FIRST ORDER LOGIC AS A FORMAL LANGUAGE: AN INVESTIGATION OF CATEGORIAL GRAMMAR

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ABSTR CT

FIRST ORDER LOGIC AS A FORMAL LANGUAGE: AN INVESTIGATION OF CATEGORIAL GRAMMAR

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Submitted to the Department of Philosophy and Linguistics August 1976 in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

This work is primarily a detailed investigation of categorial grammar--a particularly simple kind of context free phrase structure grammar with a uniform connection between the syntactic and semantic structures which it assigns. Explicitly, the work is concerned with an elaboration of the standard notion of categorial grammar due to Ajdukiewicz, Montague, Lewis, and Geach, with exposure of the limitations of such grammars, development of a more general, "extended" categorial grammar, and application of the extended grammar to the language of first order logic and to a closely related fragment of English. Implicitly, the work is concerned with clarifying the notion of logical form.

Parts 1-3 introduce the basic notions of categorial grammar, emphasizing the syntactic component. Part 4 develops a categorial grammar for a special language tailor-made to fit this kind of grammar. Parts 5 and 6 attempt to apply categorial grammar to the language of first order logic, thereby revealing limitations of the categorial framework, while also analyzing the role of variables in quantification. Since categorial grammar may be considered a formalization of Frege's theory of grammar and the language of first order logic is a close variant of Frege's Begriffsscheift, the limitations revealed are also limitations of Fregean grammatical theory. In part 7 the basic notion of categorial grammar is extended to remove the limitations while preserving the desirable features by generalizing the modes of syntactic and semantic combination allowed in the grammars. In part 8 the extended categorial framework is applied to analyze the quantificational structure of English and to compare that structure with the structure of quantification in the language of first order logic. In part 9 more complicated structures of English are dealt with and the extended categorial approach is compared with Montague's grammar for English. In part 10 attention is given to problems of the semantic interpretation of English within the categorial framework. The ideas of David Lewis are extended to provide a treatment of fully intensional contexts such as propositional attitudes. In so doing a technical problem arises in the construal of meanings (the problem does not seem limited to the specific approach to meaning of this work) the solution of which involves representing meanings by self-applicative functions.

PREFACE

In the spring of 1962 while a freshman at M.I.T. I submitted a paper entitled "Finite Automata and Linguistic Theories" for the linguistics course 23.782. The instructor, Professor Chomsky, (charitably) gave me a C+. Also attending that course was a woman named Barbara Hall.

I am afraid that I comprehended very little of what passed before my eyes and ears during that course. But the subject of transformational grammar and the excitement generated by the linguists returned frequently to my attention during the eight years I was a student at M.I.T. I remained an interested and (I hope) to some degree informed onlooker of the research in theoretical linguistics while concentrating my own efforts in the area of formal logic and philosophy of science.

Thus, I quite naturally chose the mind-body problem as my dissertation topic. I struggled, quite unhappily, with my chosen topic until fall 1974 at which time, while teaching a course on the philosophy of language at North Carolina State University, I began the research which culminated in this dissertation.

One of the sources of inspiration for my research was a talk, "Some Transformational Extensions of Montague Grammar", by Barbara Hall Partee which I had attended in April 1972 and which was my first introduction to the fascinating subject of categorial grammar. Thus it is rather fitting that mine should be the first dissertation accepted and the first degree granted

by the philosophy portion of the newly-combined department of Philosophy and Linguistics.

I have numerous debts of gratitude to my family, colleagues, and teachers acquired over the inordinately long period of nine years during which I was in the state ("process" would be misleading here) of writing my dissertation. Worthy of particular mention are my parents, for their encouragement and generosity, my wife, Connie, for her patience and encouragement, my colleagues at North Carolina State for intellectual stimulation and encouragement, and the members of my dissertation committee for making the revision and defense of my work a pleasant and rewarding experience. Special thanks are due to Julia Noell for a painstaking and expert job of typing what must have been a particularly vexing manuscript.

CONTENTS

Part 1 -- Preliminaries

It is frequently regarded as uncontentious that satisfactory grammars exist for the language of first order logic thanks to Frege and Tarski--and for the languages of other logics thanks to others. The problem now is to construct satisfactory grammars for natural languages such as English. While this may be so, I believe it worthwhile to reconsider the problem of constructing an adequate grammar for the language of first order logic (FOL); it is more complex and rewarding than usually thought.

In this paper I examine in detail a number of _categorially_ based grammars for FOL to three ends: first, to shed light on FOL (in particular, the role of variables); second, to explore the nature and adequacy of Fregean semantics; and third, to compare several proposals for the form of grammars. In doing so the notion of a categorial grammar is extended in such a way that all semantically significant structure is categorial and variables and variable-binding operators are not an essential feature of the grammatical apparatus. This extended categorial grammar appears to have promise as a means for describing the structure of natural languages, both syntactic and semantic.

For the sake of later comparison let us specify FOL as is usually done.[1] The vocabulary of FOL consists of the logical symbols $\sim$, $\&$, $\exists$, $($, $)$[2] plus infinitely many variables $x_1, x_2, \ldots$[2] plus infinitely many names $a_1, a_2, \ldots$[2] plus infinitely many predicates $F^i_1, F^i_2, \ldots$[2] of degree i, for i = 1, 2, ... The well formed expressions (WFE) of FOL can be specified by an inductive definition as follows:

(i) If P is a predicate of degree i and $n_1,\ldots,n_i$ are names, then $Pn_1\ldots n_i$ is a WFE.

(ii) If A and B are WFEs, then $\sim$A and (A&B) are WFEs.

(iii) If A is a WFE containing the name n, then $\exists$ vAv/n is a WFE, where v is any variable and Av/n is the result of replacing each occurrence of n in A by v.

(iv) A finite sequence of symbols is a WFE only if it can be shown to be a WFE by a finite number of applications of (i) - (iii).

FOL is a notational variant of the first-level portion of Frege's Begriffsschrift.[3] Notice that the above characterization does not permit WFEs with free variables. There are almost as many variants of the above presentation as there are writers on the subject. What is important about it for our purposes is that it is the sort of specification of the syntax of FOL that is given by logicians.

A domain is any non-empty set, D. An interpretation of FOL on D is a map, I, such that $I(a_j)\epsilon D$ and $I(F^i_j)\epsilon D^i$ for each i,j. The usual sort of semantics for FOL defines truth relative to an interpretation as follows:

(i) A WFE $Pn_1\ldots n_i$ is true on I if $(I(n_1),\ldots,I(n_i))\epsilon I(P)$.

(ii) A WFE $\sim$A is true on I if A is not true on I; a WFE (A&B) is true on I if A is true on I and B is true on I.

(iii) A WFE $\exists$vA is true on I if for some n not in A and some interpretation, I', which is like I, except perhaps on n, An/v is true on I', where An/v is the result of replacing each free occurrence of v in A by n.

This grammar and its variants are sufficient for the usual purposes of logicians such as defining logical truth and logical consequence. Those interested in writing grammars for natural languages make other demands upon a grammar. We shall see that imposing these other demands on a grammar of FOL leads to insight about FOL and natural languages as well.

Part 2 -- Syntax

Logicians specify the syntax of languages they study by inductive definitions of the WFEs of those languages such as the one given above for FOL. While that definition does determine the class of WFEs quite definitely, it does not assign any structure to those WFEs--at least not explicitly. The syntactic components that linguists produce are also inductive definitions, but differ in important ways from logician's syntax. First of all, the linguist's inductive definitions define not (just) the class of WFEs, but the class of structural descriptions of WFEs. Of course, a structural description can be thought of as a WFE too, but on a different (larger) vocabulary. Secondly, the linguists inductive definitions are given in a very special form, usually a context-free phrase structure grammar (with or without transformations).[4] As we shall see, FOL can provide a useful test case for general doctrines about grammar. In order to apply views formulated to deal with natural languages to FOL it is helpful to reformulate the grammar of FOL. In this section we reformulate the syntax of FOL.

Here is a context-free phrase structure grammar for FOL:[5]

(GR1) $S \to AS$ (GR2) $S \to LP + S + CON + S + RP$

(GR3) $S \to NEG + S$ (GR4) $S \to Q + V + S$

(GR5) $AS \to P^i + T + \ldots + T$ (i occurrences of 'T') for $i=1,2,\ldots$

(GR6) $T \to N$ (GR7) $T \to V$

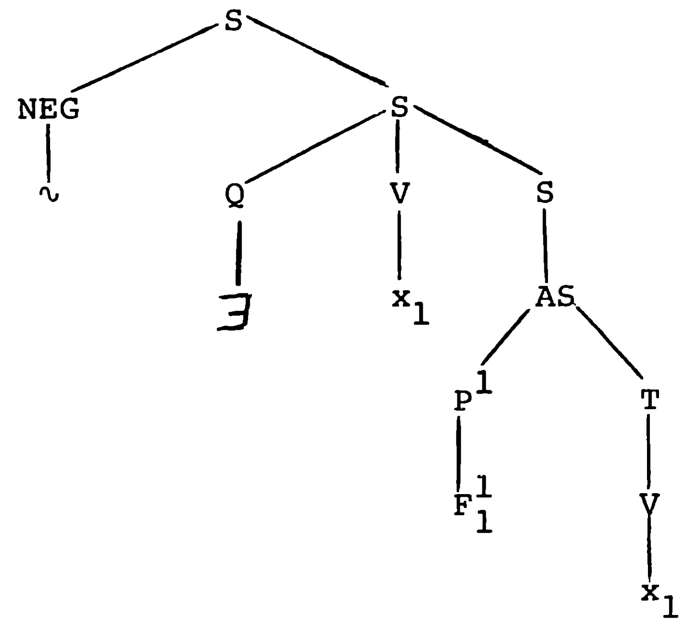
(GR8) $NEG \to \sim$ (GR9) $CON \to \&$

(GR10) $Q \to \exists$ (GR11) $LP \to ($ (GR12) $RP \to)$

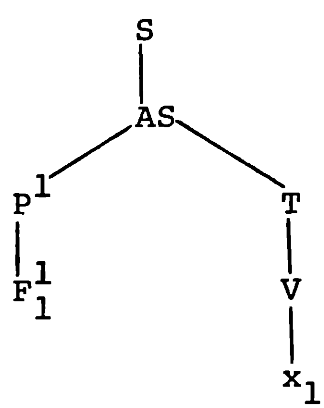
(GR13j) $N \to a_j$ for $j=1,2,\ldots$ (GR14j) $V \to x_j$ for $j=1,2,\ldots$

(GR15^i_j) $P^i \to F^i_j$ for $i,j=1,2,\ldots$

This grammar yields structural descriptions such as the following:



Indeed the grammar gives plausible structural descriptions to all the WFEs of FOL. However, it also generates expressions which our previous definition does not classify as WFEs. For example,



In general, the phrase structure grammar generates all of the usual open sentences of FOL while the inductive definition does not. Can we generate just the WFEs of the inductive definition by means of a context-free phrase structure grammar? I conjecture that the answer is no, because of the need for agreement between variables in the quantifier prefix and the sentential matrix. Variables behave like discontinuous constituents in the original definition.[6] The (conjectural) non-context free character of important, common sets of expressions has consequences even if we are solely interested in syntax.[7] And as soon as we take into account the needs of a semantic analysis of FOL we are faced with more choices and constraints.

Let us call a grammar (syntax plus semantics) categorematic with respect to a particular category (non-terminal symbol) C if the semantic component of the grammar assigns a meaning to every terminal phrase dominated by C in the structural descriptions generated by the syntactic component. A category which is not categorematic we shall call syncategorematic. A grammar is categorematic if all of its categories are categorematic. One of the questions we will be exploring in later sections is the extent to which categorematic grammars can be constructed for FOL and other languages. Even if we could somehow generate structural descriptions of just the WFEs of our original inductive definition, we will face the problem of giving a semantics that treats the category of open sentences as categorematic, thus departing from (perhaps only by elaborating) the original semantics that we gave. Furthermore, the occurrence of parentheses and the categories that dominate them stand in the way of constructing a categorematic grammar for FOL, for parentheses serve to indicate how semantic combination of meanings is to take place rather than having meanings themselves. In order to facilitate the investigation of categorematic grammars, we shall liberalize our notions of syntax to include "transformational grammars" with a context free phrase structure component that generates structural descriptions of semantically significant base structures (deep structure) plus transformations that operate to yield the final syntactic form (surface structure). This is certainly within the spirit of our attempt to treat the construction of a grammar for FOL by the principles that linguists recommend for natural languages. And, as will become clearer later on, it is not a matter of using a cannon to kill a mosquito. We can then modify our rules by deleting (GR2), (GR11), and (GR12) and by adding (GR2') and appropriate "transformations".

$$\text{(GR2')} \quad S \rightarrow CON + S + S$$

We may also, if we wish, add filtering "transformations" which always produce surface structures that contain no free variables and no vacuous quantifications. But this does not dispose of any semantic problems for us; open sentences still occur in the deep structure and must be semantically interpreted in a categorematic grammar.

Let's describe the grammar just proposed for FOL as having a phrase structure component and a functional component.[8] The functional component has two important purposes. First, it permits us to construct grammars for languages that cannot be generated by context free phrase structure grammars alone. Second, it permits the representation of a level of semantically significant structure so that we can exclude extraneous syntactic structure and construct a categorematic grammar (as we shall see later on). In fact, the only reason I can see for restricting the phrase structure component to be context free is that it is to generate the semantically significant structures together with the expectation that semantically significant structure is particularly simple. From the standpoint of the class of (surface) terminal strings generated, the restriction to grammars with a context free phrase structure component plus a functional component consisting of arbitrary recursive functions is no different than merely requiring the set of strings to be recursively enumerable and thus is the same as allowing arbitrary inductive definitions of order ω.[9]

Part 3 -- Categorial Grammars

In the rest of this paper we shall concern ourselves with a particularly simple and elegant sort of categorematic grammar based upon a context free phrase structure component, which is called a categorial grammar. The name 'categorial grammar' and the original formulation are due to Ajdukiewicz, with further developments due to Geach, Lewis, and Montague among others.[10] Though important in its own right, the notion of categorial grammar gains further interest because it gives an almost perfect reconstruction of Frege's grammatical theory. I consider categorial grammar to be a merging of modern generative grammar with Fregean grammar.

A categorial grammar is based on a finite set of basic categories, $b_1,\ldots,b_n$. If c is a category and $c_1,\ldots,c_n$ are categories, then so is the complex category $c/(c_1,\ldots,c_n)$. When $n=1$ we write c/c_1. Once vocabulary items are assigned a category, all syntactic facts (about the base structure) are determined. An assignment of a category to each of finitely many simple vocabulary items determines a context free phrase structure grammar as follows: The nonterminal symbols of the grammar are those categories which have been assigned to simple vocabulary items. To each basic nonterminal symbol there correspond phrase structure rules of the sort $b_i \rightarrow v^i_j$, where b_i is the nonterminal symbol and v^i_j is the jth vocabulary item assigned to b_i. To each complex nonterminal symbol, $c/(c_1,\ldots,c_n)$, there corresponds the phrase structure rule $c \rightarrow c/(c_1,\ldots,c_n) + c_1 + \ldots + c_n$. The categorial grammar determined by the assignment of categories to the simple vocabulary is the phrase structure grammar consisting of the rules corresponding to the assigned categories. Compound phrases generated by the grammar are assigned whatever category immediately dominates them. We shall see examples shortly.

A domain assignment for a categorial grammar is an assignment of a domain (non-empty set) to each of the basic nonterminal symbols of the grammar. Let D_i be the domain assigned to b_i; these are the basic domains. Complex categories then get assigned domains as follows: If $c/(c_1,\ldots,c_n)$ is a category such that the categories c and $c_1,\ldots,c_n$ are assigned domains $D(c)$ and $D(c_1),\ldots,D(c_n)$ respectively, then $D(c/(c_1,\ldots,c_n))$ is

$$D(c)^{D(c_1)x\ldots xD(c_n)},$$

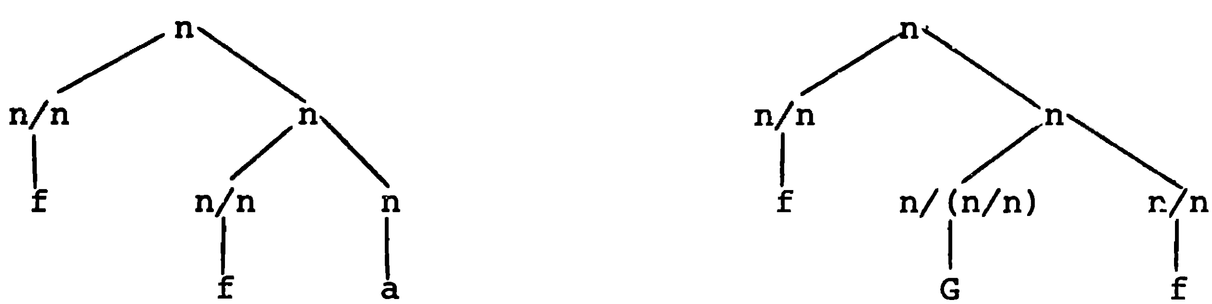
the set of functions from the set of n-tuples of elements of $D(c_1),\ldots,D(c_n)$ to elements of $D(c)$. An interpretation based on a domain assignment is a mapping of simple vocabulary items into elements of the domains assigned to the categories of the vocabulary items. Syntactically complex phrases receive interpretations according to the principle of applying functions to an appropriate set of arguments. Categories have a dual role, indicating both syntactic and semantic structure. Indeed, in a sense, categorial grammars identify syntactic (deep) structure with semantic structure. Thus if $e + e_1 + \ldots + e_n$ is a phrase generated by the grammar consisting of expressions of category $c/(c_1,\ldots,c_n)$, $c_1,\ldots,c_n$ respectively with interpretation i, $i_1,\ldots,i_n$ respectively, then the phrase is of category c with interpretation $i(i_1,\ldots,i_n)$.

As our first example of a categorial grammar let us see how congenial categorial grammars are to Frege's theory of reference.[11] There is one basic category, n, the category of names. The hierarchy of complex categories built from this base includes n/n, the category of 1-place first level function names, n/(n,n), of 2-place first level function names, n/(n/n), of 1-place second level function names and so on for each of the kinds of function names recognized by Frege. To be more concrete let us consider a simple "language" with vocabulary a, f, G of category n, n/n, n/(n/n) respectively. The categorial grammar will contain the phrase structure rules

$n \rightarrow a$, $\quad n/n \rightarrow f$, $\quad n/(n/n) \rightarrow G$

$n \rightarrow n/n + n$, $\quad n \rightarrow n/(n/n) + n/n$

and yields such structural descriptions as



A domain assignment for the language just assigns a non-empty set, D, of objects to the basic category n and the set, D^D, of functions from D to D to the complex category n/n, and the set $D^{(D^D)}$, of function from functions from D to D into D to the complex category n/(n/n). An interpretation assigns an element of D to a, an element of D^D to f, and an element of $D^{(D^D)}$ to G. And in accord with Fregean principles, f + a gets interpreted as the value of the appropriate function applied to the appropriate object.

Part 4 -- UrBegriffsschrift

In this section we present a categorial grammar for a language due to Quine[12] which is both simple and closely related to FOL. The vocabulary of the language of predicate functor logic (PFL) consists of a finite number of predicates $F_1,\ldots,F_n$ of category p one of which is the special identity predicate, I, plus the predicate functors CONJ of category p/(p,p) and NEG, EXQ, PAD, PERM all of category p/p. The language PFL is generated by the rules which categorize the vocabulary together with the phrase structure rules
$p \to p/p + p$ and $p \to p/(p,p) + p + p$.
We shall be primarily interested in special interpretations of PFL called intended interpretations. An intended interpretation of PFL consists first of an assignment of an intended domain to category p, namely a set P of all sets of n-tuples (for each integer n) of elements of an arbitrary non-empty set, S. An element of P is homogeneous if it consists entirely of n-tuples of some single length which is said to be the degree of the element, or if it is the empty set ϕ which is of degrees 0,1,2,..., or it is the set S which is of degree 0. An intended interpretation assigns to each predicate an homogeneous member of P. The predicate functors each get assigned special interpretations as follows: If a^n is an homogeneous element of P of degree n, we write $a^n x_1 \ldots x_n$ to indicate that the n-tuple of $x_1,\ldots,x_n$ taken in the order given is a member of a^n.

There are exactly two elements of degree 0, ϕ and S. CONJ is interpreted as that function <u>CONJ</u> from pairs of members of P of degrees j, k respectively to members of P of degree maximum of j and k such that $\underline{\text{CONJ}}$ $(a^j, b^k)x_1 \ldots x_{max(j,k)}$ if and only if $a^j x_1 \ldots x_j$ and $b^k x_1 \ldots x_k$ for $j, k > 0$. Also:

$$\underline{\text{CONJ}}\ (\phi, a^j) = \underline{\text{CONJ}}\ (a^j, \phi) = \phi;$$

$$\underline{\text{CONJ}}\ (S, a^j) = \underline{\text{CONJ}}\ (a^j, S) = a^j$$

To make $\underline{\text{CONJ}}$ be defined on all of PxP we can assign any arbitrary value to the other argument pairs. We will do the same in subsequent definitions without explicit mention.

NEG is interpreted as the function, $\underline{\text{NEG}}$, from members of P of degree j to members of P of degree j such that $\underline{\text{NEG}}$ $(a^j)x_1 \ldots x_j$ if and only if not $(a^j)x_1 \ldots x_j$ for $j > 1$ and $\underline{\text{NEG}}$ $S = \phi$ and $\underline{\text{NEG}}$ $\phi = S$.

EXQ is interpreted as the function, $\underline{\text{EXQ}}$, from members of P of degree j to members of P of degree maximum (0, j-1) such that $\underline{\text{EXQ}}$ $(a^j)x_2 \ldots x_j$ if and only if $a^j x_1 x_2 \ldots x_j$ for some x_1 in S for j>1 and $\underline{\text{EXQ}}$ $(a^1)=S$ if $a^1 x_1$ for some x_1 in S and $\underline{\text{EXQ}}$ $(a^1)=\phi$ if $a^1 x_1$ for no x_1 in S and $\underline{\text{EXQ}}$ $(\phi)= \phi$ and $\underline{\text{EXQ}}$ $(S)=S$.

PAD is interpreted as the function, $\underline{\text{PAD}}$, from elements of degree j to elements of degree j+1 such that $\underline{\text{PAD}}$ $(a^j)x_0 x_1 \ldots x_j$ if $a^j x_1 \ldots x_j$ for j>0 and $\underline{\text{PAD}}$ $(\phi)x$, $\underline{\text{PAD}}$ $(S)x$ for all x in S.

PERM is interpreted as the function, $\underline{\text{PERM}}$, from elements of degree j to elements of degree j such that $\underline{\text{PERM}}$ $(a^j)x_1 x_3 \ldots x_j x_2$ if $a^j x_1 x_2 x_3 \ldots x_j$ for j>2 and $\underline{\text{PERM}}$ $(a^2)x_2 x_1$ if $a^2 x_1 x_2$ and $\underline{\text{PERM}}$ $(a^1)=a^1$ and $\underline{\text{PERM}}$ $(a^0)=a^0$.

I is interpreted as the element $\underline{\text{I}}$ of degree 2 such that $\underline{\text{I}}$ $x_1 x_2$ if $x_1=x_2$.

We call an expression of PFL true on an intended interpretation over the set S if the expression receives the interpretation S and false if it receives the interpretation ϕ. What makes PFL so interesting is that it is possible to correlate expressions and interpretations of FOL (PFL) with expressions and intended interpretations of PFL (FOL) in such a way that the expression of FOL (PFL) is true on the given interpretation if the correlated expression of PFL (FOL) is true on the correlated intended interpretation. In some interesting sense FOL and PFL are intertranslatable. We have just given a categorial grammar for PFL and thereby a "truth definition" for PFL, and we have done so in a way that allows us to make use of the syntactic correlation of PFL with FOL to give a truth definition for FOL that agrees in extension with the usual one. But have we thereby also given a categorial grammar for FOL?

There is no question that in the straightforward, literal sense of the words we have not given a categorial grammar for FOL. The possibility still remains that we have done so in an implicit manner, that a categorial grammar for FOL can be "read off" the grammar for PFL. For example, we might try to get around the obvious differences in vocabulary and syntax by claiming that the paraphrase in PFL of an expression of FOL represents the "deep structure" of the expression of FOL, and thus that the difference between PFL and FOL is merely one of "surface syntax". There is an influential tradition that offers paraphrases or contextual definitions as analyses of the syntax and semantics of expressions and which is reinforced by the modern notions of deep and surface structure. The only way to deal with such claims is by detailed case-by-case analyses. In the present case the claim is dubious. Consider for example, the fact that on the most straightforward sort of translation, both

Fa and $\exists x(Ax\&Fx)$ get translated as EXQ CONJ A F via the usual sort of elimination of constants by predicates. If we have learned FOL in the normal way we have strong feelings that two syntactically and semantically very different expressions have been conflated, and further that it is the second of the two that has (most nearly) been analysed. We ought to try to give a grammar for FOL that does not do such violence to our intuitions.[13]

Part 5 -- Frege's general analysis and difficulties in its application

Let us now examine one attempt at a categorial grammar for FOL due to Frege. We start first with Frege's analysis of the sentential fragment, SL. The categories of SL are based on the single category n, which is assigned to the infinite collection of sentential symbols $s_1, s_2, \ldots$. The other vocabulary items are $\sim$ of category n/n and & of category n/(n,n). An intended interpretation of SL assigns a domain of objects to category n which includes the objects the True and the False (called truth values) and assigns a truth value to each sentential symbol. In addition $\sim$ and & are assigned functions which behave like negation and conjunction respectively on the truth values and any way that is convenient on other arguments.

The quantifier free fragment of FOL, QFFOL, adds to SL vocabulary items $a_1, a_2, \ldots$ of category n called names and symbols F^i_j, for each integer i, j, of category n/(n...n) (a total of i+1 occurrences of 'n') called predicates of degree i. The general principles of categorial grammar automatically extend an intended interpretation of SL to QFFOL. We may need to add some functions to help place parentheses and put & between its argument expressions, but basically this is an intuitively satisfactory grammar of QFFOL both from the Fregean standpoint and the modern logician's standpoint. Notice that predicates and sentential connectives are treated alike as are sentential symbols and names. If we prefer to rule out such expressions as $\sim a_1$ and $F^1_1 s_1$ we may base our grammar on two categories, n and s. We make $\sim$ and & of category s/s and s/(s,s) respectively, $s_1, s_2, \ldots$ of category s and F^i_j of category s/(n,...n) (i occurrences of n). We then assign the set consisting solely of the True and the False as the domain of s. This latter grammar gives us exactly the quantifier free portion of FOL as originally presented in Part 1.

All we need now to get FOL is to add the appropriate syntax and semantics for $\exists$. This is how it is supposed to be: $\exists$ is of category s/(s/n) and is interpreted by a second level function which yields the True as value for a given first level function as argument if the given function yields the True as value for at least one object as argument. Variables get introduced in the "surface structure" to fill in the gap in the predicate to which $\exists$ is applied. Alas, this won't do. But an analysis of the difficulties with this account of the grammar of the existential quantifier yields an understanding that is deeper than usual of the role of variables and an important extension of the notion of a categorial grammar.

Consider the quite normal and indispensible expression $\exists x_1 \sim F_1^1 x_1$. It is not generated by our categorial grammar! The obvious way of analyzing it, indeed the way that Frege intended that it should be analysed, involves treating $\sim F_1^1$ as a phrase of category s/n on a par with F_1^1 in $\exists x_1 F_1^1 x_1$. This in turn involves treating $\sim$ as of category (s/n)/(s/n). Either we must give up the categorial treatment or recategorize the vocabulary (perhaps as in Part 4) or we must assign some vocabulary to more than one (infinitely many) categories. For Frege the second of these three alternatives is really no different than the first; for him categorization of vocabulary was not just a formal matter to be chosen as most convenient, but a matter of fundamental ontological significance. While not sc strictly ruled out for us, we saw in Part 4 that resort to alternative 2 is prima facie undesirable. Depending on how strong one's intuitions are, failure to find a categorial account of FOL different than that given for PFL may undermine one's interest in categorial grammar. We thus seem to have no other alternative but to treat $\sim$ as infinitely homonymous since it must be of infinitely many categories and since the semantic domains of different categories are disjoint. Similar facts obtain for &.

In order to treat existential quantification as of category s/(s/n) we see that in addition to the entities of category s/n which get assigned to the simple vocabulary items of category s/n, there must be other entities of category s/n which get assigned to compound expressions of category s/n. Because we have expressions such as $\exists x_1 \exists x_2 F_1^2 x_1 x_2$ in FOL, existential quantification must be homonymously represented too, being of categories s/(s/n) and (s/n)/(s/(n,n)) in the present example.

Furthermore, we seem to need additional operators on predicates of all categories, which are not explicitly represented by vocabulary items, but are instead coded in the pattern of occurrences of variables and names. For example, consider $\exists x_1 F_1^2 x_1 a_1$, $\exists x_1 F_1^2 x_1 x_1$, and $\exists x_1 \exists x_2 (F_1^2 x_1 x_2 \& F_1^2 x_2 x_1)$. In each of these cases, given an interpretation of F_1^2, it is clear what entity the leftmost quantification should be interpreted as acting upon; the problem is to derive such an entity from the syntactic structure of the expression within the categorial framework. This is an additional problem, distinct from the category matching problem that leads us to suggest that the logical operators were (infinitely) homonymous.

Part 6 -- Martin's analysis and its categorial reformulation

In commenting on Frege's analysis of Begriffsschrift, Ed Martin says:[14]

> "[there is] an inconsistency in (a) the doctrine that connectives and quantifiers stand for concepts, and (b) recognizing connectives and quantifiers as meaningful parts of function names. Frege clearly holds (a) ... (b) it seems to me, is indespensible; thus either more liberal level restrictions must be instituted, connectives and quantifiers must be treated as syncategorematic, or [$\sim$, &, $\exists$] must be held to be homonymous."

Martin offers his own formulation of a grammar for Begriffsschrift in which "(a) has been sacrificed in favor of (b)." In this section I offer a categorial reformulation of Martin's account which combines his treatment of open sentences as having referents (being assigned an interpretation) with the treatment of the logical operators $\sim$, &, $\exists$ as homonymous. What follows consists of Martin's rules of interpretation, (0) - (v), (slightly altered) together with my reformulation of them and various comments.

An interpretation assigns a non-empty domain, D, to the category n, and the set of the True and the False to the category s. Predicates of degree i are of category s/(n,...,n) (i occurrences of n) and assigned functions of this category as their interpretations. Names are of category n and assigned elethat associates an expression with its interpretation by underlining. 'F' with optional subscripts will range over predicates, 'a' will range over names, 'v' with subscripts will range over variables. The logical operators will name themselves and concatenation will indicate concatenation.

(0) The interpretation of a predicate of degree i, F, followed by i distinct variables is the interpretation of F. In symbols, $I(Fv_1 \ldots v_i)=I(F)$ where $v_1 \ldots v_i$ are any i <u>distinct</u> variables. Notice that this rule assigns the same interpretation to Fv_1v_2, Fv_3v_4, and Fv_2v_1 but assigns no interpretation to Fv_1v_1. Thus one kind of occurrence of variables following predicates is semantically superfluous.

(i) If $\phi v_1, \ldots, v_m, \ldots, v_p, \ldots, v_n$ is an open sentence of at least two variables whose interpretation has been assigned, then the interpretation of $\phi v_1, \ldots, v_m, \ldots, v_m, \ldots, v_n$ is that function (of n-1 arguments) whose value for $y_1, \ldots, y_m, \ldots, y_n$ (excluding y_m a second time) as arguments is the True if the interpretation of $\phi v_1, \ldots, v_m, \ldots, v_p, \ldots, v_n$ is the True and the False in all other cases. There is a slight problem here with understanding the notation, but I believe that what Martin intended is that $v_1, \ldots, v_m, \ldots, v_p, \ldots, v_n$ be a complete list without repetition of those variables occurring free in the first open sentence and that the second open sentence be obtained from the first by replacing each free occurrence of v_p by an occurrence of v_m. Here we see the first example of a nontrivial semantic role for the variables occurring after predicates; they signal the application of a certain operation upon the interpretation of the open sentence which takes functions of degree i+1 to functions of degree

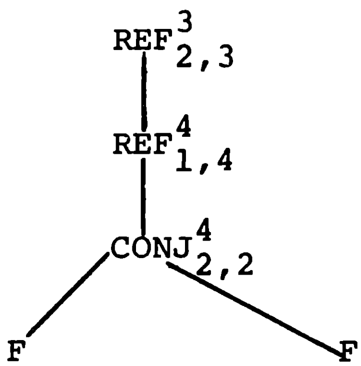
i. The exact nature of the operation depends upon the particular pattern of occurrences of variables in the open sentence. By treating variables as surface syntactic manifestations of certain operations on functions we can capture this sort of grammar within the categorial framework.

Let p^i abbreviate the category symbol s/(n,...,n) (with i occurrences of n). We introduce a collection of "deep structure" syntactic elements $REF^i_{j,k}$ and their corresponding interpretations $\underline{REF}^i_{j,k}$ of category p^{i-1}/p^i, for each integer $i \geq 2$, $1 \leq j < k \leq i$. $REF^i_{j,k}$ affects the "surface structure" by means of its associated "transformation" which fills in the jth and kth variable-taking positions of the operand of $REF^i_{j,k}$ with the same variable. And $\underline{REF}^i_{j,k}$ is the operation on functions of degree i which "identifies the jth and kth argument places", yielding a function of degree i-1.

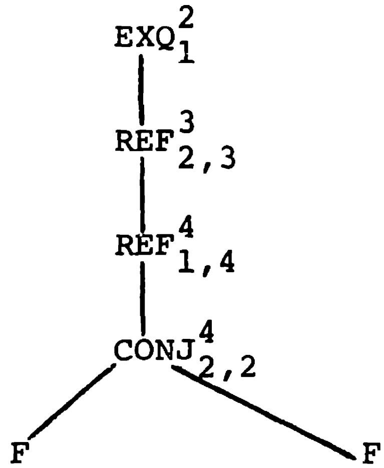
(ii) If $\phi v_1,\ldots,v_m,\ldots,v_n$ is an open sentence of at least two variables whose interpretation has been assigned, then the interpretation of $\phi v_1,\ldots,a,\ldots,v_n$ is that function whose value for $y_1,\ldots,y_n$ (excluding y_m) as arguments is the True if the interpretation of $\phi v_1,\ldots,v_m,\ldots,v_n$ for the arguments $y_1,\ldots,a,\ldots,$ y_n is the True and is the False in all other cases. Again we construe $v_1,\ldots,v_m,\ldots,v_n$ as an exhaustive, non-repeating list of the free variables of the first open sentence; and the second open sentence is obtained for the first by replacing each free occurrence of v_m by an occurrence of a. We incorporate this into our grammar by introducing the "deep structure" syntactic operations SUB^i_j and their interpretations $\underline{SUB}^i_j$ of category $p^{i-1}/$ (n,p^i), for each integer $i \geq 2$, $1 \leq j \leq i$. SUB^i_j affects the "surface structure" by means of its associated "transformation" which fills in the jth variable-taking place of the sentence operand with the name operand. $\underline{SUB}_j^i$ is the operation on elements of D and on functions of degree i which "plugs the element into the jth argument place of the function" yielding a function of degree i-1.

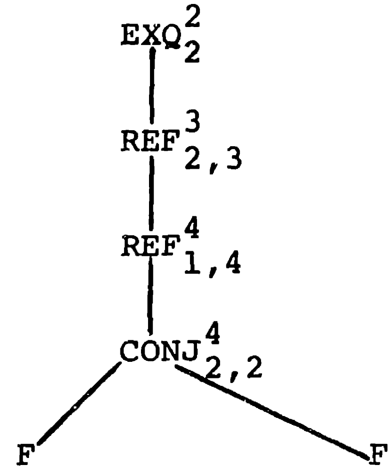
(iii) If $\phi v_1,\ldots,v_n$ is an open sentence whose interpretation has been assigned, then the interpretation of $\sim\phi v_1,\ldots,v_n$ is to be that function whose value for $y_1,\ldots,y_n$ as arguments is the True if the value of the interpretation of $\phi v_1,\ldots,v_n$ for the same arguments is the False, and whose value is the False in all other cases. We introduce NEG^i and $\underline{NEG}^i$ of category p^i/p^i for each $i\geq 1$. An appropriate "transformation" takes NEG^i into $\sim$ in the "surface structure". $\underline{NEG}^i$ is the operation on functions of degree i that replaces the True by the False and everything else by the True.

(iv) If $\phi v_1,\ldots,v_n$ and $\psi v_m,\ldots,v_p$ are open sentences whose interpretations have been assigned, then the interpretation of $\phi v_1,\ldots,v_n$ & $\psi v_m,\ldots,v_p$ is to be that function whose value for arguments $y_1,\ldots,y_n,y_m,\ldots,y_p$ is the True if the value of the interpretation of $\phi v_1,\ldots,v_n$ for the arguments $y_1,\ldots,y_n$ is the True and if the value of the interpretation of $\psi v_m,\ldots,v_p$ for the arguments $y_m,\ldots,y_p$ is the True, and whose value is the False in all other cases. It is crucial here that all the variables be distinct, that is, that the two open sentences have no free variable in common. Within Martin's framework this implicitly imposes a certain derivational history on expressions.[15] In our categorial approach these details are handled by having $CONJ_{i,j}^{i+j}$ and $\underline{CONJ}_{i,j}^{i+j}$ of category $p^{i+j}/(p^i,\ p^j)$, for $i,\ j\geq 1$. "Transformations" take $CONJ^{i+j}_{i,j}$ into & plus parentheses. $\underline{CONJ}^{i+j}_{i,j}$ operates on functions of degrees i and j yielding a function of degree i+j whose value is the True just in case both arguments yield the True (for the appropriate arguments) and which is the False otherwise. Thus one "deep structure" for $Fv_1v_2\&Fv_2v_1$ will be the following:



(v) If $\phi v_1,\ldots,v_m,\ldots,v_n$ is an open sentence of at least two free variables whose interpretation has been assigned, then the interpretation of $\exists v_m \phi v_1,\ldots,v_m,\ldots,v_n$ is to be that function whose value for the arguments $y_1,\ldots,y_n$ (excluding y_m) is the True if the value of the interpretation of $\phi v_1,\ldots,v_m,\ldots,v_n$ for the arguments $y_1,\ldots,y_m,\ldots,y_n$ is the True for some argument y_m, and is the False in all other cases. We introduce EXQ^i_j and $\underline{EXQ}^i_j$ of category p^{i-1}/p^i for each $i\geq 2$ and $1\leq j\leq i$. EXQ^i_j gets transformed into an existential quantifier whose associated variable is the jth free variable of its operand. $\underline{EXQ}^i_j$ is the existential-quantifier-on-the-jth-argument-position operation, operating on a function of degree i and yielding a function of degree i-1. Thus we have the following "deep structures" for $\exists v_1(Fv_1v_2\&Fv_2v_1)$, $\exists v_2(Fv_1v_2\&Fv_2v_1)$





Martin gives a parallel set of rules to cover the cases where the operands or results are of category s. We need not do this; we just modify our operations to apply to truth values in the straightforward way. With this assumed done, we now have a categorial grammar for FOL. This grammar is of interest because of the analysis it makes of the role of variables in FOL. Variables make possible a syntactically compact coding of what can be a complex series of operations. The usual explanation of the use of variables in FOL is that variables are used to indicate generality. Our analysis shows this to be a very incomplete explanation. In fact, on our analysis it is, taken literally false. Variables indicate the operation of certain functions on other functions and they indicate which existential quantifier is being applied. For all the complexity of our grammar, nonetheless, I believe it reveals the essence of the structure of FOL. Quine should have called his paper "Variables Explained" rather than "Variables Explained Away."[16]

We must face the fact that our categorial analysis of FOL relies on an infinite vocabulary, both terminal and nonterminal, and (because of the infinite nonterminal vocabulary) on infinitely many phrase structure rules. Infinities trouble some people. Someone might object to our analysis of FOL on the grounds that it makes the language unlearnable, since learning an infinite vocabulary and infinitely many rules is beyond our power. A more sophisticated objection might be that whatever other merits our analysis has, it is unsatisfactory as a representation of our knowledge of FOL; we may have presented a grammar for FOL, but not a grammar that represents the linguistic competence of any person. Further, if this is the best that the categorial approach can do towards providing a grammar for FOL, then we now have a demonstration of the inadequacy of the categorial approach. Both of these objections are mistaken. Before answering these objections, notice what happens if we insist upon a finite (at least as far as rules go) grammar. We can get a finite categorial grammar for FOL if we recategorize in the manner of our treatment of PFL, putting all predicates into a single category. Then various finite collections of predicate functors - any of Quine's choices will do - can give the effect of our infinitely many logical operators. As long as we sort predicates into categories according to their degree there seems to be no finite set of operators that is adequate to the demands of FOL.

Why should we be embarrassed by our infinite grammar? The crux of the matter is the question of effectiveness--in the sense of the theory of effective computation. Almost everyone assumes that the class of effectively computable functions is an upper bound to human computational abilities. So if the class of expressions, or the class of structural descriptions, or the association of structural descriptions with expressions determined by a proposed grammar is not effective, and if people learn the language, we reject the grammar as a description of the linguistic competence of human speakers. But our proposed grammar passes this test. The class of terminal strings, the class of structural descriptions, and the association of structural descriptions with terminal strings are all effective in the appropriate sense.

A related problem concerns the semantic component of the grammar, namely the specification of an interpretation for the infinite number of simple vocabulary items. How can we give a meaning to each of an infinite number of expressions? Here questions of effectiveness are out of place. Interpretations as we have construed them are not, in general, the sort of entities which effectively computable functions have as values. Even in the case of a finite vocabulary it will not, in general, make sense to talk of effectively assigning an interpretation, at least not in the same technical sense of effectiveness that we have been using. But surely there is a difference between the problem of learning a finite number of new words and the problem of learning an infinite number--the difference between what is humanly possible and what is not. In the case of a finite vocabulary, interpretation item by item is always a (theoretical, at least) possibility, in the infinite case it is not. But not all infinite vocabulary interpretation problems are the same. In some there is a definable correlation between the vocabulary items and their interpretations--'definable' being the operative word here, since there is always a correlation of some sort if each item has an interpretation. Definability not being an absolute notion, the question arises; Definable relative to what? For want of a better answer, and because we have throughout this paper defined functions in this way, let us say: Definable relative to your favorite set theory. There is no problem specifying in a reasonable set theory the appropriate interpretation functions for our infinite vocabulary. For example, the interpretation of NEG^i for variable i is just the function that maps integers, i, into the complementing operation on functions of degree i. Consequently, there is no basis for a "learnability" objection to our proposed grammar for FOL. If, as I suspect, the only case to be made against such an infinite grammar has to do with "learnability", we need not be embarrassed on that account.[17]

Not only do I claim that the infinite character of our grammar is not a serious objection to it, I want to try to convince you that it is a point in favor of the grammar. Since the class of structural descriptions is effective, there are systems of finitely many productions involving finitely many terminal and nonterminal symbols that generate the set of strings of FOL. There are "phrase structure" rules that generate the infinite vocabulary. The rules that generate our infinite vocabulary are of no semantic significance from the standpoint of categorial grammar. Certainly the vocabulary has structure, syntactic and semantic. But that structure is not, I think, best thought of as part of the grammar of FOL. Just consider for a moment the infinitely many "deep structure" vocabulary items. Our representation of them involves aspects which, I believe, have no essential connection with the language of FOL. Consequently, it seems incorrect to include those aspects in our description of FOL. The vocabulary of FOL, as described by our grammar, is infinite; our description of that vocabulary and its interpretation is not. Perhaps I have just tried to make a virtue of a necessity, but discussion of certain developments in later sections of this paper may add to the plausibility of my defense of an infinite grammar.[18]

Part 7 -- Geach's extension of categorial grammar and its semantic significance

In this section we examine a suggestion of Geach's[19] which significantly extends the power of categorial grammars in a natural way. We then consider the application of the extended notion of categorial grammar to the description of quantification in English and to the description of FOL.

Geach motivates his extension of the basic categorial framework by considering a fragment of a categorial grammar for a fragment of English. Consider the sentences 'Socrates is flying', 'Socrates is not flying', 'Every man is flying', and 'Not every man is flying'. Consider a grammar based on the categories s and n, with vocabulary assigned categories as follows:

socrates -- n,	is flying -- s/n
every man -- s/(s/n)	not -- s/s

Geach calls a terminal string of a categorial grammar SC (syntactically coherent) with respect to the grammar if the string occurs in some structural description generated by the grammar as the entire string of terminal symbols dominated by some non-terminal symbol, which is the category of the string on that derivation. (In other words, it is a constituent.) Geach points out that the four sentences are SC with respect to the fragment of a grammar given, but that the intuitively well formed and meaningful phrases 'is not flying' and 'not every man' are not SC. I take the point of this to be that an adequate grammar must do more than just generate sentences; it must give them correct structural descriptions and recognize the meaningful parts of sentences. In fact, it seems a grammar cannot even generate the sentences we want unless it also recognizes correctly other categories of phrases. Consider 'Every man is not flying'. As in the previous case of FOL, we need negation to operate on predicates--to act as an expression of category (s/n)/(s/n)--as well as on sentences.

Geach proposes the "multiplying-out rule":[20]

$$c_1/c_2 \; c_2/c_3 \rightrightarrows c_1/c_3$$

to supplement the basic categorial rule:

$$c_1/c_2 \; c_2 \rightrightarrows c_1,$$

where c_1, c_2, c_3 are any categories. This is just another way of saying that in addition to phrase structure rules of the sort

$$c_1 \rightarrow c_1/c_2 + c_2,$$

a categorial grammar shall contain phrase structure rules of the sort

$$c_1/c_3 \rightarrow c_1/c_2 + c_2/c_3,$$

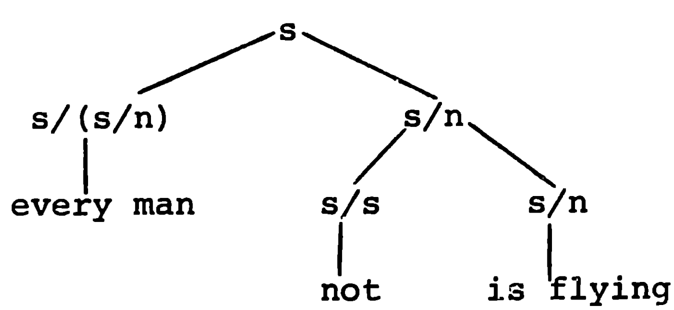
for categories used in the grammar. For example, our fragment of a grammar will contain not only

$$s \rightarrow s/s + s, \quad s \rightarrow s/n + n, \quad \text{and } s \rightarrow s/(s/n) + s/n$$

but also

$$s/n \rightarrow s/s + s/n$$

The extended grammar will then generate the desired sentence:



The new rules are definitely useful. They do not receive in

Geach's discussion any further justification than that usefulness. Until they receive more justification than convenient resolution of some problems they will appear ad hoc and be liable to suspicion because, unless we can find some semantic interpretation of the new rules the whole strength of the categorial approach, the unity of syntactic and semantic structure, will disappear. Rather than extending the categorial approach, we shall have, in effect, rejected it. Fortunately, a very good justification is close to hand.

Let us consider a simple categorial grammar with the expressions f and g of category n/n, and the expression a of category n. Let the category n be assigned a set D as its interpretation. Then the interpretation of a, <u>a</u>, must be an element of D and the interpretations of f and g, <u>f</u> and <u>g</u>, must be functions from D to D. The composition of <u>f</u> and <u>g</u> (in the order given) is that function from the domain of <u>g</u>, D in this case, to the range of <u>f</u>, also D in this case, such that its value for any argument is the result of applying <u>f</u> to the result of applying <u>g</u> to that argument. Let us write '(<u>f</u> <u>g</u>)' to denote the composition of <u>f</u> with <u>g</u>. Using this notation, the basic fact about composition can be expressed as follows:

$$(\underline{f}\ \underline{g})\ (d) = \underline{f}\ (\underline{g}(d)) \text{ for all d in D.}$$

Notice that (<u>f</u> <u>g</u>) is of category n/n; we have combined entities according to the categorial rule, n/n n/n → n/n. The semantic content of Geach's "multiplying-out" rule is that entities of appropriately related categories can combine by composition. Returning to the example above, we see that the interpretation of 'not' (of category s/s) composed with the interpretation of 'is flying' (of category s/n) yields an entity of category s/n which is the identical entity obtained by applying the predicate functor $\underline{NEG}^1$ to the interpretation of 'is flying'.

We generalize the new rules as follows: Let $f, g_1, \ldots, g_i$ be expressions of category $c/(c_1,\ldots,c_i)$, $c_1/(b^1_1,\ldots,b^1_{m_1}),\ldots,$ $c_i/(b^i_1,\ldots,b^i_{m_i})$ with interpretations $\underline{f}$, $\underline{g}_1,\ldots,\underline{g}_i$ respectively. The generalized multiplying-out rule

$$c/(c_1,\ldots,c_i)\ c_1/(b^1_1,\ldots,b^1_{m_1})\ \ldots\ c_i/(b^i_1,\ldots,b^i_{m_i}) \rightarrow c/(b^1_1,\ldots,b^i_{m_i})$$

has as its syntactic interpretation the phrase structure rule

$$c/(b^1_1,\ldots,b^i_{m_i}) \rightarrow c/(c_1,\ldots,c_i)+c_1/(b^1_1,\ldots,b^1_{m_1})+\ldots+c_i/(b^i_1,\ldots,b^i_{m_i})$$

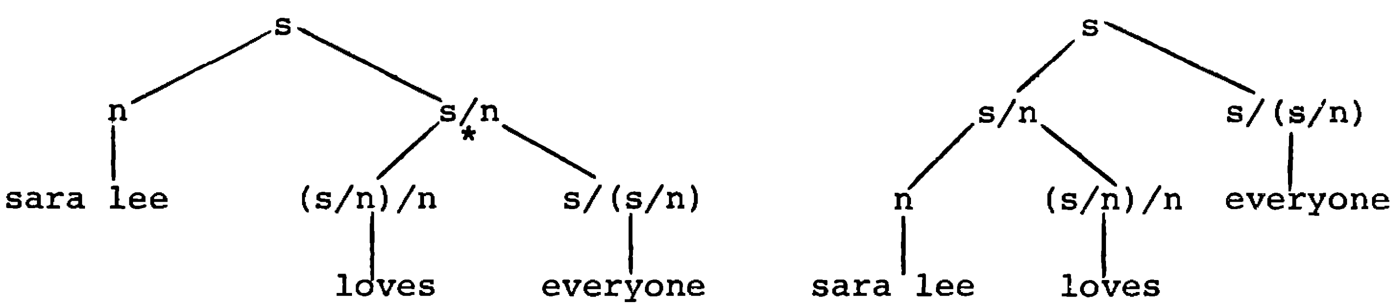
and as its semantic interpretation the operation of generalized composition, COMP, which combines $\underline{f}$, $\underline{g}_1\ldots,\underline{g}_i$ to yield a function of category $c/(b^1_1,\ldots,b^i_{m_i})$ and degree $m_1 + \ldots + m_i$, COMP $(\underline{f},\ \underline{g}_1,\ldots,\underline{g}_i)$, such that

$$\text{COMP}(\underline{f},\ \underline{g}_1,\ldots,\underline{g}_i)(x^1_1,\ldots,x^i_{m_i}) = \underline{f}(\underline{g}_1(x^1_1,\ldots,x^1_{m_1}),\ldots,\underline{g}_i(x^i_1,\ldots,x^i_{m_i})),$$

for all arguments in the appropriate domains. In what follows, we shall mark nodes of phrase structure trees corresponding to applications of these new rules with an asterisk.[21]

One application of these ideas is to simplify our categorial grammar for FOL. We no longer need to treat ∿ and & as infinitely homonymous: they are simply items of category s/s and s/(s,s) respectively. Further, since substitution can be carried out at any time, we need only as many of the operators SUB as there are categories of simple predicates. Thus the only infinite aspect of the grammar that remains is that portion which deals with the effect of variables, of which we have infinitely many even in our original naive description of FOL. I find this revised categorial grammar for FOL extremely satisfying to my syntactic and semantic intuitions.

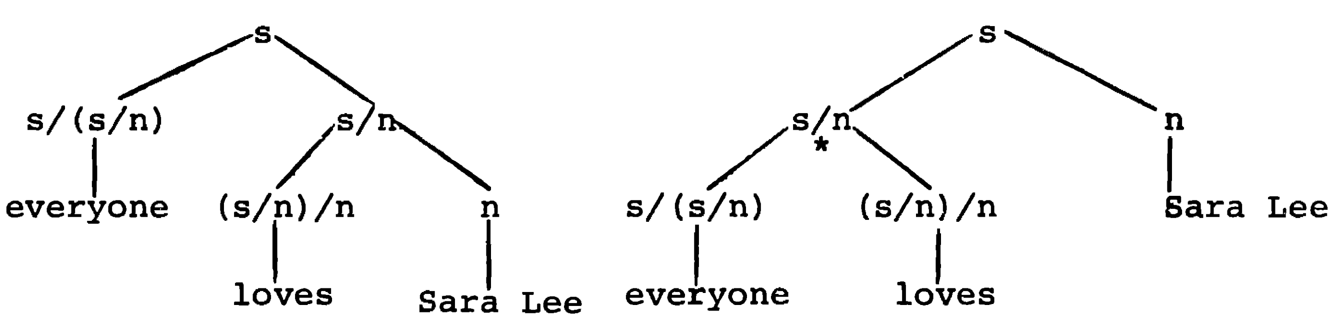
A more complex and rewarding application of extended categorial grammar concerns the treatment of quantification in a small fragment of English. Let us consider a fragment of English with some names (of category n), some intransitive verb phrases (of category s/n), and some quantifier phrases (of category s/(s/n)). Transitive verbs seem to be of category (s/n)/n. We get such reasonable structures as:



The semantic analysis of these two structures is also correct, being of the forms (everyone loves) (sara) and everyone (loves (sara)), which have the same values.[22] But problems arise again with quantifier subjects. Consider the sentence

Everyone loves Sara Lee.

We seem to be in good shape as far as the syntax goes, getting the two structures



But if we follow our rules for semantic interpretation, we shall get everyone (loves (Sara)) and (everyone loves) (Sara), which are not only identical in value (which is good), but identical in value to the interpretations of the previous sentence (which is very bad indeed). The problem is that we have been relying solely on the categories of entities to determine their semantic interpretation, while the syntax we are analysing makes use of other information as well, in this case the <u>order</u> of the expressions having the interpretations. In fact, now that the problem has been revealed to us, we can see that we have been too cavalier about the syntax as well. Our grammar must generate the distinct expressions

sara lee loves   and   loves sara lee.

A simple rule for handling the syntax of the category (s/n)/n, such as concatenate to the right or fill in blanks from left to right, will serve as long as we are combining two expressions of category n. But when we combine a single expression of category n with an expression of category (s/n)/n to form an intransitive verb phrase, we will, in general need to be able to freely specify which of the two name-taking positions is to be filled. Similar syntactic and related semantic problems face the analysis of the phrases

everyone loves   and   loves everyone.

In the next section we shall consider further extensions of categorial grammar which are adequate for an interesting fragment of English. But before doing so, some comments on the project are in order.

The attractiveness of categorial grammar depends on the interaction of its simplicity with its power. A large part of that simplicity depends upon the triviality of the phrase structure rules which it determines and the single straightforward mode of semantic composition which it requires. Our extended categorial grammars will have more complicated phrase structure components and various modes of semantic composition, not all of which depend solely upon the categories of the entities to be combined. Worse still, in recognizing a variety of modes of semantic composition we have snuck in transcategorial elements--functions whose arguments are not restricted to fixed categories. What remains of the original conception of a categorial grammar? As for the increase in complexity of our extended grammars, the justification is in an increase in power. It is important that power is understood as including not only the class of sentences generated, but also the details of constituent structure and the consequent semantic analysis. We shall see in later sections that a syntactic analysis can be "saved" by fiddling with the semantics, so we must compare grammars in both areas.

The criticism concerning the admission of transcategorial elements is more serious. It should be pointed out that even the theory of "pure" categorial grammar admits (one) transcategorial elements, for the operation of applying an arbitrarily categorized function to its arguments is without category. We might be able to get away without reifying this operation if we only considered cases of particular functions applied to particular arguments. But in specifying the semantics of "pure" categorial grammar, we must talk of a variable function applied to variable arguments and thus talk of the operation of applying a function to its arguments. Thus the difference between "pure" and extended categorial grammar is one of degree rather than one of kind. What is common to both analyses is the classification of syntactic and semantic properties of expressions by a single category classification and the use of set-theoretically definable, transcategorial modes of semantic composition which are determined in a uniform way by the structural descriptions which the grammar generates.

Another feature cited in favor of the "pure" approach is its apparent universality. The class of syntactic and semantic rules was specified once and for all languages; whatever differences exist between languages was a matter of "surface syntax". Once we allow modes of semantic composition other than functional application, where do we stop? There are (uncountably) infinitely many distinct transcategorial operations. Do we include all of them? If not, then which ones do we include? Must we include different operations for different languages? Certainly, it is a possibility that different languages make use of different modes of semantic composition within the framework of extended categorial grammar as I have expounded it so far. But then the appealing universality is lost. Notice that (under very liberal assumptions) given a collection of transcategorial modes of semantic composition, it is always possible to recategorize in such a way that only functional application is needed. We saw an example of this possibility when we considered applying the categorization of PFL to the analysis of FOL. By putting all of the vocabulary of a given categorial grammar into a single base category, previously transcategorial elements become elements of one or another complex categories. As long as the phrases of the language are of finite length (as in all our examples) we can capture all the semantic entities involved in the original grammar in a single domain. Of course, we get a quite different class of structural descriptions by so doing. Recategorization, while always possible, does not always produce a grammar "equivalent" to the original. Also, it is sometimes possible to avoid new transcategorial modes by taking expressions to be homonymous and by adding deep structure operators (usually infinitely many). We saw an example of this in our revision of Martin's treatment of FOL. Since I do not know an adequate grammar for English and since I do not know any arguments concerning the size of a universally adequate set of transcategorial modes of semantic combination, I shall content myself with giving examples which show the utility of using more than one such mode.[23]

Part 8 -- A reasonably proper treatment of quantification in ordinary English

The failure to see the way to extend pure categorial grammar has led its advocates to various clever, but misguided, attempts to handle the various well-formed combinations of transitive verbs, names, and quantifiers. Two names and a transitive verb fit happily together because the category assigned to transitive verbs was chosen precisely to allow it. A transitive verb with a name as subject fit together to give an intransitive verb phrase (of category s/n) which fits happily together with a quantifier phrase, again, by choice of the category of quantifiers. But, on the pure version, a transitive verb can only take elements of category n for subjects.[24] Various remedies have been considered. Transitive verbs could be considered homonymous, of categories (s/n)/n and (s/n)/(s/(s/n)), taking name and quantifier subjects respectively. In addition to being ad hoc, this move just deals with the most obvious of several problems. We will not consider it any further. Another attempt to deal with this situation is due to Montague;[25] it involves fiddling with the semantics of singular terms. The problem of quantifier subjects arises because there are two categories which need to combine with transitive verbs. By reducing the number of categories to one we eliminate this aspect of the problem. With brilliant disregard for the advice of Lewis Carol and others who have tried to convince us of the fundamental difference between quantifier phrases and singular terms, Montague lumps together both sorts of noun phrases in the category, s/(s/n), previously reserved for quantifiers alone. The category n survives, but turns out to be empty of members. In the absence of an alternative account, we could learn to live with the unintuitive semantic account of singular terms, especially since Montague's treatment has the additional benefit of providing a neat solution to the vexing problem of non-referring singular terms. In this section I offer an alternative treatment of singular terms which I believe compares favorably to Montague's and others. In a later section I shall consider one other alternative, due to David Lewis.

Consider the problem of generating both of the phrases

(1) sara lee loves and (2) loves sara lee

together with their interpretations as distinct functions of category s/n. Once we allow ourselves transcategorial modes beyond application, the solution is straightforward. We introduce the "phrase structure" rules

(3) $s/n \rightarrow (s/n)/n\ 1{+}n$ and (4) $s/n \rightarrow (s/n)/n\ 2{+}n$

which indicate syntactic combination of a name and a transitive verb as either subject and verb or direct object and verb. To introduce the semantic interpretation of the rules, let us use the symbol '$\breve{\ }$' as follows: where f is a function of category (s/n)/n, let $\breve{f}$ be the converse of f such that $(\breve{f}(y))(x) = (f(x))(y)$ for all x,y of category n. Let $f1(x) = f(x)$ and $f2(x) = \breve{f}(x)$. Then if v is a transitive verb and a is a name with interpretations $\underline{v}$ and $\underline{a}$ respectively, the interpretations of v 1+a and v 2+a are $\underline{v}1(\underline{a})$ and $\underline{v}2(\underline{a})$ respectively. With 'loves' interpreted by the function $\underline{\text{loves}}$, such that $(\underline{\text{loves}}(x))(y)$ is the True if x loves y, the interpretations of (1) and (2) become $\underline{\text{loves}}1\ (\underline{\text{sara}})$ and $\underline{\text{loves}}2\ (\underline{\text{sara}})$ respectively. It should be emphasized that I consider '1' and '2' to indicate modes of combination, in this case application to the first argument position and application to the second argument position respectively. They could also be construed as operations upon functions, but I do not do so.

Before continuing, I will say something about the unusual-looking rules (3) and (4). The usual sort of phrase structure rules serve two logically distinct functions when used to produce a structural description of a sentence. They indicate the grouping of expressions into phrases and they also indicate the combination of expressions by means of concatenation. Normally the distinctness of these functions is overlooked because of the fact that any (finite) linear string of expressions can be built up from its elements solely by means of concatenation to the right.[26] It is interesting to see the basic idea I have proposed above in (3) and (4) reflected quite clearly in Frege. Though Frege did not say much about syntax explicitly, it is clear that for him the structure of function and argument was both a semantic and syntactic structure. One job of a syntactic component would be to indicate which function names took which expressions as their argument names. Frege held that the way in which argument names were combined with function names was not simply concatenation to the right (or left), but by placing the argument names in gaps within the function names. This together with the fact (which Frege was prevented by other views from appreciating) that we cannot fix once and for all an order in which to fill in the gaps (as our above example shows) helps make clear the distinction between grouping and mode of syntactic combination. At this point we have sufficiently modified the original notion of a categorial grammar that a statement of the current content of 'extended categorial grammar' is needed. An extended categorial grammar consists of a categorial grammar together with 1) a system of "multiplying out rules" which specify permissible combinations of categories 2) for each permissible combination of categories, a set of pairs, called permissible modes for the combination, consisting of a syntactic mode of combination and a semantic mode of combination. The phrase structure component of the grammar is determined as follows: If $c_1 \dots c_n \rightarrow c$ is a permissible combination of categories and m is a permissible mode for the combination, then $c \underset{m}{\rightarrow} c_1 + \dots + c_n$ is a (extended) phrase structure rule of the extended categorial grammar. Derivations and structural descriptions are determined as usual except that nodes of a structure tree are labelled not only by a category, but also by a permissible mode. A simple example:

Basic categories: c

Vocabulary: a -- c, f -- c/c, g -- c/(c,c)

Permissible combinations: (i) $c/c \ c \rightarrow c$

(ii) $c/(c,c) \ c \ c \rightarrow c$

(iii) $c/c \ c/c \rightarrow c/c$

(iv) $c/(c,c) \ c \rightarrow c/c$

Permissible modes: (i) $m_1 = (m_1^{syn}, m_1^{sem})$

where $m_1^{syn}(F,A) = F \frown (\frown A \frown)$, $m_1^{sem}(\underline{F},\underline{A}) = \underline{F}(\underline{A})$.

(ii) $m_2 = (m_2^{syn}, m_2^{sem})$

where $m_2^{syn}(F,A_1,A_2) = F \frown (\frown A_1 \frown , \frown A_2 \frown)$, $m_2^{sem}(\underline{F},\underline{A}_1,\underline{A}_2) = \underline{F}(\underline{A}_1,\underline{A}_2)$.

(iii) $m_3 = (m_3^{syn}, m_3^{sem})$

where $m_3^{syn}(F,G) = (\frown F \frown G \frown)$, $m_3^{sem}(\underline{F},\underline{G}) = (\underline{FG})$.

$$\text{(iv)}\ m_4 = (m_4^{syn},\ m_4^{sem})$$

where $m_4^{syn}(F,A) = F\frown(\frown A\frown,\frown)$, $m_4^{sem}(\underline{F},\underline{A}) = \underline{SUB}_1(\underline{F},\underline{A})$.

$$m_5 = (m_5^{syn},\ m_5^{sem})$$

where $m_5^{syn}(F,A) = F\frown(\frown,\frown A\frown)$, $m_5^{sem}(\underline{F},\underline{A}) = \underline{SUB}_2(\underline{F},\underline{A})$.

Phrase structure rules:

(0) $c \rightarrow a,\ c/c \rightarrow f,\ c/(c,c) \rightarrow g$

(i) $c \underset{m_1}{\rightarrow} c/c + c$

(ii) $c \underset{m_2}{\rightarrow} c/(c,c) + c + c$

(iii) $c/c \underset{m_3}{\rightarrow} c/c + c/c$

(iv) $c/c \underset{m_4}{\rightarrow} c/(c,c) + c$

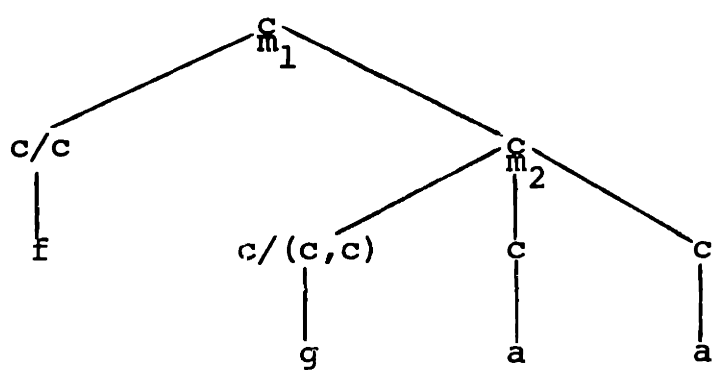
$c/c \underset{m_5}{\rightarrow} c/(c,c) + c$

For permissible combinations where there is only a single permissible mode, we will omit the mode label in phrase structure rules and structural descriptions. We will also vary the location and style of the mode label when it is supplied. For example, instead of (iv) immediately above we could also write:

$$c/c \rightarrow c/(c,c)\ 1{+}c$$

$$c/c \rightarrow c/(c,c)\ 2{+}c$$

Here is a sample structural description generated by the grammar:



The syntactic object associated with this structural description is:

$$E = m_1^{syn}(f,\ m_2^{syn}(g,\ a,\ a)) = f\frown(\frown g\frown(\frown a\frown,\frown a\frown)\frown)$$

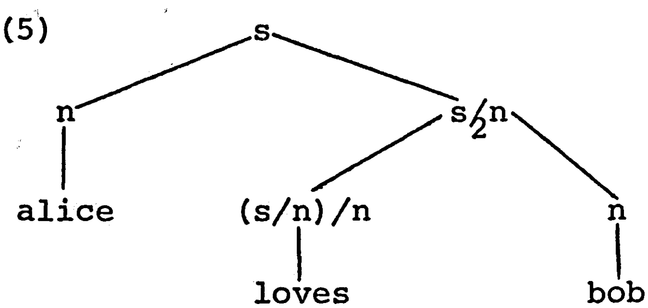
and the semantic object associated is:

$$\underline{E} = m_1^{sem}(\underline{f},\ m_2^{sem}(\underline{g},\ \underline{a},\ \underline{a})) = \underline{f}(\underline{g}(\underline{a},\ \underline{a})).$$ [27]

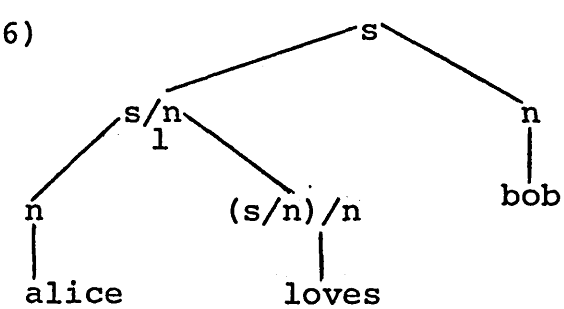
There is nothing to prevent particular grammars of this general sort from restricting the modes of syntactic combination to concatenation and restricting the semantic modes to application of a function to arguments. But there is no need to do so and there are at times reasons for not doing so. One further generalization would allow the syntactic modes to be only partially defined over the appropriate categories, effecting syntactic subcategorization. In most of the subsequent examples, we will not bother to specify details of the syntactic modes--leaving open the extent to which surface syntax is determined by deep syntactic modes or by transformations of the more usual sort, but will just write "nice" structural descriptions. Much of what follows in this paper can be taken as data to support the claim that extended grammars have promise for the description of natural languages.

We represent the structure of 'Alice loves Bob' as follows:

(5)



(6)



The numerals under the nodes labeled 's/n' indicate which of the rules (3) and (4) was applied to generate the phrase dominated by the node. As the pair of diagrams indicates, because of the two distinct ways of combining 'loves' with a name to get an intransitive verb phrase, the sentence receives two structures. If multiple structural descriptions predict that a sentence is ambiguous, and if ambiguity must be reflected in multiple meanings, then we must face the problem that the grammar predicts ambiguity where none is perceived. In our version of categorial grammar we will have to abandon the simple correspondence between the number of distinct structural descriptions and the degree of syntactically determined ambiguity. Structural ambiguity need not issue in semantic ambiguity.[28]

As an exercise we calculate the interpretations of the sentence based upon the two structural descriptions. For (5) we have, $(\underline{\text{loves2}}(\underline{b}))(\underline{a}) = (\breve{\underline{\text{loves}}}(\underline{b}))(\underline{a}) = (\underline{\text{loves}}(\underline{a}))(\underline{b})$. For (6) we have, $(\underline{\text{loves1}}(\underline{a}))(\underline{b}) = (\underline{\text{loves}}(a))(\underline{b})$. As desired, the two structures receive the same interpretation. As an additional exercise, check that both (5) and (6) are *strongly convertible*, that is, if we keep the same structure of non-terminals, interchange the two names, and replace the verb by its passive ('is loved by'), the resulting structural description receives the same interpretation as the original.

A modification of the semantic treatment of categories will greatly help us. We will identify the categories $(\ldots(c/c_1)/\ldots/c_n)$ with the categories $c/(c_1,\ldots,c_n)$, interpreting both categories semantically as we previously interpreted just the former. We could, with some reservations, eliminate the latter category altogether. We shall pass freely back and forth between equivalent category notations; for example, treating

$$((s/(s/n))/n)/n, \quad (s/(s/n))/(n,n), \quad \text{and } s/((s/n),n,n)$$

as interchangeable according to the needs of perspicuity.[29]

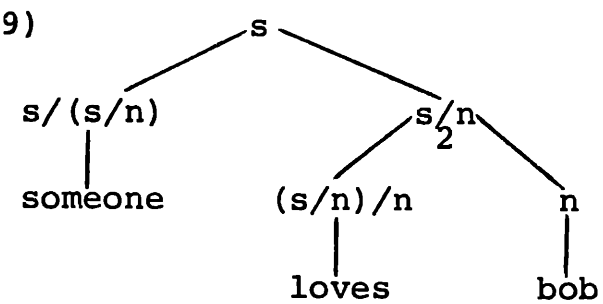
Introducing quantifiers calls for new notions. We saw earlier that the operation of composition of functions could solve some of the problems of adding quantifiers. But just as a function of two arguments may be applied to a single argument in two distinct ways, a function of two arguments may be composed with another function in several ways. Let f be a function of category s/(s/n) and g be a function of category (s/n)/n (= s/(n,n)). Then f*1g is $(f\breve{g})$ and f*2g is (fg). The two kinds of compositions can be generalized to other categories; we shall discuss generalizations later. We add two new syntactic rules

(7) s/n → (s/n)/n*1s/(s/n) and (8) s/n → (s/n)/n*2s/(s/n)

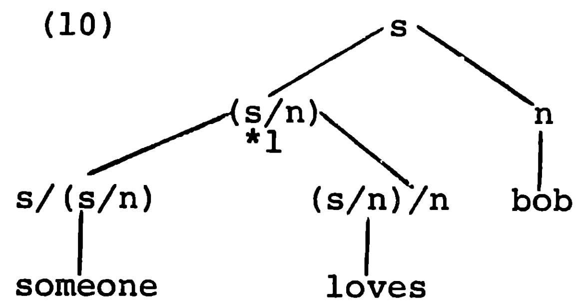
and semantic rules which interpret compounds formed by (7) using *1 and compounds formed by (8) using *2.[30]

'Someone loves Bob' has the structures:

(9)

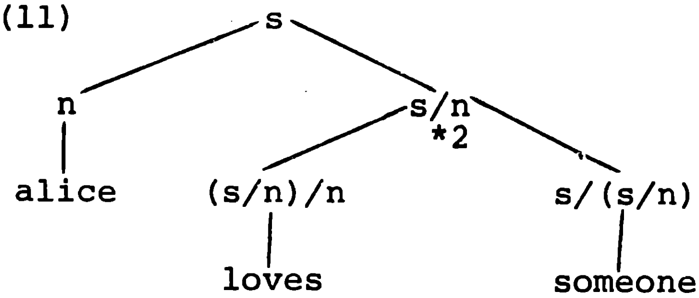


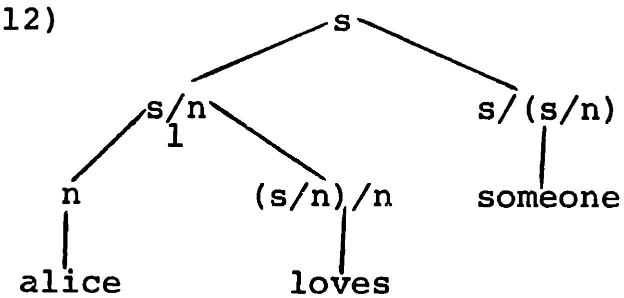
(10)



We let $\underline{\text{some}}$ be the function of category s/(s/n) such that $\underline{\text{some}}$ (f) = the True if f(x) = the True for some person x, of category n. Then (9) gets the interpretation $\underline{\text{some}}(\underline{\text{loves}}2(\underline{\text{b}}))$ = $\underline{\text{some}}$ $(\breve{\underline{\text{loves}}}$ $(\underline{\text{b}}))$. (10) gets the interpretation $(\underline{\text{some}}*1\underline{\text{loves}})(\underline{\text{b}})$ = $(\underline{\text{some}}$ $\breve{\underline{\text{loves}}})$ $(\underline{\text{b}})$ = $\underline{\text{some}}(\breve{\underline{\text{loves}}}(\underline{\text{b}}))$. Both structures receive the same interpretation, which is what is desired. Notice that (9) and (10) are each strongly convertible.

'Alice loves someone' has the structures:

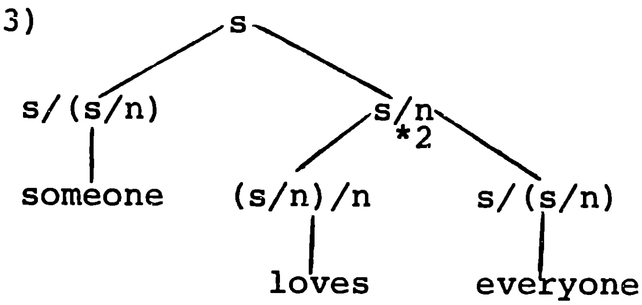
(11)


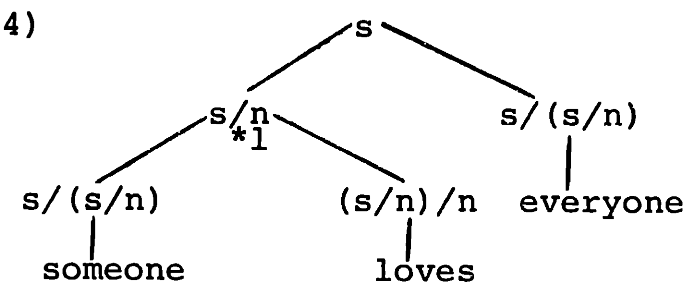
(12)


The interpretations are the same, and both structures are strongly convertible.

The first interesting test of this new machinery is a pair of sentences for which the apparatus was not specially tailored, 'Someone loves everyone' and 'Everyone loves someone'.

'Someone loves everyone' is ambiguous and so has the (non-equivalent) structures

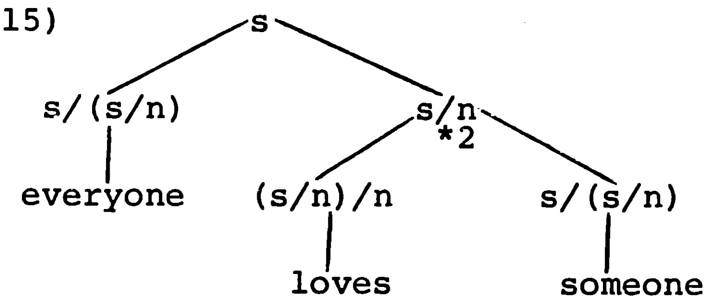
(13)


(14)


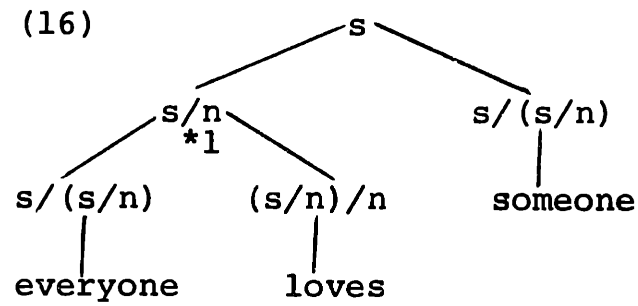
Let every be the function of category s/(s/n) such that every(f) = the True if f(x) = the True for every person, x, of category n. Then (13) gets the interpretation some(every*2 loves) = some (every loves). (14) gets the interpretation every(some*1 loves) = every(some loves). It is easy to verify that these are quite distinct interpretations, (13) corresponding to $\exists x \forall y Lxy$ and (14) to $\forall y \exists x Lxy$. In this case, the mulitplicity of structural descriptions does correspond to a genuine ambiguity in the sentence. Neither (13) nor (14) are strongly convertible. But they are weakly convertible: they are alternative structural descriptions of the same sentence such that the result of "passiving"[31] one has the same interpretation as the other. (13) passivized has the interpretation every(some*2 loves) = every(some loves), which is the same as the interpretation of (14). (14) passivized has the interpretation some(every*1 loves) = some(every loves) = some(every loves), which is the same as the interpretation of (13). The difference between strong and weak convertibility provides a sense in which actives and their corresponding passives are not equivalent and a sense in which they are.

'Everyone loves someone' has the structures

(15)



(16)



with interpretations every(some*2 loves) = every(some loves) and some(every*1 loves = some(every loves) respectively, which correspond to $\forall x \exists y Lxy$ and $\exists y \forall x Lxy$.

We introduce the following abbreviations for various categories to help shorten what follows.

v for s/n, the category of intransitive verb phrases

tv for (s/n)/n, the category of transitive verb phrases

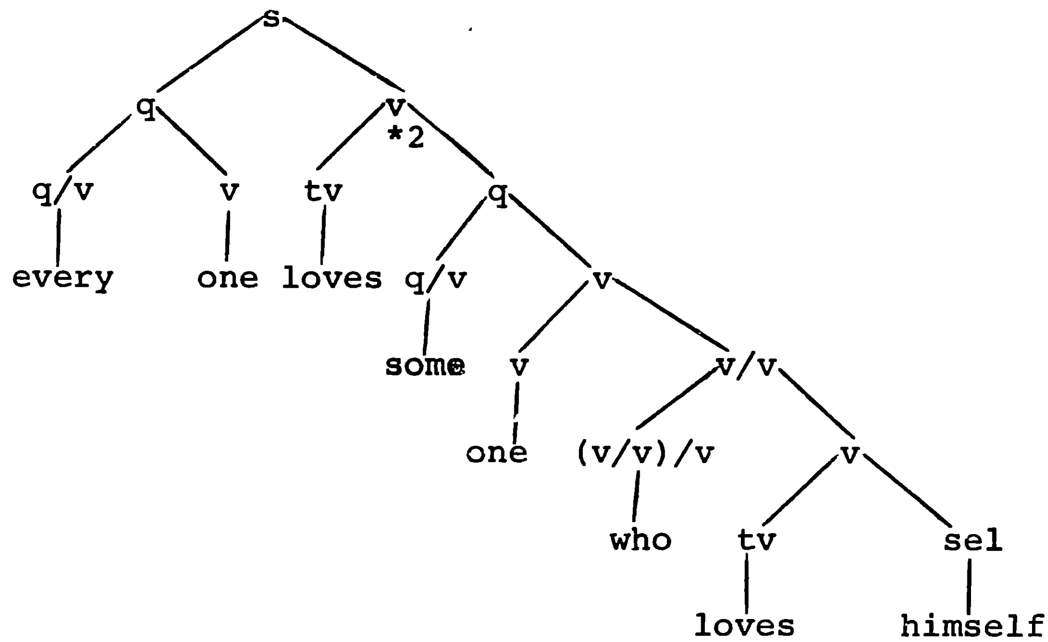
q for s/(s/n), the category of quantifier phrases

rel for (v/v)/v, the category of relative pronouns

sel for v/tv, the category of reflexive pronouns

Let us now consider more complicated sorts of quantified sentences such as 'Everyone loves someone who loves himself'. We add the vocabulary items 'who' of category rel and 'himself' of category sel.[32] One of the structures for the sentence is

(17)

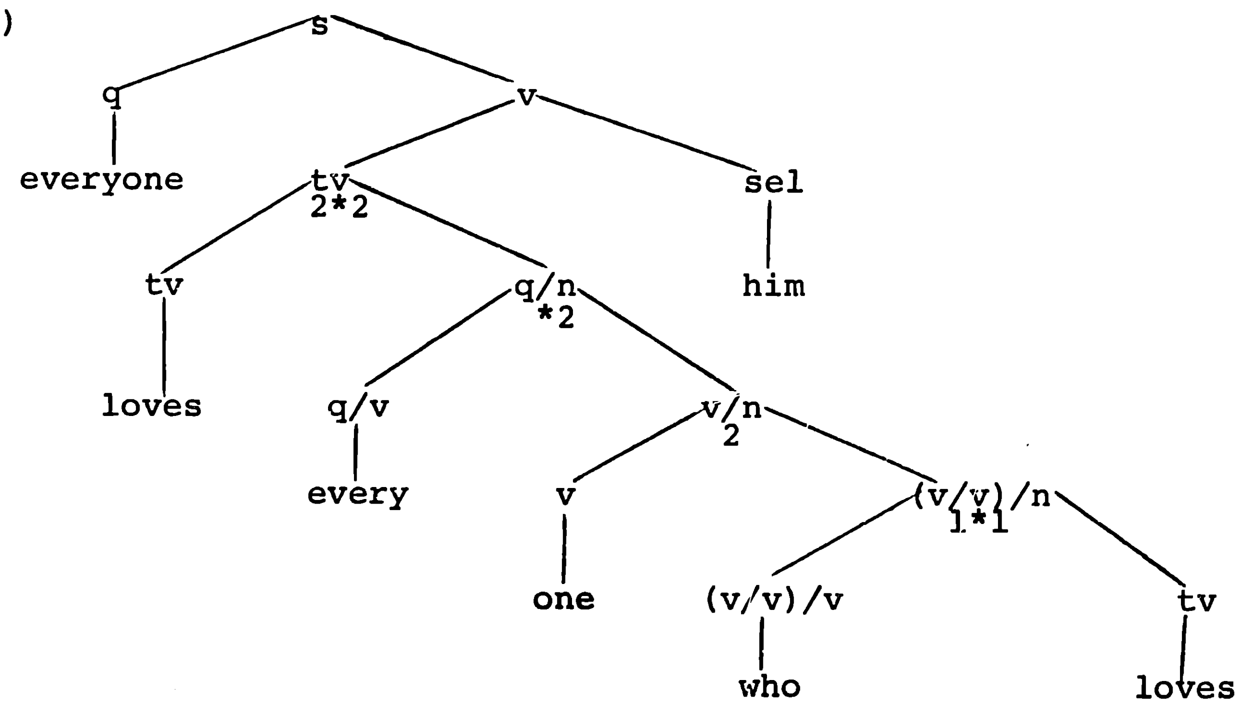


which does not involve any basically new ideas.[33] A slight variant, 'Everyone loves someone who loves him', does raise a new matter. On one of the structures for this sentence, we shall want 'loves' of category tv and 'someone who loves' of category q/n to combine to form 'loves someone who loves' of category tv. In order to get this structure we shall have to add new types of phrase structure rules and corresponding category "multiplying-out" rules as well as a semantic interpretation rule. Can we give a justification for the new rules?

Let us reconsider the operation of composing two functions. Consider, more particularly, a function f of category $c/(c_1,c_2)$ and functions g_1,g_2 of categories c_1/d_1 and c_2/d_2 respectively. We previously defined a notion of generalized composition which applied to f,g_1,g_2 yielding $\mathrm{COMP}(f,g_1,g_2)$ of category $c/(d_1,d_2)$. We now define two more "composition" operations. $\mathrm{COMP1}(f,g_1)$ is of category $c/(d_1,c_2)$ such that $\mathrm{COMP1}(f,g_1)\ (x,Y) = f(g_1(x),y)$ for all x,y of the appropriate categories. $\mathrm{COMP2}(f,g_2)$ is of category $c/(c_1,d_2)$ such that $COMP2(f,g_2)(x,y) = f(x,g_2(y))$ for all x,y of the appropriate categories. Just as filling in a term position of a transitive verb phrase with an item of category n signals the application of a function to an argument with respect to one of two argument places, so the filling in of a term position of a transitive verb with a term of category q/n (= (s/(s/n))/n) signals the composition of two functions with respect to one of two argument places--either by COMP1 or COMP2. To simplify the notation a bit, we shall write $f1{*}g_1$ for $COMP1(f,g_1)$ and $f2{*}g_2$ for $COMP2(f,g_2)$. The numeral before the asterisk tells us which argument place of the first function is being "plugged-into." Note the difference between '1*' and '2*' which were just introduced and '*1' and '*2' which were introduced earlier. '*1' indicates composition in which the second function operand is first made into its converse and '*2' indicates normal composition. We will have need of a combined notation such as '1*2' which indicates a combination of the effects of '1*' and '*2'. An intuitively convenient way of interpreting this notation (which makes easy further generalizations) involves making use of the canonical identification of categories mentioned earlier and treating the numerals flanking the asterisk as indicating certain permutations of argument places of the function on the same side of the asterisk as the numeral. Consider f of category (s/(s/n))/n (= q/n) and g of category (s/n)/n (= tv). Then f1*2g can be thought of as obtained by treating f as of category s/(s/n,n), moving the argument numbered 1 (from the left) to the rightmost argument position by a permutation operation, rewriting the resulting category as (s/n)/(s/n), permuting the function g so that the argument numbered 2 is in the rightmost position (which, in this case it is already), and then composing (in the ordinary way) the two resulting functions. Thus f1*2g is of category tv and f1*2g (x,y) = f(g(x),y) for all x,y of category n. f2*2g(x,y) = f(x,g(y)). f1*1g(x,y) = f($\breve{g}$(x),y). f2*1g(x,y) = f(x,$\breve{g}$(y)).[34]

Now we can represent one structural description of 'Everyone loves everyone who loves him':

(18)

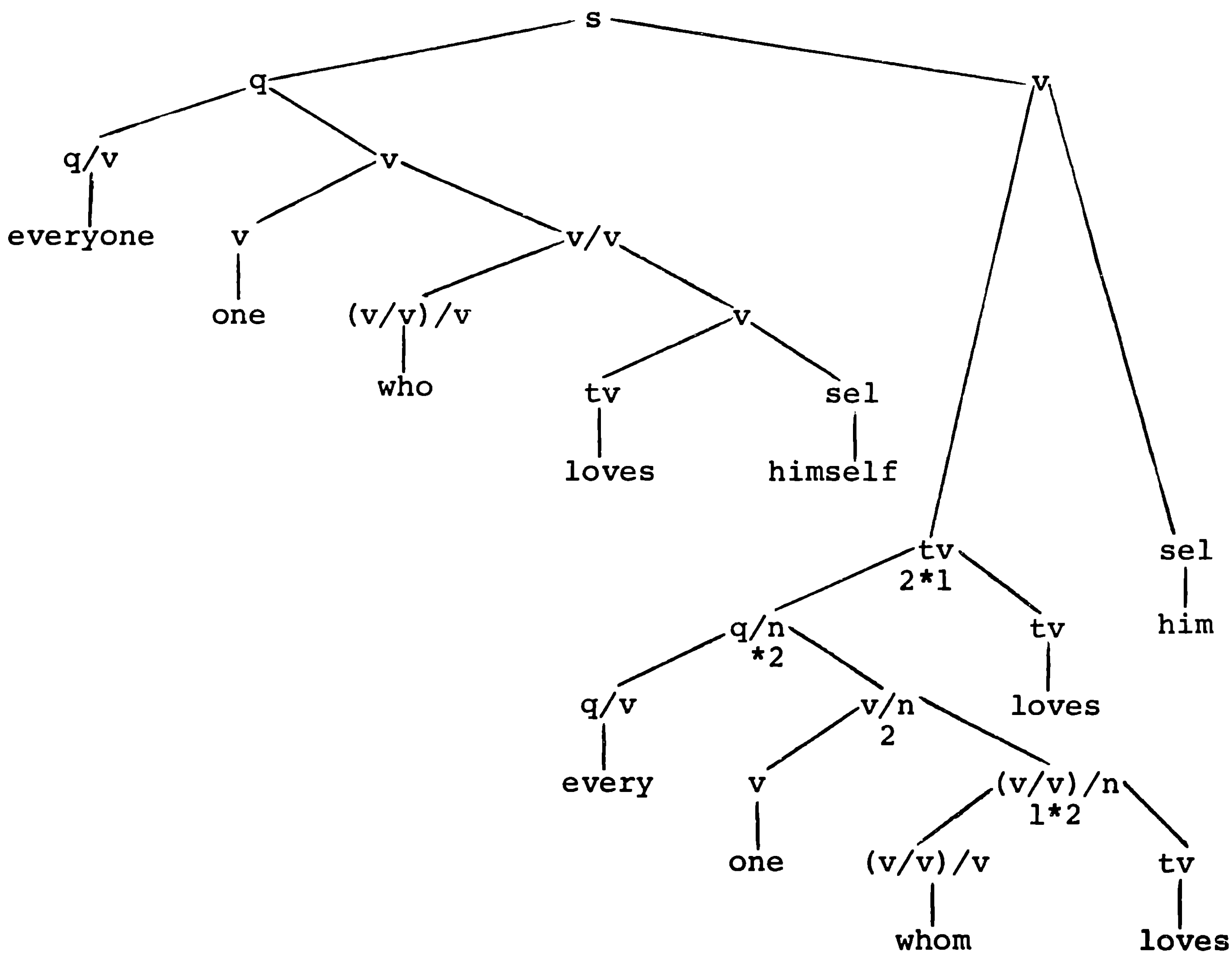


We work out the interpretation in detail. Let f(x)(y) = (who 1*1 loves) 2(one)(x)(y). f(x)(y) = T iff y is one who loves x (by P8, note 34).

So (every *2 f)(x)(g) = T iff every(f(x))(g) = T iff g(z) = T for every z such that f(x)(z) = T iff g(z) = T for every one z such that z loves x iff g(z) = T for everyone z who loves x. Now let f'(x)(y) = ((every *2 f) 2*2 loves) (x)(y). f'(x)(y) = T iff y loves everyone who loves x (by P6). So him(f')(x) = T iff x loves everyone who loves x. Finally, everyone(him(f')) = T iff everyone loves everyone who loves him.

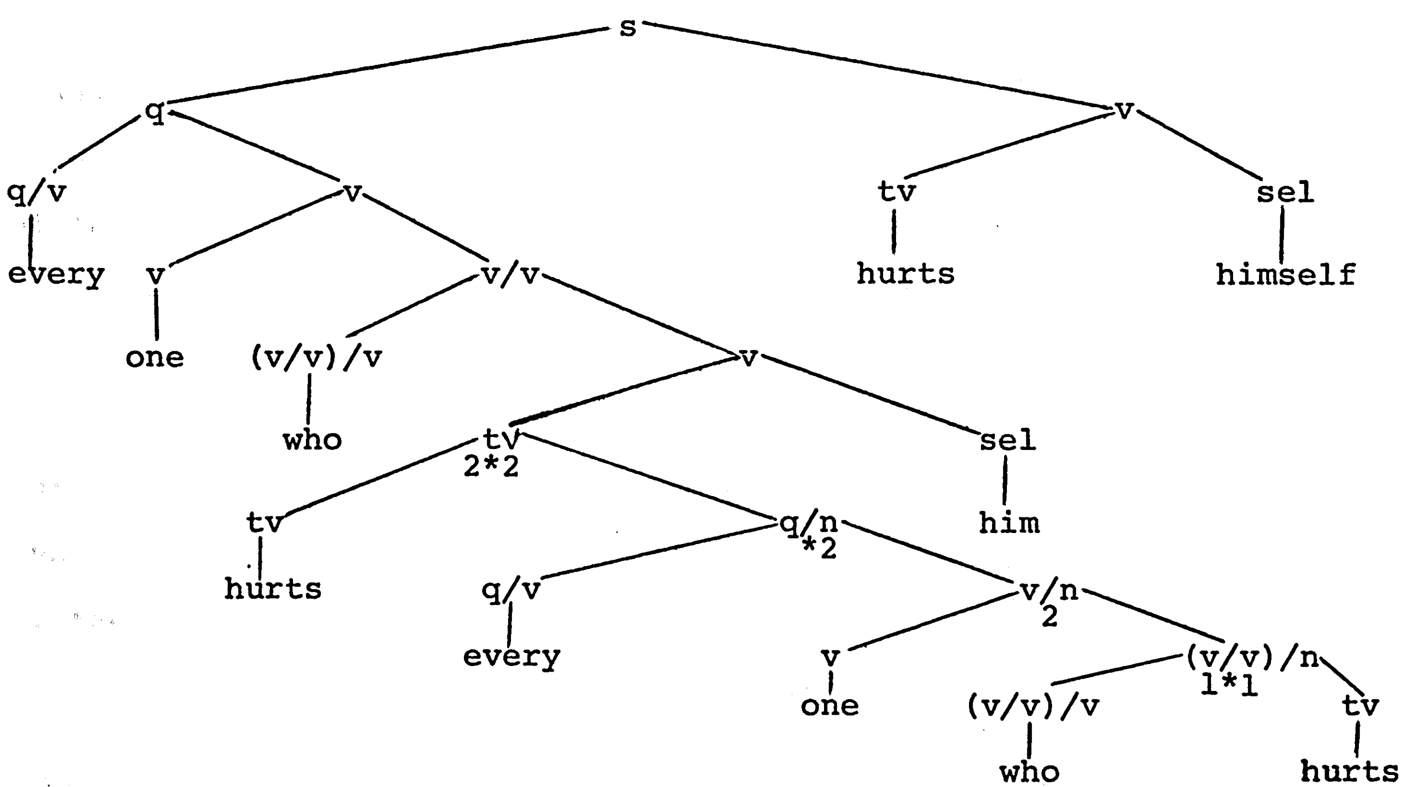
Two more examples:

(19) Everyone whom a person who loves himself loves loves him.



This example is interesting because it is our first case in which the deep structure differs significantly from the surface structure, an indication of which is the fact that we have been unable to get the desired string of terminals in the correct order by means of our practice of freely choosing which is the left and which the right branch of a 2-branch node. The deep structure subject is the quantifier phrase 'everyone who loves himself' while in the surface structure, the subject is 'everyone whom a person who loves himself loves'. The reason for this discrepancy is that the surface verb phrase 'loves him' is not a real predicate in this sentence. In particular, it is not the reflexive 'loves himself' nor can it be construed as 'loves some contextually definite person'. The pronoun here is anaphoric and is used together with the quantifier phrase 'a person who loves himself' to make a (restricted) universal generalization. The difference in surface and deep structure emphasizes the need for precisely stated "transformations" to complete our account. Two particular aspects which are involved in our examples are the rules for selecting the form of reflexive pronouns--when to use 'himself' and when 'him', and the rules for selecting universal quantifier phrases--when to use 'everyone', or 'a person', or 'anyone', or 'each person'. About the second aspect I have nothing to say here. About the first, only that in all the examples we consider the difference is simple--'himself' reflexivizes a simple (non-compound) transitive verb while 'him' is used both anaphorically and to reflexivize compound verb phrases.

(20) Everyone who hurts everyone who hurts him hurts himself.



The permissible modes of categorial combination determine the phrase structure rules. But what are the limits of permissible modes of combination? In principle, there are no limits. Consider any three categories, c_1, c_2, c_3. There will always be set theoretically definable operations that combine entities of categories c_1 and c_2 to yield entities of category c_3. So doesn't the extended categorial approach "blow up" by allowing every conceivable sort of phrase structure rule, that is all rules of the f- m $c_3 \rightarrow c_1 + c_2$ for arbitrary categories c_1, c_2, c_3? In principle yes, but not in application. The limits of permissible combination are to be determined in the case of a particular language by an analysis of that language. To illustrate in detail how this might be done, let us switch from the consideration of English to a better understood and more manageable language, FOL.

We consider a fragment of FOL with just three predicates, F^1, F^2, and F^3 of degrees 1, 2, 3 respectively and just two names, a and b and a single function symbol, f, of degree 1. The extended categorial grammar, GFOL: The base categories are s and n. We shall also mention the complex categories $c^1 = s/s$, $c^2 = s/(s,s)$, $q = s/(s/n)$, and $p^i = s/(n,\ldots,n)$ with i occurrences of 'n', for each positive integer i.

The vocabulary items are assigned categories as follows:

a -- n, b --n, f -- n/n, F^1 -- p^1, F^2 -- p^2, F^3 -- p^3,

$\sim$ -- c^1, $\&$ -- c^2, $\exists$ -- q, $REF^{i+j}_{k_1 \ldots k_j}$ -- p^i/p^{i+j}, $i,j \geq 1$

Variables along with parentheses will be inserted by appropriate syntactic modes of combination. The phrase structure component of the grammar consists of several types of rules.

Type I -- vocabulary categorization

These rules just turn the categorization into the standard form for phrase structure rules. For example, $n \rightarrow a$ and $q \rightarrow \exists$.

Type II -- pure rules

$n \rightarrow n/n + n, \quad s \rightarrow p^1 + n, \quad s \rightarrow p^2 + n + n,$

$s \rightarrow p^3 + n + n + n, \quad s \rightarrow c^1 + s, \quad s \rightarrow c^2 + s + s,$

$s \rightarrow q + p^1.$

These are the standard sort of rules, the limitations of which caused us various difficulties in our earlier accounts of FOL.

Type III -- partial application rules

$p^1 \rightarrow p^2\ 1{+}n, \quad p^2 \rightarrow p^3\ 1{+}n, \quad p^2 \rightarrow P^3\ 3{+}n,$

$p^1 \rightarrow p^2\ 2{+}n, \quad P^2 \rightarrow p^3\ 2{+}n.$

These rules make use of our canonical identification to permit the plugging-in of an argument at any of the argument places of the predicates. They could be extended to allow plugging-in more than one argument at a time, if desired. Or we could delete the Type II rules concerning p^2 and p^3, if desired. The sign '1+' and its fellows indicate which argument place is to be filled. We can easily generalize this sort of rule to the infinite set of rules $p^i \rightarrow p^{i+1}\ j + n$, for $i \geq 1$ and $1 \leq j \leq i$. Various of these will be needed to accomodate predicates of various degrees, though only finitely many as long as only finitely many predicates are in the vocabulary. Even though predicates of arbitrary degree can be formed by using & repeatedly, we need only accomodate the finitely many basic predicates with Type III rules because names can always be plugged-in at the earliest level (at least in FOL).

Type IV -- simple composition rules

$n/n \rightarrow n/n * n/n$, $\quad p^{i} \rightarrow s/s * p^{i}$, $\quad p^{i+j} \rightarrow s/(s,s)*(p^{i},p^{j})$

for $1 \leq i,j$

These rules allow us to operate on predicates as we must as a prelude to quantification.

Type V -- partial composition rules

$p^{i} \rightarrow p^{i}\ j*\ n/n$, for $i \geq 1$ and $1 \leq j \leq i$

These rules allow unevaluated functions to be plugged into argument places of predicates. For reasons similar to those given concerning Type III rules, we could get by with just a finite number of these. If we had function names of degree greater than 1, we would need rules of this sort for them too.

Type VI -- permuted composition rules

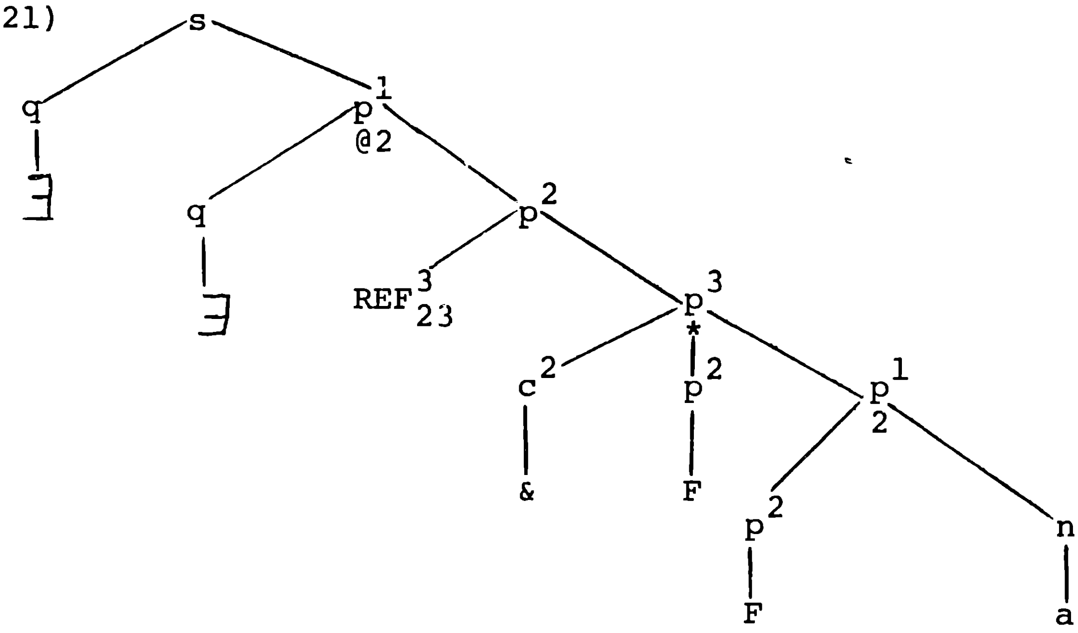
$p^{i} \rightarrow q\ @j\ p^{i+1}$, for $i \geq 1$

These rules allow applying a quantifier to the jth argument place of a predicate of degree greater than 1. In conjunction with the operators, REF, which effect the identification of argument places, this treats the full range of variable and quantifier phenomena.

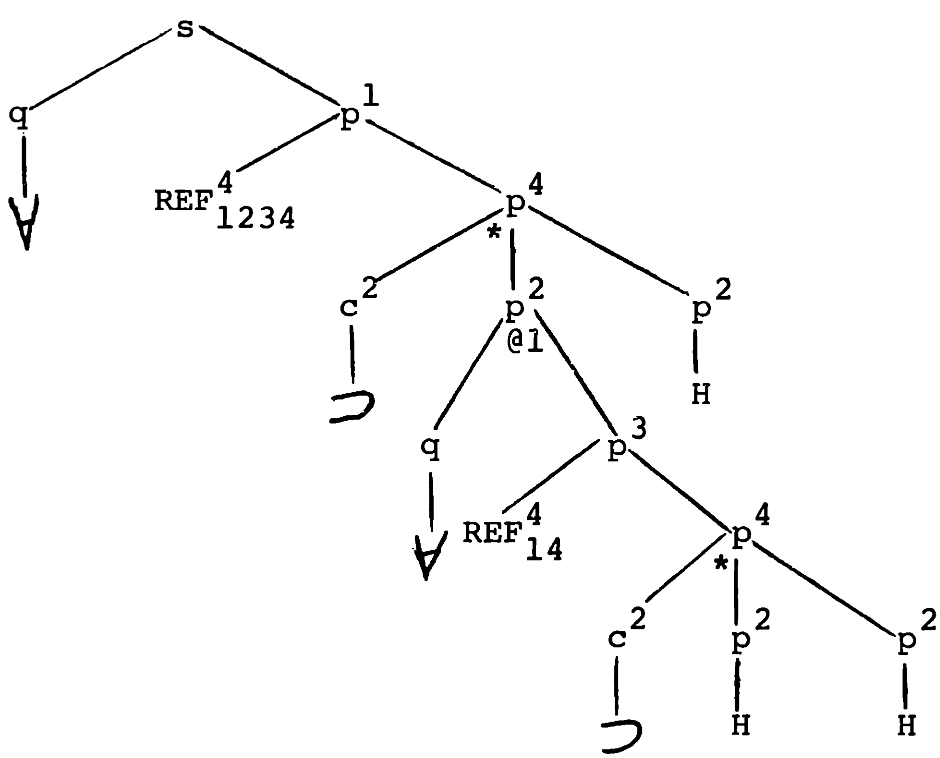
These together with syntactic rules for inserting variables and placing parentheses are all we need to generate the standard notation for FOL. We look at an example or two, present the rules of semantic interpretation, and discuss the significance of this formulation of FOL.

(21) $\exists x \exists y (Fxy\ \&\ Fya)$

(21)



(22) $\forall x(\forall y(Hyx \supset Hxy) \supset Hxx)$



(21) is just a straightforward application of GFOL. In (22) I took the liberty of adding a new quantifier and connective in order to get an interesting example. The modifications to GFOL necessary to generate (22) are trivial. The interest of (22) lies in the fact that it gives the structural description of a plausible symbolization of the English sentence of (20).

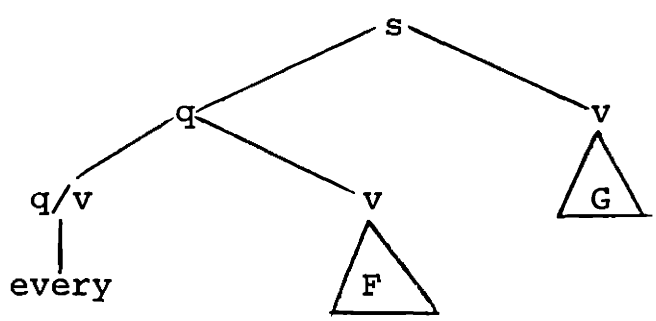
Having GFOL and even a fragmentary grammar for English along the same general principles makes possible the beginning of a comparison of the structure of the devices of quantification in both languages. It also offers the hope that we can formulate effective rules for "translating" structures such as (20) into structures such as (22), thus providing us with a means for specifying precisely what it is (or at any rate, what one of several quite different things is) that we are doing when we "translate" English into the notation of quantification theory.

We interpret GFOL semantically along conventional lines. We assign a non-empty domain D to category n and the set of the truth values to s; the complex categories receive the usual interpretation relative to this base. The names, predicates, and function names are interpreted in any way consistent with their categories. $\sim$ and & map truth values to truth values in the usual way. $\exists$ maps elements of category s/n into truth values such that $\exists(f)$ = the True if f(d) = the True for some d in D. There are numerous modes of semantic composition. Corresponding to the Type II rules is the operation of applying a function to degree i to i arguments, signaled by '+'. Corresponding to the Type III rules are the operations of partial application, j+, such that if f is of category $c/(c_1,\ldots,c_i)$ and x is of category c_j for $1 \leq j \leq i$, then fj(x) is of category $c/(c_1,\ldots,c_i)$ (with c_j excluded) and $fj(x)(y_1,\ldots,y_i)$ (with y_j excluded) = $f(y_1,\ldots,x,\ldots,y_i)$ (with x in the jth argument place). In writing the operation, we omit the '+' where convenient. Corresponding to Type IV rules is the operation of composition of a function of degree i with i functions of degrees $j_1,\ldots,j_i$ to yield a function of degree $j_1+\ldots+j_i$, signaled by the symbol '*'. Corresponding to the Type V rules are the partial composition operations, $j*$, such that if f is of category $c/(c_1,\ldots,c_i)$ and g is of category $c_j/(b_1,\ldots,b_k)$ for any i and k and any $1\le j\le i$, then f $j*$ g is of category $c/(c_1,\ldots,b_1,\ldots,b_k,\ldots,c_i)$ ($b_1\ldots b_k$ inserted to replace c_j) and $f\ j*\ g\ (x_1,\ldots,y_1,\ldots,y_k,\ldots,x_i) = f(x_1,\ldots,g(y_1,\ldots,y_k),\ldots,x_i)$. Corresponding to Type VI rules are the permuted composition operations, @j, such that if F is of category $c/(b/(b_1,\ldots,b_k))$ and if f is of category $b/(b_1,\ldots,b_k)$ for $k > 1$, $1\le j\le k$, then F@jf is of category $c/(b_1,\ldots,\cancel{b}_j,\ldots,b_k)$ with b_j excluded). Let f^j be the function such that $f^j(x_1,\ldots,\cancel{x}_j,\ldots,x_k)(x_j) = f(x_1,\ldots,x_k)$. Then $F\ @j\ f(x_1,\ldots,\cancel{x}_j,\ldots,x_k) = F(f^j(x_1,\ldots,\cancel{x}_j,\ldots,x_k))$. Finally, we specify the interpretation of the permutation and identification functions, $\underline{\mathrm{REF}}^{i+j}_{k_1\ldots k_j}$. If f is of category $c/(c_1,\ldots,c_{i+j})$ and $1\le k_1,\ldots,k_j\le i$ and $c_{k_1}=\ldots=c_{k_j}$, then $\underline{\mathrm{REF}}^{i+j}_{k_1\ldots k_j}(f)$ is of category $c/(c_1,\ldots,c_{i+j})$ (with $c_{k_2},\ldots c_{k_j}$ all deleted) and $\underline{\mathrm{REF}}^{i+j}_{k_1\ldots k_j}(f)(x_1,\ldots,x_i) = f(y_1,\ldots,y_{i+j})$, where for $i=k_1,\ldots,k_j, y_i=x_i$ and for all other i, y_i=the first x (starting at x_2) not yet picked out.

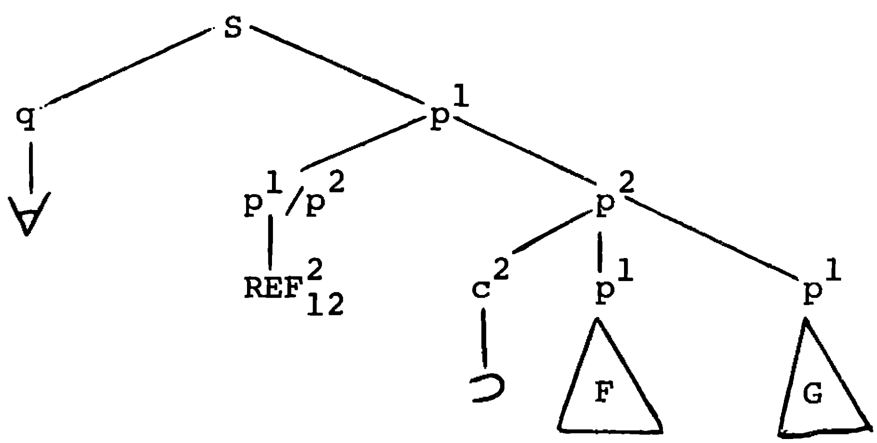
It is easy to verify that this notion of an interpretation of FOL based upon a domain, D, interpretations of the names, function names and predicates, coincides precisely with the standard sort of interpretation. In fact, we have just given the standard interpretation, expressed as is appropriate to an extended categorial grammar.

We can also see our way to a precise formulation of the difference between formulating the truth conditions of an English sentence in FOL and representing the logical form of the English sentence. On the analyses I have presented, there are several obvious differences in the way in which quantification in FOL and quantification in English differ. These include (i) the way in which quantifiers are relativized to predicates, (ii) the way in which argument positions are identified, (iii) the possibility of non-trivial structural ambiguity, and (iv) the means for controlling the scope of quantifiers. We cannot fairly conclude that English and FOL have the same structures, even if we limit our consideration to those parts of English which are easily and uniformly translated into FOL. But we can isolate structures of each language which perform similar functions. For example, these structures play similar roles in expressing restricted universal quantification:

(23)



(24)



Relative to such identifications, which can be more or less reasonable, we may establish a scheme of translation of English into FOL. Many (infinitely) expressions of FOL express the truth conditions of a sentence of English if any one does. But not all of them stand in close structural relation to the English sentence. Only those expressions which stand in close structural relation can plausibly be said to have the same logical forms. Unfortunately, there is no reason in favor and many reasons against the view that every English statement has a logical form which is adequately mirrored in the structure of some expression of FOL. We often find ourselves paraphrasing a sentence into "canonical" English in order to symbolize it.[35]

Part 9 -- Comparison with Montague grammars and further remarks on quantifiers

Because of the similarity between the extended categorial approach of the previous section and the approach of Montague in "The Proper Treatment of Quantification in Ordinary English" (sometimes called Montague grammar) it is worthwhile to note some of the differences in the two approaches. First we consider the syntactic component of a Montague grammar, (MG). MG categorizes only some of its basic vocabulary and the classes of well-formed expressions of various categories are defined by a simultaneous inductive definition determined by a set of syntactic rules. The syntactic rules consist of two parts, a categorial part and a lexical part. The categorial part of a syntactic rule specifies a collection of input categories and an output category and the lexical part specifies a function which combines any collection of well-formed expressions of the input categories to yield an expression of the output category. No general characterization of the lexical functions is given, but it seems obvious that any effective operation on the input expressions would be acceptable. The uncategorized vocabulary items are incorporated into expressions by means of various of the lexical rules. The definition of meaningful expression determines a kind of structural description for the meaningful expressions which Montague calls an analysis tree, which records how the expression was built up from the basic vocabulary items by the various lexical functions and also records the category of each phrase constructed along the way. Aside from differences in detail and emphasis, MG organizes the syntactic component of a grammar in essentially the same way as does the extended categorial approach; in particular, both types of grammar generate a phrase structure for an expression which is not expected to yield the expression simply by the operation of repeated concatenation of well-formed subcomponents. The major difference in the two approaches to syntax is that MG allows uncategorized basic vocabulary items, while the extended categorial approach does not. These uncategorized elements of Montague's grammar, such as 'every', 'the', and 'a', which function in just a single rule, relating just a single collection of input categories, i_1, $\ldots, i_n$, to a single output category, o, can be easily assigned a category, namely $o/(i_1,\ldots,i_n)$. The others, such as 'and', which relate different sets of categories can also be categorized, but only within the extended framework which allows operations other than functional application in the semantic component.[36]

As for the semantic component of the two types of grammars, there is general agreement. Categorized vocabulary elements are assigned set theoretic interpretations of a sort determined by the structure of the category and compound phrases are assigned interpretations by means of semantic compounding rules which correspond to the lexical compounding rules. But MG is more ambitious than the particular fragmentary grammar presented piecemeal in the previous sections; interpretations of MG assign more complexly structured entities to the various categories, intensions instead of merely extensions. This allows treatment of numerous semantic phenomena which do not have an analogue in FOL, such as modal operators, intensional verbs, and tense operators. In the previous sections I was concerned primarily with analyzing and representing the structure of FOL and certain aspects of English which are closely related to FOL. But the structure of quantification in English is far more complex and varied than the fragment that we have discussed so far, and it has features which are not reflected in the structure of FOL. My rejection of Montague's treatment of quantification in English was aimed at David Lewis' syntactic arguments in its favor and as a way of introducing the notion of extended categorial grammar. In the rest of this section I will discuss some of the ways in which quantification in English is much richer in structure than quantification in FOL, and reexamine Montague's treatment of names, quantifiers, and transitive verbs.

English quantificational devices are much richer than the devices of FOL, and strictly exceed them. The phrases 'everything' and 'something' together with pronouns and such bound variable substitutes as 'the first thing', 'the second thing', etc. can be used to duplicate any quantificational structure of FOL. Of course, the results are often barbarous and unintelligible. Phrases such as 'any 5, not necessarily distinct, things are such that' help with some of the problems of multiple quantification in English. English also has a wide range of semantically different quantifier-forming phrases, some of which are not expressible at all in FOL: 'fewer than 3', 'no more than 4', 'exactly 17', 'at least 7', 'more than 8', 'half of the', 'many', 'most', 'few', etc. There are additional constructions which present serious problems for analysis. Consider the sentences

(25) All of the people in this room and some of the people in the next love each other.

(26) All of the people in this room love each other and some of the people in the next room love each other.

(25) has an interpretation which entails that there is a set consisting of all the people in this room plus some of the people in the next room such that any two people in the set love each other. Since (26) has no such interpretation, we cannot take (25) to be obtained from (26) in the way that it is often claimed that (27) is obtained from (28).

(27) All of the people in this room and some of the people in the next room are fat.

(28) All of the people in this room are fat and some of the people in the next room are fat.

By considering

(29) Jack and Jill love each other.

and

(30) Jack loves Jill and Jill loves Jack.

we might be led to suspect that 'each other' is an element of category tv/tv which, in effect, conjoins a transitive verb with its converse while the compound subject provides the necessary two argument expressions. But then what do we do with

(31) Some of the people in this room love each other.?

We might try to treat (31) as coming from

(32) Some pairs of the people in this room love each other,

analyzing the verb phrase as in (30). Notice that this involves treating the quantifier 'some of the people' as homonymous or else taking 'each other' to operate on quantifiers as well as verbs. But even this won't do, because

(33) All pairs of the people in this room and some pairs of the people in the next room love each other.

does not have as an interpretation the desired interpretation of (25). One possible way of dealing with such sentences involves treating 'each other' as of category (s/tv)/q, operating on the complex quantifier phrase 'all of the people in this room and some of the people in the next room'.[37]

Furthermore, not only are there cases in which 'and' occurring between quantifier phrases seems not to be derived by a reduction of a sentence conjunction, but there are cases in which a sentence conjunction seems to be derived from a sentence with a compound verb phrase. Consider

(34) All the girls, but none of the boys, love a saxophonist in the band.

(35) All the girls love a saxophonist in the band, but none of the boys love a saxophonist in the band.

(36) All the girls love a saxophonist in the band, but none of the boys love one.

(37) All the girls love a saxophonist in the band, but none of the boys love him.

Since there is an interpretation of (34) on which it does not agree with any interpretation of (35), (34) cannot be derived from (35) by a reduction. (35) and (36) have the same interpretations. But (37), which superficially resembles (36), cannot be derived from (35). Rather, we shall see that (37) can be best thought of as being derived from

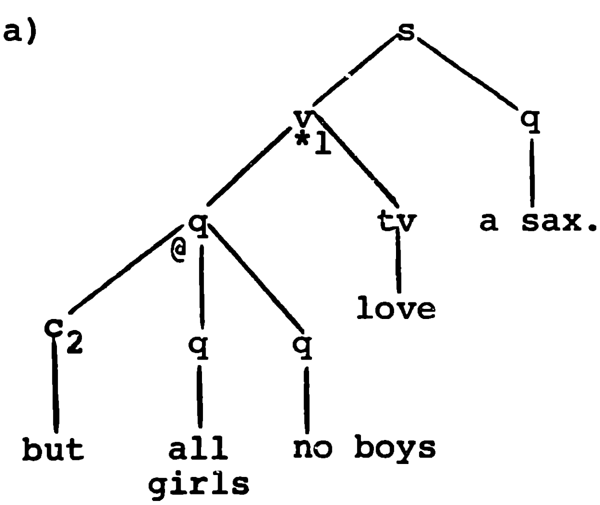
(38) All the girls love, but none of the boys love, a saxophonist

in the band.

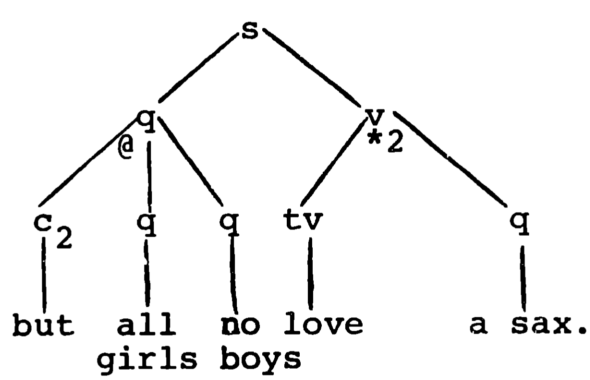
As a preliminary to an analysis of the sentences (34)-(38), let us consider how 'and' combines with two items of category v in English. The example, 'eats and wets' leads us to see that 'and' combines two intransitive verb phrases to form a new <u>intransitive</u> verb phrase. Is there a mode of semantic combination that corresponds to this rule: $s/(s,s)$ v $v \rightarrow v$? There is indeed. Remember $v = s/n$. Let '@' stand for an operation such that if f is of category $c/(c_1,c_2)$ and g_1 is of category c_1/b and g_2 is of category c_2/b then $f@(g_1,g_2)$ is of category c/b and $f@(g_1,g_2)(x) = f(g_1(x), g_2(x))$ for all appropriate x. Notice that this is quite a different mode of combination than we earlier assigned to '&' in our analysis of FOL. The treatment of 'CONJ' in the analysis of PFL was different yet, being a sort of hybrid of the other two. There are 'and's and there are '&'s. 'or' combines with verbs in a like manner. Without any additional apparatus, we can compound quantifier phrases too, since if Q_1, Q_2 are of category q, <u>and</u> is the interpretation of 'and', and f is of category v, then $\underline{and}@(Q_1,Q_2)(f) = \underline{and}(Q_1(f),Q_2(f))$.

Here are some plausible structures.

(39a)

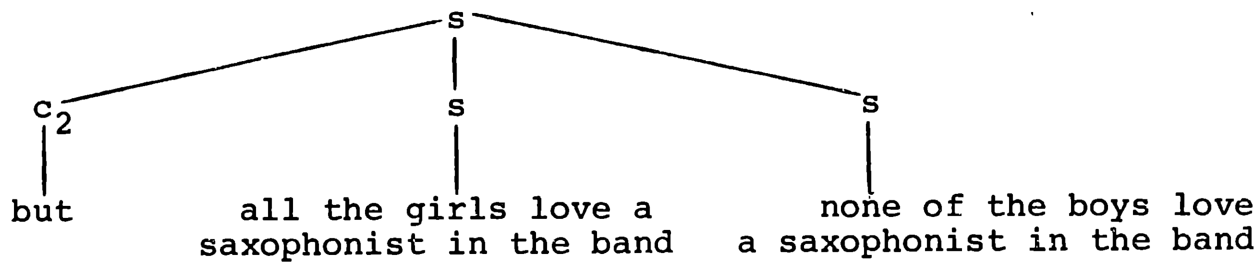


(39b)



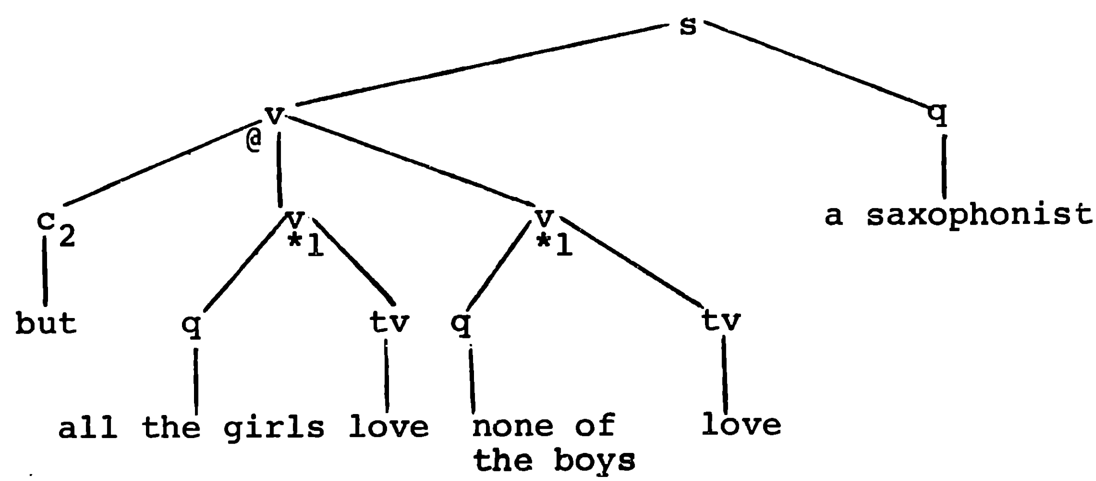
We get two structures corresponding to the two scopes of the direct object.

(40)



Each of the two sentence components of (40) has 2 interpretations, yielding a total of 4 interpretations for the whole sentence, (35). (36) has the same structure as (35) except that a pronoun of laziness has replaced the second occurrence of the shared quantifier phrase. Either (36) is 4 way ambiguous or there are some restrictions on the insertion of pronouns of laziness. The structure of (37) seems best represented by

(41)



which is also the structure for (38). The pronoun in (37) is a "bound variable" pronoun, which can fill a position governed by a preceding quantifier. This agrees nicely with the unambiguity of (37) and (38). In this case, at least, the wide variety of modes of combination and the wide variety of structures thereby permitted is well suited to the differences in the properties of the sentences considered.

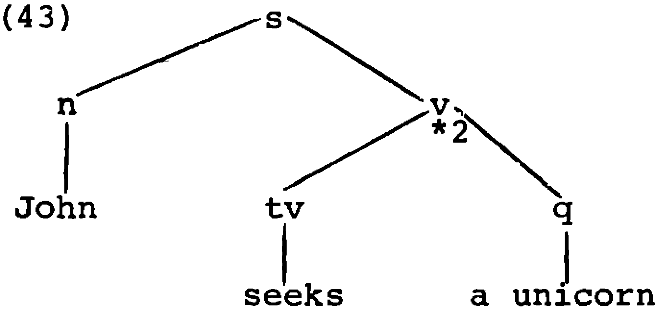
Having seen the utility of operations on quantifiers that yield quantifiers, let us now consider operations on names which yield quantifiers. First notice that there is no theoretical problem with operators of category q/(n,n), for example. Whether or not there are any English expressions which ought to be assigned to this type is another matter entirely. But before considering the propriety of doing so, let us see how much fun it is to treat certain expressions as belonging to category q/(n,n). We could, for example, generate directly sentences such as 'Jack and Jill went up the hill' by introducing yet another mode of combination, m, such that $q \underset{m}{\rightarrow} s/(s,s)\ n\ n$. Where f is of category $c/(c_1,c_1)$ and a,b are of category b let % be such that for all x of category c_1/b, f % (a,b)(x) = f(x(a), x(b)). Then we could take m^{sem} to be %. More interestingly, we could conveniently represent the difference between 'Neither Jack nor Jill went up the hill' and 'Either Jack or Jill did not go up the hill'; in the first we have negation of a quantifier and in the second we have negation of a verb. Similar considerations might lead us to introduce operations of categories q/(n,q) and q/(q,n) to handle 'Jack and a girl' and 'Every girl and Jack'. In the absence of any need to do so multiplying the modes of combination in this way is not very appealing.[38] Of course, if names and quantifiers were assigned to the same category, treating names as quantifiers in the above cases would no longer be something in need of special justification, but an automatic consequence of the assignment of categories to vocabulary. Which brings us to the consideration of Montague's proposal.

The above remarks together with earlier ones about the difficulties involved in dealing with quantifier phrases as subjects of transitive verbs, within the framework of standard treatments of categorial grammar, provide some syntactic motivation for Montague's treatment of names and quantifiers. However, the persuasive motivation concerns what is needed for an adequate semantic treatment of intensional contexts. Consider the sentence

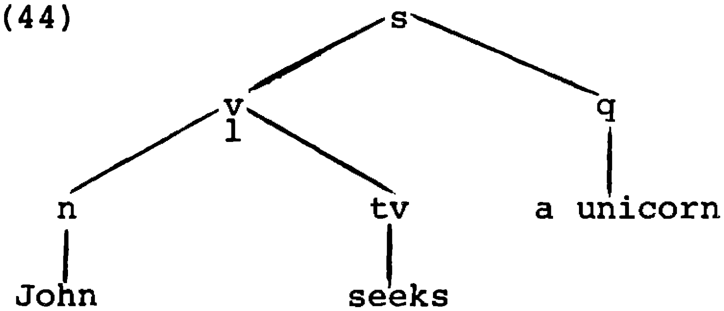
(42) John seeks a unicorn.

If we treat this sentence along the lines of all previous examples, we assign to it the two structural descriptions:

(43)



(44)



While these structures are distinct, we saw that under the principles of semantic composition appropriate for them, both structures receive precisely the same interpretation. More importantly, this remains true regardless of whether we assign just extensions to basic vocabulary items, or have a system of possible-world based intensions as interpretations. But the interesting fact about the verb 'seek' that distinguishes it from such extensional transitive verbs as 'love' is that sentences such as (42) are ambiguous and may, on one interpretation, be true even though there are no unicorns. We could incorporate this within the present framework by taking 'seek' to be ambiguous, having an extensional sense and an intensional sense and by admitting possible as well as actual objects into the domain belonging to the actual world. But this approach is not wholly satisfactory as we shall see presently. First, a brief introduction to the notion of an intensional interpretation of a categorial grammar.

Like the simpler, extensional interpretations we have so far considered, an intensional interpretation of categorial grammar assigns, via a domain function, D, certain base domains to the basic categories of the grammar, and assigns to a complex category, $c/(c_1,\ldots,c_n)$, the set of functions

$$D(c)^{D(c_1)\times\ldots\times D(c_n)}.$$

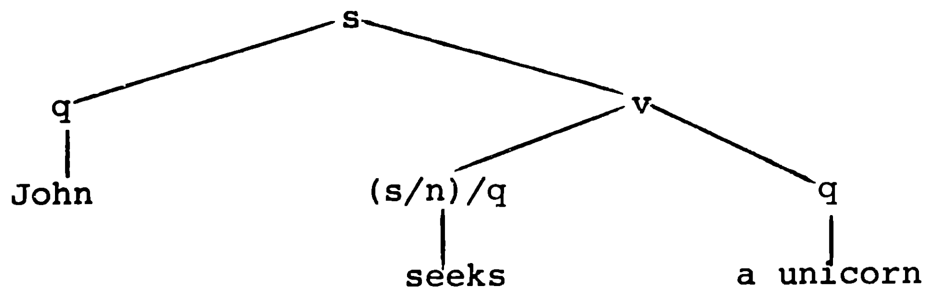
Vocabulary items are assigned elements of the domain belonging to their category, and semantic composition is determined by the syntactic structure exactly as before. The only difference is in the assignments of base domains. Let I be any set of extensional interpretations (sometimes called possible worlds, sometimes called indices) for the grammar. If i is in I and b is a basic category, let $D_i(b)$ be the domain assigned to b by the interpretation i; we call it the set of extensions of category b (b extensions) in i. Then $D_I(b)$, the set of b intensions determined by I, is the set of functions (partial functions allowed) which assign an element of $D_i(b)$ to each i in I. The b intensions for each basic category b are the basic domains of the intensional interpretations determined by I. In most cases, a member of I, $i_@$, is picked out as "the actual world". Often there are restrictions or additional structure on the set I. In the rest of this section, we will consider intensional interpretations for categorial grammars based on the categories s and n. We will require that $D_i(s)$ consist exactly of the True and the False for each i in I. $D_i(n)$ can be any non-empty set.

Thus, an s extension in a possible world is a truth value, an s intension or proposition is a function from possible worlds to truth values. An n extension or possible object in i is just a member of $D_i(n)$ and an n intension or individual concept is a function from possible worlds to possible objects in those worlds. In general, we do not talk of extensions for complex categories. An s/n intension or property is a function from n intensions (individual concepts) to s intensions (propositions). A property, p, is extensional if $(p(x))(i) = (p(y))(i)$ whenever $x(i) = y(i)$, for all possible worlds i and individual concepts x,y. Thus there is a p_i in each i of category s/n such that $p(x)(i) = p_i(x(i))$.

Montague assigns 'seek' to category (s/n)/q. In fact, he treats all transitive verbs this way because it provides a solution to the problem of quantifier subjects as well as intensional objects. Names are reassigned to category q, leaving the category n empty of basic vocabulary items. The (intensional) interpretation for 'a unicorn' being of category s/(s/n) = q, can be thought of as a set (actually, the characteristic function of a set) of properties in each possible world, the set of properties possessed by at least one unicorn in that world. Thus, 'a unicorn' and 'a centaur' are assigned different interpretations, since, even though unicorns and centaur don't exist, there are possible worlds in which at least one of these types of entity exists and in such a world, a different set of properties will be assigned to 'a unicorn' and a 'centaur'. Thus the interpretation of 'seeks' can discriminate between 'seeks a unicorn' and 'seeks a centaur' where necessary.

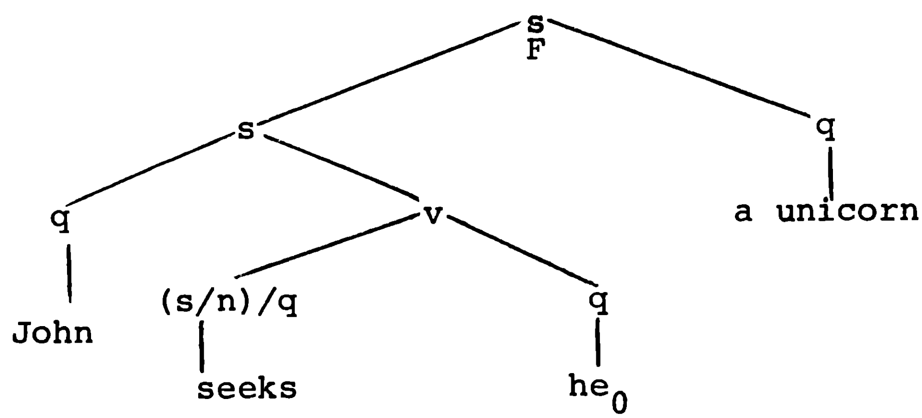
Accordingly, the structural description below represents the opague, de dicto, or narrow scope interpretation of (42).

(45)



In representing the transparent, de re, or wide scope interpretation of (42) Montague departs from the pure categorial framework, giving the following structural description:

(46)

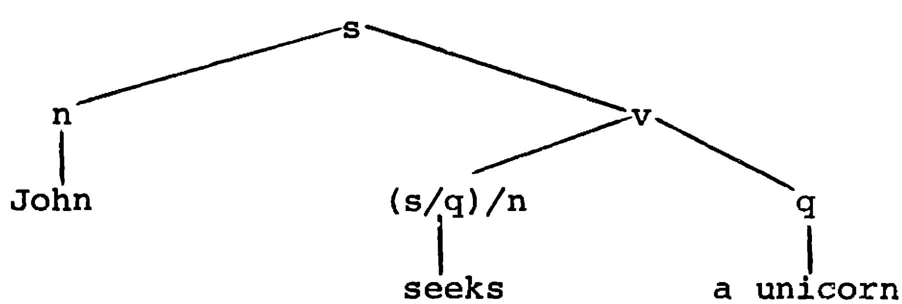


The dummy variable 'he_0' is necessary to form what is in effect the predicate 'John seeks' because the pure categorial system only admits one mode of semantic combination (application of function to argument) and this in turn requires that argument expressions be always added in the same order. To make the syntax and interpretation work out correctly, 'John seeks he_0' must be assigned to category s. But then 'a unicorn' must be combined with an entity of the wrong sort. Thus a special syntactic and semantic rule, indicated by the symbol 'F' must be invoked. The effect of this rule upon the interpretation of the structure is to extensionalize the predicate 'John seeks

he_0' and then to apply the quantifier 'a unicorn' to it in the usual way, yielding a falsehood in the case there are no unicorns. Let us call the operator that extensionalizes an intensional verb 'EXT'. EXT is semantically of category (s/n)/(s/q) and is such that if f is of category s/q and x of category n, and x =, of category s/n, is the property of being identical with x and i is any possible world index, EXT(f)(x)(i) = the True if x is the unique thing such that for some Q of category q, f(Q)(i) = the True and Q(x=)(i) = the True. Intuitively, this is a way of saying that the thing x satisfies a certain condition (definite or indefinite description) and that, whatever intentional feature corresponds to f, is intended with respect to that condition. For example, 'John seeks' when interpreted extensionally, holds of a thing, say a man, x, if there is some condition Q, holding just of x, such that 'John seeks Q' is true in the intensional sense.

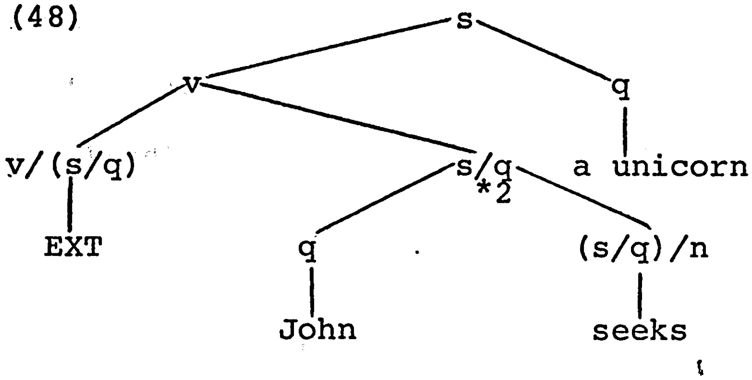
Let us try to separate what is crucial to Montague's treatment of intensional contexts from other features introduced due to limitations in the grammatical framework. To maintain uniformity with the earlier examples, we shall take 'seeks' to be of category (s/q)/n in what follows. (45) is satisfactory as a structure within the extended categorial framework. But so is

(47)

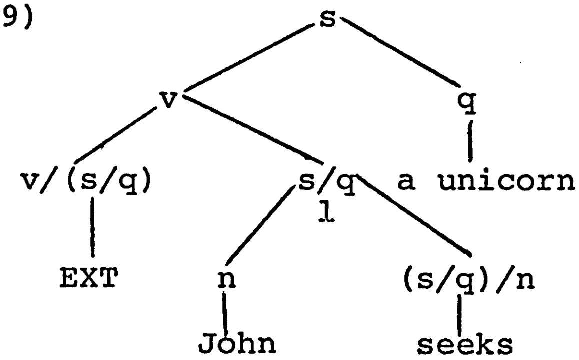


We will face the question whether to treat 'John' as belonging to category n or q later. The extensional reading can be represented in two ways also:

(48)



(49)

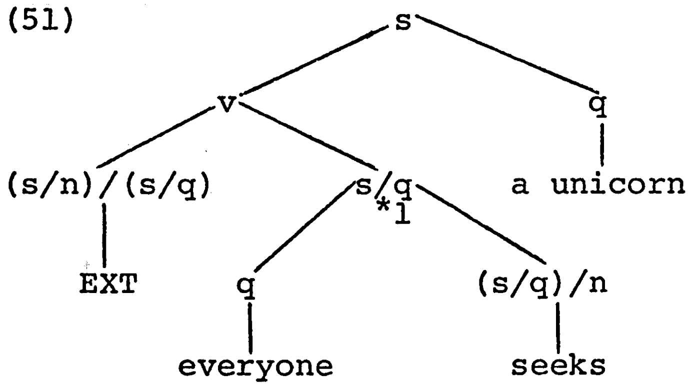


As a further example, consider

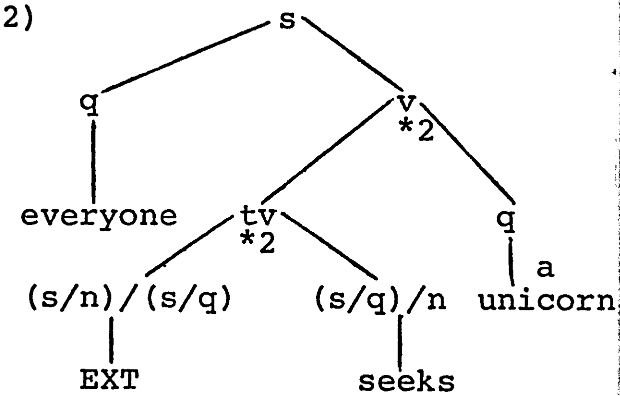
(50) Everyone seeks a unicorn.

This has the two extensional structures

(51)

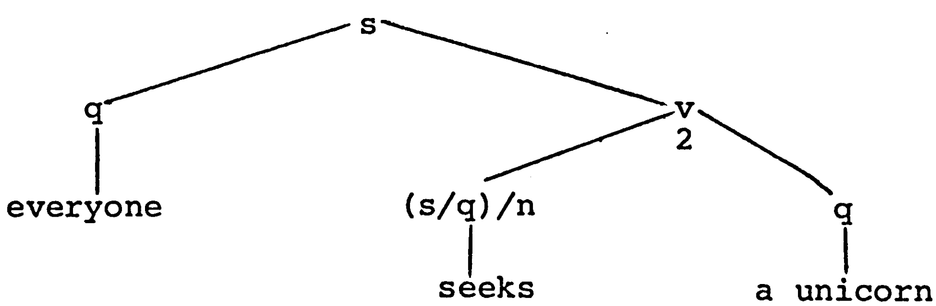


(52)



This is just the same kind of ambiguity that arises with 'Everyone loves a unicorn.' In addition, there is the ordinary intensional sense represented by

(53)



But now consider this sentence

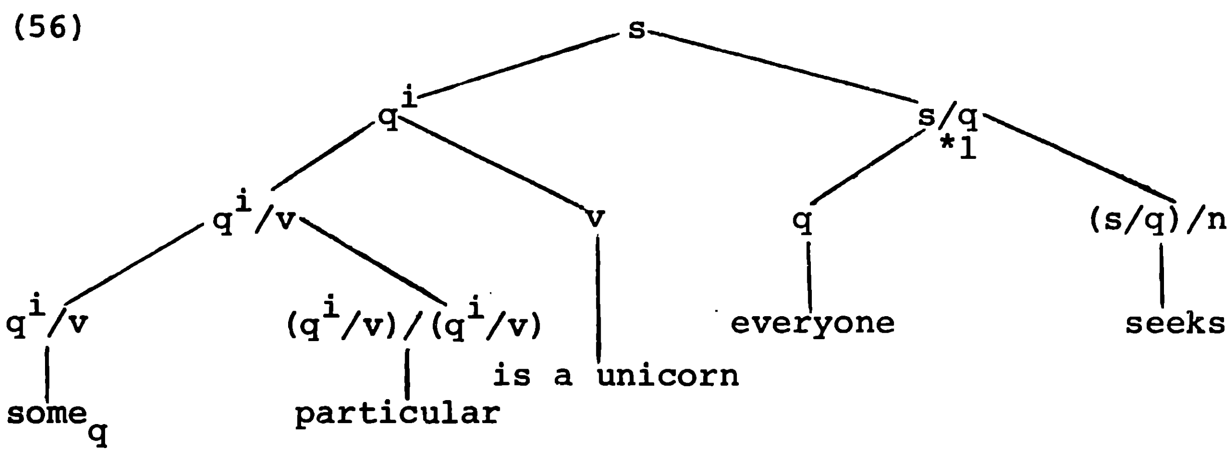
(54) John seeks a unicorn and Harry seeks it too.

It seems to me that (54) has both an extensional and an intensional interpretation.[39] If this is so, then there is yet another intensional reading of (50). Montague denies this and treats (54) as (48) only. There is room within the present framework to handle such cases. Intuitively, what we have in (54) is a higher order quantification over entities of category q there is some Q of category q which is a particular-unicorn intension such that both John and Harry seek it. We need the qualification that Q be a particular-unicorn intension, that is that it be an intension of something in particular that is a unicorn. For without this qualification, we have, in effect just existentially generalized

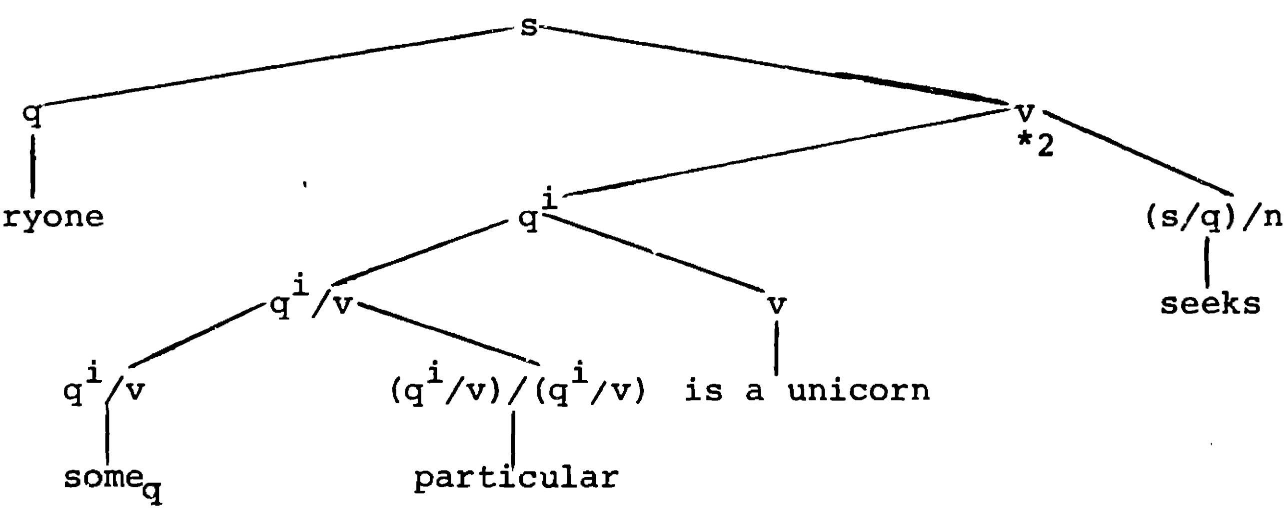
(55) John seeks a unicorn and Harry seeks one too,

a much different statement. Of course, saying what condition exactly is required for Q to be a particular-unicorn intension is not an easy matter. We introduce the new vocabulary: '$some_q$' of category (s/(s/q))/v, and 'particular' of category ((s/(s/q))/v)/((s/(s/q))/v). Also, let 'q^i' stand for (s/(s/q). We now can represent two more senses of (50):

(56)



(57)



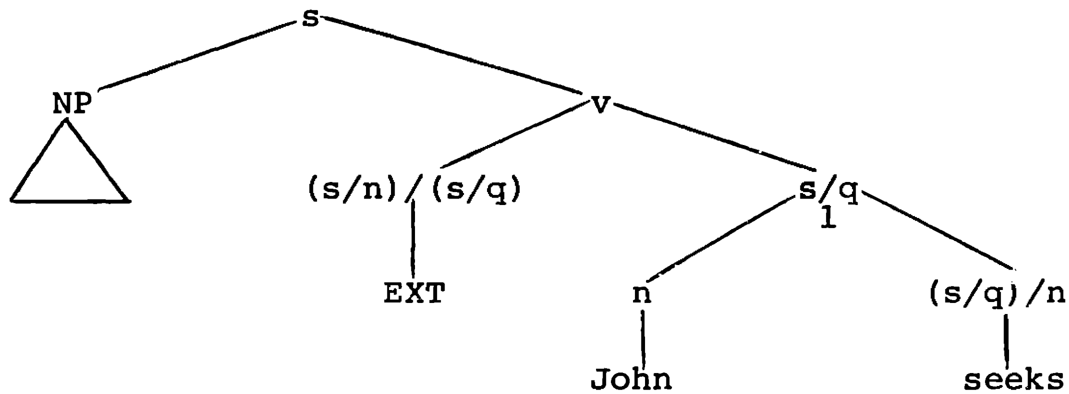
There are two extensional senses of (50), (51), and (52), and three intensional senses, two involving seeking a particular unicorn, (56) and (57), and the other, (53) involving seeking some-or-other unicorn. Regardless of whether or not entities of category q are appropriate to represent intensional objects, some such discriminations must be made. Within the framework of Montague's treatment of quantification, the treatment above makes the necessary discriminations in a straightforward way.[40]

We have seen no syntactic or semantic reason so far for putting proper names into the same category as quantifier phrases, nor have we seen any reason for putting definite descriptions in that category either. Are there any? One reason for wanting to treat proper names as expressing intensional objects is in order to deal with non-referring names, as in

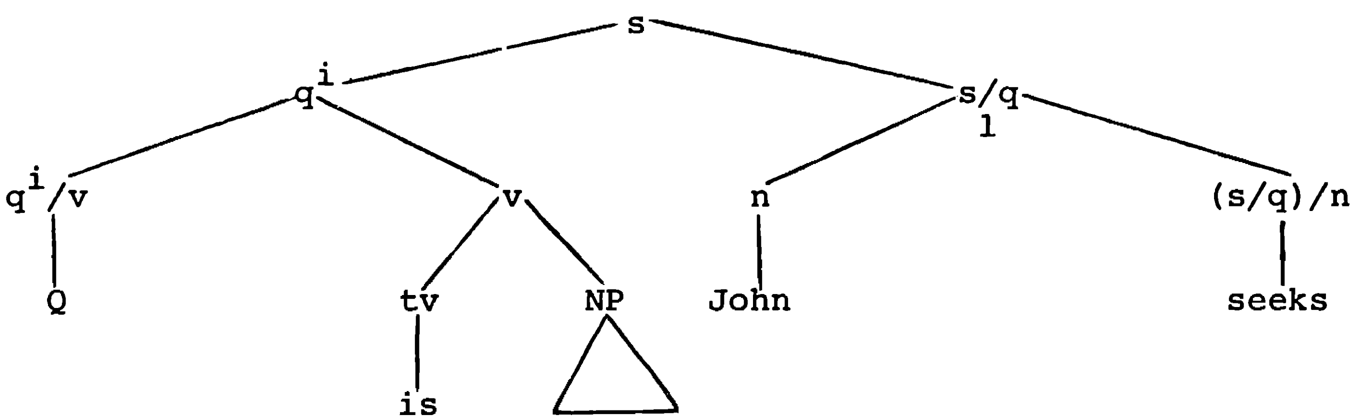
(58) John seeks Santa.

But when we remember that we are already working within an intensional semantics in which names are assigned constant individual concepts and predicates are assigned properties of individual concepts, we see that we do not need to put 'Santa' in category q; nor need we put 'Nixon' in q. In fact, all the cases so far considered can be treated uniformly on one of two patterns:[41]

(59) Extensional treatment of John seeks NP



(60) Intensional treatment of John seeks NP



where NP is either a q-phrase or an n-phrase and Q is either 'some_q' or 'some_q particular'. Both patterns do the correct thing regardless of whether proper names and definite descriptions are treated as Montague does or in the usual fashion.

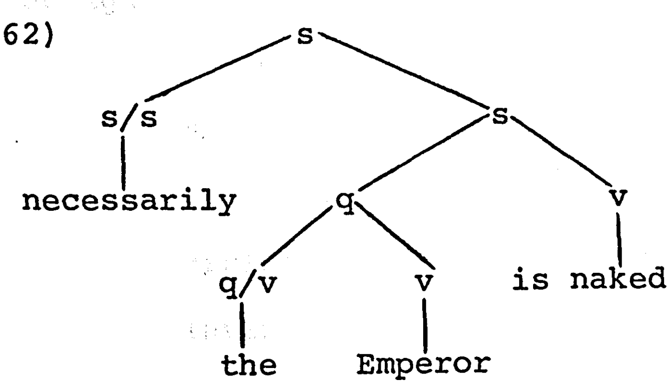
Is there any reason at all left for keeping these intensional objects of category q in the grammar; couldn't we just categorize 'seek' and 'love' both as (s/n)/n and attribute the difference between them to the fact that the extension of 'love' in a possible world is a function the value of which depends only upon the extension of both its arguments in that world, while the extension of 'seek' in a world is a function the value of which depends upon the extension of its first argument and the intension of its second? If we did revert to the standard treatment, we would lose one substantial advantage of Montague's treatment, namely the ability to distinguish between existentially indefinite and existentially particular intensional objects such as in (53) and (57).[42] But it is another matter entirely whether or not Montague's treatment is adequate. We shall return to this last question shortly, after considering how to categorize definite descriptions.

The reasons for categorizing definite descriptions as quantifiers concern the differences in the roles of definite descriptions and proper names in modal contexts, and, in particular, the problem of accommodating both de dicto and de re occurrences of definite descriptions. Entities of category q have 'scopes' while entities of category n do not. Consider the sentence

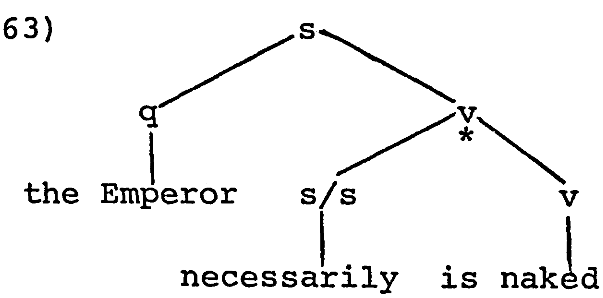
(61) The Emperor is necessarily naked.

and the structures

(62)



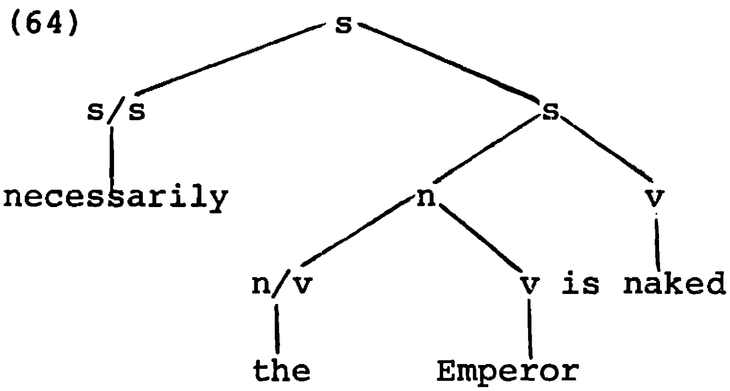
(63)



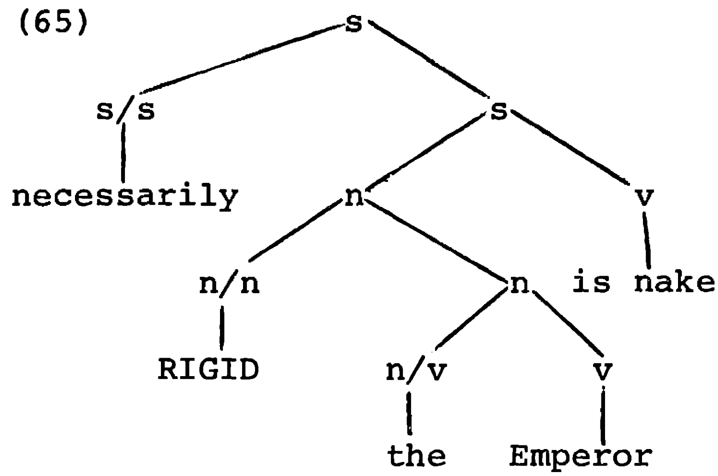
(62) gives the usual de dicto interpretation and (63) the de re interpretation. If, however, 'the' is assigned to category n/v instead of q/v, definite descriptions are of category n, with the result that there is no scope difference, at least not

of semantic significance. One way to restore the ambiguity that depends upon the scope differences in (62) and (63) while categorizing 'the Emperor' as n, is to introduce a "rigid designation" operator, RIGID, of category n/n such that for any individual concept, x, RIGID(x) is the individual concept whose extension in each possible world is that individual (if it exists) which is the extension of x in the actual world. Then, (64) and (65) correspond to the interpretations of (62) and (63) respectively.

(64)



(65)



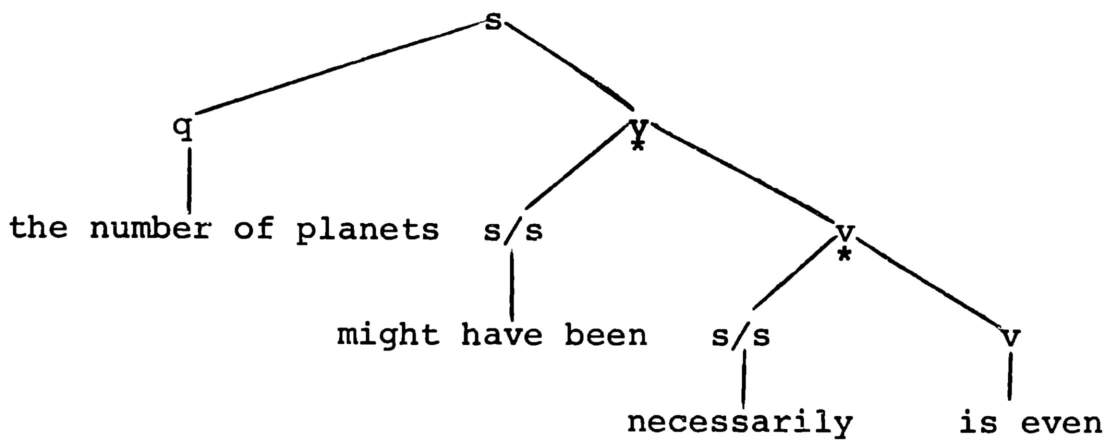
But this is still not enough to support the n categorization view against attack, as these remarks by Kripke show:[43]

> "Some philosophers think that definite descriptions, in English, are ambiguous, that sometimes 'the inventor of bifocals' rigidly designates the man who in fact invented bifocals. I am tentatively inclined to reject this view, construed as a thesis about English (as opposed to a possible hypothetical language) but I will not argue the question here.

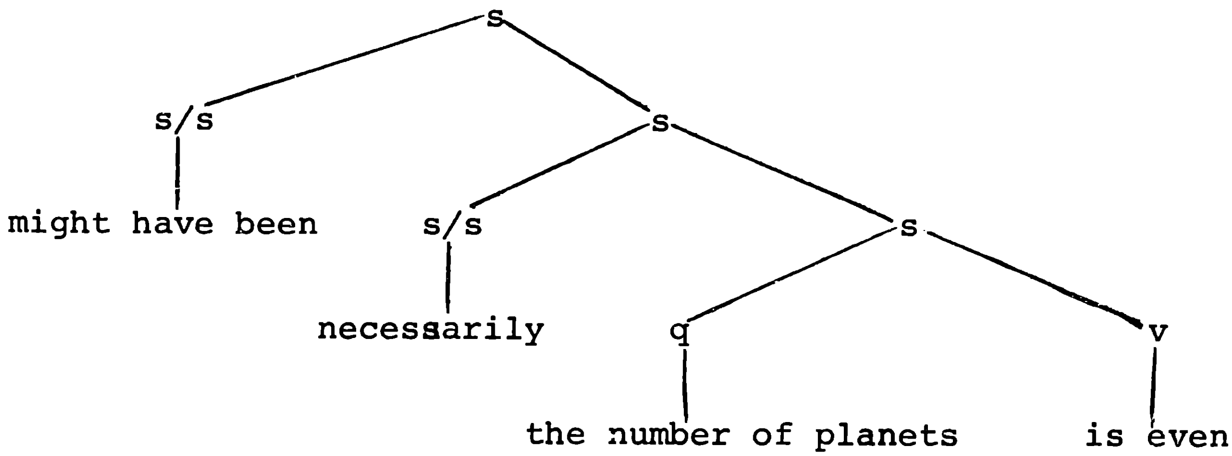
> What I do wish to note is that, contrary to some opinions, this alleged ambiguity cannot replace the Russellian notion of the scope of a definite description. Consider the sentence, "The number of planets might have been necessarily even." This sentence plainly can be read so as to express a truth; had there been eight planets, the number of planets would have been necessarily even. Yet without scope distinctions, both a 'referential' (rigid) and a non-rigid reading of the description will make the statement false. (Since the number of planets is nine, the rigid reading amounts to the falsity that nine might have been necessarily even.)"

There is a three-way ambiguity revealed here that must be dealt with, unless we wish to follow Kripke in doubting the ambiguity of definite descriptions in English. If we uphold the three-way ambiguity, we must either categorize definite descriptions as quantifiers or find some other way to introduce scope distinctions. Here are the three structural descriptions for Kripke's sentence with descriptions treated as quantifiers:

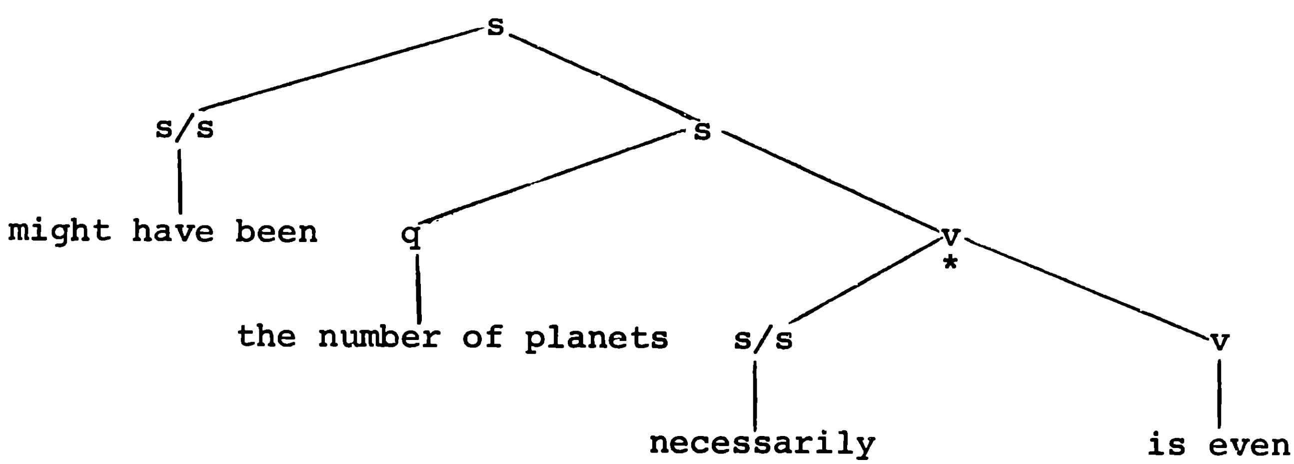
(66) The rigid reading



(67) The non-rigid reading



(68) The third reading



It appears to be possible to obtain the effect of these scope differences while categorizing definite descriptions in category n by further complicating the notion of an interpretation and properly defining RIGID. The basic formal idea is that of "two-dimensional modal logic" and allows the value of RIGID to be a constant individual concept, but to depend upon the index to determine which constant individual concept.[44] Then the two readings corresponding to (67) and (68) are constructed by making use of the difference between

for some index i, (is necessarily even)(RIGID(the number of planets))

and

for the index of the actual world @, (is possibly necessarily even)(RIGID(the number of planets)).

Not only does this treatment have the advantages of uniformity and agreement with intuition, but the complicating apparatus is useful for other quite distinct purposes as well.

The time has come to reexamine Montague's treatment of intensional objects and intensional verbs. It is important to realize that within the framework of intensional logic such as Montague uses, while there are many different possible but non-actual objects (of category n) and many different possibly but not actually instantiated properties (of category s/n) and many different possibly true but actually false propositions (of category s), there is but one impossible object, one logically uninstantiable property, and one impossible proposition. Even introducing intensional objects of category q does not help individuate impossible objects. Consider 'the round square' and 'the greatest prime'. The interpretation of 'the round square' is that property of properties whose extension at each possible world is the set of properties instantiated by the one and only thing, which, in that world, is both round and square. Since nothing in any possible world is both round and square, the set of properties instantiated by such a thing is the empty set. Thus, the interpretation of 'the round square' is that property which is true of no properties in each possible world. By a similar analysis, we determine that the interpretation of 'the greatest prime' is also the property of properties true of no properties in any possible world. Thus 'John seeks the round square' and 'John seeks the greatest prime' will receive the same truth value. Even with the introduction of intensional objects of category q, logical equivalence determines the bound on the fineness with which intensional objects can be individuated. There is an improvement over the more straightforward treatment of intensional objects as individual concepts (of category n), but as we saw earlier, the improvement concerns the ability to discriminate between definite and indefinite intensional objects. These remarks may puzzle those familiar with recent attempts to incorporate Meinongian intensional objects into set theoretic semantical systems, because such attempts claim to individuate impossible as well as possible objects and offer as a reconstruction of Meinongian objects entities of the same category as Montague's intensional objects. But while the Meinongian reconstruction of the round square is a set (or property) of properties, it is a different one than is assigned to 'the round square' by Montague, for example the set consisting solely of the two properties roundness and squareness. Two senses of predication are introduced, so that 'The round square is_1 round' can be true, 'The round square is_2 red' false, 'The president is_1 bald' false, and 'The president is_2 bald' true. Such theories are quite interesting and may be fruitful, but it is important to see that they are substantially different from Montague's treatment of intensional objects. It may be possible to graft a semantics for Meinongian objects onto the sort of semantics based upon extended categorial grammar which is presented above. But even if this can be done, and even if all the complicated details of ambiguous predication, impossible, fictional, and indefinite objects can be worked out, there will still remain problems of intensionality with regard to expressions of category other than n. Can Meinongian objects help us to interpret 'John believes that 2=1' and 'John believes that the halting problem is solvable'? We will reconsider this question later on.

Part 10 -- 'General Semantics'

David Lewis gives a pure categorial grammar for FOL (as part of a more complex family of languages) which is different from any we have so far discussed. The grammar Lewis presents is interesting in its own right; and examining it yields further insights about the workings of variables in FOL and the semantics of categorial grammars.

Consider the quantifier free part of FOL with variables, VQFFOL, with the variables $x_1, x_2, \ldots$ assigned to category n. This is just the system QFFOL of part 5 with variables added. For the time being, consider only the pure categorial grammars. An interpretation of VQFFOL on the nonempty set D is a function, I, from the vocabulary of VQFFOL to certain sets such that $I(a_j)$ is in D, $I(x_k)$ is in D, and $I(F^i_\ell)$ is in D^i for each integer k and each i, j, ℓ that index the appropriate symbols VQFFOL. Also, $I(\sim)$ and $I(\&)$ are the appropriate truth functions. Notice that both $F^1_1x_1$ and $F^1_1a_1$ are phrases of category s in this grammar of VQFFOL. Unlike the earlier grammars for FOL, variables occur as vocabulary items of the base structure. What happens if we try to extend an interpretation I to all of FOL rather than just VQFFOL? First we must add the quantifiers $\exists x_j$, of category s/s now, or some way of generating the quantifiers. This gives the proper class of expressions.[45] But there is trouble extending the semantic portion of the grammar. In many interpretations, I, we will have $I(F^1_1x_1) = I(F^1_2x_1)$, so if we stay within the categorial framework, $I(\exists x_1F^1_1x_1) = I(\exists x_1)(I(F^1_1x_1)) = I(\exists x_1)(I(F^1_2x_1)) = I(\exists x_1F^1_2x_1)$. Thus, we cannot extend I directly to give an interpretation of existential quantification given the grammatical analysis of WFEs of FOL that we are considering.[46] For similar reasons we must treat the quantifier expressions $\exists x_j$ as being basic expressions of category s/s. If we try to parse them as (s/s)/n plus n, taking $\exists$ to be of category (s/s)/n, we find interpretations I such that $I(\exists x_1) = I(\exists x_2)$ because for those interpretations $I(x_1) = I(x_2)$. And so we cannot extend such an interpretation to one in which $\exists$ plays the role of the existential quantifier. Facts such as these are what encourage people to treat predicate letters together with variables as of category s/n, s/(n,n), etc. or to banish variables from the base structure all together as syncategorematic elements indicating operations on predicates. We have seen above how this might be done in some detail and what complications ensue.

Lewis' clever proposal is to form a product structure on an index set of interpretations of the above sort rather than to try (futilely) to extend the interpretations directly. Let the index set $\mathcal{J}_I$ be the set of all interpretations of VQFFOL, I', such that the domain of I' = the domain of I and $I'(a_j) = I(a_j)$, $I'(F^i_j) = I(F^i_j)$. Thus, $\mathcal{J}_I$ is the set of all interpretations just like I except, perhaps, in what they assign to the variables x_j. The intensional or product interpretation, J, on the index set $\mathcal{J}_I$ has as its s intensions elements of $2^{\mathcal{J}_I}$, and as its n intensions elements of $D^{\mathcal{J}_I}$, with the interpretations for complex categories being determined in the usual fashion. Now we must specify the value of J on the vocabulary of FOL. For each constant, a_j, $J(a_j)(I') = I'(a_j)$, for all I' in $\mathcal{I}_I$. For each predicate, F^i_j, $J(F^i_j)(I') = I'(F^i_j)$, for all I' in $\mathcal{I}_I$. For each variable, x_j, $J(x_j)(I') = I'(x_j)$, for all I' in $\mathcal{I}_I$. Also $J(\sim)(I') = I'(\sim)$ and $J(\&)(I') = I'(\&)$ for all I' in $\mathcal{I}_I$. Thus we have an interpretation of VQFFOL on the product structure indexed by $\mathcal{I}_I$ such that the value of any expression of QFFOL at any index I', is determined solely by the values of the simple phrases of the expression at the index I'. All that is left to do is determine the value $J(\exists)$. Let I' and I'' be in $\mathcal{I}_I$ and v be in $D^{\mathcal{I}_I}$. Then $I' \underset{v}{\sim} I''$ if there is a j such that $v(I''') = I'''(x_j)$ for all I''' in $\mathcal{I}_I$ and for all k, $k \neq j$, $I'(x_k) = I''(x_k)$. $J(\exists x_j)$ is a function from $2^{\mathcal{I}_I}$ to $2^{\mathcal{I}_I}$ such that for each p in $2^{\mathcal{I}_I}$ and for each I' in $\mathcal{I}_I$, $J(\exists x_j)(p)(I')$ = the True if there is an I'' in $\mathcal{I}_I$ such that $I' \underset{J(x_j)}{\sim} I''$ and $p(I'')$ = the True. This interpretation of $\exists x_j$, depending, as it does, on the "intension" of its arguments, makes the product interpretation as a whole nontrivial. It is easy to treat $\exists$ as of category s/(n,s) as Lewis does, by letting $J(\exists)(v)(p)(I')$ = the True if for some I'', $I' \underset{v}{\sim} I''$ and $p(I'')$ = the True. There will be (in most cases) v,v',p, p',I' such that $v(I') = v'(I')$ and $p(I') = p'(I')$, but $J(\exists)(v,p)(I') \neq J(\exists)(v',p)(I')$ and $J(\exists)(v,p)(I') \neq J(\exists)(v,p')(I')$. Variables are thus fully categorematic even when "bound" by a quantifier. We can simplify the description of the class of interpretations by taking the index set to be just the set of assignments on the domain D, that is, the set of all functions which map a variable into an element of D.

Lewis' interpretations of FOL are closely related to the standard sort of interpretations. Every standard interpretation, I, of FOL determines a unique Lewis interpretation, L(I), as follows: Let $\mathcal{I}_I$ be the set of assignments to the variables of FOL of elements of the domain of I. Then $L(I)(x_j)(i) = i(x_j)$ for each i in $\mathcal{I}_I$. $L(I)(a_j)(i) = I(a_j)$ for each i in $\mathcal{I}_I$. $L(I)(F^k_j)(i) = I(F^k_j)$ for each i in $\mathcal{I}_I$. $\sim$ and & are interpreted in L(I) just as in I. $\exists$ is interpreted in L(I) as in the example J above. For each expression, e, of FOL of category s with no free variables, and any index i in $\mathcal{I}_I$, $I(e) = L(I)(e)(i)$. And if e has free variables, the value of e in I with respect to the assignment i, $I_i(e) = L(I)(e)(i)$. Each Lewis interpretation, J, also determines a unique standard interpretation, S(J) as follows: For each WFE of FOL with respect to the Lewis grammar (see below) we define the extension of the expression under J, EXT_J, $EXT_J(a_j)$ = the unique y such that $J(a_j) = \underline{y}$, where for y in the domain of J, D, $\underline{y}$ is the constant function from the index set of J to y. $EXT_J(F^k_j)$ = the unique function, f^k_j, from D^k to 2 such that $f^k_j(d_1,\ldots,d_k) = J(F^k_j)\ (d_1,\ldots,d_k)(i)$ for any i in the index set of J. If $\phi x_{j_1}\ldots x_{j_n}$ is any WFE of FOL of category s with respect to the Lewis grammar, and $x_{j_1}\ldots x_{j_n}$ is a complete list of the (distinct) variables occurring free in the expression in some fixed order, say that of their first (from left to right) occurrence in the expression, then $EXT_J(\phi x_{j_i}\ldots x_{j_n})$ = the unique function, f, from D^n to 2 such that for each $d_1,\ldots,d_n$ in D, $f(d_1,\ldots,d_n) = J(\phi x_{j_i}\ldots x_{j_n})(i)$, for some i such that $i(x_{j_i}) = d_1,\ldots,i(x_{j_n}) = d_n$. That there is indeed always a unique such function can be easily verified by proving the lemma that the value assigned by J to WFE of category s at an index i depends upon the assignment that i makes to the free variables of the expression only. The domain of S(J) is just the domain of J. $S(J)(a_j) = EXT_J(a_j) = EXT_J(a_j)$ and $S(J)(F^k_j) = EXT_J(F^k_j)$. S(J) treats the logical constants in the usual fashion. For each expression, e, of category s with respect to the Lewis grammar, $J(e)(i) = S(J)_i(e)$, for all assignments i. Since $L(S(J)) = J$ and $S(L(I)) = I$, there is a 1-1 truth-preserving correlation between Lewis interpretations of FOL under Lewis grammars for FOL and standard interpretations under standard grammars.

Here are some further facts that bear on the way in which the Lewis grammar analyzes the role of variables:

$$EXT_J(F^k_j x_{j_1} \ldots x_{j_k}) = EXT_J(F^k_j) = f^k_j$$

$$EXT_J(\phi x_{j_i} \ldots x_{j_m} \ldots x_{j_m} \ldots x_{j_{k_k}}) = \underline{REF}^k_{m,p}(EXT_J(\phi x_{j_1} \ldots x_{j_m} \ldots x_{j_p} \ldots x_{j_k}))$$

$$EXT_J(\phi x_{j_1} \ldots a_j \ldots x_{j_k}) = \underline{SUB}^k_m(J(a_j), (EXT_J(\phi x_{j_1} \ldots x_{j_m} \ldots x_{j_k}))$$

$$EXT_J(\sim\phi x_{j_1} \ldots x_{j_k}) = \underline{NEG}^k(EXT_J(\phi x_{j_1} \ldots x_{j_k}))$$

$$EXT_J(\phi x_{j_1} \ldots x_{j_k} \ \& \ \psi x_{j_{k+1}} \ldots x_{j_1}) = \underline{CONJ}^{k+1}_{k,1}(EXT_J(\phi x_{j_1} \ldots x_{j_k}),$$
$$EXT_J(\psi x_{j_{k+1}} \ldots x_{j_1}))$$

$$EXT_J(\exists x_{j_m} \phi x_{j_1} \ldots x_{j_m} \ldots x_{j_k}) = \underline{EXQ}^k_m(EXT_J(\phi x_{j_1} \ldots x_{j_k}))$$

where the operators REF, SUM, etc. are the same operators we introduced in part 6 to analyse the role of variables. Similar relations definable in terms of EXT_J exist for the other predicate operators we introduced in later sections. Thus, at some quite general level of description, Lewis analyses the role of variables in FOL just as our earlier grammars do, though without explicit appeal to predicate operators.

Lewis' treatment has a number of virtues. No "transformations" are necessary in order to preserve the categorematic character of the grammar;[47] the variables are part of the basic vocabulary and each occurrence of a variable is treated in a uniform manner by the grammar. Only a single mode of semantic and syntactic combination is needed; consequently, as we shall shortly see, the grammar can be extremely simple. The treatment of variables is quite general and extends in a straightforward manner to the other well-known cases such as iota notation for definite descriptions, lambda notation for functional abstraction, bracket notation for set abstraction, etc. The basic semantic idea of extending interpretations to "nonextensional" cases by means of product structures extends to other important operators on sentences which do not involve variables, such as modal and tense operators. It is also possible to modify Lewis' grammar very slightly so as to have a completely finite (finite vocabulary and finitely many rules) categorial grammar for FOL. Before doing this, there is one minor difficulty to consider. If we do not include any "transformations" in the grammar, then we do not generate exactly the class of WFEs previously called FOL. In particular, there are problems with vacuous quantifiers and quantifiers with an associated constant instead of an associated variable. Since variables are among the basic vocabulary and categorized n along with constants, both variables and constants behave the same from the standpoint of the syntax of a Lewis grammar.[48]

The fundamental idea involved in finitizing the Lewis grammar for FOL is simply to take the subscripts of variables to be arguments of a function denoted by 'x'. To make the presentation of a grammar that does this as simply as possible let us rewrite our subscripts in "successor" notation as: 1,S1,SS1,SSS1, etc. We also introduce a new basic category,o. The simple vocabulary items and their classifications are as follows:

a_j -- n	F^i_j -- p^i	x -- n/o
1 -- o	S -- o/o	
$\sim$ -- s/s	& -- s/(s,s)	$\exists$ -- s/(n,s)

where there are only finitely many a's and F's.[49] As phrase structure rules, we take only those pure categorial rules determined by the vocabulary--those rules corresponding to functional application with all arguments supplied in some fixed order. We interpret a_j, F^i_j, $\sim$, &, $\exists$ as above. The category o is assigned as its domain the set, N, of positive integers and '1' is interpreted as 1, 'S' is interpreted as the successor function. Reconstrue the elements of the index set as functions from the ordinal number of the variables with respect to some (the natural one, induced by the subscripts) ordering to elements of $D^{\mathcal{Q} I}$. Then we interpret J(x) as that function from N to $D^{\mathcal{Q} I}$ such that for each n in N and each i in $\mathcal{Q}_I$, J(x)(n)(i) = i(n). The trick of construing subscripts (and superscripts) as argument expressions is widely applicable as a means of converting an infinite vocabulary to a finite vocabulary within the framework of categorial grammar. This may be slightly interesting and a little clever, but the really important conceptual point about this finitizing trick is that it has no conceptual importance. There was nothing wrong or deficient or "unlearnable" about the earlier grammars insofar as they contained an infinite vocabulary or infinitely many rules. In fact, I think that the analysis of GFOL is more "intuitive" than the one presented above.

The Lewis grammars discussed above can be easily extended to modal and tense logics. Consider, for example, adding a new vocabulary item, N, to express 'it is necessary that'. The syntax runs smoothly if we add N to category s/s. But in order to have N take some true sentences to true sentences and others to false sentences (as it must to represent necessity), we need to form certain product structures over an index set of "possible worlds" as is standardly done. For tense logical operators the index sets must be time-coordinate structures. As with the case of FOL, there are two sorts of treatment that we can give to modal and tense operators: the Lewis treatment just mentioned, and an analogue to the standard Fregean treatment of quantifiers in FOL. This second sort of treatment involves taking the operators as operators on predicates as well as sentences by means of extended categorial rules, and assigning "super extensions" to predicates which code not just the distribution of truth values with respect to different assignments to the argument positions of the predicate, but also distributions of truth values with respect to possible world and temporal coordinates as well. There is no reason to prefer this latter sort of treatment of modal and tense operators to the Lewis sort, and, perhaps, some reason for the opposite preference. What is important is to appreciate the similarity of grammar for quantifiers and grammar for some of the things that are called "intensional operators". In the cases I have in mind, the basic semantic entities are the extensional ones (reference and truth value), though these may vary with respect to certain indexical factors and some operators may take the manner of their arguments' variation with indexical factors into account. There may also be different structures on the index sets which represent the indexical factors. But the essential character of such semantic features is the determination of reference and truth value of complex expressions by <u>the distribution of reference and truth values</u> of their component expressions. As we saw earlier, contexts such as 'seeks' and 'believes that' call for quite different approaches.

Lewis, unlike Montague, attempts to provide for semantic relations which individuate (sentences, for example) more finely than by logical equivalence:[50]

> Intensions, our functions from indices to extensions, are designed to do part of what meanings do. Yet they are not meanings; for there are differences in meaning unaccompanied by our differences in intension. It would be absurd to say that all tautologies have the same meaning, but they have the same intension; the constant function having at every index the value <u>truth</u>. Intensions are part of the way to meanings, however, and they are of interest in their own right.

Intensions together with structure determine meaning according to Lewis:[51]

> Differences in intension, we may say, give us *coarse* differences in meaning. For *fine* differences in meaning we must look to the analysis of a compound into constituents and to the intensions of the several constituents. For instance, 'Snow is white or it isn't' differs finely in meaning from 'Grass is green or it isn't' because of the difference in intension between the embedded sentences 'Snow is white' and 'Grass is green'. For still finer differences in meaning we must look in turn to the intensions of constituents of constituents, and so on. Only when we come to non-compound, lexical constituents can we take sameness of intension as a sufficient condition of synonymy.

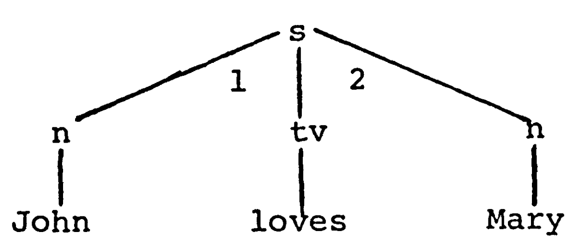
An L-meaning is[52]

> a tree such that, first, each node is occupied by an ordered pair $[c\ \phi]$ of a category and an appropriate intension for that category; and second, immediately beneath any non-terminal node occupied by such a pair $[c\ \phi]$ are two or more nodes, and these are occupied by pairs $[c_0\ \phi_0]$, $[c_1\ \phi_1]$,...,$[c_n\ \phi_n]$ (in that order) such that c_0 is $c/(c_1,\ldots,c_n)$ and is $\phi_0(\phi_1,\ldots,\phi_n^n)$.

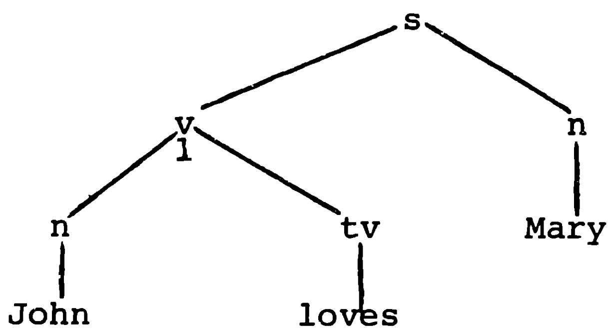
Generalizing Lewis' definition to accommodate extended categorial grammars, we allow the categories of any node and the nodes immediately dominated by it to be related by any extended categorial relation and the corresponding intensions to be combined by the corresponding mode of semantical composition. If we ignore the possible effect of transformations for the sake of simplicity here, we can define the meaning of an expression dominated by a certain node of an interpreted structural description (one in which intensions are associated with each node in the obvious way) as the subtree dominated by the node minus the terminal vocabulary items. Two questions arise about these constructs. Do they suffice for the definition of the semantic relations and properties such as synonymy, analyticity, ambiguity, anomaly, etc.? Do they suffice to treat intensional verbs such as 'seeks' and 'believes' or do we need additional intensional semantic entities to do this? Lewis considers the first question, but says nothing about the second. I shall try to deal with both, concentrating on the second question.

One problem we must face immediately is the specification of a normal form for structural descriptions and L-meanings. If L-meanings are to reconstruct meanings, they must, of course, be individuated at least as sharply as meanings. But they also must not be too much more finely individuated. Extended categorial grammar allows at least the following structures for 'John loves Mary':

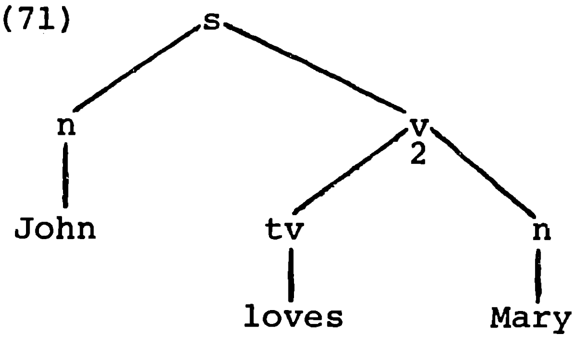
(69)



(70)



(71)

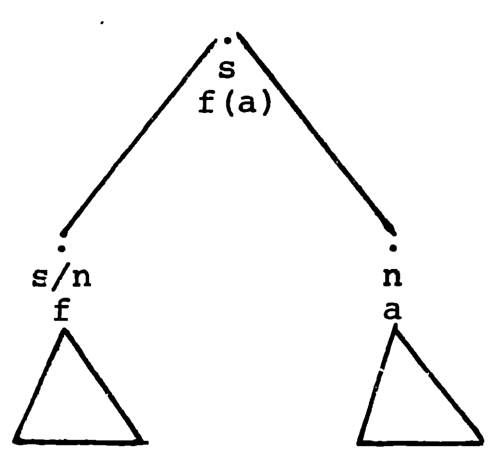


Since, considered as trees, these are three distinct, though closely related structures, and since they represent exactly the same unambiguous sentence, literal non-identity of L-meaning is not the same as non-identity of meaning or non-synonymy. This fact is not fatal to Lewis' approach to semantics combined with extended categorial grammar, but it does cause complications. We must find some other equivalence relation than identity on L-meanings to reconstruct sameness of meaning. In addition, if we want to reconstruct not just sameness of meaning, but meanings themselves, we must associate with each sameness-of-meaning equivalence class *the* meaning which is the same throughout the class. One always available way of extracting an entity from an equivalence relation is to take the equivalence classes themselves; another is to find some natural way of selecting a representative of each class. A normal form for L-meanings is a function which maps L-meanings to L-meanings such that its value is the same on all L-meanings of synonymous structural descriptions, and its value on an L-meaning is always synonymous with that L-meaning. In effect, specifying a normal form for L-meanings and specifying a definition of synonymy for L-meanings are the same thing. The details would be messy, but a normal form would discount differences in structure due to differences in the order of application of rules, where such differences were unimportant in a systematic way. One sort of reordering, for example, would be to perform all possible partial applications of a function at the same level of the tree and in some fixed order; this would select (69) as the normal form for 'John loves Mary'.[53]

A very important fact about L-meanings (and interpreted structural descriptions) is that an L-meaning of category c_1/c_2 determines a function from L-meanings of category c_2 to L-meanings of category c_1; that is, we can construe L-meanings of function names to be functions from L-meanings to L-meanings, in a manner similar to the construal of intensions of function names as functions from intensions to intensions. Consider, for example, an L-meaning of category s/n, m, with intension, f, of arbitrary structure.

For any L-meaning, m_1, of category n with intension a, there is determined an L-meaning $m(m_1)$ as follows:

$m(m_1)$ =



which is the tree obtained by joining m and m_1 dominated by a new node labelled s and with f(a) as its associated intension. For each allowable combination of categories and each mode of composition appropriate to such a combination, there is a similarly determined L-meaning function. For convenience, I shall sometimes consider complex L-meanings to be trees and sometimes functions of the above sort determined by a tree.[54]

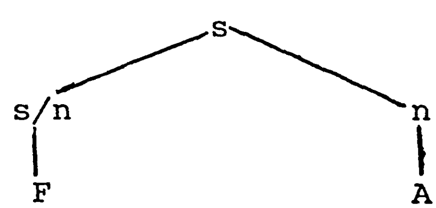
There are functions, I and M, which determine for each interpreted structural description, X, of a WFE, e, and for each WF part of e, e', an intension I(e',X) (the intension of e' on X), and an L-meaning M(e',X) (the meaning of e' on X). Since each WF part of an expression determines a unique node of a structural description of the expression (the lowest node dominating exactly that phrase), we can think of I and M as functions taking nodes as their first argument. There is also a very simple function, B, such that for all e', X I(e',X) = B(M(e',X)); in other words, intension is a function of L-meaning or the L-meaning of an expression determines the intension of that same expression. Here is a simple illustration: If

e = FA

I(F,X) = f

I(A,X) = a

X =

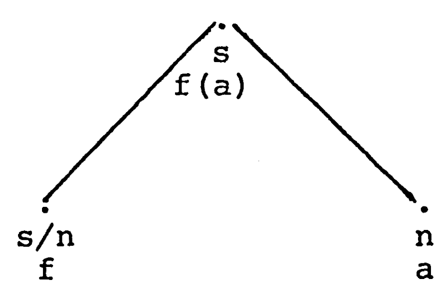


then

M(F,X) = . s/n f

M(A,X) = . n a

M(e,X) = M(F,X)((M(A,X)) =



and

$$I(e,X) = B(M(e,X)) = f(a)$$

which is also

$$I(F,X)(I(A,X)) = B(M(F,X))(B(M(A,X)))$$

in this case.

Accepting, for the moment, that L-meanings provide a means of defining the semantic properties and relations, the question still remains whether or not belief (and other "intensional") contexts can be dealt with satisfactorily. The answer must be, No, if the relationships that hold in the above example between I, B, and M hold quite generally. But there is an alteration of those relationships which preserves all the features of the analysis so far and opens the way to a treatment of belief contexts as well. The problem concerns the relationship $I(FA,X) = I(F,X)(I(A,X))$, which, on our present understanding of intensions, has as a consequence the complete interchangeability of cointensional expressions salve veritate. This is just the feature that caused us to reject Montague's analysis. We must define anew the class of intensions of category c, INT(c), and the class of L-meanings of category c, LM(c), for each category c. Let s,n be the basic categories, $\mathcal{I}$ be an index set, and D be the domain assigned to n. As before, $INT(s) = 2^{\mathcal{I}}$ and $INT(n) = D^{\mathcal{I}}$. And as before, LM(c) will consist of all interpreted structure trees with initial modes of category c and all nodes, N, interpreted so that if N is of category c', the interpretation of N is in INT(c'). Further, if in tree X mode N of category c dominates nodes $N_1,\ldots,N_j$ of categories $c_1,\ldots,c_j$ and is marked by a mode of combination, COMB, then $M(N,X) = COMB(M(N_1,X),\ldots,M(N_j,X))$. All of this is essentially as before; the alteration involves INT for complex categories. While before $INT(c/(c_1,\ldots,c_j)) =$

$$INT(c)^{INT(c_1)x\ldots xINT(c_j)},$$

we now let $INT(c/(c_1,\ldots,c_j)) = INT(c)^{LM(c_1)x\ldots xLM(c_j)}$. Let us focus on $INT(s/n) = INT(s)^{LM(n)} = (2^{\ell})^{LM(n)}$ as an example. Our previous notion of an s/n intension is preserved in essence, since a member of INT(s/n) can ignore almost all of its argument in LM(n) and just attend to the interpretation associated with the principal mode. Thus we can let F have as its new-style intension (fB). But there are also members of INT(s/n) which take into account the full L-meaning of their argument, and not just its intension at the principal node. This means that the rule of combination of L-meanings must be changed so that, for example,

$$I(e,X) = I(F,X)(M(A,X)).$$

Expressed in a slightly different, but equivalent form,

$$I(e,X) = I(F,X)(I(A,X),M(A,X)).$$ [55]

We call I(F,X) a <u>pure intension</u> if $I(F,X)(I(A,X),M(A,X)) = I(F,Y)(I(B,Y),M(B,Y))$ for all B,Y such that $I(F,X) = I(F,Y)$ and $I(A,X) = I(B,Y)$. Otherwise, I(F,X) is an <u>impure intension</u>. These notions can be straightforwardly extended to intensions of any complex category, and we consider this to be done. A pure intension of category s/n may be considered to be just a member of $INT(s)^{INT(n)}$. Pure intensions suffice for the semantics of quantification, modal operators, and tense operators. We need impure intensions or something like them to deal with belief contexts and other "intensional" verbs.

It may be helpful in understanding this treatment of semantic notions to compare it to a version of Frege's semantics. Let SENSE and REF be functions which associate with each meaningful expression of English the Fregean sense and reference (where the reference exists) of that expression. Let EXT be the function which associates with each sense its reference when that exists. Then the basic principles of Fregean semantics can be formulated as:

> there is a set D such that for any expressions F,A of categories s/n and n respectively
>
> (i) REF(F) is in 2^D, REF(A) is in D, if it exists
>
> (ii) SENSE(FA) = SENSE(F)(SENSE(A))
>
> (iii) REF(FA) = REF(F)(REF(A))
>
> (iv) EXT(SENSE(FA)) = REF(FA) = EXT(SENSE(F))(EXT(SENSE(A)))

Of course, these principles can be easily generalized to cover any complex category other than s/n. These relations are similar to those above when we let SENSE correspond to LM, REF to INT, and EXT to B. With this correspondence in mind, we can formulate our difference with the Fregean theory as the presence of the principles

> (iii') REF(FA) = REF(F)(REF(A),SENSE(A))
>
> (iv') EXT(SENSE(FA)) = EXT(SENSE(F)(SENSE(A)))

instead of (iii) and (iv).[56] (iii) and (iv) hold for special cases, but not for belief contexts. We further differ from Frege in that we allow (iii) and (iv) to hold for modal contexts. We have, in effect, given up the principle that the reference of a complex expression is a function of the reference of the component expressions--at least as a general principle, while retaining the principle that the sense of a complex expression is a function of the senses of the component expressions and also retaining the principle that the sense of an expression determines the reference of that expression. It is clear how this helps with the problem of semantics for belief contexts: the truth value (at an index) of 'John believes that the earth is flat' depends upon the L-meaning of 'the earth is flat' and thus is not, in general, to be expected to remain the same if some other sentence with the same truth value at all indices is substituted for 'the earth is flat'. But this just clears the ground; we must now see in more detail how such an account might treat belief contexts.

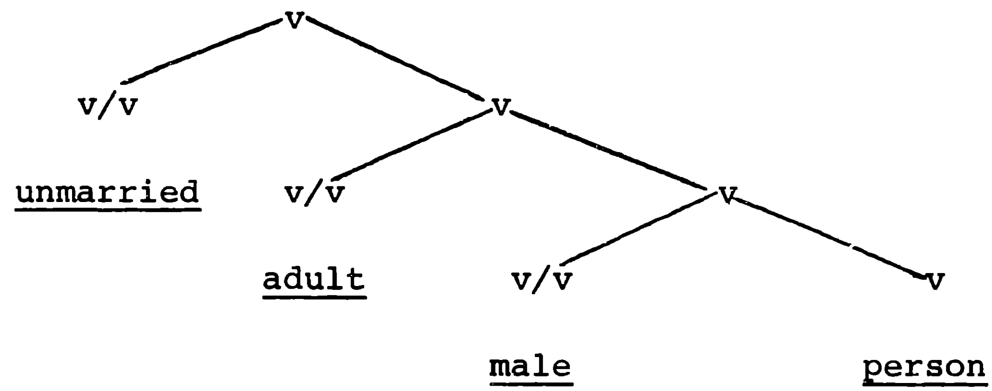
It would be unreasonable to require more of a Lewis-style semantic account of 'believes that' than of a more mundane verb such as 'is fat'. The intension of 'is fat' is that function from L-meanings of category n and indices whose value is the True if the intension of the principal node of the L-meaning evaluates to a fat thing at the index. The analogous treatment of 'believes that' is that function from L-meanings of category n, L-meanings of category s, and indices whose value is the True if the value of the intension at the principal node of the n L-meaning for the index is a thing which believes the object-of-belief represented by the s L-meaning at the index. There are (at least) two features of this specification of the intension of 'believes that' that might lead us to worry, because they complicate the traditional style of set-theoretic semantics.

First, there is the question whether or not L-meanings plus indices adequately represent the objects-of-belief, whatever they are. A minimal adequacy requirement on the representation of objects-of-belief by L-meanings plus indices is that the correspondence of L-meanings plus indices to objects-of-belief be a functional or many-one correspondence. The reason for taking the representatives to be L-meanings plus indices rather than just L-meanings, is to stand some chance to meet the requirement. The sentence 'He did it' has an L-meaning, but it is only when an index has been supplied and the indexicals 'he' and 'it' specified that we can consider a definite object-of-belief to be represented; what can be believed are the sort of things which can be true or false, and the sort of things which can be true or false are L-meanings <u>plus</u> indices. Is an adequate semantics for 'believes that' also required to specify conditions on the representatives of objects-of-belief under which substitution preserves truth value? That is, must the correspondence be shown to be one-one and not just many-one? Or, perhaps we need only define a normal form on L-meanings plus indices which yields a one-one correspondence. Perhaps. But if we are merely concerned to give reason to believe that a particular semantic framework permits an adequate treatment of belief and other intensional contexts, rather than with working out all the details of such a treatment, we need not meet this last requirement. Later on, though, we shall touch on matters which do concern the details of the relation between objects-of-belief and L-meanings plus indices.

The second, more worrisome question, concerns the effect on the set-thecretic specification of intensions and L-meanings of our revision of Lewis' original formulation. On the original formulation, intensions are defined in a straightforward and familiar way and the L-meanings are defined as certain trees with nodes labelled by categories and intensions. All this is done in a predicative or constructive fashion by means of simple indictive definitions on the structure of complex categories. But our revised version does away with the relatively straightforward hierarchical structure of the class of intensions and L-meanings, replacing it by a vague, mazelike self-applicative characterization. We need some confidence that the characterization determines a class of sets at all, and that this class contains the sort of members we would expect it to. To see the threat more clearly, let us consider the more definite specification of the class of intensions of category s/(n,s), of which one member is supposed to be the intension for 'believes that'. The intension of 'believes that' which we described above is a function, one of the arguments of which is an L-meaning of category s; further, the function is sensitive to the details of the L-meaning of its argument of category s--in particular, it is sensitive to the intensions present at various nodes other than the principal node. But, among the L-meanings of category s there are some with nodes labelled s/(n,s), and of these some have the intension of 'believes that' associated with the node. That is, we must allow for such sentences as 'John believes that Mary believes that 2 + 2 = 5'. So the specification of the intension of 'believes that' seems to involve determining the value of a certain function on complex entities, one component of which is that very function. And this causes trouble of a technical, set-theoretic sort. All we have said so far has accepted the standard account of functions as sets of ordered pairs. On this account, within standard set theory (ZF, for example), no function has itself as one of its arguments. Nor is any function defined on a domain which contains sets which are in any way built up from the function itself. All this follows from the Axiom of Foundation, which requires that no set is a member of itself, or of any member of itself, or of any member of a member of itself, etc. But the intension of 'believes that' is supposed to be a function defined on objects built up from that very function itself. How can this be? In general, isn't our previous "definition" of the class of intensions and L-meanings of this impredicative sort and thus one which determines in each case the empty set of no set at all? Fortunately, the circular characterization of INT and LM can be turned into a legitimate inductive definition, but the structure of the induction is complex. Here is a sketch of some of the details:[57] Let $INT_0(c)$, for each category c, be the set of all old-style intensions--the ones specified by Lewis. Let LM=T(c,S) be the set of L-meanings of category c with nodes labelled by intensions taken from S. Let $LM_0(c)$ be $T(c, INT_0)$, where $INT_0 = \bigcup_c INT_0(c)$. For each integer $k \geq 1$, $INT_k(s) = INT_0(s) = 2^{\mathcal{I}}$ and $INT_k(n) = INT_0(n) = D^{\mathcal{I}}$. $INT_k(c/(c_1,\ldots,c_j))$ = the class of functions from $LM_k(c_1) \times \ldots \times LM_k(c_j)$ to $INT_{k-1}(c)$. $LM_k(c) = T(c, INT_k)$, where $INT_k = \bigcup_c INT_k(c)$. The sets INT(c) and LM(c) are the "limits" of these sequences, or the "least fixed points" of the conditions: $INT(s) = 2^{\mathcal{CQ}}$, $INT(n) = D^{\mathcal{CQ}}$, $INT(c/(c_1,\ldots,c_j)) = LM(c_1)x\ldots xLM(c_j) \Rightarrow INT(c)$, and $LM(c) = T(c, INT)$, where $INT = \bigcup_c INT(c)$.

One final remark before leaving this matter: the problem above is not unique to my extension of Lewis' semantics, but would seem to arise on any reasonable account of meaning. For example, on the modified Fregean theory above, in which we gave up the principle that, in general, the reference of a compound is a function of the reference of its parts, we will have to consider cases such as SENSE(...'believes that'...'believes that'...) = F(SENSE('believes that')((...SENSE('believes that') ...))), in which the sense of a compound is determined by a certain sense applied to a compound sense, one of the components of which is that certain sense. Something like the solution sketched above seems necessary to explain how this is possible, how there can be entities like this, regardless of whether meanings are L-meanings or quite different entities.

We continue with an examination of more traditional matters such as the treatment of proper names, and of transparent or referential occurrences of noun phrases within contexts governed by intensional verbs. An element of LM(c) is a <u>simple meaning</u> if it consists of a single node labelled with c and a member of INT(c). Some (perhaps most) of the vocabulary items of English have meanings which are not simple. For example, 'bachelor' has a meaning of the sort



which makes it synonymous with the compound phrase 'unmarried adult male person'.[58] Notice that, inevitably, on this treatment of meaning in some cases (the simple meanings) the distinction we have worked so hard to establish between meaning and intension effectively vanishes. There is only 1 basic meaning with a particular intension. This is not the same as saying that there is only 1 meaning with a particular intension--which, of course, is just what we want to deny by having meanings correspond in a many-one fashion to intensions. Nor is it the same as saying that there is only 1 meaning with an intension which belongs to a given basic meaning.

When we consider how to treat proper names, we see that there are several possibilities. Assuming that our treatment is to be uniform throughout the class of proper names,[59] the possibilities are four: proper names are assigned basic meanings with an intensional component that is constant or "rigid" with respect to possible world coordinates of indices, proper names are assigned basic or complex meanings with non-rigid intensional components (at least in some cases). The basic/rigid alternative can be taken as a (partial) reconstruction within the categorial framework of the view of proper names associated with Kripke et. al. The complex/non-rigid alternative corresponds well to the Frege-Russell theory of names which takes them as synonymous with certain descriptions, or even with a "cluster of properties" modification of that view. The basic/non-rigid alternative might be palatable to someone who thought names behaved much like descriptions, but were not, strictly speaking, synonymous with any description. The complex/rigid view might seem to be nothing more than a combinatorial possibility. How could names generally be semantically complex and yet always have rigid intensions? Even if this was possible, what would be the motive for analyzing names in this fashion? As for the motive, we will consider that later; let us first demonstrate the possibility of such a view. We add to our semantical apparatus an operator, RIGID, of category n/n, which has the effect of making an n intension rigid by setting the value at all indices identical with the value at some special index--perhaps, the "actual world" index or the "present world" index. Names may have any complex structure, including that of a description, as long as RIGID is applied so that all of the structure lies in its scope. Thus names are "rigid designators" but they retain semantic structure. One result of this view is that names with identical intensions need not be "synonymous", nor need names be synonymous with any descriptions.

Consider the well-known example:

(72) Tully is Cicero.

(73) John believes that Tully is Tully.

(74) John believes that Tully is Cicero.

We cannot consistently maintain all of the following:

(i) (72) and (73) might be true and (74) false (at the same index)

(ii) replacing a phrase by a synonym always preserves meaning

(iii) synonymous sentences have the same truth value in a given circumstance

(iv) proper names have basic/rigid meaning--(or, even basic/ non-rigid meaning, with both 'Tully' and 'Cicero' having the same intension)

(v) L-meaning is an adequate reconstruction of meaning

Of course, this inconsistency does not determine a satisfactory account of the meaning of proper names nor the correct treatment of belief contexts. We might challenge (i), saying something to the effect that although John does not know that 'Tully is Tully' and 'Tully is Cicero' express the same proposition (or object-of-belief), and thus will respond differently to the two sentences, nonetheless, the two sentences express the same proposition; and since it is propositions which are believed, (73) and (74) must have the same truth value if (72) is true. This raises the question of the relation between expressing the same proposition (object-of-belief) and being synonymous. A challenge of (i) along these lines is not very satisfactory because the only support for the challenge seems to be either (a) the claim that 'Tully' and 'Cicero' are synonymous or (b) the claim that the mere truth of (72) guarantees the equivalence in truth value of (73) and (74). As far as I can see, the only way to establish (a), and hence disprove (i) is to argue convincingly for (ii) -

(v), or some similar theses that exclude (i), and rely upon a proof by contradiction, because (i) is quite plausible. So we shall have to examine (ii) - (v) in detail anyway, regardless of whether we support (i) or its denial. As for (b), it is false, unless some doctrine about proper names is implicitly appealed to, taking us back to alternative (a). (ii) and (iii) are conditions on the concept we are reconstructing--i.e., what we are calling 'meaning' is something like that. Until forced to do otherwise, we shall take it for granted that there is something which will make (ii) and (iii) true.

(iv) and (v) are the theses particular to our version of Lewis grammar, and so deserve the most scrutiny. Let us first consider (iv). Can't we solve the problem by giving up (iv)? If we are not to land back in the fire with respect to modal contexts, this means assigning proper names complex/rigid meaning. The first hurdle, how this can be done at all, we leapt over by introducing the operator, RIGID; with this device, names can have constant intensions (with respect to possible world indices) and still have features which distinguish L-meanings more finely than the identity of the intension at the principle mode. But, if we claim that names have complex/rigid meanings, we must also answer questions about the nature of that complexity in general, and also give some plausible examples. Notice that there is a condition on the sort of complexity that can be assigned to names, namely that the intension of the principal node of an otherwise arbitrary L-meaning must evaluate to the referent of the name at some particular possible world coordinate determined by RIGID (most likely, the actual world). We consider versions of the two most popular theories of names. The Frege-Russell theory takes the semantic complexity of a name to be that of some description with the two features: (a) the description serves to pick out the referent of the name by uniquely characterizing it and (b) the description characterizes by attributing properties the user of the name believes (or even knows a priori) the referent to have. A well-known criticism of this sort of view, due to Kripke and others, points out that in some cases our beliefs about the referent of, for example, 'Tully', might not uniquely characterize, or might not characterize anything, or might uniquely characterize something other than Tully and yet, so it is claimed, we might nonetheless use 'Tully' to refer to Tully. On Kripke's view, the determinants of reference are not in general (limited to) the beliefs of the user of a name about the referent of the name, but include the history (or "causal chain") of acquisition and transmission of the name. Within the present framework, we can consider this as giving up (b) while retaining (a). Lewis offers a sort of hybrid of these views (or, perhaps, just a reformulation of the Kripke view):[60]

> ...consider the suggestion... that the extension of a personal name on a given occasion depends partly on the causal chain leading from the bestowal of that name on some person to the later use of that name by a speaker on the occasion in question. We might wish to accept this

> theory, and yet to deny that the intension or meaning of the name depends, on the occasion in question, upon the causal history of the speaker's use of it; for we might not wish to give up the common presumption that the meaning of an expression for a speaker depends only on mental factors within him. We might solve this dilemma...by including a <u>causal-history-of acquisition-of-names coordinate</u> in our indices and letting the intensions of names for a speaker determine their extensions only relative to that coordinate.

I will not criticize these views or even further elaborate them here. Notice that they are primarily views about how the referent of a name is determined, and not (primarily) about the truth conditions of sentences containing names. In fact, given a device such as the operator, RIGID, none of the views need attribute a different truth value than any of the others to sentences containing only extensional or modal contexts. All three views seem compatible with a treatment of names as having complex/rigid meaning in which RIGID is applied to the L-meaning of some description or descriptive function. Thus there are alternatives to treating names as basic/rigid. But will any of these alternatives help with the problem of names in belief contexts?

By making something concerning the way in which the referent of a name is determined part of the meaning of the name, and hence part of the object-of-belief expressed by sentences containing the name, the possibility is opened up for 'Tully is Tully' and 'Tully is Cicero' to express different objects-of-belief, and hence contribute differently to the truth conditions of belief sentences. But even if we are willing to overlook the lack of worked detail to this way of giving up (iv), there are problems. Each of the above sort of treatment of proper names as complex/rigid makes the L-meaning of a proper name a very personal matter in that there is no reason to expect great uniformity in the L-meanings for different speakers even when the same name is used with the same intension. In fact, on the Kripke or Lewis view, we would expect there to be as many differences in L-meanings of 'Tully' as there are individual histories of acquisition. When _I_ say 'Tully is Cicero', the proposition that I express will depend on the meanings _I_ associate with 'Tully' and 'Cicero'. Now, John might associate a quite different meaning (though, we shall imagine, the same intension) with these names and so express a quite different proposition by 'Tully is Cicero'. This all remains true even when the sentence is embedded in a larger sentence such as (74). So, while (74) as asserted by me might be true, I would almost never be in a position to know that it is true; and what is true concerning John and the proposition expressed by _his_ assertion of 'Tully is Cicero', I would almost never be in a position to express by asserting (74). The extent to which these consequences count against the views of proper names mentioned above is not clear. It might have been the case that when we imagined that

(72) and (73) were true and (74) false, we were imagining them interpreted as John would interpret them. But it might also be that we would be willing to count (74) true, with the words interpreted as we interpret them, on evidence just concerning John's reaction to the sentence (72) (together with evidence that John assigns the same intension to 'Tully' and 'Cicero' as we do), or at any rate, without any evidence specifically concerning the meaning of 'Tully' and 'Cicero' for John. One thing does seem clear, when I seek evidence to verify (73) and falsify (74), I do not concern myself with how John (or I) acquired the names involved.[61] Unfortunately, concentrating on belief contexts obscures the point, which is really about the proper semantic representation of indirect speech. Consider the following:

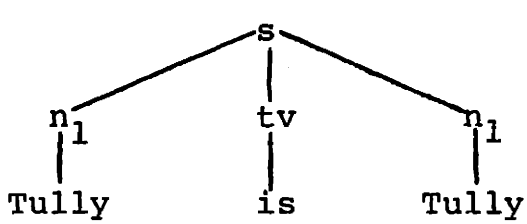
(75) John said that Tully is Tully.

(76) John said that Tully is Cicero.

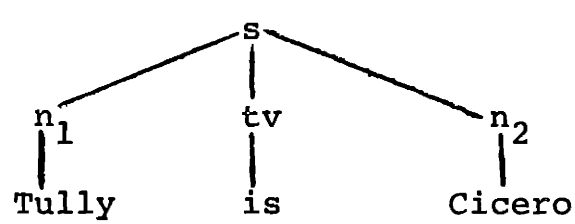
It seems that there are circumstances in which, not only is (75) true and (76) false (or vice versa); and further, that the conditions affecting the truth value of (75) and (76) depend upon what words John spoke and upon the referents of 'Tully' and 'Cicero' associated with John's speech, but not upon the history of John's coming to associate those referents with those words nor upon the descriptions that John believes to be true of the things that he calls 'Tully' and 'Cicero'. Now all this might be wrong, it might be much harder to correctly attribute beliefs and statements to others than we normally think it is, but our everyday speech habits and the sort of evidence we require to make statements such as (73) - (76) constitute a strong presumption against this view. Is there a theory that will allow us to save the phenomena, to avoid the need to deny the data?

Let us reexamine the much despised view that linguistic (symbolic) objects are to be associated with sentences in indirect discourse. There are good reasons for denying that sentences starting with 'John believes that' and even 'John says that' are about sentences. We want to be able to report the beliefs and sayings of people who speak no English and we may want to attribute beliefs to beings that speak no language at all. Besides, it is not a sentence, but the statement made by a sentence in a particular circumstance which is believed or not. But to say that we cannot always take indirect discourse to be about linguistic objects is quite different from saying that we can never do so; nor is it to say that if we do so construe indirect discourse, it must be about sentences. One attempt that will not work, is to merely mark nodes labelled n in L-meanings with indices to mark sameness or difference of the names to be placed under the node.[62] That is, we might propose the two structures:

(77)

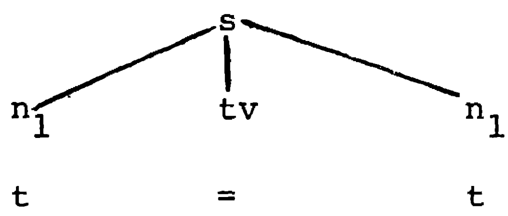


(78)

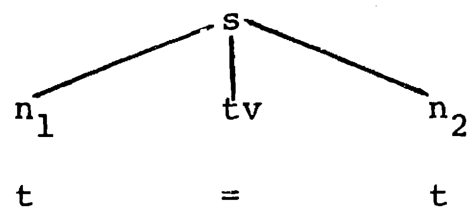


taking the L-meanings to be

(79)



(80)

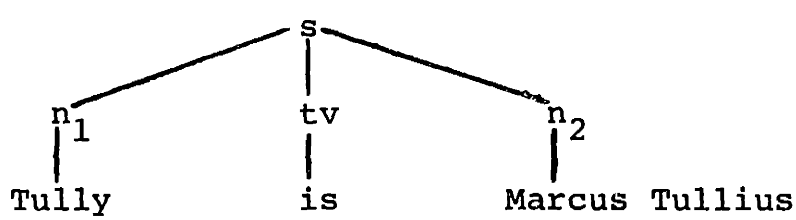


which are distinct trees (because 'n_1' $\neq$ 'n_2'). If we consider this sort of difference of trees to be a difference of L-meaning, the possibility is open that sentences with (77) embedded may differ in truth value from sentences with (78) embedded. But indexing noun phrases in this way, aside from the problems of syntactic motivation, is not a solution to our problems. Consider the case where John knows a little Roman history and a little logic, so that (73) and (74) are true, but

(81) John believes that Tully is Marcus Tullius.

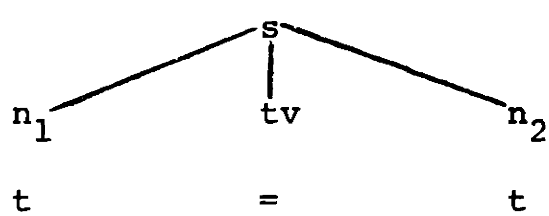
is false. Of course, Tully is Marcus Tullius. On the indexed noun phrase view mentioned above, the structure

(82)



has the L-meaning

(83)



which is identical with (80). Hence (74) and

(84) John believes that Tully is Marcus Tullius.

would have the same truth value.

A technically satisfactory modification of this approach would be to take the L-meaning of a name to consist of the single-noded tree of category n with the intension of the name together with _the name itself_ associated with the node. This has the advantage of providing sufficient individuation to allow the possibility of (73) and (74), (75) and (76) differing in truth value without making the source of the difference a highly idiosyncratic matter. But, there are also disadvantages to modifying the L-meaning of a name in this way. First, it is an ad hoc solution which does not seem to have independent motivation. There is no intuitive interpretation of the inclusion of the name itself as part of its own L-meaning; any distinguishing mark would do, including an arbitrarily assigned integer. Second, we must face the embarrassing question: Are names the only phrases which need this sort of treatment? It would seem not. The sentences

(85) John believes that Oscar is a groundhog.

(86) John believes that Oscar is a woodchuck.

(87) John said that all groundhogs are groundhogs.

(88) John said that all groundhogs are woodchucks.

present problems similar to the problems about (73) - (76) in light of the fact that 'groundhog' and 'woodchuck' name the same species of mammal. We might try to deal with this problem by drawing a distinction between names and other sorts of phrases, claiming that while it is possible to use two names in such a way that the same referent is determined for both names without knowing this fact, it is not possible to use two phrases of another sort with the same meaning and not know that the meanings are the same.[63] This might help with (85) and (86), but I don't see how it can help with (87) and (88)--whatever temptation there is to say that (75) and (76) could differ in truth value seems equally strong in this case. Is there no escape from, in effect, taking the full interpreted structural description (including terminal nodes) as semantically relevant (at least in indirect discourse)?

One alternative might be to claim that the supposed difference between the pairs (73)/(74), (75)/(76), etc. was not one of truth versus falsehood, but some other feature such as presupposition or implicature which is to be represented at a different level of "meaning" than we are concerned with. For example: When we use indirect speech to report another being's beliefs or statements we presuppose (or is it implicate?) that a certain "paraphrase" relation holds between our (indirect) speech and the other's statement or a statement he would accept as formulating his belief. The relation can vary with the circumstances from very strong (stronger than synonymy) to very weak. In the cases above, we were tacitly assuming a strong paraphrase relation and so attribute different correct utterence conditions to the paired sentences. Perhaps a case of this sort can be made out for indirect speech governed by 'believes that' and 'says that', but it does not seem to work for sentences such as

(90) It is trivially true that Tully is Tully.

(91) It is trivially true that Tully is Cicero.

of which (90) is true and (91) is false--no presuppositions or implicatures here. Notice that taking L-meanings to be full, interpreted structural descriptions will, at least, allow the possibility of correct truth values for these troublesome cases without upsetting any of the cases we discussed earlier (though complicating their treatment).[64] Also, the troubles we have had with analysing the meaning of names and indirect speech are not peculiar to the framework of extended categorial grammar, nor are they brought about by particular features of the semantics we have been detailing. They are unresolved difficulties, but not objections to the overall approach.

Let us continue now with some problems with which our framework does help. It is well known that there are two sorts of indirect speech contexts, often called referential or transparent and non-referential or opaque. Of opaque contexts we have the previous examples which do not allow the truth-preserving substitution of co-referential names or descriptions. Clear cases of referential contexts usually involve paraphrase which brings the noun phrase outside the indirect speech:

(92) John believes of Tully that he was bald.

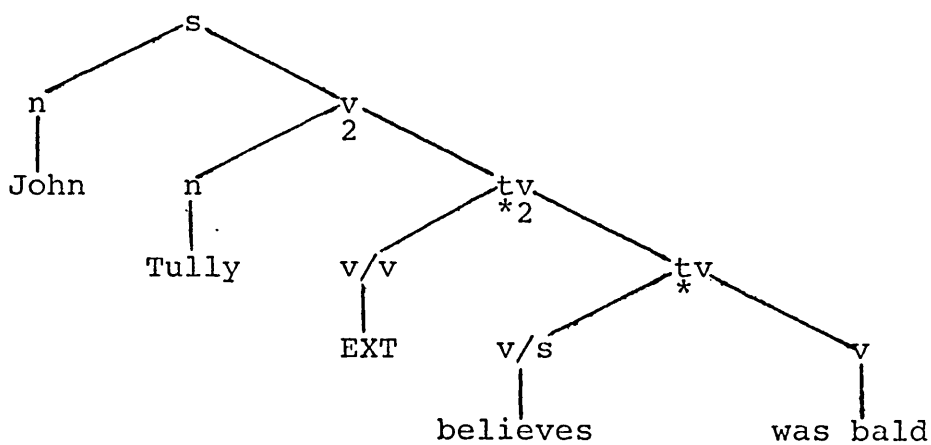
(93) John believes of Alice and Gustav that they are siblings.

Sometimes it is claimed that in order to accomodate such uses of 'believes' we shall have to take 'believes' to be not only of category v/s, but also of categories v/(n,v), v/(n,n,tv), etc. But this is not necessary; all the transparent belief contexts can be analysed as coming from the single impure operator of category v/s which also produces the opaque contexts. The idea behind the treatment is similar to that used in an earlier section to "extensionalize" the direct object position of 'seeks'. We introduce an operator, denoted by 'EXT', which operates on an L-meaning, m, of category v to give an L-meaning of category v such that for any L-meaning m_1 of category n,

> $B(EXT(m)(m_1))(i)$ = the True if there is some m_2 of category n such that $B(m_2)(i) = B(m_1)(i)$ and $B(m(m_2))(i)$ = the True.

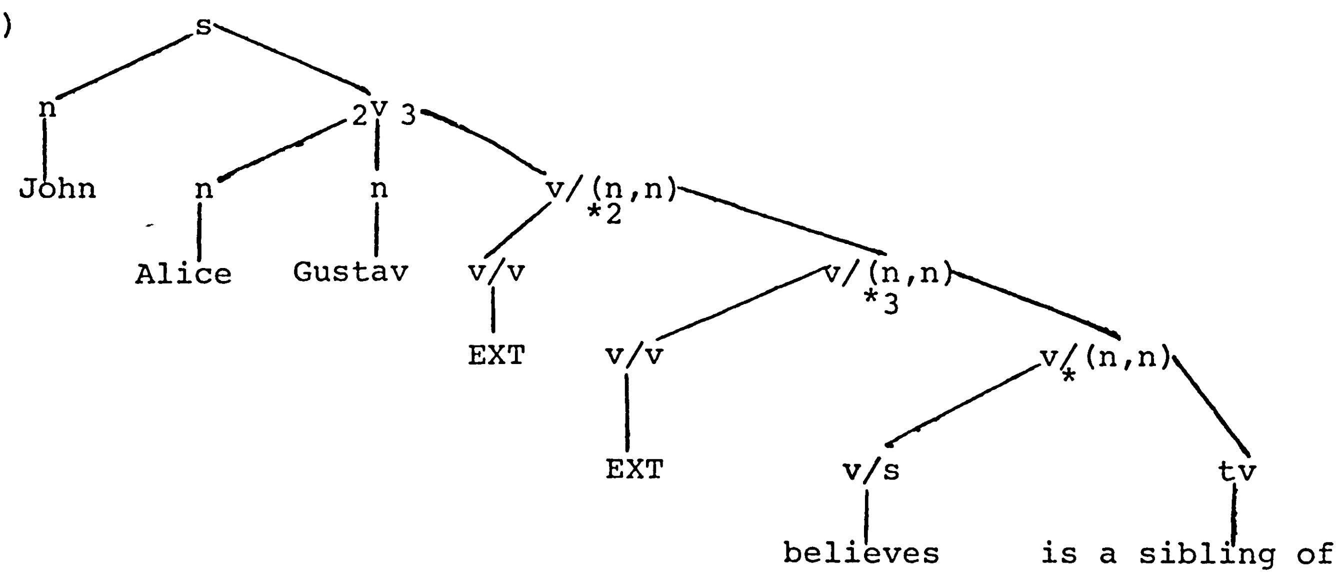
In other words, EXT makes m look only at the intension of m_1 and ignores the rest of the L-meaning. So (92) has the structure

(94)



while (93) has the structure

(95)

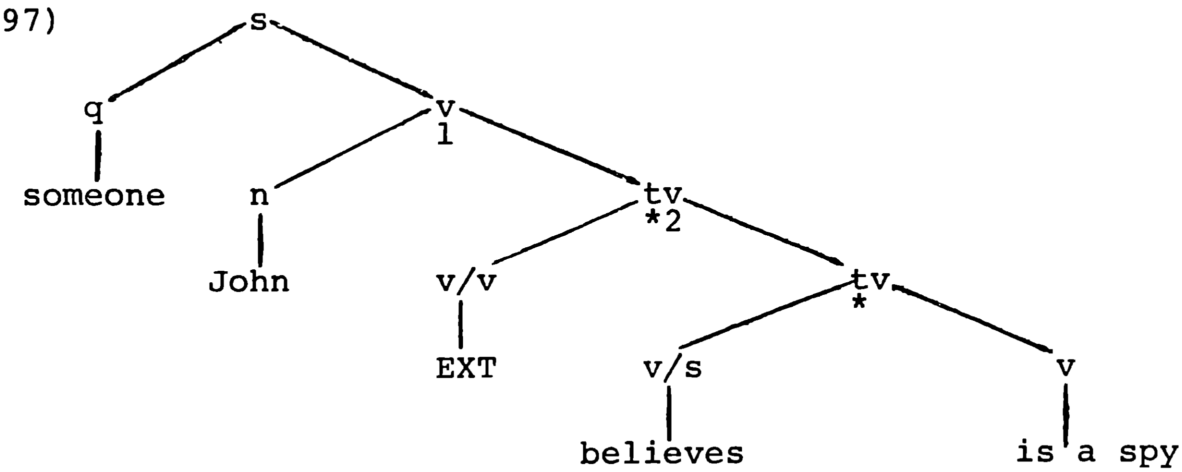


Quantifying in is also easy. The famous example

(96) There is someone who John believes is a spy.

has the structure

(97)

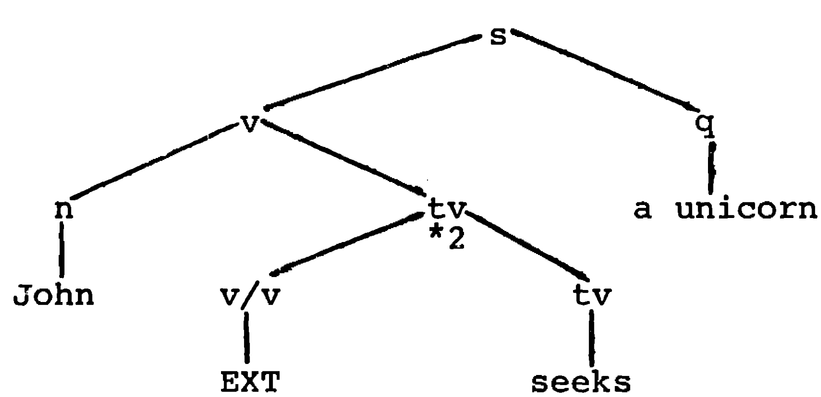


Of course, in order for this to work, we must construe quantifiers as having meanings which map v meanings into s meanings. The intension of 'someone' is true of a v meaning (at an index) if there is some n meaning whose intension at the index is a person, such that the intension of the v meaning applied to the intension of the n meaning yields the True at that index. Note that there is an n meaning whose intension at i is x if there is an n intension whose value at i is x if there is an n intension whose value at every index is x--though there may be no such n meaning which is an n meaning of some name or description of English.

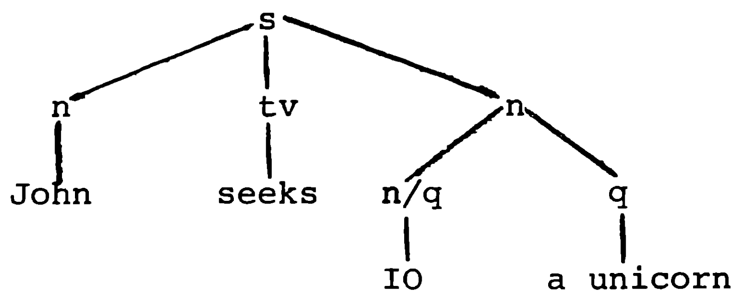
There are other "intensional" contexts than those brought about by indirect speech. Let us reconsider the verb 'seeks' as an example. We might try to modify the treatment of part 9 based upon Montague's strategy of categorizing 'seeks' as (s/n)/q. But with L-meanings and impure intensions available, it becomes possible to handle actual, possible, impossible, definite and indefinite objects while categorizing 'seeks' as an ordinary transitive verb, a (s/n)/n. Whether or not this is the best approach is another matter which will have to await further research for resolution.

How can we manage with 'seeks' classified as (s/n)/n? Remember, the problem was to distinguish 'John seeks a unicorn--any old one' from 'John seeks a particular unicorn' in either its extensional or intensional sense. The basic idea is to take as our intensional objects the intensions (rather the L-meanings) of the appropriate phrases. But, as the sentences above show, even if we have classified definite descriptions in category n there will be intensional objects specified by phrases of category q. One way around this is to introduce an operator IO (intensional object) of category n/q whose interpretation is the (properly restricted) identity function. Notice that this is possible only if we have a notion of interpretation such as that suggested above based on the λ-calculus which permits distinct categories to overlap. IO effects a special kind of nominalization. We would represent

(98) John seeks a unicorn (extensional sense)

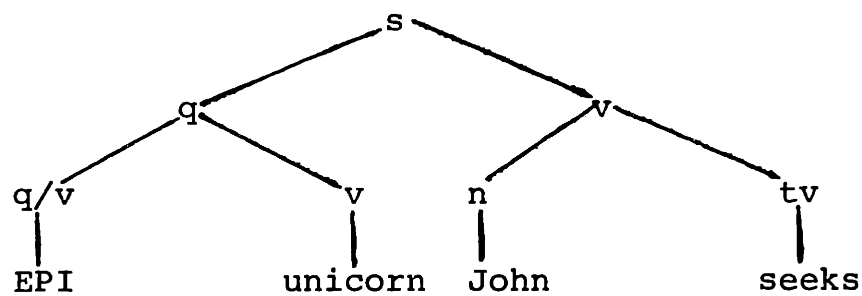


(99) John seeks a unicorn (intensional "any-old" sense)

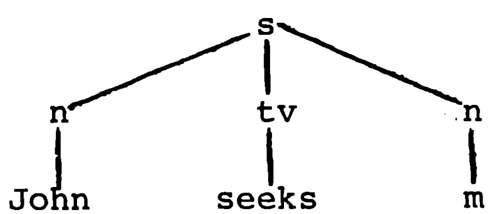


To handle the third case, the "particular" sense, let us consider a new entity of category q/v, EPI (exists a particular intension).

(100)



We can think of this as saying that there is an L-meaning, m, of a special sort ("of a particular unicorn") such that



is true.

There are a number of questions to be answered before this sketch of an account can be taken seriously. First, what is the nature of the relation between the referents of the first and second noun phrases in true sentences with 'seeks' as the verb? At first it appears strange to say that 'John seeks a unicorn' attributes a relation to John and an L-meaning. For one is tempted to say: What could that relation be but that of seeking? It is then but a short step to the strange (and false) conclusion that when John seeks a unicorn he (really--or is it in addition) seeks an L-meaning. But aside from this fallacious reasoning, the analysis is not really so strange when we realize that unlike loving or hitting, seeking a unicorn is a matter of having certain intentions, desires and beliefs--and has nothing to do with standing in a relation to certain beasts (except in very special cases). While L-meanings are far removed from beasts, they are closely related to intentions and beliefs.

The second question has to do with the definition of EPI. Specifically, when is an L-meaning a unicorn-L-meaning and when is it a particular-unicorn-L-meaning? As for being a unicorn-L-meaning, this is just a matter of the predication of 'is a unicorn' being analytic. Particularity is a more difficult matter; perhaps it is just a matter of ruling out quantifier-originated L-meanings involving IO. Obviously, this is at best a tentative beginning to a theory of intensional objects.

Of course, in addition to these constructions, there are also the constructions analogous to those for belief contexts discussed above. One pleasant feature of this treatment of 'seeks' is that it unifies the treatment of intensional noun phrases so that phenomena similar to those in (98) - (100) can also take place for any verbs. The only difference between intensional verbs and ordinary verbs is in truth conditions. It makes as much sense to say

(101) The round square is in the next room.

in either of its several senses, as to say

(102) The round square is believed by John to be in the next room.

or

(103) The round square is sought by John.

The difference is that while there is a sense of (102) and (103) in which what is said might be true--depending on John--there is no sense of (101) in which it is true.

I hope that I have at least made plausible the claim that Lewis semantics together with extended categorial grammar provides a framework adequate for the treatment of some of the more difficult "referential" parts of language--that is, adequate if any set-theoretic reconstruction is adequate.

Part 11 -- Summary and Conclusion.

In this paper I have done the following:

1) Elaborated the notion of categorial grammar in such a way that the power and delicacy of the syntactic descriptions given are increased without sacrificing the regular connection between syntactic and semantic structure.

2) Provided an account of the semantic role of variables in quantification theory and related notations and distinguished this from the structure of quantifier phrases in English.

3) Criticized and extended the work of Frege, Geach, Montague and Lewis on grammar.

4) Re-examined the relation of "intensions" as distributions of extensions in possible worlds and as meanings, and proposed a highly intensional formal surrogate for meanings.

5) Explored the treatment of some traditionally problematic aspects of reference and meaning within the extended categorial framework.

Because this paper is primarily a report of research still very much in progress, and because the originally quite limited aims of the research have developed into a "program", I think it is appropriate to end not with "conclusions" but with a short list of important areas to pursue further.

1) One large task left undone here (after all, what are linguists for?) is to work out detailed transformations and syntactic modes of combination for the structures dealt with, and to extend the analysis to as much of English as

possible. An interesting question that must await this work is to what extent rather specialized modes of combination can replace the more general notion of transformation as the means of relating deep to surface structures.

2) Only the most straightforward noun phrase constructions have been examined here--those that have analogues in FOL. Extending the analysis to cases such as mass nouns, collective and compound noun phrases is an interesting task.

3) The precise formal requirements for meanings must be determined and the appropriate function-calculus interpretations constructed in detail.

4) More detailed treatment of intensional and propositional objects must be worked out.

5) Notions of semantic metatheory--synonymy, analyticity, anomaly, semantic entailment--must be defined (where possible).

6) The relation between an abstract linguistic theory such as extended categorial grammar and a psycholinguistic account of human language performance needs to be explored. In particular: How can the infinitary, abstract entities that are L-meanings play a role in explaining or representing what is understood (or decoded or encoded) by a finite device such as the human brain?

FOOTNOTES

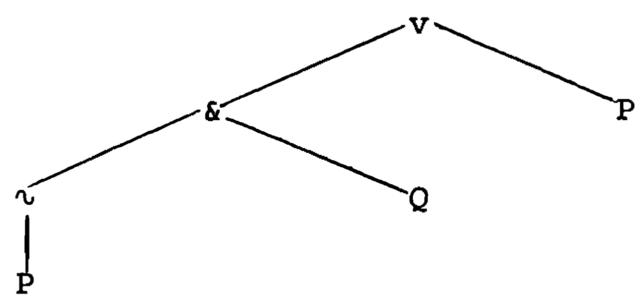
1. There are numerous variants, each of which might plausibly be called FOL. The particular version I choose counts as WFE only expressions containing no free variables. It is not important how common this version is, just that it has a right to be called a version of the language of first order logic. As a matter of fact, it is essentially the version given by Leblanc and Wisdom (23); and it is also a plausible candidate for a modern-notation version of Frege's Begriffsschrift. See note 3.

2. Of course, the symbols used here mention the symbols of FOL.

3. For a detailed account of Frege's system and citations for the claim see Martin (26).

4. See, for example, Chomsky (4).

   It is both interesting and worthwhile to compare closely the logician's and the linguist's manner of defining a class of expressions. The general form of a (finitary) inductive definition of a set S consists of 3 parts: (i) specification of a set B of basic elements (ii) specification of a class of operations (iii) the induction or closure condition to the effect that S consists of all and only those things obtainable from elements of B by a finite sequence of (zero or more) applications of the operations. While such a definition does not mention or explicitly assign structures to elements of the class defined, it is common to define the notion of a construction sequence or derivation of an element. We can even introduce trees representing "structural descriptions." Rather than do this abstractly in full generality, let us consider a simple example. The base set B is {P,Q}. & is the operation of writing '&' between its two arguments. v and ∿ are defined in the obviously similar fashion. One element of the set generated for this basis and set of operations is '∿P&QvP' and one of its structural descriptions is the tree



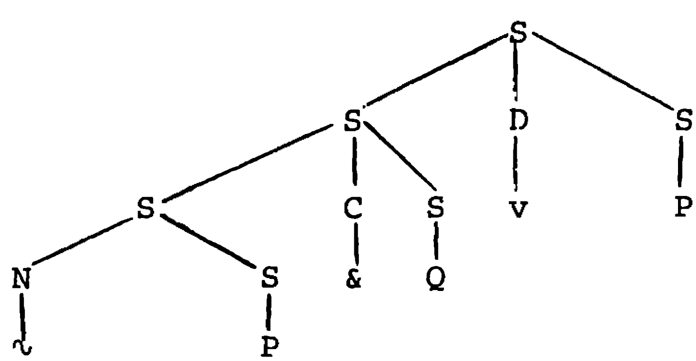
which indicates one way that the string in question can be built up from the base elements by the operations.

Here is a phrase structure grammar for the same set of expressions:

S → {P,Q}   S → NS   S → SCS   S → SDS

N → ∿   C → &   D → v

One structural description for the expression above on this grammar is

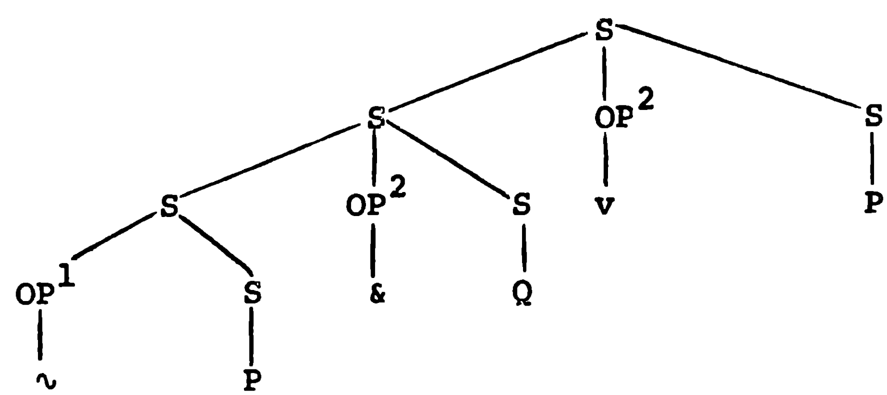


What is the relation between these two sorts of generating systems and the associated structural descriptions? First, as is well-known, the sets of expressions definable by inductive definitions exceed those specifiable by context free grammars. Second, even if we restrict attention to sets which are generated by a context free grammar, there is no unique context free grammar and hence no unique "linguist's structural description" determined by an inductive definition. Consider the alternative grammar:

$S \rightarrow \{P,Q\}$ $\qquad S \rightarrow OP^1S$ $\qquad S \rightarrow SOP^2S$

$OP^1 \rightarrow \sim$ $\qquad OP^2 \rightarrow \{\&,v\}$

On this grammar the above expression has the structural description (among others):



Phrase structure grammars are, in general, simultaneous inductive definitions of a class of sets of expressions corresponding to the strings dominated by the various nonterminal symbols of the grammar. A single inductive definition of one (or a small number) of those sets (usually the one dominated by 'S') does not uniquely determine all the others. In addition, taking the grammar as a simultaneous inductive definition, we find a very simple structure to the induction--the basic sets are the various categories of terminal vocabulary and the operations which generate the compound items are limited to concatenation alone.

There are two morals to be learned from comparing the logician's with the linguist's manner of specifying syntax. One (which the linguists appreciate) is that while the sentence may be primary in syntax, the data of syntax include many other kinds of phrases as well which must be accounted for. The second is that it is unnecessary to take concatenation as the sole phrase-forming operation. Much of this paper is concerned with showing that it is also undesirable to do so. Notice that as soon as we remove the restriction that concatenation is the only operation used in generation, the notion of context free grammar becomes superfluous, since the structure of function applied to arguments is "context free". That is, as soon as arbitrary operations are allowed, we have just another minor variation on the notion of inductive definition.

5. Strictly speaking, this is not a phrase structure grammar at all. As standardly defined, a phrase structure grammar is a finite set of rules involving a finite vocabulary (terminal and non-terminal). See for example, Chomsky (4). The example given contains an infinite terminal vocabulary. To some extent, the non-standard character of the grammar can be decreased by considering FOL to provide a source from which particular languages are taken which contain

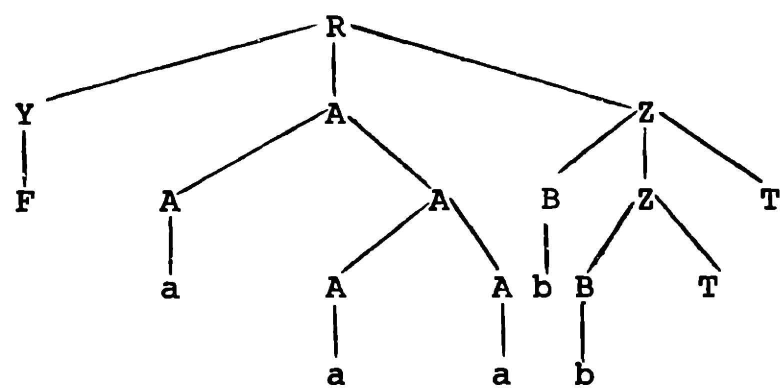
only a finite number of the F^i_j and, hence, just a finite number of non-terminal symbols. But on any of the formulations of FOL, there will be an infinite number of terminal symbols--the variables--and on my formulation, both the syntax and the semantics require an infinite number of names as well. The grammar I present might be called "a local, context-free phrase structure grammar" in that any finite subset of the rules constitutes a context-free phrase structure grammar. There is a context-free phrase structure grammar which generates exactly the same set of WFEs as does this grammar, when restricted to finitely many different degrees of predicates. But the structural descriptions of the WFEs of such a grammar will be different. See note 15.

6. I have not been able to prove (or disprove) this conjecture, though I believe the proof to be fairly simple. But in thinking a bit about this matter and in surveying the literature for useful results I have had a few relevant ideas. Consider a different, but related problem, that of generating all the atomic formulae of FOL. As an initial simplification of the problem, take predicates to be of the form--Fa^nb^m, that is, 'F' followed by n 'a's followed by m 'b's (which are subscript and superscript in unary notation). Then an atomic formula is such a predicate followed by m terms (either variables or constants in our case). Here we have a simple sort of "agreement" condition between the superscript and the number of terms following a predicate.

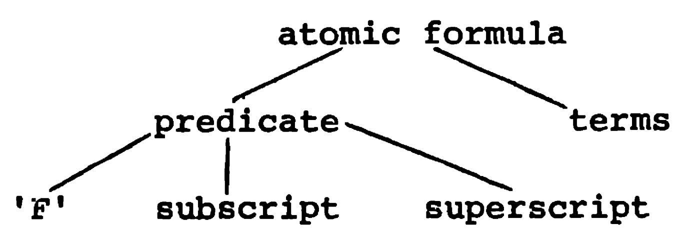
At first I thought that the set of atomic formulae was not context free, but this is wrong. Here is a simple grammar for that set.

$R \rightarrow YAZ$	$Y \rightarrow F$	
$A \rightarrow a$	$A \rightarrow AA$	
$Z \rightarrow BT$	$Z \rightarrow BZT$	$B \rightarrow b$

There are additional rules needed to generate terms from 'T'. While this grammar does generate all and only the atomic formulae of FOL (under the above simplification in notation), it does so in a strange way. Consider, for example, the expression FaaabbTT which is given the structural description (among others)



This is quite different from the intuitive structure--'F' followed by some 'a's followed by some 'b's followed by the same number of terms ('T's), for which we would want a tree something like this:



So even though the set of atomic formulae is, on one representation, context free there are two sorts of questions which arise. (i) Is the set context free on other representations. For example, what happens if we interchange the order of the sub and superscripts in the predicate? Or if we use binary or decimal numerals instead of unary ones? The trick used above to get agreement does not seem to work in these other cases. (ii) Can the context free version be generated with reasonable structural descriptions? While there are some results in the literature which seem to have bearing on questions of the first sort, I did not discover any which dealt with the subtler matters of the second sort. At any rate, I conjecture that the answer to these questions is No.

Notice that if we are restrictive enough in formulating the question, the negative result is easily proved. Lemma: If G is a context free grammar in which the only rule with 'S' on the left is of the form $S \rightarrow AB$, and if there are strings $\sigma_1 = \alpha_1\beta_1$ and $\sigma_2 = \alpha_2\beta_2$ generated by G such that α_1 and α_2 are dominated by A and β_1 and β_2 are dominated by B, then $\alpha_1\beta_2$ and $\alpha_2\beta_1$ are also generated by G. Proof: Because G is context free. In effect, this trivial lemma says that agreement cannot be enforced in certain kinds of context free grammars. As an application, we see that we cannot expect to generate sentences of FOL with the intuitively desirable structure $S \rightarrow$ Quantifier String + Matrix. Nor can we get the example above with the intuitively desirable structure.

7. One application of the inability to generate FOL by a context-free phrase structure grammar is to make clear at least one

sort of motivation for constructing grammars with a phrase structure component and a "transformational" component--namely simplicity and elegence in the resulting theory--and to separate these from psychological questions concerning the manner in which speakers produce sentences. Consider, as an even clearer example, a version of FOL in which both polish and standard infix notations for the truth functions are allowed. We could write a grammar which included two sets of rules, handling the two cases separately, but we could also generate a "base structure" (perhaps the polish version itself) which is transformed in two different ways into "surface structures". Another example; consider a variation of FOL in which there are definite descriptions treated a la Russell as well as names. We may find it desirable (and then again we may not) to give quite different "deep structures" to similar "surface structures" which differ only in the occurrence of names and descriptions.

8. There is a good deal of looseness with the term 'transformational grammar' to which I shall try not to contribute. Thus the shudder quotes in the previous paragraph and the designation 'functional component' in this one. Logicians writing about grammar have called 'transformations' many different kinds of functions. All that matters here is the recognition that it is convenient for representing both syntactic and semantic facts about FOL to divide the syntactic component of a grammar into two parts, a phrase structure part and a functional part. We shall say a bit

more about this later on.

9. Let $\{s_1, s_2, \ldots\}$ be any set of strings and let $h(s_i, s_j) = s_{i+j}$ and $g(a) = s_1$. Consider the grammar: $s \rightarrow a$, $S \rightarrow SS$, with g and h the operations which correspond to the productions. That is, we have $s \rightarrow g(a)$, $S \rightarrow h(S,S)$. Obviously, we can get any class of strings generated in this manner. Further, if the sequence s_i is recursively enumerable, g and h can be recursive.

   If we just let the function apply after the completed derivation as a "transformation" it is even easier to see that we can get any set of strings. Just let f map a^n into the nth structural description. So either by generalizing the phrase-forming operations or by allowing arbitrary "transformations" we get essentially the generative power of arbitrary inductive definitions.

10. See (1), (15), (24), (28).

11. See (26) for details on Frege's theory of grammar. In studies of Frege, attention focuses almost exclusively on his theory of reference and meaning. One hears almost nothing about Fregean syntax. Perhaps this is because the syntax is so simple and appears to be derivative from the semantics. But, of course, the two must go hand-in-hand. Analysis into function and argument can only proceed linguistically via analysis into function-name and argument-name. Geach (15) quite correctly calls his remarks about categorial based syntax 'Fregean'.

12. The earliest version is in (33). Later versions appear in (34) and (35). The version I present is that of (35).

13. Though the categorial grammar presented for PFL does not solve the problem of providing a grammar for FOL, it is of interest for two reasons. First, it is the simplest example of a categorial grammar for an interesting language. Second, it provides the key to an analysis of the role of variables.

14. (26).

15. Martin's conditions (i) - (v) and (iv) - (ix) are meant to be an inductive definition of the references of the open sentences and sentences of FOL. But though the intent is clear, the details are not worked out. The main gap is the failure to present a syntax which generates structural descriptions upon which the semantics can be based. The syntax Martin gives is the inductive definition presented in part 1 which does not even define the class of open sentences.

16. I allude to Quine here because, even though the predicate functor logic he details is a different language from FOL, and even though he uses a different set of operators to generate the various operations on predicates, the basic idea of this analysis of the role of variables is the same as that given in "Variables Explained Away".

After writing this, the paper (34) came to my attention. In this paper, Quine takes essentially the view I present, that the function of a (bindable, objectual) variable is primarily to indicate various operations on predicates rather than to quantify. He also points out the value of such a notation for the analysis of relative clauses.

The basic idea of this analysis goes back to Schoenfinkel (37) and has been elaborated by Henkin, Monk, and Tarski (18), Halmos (17), Craig (5), and Curry (7) among others.

17. Either FOL in its entirety is "unlearnable" and hence it is no defect in our grammar for FOL that it be "unlearnable" or FOL is "learnable" in spite of its infinity, in which case it seems our grammar would be "learnable" too. In the absence of a clear account of learnability suffice it to say that the vocabulary, terminal and non-terminal, and the set of productions are recursive sets.

18. Why must grammars be finite? I think that the reason such a view is widespread is historical, having to do with the derivation of phrase structure grammars from Post systems. Post systems are one of several formal reconstructions of the notion of effective computation. They have the virtue of easy application to domains other than the integers. Any recursively enumerable set of expressions can be obtained as the output of some Post system. There is no question that in order for Post systems to be successful reconstructions of the intuitive notion of effective computation, they must be restricted to finite sets of rules or "productions". Certainly we cannot allow arbitrary infinite

sets of rules, because every set of integers can be generated by some such system and because there is no connection between such systems and the intuitive ideas of effective computation. So if we allow infinite sets at all, they must be of a restricted sort. But what sort? If we say, only effectively enumerable infinite sets of rules are allowed, then the definition of Post system is not precise, relying as it does on the imprecise intuitive notion of effectively enumerable set. But if we say instead that only infinite sets of rules which can be obtained as the output of a Post system are allowed in a Post system, we have given a circular definition of the notion of a Post system. So to get started, we must restrict Post systems to a finite set of rules. Having introduced Post systems in this way, we may then propose that all effectively enumerable sets can be generated by some Post system. Part of the verification of this thesis identifying a precise, formal notion with an intuitive notion, is the proof that the precise notion satisfies certain closure principles. In particular, we can now prove that if we consider Post' systems, which are like Post systems except that Post' systems may have an infinite set of rules as long as that infinite set is generated by some Post system, the class of sets generated by Post' systems is the same as the class of sets generated by the Post systems. But this fact about Post systems does not mean that in all cases of effectively generating a set, restriction to a finite basis is either desirable or possible. For example, not every axiomatizable theory is finitely axiomatizable, though of course, the set of theorems of any axiomatizable theory can be generated by some Post system. Axiomatization of a theory is not at all the same as generating a set of theorems by a Post system. In axiomatizing a theory one is concerned not merely to generate a set of sentences, but to generate a set of sentences as the set of theorems of a set of axioms with respect to some formulation of logic. Similarly, in writing a grammar we are not just concerned to generate a set of sentences, but to generate a set of sentences as sentences with certain structural descriptions with respect to some set of grammatical rules. An infinite grammar may be necessary to do this. The case of FOL is perhaps a marginal example, since FOL can be finitely generated at the cost of including just a little unwanted structure.

19. (15).

20. I use double arrows, $\rightrightarrows$, for multiplying-out rules to distinguish them from phrase structure rules for which I use the conventional single arrow $\rightarrow$.

21. The addition of composition of functions to application of a function to its arguments as modes of semantic composition still preserves the property of categorial grammar that the semantic value of a compound expression is determined uniquely by the semantic value of the component expressions together with the syntactic mode of combination. In fact, all that is needed is the semantic value of the

components together with the category of components.

22. sara = Sara Lee.

(loves(x))(y) = the True iff x loves y.

everyone(f) = the True iff f(x) = the True, for all people x.

The left-most tree has as its associated interpretation (everyone loves)(sara), and the right-most has everyone(loves (sara)). Let us first verify the correctness of these interpretations--that is, the correctness of the resulting truth value in all cases.

(everyone loves)(sara) = everyone(loves(sara)) = T iff

loves(sara)(x) = T for all people x.

loves(sara)(x) = T iff Sara Lee loves x.

So (everyone loves)(sara) = T iff Sara Lee loves everyone. We have thus verified the correctness of the interpretations I assign to the two trees.

But one small matter remains, namely the discrepancy between the syntactic and semantic order represented by the trees. It is, at best, confusing that (everyone loves) (sara) should be the interpretation of 'Sara Lee loves everyone'. A related matter is that the multiplying-out rules

$$s/n \ n \rightrightarrows s, \qquad s/(s/n) \ s/n \rightrightarrows s$$

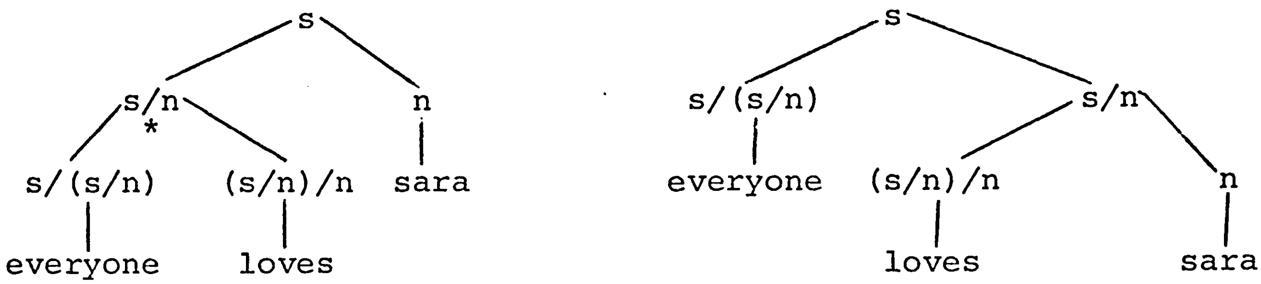
$$s/(s/n) \ (s/n)/n \rightrightarrows s/n, \qquad (s/n)/n \ n \rightrightarrows s/n$$

correspond to the phrase structure rules

$$s \rightarrow s/n + n, \qquad s \rightarrow s/(s/n) + s/n$$

$$s/n \rightarrow s/(s/n) + (s/n)/n, \qquad s/n \rightarrow (s/n)/n + n$$

respectively, and so would (if taken literally) yield the trees



All these matters will be straightened out in the next section (part 8). For now, I just remind you that we have abandoned concatenation as the only syntactic generating operation and mixed our tree notation a bit by incorporating various reordering operations without explicit mention. The aim of this sloppiness in notation is perspicuity. Of course, it is the purpose of the present section to reveal the illegitemacy of ignoring these matters in our theory of grammar. We shall, however, continue to ignore them to a certain extent in our notation even after correcting the theory. For this reason, I encourage the intrepid reader to work out the examples in the text for himself, and I apologize for the difficulty of the notation.

23. The general problem of specifying standards for the comparative evaluation of grammars is a notoriously difficult one. I do not have much to say about it; though I do compare particular grammars or types of grammars with respect to various imprecise standards.

One conjecture about the limits of the set of universally adequate transcategorial modes of semantic combination is that such modes are definable within the pure λ-calculus.

But this is not a very great restriction and is surely too large an upper bound. Probably a very small set of modes will do for any single language and even for all natural languages. Of course, from the theory of combinators we know that all λ-definable modes of combination can be generated from a small finite base, but this does not tell us how many distinct modes are needed to give a satisfactory account of the syntactic and semantic resources of natural languages.

24. Of course, which of two phrases of category n is the subject of a phrase of category (s/n)/n and which the direct object depends upon the details of the syntactic rules. For convenience and definiteness in what follows we will assume rules such as those in note 18.

25. (28). It does not seem that the sole or even chief motivation for Montague's proposal was the treatment of quantification within a pure categorial grammar, since the grammar he gives is not a pure categorial grammar. I discuss the success of his treatment with respect to other standards in part 9.

26. This is another case of the sort of lack of generality mentioned in note 15. Restriction to concatenation as the basic mode of composition is appropriate to a Post-style analysis of effective enumerability. But our goals are different and we shall not hesitate to accept this generalization of phrase structure rules.

27. There is an obvious sense in which E, the syntactic object, and <u>E</u>, the mode of presentation of the associated semantic object have the same structure. In this sense our theory identifies (deep) syntactic structure and semantic structure. However, before using this fact to place extended categorial grammar with "generative semantics", it should be observed that there is another sense in which (deep) syntactic structure is (or at any rate, can be) distinct from semantic structure. The structure of the syntactic object with respect to concatenation may be quite different from the categorial structure.

28. More on this in parts 10 and 11 where we face the problem of reconstructing ambiguity within extended categorial grammar.

29. This identification is made in order to make simpler the presentation that follows. In particular, it facilitates the "abstraction" of complex predicates of arbitrary degree. In order to accomplish this without the canonical identification, Geach must introduce a complex scheme of multiplying-out rules and categorization principles.

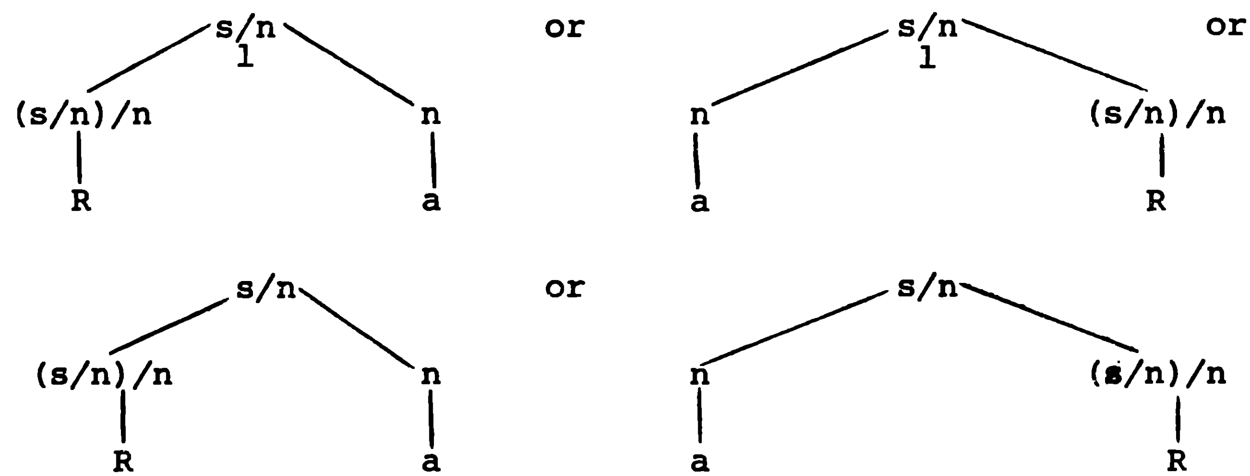
30. As an aid in following the examples in the text, I give here some simple paradigms involving a transitive verb, R, and various combinations of name and quantifier subjects and objects. R is of category (s/n)/n and takes its first noun phrase (subject) to its left and its second noun phrase (direct object) to its right. Examples: loves,

hits. Let $\underline{R}$ be that function of category (s/n)/n such that for all x,y of category n $\underline{R}(x)(y) = T$ iff x Ry. (This is not a precise use of the variables, but you know what I mean.) Let a,b be of category n with interpretations $\underline{a},\underline{b}$. Examples: Alice, Bob. Let Q,Q' be of category s/(s/n) with interpretations $\underline{Q},\underline{Q}'$. Examples: everyone, someone.

<u>P1</u> Consider phrases of the form aR. Examples: Alice loves, Bob hits. They are generated by rules of the form

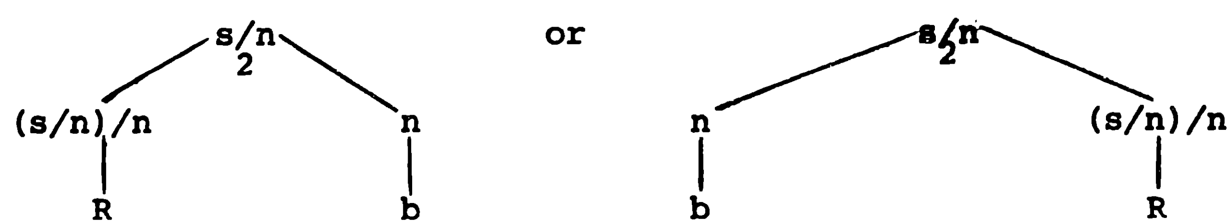
$$s/n \rightarrow (s/n)/n \; 1{+}n$$

and have structural descriptions of the sort



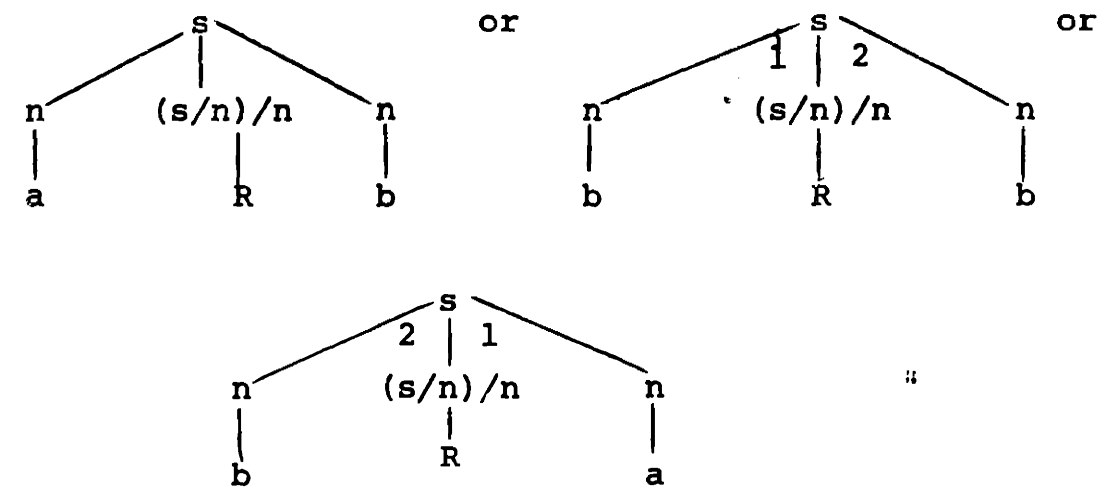
depending on taste. The interpretation is $f = \underline{R}\ 1(\underline{a}) = \underline{R}(\underline{a})$. Hence, $f(x) = \underline{R}(\underline{a})(x) = T$ iff aRx.

<u>P2</u> For Rb we have the structures



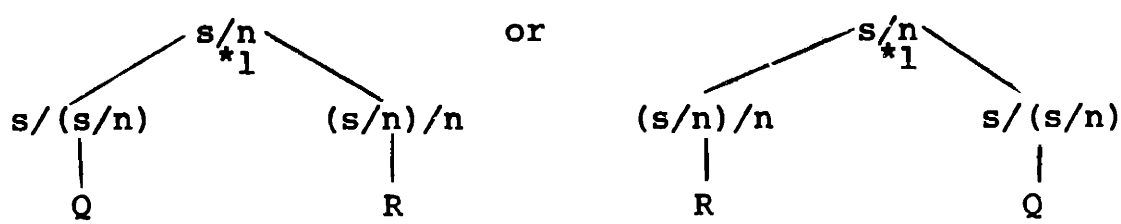
with the interpretation $f = \underline{R}2(\underline{b}) = \breve{\underline{R}}(\underline{b})$. So $f(x) = \breve{\underline{R}}(\underline{b})(x) = \underline{R}(x)(\underline{b}) = T$ iff xRb.

P3 For aRb we have



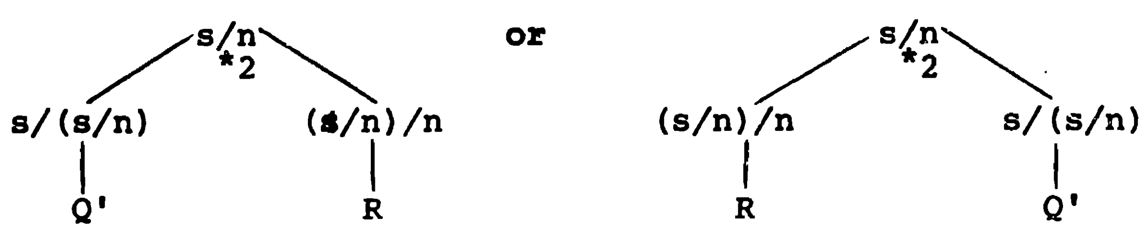
all of which have as interpretation $\underline{R}(\underline{a})(\underline{b})$.

P4 QR



$(\underline{Q}\ {*1}\ \underline{R})(x) = (\underline{Q}\ \breve{\underline{R}})(x) = \underline{Q}(\breve{\underline{R}}(x)) = T$ iff QRx.

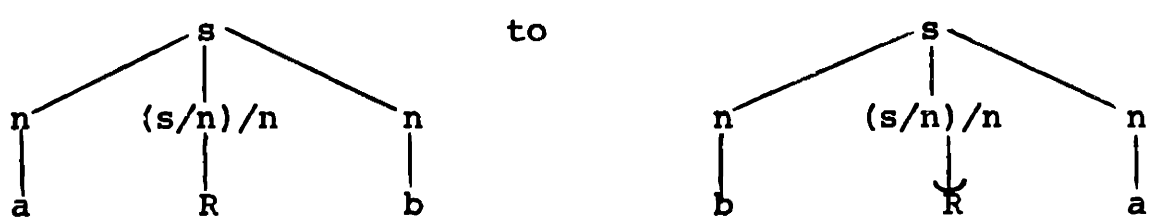
P5 RQ'



$(Q'{*2}\ R)(x) = (Q'\ R)(x) = Q'(R(x)) = T$ iff xRQ'.

QRQ' is worked out in (13) and (14).

31. Passivizing is the operation that goes from



or, more generally, replaces a transitive verb by its converse while interchanging subject and direct object. Of course, at the surface level, the operation is more complicated.

32. These are meant to be examples to illustrate various features of extended categorial grammar. No claim is made that the sample categorizations are either the best possible or the best conceivable for these vocabulary items.

33. We treat 'who' so that

$$\underline{\text{who}}\ (\underline{V}_1)(\underline{V}_2)(x) = T \text{ iff } xV_2 \text{ who } V_1.$$

For example, $\underline{\text{who}}$ ($\underline{\text{is}}$ $\underline{\text{tall}}$)($\underline{\text{is}}$ $\underline{\text{a}}$ $\underline{\text{person}}$)(x) = T iff x is a person who is tall. For convenience, we will often revert to our previous practice of writing 'someone' and 'everyone' as simple elements of category q.

34. Here are some more paradigms to aid with the complicated examples which follow in the text.

Let Q(x) be of category q/n. Also Q'(x). Examples: everyone who loves x, someone x loves.

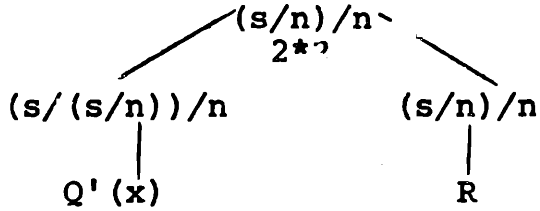
<u>P6</u> Q'(x)Ry

```
                 (s/n)/n                        or
                   2*1
        (s/(s/n))/n         (s/n)/n
             |                 |
            Q(x)               R
```

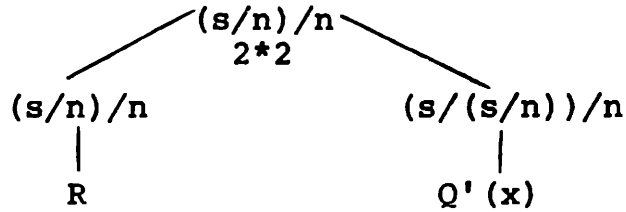
```
                 (s/n)/n
                   2*1
          (s/n)/n           (s/(s/n))/n
             |                   |
             R                  Q(x)
```

$(\underline{Q}\ 2{*}1\ \underline{R})(x)(y) = \underline{Q}(x)(\underline{R}(y)) = T$ iff Q(x)Ry.

P7 yRQ'(x)



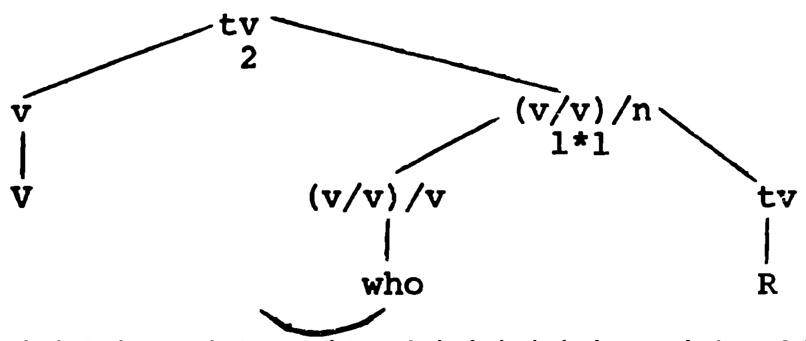
or



Examples: P6--everyone who loves x loves y

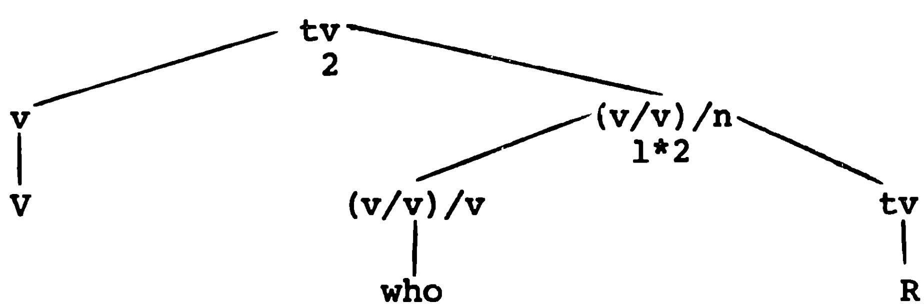
P7--y loves someone who loves x

P8 x V who Ry



(who 1*1 R) 2(V)(y)(x) = (who 1*1 R)(V)(y)(x) = (who 1*1 R)(y)(V)(x) = who(R(y),V)(x) = T iff x V who R y.

P9 x V who y R



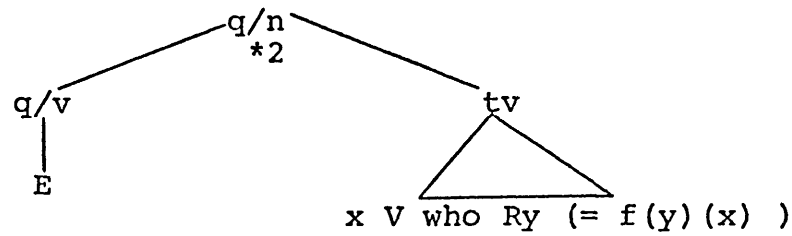
(who 1*2 R) 2(V)(y)(x) = (who 1*2 R)(y)(V)(x) = who (R(y),V)(x) = T iff x V who yR.

Examples: P8--x is one who loves y

P9--x is one who y loves

Let E be of category q/v. Example: every.

<u>P10</u> E V who Ry



Example: everyone who loves y.

(If the '*2" seems wrong to you, pay attention to the order of the arguments in P8.)

$(\underline{E}\ *2\ f)(y)(\underline{V}) = \underline{E}(f(y))(\underline{V}') = T$ iff E V who Ry V'.

35. Many approaches to elementary logic introduce the usual symbolic apparatus with talk of logical form and formally valid arguments. The examples given to introduce the symbolism often make it seem as if the notion of the form being illustrated is grammatical form. Very quickly we pass to examples which involve various equivalences and paraphrases and our aims pass from representing the quantificational form of a statement to expressing truth conditions. Perhaps we make some remarks about the difference between grammatical and logical form and claim that when it comes to symbolization we are only interested in reflecting or preserving logical form. Or perhaps we say that we have introduced a new language, L, alternative to English, and call symbolization "translation of English into L". With few exceptions, such explanations of our symbolization practices are muddled and inconsistent with the practice itself. What prevents the proper sort of

explanation is the feeling that the structure of English is too complex and too indefinite to enter into matters of logic. It is a major virtue of extended categorial grammars of the sort considered (fragmentarily) above that they can aid in explaining what symbolization is, if not as a pedagogical device, then at least to ourselves. We do at least two quite different things when symbolizing; paraphrase and represent structure. Paraphrase is by its nature unsystematic. It involves finding alternative expressions which, relative to context, "serve our purposes." This sort of paraphrase will not, in general, preserve meaning or form or produce logically equivalent statements. Representing structure is a more systematic operation upon the linguistic object. In the case of the elementary theory of quantification, it is a partially defined operation on English sentences, which, relative to certain structural equivalences, identifies an English sentence with an expression of FOL having similar structure. Extended categorial grammar provides the means for making the notions of structure involved here precise, and perhaps even mechanizing this part of the process. To the extent to which it facilitates this, an extended categorial grammar for English explains how we (might) understand English quantificational devices.

36. One minor feature of Montague's presentation is the inclusion of '//' as an additional complex category-forming expression. In general, c_1/c_2 and $c_1//c_2$ have the same

semantic interpretations, but they may correspond to different syntactic modes. This sort of syntactic subcategorization can be obtained within extended categorial grammars by making the syntactic modes sensitive to the vocabulary, or more uniformly, by operating on syntactic features other than categories. There is certainly no reason to restrict all syntactic information to be contained in the categorial classification itself.

37. Very roughly, the idea is to consider a quantifier as determining a class of sets--in this case, sets consisting of all the people in this room and some of the people in the next room--while 'each other' is true of a transitive verb thought of as expressing a relation and a set determined by a quantifier if and only if each thing in the set stands in the relation to each other thing in the set. Unfortunately, this rather straightforward approach is beset by serious technical difficulties concerning the problem of "extracting" a set from an arbitrary quantifier. See also Massey (27) for a different treatment and references.

38. Such phrases as 'each other' and 'together' provide some motivation for such operators. Consider 'Jack and Jill love each other' and 'Jack and Jill went up the hill together'.

39. See Geach (16) for a defense of this claim and a valuable discussion of some of the complexities of the data.

40. Rather than defend this tendentious proposal, we will wait until we have presented the more adequate theory of part 10 to deal seriously with intensional objects.

41. This includes the case of 'John seeks a unicorn' in the weak sense of being a unicorn-seeker, if we have an indefinite "intensional" quantifier--some$_q$ indefinite. In this case, we need not categorize 'seeks' as (s/n)/q, but may treat it uniformly with other transitive verbs.

42. The more adequate treatment of intensions in part 10 will allow this final simplification and unification of the treatment of transitive verbs.

43. (21), p. 149n.

44. I first became aware of this possibility in conversation with Hans Kamp. See Kamp (20), Segerberg (44), and Stalnaker (45).

45. Of course, the class of expressions is not that described in part 1.

46. Suppose, for example, that $D = \{0,1\}$, $I(F_1^1) = \phi$, $I(F_2^1) = \{1\}$, $I(x_1) = I(x_2) = 0$.

47. Though they are still needed to introduce parentheses.

48. There are formulations of FOL which do not distinguish variables from constants as vocabulary items, but treat free variables semantically as constants. The grammar described would be suitable for such a formulation of FOL if the constants, a_j, were removed from the vocabulary.

49. It is obvious how to extend the grammar to accomodate infinitely many constants. Infinitely many predicates can also be accomodated in such a finite grammar as long as only finitely many different degrees are required. But if infinitely many predicates of infinitely many degrees are called for, the grammar must be infinite. Notice that it is Lewis' method of taking each occurrence of a variable as categorematic that makes this finitizing trick possible.

50. (24), p. 176.

51. op. cit. p. 182.

52. op. cit. p. 184.

53. One promising line for the definition of normal forms for L-meanings is via the established notion of normal form for the terms of the λ-calculus. For the relevance of the λ-calculus to L-meanings see note 57, especially the latter portions. For discussion of normal forms for the λ-calculus, see (19) and (46).

    We will not explore further in this work the definition of metalinguistic semantic notions in terms of L-meanings.

54. There is more than notational convenience to be obtained from the fact that L-meanings as trees uniquely determine L-meanings as functions. It is this fact which opens the door to construing L-meanings not only as representations of abstract, infinitary structures which behave like mean-

ings, but also as representations of human speakers' linguistic knowledge. This will be elaborated later.

55. That is, the intension of a compound expression depends upon not only the intension of its components, but also upon the meaning of the components.

56. In order to avoid a questionable claim about the nature of Frege's theory of sense and reference, I present the standard formulation (i) - (iv). But it seems to me that my revised theory, incorporating (iii') and (iv') is not so much a revision of Frege as it is the isolation and clarification of an aspect of Frege's own views. As is well known, when discussing natural language (e.g.(14)), Frege supplements the principles (i) - (iv) by introducing the notion of indirect reference and indirect sense. For certain expressions, FA, the following principles hold instead of (ii) and (iii):

(v) REF(FA)= REF(F)(INDREF(A))

(vi) SENSE(FA) = SENSE(F)(INDSENSE(A))

(vii) INDREF(A) = SENSE(A)

(viii) INDREF(A) = EXT(INDSENSE(A))

In other words, REF(FA) = REF(F)(SENSE(A)). Since, in all cases, REF(A) = EXT(SENSE(A)), (iii') really is, for Frege, the general form of the relation governing the reference of a compound expression. (iii) is not a different principle from (iii'), but a special case of it. The notion of indirect reference, and with it the notion of indirect sense, play no role in the theory as thus formulated and can be dropped altogether. The effects of context then become features of the function which is the reference of the function name, F.

57. The mathematical basis for these claims is in (38), (39), (40), (41), (42), and (43). See also (10). It is interesting that the complexity of structure of "meaning algebras" has gone unnoticed. The usual characterization (when any is given) is that the class of sentence meanings or propositions is a lattice with respect to various operations. But that the class of meanings of all categories is best thought of (or even that it can be thought of) as a type-free function algebra has not, to my knowledge, been previously noted. There are, however, two recent developments in semantics which seem to involve special cases of the full structure of meaning algebras. The first is in Cresswell (6). If, as seems desirable, we allow that any entity may be named by an expression of category n, the base domain, D, must contain all the other domains associated with complex categories. In particular, the domain of s/n must be contained in D. But under the standard categorial semantics, this would mean $2^D \subset D$, which is impossible. Cresswell points out that if we take the domain of s/n to consist only of partial functions from D to 2, and similarly for other categories, it becomes possible to maintain such inclusions. This solution to the problem is not ideal (even if it were worked out) because it not only excludes total functions, but also

excludes functions which are in their own domain. Thus, while avoiding difficulties with 'the concept horse', we still have problems with 'the concept concept', which intuitively ought to be a term of category n which denotes an entity of category s/n and which is true of itself. I have recently heard of a second development along these lines in which Kripke deals with a special case of this sort of situation as part of a theory of truth. In a later paper I will deal with the technical details of the necessary constructions and the applications to semantics.

Cresswell, (6) also, develops an elaboration of categorial grammar which deals with many of the problems of part 8 by adding the 'λ' and bindable variables of Church's lambda calculus. With respect to syntax, this is very much like Lewis' grammar in (24) with the following exceptions: 1) Lewis treats his abstraction operator and variables categorematically while Cresswell does not. 2) Lewis assigns tree-structured syntactic analyses to sentences while Cresswell assigns a linear parenthesized string. In this comment I want to suggest that the lambda calculus is indeed a useful tool in syntax though not in quite the way that Cresswell (or Lewis, in effect) recommends, and to show that a reformulation of the role of the lambda calculus leads naturally to extended categorial grammar.

Consider the familiar sentence

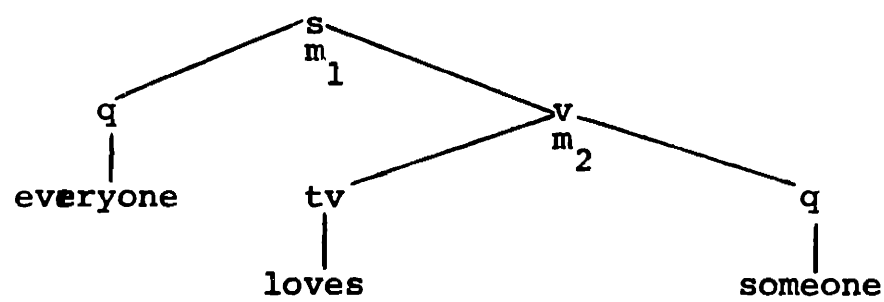
Everyone loves someone.

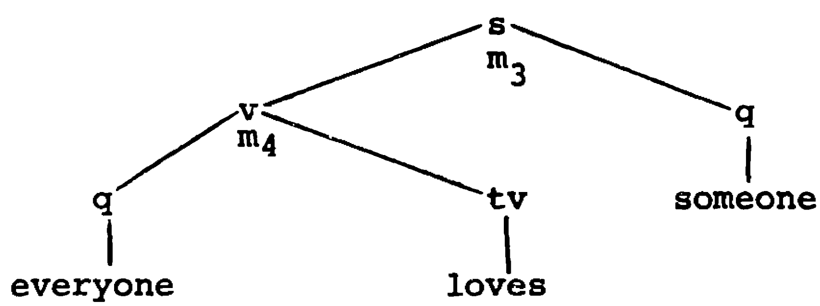
Cresswell assigns it (with minor notational differences) the two deep structures

$$\text{everyone}(\lambda x \cdot \text{someone}(\lambda y \cdot \ x \text{ loves } y))$$

$$\text{someone}(\lambda y \cdot \ \text{everyone}(\lambda x. \ x \text{ loves } y))$$

while I have assigned the two structural descriptions





I have purposely left the modes of combination m_1, m_2, m_3, m_4 unspecified. Now we can use the lambda calculus notation to specify the semantic modes as follows:

$$m_1 = \lambda zw \cdot \ z(w)$$

$$m_2 = \lambda zw \cdot \ \lambda x. \ \ w(\lambda y. \ z(x,y))$$

$$m_3 = \lambda zw. \ w(z)$$

$$m_4 = \lambda zw. \ \lambda y. \ z(\lambda x. \ w(x,y))$$

It is easy to verify that these are notational variants. of the definitions given in part 8, and that they yield the same semantic structure as Cresswell's analyses. The differences are that the extended categorial grammar approach allows intuitively plausible syntactic descriptions and categorematic semantics by keeping the metalinguistic notions and notations separated from the language being described. Whether this difference is of any great conceptual importance is another matter.

58. Not much weight should be put on the details of the example. I neither claim that 'bachelor' is synonymous on a reading with 'unmarried, adult, male person' nor that v/v is the proper category for adjectival modifiers. There are obviously both compound and simple L-meanings on Lewis' account. What is not quite so obvious is that some syntactically simple phrases, e.g., 'bachelor', have compound L-meanings of particular sorts.

59. This assumption is questionable. Perhaps some "proper names" are really disguised (abbreviated) definite descriptions while others are not. Perhaps contextual factors affect the interpretation of proper names. Perhaps names are semantically ambiguous. The assumption is made to simplify the discussion that follows and could be removed at the cost of modest complications involving considering cases.

60. (24) pp. 214-215.

61. Strictly speaking, this is not correct. For often the manner of acquisition of names is the best data concerning sameness of their referents.

62. There are reasons concerning both syntactic and semantic representation which favor some sort of indexing or co-

reference indication for noun phrases. But whatever the virtues of so doing, it is not a solution to the present difficulties. Of course, rather than the usual sort of local indexing, we might suggest a global indexing. But it is not clear how this is any different, let alone better, than taking the syntactic form of a name together with its intension as the "index".

63. Many of the views about proper names which Kripke offers have been applied by others to common names and substance as well.

64. I only suggest here what I defend elsewhere, that a certain "confusion" of use with mention may probe semantically fruitful. It is certainly intelligible (though perhaps unnecessary) to take as L-meanings full structural descriptions including lexical items. Once this is done, there can be operators on such entities which notice the lexical level as well as others which ignore it. Of course, L-meanings so construed are not suitable reconstructions for propositions in general--only for linguistically embodied propositions. But one way to take the problems we have had with indirect speech is as a lesson that we take seriously its claim to be indirect <u>speech</u>.

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