DUAL MODELS

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CERN

1. Introduction

First I wish to explain why the dual theories provide such an interesting and promising new theoretical framework for elementary particle physics.[1] The original motivation was that it (and it alone) was capable of incorporating in a natural way so many features of hadronic physics.

It is a Lorentz invariant theory with a vast ghost-free spectrum of particle states lying on linear Regge trajectories. There are three sectors to the theory as tabulated below.

	Leading trajectory	Massless state
Fermion	$\frac{1}{2} + \alpha' S$	"electron"
Meson	$1 + \alpha' S$	"photon"
Pomeron	$2 + \frac{\alpha'}{2} S$	"graviton" and "dilaton"

The daughter trajectories are parallel to the leading trajectories and spaced below by integral multiples of one half.

All the mutual couplings between these states are specified in terms of one parameter. Thus in a sense all the particles lie in one huge supermultiplet - this is the ultimate possibility in symmetry schemes. As we shall see it is the extreme and very beautiful degree of symmetry which makes the theory work so consistently.

The couplings are just such as to ensure both the ghost-free nature of the spectrum already mentioned and the Regge behaviour of the tree amplitudes (obtained by summing all the tree diagrams).

In addition there are tantalising hints of a quark structure (so that we maybe have a relativistic quark model) together with a chiral symmetry. These are not really understood yet but will be elaborated upon below.

The viability of dual theory is further enhanced by the remarkable fact that almost all the classical field theories can be obtained as special cases:

- electrodynamics of photons and massless electrons
- Yang Mills theory[2]
- Einsteinian theory of gravity.[3]

From the table we see that the slope parameter α' is variable while the intercepts are fixed. As $\alpha' \to 0$ the masses of all states move to infinity except for the massless states which survive and are listed in the third column of the table. When the couplings are checked the above results follow at least at the tree level - (There is some subtlety about precisely which coupling we hold fixed and whether or not we include isotopic spin.)

Notice that the field theories so obtained have a high degree of invariance probably necessary for renormalisability in the presence of spinning particles. In fact since the Yang-Mills theory is the unique theory of self-interacting spin -1 particles in which the tree diagrams in toto have a certain "good" asymptotic behaviour[4] (the $\alpha' \to 0$ limit of Regge), necessary for the renormalisation of loops the above result appears very natural and augurs well for the renormalisability of dual loops (but see later).

Originally designed for hadrons, the dual theory seems to be capable of describing the weak electro-magnetic and gravitational interactions. Thus we have the tantalising possibility that there may emerge a unified theory of all the interactions.

During the last two years there has been much interest in working backwards and deriving features of dual theories from Yang-Mills gauge theories.[5]

As presently formulated the dual theories have some very definite faults which preclude any detailed comparison with experimental data - the trajectory intercepts are systematically too high and a few low lying states are therefore tachyons. Further, space time has to have 10 or 26 dimensions depending on whether or not fermions are included. As we shall see there are possible ways out of these difficulties.

2. Formulations of the Theory

The theory is dual in many ways - it exists in two versions (apart from minor variations) and can be formulated in two distinct approaches.

The starting point was Veneziano's four meson amplitude[6]

$$\frac{\Gamma(-\alpha s)\ \Gamma(-\alpha t)}{\Gamma(-\alpha s-\alpha t)}$$

which developed over the years into the theory without fermions. The theory with fermions is naturally more complicated and has developed more recently. Its four point function was written down (unknowingly) by Lovelace[7]

$$\frac{\Gamma(1-\alpha s)\ \Gamma(1-\alpha t)}{\Gamma(1-\alpha s-\alpha t)}$$

who already noted the hints of chiral symmetry. The two formulations of the theory - the operator and string approaches - each have their own advantages.

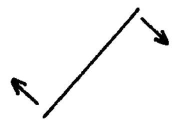
The operator approach is the historical method and still the most powerful but is difficult to communicate to an outsider because of the technicalities of an unfamiliar formalism. Nevertheless it has its own mathematical beauty and simplicity which constantly spurs on its addicts. It is an on-mass-shell S-matrix approach. The S-matrix is given as a sum of diagrammatic contributions with well defined operator expressions for the vertices and propagators of the diagram. The topology of the diagrams differs from that of ordinary Feynman diagrams and because of this and the fact there was no underlying Lagrangian the inner consistency of this expansion had always to be checked.

The string picture was just that -an intuitive way of understanding visually all of the above mentioned inner consistency. Since it is a space time picture there should be an underlying field theory of a new non local type which is nevertheless causal. During the past eighteen months there has been much progress in translating this promise into a well-defined mathematical reality as we shall discuss. One of the problems is the lack of manifest Lorentz convariances.

3. Serenade for Strings

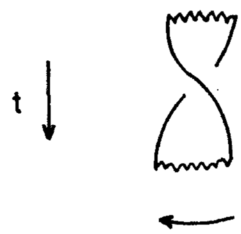
Because of its pedagogic and conceptual advantages we start with the string approach - but bearing in mind that most of the results mentioned were first discovered by the operator methods. The idea is quite old,[8] dating from 1969 but has been refined and developed over the years. Now it is on the verge of a complete, mathematically rigorous formulation.

A meson is thought of as an open string with free ends, moving under the action of a tension along its length and a centrifugal force due to rotation. For states of given energy (or rather mass in the relativistic case) the maximum angular momentum occurs when the string is effectively rigid, merely rotating bodily about an axis perpendicular to itself.



This corresponds to the leading trajectory. Superimposed on this motion can be many modes of vibration which are musical in the sense that, as in a violin string, the possible eigenfrequencies are integer multiples of a fundamental, in this case the rotational frequency. These motions add energy or mass and so give rise to "daughter states".

Just as a point particle traces out in space time a "worldline" so, as time elapses, the string traces out a surface called the "worldsheet" which would resemble a corkscrew - because of the rotation.



Let $X^\mu(\sigma,\tau)$ be the space time co-ordinate vector of a point of the string world-sheet specified by the parameters (σ,τ), which are co-ordinates in the world-sheet. In the original string picture[8] all the components of $X^\mu(\sigma,\tau)$, including the time components, were treated as equivalent dynamical variables as a way of ensuring Lorentz convariance. Just as with the anolagous situation with the photon field in QED this leads to the difficulty that the time components could give rise to ghost states. As discussed later these could be eliminated by imposing subsidiary conditions.

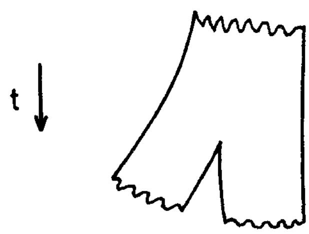
A more elegant solution was to suppose that the only observable oscillations of the string be transverse to the worldsheet since oscillations tangent to it, i.e longitudinal or time-like oscillations could be transformed away by redefining the parameters σ and τ. This could be achieved by choosing an action, such as the following suggested by Nambu and others[9,10], which depends only on the position of the worldsheet in space time and not on the particular choice of co-ordinates σ,τ

$$A = -\frac{1}{2\pi\alpha\hbar c^2}\int d\sigma\, d\tau \sqrt{\left(\frac{\partial X^\mu}{\partial\sigma}\frac{\partial X_\mu}{\partial\tau}\right)^2 - \left(\frac{\partial X}{\partial\sigma}\right)^2\left(\frac{\partial X}{\partial\tau}\right)^2}$$

The invariance groups of reparametrisations is an infinite parameter group and is enormously important in all that follows. It plays a role similar to gauge invariance in Yang-Mills theory or general relativity (and indeed reduces to these invariances in the limiting cases mentioned earlier [2,3]).

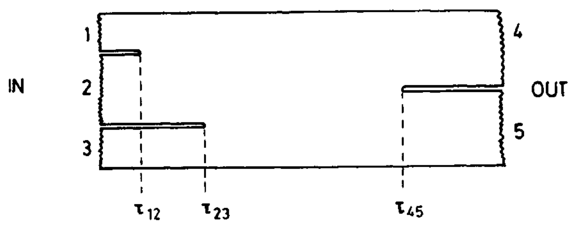
Starting from the above action G G R T[10] developed the whole canonical procedure. When the transverse degrees of freedom, which are suitable independent variables, are quantised Lorentz invariance can only be achieved if the leading trajectory intercept $\alpha(0) = 1$ and if space time has 26 dimensions. The peculiar structure of the Lorentz generators previously known from the operator formalism[11] now had a direct physical interpretation. The no-ghost theorem was automatic since only space-like and hence positive definite degrees of freedom were excited.

The next problem was to extend this beautiful theory of free propagation to include interaction while respecting the transversality and hence the no-ghost theorems. The solution is to imagine that strings can split to form shorter strings or rejoin.[12]

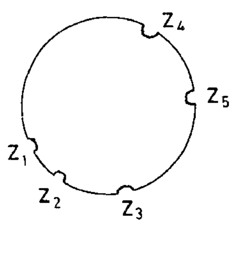


The resultant transition vertex can be expressed as an overlap integral between the strings concerned. Much work has been done this year in clarifying this calculation and checking it with the previously known result from the operator formalism(13,14) .

When considering several interacting strings one can choose a (σ,τ) co-ordinate system(10,13) on the worldsheet of the process such the (i) $\tau = \frac{t+x}{\sqrt{2}}$ and (ii) the σ length of each string is equal to its + component of momentum $\frac{E+p_x}{\sqrt{2}}$. This means that the total σ- length of the strings is conserved throughout and that the (σ,τ) picture indicates the temporal development.



The interior of this worldsheet can be mapped by a Schwartz-Christoffel transformation(13) into the interior of the Koba Nielsen circle - now each string is on an equivalent footing and so duality appears manifest(11).

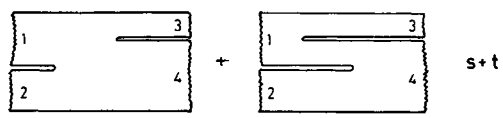


Thus the interaction times $\tau_{12}\ \tau_{23}\ \tau_{45}$ are related to the Koba Nielsen variables Z_i.

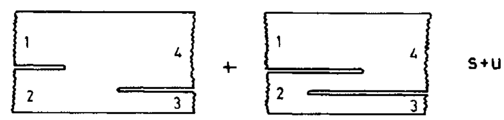
Mandelstam(13) suggested that the corresponding string scattering amplitude be expressed as an integral of $e^{iA/\hbar}$ over the possible histories of the string with the given initial and final configurations.

These histories are parametrised in terms of the τ's (or better, their differences) and the components of $X^\mu(\sigma,\tau)$ (other than the t and x components) considered as fields on the worldsheet. Mandelstam discusses the problem of choosing the measure of integration and evaluating the resultant integrals which yield exponentials of Neumann functions, as predicted by the analogue model.(15)

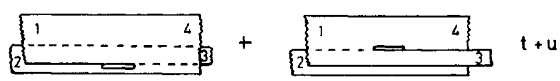
It is instructive to consider the scattering of strings 1 and 2 into strings 3 and 4. The (s-t) term is the sum of the following two contributions separated according to the time ordering of the string interactions which also correspond to whether the intermediate string propagates in the s or t channel.



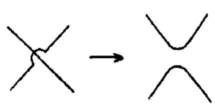
The (s-u) term works similarly.



The (t-u) term does not since we findcontributions.



Now the string worldsheet is a two -sheeted Riemann surface. Further, unlike the previous cases the two diagrams do not coincide when the interaction times do. What is missing is a third contribution due to a four string interaction describing the process(16,17)



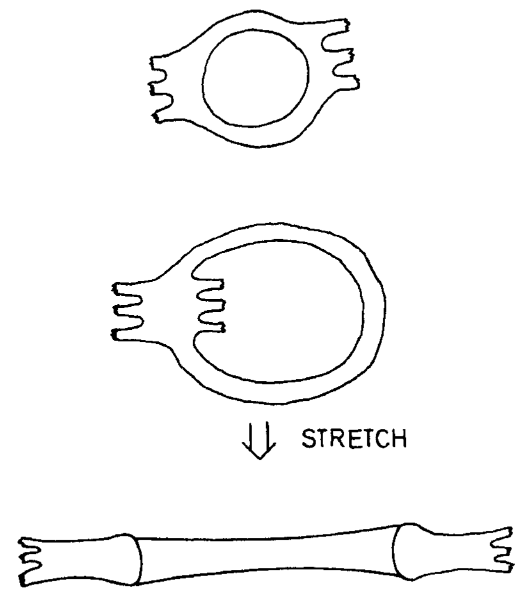
Kaku and Kikkawa[17] also noted the similarity of the above diagrams to individual contributions from a non relativistic perturbation theory - or better - a light cone or infinite momentum frame expansion. In fact the authors succeeded in defining a Lagrangian with tri and quadrilinear interactions in terms of a non-local second-quantised field operator describing the strings. The perturbative expansion resulted naturally because of the light cone quantisation necessitated by the choice of (σ,τ) parametrisation mentioned earlier. The importance of this is that it clarifies the counting of graphs - particular loop graphs and the checking of unitarity. Further discussions have been made by these authors and Cremmer and Gervais[18].

A delicate feature of the approach is that the exploitation of transversality implies a lack of manifest Lorentz covariance. In his talk Mandelstam discussed new methods of establishing this Lorentz covariance.[19] The introduction of a Lorentz covariant non-local field operator has been discussed by Ramond.[20]

4. Pomerons are closed strings

Isotopic spin can be introduced by imagining that quarks carrying isotopic spin move along the boundary of the worldsheet. This is equivalent to the Chan Paton factors[1]. Then the processes discussed so far lead to amplitudes with just poles in $q\bar{q}$ channels (if the external particles are $q\bar{q}$) and so can be regarded as the tree amplitudes of a perturbative expansion.

Unitarity of the S matrix demands the presence of branch cuts due to two (or more) body intermediate states. In ordinary Lagrangian perturbation theory this is achieved by adding loop diagrams and so at the single loop level we shall consider



The first diagram represents the worldsheet of a process in which strings join, separate and then rejoin before finally splitting. This is called the planar loop. When calculated (by operator formalism methods[1] its real part diverges. Renormalisation procedures have been found which unfortunately fail for the case a = 1, d = 26.[21] Since this is the most interesting case this is a defect of the theory.

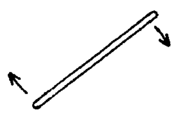
The first diagram can also be thought of as a string circulating in a loop, emitting strings from one end. The second diagram represents the extra possibility that the circulating string can emit strings from either end. This is called the non planar loop.

A remarkable feature of this second diagram is that it can be stretched into the form indicated in which a tube appears. Since a tube is the worldsheet of a closed string we see that a closed string appears as an intermediate state between the open strings emitted from each end of the circulating string. This argument is confirmed by detailed calculation with the operator formalism.[22] The excited state of the closed string indicate a whole new spectrum of excited resonances which are interpreted as "Pomeron" states for the following reasons.

1) The channel has zero quark quantum numbers since the quarks can flow only along the boundary of the sheet

2) The leading Pomeron trajectory is $J = 2 + \frac{\alpha'}{2}p^2$

This is because the closed string states with maximum angular momentum occur when the closed string behaves as two rigid open strings stuck together.[23]



Since the total momentum and angular momentum are shared between the two open strings

$J = 2j \quad P^{\mu} = 2p_{\mu}$

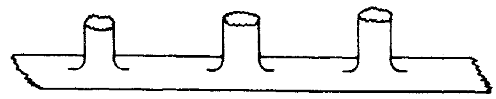
and the leading open string trajectories satisfy

$j = 1 + \alpha' p^2$

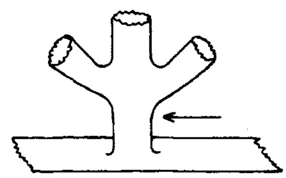
There is phenomenological support for the fact that the leading Pomeron trajectory should have twice the intercept and half the slope of the leading meson trajectory.

Of course the intercept 2 is wrong and violates the Froissart bound but at least fits in with the meson intercept 1, already agreed to be wrong, in that both double the physical value. It could be that this defect will be righted either as a consequence of finding a Born term with physical intercepts or by performing the perturbation sum in the present theory.

The dual theory has made the specific prediction that there be a whole spectrum of Pomeron states or resonances. For theoretical consistency one must check that these resonances have real couplings (the no ghost theorem) and that a self consistent theory of Pomeron scattering emerges from the multiloop diagrams (which we have not discussed). The no-ghost theorem (for couplings to open strings) has been proved[22] and recently much progress has been made in answer to the second question. There now exists a unified and consistent theory of amplitudes describing the scattering of open and closed strings.[24] Thus the amplitude for the process below can be written down using an operator formalism very like the original formalism for multiperipheral trees:



This can be stretched (dualised) into the form



By factorising on the tube indicated the four Pomeron amplitude is found and yields the Virasoro Shapiro amplitude. Since in this formalism a closed strong exchanged between open strings does yield the loop graph discussed above the results finally resolve a protracted controversy about whether the Virasoro Shapiro amplitudes[25] do describe Pomeron scattering.

The graviton scattering amplitudes, in the zero slope limit, coincide with the graviton tree amplitudes arising from the Einstein gravitational Lagrangian considered as a non-polynomial expansion in flat space-time. This is the sense in which Einsteinian gravitation is a special case of the dual theory.[3]

We should like to emphasise that the results mentioned depended on very detailed and difficult calculations in the operator formalism. It is the hope of the string picture that the intuitive pictorial arguments given here represent directly a rigorous mathematical treatment but it is too early to be certain that this has been achieved because of the mathematical problems inherent in functional integrations.

Asymptotic behaviour of multiloop proper have been studied recently.[26]

5. Generalisations of the string

Given that the string is an extended object in space time whose dynamics is consistent with relativity, causality and quantum theory, one can wonder about further possibilities. Two proposals have been discussed, - Dirac's bubble which predates the string by some years and the MIT bag. These are both three dimensional objects whose natural tendency is to contract because of a surface or body tension. Because of the three dimensional nature, rotation can no longer provide the counterbalance to this contractive force since it has no effect parallel to the axis of rotation. Hence extra forces are needed.

In order to eliminate the infinite self energy of a point particle Dirac[27] wanted to picture the electron (with spin 0 for simplicity) as a bubble with the charge distributed over the surface. The surface tension of the bubble then balanced the electrostatic repulsion. Oscillating modes were also possible.

The MIT bag is intended as a model for hadrons and consists of a variable three dimensional volume containing ordinary quantum fields-quarks and gluons, say[28]. These are effectively free if the couplings are weak except when they are reflected at the walls of the bag. This is sufficient to give the parton structure desired for scaling. Gauss' theorem together with the assumed confinement of the gluon fields (supposed to be colour octets) implies that hadrons must be colour singlets. Approximate wave functions for spherically symmetric quark fields can be found and yield surprisingly good values for the proton magnetic moment, g_A/g_V and other static quantites. Thus as a relativistic shell model this approach is highly promising.

A quantisation in 4 rather than 2 space time dimensions is still lacking and there are problems with rotating states but there is certain to be much more work on this model.

6. Subsidiary conditions and the operator formalism

We mentioned previously that if $X^\mu(\sigma,\tau)$ is quantised covariantly it is necessary to eliminate possible ghosts states by imposing a subsidiary condition. The following is a consequence of the formalism:-

$$\langle P^\mu(\sigma)^2 - m^2 \rangle_n \ | \psi \rangle = 0, \ n = 0, 1, 2, 3.....$$

$P^\mu(\sigma)$ is the quantised momentum flux of a point in the string at $\tau=0$

$$P^\mu(\sigma) = P^\mu + \sum_{n=1}^{\infty} \sqrt{n}\ (a_n^\mu e^{in\sigma} + a_n^{\mu\dagger} e^{-in\sigma})$$

$\langle\rangle_n$ denotes that the nth Fourier component in σ is taken and $|\psi\rangle$ is a state in the Fock space of the creation operators $a_n^{\mu\dagger}$ where

$$\left[a_n^{\nu},\ a_m^{\mu\dagger}\right] = -\ g^{\mu\nu}\ \delta_{nm} \qquad n,m = 1,2,3 \qquad \mu,\nu = 0,1 ... D-1$$

so that the $\nu = \mu = 0$ commutator yields $-\delta nm$, and hence possible ghosts.

The above subsidiary condition is common between the two formalisms-string[10] and operator - it was originally derived in the latter.[29,1]

The quantities $L_n = -\frac{1}{2} \langle P^2 \rangle_n$ close under commutation producing the Virasoro algebra[30] - a very important structure in the operator formalism and the analogue of the group of co-ordinate reparametrisations in the string world sheet.

We shall mention the following steps in the developments of the operator formalism.

(1) The vertices and propagators mentioned earlier can be expressed in terms of L_0, L_1 and L_{-1}

(2) Because of this and the Virasoro algebra, Ward identities can be derived which yield the subsidiary condition above.[1]

(3) Solutions to the subsidiary condition were constructed with DDF (transverse operators) $A_i^{\dagger n}$ $i = 1,\ldots D-2$ which were complicated functions of $a_i^{\dagger n}$ and which were later seen to correspond to the transverse string vibrations.[31]

(4) The proof of the no ghost theorem:– $\langle\psi|\psi\rangle \geq 0$ for any state $|\psi\rangle$ satisfying the subsidiary condition. This followed from the proof that the Fock space of DDF oscillators (which is positive definite) provides, modulo zero norm states, a complete set of solutions to the subsidiary condition,[32] if D=26.

(5) Construction of the operator projecting from the "a" Fock space to the "A" Fock space.[33]

(6) Construction of unitary single loops using this – this also corresponds to achieving a partial second quantisation.[34] and lead to the Pomeron previously mentioned.

7. Dual Fermions

We have presented the above subsidiary condition so that it very much resembles a generalised Klein-Gordon equation. This suggests that the fermions which hitherto have been lacking could be incorporated by considering a correspondingly generalised Dirac equation as subsidiary condition:-

$$\langle\Gamma(\sigma).P(\sigma)-m\rangle_n|\psi\rangle = 0 \quad n = 0,1,2..$$

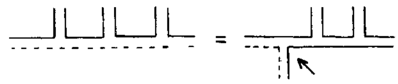
where $\Gamma^\mu(\sigma)$ is a generalised Dirac gamma matrix, its zero Fourier component being γ^μ and its higher Fourier components involving anticommuting annihilation creation operators (since γ^μ's anticommute)

$$\Gamma^\mu(\sigma) = \gamma^\mu + \sqrt{2}\,\gamma_5 \sum_{n=1}^{\infty} (d_\mu^{n\dagger} e^{in\sigma} + d_\mu^n e^{-in\sigma})$$

The idea of imitating Dirac's original argument was due to Ramond[35] and does indeed lead to a satisfactory fermion theory once a vertex for meson emission is found which respects the fermion subsidiary condition.[36]

The anticommutators of the quantitive $\langle\Gamma.P\rangle_n$ yield new Virasoro generators which in turn generate the Virasoro algebra.[35] This new mathematical structure, a closed algebra with both anticommutators and commutators, is called the supergauge algebra and effectively puts mesons and fermions into the same multiplet as mentioned earlier. The application of supergauges to four dimensional local field theories has received much attention recently.[37]

The amplitude for a fermion emitting mesons can be constructed[36] and then dualised to exhibit an internal meson channel.[36]

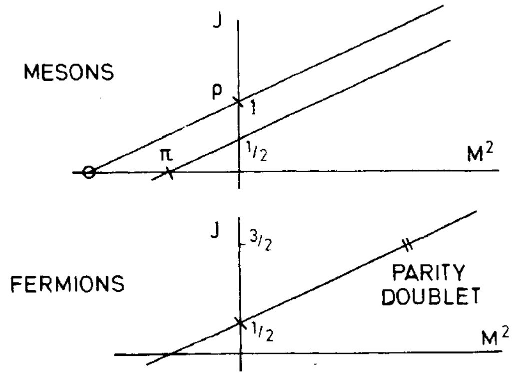


This gives two new pieces of information

(a) a new purely mesonic amplitude, obtained by factorising the residue of the pole indicated: This is the "Neveu-Schwarz" amplitude[38] – which for four mesons coincides with the Lovelace amplitude written down earlier.

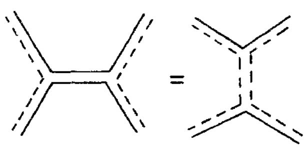
(b) a new vertex: that for fermion emission[39,40]

The positive definiteness of the mesonic and fermionic spectra so obtained requires m = 0 (for the ground state fermion)[41,42] and D = 10. This is encouraging in that the critical space-time dimension reduces as more degrees of freedom are incorporated into dual hadrons.[43] In fact the spectra look altogether more promising.



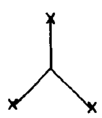
The leading meson trajectories resemble the ρ and π in that they are separated by ½ unit and that the spin 0 state on the ρ trajectory has decoupled. The leading fermion trajectory is parity doubled except for the massless ground state. It seems likely that the MacDowall symmetry would always be satisfied in this way with low lying states parity singlets.

The gauge properties of the fermion emission vertex lead to a supergauge[42,44] structure in the meson sector, originally discovered independently,[45] which play an important role in the construction of the four fermion amplitude by summing over meson exchanges[46]. The snag is that the physical state projection operator must be used to prevent the exchange of spurious mesons. The result of some quite involved calculations[47] is that the four fermion amplitude is dual:



but that the resultant t-channel meson spectrum differs from the s channel in that there is a parity change on those trajectories like the π which had half integral intercept.[48]

The interpretation of this result is not entirely clear. One possibility is that the dual fermion is really a dual quark consisting of a string with a point quark at one end. That end emits $q\bar{q}$ mesons and the other 0q mesons or gluons. In the figures the ends with and without the quarks are indicated by solid and dotted lines respectively. The baryons should then be constructed by tying together the quark free ends of three dual quarks[48], but noone has succeeded in doing this.



It is also interesting that this fermion dual model appears to possess a chiral ($\gamma 5$) symmetry at least for the ground states but probably for all[49]: the π and σ form a parity doublet with identical couplings to a massless fermion. This is a σ-model in which the chiral symmetry is spontaneously broken by the $\pi\sigma$ doublet being tachyonic. Unfortunately with present techniques it is not possible to translate the fermionic dual model to the true vacuum but it is an attractive possibility that some of the ills of the present theory are due to a misinterpretation of this kind.[50] At least the fermion model unlike the non-fermionic one does automatically provide a spontaneously broken symmetry.

Another possible interpretation (a weak interaction interpretation as opposed to a strong interaction interpretation?) is to impose a generalised Weyl equation on the dual fermions. (This is compatible with the subsidiary condition since $m = 0$). Then the fermions decouple from all the meson trajectories with half integral intercept and the four fermion amplitude is dual having identical spectra and couplings in s and t channels.[51] This model is tachyon free but the price is parity violation.

I should emphasise that the mathematics behind the four fermion amplitude is very complicated and has been derived by two independent but presumably equivalent routes, via oscillators and via strings.[49] The first approach makes the Lorenz covariance[46,47] more evident and the second the duality.[49] It would be nice to synthesise the best features of each approach.

8. Conclusions

Amongst the problems left we list

i) tachyons

ii) $D = 26, 10$

iii) no currents

iv) renormalisation

v) baryons?

vi) ?

Possible solutions to (i) and (ii) have been mentioned spontaneously broken symmetries, also Higgs mechanisms and the incorporation of internal degrees of freedom. There is a suggestion that (iii) will be solvable when i) is solved and the intercepts acquire their physical values.[52] In fact there **exist** ideas to solve all the problems mentioned, the problem is to find the mathematical technique with which to realise the idea. Historically dual theories have always been like this, there have always been mathematical barriers which one by one have been breached only to reveal new obstacles. One must therefore be patient, dual theories are complicated but hopefully this is the complication of physical reality - witness the hadron spectrum. The dual theories are very rich; remember that general relatively is a special case with new results emerging after 50 years, yet progress has really been rapid since Veneziano wrote down his beta function 6 years ago.

Finally I shall mention what, on the evidence of the last year will, be the future trends

1) reinterpretation of the existing model (by translating the vacuum and/or incorporating more internal degrees of freedom)

2) Modification

3) generalisation to consider new extended objects (e.g. the MIT bag)

4) solution of a conventional, probably gauge field theory to find the dual theory as a certain approximation.[5]

I wish to thank Edward Corrigan for his help in preparing this review.

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