GENERAL PROBLEMS OF FIELD THEORY

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K. Hepp

According to the rule that everybody gets promoted to his level of incompetence, I have the honour to report on recent results in quantum field theory in perturbation theory, in axiomatic and in constructive quantum field theory. It requires a superhuman mind to do justice to all the fine points of the numerous contributions in the two parallel sessions on field theory which have lasted for more than eight hours. With due reference to the creativity of all people present I start my talk with my sincere apology for all the omissions and distortions which I am going to make. My language will appear to some of you unduely mathematical. My justification is the experimental fact that physical observations can be cast in mathematical formulae which have predictive power. It is also well-known that we, human beings, are very incomplete: in the same way as my experimental colleagues need more sophisticated measuring instruments to extend their senses, similarly we theorists need the most complicated mathematical structures in order not to be led astray in our exploration of Nature.

Most of our present knowledge about quantum field theory has been first guessed from formal **perturbation theory** or by using functional «integration» for a closed representation of the «solutions» of the field equations. Significant progress has been made at this conference on two interesting topics, on the definition of the S-operator for non-polynomial Lagrangians and on the $m \to 0$ limit in massive Yang — Mills theories. In my review on non-polynomial Lagrangians I shall restrict myself to interaction densities $V_0(x)$ which are functions of a free neutral scalar field $\Phi_0(x)$. The ill-defined formal perturbation expansions of quantum field theory are in this case

$$S(h) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int dy_1 \ldots dy_n h(y_1) \ldots h(y_n) T(V_0(y_1) \ldots V_0(y_n)) \qquad (1)$$

for the space-time cut-off scattering operator,

$$\langle T(\Phi(x_1) \ldots \Phi(x_m))\rangle^T = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int dy_1 \ldots dy_n \langle T(\Phi_0(x_1) \ldots V_0(y_n))\rangle_0^T \quad (2)$$

for the truncated Green's functions (where $\langle \ldots \rangle_0^T$ is equivalent to taking only connected graphs) and

$$R(\Phi(x_1) \ldots \Phi(x_m)) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int dy_1 \ldots dy_n R(\Phi_0(x_1) \ldots V_0(y_n)). \quad (3)$$

The interaction density $V_0(y)$ is assumed to be a Wick ordered formal power series in $\Phi_0(y)$,

$$V_0(y) = \sum_{n=0}^{\infty} \frac{a_n}{n!} : \Phi_0(y)^n : a_n^* = a_n. \quad (4)$$

It is a plausible extrapolation from the careful analysis by Jaffe [1] on the entire functions of the free field to conjecture that the space of generalized functions, in which (1), (2) and (3) should have non-formal meaning in every order of V_0 («major coupling constant») is determined by the 2-point function

$$E_+(x_1 - x_2) = \sum_{n=1}^{\infty} \frac{a_n^2}{n!} \Delta_+(x_1 - x_2)^n. \quad (5)$$

Let me start with the Jaffe class [2] of **strictly localizable interactions,** where the two-point function belongs to $\mathfrak{c}_g'(R^4)$. The test function space $\mathfrak{c}_g(R^4)$ is the Fourier transform of the space $\mathfrak{M}_g(R^4)$ of all C^∞ functions f with finite semi-norms

$$\|f\|_{mnA} = \sup_p g(A\|p\|^2)(1 + \|p\|^2)^m |D^n f(p)|, \quad (6)$$

$$g(t) = \sum_{r=0}^{\infty} C_{2r} t^r, \quad C_{2r} \geqslant 0, \quad C_0 > 0,$$

$$\int_0^{\infty} dt \ln g(t^2)/(1 + t^2) < \infty$$

for all integers $m, n \geqslant 0$ and $A \geqslant 0$. The indicator function g is entire and chosen consistently with the nuclear theorem and characterizes the growth of the Wightman functions in momentum space. In the abstract manner of Bogolubov and co-workers [3] we shall define a renormalization as a mapping which maps every n-tuple $(V_1(y_1) \ldots V_n(y_n))$ of entire functions of $\Phi_0(y)$ of type g (i. e. defined as operator valued generalized functions with test functions in $\mathfrak{c}_g(R^4)$ into operator-valued generalized functions $T(V_1(y_1) \ldots V_n(y_n))$ and $\overline{T}(V_1(y_1) \ldots \ldots V_n(y_n))$ over $\mathfrak{c}_g(R^{4n})$ which satisfy the following three axioms

$$(A)\ T(V_1(y_1) \ldots V_n(y_n)) = T(V_{P(1)}(y_{P(1)}) \ldots V_{P(n)}(y_{P(n)})) = \overline{T}(V_1(y_1)^* \ldots V_n(y_n)^*)^* =$$
$$= U(a, \Lambda) T(V_1(\Lambda^{-1}(y_1 - a)) \ldots V_n(\Lambda^{-1}(y_n - a))) U(a, \Lambda)^{-1} \quad (7)$$

where $(a, \Lambda) \in iL_+^\uparrow$ is a Lorentz transformation and P a permutation of $(1, \ldots, n)$.
(B) Unitarity

$$0 = \sum_{k=0}^{n} (-)^k \sum_{i_1 < \ldots < i_k} \overline{T}(V_{i_1}(x_{i_1}) \ldots V_{i_k}(x_{i_k})) T(V_{i_{k+1}}(x_{i_{k+1}}) \ldots V_{i_n}(x_{i_n})). \quad (8)$$

(C) Causality: The support of

$$\sum_{k=0}^{n-1} (-)^k \sum_{1<i_1<\ldots<i_k} \overline{T}(V_1(x_1) V_{i_1}(x_{i_1}) \ldots V_{i_k}(x_k)) \times \times T(V_{i_{k+1}}(x_{i_{k+1}}) \ldots V_{i_{n-1}}(x_{i_{n-1}})) \quad (9)$$

lies in $\{x_1 - x_k \in \overline{V}_+,\ 1 < k \leqslant n\}$.

Of course, the $\overset{(-)}{T}(V_1(x_1) \ldots V_n(x_n))$ should have a common dense invariant domain in the Fock space F of $\Phi_0(x)$ and then (8) and (9) are well-defined in the sense of operator-valued generalized functions.

Let us require that the time-ordering T is not only defined for entire functions of $\Phi_0(y)$ of type g, but also for series of type g in generalized Wick monomials of the type

$$\left(D \prod_{i=1}^{n-1} \delta(y_1 - y_{i+1})\right) : D_1\Phi_0(y_1) \ldots D_n\Phi_0(y_n) : \quad (10)$$

(quasi-local operators of the type g).

The very heated discussion about the «almost unique» renormalizability of non-polynomial Lagrangians at this conference can be supplemented by the following.

Conjecture. For every g of strictly localizable type, there exist definitions of time-ordered products satisfying the axioms (A, B, C) of renormalization. Every two renormalizations differ by a finite renormalization, i. e. by the recursive addition of a quasi-local operator of type g, when going from $n - 1$ to n in the definition of $\overset{(-)}{T}(V_1(y_1) \ldots V_n(y_n))$. Within the class of polynomial interactions this conjecture is true, as we have learnt from the work of Bogolubov and Parasiuk [4]. Epstein and Glaser [5] have recently given a concrete construction of time-ordered products in the polynomial case which is based on the structural properties (A, B, C). One can verify that very different renormalization prescriptions for Feynman graphs, as the R-operation, analytic renormalization [6] and the construction based on Hörmander's solution of the division problem by polynomials within S' (presented by Stepanov at this conference), that all these seemingly very different analytical operations lead to time-ordered products with the (A, B, C) structure. Therefore [7] there exists an algorithm for computing the finite renormalizations by which any two axiomatic renormalizations differ.

As indicated before, the above conjecture is not yet proved, but the Epstein — Glaser method does not need serious modifications in order to become applicable. Let me indicate this idea for $V_0(x) = :\exp \Phi_0(x):$, where the two-point function has been treated by many different methods in the work of Volkov [8], Arbuzov, Atakishiev and Filippov [9], Lehmann and Pohlmeyer [10], Constantinescu [11] and by Christ [12].

Here a renormalization amounts to defining

$$T(:\exp\Phi_0(x_1): \ldots :\exp\Phi_0(x_n):) = \exp\left(\sum_{i<j} \Delta_F(x_i - x_j)\right) : \Pi \exp \Phi_0(x_i): \quad (11)$$

which is to start with only given for arguments $(x_1, \ldots, x_n)$ where $(x_i - x_j)^2 \neq 0$ for all $i \neq j$. With the notation $\overset{(-)}{E}_F(x) = \exp(\overset{(-)}{\Delta}_F(x)) - 1$ one has for instance to define $\prod_{i<j} \overset{(-)}{E}_F(x_i - x_j)$ as $iL_+^\uparrow$ — invariant symmetric generalized functions of type g, where g can be taken of the form $C_{2r} = 1/(3r - \varepsilon r)!$, $0 < \varepsilon < 1$ [2]. The requirements (A, B, C) become besides symmetry and Lorentz invariance

$$\prod_{i<j} \overline{E}_F(x_i - x_j) = \left(\prod_{i<j} E_F(x_i - x_j)\right)^*, \quad (12)$$

$$0 = \sum_{X,Y} (-)^{|X|} \Big(\prod_{\substack{i<j \\ i,j \in X}} \overline{E}_F(x_i - x_j) \Big) \prod_{\substack{i \in X \\ j \in Y}} E_+(x_i - x_j) \Big(\prod_{\substack{i<j \\ i,j \in Y}} E_F(x_i - x_j) \Big) \tag{13}$$

(where Σ extends over all disjoint partitions X, Y of $\{1, \ldots, n\}$ and $|X|$ is the number of elements in X). Finally one needs that the support of

$$\sum_{X \ni 1} (-)^{|X|} \Big(\prod_{\substack{i<j \\ i,j \in X}} \overline{E}_F(x_i - x_j) \Big) \prod_{\substack{i \in Y \\ j \in X}} E_+(x_i - x_j) \Big(\prod_{\substack{i<j \\ i,j \in Y}} E_F(x_i - x_j) \Big) \tag{14}$$

lies in $\{x_1 - x_k \in \overline{V}_+,\ 1 < k \leqslant n\}$.

For $n = 2$ Jaffe [2] has given a definition of $\overset{(-)}{E}_F(x)$ satisfying (12), (13) and (14). In this construction the ambiguity is

$$\Delta E_F(x) = \sum_{n=0}^{\infty} b_n \,\square^n \delta(x), \qquad \overset{*}{b}_n = - b_n \tag{15}$$

which has to be a generalized function in $\mathfrak{c}_g\,(R^4)$ with support $\{0\}$. The Epstein—Glaser construction can be viewed as follows: By the causality requirement (14) the expressions

$$\begin{aligned} R(x_1 - x_2) &= E_F(x_1 - x_2) - E_+(x_2 - x_1), \\ A(x_1 - x_2) &= E_F(x_1 - x_2) - E_+(x_1 - x_2) \end{aligned} \tag{16}$$

should have the property of retarded (support: $x_1 - x_2 \in \overline{V}_+$) and advanced (support: $x_2 - x_1 \in \overline{V}_+$) generalized functions. The difference

$$C(x_1 - x_2) = R(x_1 - x_2) - A(x_1 - x_2) = E_+(x_1 - x_2) - E_+(x_2 - x_1) \tag{17}$$

is well-defined and has support in $\{(x_1 - x_2)^2 \geqslant 0\} = \overline{V}_+ U \overline{V}_-$, in a union of opposite closed convex cones. Thus R and A are obtained by giving non-formal meaning to $\theta\,(x_1^0 - x_2^0)\, C\,(x_1 - x_2)$ which has to be consistent with (12), (13) and (14). This is a well-known operation on strictly localizable generalized functions, which can be performed in p-space by a subtracted dispersion integral. Here the vanishing of $\tilde{C}\,(p)$ for $p^2 < m^2$ is very helpful. By induction, the general case can be treated in the same way. Here one needs retarded and abvanced solutions

$$\begin{aligned} R(x_1 - x_2, \ldots, x_1 - x_n) &= \prod_{i<j} E_F(x_i - x_j) + R'(x_1 - x_2, \ldots, x_1 - x_n), \\ A(x_1 - x_2, \ldots, x_1 - x_n) &= \prod_{i<j} E_F(x_i - x_j) + A'(x_1 - x_2, \ldots, x_1 - x_n) \end{aligned} \tag{18}$$

where R' and A' are already well — defined lower — order contributions and

$$C = R - A = R' - A' \tag{19}$$

has support in $\overline{\Gamma}_+ U \overline{\Gamma}_-$,

$$\overline{\Gamma}_\pm = \{x_1 - x_k \in \overline{V}_\pm,\quad 1 < k \leqslant n\}. \tag{20}$$

A consistent cutting in $\mathfrak{c}_g'$ is possible and defines $\prod_{i<j} E_F\,(x_i - x_j)$ non-uniquely up to a quasi — local generalized function of the type g.

What is the relation of this definition of superpropagators to the ones discussed at this conference? The almost uniqueness of $E_F\,(x)$ in the work of the above mentioned authors stems from additional regularity requirements [10] or from seeking the solution of the cutting problem within a somehow physically motivated regularization scheme [8, 9]. A typical regularization is the transition to the Euclidean region, analytic interpolation and integral transformations, while a typical regularity requirements has been given by Filippov [13]:

$$\frac{\operatorname{Re} \tilde{E}_F(p^2)}{\operatorname{Im} \tilde{E}_F(p^2)} \to 0 \text{ for } p^2 \to +\infty. \tag{21}$$

It is clear that such postulates are physically very important. In the polynomial case one only considers renormalizations, which in addition to (A, B, C) are minimal in the sense that the power counting index is preserved in the renormalized amplitudes. Only in the minimal renormalization schemes the distinction between finitely renormalizable theories (with finitely many different counterterms in $V_0(y)$ for all orders n) and finitely non-renormalizable theories makes sense. A further clarification of the minimality conditions for non-polynomial interactions is highly desirable.

The advantage of the general distribution theoretical setting for the definition of superpropagators is that one automatically obtains the unitarity of $S(h)$ for $h \in \mathfrak{C}_g(R^4)$ in every order of h, and the definition of the terms in the Gell-Mann — Low series (2) as $iL_+^\uparrow$ — invariant generalized functions. However, without proper care of the vacuum diagrams and the finite mass renormalizations the adiabatic limit

$$S = \lim_{h \to 1} S(h) \tag{22}$$

and the interpolating fields (3) have no hope to exist. It is a shame that a complete and rigorous proof of the adiabatic theorem in all orders in V_0 has not yet been given, even for polynomial interactions, although some progress has recently been made [7]. At any rate it has to be stressed that even in the most well-defined approach to superpropagators the finite mass- and amplitude renormalizations have to be carried out.

Papers at this conference (see Efimov [14], Salam and Strathdee [15] and Keck and Taylor [16]) have discussed the definition of time-ordered products for **non-localizable interactions,** with the following clear motivation:

(a) occurence of these interactions in a non-linear realization of the chiral group or in quantum gravity,

(b) existence of entire functions of non-localizable type, which decrease as cut-offs in propagators in the Euclidean infinity $p^2 \to -\infty$.

Clearly, the locality axioms (A) has to be weakened in this case to macroscopic locality, saying that the local support properties should be rapidly approximated at infinity for large translations. One has to operate in a function space compatible with the singularities of the 2-point function $E_+(x)$ in (5). The construction of superpropagators $E_F(x)$ in second order and the proof of unitarity can now meet any standard of mathematical rigor, and $E_F(x)$ can show many interesting properties. In higher orders the usual Euclidean approach meets the difficulty of analytic continuation. The observation of Efimov [14] is interesting: the local singularities in p-space for certain non-localizable interactions are the same as for the renormalized Feynman amplitudes of polynomial interactions. Since we have only little control about these analyticity properties in higher orders of perturbation theory, a distribution—theoretic proof of unitarity in the sense of (8) is desirable.

Let me turn to the Yang — Mills theory of massive vector mesons. Here the contributions by Slavnov and Faddeev [17], Fradkin and Tyutin [18], Kallosh [19] and Khriplovich and Vainshtein [20] at this conference present the following picture:

(a) The hope that in the massive theory the current conservation leads to a significant cancellation of diagrams and makes the S-operator finitely renormalizable has not been materialized.

(b) The passage to the limit $m \to 0$ is singular in perturbation theory, where in transitions involving k longitudinal external particles the behavior is of the type m^{2-k}.
(c) In the functional integral representation of the S-operator a continuous transition from the massive to the massless theory of Faddeev and Popov [21] can be seen with non-analyticity in the coupling constant around $m = 0$. Here some reservations are necessary, since the infinite renormalizations have not been incorporated in the functional representation.

Unfortunately there is no time to do justice to the contributions by H. P. Dürr, Yu. A. Gol'fand, D. D. Ivanenko, V. N. Melnikov, M. B. Mensky, V. Pavlov, D. Petrina, J. C. Polkinghorne, J. Rayski and A. E. Shabad at this conference.

The second part of my talk is devoted to **axiomatic quantum field theory.** The Wightman axioms require the existence of field operators $\Phi(x)$ as operator-valued tempered distributions on a common dense invariant domain D in a Hilbert space $\mathcal{H}$, with a unitary representation $U(a, \Lambda)$ of $iL_+^\uparrow$, energy-momentum spectrum in $\overline{V}_+$, a vacuum $\Omega \in D$ and satisfying for a scalar field $\Phi(x)$:

$$U(a, \Lambda)\Phi(x)U(a, \Lambda)^{-1} = \Phi(\Lambda x + a), \tag{23}$$

$$[\Phi(x), \Phi(y)]_- = 0 \text{ for } (x-y)^2 < 0. \tag{24}$$

It was realized by Jaffe [22] that all consequences of the Wightman axioms can be obtained, if one only requires the fields to be strictly localizable operator-valued generalized functions. Parallel to the investigation of non-polynomial interactions in perturbation theory the question was discussed at this conference, how to generalize the Wightman axioms to strictly non-locallizable interactions and which of the main consequences of axiomatic quantum field theory can be retained:

(α) TCP-invariance and weak locality,
(β) relation between spin and statistics,
(γ) asymptotic condition,
(δ) dispersion relations,
(ε) bounds on scattering amplitude.

A recent result by Epstein, Glaser and Martin [23] shows that for the proof of dispersion relations and the Froissart bound $s(\ln s)^2$ on the forward scattering amplitude locality is not needed in the sense of the sharp local commutativity (24). What is needed is a local field operator B or a local observable in the sense of Haag and Araki, such that $B\Omega$ is a 1-particle state and

$$[U(x)BU(-x), B] = 0 \tag{25}$$

for all x such that $(x+y)^2 < 0$ for all $y \in R^4$ with $\| y \| \leqslant \rho$. Here ρ can be as large as our galaxy, and still the S-operator computed from B and its Lorentz transforms satisfy twice-subtracted dispersion relations (for favorable mass spectra) and the Froissart bound.

Fainberg, Iofa and Soloviev (see [24]) at this conference relax strict localizability further to **localizable** quantum field theories, by admitting as indicator functions $g(t^2)$ for the test function space those with growth of order $< 1/2$:

$$g(t^2) < C(\varepsilon)\exp \varepsilon\sqrt{t^2} \tag{26}$$

(with $C(\varepsilon) < \infty$ for any $\varepsilon > 0$), and by requiring locality as a topological support property of the Wightman functionals. In such a framework, (α), (β) and (γ) continue to hold. Although a cut-plane analyticity domain for $T(s, t)$ is still unknown, the authors obtain by real methods that $\overline{T}(s, 0)$ (the averaged forward scattering amplitude) does not grow faster than $s^2(\ln g(s))^2$.

It is very interesting that one can further relax in a sensible way the Wightman axioms to **l-localizable interactions,** where the above weak localization only holds up to an elementary length l and where technically

$$g(t^2) \sim \exp l\sqrt{t^2} \text{ for } t \to \infty. \tag{27}$$

Again [24] *TCP* is equivalent to weak local commutativity, spin and statistics are correctly related, the asymptotic condition holds and the bounds on the averaged scattering amplitude are proportional to $s^3 \sim s^2 (\ln g(s))^2$. A further generalization of locality would be **macroscopic causality** in the sense that the truncated vacuum expectation values decrease in space-like direction faster than every power of the inverse radius of the point configuration. Under this assumption on the short range of the forces the asymptotic condition can again be proved and, if the indicator function $g(t^2)$ increases not faster than $\exp(ct^2)$ for $t \to \infty$, the asymptotic bound on the scattering amplitude would be $c \cdot (\ln g(s))^2 \cdot s^2$ with $c < \infty$ [25].

At this conference, Taylor has proposed another axiomatic generalization of strict locality. In his framework the n-point Wightman function $W_n \in (S^{\alpha})'$ [26] can have in momentum space a growth as $\exp \| p \|^{1/\alpha}$ with any $\alpha > 0$. There should exist a family of tempered Wightman fields Φ^{ν} with vacuum expectation values W_n^{ν} and an energy spectrum independent of ν, such that uniformly for translations $\boldsymbol{a} \in R^{4(n-1)}$

$$\lim_{\nu \to \infty} \boldsymbol{a}^m (\partial/\partial \boldsymbol{a})^n [\tau_{\boldsymbol{a}} (W_n - W_n^{\nu})] = 0 \tag{28}$$

in the topoly of $(S^{\alpha})'$. In this generalized local quantum theory TCP, spin and statistics and the asymptotic condition hold, while bounds on the scattering amplitudes have not yet been obtained. The condition (28) is satisfied for the Wightman functions of the field: $\Phi_0(x)(1 - \lambda\Phi_0(x))^{-1}$: and this observation merits certainly interest among the experts.

Let me take the rapporteur's liberty of collecting exotic butterflies in his field: Unitarity in the sense of asymptotic completeness,

$$\mathcal{H} = \mathcal{H}_{\text{in}} = \mathcal{H}_{\text{out}}, \tag{29}$$

has been often involved as a necessary ingredient of any physically reasonable theory. The following theorem by Rinke [27] will certainly trouble all those who pretend to understand what unitarity means in a quantum field theory:

Theorem: Let $\{A(x), \Omega, U(a, \Lambda)\}$ be a tempered local quantum field with a physical mass spectrum. Let $A(x)$ be asymptotically complete and S its scattering operator.

Define $\hat{A}(x) = (A(x) \otimes 1 + 1 \otimes A(x))\, 2^{-1/2}$, $\hat{\Omega} = \Omega \otimes \Omega$ and $\hat{\mathcal{H}}$ as the closure of the polynomial algebra in $\hat{A}$ applied to $\hat{\Omega}$ and $\hat{U}(a, \Lambda)$ as the restriction of $U(a, \Lambda) \otimes U(a, \Lambda)$ to $\hat{\mathcal{H}}$. Then $\{\hat{A}(x), \Omega, \hat{U}(a, \Lambda)\}$ is again a tempered local quantum field with a physical mass spectrum, and $\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\text{in}}$ if and only if $A(x)$ is a free field,

$$\hat{\mathcal{H}}_{\text{in}} = \hat{\mathcal{H}}_{\text{out}} \text{ if and only if } S = 1. \tag{30}$$

The energy-momentum spectrum is called physical for a scalar theory of particles of mass m, if $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_c$, where $\mathcal{H}_0$ is the 1 — dimensional vacuum subspace, $\mathcal{H}_1$ carries an irreducible representation of $iL_+^{\uparrow}$ with mass m and spin zero, induced by $U(a, \Lambda)$, and $U(a, \Lambda)$ restricted to $\mathcal{H}_c$ has a continuous mass spectrum above 2 m.

It is challenge to supplement the Wightman axioms and the spectrum condition by a further physical axiom which allows us to understand and to prove unitarity

$$SS^* = 1 = S^*S \tag{31}$$

which is different from the partial isometry which is implied by the asymptotic condition.

Another interesting contribution at this conference has been made by Korushy and Sushko. The authors imbed the Haag — Kastler framework of local observables into a Hilbert space formalism with a very aesthetic structure of superselection rules. A Haag — Kastler local quantum theory [28] is a rigorization of the concept of a local Heisenberg field: To every open bounded region $B \subset R^4$ one associates a C^*-algebra $\mathfrak{A}(B)$. Furthermore on the C^*-algebra $\mathfrak{A}$ generated by $U_B\, \mathfrak{A}(B)$ there is a representation $(a, \Lambda) \to \alpha_{(a,\Lambda)} \in \mathrm{Aut}(\mathfrak{A})$ of the Lorentz group $iL_+^\uparrow$ such that

$$\begin{gathered} \mathfrak{A}(B_1) \subset \mathfrak{A}(B_2) \text{ for } B_1 \subset B_2, \\ \mathfrak{A}(B_1) \subset \mathfrak{A}(B_2)' \text{ for } B_1,\ B_2 \text{ space-like}, \\ A \to \alpha_{(\alpha,\Lambda)}(A) \text{ is continuous}, \\ \alpha_{(a,\Lambda)}(\mathfrak{A}(B)) = \mathfrak{A}(B_{(a,\Lambda)}). \end{gathered} \tag{32}$$

Finally one requires that $\mathfrak{A}$ has at least one faithful irreducible representation.

If $\mathfrak{A}(B)$ is the algebra generated by all observables which can be measured in B, then all unobservable local quantities, as charged or fermion fields, should be constructed from the observable algebra $\mathfrak{A}$, if $\mathfrak{A}$ has the correct structural properties. This program has been pursued by Borchers [29]. Recently Doplicher, Haag and Roberts [30] have investigated the structure of $\mathfrak{A}$ implied by the existence of a local field algebra $\{F(B)\}$. With this program in mind the structure proposed by Korushy and Sushko at this conference is very reasonable. A global von Neumann algebra R acts in a Hilbert space $\mathcal{H} = \bigoplus \mathcal{H}_\alpha$ with superselection sectors $\mathcal{H}_\alpha$ in such a way that for every $\Psi_i \in H_{\alpha_i}$ $(i = 1, 2, \alpha_1 \neq \alpha_2)$ the state $\omega_{\Psi_1+\Psi_2}$ on R, determined by $\Psi_1 + \Psi_2$, is a mixture. Furthermore, the set of all states $\Psi \in \mathcal{H}$ for which ω_Ψ is a pure state on R, is assumed to be dense in $\mathcal{H}$. Then one obtains naturally by restriction to $\mathcal{H}_\alpha$ that $R = \bigoplus R_\alpha$, where all R_α are type I factors and are Haag — Araki theories, if R has this structure. Furthermore all superselection rules, which here are affiliated with the centre $\mathfrak{Z} = R \cap R'$, commute and, if the subspaces $\mathcal{H}^R_{\varphi_\alpha(i)}$ and $\mathcal{H}^{R'}_{\varphi_\alpha(i)}$ generated by R and R' applied to a pure state $\varphi_{\alpha(i)} \in \mathcal{H}_{\alpha(i)}$ $(i = 1, 2)$ have equal dimension for $i = 1, 2$, then the coherent sectors $\{R_{\alpha(1)}, \mathcal{H}_{\alpha(1)}\}$

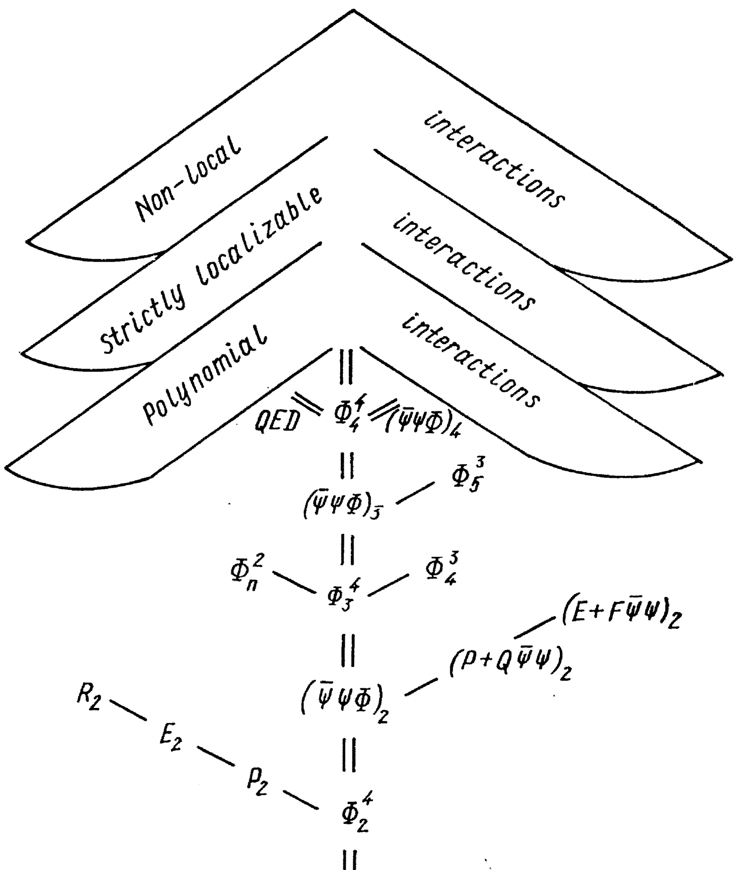

Fig. 1. The field-theory tree (after A. M. Jaffe [31]).

and $\{R_{\alpha(2)}, \mathcal{H}_{\alpha(2)}\}$ are locally unitarily equivalent. This is an interesting step forward towards the solution of the Fermi field problem within algebraic quantum field theory.

Again the time is lacking to characterize the contributions by V. P. Gachok, L. Sh. Knodzaev, J. Lukierski, M. E. Perel'man, Yu. M. Shirokov and B. L. Voronov at this conference. However, the names of all omitted authors should be sufficient to call to your attention their unbroken creativity.

The conclusion of my talk is devoted to **constructive quantum field** theory, where many interesting new results have been obtained in the past two years. The variety of possible models is seen in a drawing by A. Jaffe called «the field-theory tree» (Fig. 1). The notation is as follows: upper indices — exponents; lower indices — space-time dimension, Φ is a scalar, Ψ a Dirac spinor field over an appropriate dimensional momentum space.

The main stem (in double lines) has many intermediate steps. Each step is characterized by a new serious difficulty. One can proceed along the side lines, once the branching point has been climbed. P and Q are polynomials, E and F entire and R a rational function in Φ.

Let me start with an open question: For none of the models on the field theory tree (except for Φ_n^2) the existence of solutions have been shown which satisfy **all** Wightman or Haag — Araki axioms. The greatest difficulty still to overcome is the control of the infinite volume limit, the proof of Lorentz covariance in the physical representation with a physical mass spectrum. In view of the celebrity of this problem I propose that we start collecting founds for a prize for the first climber who reaches a non-trivial S-operator.

For Φ_2^4 the present status is the following

Theorem: For every $\lambda \geqslant 0$ and $m > 0$ there exist operator — valued tempered solutions of the field equations

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + m^2\right) \Phi(x, t) + \lambda \Phi(x, t)^3 = 0 \tag{33}$$

in Fock space F, with the initial conditions

$$\Phi(x, 0) = \Phi_0(x, 0), \qquad \frac{\partial}{\partial t} \Phi(x, 0) = \frac{\partial}{\partial t} \Phi_0(x, 0) \tag{34}$$

which are local and satisfy canonical commutation relations. Here

$$\Phi(x; t)^k = \exp(iH(g)t) : \Phi_0(x, 0)^k : \exp(-iH(g)t) \tag{35}$$

where $H(g) = H_0 + V(g)$,

$$V(g) = \lambda \int_{-\infty}^{+\infty} dxg(x) : \Phi_0(x, 0)^4 : \tag{36}$$

and $g \in S(R^1)$ is any space-cut-off with $g = \bar{g}$ and $g(y) \equiv 1$ for $y \in [x - |t|, x + |t|]$. The Heisenberg fields $\Phi(f) = \int dxdt\, \Phi(x, t) f(x, t)$, $f \in D(B)$ are affiliated with local C^*-algebras $\mathfrak{A}(B)$ with a normcontinuous representation $\alpha_{(a,\Lambda)}$ of $iL_+^\uparrow$, $\{\mathfrak{A}(B), \alpha_{(a,\Lambda)}\}$ satisfies all Haag — Kastler axioms. There exists a physical representation π of $\{\mathfrak{A}(B), \alpha_{(a,1)}\}$ (with only translation invariance) in a separable Hilbert space with a unitary continuous representation $U_\pi(a, 1)$ of the translations with positive energy $H_\pi \geqslant 0$ and a cyclic invariant vacuum Ω_π. The vacuum expectation values $(\Omega_\pi, \pi(A)\Omega_\pi)$ are limits of a sequence of expectation values $(\Omega_n, A\Omega_n)$ of smeared-out ground states of $H(g_n)$, $g_n \to 1$. For every bounded open $B \subset R^2$, $\mathfrak{A}(B)$ and $\pi(\mathfrak{A}(B))$ are unitarily equivalent.

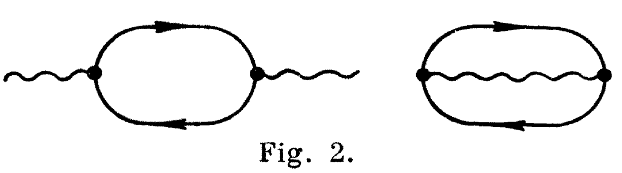

Fig. 2.

These now almost classic results are mainly due to Glimm and Jaffe (see [32]). The main progress in the past two years has been

(a) the proof of the Lorentz covariance of $\{\mathfrak{A}(B)\}$ [33],

(b) a great simplification in the treatment of the local time evolution [34],

(c) the extension of most of the Φ_2^4 results to polynomials $P = \sum_{m=0}^{2n} a_m :$ $:\Phi_0(x)^n:$ where $a_{2n} > 0$ and to certain entire functions [34], [35],

(d) for a class of anharmonic oscillators with N degrees of freedom and probably for the space cut-off Φ_2^4 — interaction the perturbation series for the ground-state energy $E(\lambda)$ determines $E(\lambda)$ uniquely by an Abelian summation [36].

The present state of $(\overline{\Psi}\Psi\Phi)_2$ is rapidly approaching that of Φ_2^4, in spite of a considerable increase of difficulty. For the S-operator one has logarithmic divergencies (Fig. 2). Thus perturbation theory suggests to study instead of $H_0 + V_\varkappa(g)$, where with an ultraviolet cut-off $\varkappa$

$$V_\varkappa(g) = \lambda \int dx g(x) : \overline{\psi}_\varkappa(x)\psi_\varkappa(x) : \Phi_\varkappa(x), \tag{37}$$

the renormalized Hamiltonian

$$H_\varkappa(g) = H_0 + V_\varkappa(g) + R_\varkappa(g). \tag{38}$$

The renormalization part $R_\varkappa(g)$ should comprise the second order mass renormalization and the self-energy of Fig. 2.

Theorem: There exists a choice of $R_\varkappa(g) = R_\varkappa(g)^*$ and a positive self-adjoint $H(g) = H(g)^* \geqslant 0$ with a ground state $\Omega(g)$, which is approximated by $H_\varkappa(g)$ for $z \in [0, \infty)$ as

$$n - \lim_{\varkappa\to\infty} (z - H_\varkappa(g))^{-1} = (z - H(g))^{-1}. \tag{39}$$

The proof of this theorem is due to Glimm and Jaffe [32] as well as the following remarkable result:

Theorem: $H(g)$ defines a local propagation in Fock space with a velocity not exceeding the speed of light.

These results lead to the existence of solutions of the field equations

$$\begin{aligned} &\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + m^2\right)\Phi(x, t) + J(x, t) = 0, \\ &\left(\gamma^0 \frac{\partial}{\partial t} - \gamma^1 \frac{\partial}{\partial x} + M\right)\Psi(x, t) + j(x, t) = 0, \end{aligned} \tag{40}$$

where $j(x, t)$ is defined as in (35) in the form $\lambda(\Psi\Phi)(x, t)$, while $J(x, t)$ is the weak limit [37] of $\lambda(\overline{\Psi}_\varkappa\Psi_\varkappa)(x, t) - \delta m_\varkappa^2\Phi_\varkappa(x, t)$.

The lesson to learn from Φ_2^4 and from $(\overline{\Psi}\Psi\Phi)_2$ is the following: In Φ_2^4 one has presumably a 1—1 correspondence over a continuous range between the parameters of the theory m, λ and the physical masses and coupling constants $\hat{m}$, $\hat{\lambda}$ in the physical representation without space cut-off (as it is suggested by perturbation theory). For $(\overline{\Psi}\Psi\Phi)_2$ one has to specify m, M, λ and a family of mass renormalizations $\delta m_\varkappa^2(g)$ in order to obtain in the no cut-off limit the physical values $\hat{m}$, $\hat{M}$, $\hat{\lambda}$. There is necessarily an infinite mass renormalization in $\delta m_\varkappa^2(g)$, while one can shift a finite mass renormalization from $\delta m_\varkappa^2(g)$ to m, M and λ. The «bare» and the physical parameters are no longer equally well-defined. Quantum field theory as it emerges here in a completely rigorous and non-perturbative setting gives no theory of the coupling constants which distinguishes between $\hat{m}$, $\hat{M}$ and $\hat{\lambda}$ over a continuous range. However, once these three parameters have been fixed, the predictive power of Yukawa models should be phantastic:

all nuclei in a two-dimensional world would emerge and hadron dynamics for all energies! The next level in the field theory tree is scarcely populated, in spite of the following beautiful result by Glimm [38]:

Theorem: Let $V_\varkappa(g) = \int d^2x g(x) : \Phi_\varkappa^4(x, 0) : g = g^* \in S(R^2)$. There exist renormalizations $R_\varkappa(g) = R_\varkappa(g)^*$ in the form predicted by the divergent graphs (Fig. 3) and a truncation $T_\varkappa(g)$ of the perturbation series of the wave operator $T_\varkappa^\pm(g)$, such that for all φ, Ψ from a dense domain $D \subset F$

$$\lim_{\varkappa\to\infty} (T_\varkappa(g)\varphi, T_\varkappa(g)\Psi) = \langle T(g)\varphi, T(g)\Psi\rangle,$$
$$\lim_{\varkappa\to\infty} (T_\varkappa(g)\varphi, H_\varkappa(g) T_\varkappa(g)\Psi) = \langle T(g)\varphi, H(g) T(g)\Psi\rangle. \quad (41)$$

Here $T_\varkappa(g): D \to F$ is invertible for $\varkappa < \infty$, $T(g): D \to \mathcal{H} = \overline{T(g) D}$ is an invertible mapping into a new Hilbert space $\mathcal{H}$ (with scalar product $\langle\cdot, \cdot\rangle$) and $H(g)$ is a real, symmetric operator on $T(g) D$. From Φ_3^4 on upwards the van Hove phenomenon forces us out of Fock space even for a space cut-off $g \in S$: Only on states with infinitely many high momentum particles (with probability one) the singularities of H_0, $V_\varkappa(g)$ and $R_\varkappa(g)$ cancel for $\varkappa \to \infty$. $H(g)$ is expected to be bounded from below, but the proof of this highly non-trivial result has not yet been given. Osterwalder [39], however, has constructed the Φ_4^3-Hamiltonian by similar methods and has shown that its unboundedness from below is not destroyed by the rather violent positive infinite mass and energy renormalizations and by the multiplication with zero in the wave function renormalization. Hence beware of classically non — sensical boson Hamiltonians!

If one climbs further the field-theory tree towards $(\overline{\Psi}\Psi\Phi)_3$, one encounters the following obstacle: while the theory is still super-renormalizable and while the counterterms $R_\varkappa(g)$ for the S-operator can be easily found, not all divergent graphs cancel for $\varkappa \to \infty$ in the wave-operators $T_\varkappa^\pm(g)$ in perturbation theory. One has reached the frontier of the «Stueckelberg divergencies» [40] associated with an inappropriate sharp time propagation. Since one has always the identities (see [7])

$$H_\varkappa(g) T_\varkappa^\pm(g) = T_\varkappa^\pm(g) H_0, \quad (42)$$
$$T_\varkappa^\pm(g)^* T_\varkappa^\pm(g) = 1$$

in every order in $V_\varkappa(g) + R_\varkappa(g)$, one is still tempted to generalize the Φ_3^4 — results, by finding a truncation $T_\varkappa(g)$ of $T_\varkappa^\pm(g)$ which (a) is given by a convergent series (b) and compensates all divergencies in the weak limit (41). However, (a) necessitates a rapid truncation in order to cope with the particle number divergence of quantum field theory, while (b) requires a very weak truncation. It might be that (a) and (b) are incompatible.

Thus there is not yet any indication how to generalize the present techniques to Φ_4^4, $(\overline{\Psi}\Psi\Phi)_4$ and quantum electrodynamics. Nevertheless, the past decade in constructive quantum field theory has brought to light many unexpected saving graces due to the interplay of locality and spectrum conditions and very physical results, which give us today more hope than ever that quantum field

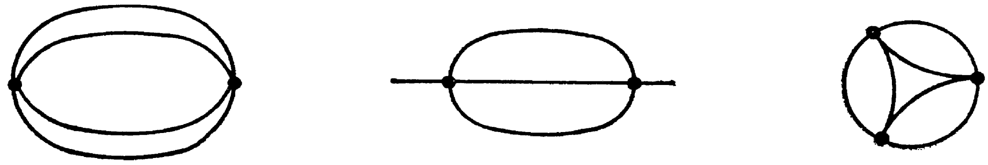

Fig. 3.

theory is a good framework for relativistic quantum mechanics. However, 4-dimentional quantum field theories might remain difficult to reach and to analyse for quite a long time.

There is the wild hope among some of you that the difficulties on the field-theory tree will decrease again, if one climbs higher in the non-polynomial interactions. This is an interesting idea which will should be tested with the full power of functional analysis.

Diffusely-optimistically I see many fundamental and not completely unsolvable problems for the next decade.

DISCUSSION

M o r a v c s i k:

You said that perturbation theory did not contain a theory of coupling constants, unless one fixed of the parameters in the theory. Does this imply something about the existence or non-existence of bootstrap type theories?

H e p p:

The two-dimensional local quantum field theories φ_2^4 and $(\overline{\Psi}\Psi\varphi)_2$ do not show any «bootstrap behaviour» which would single out discrete values for the masses and coupling constants.

F a i n b e r g:

I should like to ask you about Rinke's result. If we have the Wightman formulation of the theory with the asymptotic condition and the completeness ($H = H_{\text{in}}$), is then Wightman's scheme empty, i. e. is it true that $S = 1$?

H e p p:

No. From any Wightman quantum field theory with a non-trivial unitary S-operator we can construct a new theory (which according to our present criteria looks as good as the old one), where $\hat{S}$ is not unitary. It would be extremely satisfying to prove the converse of Rinke's theorem, that the equality of H_{in} and H_{out} and hence the unitarity of S on H_{out}, can be obtained by simple algebraic changes within the class of local quantum field theories.

P o l i v a n o v:

My question is again on the Rinke theorem, which seems to be very interesting in the respect of triviality criteria and model producing. Is the S-matrix in it unitary or only a partially isometric operator with respect to the product Hilbert space?

H e p p:

The S-operator is always partially isometric whenever the Haag-Ruelle construction of scattering states works. However, S needs not to be unitary.

T o d o r o v:

I have one question and one remark.

1. I would like to ask about the relation between the work on non-localizable theories presented at this conference (and covered in the rapporteur's talk) and the published work of Hoegh — Krohn. It was rigorously shown in this latter work that a non-polynomial interaction with bounded Lagrangian density leads to a trivial S-matrix in the canonical quantization scheme. I think that this paradox is explained by the fact that Efimov et al. postulate quite arbitrarily the form of the superpropagator and do not deduce it from the non-polynomial Lagrangian. I would like to know whether Prof. Hepp would agree with this interpretation.

2. My comment is concerned with the discussion of the generalizations of the Wightman axioms. It seems to me that most natural extension of the class of distributions which still gives room for local fields is the class of localizable theories in the terminology of Prof. Hepp (or «nonrenormalizable theories of the first kind» in the terminology of B. Schroer, J. Math. Phys. 5, 1361 (1964). I think that this type of theories, whose significance seems to have been underestimated untill recently, is both general and simple enough to apply in concrete problems. One such application was made by A. I. Oksak and myself «Degeneracy of the mass spectrum for

local infinite component fields». Proceedings of the Coral Gables Conference (1970) and Phys. Rev. D (1970) (to appear). It extends the Grodsky and Streater «no-go» theorem to nonrenormalizable theories. The theorem says that an irreducible infinite-component local field yields an infinitely degenerate mass spectrum.

H e p p:

The work reported here on nonpolynomial interactions is a natural generalization of the perturbative treatment of polynomial interactions. In the same sense as there, the S-operator is nontrivial.

E f i m o v:

Firstly I would like to answer the question of Prof. Todorov. The result of Hoegh — Krohn follows from the assumption that the interaction Lagrangian is chosen in nonnormally ordered form. It is not valid in the usual case when we choose the normally ordered interaction Lagrangian. Secondly, I want to say something about the ambiguity in the construction of the S-matrix with nonlinear interaction Lagrangians. The main problem is to find such methods which concentrate all the ambiguity in the second order of the perturbation theory and secure unitarity and causality of our theory in the highest orders. There are not so many methods of such a kind and it is not so simple to find them. These are methods suggested by E. Fradkin and myself in 1963, nonlocal methods and Volkov's methods. Now the problem is to give the physical meaning of these methods.

I would like to say several words about Christ's work. This model was considered by M. Volkov several years ago. Dr. Christ have to introduce the new prescription in the highest orders of the perturbation theory to secure unitarity.

J. G. T a y l o r:

I have a comment to make about what it could be like at the top of the Jaffe tree. Whilst it is expected to be greener up there it could also be a little dizzy up there. In particular, not all nonlocalizable interactions give finite results, and in some cases even have an infinity of different divergences, or ambiguities, even though they are all at most quartic or cubic, etc., according to the model. This means that in order to get a theory of the renormalization constants it is necessary to make a choice of theory, for example by using general relativity along the lines suggested by Markov and Salam. It is here that physics enters; I hope that those working at the top of the tree, doing physics and those at the bottom, doing functional analysis, keep good contact and remain good friends!

B o g o l u b o v:

It is interesting to note that the profound remarks of Stueckelberg about the particular type of divergences which was made about 20 years ago, was not appreciated for a long time after and now in the report of Prof. Hepp we have seen a concrete example of «Stueckelberg divergence».

A. T. F i l i p p o v:

I would like to make a comment on the problem of the unique prescription for constructing the superpropagator. The two essentially different approaches to this problem were developed simultaneously and independently by M. K. Volkov and by B. A. Arbuzov, N. Atakishiev and myself (Yadern. Fiz. 8, (1968) 385), both of them leading to the same expression for the superpropagator. The essential feature of the expression is its non-analytic behaviour in the coupling constant g for $g = 0$ (in fact it has the logarithmic branch point). This fact has the close analogy in the behaviour of the Green's functions for nonrelativistic scatte ring on singular potentials and we have obtained the superpropagator by the method which is closely related to one used in the singular potential case. In fact, we have directly summed all the Feynman diagrams in Euclidean momentum space (with a cut-off) by converting the integral equation for the superpropagator into the differential one. It is very important that there exists a class of problems (e. g. the summation of the ladder-type diagrams for the vertex functions or scattering amplitudes), for which our method gives the unique result, rigorously obtainable by the direct summation of diagrams in the Minkovsky momentum or coordinate space. So, I would like to conclude that the prescription used to obtain the unique (up to one arbitrary real constant) superpropagator is indeed the most natural one from the physical point of view.

P o l i v a n o v:

You mentioned a demonstration by Jaffe that his models satisfy Haag — Araki axioms. Within this formalism, translation invariance and Lorentz invariance differ because only for translation invariance it is known that it is affiliated with the algebra of observables.

Is Lorentz covariance in Jaffe models proved by usual means of the Haag — Araki approach or is it directly demonstrated by construction from Lorentz-covariant fields?

H e p p:

In the proof of Lorentz covariance one constructs for every bounded region $B \subset R^2$ a Lorentz boost M which satisfies $\Phi(x \operatorname{ch} \beta + t \operatorname{sh} \beta, t \operatorname{ch} \beta + x \operatorname{sh} \beta) = \exp(i\beta M)\, \Phi(x, t) \exp \times \times (-i\beta M)$.

P o l i v a n o v:

It is very instructive in my opinion that cluster decomposition proves to be such a loose condition that it is satisfied even by the theories having no vestige of locality as is shown by an example given by Martin et al. From the light-minded point of view that is good, because it seems we can abandon the notion of field and preserve in a sense S-matrix and Haag — Ruelle theory. But we lose in this way all the space-time structure which is so characteristic of a field theory and it seems to me to lose also a possibility of any real description of processes in which we are interested and which place in the same space-time.

C h r i s t:

I would like to reply to Efimov's criticism of the work which I presented in the Conference. Although my work in 2nd order corresponds to that which Volkov performed several years ago, I extend his method to give unitary results in higher order. I believe that I do not introduce new methods to overcome new difficulties that occur in higher order but use to same method to define higher order amplitudes that was used in 2nd order. The result is a prescription to define a unique scattering matrix 2nd, 3rd and 4th orders, and perhaps higher order.

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