Integrated Analysis Procedure of Aerospace Composite Structure

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Abstract

The emergence of composite material application in major commercial aircraft design, represented by the Boeing 787 and Airbus A350-XWB, signals a new era in the aerospace industry. The high stiffness to weight ratio of continuous fiber composites (CFC) makes CFCs one of the most important materials to be introduced in modern aircraft industry. In addition to inherent strength (per given weight) of CFCs, they also offer the unusual opportunity to design the structure and material concurrently. The directional properties (and the ability to change these properties through the design process) of composite materials can be used in aeroelastically tailored wings, the fuselage and other critical areas. Due to the longer lifecycle (25-30 years) of a commercial airliner and the tools and processes developed for the airplane of previous product development cycles, new technology often ends up being deployed less effectively because of the mismatch in the technical potential (what can be done) vs. design tools and processes (what was done before). Tools and processes need to be current to take advantage of latest technology, and this thesis will describe one possible approach in primary composite structural design area using integrated structural analysis.

Acknowledgements

Started in 2002 as a student of SDM certificate program, this journey has been long and lonely, but equally satisfying, and a once in a lifetime opportunity. I would like to thank everybody whom I had privilege to meet, share knowledge, and most importantly, share time with. Completion of this thesis means closure of my long education saga in USA, started in fall of 1994 as a master and Ph.D. student in aerospace engineering program at University of Michigan, Ann Arbor, but also as a new career start of a system architect. Insight learned during SDM program cannot be described in one sentence, neither is it easy to define. However, SDM disassembled the entire technical knowledge in my area and completely rearranged it in such a way that I can see the problem more holistic way and attack any kind of difficult problem in a systematic, integrated and framework based view. The impact is profound and hard to describe. I can only thank SDM so much and value the many precious encounters I had during this 6 year period.

Paramount appreciation goes to my certificate program director, advisor, now thesis advisor, and SDM program director Mr. Patrick Hale, without whom I would have never been able to start this program, let alone completing this thesis. My professor in Ann Arbor, Professor Anthony M. Waas, was (and is) always there for me whenever I am in trouble and need of help. I owe these two too much in my life, not even sure how to pay this back anymore.

Eternal thanks goes to my friends at Pratt & Whitney, especially Mr. Paul Rembish, Mr. James Tagg, and Mr. Logan Do, without them I would not be where I am today. My mentor, Mr. Cliff Chen at Boeing, has helped me in every possible way throughout my tenure at Boeing, and I thank all of them with greatest appreciation. I am grateful to my precious wife Moonjung and daughter Yueun, who have been endlessly patient and supportive throughout my journey. My Parents in Korea were always there for me throughout entire journey in USA, and I owe everything to them. Finally, I give greatest thanks to God, who guided me throughout my entire life, as well as 14 years of education in USA.

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CHAPTER 1

INTRODUCTION

1.1. Introduction

Continuous fiber reinforced composite laminates offer several superior attributes when compared to metals on a pound for pound basis. Because of this, these laminates are increasingly utilized in weight critical aerospace applications. Although the utilization of composite laminates in structural application is relatively recent, the concepts and basic ideas that are central to the notion that a composite material exhibits superior properties than the constituents by themselves are as old as the straw-reinforced clay bricks in ancient Egypt.

In more recent times, iron rods were used to reinforce masonry in the nineteenth century, leading to steel-reinforced concrete. Phenolic resin reinforced with asbestos fibers was introduced in the beginning of the twentieth century. The first fiberglass application was made in 1942, and reinforced plastics were also used in aircraft and electrical components. Filament winding was invented in 1946 and incorporated into the manufacturing of missiles applications in the 1950s. The first boron and high strength carbon fibers were introduced in the early 1960s, with applications of advanced composites to aircraft components by 1968. Metal matrix composites such as boron/aluminum were introduced in 1970. DuPont developed Kevlar (aramid) fibers in 1973. Starting in the late 1970s, applications of composites expanded widely to the aircraft, automotive, sporting goods, and biomedical industries.

Continuous fiber composites (CFCs) are one of the most important materials to be introduced into aircraft structures in the last 30 years. CFCs consist of strong fibers set in a matrix of epoxy resin that is mechanically and chemically protective. They were developed at the RAE Farnborough and announced in 1966. Not only do CFC's possess excellent strength/weight and stiffness but also they offer the unusual opportunity to design the structure and the material simultaneously. The directional properties of composite materials can be used to aeroelastically tailored wing structures in order to obtain, under load, specified twist and camber. This has beneficial effects on aerodynamic drag, control effectiveness and air load distribution, leading to increases in range capability and load carrying capacity. Such tailoring can be used to obtain a lower weight design that satisfies all of the applicable design constrains such as strength, flutter and divergence. Compared to 2000 and 7000 series aluminum alloys, CFC's offer weight savings of 20%. A further advantage is the ability to mould complex shapes.

Still, CFC material remains expensive and requires labor intensive structural fabrication methods. Further drawbacks include significantly reduced strength when there is undetected damage, reparability problems, and environmental difficulties. The first major application of CFC was demonstrated in the design of AV-8B Harrier II by the then McDonnell Douglas (Boeing) and British Aerospace. It took about 10 years to get CFC's into the production cycle.

Carbon fiber based CFC's have been used extensively in recent aerospace applications. Many airplane surface components are being replaced by CFC material except in primary load bearing members (landing gear, main spar), or thermal resistance members (engine mount, nozzle, firewall, etc). The most aggressive application of composite structure in an aerospace vehicle can be seen at Scaled Composite Corporation [Ref. 1.7], where all composite vehicles are being developed and tested. Typical CFC-metal main wing structure along with conventional types is shown in Figure 1.1. Shown (e) is the main wing of the British Aerospace Experimental Aircraft Prototype. Dark areas illustrate the use of CFC and light areas show metal usage, including three titanium-made wing attachment joints.

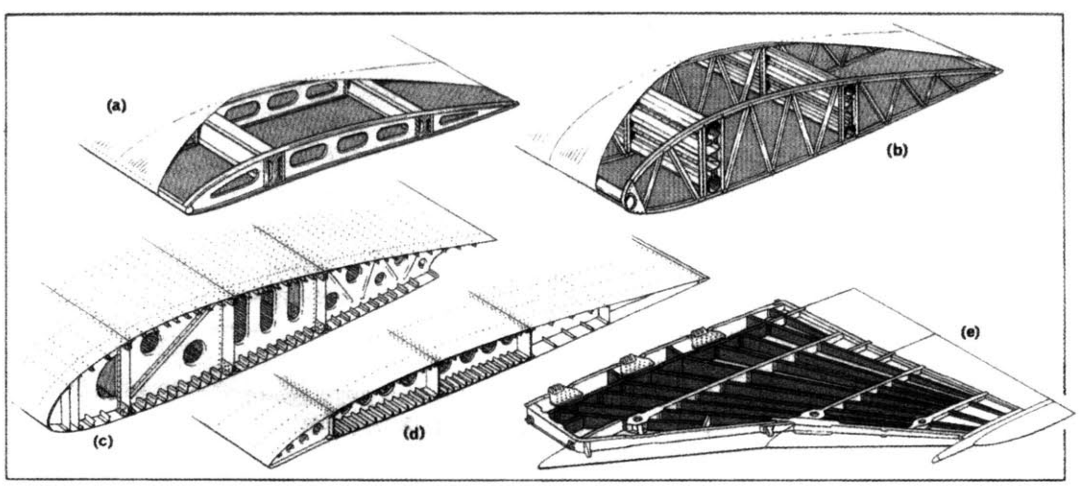

Figure 1.1 Evolution of wing structure in chronological order (a)-(e) (Ref. 1.2)

One particular accomplishment in how the CFC application to structures could be stretched in aero-vehicle design was the Boeing-Sikorski RAH-66 Comanche helicopter (Figure 1.2). In addition to conventional composite application to save weight in secondary, non-load-bearing structures, the Comanche airframe had load-bearing members made of Hercules IM7 graphite in thermosetting epoxy resin.

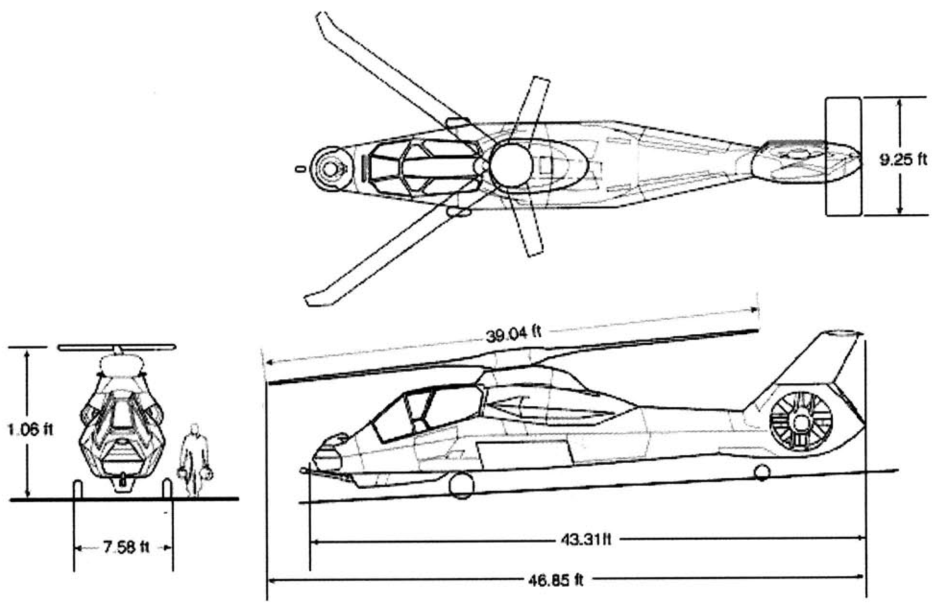

Figure 1.2 RAH66 Comanche (Ref. 1.2)

The RAH-66 was built around a composite box (Figure 1.3) beam running the length of the forward fuselage. The beam also provides space for the fuel. Composites also had opened new opportunities for crashworthy design. Cockpit floors had frangible panels to let the crew seats stroke down in a crash, and the entire tail boom was designed to break away when impacts greater than 20 ft per second occur, to relieve crash loads on the retractable landing gear. The main rotor was an all-composite bearingless design.

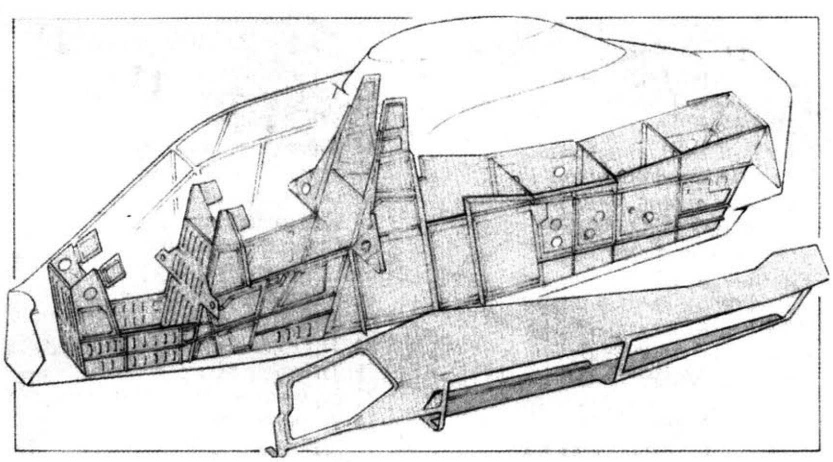

Figure 1.3 CFC Fuselage member of RAH66 (Ref. 1.2)

In the commercial aircraft industry, composite application in primary load bearing structure has been spearheaded by gas turbine engine industry. The latest GeNX engine for the 787 Dreamliner has composite fan blades as well as a composite fan case (Figure 1.4), and airframe companies are not far behind in taking advantage of composite material. Figure 1.5 and 1.6 are showing various composite applications in their latest aircraft design (A-380 & 787).

Figure 1.4 GeNX Composite blades and fan case (Ref. 1.3)

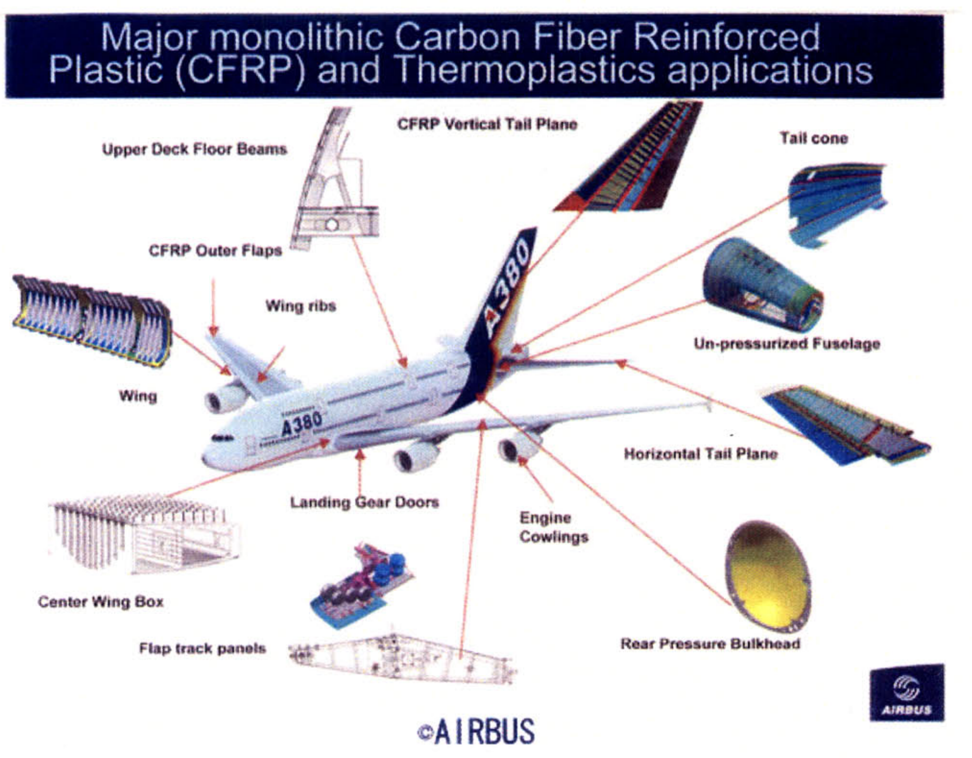

Figure 1.5 A-380 Composite Applications (Ref. 1.4)

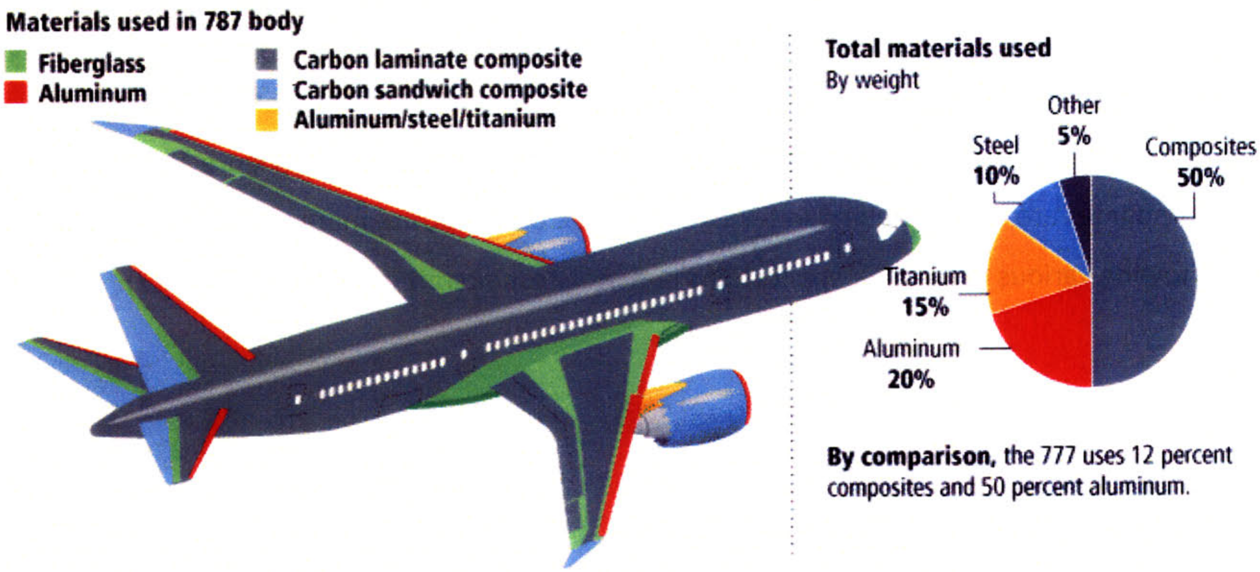

Figure 1.6 787 Composite Applications (Ref. 1.5)

In this paper, the overall design process of aircraft systems will be investigated to lay the foundation for the following research work in Chapter 2. Chapter 3 will present a generic composite material overview. Chapter 4 will discuss in detail the current design process for aircraft structure and 'proposed' design and analysis procedures for composite primary structures. Chapter 5 will discuss on organizational perspective to achieve tighter design integration team followed by example case. Chapter 6 includes a summary of the contribution from this thesis, with recommendation for future investigations.

1.2. References

1.1 Issac M. Daniel and Ori Ishai, 1994, "Engineering Mechanics of Composite materials", Oxford University Press.

1.2 Ray Whitford, Fundamentals of Fighter Design, 1999,"Air International", Key Publishing

1.3 http://www.aviatorchat.com/forum/attachment.php?attachmentid=1764&stc=1&d=1179441137

1.4 http://www.carbonfiber.gr.jp/english/tanso/use01.html

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CHAPTER 2

THE COMMERCIAL AIRLINER INDUSTRY AND PRODUCT DEVELOPMENT PROCESS

2.1. Aircraft Design Process

The commercial airliner product development process generally takes about 48 months from the authority to proceed (ATP) to initial delivery. The general process of overall product design is shown in Figure 2.1.

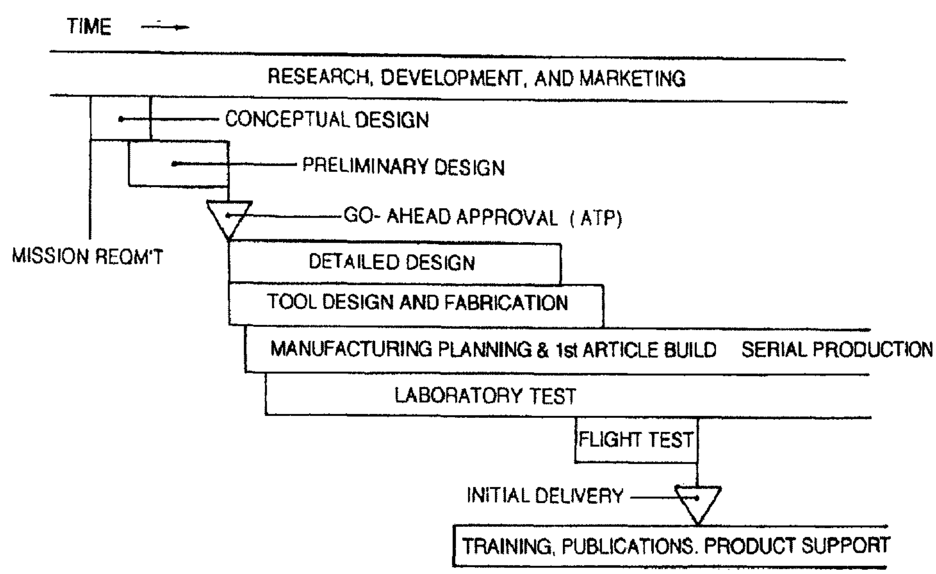

Figure 2.1. Aircraft Product Development Process (Ref. 2.3)

Prerequisites for product development are ongoing R&D and marketing (economics) activities. Especially for the commercial airliner industry, economic validity of new product to a target market segment is such a crucial aspect of a delivering a successful new airplane (which requiries maintaining a continuous dialog with airline planning in order to keep current regarding projected future market needs) that it will be discussed in separate section of this chapter. R&D activities in important technological areas such as aerodynamics, structures, materials, propulsion, avionics and integration of the aircraft as a system are categorized according to their 'technical readiness' for insertion of new technologies and decisions are made for incorporation in a new aircraft design according to their 'maturity level'.

Mission specification is a statement of the basic performance objectives and related criteria which should be met by the new design. The mission specification consists of the following typical information;

1. Objectives of the aircraft
2. Design payload, range and radius
3. Normal cruise / maximum speed and normal operational altitude
4. take off / landing distance at maximum weight
5. direct operating cost / flight
6. airport noise levels

Mission specifications can come from difference sources, for commercial airliner industry, they come directly from an airline in collaboration with aircraft manufacturers, where internal studies of future operation creates new concept (mission) for new aircraft. Once mission specification is frozen, the starting point of designing new aircraft is a design mission specification with representative mission profile.

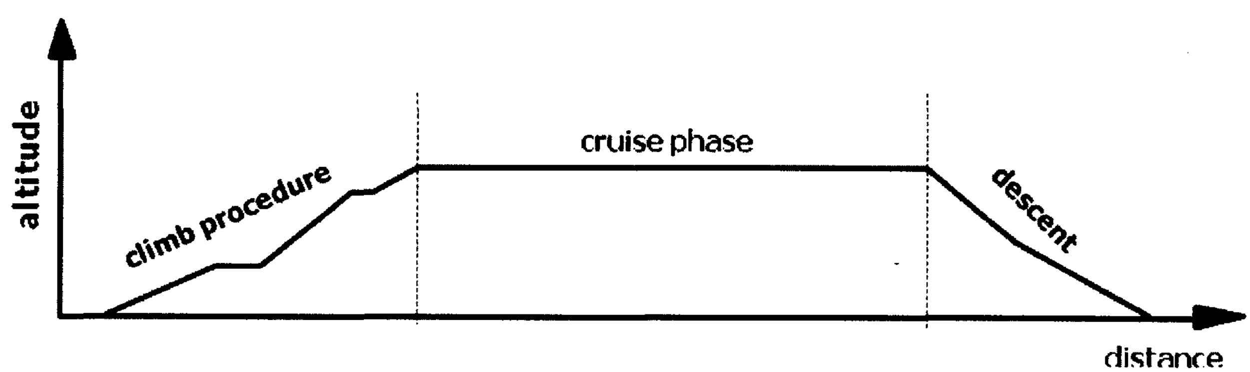

Figure 2.2. Mission Profile for Commercial Aircraft (Ref. 2.3)

Purpose: Competitive alternative to MD-90, A-320, and B 737-400

Payload: 150 passengers at 175 lbs each plus 30 lbs of baggage each

Crew: 2 pilots at 175 lbs each plus baggage at 30 lbs each
3 cabin attendants at 130 lbs each plus baggage at 30 lbs each

Range: 1500 nautical miles

Reserve fuel: 150 nautical mile flight to alternate, followed by 45 minute loiter

Cruise Altitude: 35,000 feet

Cruise Speed: Mach number = 0.82

Climb: Direct to 35,000 feet at maximum takeoff weight

Takeoff Field Length: 5.000 feet at sea level, 90°F, at maximum takeoff weight

Landing Field Length: 4,500 feet at sea level, 90°F, at maximum landing weight

Powerplants: 2 Turbofans

Pressurization: 5,000 foot cabin altitude at 35,000 feet

Certification Basis: FAR Part 25

Mission Profile:

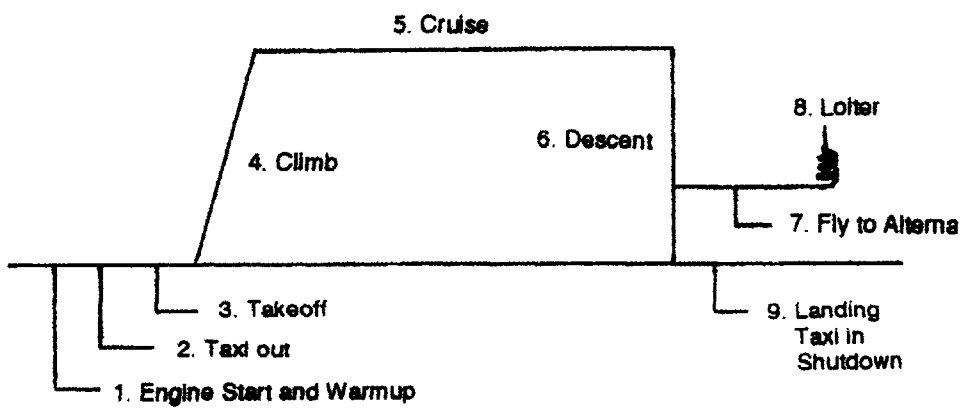

Figure 2.3. Design Mission Profile for Commercial Airliner (Ref. 2.3)

The resulting design mission profile above will become the starting point of conceptual/preliminary design for a new airplane. After the basic requirements have been set, it is in the conceptual design phase that the basic questions of configuration arrangement, size and weight, and performance are answered.

Conceptual Design Phase answers following questions;

1. Will it work?
2. What does it look like?
3. What requirements drive the design?
4. What trade-offs should be considered?
5. What should it weigh and cost?

Each time a design iteration is analyzed, design parameters such as gross weight, fuel weight, wing size, engine size and overall arrangements should be refined and updated. The preliminary design phase starts when the major changes are not seen in succeeding iterations. The basic configuration arrangement can be expected to remain stable. During preliminary design, the designers in areas such as structures, landing gear, and control systems will design and analyze their portions of the aircraft. Testing is initiated in areas such as aerodynamics, propulsion, structure, stability and control. A key activity during preliminary design is lofting, which is the mathematical modeling of the outside skin of the aircraft with sufficient accuracy to insure proper fit between its different parts. The ultimate objective of preliminary design is to ready the company for the execution of detail design stage (full scale development proposal: ATP), with confidence that major issues have been exposed and settled prior to the major investments in full scale development.

Preliminary Design output and boundaries;

1. Configuration (architecture?) Freeze
2. Lofting Development
3. Test and Analysis Baseline
4. Design Major Items
5. Cost Estimates

The economics of a new aircraft for a specified mission profile are a key factor for successful design. Next chapter will describe the basic concepts of commercial aircraft economics.

2.2. Economics of Commercial Aircraft

An airliner's **Productivity** is defined as its capability to produce useful and profitable transportation in a specific operational situation. The productivity can be expressed in revenue dollars per seat-km, passenger-km, and ton-km.

Productivity = Payload (Revenue) x Block Speed x Block Hours

Block speed is average speed for the block distance. A common trend is that total productivity is reduced for short flight, when each of the three factors suffers as stage lengths become shorter.

Utilization refers to the time that an aircraft is employed in revenue flights. It is not related to how well the capacity of aircraft is filled in with revenue passengers (load factor). It may be poor managements to have all flights with high utilization but low **Load Factors**. Flight scheduling plays a central role in the optimal use of an airline's resources. For each segment to be flown, scheduler must consider how large a market is expected and how it will fluctuate by the day of the week and hour of the day. Aircraft must be scheduled to end its day's flying at the point of origin of the next day's flight, and that pattern must bring the aircraft into maintenance shops on a predetermined schedule. Aircraft depreciation is severe, and it applies whether the aircraft is flying or idle. At the end of route it often has to lay-over, because passengers want departures and arrivals during certain periods of the day. Experience shows 9-10hr utilization per day over 25-year lifetime of a long range airliner is good.

Passenger Load Factor is the measure of an operator's skill (aircraft choice, pricing, pricing and general service quality). Load Factor is the percentage of available seat-km converted to revenue paying passenger-km (US cent/seat-km). The break-even load factor must be exceeded at the end of the year if the aircraft to be profitable. Because profitable passenger load factor alone is not enough, and most of commercial airliners carry some cargo/mail, the total unit of production is often described in terms of capacity tonne-km.

Fares may be divided into two concepts; one is the 'value' of the service, the other is the 'cost' of the service. The value of the long range flight is greater than short one, and business travel is more valuable than tourist flight, and business flight is less sensitive to fare change.

With all above variables defined, it is possible to convert productivity into actual revenue required to overcome operating cost. The breakup of cost is in table 1.

Direct Operating Cost (DOC)	Indirect Operating Cost (IOC)
<u>Fixed Cost</u> 1. Interest 2. Depreciation 3. Insurance (2 ~ 5%) <u>Variable Cost</u> 1. Fuel (30 % ~ 50 %) 2. Crew (2 ~ 5%) 3. Maintenance and Overhaul (~15%) 4. Landing and Maintenance Fees (~11%)	1. Airport / baggage handling fee (~20%) 2. Passenger service (~ 10%) 3. Ticketing & Sales (~15%) 4. Administration (~5%)

Table2.1. Total Operating Cost (Ref. 2.1)

Since the appearance of the jet transport, the impact of technological advances on operating costs has come not from faster aircraft but from efficiency improvement through increased aircraft size and higher bypass ratio turbofan engines. Improvement of airframe and engine technology extended the period between maintenance and overhauls. Improved reservation systems, maintenance equipment and other airplane operations and customer service related technologies has also reduced the operations costs of airline industry.

Selection of new aircraft (types and numbers) by airliner industry is more delicate art than science. Out of many considerations to make a selection of new plane, the representative 5 factors are; price, performance, after-sales support, residual value and transition costs. Performance characteristics must be matched against the carrier's existing and future routes, the stage lengths and the projected flow of current and future traffic. The aircraft must meet the airworthiness standards with respect to safety, noise, and air pollution, and should have passenger appeal. Fleet planning models, prepared by airlines' research personnel with the airframe manufacturer, should answer the following question at the end of day; how many seat-km will it deliver per kg of fuel? This is the decisive factor in selecting new aircraft, since most other factors will lie in a narrow band for any good design.

2.3. References

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2.2. Daniel Raymer: Aircraft Design: A Conceptual Approach

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CHAPTER 3

CHARACTERISTICS OF COMPOSITE MATERIALS AND IMPACT ON AIRCRAFT DESIGN

3.1. Impact of New Materials in Aerospace Application

Weight saving through increased specific strength or stiffness is a major driver for the development of materials for aircraft structures. A crucial issue in changing to a new material, even when there are performance benefits such as weight savings to be gained, is affordability. Affordability includes procurement cost and life cycle support cost (ownership, maintenance and repair). Thus the benefits of weight savings must be balanced against the costs.

• Weight Reduction - increased range - reduced fuel cost - higher payload - increased maneuverability	• Improved Performance - smoother, more aerodynamic form - special aeroelastic properties - improved damage tolerance
• Reduced Acquisition Cost - reduced fabrication cost - reduced assembly cost	• Reduced Life Cycle Cost - improved fatigue and corrosion resistance - improved damage tolerance

Table 3.1 Drivers for Improved Material for Aerospace Application (Ref. 3.1)

The cost benefit on weight savings is particularly sensitive in aerospace applications; 1 % of saving on empty weight usually generates about 5 % or so maximum takeoff weight (MTOW) savings, which is directly proportional to overall life cycle operating cost. Approximate values that may be placed on saving 1 kilogram of weight on a range of aircraft types are listed in Table 3.2.

• Small Civil: \$80 • Civil Helicopter: \$80 - \$200 • Military Helicopter: \$400 • Large Transport: \$300 • Large Commercial: \$500	• Advanced Fighter: \$500 • VTOL: \$800 • SST: \$1500 • Space Shuttle: \$45,000

Table 3.2 Approximate Actual (US \$/kg) Values of Saving One Unit of Weight: (Ref. 3.1)

In choosing new materials for airframe applications, it is essential to ensure that there are no compromises in the levels of safety achievable with conventional alloys. Retention of high levels of residual strength in the presence of typical damage for the particular material is a critical issue for (damage tolerance). Durability, the resistance to cyclic stress or environmental degradation and damage through the service life is also a major factor in determining through-life support costs. The rate of damage growth (for example, crack propagation) and tolerance to damage determine the frequency and cost of inspections and the need for repairs throughout the life of structure.

3.2. Carbon Fiber Composites (CFC)

CFC is comprised of strong, high-modulus small diameter fibers set in a matrix of epoxy resin that is mechanically and chemically protective. The fibers provide the basic strength, while the matrix stabilizes the fibers and acts to redistribute the load in shear between fibers in the case of fiber failure. At the level of design strains for these materials (~ 0.4 %), fatigue is not a problem, and designs are based on their static properties. CFC offers weight savings of 20% or more even when allowances are made for hot/wet conditions and notch effects, compared with 2000- and 7000- series aluminum alloys. However, the resulting structures have been much more expensive than their metal counterparts, due in part to the expensive raw material and the fact that the major emphasis is on maximum weight reduction. To accomplish this objective the design approaches have concentrated on structural simplification, reduced part count and the elimination of costly design features. The ability to mould complex shapes reduces waste material and reduces the parts by a factor of three, thereby reducing joining costs.

For composites to become more competitive with traditional aluminum alloys, the costs of using them must drop significantly. Central to cutting those costs will be improvements in maintainability, reliability and reparability. The performance benefits can be outweighed by the higher cost of manufacturing and maintenance in the field. They have a better initial service record, mainly because of their corrosion resistance and fatigue properties, but composites are more prone to impact damage, the economic repair of which has typically been limited to minor damage. Current evidence shows that repair costs for composite structures can exceed those for conventional metal by a factor of at least two. Parts with substantial damage must be replaced, with the cost and out-of-service time for such work, which, combined with special facilities required, makes major repairs impractical. In the event of the need to replace entire items with significant damage, the reinvestment required to replace the damaged item does not appear to offset the relatively small fuel-burn reduction. Environmental consideration in disposal of carbon fiber components and fire hazard also come into equation.

Characteristics	Composite	Metal
Fatigue	Much better than metals	Problems
Corrosion	Much better than metals	Problems
Load/Strain relationship	Linear strain to failure	Yield before failure
Failure Mode	Many	Few
Transverse Properties	Anisotropic (weak)	Isotropic (same)
Notch Sensitivity/ Static Fatigue	More sensitive/less sensitive	Sensitive / Very Sensitive
Mechanical Properties Variation	High, in compression/transverse direction	Normal
Sensitivity to hygrothermal environment	Sensitive to hot/wet condition	Less sensitive
Through-thickness crack growth	Growth/no growth	Slow growth
Delamination	Problem	No problem
Initial and in-service flaw/damage size	Not well defined	Defined
Damage inspectability	Problem	Adequate

Table 3.3. Composite vs. Metal (Ref. 3.5)

3.3. Impact on Aircraft Design

The use of composites has a significant impact on the aircraft design process. Metal parts start as a solid piece, then are machined down to a specified size, shape and thickness. Multiple parts are fastened or riveted together to form structure. Using composite, a designer has much greater flexibility because the strength and stiffness of structures can be tailored. The material can be stacked in various layup angles to tailor thickness and stiffness according to design requirement of specific parts.

To increase strength or stiffness in a localized area, a larger number of plies may be overlaid, each with a different shape and orientation. Tailorable strength enables designers to optimize aerodynamics such as in forward-swept wing aircraft design. By manipulating the anisotropic nature of composite material, local stiffness/strength can be tailored to meet the specific requirement of aeroelasticity (vibration, flutter).

Fiber reinforced thermoplastic composites produced by molding CFCs to complex shapes under high pressure are highly resistant to damage and can be reshaped and quickly fabricated. Compared to carbon epoxy, fiber-reinforced thermoplastics are equal in density, equivalent in strength and part production is less expensive. Other area where composites have a significant advantage over metallic structures is in radar cross-section reduction. Aircraft can be formed with smoother lines, fewer areas where different materials merge and into the complex shapes required for reduced RADAR signature.

Arrival of the 787 in the commercial aircraft industry has established a new standard in terms of composite application in primary structure and a full composite fuselage in segmented barrel structure brings composite application to about 50% of airframe weight. Although it has already been an early success as an aircraft program, the long-term success of the program will depend on validated cost savings in maintenance and overall operating cost. It is worth noting that the Airbus counterpart A350XWB is not following the same design path chosen by Boeing for a composite barrel fuselage, but is using an evolutionary approach in designing a section-based metal/composite hybrid fuselage and new materials (above 60 %), promising 30 % reduction in maintenance cost.

Overall, the composite material technology is still relatively young (the first CFC was announced in 1966 by RAE Farnborough), and the properties achieved so far are modest compared with theoretical full potential. The general lack of sufficient toughness and damage tolerance is still a major problem for most of composites. The improvements in resin material, fiber material, fiber/matrix architecture and general design process (in next chapter) are continuing in all aerospace fields.

3.4. References

1. Ray Whitford; Evolution of Airliner
2. Daniel Raymer: Aircraft Design: A Conceptual Approach
3. Roger D. Schaufele: The Elements of Aircraft Preliminary Design
4. Klaus Huenecke: Modern Combat Aircraft Design
5. Ray Whitford: Fundamentals of Fighter Design

CHAPTER 4

INTEGRATED ANALYSIS PROCESS OF AIRFRAME COMPOSITE STRUCTURE & ORGANIZATION ASPECTS

4.1. Integrated Design Process using Continuous Load/Design Refinement

In previous chapters, effect of composite material on traditional aircraft design was described. In this chapter, a detailed process changes for composite structural design and analysis will be explained, followed by an example of composite bolted joint analysis in the next chapter. As discussed in chapter 3, having composite material as a design object calls for an integrated, multi-disciplinary perspective. Material no longer arrives in given, pre-existing condition and properties, but becomes a variable of structural design itself. Starting from simple lamina plies, the design process includes tailoring the composite material itself by controlling number of plies, ply angle, selection of fiber and matrix as well as general optimization of structural design to meet structural requirement of components, sub-systems, and the entire system (durability, strength, weight under loading requirement).

The traditional aircraft design process started with aeroelasticity studies (after requirements definition, general configuration, and initial sizing process) for aerodynamic loading analyses to define the overall loading requirement (rigid aerodynamic body and/or simplified [beam and plate] compliant aircraft model based), and this aerodynamic loading (external loading in general term) definition is mapped into aircraft system structural models (internal loading) to define loading of system/sub-system (fuselage section, wing section, landing gear, etc) structures. Once this loading data requirement is defined, each design team's leader initiates the design process to satisfy design target weight while meeting all loading requirements (strength, toughness, durability, fatigue, corrosion etc, Figure 4.1).

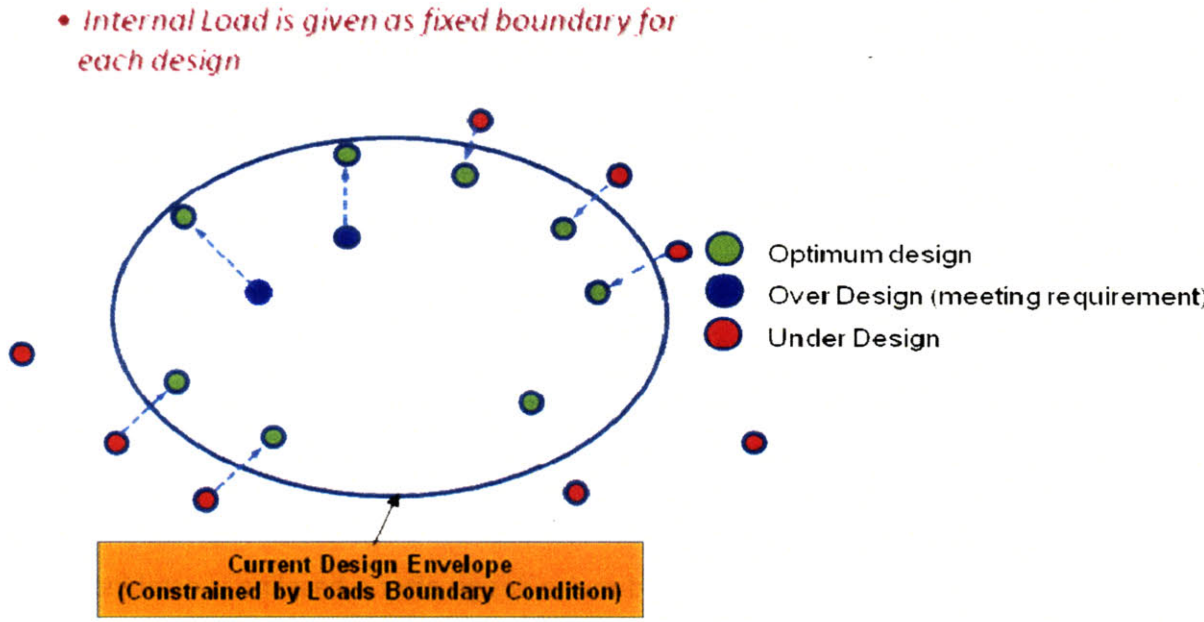

Figure 4.1. Traditional Structural Design Process

Figure 4.1 indicates a typical design process employed to meet the requirements. Optimum, over, and under design are the status of the initial design points (on target, overweight, and underweight). All the design effort is to be on target (arrow) while meeting cost and schedule. There may be a requirement refinement (more accurate loading refinement, typically lowering the initial study requirements 1-2 times before flight testing), but largely the requirements are a fixed target and each design evolves given that target requirement. This approach has been generally acceptable because of the nature of the module based assembly process of metal airframe structure (bolted assembly and system), and experience accumulated throughout metal aircraft design history (large commercial airliners -- around 50 years since De Havilland Comet). Weight reduction (design improvement) effort can be incorporated later stages relatively easily by revising the design, maintaining the interface (bolted/riveted joints), and inserting the improved design features into the system.

This approach presents a particular problem in designing composite aircraft, where the design configuration is integrated in nature (fewer components, bigger, integrated one-piece components). The conventional 'Let's over-design first and cut weight later' type of trial and error approach will not work as well as it does in designing metal aircraft. Previous experiences in metal design mixed with new composites in aircraft have sometimes yielded weight savings far less than hoped for, even to the extent of increases in weight in some extreme cases. From the initial aerodynamic loading calculations, which are done based on equivalent stiffness flutter models (wind tunnel testing, flutter analysis), the correlation relationship (scale factor) gained from existing metal design process will not work, and more unknowns generally calls for more conservative nature of loading estimation. The analyses and models are built around assumptions which do not hold true for composite designs. Therefore, to use these models, more frequent system loading requirement updates during design phase are necessary, as well as more (and more frequent) system design reviews. Composite structure has a lot of advantages over traditional metal design, but it requires more precise analyses and design processes because it is harder to adjust and revise design after a baseline is established due to the high interdependencies of the composite structural properties. Trial and error would work only given unlimited time and resources, at the expense of cost and weight penalties.

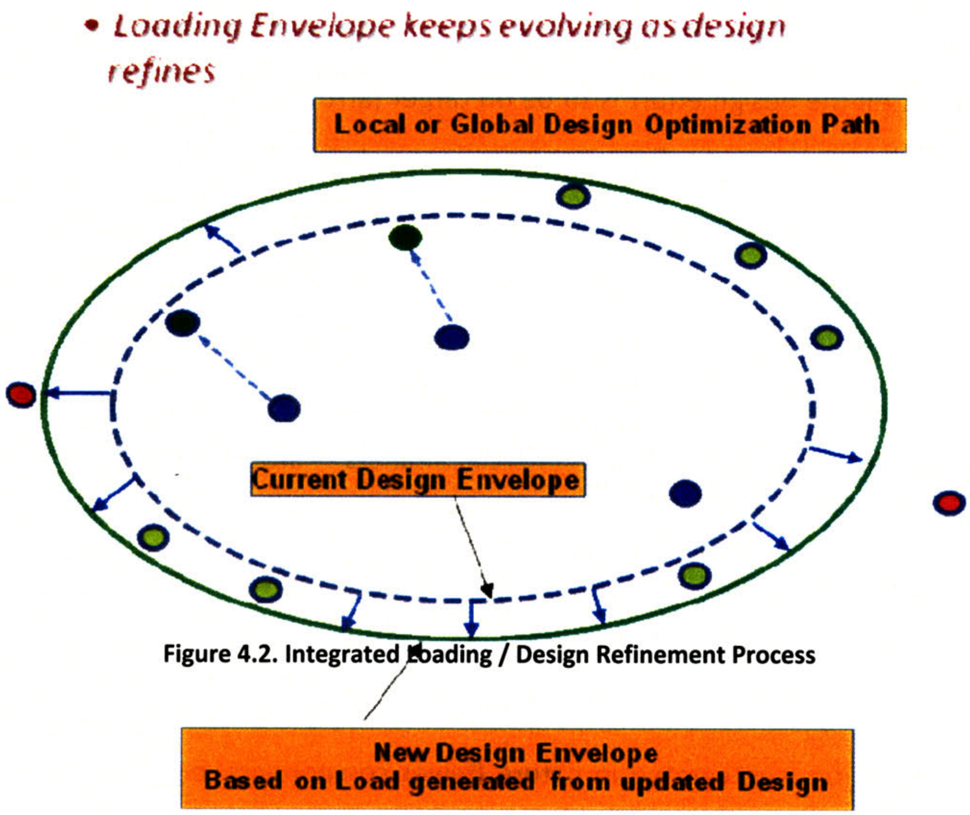

Figure 4.2. Integrated Loading / Design Refinement Process

As shown in Figure 4.2, continuous (or at the least more frequent than before) load revision as design evolves, and concurrent design refinement with updated loading, would minimize overdesigning the system under unrealistically high loading requirements. Overdesigned components can be further optimized given updated loading condition (usually lower than before), and designs not meeting original loading requirement will have more room to improve than before.

Current design iteration efforts (at the IPT level) should be able to communicate to the loads group (external/internal load team) to refine loading based on new design information (Figure 4.3), represented by higher resolution structural analysis models.

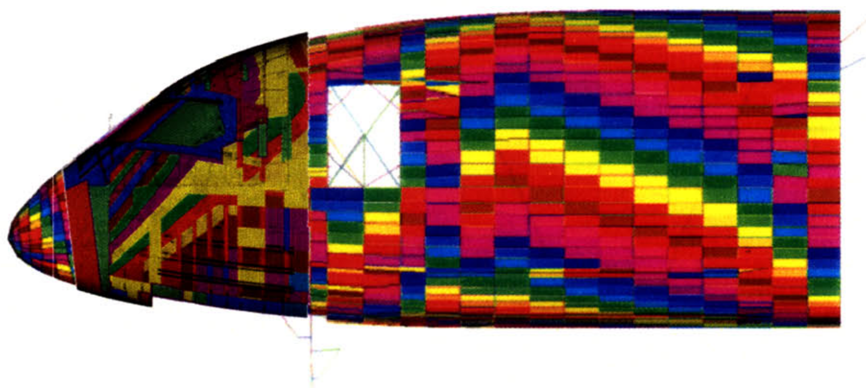

Figure 4.3. New Load Regeneration Model using updated Design

Usually, the more design information is available, the mapped external load to internal loading model yields lower requirements because of the compliant nature (more information as design evolves) of higher resolution models (Figure 4.4). After completing first pass design based on original loading, it can be further optimized if the process of using re-mapped internal loading based on latest design at the IPT level is applied. The ideal scenario would be that the loads group is more tightly involved in module-level IPT, providing the latest internal load information as design milestone (preliminary-phase 1-phase 2-final design etc) and following the IPT schedule, instead of working as independent organization. In this scenario, changes to the loading information flow down less frequently.

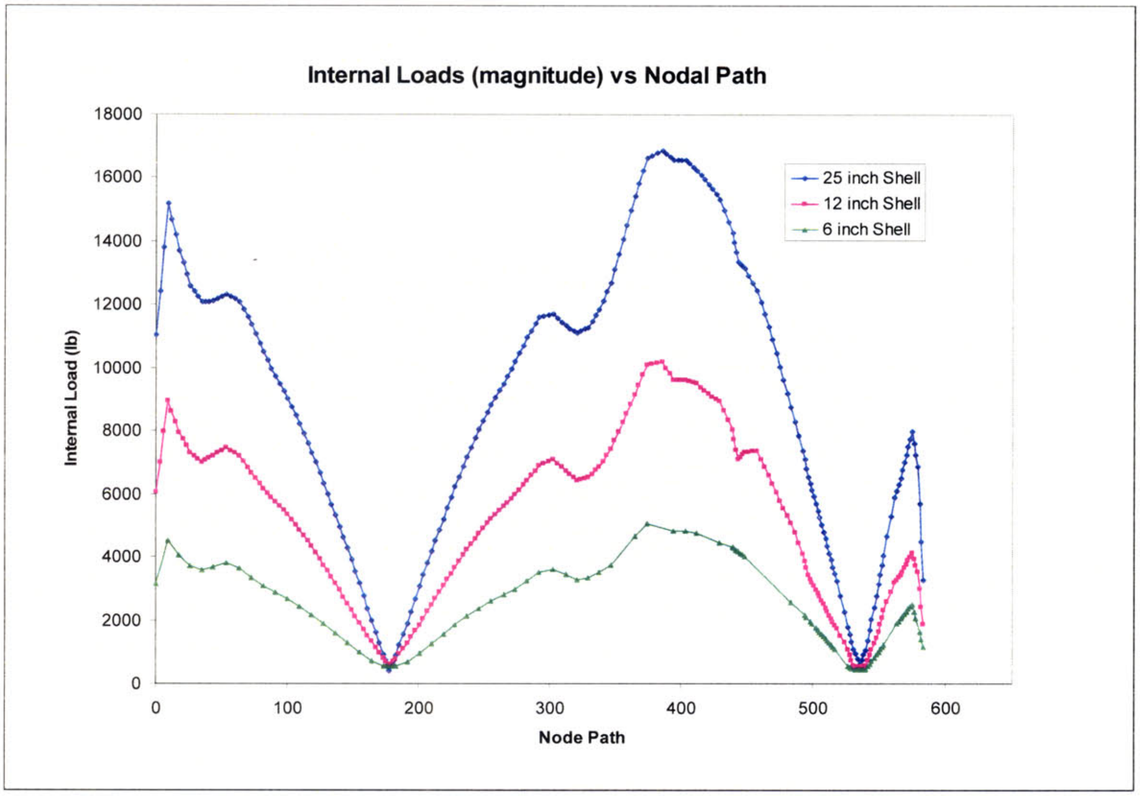

Figure 4.4. Loading Reduction as Design Evolution

The process of improving design by employing concurrently refined loading information needs to be established to optimize design during component level design stage instead of waiting for updated loading flow down later (Figure 4.5)

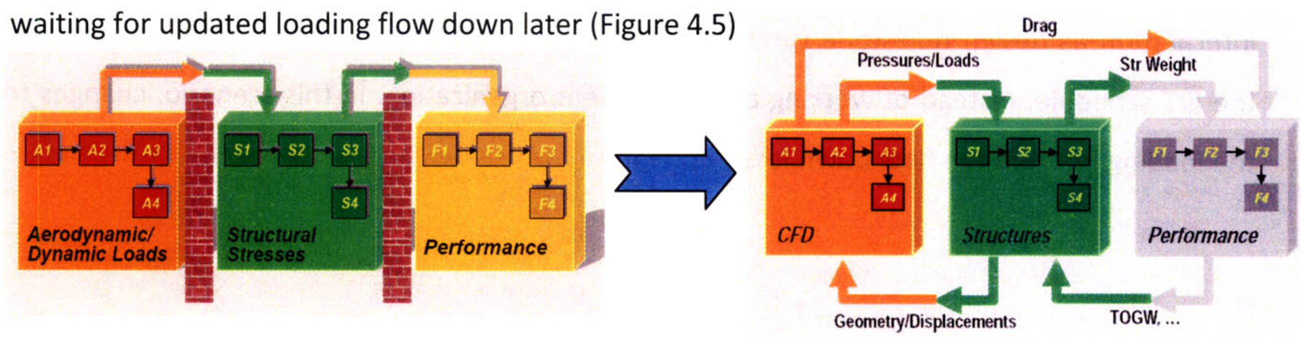

Figure 4.5. Traditional Design Process vs. Integrated Design Process (Ref. 4.5)

4.2. Organizational Perspectives

To have a design team which can perform the tasks describe in previous section, seamless integration between design and structural analysis teams is critical. The nature of integration in composite structure calls for more frequent design-structural analysis interaction in addition to refined loading information. The traditional stress analyst, who assesses 'finished' design based on a 'given' loading condition, in isolation, is not going to be effective, and neither is the designer who works on refinements based on that feedback; both are attempting to apply iterative discrete process to a tightly-coupled, near continuous design evolution. In the conventional scenario, the designer will over-design to pass stress checks, and the analyst will say 'ok' for it during this design pass. Weight will become higher than target, and by following this process, the necessary degree of design improvement cannot be achieved in time.

In a more 'integrated' composite design team, basic stress checks will be performed by designer using integrated structural analysis tools (most of the latest design software includes an integrated stress analysis module), and the structural analyst will perform design optimization under refined loading from the start. Design tools (CATIA, Unigraphics, AutoCad, etc) will import design optimization results from structural analysis tools (NASTRAN, ABAQUS, LS_DYNA, etc), and the resulting continuous update/revision of design with updated loading results in faster, more agile design evolving process. The basic requirement is for the organization to have integrated composite design team -- the following objectives should be achieved:

1. Merging design / structural analysis disciplines (Designer/Structural Analyst becomes one)
2. Analysis tool modernization: when design community is using fully digitized CATIA system, the traditional hand-calculation methodology will not be fast enough.
3. Common models between design and structural analysis communities (or at the least, seamlessly compatible)
4. Team work environment as in "Wolf Pack", not like a "Baseball Team"

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CHAPTER 5

INTEGRATED ANALYSIS APPLICATION (COMPOSITE BOLTED JOINT)

5.1. Introduction

Mechanical fastening is still the primary means of joining multiple components in modern aircraft structures. Smaller, more aggressive technology-driven fields such as military aircraft and UAVs are taking full advantage of integrated nature of composite material using all in one piece structure and bonded joints. However, the first generation all composite fuselage aircraft such as 787 and A-350 XWB are still using mechanical fastening to join major structures (fuselage skin-frame-stringer, wing to body joint, empennage joint, etc), the same as metal-based aircraft, indicating the hybrid, evolutionary nature of airframe design (metal design procedures with metal replaced by composite materials). Current industry design methods are largely based on design charts and stress handbooks. Advanced 3D Finite Element Analysis (FEA) plays a limited role, indicating the technology gap between last major civil airliner development (Airbus A310 (80s), A320 (80s), A330 (90s), A340 (90s) & Boeing 737 (60s), 747 (60s), 757/767 (80s), 777 (90s)), when computation mechanics (FEA) was still relatively new to the industry. Boeing (between 777 & 787) and Airbus (A350 & A380) have around 14 years of development gap since the last major twin aisle airliner development programs.

Based on these facts, development problems currently experienced in the Boeing 787 program (weight, fastener problems and supply chain issues) and A-380 (weight and production) demonstrate how difficult it is to establish new design procedures while developing new product at the same time. In this chapter, most fundamental design procedures for mechanical fasteners in composite structures will be described and proposed new design/analysis procedures will be presented.

5.2. Composite Bolted Joint design

Composite materials, if properly used, offer many advantages over metal alternatives. Examples of such advantages are: high strength and high stiffness-to-weight ratio, good fatigue strength, corrosion resistance and low thermal expansion. Nevertheless, conventional composites made of tape or fabric also have some disadvantages, such as poor transverse properties, inability to yield elastically and sensitivity to moisture and high temperatures, which must be accounted for during design process.

Among the most important elements in aircraft structures in general and in composite structures in particular are mechanically fastened joints. Improper design of joints may lead to structural problems or overly conservative design, leading to overweight and high life-cycle cost of the aircraft. Typical examples of mechanically fastened joints in composite aircraft structures are: the skin-to-spar/rib connections in wing structures, the wing-to-fuselage (main, empennage) connection and attachment of fittings, fuselage stringer frame-to-fuselage skin.

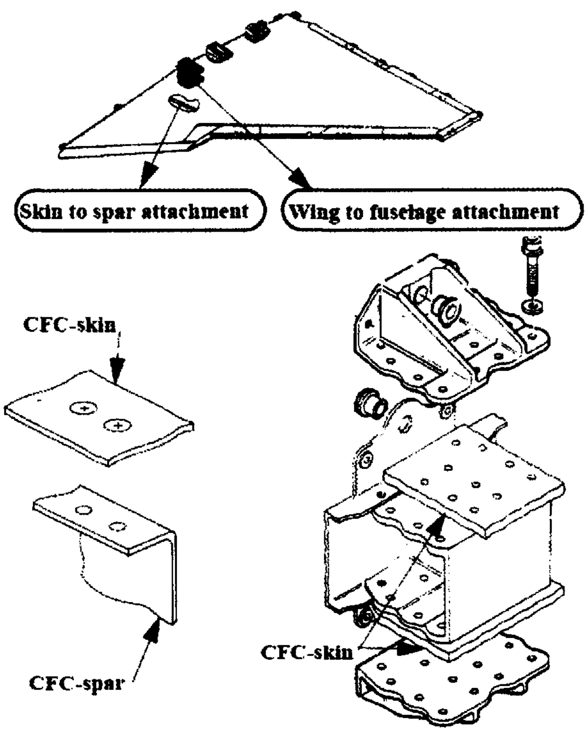
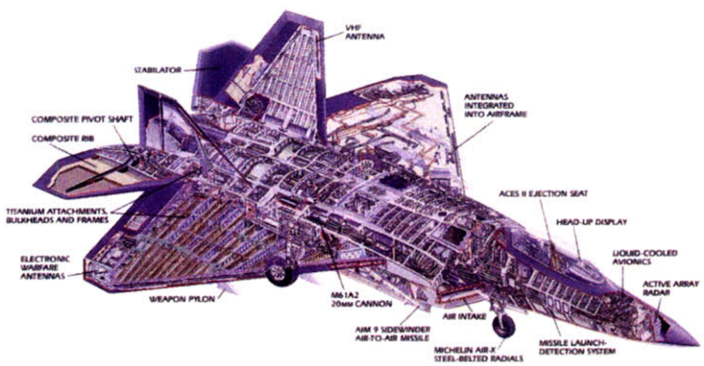


Figure 5.1. Bolted Joint in Wing structure (Ref. 5-1)

All the structural parts are designed to be able to withstand a high level of structural loads in order to provide efficient aircraft design; major structural parts are joined by means of mechanically fastened joints to provide an equally efficient assembly method. Although there are many different joint configurations available, their applications are driven by service requirements applied to the particular structures to be joined. A key advantage of mechanically fastened bolted joints is enabling the connected structural components to be disassembled to access to the interior of the structure for inspection and repair purposes, in contrast to bonded joints. The functioning principle of the bolted joint is based on micro and macroscopic mechanical interference such as friction between joined parts, shear or tensile transfer forces in fasteners, and contact forces between the joined components with similar or dissimilar materials. Mechanical joining is used extensively in the aircraft industry to join titanium or aluminum components with composite structures. For example, in the F-22 fighter (Figure 5.2), the upper composite wing skin is attached by mechanical fastening to the internal wing sub-structure, which is in the form of composite and titanium spars.



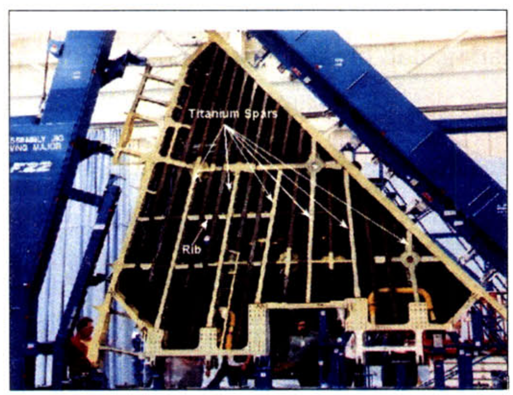


Figure 5.2. F/A-22 Wing Structure

Although there are several advantages, mechanically fastened joints have several disadvantages. The major joints introduce high stress concentration around the bolt hole, often becoming the starting point for damage initiation. Secondly, aluminium and stainless steel fasteners result in potential for galvanic corrosion when installed in carbon fiber based laminates. Hole generation requires specific drilling techniques, taking into account the possibility of mechanically and thermally induced defects and, finally, numerous metal fasteners and surrounding area reinforcement to join aircraft structural components result in large weight penalty. Because of this conflicting aspect of good and bad facts about bolted joints, extra careful consideration should be put into their design process.

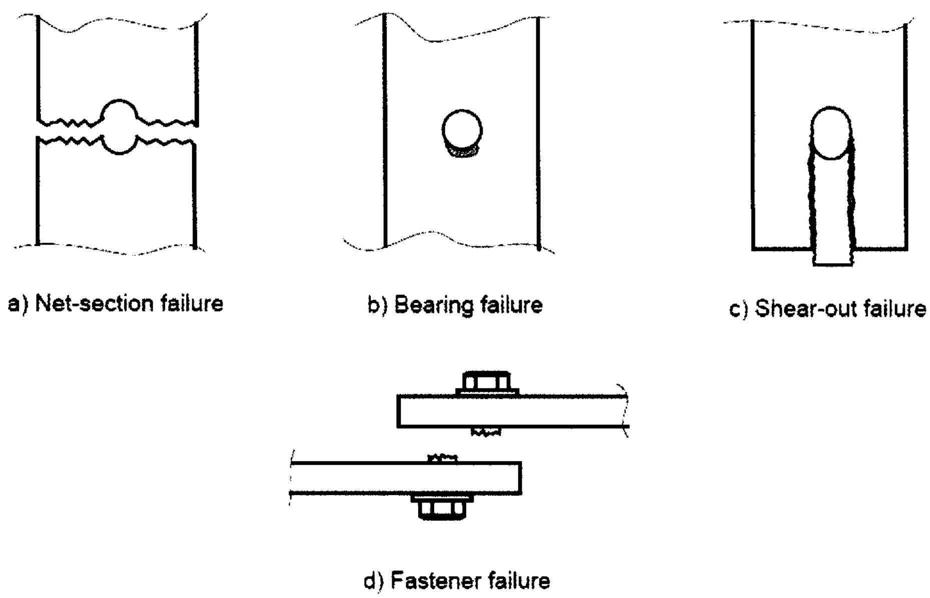


Figure 5.3. Failure Modes of Composite Bolted Joints (Ref. 5.1)

5.3. Design Process Improvement

Traditional composite bolted joint design is based on late 1980s research work (also the time of 777 and 330 development periods) at NASA, mainly focusing on failure modes in the 2D plane using bypass-bearing load breakup (Ref. 5.3). Bearing-Bypass load break up based design procedure assumes that the composite joint fails by a unique combination of bearing/bypass load, where bearing load is contact force due to bolt interaction, and bypass load is the loading passing through net section area (Figure 5.4).

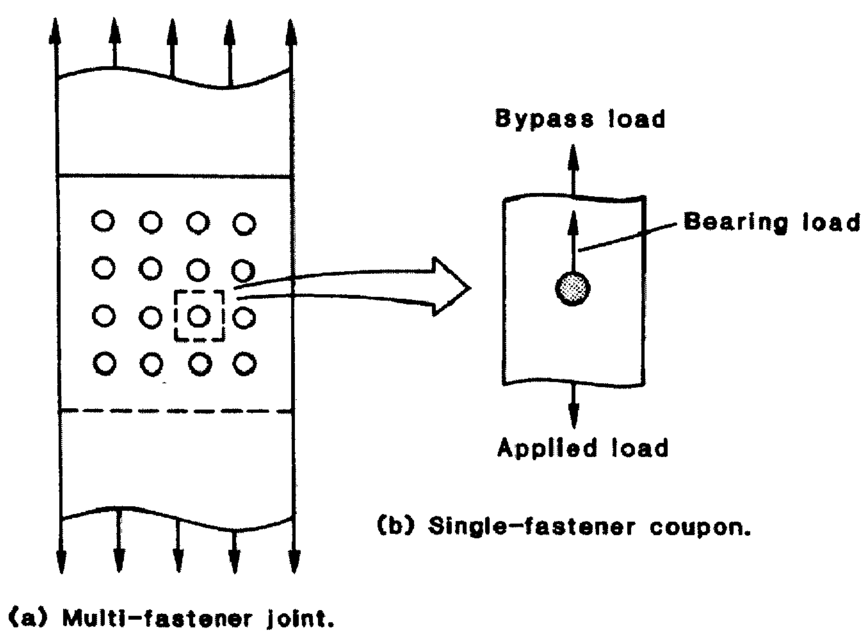


Figure 5.4. Bearing/Bypass loading definition (Ref. 5.3)

Using this approach, engineers were able to isolate particular 'unit-cell' joint sections from various, multiple fastener configurations, which can be tested (single fastener coupon based tension test) relatively easily. Once break-up and remapping back to original configuration scheme is defined, the next step is to generate the failure envelope by performing test on various loading and different hole size, composite layup configurations (Figure 5.5)

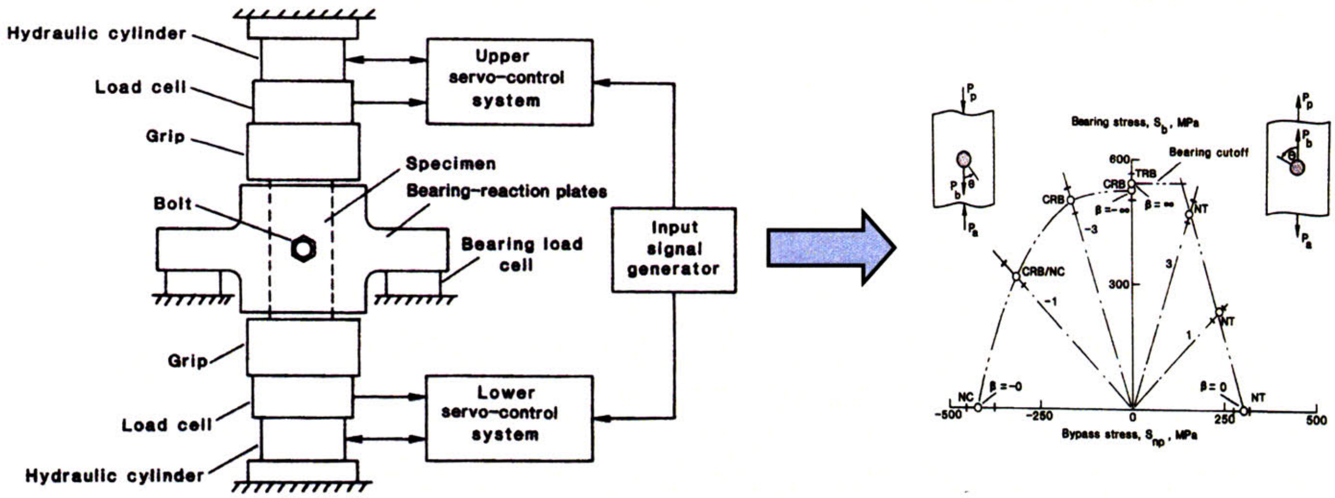


Figure 5.5. Configuration for Bearing/Bypass Test (Ref. 5.3)

Once the data are collected and the failure envelope is defined for particular set of configurations, engineers can calculate the bearing/bypass loading of each and every 'unit-cell' of composite joint structure, compare it with the failure envelope from the test results, and generate a margin of safety for each joint. The process is shown in Figure 5.6 illustrates this typical design process. The component is isolated and boundary loading is calculated (**a. load path analysis**), then individual unit cells are broken up to calculate bearing/bypass load of each unit cell (**b. joint load share analysis**), and then each unit cell bearing/bypass loading is compared with pre-determined design curve (Figure 5.5) to generate the margin of safety (**c. margin of safety calculation**). This process is the brief summary of traditional composite bolted joint design, which is the combination of three individually separated procedures.

The new composite design approach starts from the premise that the composite bolted joint problem is a part of the whole, a system problem, instead of looking at it as a linear combination of bearing and bypass loading breakup. Composite material does not differentiate the loading combination at the far field edge of "unit cell" but only responds to the local stress field directly applied to the area of failure initiation. Complex 3D stress behavior for a random design configuration of composite bolted joints cannot be accurately described using 2D stress field, classical laminated theory and linear superposition based bearing/bypass loading approach.

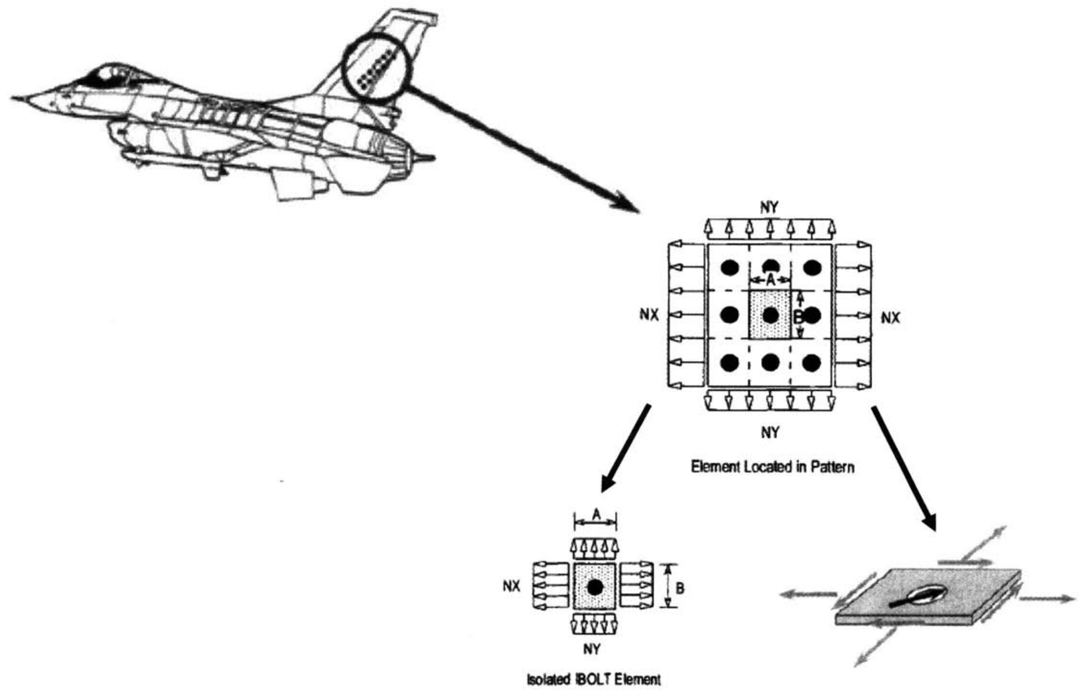


Figure 5.6. Typical Bolted Joint Analysis Process (Ref. 5.7)

The new procedure begins with the same global analysis model for load extraction of sub-system level components (Figure 5.7), however there are no steps b and c of bearing-bypass breakup analysis.

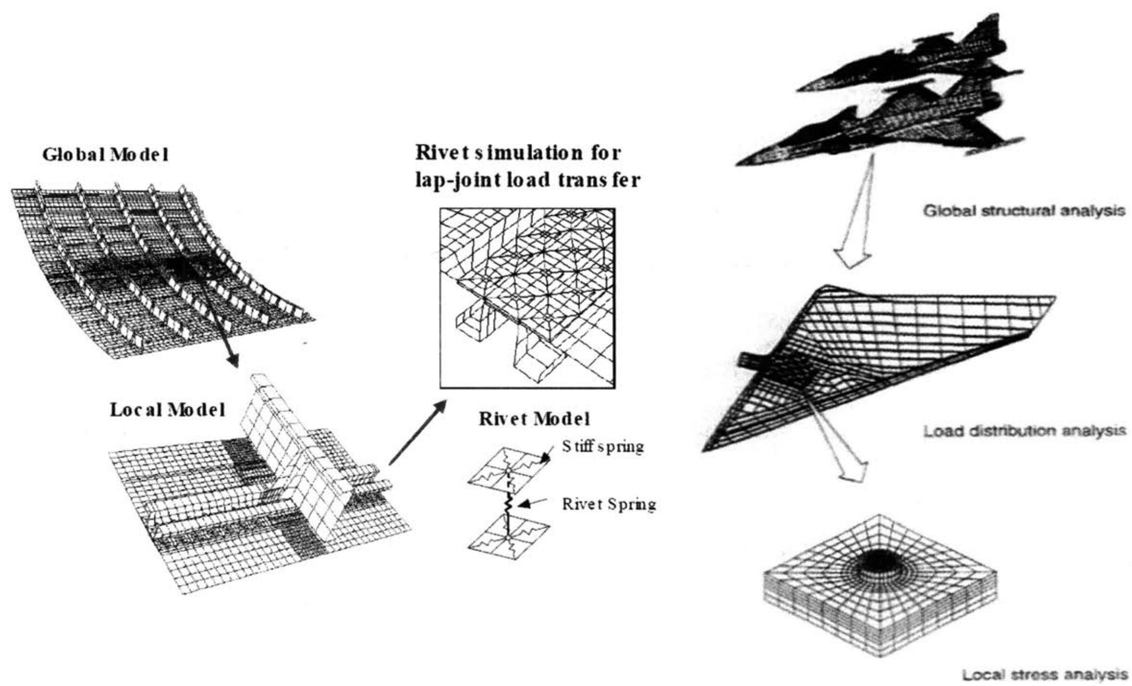


Figure 5.7. FEA based Bolted Joint Analysis Process (Ref.5.2 & 5.4)

Composite joints are no longer treated as independent problems where whole sets of failure envelopes need to be established for each and every design configuration from test, but are rather treated as another composite material problem with different sets of boundary conditions and loading conditions (though more complex and difficult). If the joint configuration behaves the certain way that the local stress field is exceeding failure load of particular mechanism of composite material, it is declared as 'failed'. There is no longer isolation of 'unit cell' from multiple fastener configurations -- the entire design is analyzed to assess the integrity of the design as a system, not just sum of individual unit cells (Figure 5.8).

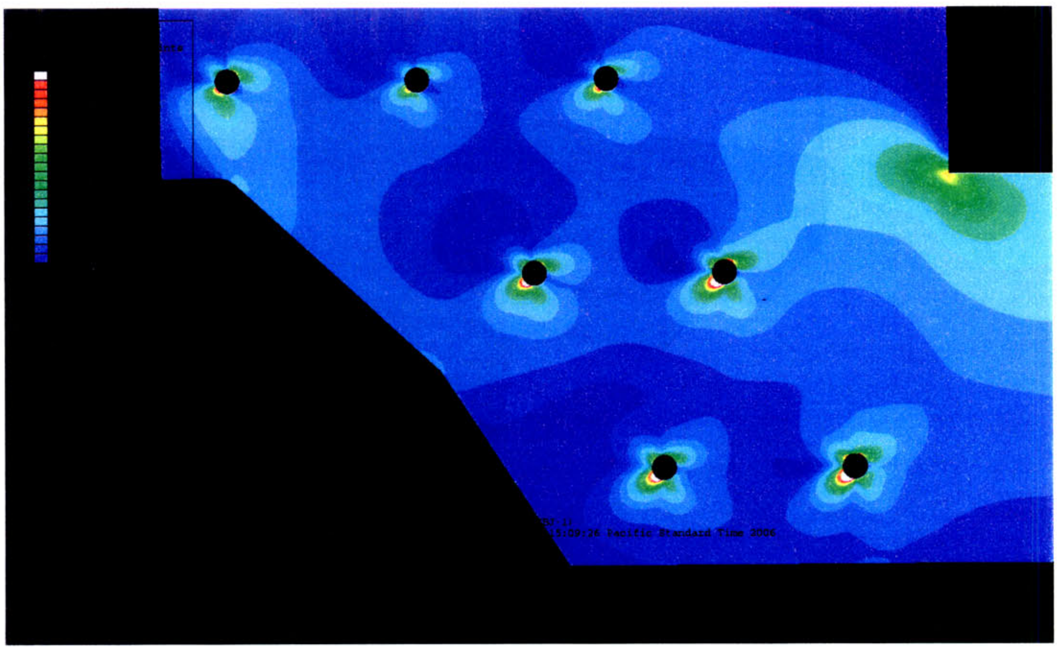


Figure 5.8. Stress Field of Multiple Fastener Configurations

The reason this system-level approach to solving composite joints problems is not being used is partially due to the time gap from 1980s bearing-bypass approach to 2000s environment, with several orders of magnitude increase in computation power and latest development of modern FEA based design and analysis tools. Production life cycle of airliners is usually 20-30 years, and hence the new product development project happens in about the same time (15-20 yrs between major product developments). Also, long product life cycles (20-30 years) and the conservative nature of the aerospace industry usually means that a new product development project is using tools and processes established during 'last' major product development cycle, which is 20 years behind for most of the case. The new initiative in analyzing composite bolted joint is currently being proposed and demonstrated within Boeing technology organizations.

In summary, the new composite bolted joint analysis procedure states:

1. It is a "**composite**" joint system problem with bolted joint boundary conditions, not independent 'composite bolted joint' problems.
2. Composite material does not have an 'intelligence' of breaking up bearing/bypass load at the far field edge, then fail according to pre-defined far field loading combination, but it just fails when local stress field reaches failure stress/strain for most susceptible failure mechanism (material property).
3. Load share and failure analysis should be performed as single problem, because failure progress and load share state keeps interacting each other (Figure 5.9)
4. The entire joint configuration with multiple fasteners should be looked at, not just single fastener of test coupon configuration (Fig. 5.7). It is not the problem of failure of single bolt, but degradation of system stiffness as a whole.

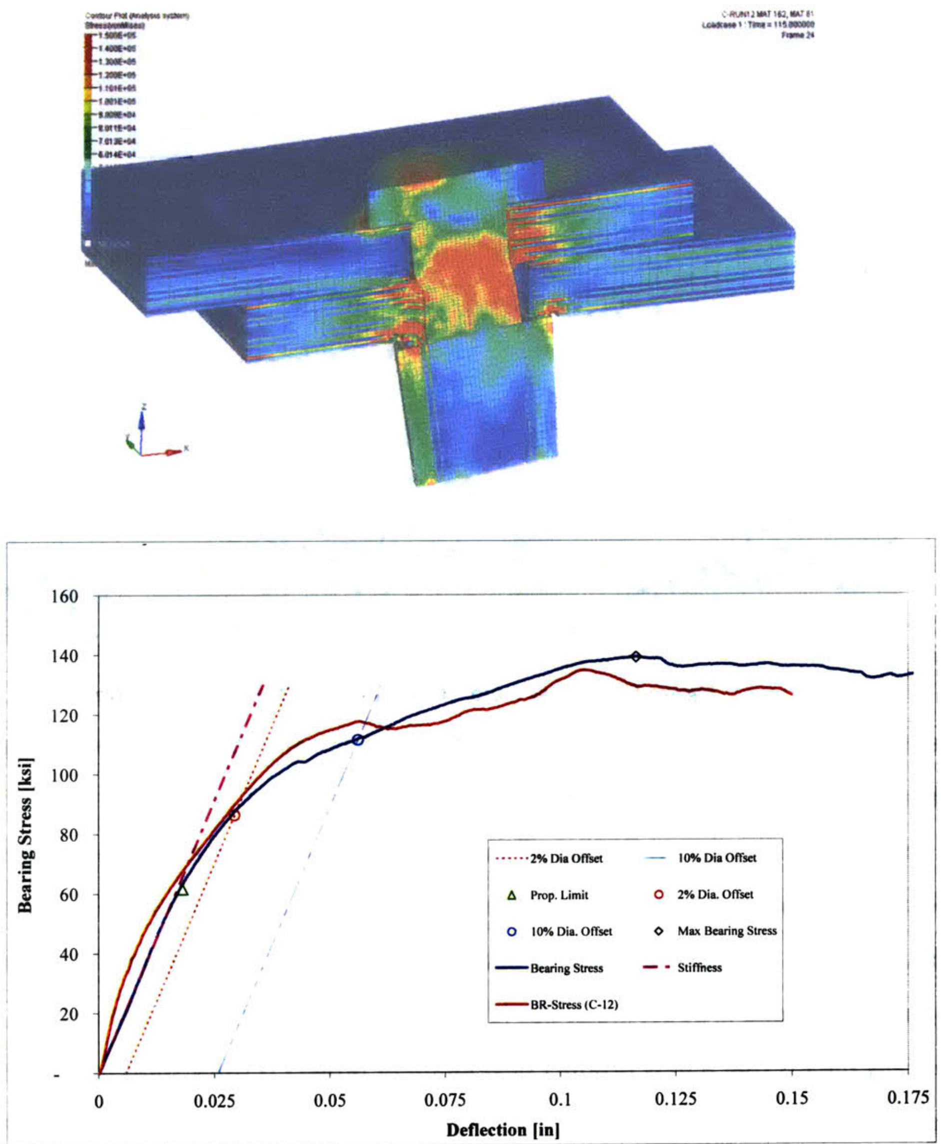


Figure 5.9. Integrated Composite Bolted Joint Analysis

(load share, bolt bending, composite failure data from single analysis)

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CHAPTER 6

CONCLUSION

6.1. Solving today's problem using yesterday's knowhow

Almost all new products in aerospace industry are evolutionary improvements on previous developments. A few exceptional breakthroughs in technology such as the jet engine (turbo jet, turbo fan, turbo prop engines), supersonic aerodynamics (swept wing), stability and control (Fly by Wire, Relaxed Stability), Material (semi-monocock, Aluminium, Titanium, Carbon Fiber Composite) and avionics boosted aircraft performance and economy. However, the overall system configuration remained the same after World War II, especially in commercial airliner sector, starting from the ground breaking De Havilland Comet and Boeing 707. Long product lifecycles (around 30 years) and fluctuation of airline industry economics means all new development effort comes in 15-20 year intervals, depending on remaining life for current products and market/competitive pressure. During these intervals, the industry is largely in 'production mode', providing derivative (stretched, shortened, new avionics, new cabin arrangement, new engines etc) aircraft to answer market demand. When new product demand arises within 5 years from the first increment of in-service aircraft reaching end of lifecycle, the manufacturer starts developing new airliners, gathering all current available technology, predicting market demand, perform initial sizing based on those specifications, and initiates a new product development cycle.

Once the development project goes into high gear in the detail design stage, the technology gap between old generation processes based on previous product development project (baseline) and current design requirement becomes the problem. Most of the sizing relationships, engineering organizational structure, task division and integration aspects of the entire system, valuable 'lessons learned', and, most importantly, technical leadership structures are all based on the 'last operational' airframe project experience. Combined with lack of market competition (there are two in commercial airframe, 2 in commercial jet engine industry, and 3 in military aircraft sector), complacency acts as a barrier to technology insertion for the new design.

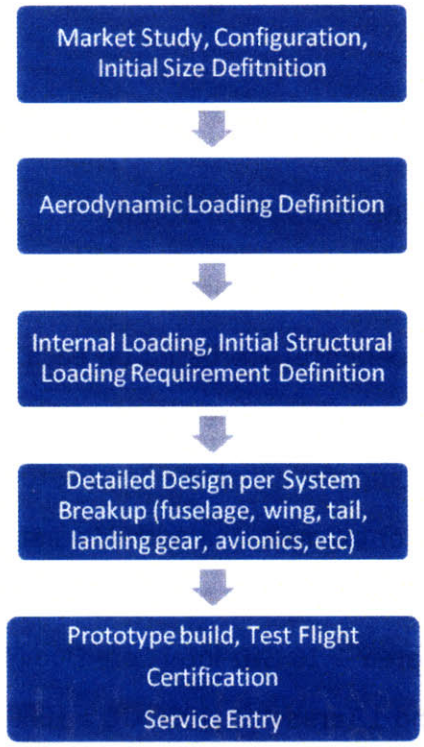

Figure 6.1. Typical Design Process of Commercial Aircraft Product

Solving today's problem, which requires faster, more efficient, leaner and more integrated processes with the old processes creates a new set of challenges (in addition to technical challenges associated with solving new problems never before attempted). Current problems experienced in the Airbus A-380 (integration, production issue), A-400M (engine program delay), Boeing 787 (supply channel, fastener problem), KC-767 (integration issues) are in effect, the outcome of trying to solve new problems using old procedures and mindset. Aggressive scheduling, ambitious technology insertion (20 years in-between product, lots of new technology available), and impressive business models and slogans (lean, global sourcing, JIT, six sigma, kaizen, ACE, etc) are ahead, largely lead by management and semi-technical leaders, but on the actual battlefront, the tools and processes are outdated, and people executing them are equally behind in conceptual frameworks.

Changing engineering resources and organizations accustomed to 'evolutionary design' task into 'revolutionary breakthrough design' teams and methods is not something that can be achieved overnight, let alone through buzzwords and management slogans. Something else must be done within the engineering community to cope with new tasks and challenge for the 21st century engineering projects.

6.2. Integrated Analysis solving today's problem

As described in previous chapter, the traditional aerospace design processes (from gas turbine and airframe industry experience) are based on each firm's last projects. One of the biggest changes between engineering environment in the late 80s, early 90s and late 90-2000s is the extreme advancement in computer aided engineering tools and methods with advent of high speed computation power. For example, the Author's first PC in 1987 was an 'amazing' 33 MHz Intel 486 CPU, 16 MB video memory, 100 MB main memory, and 100 MB hard disk, and current (2007) PC is a little 'outdated' 2.16 GHz Intel Core Duo (4.32 GHz thorough output), 256 MB video memory, 2GB RAM, and 1 Terabytes (1000 GB = 1000 x 1000 MB) hard disk space. It was taking days to run my research problem in 1999 using Finite Element Analysis solver, and now the same problem takes less than an hour to finish.

Advances in computational performance and especially structural analysis solver technologies since the late 1990s made it possible to perform not only larger scale analysis, but also to solve multidisciplinary, multi-domain problem in a single set of analyses. Traditional structural analysis categories are:

1. External Loading Analysis: Aeroelasticity, Flutter Analysis
2. Internal Loading Mapping: Linear Static, Linear Dynamic (Modal Superposition)
3. Durability/Strength/Stability: Linear Static, Nonlinear Static, Buckling, Post Buckling, Fatigue, Damage Tolerance Analysis
4. Kinematics, Motion Solution: Rigid Body Dynamics
5. Numerous hand calculation procedure based on pre-FEA era (pre-80s) for analysis category 1-4

Each category of problems will further be divided according to assigned sub-system (fuselage, empennage, wing, landing gear, propulsion, control and internal subsystem [interior, fuel, electronics, etc]), solving the same class of problem, but for different design objects. Usually different analysis models and processes are executed for different classes of problems (model size and resolution), resulting in numerous analysis models representing same design components, but solving different problems.

Using an integrated analysis approach, many traditional analysis work breakdown structures can be streamlined as sub-system and system level analyses consisting of many components, dispersing component level structural analyses as separate tasks. Sub-system level analysis is based on larger, higher resolution system analysis models with enough resolution to generate the required information for detailed component analysis.

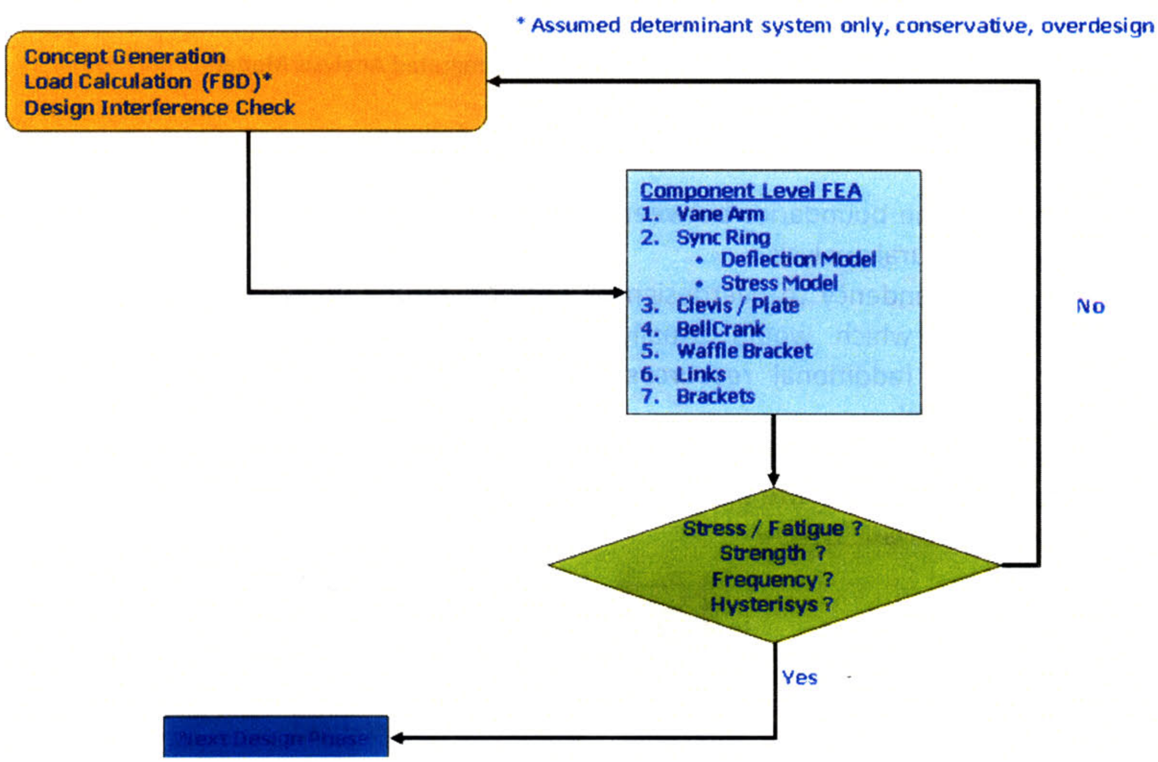

Figure 6.2. Typical Design Iteration Process for Sub-System Level Analysis

Each component level analysis points to separate structural analyses to answer individual component requirements. Once the integration/streamlining completed, the process becomes as Figure 6.3.

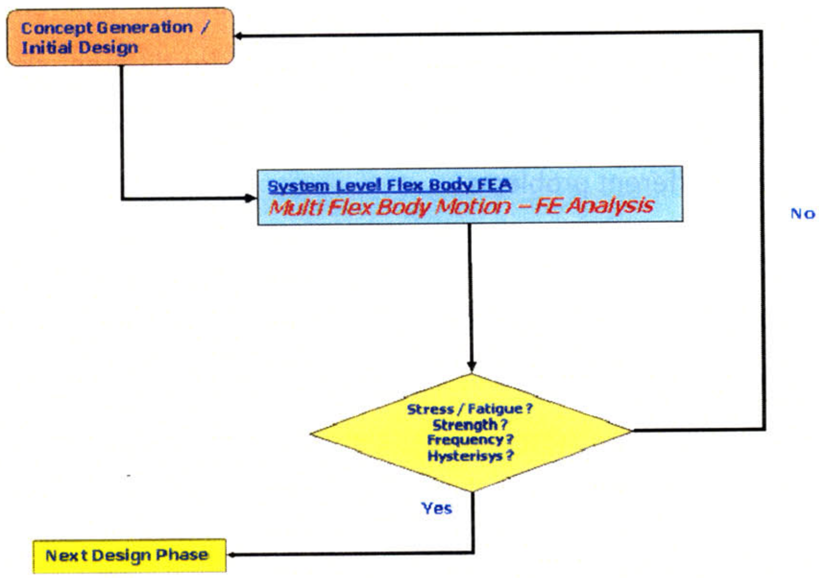

Figure 6.3. Design Process based on Integrated Analysis Model

Advantages of this approach can be summarized as follows:

1. Eliminates the boundaries between traditional load generation system analysis and FE based structural analysis.
2. Minimizes tendency to overdesign by performing one analysis with minimum system decoupling, which would usually require separate analyses with separate load calculations (additional resources), hence reduces successive additions of analyst 'conservatism'
3. One system analysis model will provide all design information (structural integrity) of most of components within particular system.

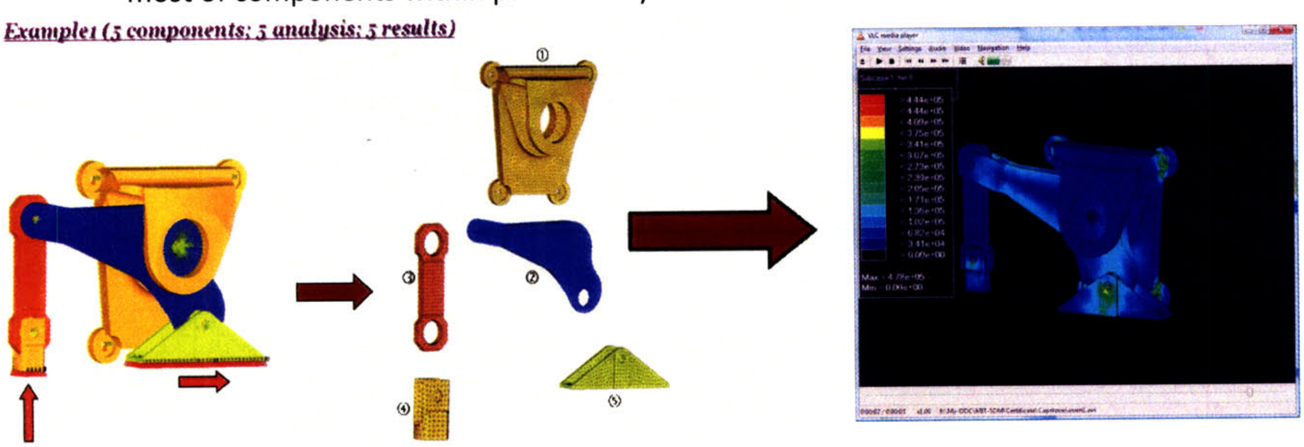

Figure 6.4. Integrated FE Analysis (one model, one analysis, 5 components

The same approach can be applied in composite bolted joint design analysis by solving the problem as a whole instead of breaking up individual components as is done for the current process, which was shown in chapter 5. One drawback of this approach is that the analyst who is working on this approach must be exceptionally skilled to correctly set up the model. Systematic training of engineering resource to become fully capable of performing larger system level analysis takes some time (6 months to 1 year).

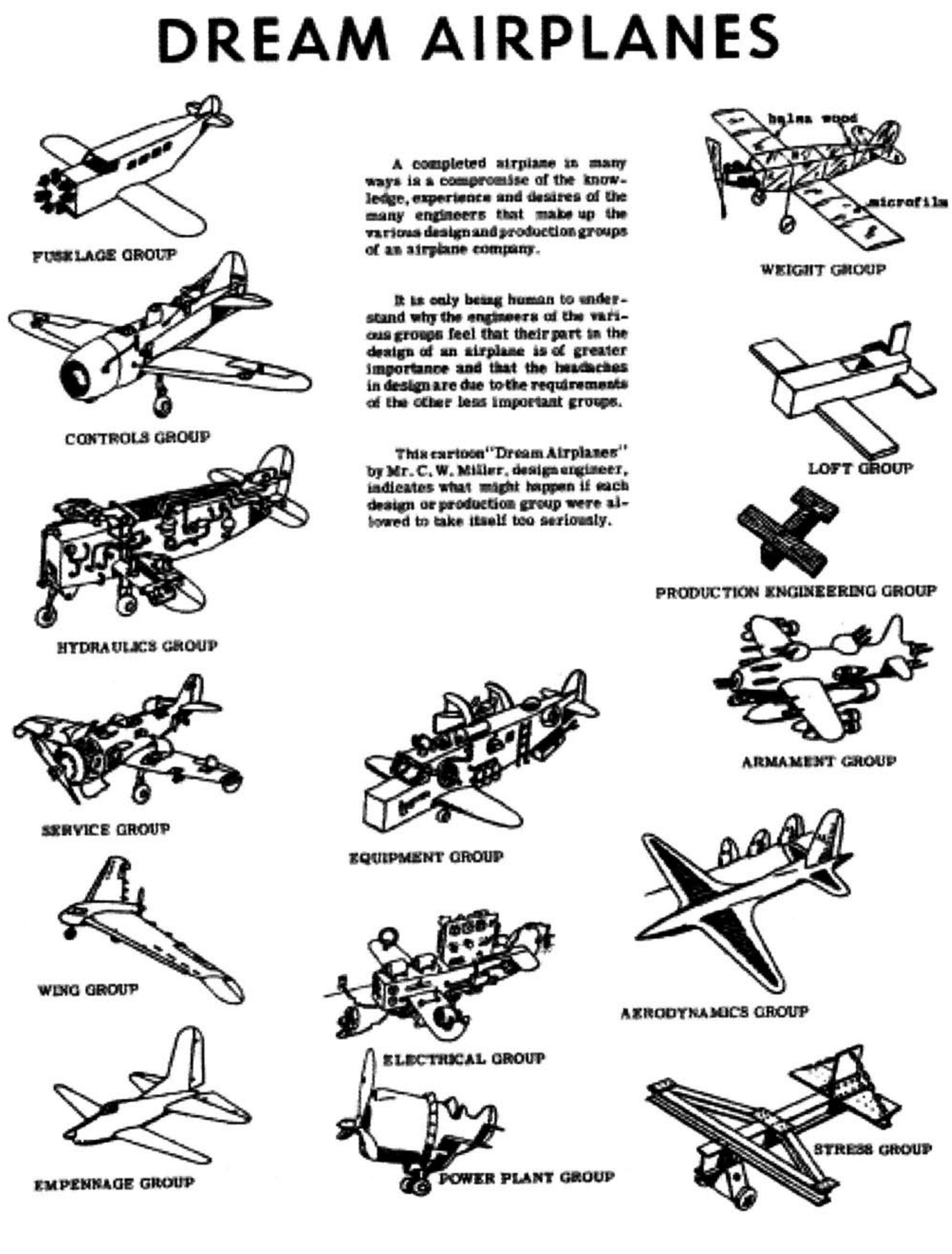

Figure 6.5. Different Aspect of Aircraft System Design

6.3. Preparing for the next Challenge

Considering the time gap from the last product development cycle of the aerospace industry, particularly in the airframe sector, it is understandable to see the general culture of resistance to change. "This is how we did it", "If it worked well before, why change?", "We know how to do this, you don't", and "It takes too much time and resource to implement change" were the typical response from senior technical leadership, especially in the company doing well or formerly dominant. Only when there is a serious risk emerging does the engineering community mobilize a task force to resolve problems with whatever it takes. However, once the product finally settles in and stable maturing stage starts, the program lapses into 'supporting production' mode, providing derivative product from time to time for the next 30 years of program lifecycle. As the development cycle gets longer and with fewer competitors in the overall aerospace industry sector, this tendency has grown even worse.

The US aerospace industry in defense sector is still dominant in the world, but in the commercial aircraft industry, especially large airliner manufacturers, only Boeing remains in the U.S.. The A-380 from EADS appears to be under control, and the 787 will eventually become a successful product. While Boeing and Airbus swap position in each market segment (very large aircraft to Airbus A380, 767 sector to 787), the next big competition will come in the form of A-350 900/1000, 787-10 or 777 derivative. Japanese manufacturers will unlikely stay as a loyal supplier (partners) as all of them are developing regional airliners in the segment of 737/A-320. China is developing their own regional airliners, as well as Bombardier of Canada, and the Sukhoi super 100 program will challenge the market position of two market leaders (Boeing & EADS).

The engineering community in the aerospace industry should start streamlining their processes to respond to challenges from all market segments instead of growing complacent in the 787 and its derivative products, which is in the medium size double aisle commercial airliner market segment. Without establishing more agile and efficient engineering processes, it will be very difficult to develop multiple platforms in relatively short period of time, forcing the company to form an alliance (Japan, China, etc), resulting in concession of our market leading position. No, brilliant management will not fix the inefficient engineering processes -- it's an engineering communities responsibility to win the competition in the form of superior and most efficient product in the final product performance and competitive engineering execution to make it happen.

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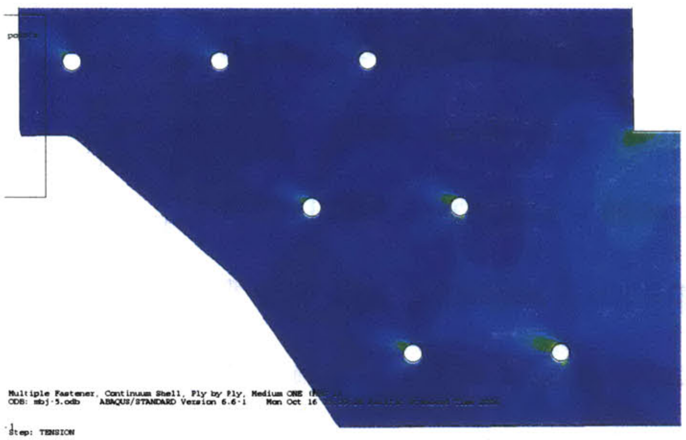
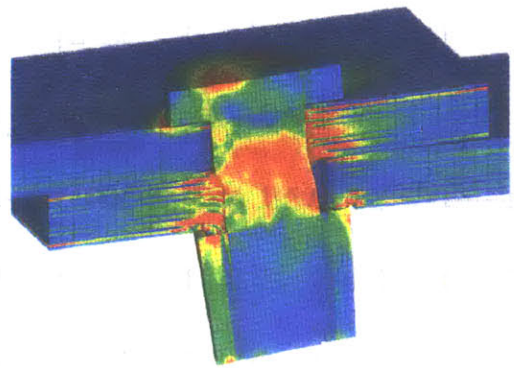
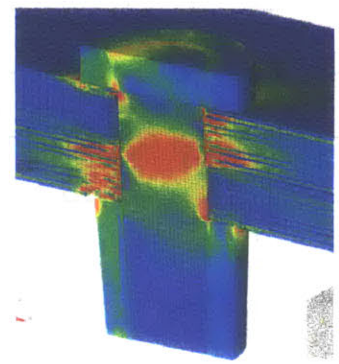
Appendix

Composite Failure Modeling and Analysis (CFMA) using FEA (V.10)







Background, Theory, Framework and Application

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Some Thoughts….

- Technical leading/managing consists with 'being technical' first and then 'managing' because;
  - You can not hire good technical engineers if you do not know technical detail.
  - You can not train new engineers if you do not know the detail.
  - You can not transfer procedure if you do not know the detail.
  - You can not outsource task you do not know how to execute.
  - You can not delegate task you do not know how to.
  - You can not integrate stuff if you do not know the detail.
- You can not 'lean out' unless you are willing to break out your 'comfort zone'.
- Talking about 'LEAN' has nothing to do with "Being LEAN"
- "**Shooting in the dark, Hoping for the best**" approach works well, as long as one can afford and customers are patient…..but money is limited and customers are not patient…

Background of CFMA

- Failure analysis of composite material using Finite Element Analysis
- High end FE based stress analysis is given (**No Excel, Not Hand Calc, No Linear Static**)
- Test data is used for initial model definition, calibration and validation
- It is not fully **deterministic** (no need test data) predictive analysis
- Initial Model is calibrated (correlated) against simple set of test configurations, then robustness is tested using complex configuration. Established model is '**extrapolated** (aka **predict**)' to generate data for realistic condition using "**similarity**"

Composite Material

- Revisiting Generalized Hook's Law $\sigma_{ij} = D_{ij}\varepsilon_j$ I, j = 1,2,….6

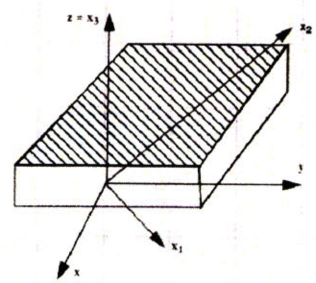
If following strain energy density function exists, $W = \frac{1}{2} D_{ij}\varepsilon_i\varepsilon_j$ $\sigma_i = \frac{\partial W}{\partial \varepsilon_i}$ Then, $D_{ij} = D_{ji}$

$$\begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ & & D_{33} & D_{34} & D_{35} & D_{36} \\ & & & D_{44} & D_{45} & D_{46} \\ & & & & D_{55} & D_{56} \\ & & & & & D_{66} \end{bmatrix}$$

Anisotropic Material
(No Symmetry)

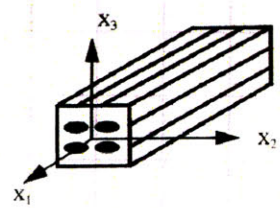
$$\begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & D_{16} \\ & D_{22} & D_{23} & 0 & 0 & D_{26} \\ & & D_{33} & 0 & 0 & D_{36} \\ & & & D_{44} & D_{45} & 0 \\ & & & & D_{55} & 0 \\ & & & & & D_{66} \end{bmatrix}$$

Monoclinic Material
Symmetry in X1-X2 plane
(Unidirectional Lamina)



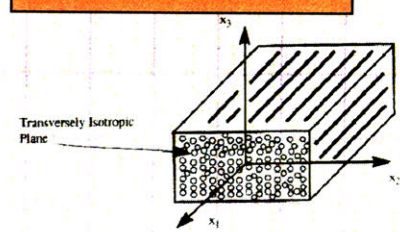
$$\begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ & D_{22} & D_{23} & 0 & 0 & 0 \\ & & D_{33} & 0 & 0 & 0 \\ & & & D_{44} & 0 & 0 \\ & & & & D_{55} & 0 \\ & & & & & D_{66} \end{bmatrix}$$

Orthotropic Material
Symmetry in X1-X2, X2-X3, X1-X3 plane



$$\begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ & D_{22} & D_{23} & 0 & 0 & 0 \\ & & D_{33} & 0 & 0 & 0 \\ & & & \frac{D_{22}-D_{23}}{2} & 0 & 0 \\ & & & & D_{66} & 0 \\ & & & & & D_{66} \end{bmatrix}$$

Transversely Isotropic
Material (isotropic in one plane)



$$\begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ & D_{22} & D_{23} & 0 & 0 & 0 \\ & & D_{33} & 0 & 0 & 0 \\ & & & \frac{D_{11}-D_{12}}{2} & 0 & 0 \\ & & & & \frac{D_{11}-D_{12}}{2} & 0 \\ & & & & & \frac{D_{11}-D_{12}}{2} \end{bmatrix}$$

Fully Isotropic

Composite Material

- Orthotropic Material $\gamma_{ij} = 2\varepsilon_{ij}$ Engineering Shear Strain

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{13}\\ \gamma_{12}\end{Bmatrix} = \underbrace{\begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ & & & C_{44} & 0 & 0\\ & & & & C_{55} & 0\\ & & & & & C_{66}\end{bmatrix}}_{C_{ij}} \begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{Bmatrix}$$

$$\varepsilon_1 = \frac{\sigma_1}{E_1} - \frac{\nu_{21}}{E_2}\sigma_1 - \frac{\nu_{31}}{E_3}\sigma_3$$

$$\varepsilon_2 = -\frac{\nu_{12}}{E_1}\sigma_1 - \frac{\sigma_2}{E_2} - \frac{\nu_{32}}{E_3}\sigma_3$$

$$\varepsilon_3 = -\frac{\nu_{13}}{E_1}\sigma_1 - \frac{\nu_{23}}{E_2}\sigma_2 + \frac{\sigma_3}{E_3}$$

$$\gamma_{23} = \frac{\tau_{23}}{G_{23}}, \gamma_{13} = \frac{\tau_{13}}{G_{13}}, \gamma_{12} = \frac{\tau_{12}}{G_{12}}$$

$$\nu_{ij} = \frac{-\varepsilon_j}{\varepsilon_i} \quad \sigma_i \neq 0, i \neq j, i, j = 1,2,3$$

$$\begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{21}}{E_2} & \frac{-\nu_{31}}{E_3} & 0 & 0 & 0\\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu_{32}}{E_3} & 0 & 0 & 0\\ \frac{-\nu_{13}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

Orthotropic

$$\begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{12}}{E_2} & \frac{-\nu_{12}}{E_3} & 0 & 0 & 0\\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu_{23}}{E_3} & 0 & 0 & 0\\ \frac{-\nu_{12}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{2(1+\nu_{23})}{E_2} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G_{12}} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

Transversely Isotropic

$$D_{ij} = C_{ij}^{-1}$$

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{Bmatrix} = \underbrace{\begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0\\ D_{12} & D_{22} & D_{23} & 0 & 0 & 0\\ D_{13} & D_{23} & D_{33} & 0 & 0 & 0\\ & & & D_{44} & 0 & 0\\ & & & & D_{55} & 0\\ & & & & & D_{66}\end{bmatrix}}_{D_{ij}} \begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{13}\\ \gamma_{12}\end{Bmatrix}$$

$$[D] = \begin{bmatrix} \frac{1-\nu_{23}\nu_{32}}{E_2E_3\Delta} & \frac{\nu_{21}+\nu_{23}\nu_{31}}{E_2E_3\Delta} & \frac{\nu_{31}+\nu_{21}\nu_{32}}{E_2E_3\Delta} & 0 & 0 & 0\\ \frac{\nu_{21}+\nu_{23}\nu_{31}}{E_2E_3\Delta} & \frac{1-\nu_{13}\nu_{31}}{E_1E_3\Delta} & \frac{\nu_{32}+\nu_{12}\nu_{31}}{E_1E_3\Delta} & 0 & 0 & 0\\ \frac{\nu_{31}+\nu_{21}\nu_{32}}{E_2E_3\Delta} & \frac{\nu_{32}+\nu_{12}\nu_{31}}{E_1E_3\Delta} & \frac{1-\nu_{12}\nu_{21}}{E_1E_2\Delta} & 0 & 0 & 0\\ 0 & 0 & 0 & G_{23} & 0 & 0\\ 0 & 0 & 0 & 0 & G_{13} & 0\\ 0 & 0 & 0 & 0 & 0 & G_{12} \end{bmatrix}$$

$$\Delta = (1-\nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{21}\nu_{32}\nu_{13})/(E_1E_2E_3)$$

Displacement, Strain, Compatibility & Equilibrium

Displacement (3 variables) u_1, u_2, u_3

Strain (6 equations, 3 unknowns) $\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ $u_{i,j} = \frac{\partial u_i}{\partial x_j}$ $\varepsilon_{ij} = \varepsilon_{ji}$

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} \qquad \varepsilon_{12} = \varepsilon_{21} = \frac{1}{2}\gamma_{12} = \frac{1}{2}\gamma_{21} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$$

$$\varepsilon_{22} = \frac{\partial u_2}{\partial x_2} \qquad \varepsilon_{13} = \varepsilon_{31} = \frac{1}{2}\gamma_{13} = \frac{1}{2}\gamma_{31} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right)$$

$$\varepsilon_{33} = \frac{\partial u_3}{\partial x_3} \qquad \varepsilon_{23} = \varepsilon_{32} = \frac{1}{2}\gamma_{23} = \frac{1}{2}\gamma_{32} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right)$$

Bound by Compatibility Equation (St. Venant)

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \qquad 2\frac{\partial^2 \epsilon_x}{\partial y \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}\right)$$

$$\frac{\partial^2 \epsilon_y}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y \partial z} \qquad 2\frac{\partial^2 \epsilon_y}{\partial x \partial z} = \frac{\partial}{\partial y}\left(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}\right)$$

$$\frac{\partial^2 \epsilon_x}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial x^2} = \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} \qquad 2\frac{\partial^2 \epsilon_z}{\partial x \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right)$$

Constitutive Equations

$$\sigma_{ij} = D_{ij}\varepsilon_j$$

Equilibrium Condition
(6 stress components)

$$\sigma_{ij,j} + F_i = 0$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

Equilibrium, Plane Stress and Plane Strain Condition

Equilibrium Condition

$$\sigma_{ij,j} + F_i = 0$$

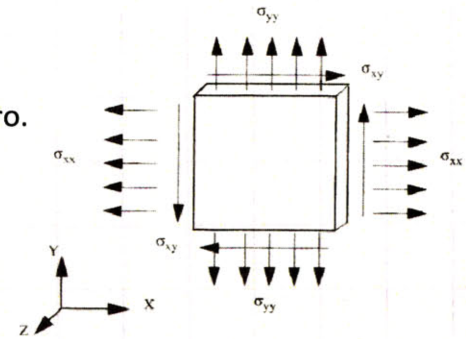
$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

Plane Stress (thin plate)

All out-of-plane stress components are zero.



$$\sigma_{zz} = 0, \varepsilon_{zz} \neq 0$$

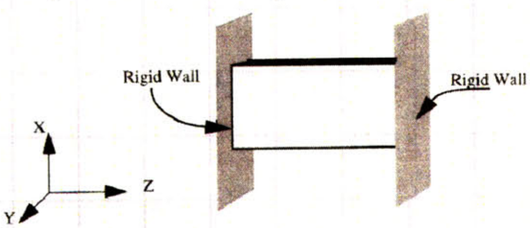
$$\sigma_{zx} = \sigma_{zx} = 0$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

Plane Strain (thick plate)

All out-of-plane strain components are zero.



$$\varepsilon_{zz} = 0, \sigma_{zz} = f(x,y) \neq 0$$

$$\varepsilon_{zx} = \varepsilon_{zx} = 0$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

Generalized Plane Problem (Lekhnitskii, 1963)

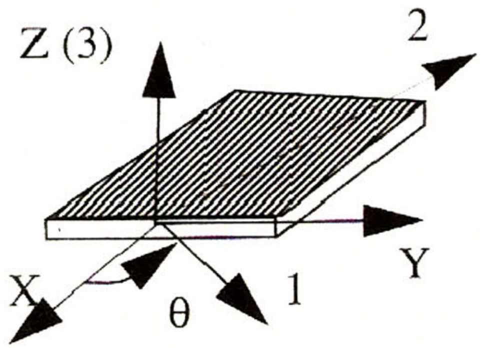
Stresses and strains do not vary along prescribed direction (z)

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} = 0$$

Coordinate Transformation (plane stress condition)



$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{Bmatrix}=\begin{bmatrix}\bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & 0 & 0 & \bar{C}_{16}\\ \bar{C}_{12} & \bar{C}_{22} & \bar{C}_{23} & 0 & 0 & \bar{C}_{26}\\ \bar{C}_{13} & \bar{C}_{23} & \bar{C}_{33} & 0 & 0 & \bar{C}_{36}\\ 0 & 0 & 0 & \bar{C}_{44} & \bar{C}_{45} & 0\\ 0 & 0 & 0 & \bar{C}_{45} & \bar{C}_{55} & 0\\ \bar{C}_{16} & \bar{C}_{26} & \bar{C}_{36} & 0 & 0 & \bar{C}_{66}\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{Bmatrix}$$

$\bar{S}_{ij}$ Transformed Compliance Matrix

$$\sigma_z = \tau_{yz} = \tau_{zx} = 0$$

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{bmatrix}\bar{C}_{11} & \bar{C}_{12} & \bar{C}_{16}\\ \bar{C}_{12} & \bar{C}_{22} & \bar{C}_{26}\\ \bar{C}_{16} & \bar{C}_{26} & \bar{C}_{66}\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}$$

Stiffness Matrix

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

Orthotropic

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & 0\\ \bar{Q}_{12} & \bar{Q}_{22} & 0\\ 0 & 0 & \bar{Q}_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

$$\bar{Q}_{ij} = \bar{D}_{ij} - \frac{\bar{D}_{i3}\bar{D}_{3j}}{\bar{D}_{33}} \quad \text{I, j = 1,2,6}$$

$$[\bar{Q}] = [T_1]^{-1}[Q][T_2]$$

$$[T_1] = \begin{bmatrix} m^2 & n^2 & 2mn\\ n^2 & m^2 & -2mn\\ -mn & mn & m^2-n^2\end{bmatrix}$$

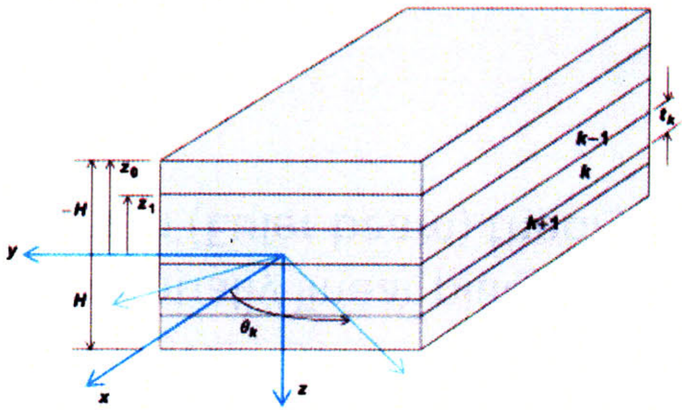
$$[T_2] = \begin{bmatrix} m^2 & n^2 & mn\\ n^2 & m^2 & -mn\\ -2mn & 2mn & m^2-n^2\end{bmatrix}$$

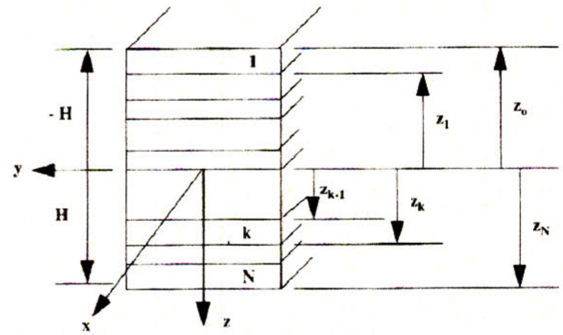
$$m = \cos\theta, n = \sin\theta$$

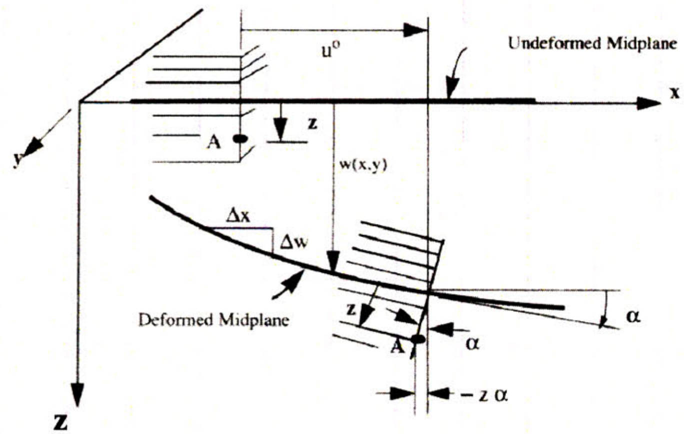
Classical Lamination Theory

- The laminate consists of perfectly bonded layers (laminae)
- Each lamina is a homogeneous material with known effective properties
- Individual lamina can be isotropic, orthotropic, or transversely isotropic
- Each lamina is in a state of plane stress (stress in z is zero)
- Laminate deforms according to the "Kirchhoff-Love Assumptions" for bending and stretching of thin plates:
  - Normals to the midplane remain straight and normal to the deformed midplane after deformation (out of plane shear stresses are zero)
  - Normals to the midplane do not change length (z displacement is a function of (x,y) only)
- Essentially linear plate theory (pg. 17), which is itself extension of linear beam (Euler beam) theory (pg. 15)

Classical Lamination Theory (continues)







$$u = u^o - z\tan\alpha = u^o - z\alpha = u^o - z\frac{\partial w}{\partial x}$$

$$v = v^o - z\frac{\partial w}{\partial y} \qquad w(x,y) = w^o(x,y)$$

$$\sigma_{zz} = 0, \gamma_{zx} = \gamma_{zy} = 0$$

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u^o}{\partial x} - z\frac{\partial^2 w}{\partial x^2} = \varepsilon_x^o + z\kappa_x$$

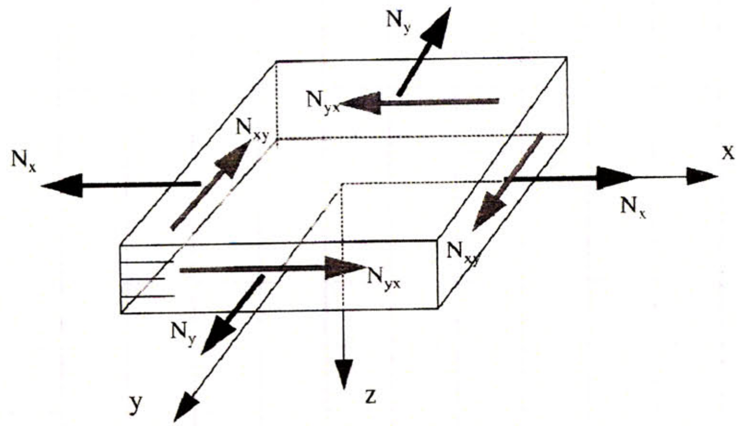
$$\varepsilon_y = \frac{\partial v}{\partial y} = \frac{\partial v^o}{\partial y} - z\frac{\partial^2 w}{\partial y^2} = \varepsilon_y^o + z\kappa_y$$

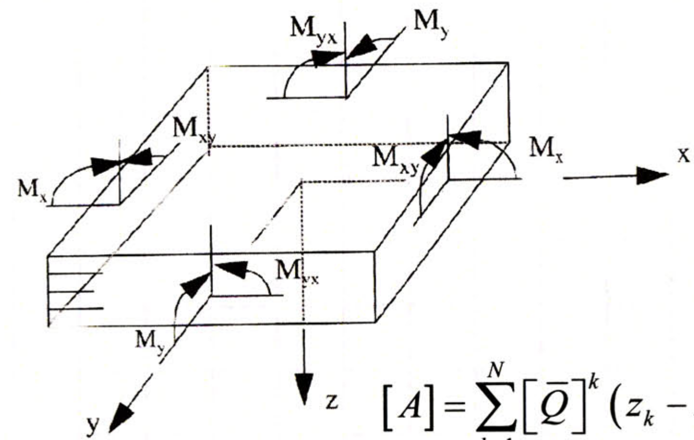
$$\gamma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{\partial u^o}{\partial y} - 2z\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial v^o}{\partial x} = \gamma_{xy}^o + z\kappa_{xy}$$

$$\{\sigma\}^k = [\bar{Q}]^k \{\varepsilon^o\} + [\bar{Q}]^k z\{\kappa\} \Longleftarrow \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

Independent of Material

Classical Lamination Theory (continues)





$$\{N\} = \int_{-H}^{H} \{\sigma\} dz, \{N\} = [A]\{\varepsilon^o\} + [B]\{\kappa\}$$

$$\{M\} = \int_{-H}^{H} \{\sigma\} z dz, \{M\} = [B]\{\varepsilon^o\} + [D]\{\kappa\}$$

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \varepsilon^o \\ \kappa \end{Bmatrix}$$

$$[A] = \sum_{k=1}^{N} [\bar{Q}]^k (z_k - z_{k-1})$$

$$[B] = \frac{1}{2} \sum_{k=1}^{N} [\bar{Q}]^k (z_k^2 - z_{k-1}^2)$$

$$[D] = \frac{1}{3} \sum_{k=1}^{N} [\bar{Q}]^k (z_k^3 - z_{k-1}^3)$$

[A]: in-plane stiffness matrix

[B]: Bending Stretching Coupling Matrix

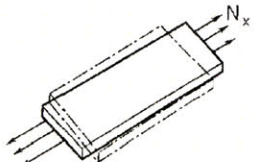
[D]: Bending Stiffness Matrix

- [A] is a function of layer thickness, independent of stacking sequence
- [B], [D] is dependent on the stacking sequence

Classical Lamination Theory (continues)

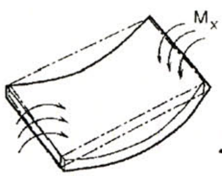
Classical Orthotropic Laminates

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$



Stretching

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$



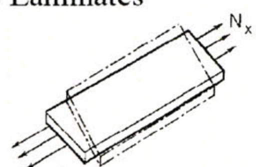
Bending

- Laminate Composed of 0° and 90° Plies
- Symmetric About its Mid-Plane

[0, 90, 90, 0]

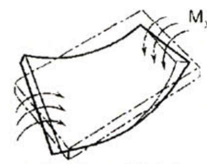
Classical Anisotropic Laminates

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$



Stretching and Shearing

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$



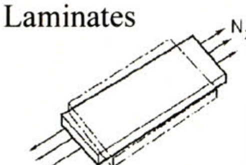
Bending and Twisting

- Laminate Composed of Plies With $-\theta_1, +\theta_2, +\theta_3, +\ldots$ But No $-\theta_1, -\theta_2, -\theta_3, -\ldots$
- Symmetric About Its Mid-Plane But Unbalanced

[45, 22•5, 22•5, 45]

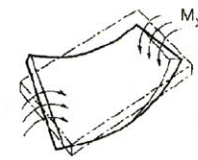
Pseudo-Orthotropic Laminates

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$



Stretching

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$



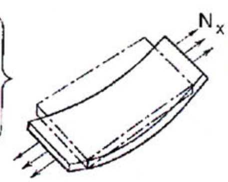
Bending and Twisting

- Laminate Is Symmetric About its Mid-Plane
- For Every $+\theta$ There Is a $-\theta$ (Balanced)
- D_{16} and D_{26} Become Small for $>\sim 10$ Plies

[0, 90, 45, –45, –45, 45, 90, 0]

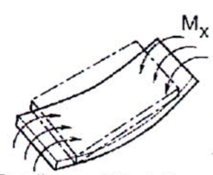
Unsymmetric Cross-Ply Laminates

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$



Stretching and Bending

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$



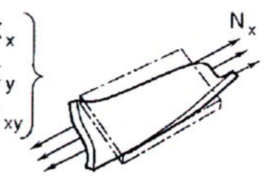
Bending and Stretching

- Laminate Composed of 0° and 90° Plies
- Anti-Symmetric About the Mid-Plane

[90, 0, 90, 0]

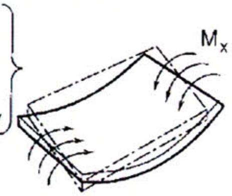
Unsymmetric Angle Ply Laminates

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & B_{16} \\ 0 & 0 & B_{26} \\ B_{16} & B_{26} & 0 \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$



Stretching and Twisting

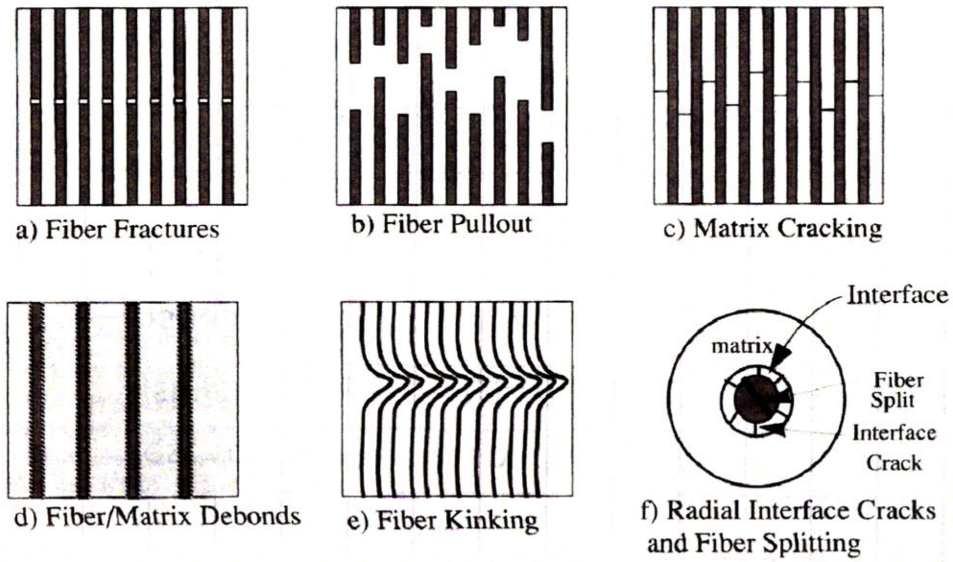
$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & B_{16} \\ 0 & 0 & B_{26} \\ B_{16} & B_{26} & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$



Bending and Shearing

- Laminates Composed of $+\theta$'s and $-\theta$ s' Pies
- Anti-Symmetric About its Mid-Plane
- Balanced [45, –45, 45, –45]

Failure Mechanisms of Composite



- Fiber Fracture (a): maximum tensile fiber strength
- Fiber Pullout (b): Fiber Fracture (a) + Debond (d)
- Matrix Cracking (c): Matrix strength exceeds
- Fiber Kink (e): Microbuckling due to fiber/matrix/interface interaction (material + geometry)
- Fiber Splitting and radial interface cracks (f) occur when the transverse or hoop stresses in the fiber or interphase region between the fiber and matrix reaches its ultimate value

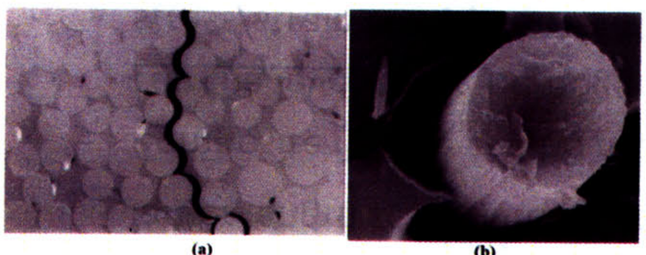
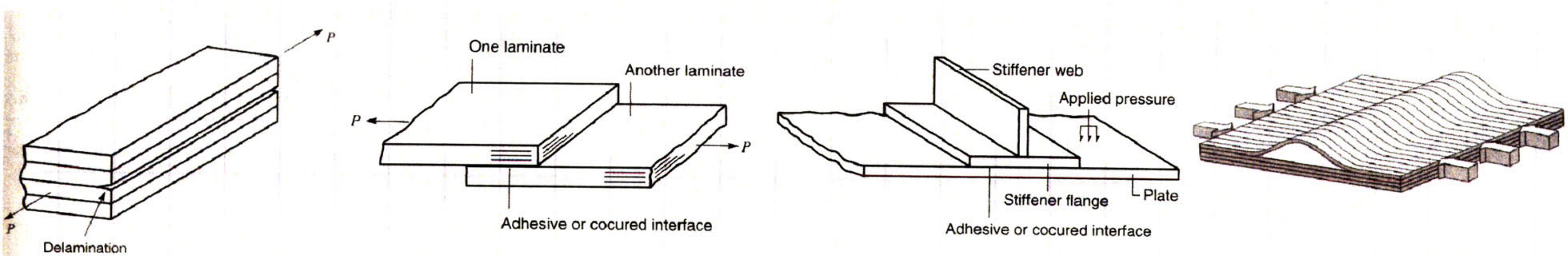


Fig. 2. Transverse crack initiation from fiber/matrix interface failure.

- At the laminate level, micro-level mechanisms manifest themselves as lamina failures in the form of **transverse cracks in planes parallel to the fibers**, **fiber dominated failures in planes perpendicular to the fibers** and **delaminations between layers of the laminate**



Composite Failure Modeling and Analysis (CFMA)

- Line of thought;
  - Composite Failure is a function of interaction at the fiber and matrix length scale. Therefore we model that length scale and analyze failure behavior, then translate failure effect to larger length scale (lamina and laminate): **Micromechanics based Failure Modeling**
  - Composite Failure can be modeled more efficiently if the failure can be represented in **Lamina level**, using homogenized (rule of mixture) lamina material property and stress/strain components: **Mesoscale, lamina level Failure Modeling**
  - Composite Failure is best modeled in **Laminate level**, which is same length scale as common test configuration, so identify global failure using test data and/or resolve into lamina level using CLT: **Common Laminate Level Failure Analysis**
  - Usually trying to make one simpler, less cumbersome and elegant, such as Von-Mises Yield Criteria

$$f\left(\sigma_{ij}\right)=1, f\left(\varepsilon_{ij}\right)=1, f\left(E \propto g\left(\sigma_{ij} \cdot \varepsilon_{ij}^{2}\right)\right)=1$$

CFMA Framework (General Composites)

A. Failure Initiation Criteria (Declare)
- Hill's General Failure Function
- Tsai Family (Tsai-Hill, Tsai-Wu)
- Strain Invariant Failure Theory (SIFT)
- Hashin Family (Stress/Strain Invariant)

Damage Mapping (f1)

B. Damage Accumulation (Degradation)
- Direct Material Stiffness Reduction
  - Exponential Decay (Continuum Damage Mechanics)
  - Linear/Quadratic Decay (Hashin/Rotem)
- Nodal Force Modification
  - EFM (customizable force application)
- LEFM Family: VCCT
- Cohesive Element Based: DCZM
- Script based, indirect material swapping (Python)

Update Damage State (f2)

C. Damage Representation (optional except BJ problem)
- Contact Based
  - Delamination Opening/Closure
- Void Elements
  - EFM (elements are there, but no effective stiffness)
  - Script based (Python)
- Real Time Element Removal (tension/compression)

Corresponding Metal basis FEA Process

Linear Analysis
Conventional Linear Analysis bound by allowable or Failure Index

A. Failure Initiation Criteria

Nonlinear Analysis
Material Degradation using Progressive Damage Model

B. Damage Accumulation Mechanics

Nonlinear Failure Analysis
Using Composite Damage/Fracture Model

C. Damage Representation (rupture)

CFMA

Linear Analysis
Conventional Linear Analysis bound by Yield Stress

A. Yield Criteria

Nonlinear Analysis
Material Degradation using plasticity model

B. Elasto-Plastic Analysis

Nonlinear Failure Analysis
Using Metal Damage Mechanics (plasticity-rupture)

C. Damage/Fracture Mechanics

Conventional Metal FEA

Defining Failure Initiation Criteria (A) (Lamina Length Scale)

- There are 7 major failure mechanisms, and possibly more. Currently most of attempts are focusing on representing few dominant failure modes manifesting in lamina level (hence laminate), also known as **Mesoscale** failure description (accumulated effect of **Micro-Level** failure modes (fiber-matrix length scale) as in pg. 40
  - Fiber Tension **(1)**, Fiber Compression (crushing, buckling) **(2)**, and Fiber Shear **(8),** Interface debonding **(9)**
  - Matrix Transverse Failure (tension, compression)**(3)**, Delamination (tension, compression)**(4)**
  - Fiber/Matrix interface interaction **(10)**
  - Out-of-plane Failure (direct loading in Z) **(5)**
  - Pinching, Crushing within confined volume **(6)**
  - Coupling with loading source (compliant loading source such as bolt, bird, blade, ice, etc) **(7)**
  - Global buckling (geometry, loading dependant), followed by or initiated by composite failure **(11)**
- Typical Laminate Failure (in-plane, simple bending, smooth response): (1),(2),(3),(4)
- VCCT: (4)
- Typical BJ: (1),(2),(3),(4),(5),(6),(7)
- Ballistics Problem: (1),(2),(3),(4),(5),(6),(7),(11)

Quadratic Failure Function, Tsai-Hill Family (A)

- Represents an attempt to apply **Hill's anisotropic plasticity** to failure of homogeneous, anisotropic materials (quadratic function)
  - The quadratic criteria are based upon the mathematical premise that a second-order curve has more parameters with which to fit experimental data, but not as complicated as higher order approximation.
  - These criteria is generally not based on physics of the failure mechanisms, but assumption that composite will follow plasticity characteristics, and better/robust correlation feature.
  - The sign of the normal stress components must be known before if the positive and negative strengths are different

$$(G+H)\sigma_1^2+(F+H)\sigma_2^2+(F+G)\sigma_3^2-2H\sigma_1\sigma_2-2G\sigma_1\sigma_2\ldots$$
$$-2F\sigma_2\sigma_3+2L\tau_{23}^2+2M\tau_{13}^2+2N\tau_{12}^2=1$$

3 pure shear cases yield; $\tau_{12}\neq 0\rightarrow 2N=1/S^2$

$\tau_{23}\neq 0\rightarrow 2L=1/Q^2$

$\tau_{13}\neq 0\rightarrow 2M=1/R^2$

3 Uniaxial cases yield; $2H=1/X^2+1/Y^2-1/Z^2$

$2G=1/X^2-1/Y^2+1/Z^2$

$2F=-1/X^2+1/Y^2+1/Z^2$

Plane stress (1-2) $\sigma_3=\tau_{13}=\tau_{23}=0$
Transversely Isotropic (Y=Z)

$$\frac{\sigma_1^2}{X^2}-\frac{\sigma_1\sigma_2}{X^2}+\frac{\sigma_2^2}{Y^2}+\frac{\tau_{12}^2}{S^2}\geq 1$$

Tensor Polynomial Failure Criteria, Tsai-Wu Family (A)

- To overcome original Tsai-Hill criteria, adding feature to differentiate between tension and compression

$$f(\sigma_i) = F_i\sigma_i + F_{ij}\sigma_i\sigma_j \geq 1$$

$$f(\sigma_i) = F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 \ldots$$
$$+ 2F_{12}\sigma_1\sigma_2 + 2F_{13}\sigma_1\sigma_3 + 2F_{23}\sigma_2\sigma_3 \geq 1$$

Plane stress (1-2) $\sigma_3 = \tau_{13} = \tau_{23} = 0$
Transversely Isotropic (Y=Z)

$$f(\sigma_i) = F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2 \geq 1$$

$$F_1 = \frac{1}{X_t} + \frac{1}{X_c}, \quad F_2 = \frac{1}{Y_t} + \frac{1}{Y_c}, \quad F_{11} = -\frac{1}{X_t X_c}, \quad F_{22} = -\frac{1}{Y_t Y_c}, \quad F_{66} = \frac{1}{S^2}.$$

$$F_{12} = \frac{1}{2\sigma_{biax}^2}\left[1 - \left(\frac{1}{X_t} + \frac{1}{X_c} + \frac{1}{Y_t} + \frac{1}{Y_c}\right)\sigma_{biax} + \left(\frac{1}{X_t X_c} + \frac{1}{Y_t Y_c}\right)\sigma_{biax}^2\right]$$: Interaction Parameter, set to zero typical

Strain Invariant Failure Theory (A)

- Failure '**INITIATION**' theory based on observation (compare with pg. 7) of continuous fiber lamina failure.
  - Damage initiation is defined as intra-lamina (within) and inter-lamina (between) crack
  - Interlamina failure (out of plane opening) is dominated by volume increase of matrix material, hence hydrostatic strain invariant: $J_1 = \varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \geq J_1^*$
  - Other failure initiation (distortion, deviatoric) follows by second deviatoric strain invariant

$$J_2' = \frac{1}{6}\left[(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_x - \varepsilon_z)^2\right] - \frac{1}{4}\left(\varepsilon_{xy}^2 + \varepsilon_{yz}^2 + \varepsilon_{xz}^2\right) \geq J_2'^*$$

$$\varepsilon_{vm} = \sqrt{3J_2'} = \left\{1/2 \cdot \left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2\right]\right\}^{1/2}$$

- Von-Mises equivalent strain is tracking variable for deviatoric failure initiation

Hashin Family, Original (2D) (A)

- Break up each identified failure mode, based on observation that each failure modes are competing each other during loading phase (1)~(7) and propose failure function of each modes (Plane Stress Lamina)
- Originally incorporated failure function modes are;

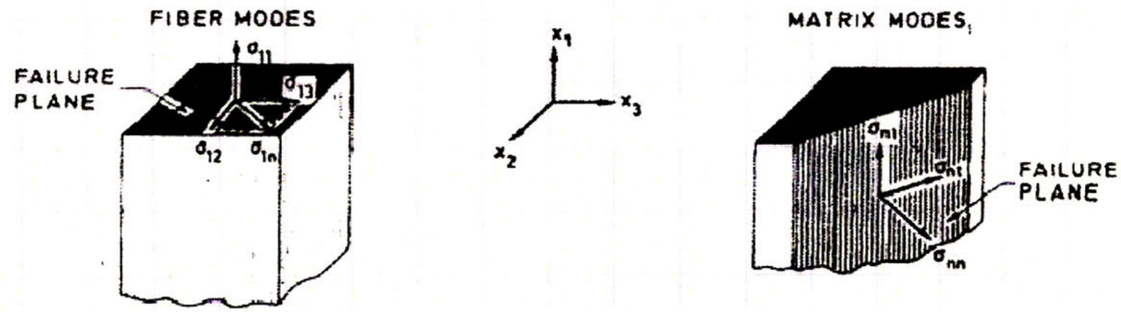
$$\left(\frac{\hat{\sigma}_{11}}{X^T}\right)^2 + \alpha\left(\frac{\hat{\tau}_{12}}{S^L}\right)^2 \geq F_f^T, \hat{\sigma}_{11} \geq 0 \quad \text{Fiber Tension}$$

$$\left(\frac{\hat{\sigma}_{22}}{Y^T}\right)^2 + \left(\frac{\hat{\tau}_{12}}{S^L}\right)^2 \geq F_m^T, \hat{\sigma}_{22} \geq 0 \quad \text{Matrix Tension}$$

$$\left(\frac{\hat{\sigma}_{11}}{X^T}\right)^2 \geq F_f^C, \hat{\sigma}_{11} \leq 0 \quad \text{Fiber Compression}$$

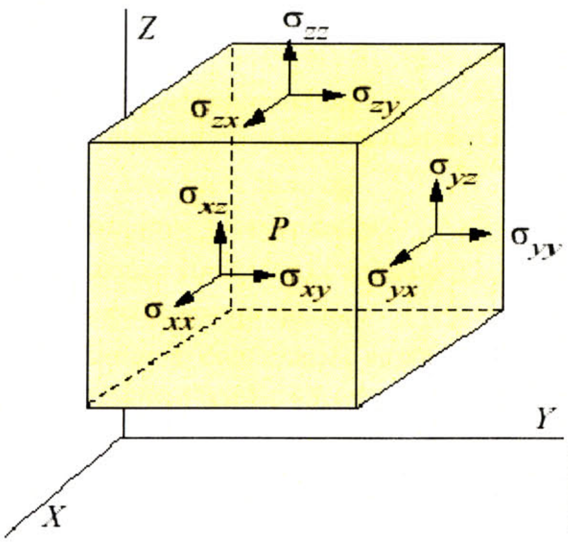
$$\left(\frac{\hat{\sigma}_{22}}{2S^T}\right)^2 + \left[\left(\frac{Y^C}{2S^T}\right)^2 - 1\right]\left(\frac{\hat{\sigma}_{22}}{Y^C}\right) + \left(\frac{\hat{\tau}_{12}}{S^L}\right)^2 \geq F_m^C, \hat{\sigma}_{22} \leq 0 \quad \text{Matrix Compression}$$

X^T longitudinal tensile strength;
X^C longitudinal compressive strength;
Y^T transverse tensile strength;
Y^C transverse compressive strength;
S^L longitudinal shear strength;
S^T transverse shear strength;
α the contribution of the shear stress to the fiber tensile initiation criterion



Hashin Family, Generalized 3D (A)

- Original Hashin Theory extended to 3D (Stress Description)



$$(x, y, z, xy, yz, zx) = (a, b, c, ab, bc, ca) = (1, 2, 3, 12, 23, 31)$$

$$\left(\frac{\langle \sigma_a \rangle}{S_{aT}}\right)^2 + \left(\frac{\tau_{ab}^2 + \tau_{ca}^2}{S_{FS}^2}\right) = f_1 \geq 1$$

Fiber Tension

$$\left(\frac{\langle \sigma_a' \rangle}{S_{aC}}\right)^2 = f_2 \geq 1, \sigma_a' = -\sigma_a + \left\langle -\frac{\sigma_b + \sigma_c}{2} \right\rangle$$

Fiber Compression

$$\left(\frac{\langle p \rangle}{S_{FC}}\right)^2 = f_3 \geq 1$$

Fiber Crushing

$$\left(\frac{\langle \sigma_b \rangle}{S_{bT}}\right)^2 + \left(\frac{\tau_{bc}}{S_{bc}'}\right)^2 + \left(\frac{\tau_{ab}}{S_{ab}}\right)^2 = f_4 \geq 1$$

Transverse Matrix Mode

$$S^2 \left\{ \left(\frac{\langle \sigma_c \rangle}{S_{cT}}\right)^2 + \left(\frac{\tau_{bc}}{S_{bc}''}\right)^2 + \left(\frac{\tau_{ca}}{S_{ca}}\right)^2 \right\} = f_5 \geq 1$$

Delamination Mode

$$S_{ab} = S_{ab}^{(0)} + \tan(\varphi)\langle -\sigma_b \rangle$$

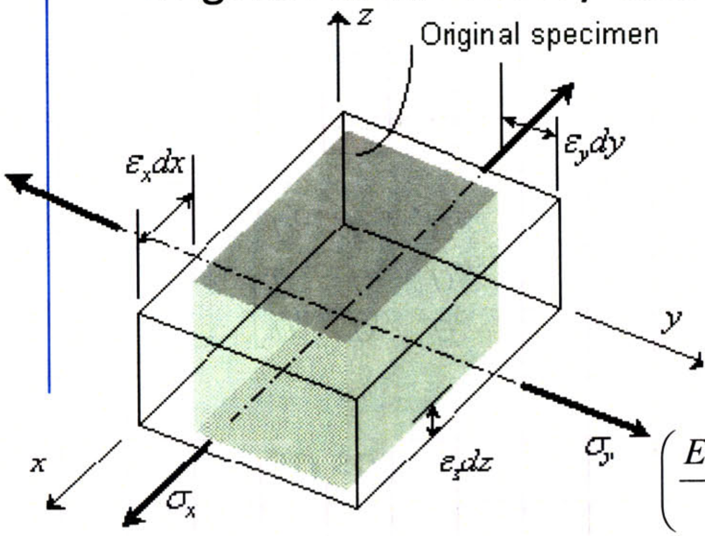
$$S_{bc}' = S_{bc}^{(0)} + \tan(\varphi)\langle -\sigma_b \rangle$$

$$S_{ca} = S_{ca}^{(0)} + \tan(\varphi)\langle -\sigma_c \rangle$$

$$S_{bc}'' = S_{bc}^{(0)} + \tan(\varphi)\langle -\sigma_c \rangle$$

Hashin Family, Generalized 3D (A)

- Original Hashin Theory extended to 3D (Strain Description)



$$\left(\frac{E_a\langle\varepsilon_a\rangle}{S_{aT}}\right)^2+\left(\frac{G_{ab}^2\varepsilon_{ab}^2+G_{ca}^2\varepsilon_{ca}^2}{S_{FS}^2}\right)=f_1\geq 1$$

Fiber Tension

$$\left(\frac{E_a\langle\varepsilon_a'\rangle}{S_{aC}}\right)^2=f_2\geq 1,\ \varepsilon_a'=-\varepsilon_a+\left\langle-\frac{E_b\varepsilon_b+E_c\varepsilon_c}{2E_a}\right\rangle$$

Fiber Compression

$$\left(\frac{E_c\langle\varepsilon_c\rangle}{S_{FC}}\right)^2=f_3\geq 1$$

Fiber Crushing

$$\left(\frac{E_b\langle\varepsilon_b\rangle}{S_{bT}}\right)^2+\left(\frac{G_{bc}\varepsilon_{bc}}{S_{bc0}+S_{srb}}\right)^2+\left(\frac{G_{ab}\varepsilon_{ab}}{S_{ab0}+S_{srb}}\right)^2=f_4\geq 1$$

Transverse Matrix Mode

$$S^2\left\{\left(\frac{\langle\sigma_c\rangle}{S_{cT}}\right)^2+\left(\frac{G_{bc}\varepsilon_{bc}}{S_{bc0}+S_{src}}\right)^2+\left(\frac{G_{ca}\varepsilon_{ca}}{S_{ca0}+S_{src}}\right)^2\right\}=f_5\geq 1$$

Delamination Mode

$$S_{srb}=E_b\tan(\varphi)\langle-\varepsilon_b\rangle$$

$$S_{src}=E_c\tan(\varphi)\langle-\varepsilon_c\rangle$$

Damage Accumulation, Progress Damage Modeling (B)

- Based on observed failure phenomena of certain system, how are we going to describe this in mathematical form ?
- 2D laminate basis and full 3D solid composite basis length scale (Mesoscale), VCCT is direct modeling of fracture interface (Microscale).
- Failure Initiation Criteria (A) is the process of 'detection', Damage accumulation modeling (B) is the process of 'tracking' effect of (A) to the material using mapping process (f1) and updating (A) by updating state (f2)
- (B) has specific length scale to be represented, and they are based on observation and test result (1)~(11), under the setting of using FEA (AKA Generalized Hook's Law) and 3D solid composite, variables representing damage are confined to use;

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{Bmatrix} = \underbrace{\begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0\\ D_{12} & D_{22} & D_{23} & 0 & 0 & 0\\ D_{13} & D_{23} & D_{33} & 0 & 0 & 0\\ & & & D_{44} & 0 & 0\\ & & & & D_{55} & 0\\ & & & & & D_{66}\end{bmatrix}}_{D_{ij}} \begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{13}\\ \gamma_{12}\end{Bmatrix} \Leftarrow \begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{13}\\ \gamma_{12}\end{Bmatrix} \Leftarrow \begin{Bmatrix}u_1\\ u_2\\ u_3\end{Bmatrix} \Leftarrow \{P\} = [K(u, bc, E)]\cdot\{u\}$$

Orthotropic

Typical NL System

- How do we incorporate effect of the system damage under above relationship (equilibrium) ?

Damage Accumulation, Progress Damage Modeling (B)

$$\underbrace{\begin{bmatrix} \frac{1}{(1-\varpi_1)E_a} & \frac{-\nu_{ba}}{E_b} & \frac{-\nu_{ca}}{E_c} & 0 & 0 & 0 \\ \frac{-\nu_{ab}}{E_a} & \frac{1}{(1-\varpi_2)E_b} & \frac{-\nu_{cb}}{E_c} & 0 & 0 & 0 \\ \frac{-\nu_{ac}}{E_a} & \frac{-\nu_{bc}}{E_b} & \frac{1}{(1-\varpi_3)E_c} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{(1-\varpi_4)G_{bc}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{(1-\varpi_5)G_{ac}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{(1-\varpi_6)G_{ab}} \end{bmatrix}}_{C_{damage}}$$

Compliance with damage

$$E_i = (1-\varpi_i)E_{io}$$

$$G_i = (1-\varpi_i)G_{io}$$

Material Form

ω is a variable describing damage (DV)
ω = 0, no damage
ω = 1, complete damage

$$\{\varepsilon\} = \left[C_{damage}\right]\{\sigma\}$$

$$\{\sigma\} = \left[D_{damage}\right]\{\varepsilon\} = \left[C_{damage}\right]^{-1}\{\varepsilon\}$$

How to define DV ?
What is correct functional shape of DV ?
Is this material property ?
Is this loading dependent ?

Line of thought (B) on damage variable

- Linear degrading: make it drop as quickly as possible, constrained by solver robustness (convergence issue when material degrades too fast for implicit solver)
- Quadratic degrading: use second order smoother degrading, which has more 'benign' characteristics using 'smoother decaying', again limited by solver performance
- Script based, in-between increment material swapping does not require direct material degradation during iteration, but with limit. Using Linear solver (NL FEA implies stiffness update using iteration), while updating material (ok state and failed state)
- EFM applies nodal force to affected elements, using linear solver, requires additional mapping (explained later) using generalized micromechanics level library to resolve failure criteria index into corresponding failure mode, and resolving into corresponding nodal force
- Explicit solver based degradation does not have limitation of Implicit solver based approach (**full nonlinear solver by definition**, **any kind of degradation applicable** limited by stable time increment condition), however, Explicit solver has its own advantages and disadvantages
- All DV implementation to FEA implies very high-end FEA deployment, which requires significant care to establish process (Boeing VCCT level process maturity minimum)

MLT, Continuum Damage Mechanics (B) (In Works)

- Proposed by A. Matzenmiller, J. Lubliner, R.L Taylor
- Homogenized continuum is adopted for the constitutive theory of anisotropic damage and elasticity.
- Exponential decay function is proposed, in which several 'degrading characteristics' are treated as 'material property'
  - Fiber damage behavior (brittle), in-plane
  - Fiber crushing behavior (incompressible), out of plane
  - Matrix damage behavior (compliant)
- All (included) failure(damage) state, provided by failure initiation criteria, are evaluated in each increment, picking up the maximum damage parameters to degrade corresponding material property.

$$\phi_j = 1 - \exp\left\{ \frac{1}{m_j} \left(1 - r_j^{m_j} \right) \right\}$$

$\phi_j =$ Degradation Function (DF)

$m_j =$ Softening Variable

$r_j =$ Damage Threshold (DT)

$j =$ Failure Modes

MLT, Continuum Damage Mechanics (B) (In Works)

- If the generalized Hashin criteria is used (j=1~5)

$$E_i = (1-\varpi_i)E_{io}, 1 \le i \le 6 \qquad \phi_j = 1-\exp\left\{\frac{1}{m_j}\left(1-r_j^{m_j}\right)\right\}$$

$$f_j - r_j^2 = 0, r_j \ge 1, 1 \le j \le 5$$

- **How to map effect of 5 damage modes to 6 material coefficients ?**
- f_1~f_3 are fiber damage related
- f_4~f_5 are matrix related, and coupled, therefore following mapping relation is used

$$\varpi_i = \max\left\{q_{ij}\phi_j\right\}, i = 1\ldots 6, j = 1\ldots 5$$

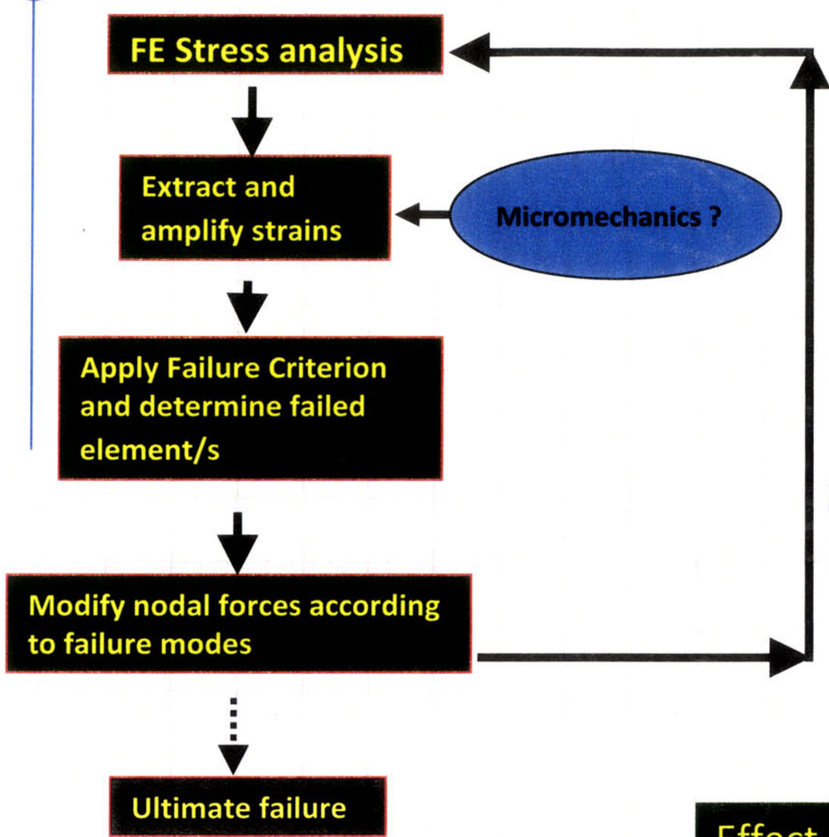
No Summation

$$[q] = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

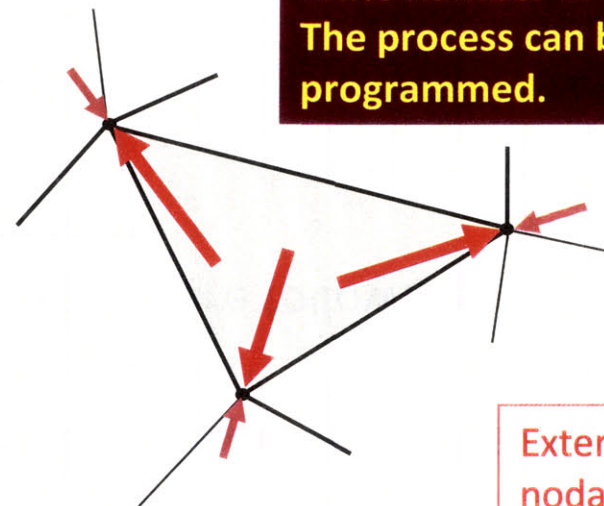
Additional Damage Characteristics to add(B) (In Works)

- When fiber fails in tension, affected material property degrades to zero
- After damage initiated in fiber compression, the residual compressive strength will still take the load, compressed to down to very small volume in FEA model
- Both matrix crack (transverse crack, delamination) are behaving differently depends on 'opening' and 'closing/sliding' state
- This is common for all CFMA, as long as they are following;
  - 3D solid/2D laminate Composite description
  - Equilibrium condition (Hook's Law)
  - Identify failure modes (A), then mapping(f2) damage effect(B) to compliance (stiffness) matrix

Element Failure Method, Tay, T.E. & Tsai (B), from Prof. Tay's slides



This is achieved in a finite number of steps. The process can be programmed.



External applied nodal forces

Net nodal forces of adjacent elements

Effect of damage through *modification of nodal forces only*. The stiffness matrix of the finite element is not altered.

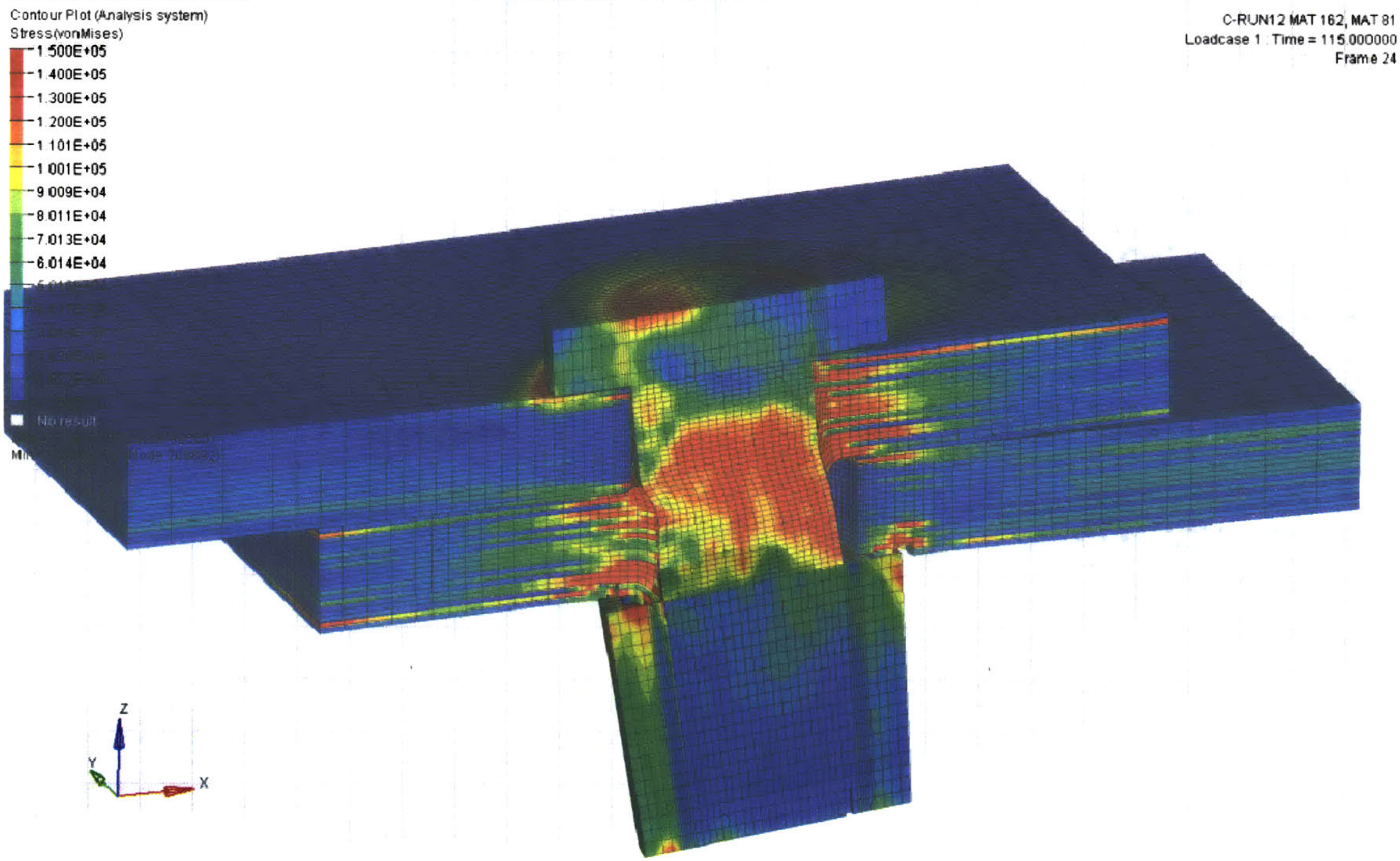
Physical Representation of Damage (C)

- In tension failure, when effective material property of damaged elements becomes zero or near zero (FEA elements likes to extend), as long as solver can take it.
- In compression failure, when effective material property of damaged elements becomes near zero, most of current solver/element (except specialized zero thickness element such as cohesive zone element) does not like it at all (zero/negative volume singularity situation).
- Most of simple lamina failure analysis such as simulating tension dominant, failure initiation in compression does not need to concern much about this domain.
- Problems when failure is governed by high bearing load and direct penetration such as BJ and Thick Armor (hybrid composite armor of tank), this can not be ignored.

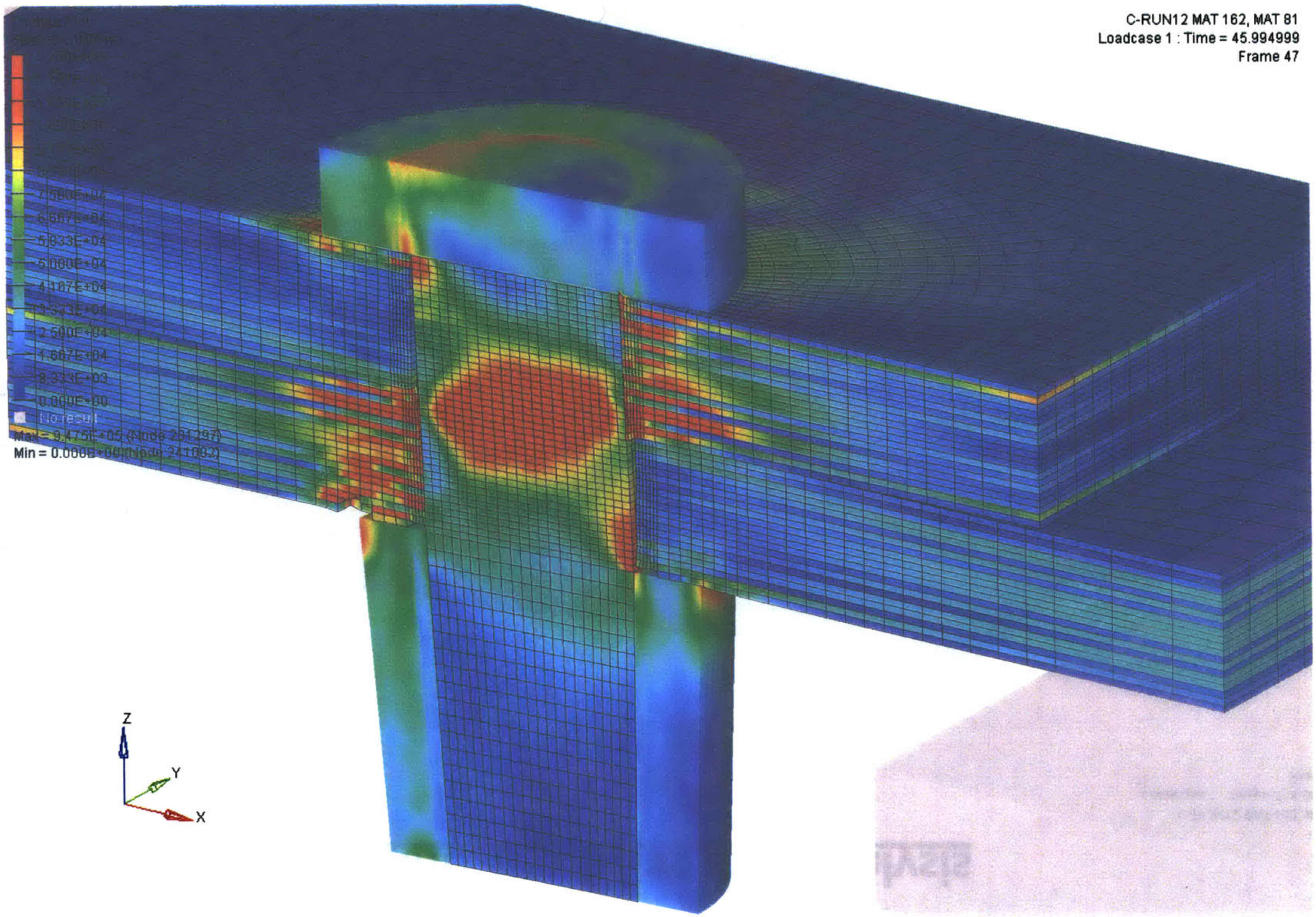
Composite Bolted Joints Modeling and Analysis

- Worst of all, BJ problem happens to have all the possible failure modes acting up almost at the same time, competing each other.
- 3D out-of-plane mode is significant (bolt head/nut pinching)
- Crushing failure is there, manifested by contact interaction.
- Delamination type failure mode (blooming) is significant
- Bolt compliance (tension, bending, compression, shear) affects the entire system, creating fully coupled analysis, by definition full Nonlinear problem.

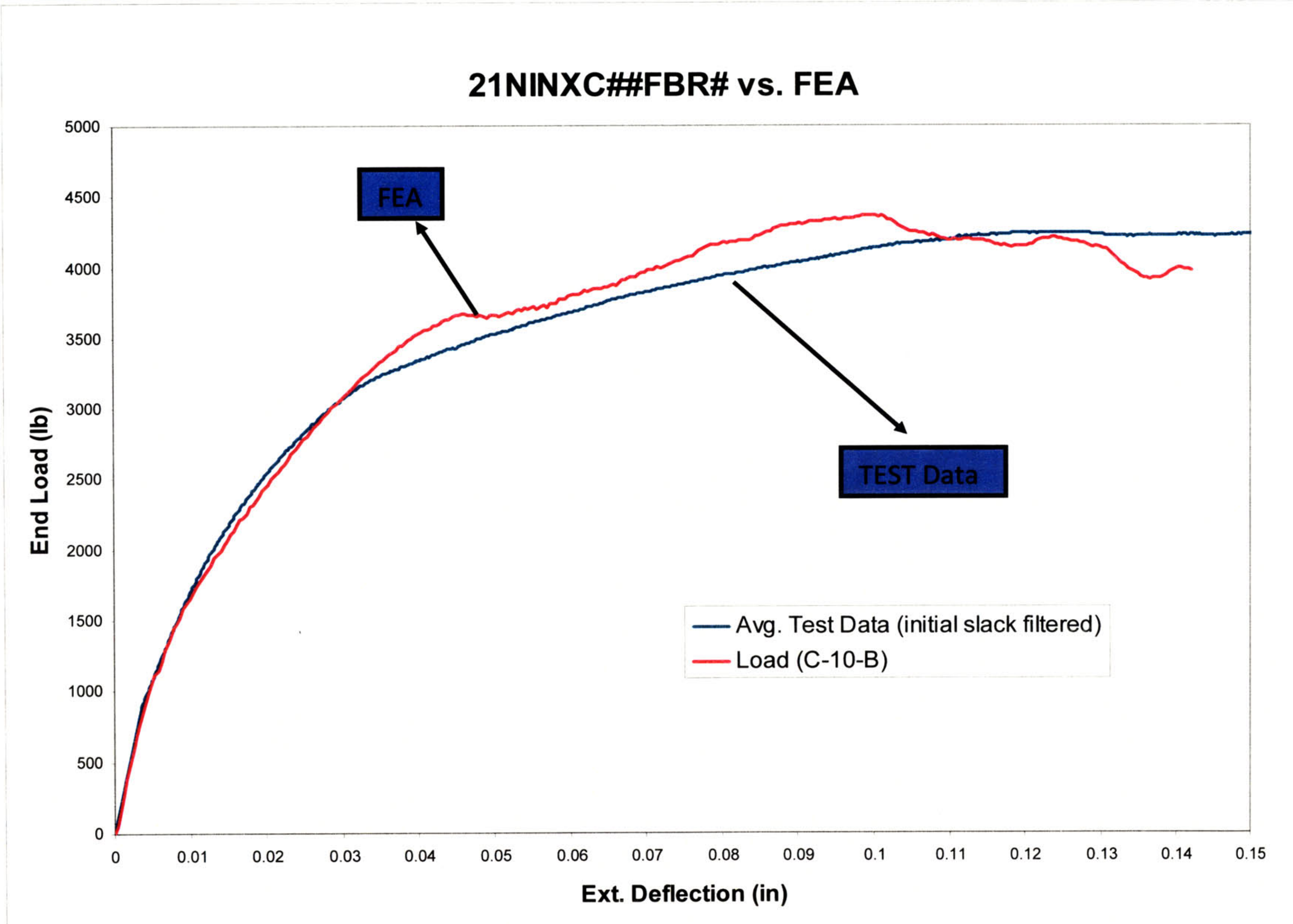
Composite Bolted Joints Modeling and Analysis



Animation


C-RUN12 MAT 162, MAT 81
Loadcase 1 : Time = 45.994999
Frame 47
7.500E+04
6.667E+04
5.833E+04
5.000E+04
4.167E+04
3.333E+04
2.500E+04
1.667E+04
8.333E+03
0.000E+00
No result
Min = 0.000E+00
Z
Y
X

Composite Bolted Joints Modeling and Analysis



Additional Slides

Governing Equation of Motion (Newtonian Mechanics)

$$[M]\cdot[\ddot{u}(X,t)]+[B]\cdot[\dot{u}(X,t)]+[K]\cdot[u(X,t)]=[P(X,t)]$$

[M] = Mass Matrix

[B] = Damping Matrix

[K] = Stiffness Matrix

U = Displacement

P = External Force

External Force will be resolved into;

Inertia (mass x acceleration), Damping (velocity)

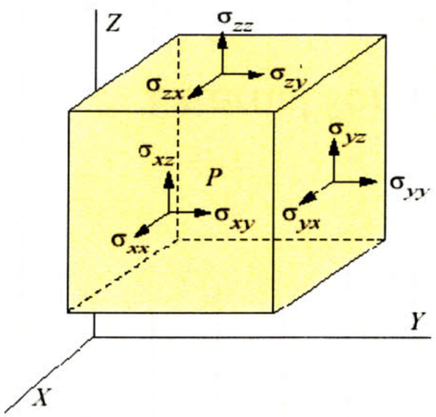
Internal Energy (Stiffness x Deformation)

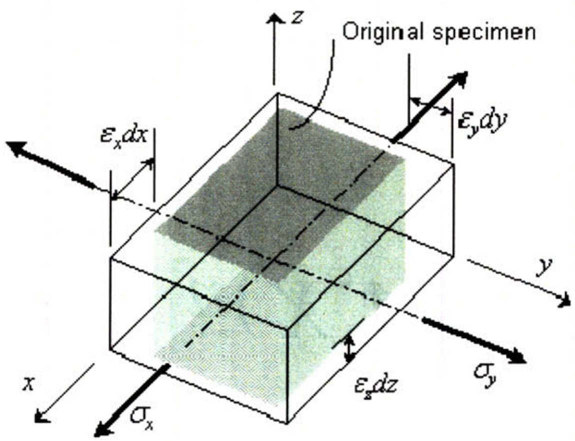
Generalized Hook's Law

- "The power of any springy is the same proportion with the extension" (1676)
- "As the extension, so the force"

$$F = K \cdot u$$

Stiffness K is a function of material and geometry characteristics



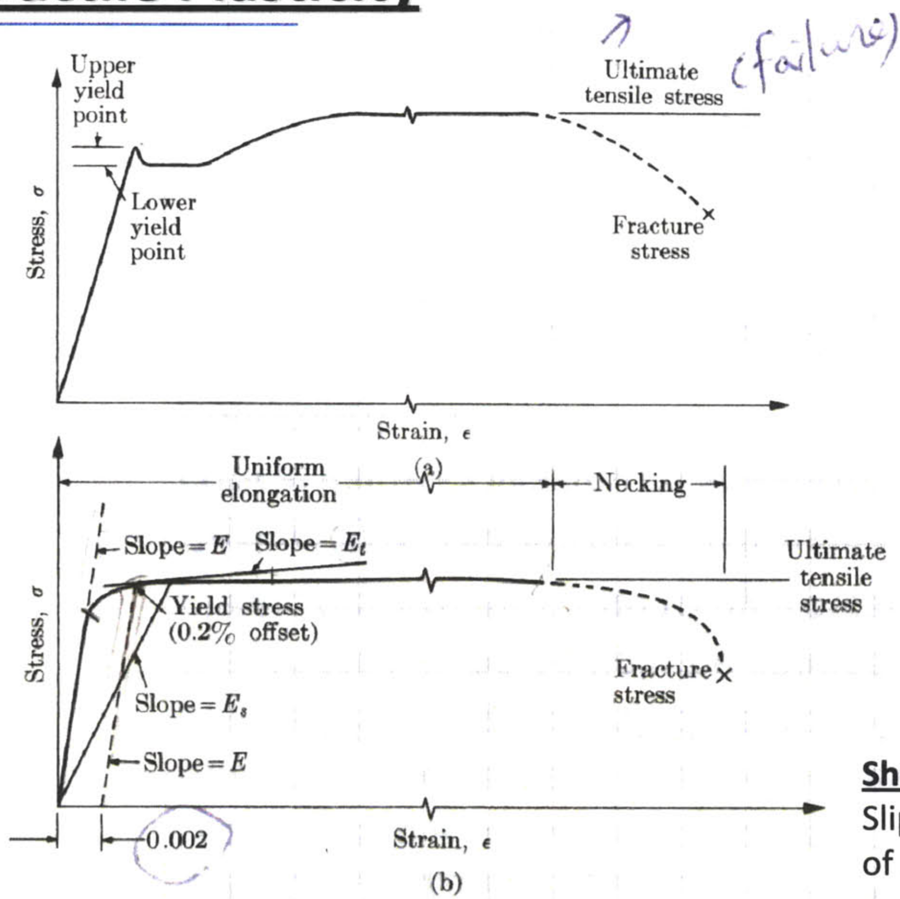


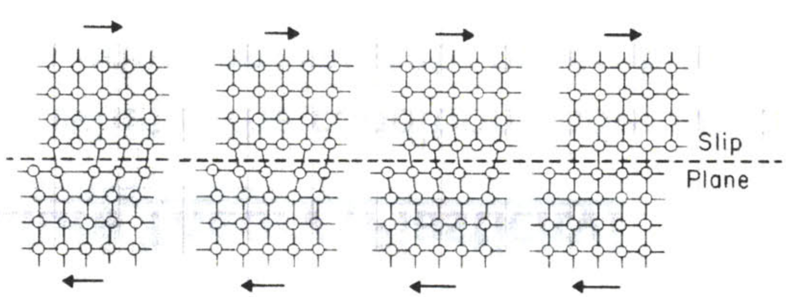
$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{34} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{35} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{36} & D_{64} & D_{65} & D_{66} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix}$$

$$[\varepsilon] = [C] \cdot [\sigma]$$ Compliance Matrix

$$[\sigma] = [C]^{-1} \cdot [\varepsilon] = [D] \cdot [\varepsilon]$$ Stiffness Matrix

Ductile Plasticity



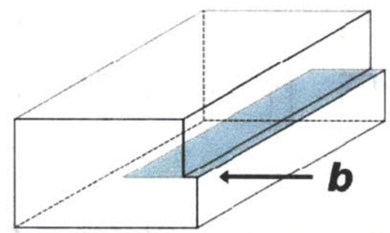


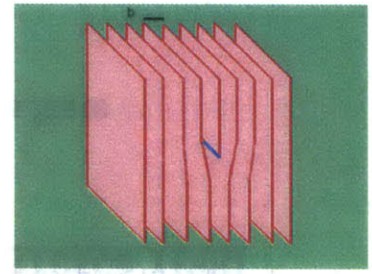
Deformation Theory (Hook's Law)

Stress tensor is a function of strain tensor

Flow Plasticity Theory (slipping)

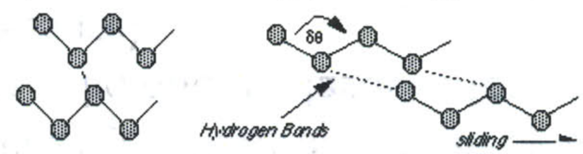
Based on theory of dislocation (physically consistent)





Polymer Plasticity

Polymer chain sliding (above Tg)

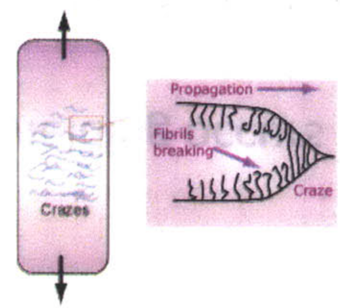


Shear Bands (below Tg)

Slip at 45 degree (max shear) of stress direction

Crazing (below Tg)

Formation of micro-crack bridged by polymer fibrils

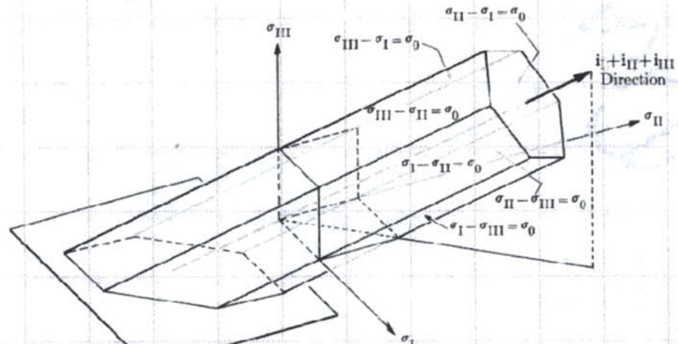


Failure Theory (Plasticity)

- What is the condition under which plastic deformation or yielding occurs under general state of combined stress ?
  - **Observation/Assumption**: plastic deformation of crystalline materials are the result of slip along atomic planes or twin glide displacement. The deformation does not involve permanent changes in the in the distance between lattice planes (shearing action), thus no volume change. Therefore, it can be assumed that there will be no volume change in the solid due to plastic deformation (no plastic deformation due to hydrostatic pressure).
  - **Declaration**: The material will yield when the maximum shear stress reaches critical value -> Maximum Shear-Stress Criterion for Yielding (**Tresca Yield Condition**)

$$(\tau_{\max})_{yield} = \max\left(\frac{1}{2}|\sigma_I - \sigma_{II}|, \frac{1}{2}|\sigma_{II} - \sigma_{III}|, \frac{1}{2}|\sigma_{III} - \sigma_I|\right)$$

3D Stress State

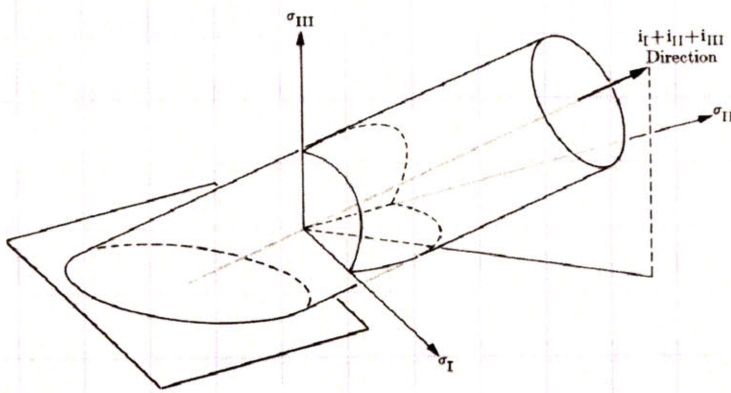


Tresca Yield Surface

Failure Theory (Plasticity), Continues

- What is the condition under which plastic deformation or yielding occurs under general state of combined stress ?
  - **Observation/Assumption**: Observation implies that the yield condition will depend only on the **distortion energy** and be independent of the dilatation energy
  - **Declaration**: Plastic flow will occur when the distortion-energy in the material reaches the value corresponding to the yielding of a simple tensile specimen (**Huber-von Mises-Hencky or Mises-Hencky Theory**)

$$\hat{U}^* = \frac{1}{12G}\left[(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2\right] = \frac{1}{6G}(\sigma_o)^2$$



$$G = \frac{E}{2(1+\nu)}$$

Mises-Hencky Yield Surface

Generalized Failure Criteria (Hill's Stress Function)

- Extended Mises Stress Potential to allow anisotropic behavior

$$A\cdot(\sigma_{11}-\sigma_{22})^2+B\cdot(\sigma_{22}-\sigma_{33})^3+C\cdot(\sigma_{33}-\sigma_{11})^2+2D\cdot\sigma_{12}^2+2E\cdot\sigma_{23}^2+2F\cdot\sigma_{31}^2=1$$

- Each coefficient can be generated based on testing value with controlled loading condition
- Alternate Form; "Volume remains unchanged during plastic deformation"

$$d\varepsilon_{ij}^{pl}=\hat{\sigma}_{ij}d\lambda=\frac{\partial f\left(\sigma_{ij}\right)}{\partial\sigma_{ij}}d\lambda$$

During plastic deformation, the ratio between plastic strain increment and corresponding deviatoric stress components remains constant (dλ = positive scalar factor of proportionality)

$$f(\boldsymbol{\sigma})=\sqrt{F(\sigma_{22}-\sigma_{33})^2+G(\sigma_{33}-\sigma_{11})^2+H(\sigma_{11}-\sigma_{22})^2+2L\sigma_{23}{}^2+2M\sigma_{31}{}^2+2N\sigma_{12}{}^2},$$

$$F=\frac{\sigma_0^2}{2}\left(\frac{1}{\bar{\sigma}_{22}^2}+\frac{1}{\bar{\sigma}_{33}^2}-\frac{1}{\bar{\sigma}_{11}^2}\right),$$

$$G=\frac{\sigma_0^2}{2}\left(\frac{1}{\bar{\sigma}_{33}^2}+\frac{1}{\bar{\sigma}_{11}^2}-\frac{1}{\bar{\sigma}_{22}^2}\right),$$

$$H=\frac{\sigma_0^2}{2}\left(\frac{1}{\bar{\sigma}_{11}^2}+\frac{1}{\bar{\sigma}_{22}^2}-\frac{1}{\bar{\sigma}_{33}^2}\right),\qquad \tau^0=\sigma^0/\sqrt{3}\qquad d\boldsymbol{\varepsilon}^{pl}=d\lambda\frac{\partial f}{\partial\boldsymbol{\sigma}}=\frac{d\lambda}{f}\mathbf{b},\qquad \mathbf{b}=\begin{bmatrix}-G(\sigma_{33}-\sigma_{11})+H(\sigma_{11}-\sigma_{22})\\ F(\sigma_{22}-\sigma_{33})-H(\sigma_{11}-\sigma_{22})\\ -F(\sigma_{22}-\sigma_{33})+G(\sigma_{33}-\sigma_{11})\\ 2N\sigma_{12}\\ 2M\sigma_{31}\\ 2L\sigma_{23}\end{bmatrix}$$

$$L=\frac{3}{2}\left(\frac{\tau_0}{\bar{\tau}_{23}}\right)^2,$$

$$M=\frac{3}{2}\left(\frac{\tau_0}{\bar{\tau}_{13}}\right)^2,$$

$$N=\frac{3}{2}\left(\frac{\tau_0}{\bar{\tau}_{12}}\right)^2,$$

Few Additional Relationship

Characteristic equation

$$\sigma_p^3 - I_1\sigma_p^2 + I_2\sigma_p - I_3 = 0$$

Stress Invariants

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$$

$$I_3 = \sigma_1\sigma_2\sigma_3$$

Maximum Shear Stress

$$\tau_{max} = \left| max\left(\frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2 - \sigma_3}{2}, \frac{\sigma_3 - \sigma_1}{2}\right) \right|$$

Characteristic equation

$$\varepsilon_p^3 - I_1\varepsilon_p^2 + I_2\varepsilon_p - I_3 = 0$$

Strain invariants

$$I_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

$$I_2 = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{vmatrix} + \begin{vmatrix} \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zy} & \varepsilon_{zz} \end{vmatrix} + \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xz} \\ \varepsilon_{zx} & \varepsilon_{zz} \end{vmatrix} = \varepsilon_1\varepsilon_2 + \varepsilon_2\varepsilon_3 + \varepsilon_3\varepsilon_1$$

$$I_3 = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{vmatrix} = \varepsilon_1\varepsilon_2\varepsilon_3$$

Maximum shear strain

$$\frac{\gamma_{max}}{2} = \left| max\left(\frac{\varepsilon_1 - \varepsilon_2}{2}, \frac{\varepsilon_2 - \varepsilon_3}{2}, \frac{\varepsilon_3 - \varepsilon_1}{2}\right) \right|$$

Few Additional Relationship (Plasticity)

Under Maximum Distortion Energy Theory

$$(\sigma_I)^2 - \sigma_I \sigma_{II} + (\sigma_{II})^2 = (\sigma_0)^2$$

$$\sigma_I = \sigma/2 + \sqrt{(\sigma/2)^2 + (\tau)^2}, \sigma_{II} = \sigma/2 - \sqrt{(\sigma/2)^2 + (\tau)^2}$$

$$(\sigma)^2 + 3(\tau)^2 = (\sigma_0)^2 \rightarrow \sigma = \sqrt{3}\tau$$

$$\sigma_{vm} = \sqrt{\frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2\right] + 3(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)} = \sigma_{yp}$$

Deformation Theory

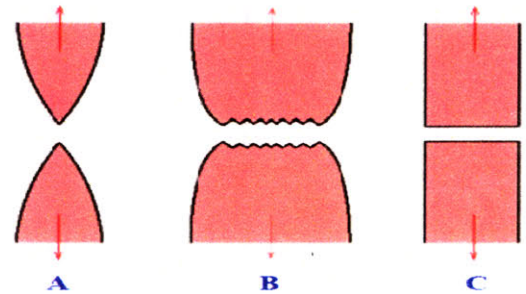
$$\varepsilon_{total} = \varepsilon_{elastic} + \varepsilon_{plastic}$$

$$d\lambda = \frac{d\varepsilon_{ij}^p}{\hat{\sigma}_{ij}} = \frac{3}{2}\frac{d\bar{\varepsilon}^p}{\bar{\sigma}}$$

$$d\varepsilon_{ij}^p = \frac{3}{2}\hat{\sigma}_{ij}\frac{d\bar{\varepsilon}^p}{\bar{\sigma}}$$

$$\varepsilon_{ij}^p = \frac{3}{2}\frac{\hat{\sigma}_{ij}}{E_s^p}$$

$$d\bar{\varepsilon}^p = \sqrt{\frac{2}{9}\left[(d\bar{\varepsilon}_I^p - d\bar{\varepsilon}_{II}^p)^2 + (d\bar{\varepsilon}_{II}^p - d\bar{\varepsilon}_{III}^p)^2 + (d\bar{\varepsilon}_I^p - d\bar{\varepsilon}_{III}^p)^2\right]}$$

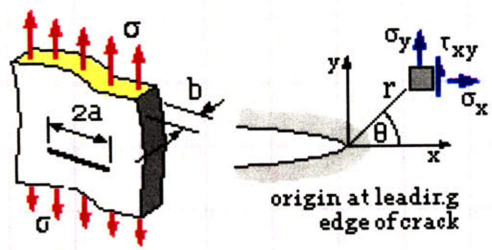


- **A. Very ductile**, soft metals at room temperature, other metals, polymers, glasses at high temperature.
- **B. Moderately ductile fracture**, typical for ductile metals
- **C. Brittle fracture,** cold metals, ceramics.

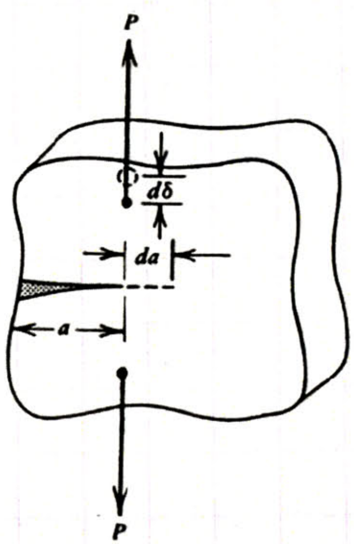
Linear Elastic Fracture Mechanics (L.E.F.M. Damage Tolerance)

- Given initial damage (after fatigue crack initiation, manufacturing defect, local damage due to certain event (impact for example), how much time does it take for crack to become diverging (stable crack growth ?, catastrophic failure ?, or no growth ?)

$$Pd\delta = dU + Gda + dE_p + dE_k$$



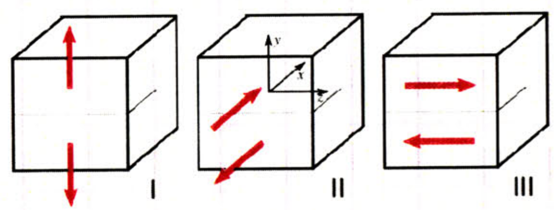
Assumption
- Crack is already initiated
- Material is Linear Elastic (Hooks Law)
- Quasi-Static
- Plasticity zone near crack tip is very small



$$Gda = Pd\delta - dU = Pd\delta - \frac{1}{2}Pd\delta$$

$$G = \frac{1}{2}P\frac{d\delta}{da} = \frac{1}{2}P^2 d\left(\frac{\delta}{P}\right)\Big/da = \frac{1}{2}P^2 dC\Big/da$$

G = Energy Release Rate
C = Compliance = δ / P



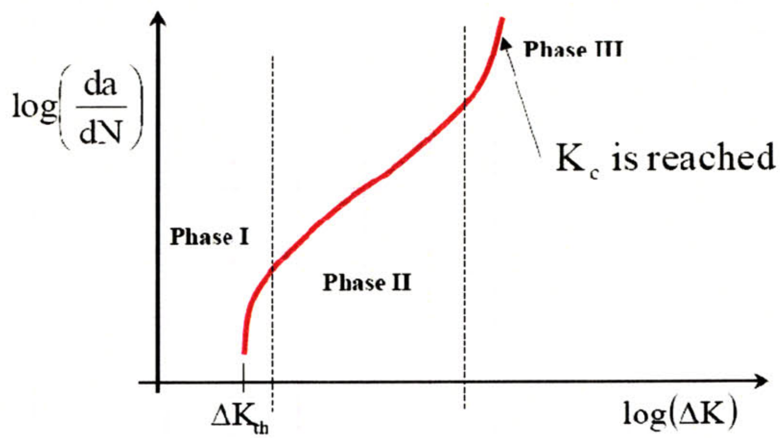
Mode I, II, III Energy Release Rate: G_I, G_{II}, G_{III}

$K_{1C}{}^2 = E^* \cdot G_{1C}$	K = Fracture Toughness
$E^* = E$	Plain Stress
$E^* = E/(1-\nu^2)$	Plain Strain

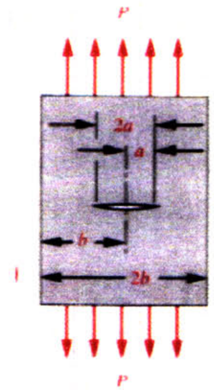
Linear Elastic Fracture Mechanics (L.E.F.M. Damage Tolerance)

- Stress field around crack tip described by K (stress intensity factor)
- Crack grows when stress field reaches critical dimension K_{1C} (fracture Toughness)

Crack Growth Rate Curve



Phase I: Crack initiation
Phase II: Stable crack-growth
Phase III: Unstable crack-growth (fracture)



Stress Intensity Equation

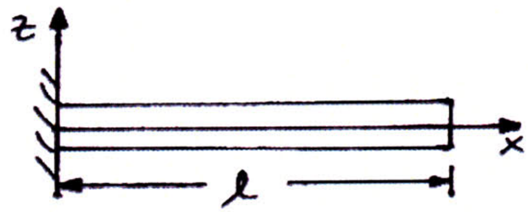
$$K = Y\sigma_{nom}\sqrt{\pi \cdot a}$$

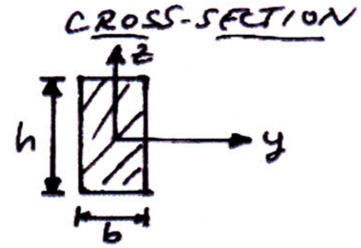
$$da = f(\alpha, \Delta K^m) \cdot dN$$

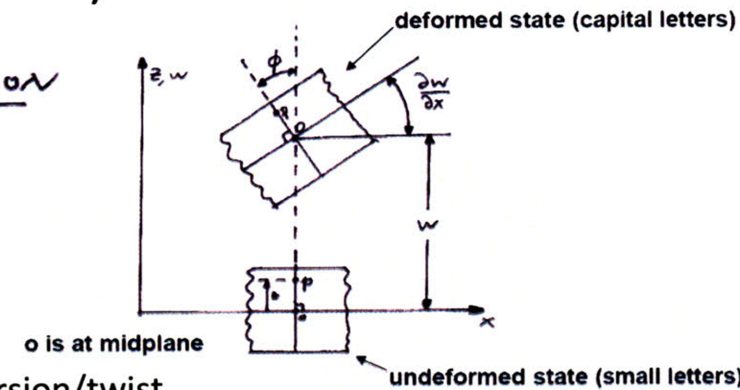
Then integrated to calculate cycle to critical crack length

Beam Theory

- Euler-Bernoulli Beam Theory (Simple Beam)







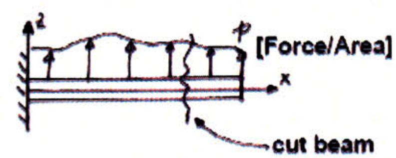
  - Long and thin (ℓ >> b, h), Loading in z-direction, no torsion/twist
  - No stress in y-direction (σ_{yy}, σ_{yz}, σ_{xy} = 0), $\sigma_{xx} >> \sigma_{zz}$, $\sigma_{xz} >> \sigma_{zz}$
  - Plane sections remain plane and perpendicular to the mid-plane (Bernoulli-Euler Hypothesis)

$$u(x,y,z) \approx -z\phi \approx -z\frac{dw}{dx}$$

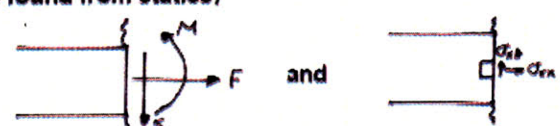
$$v(x,y,z) = 0$$

$$w(x,y,z) \approx w(x)$$

$$\sigma_{xx} = E\varepsilon_{xx} = -E\cdot z\cdot u' = -E\cdot z\cdot w''$$



(F, S, M found from statics)



$$M = EIw''$$

$$\sigma_{xx} = -\frac{Mz}{I}$$

$$F = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xx} b\, dz$$

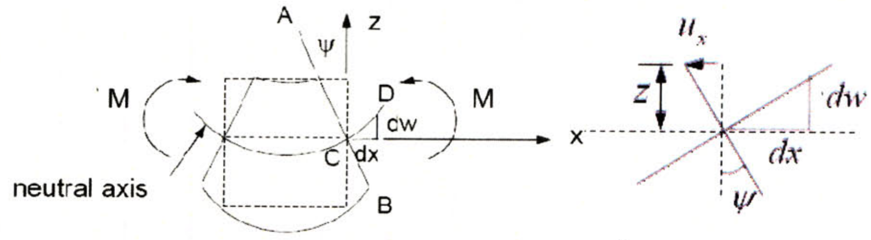
$$S = -\int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xz} b\, dz$$

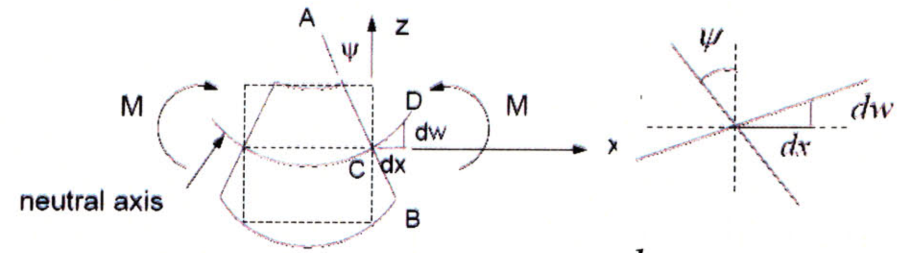
$$M = -\int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_{xx} b\, dz$$

$$I = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 b\, dz$$

General Beam Theory (Timoshenko Beam)

- For shorter (deeper) beam, shearing deformation needs to be incorporated
- Deformed plane is no longer perpendicular to neutral axis





$$u_x = -z\psi(x) = -z\frac{dw}{dx}$$

$$\sigma_{xx} = -E\,z\,\frac{d^2w}{dx^2}$$

$$\sigma_{xz} = G\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) = G\left(\frac{dw}{dx} - \frac{dw}{dx}\right) = 0$$

Euler Beam

$$u_x = -z\psi(x) \neq -z\frac{dw}{dx}$$

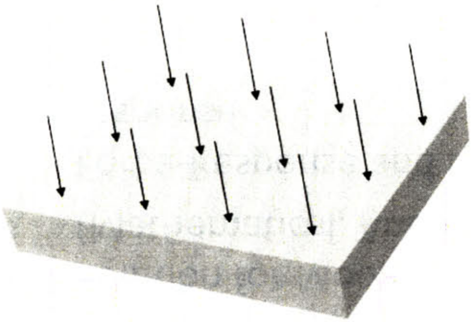
$$\sigma_{xx} = -E\,z\,\frac{d\psi}{dx}$$

$$\sigma_{xz} = G\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) = G\left(-\psi(x) + \frac{dw}{dx}\right)$$

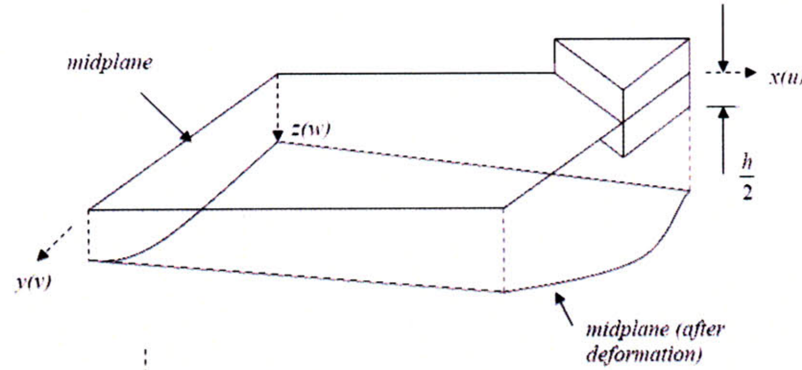
Timoshenko Beam

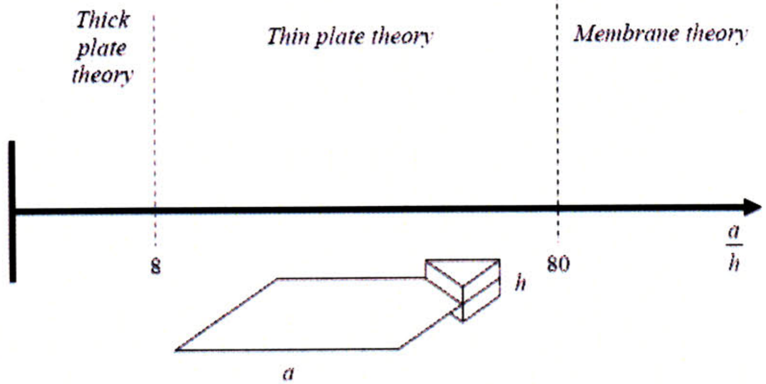
- Shear Deformation is included, suitable for description of sandwich composite beams

Plate Theory (Classical, Kirchhoff's Plate Theory)



$$\nabla^4 w = \frac{p(x, y)}{D}$$





$$\left(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}\right) \cdot w = \frac{P}{D}$$

$$D = \frac{Et^3}{12(1-v^2)}$$

- Linear, elastic, small-deflection theory for the bending of thin plates
- Slope of the deflected surface is small
- Plane normal to midplane remains normal and straight after deflection (no shear in xy & yz), midplane surface remains unstrained
- σ_{zz} is small compared with other stress components
- Two dimensional generalization of beam theory

Stability of Structure

$$[M]\cdot[\ddot{u}(X,t)]+[K]\cdot[u(X,t)]=[P(X,t)]$$

- Conventional terms: (Global/Local) Buckling, Natural Frequency, Resonance, Flutter, etc
- Refers condition when there is no unique displacement solution for a given loading condition (AKA Eigenvalue Problem)

$$[A-\lambda I]\{u\}=0$$
$$\det[A-\lambda I]=0$$

Eigenvalue Problem

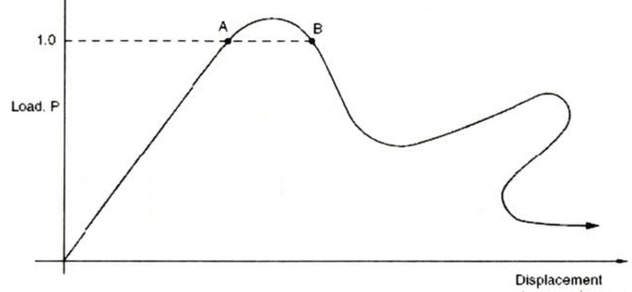
$$[K]\{u\}=\{P\}$$
$$([K]-\lambda[I])\{u\}=0$$
$$\det\left\|[K]u-\lambda[I]\right\|=0\rightarrow\lambda_i=P_{cr,i}$$

Buckling Problem

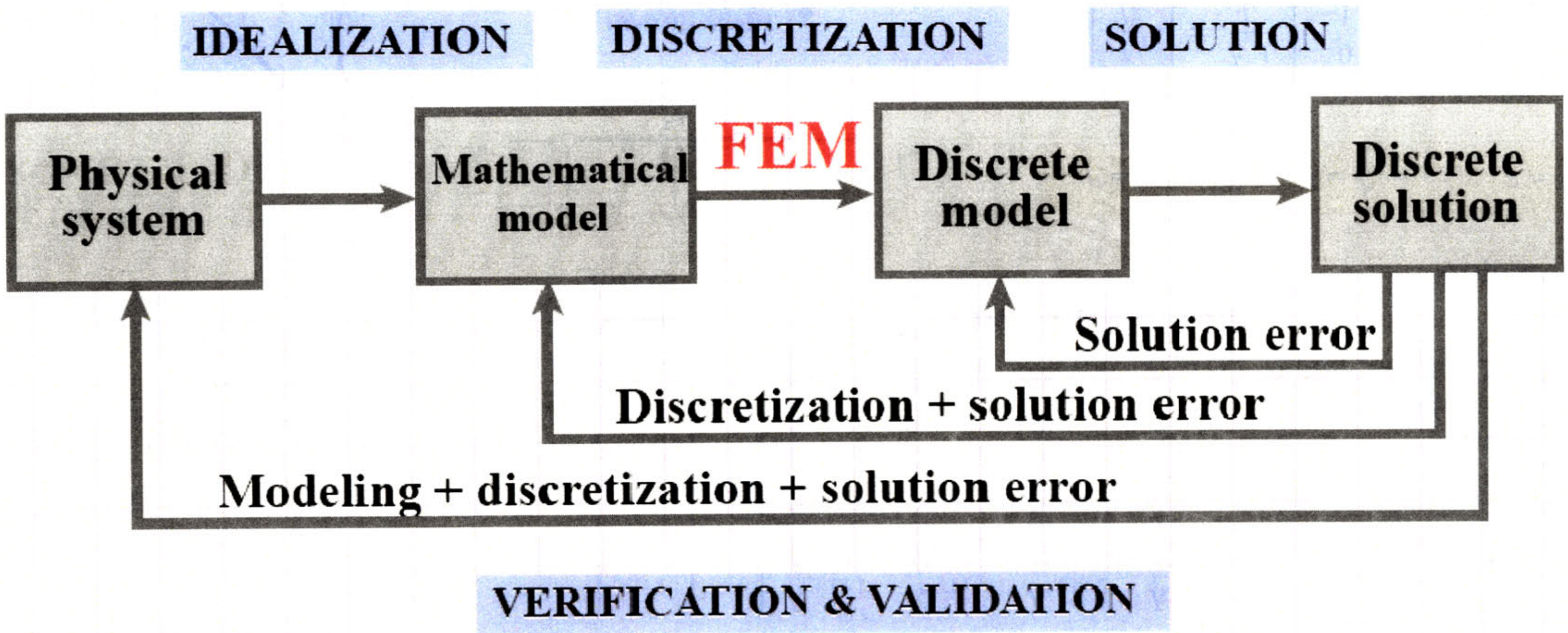
$$[M]\{\ddot{u}\}+[K]\{u\}=0$$
$$([K]-\omega^2[M])\{\phi\}=0$$
$$\det\left\|[K]-\omega^2[M]\right\|=0\rightarrow f_i=\frac{\omega_i}{2\pi}$$

Natural Frequency

- Solution for Eigenvalue (Linear Buckling Analysis) gives Eigenvalue Buckling Mode (Mathematical, Anti-Conservative)
- Solution for Modal Analysis (Natural Frequency Analysis) gives Frequency Mode (Mathematical, Anti-Conservative)
- Force-Response and Full Nonlinear Stability Analysis give full solution of given system response



Understanding FEA (solving for displacement)



$$[F] = [K] \cdot [u]$$

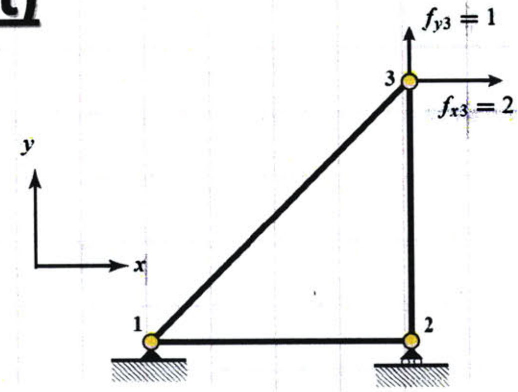
$$[u] = [K]^{-1} [F]$$

Linear FEA (solving for displacement)

- Linear Static

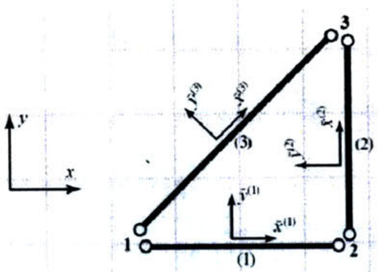
$$[F]=[K]\cdot[u]$$

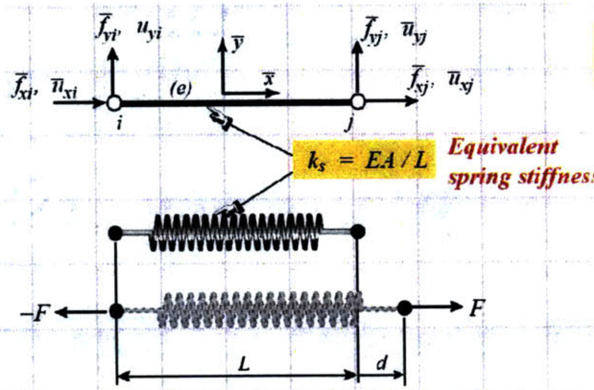
$$[u]=[K]^{-1}\cdot[u]$$



$$\begin{bmatrix} f_{x1}\\ f_{y1}\\ f_{x2}\\ f_{y2}\\ f_{x3}\\ f_{y3}\end{bmatrix}=\begin{bmatrix} K_{x1x1} & K_{x1y1} & K_{x1x2} & K_{x1y2} & K_{x1x3} & K_{x1y3}\\ K_{y1x1} & K_{y1y1} & K_{y1x2} & K_{y1y2} & K_{y1x3} & K_{y1y3}\\ K_{x2x1} & K_{x2y1} & K_{x2x2} & K_{x2y2} & K_{x2x3} & K_{x2y3}\\ K_{y2x1} & K_{y2y1} & K_{y2x2} & K_{y2y2} & K_{y2x3} & K_{y2y3}\\ K_{x3x1} & K_{x3y1} & K_{x3x2} & K_{x3y2} & K_{x3x3} & K_{x3y3}\\ K_{y3x1} & K_{y3y1} & K_{y3x2} & K_{y3y2} & K_{y3x3} & K_{y3y3}\end{bmatrix}\begin{bmatrix} u_{x1}\\ u_{y1}\\ u_{x2}\\ u_{y2}\\ u_{x3}\\ u_{y3}\end{bmatrix}$$

Nodal forces **Master stiffness matrix** **Nodal displacements**



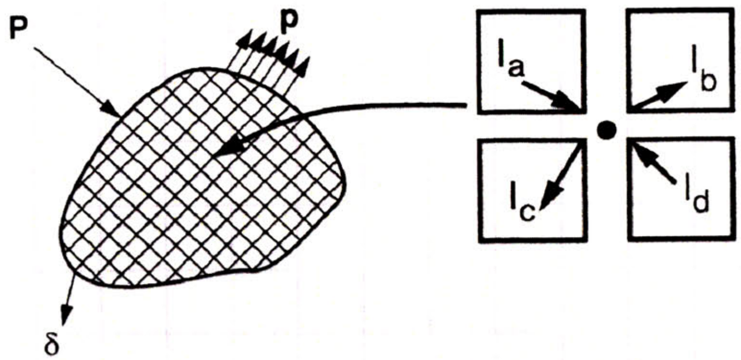


$$F=k_s d=\frac{EA}{L}d,\quad F=\bar{f}_{xj}=-\bar{f}_{xi},\quad d=\bar{u}_{xj}-\bar{u}_{xi}$$

$$\begin{bmatrix}\bar{f}_{xi}\\ \bar{f}_{yi}\\ \bar{f}_{xj}\\ \bar{f}_{yj}\end{bmatrix}=\frac{EA}{L}\begin{bmatrix}1 & 0 & -1 & 0\\ 0 & 0 & 0 & 0\\ -1 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\bar{u}_{xi}\\ \bar{u}_{yi}\\ \bar{u}_{xj}\\ \bar{u}_{yj}\end{bmatrix}$$

Nonlinear FEA

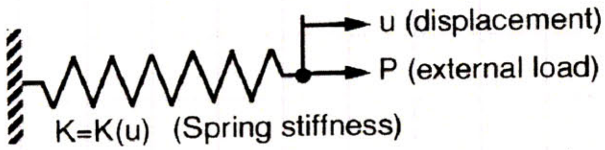
- Nonlinear Static



$$P = K(u, \varepsilon, ...) \cdot u$$

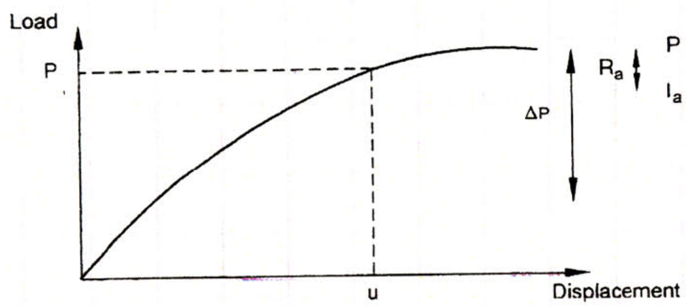
$$dP = K \cdot du + u \cdot dK$$

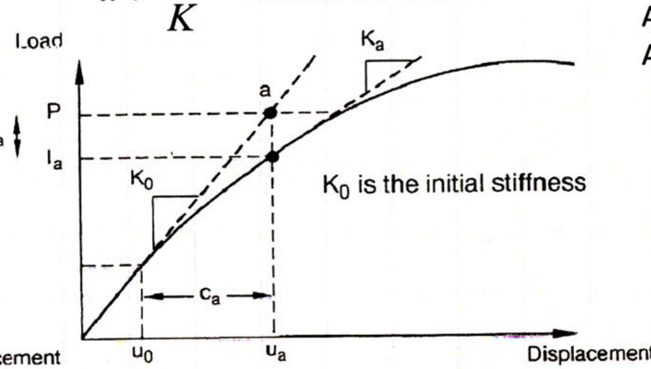
Needs to be solved Iterative (Newton-Raphson)



$u = \frac{P}{K}$ Linear Static

$u \neq \frac{P}{K}$ Nonlinear Static





Assuming smooth, small change in stiffness
Applying incremental force dP

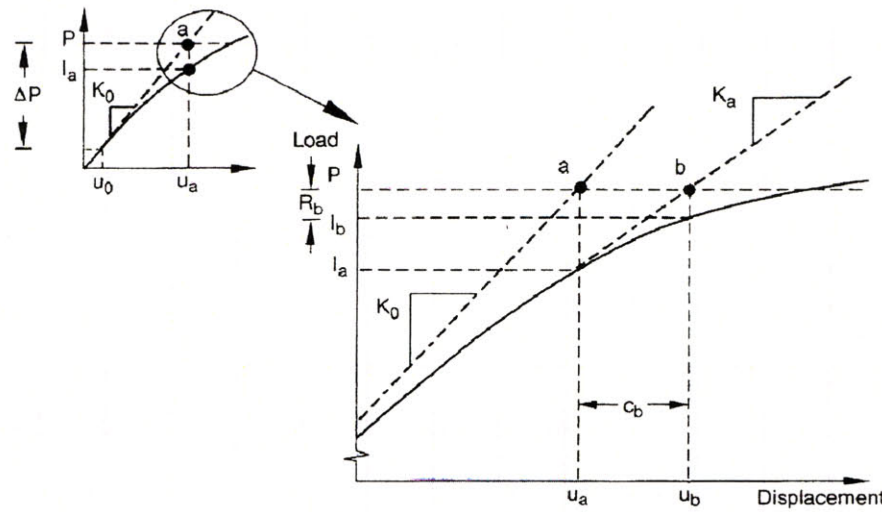
$u_a = u_o + c_a$ C_a = Displacement Correction

$$K_a = \frac{\partial P}{\partial u} - \frac{\partial I}{\partial u}$$

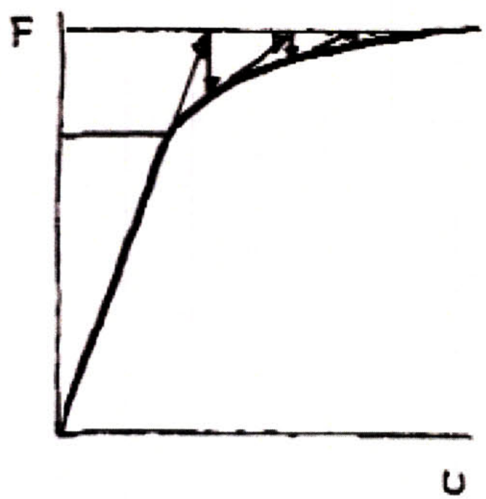
$$I_a = I_a(u_a)$$

$R_a = P - I_a$ R_a is force residual used as convergence criteria

Nonlinear FEA (Continues)



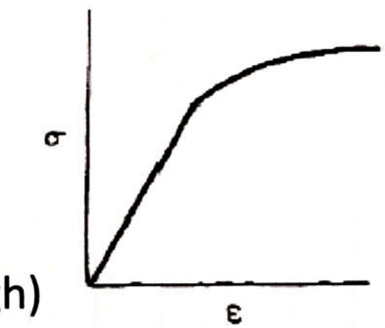
- If R_a is not small, do second iteration
  - $K_a\, c_b = R_a$
  - New configuration of spring, u_b is based on c_b
  - The stiffness of updated configuration K_b, and updated spring force I_b
  - New force residual R_b is then calculated



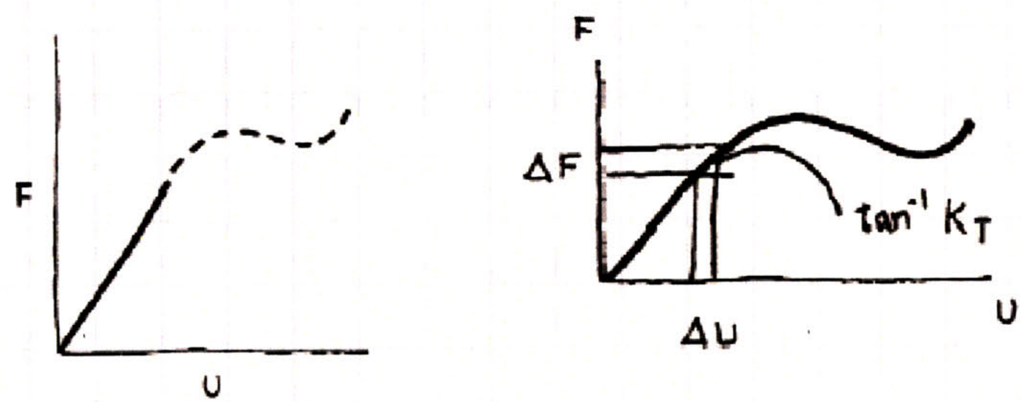
- Convergence criteria for equilibrium iteration is based on residuals (specific number depends on solver and/or can be specified: **convergence criteria is key pointer of solver robustness and maturity**), balance between accuracy and computation cost (**ABAQUS & MARC**)
- If the equilibrium iteration does not meet convergence criteria after certain iteration, incremental load dP is cutback in specified scheme (**solver maturity**)

Source of Nonlinearity (In Progress)

- Geometrical Nonlinearity
  - Large Displacement
  - Large Deformation
  - Structural instability (buckling, post buckling, snap-through)
- Material Nonlinearity
  - Plasticity, Hyperelasticity (rubber), Hypoelasticity
  - Creep
  - Viscoelasticity
  - Viscoplasticity
- Boundary Nonlinearity
  - Gap
  - Contact
  - Follower Force



Nonlinear Stress-Strain



Nonlinear Force-Displacement

Is there any "Linear Static Problem" in a real world ?
Everything is "Nonlinear Dynamic"

Implicit Solver (ABAQUS Standard, MARC, NASTRAN, LS-DYNA Implicit)

- Newmark Integration Method

$$\{\dot{u}\}_{t+\Delta t} = \{\dot{u}\}_t + \Delta t\left[(1-\gamma)\{\ddot{u}\}_t + \gamma\{\ddot{u}\}_{t+\Delta t}\right]$$

$$\{u\}_{t+\Delta t} = \{u\}_t + \Delta t\{\dot{u}\}_t + \Delta t^2\left[\left(\frac{1}{2}-\beta\right)\{\ddot{u}\}_t + \beta\{\ddot{u}\}_{t+\Delta t}\right]$$

- Implicit (unconditionally stable)

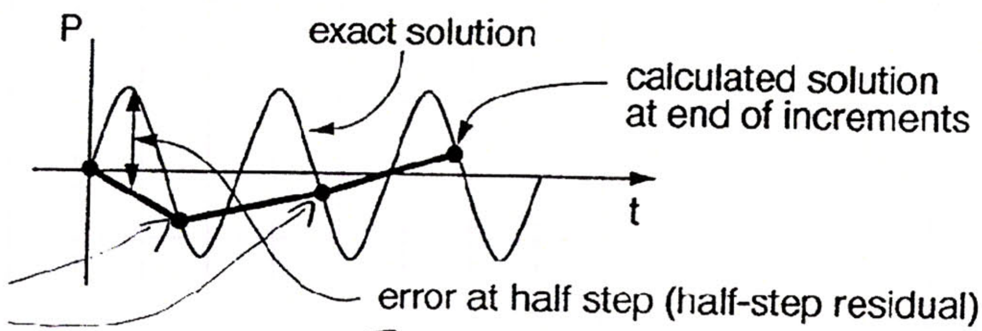
$$\{\dot{u}\}_{t+\Delta t} = \{\dot{u}\}_t + 1/2\cdot\Delta t\left[\{\ddot{u}\}_t + \{\ddot{u}\}_{t+\Delta t}\right]$$

$\beta = 1/4, \gamma = 1/2$

$$\{u\}_{t+\Delta t} = \{u\}_t + \Delta t\{\dot{u}\}_t + 1/4\cdot\Delta t^2\left[\{\ddot{u}\}_t + \{\ddot{u}\}_{t+\Delta t}\right]$$

$$[M]\{\ddot{u}\}_{t+\Delta t} = P - [C]\{u\}_{t+\Delta t} - [K]\{u\}_{t+\Delta t}$$

- Above equations are solved using Newton-Raphson Iteration to solve for displacement at each time of increment. (expensive matrix inversion for tangential stiffness)
- Solution is unconditionally stable with any time step increase (**advantage**)
- Solution accuracy and time stepping is based on **half-step residual control**
- Does not mean the solution is accurate for extremely transient problem (AKA failure, impact)
- Additional error checking scheme in-between iteration is necessary to ensure solution accuracy (analyst experience)



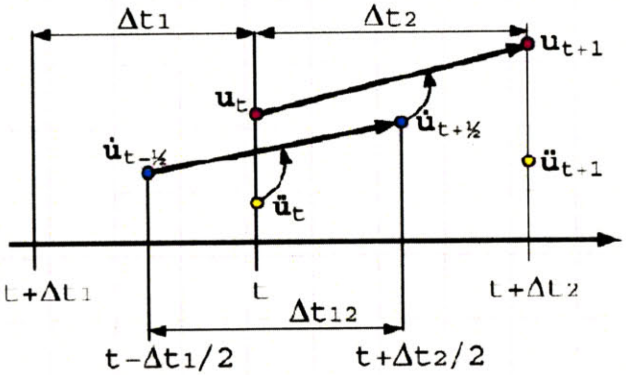
Explicit Solver (LS-DYNA, RADIOS, ABAQUS Explicit, MARC Explicit)

- Newmark Integration Method $\{\dot{u}\}_{t+\Delta t} = \{\dot{u}\}_t + \Delta t\left[(1-\gamma)\{\ddot{u}\}_t + \gamma\{\ddot{u}\}_{t+\Delta t}\right]$

$$\{u\}_{t+\Delta t} = \{u\}_t + \Delta t\{\dot{u}\}_t + \Delta t^2\left[\left(\frac{1}{2}-\beta\right)\{\ddot{u}\}_t + \beta\{\ddot{u}\}_{t+\Delta t}\right]$$

- Explicit (**conditionally stable**), Central Difference Integration Rule

$\beta = 0, \gamma = 1/2$



$$\{\dot{u}\}_{t+1/2} = \{\dot{u}\}_{t-1/2} + \Delta t_{12}\left[\{\ddot{u}\}_t\right]$$

$$\{u\}_{t+1} = \{u\}_t + \Delta t_2\{\dot{u}\}_{t+1/2}$$

$$[M]\{\ddot{u}\}_{t+1} = P - [C]\{u\}_{t+1/2} - [K]\{u\}_{t+1}$$

Assuming, $[C]\{\dot{u}\}_{t+1} \approx [C]\{\dot{u}\}_{t+1/2}$

$$[M]\{\ddot{u}\}_{t+1} = P - \underbrace{[C]\{u\}_{t+1} - [K]\{u\}_{t+1}}_{I}$$

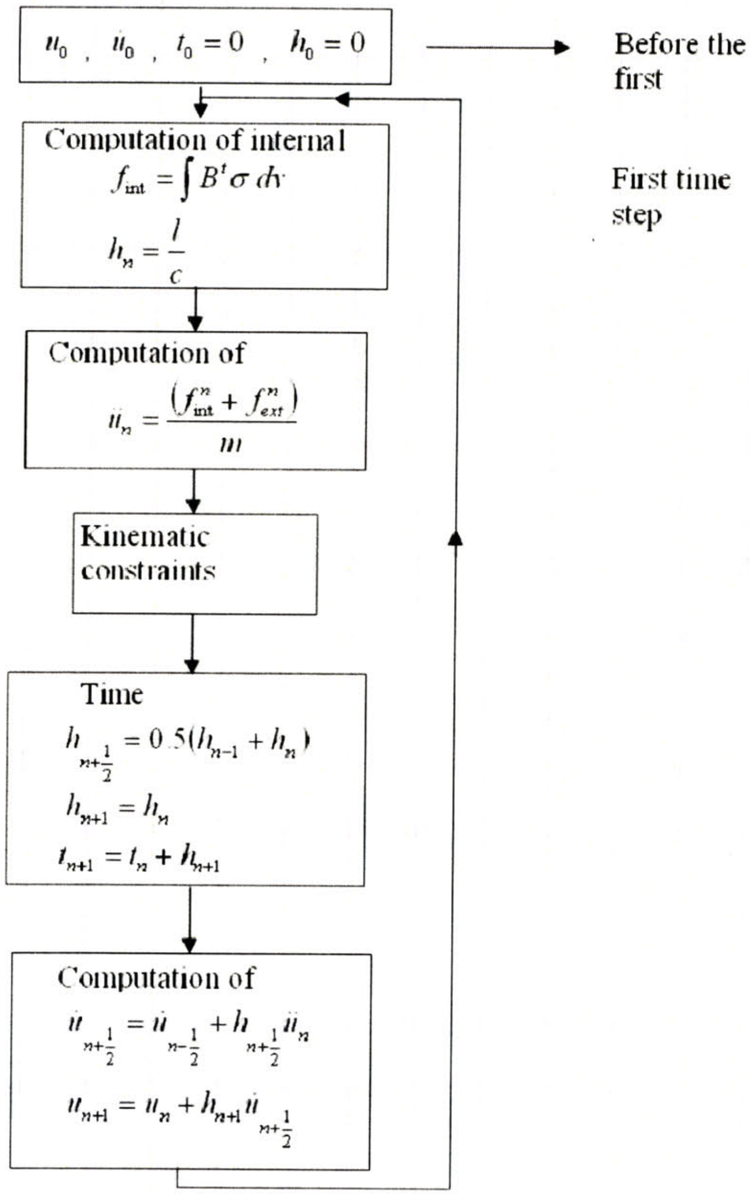
$$\{\ddot{u}\}_{t+1} = [M]^{-1}\cdot(P-I)_{t+1}$$ **Calculate Acceleration at the beginning of increment**

General Form $M\ddot{u}_{t+1} = P - \int B_{t+1}^T\sigma_{t+1}dV$

$B = [\partial\phi/\partial x]$ **Element Shape Function**

- Explicit dynamics procedure does not require a tangent stiffness matrix, therefore no iteration or tolerance are associated to the process
- As long as conditional stability condition is met (minimum time step), each time increment is relatively inexpensive because there is no solution necessary for a set of simultaneous equations
- Most of computation expense is from element nodal force calculation (I)

Explicit Solver, Stability Limit



- The explicit dynamics gives a solution only when the time increment (Δt)is less than the stable time increment (Δt_{min})
- The stability limit is defined as a function of highest eigenvalue of the model and the fraction of damping in the highest mode

$$\Delta t_{\min} \le \frac{2}{\varpi_{\max}}\left(\sqrt{1+\xi^2} - \xi\right)$$

- The stable time increment is the minimum time that a dilatational wave takes to move across any element in the model (volume expansion/contraction)

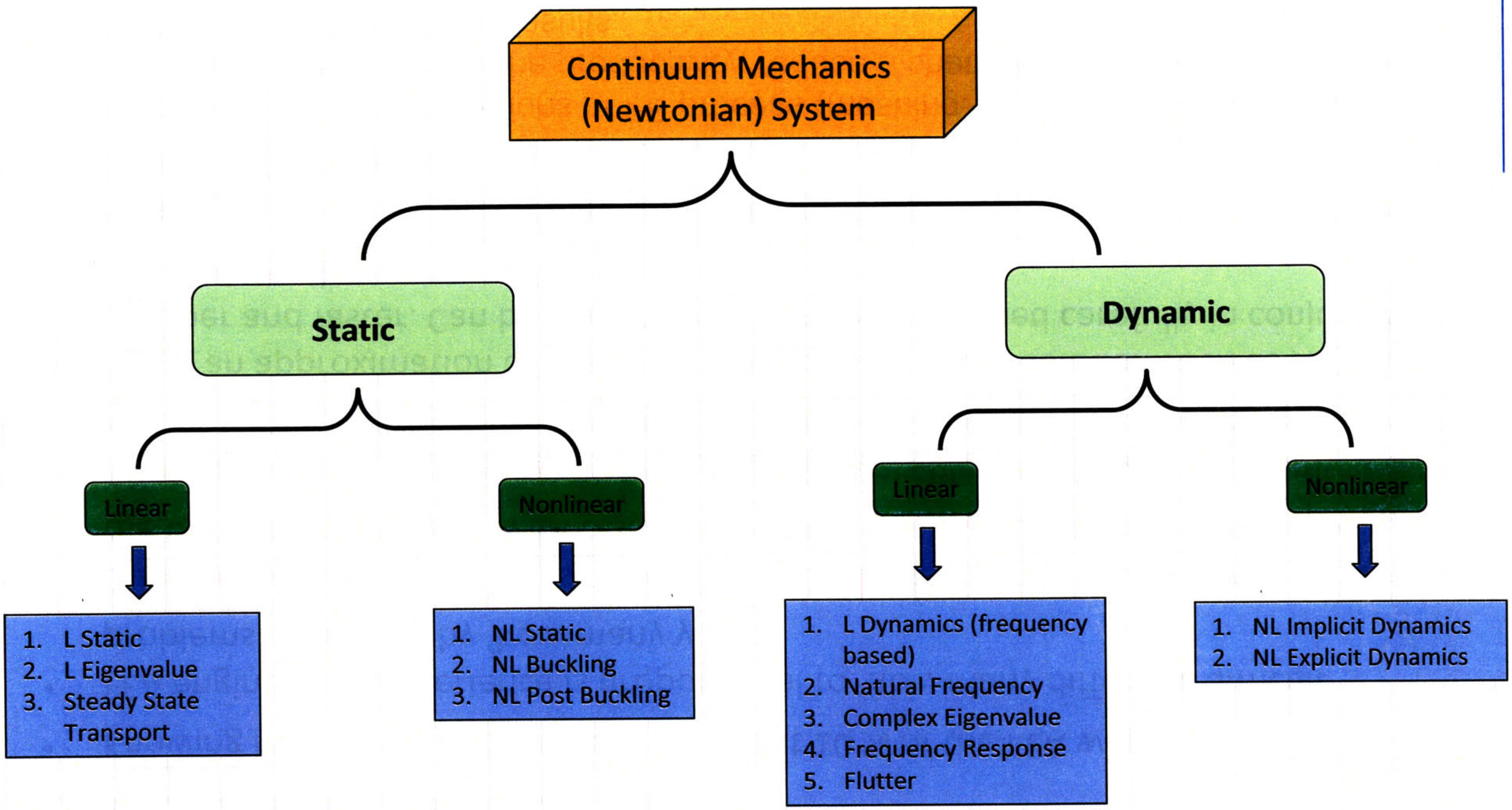
$$C_d = \sqrt{\frac{E}{\rho}}$$

L^e **Element Characteristic Length**

$$\Delta t_{stable} = \frac{L^e}{C_d}$$

- Smallest element size will govern system time step (takes lot of time to get used to, and generating optimal model)
- Stiffer the material, stable time step reduces
- Heavier the material, stable time step increases

World of FEA


Continuum Mechanics
(Newtonian) System
Static
Dynamic
Linear
Nonlinear
Linear
Nonlinear
1. L Static
2. L Eigenvalue
3. Steady State Transport
1. NL Static
2. NL Buckling
3. NL Post Buckling
1. L Dynamics (frequency based)
2. Natural Frequency
3. Complex Eigenvalue
4. Frequency Response
5. Flutter
1. NL Implicit Dynamics
2. NL Explicit Dynamics

Good FEA based Engineering

- Knowing how to use certain solver is about 10 % of the FEA work.
- FEA engineering experience is proportional to 'how many different/difficult problems tried', not by 'how many years'
- Duplicatable, transferrable, solver (tool) independent, and flexible FEA procedure is the ultimate outcome of FEA task, not isolated analyst doing individual analysis.
- Establish process **how to produce desired deliverables** for design requirement is the key, not limiting procedure itself by tools (pre-post, solver, other in-house software).
- FEA is an approximation of analytical solution, not exact solution such as test, but cheaper and faster. Can be extremely effective when used carefully in conjunction of test process (less, more efficient testing)
- Hardly deterministic prediction, mostly extrapolation using calibrated/correlated baseline FEA process.
- Automation of certain portions of FE process (meshing, connecting, load application) can significantly improve the productivity, reduce a chance of a modeling error, and produce more repeatable results.
- 2 'ok' engineers working as a team is always better than one big shot doing analysis alone (human error always happens)

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