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Research on fuzzy control charts for fuzzy multilevel quality characteristics

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Abstract. Fuzzy control charts are proposed to solve the problem that traditional control charts cannot be applied to fuzzy quality characteristics. First, fuzzy quality characteristics are converted to representative statistics, which are fuzzy mode transformation, fuzzy level midrange transformation and fuzzy level median transformation. Control charts are designed based on the Poisson distribution. Second, the effects of the different statistics are analysed. Direct Fuzzy Control Charts are designed to avoid some information omission when translating fuzzy quality characteristics into representative statistics. The area ratio that falls within the control limits is used to conclude whether the process is relative out of control or in control. The performance of the control charts is analysed by MATLAB simulation. Finally, an example of an energy metre assembly is given to prove the proposed method.

Keywords: Fuzzy hierarchical quantitative characteristics / fuzzy theory / control chart / Direct Fuzzy Control Chart (DFCC)

1 Introduction

Due to the complexity of manufacturing systems and the limitations of detection technology, it is difficult to measure the accuracy of some product or process characteristics. For this kind of product or process characteristics, most enterprises use multilevel fuzzy quality characteristics for characterization. For example, in the machining process, when the comparison method is used to evaluate the work-piece with large surface roughness, three fuzzy conclusions are often given: very rough, general and good. Food is often described as very fresh, relatively fresh, not very fresh, and not fresh on four levels of fuzzy conclusion. To control this kind of characteristic, the traditional control chart first uses dichotomy to transform it into binary values of 0 or 1, and then uses attribute control charts (p, np, c and u charts) to control. There are three disadvantages:

- When information is simply classifieds to "qualified" or "unqualified", information is lost. Therefore, it is difficult to describe the variance or average value to achieve fine control of the characteristic when classifying the quality characteristics of a product or process into such binary categories.
- The sample size requirements are particularly large. To make a p-chart or np-chart requires a subgroup sample size of $1 \leq n\overline{p} \leq 5$. If $\overline{p} = 2\%$, then n$\geq$50, at least 1000 samples are needed to make the control chart for analysis.
- Insufficient detection capability.

At present, there is little research on fuzzy multilevel control charts. Hou et al. [1–3] proposed constructing a fuzzy function of product quality by quality expert score, drawing a control chart with a fuzzy function, and monitoring the fuzzy degree by using possibility measures and necessity measures. Faraz et al. [4] designed a fuzzy control chart with an alarm limit to control the process mean value. Shu et al. [5] designed a fuzzy $\tilde{\bar{X}} \sim \tilde{S}$ control chart. Gubay and Kahraman [6,7] proposed using the percentage of sample fuzzy data points falling into the control limit to determine whether the process is in control. Wibawati et al. [8] proposed a kind of fuzzy binary Poisson (FBP) control chart based on C-chart. The above mentioned literature proposed good ideas for the treatment of fuzzy multilevel quality characteristics, but their common point is that the processing and evaluation of fuzzy multilevel quality characteristics can be realized by changing control limits of the control chart, which is not sufficient for solving the current situation of dichotomy quality control with multilevel fuzzy quality characteristics. In this paper, fuzzy theory is introduced. The "fitness criterion" of quality is expressed in the form of fuzzy sets. The real number set or the corresponding subset is taken as

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the universe of discourse. The fuzzy membership function and its semantic rules are used to quantitatively describe the gradual change level, and a new fuzzy control chart is designed. Through MATLAB simulation, the performance of the new design control chart is explained, and it is successfully applied to the control of the electric energy metre assembly process.

2 Characterization and calculation of fuzzy multilevel quality characteristic statistics

Assume that a quality feature is divided into $i\,(i=1,2,\ldots,m)$ levels; for example, the electric energy meter starting characteristic can be divided into several levels of fuzzy characteristics, such as {very good, good, common, bad, very bad}. According to the real number set or corresponding subset, as the domain fuzzy membership function and semantic rules are used to quantitatively describe the gradual level of the fuzzy quality characteristics, the fuzzy function is important in the decision-making process.

Definition 1: Let U the universe of discourse, and the fuzzy set A on U is represented by a $U\to[0,\,1]$ function: $u\to\mu_A(x)$. The function value $\mu_A(x)$ is called the membership degree U of x in A, and the function $\mu_A(x)$ is referred to as a membership function of A. All the fuzzy sets on U are called $F(U)$ and referred as the fuzzy power set [9].

Definition 2: Let $A\in F(U)$, take any one $\alpha\in[0,\,1]$, denoted $A_\alpha=\{u\in U/A(u)\geq\alpha\}$, $0\leq\alpha\leq1$, where A_α is the α cut set of A, and α is the threshold or cut set level [10].

Theorem 1: (Fuzzy sets decomposition theorem) Provided $A\in F(Z)$, $A=U_{\alpha\in[0,1]}\alpha A_\alpha$.

Theorem 2: (Expansion theorem) Set f is a mapping of normal sets from X to Y, which is denoted by fuzzy sets on X and expressed as $A=\{(a,\,F_A(a))\,|\,a\in X\}$; then, the fuzzy set B on Y is obtained by extending f, and $B=\{(f(a),\,F_A(a))\,|f(a)\in Y\}$. The membership function of B is expressed as

$$F_B(f(a))=\begin{cases}V_{f(a)=b}F_A(a), b\in Y\\ 0, b\notin Y\end{cases}\tag{1}$$

Definition 3: Let R be a real number domain; if $A\in F(R)$, its membership function can be expressed as equation (2).

$$A(u)=\begin{cases}L\left(\dfrac{b-u}{b-a}\right), a\leq u<b\\ 1, b\leq u<c\\ T\left(\dfrac{u-c}{d-c}\right), c\leq u<d\\ 0,\ \text{else}\end{cases}\tag{2}$$

LR is called the fuzzy number, where $[b,\,c]$ is the kernel value interval of fuzzy function A. L and R are continuously increasing or decreasing shape functions on the left and right, respectively, and satisfy the requirements of

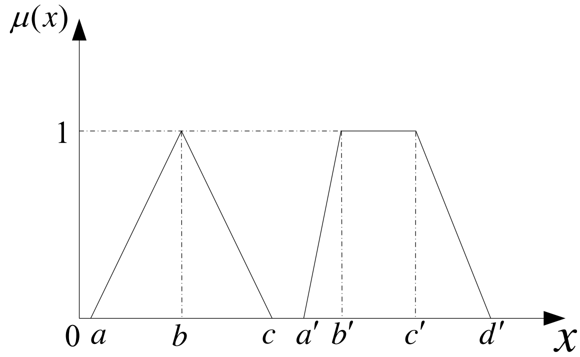

Fig. 1. Triangular membership function and trapezoidal membership function.

$L(a)=L(d)=0$ and $L(b)=L(c)=1$. Then, $\alpha\ (0\leq a\leq1)$ is the horizontal cut set and can be expressed as equation (3)

$$A_\alpha=\left[b-(b-a)L^{-1}(\alpha), c+(d-c)T^{-1}(\alpha)\right], \forall\alpha\in[0,1]\tag{3}$$

The triangular membership function and trapezoidal membership function are the two most common fuzzy numbers. As shown in Figure 1, the triangular membership function is represented as $A=(a,\,b,\,c)$, and the trapezoidal membership function is expressed as $A=(a',\,b',\,c',\,d')$.

Then, α horizontal truncation sets are:

$$A_\alpha=[a+(b-a)\alpha, c-(c-b)\alpha]\tag{4}$$

$$A_\alpha=[a'+(b'-a')\alpha, d'-(d'-c')\alpha]\tag{5}$$

$A=(a',\,b',\,c',\,d')$ is a trapezoidal membership function. Then:

$$\overline{M}_w(A)=\frac{1}{2}\left(M_w^-(A)+M_w^+(A)\right)\tag{6}$$

where:

$$M_w^-(A)=b-(b-a)\int_0^1 L^{-1}(\alpha)d\alpha\tag{7}$$

$$M_w^+(A)=c+(d-c)\int_0^1 T^{-1}(\alpha)d\alpha\tag{8}$$

Let a fuzzy quality characteristic expressed as multiple levels {good, general, bad, very bad}; each term set corresponds to a fuzzy function $\mu(x)$. Fuzzy functions are expressed by trapezoidal membership functions. If other types of fuzzy functions are selected, the processing methods are similar.

Fuzzy mode f_{mode}, fuzzy interval value f_{mr}^{α} of cut set α, fuzzy median f_{med}, and fuzzy mean value f_{avg} are used to characterize the concentration degree of the fuzzy data. Their interrelationships are shown in Figure 2.

$$f_{\text{mode}}=\{x|\mu_F(x)=1\}, \forall x\in F\tag{9}$$

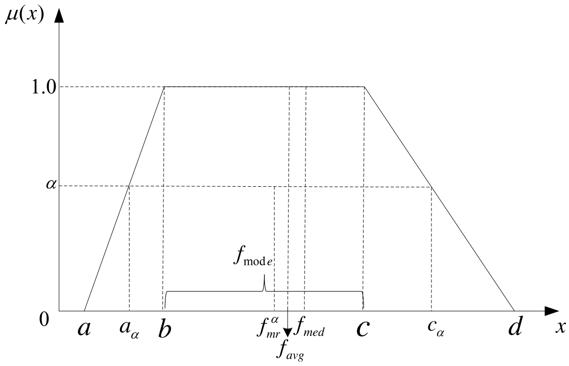

Fig. 2. Correlation between fuzzy data concentration degree statistics.

When the ambiguity functions as a single peak, the mode is unique.

Definition 3: Let F as a fuzzy set on the domain; then, the α horizontal truncation set of F is denoted as F_α, and $F_\alpha = \{x|\mu_F(x) \geq \alpha\}$. Assume a_α and c_α are both ends of F_α; for $a_\alpha = \mathrm{Min}\{F_\alpha\}, c_\alpha = \mathrm{Max}\{F_\alpha\}$, then $f_{\mathrm{mr}}^{\alpha} = \frac{1}{2}(a_\alpha + c_\alpha)$.

The α takes different values, and some common sets can be obtained by original fuzzy set A. The smaller α is, the lower the number of classes is, and the more elements A_α contains; the larger α is, the more classes are separated, and the fewer elements A_α contains. In other words, a series of classifications are obtained by taking different values of α.

$$\int_a^{f_{\mathrm{med}}} \mu_F(x)dx = \int_{f_{\mathrm{med}}}^{c} \mu_F(x)dx = \frac{1}{2}\int_a^c \mu_F(x)dx \quad (10)$$

$$f_{\mathrm{avg}} = \frac{\displaystyle\int_{x=0}^{1} x\mu_F(x)dx}{\displaystyle\int_{x=0}^{1} \mu_F(x)dx} \quad (11)$$

The fuzzy representative value methods proposed in the existing research can be summarized into three categories:

- Emphasizing the information of a section set of fuzzy data, such as fuzzy mode f_{mode} and α cut set fuzzy interval value f_{mr}^{α};
- Focusing on the information equal to or higher than α cut set, such as fuzzy median f_{med};
- The weights of all α cut sets are the same, such as fuzzy average f_{avg}.

When fuzzy data are transformed into fuzzy representative values, the problem of data information loss inevitably occurs. I. Kaya et al. [11–13] discussed the problem of information loss in the process of transforming fuzzy data into representative numerical values. The above four fuzzy representative values may lead to the loss of some important sample data information, which is due to the different information contained in different α level cut sets not being fully and accurately utilized.

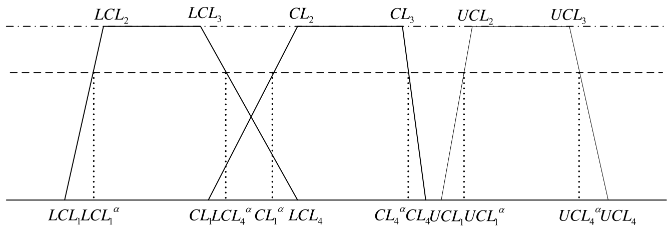

Fig. 3. Control limits of the fuzzy control charts.

3 Design of the fuzzy control chart

3.1 Design of fuzzy control limits

Normally, data are represented as trapezoid membership function (a, b, c, d) or triangular membership function (a, b, c). Triangular membership function is a special case of trapezoid membership function $c = d$. This paper takes trapezoid membership function as an example to study the control chart design under different fuzzy statistics.

When fuzzy samples are represented as (a_i, b_i, c_i, d_i), samples of m were taken on the centre line of the fuzzy control chart:

$$CL = \left(\frac{\sum_{i=1}^{m} a_i}{m}, \frac{\sum_{i=1}^{m} b_i}{m}, \frac{\sum_{i=1}^{m} c_i}{m}, \frac{\sum_{i=1}^{m} d_i}{m}\right) = \left(\overline{a}, \overline{b}, \overline{c}, \overline{d}\right) \quad (12)$$

The control centreline is a fuzzy set, which can be expressed by the fuzzy majority in the closed interval of $\left[\overline{b}, \overline{c}\right]$. Then, the centre line and upper and lower control limits of the fuzzy control chart are as follows:

$$CL = \left(\overline{a}, \overline{b}, \overline{c}, \overline{d}\right) = (CL_1, CL_2, CL_3, CL_4) \quad (13)$$

$$UCL = \left(\overline{a} + 3\sqrt{\overline{a}}, \overline{b} + 3\sqrt{\overline{b}}, \overline{c} + 3\sqrt{\overline{c}}, \overline{d} + 3\sqrt{\overline{d}}\right) \quad (14)$$

$$LCL = \left(\overline{a} - 3\sqrt{\overline{d}}, \overline{b} - 3\sqrt{\overline{c}}, \overline{c} - 3\sqrt{\overline{b}}, \overline{d} - 3\sqrt{\overline{a}}\right) \quad (15)$$

The control limits of fuzzy control charts based on fuzzy interval values of α cut sets (FCC-Midrange) and fuzzy control charts based on the fuzzy median of α cut sets (FCC-Median) can be expressed as (Fig. 3):

$$CL^\alpha = \left(\overline{a}^\alpha, \overline{b}, \overline{c}, \overline{d}^\alpha\right) = \left(CL_1^\alpha, CL_2, CL_3, CL_4^\alpha\right) \quad (16)$$

$$\begin{aligned} LCL^\alpha &= CL^\alpha - 3\sqrt{CL^\alpha} = \left(\overline{a}^\alpha, \overline{b}, \overline{c}, \overline{d}^\alpha\right) - 3\sqrt{\left(\overline{a}^\alpha, \overline{b}, \overline{c}, \overline{d}^\alpha\right)} \\ &= \left(\overline{a}^\alpha - 3\sqrt{\overline{d}^\alpha}, \overline{b} - 3\sqrt{\overline{c}}, \overline{c} - 3\sqrt{\overline{b}}, \overline{d}^\alpha - 3\sqrt{\overline{a}^\alpha}\right) \\ &= \left(LCL_1^\alpha, LCL_2, LCL_3, LCL_4^\alpha\right) \end{aligned} \quad (17)$$

$$UCL^{\alpha} = CL^{\alpha} + 3\sqrt{CL^{\alpha}} = \left(\overline{a}^{\alpha}, \overline{b}, \overline{c}, \overline{d}^{\alpha}\right) + 3\sqrt{\left(\overline{a}^{\alpha}, \overline{b}, \overline{c}, \overline{d}^{\alpha}\right)}$$
$$= \left(\overline{a}^{\alpha} + 3\sqrt{\overline{d}^{\alpha}}, \overline{b} + 3\sqrt{\overline{c}}, \overline{c} + 3\sqrt{\overline{b}}, \overline{d}^{\alpha} + 3\sqrt{\overline{a}^{\alpha}}\right)$$
$$= \left(UCL_1^{\alpha}, UCL_2, UCL_3, UCL_4^{\alpha}\right) \tag{18}$$

The fuzzy mode satisfies $f_{\text{mode}} = \{x|\mu_F(x) = 1\}$, $\forall x \in F$. When membership functions are unimodal, this value is unique.

The fuzzy number of sample j is as follows:

$$S_{\text{mod},j} = \left[b_j, c_j\right] \tag{19}$$

The control centre line and upper and lower control limits of FCC-Mode are as follows:

$$CL_{\text{mod}} = (CL_2, CL_3) \tag{20}$$

$$UCL_{\text{mod}} = CL_{\text{mod}} + 3\sqrt{CL_{\text{mod}}} \tag{21}$$

$$UCL_{\text{mod}} = \left(CL_2 + 3\sqrt{CL_2}, CL_3 + 3\sqrt{CL_3}\right) \tag{22}$$

$$LCL_{\text{mod}} = CL_{\text{mod}} - 3\sqrt{CL_{\text{mod}}} \tag{23}$$

$$LCL_{\text{mod}} = \left(CL_2 - 3\sqrt{CL_2}, CL_3 - 3\sqrt{CL_3}\right) \tag{24}$$

Control limits of the fuzzy control charts are shown in Figure 4.

3.2 Design of the direct fuzzy control chart (DFCC)

Direct Fuzzy Control Charts are referred as DFCC (Direct Fuzzy Control Chart). FCC-Mode control chart, FCC-Midrange control chart and FCC-Median control chart all convert fuzzy data into representative values. In the conversion process, data information is inevitably lost. The basic principle of the control chart is hypothesis testing and concluding whether uncontrolled or controlled are based on sample information. Therefore, the most scientific conclusion about controlled or uncontrolled should be at a certain significance level.

Based on this, this paper proposes a Direct Fuzzy Control Chart method. This method no longer converts the fuzzy information of samples into representative values, so it avoids the problem of information loss in the process of data transformation. The DFCC control chart determines the probability of the process being controlled or out of control according to the percentage of the sample statistic area falling into the upper and lower control limits. When the area of sample statistics falls into the fuzzy control limit, the process is in control; when the area of sample statistics exceeds the fuzzy control limit, the process is out of control; when the area of sample statistics exceeds the fuzzy control limit, that is, the proportion of area $\beta_j > \beta$, the process is in control; otherwise, the process is relatively out of control. Here, β can be given according to the actual needs.

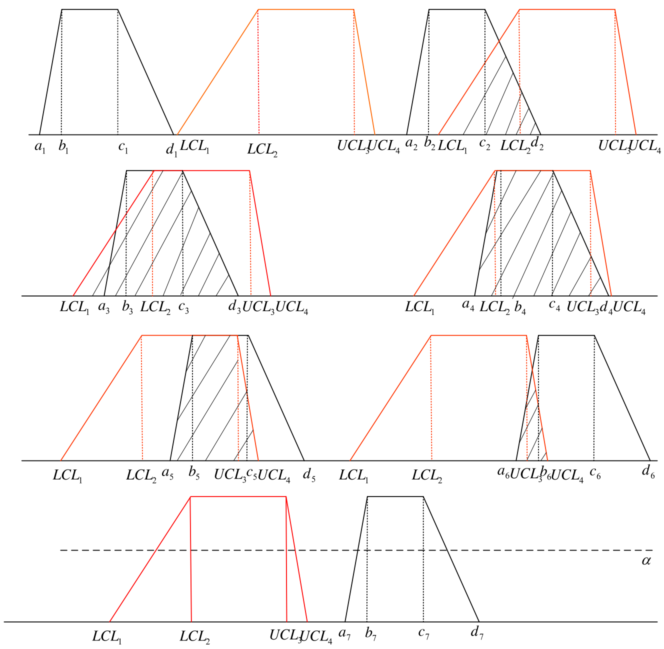

Fig. 4. Relationship between the direct fuzzy statistics and the control limit.

The area within the control limit is determined by the control limit and the relationship between samples. Figure 4 shows the calculation method of the area of fuzzy samples falling within the control limit. The area of the sample falling within the control limit is recorded as A_{IN}^j.

$$\beta_j = \frac{A_{IN}^j}{S_j} \tag{25}$$

The area of the sample is divided into the following situations:

$$\text{Type 1: } \beta_1 = 0 \tag{26}$$

$$\text{Type 2: } A_{IN}^2 = \frac{1}{2} \times \frac{d_2 - LCL_1}{LCL_2 - LCL_1 + d_2 - c_2} \times (d_2 - LCL_1) \tag{27}$$

$$A_{IN}^2 = \frac{1}{2} \times \frac{(d_2 - LCL_1)^2}{(LCL_2 - LCL_1 + d_2 - c_2)} \tag{28}$$

$$\beta_2 = \frac{A_{IN}^2}{S_2} = \frac{\frac{1}{2} \times \frac{(d_2 - LCL_1)^2}{(LCL_2 - LCL_1 + d_2 - c_2)}}{\frac{1}{2} \times (c_2 - b_2 + d_2 - a_2)} \tag{29}$$

$$\beta_2 = \frac{(d_2 - LCL_1)^2}{(c_2 - b_2 + d_2 - a_2) \times (LCL_2 - LCL_1 + d_2 - c_2)} \quad (30)$$

$$\text{Type 3: } \beta_3 = \frac{A_{IN}^3}{S_3} = 1 - \frac{(LCL_2 - b_3)^2}{(a_3 - LCL_1 + LCL_2 - b_3)(c_3 - b_3 + d_3 - a_3)} \quad (31)$$

$$\text{Type 4: } \beta_4 = 1 \quad (32)$$

$$\text{Type 5: } \beta_5 = \frac{UCL_3 - b_5 + UCL_4 - a_5}{d_5 - a_5 + c_5 - b_5} \quad (33)$$

$$\text{Type 6: } \beta_6 = \frac{(UCL_4 - a_6)^2}{(d_6 - a_6 + c_6 - b_6)(b_6 - UCL_3 + UCL_4 - a_6)} \quad (34)$$

$$\text{Type 7: } \beta_7 = 0 \quad (35)$$

The process control determination is shown as follows:
- $\beta_j = 1$, the process is in control;
- $\beta_j = 0$, the process is out of control;
- $\beta_j \geq \beta$, the process is relatively in control;
- $\beta_j \leq \beta$, the process is relatively out of control.

4 Simulation analysis of fuzzy single variable control chart performance

This section shows the performance of the control chart by MATLAB simulation. The first 25 sets of data in Section 5 are used to estimate the mean value of the population. Assuming the process mean under the controlled state:

$$[u_a, u_b, u_c, u_d] = [38.48, 42.14, 44.52, 49.57]$$

When a, b, c, d obeys a Poisson distribution, the process variance is:

$$[\sigma_a^2, \sigma_b^2, \sigma_c^2, \sigma_d^2] = [38.48, 42.14, 44.52, 49.57]$$

First, 25 groups of data from the Poisson distribution are randomly generated by MATLAB and recorded as follows: $P(38.48)$, $P(42.14)$, $P(44.52)$, $P(49.57)$. The first 25 groups of data are process stable data. The control limits of different types of control charts are calculated with 25 groups of randomly generated numbers. Then, random data with a sample mean offset ξ are generated randomly, and the simulation process is out of control. The statistics of the data are calculated, and the value of i when the No. i Sample point gives an alarm is output. The above process is repeated 5000 times and the average value of i is calculated as

$$n = \frac{\sum_{j=1}^{5000} i}{5000} \quad (36)$$

The sensitivity of the control chart can be explained by the size of n. The simulation output results are shown in Tables 1–5.

Tables 1–5 show the following:
- When the parameter is constant, DFCC control chart sensitivity is highest, the sensitivity of FCC-Mode control chart is second, and the sensitivity of FCC-Median control chart is lowest. The sensitivities of the four control charts increase with increasing migration volume.
- The sensitivity of FCC-Mode control chart increases with increasing value, but this effect is insignificant.
- When the offset ξ is constant, the sensitivity of FCC-Midrange control chart decreases with increasing value, but this effect is significant. It can be seen that the control diagram of FCC-Midrange is insensitive to small offsets ξ and cannot recognize small offsets ξ.

Table 1. $\alpha = 0.5$, $\beta = 0.5$, the relationship between n and ξ.

	FCC-Mode	FCC-Midrange	FCC-Median	DFCC
ξ	n	n	n	n
5	18.65	234.56	501.52	9.23
10	5.66	75.27	346.60	4.75
15	3.82	23.17	208.24	2.21
20	1.81	8.69	42.09	1.45
25	1.40	3.35	10.50	1.30
30	1.26	1.97	3.81	1.21
35	1.21	1.48	1.96	1.09

Table 2. The relationship between n and ξ when β is different (FCC-Mode control chart).

	$\beta = 0.2$	$\beta = 0.5$	$\beta = 0.8$
ξ	n	n	n
5	18.58	18.65	18.62
10	9.99	5.66	5.47
15	6.15	3.82	3.80
20	4.08	1.81	1.75
25	3.03	1.40	1.34
30	2.68	1.26	1.18
35	2.44	1.21	1.15

Table 3. The relationship between n and ξ when α is different (FCC-Midrange control chart).

	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$
ξ	n	n	n
5	111.85	234.56	288.98
10	31.61	75.27	112.65
15	10.74	23.17	38.11
20	4.91	8.69	14.97
25	2.51	3.35	5.85
30	1.68	1.97	3.51
35	1.21	1.48	2.74

Table 4. The relationship between n and ξ when α is different (FCC-Media control chart).

	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.8$
ξ	n	n	n
5	536.91	501.52	475.63
10	368.31	346.60	342.68
15	210.27	208.24	178.65
20	80.54	42.09	39.10
25	20.26	10.50	9.07
30	4.39	3.81	2.90
35	3.06	1.96	1.79

Table 5. The relationship between n and ξ When β is different (DFCC control chart).

	$\beta = 0.2$	$\beta = 0.5$	$\beta = 0.8$
ξ	n	n	n
5	11.43	9.23	5.27
10	10.46	4.75	4.22
15	6.07	2.21	2.02
20	4.06	1.45	1.27
25	2.93	1.30	1.20
30	2.59	1.21	1.12
35	2.37	1.09	1.05

– When the offset ξ is constant, the sensitivity of FCC-Median control chart increases with increasing value, but this effect is general. FCC-Median is insensitive to small offsets ξ and cannot recognize small offsets ξ.
– When the offset ξ is constant, the sensitivity of DFCC control chart increases with increasing value, but this effect is general.

5 Case study

The accurate measurement of energy metres is very important for production safety, maintenance, and supply with the legitimate interests of the three areas. The manufacturing process of a single-phase energy metre produced by an enterprise is shown in Figure 5. Assembly is the key process that affects the accuracy of energy metre measurements.

Key process parameters for the output assembly step: assembly clearance between the voltage and current cartridge assembly; the assembly gap has an important influence on the accuracy of the measurement. The component drawing after assembly is shown in Figure 6. The size specification after the assembly procedure is shown in Figure 7.

In the process design, the clearance gauge is a special clearance plug gauge. When the clearance plug gauge is used for inspection, the influence of tightness, measuring position and elastic clearance after assembly is considered, and the inspection results are expressed as linguistic data. The samples from energy metres are taken every 3 hours to control number of nonconformities. Because it is still in the trial production stage, the number of nonconformities may be large. The data collected from 21 subgroups are linguistic, and the linguistic expressions are represented by fuzzy numbers.

Table 6 shows the statistics and calculation table of unqualified numbers after fuzzy judgement.

The results of designing different fuzzy control charts by MATLAB software are shown in Figures 8–11.
– Fuzzy control chart based on the fuzzy mode.
– Fuzzy control chart based on the fuzzy interval value of the α cut set.
– Fuzzy control chart based on the fuzzy median of the α cut set.
– Direct Fuzzy Control Chart.

In Figures 8–11, the results shown in Table 7 are obtained when β is 0.8. Based on the judgement result of Table 7, we can draw the following conclusions:
– The method of transforming fuzzy data into representative statistics and DFCC Control Chart is more sensitive than an attribute control chart, which is directly transformed into 0 or 1, and further utilizes different intermediate state information, so it has better sensitivity.
– Because different statistics represent different data characteristics, the control charts designed based on different statistics have different effects. For example, the judgement results of FCC-Mode control chart, FCC-Midrange control chart and FCC-Median control chart at sample No. 3, sample No. 4, sample No. 10 and sample No. 11 are different, which further verifies the

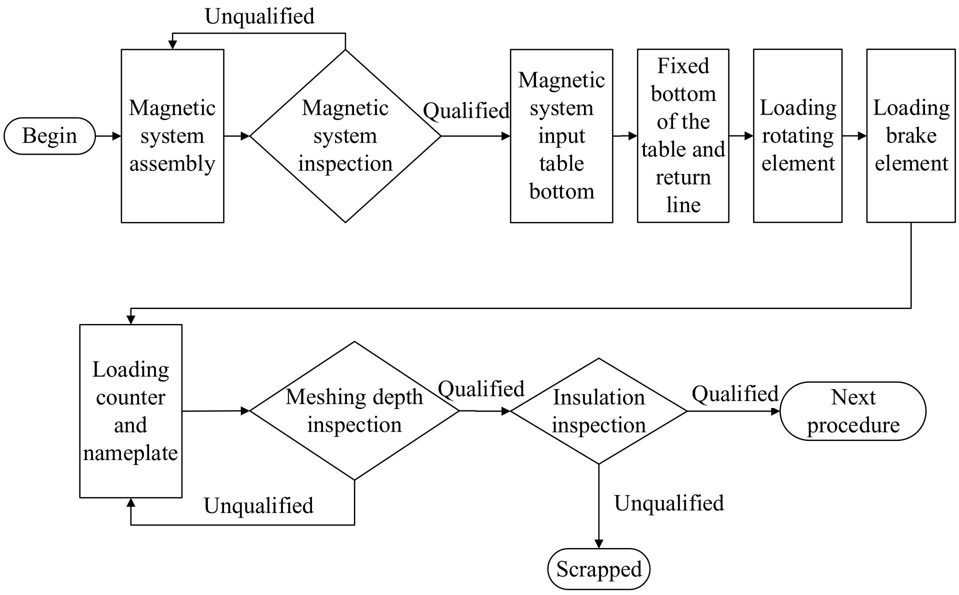

Fig. 5. Process flow diagram.

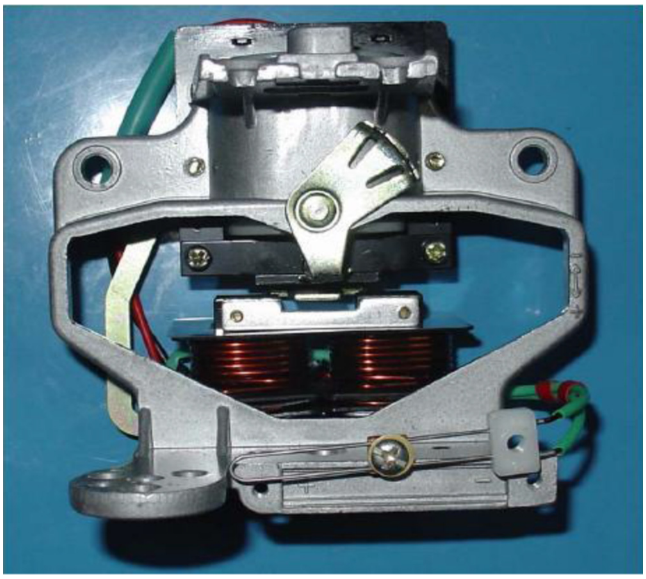

Fig. 6. Component drawing after assembly.

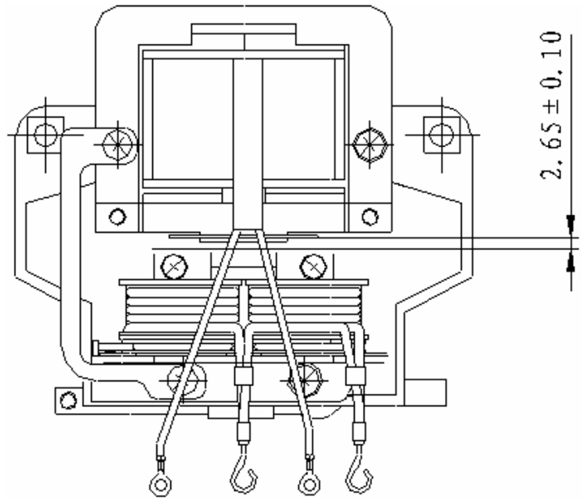

Fig. 7. Dimension requirement diagram after assembly.

information loss problem of transforming fuzzy data into representative statistics; however, the converted statistics with different representativeness are easier to understand in practical use.

– The sensitivity of DFCC control chart is highest, FCC-Mode control chart is second, and FCC-Median control chart is lowest, which is consistent with the simulation analysis conclusion.

– DFCC control chart is judged based on a certain level and consistent with hypothesis testing. Different requirements can be selected according to different β in practice to make results more scientific.

6 Conclusions

With the help of fuzzy theory, control charts for fuzzy quality characteristics are designed. The "applicability" standard of quality is expressed in the form of fuzzy sets. Taking the real number set or corresponding subset as the universe of discourse, the fuzzy membership function and its semantic rules are used to quantitatively describe the

Table 6. The number of unqualified statistical measurement gaps.

样本号	a	b	c	d	S^j_{mod}	$S^\alpha_{\text{mr},j}$	$S^\alpha_{\text{med},j}$	β_j
1	35	37	38	46	(37, 38)	38.7	38.1	1
2	37	50	54	58	(50, 54)	50.2	51.1	1
3	13	15	23	30	(15, 23)	20	19.5	0.269
4	17	20	23	25	(20, 23)	21.3	21.4	0.768
5	33	36	36	40	(36, 36)	36.2	36.1	1
6	34	35	42	45	(35, 42)	38.9	38.7	1
7	35	38	38	45	(38, 38)	38.8	38.4	1
8	40	44	44	45	(44, 44)	43.4	43.7	1
9	41	43	43	47	(43, 43)	43.4	43.2	1
10	60	65	70	80	(65, 70)	68.5	68	0.409
11	58	60	64	78	(60, 64)	64.4	63.2	0.718
12	35	38	39	49	(38, 39)	39.9	39.2	1
13	38	41	44	48	(41, 44)	42.7	42.6	1
14	43	45	45	47	(45, 45)	45	45	1
15	43	47	47	48	(47, 47)	46.4	46.7	1
16	36	38	45	49	(38, 45)	41.9	41.7	1
17	44	48	52	56	(48, 52)	50	50	1
18	36	39	40	49	(39, 40)	40.7	40.1	1
19	39	45	47	49	(45, 47)	45.2	45.6	1
20	42	47	47	48	(47, 47)	46.2	46.6	1
21	49	54	54	59	(54, 54)	54	54	1

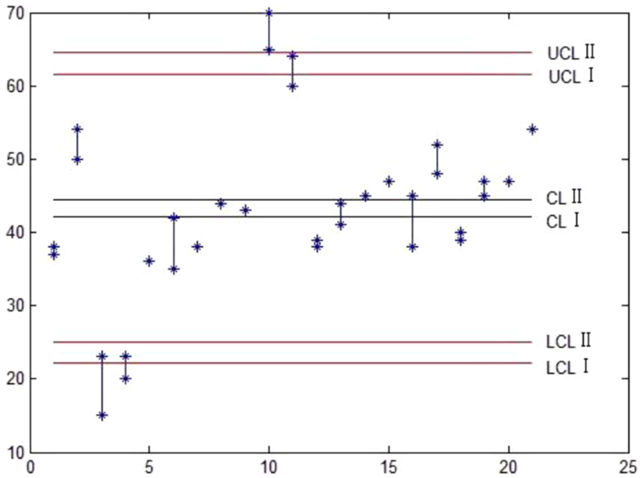

Fig. 8. FCC-Mode control chart based on the fuzzy mode.

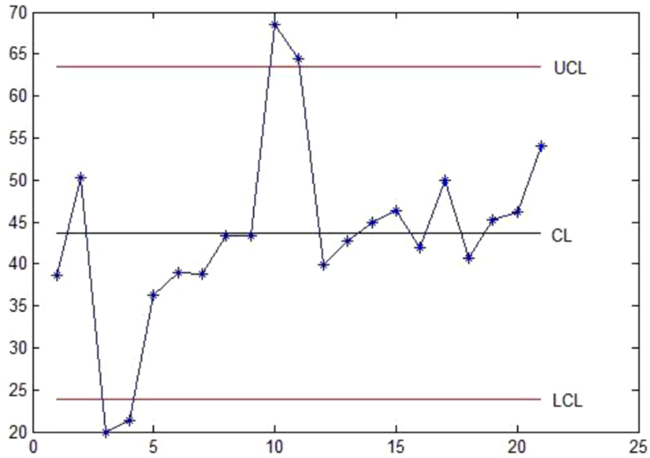

Fig. 9. Fuzzy control chart based on the fuzzy interval value of the α cut set.

gradual change level of product or process characteristics. The fuzzy quality characteristics are transformed into typical statistics, and the characteristics of different statistics are analysed. On this basis, a fuzzy control chart is designed based on the Poisson distribution. This control chart uses the area ratio falling within the control limit to judge the relative controlled and relatively uncontrollable problem and solves the problem of low sensitivity when the fuzzy quality characteristic is applied in statistical process control technology.

Fuzzy control charts algorithm is designed by MATLAB programming.

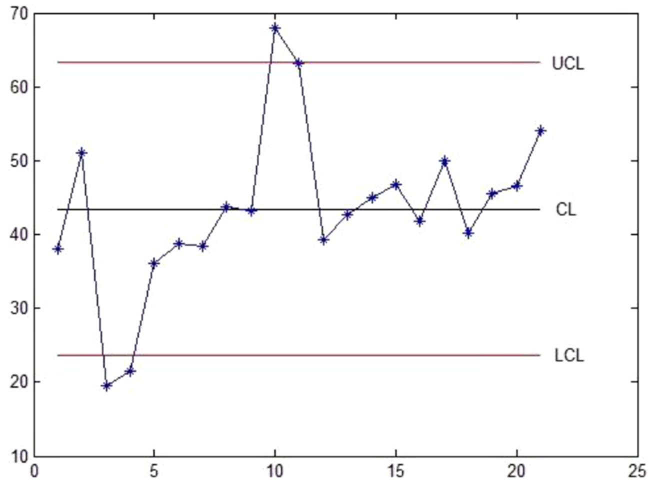

Fig. 10. Fuzzy control chart based on the fuzzy median of the α cut set.

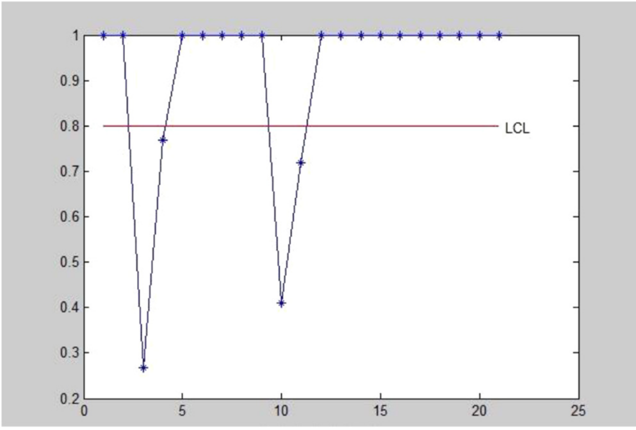

Fig. 11. DFCC fuzzy control chart.

Table 7. Judgement results of different fuzzy control charts.

Sample	FCC-Mode	FCC-Midrange	FCC-Median	DFCC
1	Controlled	Controlled	Controlled	Controlled
2	Controlled	Controlled	Controlled	Controlled
3	Relatively out of control	Out of control	Out of control	Relatively out of control
4	Relatively out of control	Out of control	Out of control	Relatively out of control
5	Controlled	Controlled	Controlled	Controlled
6	Controlled	Controlled	Controlled	Controlled
7	Controlled	Controlled	Controlled	Controlled
8	Controlled	Controlled	Controlled	Controlled
9	Controlled	Controlled	Controlled	Controlled
10	Out of control	Out of control	Out of control	Relatively out of control
11	Controlled	Out of control	Controlled	Relatively out of control
12	Controlled	Controlled	Controlled	Controlled
13	Controlled	Controlled	Controlled	Controlled
14	Controlled	Controlled	Controlled	Controlled
15	Controlled	Controlled	Controlled	Controlled
16	Controlled	Controlled	Controlled	Controlled
17	Controlled	Controlled	Controlled	Controlled
18	Controlled	Controlled	Controlled	Controlled
19	Controlled	Controlled	Controlled	Controlled
20	Controlled	Controlled	Controlled	Controlled

The sensitivity of the control chart is analysed by MATLAB simulation. The sensitivity of DFCC control chart is highest, FCC-Mode control chart is second, and FCC-Median control chart is lowest.

Through the example of the energy metre assembly, the scientificity and effectiveness of the proposed method are further explained.

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