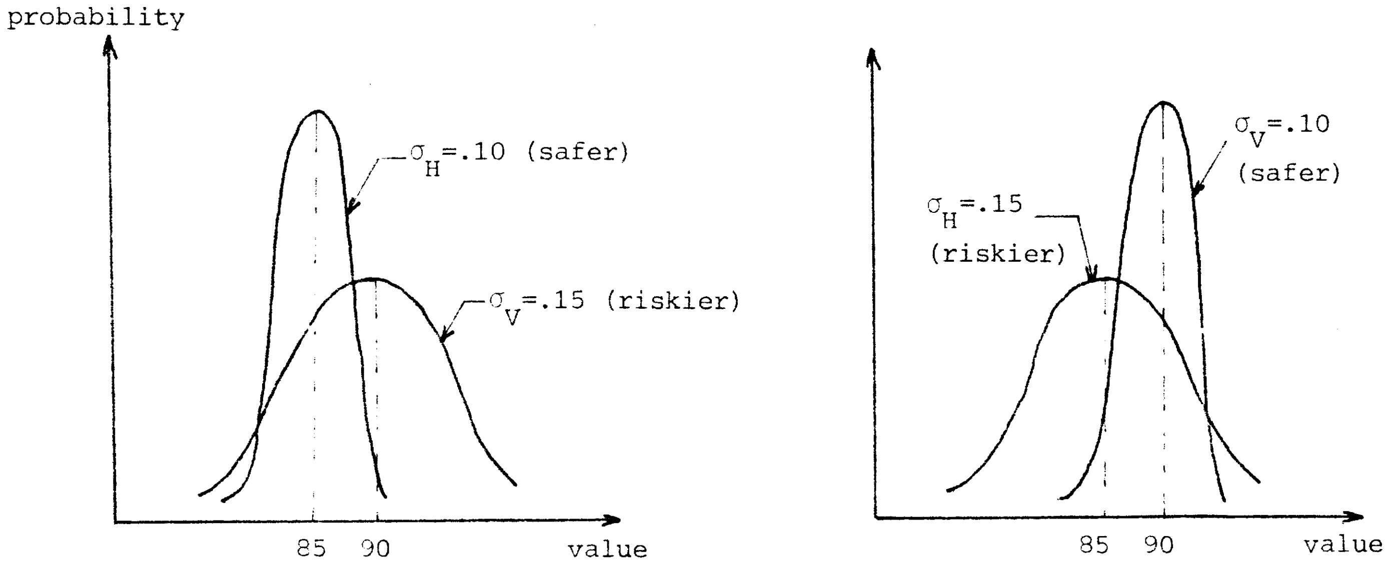


Figure 6.6: Two Configurations for a Safe and a Risky Project

The calculations for these two configurations gave surprisingly the same result (S = \$79.994). This can probably be generalized to other values of σ_V, σ_H, and ρ. This identity may be attributable to the fact that these options are "way in the money," so that the whole distributions are evaluated (their symmetry may intervene). To determine conclusively the reason for this result, more research is needed on the problem, but is beyond the scope of this current thesis. This question will be addressed again in section 4 for a higher exercise price.

Finally, Figure 6.7 shows the results of a sensitivity analysis with respect to the parameters H (value of apartment alternative) and τ (design and construction phases durations).

τ \ H	80	85	90	95	10	call option on V only (1)	comparison with H=85(2)
2	75.502	77.279	79.664	82.635	86.126	72.957	5.92%
3	78.017	79.994	82.476	85.437	88.828	74.267	7.71%
4	80.288	82.398	84.949	87.913	91.252	75.477	9.17%
5	82.363	84.572	87.178	90.150	93.459	76.594	10.42%

other parameters: V = 90 $\sigma_V = .15$ F = 20

$\sigma_H = .10$ $\rho = .5$ r = .08

(1): this column shows the value of the call option on asset V only

(2): this column shows the percentage additional value attributable to the choice between two uses for H = 85, V = 90, and different values of τ (expressed in % of the call option on V)

Figure 6.7: Sensitivity Analysis with Respect to H and τ

A longer time to maturity clearly increases the value of the option. As this effect is partly attributable to the increase in the value of the dominant project V, we have indicated in column (1) the value of this simple call option. A calculation of the percentage increase attributable to the dual alternative when V = 90 and H = 85 shows however also an increase from 5.92% (2 years) to 10.42% (5 years). The choice between the two assets also becomes more valuable as time increases.

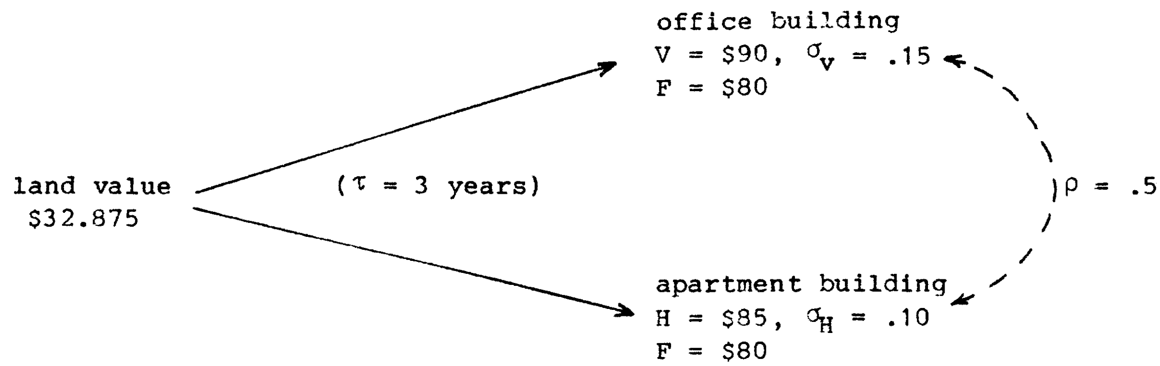
The effect of changes in H can be best interpreted for H between $80 and 90, when the dominant project is still the office alternative. The option value always increases when H increases, showing that the dual alternative becomes best valuable when H and V are equal.

6.4 The Design Option When Two Development Schemes are Possible

Besides the valuation of the use option when a common design allows two uses, the model of valuation of an option on the maximum of two assets can also be applied to the design option, when two totally different development schemes are possible. This case is different from the study in section 3 since the construction cost (F) is here in the same order of magnitude as the value of the two development alternatives. It may even be higher than these values if no development is profitable in the immediate future. As a consequence, the exercise decision is not as immediate as in the preceding case. In other words, a third alternative is possible, the alternative not to exercise the option, whose value is generally known as the "right to default."

In this study, we will use most of the data discussed in section 1. The two development alternatives are an office building (value V = \$90 p.s.f., variance σ_V = .15) and a residential building (H = \$85 p.s.f., σ_H = .10). Unlike the preceding case, these buildings are assumed to be totally different. Their cost is assumed however to be the same, or \$80 p.s.f. in the base case. As we focus here on the design option, the next steps of the construction process have been disregarded. \$85 and \$90 can be considered as the values provided by the use options for both developments, or simply the values of the income produced by these projects.

The results in the base case are presented in Figure 6.8. The value of the land is found to be \$32.875 p.s.f., or 18% more than if the dominant development scheme is considered alone.



call option on V = $27.819

additional value for two development schemes: $5.056
(or 18.2% of the initial value)

Figure 6.8: <u>The Design Option When Two Development Schemes are Possible</u>

σ_H \ F	70	80	90	100	110
.05	40.663	32.797	24.960	17.532	11.590
.10	40.670	32.875	25.363	18.557	12.892
.15	41.745	34.082	26.825	20.291	14.750
.20	43.565	35.994	28.883	22.499	17.044
.25	45.808	38.296	31.276	24.992	19.607

Figure 6.9: <u>Sensitivity Analysis with Respect to F and σ_H</u>

The sensitivity analysis with respect to most parameters does not differ significantly from the results presented in section 3. The same tables as in Figures 6.5 and 6.7 could be drawn concerning the effect of changes in ρ, σ_H, τ, and H.

The most interesting results are found when calculations are done with different exercise prices. As stated in Appendix 1, the construction cost, F, is equal for both projects since the study of these types of options by R. Stulz (reference [9]) does not provide a valuation formula for two different exercise prices. In real cases, this will generally not be true. Figure 6.9 shows option values obtained when F varies from $70 to $110 p.s.f. A second dimension is provided by variations of σ_H from .05 to .25.

We can notice in this table that the option has value even if the construction cost, F, is much higher than both project's values. In that case, the variances of the two projects have an increasing effect on the land's value. When F = $110, the option value almost doubles when σ_H varies from .05 to .25. When $\sigma_H = .25$, project H provides even more value to the option than project V, since the call option on H alone is worth $13.95 while the option on V is only worth $10.96.

Finally, exchanging the variances in the base case (that is, assuming $\sigma_V = .10$ and $\sigma_H = .15$) provides $32.846 as option value, instead of $32.875. A dominant risky project supported by a safer alternative is slightly more valuable than the contrary. This result seems to be confirmed by calculations for other exercise prices and correlations. The difference is not large, although other tradeoffs between V and H and σ_V and σ_H have not been studied.

6.5 An Example of Renovation Option

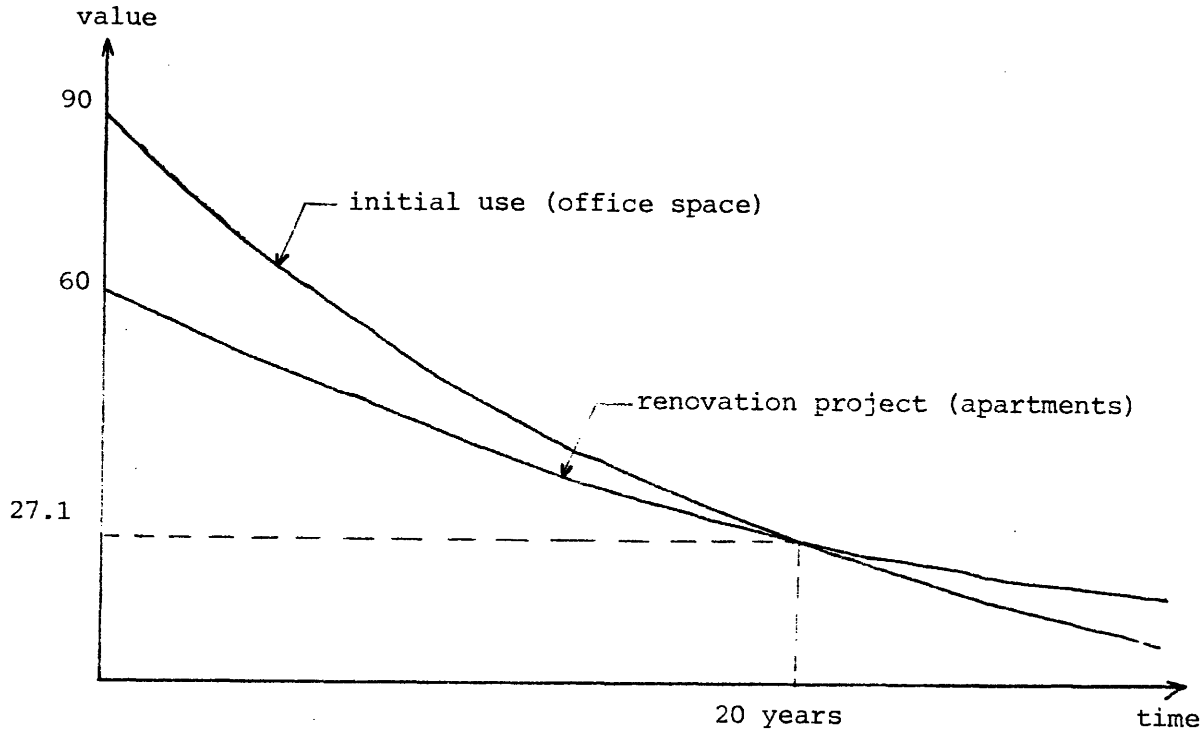
Although no emphasis will be put on analysing the value of a renovation operation, a numerical example is presented here in order to show how the model of infinite put option can be used in this situation (This model is described in section 2.3.3 and Appendix 2).

We assume here that the developer has finally chosen the office space use in our basic example, for an expected value of $90 p.s.f. This value is expected to decrease with time as rents will be perceived, with a payoff ratio $\gamma_V = .06$. Therefore, the value of this use is $V = 90e^{-.06t}$.

The developer still has the possibility to renovate his building later, for a residential use. At the beginning of the operation phase, the conversion cost can be evaluated at $25 per square foot (finishes $20 + fees), so that the immediate value of this use is $60 p.s.f. This value is expected to decrease with a .04 payoff ratio. Therefore, $H = 60e^{-.04t}$.

The curves of V and H over time are represented in Figure 6.10. They intersect when H = V, or $60e^{-.04t} = 90e^{-.06t}$, that is $t \simeq 20$ years. At that moment, the non renovated office building and the renovation project have the same value, or $27.1 p.s.f. This is the expected optimal date for the renovation operation.

Using the variances $\sigma_V = .15$ and $\sigma_H = .10$, and the correlation $\rho = .5$ as in the other examples, the value of this option can be derived, using the expressions presented in Appendix 2.



office use: $V = 90e^{-\gamma_V t}$, $\gamma_V = 6\%$, $\sigma_V = 15\%$

apartment use: $H = 60e^{-\gamma_H t}$, $\gamma_H = 4\%$, $\sigma_H = 10\%$

correlation between H and V: $\rho = .5$

value of the renovation option = $9.43 p.s.f.
(or 10.5% of the office project)

Figure 6.10: Model and Value of a Renovation Option from Office Space to Residential Use

$$X = \frac{V}{H} = 1.5$$

$$\sigma_x = \sqrt{\sigma_V^2 - 2\rho\sigma_V\sigma_H + \sigma_H^2} = .1323$$

$$\alpha = \frac{(\gamma_H - \gamma_V - \frac{1}{2}\sigma_x^2) + \sqrt{(\gamma_H - \gamma_V - \sigma_x^2/2)^2 + 2\gamma_H\sigma_x^2}}{\sigma_x^2} = 1.0535$$

$$\text{value} = \frac{H}{1+\alpha}\left[\frac{1+\alpha}{\alpha}\frac{V}{H}\right]^{-\alpha} = \$9.43 \text{ p.s.f.}$$

Thus, $9.43 (10.5% of V) is the value of the renovation option.

At this point, it is now possible to calculate the value of a project in which a building can accomodate offices or apartments, through a whole design-use-renovation valuation process. This design offers the following opportunities:

- a choice between offices or apartments at the end of the construction phase (see section 3)
- if the office use is selected, the later renovation of the building for a residential use if the market is favorable
- if the residential use is selected, the later renovation of the building for use as offices.

This process is described in Figure 6.11. These three options add value to this project in comparison with a project with a single use. The value of these opportunities may make this development scheme attractive enough to justify a more elaborated design at a higher cost, i.e., if the value of the option exceeds the additional cost.

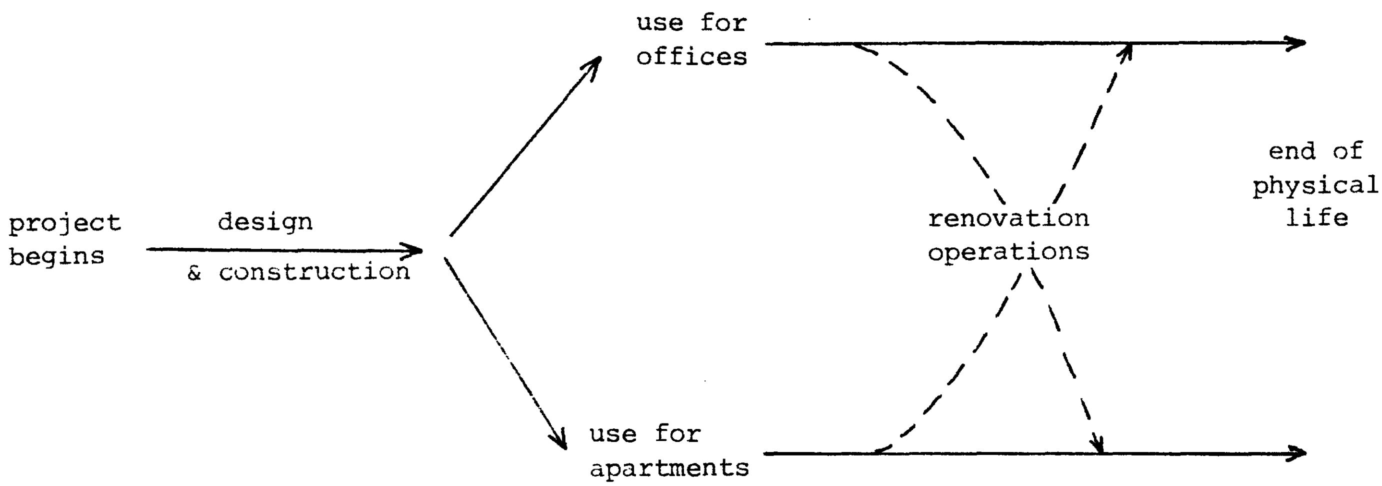


Figure 6.11: Development Opportunities of an Office/Apartment Building as Described by a Design-use-renovation Model for the Construction Process

CONCLUSION

When this study began, in early 1984, its subject was easy to express: what can be done with the option valuation model in the area of the construction industry? No former research or similar approach had investigated this field of applications. This subject was therefore new and open-ended.

The points discussed in this thesis provide a partial answer to this question. It is possible to define a construction project by means of a series of options. The decision on the design, use, or renovation of a facility take a new significance when viewed as options. Other more specific applications can be found for certain types of projects. The option model may be applied to a variety of purposes such as buildings, plants, or infrastructure projects.

The practical use of the option model however raises more questions than it provides answers. For example, a piece of land is not viewed realistically by an infinite call option model. One may also question whether a developer taking advantage of the best opportunities offered to him would necessarily gain the 5 to 20% additional value promised when options are identified. On the other hand, the numerical results are sensitive to parameters such as variances and correlation in accordance with intuitive judgement.

All these points would be interesting to investigate in more detail. This study is surely far from exhausting all the resources of this model in the construction environment. Finite judgements on its applications are not appropriate. More investigations are needed before any realistic application can be done, and it is hoped that this study will be useful in this effort.

APPENDIX 1: OPTION ON THE MAXIMUM OF TWO RISKY ASSETS

The option on the maximum of two risky assets was studied by René Stulz. In his article in the Journal of Financial Economics (reference 9), a solution is derived as a function of the following parameters:

V = value of the first asset

H = value of the second asset

F = exercise price (identical for both assets)

τ = time to maturity

r = risk-free interest rate

σ_V = variance of the rate of return of V

σ_H = variance of the rate of return of H

ρ = correlation coefficient between V and H

The value of the option, noted M, is found to be:

$$M = C(v,F,\tau,r,\sigma_V) + C(H,F,\tau,r,\sigma_H) - m$$

$$m = H\, N_2\left(\gamma_1 + \sigma_H\sqrt{\tau},\ \left(\ln(V/H) - \tfrac{1}{2}\sigma^2\tau\right)/\sigma\sqrt{\tau},\ (\rho\sigma_V - \sigma_H)/\sigma\right)$$

$$+ V\, N_2\left(\gamma_2 + \sigma_V\sqrt{\tau},\ \left(\ln(H/V) - \tfrac{1}{2}\sigma^2\tau\right)/\sigma\sqrt{\tau},\ (\rho\sigma_H - \sigma_V)/\sigma\right)$$

$$- F\, e^{-rt}\, N_2(\gamma_1,\gamma_2,\rho)$$

in which:

$C(S,E,\tau,r,\sigma)$ is the value of a simple call option (see section 1.2)

m is the value of the option on the minimum of assets V and H

$N_2(\alpha,\beta,\Theta)$ is the bivariate cumulative standard normal distribution with upper limits of integration α and β, and coefficient of correlation Θ

$$\gamma_1 = \frac{\ln(H/F) + \left(r - \frac{1}{2}\sigma_H^2\right)\tau}{\sigma_H\sqrt{\tau}}$$

$$\gamma_2 = \frac{\ln(V/F) + \left(r - \frac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}}$$

$$\sigma^2 = \sigma_V^2 + \sigma_H^2 - 2\rho\sigma_V\sigma_H$$

A careful reader of R. Stulz may notice a difference with the expression presented in his article. Two typing errors were found in the second parameters of the first two bivariate function expressions ($\sigma^2\tau$ instead of $\sigma^2\sqrt{\tau}$, for reasons of homogeneity)

When F=0, the expression is simpler:

$$M = H + VN(d_1) - HN(d_2)$$

where $$d_1 = \frac{\ln(V/H) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

$$\sigma^2 = \sigma_V^2 - 2\rho\sigma_V\sigma_H + \sigma_H^2$$

$N(\cdot)$ = normal distribution function

Both expressions have been used for the computer program in FORTRAN presented at the end of this appendix. The calculation of option values are possible for both F>0 and F=0, although this last case was not studied in the numerical applications. The program can also provide as outputs the value of an option on the minimum of two assets and, when F=0, the value of an option to exchange asset H for asset V.

This program requires the use of an external function for calculations of bivariate normal probabilities. This function may be included in scientific subroutine packages accessible on the computer. As this was not the case for this study, a program was written using Roy C. Milton article in Technometrics (reference [25]).

Finally, two properties of the option on the maximum of two assets are demonstrated in René Stulz article:

- $M \geq C(V,F,\tau,r,\sigma_V)$ as well as $M \geq C(H,F,\tau,r,\sigma_H)$. The option value is always superior or equal to the values of the simple call options on each of the assets.
- M is a decreasing function of the correlation coefficient ρ. M is maximal when $\rho=-1$ and minimal when $\rho=1$.

The value of this option has not been derived when the exercise price for asset V is different from the exercise price for H. There is probably no theoretical difficulty in addressing this problem. The expression of the option value may however be even larger than the formulas presented here.

```
c       Emmanuel BAR-OR                May 18, 1984
c
c       Option on the maximum of two assets
c       (R. Stulz, Journal of Financial Economics No 10, 1982)
c
c
c       Inputs :      v        value of first asset
c                     h        value of second asset
c                     f        exercise price
c                     t        time to maturity (in years)
c                     r        risk-free interest rate
c                     sigv     variance of the rate of return on v
c                     sigh     variance of the rate of return on h
c                     rho      correllation coefficient between v and h
c             (r, sigv, sigh, and rho in units, e.g. r=.10 for 10%)
c
c       Output :      vmax     value of option on the maximum of v and h
c
c       Note : calculations for f > 0 and f = 0
c              binorm(a,b,rho) external function (bivariate normal distr
c
c=====================================================================
c
        subroutine maxopt(v,h,f,t,r,sigv,sigh,rho,vmax)
c
cx        write(1,150)v,h
cx        write(1,151)f,t,r
cx        write(1,152)sigv,sigh,rho
c
        if(t.eq.0.) go to 120
        if(sigv.le.0.)return
        if(sigh.le.0.)return
        if(sigv.eq.sigh.and.rho.eq.1) go to 125
        if(f.eq.0.) go to 130
c
        tsr=t**.5
        g1=(alog(h/f)+(r-.5*sigh*sigh) t)/(sigh*tsr)
        g2=(alog(v/f)+(r-.5*sigv*sigv) t)/(sigv*tsr)
        sig2=sigv*sigv+sigh*sigh-2.*rho sigv*sigh
        sig=sig2**.5
c
        a1=g1+sigh*tsr
        a2=(alog(v/h)-.5*sig2*t)/(sig*tsr)
        a3=(rho*sigv-sigh)/sig
c
        b1=g2+sigv*tsr
        b2=(alog(h/v)-.5*sig2*t)/(sig*tsr)
        b3=(rho sigh-sigv)/sig
c
        pa=binorm(a1,a2,a3)
        pb=binorm(b1,b2,b3)
        pg=binorm(g1,g2,rho)
        call copt(v,f,t,r,sigv,cv)
        call copt(h,f,t,r,sigh,ch)
c
        vmin=h*pa+v*pb-f*exp(-r*t)*pg
        vmax=cv+ch-vmin
```

```
110    continue
cx        write(1,160)cv
cx        write(1,161)ch
cx        write(1,146)vmin
cx        write(1,153)vmax
cx        write(1,165)
       return

120    v1=v-f
       h1=h-f
       cv=amax1(0.,v1)
       ch=amax1(0.,h1)
       vmin=amin1(cv,ch)
       vmax=amax1(cv,ch)
       go to 110
c
125    call copt(v,f,t,r,sigv,cv)
       call copt(h,f,t,r,sigh,ch)
       vmin=amin1(cv,ch)
       vmax=amax1(cv,ch)
       go to 110
c
130    tsr=t**.5
       sig2=sigv*sigv+sigh*sigh-2.*rho*sigv*sigh
       sig=sig2**.5
       d1=(alog(v/h)+.5*sig2*t)/(sig*tsr)
       d2=d1-sig*tsr
       call ndtr(d1,p1,den1)
       call ndtr(d2,p2,den2)
       cv=v
       ch=h
c
       vexch=v*p1-h*p2
       vmin=v-vexch
       vmax=h+vexch
cx        write(1,170)
cx        write(1,171)vexch
       go to 110
c
146    format(5x,'option on the minimum      vmin=',f10.5)
150    format(1x,'v=',f10.5,5x,'h=',f10.5)
151    format(1x,'f=',f10.5,5x,'t=',f10.5,5x,'r=',f10.5)
152    format(1x,'sigv=',f10.5,5x,'sigh=',f10.5,5x,'rho=',f10.5)
153    format(5x,'option on the maximum      vmax=',f10.5)
160    format(5x,'call option on v           cv=',f10.5)
161    format(5x,'call option on h           ch=',f10.5)
165    format('===========================================================
170    format(1x,'simplified calculations for a zero exercise price')
171    format(5x,'option to exchange v for h         vexch=',f10.5)
       end
c
```

```
c
c       Simple call valuation
c
        subroutine copt(s,e,t,r,sig,v)
        tsr=t**.5
        d1=(alog(s/e)+(r+sig*sig*.5)*t)/(sig*tsr)
        d2=d1-sig*tsr
        call ndtr(d1,p1,den1)
        call ndtr(d2,p2,den2)
        v=s*p1-e*exp(-r*t)*p2
        if(v.lt.0.00) v=0.00
        return
        end
c
c
c       Cumulative normal distribution function
c
        subroutine ndtr(x,p,d)
        ax=abs(x)
        if(ax.gt.7.)go to 9
        t=1./(1.+.2316419*ax)
        d=.3989423*exp(-x*x/2.)
        p1=(1.330274*t-1.821256)*t+1.781478
        p=1.-d*t*((p1*t-.3565638)*t+.3193815)
        goto 10
9       d=0.
        p=.9999999
10      if(x.lt.0.) p=1-p
        if(p.le..0000001)p=.0000001
        return
        end
```

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APPENDIX 2: PERPETUAL AMERICAN PUT OPTION

This appendix derives the value of a perpetual put option on a dividend paying project for a deterministic or a stochastic salvage value. For a more detailed discussion of this problem and more sophisticated resolution methods, see Stewart Myers and Saman Majd paper (reference [3]). The solution presented here was suggested by Saman Majd as a satisfactory approximation to his calculations for a finite time to maturity (error inferior to 10%). It can be traced back to Robert Merton article in the Bell Journal of Economics (reference [5]).

Case of a Deterministic Salvage Value

The problem can be defined with the following variables:

P: project's value at t=0 (the notation P(t) is used for t>0)

S: salvage value

σ: standard deviation of the rate of return of P

γ: payout ratio of the project

r: risk-free interest rate

The project's value is assumed to follow stochastic movements. Its value is forecasted to decrease over time as cash flows are received. The constant payout ratio hypothesis means that at time t, the project will produce a cash flow equal to γ times the project's value: $CF(t) = \gamma P(t)$. As a consequence, project's value and cash flows are expected to decline by γ percent per year (the expected values are exponential function, e.g. $P(t)=P\exp(-\gamma t)$). In this section, the salvage value is supposed to be constant with a value S for any t.

The stochastic process describing the project value is

$$\frac{dP}{P} = \left(\alpha_p - \gamma\right) dt + \sigma\, dz$$

where α_P is the expected rate of change of P.

Let F(P) be the value of the put option. Since the time to maturity is infinite, this function does not depend on the time variable. If $\overline{P}$ is the chosen level to exercise the put option, the differential equation satisfied by F(P) is:

$$\frac{1}{2}\sigma^2 P^2 \frac{d^2F}{dP^2} + (r-\gamma)P\frac{dF}{dP} - rF = 0$$

subject to $F(+\infty) = 0$ (1)

$F(\overline{P}) = K - \overline{P}$ (2)

This homogeneous differential equation admits as solutions the functions $F(P) = h_1(\overline{P})P^{-\alpha_1} + h_2(\overline{P})P^{-\alpha_2}$, where $(-\alpha_1)$ and $(-\alpha_2)$ are solutions of the associated second degree equation. (1) implies that α_1 or α_2 have to be positive. The solution can therefore be rewritten as

$$F(P) = h(\overline{P})P^{-\alpha}$$

with $$\alpha = \frac{(r-\gamma-\frac{1}{2}\sigma^2) + \sqrt{(r-\gamma-\frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}$$

The second boundary condition allows to calculate $h(\overline{P})$ since $h(P)\overline{P}^{-\alpha} = S-\overline{P}$. Replacing $h(\overline{P})$ by its value, we find the expression F(P): $F(P) = (S\overline{P}^{\alpha} - \overline{P}^{1+\alpha})\, P^{-\alpha}$

The best exercise level, $\overline{P}$, can now be computed to optimize the option value.

$$\frac{\partial F}{\partial \overline{P}} = 0 \quad \text{leads to } \overline{P} = \frac{\alpha S}{1+\alpha}$$

Replacing in the expression of F(P), we finally obtain

$$F(P) = \frac{S}{1+\alpha}\left[\frac{1+\alpha}{\alpha}\,\frac{P}{S}\right]^{-\alpha} P^{-\alpha}$$

α defined as above

Case of a Stochastic Exercise Price

In the case where S follows the same type of movements as P (variance σ_S^2, payout ratio γ_S), and when S and P are correlated with the coefficient P, a simple transformation allows to derive the value of the put option. A new project X, and a constant salvage value K are defined by the following relations:

$X = P/S$ (new project's value)

$\sigma_x^2 = \sigma_P^2 - 2\rho\sigma_P\sigma_S + \sigma_S^2$

$\gamma_x = \gamma_P$

$K = 1$ (new salvage value)

$r = \gamma_S$ (the riskless rate is substituted for γ_S)

The value of the put option on project X can be computed by using the expression derived in section 1. This is the value of the put option on P expressed in percentage of the salvage value S. The final expression is therefore:

$$F(P) = \frac{S}{1+\alpha}\left[\frac{1+\alpha}{\alpha}\,\frac{P}{S}\right]^{-\alpha}$$

$$\alpha = \frac{\left(\gamma_S - \gamma_P - \frac{1}{2}\sigma_x^2\right) + \sqrt{\left(\gamma_S - \gamma_p - \frac{1}{2}\sigma_x^2\right)^2 + 2\gamma_S\sigma_x^2}}{\sigma_x^2}$$

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