

IDENTITY AND QUANTIFICATION

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For my Parents

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ABSTRACT

Chapter 1 is a study of Quine's well-known thesis that it is illicit to quantify into opaque constructions. In Section 1.1, I attempt to distinguish between the notions of a purely referential occurrence of a singular term and the referential transparency of the position of the occurrence of a singular term, and characterize Quine's thesis in the light of this distinction. It appears that if Quine's thesis is true, then sentences such as

($\exists x$) It is necessary that x is odd,
 ($\exists x$) Ralph believes that x is a spy,
 and

($\exists x$) Sometime in the past x was a Catholic,
 involve illicit quantification into referentially opaque constructions. In Section 1.2, I examine proposals for formulating, without violating Quine's thesis, the thoughts which are apparently formulated in these sentences. I argue that though some of these proposals are successful, they have consequences which strongly suggest that Quine's thesis is false. In Section 1.3, I examine two types of argument in defence of Quine's thesis, and attempt to show that these arguments are unsuccessful in establishing Quine's thesis.

Chapter 2 is a study of the notion of parthood. I examine three characterizations of this notion which are found in the intended interpretations of the Leonard-Goodman Calculus of Individuals, a tensed analogue of the Leonard-Goodman Calculus of Individuals, and Thomson's Cross-temporal Calculus of Individuals respectively; and I discuss problems arising from the loss or acquisition of parts which these three characterizations face.

Thesis Supervisor: George Boolos
 Professor of Philosophy

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QUANTIFICATION AND OPACITY

Over the past forty years, Quine has repeatedly claimed that it is illicit to quantify into opaque constructions. In a paper of 1943, Quine wrote: 'No pronoun (or variable of quantification) within an opaque context can refer back to an antecedent (or quantifier) prior to that context.'¹ This is one of the central claims of many of Quine's papers, including 'The Problem of Interpreting Modal Logic', 'Reference and Modality', and 'Three Grades of Modal Involvement'; and it occupies a large part of the discussion in chapters IV, V, and VI of *Word and Object*.

Quine's claim, that it is illicit to *quantify into* opaque constructions, should not be confused with his misgivings about essentialism, i.e. the thesis that among the traits of an object some are essential to it, and others not, which he thinks quantified modal logic is committed to; nor should Quine's claim be confused with his more recent doubts about certain epistemological doctrines that he thinks that quantified logic of belief is committed to.

Quine's claim is a claim about quantification in general. It seems to be his view that there is a purely technical

¹ 'Notes On Existence & Necessity', *Journal of Philosophy*, 1943

difficulty — a difficulty that can be established on purely logical and semantic considerations — that quantification into opaque constructions faces. Thus in 'Quantifiers and Propositional Attitudes', after Quine has distinguished the relational senses of propositional attitudes from their corresponding notional senses, he notes:

'However, the suggested formulations of the relational senses — viz.,

$(\exists x)(x \text{ is a lion. Ernest strives that Ernest finds } x)$

$(\exists x)(x \text{ is a sloop. I wish that I have } x)$

$(\exists x)(\text{Ralph believes that } x \text{ is a spy})$

$(\exists x)(\text{Witold wishes that } x \text{ is a president})$

all involve quantifying into a propositional-attitude idiom from outside. This is a dubious business.'²

The rest of that paper is an attempt to offer reconstructions of the relational senses of propositional attitudes which do not

² 'Quantifiers and Propositional Attitudes', reprinted in *Reference & Modality*, ed. Leonard Linsky (Oxford, 1971), pp. 102-3

involve *quantifying into* opaque constructions. And in his more recent paper 'Intensions Revisited', having noted that quantified modal logic also involves the allegedly illicit *quantifying into* opaque constructions, he offers a reconstrual of quantified modal discourse which is free from this alleged defect.

In section (1.1) of this paper I present a general characterization of referential opacity and a statement of Quine's thesis. In section (1.2) I examine some attempts to render *quantification into* modal, epistemic, and temporal constructions compatible with Quine's thesis. In section (1.3), I examine some arguments for Quine's thesis. I argue that these arguments are inconclusive and that though some of the attempts at reconstrual examined in section (1.2) are successful, these attempts also provide reasons for rejecting Quine's thesis.

1.1 RÉFERENTIAL OPACITY

Quine's characterization of referential opacity employs the notion of a purely referential position which, in turn, is explained by appeal to a principle that Quine has called 'the principle of substitutivity'.³ Quine formulates this principle in these words: '..., given a true statement of identity, one of its two terms may be substituted for the other in any true statement and the result will be true.'⁴ Making allowances for Quine's use of the word 'statement', this principle may be understood as the claim that

- (A) for all expressions α and β , if $\ulcorner \alpha = \beta \urcorner$ expresses a true proposition, then for any sentences S and S' , if S contains an occurrence of α and S' is the result of substituting β for some occurrence of α in S , then S expresses a true proposition only if S' expresses a true proposition.⁵

It should be recognized, as Quine has frequently stressed, that

³ See *Word and Object*, (The M.I.T. Press, 1970), pp. 142,144

⁴ *From a Logical Point of View*, second edition, (Harper and Row, 1963), p. 139

⁵ This formulation is derived from one of Richard Cartwright's in 'Identity and Substitutivity' in *Identity and Individuation*, ed. Milton K. Munitz, (New York, 1971)

(A) is false. For example, the propositions expressed by

(1) Giorgione = Barbarelli,

and

(2) 'Giorgione' contains nine letters,

are true, whereas

(3) 'Barbarelli' contains nine letters,

which is the result of substituting 'Barbarelli' for an occurrence of 'Giorgione' in (2), expresses a false proposition.

Similarly,

(4) Giorgione was so-called because of his size,

expresses a true proposition, but

(5) Barbarelli was so-called because of his size,

does not, even though (5) is the result of substituting 'Barbarelli' for an occurrence of the co-referential 'Giorgione' in (4). Counterexamples to (A) are not confined to those cases which involve substitution within contexts of quotation. For instance, though

(6) 9 = the number of planets,

and

(7) It is necessary that 9 is odd,

both express true propositions,

(8) It is necessary that the number of planets is odd,

does not; and likewise, though

(9) Hesperus = Phosphorus,

and

(10) It is an astronomical discovery that Hesperus =
Phosphorus,

both express true propositions,

(11) It is an astronomical discovery that Hesperus =
Hesperus

does not express a true proposition.

The temptation to think that (A) is true might arise from a failure to distinguish (A) from another principle, i.e.

(B) $(\forall z)(\forall x)(\forall y)[x=y \rightarrow (z \text{ is a property of } x \rightarrow z \text{ is a property of } y)]$.

But it is important to recognize that whereas the proposition

that sentences (1) and (4) express true propositions and (5) fails to express a true proposition falsifies (A), it does not falsify (B); and therefore, (B) does not entail (A).⁶

Although Quine has frequently argued against (A), the tendency to think that (A) is, after all, true may be encouraged by some of his remarks. For instance, in 'Reference and Modality', Quine writes: 'The principle of substitutivity should not be extended to contexts in which the name to be supplanted occurs without referring simply to the object'.⁷ Now (A), the principle of substitutivity, makes a claim about *all* expressions and *all* sentences, and it simply makes no sense to say that the principle should not be extended to such and such contexts. Presumably it is thought that some principle of substitutivity the range of whose variables is suitably restricted is true. But it is worth stressing that the existence of such a principle is not relevant to the status of (A).

Some seem to find reason for affirming (A) in the observation that first-order logic with identity licenses intersubstitutability of co-referential terms. For example, Ruth Marcus claims: 'Substitutivity may be taken as a rule of derivation in first-order logic with identity. In the absence of quantifi-

⁶ A proposition *x* falsifies a proposition *y* if and only if *x* is true and *x* entails the denial of *y*.

⁷ cf. Quine, loc. cit., p. 139

cation over properties, the principle of indiscernibility⁸ comes to the set of those valid sentences that are the associated conditionals of the rule of substitutivity for each of the predicates of the theory. Alternatively, (Ind) may be taken as definitive of identity in second-order logic, and the principle of substitutivity falls out metatheoretically. Anyone who claims that substitutivity does not govern identity but that indiscernibility does must wonder why this is not so in formal languages.'⁹

If α and β are singular terms and S and S' are sentences of a first-order language such that S contains an occurrence of α and S' is the result of substituting β for an occurrence of α in S , then a universal closure of $\ulcorner \alpha = \beta \rightarrow (S \rightarrow S') \urcorner$ is a sentence of a first-order language which is *associated* with the rule of substitutivity. Counterexamples to (A) show that not all of these sentences of a first-order language are true. For example,

- (12) Hesperus = Phosphorus \rightarrow (It is an astronomical discovery that Hesperus = Phosphorus \rightarrow It is an astronomical discovery that Hesperus = Hesperus)

⁸ Marcus formulates what she calls 'the principle of indiscernibility' or '(Ind)' as:

If $a=b$ then every property of a is a property of b .

⁹ Ruth B. Marcus: 'Does the Principle of Substitutivity Rest on a Mistake?'

is a false sentence of a first-order language which is associated with the rule of substitutivity.

There is, however, a class of these sentences which deserves attention. It consists of those universal closures of $\lceil \alpha = \beta \rightarrow (S \rightarrow S') \rceil$ where α , and β are variables, S contains a free occurrence of α and S' is the result of replacing some free occurrence of α in S with a free occurrence of β . (B), i.e.

$$(\forall z)(\forall x)(\forall y)(x=y \rightarrow (z \text{ is a property of } x \rightarrow z \text{ is a property of } y))$$

is a member of this class; so is

$$(\forall z)(\forall x)(\forall y)(x=y \rightarrow (z \text{ is a friend of } x \rightarrow z \text{ is a friend of } y)).^{10}$$

If

$$(13) \quad (\forall x)(\forall y)(x=y \rightarrow (Fx \rightarrow Fy))$$

is thought of as a schema, then members of this class are the universal closures of instances of (13) or a notational variant of (13). Quine describes (13) as a principle of substitutivity of variables and argues in its defence as follows:

¹⁰ These examples are from Richard Cartwright: 'Indiscernibility Principles', in *Midwest Studies in Philosophy*.

(13) does have the air of a law; one feels that any interpretation of 'Fx' violating (13) would be simply a distortion of the manifest intent of 'Fx'. Anyway I hope one feels this, for there is good reason to. Since there is no quantifying into opaque construction, the positions of 'x' and 'y' in 'Fx' and 'Fy' must be referential if 'x' and 'y' in these positions are to be bound by the initial ' $(\forall x)$ ' and ' $(\forall y)$ ' of (13) at all. Since the notation of (13) manifestly intends the quantifiers to bind 'x' and 'y' in all four shown places, any interpretation of 'Fx' violating (13) would be a distortion.¹¹

Now, even if one were to disagree with the details of Quine's argument, it should be recognized that no instance of (13) is false. The temptation to think that (13) has false instances might arise, again from a failure to distinguish (A) from what is intended to be expressed by (13). But notice that unlike (A), an instance of (13) is false only if there is a sequence, and an expression F such that the relevant elements in the sequence (i.e. those assigned to 'x' and 'y' respectively) are

¹¹ *Word and Object*, (The M.I.T. Press, 1970), pp. 167, 168. I have changed the notation and the numbering to conform to this essay.

identical, the sequence satisfies $\lceil Fx \rceil$, and it fails to satisfy $\lceil Fy \rceil$. Clearly it does not follow from every proposition which falsifies (A) that there is such a sequence. Indeed, it is difficult to see that there can be such a sequence. For suppose that a sequence satisfies $\lceil Fx \rceil$. Then, the element of the sequence assigned to 'x' is in the extension of F. But if the element of the sequence assigned to 'x' is identical with the element of the sequence assigned to 'y', then the element of the sequence assigned to 'y' is in the extension of F, and hence the sequence satisfies $\lceil Fy \rceil$.

Now it is not clear to me as to how one is to understand Marcus' remark that 'in the absence of quantification over properties, the principle of indiscernibility comes to the set of those valid sentences that are the associated conditionals of the rule of substitutivity for each of the predicates of the theory.' Marcus formulates what she calls 'the principle of indiscernibility' or '(Ind)' as

If $a=b$ then every property of a is a property of b.

Assuming that 'a' and 'b' are being used here as variables, one would quite naturally see the universal closure of this sentence as articulating the proposition expressed in (B). If we are right in understanding Marcus' principle of indiscernibility in this way, we may reformulate her remark as:

- (i) In a first-order theory, (B) comes to the set of those sentences of the theory which are the universal closures of instances of (13).

It is still far from clear what (i) means. Marcus does not give any indication about how the expression 'comes to' is to be interpreted. Even if sense is given to this expression which renders (i) true, I do not see how (i)'s being true is any evidence for (A) or for Marcus' suggestion that substitutivity governs identity in formal languages. Perhaps, after all, the thought is that (13) implies (A); for, if that is true, we have a strong argument in defence of (A). But that thought, I have tried to argue, is simply false.¹²

Marcus' remark that 'alternatively, (Ind) may be taken as definitive of identity in second-order logic, and the principle of substitutivity falls out metatheoretically' suggests that it was wrong to construe her principle of indiscernibility (Ind) as (B), since (B) is not even a second order sentence. Notwithstanding her own formulation, perhaps, Marcus should be seen as drawing attention to either

- (ii) $(\forall F)(\forall x)(\forall y)(x=y \rightarrow (Fx \rightarrow Fy))$

¹² Also see Richard Cartwright: 'Indiscernibility Principles' in *Midwest Studies in Philosophy*.

or the schema

$$(iii) \quad (\forall F)(\alpha=\beta \rightarrow (F\alpha \rightarrow F\beta))$$

in which 'F' as in (ii) is a second-order variable, but ' α ' and ' β ' are schematic letters to be replaced by names in an instance of (iii). Now (ii), as well as every instance of (iii), is a valid second-order sentence, but it seems that (ii) cannot be taken as definitive of identity since the range of the first-order variables in (ii) must constitute a set or a class.¹³ Secondly, no general principle of substitutivity, such as (A), falls out, metatheoretically, of either (ii) or (iii). (ii) indeed licenses instances of (13), but as we have already seen, that instances of (13) are true is irrelevant to (A).

Quine takes the falsity of (A) as evidence that an occurrence of some singular term in a sentence is not purely referential. For instance, in 'Reference and Modality', he writes: 'Failure of substitutivity reveals merely that the occurrence to be supplanted is not purely referential, that is, that the statement depends not only on the object but on the form of the

¹³ See George Boolos: 'Second-Order Logic', *Journal of Philosophy*, 72 (1975), pp. 509-526.

name.'¹⁴ And, in the same essay, he notes: the failure of substitutivity shows that the occurrence of the personal name in (4) is not purely referential. These remarks would indicate that Quine takes the following principle (C) to be true:

- (C) For any sentence S , and any singular term α , and any z , z is a purely referential occurrence of α in S , only if, for any sentence S' , and any singular term β , if S' is the result of substituting β for z in S and $\alpha = \beta$ expresses a true proposition then S expresses a true proposition if and only if S' expresses a true proposition.

Since (1) and (4) express true propositions, and (5) does not express a true proposition, if (C) is true, then the occurrence of 'Giorgione' in (4) *and* the occurrence of 'Barbarelli' in (5) are not purely referential. Similarly, since (6) and (7) express true propositions and (8) does not express a true proposition, if (C) is true, then the occurrences of '9' and 'the number of planets' in (7) and (8) respectively are *both* not purely referential. It is worth stressing that (C) is a very strong principle — that if (C) is true, then it is also true that

¹⁴ *From a Logical Point of View*, (Harper & Row, New York) p. 140.

- (D) for any sentence S and S' , and any singular terms α and β , and any z , if z is an occurrence of α in S and S' is the result of substituting β for z in S , then, $\lceil \alpha = \beta \rceil$ expresses a true proposition and $\lceil S \leftrightarrow S' \rceil$ does not express a true proposition then z and the corresponding occurrence of β in S' are *both* not purely referential.

Though there is evidence that Quine would endorse (C), there is also evidence that he does not intend (C) to be taken as part of a definition of 'a purely referential occurrence'. But if (C) is not to be taken as defining 'a purely referential occurrence' then we are owed an account of what this expression means. It seems to me that in discussions of referential opacity it is often too readily granted that we know what it is for an occurrence of a singular term in a sentence to be purely referential. Quine has, at times, described a purely referential occurrence of a singular term in a sentence as an occurrence of a singular term 'used in a sentence purely to specify its object.'¹⁵

¹⁵ *Word and Object*, (The M.I.T. Press, 1970), p. 142

And Kaplan offers the following as a definition:

- '(1) A purely designative occurrence of a singular term α in a formula ϕ is one in which α is used *solely* to designate the object.'¹⁶

But if (i) is a definition, then there simply are no purely designative (referential) occurrences of any singular term in a sentence, for no term in a sentence is used *solely* to designate (refer to) its object. After all, singular terms are *also* used in a sentence to complete the sentence.

Quine's remark that 'failure of substitutivity reveals merely that the occurrence to be supplanted is not purely referential, *that is, that the statement depends not only on the object but on the form of the name*'¹⁷ may appear more helpful. Presumably the thought is that the only contribution that a purely referential occurrence of a singular term in a sentence makes towards determining the truth-value of that sentence is the specification of the object that it refers to. One might then propose to define a *purely referential occurrence* of a singular term in a sentence as follows:

¹⁶ 'A Historical Note on Quine's Argument Concerning Substitution and Quantification'

¹⁷ *From a Logical Point of View*, (Harper & Row, 1963), p. 140

(E') For any sentence S , and any non-vacuous singular term α , and any z , z is a purely referential occurrence of α in S , if and only if, for any S' , if S' is the result of substituting, for z in S , a variable which does not occur in S , then S expresses a true proposition if and only if whatever z refers to satisfies S' .

(E') accords with some of our intuitions about the concept of a referential occurrence. If (E') is true then the first occurrence of 'Giorgione' in 'Giorgione was called 'Giorgione' because of his size' is purely referential since 'Giorgione was called 'Giorgione' because of his size' expresses a true proposition if and only if for any variable α , Giorgione satisfies $\ulcorner \alpha$ was called 'Giorgione' because of his size \urcorner . On the other hand, given (E'), the occurrence of 'Giorgione' in

(4) Giorgione was so-called because of his size

is not purely referential. Surely we want to say that for any variable α , Giorgione does not satisfy $\ulcorner \alpha$ was so-called because of his size \urcorner . $\ulcorner \alpha$ was so-called because of his size \urcorner , for any variable α , is not a kind of sentence that anything satisfies, and hence Giorgione does not satisfy it; but (4) expresses a true proposition and hence if (E') is true, the occurrence of 'Giorgione' in (4) is not purely referential.

However, it should be noted that an advocate of (C) is in no position to endorse (E'). Surely we also want to say that for any variable α , Barbarelli does not satisfy $\ulcorner \alpha$ was so-called because of his size \urcorner . $\ulcorner \alpha$ was so-called because of his size \urcorner , for any variable α , is not a kind of sentence that anything satisfies, and hence Barbarelli does not satisfy it; but

(5) Barbarelli was so-called because of his size

does not express a true proposition, and hence if (E') is true, the occurrence of 'Barbarelli' in (5) is purely referential. But since unlike (5), (4) and

(1) Giorgione = Barbarelli

express true propositions, if (C) is true, the occurrence of 'Barbarelli' in (5) is not purely referential; and hence, if (E') is true, (C) is not true.

It would seem that our present difficulty arises because for any variable α , $\ulcorner \alpha$ was so-called because of his size \urcorner is not an open sentence.¹⁸ This suggests that we should revise (E')

¹⁸ If for any variable α , $\ulcorner \alpha$ was so-called because of his size \urcorner is not an open sentence, then, either the usual characterization of an open sentence as a sentence which contains an unbound (free) occurrence of a variable is incorrect, or the

as follows:

(E'') For any sentence S , and any non-vacuous singular term α , and any z , z is a purely referential occurrence of α in S , if and only if, for any S' , if S' is the result of substituting, for z in S , a variable which does not occur in S , then S' is an open sentence, and, S expresses a true proposition if and only if whatever z refers to satisfies S' .

Now, unlike (E'), (E'') is not in conflict with (C). It is not true that if (5) does not express a true proposition and Barbarelli does not satisfy $\ulcorner \alpha$ was so-called because of his size \urcorner , for any variable α , then (E'') is true only if the occurrence of 'Barbarelli' in (5) is purely referential. A further condition needs to be met in order for the occurrence

usual syntactic characterization of bondage, i.e. that an occurrence of a variable α in a sentence S is bound if and only if there is a variable-binding operator I such that this occurrence of α is in a part of S of the form $\ulcorner (I\alpha)S' \urcorner$, is incorrect. But, if these syntactic characterizations are incorrect, how are we to understand the notion of an open sentence and the related notion of a bound occurrence of a variable? I shall argue that Quine's thesis, i.e. H, offers an answer, though one which is incorrect, to these questions.

of 'Barbarelli' in (5) to be purely referential, i.e. that for any variable α , $\lceil \alpha \text{ was so-called because of his size} \rceil$ is an open sentence; - and surely we are inclined to say that that is not the case.

Now though (E'') is not in conflict with (C), it has another consequence which deserves attention. Consider for instance the following sentence:

- (i) It is possible that the number of planets
is odd.

We are inclined to say that the occurrence of 'the number of planets' in (i) is not purely referential. But if (E'') is true, and the occurrence of 'the number of planets' in (i) is not purely referential then

- (ii) It is possible that x is odd

is not an open sentence. For, suppose, (ii) is an open sentence. Then surely, 9, the number of planets, satisfies it, and since (i) expresses a true proposition, if (E'') is true, the occurrence of 'the number of planets' in (i) is purely referential.

Hence if we are inclined to say that (ii) is an open sentence, and also that the occurrence of 'the number of planets' in (i) is not purely referential, we had better reject (E''). But even

if here we have grounds for rejecting (E'), these are not grounds for rejecting the following principle which is implied by (E').

- (E) For any sentence S, and any non-vacuous singular term α , and any z, z is a purely referential occurrence of α in S, *only if*, for any S', if S' is the result of substituting, for z in S, a variable which does not occur in S, then S' is an open sentence, and, S expresses a true proposition if and only if whatever z refers to satisfies S'.

It is worth noting that (E) is a weaker principle than (C). Unlike (C), (E) does not guarantee the truth of (D). However, if (E) is true, then it is also true that

- (D') for any sentences S and S', and any singular terms α and β , and any z, if z is an occurrence of α in S, and S' is the result of substituting β for z in S, then $\lceil \alpha = \beta \rceil$ expresses a true proposition and $\lceil S \Leftrightarrow S' \rceil$ does not express a true proposition, then *either z or* the corresponding occurrence of β in S' is not purely referential.

Consider, for instance, sentences (6), (7), and (8). Since (6) expresses a true proposition, for any variable α , α satisfies 'It is necessary that α is odd' if and only if the number of planets satisfies it. But since (7) expresses a true proposition, and (8) does not express a true proposition, if (E) is true then *either* the occurrence of ' α ' in (7) *or* the occurrence of 'the number of planets' in (8) is not purely referential. Of course, if 'It is necessary that α is odd', for any variable α , is not an open sentence then the occurrences of ' α ' in (7) and of 'the number of planets' in (8) *both* fail to be purely referential. But it does not follow from (E) (or from (E) and the fact that (6) and (7) express true propositions and (8) does not) that 'It is necessary that α is odd', for any variable α , is not an open sentence.

Now (E) is in conflict with some of Quine's remarks about the concept of a purely referential occurrence. Apparently Quine¹⁹ thinks that not only (C) is true, but the following stronger principle (C') is true as well:

(C') For any sentence S , and any singular term α , and any z , z is a purely referential occurrence of α in S , *if and only if*, for any sentence S' , and any

¹⁹ *From a Logical Point of View*, (Harper & Row, 1963), pp. 140, 141

singular term β , if S' is the result of substituting β for z in S and $\ulcorner \alpha = \beta \urcorner$ expresses a true proposition, then S expresses a true proposition if and only if S' expresses a true proposition.

If (C') is true, then the occurrence of 'Giorgione' in

(i) 'Giorgione' names a chess player

is purely referential. But surely we want to say that for any variable α ,

' α ' names a chess player

is not an open sentence, since, α occurs here within quotation marks. But if, for any variable α ,

' α ' names a chess player

is not an open sentence, and (E) is true, then, the occurrence of 'Giorgione' in (i) is not purely referential. And, therefore, if, for any variable α ,

' α ' names a chess player

is not an open sentence, then, (E) is true only if (C') is not true.

Now, (E) identifies a concept which, I think, is of great interest in discussions of referential opacity, and if we were to reject (E), we ought to introduce a new term to characterize that concept. I propose that, instead, we accept (E) as explicative of the concept of a purely referential occurrence and that, therefore, (C') should be rejected. (C), on the other hand, is an important principle, and, I think that it will pay us to examine its consequences. As for Quine's remarks about (i), the intuitions which underlie it are captured by another distinction that Quine draws attention to.

Quine writes:

In sentences there are positions where the term is used as means simply of specifying its object, or purporting to, for the rest of the sentence to say something about, and there are positions where it is not. An example of the latter sort is the position of 'Tully' in:

(a) 'Tully was a Roman' is trochaic.

When a singular term is used in a sentence purely to specify its object, and the sentence is true of the object, then certainly the sentence will stay true when any other singular term is substituted that designates

the same object. Here we have a criterion for what may be called purely referential position: the position must be subject to the substitutivity of identity. That the position of 'Tully' in (a) is not purely referential is reflected in the falsity of what we get by supplanting 'Tully' in (a) by 'Cicero'.²⁰

This passage presents a two-fold distinction: one, a distinction among positions occupied by singular terms in a sentence, and two, a distinction among uses of singular terms in a sentence. Substitutability *salva veritate* of co-referential singular terms is offered as a criterion for distinguishing those *positions* of a singular term in a sentence which are purely referential from those which are not; but what is apparently given as a justification for this criterion is a claim which involves distinguishing those *uses* of a singular term in a sentence which are means simply (purely) of specifying its object from those uses which are not. Quine has frequently referred to the latter distinction as a distinction between a purely referential occurrence of a singular term in a sentence and other kinds of occurrence. To avoid confusion between Quine's distinction among positions and the associated

²⁰ *Word and Object*, (The M.I.T. Press, 1970), p. 142

distinction among occurrences which is characterized in (E), let us agree to use the phrase 'referentially transparent position' in place of Quine's 'purely referential position'. I shall understand by \ulcorner a position of an occurrence of a singular term α in $S \urcorner$ the result of deleting that occurrence of α from S .²¹ Thus, the position of the occurrence of '9' in '9 is odd' is '- is odd', the position of the first occurrence of 'x' in 'x=9·x is odd' is '-=9·x is odd', and the position of the second occurrence of 'x' in 'x=9·x is odd' is 'x-9·-- is odd'. It should be noted that each occurrence of a singular term in a sentence has exactly one position in that sentence; and that occurrences of two or more singular terms in different sentences may have the same position in those sentences, as, for instance, '- is odd' is the position of the occurrence of '9' in '9 is odd', and also, the position of the occurrence of 'The number of planets' in 'The number of planets is odd'. Following Quine, I shall define referential transparency of the position of an occurrence of a singular term in a sentence thus:

(F) For any sentence S , any singular term α , and any

²¹ The use of the word 'position' here, corresponds to the way Quine, at times, uses 'context'.

z, if z is the position of an occurrence, w, of α in S, then z is referentially transparent if and only if for any sentence S', and any singular term β , if S' is the result of substituting β for w (i.e. the relevant occurrence of α) in S, and $\ulcorner \alpha = \beta \urcorner$ expresses a true proposition then S expresses a true proposition if and only if S' expresses a true proposition.²²

And, following Quine, I shall say that the position of an occurrence of a singular term in a sentence is referentially opaque if and only if it is not referentially transparent.

The position of the occurrence of '9' in '9 is odd', i.e. '- is odd' is presumably referentially transparent, because, for any singular term β , if $\ulcorner \alpha = \beta \urcorner$ expresses a true proposition then $\ulcorner \beta \text{ is odd} \urcorner$ expresses a true proposition if and only if '9 is odd' expresses a true proposition. Similarly, the position of the occurrence of 'Giorgione' in "'Giorgione' names a chess player' is referentially transparent. However, (A) is false if and only if the position of some occurrence of a singular term in a sentence is referentially opaque. For (A)

²² If S and S' contain some *free* variables, read the consequent as: 'the universal closure of $\ulcorner S \Leftrightarrow S' \urcorner$ expresses a true proposition'.

is false if and only if there are sentences S and S' , and expressions α and β , such that $\lceil \alpha = \beta \rceil$ expresses a true proposition, S' is the result of substituting β for some occurrence of α in S , and S expresses a true proposition but S' does not express a true proposition. But then, given (F), the position of some occurrence of α in S is not referentially transparent, i.e. it is referentially opaque. Thus, since (1) and (2) express true propositions, but (3) does not express a true proposition, the position of the occurrence of 'Giorgione' in (2), i.e. '- contains nine letters' is referentially opaque. Similarly, the position of the occurrence of 'Giorgione' in (4, i.e. '- was so-called because of his size' and the position of the occurrence of '9' in (7), i.e. 'It is necessary that - is odd' are both referentially opaque.

I shall say that a (one-place) sentential operator, I , is referentially transparent if and only if any position z of an occurrence of a singular term in a sentence is referentially transparent only if $\lceil Iz \rceil$ is referentially transparent; and that a sentential operator is referentially opaque if and only if it is not referentially transparent. The sentential operators 'It is true that', and 'It is not the case that' are referentially transparent; but 'It is necessary that' is referentially opaque, since '- is odd' is referentially transparent, but 'It is necessary that - is odd' is not. Similarly, since 'Hesperus

= -' is referentially transparent but 'It is an astronomical discovery that Hesperus = -' is not referentially transparent, the sentential operator 'It is an astronomical discovery that' is referentially opaque.

The concept of referential transparency of a position, as one would expect, is closely connected with that of purely referential occurrence. Suppose that the position of an occurrence, w , of a singular term α in a sentence S , is not referentially transparent. Given (F), there is, then, a sentence S' , and a singular term β such that S' is the result of substituting β for w (i.e. the relevant occurrence of α in S), $\ulcorner \alpha = \beta \urcorner$ expresses a true proposition, and $\ulcorner S \Leftrightarrow S' \urcorner$ does not express a true proposition. But then, given (D'), either w is not purely referential, or the occurrence of β in S' which corresponds to w (in S) is not purely referential. Consider, for instance, (7). Since the position of the occurrence of '9' in (7) is not referentially transparent, given (F), and (D'), it follows that there is *some* singular term α , such that $\ulcorner 9 = \alpha \urcorner$ expresses a true proposition, and the occurrence of α in $\ulcorner \text{It is necessary that } \alpha \text{ is odd} \urcorner$ is not purely referential. Given the referential opacity of the position of the occurrence of '9' in (7), and (F) and (D'), it also follows that for *any* singular term α , such that $\ulcorner 9 = \alpha \urcorner$ expresses a true proposition, if the

proposition expressed by 'It is necessary that α is odd' differs in truth-value from the proposition expressed by (7) then either the occurrence of '9' in (7) is not purely referential or the occurrence of α in 'It is necessary that α is odd' is not purely referential. However, it is important to appreciate that it does not follow from the referential opacity of the position of the occurrence of '9' in (7), and (F) and (D') that the occurrence of '9' in (7) is not purely referential.

Quine notes that the existence of referentially opaque positions shows not only that (A) is false, but that existential generalization is unwarranted, as well. Quine's remarks suggest that existential generalization is the principle that

- (G) for any sentences S , and S' , any singular term α , and any variable β , if β does not occur free in S , and S' is the result of substituting β for one or more occurrences of α in S , then, S expresses a true proposition only if ' $(\exists\beta)S'$ ' expresses a true proposition.²³

²³ *From a Logical Point of View*, second edition, (Harper & Row, 1963), pp. 120, 145.

As Quine notes, the existence of vacuous singular terms falsifies (G); 'There is no such thing as Pegasus' expresses a true proposition but ' $(\exists x)$ There is no such thing as x ' does not express a true proposition. (G) is also falsified by some pairs of sentences whose members are (i) a sentence containing an occurrence of a singular term which is not purely referential, and (ii) an existential generalization of such an occurrence of a singular term in that sentence. Consider, for instance, (4). (4) expresses a true proposition, but if (G) is true, then

(4') $(\exists x)$ x was so-called because of his size

expresses a true proposition as well. But surely we would say that (4') does not express any proposition, and that, therefore, it does not express a true proposition; and hence, (G) is false. And consider (2). Since (2) expresses a true proposition, if (G) is true, then

(2') $(\exists x)$ ' x ' contains nine letters

expresses a true proposition as well. Now it is not clear what sense is to be made of (2'). Perhaps, one is to think of (2') as expressing the proposition that ' x ', the 24th letter of the English alphabet contains nine letters. If so, (G) is false, and the initial quantifier in (2') does not *bind* the second occurrence of ' x ' in (2'), since in its second occurrence ' x ' is not being used as a variable.

From considerations such as these, Quine appears to conclude that 'if to a referentially opaque context of a variable we apply a quantifier, with the intention that it govern that variable from outside the referentially opaque context, then what we commonly end up with is unintended sense or nonsense.... In a word, we cannot in general quantify into referentially opaque contexts.'²⁴ Making allowances for Quine's allusion to unintended sense, I think that Quine's claim in this passage may be formulated as:

- (H) An occurrence of a variable in a sentence may be bound by a quantifier outside of that sentence only if the position of that occurrence (of the variable) in the sentence is referentially transparent.

Since the position of the occurrence of 'x' in "'x' contains nine letters', and the position of the occurrence of 'x' in 'x was so-called because of his size' are both referentially opaque, if (H) is true, the second occurrence of 'x' in (2'), and the second occurrence of 'x' in (4') both fail to be bound by the initial quantifiers in (2') and (4') respectively.

(H) is to be distinguished from the claim that if an occurrence of a singular term in a sentence is not purely

²⁴ *From A Logical Point of View*, (Harper & Row, 1963), p. 148

referential then existential generalization on that occurrence is unwarranted. The latter is suggested by the pairs of sentences (2), and (2'), and (4), and (4'), and Quine, I think, endorses it; but, it is the stronger (H), which I think, articulates Quine's frequently repeated assertion that there is no *quantification into* referentially opaque contexts. It is (H), then, that I shall describe as 'Quine's Thesis'.

1.2 AMBIGUITY

Since

(6) 9 = the number of planets,

and

(7) It is necessary that 9 is odd,

express true propositions, and

(8) It is necessary that the number of planets is odd,

does not express a true proposition, the position of the occurrence of '9' in (7) is referentially opaque. And since the position of the occurrence of 'x' in

(14) It is necessary that x is odd

is identical with the position of the occurrence of '9' in (7), the position of the occurrence of 'x' in (14) is referentially opaque. And if Quine's thesis is true,

(15) $(\exists x)$ It is necessary that x is odd

is an instance of illicit quantification, where, as in the case of

(4') $(\exists x)$ x was so-called because of his size,

the initial quantifier fails to bind the second occurrence of 'x' in that sentence. Barring some unintended interpretation, (15) is then as unintelligible as (4'); -- it expresses no proposition.

The claim that (15) is as unintelligible as (4') strikes one as puzzling. It seems that there is such a thing as the proposition that something is such that *it* is necessarily odd, and that (15) is as good a candidate to express it as any. The point is not that the proposition that something is such that *it* is necessarily odd is true, though one is inclined to think that it is, but rather that **it** seems that there is such a proposition and that there appears to be no reason to think that (15) does not express it.

Now, if the idea of necessity was an idea of something which was an attribute or a characteristic only of closed sentences, and we looked upon, for instance, (7) as a deficient way of expressing what is expressed by

'9 is odd' is necessary

there would be no temptation to think that there is such a thing as the proposition that something is such that it is necessarily odd; and, presumably, we would not be puzzled by the suggestion that (15) is unintelligible, for (15) could then, at best, be viewed as a careless rendering of

(15') $(\exists x)$ 'x is odd' is necessary.

Though the position of the occurrence of 'x' in

'x is odd' is necessary

is referentially opaque, it would seem that (15') does not provide any reason against Quine's thesis; for one is inclined to think that since the second occurrence of 'x' in (15') is within quotation marks, it is not bound by the initial quantifier in that sentence.

Again, if the idea of necessity was an idea of something which was an attribute or a characteristic only of those things that closed sentences express and which are affirmed or denied, i.e. propositions, and we looked upon, for instance, (7) as expressing what is expressed by

The proposition that 9 is odd is necessary,

there would be no temptation to think that there is such a thing as the proposition that something is such that it is necessarily odd; and, presumably, we would not be puzzled by the suggestion that (15) is unintelligible, for (15) could then, at best, be viewed as a careless rendering of

(15'') $(\exists x)$ The proposition that x is odd is necessary,

and it is not clear whether sense can be made of (15'').

However, it is I think false, though I have no proof that it is false, that the idea of necessity is an idea of something which is an attribute or a characteristic only of closed sentences or propositions. It appears to make sense to say such things as

(i) 9 is odd and 9 could not have failed to be odd,

and

(ii) The number of planets is odd and the number of planets, whatever it is, could not have failed to be odd.

In saying (i) and (ii) I intend to say, not of any sentence or a proposition that it is true and that it could not have failed to be true; but rather of a certain number that *it* is such and such, and that *it* could not have failed to be such and such. And it seems to me that if it makes sense to say such things as (i) and (ii) (regardless of whether they are true or not) then it also makes sense to say such things as

(iii) Something is odd and *it* could not have failed to be odd,

and

(iv) Something is such that *it* is necessarily odd.

But if it does make sense to say such things as (iii) and (iv) then it would seem that the claim that (15) is unintelligible is false, for (15) appears to express the same proposition as (iv).

Verbs of propositional attitudes and temporal modifiers give rise to similar problems. Quine draws attention to what he describes as 'the relational and notional senses of believing in spies' in these words:

(16) $(\exists x)$ Ralph believes that x is a spy

and

(16') Ralph believes that $(\exists x)$ x is a spy

both may perhaps be ambiguously phrased as 'Ralph believes that someone is a spy', but they may be unambiguously phrased respectively as 'There is someone whom Ralph believes to be a spy' and 'Ralph believes that there are spies'. The difference is vast; if Ralph is like most of us (16') is true and (16) false.²⁵

But, now it will surely be granted that if Ralph is like most of us, the position of the occurrence of 'x' in

²⁵ 'Quantifiers and Propositional Attitudes' reprinted in *Reference and Modality*, ed. Leonard Linsky (Oxford, 1971) p. 102. I have changed the numbering to conform to that of the present essay.

Ralph believes that x is a spy

is referentially opaque; that is, there are expressions α and β such that $\lceil \alpha = \beta \rceil$ and $\lceil \text{Ralph believes that } \alpha \text{ is a spy} \rceil$ express true propositions, and $\lceil \text{Ralph believes that } \beta \text{ is a spy} \rceil$ does not express a true proposition. But then, if Quine's thesis is true, the second occurrence of ' x ' in (16) is not bound by the initial quantifier in that sentence; and therefore, barring some unintended interpretation, (16) is unintelligible. And if (16) is unintelligible, it would seem that

(v) There is someone whom Ralph believes to be a spy

is unintelligible as well, for (16) appears to express the same proposition as (v). And again, if Quine's thesis is true, barring some unintended interpretation,

(17) $(\exists x)$ Sometime in the past x was a Catholic

must be counted as unintelligible, since the position of the occurrence of ' x ' in

Sometime in the past x was a Catholic

is referentially opaque; for surely

Reagan = the president of the U.S.

and

Sometime in the past the president of the U.S. was
a Catholic

express true propositions, but

Sometime in the past Reagan was a Catholic
does not express a true proposition. But if (17) is unintelligible, then it seems that

(vi) There is someone who sometime in the past was a
Catholic

must be unintelligible as well, for (16) appears to express the same proposition as (vi). And yet it seems that there is no difficulty in understanding either (17) or (vi), and indeed no special difficulty in identifying an individual who sometime in the past was a Catholic.

How is one to resolve these difficulties? In 'Quantifiers and Propositional Attitudes', where Quine raises the second of these difficulties, he suggests:

As we are scarcely prepared to sacrifice the relational construction 'There is someone whom Ralph believes to be a spy' which (16) as opposed to (16') was supposed to reproduce, the obvious next move is to try to make the best of our dilemma by distinguishing two senses of belief:

belief₁, which disallows conjunctions of the form

(18) w sincerely denies '---'. w believes that ---

and belief₂, which tolerates (18) but makes sense of (16).

For belief₁, accordingly, we sustain

(19) Ralph believes that the man in the brown hat is a spy

and

(20) Ralph does not believe that the man seen at the
beach is a spy

(even though the man in the brown hat is the man seen at the beach) and ban (16) as nonsense. For belief₂, on the other hand we sustain (16); and for this sense of belief we must reject (20) and acquiesce in the conclusion that Ralph believes₂ that the man at the beach is a spy even though he also believes₂ (and believes₁) that the man at the beach is not a spy.²⁶

It might seem that here we have a general strategy for dealing with any apparent counterexample to Quine's thesis. For instance, someone might suggest:

²⁶ 'Quantifiers and Propositional Attitudes' reprinted in *Reference and Modality*, ed. Leonard Linsky (Oxford, 1971) p. 103-104. I have changed the numbering to conform to that of the present essay.

As we are scarcely prepared to sacrifice the *de re* construction 'Something is such that *it* is necessarily odd' which (15) was supposed to reproduce, the obvious next move is to try to make the best of our dilemma by distinguishing two senses of necessity: necessity₁, which disallows conjunctions of the form

(18') '---' is not analytic. it is necessary that ---

and necessity₂, which tolerates (18') but makes sense of (15). For necessity₁, accordingly we sustain

(19') It is necessary that 9 is odd

and

(20') It is not necessary that the number of planets is odd

(even though 9 is the number of planets) and ban (15) as nonsense. For necessity₂, on the other hand, we sustain (15); and for this sense of necessity we must reject (20') and acquiesce in the conclusion that it is necessary₂ that the number of planets is odd even though it is not necessary₁ that the number of planets is odd.

These suggestions are incomprehensible. We do not know what 'belief₁' and 'belief₂', and similarly what 'necessity₁'

and 'necessity₂', mean. That these expressions resemble some expressions of English cannot be regarded as more than merely a coincidence. We are told that for *belief*₁ we ban (16) as nonsense, but for *belief*₂ we sustain (16). Presumably, that is to say that

(i) $(\exists x)$ Ralph believes₁ that x is a spy

is to count as unintelligible, but

(ii) $(\exists x)$ Ralph believes₂ that x is a spy

will count as intelligible. If so, that would, indeed, distinguish *belief*₁ and *belief*₂; but there is no reason to suppose that it distinguishes two senses of *belief*. Similarly, to suppose that

$(\exists x)$ It is necessary₂ that x is odd

is intelligible is irrelevant to whether or not (15) is intelligible; we would first need to know what concept, if any, is the concept of *necessity*₂. The contrast between *belief*₂ and *belief*₁ is presumably meant to reflect some contrast between what Quine calls 'the transparent sense of belief' and 'the opaque sense of belief'.²⁷ But even if these senses, whatever they are,

²⁷ *Word and Object*, (The M.I.T. Press, 1970), p. 145

are adequately characterized, I shall argue that there is strong evidence to suggest that if the expressions 'It is necessary₂ that' and 'Ralph believes₂ that' are referentially transparent then they do not express any sense of 'It is necessary that', or 'Ralph believes that' respectively.

Let us say that sentences S , and S' are *weakly equivalent* if and only if $\lceil S \Leftrightarrow S' \rceil$ is true in every interpretation in which the class abstraction operator ' \wedge ' and the sign of class membership receive their intended interpretations. Sentences which are (logically) equivalent are, obviously, weakly equivalent; but the converse is not true. For instance

$$(\hat{x})(x=x) = (\hat{x})(x \notin x)$$

and

$$(\forall x)(x=x)$$

are weakly equivalent but not logically equivalent. And let us say that a sentential operator I is *normal* if and only if, for any sentences S and S' , if S and S' are weakly equivalent, then $\lceil IS \Leftrightarrow IS' \rceil$ is true. Following an argument of Quine,²⁸ it can be shown that if a normal sentential operator of a language, which has ' \wedge ' and ' \in ' in its vocabulary, is referentially transparent

²⁸ *From A Logical Point of View*, (Harper & Row, 1963), p. 159

then it is truth-functional.

Suppose that I is a normal and a referentially transparent sentential operator of a language in which ' \wedge ' is available. Given any sentence S, since S is weakly equivalent to

$$\lceil (\hat{x})(x=x \cdot S) = (\hat{x})(x=x) \rceil,$$

$$\lceil IS \Leftrightarrow I(\hat{x})(x=x \cdot S) = (\hat{x})(x=x) \rceil$$

is true. But given any sentence S' which has the same truth-value as S,

$$\lceil (\hat{x})(x=x \cdot S) = (\hat{x})(x=x \cdot S') \rceil$$

is true. But then, since I is referentially transparent,

$$\lceil IS \Leftrightarrow I(\hat{x})(x=x \cdot S') = (\hat{x})(x=x) \rceil$$

is true. Now S' is weakly equivalent to

$$(\hat{x})(x=x \cdot S') = (\hat{x})(x=x)$$

and since I is normal,

$$\lceil IS \Leftrightarrow IS' \rceil$$

is true; and hence, I is truth-functional.

The proposed sentential operator 'It is necessary₂ that' is by hypothesis, referentially transparent. But then, if a language in which it is introduced has ' \wedge ' and ' ϵ ' in its

vocabulary, 'It is necessary₂ that' is either truth-functional, or it is not normal - that is, substitution of some weakly equivalent sentence for S in \lceil It is necessary₂ that S \rceil does not preserve truth-value. Now, it would seem that any sentential operator whose sense is a sense of 'It is necessary that' must be normal and it must not be truth-functional. But if this is right, and 'It is necessary₂ that' is referentially transparent, then, whatever its sense, it is not a sense of 'It is necessary that'.

The preceding result that any normal and referentially transparent sentential operator of a language, which includes ' \wedge ' and ' ϵ ' in its vocabulary, is truth-functional, also supports the claim that there is no sentential operator in this language which is referentially transparent and whose sense is a sense of 'sometime in the past'; for presumably, any sentential operator whose sense is a sense of 'sometime in the past' is normal and non-truth-functional. On the other hand, 'Ralph believes that' is not a normal sentential operator, - for whatever the logical acumen of Ralph, we do not expect that for any weakly equivalent sentences S and S', if \lceil Ralph believes that S \rceil expresses a true proposition then \lceil Ralph believes that S' \rceil expresses a true proposition as well. Hence it would not be reasonable to suppose that 'Ralph believes₂ that' is a

normal operator, if the sense of 'believes₂' is a sense of 'believes'. However, as Quine remarks, the stipulation that 'Ralph believes₂ that' is a referentially transparent sentential operator whose sense is a sense of 'Ralph believes that' leads to unwelcome results, even though it is not a normal operator. Let us say that 'α believes₂ that' is a quasi-normal operator if and only if the referent of α satisfies every instance of the following schema:

- (i) x believes₂ that S if and only if x believes₂ that the truth-value of the proposition that S is the true.

It appears reasonable to suppose that if the sense of 'believes₂' is a sense of 'believes', then it is possible that there is some individual who satisfies every instance of (i). Suppose that Ralph does. But since 'Ralph believes₂ that' is, by hypothesis, referentially transparent, given any sentences S and S', if

[the truth-value of the proposition that S =
the truth-value of the proposition that S']

expresses a true proposition,

[Ralph believes₂ that the truth-value of the proposition that S is the true ↔ Ralph believes₂ that the truth-value of the proposition that S' is the true']

expresses a true proposition. And since 'Ralph believes₂ that' is supposed to be quasi-normal,

⌈Ralph believes₂ that S ↔ Ralph believes₂ that S'⌋

expresses a true proposition. Hence if the sense of 'believes₂' is a sense of 'believes' and 'Ralph believes₂ that' is referentially transparent and quasi-normal, then Ralph believes every proposition if he believes at least one true proposition and one false proposition.

A proponent of the view that 'It is necessary₂ that' is a referentially transparent sentential operator whose sense is a sense of 'It is necessary that' is likely to reject the assumption that any sentential operator whose sense is a sense of 'It is necessary that' is normal. Similarly, a proponent of the view that 'Ralph believes₂ that' is a referentially transparent sentential operator whose sense is a sense of 'Ralph believes that' is likely to reject the assumption that 'Ralph believes₂ that' is quasi-normal. Suppose that there is a unique sense of 'It is necessary that' which is customarily thought to be the sense of 'It is necessary that'. One might propose that we understand 'It is necessary₁ that' as unambiguously expressing the sense which is customarily thought to be the sense of 'It is necessary that', and define a new sentential operator 'It is necessary₂ that' as follows:

- (ii) For any sentences S and S' , and any singular terms $\alpha_1, \dots, \alpha_n$, if the set of singular terms which occurs in S is $\{\alpha_1, \dots, \alpha_n\}$ or empty, and S' is the result of replacing all occurrences of $\alpha_1, \dots, \alpha_n$ in S respectively with the variables x_1, \dots, x_n which do not occur in S , then 'It is necessary₂ that S ' expresses a true proposition if and only if \langle the referent of $\alpha_1, \dots, \alpha_n \rangle$ satisfies 'It is necessary₁ that S' '.

If 'It is necessary₂ that' is to be understood as defined in (ii), then clearly 'It is necessary₂ that' is a referentially transparent operator but it is also non-normal. For, given (ii),

It is necessary₂ that the number of planets is odd
expresses a true proposition, since, presumably 9, the number of planets, satisfies

It is necessary₁ that x is odd.

On the other hand, given (ii),

It is necessary₂ that there is a unique number of planets and it is odd

does not express a true proposition, since, presumably

It is necessary₁ that there is a unique number of
planets and it is odd

is not satisfied by any sequence. But,

The number of planets is odd

is weakly equivalent to

There is a unique number of planets and it is odd,

and, therefore, 'It is necessary₂ that' is not a normal sentential operator.

Similarly, suppose that there is a unique sense of 'Ralph believes that' which is customarily thought to be the sense of 'Ralph believes that'. It might be proposed that we understand 'Ralph believes₁ that' as unambiguously expressing the sense which is customarily thought to be the sense of 'Ralph believes that', and define a new sentential operator 'Ralph believes₂ that' as follows:

- (iii) For any sentences S and S' , and any singular terms $\alpha_1, \dots, \alpha_n$, if the set of singular terms which occur in S is $\{\alpha_1, \dots, \alpha_n\}$ or empty, and S' is the result of replacing all occurrences of $\alpha_1, \dots, \alpha_n$ in S respectively with the variables x_1, \dots, x_n

which do not occur in S , then \lceil Ralph believes₂ that S \rceil expresses a true proposition if and only if \langle the referent of $\alpha_1, \dots, \text{the referent of } \alpha_n \rangle$ satisfies \lceil Ralph believes₁ that S \rceil .

If 'Ralph believes₂ that' is to be understood as defined in (iii) then, clearly, 'Ralph believes₂ that' is a referentially transparent sentential operator, but given (iii), it would also be unreasonable to suppose that 'Ralph believes₂ that' is quasi-normal. For if Ralph is like any of us, it would be very unreasonable to insist that if there are truth-values, a , and the true, such that $\langle a, \text{the true} \rangle$ satisfies

Ralph believes₁ that x is y

then, provided that a is the truth-value of the proposition that the man in the brown hat is a spy, the man in the brown hat satisfies

Ralph believes₁ that x is a spy.

But if \langle the truth-value of the proposition that the man in the brown hat is a spy, the true \rangle satisfies

Ralph believes₁ that x is y ,

and the man in the brown hat does not satisfy

Ralph believes₁ that x is a spy,

the, given (iii),

Ralph believes₂ that the truth-value of the
proposition that the man in the brown hat is
a spy is the true

expresses a true proposition, though,

Ralph believes₂ that the man in the brown hat
is a spy

does not express a true proposition. But, then, 'Ralph believes₂
that' is not a quasi-normal operator.

Now, anyone, who like Quine, thinks that since the positions
of the occurrences of 'x' in

It is necessary₁ that x is odd,

and

Ralph believes₁ that x is a spy

are referentially opaque, these are not the kind of sentences of
which one could intelligibly speak as being satisfied by any ob-
ject, would be inclined to reject (ii) and (iii) as incoherent.
Thus, if the purpose of introducing definitions (ii) and (iii)
was to find a way of articulating thoughts such as those intended
to be expressed by

(15) $(\exists x)$ It is necessary that x is odd,

and

(16) $(\exists x)$ Ralph believes that x is a spy,

without violating Quine's thesis, then (ii) and (iii) do not achieve this purpose. But even if these Quinean considerations against (ii) and (iii) are to be disregarded, it seems to me unlikely that the sense of 'It is necessary₂ that' as defined in (ii) is a sense of 'It is necessary that', or that the sense of 'Ralph believes₂ that' as defined in (iii) is a sense of 'Ralph believes that'.

Presumably, the ideas which underlie (ii) and (iii) are that

(a) there is a sense of 'It is necessary that' such that, given that sense, any occurrence of a singular term in the scope of 'It is necessary that' is purely referential,

and

(b) there is a sense of 'Ralph believes that', such that, given that sense, every occurrence of a singular term in the scope of 'Ralph believes that' is purely referential.

Now, if, for instance, (a) is true, then there is a sense of 'It is necessary that' such that construed in that sense, 'It is necessary that' is both referentially transparent and non-normal. But if there are any reasons to suppose that (a) is true, then there are equally good reasons to suppose that there is a sense of 'It is necessary that', such that, given that sense, only the first occurrence of a singular term in the scope of 'It is necessary that' is purely referential. And again, if there are reasons to suppose that (a) is true, then there are equally good reasons to suppose that there is a sense of 'It is necessary that', such that, given that sense, only the second occurrence of a singular term in the scope of 'It is necessary that' is purely referential; and so on. Now, though, it is reasonable to suppose that, for instance,

(8) It is necessary that the number of planets is odd

is ambiguous, it is, I think a mistake to locate the source of this ambiguity in a multiplicity of senses of 'It is necessary that'. Similarly, though it is reasonable to suppose that

(19) Ralph believes that the man in the brown hat is
a spy

is ambiguous, it is, I think, a mistake to locate the source of this ambiguity in a multiplicity of senses of 'Ralph believes

that'. But if, as I have suggested, (ii) and (iii) do not succeed in identifying senses of 'It is necessary that' and 'Ralph believes that' respectively, I see no reason to suppose either that there is a non-normal and a referentially transparent sentential operator whose sense is a sense of 'It is necessary that', or that no sentential operator whose sense is a sense of 'α believes that' can be quasi-normal.

The problem that I have been examining in this section is this: If Quine's thesis is true, then in each of

(15) $(\exists x)$ It is necessary that x is odd,

(16) $(\exists x)$ Ralph believes that x is a spy,

and

(17) $(\exists x)$ Sometime in the past x was a Catholic.

the initial quantifier fails to bind the second occurrence of 'x', and therefore, barring some unintended interpretations, none of these sentences express any proposition. In Quine's words, these are to be counted as nonsense. But not only do these sentences appear intelligible, it would also seem that if these sentences were unintelligible, then the corresponding English sentences

(iv) Something is such that it is necessarily odd,

(v) There is someone whom Ralph believes to be a spy,

and

(vi) There is someone who sometime in the past was a
Catholic

would be unintelligible as well, for the latter are apparently synonymous with (15), (16), and (17) respectively. The alleged failure of (15), (16), and (17) to express the thoughts that one intends to express by them would indeed be taken as evidence that these thoughts themselves are incoherent. Are we then to reject Quine's thesis, or to acquiesce in the conclusion that (15), (16), and (17) are unintelligible?

We have seen that if the expressions 'necessary', 'believes', and 'sometime in the past' were ambiguous in such a way as to allow interpretations of 'It is necessary that', 'Ralph believes that', and 'Sometime in the past' which would render them referentially transparent, this dilemma would be resolved. Following Quine I have argued that the project of locating such an ambiguity in these expressions must fail since any sentential operator of a suitably rich language which is referentially transparent and normal cannot express necessity or temporality, and cannot express belief if it is quasi-normal.

In discussions of Quine's views on quantified modal logic, attention is sometimes drawn to what is apparently a purely syntactic ambiguity in some modal sentences. For instance,

(8) It is necessary that the number of planets is odd,

may be understood, *de re*, as formulating the proposition that the number of planets, whatever it is, is necessarily odd; but it may also be understood, *de dicto*, as formulating the proposition that the number of planets is odd is necessary. Though the ambiguity of sentences such as (8) has frequently been noticed, it has not been always adequately characterized. Plantinga, for instance, describes a *de re* ascription of necessity as an ascription to an object of having a property necessarily, and a *de dicto* ascription of necessity as an ascription to a proposition of having the property of being necessarily true.³⁰ A *de dicto* ascription of necessity is then seen as an instance of a *de re* ascription of necessity. Thus, on Plantinga's view, (8) may be understood as formulating the proposition that the number of planets has the property of being necessarily odd, but it may also be understood as formulating the proposition that that the number of planets is odd has the property of being necessarily true. This claim, I think is false. (8) does not admit of a reading under which it says or

³⁰ *The Nature of Necessity*, (Oxford University Press, 1974), pp. 9-13.

entails that the number of planets (whatever it is) has *the property* of being necessarily odd. Not that I think that there is no such property, nor that the number of planets lacks it; but rather that (8) does not admit of a reading under which it says or entails that there is such a property or that the number of planets has it. What I see as a difficulty in Plantinga's exposition of the availability of *de re/de dicto* readings of a sentence such as (8) arises from a more general consideration concerning Russell's Paradox. It is a lesson to be learned from what is essentially Russell's Paradox that not every open sentence determines a property. We need then an additional argument other than simply an observation of some structural ambiguity in (8) to support the claim that (8) admits of a reading under which it says of 9 that it has the property of being necessarily odd.

Smullyan characterizes the ambiguity of sentences such as (8) using Russell's notions of scope and contextual definition of definite descriptions.³¹ (8), on his view, may be understood as expressing the proposition expressed by

$(\exists x)((\forall y)(y \text{ numbers the planets} \Leftrightarrow y=x) \cdot \text{it is necessary that } x \text{ is odd})$

³¹ 'Modality & Description', *The Journal of Symbolic Logic*, 1948, pp. 31-7

or, it may be understood as expressing the proposition expressed by

It is necessary that $(\exists x)(\forall y)(y \text{ numbers the planets} \Rightarrow y=x) \cdot x \text{ is odd}$

Now, I think that the question of whether definite descriptions are contextually defineable is quite independent of any considerations of scope; and if we were presently to ignore Quine's thesis, the ambiguity of (8) may be depicted by resorting to quantification. (8) may be seen as formulating the proposition formulated in either

(21) $(\forall x)(x = \text{the number of planets} \rightarrow \text{it is necessary that } x \text{ is odd})$

or

(21') It is necessary that $(\forall x)(x = \text{the number of planets} \rightarrow x \text{ is odd})$.

Since the ambiguity in (8) is a structural ambiguity, there is no reason to think that a similar ambiguity is not present in

(7) It is necessary that 9 is odd

as well. Like (8), (7) admits of a reading under which it expresses what is presumably expressed by

(22) $(\forall x)(x=9 \rightarrow \text{it is necessary that } x \text{ is odd}),$

but it also admits of a reading under which it expresses what is expressed by

(22') It is necessary that $(\forall x) (x=9 \rightarrow x \text{ is odd}).$

That both (22) and (22') express true propositions is irrelevant to the issue of structural ambiguity.³²

The claim that the ambiguity in (7) and (8) arises out of distinctions of scope of the various expressions in these sentences suggests that similar ambiguity may be found in

(19) Ralph believes that the man in the brown hat is
a spy,

and

(23) Sometime in the past it was the case that the president
of the U.S. was a Catholic.

(19) admits of a reading under which it expresses what is presumably expressed by

³² The misconception that *de dicto/de re* ambiguity requires difference of truth-value may be found in Linsky's remark: "Kripke cannot admit that these names 'St. Anne', 'Homer', or any others, do induce *de dicto/de re* ambiguity... . His principal thesis about proper names - that they are rigid designators, just is logically equivalent, in his *Semantic for Quantified Modal Logic*, to the thesis that they *cannot* induce *de dicto/de re* ambiguity." *Names and Descriptions*, pp. 56-7.

- (24) $(\forall x)(x = \text{the man in the brown hat} \rightarrow \text{Ralph believes that } x \text{ is a spy}),$

but it also admits of perhaps the more natural reading under which it expresses what is expressed by

- (24') Ralph believes that $(\forall x)(x = \text{the man at the beach} \rightarrow x \text{ is a spy}).$

Similarly, the ambiguity in (23) is characterized by

- (25) $(\forall x)(x = \text{the president of the U.S.} \rightarrow \text{sometime in the past it was the case that } x \text{ was a Catholic})$

and

- (25') Sometime in the past it was the case that $(\forall x)(x = \text{the president of the U.S.} \rightarrow x \text{ was a Catholic})$

Now, though I think that it ought to be acknowledged that sentences (7), (8), (19), and (23) are structurally ambiguous, it is not easy to see what, if any, relevance the appeal to the ambiguity in these sentences has to the problem at hand.

Rather than guiding us how to formulate a thought such as that the number of planets is necessarily odd without apparently violating Quine's thesis, our characterization of the ambiguity in these sentences suggests that Quine's thesis is false. Each of the sentences (21), (22), (24), and (25) contains an

occurrence of a variable in the scope of a referentially opaque operator which is intended to be bound by a quantifier outside; but these are the very kinds of sentences which are proclaimed unintelligible in the light of Quine's thesis.

Perhaps the proposal, after all, is not simply that we pay attention to the structural ambiguity in sentences such as (8), (19), and (23), but that the definite descriptions in these (and other) sentences are to be contextually defined as well. Thus Smullyan writes:

'In the light of our discussion so far, it may suggest itself to the reader that the modal paradoxes arise not out of any intrinsic absurdity in the use of the modal operators but rather out of the assumption that descriptive phrases are names. It may indeed be the case that the critics of modal logic object primarily not to the use of modal operators but to the method of contextual definition as employed, e.g. in Russell's theory of definite descriptions.'³³

Now it is true that in a language which does not contain any definite descriptions the sentential operator 'It is necessary that' will be referentially transparent, and hence Quine's thesis will not rule against quantified modal sentences of such a

³³ 'Modality & Description', *The Journal of Symbolic Logic*, 1948.

language; but it should be noticed that Quine's thesis is not directed against quantified modal sentences alone. If Quine's thesis is true then a variable in the scope of *any* referentially opaque operator cannot be bound by a quantifier outside. But now it seems that the referential opacity of epistemic operators such as 'Ralph believes that' does not depend upon the presence of a definite description in a sentence. It seems reasonable to suppose that, for instance, 'Ralph believes that Hesperus is a planet' does not express a true proposition even though 'Ralph believes that Venus is a planet' may express a true proposition. And hence the eliminability of definite descriptions does not ensure that, for instance, ' $(\exists x)$ Ralph believes that x is a planet' is intelligible.

No doubt it will be suggested that names are not genuine singular terms either, and that following Quine's proposal names are to be contextually defined as well.³⁴ It seems to me that Quine's proposal about contextual definability of names faces serious objections. It is not my intent to pursue these objections here, but it should be noted that even if all singular terms other than variables are contextually defined, the difficulty which is raised by Quine's thesis is not fully resolved. I formulated Quine's thesis as a thesis about first-

³⁴ *From A Logical Point of View*, (Harper & Row, 1963), pp. 7,8

order variables, but if Quine's thesis is true there is no reason to think that the following corresponding thesis about second-order variables is not true:

An occurrence of a second-order variable in a sentence is bound by a quantifier outside of that sentence only if the position of the occurrence of the variable in that sentence is *extensional*.

But if this thesis is true, then in

$(\exists X)$ It is necessary that 9 is X

the second occurrence of 'X' is not bound by the initial quantifier since its position in

It is necessary that 9 is X

is clearly not extensional. Evidently no strategy for contextually defining those predicates which are responsible for the non-extensionality of 'It is necessary that' is available. It seems to me then that a resolution of the general problem raised by Quine's thesis is not to be found in the contextual definability of those singular terms which are responsible for the referential opacity of operators such as 'It is necessary that', and 'Ralph believes that'.

Let us then re-examine the notion of ambiguity. I

described the ambiguity in (8) as a structural ambiguity and depicted it by adverting to the idiom of quantification. The contrast between the two readings of (8) is thus seen as a contrast between the role of the modal operator 'It is necessary that' as a modifier of closed sentences as in (21') and its role as a modifier of open sentences as in (21). But if, as I have suggested, for any name or a definite description α , 'It is necessary that α is odd', or equivalently 'Necessarily α is odd' is structurally ambiguous, then there is no reason not to suppose that 'Necessarily α is odd' admits of a similar structural ambiguity where α is a variable. It would seem that in 'Necessarily x is odd', 'Necessarily' may be construed as modifying the open sentence ' x is odd', or it may be construed as modifying the verb phrase 'is odd', or simply the expression 'odd'. This ambiguity in 'Necessarily x is odd' obviously cannot be illustrated by the use of quantification, but one may use some arbitrary convention such as the one employed in (26) and (26') below to disambiguate the sentence. Thus in

(26) Necessarily (x is odd)

one is to understand 'Necessarily' as unambiguously modifying the open sentence ' x is odd', whereas in

(26') x is necessarily-odd

it is taken as modifying the expression 'odd'. Now it should be noticed that the occurrence of 'x' in (26'), unlike its occurrence in (26) is not in the scope of a referentially opaque operator, and that its position in (26'), unlike its position in (26) is referentially transparent. Thus,

(27') $(\exists x)$ x is necessarily-odd

unlike (15) and

(27) $(\exists x)$ Necessarily (x is odd)

does not purport to violate Quine's thesis.

Quine notices an analogous structural distinction in epistemic sentences.³⁵ He contrasts

(28) Tom believes that Cicero denounced Catiline

with

(28') By Tom Cicero is believed to have denounced Catiline

and points out that whereas the position of the occurrence of 'Cicero' in (28) is referentially opaque, its position in (28') is referentially transparent. Thus, if Quine's thesis is true,

³⁵ *Word and Object*, (The M.I.T. Press, 1970), p. 149-151

(29) $(\exists x)$ Tom believes that x denounced Catiline
 fails to express a proposition, but

(29') $(\exists x)$ By Tom x is believed to have denounced Catiline
 is unobjectionable.

Now, though I think that the structural ambiguity in
 'Necessarily x is odd' (and analogous belief sentences etc.)
 ought to be recognized and consequently the structural difference
 between (27) and (27') (and analogously between (29) and (29'))
 ought to be acknowledged, it would not be a satisfactory response
 to the issue raised by Quine's thesis that (27') (or (29')) does
 not violate Quine's thesis and that it succeeds in expressing
 what was intended to be expressed by (27) (or (29)). For
 surely if the variables in (27) and (27') (and (29) and (29'))
 are variables of objectual quantification, it would seem that
 (27) (or (29)) expresses a proposition if and only if (27') (or
 (29')) does. An advocate of Quine's thesis who claims that (27)
 is unintelligible, but (27') is not, owes us an explanation.

1.3 QUINE'S THESIS

Is Quine's thesis true?

In a recent article Kaplan writes:

I have concluded that in 1943 Quine made a mistake. He believed himself to have given a proof of a general theorem³⁶ regarding the semantical interpretation of any language that combines quantification with opacity. The purported theorem says that in a sentence, if a given position occupied by a singular term, is not open to substitution by co-designative singular terms *salva veritate*, then that position cannot be occupied by a variable bound to an initially placed quantifier. The proof offered assumes that quantification receives its standard interpretation. But the attempted proof is fallacious. And what is more, the theorem is false.

Kaplan then goes on to reconstruct the alleged proof as follows:

Step 1: A purely designative occurrence of a singular term α in a formula ϕ is one in which α is used *solely* to designate the object. [This

³⁶ Kaplan calls 'Quine's Theorem' the claim that I have described as Quine's Thesis. Kaplan uses 'Quine's Thesis' for a different claim, presumably also endorsed by Quine.

is a definition.}

Step 2: If α has a purely designative occurrence in ϕ , the the truth-value of ϕ depends only on *what* α designates, not on *how* α designates. [From 1]

Step 3: Variables are devices of pure reference; they cannot have non-purely designative occurrences. [By standard semantics]

Step 4: If α and β designate the same thing, but $\phi\alpha$ and $\phi\beta$ differ in truth-value, then the occurrences of α in $\phi\alpha$ and β in $\phi\beta$ are not purely designative. [From 2]

Now assume (5.1): α and β are co-designative singular terms, and $\phi\alpha$ and $\phi\beta$ differ in truth-value, and (5.2): γ is a *variable* whose value is the object designated by α and β .

Step 6: Either $\phi\alpha$ and $\phi\gamma$ differ in truth-value or $\phi\beta$ and $\phi\gamma$ differ in truth-value. [From 5.1 since $\phi\alpha$ and $\phi\beta$ differ.]

Step 7: The occurrence of γ in $\phi\gamma$ is not purely designative. [From 5.2, 6, and 4]

Step 8: $\phi\gamma$ is semantically incoherent. [From 7 and 3]

Kaplan notes:

All but one of these steps seem to me to be innocuous. That one is step 4 which, of course, does not follow from 2. All that follows from 2 is that at least one of the two occurrences is not purely designative. When 4 is corrected in this way, 7 no longer follows. The error of 4 appears in later writings in a slightly different form. It is represented by an unjustified shift from talk about *occurrences* to talk about *positions*. Failure of substitution does show that some occurrence is not purely referential. (Shifting now from the 'designative' language of 'Notes on Existence and Necessity' to the 'referential' language of 'Reference and Modality'). From this it is concluded that the *context* (read 'position') is referentially opaque. And thus that what the context expresses 'is in general not a trait of the object concerned, but depends on the manner of referring to the object.' Hence, we cannot properly quantify into a referentially opaque context.³⁷

Though Kaplan claims that the only step in his reconstruction of Quine's alleged proof to which he takes exception is Step 4,

³⁷ All quotations from Kaplan are from his 'A Historical Note On Quine's Argument Concerning Substitution and Quantification'.

there are a number of difficulties presented by other steps in this argument which deserve attention. If, as Kaplan indicates, the proposition expressed in Step 1 is a definition of 'a purely referential occurrence of a singular term',³⁸ then it seems to me that there simply are no purely referential occurrences of a singular term, since no singular term is used (in a sentence) *solely* to refer. There are obviously numerous other uses that an occurrence of a singular term has in a sentence. Hence, if the proposition expressed in Step 1 is a definition then the proposition expressed in Step 3 is not a truth of standard semantics, it is simply false.

Apparently, the proposition expressed in Step 2 is false as well. Consider, for instance,

Giorgione was called 'Giorgione' because of his size.

Notwithstanding Step 1, it would, I think, be granted that 'Giorgione' has *a* purely referential occurrence in this sentence. However, it is not true that the truth-value of this sentence depends only on what 'Giorgione' designates; for if it did

Giorgione was called 'Barbarelli' because of his size,

³⁸ Following 'Reference & Modality', I have used the word 'referential'.

would be true as well. Perhaps what is intended as Step 2 is this:

If α has *only* a purely referential occurrence in ϕ , then the truth-value of ϕ depends only on *what* α refers to, not on *how* α refers.

But this will not do either, since, obviously, the truth-value of a sentence depends *also* on what other terms in that sentence mean or refer to.

Now if we understand the notation ' $\phi\alpha$ ' in Step 4 as standing for any sentence S which contains one or more occurrences of α , and ' $\phi\beta$ ' as the result of replacing an occurrence of α in S by β , then the proposition expressed in Step 4 is equivalent to (D) of Sec. 1.1. (D) is the principle that

for any sentences S and S' , any singular terms α and β , and any z , if z is an occurrence of α in S and S' is the result of substituting β for z in S , then if ' $\alpha=\beta$ ' expresses a true proposition and ' $S \Leftrightarrow S'$ ' does not express a true proposition, then z and the corresponding occurrence of β in S' are *both* not purely referential.

As Kaplan emphasizes, if the proposition expressed in Step 4,

or equivalently (D), is true, then there is a strong argument for Quine's thesis. Suppose that a sentence S_1 contains an occurrence of a singular term α . Let us agree to represent S_1 as follows:

$$S_1: \quad \text{_____} \alpha \text{_____}$$

Suppose, moreover, that the position of the displayed occurrence of α in S_1 is not referentially transparent. Then there is a sentence S_2 and a singular term β such that S_2 is the result of replacing the displayed occurrence of α in S_1 with β , $\lceil \alpha = \beta \rceil$ expresses a true proposition and $\lceil S_1 \Leftrightarrow S_2 \rceil$ does not express a true proposition.

$$S_2: \quad \text{_____} \beta \text{_____}$$

Consider now a sentence S_3 and a variable γ (which does not occur in S_1) such that S_3 is the result of replacing the displayed occurrence of α in S_1 with γ and the value of γ in S_3 is the object designated by α and β . Since the position of the occurrence of γ in S_3 is identical with the position of the displayed occurrence of α in S_1 and since the position of the displayed occurrence of α in S_1 is not referentially transparent, the position of the occurrence of γ in S_3 is not referentially transparent. But now, since $\lceil S_1 \Leftrightarrow S_2 \rceil$ does not express a true proposition, either $\lceil S_1 \Leftrightarrow S_3 \rceil$ does not express a true proposition

or $\lceil S_2 \Leftrightarrow S_3 \rceil$ does not express a true proposition. But since $\lceil \gamma = \alpha \rceil$ and $\lceil \gamma = \beta \rceil$ both express true propositions, if (D) is true, the occurrence of γ in S_3 is not purely referential. But if

(J) the occurrence of a variable in a sentence may be bound by a quantifier outside that sentence only if that occurrence is purely referential,

is true, then the occurrence of γ in S_3 may not be bound by a quantifier outside of S_3 . Hence if (D) and (J) are true and the position of the occurrence of a variable in a sentence is not referentially transparent, then the occurrence of that variable in the sentence may not be bound by a quantifier outside the sentence.

Are (D) and (J) true? To answer this question we need to know what it is for an occurrence of a singular term in a sentence to be purely referential. Now Quine remarks: 'Failure of substitutivity reveals merely that the occurrence to be supplanted is not purely referential.'³⁹ I formulated this claim in sec. 1.1 as

(C) For any sentence S , and any singular term α , and any z , z is a purely referential occurrence of α in S ,

³⁹ *From a Logical Point of View*, (Harper & Row, New York), p. 140

only if, for any sentence S' and any singular term β , if S' is the result of substituting β for z in S and $\lceil \alpha = \beta \rceil$ expresses a true proposition, then S expresses a true proposition if and only if S' expresses a true proposition.

It is easily seen that if (C) is true then (D) is true as well. For suppose that (D) is false. Then there is a sentence S_1 which contains an occurrence of a singular term α ,

$$S_1: \text{_____} \alpha \text{_____},$$

and a sentence S_2 which is the result of substituting β for the displayed occurrence of α in S_1 ,

$$S_2: \text{_____} \beta \text{_____};$$

$\lceil \alpha = \beta \rceil$ expresses a true proposition, $\lceil S_1 \Leftrightarrow S_2 \rceil$ does not express a true proposition and either the displayed occurrence of α in S_1 or the displayed occurrence of β in S_2 is purely referential.

But then (C) is false.

Perhaps it would be thought that (C) is merely one half of a definition of 'a purely referential occurrence of a singular term'. It would then be argued that if (C) is a truth of definition, (D) must be true. But, as we have seen, if (D) and (J) are true, Quine's thesis is true; and surely, the argument

would go on, (J) is a truth of standard semantics;⁴⁰ hence Quine's thesis is true. Now I think that if (J) is to appear as a premise in any argument for Quine's thesis, we had better not construe (C) as partly defining 'a purely referential occurrence'. Notice that (C) states that an occurrence of a singular term in a sentence is purely referential only if its position in that sentence is referentially transparent. But if (C) is taken as defining 'a purely referential occurrence', then

(J) the occurrence of a variable in a sentence may be bound by a quantifier outside that sentence only if that occurrence is purely referential,

means that

(J') the occurrence of a variable in a sentence may be bound by a quantifier outside that sentence only if its position in that sentence is referentially transparent.⁴¹

And (J') is Quine's thesis. Hence, if (C) is taken as defining

⁴⁰ See Kaplan's Step 3.

⁴¹ See the definition of 'a referentially transparent position', sec. (1.1).

'a purely referential occurrence', (J) can appear as a premise in an argument for Quine's thesis only on pain of circularity.

In Section (1.1) I proposed that we take the following principle (E) as explicative of the notion of a purely referential occurrence of a singular term.

- (E) For any sentence S , any non-vacuous singular term α , and any z , z is a purely referential occurrence of α in S , only if, for any S' , if S' is the result of substituting, for z in S , a variable which does not occur in S , then S' is an open sentence, and S expresses a true proposition if and only if whatever z refers to satisfies S' .

I argued that (E) is a weaker principle than (C); that though (D) is a consequence of (C), it is not a consequence of (E).

If (E) is true then it is true that

- (i) if a sentence S_1 contains an occurrence of a singular term α , and
- (ii) if S_2 is the result of substituting β for an occurrence z of α in S_1 , and
- (iii) $\ulcorner \alpha = \beta \urcorner$ expresses a true proposition, but
- (iv) $\ulcorner S_1 \neq S_2 \urcorner$ does not express a true proposition, then
- (v) either z (i.e. the specified occurrence of α in S_1)

or the corresponding occurrence of β in S_2 is not purely referential.

But it is not a consequence of (E) that given (i) - (iv), *both* z and the corresponding occurrence of β in S_2 are not purely referential. Thus, it is not incompatible with (E) that the occurrence of '9' in

(7) It is necessary that (9 is odd)

be purely referential, and

It is necessary that x is odd

be an open sentence, even though the occurrence of 'the number of planets' in

(8) It is necessary that (the number of planets is odd)

is not purely referential. Note that even if the occurrence of '9' in (7) is purely referential, the position of the occurrences of '9' in (7) is *not* referentially transparent. What Kaplan describes as 'the error of Step 4' is presumably the error of thinking that (D) is a consequence of (E). In my opinion it is not clear from Quine's writings that he is guilty of this error; Quine endorses (C), and (D) is a consequence of (C). Kaplan writes that the error of Step 4 'is represented (in

later writings) by an unjustified shift from talk about *occurrences* to talk about *positions*.' But notice that (C) does in fact license this *shift*, for (C) states that an *occurrence* of a singular term in a sentence is *purely referential* only if its *position* in that sentence is *referentially transparent*. If this *shift* from talk about occurrences to talk about positions is unjustified then (C) is unjustified.

It is, I think worth emphasizing that if (E) is true and (D) is false then the following principle (K) is false.

(K) For any S and S' and any (non-vacuous) singular term α , if S is an open sentence in one variable and S' is the result of substituting α for the free occurrences of that variable in S , then the referent of α satisfies S if and only if S' expresses a true proposition.

Suppose that (E) is true, and (D) is false. There are then sentences S_1 and S_2 , and singular terms α and β

S_1 : _____ α _____
 S_2 : _____ β _____

such that

- (i) S_1 contains an occurrence of α ,
- (ii) S_2 is the result of substituting β for the displayed occurrence of α in S_1 ,
- (iii) $\alpha=\beta$ expresses a true proposition,
- (iv) $S_1 \Leftrightarrow S_2$ does not express a true proposition, and
- (v) either the displayed occurrence of α in S_1 or the displayed occurrence of β in S_2 is purely referential.

Suppose that the displayed occurrence of α (β) in S_1 (S_2) is purely referential. Since (E) is true, there is an open sentence S_3 and a variable γ which does not appear in S_1 ,

S_3 : _____ γ _____

such that

- (i) S_3 is the result of substituting γ for the displayed occurrence of α in S_1 , and
- (ii) the referent of α (β) satisfies S_3 if and only if $S_1(S_2)$ is true.

But since $\alpha=\beta$ expresses a true proposition, the referent of β (α) satisfies S_3 if and only if $S_1(S_2)$ is true. But since $S_1 \Leftrightarrow S_2$ does not express a true proposition, (K) is false. Hence, if (E) is true, and (K) is true, then (D) is true.

It is (K) then that we need to prove, in order to prove Quine's thesis.

Quine has frequently observed that if we try to apply existential generalization to

(7) It is necessary that 9 is odd,

we obtain

(15) $(\exists x)$ It is necessary that x is odd.

But, he asks rhetorically, what is this object which is necessarily odd? In the light of (7) it is 9, but in the light of

(6) $9 =$ the number of planets,

and

(30) It is not necessary that the number of planets is odd,

it is not. Now, it is not clear to me why these observations are relevant to Quine's thesis. Perhaps, as Cartwright says, we should construe Quine as pointing out that a double application of existential generalization on the conjunction of (6), (7), and (30) yields

(31) $(\exists x)(\exists y)(x=y \cdot$ it is necessary that x is odd \cdot
it is not necessary that x is odd).⁴²

⁴² 'Indiscernibility Principles' in *Midwest Studies in Philosophy*, vol. pp. 302-303.

But now consider the schema

$$(13) \quad (\forall x)(\forall y)(x=y \rightarrow (Fx \rightarrow Fy)).$$

If

$$(31') \quad (\forall x)(\forall y)(x=y \rightarrow (\text{it is necessary that } x \text{ is odd} \rightarrow \text{it is necessary that } y \text{ is odd}))$$

is an instance of (13), then (31) is in conflict with the claim that every instance of (13) is true. I have argued that the validity of (13) is fundamental to the intent of identity and quantification. I would, therefore, argue that if (31') is an instance of (13) then (31) is not true. Now, presumably the principle of existential generalization whose double application to the conjunction of (6), (7), and (30) yields (31) is this:

(G') For any sentences S , and S' , any non-vacuous singular term α , and any variable β , if β does not occur free in S , and S' is the result of substituting β for one or more occurrences of α in S , then S expresses a true proposition only if $\lceil (\exists \beta)S' \rceil$ expresses a true proposition.

Since the conjunction of (6), (7), and (30) is true, either (G') is false, or (31') is not an instance of (13). Now, I think that it should be granted that (31') is an instance of

(13) if and only if 'It is necessary that x is odd' is an open sentence. Hence, it should be granted that either (G') is false, or 'It is necessary that x is odd' is not an open sentence. Now, I do not see why this is any evidence for Quine's thesis. That (G') is false is established by the facts that

(4) Giorgione was so-called because of his size

expresses a true proposition, but

(4') $(\exists x)$ x was so-called because of his size

does not express any proposition, and hence does not express a true proposition. What is obviously needed is an argument which shows that (31) is obtained from the conjunction of (6), (7), and (30) by the application of a true principle of existential generalization.

Cartwright notes:

Perhaps Quine is to be understood, rather, as follows.

It would be counter to astronomy to deny

(32) $(\forall y)(y=\text{Phosphorus} \Leftrightarrow y=\text{Hesperus})$

and an application of existential generalization to the conjunction of (32) with

(33) astro Hesperus = Phosphorus

would yield

$$(34) \quad (\exists x)((\forall y)(y=\text{Phosphorus} \Leftrightarrow y=x) \cdot \\ \text{astro } x = \text{Phosphorus}.$$

Again, no-one could reasonably deny

$$(35) \quad (\forall y)(y=\text{Phosphorus} \Leftrightarrow y=\text{Phosphorus}),$$

and an application of existential generalization to the conjunction of (35) with

$$(36) \quad - \text{astro Phosphorus} = \text{Phosphorus}$$

would yield

$$(37) \quad (\exists x)((\forall y)(y=\text{Phosphorus} \Leftrightarrow y=x) \cdot \\ - \text{astro } x = \text{Phosphorus}).$$

Consider, then, the thing identical with Phosphorus. Is it a thing such that it is a truth of astronomy that it is identical with Phosphorus? In view of (34) and (37), no answer could be given. There is some one thing identical with Phosphorus. But there is no settling the question whether it satisfies 'astro $x = \text{Phosphorus}$ '. To permit quantification into opaque constructions is thus at odds with the

fundamental intent of objectual quantification.⁴³

Cartwright sees in this reasoning an argument in defence of the validity of (13). Surely the conjunction of (34) and (37), he suggests, is not true; — for if it were, the question: 'Is the thing identical with Phosphorus such that it is a truth of astronomy that it is identical with Phosphorus?' would be intelligible, but no answer could be given to it. Hence, the conjunction of (34) and (37) is either unintelligible or false.

However, seen as an argument for Quine's thesis, this reasoning, I believe, is invalid. The last sentence, i.e.

'To permit quantification into opaque constructions is thus at odds with the fundamental intent of objectual quantification.'

does not follow from the rest. Consider, for instance, the following argument:

Perhaps Quine is to be understood, rather, as follows.

It would be counter to history to deny

(32') $(\forall y)(y = \text{Reagan} \Leftrightarrow y = \text{the president of the U.S.}),$

⁴³ 'Indiscernibility Principles' in *Midwest Studies in Philosophy*, vol. , pp. 302-3. I have changed the numbering to conform to that of the present essay.

and an application of existential generalization to the conjunction of (32') with

(33') It was not the case in 1972 that the president of the U.S. was identical with Reagan,

would yield

(34') $(\exists x)((\forall y)(y = \text{Reagan} \Leftrightarrow y = x) \cdot \text{it was not the case in 1972 that } x \text{ was identical with Reagan})$.

Again, no-one could reasonably deny

(35') $(\forall y)(y = \text{Reagan} \Leftrightarrow y = \text{Reagan})$

and an application of existential generalization to the conjunction of (35') with

(36') It was the case in 1972 that Reagan was identical with Reagan,

would yield

(37') $(\exists x)((\forall y)(y = \text{Reagan} \Leftrightarrow y = x) \cdot \text{it was the case in 1972 that } x \text{ was identical with Reagan})$.

Consider then the thing identical with Reagan. Is it a thing such that it was the case in 1972 that it was identical with Reagan? In view of (34') and (37'), no answer could be given. There is some one thing identical with Reagan. But there is no settling the question whether it satisfies 'It was the case in 1972 that x was identical with Reagan.' To permit quantification into opaque constructions is thus at odds with the fundamental intent of objectual quantification.

Surely we must resist the suggestion that no answer could be given to the question: 'Is the thing identical with Reagan such that it was the case in 1972 that *it* was identical with Reagan?'. The question is intelligible; there is indeed a thing identical with Reagan; and there is little doubt that this thing is such that it was the case in 1972 that *it* was identical with Reagan. The conjunction of (34') and (37') is, therefore, not unintelligible; it is false. Now it ought to be noted, as both Quine and Cartwright emphasize, that the intelligibility of this question or the intelligibility of the conjunction of (34') and (37') is not guaranteed simply by the intelligibility of quantification and the intelligibility of the role of 'It was the case in 1972 that' as an operator on closed sentences. Cartwright writes:

The symbol ' \Box ' is sometimes so used that ' $\Box\phi$ ' counts as true if and only if ϕ itself is necessary. If that is all there is to go on, we have no option but to count the ' \Box '-construction opaque and hence

$$(i) \quad (\forall x)(\forall y)(x=y \rightarrow (\Box x=x \rightarrow \Box x=y))$$

unintelligible. But (i),

$$(ii) \quad (\forall x) \Box (x=x)$$

and

$$(iii) \quad (\forall x)(\forall y)(x=y \rightarrow \Box x=y)$$

are witnesses to a contemplated *transparent* ' \Box '-construction. Now, the intelligibility of such a construction is not guaranteed simply by an antecedent understanding of quantification and of the opaque ' \Box '-construction.⁴⁴

And Quine remarks:

The important point to observe is that granted an

⁴⁴ 'Indiscernibility Principles' in *Midwest Studies in Philosophy*, vol. , p. 304. I have changed the numbering to conform to that of the present essay.

understanding of the modalities (through uncritical acceptance, for the sake of argument, of the underlying notion of analyticity), and given an understanding of quantification ordinarily so-called, we do not come out automatically with any meaning for quantified modal sentences.⁴⁵

I think that it ought to be conceded that for any referentially opaque operator I , if all there is to go on about I , is that for any closed sentence S , $\lceil IS \rceil$ is true if and only if S is such and such, then we do not thereby gain any understanding of $\lceil IS' \rceil$, where S' is an open sentence. The point, I think, is a perfectly general one; one which is independent of any considerations about referential opacity. Indeed, it ought to be conceded that for *any* operator I , if all there is to go on about I is that for any closed sentence S , $\lceil IS \rceil$ is true if and only if S is such and such, then we do not thereby gain any understanding of $\lceil IS' \rceil$, where S' is an open sentence. Consider, for instance, the operator 'It is not the case that'. If the only available rule for understanding 'It is not the case that' is that

- (1) $\lceil \text{It is not the case that } S \rceil$ is true if and only if
 S is not true,

⁴⁵ *From A Logical Point of View*, (Harper & Row, 1963), p. 150

and quantification is understood, we are not guaranteed any understanding of

(ii) $(\exists x)$ It is not the case that x is odd.

For surely,

(iii) $(\exists x)$ 'x is odd' is not true

does not count as an explanation of (ii). What is obviously needed is an explanation of the role of 'It is not the case that' as an operator on open sentences.

But, now, suppose that

(38) It is not the case that x is odd,

is specified as an open sentence, and the problem of determining which sequences, if any, satisfy this open sentence is somehow to be settled. It seems to me that it would not be a necessary condition for settling this problem that the position of the occurrence of 'x' in (38) be counted as referentially transparent; for, I am inclined to think that this problem is to be settled independently of any considerations about what singular terms (other than the variables) or what kinds of singular terms (other than the variables) are available. The point is not that there is some doubt about the referential transparency of the position of the occurrence of 'x' in (38); — it is rather that

the referential transparency of this position is not a necessary condition for settling the problem of determining which sequences, if any, satisfy (38). Similarly, suppose that

(39) Necessarily x is odd,

and

(40) It was the case in 1972 that x was identical with
Reagan

are specified as open sentences, and the problem of determining which sequences, if any, satisfy these open sentences is somehow to be settled. It is not a necessary condition for settling this problem that the position of the occurrences of 'x' in (39) and (40) respectively be counted as referentially transparent. But surely if (39) and (40) are open sentences, then the free occurrences of the variable 'x' in these sentences may be bound by quantifiers outside of these sentences. Why is it, then, claimed, as Quine apparently does that 'to permit quantification into opaque constructions is *thus* at odds with the fundamental intent of objectual quantification.'

One cannot help but think that at issue are some principles of instantiation and generalization. If (39) is an open sentence and the position of the occurrence of 'x' in (39) is not referentially transparent then the following principle of

existential generalization is not true.

- (L) For any sentences S and S' , any non-vacuous singular term α , and any variable β , if β does not occur free in S , and S' is an open sentence, which is the result of substituting β for one or more occurrences of α in S , then S expresses a true proposition only if $\lceil (\exists \beta) S' \rceil$ expresses a true proposition.

If the position of the occurrence of 'x' in (39) is not referentially transparent, then there are singular terms α and β such that

$$\lceil (\alpha = \beta \cdot \text{necessarily } \alpha \text{ is odd} \cdot \neg (\text{necessarily } \beta \text{ is odd})) \rceil$$

expresses a true proposition. But if (39) is an open sentence, then surely

$$x = y \cdot \text{necessarily } x \text{ is odd} \cdot \neg (\text{necessarily } y \text{ is odd})$$

is an open sentence as well. And if (L) is true, then

$$(\exists x)(\exists y)(x = y \cdot \text{necessarily } x \text{ is odd} \cdot \neg (\text{necessarily } y \text{ is odd}))$$

expresses a true proposition. But that conflicts with the validity of (13). Granted that (13) is valid, then either (L)

is not true or (39) is not an open sentence in which the position of the occurrence of 'x' is referentially opaque.

(L) is closely related to a principle of universal instantiation.

- (M) For any sentences S and S' , any non-vacuous singular term α , and any variable β , if β does not occur free in S , and S' is an open sentence which is the result of substituting β for one or more occurrences of α in S , then $\lceil (\forall\beta)S' \rceil$ expresses a true proposition only if S expresses a true proposition.

Again, if (39) is an open sentence in which the position of the occurrence of 'x' is not referentially transparent then (M) is false. If the position of the occurrence of 'x' in (39) is not referentially transparent, then there are singular terms α , and β , such that

$$\lceil (\alpha=\beta \cdot \text{necessarily } \alpha \text{ is odd} \cdot - (\text{necessarily } \beta \text{ is odd})) \rceil$$

expresses a true proposition. But granted that (13) is valid, if (39) is an open sentence, then

$$(\forall x)(\forall y)(x=y \rightarrow (\text{necessarily } x \text{ is odd} \rightarrow \text{necessarily } y \text{ is odd}))$$

expresses a true proposition. But if (M) is true, then for any

non-vacuous singular terms α , and β ,

$$\ulcorner \alpha = \beta \rightarrow (\text{necessarily } \alpha \text{ is odd} \rightarrow \text{necessarily } \beta \text{ is odd}) \urcorner$$

expresses a true proposition. But that conflicts with the claim that there are singular terms α and β such that

$$\ulcorner \alpha = \beta \cdot \text{necessarily } \alpha \text{ is odd} \cdot - (\text{necessarily } \beta \text{ is odd}) \urcorner$$

expresses a true proposition. Hence, granted that (13) is valid, either (M) is false, or (39) is not an open sentence in which the position of the occurrence of 'x' is not referentially transparent.

It should be noticed that (M) is true if and only if the principle (K) of a few pages back is true. (K) is the principle that

for any S and S', and any non-vacuous singular term α , if S is an open sentence in one free variable and S' is the result of substituting α for the free occurrences of that variable in S, then the referent of α satisfies S if and only if S' expresses a true proposition.

Suppose that (M) is false. Then there is a sentence S, and a non-vacuous singular term α , and a variable β which does not occur free in S, and S' is the result of substituting β for one or more occurrences of α in S, $\ulcorner (\forall \beta) S' \urcorner$ expresses a true

proposition, but S does not express a true proposition. But if $\lceil (\forall\beta)S' \rceil$ expresses a true proposition then every thing satisfies S' ; and if everything satisfies S' , then presumably the referent of α satisfies S' . But since S does not express a true proposition (K) is false. Hence, if (K) is true, (M) is true. On the other hand, suppose that (M) is true. Granted that (13) is valid, for any open sentences S_1 and S_2 , each in one free variable, if S_1 contains one or more free occurrences of a variable α , and S_2 is the result of substituting a variable β for the free occurrences of α in S_1 , then

$$\lceil (\forall\alpha)(\forall\beta)(\alpha=\beta \rightarrow (S_2 \Leftrightarrow S_1)) \rceil$$

expresses a true proposition. But given some non-vacuous singular term α' , if S' is the result of substituting α' for the free occurrences of the variable α in S_1 , then if (M) is true,

$$\lceil (\forall\beta)(\alpha'=\beta \rightarrow (S_2 \Leftrightarrow S')) \rceil$$

expresses a true proposition. But in that case the referent of α' satisfies

$$\lceil S_2 \Leftrightarrow S' \rceil.$$

But since S_2 is an open sentence in one free variable and S' is a closed sentence, if the referent of α' satisfies

$$\lceil S_2 \Leftrightarrow S' \rceil,$$

the referent of α' satisfies S_2 if and only if S' is true.

Hence, if (M) is true, (K) is also true.

I had argued that we need to prove (K) in order to prove Quine's thesis. It is easily seen that granted that (13) is valid, if (M) (or equivalently (K)) is true then Quine's thesis is true. For if (13) is valid then for any open sentences S_1 and S_2 , each in one free variable, if a variable α occurs free in S_1 and S_2 is the result of substituting β for free occurrences of α in S_1 , then

$$\lceil (\forall \alpha) (\forall \beta) (\alpha = \beta \rightarrow (S_1 \rightarrow S_2)) \rceil$$

expresses a true proposition. But if S_1' is the result of substituting any singular term α' for all free occurrences of α in S_1 , and S_2' is the result of substituting any singular term β' for all free occurrences of β in S_2 , and, if (M) is true, then

$$\lceil (\alpha' = \beta' \rightarrow (S_1' \rightarrow S_2')) \rceil$$

expresses a true proposition. But then the positions of the occurrences of any free variable in S_1 are referentially transparent. Hence, granted the validity of (13), if (M) is true, then the positions of the occurrences of any free variable in any open sentence are referentially transparent. But, then,

Quine's thesis is true, since, surely, it is only the free occurrences of a variable in an open sentence which may be bound by a quantifier outside of that open sentence.

But is (M) true? It seems to me that (M) is not a principle which is fundamental to the intent of objectual quantification. Objectual quantification is best understood in terms of satisfaction of open sentences, and it appears to me that the problems of determining what it is for a sequence to satisfy an open sentence are to be settled independently of any considerations about what kinds of singular term other than the variable are available. It seems, then, that it is not required for an understanding of objectual quantification that the principle that the position of an occurrence of a free variable in an open sentence is referentially transparent be regarded as true. But since, this principle is true if (M) is true, a defence of (M) is, apparently, not to be found in any appeal to the fundamental intent of objectual quantification. Indeed, one may find, in the facts that there are singular terms α and β , such that

$$\lceil \alpha = \beta \rightarrow (\text{it is necessary that } \alpha \text{ is odd} \rightarrow \text{it is necessary that } \beta \text{ is odd}) \rceil$$

does not express a true proposition, whereas

$$(\forall x) (\forall y) (x = y \rightarrow (\text{it is necessary that } x \text{ is odd} \rightarrow \text{it is necessary that } y \text{ is odd}))$$

expresses a true proposition, evidence that (M) is not true.

IDENTITY AND TIME

It is a compelling idea that bodies, or material objects, are distinct only if they do not share all their parts. The idea is elegantly economical; and, if it is true, then we also know what relation the expression 'is constituted of' in Wiggins' example 'The jug is constituted of a collection of china-bits' stands for.¹ Surely, one wants to say that for any material objects x , and y ,

(1) x is constituted of $y \rightarrow x$ and y have the same parts.

But, then, given the idea that for any material objects x , and y ,

(2) x is distinct from $y \rightarrow x$ and y do not have the same parts,

it follows that for any material objects x , and y ,

(3) x is constituted of $y \rightarrow x$ is identical with y .

On the other hand, if (3) is true, and for any material object x ,

(4) $(\exists z)$ x is constituted of z ,

then, for any material objects x , and y ,

¹ *Sameness and Substance*, (Basis Blackwell, 1980), p. 31

(5) x is identical with $y \rightarrow x$ is constituted of y .

For suppose that x is identical with y . Given (4), there is something, z , such that x is constituted of z ; and if (3) is true, x is identical with z . But, then y is identical with z ; and since x is constituted of z , x is constituted of y . Hence, if (1), (2), and (3) are true, then it is also true that for any material objects x , and y ,

(6) x is constituted of $y \Leftrightarrow x$ is identical with y .

There is another, and, I think, a more urgent consideration that makes the idea that distinct material objects do not share all their parts attractive. Consider, for instance, this silver chain which is on my table now. It would be natural to think that there is such a thing as the mass of silver which is on my table now, and that this silver chain which is on my table now is made of it. But if there is indeed such a thing as the mass of silver which is on my table now, and it is distinct from this silver chain which is on my table now, then either there are at least two silver chains which occupy exactly the same place on my table now, or the mass of silver which is on my table now is not a silver chain. The former alternative is contrary to the plainest common sense; and the latter leaves one wondering why it is that the mass of silver which is on my table now is not a

silver chain. It has, after all, exactly the same parts as a silver chain, and these parts are organized in exactly the same way as the parts of a silver chain. What is it then that prevents the mass of silver which is on my table now from being a silver chain?

In this essay I examine the claim that material objects are distinct only if they do not share all their parts, and I argue that, notwithstanding its attractiveness, the claim is false.

2.1 PARTHOOD AND PERSISTENCE

It is tempting to think that the logic of parthood is the Leonard-Goodman Calculus of Individuals.² This calculus³ takes a two-place predicate 'D' as primitive, where under the intended interpretation 'Dxy' is to be read as 'x is discrete from y', and defines 'Pxy' (read as: x is part of y), 'Oxy' (read as: x overlaps y), and 'FuxS' (read as: x fuses a set S) as follows:

$$(CI-Df.1) \quad Pxy =_{df.} (\forall z)(Dzy \rightarrow Dzx)$$

$$(CI-Df.2) \quad Oxy =_{df.} (\exists z)(Pzx \cdot Pzy)$$

$$(CI-Df.3) \quad FuxS =_{df.} (\forall y)(Dyx \Leftrightarrow (\forall z)(z \in S \rightarrow Dyz))$$

The calculus contains the following distinctive axioms:

$$(CI-Ax.1) \quad (Pxy \cdot Pyx) \rightarrow x=y \quad \text{identity axiom}$$

$$(CI-Ax.2) \quad Oxy \Leftrightarrow \neg Dxy \quad \text{overlap axiom}$$

$$(CI-Ax.3) \quad (\exists x)x \in S \rightarrow (\exists y)FuyS \quad \text{fusion axiom}$$

Since the logic of identity guarantees that

$$x=y \rightarrow (Pxx \rightarrow Pxy),$$

² I owe this thought and many others in what follows to Prof. J. J. Thomson.

³ Henry S. Leonard and Nelson Goodman, 'The Calculus of Individuals and Its Uses', *Journal of Symbolic Logic*, v, 2 (June, 1940). I have changed the notation to conform to that of the present essay.

and

$$x=y \rightarrow (P_{yx} \rightarrow P_{yy}),$$

and, the definition of parthood (CI-Df.1) implies that

$$(\forall x)P_{xx},$$

it is provable in this calculus that

$$x=y \rightarrow (P_{xy} \cdot P_{yx}).$$

The fusion axiom says that for any non-empty set there is an object which fuses it; — that is, given any non-empty set, there is an object which is discrete from all and only those things which are discrete from every member of that set. But it is provable in this calculus that given any non-empty set, there is exactly one such object. Suppose that S is a non-empty set. Then given the fusion axiom there is some object, y , which fuses S . But suppose, there is, also, an object, z , which fuses S . Given the definition (CI-Df.3), then, all and only those things are discrete from y which are discrete from every member of S . But since z fuses S , as well, all and only those things are discrete from z which are discrete from every member of S . And hence, all and only those things are discrete from y which are discrete from z . But then, y is part of z , and, z is part of y , and given the identity axiom, y is identical with z .

Hence, for any set S ,

$$(\exists x)x \in S \rightarrow (\exists y)(\forall z)(FuzS \Leftrightarrow z=y).$$

It is also provable in this calculus that every object fuses the set of which it is the sole member. For given any object x , and given the fusion axiom, there is an object, y , which fuses $\{x\}$. But then since all and only those things are discrete from y which are discrete from x , given the identity axiom, y is identical with x ; and hence x fuses $\{x\}$.

Under the intended interpretation of this calculus, the variables ' x ', ' y ', etc., are taken as ranging over material objects. Parts of material objects are themselves construed as material objects. The variable ' S ' ranges over sets of material objects, and the predicates ' D ', ' P ', and ' O ', are seen as expressing the relations of discreteness, parthood, and overlap respectively. Notice that since it is a theorem of this calculus that

$$(Pxy \cdot Pyx) \Leftrightarrow (\forall z)(Pzx \Leftrightarrow Pzy)$$

the thesis articulated at the beginning of this essay that material objects are distinct only if they do not share all their parts is preserved under the intended interpretation of this calculus. Consider, for instance, the silver chain which is on my table now. It is made of thirty links, and a clasp.

Let us call this set of thirty-one objects 'A', and let us suppose that the time now is t_1 . Since A is a non-empty set, if the axioms of the calculus of individuals are true under their intended interpretation, then there is such an object as *the fusion of A*; and since it is true that

- (7) $(\forall z)(z \text{ is discrete from the silver chain on my table at } t_1 \Leftrightarrow (\forall w)(w \in A \rightarrow z \text{ is discrete from } w)),$

if the axioms of the calculus of individuals are true under their intended interpretation, then

(8) the silver chain on my table at $t_1 =$ the fusion of A. Again, if there is such a thing as the mass of silver on my table at t_1 , and the axioms of the calculus of individuals are true under their intended interpretation, then since it is true that

- (9) $(\forall z)(z \text{ is discrete from the mass of silver on my table at } t_1 \Leftrightarrow (\forall w)(w \in A \rightarrow z \text{ is discrete from } w)),$

(10) the mass of silver on my table at $t_1 =$ the fusion of A,

and, hence,

(11) the silver chain on my table at $t_1 =$ the mass of silver on my table at t_1 .

Notwithstanding the obvious appeal of the calculus of individuals, there is, I think, reason to suppose that the axioms of this calculus are not true under their intended interpretation. Imagine, for instance, that between t_1 and t_2 I remove a link from the silver chain which was on my table at t_1 and drop that link on the floor. Considering that this is an ordinary silver chain, and considering that the link I have removed is not of any special significance, it would not be unreasonable to suppose that the silver chain which was on my table at t_1 has survived this change, and that *it* continues to exist in non-scattered form. The idea is not that any silver chain would survive the loss of some of its links, nor that any ordinary silver chain would survive the loss of any number of insignificant links, but that some silver chains do survive the loss of some of their links, and the silver chain which was on my table at t_1 is one of these. There is, now, at t_2 , a silver chain on my table, and it consists of the clasp and twenty-nine of the thirty links which formed the silver chain which was on my table at t_1 . Considering the circumstances, it seems to me that it would be unreasonable to deny that

- (12) the silver chain on my table at t_1 =
the silver chain on my table at t_2 .

But notice that a member of the set A is now, at t_2 , on the

floor; and so, if there is such a thing as the fusion of A, a part of this fusion is now on the floor. But no part of the silver chain which is on my table at t_2 is, now, at t_2 on the floor. Hence, if there is such a thing as the fusion of A, then

(13) the silver chain on my table at $t_2 \neq$ the fusion of A.

On the other hand, as we saw earlier, since A is a non-empty set, if the axioms of the calculus of individuals are true under their intended interpretation, then there is such a thing as the fusion of A, and given (7),

(8) the silver chain on my table at $t_1 =$ the fusion of A.

The difficulty is that the set of (8), (12), and (13) is inconsistent.

And there are other inconsistent sets in the offing. For suppose that between t_2 and t_3 , I pick up the link on the floor and replace it in its original position in the silver chain which is on my table at t_2 . It would be unreasonable to deny that

(14) the silver chain on my table at $t_2 =$ the silver chain on my table at t_3 .

But since the set A is non-empty, if the axioms of the calculus of individuals are true under their intended interpretation,

there is such a thing as the fusion of A; and since it is also true that

(15) $(\forall z)(z \text{ is discrete from the silver chain on my table at } t_3 \Leftrightarrow (\forall w)(w \in A \rightarrow z \text{ is discrete from } w)),$

(16) the silver chain on my table at $t_3 =$ the fusion of A.

But now, the set of (13), (14), and (16) is inconsistent.

And again, if instead of removing just one link between t_1 and t_2 , as I supposed I did, if all the links from the silver chain on my table at t_1 were separated from one another, and scattered on the floor between t_1 and t_2 , one would be inclined to say that

(17) the silver chain on my table at t_1 does not exist at t_2 .

But since every member of A exists at t_2 , if the axioms of the calculus of individuals are true under their intended interpretation, then

(18) the fusion of A exists at t_2 .

However, the set of (8), (17), and (18) is inconsistent.

It will be immediately suggested that these difficulties arise because of our failure to take time into account.

Consider, for instance, the set of (8), (12), and (13). It is going to be suggested that the propositions expressed by these sentences must be distinguished from those expressed by

(8') It is the case at t_1 that the silver chain on my table at t_1 = the fusion of A,

(12') It is the case at t_2 that the silver chain on my table at t_1 = the silver chain on my table at t_2 ,

and

(13') It is not the case at t_2 that the silver chain on my table at t_2 = the fusion of A,

respectively; and that it is the set of these latter propositions rather than those expressed by (8), (12), and (13) which correctly describe the case at hand. But unlike the set of (8), (12), and (13), it would be said that the set of (8'), (12'), and (13') is not inconsistent. Similarly, the propositions expressed by (14), and (16), according to this proposal, are to be distinguished from those expressed by

(14') It is the case at t_3 that the silver chain on my table at t_2 = the silver chain on my table at t_3 ,

and

(16') It is the case at t_3 that the silver chain on my table at $t_3 =$ the fusion of A,

respectively. And though the conjunctior of (13), (14), and (16) is inconsistent, it would be suggested that that is not a problem for the intended interpretation of the axioms of the calculus of individuals since the case in question is correctly described instead by the conjunction of (13'), (14'), and (16'), which is not inconsistent.

Now, I think that it is not clear how, for instance, (8'), (12'), and (13') are to be understood. (8') admits of a reading under which it expresses the proposition which is expressed by

(8.1) It is the case at t_1 that $(\exists x)(\exists y)(x = \text{the silver chain on my table at } t_1 \cdot y = \text{the fusion of A} \cdot x=y)$.

But it also admits of a reading under which it expresses the proposition which is expressed by

(8.2) $(\exists x)(\exists y)(x = \text{the silver chain on my table at } t_1 \cdot y = \text{the fusion of A} \cdot \text{it is the case at } t_1 \text{ that } x=y)$.

The ambiguity in (8') is, I think, due not to a semantic ambiguity in any of the expressions in that sentence; it is, instead, a syntactic ambiguity which arises from assigning different scopes to the occurrence of 'It is the case at t_1 that' in (8').

Similarly, (12') admits of a reading under which it expresses the proposition which is expressed by

(12.1) It is the case at t_2 that $(\exists x)(\exists y)(x = \text{the silver chain on my table at } t_1 \cdot y = \text{the silver chain on my table at } t_2 \cdot x=y)$.

But, it also admits of another reading under which it expresses the proposition which is expressed by

(12.2) $(\exists x)(\exists y)(x = \text{the silver chain on my table at } t_1 \cdot y = \text{the silver chain on my table at } t_2 \cdot \text{it is the case at } t_2 \text{ that } x=y)$.

Likewise, (13') admits of a variety of readings among which are those identified by

(13.1) It is not the case at t_2 that $(\exists x)(\exists y)(x = \text{the silver chain on my table at } t_2 \cdot y = \text{the fusion of } A \cdot x=y)$

and

(13.2) $(\exists x)(\exists y)(x = \text{the silver chain on my table at } t_2 \cdot y = \text{the fusion of } A \cdot \text{it is not the case at } t_2 \text{ that } x=y)$.

It is, I think worth stressing that the conjunction of (8.2), (12.2), and (13.2) expresses a proposition which is not

true. The conjunction of these sentences expresses a true proposition only if

- (19) $(\exists x)(\exists y)(\exists z)(x = \text{the silver chain on my table at } t_1 \cdot$
 $y = \text{the fusion of } A \cdot z = \text{the silver chain on my table}$
 $\text{at } t_2 \cdot \text{it is the case at } t_1 \text{ that } x=y \cdot \text{it is the case}$
 $\text{at } t_2 \text{ that } x=z \cdot \text{it is not the case at } t_2 \text{ that } y=z)$

expresses a true proposition. And, obviously, (19) expresses a true proposition only if

- (20) $(\exists x)(\exists y)(\exists z)(\text{It is the case at } t_1 \text{ that } x=y \cdot \text{it is the}$
 $\text{case at } t_2 \text{ that } x=z \cdot \text{it is not the case at } t_2 \text{ that}$
 $y=z)$

expresses a true proposition.

Now, some people seem to suggest that (20), in fact, does express a true proposition and that the cases of fusion and fission are evidence for it. This, I think, is simply a confusion; and it ought be recognized that provided that the quantifiers in (20) are interpreted objectually, the claim that (20) expresses a true proposition is false. For suppose that the proposition expressed by (20) is true, and suppose that the time now is t' . There is then a sequence S , such that

- (i) S satisfies at t' the open sentence
 $(\text{It is the case at } t_1 \text{ that } x=y \cdot$

it is the case at t_2 that $x=z$.

it is not the case at t_2 that $z=y$).

But then,

(ii) S satisfies at t' the open sentence

It is the case at t_1 that $x=y$,

and

(iii) S satisfies at t' the open sentence

It is the case at t_2 that $x=z$,

and also,

(iv) S satisfies at t' the open sentence

It is not the case at t_2 that $z=y$.

But surely, for any sequence x , any open sentence ϕ , and any times m and n ,

(21) x satisfies at m the open sentence ϕ if and only if

x satisfies at n the open sentence \ulcorner It is the case at m that $\phi \urcorner$.

But given (21), and (ii)

(v) S satisfies at t_1 the open sentence

$x=y$,

and given (21), and (ii)

- (vi) S satisfies at t_2 the open sentence
 $x=z$.

But now given (v),

- (vii) S satisfies at t_1 every instance of the schema
 $Fx \rightarrow Fy$,

but, since

It is the case at t_2 that $x=x \rightarrow$ it is the case
 at t_2 that $x=y$

is an instance of ' $Fx \rightarrow Fy$ ', given (vii),

- (viii) S satisfies at t_1 the open sentence
 It is the case at t_2 that $x=x \rightarrow$
 It is the case at t_2 that $x=y$.

But since

- (ix) S satisfies at t_1 the open sentence
 It is the case at t_2 that $x=x$,

given (viii),

- (x) S satisfies at t_1 the open sentence
 It is the case at t_2 that $x=y$.

And given (21) and (x),

(xi) S satisfies at t_2 the open sentence
 $x=y$

But now given (vi) and (xi)

(xii) S satisfies at t_2 the open sentence
 $z=y$

and given (21) and (xii),

(xiii) S satisfies at t' the open sentence
 It is the case at t_2 that $z=y$.

But (xiii) contradicts (iv); and hence the proposition expressed by (20) is not true.

Advocates of the view that (20) expresses a true proposition would perhaps object to my claim that if a sequence satisfies at t_1 the open sentence ' $x=y$ ', then it satisfies at t_1 every instance of the schema ' $Fx \rightarrow Fy$ '. In discussions of this issue one frequently finds the remark that the substitution class of 'F' in

$$x=y \rightarrow (Fx \rightarrow Fy)$$

must be suitably restricted in order *to rule out false instances*. This remark calls for explanation. If the restriction on admissible substituends for 'F' in this schema is used to fix the

meaning of '=', then '=' would not express the relation of identity, since the relation may not be a *total* indiscernibility relation; i.e., objects which are related by this relation may not be indiscernible with respect to every open sentence. On the other hand, if the object of the restriction is to rule out what are claimed to be false instances of this schema, then it is pointless, since there simply are not false instances of this schema.⁴

Some advocates of the view that not all identities are permanent may object to my treating

It is the case at t_1 that $x=y$

as an open sentence. It should be noted that the sentential operator 'It is the case at t_1 that' is referentially opaque. If t_1 is sometime during 1964, then

It is the case at t_1 that the president of the U.S.
is a Democrat

expresses a true proposition, but

It is the case at t_1 that Reagan is a Democrat

does not express a true proposition; even though

⁴ See Richard Cartwright, 'Indiscernibility Principles', *Midwest Studies in Philosophy*, vol.

Reagan = the president of the U.S.

expresses a true proposition, and

Reagan is a Democrat,

and

The president of the U.S. is a Democrat

both express false propositions. Now, Quine has claimed that it is illicit to put a variable of quantification in the scope of an opaque operator to be bound by a quantifier outside of that scope. Since, 'It is the case at t_1 that' is a referentially opaque operator, if Quine is right then

It is the case at t_1 that $x=y$

is not an open sentence. I have argued in chapter 1 that Quine's claim is false. But it should be noticed that even if Quine's claim is true, it is of no help to an advocate of the view that some identities are temporary, since an identity is temporary only if there is a sequence which satisfies *the open sentence*

It is the case at t_1 that $x=y$

and fails to satisfy *the open sentence*

It is the case at t_2 that $x=y$

where t_1 and t_2 are distinct times.

The temptation to think that (20) expresses a true proposition perhaps arises from a failure to distinguish it from the claim that not all true statements of identity are permanently true. But it ought to be recognized that, though the latter claim is true, it does not imply (20).

I have argued that the proposition expressed by (20) cannot be true. But since the proposition expressed by the conjunction of (8.2), (12.2), and (13.2) can be true only if the proposition expressed by (20) can be true, the proposition expressed by the conjunction of these three sentences cannot be true either. But, I do not think that this is a result which should dismay a friend of the calculus of individuals. A friend of the calculus of individuals would be content to affirm that the proposition expressed by the conjunction of (8.1), (12.1), and (13.1) is true. Since the conjunction of (8.1), (12.1), and (13.1) does not imply the conjunction of (8.2), (12.2), and (13.2), the falsity of the proposition expressed by the latter conjunction is not a reason against the intended interpretation of the axioms of the calculus of individuals. However, I shall argue that the conjunction of (8.1), (12.1), and (13.1) faces an independent difficulty of its own - one which a friend of the calculus of individuals ought to try

to resolve.

2.2 TENSE

I have suggested that the claim that the proposition expressed by the conjunction of (8.1), (12.1), and (13.1) is true presents a difficulty for the intended interpretation of the axioms of the calculus of individuals. The difficulty I have in mind is this. If (8.1) expresses a true proposition, then, at t_1 , there is such a thing as the fusion of A and it is identical with the silver chain on my table at t_1 ; and if (12.1) expresses a true proposition as well, then, at t_2 , there is such a thing as the silver chain on my table at t_2 and it is identical with the thing which, at t_1 , fused A. But now, if (13.1) also expresses a true proposition then either at t_2 there is no such thing as the fusion of A, or at t_2 the thing which fuses A is not identical with the silver chain on my table at t_2 . However, if the axioms of the calculus of individuals are true under their intended interpretation, then, since every member of A exists at t_2 , at t_2 there is such a thing as the fusion of A; and hence, at t_2 the thing which fuses A is not identical with the silver chain on my table at t_2 . But since the silver chain on my table at t_2 is the thing which fused A at t_1 , the thing which fuses A at t_2 is not identical with the thing which fused A at t_1 ; — and hence, if the conjunction of (8.1), (12.1), and (13.1) expresses a true proposition and the

axioms of the calculus of individuals are true under their intended interpretation, then distinct objects fuse A at t_1 and t_2 .

Now, I think that it is not easy to see how the idea that distinct objects may fuse the same set at different times may be reconciled with the intended interpretation of the calculus of individuals. If the axioms of this calculus are true under their intended interpretation, then for any non-empty set of material objects there is exactly one thing which fuses that set; and hence for any non-empty set of material objects there is such a thing as *the* fusion of that set. But the axioms of the calculus make no explicit reference to time and under their intended interpretation they are not construed as asserting that any non-empty set of material objects is *fused at* a time. Indeed, the calculus is most naturally interpreted as making untensed and non-dated claims. Now what I envisage as a difficulty for the intended interpretation of this calculus is not that if distinct objects fuse A at different times, then there is no such thing as *the* fusion of A. For surely distinct individuals have been presidents of the U.S. at different times, but it is false that there is no such thing as the president of the U.S., and similarly, though different numbers may have numbered the planets at different times, it is false that there is no such thing as the number of the planets. The difficulty

is that if the conjunction of (8.1), (12.1), and (13.1) expresses a true proposition and there is such a thing as the fusion of A, then, as in the case of 'The president of the U.S.' and 'The number of the planets', 'the fusion of A' must be understood as containing an indexical element or a tense, which in a suitable sentential context contributes towards fixing the reference of that expression. And if 'the fusion of A' is construed in this way, then the axioms of the calculus must themselves be reinterpreted as tensed statements; and, hence, the intended interpretation of the axioms of the calculus of individuals must be abandoned.

Though the reinterpretation of the axioms of the calculus of individuals as tensed statements is a radical departure from the original intended interpretation of these axioms, it is not unnatural. Indeed if the axioms of this calculus are construed as statements about objects which survive the loss of some of their parts, it quite readily suggests itself that the expression 'is part of' be so understood that it may be true, at one time to say 'x is part of y', and not at another. Now a very natural way in which the calculus of individuals may be revised as a tensed calculus of individuals is to take the axioms of this calculus as being modified by the temporal sentential operator 'It is always the case that'. The operator 'It is always the case that' is analogous to the modal operator 'It is necessary

that'. Where S is a closed sentence, intuitively, 'It is always the case that S ' is true if and only if S is true at all times; and if S is an open sentence in one variable, then, intuitively, 'It is always the case that S ' is true of an object x if and only if S is true of x at all times. Thus,

It is always the case that Reagan = Reagan,

is true, but

It is always the case that Reagan = the president
of the U.S.

is not. Similarly, Reagan satisfies the open sentence

It is always the case that x = Reagan,

but he does not satisfy the open sentence

It is always the case that x = the president of
the U.S.

Formally, the semantics for 'It is always the case that' (to be abbreviated as 'L') may be given following Kripke semantics for ' \Box '.⁵ A model structure is defined as a triple $\langle G, K, R \rangle$

⁵ Saul Kripke, 'Semantical Considerations on Modal Logic' in *Acta Philosophica Fennica*, 16 (1963), pp. 83-94.

together with a function, ψ , where K is a set (the set of instants or moments), R is a reflexive relation on K , G (the present moment) is a member of K , and $\psi(H)$ is a set for each $H \in K$. Intuitively, $\psi(H)$ is the set of things which exist at H . Let $U = \bigcup_{H \in K} \psi(H)$, and let U^n be the n th cartesian product of U with itself. A model on a model structure $\langle G, K, R \rangle$ is a binary function $\phi(P^n, H)$, where ' P^n ' ranges over n -adic predicate letters, ' H ' ranges over members of K , and $\phi(P^n, H) \subseteq U^n$, where $n \geq 1$, otherwise $\phi(P^n, H) = T$ or F .

The clauses of the inductive definition are as follows:

- (i) For an atomic formula
 $\phi(P^n(x_1, \dots, x_n), H) = T$ with respect to an assignment
 a_1, \dots, a_n of elements of U to x_1, \dots, x_n , if and only
 if $\langle a_1, \dots, a_n \rangle \in \phi(P^n, H)$.
- (ii) $\phi(\neg A(x_1, \dots, x_n), H) = T$ with respect to an assignment
 a_1, \dots, a_n of elements of U to x_1, \dots, x_n , if and only
 if $\phi(A(x_1, \dots, x_n), H) \neq T$ with respect to that assignment.
- (iii) $\phi((A(x_1, \dots, x_n) \wedge B(y_1, \dots, y_n)), H) = T$ with respect to an
 assignment of a_1, \dots, a_n of elements of U to x_1, \dots, x_n ,
 and b_1, \dots, b_n of elements of U to y_1, \dots, y_n , if and
 only if both $\phi(A(x_1, \dots, x_n), H) = T$ and $\phi(B(y_1, \dots, y_n), H) = T$
 with respect to that assignment.

- (iv) $\phi(LA(x_1, \dots, x_n), H) = T$ with respect to a given assignment if and only if $\phi(A(x_1, \dots, x_n), H') = T$ with respect to that assignment, for every H' such that HRH' .
- (v) $\phi((\forall x) (x, y_1, \dots, y_n), H) = T$ with respect to an assignment a_1, \dots, a_n of elements of U to y_1, \dots, y_n if and only if for every $a \in \psi(H)$ $\phi(A(x, y_1, \dots, y_n), H) = T$ with respect to that assignment of a_1, \dots, a_n to y_1, \dots, y_n .

For the intended interpretation of 'L', we take R to be a transitive and a symmetric relation; thus intuitively, all accessible moments of time are accessible from one another.

The Revised Calculus of Individuals takes the two-place predicate 'D' as primitive, where under the intended interpretation, 'Dxy' is to be read as 'x is discrete from y', and defines 'Pxy', 'Oxy', and 'FuxS' as in (CI-Df.1), (CI-Df.2), and (CI-Df.3) respectively. The calculus contains the following distinctive axioms:

- | | | |
|------------|---|----------------|
| (RCI.Ax.1) | $L((Pxy \cdot Pyx) \rightarrow x=y)$ | identity axiom |
| (RCI.Ax.2) | $L(Oxy \leftrightarrow \neg Dxy)$ | overlap axiom |
| (RCI.Ax.3) | $L((\exists x)x \in S \rightarrow (\exists y)FuyS)$ | fusion axiom |

Under the intended interpretation of the axioms of the revised calculus, the free individual variables are taken as ranging over material objects which exist now, whereas variables which

are bound by a quantifier inside the scope of 'L' range over material objects which exist at any time past, present, or future. Under the intended interpretation, the fusion axiom says that for any set of material objects, S, it is always the case that if there *exists* a member of S, then there also *exists* something which *fuses* S; or equivalently, that for any set of material objects, S, if at any time there exists a member of S, then there also exists at that time something which at that time fuses S. And, under the intended interpretation the identity axiom says that for any material objects x, and y, it is always the case that if x is part of y and y is part of x, then x is identical with y, or equivalently, that for any material objects x, and y, if at any time x is part of y and y is part of x, then at that time x is identical with y.

Now, though under the intended interpretation of the revised calculus of individuals the verb 'to fuse' is interpreted as tensed and thus the question 'Does a material object always fuse the same sets?' becomes intelligible, it may, nevertheless, be thought that if the axioms of the revised calculus of individuals are true under their intended interpretation then material objects cannot survive the loss or removal of any of their parts. Consider again the silver chain which was on my table at t_1 . I had supposed that between t_1 and t_2 I had removed a link from the silver chain on my table at t_1 and

dropped that link on the floor, and moreover, that the silver chain on my table at t_1 was identical with the silver chain on my table at t_2 . Now, if the axioms of the Revised Calculus of Individuals are true under their intended interpretation then there is exactly one object, x , which, at t_1 , fuses the set A , i.e. the set of thirty links and a clasp which at t_1 formed the silver chain on my table at t_1 , and since at t_1 , all and only those things are discrete from the silver chain on my table at t_1 which are discrete from every member of A , the silver chain on my table at t_1 is identical with x . Let us call the set of twenty-nine links and a clasp which remain on my table at t_2 , 'B'. If the axioms of the revised calculus are true under their intended interpretation then there is exactly one object which fuses B at t_1 . Let us call this object 'Alpha'. Again, if the axioms of this calculus are true under their intended interpretation, then there is exactly one object, y , which fuses B at t_2 ; and since at t_2 all and only those things are discrete from the silver chain on my table at t_2 which are discrete from every member of B , the silver chain on my table at t_2 is identical with y . But now, since the silver chain on my table at t_2 fuses the same set at t_2 that Alpha fuses at t_1 , namely B , whatever is part at t_2 of the silver chain on my table at t_2 is part of Alpha at t_1 . But one of the things which at t_2 is part of the silver chain on my table at t_2 is the silver chain on my table

at t_2 itself. But then the silver chain on my table at t_2 is, at t_1 , part of Alpha. However, the silver chain on my table at t_2 , by hypothesis, is identical with the silver chain on my table at t_1 , and, therefore, the silver chain on my table at t_1 is, at t_1 , part of Alpha. But that is false, since Alpha, at t_1 , is a proper part of the silver chain on my table at t_1 .

However, this argument is fallacious. The step that since the silver chain on my table at t_2 fuses *the* same set at t_2 that Alpha fuses at t_1 , whatever is at t_2 part of the silver chain on my table at t_2 is part of Alpha at t_1 does not follow from the Revised Calculus of Individuals. In general, it is not provable in the Revised Calculus of Individuals that if an object x fuses a set S at t_1 , and y fuses S at t_2 then whatever is part of x at t_1 is part of y at t_2 .

Though the intended interpretation of the Revised Calculus of Individuals is not in conflict with the view that the silver chain on my table at t_1 is identical with the silver chain on my table at t_2 , it should be noted that it has a consequence which many would find perplexing. Let us suppose that the members of B do not lose or acquire any parts between t_1 and t_2 . Given that Alpha fuses B at t_1 , and the silver chain on my table at t_2 fuses B at t_2 , Alpha presumably has the same atomic parts (or even proper parts) at t_1 that the silver chain on my table at t_2 has at t_2 ; and yet Alpha is not

identical with the silver chain on my table at t_2 : - this is so even if the atomic parts of Alpha retain their organizational structure at t_2 . Now I think that it would not be reasonable to reject the intended interpretation of the Revised Calculus of Individuals on the ground that it has this consequence, because I think that there are strong independent reasons for holding that it is possible for distinct objects to have exactly the same atomic parts at different times.

There is, however, another consideration that weighs against the intended interpretation of the Revised Calculus of Individuals. Alpha, it will be remembered, fused B at t_1 . It would be reasonable to think that Alpha goes on existing after t_1 . After all, no part of Alpha was removed between t_1 and t_2 . But if it is reasonable to think that Alpha exists at t_2 , then it is also equally reasonable to think that Alpha fuses B at t_2 . But if Alpha fuses B at t_2 then if the axioms of the Revised Calculus of Individuals are true under their intended interpretation, Alpha is identical with the silver chain on my table at t_2 . But that is false, since the silver chain on my table at t_2 is identical with the silver chain on my table at t_1 which is distinct from Alpha. So, though under their intended interpretation the axioms of the Revised Calculus of Individuals are not in conflict with the judgement that the silver chain on my table at t_1 is identical with the silver chain on my table at t_2 , they are

in conflict with the judgement that the silver chain on my table at t_1 is identical with the silver chain on my table at t_2 and that Alpha fuses B at t_2 .

Some at this stage would perhaps propose to reject my assumption that the silver chain on my table at t_1 is identical with the silver chain on my table at t_2 . But notice that the intended interpretation of the Revised Calculus of Individuals gives rise to difficulties parallel to the one I have been considering if it is assumed that some objects acquire some parts, that some objects do not survive the removal of some of their parts, that some objects do not survive certain changes in the organization of their parts, or that some objects do not perish if some of their parts perish. I think that we should try to find a more reasonable alternative before we are forced to reject all of these assumptions.

Some would perhaps be inclined to reject the fusion axiom. They might have felt that the fusion axiom is excessively strong, and that the problem that I have been considering for the intended interpretation of the Calculus of Individuals reinforces their claim that the fusion axiom, as it is interpreted, is not true. However, it should be noted that an analogous problem arises even if the fusion axiom is false

under its intended interpretation.⁶ For even if it is not true that any time at which a member of a set exists, there also exists something which fuses it, it seems reasonable to suppose that there does exist something which fuses B at t_1 and that it also fuses B at t_2 . But then given the identity axiom, this object is identical with the silver chain on my table at t_2 . But that must be false, since this object is, at t_1 , a proper part of the silver chain on my table at t_1 which, in turn, is identical with the silver chain on my table at t_2 .

And even if no material objects are fusions, it seems that we can still reconstruct our problem. Following Thomson, it may be argued that if there is such a thing as the mass of silver on my table at t_1 , then it very naturally suggests itself that

- (11) the silver chain on my table at t_1 = the mass of silver on my table at t_1 .

But we can also truly say that

- (22) The silver chain on my table at t_1 is on my table at t_2 .

⁶ See J.J. Thomson: 'Parthood and Identity Across Time', *Journal of Philosophy*, vol. LXXX, pp. 201-220.

Now, the conjunction of (11) and (22) entails that

(23) The mass of silver on my table at t_1 is on my
table at t_2

which is not true, for the mass of silver on my table at t_1 is only partly on my table at t_2 — a part of it is on the floor at t_2 . Hence, the identity sentence (11) is false. I think that this argument does not settle that (11) is false. Notice that our confidence in asserting (22) comes from the thought that the silver chain on my table at t_1 survived the removal of a link. But then we, presumably, think that some objects do not always have the same parts. Now we are given that something which at t_1 was a part of the mass of silver on my table at t_1 is at t_2 not on my table, but that does not give us reason to believe that the mass of silver on my table at t_1 is not on my table at t_2 , unless we are given reason to believe that the mass of silver on my table at t_1 has the same parts at t_2 that it had at t_1 .

However, I think that there is still trouble for (11).

It seems to me natural to think that there is not only the mass of silver on my table at t_1 , but there are also several other quantities (or portions) of silver on my table at t_1 , each of which is a proper part of the mass of silver on my table at t_1 . There is among these quantities of silver on my table at t_1 one

which overlaps with all and only those quantities of silver on my table at t_1 which do not overlap with the link which is on the floor at t_2 . Let us call this quantity of silver Beta. It seems to me very reasonable to assume that Beta exists at t_2 and has at t_2 the same parts that it had at t_1 . But now if the link which at t_2 is on the floor is, at t_2 , not part of the mass of silver on my table at t_1 , then, at t_2 , Beta and the mass of silver on my table at t_1 have the same parts. But, then, it would very naturally suggest itself that Beta is identical with the mass of silver on my table at t_1 . But that must be false since Beta and the mass of silver on my table at t_1 do not have the same parts at t_1 . Hence (23) is after all false; but then given that (22) is true, (11) is also false.

I have argued that the problem that I am raising arises even if the fusion axiom is false under its intended interpretation, and secondly that this problem arises even if it is true that a material object fuses a set at one time and fails to fuse it at another time (even though it exists at that other time). One of the things which is common to my various reconstructions of the problem is the assumption that if at any time some material objects, x , and y , have the same parts then x is identical with y , or equivalently, given that parthood is, by definition, a reflexive and a transitive relation, that if at

any time some material objects x , and y are parts of each other then x is identical with y . This is indeed guaranteed by the axiom of identity under its intended interpretation of the Revised Calculus of Individuals. I have argued that this axiom does not imply that an object has the same parts at all times at which it exists and secondly that it is compatible with the idea that distinct objects have the same atomic (or even proper) parts at *different* times.

It would be instructive to compare the issue that I have raised here concerning the intended interpretation of the axiom of identity with an analogous issue concerning the axiom of extensionality for sets or classes. I suppose that most of us are inclined to think that sets do not undergo change of membership. But one may wonder what underlies our confidence in this thought. It would not do to appeal to the principle that for any sets, x , and y

$$x=y \rightarrow (\forall z)(z \in x \rightarrow z \in y);$$

for that does not establish that sets do not lose or gain members any more than an appeal to the principle that for any material objects x , and y ,

$$x=y \rightarrow (\forall z)(z \text{ is a property of } x \rightarrow z \text{ is a property of } y)$$

establishes that material objects do not lose or gain properties.

On the other hand, the axiom of extensionality, i.e. that for any sets x , and y ,

$$(\forall z)(z \in x \Leftrightarrow z \in y) \rightarrow x=y$$

does not establish either that sets do not lose or gain members. Suppose that a set S_1 has at t_1 exactly three members, x , y , and z . Suppose, moreover, that between t_1 and t_2 , z goes out of existence. At t_2 there is a set, call it ' S_2 ', which has, at t_2 , exactly two members, x and y . Someone who thought that S_1 survived the loss of z would be inclined to say that

$$(24) \text{ It is the case at } t_2 \text{ that } S_1=S_2.$$

Now if we are to consider the question of whether sets survive the loss or gain of members at all seriously, we had better construe the axiom of extensionality as a tensed statement or a statement with temporal qualifications which asserts that for any sets x , and y , if at any time, x , and y have the same members then x is identical with y ; i.e.,

(25) It is always the case that

$$[(\forall z)(z \in x \Leftrightarrow z \in y) \rightarrow x=y]$$

But now notice that the observation that S_1 does not have the same members at t_1 as S_2 has at t_2 does not imply that the

conjunction of (24) and (25) is false. What seems to be missing is the claim that S_2 has the same members at t_1 as it had at t_2 . But we cannot assume that without begging the question.

There is, however, still trouble for (24); the claim that $S_1=S_2$. Given that S_1 exists at t_1 , there is another set, call it 'S', which has as members at t_1 just x and y. I think that we are entitled to ask: what happened to S' at t_2 ? It would be unreasonable to deny that

(26) S' has at t_2 the same members as it had at t_1 .

But if (26) expresses a true proposition then, at t_2 , S' has the same members that S_2 has at t_2 . And given that (25) expresses a true proposition, at t_2 , S' is identical with S_2 . But if (24) expresses a true proposition then, at t_2 , S' is identical with S_1 . But then, at t_1 , S' is identical with S_1 . However, that is false, since, at t_1 , S' is a proper subset of S_1 . Now, I think that we simply cannot deny (25), i.e. the axiom of extensionality; it defines the concept Set; and we are not really in a position to say that (26) expresses a false proposition. Hence, it is reasonable to deny the identity statement (24) – and, in general, it is reasonable to deny that sets undergo changes of membership. Now, my argument against the intended interpretation of the axiom of identity of the

Revised Calculus of Individuals has a similar form. I have argued that the conjunction of

(12) The silver chain on my table at t_1 = silver chain on my table at t_2 ,

(27) $L[(\forall z)(Pzx \leftrightarrow Pzy) \rightarrow x=y]$,

and

(28) There is something which fuses B both at t_1 and t_2

expresses a proposition which cannot be true, if the silver chain on my table at t_2 and the thing which fuses B at t_1 do not have the same parts at t_1 but the silver chain on my table at t_2 and the thing which fuses B at t_2 have the same parts at t_2 . I think that we cannot reasonably deny (12). Material objects do, after all, lose and gain parts - and, in any case, there would still be trouble even if we denied (12); and we are not really in a position to deny (28). Hence, it is reasonable to believe that (27) (or, equivalently, the identity axiom of the Revised Calculus of Individuals) is not true under its intended interpretation. If (27) is not true under its intended interpretation, then, I think it shall pay us to try to find a replacement for it - one which preserves at least some of the intuitions which make (27) so compelling. Professor Thomson,

in her paper, 'Parthood and Identity Across Time', offers such a proposal.⁷

⁷ *Journal of Philosophy*, vol. LXXX, pp. 201-220

2.3 THOMSON'S CROSS-TEMPORAL CALCULUS OF INDIVIDUALS

Thomson proposes that we take a three place predicate 'Dxy@t' as primitive and read it as: 'x is discrete from y at t'. For the intended interpretation, the individual variables, 'x', 'y', etc. are taken as ranging over objects which exist at one time or another, and the temporal variable 't' is taken as ranging over time-points. Thomson's cross-temporal Calculus of Individuals defines 'Ex@t' (read as: x exists at t), 'Pxy@t' (read as: x is part of y at t), 'Oxy@t' (read as: x overlaps y at t) and 'FuxS@t' (read as: x fuses a set S at t) as follows:⁸

$$(CCI-Df.1) \quad Ex@t =df. \neg (\forall y)Dxy@t$$

$$(CCI-Df.2) \quad Pxy@t =df. Ex@t \cdot Ey@t \cdot (\forall z)(Dzy@t \rightarrow Dzx@t)$$

$$(CCI-Df.3) \quad Oxy@t =df. (\exists z)(Pzx@t \cdot Pzy@t)$$

$$(CCI-Df.4) \quad FuxS@t =df. Ex@t \cdot (\forall y)(Dyx@t \Leftrightarrow (\forall z)[(z \in S \cdot Ez@t) \rightarrow Dyz@t])$$

The Calculus contains the following distinctive axioms:

$$(CCI-Ax.1) \quad \text{The Overlap Axiom:}$$

$$Oxy@t \Leftrightarrow \neg Dxy@t$$

⁸ I have changed the notation to conform to the notation in the rest of this paper.

(CCI-Ax.2) The Identity Axiom:

$$x=y \Leftrightarrow (\forall t) [(Ex@t \vee Ey@t) \rightarrow (Pxy@t \cdot Pyx@t)]$$

(CCI-Axs.3) The Fusion Axioms:

For any set of n sets, S_1, \dots, S_n , if $n=1$,

$$(3.1) \quad (\exists x)(x \in S \cdot Ex@t) \rightarrow (\exists y)FuyS@t, \text{ if } n=2$$

$$(3.2) \quad t_1 \neq t_2 \cdot (\exists x)(x \in S_1 \cdot Ex@t_1) \cdot (\exists y)(y \in S_2 \cdot Ey@t_2) \rightarrow$$

$$(\exists z)(FuzS_1@t_1 \cdot FuzS_2@t_2),$$

(and so on for $n > 2$)

The identity axiom of Thomson's Cross-Temporal Calculus of Individuals marks the most significant departure from the Revised Calculus of Individuals I discussed in the previous section; in particular, its consequence that

$$(29) \quad (\forall t) [(Ex@t \vee Ey@t) \rightarrow Pxy@t \cdot Pyx@t] \rightarrow x=y$$

is noteworthy. Under its intended interpretation, it says that any material objects x , and y are identical if at *all* times at which either x or y exists, x and y are parts of each other. In contrast, an analogue, in the language of the Cross-Temporal Calculus, of the old axiom of identity of the Revised Calculus is:

$$(30) \quad (\forall t) [[(Ex@t \vee Ey@t) \rightarrow (Pxy@t \cdot Pyx@t)] \rightarrow x=y]$$

or equivalently:

$$(31) \quad (\exists t)[(\exists x@t \vee \exists y@t) \rightarrow (Pxy@t \cdot Pyx@t)] \rightarrow x=y$$

Clearly, the weaker (29) does not entail (30). Unlike (30), (29) does not have the consequence that distinct objects are not part of each other at any time. I argued in the previous section that the intended interpretation of the Revised Calculus of Individuals is not in conflict with their being distinct individuals which fuse the same set at *different* times. The intended interpretation of Thomson's Cross-Temporal Calculus of Individuals is not in conflict with there being distinct individuals which fuse the same set at *a* time. It is this feature of Thomson's Cross-Temporal Calculus of Individuals which allows us to resolve our problem of identity across time. Our problem, it will be remembered, arose as follows: We supposed that A, the set of thirty links and a clasp, is at t_1 fused by the silver chain on my table at t_1 . At t_1 , another object, Alpha, fused a proper subset of A, i.e. B, the set of twenty-nine links and a clasp which remain on the table from t_1 through t_2 . We are inclined to say that the silver chain on my table at t_1 survived the removal of a link between t_1 and t_2 , and that at t_2 it fuses B. But we are also inclined to say that Alpha fuses B at t_2 . However, if the axioms of the Revised Calculus of Individuals are true under their intended interpre-

tation and Alpha and the silver chain on my table at t_1 each fuses B at t_2 , then they are identical. But that is not true, since they fuse different sets at t_1 .

Now, unlike the Revised Calculus of Individuals, the Cross-Temporal Calculus of Individuals does not have the consequence that if any objects x , and y fuse the same set at some time then x is identical with y . Hence, if the logic of parthood is the Cross-Temporal Calculus of Individuals, we may consistently affirm that Alpha and the silver chain on my table at t_1 are distinct but that they fuse the same set at t_2 .

It should be noticed that for the logic of parthood to be the Cross-Temporal Calculus of Individuals it is not sufficient that the axioms of this calculus, and in particular the axiom of identity, be true under their intended interpretation; it is also required that the stronger axiom of the Revised Calculus of Individuals, i.e.

$$(31) \quad (\exists t)[(Ex@t \vee Ey@t) \rightarrow (Pxy@t \cdot Pys@t)] \rightarrow x=y$$

or the theorem that

$$(32) \quad (\exists t)[(Ex@t \vee Ey@t) \rightarrow (FuxS \cdot FuyS)] \rightarrow x=y$$

be false under their intended interpretation.

Now, the claim that (31) or (32) are not true under their intended interpretation gives rise to a difficulty which, I

think, deserves our attention. For if (31) or (32) are not true (under their intended interpretation) then there are distinct individuals which fuse the same set at a given time. Alpha and the silver chain on my table at t_1 , according to the proposal we are considering, are such individuals. Now I have frequently used the expression 'the silver chain on my table at t_1 ', implying that at t_1 there was only one silver chain on my table. Considering the case that I have described, it would be very natural to say, as indeed I do, that there is only one silver chain on my table at t_2 as well. But if I am right in thinking that there is only one silver chain on my table at t_2 , then, since Alpha is on the table at t_2 , Alpha is not a silver chain. However, Alpha occupies exactly the same place as a silver chain, it has the same parts as a silver chain, and these parts are put together in exactly the same way as the parts of a silver chain. What, then, prevents Alpha from being a silver chain?

It seems that if (32) is false under its intended interpretation then a problem of a very general nature arises. For if (32) is false under the intended interpretation then presumably it is true of a very large class of count nouns that either it is not sufficient for a member of this class to correctly apply to an individual that the individual has a specified shape, or size or parts of a specified kind; or it correctly applies to two or more individuals which occupy exactly the

same place at a given time. Now, it might be suggested that, for instance, what prevents Alpha from being a silver chain at t_2 is its history. It would be said that Alpha is not a silver chain at t_2 because it was not a silver chain at t_1 ; and that it was not a silver chain at t_1 because at t_1 it was a proper part of something which at t_1 was a silver chain. I think that this is not a satisfactory explanation. Evidently, not every substitution instance of the schemas

- (i) $(\forall x)(x \text{ is an } F \text{ at } t \rightarrow (\forall t')(x \text{ exists at } t' \rightarrow x \text{ is an } F \text{ at } t'))$

and

- (ii) $(\forall x)(x \text{ is an } F \text{ at } t \rightarrow x \text{ is not, at } t, \text{ a proper part of anything which is an } F \text{ at } t),$

where 'F' is replaced by a count noun, is true. (ii) yields a falsehood on substitution of 'building' for 'F', and (i) yields a falsehood on substitution for 'F' of 'philosopher' and 'long silver chain'. But then what reason is there for thinking that both (i) and (ii) yield truths on substitution for 'F' of 'silver chain'? I am inclined to think that no satisfactory answer to this question is available. Therefore I am inclined to think that if we are prepared to say that the silver chain

on my table at t_1 survived the removal of a link, then we had better accept that Alpha is also a silver chain.

BIOGRAPHICAL NOTE

Ali Akhtar was born on 18 September, 1947, in the state of Jammu and Kashmir, the eldest son of Syed Mohammad Akbar Kazmi and Begum Mumtaz Jafri. He received his early education in Pakistan, and graduated from the Forman Christian College of the Panjab University in 1966. He received master's degrees in English and Philosophy from the Panjab University in 1968 and 1969 respectively, and an M.A. in Philosophy from Dalhousie University in 1972. He commenced graduate studies in the philosophy department at M.I.T. in 1972. In 1976, he joined the philosophy department of the University of Calgary as a visiting assistant professor. Since then, he has served as assistant professor at a number of Canadian universities including McGill, the University of British Columbia, and Simon Fraser University. Ali Akhtar is married to Ghazala Kazmi, and they have two children, Mustapha and Batool.