INTERMEDIATE AND HIGH-ENERGY COLLISIONS

Theoretical

Chairman C.N. YANG

Rapporteur CHAN HONG-MO

Discussion Leader M. JACOB

Secretaries G. COHEN-TANNOUDJI
C. SCHMID

ADVANCES IN THE THEORY OF COLLISIONS AT HIGH ENERGY

Chan Hong Mo

CERN, Geneva

1. INTRODUCTION

In high-energy collisions, perhaps even more than in other branches of particle physics, the lack of a general theory is keenly felt. One has mostly to rely on models for the correlation and understanding of a massive collection of data. As a consequence, the diversity of approaches to the problem is only equalled by the diversity of empirical facts to be explained. It would be quite impossible for me to try to summarize and do justice to all the interesting work done in this field. Therefore, rather than attempt such a task, I propose to concentrate on only a few areas of recent development which have not been covered adequately by earlier international conferences. Only the phenomenological aspects of high-energy collisions will be considered in this report; the more theoretical questions will be dealt with by Professor Frazer in a separate session.

Among the subjects not discussed in this report, I wish to mention in particular:

i) symmetries and quarks,

ii) particle reactions with nuclei,

in which interesting work continues. Reluctantly, I have to contend with quoting earlier reviews and some contributions to this Conference[1,2].

Since Reggeism is still the favourite language among high-energy phenomenologists, I shall start by summarizing briefly the experimental status of Regge poles.

2. REGGE-POLE MODELS

The relevance of exchange quantum numbers to high-energy collisions has long been recognized. Probably the best illustrations for this are the following experimental facts:

i) In two-body collisions at energies $\gtrsim$ 3 GeV/c

$$A + B \to C + D \qquad (1)$$

the differential cross-section exhibits peaks at small t_{AC} if, and only if, there exist particle or resonance states in the exchange channel: $A + \overline{C} \to \overline{B} + D$. Up to the present, no exceptions have been found.

ii) The energy-dependence of such cross-sections are strongly correlated with the exchange quantum numbers. Thus, assuming a power dependence on the laboratory momentum:

$$\sigma = \sigma_0 \, p_L^{-n} . \qquad (2)$$

Morrison, in his latest compilation of data, found that the values of n fall naturally into four groups depending on the exchange quantum numbers[3],

Vacuum exchange	$n \sim 0$
Charge or isospin exchange	$n \sim 2$
Strangeness exchange	$n \sim 2.5$
Baryon-number exchange	$n \sim 3 - 4$.

In view of this, a convenient framework to describe high-energy collisions is the Regge model in which the exchanged systems are represented by the leading singularities on the complex angular momentum plane. Although the existence of branch-points has been known theoretically for some time, the early analyses of data assumed that they were quantitatively unimportant. Indeed, such analyses with Regge poles alone have been successful in reactions dominated by one single exchange. Two such examples:

i) $\pi^- p \to \pi^0 n$ (ρ exchange)

ii) $\pi^- p \to \eta n$ (A_2 exchange)

were reported already in the Berkeley Conference (1966)[4]. The ρ and A_2 trajectories so determined extrapolate comfortably through the corresponding resonances on the Chew-Frautschi plot.

More recently, using the accurate backward scattering data on $\pi^{\pm}p \to p\pi^{\pm}$ by the BNL-Cornell group, Barger and Cline made a successful fit with linear N_α(nucleon) and Δ_δ(N^*_{33} resonance) trajectories[5]. Only Δ_δ exchange contributes to π^-p scattering, whereas in π^+p both N and Δ can be exchanged, but N is found to dominate. The fitted trajectory parameters are:

$$\text{i)}\quad \alpha = -0.38 + 0.88\,u \quad \text{for} \quad N_\alpha \qquad (3)$$

$$\text{ii)}\quad \alpha = +0.19 + 0.87\,u \quad \text{for} \quad \Delta_\delta \qquad (4)$$

in units of 1 GeV^2. At the same time, for $u > 0$, they made a fit for the same trajectories using as input the known nucleon resonances, obtaining the values[6]:

$$\text{i)}\quad \alpha = -0.39 + 1.0\,u \quad \text{for} \quad N_\alpha \qquad (5)$$

$$\text{ii)}\quad \alpha = +0.15 + 0.9\,u \quad \text{for} \quad \Delta_\delta\,. \qquad (6)$$

The agreement between the two sets of values is astounding.

The simple pole model has also been successful in predicting dips in the differential cross-section at certain points where trajectories go through integer (boson) or half-integer (fermion) values. From the analyticity and factorizability of Regge-pole residues, it can be shown[7] that at wrong-signature points α_W of a trajectory α, a helicity amplitude $f^{\alpha}_{\lambda_A\lambda_B,\lambda_C\lambda_D}$ due to the exchange of α must vanish if it involves a nonsense transition, i.e. if either $|\lambda_A - \lambda_C| > \alpha_W$ or $|\lambda_B - \lambda_D| > \alpha_W$. This may produce dips in the differential cross-section at $\alpha = \alpha_W$, whose position is independent of the incoming energy. Two well-known examples are:

i) $\pi^-p \to \pi^0 n$ (ρ exchange). $\alpha_\rho = 0$ corresponds to a wrong-signature point where the spin-flip amplitude must vanish. This may be associated with the dip in $d\sigma/dt$ at $t = -0.6$ GeV^2.

ii) $\pi^+p \to p\pi^+$ (N_α and Δ_δ exchange). $\alpha_{N\alpha} = -1/2$ is a wrong-signature point where all N_α exchange amplitudes must vanish. The position of the observed dip in $d\sigma/du$ at $u = -0.2$ GeV^2 agrees very well with the value of u at $\alpha_N = -1/2$, as determined from Barger and Cline's trajectory parameters just quoted. Since at the dip, only Δ_δ exchange is supposed to contribute, the value of $d\sigma/du$ should be related simply by isospin Clebsch-Gordan coefficients to that for π^-p scattering at the same u. The ratio 1:9 so predicted agrees very well with the experimental value[5]. In addition, it has been shown by Contogouris et al.[8] that in the reactions

$$\pi^{\pm}p \to \rho^{\pm}p\,;\quad \pi^-p \to \rho^0 n\,,$$

the combination

$$X(s,t) = (d\sigma_+/dt) + (d\sigma_-/dt) - (d\sigma_0/dt) \qquad (7)$$

receives contribution only from the exchange of trajectories with the quantum numbers of the ω. Experimentally, a dip is observed in $X(s,t)$ at $t = -0.5$ GeV^2, which may correspond to the wrong-signature point at $\alpha_\omega = 0$.

The prediction of nonsense zeros at wrong signature points is, of course, only valid when one can neglect the contribution of cuts and of the Mandelstam-Wang fixed poles[7]. The experimental verification of the prediction thus indicates that, at least in the cases discussed, the cuts and fixed poles indeed give only small effects.

At the right-signature points α_R, and for sense-sense transitions at wrong-signature points α_W, the prediction of the pole model on the behaviour of the amplitudes is less definite. The possible alternatives for either case are listed in Table 1, where each alternative may further be multiplied throughout by additional powers of $\alpha - \alpha_{R(W)}$. However, the ambiguity can in principle be resolved by experiment in some reactions, and then hopefully be applied to make definite predictions in other reactions by means of the factorization theorem. Attempts in this direction have been made on:

i) the A_2 trajectory at the right-signature point $\alpha_{A_2} = 0$. The absence of any significant dip in $d\sigma/dt$ for the reaction $\pi^-p \to \eta n$ favours the nonsense-choosing (Gell-Mann) mechanism with no extra power of α. A recent analysis by Krammer and Maor[9] of the data on $\pi^+p \to \eta\Delta^{++}$ (A_2 exchange) and $K^+p \to K^0\Delta^{++}$ (ρ and A_2 exchange) also favours the same mechanism.

ii) the Δ_δ trajectory at the wrong-signature point $\alpha_\Delta = +1/2$. In $\pi^-p \to p\pi^-$ (backward) scattering, all amplitudes are sense-sense at $\alpha_\Delta = +1/2$. From the

TABLE 1

	Mechanisms	T_{ss}	T_{sn}	T_{nn}
Right signature	i) choose sense (Chew)	const.	$(\alpha - \alpha_R)$	$(\alpha - \alpha_R)^2$
	ii) choose nonsense (Gell-Mann)	const.	const.	const.
Wrong signature	i) choose sense (Chew)	const.	$(\alpha - \alpha_W)$	$(\alpha - \alpha_W)^2$
	ii) choose nonsense (Gell-Mann)	$(\alpha - \alpha_W)$	$(\alpha - \alpha_W)$	$(\alpha - \alpha_W)$

The designation of the various mechanisms which is used consistently in the text, differs somewhat from that normally used (see, for example, L. Bertocchi, Ref. 7). I apologize if this should cause some confusion. It seems to me, however, that the normal designation is excessively clumsy. Ours represents a feeble attempt at its simplification. It is earnestly hoped that experts on this subject could agree on a simpler and more rational notation.

depression in $d\sigma/du$ near $u = 0$, and the comparison of the extrapolated residue with the actual $N^*_{3,3}$ resonance width, Igi et al.[10] concluded in a recent analysis that the following alternatives are favoured: either i) sense-choosing (Chew) mechanism with one extra power of $(\alpha - 1/2)$; or ii) nonsense-choosing (Gell-Mann) mechanism with no extra power of $(\alpha - 1/2)$. However, neither of the above examples can yet be considered as conclusive.

Although simple pole models were reasonably successful in describing reactions dominated by a single exchange, more general analyses were hampered by two major complications:

i) the difficulty of fixing the many parameters left undetermined by the model;

ii) the existence of kinematical constraints on the helicity amplitudes arising from analyticity requirements.

The first question is purely technical. It arises from the lack of sufficiently accurate data at high energy where experiments give normally only differential cross-sections and not polarization. Recently, however, a new technique has been developed which exploits analyticity in the form of sum rules relating the asymptotic expansion of an amplitude to its behaviour in the low-energy region. Assuming the Regge form of amplitudes at high energy, this then allows one to determine Regge parameters using low-energy data to a greater accuracy than was previously possible. This technique will form the subject of our next section.

The other problem of kinematical constraints is a more intriguing one and has attracted a great deal of effort by many physicists. It arises in the following manner. Let $\tilde{F}^{(s)}_{\{\lambda\}}$ and $\tilde{F}^{(t)}_{\{\lambda'\}}$ be, respectively, the s- and t-channel helicity amplitudes free from kinematical singularities--the so-called regularized helicity amplitudes. They are related by the crossing matrix $X^{\{\lambda'\}}_{\{\lambda\}}$, thus

$$\tilde{F}^{(s)}_{\{\lambda\}} = \sum_{\{\lambda'\}} X^{\{\lambda'\}}_{\{\lambda\}} \tilde{F}^{(t)}_{\{\lambda'\}} . \tag{8}$$

However, the crossing matrix X itself has kinematical singularities at $t = 0$ and at thresholds and pseudo-thresholds, i.e. $t = (m_i \pm m_j)^2$. Thus in order to have $\tilde{F}^{(s)}_{\{\lambda\}}$ free of such singularities, $\tilde{F}^{(t)}_{\{\lambda'\}}$ must satisfy certain constraints at these t-values. So far, the arguments are entirely general and independent of the Regge model.

If now, in addition, one imposes the Regge-pole hypothesis and the factorization requirement of pole residues, the constraints on the amplitudes are relegated to the trajectories and their couplings. These constraints can be satisfied either by: (Leader)

i) conditions on the residue functions but not on the trajectories (Evasion);

or by:

ii) conditions also on the trajectories (Conspiracy).

Obviously, conditions on the trajectories themselves, being then independent of the particular reactions considered, will have very far-reaching consequences.

An excellent review on the earlier work on this subject has been given by Bertocchi at the Heidelberg Conference (1967)[7)]. The more recent developments using group theoretical techniques initiated by Toller is treated in this Conference by Frazer[11)]. Many of the results so derived, such as the Toller classification, even for unequal mass scattering, can also be obtained from analyticity and the factorization hypothesis without the use of group theory[12)]. Here, however, I shall confine myself to only a few remarks on the present experimental status of conspiracy.

Probably the best test case of conspiracy versus evasion is the reaction $\gamma p \to \pi^+ n$. The quantum numbers are such, that of the known trajectories, only the pion can be exchanged. If the amplitude at $t = 0$ is indeed dominated by pion exchange (or, in fact, by any single trajectory with definite parity), evasion is the only possible solution and implies a zero (dip) in $d\sigma/dt$ at $t = 0$. If, however, the pion conspires, then there may be another trajectory π' with opposite parity but otherwise the same quantum numbers. The amplitude resulting from the exchange of both π and π' may then satisfy the kinematical constraints without requiring a zero at $t = 0$. The existence or otherwise of a dip at $t = 0$ for this reaction is thus a clear test of evasion versus conspiracy within the framework of the pole model. Accurate measurements of $d\sigma/dt$ at small intervals down to $t \sim 10^{-3}$ GeV^2 have now established beyond doubt the existence of a peak instead of a dip at $t = 0$, thus clearly favouring conspiracy[13)]. As we shall see, analyses with superconvergence relations give further support to this conclusion.

A similar test can be made on the reaction $pn \to np$ (charge exchange). The experimental evidence here of a peak near $t = 0$ again favours conspiracy against evasion[14)].

However, it should be stressed that the preceding discussion is based on the assumption that the leading singularities on the complex J-plane are simple poles with factorizable residues. If it happens that cuts or other mechanisms are important, as seems likely from present experimental evidence, then the significance of these peaks and dips as regards pole conspiracy has to be reconsidered.

To summarize the situation for the simple pole model, the following statements seem in order:

a) in reactions dominated by one single exchange, the model is successful, at least to first approximation;

b) where several trajectories are necessary, because of conspiracy or otherwise, the model can give an adequate description but has in general too much freedom. In certain cases, however, factorization gives definite predictions which can be tested with experiment.

Although the general picture is reasonable, there are some important discrepancies to be noted. I shall give some outstanding examples:

i) The slope α'_p of the Pomeranchuk trajectory as determined by fitting high-energy data lies in the range $0 < \alpha'_p < 0.5$ GeV^{-2}, with the smaller values being preferred[15)]. This is much smaller than the slopes of other trajectories which are typically $\sim (1 \pm 0.1)$ GeV^{-2}.

ii) Experiment gives polarization $\sim$ 15 p.c. in the reaction $\pi^- p \to \pi^0 n$ even at high energy (11 GeV/c)[16)], whereas a simple ρ-exchange model predicts zero polarization.

iii) The difference between $d\sigma/dt$ of elastic $\bar{p}p$ and pp scattering changes sign around $t \sim -0.15$ GeV^2, which is roughly independent of the energy. Normal explanations of this effect in the Regge-pole model require the residue of the ω trajectory to vanish at $t \sim$ ~ -0.15[17)]. The factorization theorem then implies a dip in $d\sigma/dt$ at this t-value for all reactions dominated by the ω trajectory. However, no sign of such a dip is found experimentally in either a) $\pi p \to \rho N$ (see, for example, Ref. 8), or b) $\gamma p \to \pi^0 n$ [18], both supposed to be dominated by ω exchange.

iv) Assuming that the π trajectory conspires with a parity-doublet as in π^+ photoproduction, Le Bellac has shown that factorization then implies that for the reaction $\pi^+ p \to \rho^0 \Delta^{++}$, $d\sigma/dt$ should show a dip at $t = 0$ [19]. Experimentally, in contrast, a peak is observed[20].

Whilst objections to the flat slope of the Pomeranchuk (i) are mainly aesthetic, the other discrepancies (ii), (iii), and (iv) are more serious, since they mean the failure of the few existing "clean" tests of the phase of Regge amplitude (ii) and the factorization theorem (iii) and (iv). The difficulties can indeed be removed by introducing secondary trajectories such as the ρ' [Høgaasen and Fischer[21]], the $\bar{\omega}$ [Barger and Durand[17]] or the A_1 [Arbab and Brower[22]]. However, these secondaries are otherwise unknown; they should thus be regarded as representing these difficulties rather than explaining them.

3. SUPERCONVERGENCE RELATIONS

Over the last year or so, a new technique has been developed which exploits the analytic properties and asymptotic behaviour of scattering amplitudes in the form of dispersion sum rules relating the low-energy and high-energy regions. For lack of a better name, I shall follow de Alfaro et al.[23] and call the whole class of such sum rules "superconvergence relations" to distinguish them from those which are deduced from other sources, such as current algebra. An example of a superconvergence relation was first considered by Igi in 1962 [24]. It is only recently, however, that their generality and usefulness became fully recognized[25].

Let $f(\nu,t)$ be a scattering amplitude odd under crossing, and analytic as usual in the energy variable ν for fixed momentum transfer t. [Here we follow the notation and derivation of Logunov et al., Ref. 25.] Assume, further, that at high energy $f(\nu)$ can be represented as

$$f(\nu) = \sum_i C_i \, \nu^{\alpha_i} + \varepsilon(\nu) , \tag{9}$$

where the function $\varepsilon(\nu)$ decreases rapidly as $\nu \to \infty$ and is negligible for $\nu > A$. Then applying the Cauchy theorem to $\varepsilon(\nu)$, using the crossing relation, and neglecting the integral

$$\int_A^\infty \mathrm{Im}\, \varepsilon(\nu)\, d\nu \tag{10}$$

we get immediately the sum rule

$$\int_{\nu_0}^{A} \mathrm{Im}\, f(\nu)\, d\nu = \sum_i \frac{\mathrm{Im}\, C_i}{\alpha_i + 1} \, A^{\alpha_i + 1} \tag{11}$$

which is the simplest example of a "superconvergence relation". Clearly, a whole class of such sum rules can be derived by considering various moments $\nu^\gamma f(\nu)$ instead of $f(\nu)$ itself, where γ need not even be an integer. The resultant sum rules will emphasize different regions in ν, depending on the value of γ, and will in general involve both the real and imaginary parts of the amplitude[26]. We quote here as an example one form of such "continuous moment sum rules"--a form proposed by Della Selva et al. (see Ref. 26):

$$-\frac{1}{A^{\gamma+1}} \int_0^A d\nu\, \nu^\gamma\, \mathrm{Im}\left\{\exp\left[-i\,(\pi/2)\,\gamma\right] f(\nu)\right\} = \sum_i \frac{\beta_i}{\cos(\pi/2)\,\alpha_i} \cdot \frac{\sin(\pi/2)(\alpha_i + \gamma + 1)}{\alpha_i + \gamma + 1} \, A^{\alpha_i} . \tag{12}$$

All these sum rules just express the simple fact that the function $f(\nu)$ is analytic and has the Regge asymptotic behaviour.

Equations (11) and (12) are consistency conditions relating the amplitude in the energy range $\nu_0 < \nu < A$ to the amplitude in the region $\nu > A$. As such they may be used to check the validity of our assumptions in the same way as one uses ordinary dispersion relations. However, since existing high-energy data are normally much less accurate than those at lower energies, these equations are more conveniently regarded as a means of determining Regge parameters from low-energy data.

The left-hand side of Eq. (11) or Eq. (12) can be evaluated directly in cases where phase-shift analysis exists for the low-energy region. Otherwise the integrals may be evaluated by saturation with known resonances. Needless to say, results obtained with phase shifts are far more accurate and reliable. In addition, we note the following points:

i) The cut-off value A at which Regge asymptotic behaviour is supposed to set in, is normally taken rather low, being limited by the energy of presently available phase-shift analyses ($E_{lab} \sim$ 1-2 GeV).

ii) Results at t = 0 are in general more reliable, since many sum rules there involve only the total cross-sections for which accurate measurements exist.

iii) Wherever good high-energy data are available, the accuracy of parameters is increased by a simultaneous fit to both low and high energies.

Work in this direction is still developing rapidly. The main results which have come to my notice up to the present are listed below:

3.1 Vacuum exchanges in πN scattering

Here one has the detailed phase-shift analyses of, for example, Lovelace et al., and the accurate measurements of total cross-sections. The main results are:

i) intercept of the Pomeranchuk trajectory[27]:

$$\alpha_P = 1 \pm 0.02 \qquad (13)$$

(Della Selva, Masperi, and Odorico);

ii) slope of the Pomeranchuk trajectory[28]:

$$\alpha'_P = 0 \pm 0.1 \ (\mathrm{GeV})^{-2} \qquad (14)$$

(Barger and Phillips);

iii) the necessity of three vacuum trajectories (P,P',P'') to describe adequately the variation of total cross-sections above E_{lab} = 6 GeV [Della Selva, Masperi and Odorico[27]; Barger and Phillips[28]; Olsson and Yohd[29]]. The variation of P' and P'' parameters with the method of determination probably indicates that these are not simple pole trajectories but more complicated singularities, e.g. cuts.

3.2 The nonsense zero for ρ exchange in πN scattering

Evaluating the spin-flip, I = 1 exchange amplitude $B^{(-)}$ as a function of the momentum transfer t, Dolen, Horn and Schmid[30] found a zero at $t \sim 0.5$ GeV^2, as required by Regge ρ exchange, and as indicated by the dip in $d\sigma/dt$ at the same t-value for the reaction $\pi^- p \to \pi^0 n$ (Section 2).

3.3 Ghost-killing mechanism at right-signature points

The cases actually studied are the points $\alpha = 0$ for i) P' exchange in πN scattering, and ii) A_2 exchange in KN scattering and π photoproduction. Here the input data are less favourable. Although phase-shift analysis exists for πN scattering, the P' trajectory is heavily shielded by the Pomeranchuk and is hard to study. For KN scattering one has to rely on resonance saturation, and for π photoproduction on not very accurate phase-shift analysis extending only up to E_{lab} = 1.2 GeV. The results are thus less impressive.

i) P' exchange favours the nonsense-choosing (Gell-Mann) mechanism at $\alpha = 0$ (see Table 1) with no additional power of α [Barger and Phillips[28]; Gilman, Harari and Zarmi[31]].

ii) Results for A_2 exchange at $\alpha_A = 0$ are contradictory. Whereas the nonsense-choosing (Gell-Mann) mechanism is favoured by Matsuda and Igi[32] in their analysis of KN scattering, Chu and Roy[33] prefer the sense-choosing (Chew) mechanism in analysing pion-photoproduction data. It may be noted here that in high-energy fits of several reactions (see Section 2), the Gell-Mann mechanism is preferred at $\alpha = 0$ for A_2 exchange. SU(3) and the result (i) for P' also favours the Gell-Mann mechanism.

3.4 Cross-over effect in ω exchange

In analysing KN scattering with superconvergence relations, it was found that both the spin-flip and spin-non-flip amplitudes in ω exchange cross zero around $t \sim -0.15$ GeV^2 [34]. This confirms the result from high-energy fits, and underlines the apparent failure of factorization in the ω residue discussed earlier (Section 2).

3.5 Pion-conspiracy in π^+ photoproduction

Using the phase shifts of Walker, a group at Trieste[35] has found that the superconvergence relations can be satisfied at present accuracy with just a pion pole π together with a conspirator, its parity-

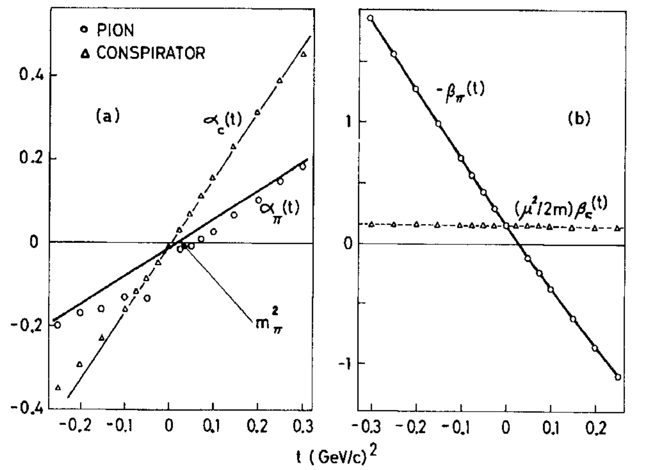

Fig. 1 Trajectory and residue of the pion and the conspirator as result from the one-pole best fit with superconvergence relations (discrete points) (Ref. 35).

doublet π'. They then determine from the sum rules the trajectory parameters α(t) and the residue functions β(t) for both π and π' over a range of t-values. Their result is shown in Fig. 1. The values of α and β so found agree very well with those obtained in the high-energy fit of Ball, Frazer and Jacob[13] and reproduce the existing high-energy data, as can be seen in Fig. 2.

In certain cases where the Regge description of the high-energy region is reasonable, the same technique can be turned around and used to resolve ambiguities in the low-energy region by means of high-energy data. One interesting example of this is the work of Martin and Ross[36], which applies the superconvergence relations to KN scattering to determine the ΛKN and ΣKN coupling constants. They obtained

$$g_\Lambda^2 + 0.79\ g_\Sigma^2 = 6.1 \pm 4.7 \qquad (15)$$

which lies just outside the limit allowed by pure SU(3) invariance:

$$g_\Lambda^2 + 0.79\ g_\Sigma^2 > 13.6, \qquad (16)$$

thus contradicting a previous result of Kim's. A parallel analysis by Logan and Razmi, also with superconvergence relations, came to similar conclusions[37].

The success of superconvergence relations in determining Regge parameters, and the fact that saturation with resonances alone often gives already a qualitative adequate description, have led to a new concept which may subsequently prove far-reaching. This is the so-called Dolen-Horn-Schmid duality[30]. [See also G.F. Chew[38].] We note first the empirical fact that the superconvergence relations (11) or (12) are quite well satisfied even down to low energies (∿ 1 GeV), assuming on the right-hand side only a few leading Regge poles. If, in addition, the integral on the left is approximately saturated by resonances, one can consider the Regge poles as being "built up" by summing a series of direct channel resonances. Moreover, since the relations (11) and (12) are supposed to be valid for all A, the equivalence is "semi-local", meaning that the Regge pole approximates the contributions of resonances averaged over a small region in energy. This duality is even more dramatically illustrated by Schmid[39], who by projecting out partial wave contributions from the ρ-exchange amplitude in πN scattering, obtains circles in the Argand diagram, which correspond well in position and J^P assignments to the known nucleon resonances.

This simple observation has important repercussions both in bootstrap dynamics and in the study of resonances, which are dealt with in the appropriate sessions. What concerns us here as regards high-energy phenomenology is the correct parametrization of the intermediate region, say from 2 to 6 GeV/c. Above 6 GeV/c, the Regge model is reasonably successful.

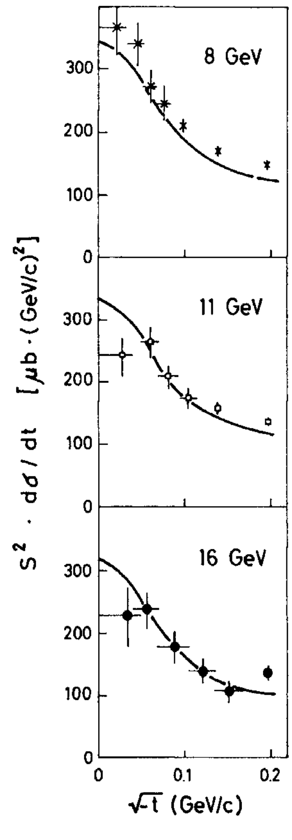

Fig. 2 Comparison of the predicted differential cross-sections with experimental data (Ref. 35).

Below 2 GeV/c, reactions are known to be dominated by resonances and can be studied by the powerful machinery of Lovelace and others for phase-shift analysis. Clearly, in the intermediate region, one needs a model which can interpolate smoothly between direct-channel resonances and Regge-pole exchange. The simple interference model which adds resonances to the Regge amplitude, thus: $A = A_{res} + A_{Regge}$, though qualitatively correct in certain cases[40], is untenable when taken literally. This is clear from the discussion of duality given above: one cannot just add A_{res} to A_{Regge}, since A_{Regge} already contains part of the resonance contributions.

This problem of parametrizing the intermediate energy region is not yet entirely solved. However, I should like to mention an interesting model of Veneziano, which has gone a long way towards its resolution[41]. He studied, in particular, the reaction:

$$\pi + \pi \rightarrow \pi + \omega \qquad 17)$$

which has the nice property of being symmetric in all three channels. The amplitude can be written as

$$T = \varepsilon_{\mu\nu\rho\sigma}\ \varepsilon_{\mu}\ p^1_{\nu}\ p^2_{\rho}\ p^3_{\sigma}\ \ A(s,t,u) \qquad (18)$$

with A(s,t,u) symmetric in s, t, and u. Veneziano observed that if one takes for A the form:

$$A = \frac{\bar{\beta}}{\pi^2}\ \Gamma[1-\alpha(s)]\Gamma[1-\alpha(t)]\ \Gamma[1-\alpha(u)] \times \times[\sin\pi\,\alpha(s) + \sin\pi\,\alpha(t) + \sin\pi\,\alpha(u)] \qquad (19)$$

with

$$\alpha(s) + \alpha(t) + \alpha(u) = 2\ , \qquad (20)$$

then one has:

i) explicit crossing symmetry,

ii) Regge asymptotic behaviour in all three channels,

iii) poles at points corresponding to resonances,

iv) validity of all superconvergence relations at all values of t, thus avoiding the difficulties of the old interference model as regards "duality".

The present form (19) with α real is insufficiently realistic for phenomenological analysis. However, with appropriate modification, it may well prove to be the answer for the intermediate energy region.

4. DIFFRACTION MODELS AND UNITARITY CORRECTIONS TO REGGE POLES

The existence of cuts in the angular momentum plane associated with multiple exchanges of Regge poles has been known theoretically for many years. In practical applications also, unitarity corrections to exchange models in the form of absorption factors have been considered already by several authors, notably by Jackson et al.[42] for particle-exchange models, and by Cohen-Tannoudji et al.[43] for Regge-pole exchange. The only reasons therefore for persisting in simple pole models are:

i) the apparent success it has enjoyed in earlier analyses;

ii) the theoretical difficulty in predicting the corrections due to cuts.

Recently, however, experiment has brought to light a few inconsistencies with the simple pole model, such as the apparent failure of the factorization theorem discussed in Section 2. These have prompted many theorists to make renewed and fruitful attempts to solve the cut problem. Another incentive in this direction is the experimental discovery of interesting structures in dσ/dt for elastic scattering at large momentum transfers, structures which are highly reminiscent of diffraction phenomena[44].

One can distinguish two general classes of approaches to this problem, depending on the existence or otherwise of the limit: lim (dσ/dt) for elastic scattering at infinite energy. This question is, of course, a fundamental one, and is quite independent of the Regge model. In the Regge language, however, it hinges on whether the slope of the Pomeranchuk trajectory $\alpha'_P = 0$. Now all Regge fits up to the present, either directly of high-energy data, or of low-energy data via superconvergence relations, have yielded α'_P considerably lower than 1 GeV^{-2}, in contrast to all other trajectories. The best limit, obtained by Barger and Phillips[28] from superconvergence relations, gives $\alpha'_P = 0 \pm 0.1$ (see Section 2). This means that all present data are consistent with a flat Pomeranchuk trajectory and favour the existence of the limit: $\lim_{s\to\infty}$ (dσ/dt) for elastic scattering.

The likelihood of a non-vanishing asymptotic limit for elastic cross-sections, together with the fact that all reactions well described by simple Regge exchanges (such as $\pi^- p \to \pi^0 n$) vanish rapidly with increasing energy, has prompted many authors to suggest essentially non-Regge mechanisms for reactions in the asymptotic region. A particularly elegant and simple example of this is the diffraction model of Chou and Yang[45] which I shall describe. Others, such as Arbarbanel, Drell and Gilman[46] prefer to interpret the asymptotic limit as being related to a new current-current contact interaction which dominates at large t over the ordinary strong interactions described by Regge poles. This new current may be assigned symmetry properties which will lead to relations between asymptotic $d\sigma/dt$ at large t for various reactions. In particular, Ne'eman wants to identify this contact term with the fifth interaction responsible for SU(3) breaking, and gives some predictions of this hypothesis that are available to experimental tests[47].

Based on these non-Regge asymptotic models, a number of authors have then added finite energy corrections in the form of Regge poles and cuts to describe existing data[48]. In the Regge language, the Pomeranchuk trajectory in such hybrid models appears as a fixed pole on the complex angular momentum plane, and is fundamentally different from other trajectories.

Other authors, however, such as Anselm and Dyatlov, and Frautschi and Margolis[49], prefer not to excommunicate the Pomeranchuk trajectory, and allow it to retain a finite slope. In these models, the diffraction peak will continue to shrink at asymptotic energies leading to vanishing elastic cross-sections. They then have to rely on the effects of cuts and non-Pomeranchuk trajectories in order to explain the general lack of shrinkage observed at present experimental energies.

Most of the models suggested in either class discussed above use a technique known as <u>the eikonal approximation,</u> which has a lot of formal similarity to the Glauber theory of scattering from nuclei (see, for example, Glauber, Ref. 2). To derive this, we write first the differential cross-section as (we use the notation of Ref. 45):

$$d\sigma/dt = \pi |a|^2 , \tag{21}$$

where

$$a = \lambda\!\!\!^{-2} \sum_{\ell} (2\ell + 1)\, P_\ell(\cos\theta)\, (1 - S)/2 . \tag{22}$$

The eikonal approximation, which is supposed to be valid at large energies and small angles, consists of the following replacements:

$$P_\ell(\cos\theta) \to J_0(b\sqrt{-t}) , \tag{23}$$

where the impact parameter

$$b = \lambda\!\!\!^{-} (\ell + \tfrac{1}{2}) , \tag{24}$$

while the sum over ℓ is replaced by an integral over b, thus

$$a \to \int_0^\infty [1 - S(b)]\, J_0(b\sqrt{-t})\, b\, db . \tag{25}$$

This expression is more convenient to use in terms of a two-dimensional Fourier transform. This can be arrived at by writing first

$$a = \int_0^\infty [1 - S(b)] \int_0^{2\pi} \exp(ib\sqrt{-t}\cos\phi)\, d\phi\, b\, db \tag{26}$$

using a known integral representation of J_0. Then introducing two-dimensional vectors

$$\begin{aligned} &\underline{K} = (K_x, K_y) \qquad && \underline{K}^2 = -t \\ &\underline{b} = (b_x, b_y) && \underline{b}^2 = b^2 \\ &\underline{K}\cdot\underline{b} = K\, b \cos\phi && \end{aligned} \tag{27}$$

we have

$$\begin{aligned} a &= \langle 1 - S \rangle \\ &\equiv \iint [1 - S(b)] \exp(i\, \underline{b}\cdot\underline{K})\, d^2\underline{b} , \end{aligned} \tag{28}$$

where one has introduced the notation ⟨ ⟩ for the two-dimensional Fourier transformation.

As the simplest example, we turn to the <u>Chou-Yang model</u>[45] for elastic scattering at asymptotic energies. We shall describe it in some detail since its formal features are shared by many others. We note first that the transmission coefficient S(b) can be given a physical interpretation as follows. Consider a uniform slab of thickness g. If the slab absorbs and disperses an incoming wave, the transmission coefficient for the wave through the slab would be:

$$S = \exp(-\alpha g) . \qquad (29)$$

The quantity $-\log S = \alpha g$ can be conveniently termed the opaqueness of the slab as it appears to the wave. Similarly, for the scattering of waves by a spherically symmetric object, the quantity $-\log S(\underline{b})$ is the opaqueness at the impact parameter $\underline{b}$.

Pursuing then a picture suggested earlier[50], Byers and Yang then proposed that the hadrons be taken as extended objects with internal structures given by spherically symmetric density functions $\rho(x,y,z)$. In a collision of two hadrons, say A and B, B will then appear to a point in A as a disc with a two-dimensional opaqueness density,

$$D_B(x,y) = \int_{-\infty}^{+\infty} \rho_B(x,y,z)\, dz . \qquad (30)$$

They then argued that for the two hadrons passing through each other, the resultant opaqueness at impact parameter $\underline{b}$ will be

$$-\log S(\underline{b}) = K_{AB} \iint D_A(\underline{b}-\underline{b}')D_B(\underline{b}')\, d^2\underline{b}' , \qquad (31)$$

where K_{AB} is some constant absorption coefficient depending only on the type of particles. At this point it is convenient to introduce the notation $\otimes$ for the convolution integral; thus (31) reads

$$-\log S = K_{AB}\, D_A \otimes D_B . \qquad (32)$$

Substituting Eq. (32) into Eq. (28) then yields immediately the relation

$$a_{AB}(\underline{K}) = \Delta_{AB}(\underline{K}) - \frac{1}{2!}\Delta_{AB}(\underline{K}) \otimes \Delta_{AB}(\underline{K}) + $$
$$+ \frac{1}{3!}\Delta_{AB}(\underline{K}) \otimes \Delta_{AB}(\underline{K}) \otimes \Delta_{AB}(\underline{K}) - \cdots , \qquad (33)$$

where

$$\Delta_{AB}(\underline{K}) = \langle -\log S \rangle = K_{AB} \langle D_A \rangle \langle D_B \rangle . \qquad (34)$$

Conversely, one can easily express Δ_{AB} also in terms of a_{AB}, thus

$$\Delta_{AB}(\underline{K}) = a_{AB}(\underline{K}) + \frac{1}{2} a_{AB}(\underline{K}) \otimes a_{AB}(\underline{K}) + $$
$$+ \frac{1}{3} a_{AB}(\underline{K}) \otimes a_{AB}(\underline{K}) \otimes a_{AB}(\underline{K}) + \cdots . \qquad (35)$$

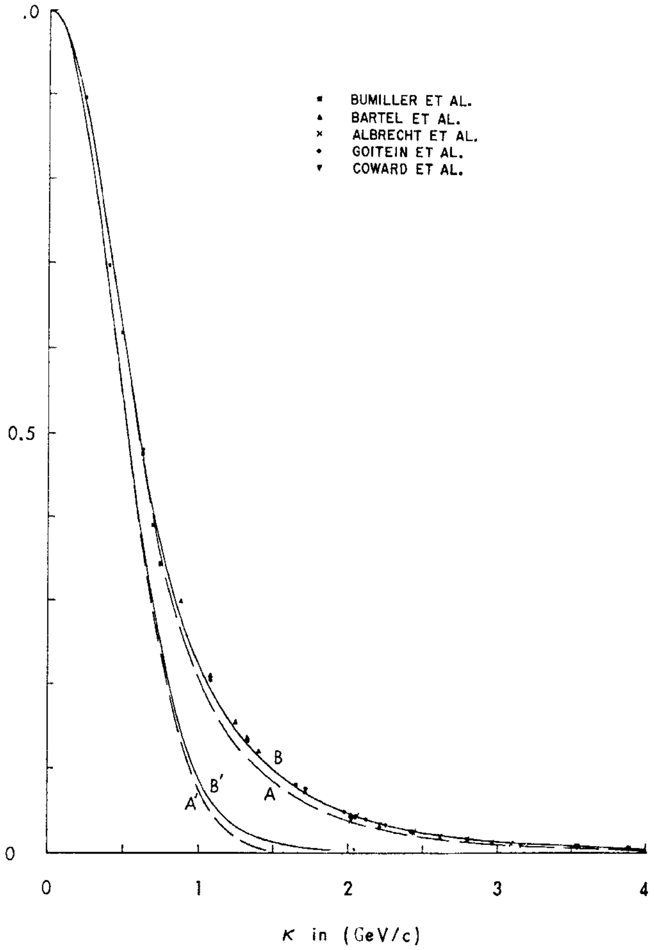

Fig. 3 The charge form factor F_1 of proton versus K. The curves A and B are the predictions of the Chou-Yang model with
(A) $\lim (d\sigma/dt)_{pp} = 79.04 \exp(10.3\ t)$ mb $(\mathrm{GeV}/c)^{-2}$.
(B) $\lim (d\sigma/dt)_{pp} = 79.04 [\exp(5.15\ t) + 0.015 \exp(2\ t)]^2$ mb $(\mathrm{GeV}/c)^{-2}$.
A' and B' correspond to the single scattering terms alone (Ref. 45).

The relations (33) and (35) which relate the asymptotic scattering amplitude between A and B to their internal structures, have close connections to the Glauber theory of nuclear collisions. Thus, in Eq. (33), the n^{th} term in the series corresponds to n-tuple scattering in Glauber's theory.

In order to test the model with experiment, Chou and Yang made the following tentative assumptions:

i) Identification of the density function $\rho(x,y,z)$ in protons with the charge distribution as measured by electron-proton scattering. This means

$$\langle D_p \rangle = \text{const}\ F_{1p}(K^2) . \qquad (36)$$

ii) Exponential form in t for the asymptotic differential cross-section in pp elastic scattering, thus:

$$d\sigma/dt = \sigma_0 \exp(At) , \qquad (37)$$

where A is fitted to present data for $|t| < 1$ GeV^2. Their result for the form factor $F_{1p}(t)$ as calculated from Eqs. (35), (36), and (37) is shown in Fig. 3. The agreement with data over several orders of magnitude and a large range in t is quite impressive.

One notes, in addition, two interesting points:

i) If for pp scattering, one keeps in Eq. (33) only the first term, i.e. if one neglects all effects of multiple scattering, one has

$$d\sigma/dt \propto |a(\underline{K})|^2 \propto [F_1(\underline{K})]^4, \qquad (38)$$

a relation first proposed by Wu and Yang[50] and noted by several authors to be approximately valid for small t values[51].

ii) Multiple scattering terms in Eq. (33) have gentler dependence on t than the single scattering term Δ_{AB}. This can be checked explicitly for $\Delta_{AB} \propto \exp(at)$, which gives for the n-tuple scattering term a t-dependence $\sim \exp(at/n)$. The general picture then is that at small $|t|$ values, the single scattering term dominates. As $|t|$ increases however, multiple scattering becomes more important and ultimately takes over completely. Now the terms in Eq. (33) alternate in sign. Thus at the value of t where, for example, the single scattering term becomes comparable to the double scattering correction, they tend to cancel, giving rise to a pronounced dip in the differential cross-section[52]. In the asymptotic model, these dips are actual zeros in $d\sigma/dt$. However, as Durand and Lipes have shown, finite energy corrections, for example, in the form of a real part to the amplitude, will fill the dips partially[52]. This phenomenon of dips due to interference between single and double scattering terms is well-known already in particle-nuclei scattering (see, for example, Ref. 7, and Glauber in Ref. 2). It arises simply from the shadow effect of nucleons in the front of the nucleus on those in the back.

The picture for asymptotic scattering offered by the Chou-Yang model is an attractive one. Unfortunately, it gives us as yet no hint of what is to happen at finite energies. On the other hand, one has the Regge model which has proved most successful in describing the energy dependence of reaction cross-sections. It seems natural, therefore, to try to combine these models in some way so as to keep the virtues of both. Attempts in this direction have been made, for example, by Arnold and Blackmon and by Chiu and Finkelstein[48]. The result is what are known as hybrid models.

Using again the eikonal approximation of Eq. (25), one has in Hybrid Models as in Eq. (28)

$$a(s,t) = \langle 1 - S(s,b)\rangle = \iint [1 - S(s,\underline{b})] \exp(i\,\underline{b}\cdot\underline{K})\, d^2\underline{b}\,, \qquad (39)$$

the only difference so far being that a and S are now functions also of the energy s. To the opaqueness factor Δ_{AB}, which represents the point interactions between volume elements of the colliding hadrons, one adds now a finite energy (or finite range) correction in the form of Regge poles; thus

$$\Delta'_{AB}(s,t) = \langle -\log S(s,\underline{b})\rangle = \Delta_{AB}(t) + \sum_j R_j(s,t)\,, \qquad (40)$$

where

$$R_j(s,t) = -i\ \beta_j(t)\,(s/s_0)^{\alpha_j(t)-1}\,. \qquad (41)$$

In the sum of Eq. (40), the Pomeranchuk trajectory is excluded, since diffraction scattering which the Pomeranchuk pole is supposed to represent is already contained in the Chou-Yang factor $\Delta_{AB}(t)$. Obviously, since all trajectories other than the Pomeranchuk has $\alpha < 1$, the correction terms (41) will all vanish as $s \to \infty$, yielding again the Chou-Yang model at asymptotic energies.

Physically, there is little justification for the choice of Eq. (40) as finite corrections to the Chou-Yang model. Formally, however, the expression (40) has the attractive feature of generating automatically in the amplitude a series of cuts corresponding exactly to multiple exchanges of Regge poles. Thus, for example, in the double-scattering term in Eq. (33) (with Δ' substituted for Δ):

$$\Delta'_{AB} \otimes \Delta'_{AB} = \Delta_{AB} \otimes \Delta_{AB} + \Delta_{AB} \otimes \sum_j R_j + \sum_j R_j \otimes \Delta_{AB} + \sum_j R_j \otimes \sum_j R_j\ , \qquad (42)$$

the term $R_j \otimes \Delta_{AB}$ has a cut corresponding to the exchange of the trajectory α_j together with a flat Pomeranchuk represented by Δ_{AB}. Moreover, it has been shown by Arnold[53] that the cuts so generated have all the known properties of cuts in the Regge theory, such as the correct position and the correct behaviour of the discontinuity near the branch point. Thus although the cuts so generated are a very special case of all possible solutions, they are a convenient tool for studying their general properties.

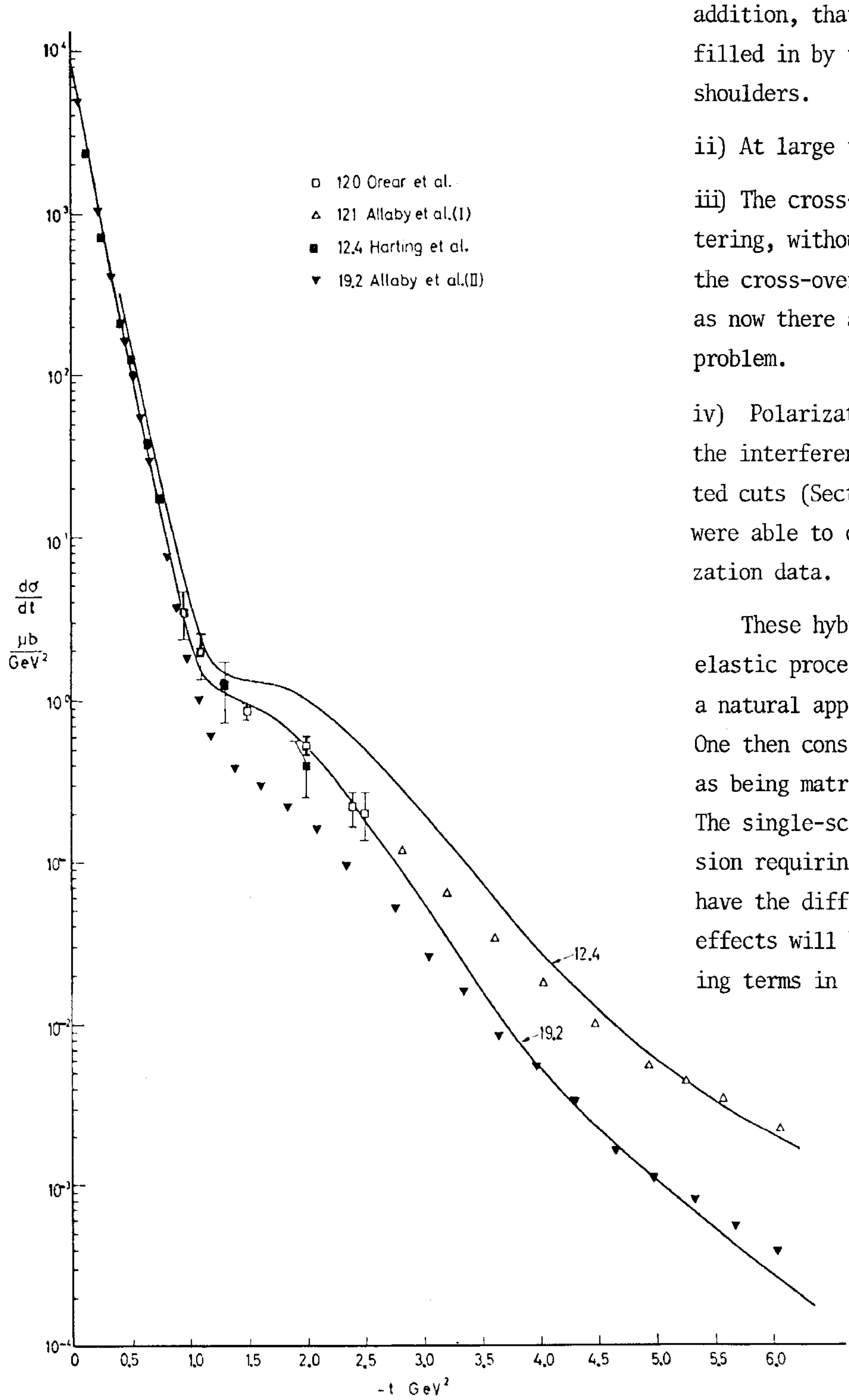

Fig. 4 The pp elastic differential cross-section versus $-t$. Calculated curves are at p_{lab} = 12.4 and 19.2 GeV/c (Chiu and Finkelstein, Ref. 48).

Such hybrid models have been applied, for example, by Chiu and Finkelstein[48] to pp and $p\bar{p}$ elastic scattering, and by Arnold and Blackmon[48] to πN elastic and charge exchange. The details of the models differ, mainly in the Regge inputs R_j. However, the main results are not dependent on such details. One obtains, in particular, the following features:

i) A qualitatively correct dependence of $d\sigma/dt$ on t and on the incoming energy s. An example from Chiu and Finkelstein is shown in Fig. 4. One notes, in addition, that the diffraction dips are partially filled in by the Regge corrections and appear only as shoulders.

ii) At large t, $d\sigma/dt \sim \exp(-B\sqrt{-t})$.

iii) The cross-over effect in $\bar{p}p$ and pp elastic scattering, without requiring the ω residue to vanish at the cross-over point (see Section 2). In any case, as now there are cuts, factorization is no longer a problem.

iv) Polarization in πN charge exchange scattering via the interference of the ρ trajectory with the generated cuts (Section 2). In fact, Arnold and Blackmon[48] were able to obtain good fits to all existing polarization data.

These hybrid models can also be extended to inelastic processes, and yield the absorption model as a natural approximation[54] (see also Refs. 48 and 49). One then considers Eq. (40) and all related equations as being matrix equations connecting various channels. The single-scattering term Δ' for an inelastic collision requiring exchanges of quantum numbers will not have the diffraction term Δ. However, diffraction effects will be brought in via the multiple scattering terms in the series (33). Now terms in (33) involving more than one exchange of non-Pomeranchuk poles will decrease rapidly with energy. Neglecting all such higher terms in R_j, one has then for the inelastic amplitude simply the single Regge exchange modified by corrections due to diffraction scattering, which is identical to the "absorbed" Regge amplitude.

Such "absorbed" Regge models have been applied with success (see, for example, Refs. 43 and 54). One notes, in particular, that the forward peak in π^+ photoproduction which was ascribed (Section 2) by simple pole models to pion-conspiracy, can be explained here without requiring a pion conspirator. The reason is that the cuts generated by multiple Regge exchange have in general both parities, and, conspiring with themselves, require no evasive zero at $t = 0$.

Moreover, it may be noted that early successes of the simple pole models, namely

i) energy dependence of cross-sections,
ii) occurrence of nonsense dips,

are to a certain extent preserved in the new formalism. Point (i) is approximately correct, at least in the forward direction where cut corrections are small. For point (ii), one notes that since the Regge term changes sign near the nonsense dip, the convolution integral in the double-scattering term largely cancels, yielding a small "absorption" correction[48].

There is, however, one theoretically unpalatable feature in the hybrid models, namely the artificial grafting of an essentially non-Regge mechanism on to a Regge model. Therefore, other authors, such as Frautschi and Margolis[49], and Anselm and Dyatlov[49] prefer models in which the Pomeranchuk trajectory is no different from others and has a finite slope ~ 1 GeV^{-2}. The techniques used are similar, the only difference being that in Eq. (40) the Chou-Yang diffractive factor $\Delta(t)$ is replaced by a Regge term:

$$R_P(s,t) = -i\ \beta_P(t)\ (s/s_0)^{\alpha_P(t)-1} \quad . \qquad (43)$$

Most of the attractive results of the hybrid models are retained. However, it turns out that terms coming from multiple Pomeranchuk exchanges have the following features:

i) positive real part to the amplitude,
ii) total cross-sections increasing to the asymptotic values.

Both these are opposite to what is observed at present experimental energies. Frautschi and Margolis ascribed these apparent discrepancies to the effects of non-Pomeranchuk poles. However, at higher energies, points (i) and (ii) are still expected to be valid.

In this connection, it is interesting to note that the same predictions (i) and (ii) on the asymptotic behaviour have also been obtained by Gribov and Migdal in papers contributed to this Conference[55]. Using the Reggeon graph technique developed by Gribov and collaborators, which is independent of the model assumptions just discussed, they derived an expansion of the amplitude in powers of (1/log s), where the n^{th} term corresponds to the cut with n Pomerons exchanged. They were able, in addition, to give the sign and a lower bound on the size of the first cut contribution, thus yielding the previous conclusion. Unfortunately, their expansion as yet is expected to be valid only at superhigh energies ($\sim 10^{15}$ eV).

A remark at this point on the dips and other structures in the differential cross-section $d\sigma/dt$ may be appropriate. All the models described above use the "multiple scattering" expansion, and will thus give diffraction minima. On the other hand, kinematic zeros in the residues of non-Pomeranchuk trajectories may also give rise to dips of the type discussed in Section 2. As to which of these the experimentally observed dips should correspond, experts in this field are not unanimous. Thus, for example, Henyey et al.[54] observed that the dip in $d\sigma/dt$ at $t \simeq -0.6$ GeV^2 for $\pi^- p \to \pi^0 n$ (Section 2) can be equally well-fitted as a diffraction minimum without assuming a zero in the ρ-exchange residue; whereas Barger and Phillips have gone to the other extreme, and suggest that even the structures seen in large-angle elastic scattering may be due to zeros of pole residues[56]. This difference in opinion can, in principle, be settled by studying the energy dependence of the dip-bump structures. Whereas diffraction minima are expected to move forward and deepen with increasing energy, dips of the other type are fixed in position and will eventually disappear at higher energies.

To conclude this section, I should mention also several other interesting models which have not been included in the general stream of development outlined above:

i) The model of Arbarbanel, Drell and Gilman[46]. In this model, as already mentioned above, scattering at large t and s is pictured as being due to a contact current-current interaction. To this is added the normal strong interactions, say in the form of Regge poles, thus yielding again a hybrid model in the sense used above. The main physical difference is that here, asymptotically, $d\sigma/dt$ at large t approaches $[F_1(t)]^4$. Unitarity effects which are included by means of an N/D method developed by Baker and Blankenbecler are shown not to alter this substantially. Experimentally, this prediction is probably readily checked with the new generation of machines, since at 30 GeV $d\sigma/dt$ is already quite close to the limit. Also, the model gives no sharp diffraction minima which are common to all models described above.

ii) The extended Chou-Yang model, which has been suggested independently by Chou and Yang, and by Byers and Frautschi, in papers contributed to this Conference[57]. This replaces the c-number densities $\rho(x,y,z)$ by q-number quantities, in second quantized notation: $\rho(\underline{x}) = \Phi^+(\underline{x})\Phi(\underline{x})$, where $\Phi(\underline{x})$ is a quantized field. If the structure of the hadrons is fine-grained, this model reduces to the original Chou-Yang model for elastic scattering. However, the new formulation allows the hadrons to be excited, giving rise to diffractive dissociation processes non-vanishing at asymptotic energies. Assuming only spatial interaction between elemental matter, diffractive dissociation can occur only when no internal quantum numbers are exchanged, and when the change in spin-parity corresponds to an exchange of orbital angular momentum. This prediction agrees well with present experimental evidence (see, for example, Ref. 3).

iii) Possible backward peak due to diffractive scattering in analogy to the Glory effect in optics [Arnold[58]].

5. THE MULTI-REGGE MODEL

The problem of diffraction scattering and unitarity corrections to exchange models is, of course, intimately connected with that of inelastic channels having many particles in the final state. Indeed, it may not be possible to understand fully even the simplest reactions, such as elastic scattering, without some basic knowledge of multi-particle processes. This point of view has, for example, been emphasized by Van Hove and his collaborators in their study of the overlap functions[59]. Unfortunately, however, due to the complexity of the problem, our experimental knowledge and theoretical understanding of multiparticle reactions have remained at a considerably lower level than that for two-body collisions. Nevertheless, some progress has been made over the last year via the multi-Regge model, and I wish to report here the main results.

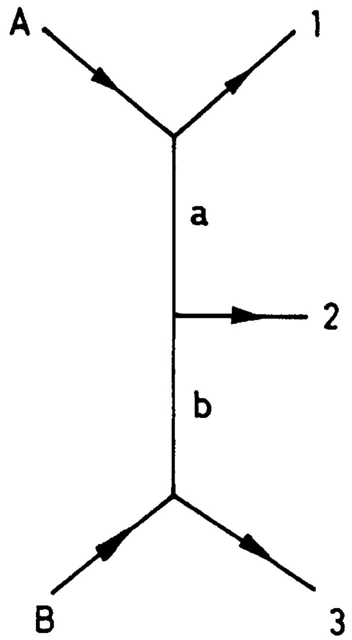

Fig. 5 Example of a double-Regge graph.

Before I proceed, I should first make clear a point in semantics. In accordance with current practice, I shall use multi-Regge (hyphenated) to denote exchanges of Regge poles "in series". This is to be distinguished from multiple Regge exchanges (without hyphen) of Regge poles "in parallel" which give rise to cuts, as discussed in the previous section. An example of a multi-Regge diagram is shown in Fig. 5.

The multi-Regge model is not a new idea. It is an off-shoot of the multiperipheral model[60] and has been considered by Ter-Martirosyan and Kibble as early as 1963 [61]. Since then it has been developed theoretically by many authors[62]. Its recent revival has been mainly due to the data which have become available for detailed analysis.

Consider first the simple case:

$$A + B \to 1 + 2 + 3 \qquad (44)$$

at a high incoming energy s. The model predicts that in the region of phase space where all $s_{ij} = (p_i + p_j)^2$ are large, the process (44) will be dominated by the graph in Fig. 5 and by similar graphs with the final particles permuted. The question of which graphs are actually admissible will be determined by the quantum numbers of the external particles and of the exchanged Regge poles. Assuming some analytic properties of the amplitude, and performing a double Sommerfeld-Watson transformation, it can be shown that for the graph in Fig. 5 [63]:

$$A \sim \beta_a(t_a)\,\beta_b(t_b)\,\beta(t_a,t_b,\omega)\,\zeta_a(t_a)\,\zeta_b(t_b)\,s_{12}^{\alpha_a(t_a)}\,s_{23}^{\alpha_b(t_b)} \qquad (45)$$

as expected. We need only note the vertex function $\beta(t_a,t_b,\omega)$ which represents the coupling of two Regge poles to a particle. In addition to the masses of the Reggeons t_a and t_b, it depends on a Toller variable ω which may be defined here as the azimuthal angle between the planes $\underline{a} \times \underline{1}$ and $\underline{b} \times \underline{3}$ in the rest frame of $\underline{2}$. Theoretically, little is known about the dependence of $\beta(t_a,t_b,\omega)$ on ω. Blankenbecler and Sugar[64], and also Drummond[65] using a different method, have made some predictions of this dependence, which are however model dependent. Unfortunately, the experimental data are as yet insufficient to test these predictions.

The general features of formula (45) are quite obvious. The amplitude is appreciable only when both t_a and t_b are small and it has a dependence on s_{12} and s_{23} characteristic of the exchange quantum numbers, and of the intercepts of the Regge poles α_a and α_b.

By restricting oneself to those events in the central region of the Dalitz plot with all s_{ij} large, the model can be systematically tested by fitting data with the formula (45) in the same way as one does in Regge analyses of two-body reactions. The accuracy of such tests is limited by the available data. However, from the contributions to this Conference reviewed by Czyzewski[66], it appears that fits in this direction are becoming quantitative. At present, one can claim in decreasing order of certainty[67] that:

i) exchange quantum numbers forbidden in two-body reactions are also forbidden here;
ii) the intercepts of Regge poles are similar to those observed in two-body reactions;
iii) vertex functions are approximately exponential in t and weakly dependent of the Toller variable ω.

In the near future, the following analyses should be feasible:

i) quantitative determination of the Regge intercepts;
ii) observation of nonsense dips, e.g. in ρ exchange;
iii) detection of shrinkage in peaks;
iv) test of the Regge phase, for example by interference with known resonances.

Such direct tests of the multi-Regge model, however, being restricted to events with all s_{ij} large, are applicable only to three-body events; in fact only to a small fraction of such, at present experimental energies. In order to extend our study to reactions with more than three particles in the final state, one needs to generalize formula (45) not only to arbitrary multiplicity in the multi-Regge region, but also to the regions where some particles emerge in clusters, each with a low effective mass. The first question to settle is: what are the proper variables for Reggeization? This has been settled in a paper by Bali, Chew and Pignotti[68], applying an elegant technique for Reggeization developed by Toller. However, the Regge model by itself gives no information on the structure of low-mass clusters. Any attempts at a general analysis must therefore supplement the multi-Regge model by further assumptions concerning these clusters.

One example of such attempts by a CERN group[69] assumes that, except for sharp resonances, the structure of low-mass clusters is governed only by phase space. In other words, one makes here the statistical assumption for low-mass clusters in the same way as Fermi did for low-energy production processes. A parametrization was suggested which interpolates between the multi-Regge region and the region where clusters are formed. Then, with further simplifying assumptions, such as the neglect of differences in charge of final pions, a model was constructed which allows one to calculate cross-sections and single-particle distributions for varying multiplicities and energies, in terms of a few constant parameters.

This model, and variants of it, have now been used to study a number of reactions, in particular from πN collision, for details of which I refer to the review by Czyzewski[66] in this Conference. It seems that in general such a model is able to give a qualitatively correct description of the following features of the data[66,69]:

i) for fixed s and increasing multiplicity n, the gradual transition of single-particle distributions from ones showing strongly multiperipheral features to ones approximating phase space;
ii) the opposite transition for fixed n and increasing s;
iii) the dependence of the cross-section on energy for not too high multiplicities ($n \lesssim 7$);
iv) the dependence of the average transverse momentum on multiplicity;
v) the dependence of the average transverse momentum on the longitudinal momentum for fixed n and s;
vi) the dependence of final-particle distributions on quantum numbers.

In particular, it was found that cross-sections and particle distributions are sensitive to exchange quantum numbers and to intercepts of exchanged Regge poles. The application to these of knowledge gained from two-body reactions gives definite predictions which are in agreement with data[69,70].

From a purely descriptive point of view, therefore, it appears that the multi-Regge model is reasonably successful. However, present calculations rely too strongly on the Monte Carlo technique, the efficiency of which decreases rapidly with increasing energy. A better technique for calculation has to be developed before one can make the analyses more quantitative.

We turn next to the much deeper problem of consistency with unitarity. Consider first the simple case for elastic scattering, represented by the amplitude $\langle i|T|f\rangle$. The unitarity condition reads:

$$\mathrm{Im}\,\langle i|T|f\rangle = \sum_n \langle i|T^*|n\rangle\langle n|T|f\rangle\,, \qquad (46)$$

where n runs over all elastic and inelastic states. If one assumes further that the elastic amplitude is purely imaginary at high energy, Eq. (46) will give elastic scattering in terms of multi-particle final states. Thus, given a model for inelastic processes, one should by Eq. (46) be able to calculate its shadow on the elastic channel and obtain agreement with elastic data. Thus, for example, the slope of the diffraction peak at small t must agree with the slope of the overlap function, as defined by Van Hove.

The argument can obviously be extended to cases where $\langle i|T|f\rangle$ is itself inelastic. The result is a large number of consistency relations that inelastic amplitudes have to satisfy. These conditions may represent an enormous source of physical information once we have a reliable model for inelastic collisions.

Work in this direction is still quite primitive, being limited by the crudeness of present inelastic models. Nevertheless, a beginning has been made, and I shall quote a few examples connected with the multi-Regge model from among the contributions to the Conference:

i) Barger and Cline[71] made the observation that the isospin independence predicted by meson exchange models for both the elastic and total pp and $\bar{p}p$ cross-sections at high energy implies that the total inelastic cross-sections must also be isospin independent. From the multi-Regge point of view, this requirement is by no means obvious, especially in the case of $\bar{p}p$ inelastic which includes annihilation channels supposedly described by multi-baryon exchange. Satisfying these constraints presumably implies complicated conditions on the various exchanges involved.

ii) It has been shown, in a general analysis by Koba and Namiki[72], that the slope in t near t = 0 of the Van Hove overlap function is expressible as a sum of two positive terms: $\Gamma = \Gamma_1 + \Gamma_2$, where Γ_1 depends only on the absolute value of the inelastic amplitude, whilst Γ_2 is sensitive to the momentum-dependence of the amplitude's phase. Thus, only Γ_1 and not Γ_2 can be calculated from experimental measurements of particle distributions in, for example, bubble chamber experiments. Now the unitarity condition (46) requires that Γ be approximately equal (say, to within 10 per cent) to the diffraction slope of elastic scattering, Γ_{el}. Michejda et al.[73] have calculated Γ_1 for the reaction $\pi p \to p + n\pi$ at 8 GeV/c using the multi-Regge model of Chan et al.[69] discussed above, and found $\Gamma_1 < (1/4)\Gamma_{el}$, which is much too small to explain the elastic slope. However, taking the phase of the amplitude as given by the signature factors of the Reggeons exchanged, they obtained a Γ_2 of the right magnitude. A quantitative comparison is not possible at present without a better knowledge of the phase in the low mass regions. Nevertheless, the observed strong dependence on the Regge phase is interesting.

iii) Chew and Pignotti[74] went further, and suggested a new bootstrap mechanism with the multi-Regge model. They started with a simplified version of the model in which there are only two meson trajectories, the Pomeranchuk P, and another one M, which represents the average of all non-Pomeranchuk poles. With some approximations to phase space, they then made a crude estimate of the total cross-section in terms of the trajectory parameters α and the internal couplings g of the Regge poles. Then by requiring that the total cross-section does not violate the Froissart bound at asymptotic energies, they obtained the conditions:

$$\begin{aligned} g_M^2 &\lesssim 2(1-\alpha_M) \\ g_P^2 &\lesssim 2(1-\alpha_P)\,, \end{aligned} \tag{47}$$

where g and α represent constant averages of the corresponding quantities. The inequalities are converted to near equalities if the total cross-section is required to go to a constant as $s \to \infty$. One notes that by formula (47) the constant g_P, which represents the coupling of P to M and an external particle, is required to be small inasmuch as α_P is close to 1. This result is similar to an earlier result of Ter-Martirosyan, and of Finkelstein and Kajantie[75]. However, α_M being ~ 0.5, the constant g_M can be large, and will thus dominate in the high-energy region. The application of their results to the study of pp inelastic cross-sections above 6 GeV/c yields good agreement with experiment. During the discussion in the parallel sessions, Professor Chew reported a significant advance by Low and Goldberger in generalizing the Amati-Fubini-Stanghellini method of dealing with multiparticle unitarity. This will be reviewed by Professor Frazer in another session[11].

Acknowledgements

I am grateful to Dr. M. Jacob for helpful advice and for reading a draft of the manuscript, to Drs. C.B. Chiu, J. Finkelstein, G. Kane, C. Lovelace and C. Schmid for discussions, and to Drs. R.C. Arnold, N. Byers, F. Gilman and R.F. Peierls for pointing out errors and omissions.

REFERENCES AND FOOTNOTES

1. See, for example, V. Barger and A. Białas, review papers in the Proc. of the Topical Conference on High-Energy Collisions of Hadrons, CERN (1968), CERN 68-7, pp. 3-60 and 218-252.
Y. Ne'eman, paper 806.
H.D.D. Watson et al., paper 828.
Bruxelles-CERN Collaboration, paper 444.
A. Białas et al., paper 687.

2. See, for example, R.J. Glauber, Proc. Conf. on High-Energy Physics and Nuclear Structure (edited by G. Alexander) (North-Holland Publ. Co., Amsterdam, 1967), pp. 311-338.
K.S. Kölbig and B. Margolis, paper 3.
I.I. Levintov and K.A. Ter-Martirosyan, abstract 876.
V.N. Gribov, abstract 885.

3. D.R.O. Morrison, private communication during this Conference; also Proc. Conf. on High-Energy Two-Body Reactions, Stony Brook, N.Y. (1966).

4. See, for example, L. Van Hove, review paper in the Proc. Int. Conf. on High-Energy Physics, Berkeley (1966), p. 253.

5. V. Barger and D. Cline, paper 53.

6. V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966); Phys. Rev. 155, 1792 (1967).

7. See, for example, L. Bertocchi, review paper in the Proc. Int. Conf. on Elementary Particles, Heidelberg (1967), pp. 198-250. This gives also references to earlier work.

8. A.P. Contogouris, H.J. Lubatti and J. Tran Thanh Van, Phys. Rev. Letters 19, 1352 (1967).

9. M. Krammer and U. Maor, paper 249.

10. K. Igi et al., paper 780.

11. W.R. Frazer, Rapporteur's talk.

12. A. Capella, A.P. Contogouris and J. Tran Thanh Van, Orsay preprint (1968).
P. di Vecchia, F. Drago and M.L. Paciello, Rome preprint (1968), and others.

13. See, for example, J.S. Ball, W.R. Frazer and M. Jacob, Phys. Rev. Letters 20, 518 (1968).

14. See, for example, R.J.N. Phillips, Nucl. Phys. B2, 394 (1967).

15. See, for example, V. Barger, review paper in the Proc. of the Topical Conference on High-Energy Collisions of Hadrons, CERN (1968), CERN 68-7, p. 3-60.

16. Saclay-Orsay-Pisa Collaboration, Proc. Int. Conf. on Elementary Particles, Heidelberg (1967).

17. For example, V. Barger and L. Durand III, Phys. Rev. Letters 19, 1295 (1967).

18. See, for example, Ref. 17; also H. Rollnik and E. Seitz, paper 503.

19. M. Le Bellac, Phys. Letters 25 B, 524 (1967).

20. Aachen-Berlin-CERN Collaboration, CERN preprint (Ref. D.Ph.II/DROM/mm); paper 452.

21. H. Høgaasen and W. Fischer, Phys. Letters 22, 516 (1966).

22. F. Arbab and R.C. Brower, UCRL-18291 (1968).

23. V. de Alfaro, S. Fubini, G. Rossetti and G. Furlan, Phys. Letters 21, 576 (1966).

24. K. Igi, Phys. Rev. Letters 9, 76 (1962).

25. A. Logunov, L.D. Soloviev and A.N. Tavkhelidze, Phys. Letters 24 B, 181 (1967).
K. Igi and S. Matsuda, Phys. Rev. Letters 18, 625 (1967).
R. Dolen, D. Horn and C. Schmid, Phys. Rev. 166, 1768 (1968).
R. Gatto, Phys. Letters 25 B, 140 (1967).

26. Y. Liu and S. Okubo, Phys. Rev. Letters 19, 190 (1967).
M.G. Olsson, Phys. Letters 26 B, 310 (1968).
A. Della Selva, L. Masperi and R. Odorico, Nuovo Cimento 54 A, 979 (1968).
V. Barger and R.J.N. Phillips, Phys. Letters 26 B, 730 (1968).

27. A. Della Selva et al., paper 621.

28. V. Barger and R.J.N. Phillips, paper 51.

29. M.G. Olsson and G.B. Yodh, Maryland preprint (1968).

30. R. Dolen, D. Horn and C. Schmid, Phys. Rev. Letters 19, 402 (1967).

31. F.J. Gilman et al., paper 631.

32. S. Matsuda and K. Igi, Phys. Rev. Letters 19, 928 (1967), and paper 178.

33. S.Y. Chu and D.P. Roy, paper 592.

34. P. di Vecchia, F. Drago and M.L. Paciello, Phys. Letters 26 B, 530 (1968).
G.V. Dass and C. Michael, Phys. Rev. Letters 20, 1066 (1968).

35. P. di Vecchia et al., paper 622; also Trieste preprint, Aug. 1968.

36. A.D. Martin and G.G. Ross, Phys. Letters 26 B, 527 (1968).

37. R.K. Logan and M.S.K. Razmi, Toronto preprint (1968).

38. G.F. Chew, Comments on Nuclear and Particle Physics 2, 74 (1968).

39. C. Schmid, Phys. Rev. Letters 20, 689 (1968).

40. V. Barger and L. Durand III, paper 481.

41. G. Veneziano, paper 667.

42. J.D. Jackson, J.T. Donohue, K. Gottfried, R. Keyser and B.E.Y. Svensson, Phys. Rev. 139 B, 428 (1965).

43. G. Cohen-Tannoudji, A. Morel and H. Navelet, Nuovo Cimento 48 A, 1075 (1967).

44. G. Bellettini, Rapporteur's talk.

45. T.T. Chou and C.N. Yang, Phys. Rev. 170, 1591 (1968).

46. H.D.I. Arbarbanel, S.D. Drell and F.J. Gilman, Phys. Rev. Letters 20, 280 (1968); also paper 682.

47. Y. Ne'eman, paper 683.

48. C.B. Chiu and J. Finkelstein, CERN Preprint TH 892 (1968); also paper 176.
R.C. Arnold, The next step in high-energy phenomenology, Argonne preprint (1968).
R.C. Arnold and M.L. Blackmon, paper 60.

49. S. Frautschi and B. Margolis, paper 727.
A.A. Anselm and I.T. Dyatlov, paper 851.

50. T.T. Wu and C.N. Yang, Phys. Rev. 137, B 708 (1965).
N. Byers and C.N. Yang, Phys. Rev. 142, 976 (1966).

51. For example, L. Van Hove, Proc. Conf. on High-Energy Two-Body Reactions, Stony Brook, N.Y. (1966).

52. L. Durand III and R. Lipes, Phys. Rev. Letters 20, 637 (1968).
T.T. Chou and C.N. Yang, Phys. Rev. Letters 20, 1213 (1968).

53. R.C. Arnold, Phys. Rev. 153, 1523 (1967).
A.A. Anselm and I.T. Dyatlov, Phys. Letters 24 B, 479 (1967).
A.P. Contogouris, Phys. Letters 23, 698 (1966).

54. F. Henyey et al., paper 1.

55. V.N. Gribov and A.A. Migdal, papers 943 and 944.

56. V. Barger and R.J.N. Phillips, Phys. Rev. Letters 20, 564 (1968).

57. T.T. Chou and C.N. Yang, Stanford preprint (1968).
N. Byers and S. Frautschi, UCLA preprint (1968), and abstract 736.

58. R.C. Arnold, paper 61.

59. See, for example, R. Henzi, paper 511.

60. D. Amati, S. Fubini and A. Stanghellini, Nuovo Cimento 26, 896 (1962).

61. D.A. Ter-Martirosyan, Soviet Phys. JETP 44, 341 (1963).
T.W.B. Kibble, Phys. Rev. 131, 2282 (1963).

62. For a list of the early references see, for example, Chan Hong-Mo, review paper in the Proc. of the Topical Conference on High-Energy Collisions of Hadrons, CERN (1968), CERN 68-7, p. 380-415.

63. See, for example, Chan Hong-Mo, K. Kajantie and G. Ranft, Nuovo Cimento 49 A, 157 (1967).

64. R. Blankenbecler and R.L. Sugar, Phys. Rev. 168, 1597 (1968).
J.M. Kosterlitz, Oxford University preprint (1968).

65. I.T. Drummond, paper 512.

66. O. Czyzewski, Rapporteur's talk.

67. Chan Hong-Mo, Proc. Int. Conf. on Particles and Fields, Rochester (1967), p. 592-602.

68. N.F. Bali, G.F. Chew and A. Pignotti, Phys. Rev. 163, 1572 (1967).

69. Chan Hong-Mo, J. Łoskiewicz and W.W.M. Allison, CERN preprint TH 866 (1968), to be published in Nuovo Cimento.

70. G. Ranft, paper 798.

71. V. Barger and D. Cline, paper 48.

72. Z. Koba and M. Namiki, paper 747.

73. L. Michejda et al., paper 576.

74. G.F. Chew and A. Pignotti, paper 121.

75. I.A. Verdiev, O.V. Kancheli, S.G. Matinyan, A.M. Popova and K.A. Ter-Martirosyan, Soviet Phys. JETP 19, 1148 (1964). J. Finkelstein and K. Kajantie, Phys. Letters 26 B, 305 (1968).

DISCUSSION

BERTOCCHI: You quoted a contradiction between the determination of the sense-choosing mechanism of the A_2 trajectory as determined from the sum rules. As you said, Roy and Stein used photoproduction, whilst Igi and Matsuda used KN scattering.

The apparent contradiction can be understood if you notice that from continuous moment sum rules*) it turns out that if you look to the amplitude which contains the A_2, it contains also the π', the π conspirator. For positive t, the A_2 is indeed dominant, while for negative t where the A_2 trajectory vanishes the π_c is dominant, so that if you use only the A_2 exchange you will not find the correct answer.

RATTI: In the study of vector boson production in π^- collisions at 11 GeV/c, the introduction of absorptive corrections improves the fit of the RP model to the data representing the t-dependence of the differential cross-section. However the same happens to the old OPE, both in the form factors and the absorptive versions.

On the other hand, the RP model fails in predicting the t-dependence of the spin density matrix elements.

My question is whether you see any possibility of overcoming the difficulty found by the RP model in reproducing the ρ_{ij}'s.

FINKELSTEIN: I think that these diffractions or cut models can shed some light on the question of the slope of the Pomeranchon. In the first place, since in these models the 2-Pomeranchon cut contributes to the amplitude with a sign opposite to that of the Pomeranchon pole, then for given slope of the Pomeranchon, there is more shrinkage than there would be if the cuts were absent; this means that in order to fit data which show very little shrinkage, the slope of the Pomeranchon must be very small indeed. Secondly, if we expect that cuts as well as poles be exchange degenerate--for example, in pp scattering, where we know the total cross-section is very nearly constant with energy--thus the Pomeranchon would have to be flat.

CHEW: Fox has observed that all Regge-pole difficulties in two-particle I = 1 exchange reactions near t = 0 can be resolved by assigning M = 0 to the pion trajectory and having a separate M = 1 trajectory pair. This combination reproduces the usual absorptive model. The new Gell-Mann - Zweig model, developed from entirely different arguments, contains an M = 1 exchange-degenerate trajectory (passing through the A_2) of precisely the required nature. Here we may have an example of a new kind of duality: several different Regge-poles being correlated so as to duplicate the effects of absorption.

Relevant to the same general question is the Regge cut-pole relationship exhibited by the multiperipheral equation mentioned at the end of Chan's report. This equation, being based on unitarity, contains absorptive effects and correspondingly generates cuts. The Regge-cut discontinuities are large, however, only when there are Regge poles lying nearby on an "unphysical" J-sheet. This situation is analogous to that in the energy-plane, where cuts are small except in the presence of resonances. Just as

*) P. di Vecchia, F. Drago, F. Ferro-Fontan, R. Odorico, paper not presented to the Conference.

in the energy-plane, J-cuts can be approximated by the underlying poles--inclusion of secondary trajectories and cuts constitute double-counting.

TER MARTIROSYAN: I would like to make some remarks. Firstly, I want to say that Gribov, in a paper published elsewhere and in two papers presented at this Conference (in collaboration with Migdal), has developed a very simple and nice technique allowing him to evaluate the contribution of any graph containing Reggeon lines and corresponding to the so-called rescattering processes. His approach is very general and includes, as a special case, a number of models mentioned in Dr. Chan's report. For instance, the Yang optical model, or an approach developed by Arnold, can be obtained as a special case (in the framework of Gribov's technique), of the values of parameters, or vertices, which enter. Secondly, I want to state that this technique can be used as a basis for complex angular momentum theory. As a result, the theory can be put in the form of a power series in a small parameter $1/\xi$, where $\xi = \ln s/s_0$. The zero-th order term in $1/\xi$ corresponds to the Regge-pole contribution, higher orders to the rescattering processes.

Thirdly, I want to say that evaluation of the rescattering corrections (i.e. Mandelstam cut contribution) has shown that they are very important at large $|t|$ and at $|t| \sim m^2$ inside of the scattering cone. On the contrary, at $t = 0$ (and for $\alpha'|t|\ln s/s_0 \ll 1$) their effect on the phase of the scattering amplitude and on its energy dependence turns out to be negligible. Using the values of parameters for the Regge-pole residues and trajectories, which are now very well known, I have estimated (in the paper presented to the Conference) that at $t = 0$ the rescattering effect is always of the order of 2-4%.