DYNAMICS OF MANY BODY PROCESSES

Chairman	D. Blokhintsev
Rapporteur	D. Chernavsky
Discussion leader	A. Pignotti
Secretaries	I. Roizen
	S. Goloskokov
	I. Sisakyan

DYNAMICS OF MANY BODY PROCESSES

D. S. Chernavsky

1. Introduction

In the last two years much attention has been paid to high multiplicity processes, and a tendency to develop ideas already discussed may be noticed. The following three trends are at present under investigation.

I. Multiperipheral schemes
II. Statistical Theory
III. Quasiclassical models

In this connection new ideas have been developed: for example the Feynman parton model and scale-invariance [1].

Before going over to the discussion of the concrete results it is not out of place to mention here what can be expected from the multiple production theory.

First, the multiple production theory should, of course, describe the process under observation.

Secondly, it should lead to the right imaginary part of the elastic scattering (i. e. shadow scattering).

Thirdly, it should be connected with the other fields of elementary particle physics. Otherwise this theory would be of a rather limited value.

Multiperipheral schemes satisfy the requirements enumerated; they are now a centre of attention; the majority of the reports submitted to the conference were devoted to these schemes.

2. Multiperipheral theory

Different versions of multiperipheral theory exist. Let us first discuss the properties common to all of the versions such as a logarithmic increase of the multiplicity with the energy, small transverses momenta etc.

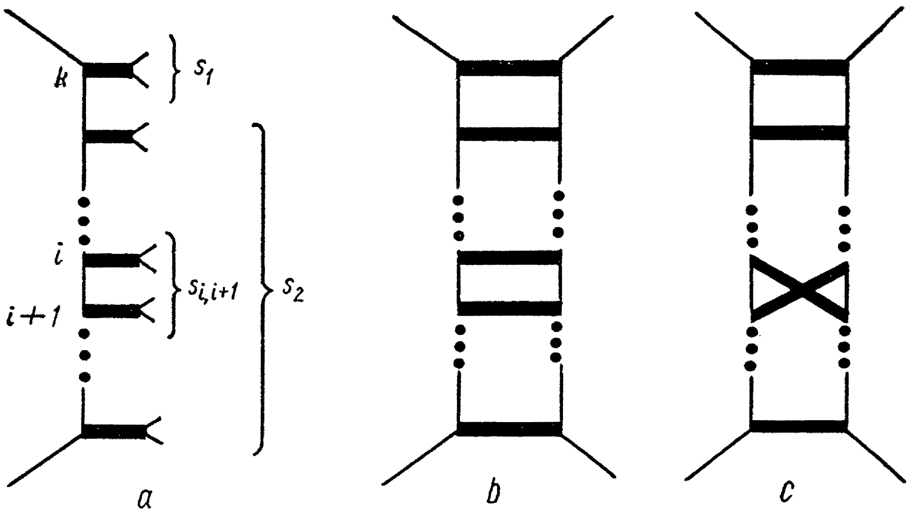

Fig. 1. Multiperipheral graphs *a*) AFST inelastic process; *b*) shadow elastic scattering; *c*) interference graph.

An equation of multiperipheral type was initially proposed by Amati, Fubuni, Stanghellini and Tonin [2] (AFST). It was based on summation of diagrams of the type shown on Fig. 1a. This equation is of the form

$$\sigma = \bar{\sigma} + \frac{1}{8\pi^3} \int \bar{\sigma}\sigma D^2(k^2) H \, ds_1 ds_2 dk^2 \tag{1}$$

where H is the kinematical factor and $D(k^2)$ is propagator. The other symbols are seen in Fig. 2. In the original works of AFST it was supposed that the quantity $\bar{\sigma}$ is simply equal to the total cross section at low energies (for pions, in fact, in the ρ-resonance region).

A new important step was made after exploration [3] of the Bethe — Salpeter (B. S.) equation. It was shown that one can deduce an equation for the s-channel imaginary part A_1,

$$A_1 = \bar{A}_1 + \frac{1}{8\pi^3} \int \bar{A}_1 A_1 D^2(k^2) \, ds_1 ds_2 dk_2 \tag{2}$$

unambigously from the exact B. S. equation. Here $\bar{A}_1$ is equal to the overall sum of the imaginary parts of irreducible diagrams. The equations (1) and (2) may be transformed each to the other by using the optical theorem. However, the latter one is directly applicable only to the amplitude A_1, and one has to make an additional assumption on the absence of interference terms [4] to use this theorem for the function $\bar{A}_1$ as well. In other words, this means the neglection of contributions of diagrams of type (1c) by comparison with the ones of diagrams of type (1b)*. There exist plausible physical reasons for making such an assumption [5]: the particles (or groups of them) «i» and «$i+1$» occupy different parts of phase space, so that the overlapping of corresponding amplitudes cannot be essential.

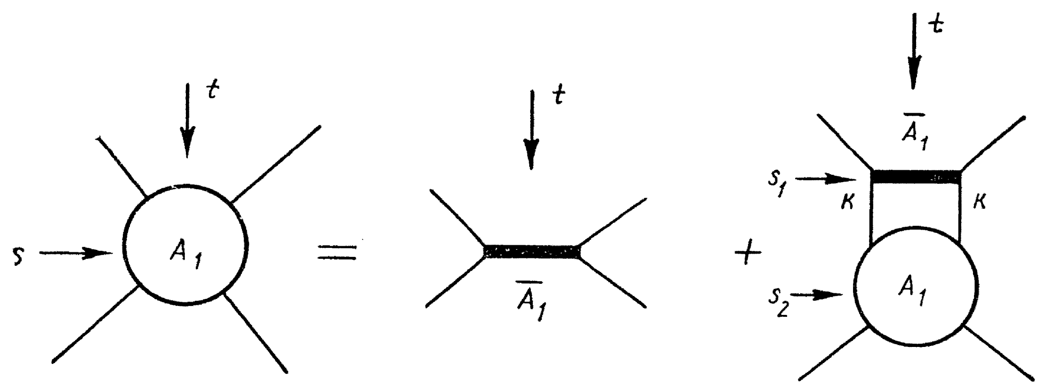

Fig. 2. Graphical representation of the integral equation for the imaginary part of the elastic scattering amplitude.

Let us note, that, in fact, a similar assumption has to be made in the AFST method of deriving equation (1) as well.

As a result of such considerations one can extend the framework of the multiperipheral scheme and understand the precise meaning of the quantity $\bar{\sigma}$ (i. e. blobs in the diagrams) in equation (1).

The inhomogeneous term, i. e. the quantity $\bar{\sigma}$, is a cross section of nonperipheral (central) interactions (elastic and inelastic) including both resonances and low energy statistical processes *. Thus, we see that the diagrams of inelastic processes are like AFST ones, however many secondaries can be emitted from each blob. We may determine the irreducible part $\bar{A}_1$ by the duality hypothesis and the finite energy sum rules — *FESR* [5]. This approach is discussed in a large number of the reports.

Now I should like to say some words as to what particles are exchanged.

In the original AFST version as well as in the BS equation [3, 17] pions were supposed to be such particles. The reasoning was purely qualitative: pions are the lightest hadrons; they should play the main part in peripheral interactions. Then the opinion arose that exchanges of the highest poles (P-pomeron, ρ-meson, P'-trajectory) played the main part. This would be altogether valid if the total energies of two neighbouring blobs $s_{i,i+1}$ increased significantly when the energy increased, however, this is not the case [11].

Nevertheless a multireggeon scheme appeared and it was important for later developments.

We owe great credit to the pioneering of Ter-Martirosyan's group [6] and Chew's group [7] for these developments. Later it was recognized impossible to build an internally consistent scheme with the exchange of one right-hand pole, i. e. Pomerons only [8, 9, 10]. As to the other mesons, the general qualitative properties of the scheme do not depend strongly on the type of meson exchanged or even on whether it is Reggeized or not ** [11].

At the same time the calculations with non-reggeized mesons are much simpler.

In many papers (see for example [7, 12]) the so-called generalized mesons are considered and instead of the propagator (for non-Reggeized mesons) or signature factor (for Reggeized mesons) an appropriate function is used.

Recently the opinion has spread that the exchange takes place mainly by pions in all the cases where a pion exchange is not forbidden by conservation laws of spin, isospin etc. In favour of this is the fact that the pion coupling constant $g_{\pi\pi}$ (and therefore the cross section $\bar{\sigma}_{\pi\pi}$) as well as its propagator are a bit larger than for the other mesons. We should emphasize that we speak of a numerical and perhaps not very big superiority.

Of course in the exact B. S. equation the exchange of both pions and other mesons is taken into account. The exchange of single pions enters into the reducible part (described by the integral) and is exactly considered while the many pion exchange and the exchange of other particles enter into the irreducible part, which is the unknown input of the theory and should be taken into account phenomenologically.

Now we turn to the technique of calculation. It is convenient to go from the elastic amplitude to partial waves in the t-channel f_l. If the exchanged particles

---

* In this connection there appears a natural separation of the interactions into peripheral and central ones which was previously discussed [13, 14, 15]. (By peripheral processes we mean inelastic collisions described by one particle exchange graphs, which produce the two particle exchange graphs appearing in shadow elastic scattering).

** At present it appears that this point of view can be considered common for all the theorists involved, as a significant achievement.

have no spin the equation for f_l at $t = 0$ has the form [3]:

$$f_l = \bar{f}_l + \frac{2^{2l}\sqrt{\pi}\Gamma(l+3/2)}{8\pi^3\Gamma(l+2)} \int (k^2)^{l+1} D^2(k^2) \bar{f}_l f_l dk^2. \tag{3}$$

In the report [16] on the basis of the group theory there was developed a technique of obtaining analogous equations for exchange of particles with spin. At the poles, where $f_l \simeq \frac{R_l}{l-l_i}$ the equation becomes a homogeneous one for the residues

$$R_{l_i} = \frac{2^{2l_i}\sqrt{\pi}\Gamma\left(l_i + \frac{3}{2}\right)}{(2\pi)^3\Gamma(l_i+2)} \int (k^2)^{l_i+1} D^2(k^2) \bar{f}_{l_i} R_{l_i} dk^2. \tag{4}$$

Eigenvalues in Eq. (4) define the position of the amplitude singularities l_i in the l-plane. The least eigenvalue corresponds to the farthest right — hand pole. By using a well known estimation for the lowest eigenvalue we can connect the most right-hand pole position with the quantity $\bar{A}_1$ (or $\bar{\sigma}$)

$$1 \simeq \frac{1}{8\pi^3(l_0+1)} \int_0^\infty dk^2 \int_{4\mu^2}^\infty ds \left(\frac{k^2}{s+2k^2}\right)^{l_0+1} \frac{\bar{A}_1(s, k^2, k^2)}{(k^2+m_\pi^2)^2}. \tag{5}$$

Specifically, in the simplest case when $\bar{A}_1$ decreases with the increase of s and does not depend on k^2, from (5) we obtain the expression discussed by *AFST*

$$l_0(l_0+1) \simeq \frac{1}{8\pi^3} \int_{4m_\pi^2}^\infty \bar{\sigma}(s)\, ds. \tag{6}$$

The use of the l-plane greatly simplifies the investigation of multiperipheral schemes, both the search for singularities of the amplitude f_l, and the description of the process at asymptotic energies.

In the pre-asymptotic region it is more convenient to solve equation (1) and use an iteration procedure.

In this representation it might seen that no hypothetical element is present in the multiperipheral scheme. However, even after neglecting interference graphs (Ic) this element still exists.

Hypotheses (and arbitrariness) appear with the assumption of a specific form of the theoretically unknown kernel of the integral equation, i. e. the irreducible part. The arbitrariness is limited by a number of conditions. First, equations (3) and (4) should have solutions and be of Fredholm type.

For this purpose the kernel $\bar{\sigma}(s, p^2, k^2)$ should necesserily decrease with the increase of energy. Secondly, if one assumes that the total cross section is almost constant asymptotically, then the right-most vacuum singularity should at $t = 0$ be close to $l_0 = 1$. This determines the range of values of s making important contributions to the integral (6): they should be high enough $s_{max} \geqslant 10$ $(GeV)^2$.

Not all of the schemes proposed satisfy the latter condition.

We should enumerate the principal qualitative results of the multiperipheral scheme in the region of asymptotically high energies [17] resulting from the general properties of the kernel without using its detailed structure.

1. Properties of elastic processes: Equation (4) has a number of solutions with different eigenvalues connected with the amplitude poles in the l-plane. The extreme right-hand pole should naturally be identified with the Pomeron. A new point was discovered in [18]; it is shown there that the next eigenvalue can be identified with the P'-trajectory; it is also shown there that investigation of the equation at $t = \varepsilon$, $\varepsilon \to 0$ gives the possibility to determine the slopes of the trajectories (depending only on the kernel of the equation, i. e. on $\overline{A}$ or $\overline{\sigma}$).

Thus, a multiperipheral scheme gives a dynamical picture of the origin of vacuum singularities. In other words, the substructure of the Pomeron is displayed here *.

Here the physical nature of the increase of interaction radius and transparency also becomes clear: they both are connected with the logarithmic growth of the number of blobs in the graphs ($N \sim \ln s$) making the main contribution [19].

Now let us present the picture explaining this statement (see Fig. 3). Let us consider the momenta of emitted particles in the plane perpendicular to the collision for the process with N blobs. Each line is characterized by a transverse component of the transferred momentum $k_{\perp i}$ and by the impact parameter $r_{\perp i} \simeq 1/k_{\perp i}$ connected with this component. In the plane mentioned directions of $r_{\perp i}$ are stochastic and we come to a picture resembling the Brownian motion. The total impact parameter R is given by

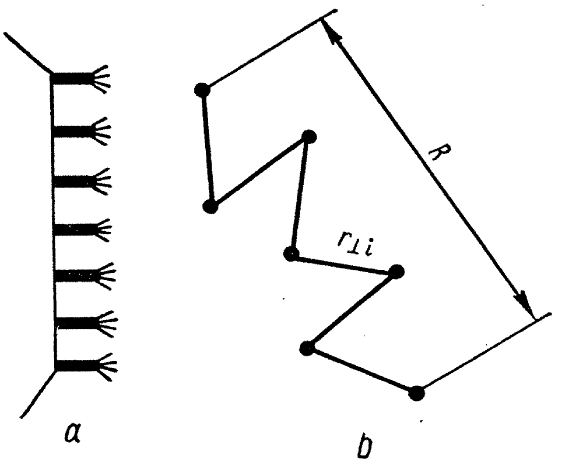

Fig. 3. Physical origin of the logarithmic increase of interaction radius *a*) multiperipheral graph; *b*) «Brownian motion» of impact parameters in the plane perpendicular to the collision line.

$$R = \sqrt{\sum_{i=1}^{n} r_{\perp i}^2} \simeq \overline{r_{\perp}^2}\sqrt{N} \simeq \overline{r_{\perp}^2}\ln s$$

which gives rise to shrinkage of the elastic diffraction cone.

2. *Properties of inelastic processes:* The transverse momentum distribution of secondary particles does not depend significantly on the energy.

The squared 4-momentum transferred between the blobs on the average depends neither on the energy nor on the position of the link.

There is a number of consequences of this important conclusion.

First, the Lorentz-factors of a relative movement of adjacent blobs $\gamma_{i,i+1}$ also on the average depend neither on the energy nor on the position of a blob in the link.

Secondly, the distribution of the transverse momenta of secondary particles at $s \to \infty$ has the form **

$$\frac{dN}{d\ln p_{\parallel}} = \text{const, or } dN = \text{const} \cdot \frac{dp_{\parallel}}{p_{\parallel}} \qquad (7)$$

$$\text{at} \quad m \sim \overline{p}_{\perp} \ll p_{\parallel} \ll E_c,$$

E_c is the energy in c. m. s.

Thirdly, the invariant mass of any blob and the number of emitted from this blob particles n_0 do not on the average depend on the energy. That is why the multiplicity as well as the number of blobs increases logarithmically with the energy, $\overline{n} \simeq Nn_0 \sim \ln s$. This means that for given s the main contribution is made by a

* The appearance and properties of non-vacuum bipion Reggeons and in particular of the ρ-meson may be considered in the same way. The kernel of the equation should in this case represent an irreducible part possessing corresponding quantum numbers in the t-channel.

** With the restrictions mentioned ($p_{\perp}$ = const) $d\ln p_{\parallel} = 2{,}3\ d\lambda$; $\lambda = -\log \operatorname{tg} \theta_L$, variable λ has been for a long time used in the treatment of the data in consmic rays. Note, that the distribution $dN/d\lambda$ is connected with the transferred momenta k_i^2. The highest k_i^2 correspond to the maxima of $dN/d\lambda$, while the lowest k_i^2 correspond to the minima [20].

process with a definite number of blobs. With the increase of the energy processes, a small number of blobs is replaced by those with a greater number.

These results are simple kinematic consequences of the main result of the multiperipheral scheme, i. e. constancy of $\overline{k_i^2}$.

It is not out of place here to discuss the correspondence of these conclusions with scale-invariance. Feynman [1] has recently proposed that in a multiple production the distribution of transvers and longitudinal momenta is described by the expression

$$dN = dp_{\perp} \frac{dx}{x} f(p_{\perp}, x)$$

where $x = p_{\parallel}/\sqrt{s}$ and $f(p_{\perp}, x)$ is a smooth function, which does not depend on the energy s at $s \to \infty$ and has the limit: $f(p_{\perp}, x) = \text{const} \neq 0$ at $x \to 0$. This property is called Feynman scaling. It is clear from the above that multiperipheral schemes have a scaling behaviour. Moreover, for the secondary nucleons scaling means independence of energy for the inelasticity coefficient (K) distribution. The value of x for nucleons is connected with the inelasticity coefficient K by the equality: $x = 1 - K$.

As to the secondary pions the situation is a bit more complicated, and it is of interest to discover the details of correspondence and to find the form of the function $f(x)$. This is an urgent problem to which a number of reports were devoted.

It is shown in reports [21] that scaling occurs in multiperipheral schemes if their cross section is asymptotically constant. From experimental data at the energy $E_{\text{Lab}} \simeq (10 \div 30)$ GeV there was also found the form of the function $f(x) \simeq \simeq \exp(-10x^2)$. The data at $E_{\text{Lab}} \simeq (8 \div 16)$ GeV were used in [22a] and the form $f(x) \sim e^{-3x}$ was obtained there. The dependence $f(x) \sim e^{-5x}$ was also considered [22b].

Below, however, I shall try to show that the asymptotic form of the function $f(x)$ differs from those discussed and is achieved only at very high energies ($E_{\text{Lab}} \simeq \simeq 10^{13}$ eV). In the intermediate energy region the function $f(x)$ can undergo a number of unexpected changes.

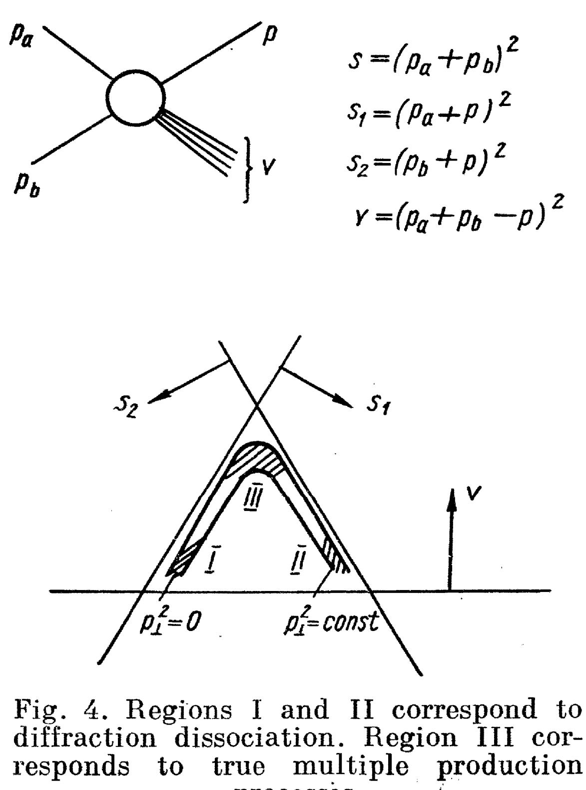

Fig. 4. Regions I and II correspond to diffraction dissociation. Region III corresponds to true multiple production processes.

Considering general properties of the peripheral processes I should mention the report [23]. The main results similar to those considered above were obtained here on the basis of the original method and general assumptions of the Regge method. The case of diffraction dissociation — exchange of the Pomeron — was especially investigated. The above discussed conclusion was also confirmed that the cross section for diffraction production should decrease (though very slowly — logarithmically) with the energy.

In addition this report also presented an elegant method of differentiating the diffraction dissociation processes from those with large multiplicity (including multiperipheral ones). According to this method one of the secondary particles is isolated and the invariants s_1, s_2, v are considered (the meaning of these is clear from Fig. 4). These quantities are connected by the condition that $s_1/s + s_2/s + v/s =$

$= 1 + \frac{m_a^2 + m_b^2 + 3m^2}{s}$ and correspond to a point in a three-corner diagram. Cases of diffraction production of a definite particle appear in regions I or II while the high multiplicity events are found in region III.

We have considered the properties common to all of the multiperipheral schemes. At the same time there are differences in the versions consisting in the formulation of the properties of the irreducible part. We mean the dependence of $\bar{\sigma}(s)$ on the energy s, the effective values of s making the main contribution and the processes taking place in the diagram vertices.

These problems are closely connected with clusterization i. e. with the question whether the pions are produced in groups, pairs or separately.

In the original AFST model ρ-mesons decaying into two pions were assumed to form the blobs. In the paper by C. L. A. [12] there is only one pion in every vertex. This scheme is often called a «comb scheme».

When using these suppositions the multiperipheral scheme faces a number of difficulties discussed in 1962—1964 [68, 19]: if the blob mass is small ($s_{\text{eff}} \sim 0,5\ GeV^2$), the extreme right-hand pole of the amplitude is far to the left of unity, i. e. the scheme leads to a total cross section decreasing with incident energy. It is shown in the paper [24] that if we assume the blob energies are small but keep the assumption that not more than two pions are formed in a blob then the multiplicity is unreasonably small. It is easy to avoid these difficulties if we do not agree with the supposition that the multiplicity in the blobs is small and assume the production of many particles in a blob.

Within the framework of the B. S. equation the question is treated differently.

The condition of asymptotic constancy of the cross section (i. e. the condition that $l_0 \simeq 1$) is initial and the irreducible part should be such that this condition is satisfied. For this purpose it is necessary that the integral in (6) be rather large in order to compensate the numerical factor $\frac{1}{8\pi^3}$. Integration over s should be spread up to high values of $s_{\text{eff}} \simeq 10\ GeV^2$. This means that the effective blob masses are high:* $\mathfrak{M}_{\text{eff}} \simeq \sqrt{s_{\text{eff}}} \simeq (3 \div 3,5)\ GeV$. This is a very important condition; it defines the character of inelastic processes.

It is evident that the main contribution to the irreducible blob at high values of $\mathfrak{M}$ is made by highmultiplicity inelastic processes. Distribution over $\mathfrak{M}_i$ or over s_i-$\mathfrak{M}_i^2$ should be rather wide. Therefore, the contribution to the irreducible part is made simultaneously by several many particle resonances but not a separate one. The question of the number of particles produced in a blob and their distribution in c. m. s. of the blob is beyond the framework of the B. S. equation. One may think that the process is developing here by analogy with that of nucleon annihilation at $E_c \simeq (3 \div 4)\ GeV$ since in both cases we deal with a system without baryon charge. Then we can use statistical theory (which is considered below) and estimate the number n_0 of pions formed in the blob:

$$n_0 \simeq \mathfrak{M}_{\text{eff}}/0,5\ GeV \simeq 6 \div 7.$$

Formation of heavy blobs leads to a substructure in the distribution over angles and that over $p_{\parallel}$ and λ.

The transferred momenta inside the irreducible blobs are on the average a bit larger than the transfers between them. Hence, such group particle radiation should lead to a non-monotonic angular distribution in the co-ordinate λ: every blob

---

* The origin of the important factor $(8\pi^3)^{-1}$ can be explained in the following way. In the t-channel the denominator of $(8\pi^3)^{-1}$ is the normalization of the intermediate exchanged particle phase space volume; it is compensated by integration over angles. In the s-channel the phase space is transformed to the form $ds_1 ds_2 dk^2$. To compensate the factor one should integrate over wide region of s_2.

should produce a bump in the distribution, the minima between the bumps corresponding to one meson exchange *.

Let us discuss the correspondence with experiment at high energies.

In the cosmic ray experiment at the energies $E_{\text{Lab}} \geqslant 10^{12}$ eV the pions were observed long ago to appear in groups or, more specifically, in so-called fireballs [25a].

In the Miesowicz group [25b] (Cracow) two fireballs were shown to form at the energies $E_{\text{Lab}} \simeq (10^{12} \div 10^{13})$ eV.

In the Dobrotin group 25c (Lebedev Physical Inst., Moscow) cases with $E_{\text{Lab}} \simeq (10^{11} \div 10^{12})$ eV were investigated where one fireball is formed.

Fireball masses were approximately estimated as $\mathfrak{M}_{\text{f.b.}} \simeq 3$ GeV and the number of particles produced $\bar{n}_{\text{f.b.}} \simeq (6 \div 8)$.

Here we would go back to the problem of the function $f(x)$ and its asymptotic form.

Knowing the distributions with respect to λ $\left(\text{or with respect to } \eta = \log \operatorname{tg} \frac{\vartheta_c}{2}\right)$ it is possible to find the distribution with respect to x in a given interval of energies;

$$\frac{dx}{x} = 2.3\, d\lambda \ (\text{sign «—» for } x > 0\, (\eta < 0)).$$

$$x = \frac{\bar{p}_\perp}{2m\gamma_c} e^{\mp 2.3\eta} \ (\text{sign «+» for } x < 0\, (\eta > 0)).$$

The forms of distributions with respect to λ for energy intervals: $E_L = (10^{11} \div 10^{12})$ eV; $E_L = (10^{12} \div 10^{13})$ eV; $E_L \simeq (10^{13} \div 10^{14})$ eV are presented in Fig. 5. They qualitatively indicate available results from cosmic ray experiments **. These distributions qualitatively agree with the schemes with heavy blobs.

Experimental data on the processes at $E_L > 10^{14}$ eV are rather poor. From the theoretical point of view the substructure is expected to be smoothed in the averaged distribution which should be monotonic; it can be considered as already asymptotic.

In the right-hand part of Fig. 5 the corresponding forms of the function $f(x)$ are presented. The asymptotic form is comparatively simple: $f(x)$ is almost constant up to $x \simeq 0{,}4$ and then it is sharply cut off because of conservation laws ***.

In the pre-asymptotic region oscillation of the distribution with respect to λ implies a non-monotonic form of $f(x)$ which is not asymptotic. At $E_L \simeq (10^{11} \div 10^{12})$ eV $f(x)$ is monotonic, however, this may be an «accident» connected with the formation of only one fireball.

The formation of fireballs at accelerator energies is not yet considered proved, however there was considered the idea of group formation of particles; the name «clusters» [12] was suggested for such groups.

Several methods may be tried in order to decide whether particle grouping takes place and what is the nature of these groups (clusters). Note that all the methods require «exclusive» experiments with complete information; «inclusive» experiments cannot help here.

---

* The values of the transferred k^2 inside the blobs and between them differ but not very much, less than an order of mangnitude. However, as was shown in [20] this difference in enough to influence the distribution in the co-ordinates λ.

** As to the presence of a three bump distribution at $E_L \simeq (10^{13} \div 10^{14})$ eV there are only some indications [26] which cannot be considered firmly established.

*** Since we speak of the pion distribution the conservation laws should be used with proper account taken of inelasticity coefficients. Therefore, the maximum pion energy in c. m.s. is $\varepsilon_{\max} = E_N K$, where $x_{\max} \simeq K = 0{,}4$.

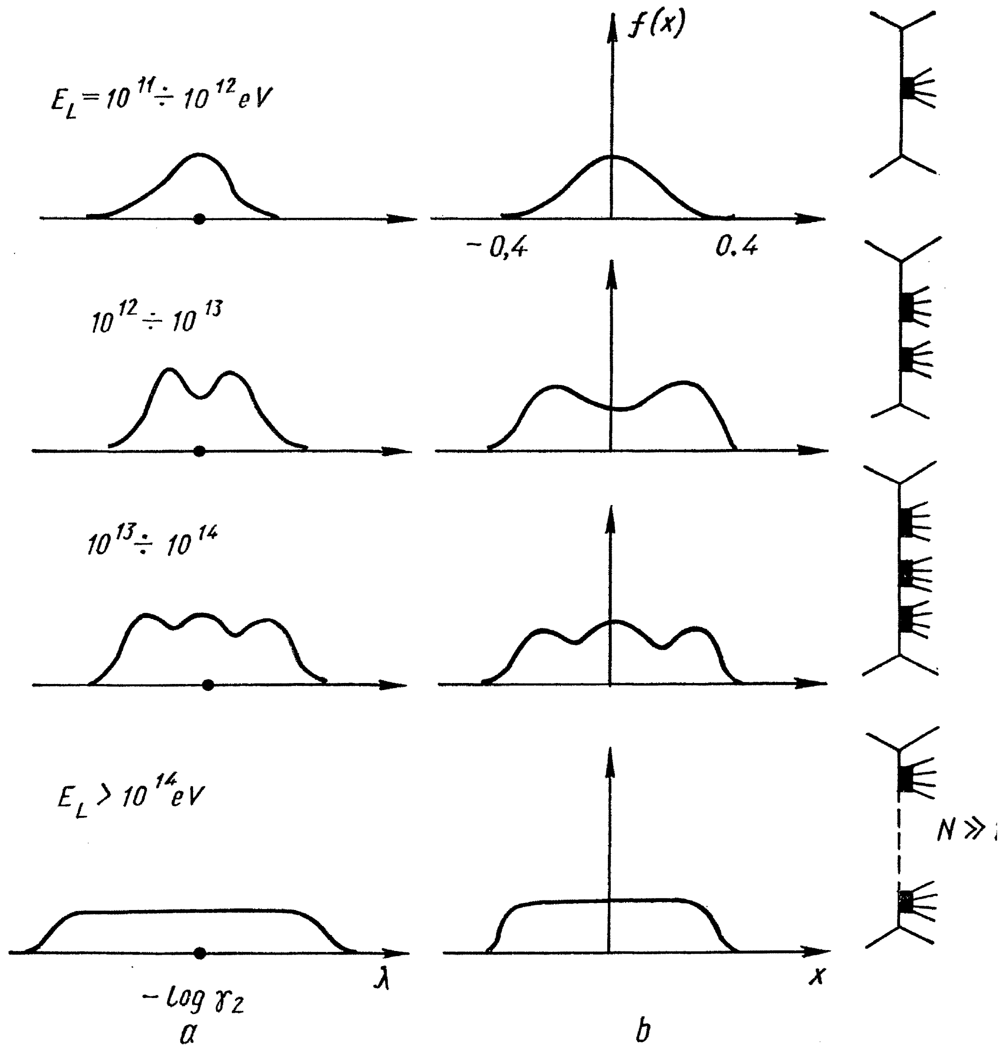

Fig. 5. *a*) Secondary pions distribution in λ-coordinates ($\lambda = -\log \text{tgv}_L$) for different energy intervals. *b*) Feynman function $f(x)$ for the same intervals. *c*) Multiperipheral graphs with heavy blobs (BS approach) for the same intervals.

Van Hove's method [27] suggested in 1968 is effective and was successfully used only when the number of secondary particles is not large. The application of this method showed that clustering takes place even in low prong events. As was shown in [28] in the case of four secondary particles clustering is connected with resonance formation.

In the report [29] a method is suggested, which is based on calculating the correlation coefficients between the quantities

$$x_i = k_i^2 \, *$$

For an event with n particles the correlation coefficient is calculated in the usual way:

$$r_\alpha^\nu = \frac{\sum\limits_{n}^{n-1} (x_i - \bar{x})(x_{i+\alpha} - \bar{x})}{\sum\limits_{i}^{n-1} (x_i - \bar{x})^2} .$$

* Here index i is the number of the interval between secondary particles placed in order of increasing longitudinal momenta.

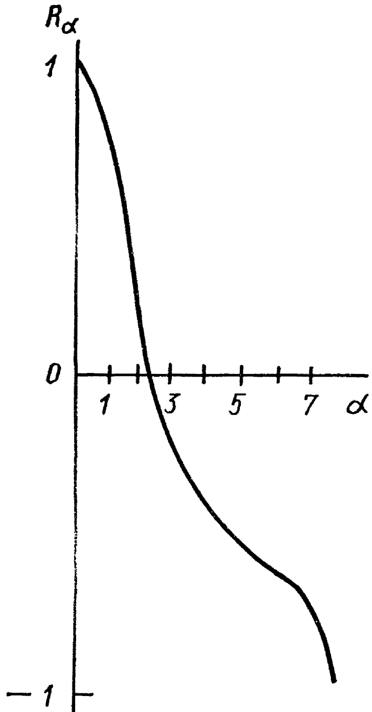

Fig. 6. Dependence of the average correlation coefficient R_α on the index α (see [30]).

The mean correlation coefficient R_α for the observed ensemble of N events is equal to

$$R_\alpha = \frac{1}{N}\sum_{\nu}^{N} r_\alpha^\nu.$$

In the case when the transferred 4-momenta do not correlate (for example in the case when one particle is formed in each blob), $R_\alpha = 0$ at any α. When n-pion clusters are formed $R_1 > 0$; $R_\alpha = 0$ at $\alpha \simeq \frac{\bar{n}_0}{4}$; $R_\alpha <$ < 0 at $\alpha \simeq \frac{\bar{n}}{2}$. The results of testing this method in the cosmic ray data are represented in [30]. They lead to a rather clear picture (fig. 6). The curve R_α obtained from 20 jets crosses the axis $R_\alpha = 0$ in the region $\alpha \simeq 1{,}5 \div$ $\div\, 2$. This is an indication of the existence of clusters of $\bar{n} \simeq (6 \div 8)$ particles which agrees with the heavy blob version.

However, the quantities k_i^2 were calculated approximately and it is desirable to test the method at the accelerator energies and with computer-simulated jets.

Thus cosmic ray data do not contradict the scheme of heavy blobs — fireballs but rather confirm it. It should be noted here that the main experimental data were obtained long before the creation of the theory.

The whole previous discussion of BS scheme was concerned with the asymptotic or pre-asymptotic energy region when energy is high enough for the fireballs to be produced. At lower energies the clusters (if they are formed) are comparatively small in mass.

Thus let us discuss the correspondence with experiment at the accelerator energies. Many reports were made on this subject.

As a rule Reggeized propagators are used here and the data on Reggeon vertices known from the fit of other more simple processes are sometimes applied.

In [28] the reactions $\pi^- p \to \pi^-\pi^-\pi^+ p$ and $\pi^+ p \to \pi^+\pi^+\pi^- p$ at $E_L = 16$ *GeV* were described by OPE diagrams (fig. 7).

Not only the comparison of distribution with respect to $p_\perp$ and $p_\parallel$ but also the analysis by Van Hove's method (to separate resonance production) was carried out. The agreement of theory and experiment is good.

In [31] a literal scheme of the AFST model (fig. 8) is used for the description of many prong events in $\pi^+ p \to p\, 3\pi^+ 2\pi^-$ at 8 *GeV/c*. The quality of this work

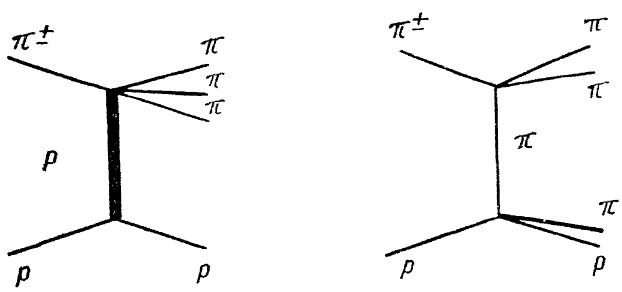

Fig. 7. Graphs of the reactions $\pi^- p \to$ $\to \pi^-\pi^-\pi^+ p$ and $\pi^+ p \to \pi^+\pi^+\pi^- p$ at $E_L = 16$ *GeV* (see [28]).

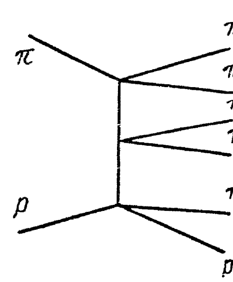

Fig. 8. AFST-graph for the reaction $\pi^+ p \to p 3\pi^+ 2\pi^-$ at $E_L = 8$ *GeV* (see [31]).

consists in the fact that all the parameters are borrowed from other works; there were no free parameters. In spite of this a satisfactory agreement of distributions was obtained (though a bit worse than that in other works using fitted parameters). As to the values of cross sections, a more serious contradiction arose: the cross section calculated is five times less than experimental ($\sigma_{\exp} = 0{,}4\,mb$; $\sigma_{\text{theor}} = 0{,}083\,mb$). The latter is quite natural since, as was already mentioned, this version of the model leads to a small cross section, decreasing with energy. The authors could estimate the interference of the two upper ($\pi\pi$)-blobs. It proved to be significant and constructive. This result is important in connection with the discussion at the beginning of § 2.

It seems to us both these results testify to the necessity to take into account greater blob masses than those in the initial *AFST* model.

In [32] it was shown that the data on *pp*-interactions at 10 *GeV* and *np* at 8 *GeV* are well described by a rougher model in which the amplitude of an inelastic process is given in the form

$$A = F(q_{1a}^2)\, F(q_{2b}^2) \prod_{i=3}^{n} [F(q_{ia}^2) + F(q_{ib}^2)]$$

where q_{ia}, q_{ib} is a transferred momentum between the *i*-th and primary (*a* or *b*) particles; $F(x) = \left[1 + \frac{x}{0{,}7\, GeV^2}\right]^{-2}$.

In [33] it was shown that at 8 *GeV* and 16 *GeV* the data can be satisfactorily described in the framework of a multiperipheral scheme of the type fig. 9b (graphs of a «comb»-type) in which the expression exp (at_i) is used for the propagator.

In [34] (see section 11) it was shown that the distribution of secondary particles with respect to $p_\perp$, $p_\parallel$ and the invariant masses is equally well described by both the model CLA [12] (diagram of a «comb»-type) and the scheme with one heavy ($\pi\pi$)-blob (a fireball model) (see figs. 9*a*, 9*b*).

Many analogous studies were discussed in Wroblewski's report.

In a number of papers multiperipheral schemes were applied to the description of the production spectra of most energetic particles at accelerator energies.

In [35] graphs of the type Fig. 10 are used. The spectra obtained for π^-, K^- and $\bar{p}$ production agree well with experiment. The calculations carried out in [36] on the basis of the graph (Fig. 10), where statistical theory was used for the description of the blob (I shall further speak of this approach) are also in good agreement with experiment.

I think one can draw a general conclusion:

Summary distributions with respect to $p_\perp$, $p_\parallel$ in inclusive experiments are not sensitive to the details of the scheme.

The principle of multiduality predominates here: all the experiments are explained by all the theories (both the term itself and the explanation of its meaning belong to Morrison).

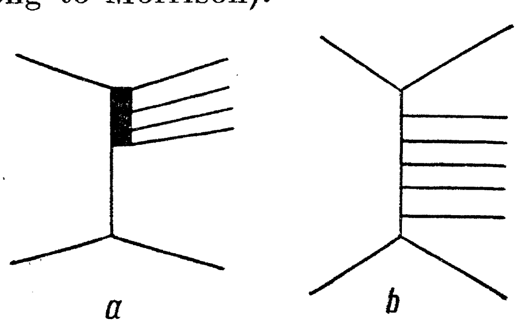

Fig. 9. Multiple production graphs (see [34]) *a*) fireball model; *b*) «comb»-model.

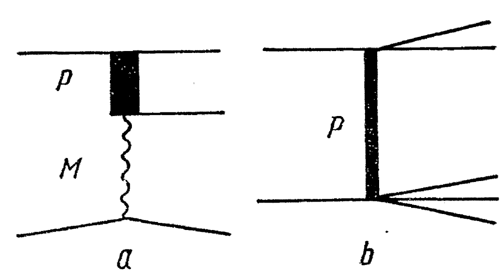

Fig. 10. Graphs used in [35] to describe the production spectra *a*) double exchange of a meson and a Pomeron; *b*) Pomeron exchange.

Thus the experiments in cosmic rays give more clear but less reliable information whereas the accelerator data are more reliable but less clear. We believe that a thorough investigation of multiple production processes at $E_L \sim 70\ GeV$ will give information that is both clear and reliable.

It would be desirable in our opinion to carry out the following: on one hand to apply a careful treatment of the experimental data available with the use of a correlation analysis (for example, of the type mentioned above);

on the other hand to impose stricter limitations on the vertex functions of multiperipheral graphs. In fact schemes satisfying asymptotic conditions (such as the BS approach, for example) should be applied at accelerator energies. To compare predictions with experimental data at accelerator energies one must make quantitative predictions, i. e. assume a definite form of the irreducible part of the BS equation which satisfies asymptotic requirements. For this purpose it is tempting to use duality and FESR.

There is such an attempt in [18]. The kernel of the BS equation was taken in the form

$$\overline{A}_1 = \beta_0 (\gamma_{p'} s)^{\alpha_{p'}} \theta (s_f - s) + \beta_0 (\gamma_{p'} s_f)^{\alpha_{p'}} \frac{s_f}{s} \theta (s - s_f),$$

where $\alpha_{p'}$ and $\gamma_{p'}$ are the position and slope of the trajectory. If $t = 0$, this means that the complete set of resonances up to the energy s_f can be «on the average» described by P' exchange.

A «selfconsistent» scheme was built: the input — trajectory $\alpha_{p'}$, which defined the behaviour of the irreducible part, was required to coincide with the solution of the equation, i. e. the output — trajectory $l_{p'}(0)$, corresponding to the second eigenvalue of the B. S. equation for the amplitude with vacuum quantum numbers in the t-channel. Thus, for determination of four free parameters: β_0, $\gamma_{p'}$, $\alpha_{p'}$ and s_f there were four asymptotic conditions:

$$1)\ l_p(0) = 1; \quad 2)\ l_{p'} = \alpha_{p'}; \quad 3)\ (\gamma_{p'})_{in} = (\gamma_{p'})_{out}; \quad 4)\ \sigma_\infty^{\pi\pi} \simeq \frac{(\sigma_\infty^{\pi N})^2}{\sigma_\infty^{NN}}.$$

Eigenvalue solutions of the BS equation were computed. The results of this version of the scheme are as follows:

$$l_{p'} = \alpha_{p'} \simeq 0{,}25; \quad \gamma_p = \gamma_{p'} \simeq 1\ GeV^{-2}; \quad (s_f)_{\pi\pi} \simeq 12\ GeV^2;$$

$$\overline{n} = 0{,}5 \ln \frac{s}{s_f}; \quad \mathfrak{M}_{f.b.} \simeq \sqrt{s_f} \simeq 3{,}5\ GeV.$$

Thus, the orders of magnitude of the P and P'-trajectory parameters should be considered as satisfactory in this scheme.

At the same time to obtain nearly exact quantitative results, corrections made by high energy, non-peripheral processes are likely to be important.

Let us briefly consider the role of diffraction dissociation. This process is not multiperipheral and is not described by the integral term of equation (1). According to the above scheme it is a process «of the third order» (an inelastic multiperipheral process gives rise to an elastic scattering which in its turn produces an inelastic diffraction).

So the diffraction dissociation is due to exchange of a Pomeron (see Fig. 10*b*). The cross section of such processes decreases with increasing energy at least logarithmically.

It was shown in [37] that in the version of the so-called «weak coupling» (see 38) only resonances of limited energy can be produced in the graph vertices.

Thus diffraction dissociation contributes to low prong, strongly collimated jets. The cross section of the diffraction production of a definite number (n) of particles $\sigma_n^{(d)}$ decreases but very slowly at high energy (this differentiates the diffraction production from multiperipheral processes where the n-particle cross section decreases as a power of energy).

At accelerator energies the diffraction dissociation cross section is a small part of the total (about 5%). Opinions were expressed [27] that diffraction processes at high energies would be prevalent. This opinion however does not agree with the considerations mentioned above. The statement that the cross-section of diffraction dissociation is rather small at all energies seems more probable.

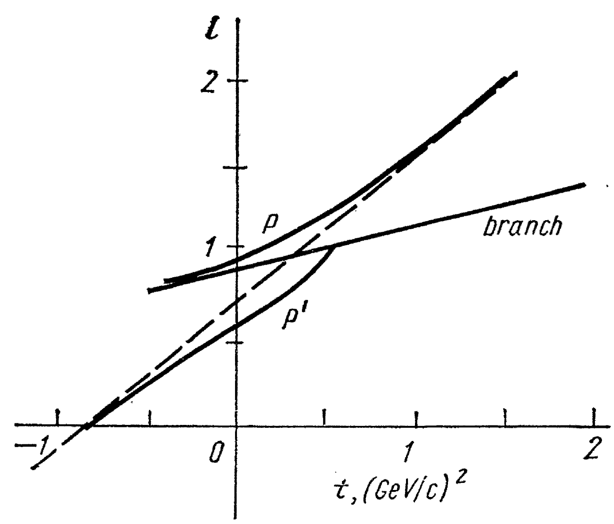

Fig. 11. Splitting of P and P'-trajectories because of the cut (see [39]).

However, an interesting question arises as to how the properties of the elastic scattering amplitude change within the framework of a multiperipheral scheme if we take diffraction processes into account as a small perturbation. Ref. [39] is devoted to this problem. The trajectories were shown to be easily distorted greatly (see Fig. 11): the main trajectory of a non-perturbed equation and that of a cut present in the diffraction dissociation term together produce a new corrected Pomeranchuk trajectory on the physical sheet and a P'-trajectory on the second sheet.

Now we consider the Veneziano model (it was discussed in detail at session 13).

It proved to be applicable for the low energy region. It is likely to be fruitful at energies in the cms $E_c \simeq (3 \div 4)$ GeV and multiplicities $n \sim 7$ (though at present there are some difficulties in describing even four secondary particles). It seems to me that it is extremely difficult to apply this model to the region of asymptotically high energies and high multiplicities. Probably, it is not necessary. It will not be out of place here to mention the optimistic utterance of a well-known Ukrainian philosopher Grigori Savvich Skovoroda: «Thank God for making everything difficult unnecessary and everything necessary easy».

One may espect that in the energy region $E_c \simeq (3 \div 4)$ GeV the Veneziano model gives the possibility to determine the irreducible part — this is exactly what we need for the construction of a completely quantitative multiperipheral scheme.

3. Statistical Theory

It will be recalled that in this approach the statistical equilibrium of hadrons generated in some volume V with temperature T is assumed to be set, total energy of the statistical system being $W = VT^4$. It is enough for the calculation of the number of particles, their composition and energy and angular distributions (the last one is almost, though not exactly isotropic). The parameter determining the results is the temperature of equivalently is the volume. There are different assumptions in different versions of the theory. In the initial Fermi version [40] it was considered that V is a Lorentz-contracted nucleon volume

$$V = V_0 \frac{2M}{E_0}; \quad V_0 = \frac{4}{3}\pi\left(\frac{\hbar}{m_\pi c}\right)^3; \quad E_0 = \sqrt{s}.$$

Then the temperature increased with the energy $T \sim W^{1/2}$.

Pomeranchuk showed [41] the inconsistency of this assumption. In fact, the interaction of secondary particles should be taken into account. It stops only when the particles are at a distance of an interaction radius and the system volume is equal to $V = nV_0$ where n is a number of secondary particles.

At the end of the process the temperature is low and, what is important, it is independent of the energy: $T_c \sim m_\pi$*.

Generalization of this idea lead Landau [43] to the creation of a consistent hydrodynamical theory considering particle acceleration due to the pressure. This effect is important at high W.

In [44] again arises the question of thermodynamical theory at superhigh energies in connection with quark production. It is shown that if we take the Fermi version (and neglect hydrodynamical pressure) then at the energy $E_L \sim 2 \cdot 10^{14}$ eV the temperature is $T \simeq 15m_\pi$ and the cross-section of the production of heavy particles (quarks with $m_q \simeq 10$ GeV) is large: $\sigma_{q\tilde{q}} \simeq 6$ mb. This quantity is in good agreement with McCusker's data [45]. However, some difficulties arise in this connection: besides the internal inconsistency of the scheme (which was already mentioned) within its framework there are too large transverse momenta ($\bar{p}_\perp \sim \sim \frac{3}{2} \cdot 15$ $m_\pi \sim 3$ GeV) and a large number of nucleon pairs. If the approach is more realistic, then within the framework of both statistical and hydrodynamical theories the cross-section of heavy particle formation is small [46] (which is in agreement with the data [47]). At present there are no definite experimental indications of the existence of hydrodynamical or thermodynamical processes at high energies.

Then interest switched to the statistical theory at moderate energies where the mass of a statistical system is not high and the hydrodynamical process does not develop. Apparently it has a region of applicability. Three points may be mentioned in this direction:

I. Apparently the question of the volume of the statistical system, i. e. of the choice of the most realistic model may be considered resolved.

It is shown in [48] that a consistent treatment of the Pomeranchuk approach gives the possibility to understand and describe quantitatively (with a corresponding statistical accuracy) many features of multiparticle processes. First of all the composition of produced particles in a wide mass region up to 3 GeV ($\widetilde{He}^3$), is described correctly. Because of the low temperature pions are mainly produced while the admixture of heavier particles decreases exponentially with the increasing mass.

I should say that particle composition gives the clearest test of the different versions of statistical theory. It is very important that independence of T on the energy of the process leads to independence of particle composition on the energy.

It will be recalled that in the use of the Fermi version in order to explain the observable composition we should have made additional artificial assumptions, for example, to use different volumes for different kinds of particles, to introduce large coefficients with no physical meaning.

The variant under consideration has only one adjustable parameter — the decay temperature T_c known in advances to have order of magnitude $T \sim m_\pi$ — and correctly gives the multiplicity, distribution in the transverse momenta, dependence of this distribution on the particle mass and even such a fine effect

* Then Hagedorn [42] introduced the statement of the small and constant temperature though he had somewhat different considerations. In his approach the temperature is small at any stage of the process.

as dependence of $p_\perp$ on the number of produced particles at a fixed energy. The cause of the latter effect is that the decay of a statistical system into a small number of particles (less than average) takes place at early stages of the expansion, when the temperature and the transverse momentum is above average values.

II. The range of this theory has been determined. It is valid only provided the mass of the system is not too small ($n \gg 1$) but not too high ($n \leqslant 10—15$) so that hydrodynamical acceleration is not important.

This theory cannot be applied to all interactions but only to part of them, the so-called central interactions, or to statistical subsystems appearing for example in peripheral processes. Such are the annihilation $N\bar{N} \to$ hadrons (or $e^+e^- \to$ hadrons) at $W \leqslant 5$ *GeV*; $\bar{n} \geqslant 5$; the decay of clusters or fireballs in peripheral or multiperipheral processes; nucleon collisions with high inelasticity coefficients $K \to 1$.

The reports [36] mentioned above were based on the application of statistical theory to the subsystem-fireball decay. Distribution of the fireballs themselves along the longitudinal velocities is taken from the peripheral system and momentum distribution of the products of the fireball decay — from Hagedorn's thermodynamical theory [42] close in this case to statistical theory. The description of the secondary particle spectra in the energy region of initial particles 19 — 70 *GeV* turns out to be quite satisfactory.

The problem of nucleons is of particular interest. In peripheral collisions excitation of the nucleons is small and they do not form a statistical system. But if they lose so much energy that at the end of the process their energy is the same as that of pions in a statistical system, they should be included in this system and can be treated statistically. In the work of the Argonne group [49] at $E_L = 12{,}5$ *GeV* the nucleon distributions in transverse momenta were measured separately for different inelasticities. These distributions differ greatly for $K \leqslant 0{,}70$ and $K \geqslant 0{,}88$ (Fig. 12). It is shown in [48] that it is the latter case that corresponds to the «central collisions» (the proton is a part of a statistical system and the distribution is in good agreement with statistical theory). Hence, one can also estimate the share of the statistical «central» nucleon collisions, about 10%.

Thus, in its region of applicability, the statistical system agrees with experiment. Moreover,

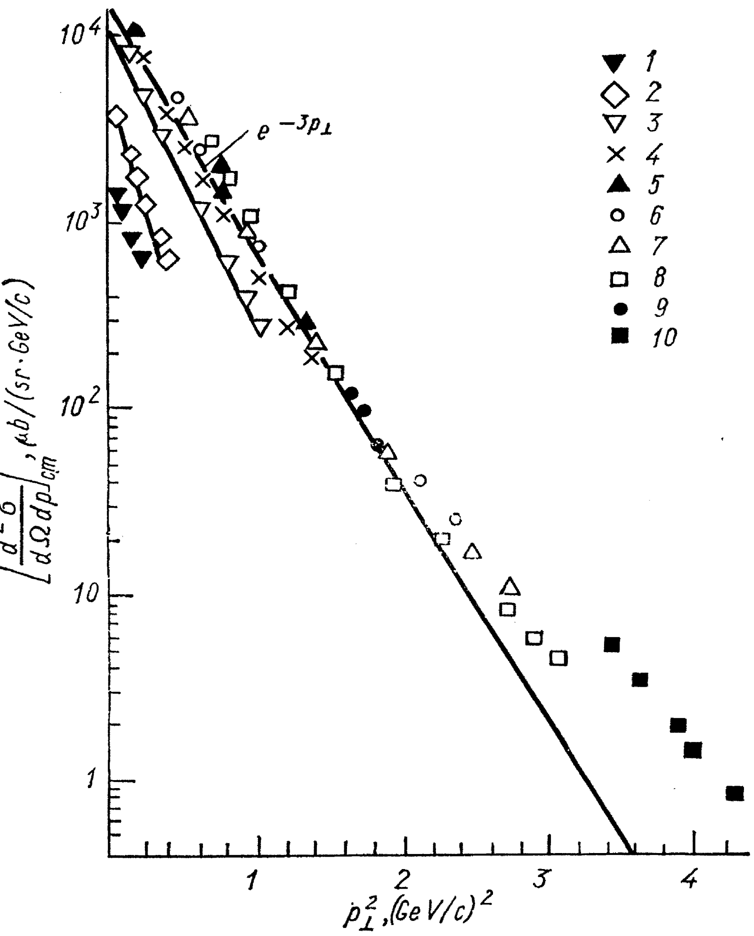

Fig. 12. Secondary nucleons distributions in transverse momenta at different inelasticity coefficients (see [49]). $p + p \to p +$ anything. The lines are straight-line fits to the data. Different symbols are used to designate the different inelasticities or energy losses (ΔE_{cm} (*GeV*), Inelasticity): 1. (1.47, 94%), 2. (1.37, 87%), 3. (1.10, 70%), 4. (0.95, 61%), 5. (0.77, 49%), 6. (0.41, 26%), 7. (0.29, 19%), 8. (0.13, 8%). This experiment, 12.4 *GeV/c*. 9. (0.83, 53%), 10. (0.22, 14%). Asbury, et al., 12.5 *GeV/c*.

it successfully combines with the multiperipheral scheme to describe cluster decay.

III. The third point which should be considered is the question of principle: «What is statistical theory»?

There is an opinion that statistical theory is only a rough approximation to the exact description of a process given by the amplitude of dynamical theory.

This opinion is not necessary. The statistical system strictly speaking is described not by a state vector but by a density matrix. The development with time is irreversible here. It is given by kinetical but not dynamical (T-invariant) equations.

In particular the requirement of probability normalization at any moment is significant here. It does not agree with the S-matrix formalism in which the

Table

Dynamic system		Instability $\longrightarrow$		Statistic system
Classical mechanics. Mechanic equations for trajectory (t-invariant)	$\rightarrow$	Averaging of the solutions with respect to unstable parameters	$\rightarrow$	Kinetic equations for density of probability (irreversible)
Quantum mechanics. Quantum-mechanical eguations for the amplitude	$\rightarrow$	Phase instability. Averaging with respect to the phases	$\rightarrow$	Kinetic equations for the density matrix

«S»-matrix at definite time is absent and there is no question of normalization in the intermediate states.

One can retrace in detail the change from a dynamical description (with the help of the S-matrix or the amplitude) to a statistical one.

In classical physics, the statistical features appear if the dynamical quantities are unstable, i. e. supersensitive to the external conditions. It becomes impossible for this reason to consider the system as a quite isolated one; the dynamical quantities should necessarily be averaged (See table, upper part).

It is shown in [50] in a non-relativistic model that the same situation can be realized in quantum mechanics: at some values of the system parameters the scattering phases can be unstable, supersensitive to the external conditions. Then it is necessary to carry out phase averaging which is equivalent to a change from the dynamical description to the statistical one (See table 1, lower part).

Apparently we must still determine whether a description of particle collision with the help of an amplitude (or state vector) is the only possibility or not.

4. Phenomenological Models

Let us consider a group of models which can conditionally be called quasiclassical. As is well known Feynman [1] recently supposed that hadrons (or their fields) consist of a large number of virtual particles — partons (possibly quark-antiquark pairs).

Multiple production in this model is treated as a «shaking off» of some partons during the interaction of two hadrons. The physical sense of this phenomenon is most clear in the coordinate representation.

During the breaking or bending of the path the weakly bound external parts of the field are shaken off and radiated in the form of particles. This is the very way this idea was formulated in the works of Lewis, Oppenheimer and Wouthuysen [51] and in works using the Weizsäcker — Williams method [14]. The process is analogous to the bremsstrahlung of soft quanta. The features of the elastic scattering resulting from such a process were recently considered [52].

The model is somewhat inconsistent: in concrete calculations one has to introduce a cut off parameter (or an upper momentum) depending on the energy; with increasing energy the deeper parts of the field can be shaken off. At the conference this approach was represented by the reports [53, 54].

In these works the concept of a system with many internal degrees of freedom is realized with the use of modern techniques. In the first work the state vector of the excited nucleon is assumed to be a coherent state of the 4-dimensional oscillator. In the second work this vector is calculated on the basis of a field-theory approach in the soft quantum approximation. The functional integration technique suggested by Fradkin [55] and developed by Fradkin, Milekhin [56] and Barbashov [57] is used. It is shown that the state can really be considered coherent. From the viewpoint of multiple production theory the main difficulty is nevertheless not yet overcome: a cut-off parameter of the spectrum must be introduced and the results depend essentially on this cut-off.

From the formal point of view this model is close to the method of uncorrelated jets (Van Hove [58]). In both cases the state vector of an excited system is factorized which means the absence of correlations and clustering.

In this connection I should mention the phase problem. If we neglect the phases of the matrix elements of inelastic processes as was done in the majority of these papers or consider them independent on the momenta of secondary particles, too large a width of the elastic scattering cone is obtained from the amplitude for inelastic processes where one uses the unitarity condition to reconstruct the elastic amplitude. The inconsistency of such an approach of constant phases is shown in a number of works [59, 60, 61].

Very close to this uncorrelated jets approach are the phenomenological models based on the hypothesis of limiting fragmentation (hereafter LF) [62, 63].

The LF hypothesis was already discussed at the conference in the report by Yang [63] and in the rapporteur talk given by Tavkhelidze. That is why I shall speak of it rather briefly and emphasize only the comparison of its results with those of the multiperipheral scheme and the possibilities of experimental verification.

The physical picture is as follows: while flying by, the hadrons get excited and radiate secondary particles. The spectrum $\rho(\vec{p})$ of the radiated particles was assumed independent of the energy, the transverse momenta being small. The physical picture is close to the models of Zatsepin [13], Takagi [64] and others [14] discussed at the beginning of the fifties.

In the initial version the spectrum $\rho(\vec{p})$ was supposed to be integrable, the multiplicity being constant. It is difficult to relate this version to experiment.

In a new version reported by Yang the model was considerably improved: at $p_{\parallel} \to \infty$ the spectrum has the form $\rho(p_{\parallel}) \sim 1/p_{\parallel}$, the multiplicity is proportional to the logarithm of the energy, Feynman's scaling law is satisfied. On the whole the results of the model are close to those of a multiperipheral scheme *.

The above mentioned inclusive-experiments do not contradict the LF hypothesis. On the basis of LF a calculation of the production spectra at $E_L \simeq \simeq 70$ GeV was carried out in [65]. The general trend of the curves is described satisfactorily.

* The properties of the elastic scattering amplitude due to inelastic processes are not considered within the framework of the LF model.

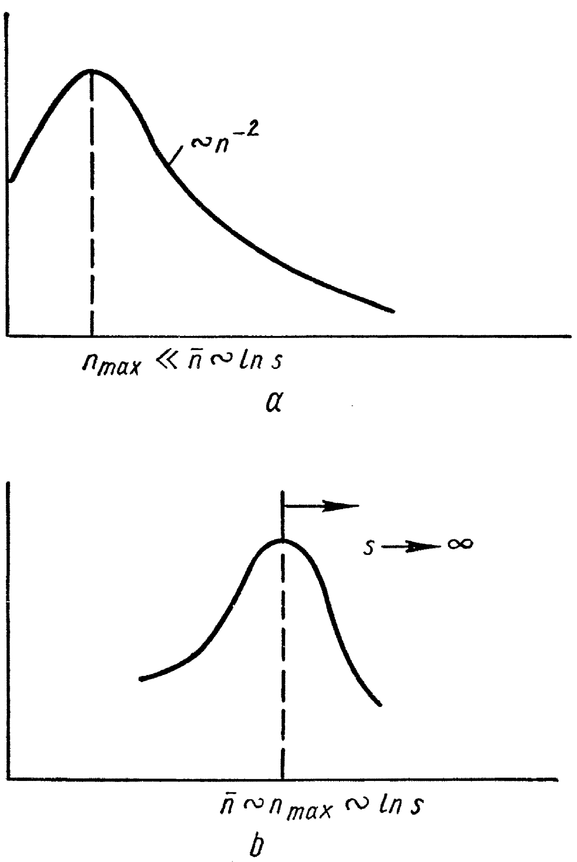

Fig. 13. Multiplicity distribution at high energy a) limiting fragmentation model; b) multiperipheral models.

The difference between the LF model and a multiperipheral scheme can be found in two points:

I. In the LF model the partial cross-sections of the production of n particles are asymptotically constant $\sigma_n(s) \to \text{const}$ at $s \to \infty$ and $n \gg 1$.

From the conditions $\sigma_{tot} = \sum_{n=2}^{n_{max} \sim \sqrt{s}} \times \sigma_n(s) = \text{const}$ and $\bar{n} = \sum_{n=2}^{n_{max} \sim \sqrt{s}} n\sigma_n \sim \ln s$ it follows that $\sigma_n \sim n^{-2}$ at $\bar{n} \gg 1$. This means that the distribution with respect to a number of particles in the jets does not depend on the energy asymptotically and the mean value $\bar{n} \gg n_{extr}$ (where n_{extr} is the quantity corresponding to the distribution maximum) (see Fig. 13*a*).

As to the multiperipheral scheme the situation is different: the maximum in the distribution with respect to a number of particles shifts to the right with increasing energy (see Fig. 13*b*) and the mean value is always close to n_{extr}.

The experimental situation represented by Wróblewski consists in the following. The data of this work [66] show that within the energy interval $E_L = (100 \div 700)$ *GeV* the quantities $\sigma_n(s)$ practically do not change. Correspondingly, the distributions with respect to n are also almost constant. This information is in good agreement with the LF hypothesis. It does not however contradict the multiperipheral scheme with heavy blobs, for in the latter one and the same graph makes the main contribution in a wide energy region, replacement of the main graph occurs slowly and asymptotic values are achieved only at $E_L \sim 10^{14}$ *eV*.

On the other hand the empirical formula of Czyzewski and Rybicki [67] which agrees with all the data available predicts a change of the distribution over n with increasing energy. According to this formula the distribution with respect to n has a bell-like form and $\bar{n}$ is close to n_{max}.

I suppose that on the whole this formula is close to what can be expected within the framework of the multiperipheral scheme.

Thus as far as this question is concerned we cannot yet come to the final conclusion on the basis of experiment. The difficulty is that the theory gives clear but asymptotic predictions whereas the asymptotic energy region is far off.

2. In the LF model, as distinct from the multiperipheral scheme, particle clustering and heavy blob formation is absent. From the experimental point of view this question is reduced to the investigation of correlations and to the detailed study of fireball production which was already mentioned.

It seems to me that on the whole the idea of a great number of particles constituting the field of elementary particles and a quasiclassical character of this field is rather attractive. However, before applying it to a concrete process it is necessary to solve a number of questions of principle. One of them is how to describe a system (by the state vector or the density matrix). In the latter case the description may be simpler and will make possible the solution not only of the problem of multiple production but also that of elementary particle structure, which is the final goal of all our efforts.

In conclusion I would like to express my deep gratitude to E. L. Feinberg, I. I. Royzen and I. M. Dremin for numerous and fruitful discussions, and their assistance in the preparation of this report.

DISCUSSION

Micheida:

I would like to comment on the clusters of pions which you called «fire-balls» in one of your drawings. You say that in the 100 to 1000 *GeV* region there should be one fire-ball, for higher energies 2, etc.

From the point of view of experimentalists one would like to have a definition of fireball and the most natural and clear one is that used in studies of cosmic ray interactions: this system of pions should be isotropic in its rest frame.

Now if we agree with this definition the bubble-chamber data at 16 *GeV/c* for average multiplicities are already inconsistent with the picture of the leading pion and nucleon plus an isotropic fireball. The cluster of pions is elongated along the initial momenta direction and it is quite clear that it will become more elongated for the 100 to 1000 *GeV* region.

On the other hand if we give up the requirement of isotropy it seems to me almost impossible to look for those objects in a quantitative way or to distinguish whether at a given energy we observe one or two fireballs.

Yang:

Since the exact locations of the fireballs in the many fireball model may fluctuate from event to event, the single particle distribution may have no peaks and valleys. If that is the case, to check the fireball model one would have to study 2 body (maybe 3 body too) correlations.

Chernavsky:

As to angular distribution of pions in the c. m. s. of the fire-ball one can say the following. According to the statistical theory no angular distribution should and can be rigorously isotropic. It is connected with the fact that many angular moments are represented in the fire-ball. Therefore, in the statistical theory the angular distribution is close to isotropic, but is not exactly isotropic.

To answer the question of Prof. Yang I would like to note that a double-humped fire-ball structure peculiar for every shower is smeared out in averaging but not totally (according to the data of Cracow group). Double-humped structure in the λ-distribution is vividly seen even in the sum angular distribution.

Rosental:

1. Increasing the energy of colliding particles the number of the channels of reaction increases and, consequently, the probability of «guessing» the correct diagrams which describe the multiplication process decreases. Therefore the part of statistical theories enhances.

2. The persistent investigations of multiplication processes in cosmic rays don't indicate the existence of many clusters-fireballs because the angular distributions (in λ-coordinates) with two maxima are explained by the boundedness of $p_{\perp}$.

There are indications that many-particle correlations exist at the available on accelerators energies. It is possible that the only cluster formed is the one which should decay at large energies according to the hydrodynamical theory (the statistical theory predicts too large $p_{\perp}$). The question on the methods of including the modern apparatus of the field theory into the hydrodynamic theory arises.

Efremov:

As I understand from the report for a description of phenomena of the multiple production we have two different mechanisms a) multiregge exchange, b) classical current or impact picture approach.

Both of them are in a good agreement with an experiment. From the point of view of field theory there is no contradiction at all. We know that the first picture can be obtained in the theories of the type Φ^3 or Φ^4, from the region of hard pions. The second picture was generated by electrodynamics where the main role for high energies plays soft photons. In fact there is the only renormalizable theory which can pretend for the discription of hadron phenomena. It is γ^5 mesodynamics $g\bar{\psi}\gamma^5\psi + h\Phi^4$. If the coupling constant were small the main contribution in this theory comes from hard pion region. If gives Reggepole term. The rest soft pion region of diagrams could give an impact picture. But in hadron physics the coupling constants are large and both contributions become comparable. That is why we deal with rather complicated interference of both pictures.

S e l o v e:

Regarding the distribution of multiplicities and its possible change with s: First, are there any predictions from the parton theory; and second, would you comment further on how the present experimental data compare with the various models?

C h e r n a v s k y:

I've already said that the parton model contains the parameter — the cut-off radius — on which concrete results of the model depend strongly. In the discussed papers the model is not yet brought to the state in which it can be compared with experiment. No concrete predictions of multiplicity of the angular distribution etc. are put forward.

Comparing various models with experiment the following situation arizes: inclusive experiments can't clarify the fact, practically, all the models can satisfy them. Exclussive experiments and the correlation analysis are necessary at higher energies. I think that studying deep inelastic processes at $E_{\mathrm{lab}} \simeq 70$ GeV will be an experimentum-crucis.

F e i n b e r g:

1. (Comment to Rozenthal's comment). Reality of fire-balls is an experimental problem, concerning which there is a controversy among experimentalists. However, seemingly more data accumulate supporting the idea of the pion production via clusters. It is essential that the Bethe — Salpeter equation theory authomatically leads to a prediction of fire balls, with the same mass as is claimed by experimentalists.

2. The hydrodynamical theory by Landau has a rather tragical history. Being a very elegant theoretical piece of work (classical relativistic hydrodynamics within a nucleon and this is strictly and consequently founded), far developed, it, may be, has no object to be applied: the only essential assumption of this theory is the existence of a large mass lump of meson-nucleon substance, arising in collision of high energy hadrons. However, the experimental data seem to show that such lumps ($N\bar{N}$ annihilation, fire balls etc.) do not exceed in mass some 5 GeV. In such clusters the hydrodynamical pressure does not yet tell and the statistical theory by Pomeranchuk — the limiting case of hydrodynamical theory, valid for small mass lumps, is adequate.

Similar lumps arise in the process of electron-hadron deeply inelastic collision. Here energy transfer to hadrons up to ~ 5 GeV was already studied. May be here the problem of existence of heavy hadronic clusters, for which hydrodynamics is necessary, will be solved?

Y a n g:

In connection with Selove's question, I thought in his report yesterday Dr. Wróblewski summarized the experimental data as showing that the cross-sections for finite number of multiplicities seem to stay flat. Dr. Wróblewski, is that correct?

W r ó b l e w s k i:

Yes.

C h e r n a v s k y:

I agree with Dr. Efremov's comment. Realy, there is an ideological contradiction (I spoke about in my report) between the multiperipheral scheme based on the field theory and statistical theory. I spoke also about the possibility of solving it by the stability analysis. As it seems to me there is no practical contradiction when each theory is applied in the area of its applicability in the multiperipheral scheme.

The Bethe — Salpeter equation allows to solve the question of the block masses but the question of the decay of blocks goes outside the framework of its competence, in this case namely the statistical theory is used.

As to the comments of Prof. Yang and Prof. Wróblewski on the approximate constancy of partial cross-sections $\sigma_n(s)$ in the region $E_{\mathrm{lab}} \sim 10^{12}$ it is possible to note that in the multiperipheral scheme with heavy blocks the energy region in which the one block diagrams give the main contribution is large; $\sigma_n(s)$ will be also approximately constant in this region but it does not mean the asymptotic constancy $\sigma_n(s)$.

P i g n o t t i:

I want to make a comment on the energy behavior of partial cross sections in the multiperipheral model, in connection with the question of Professor Selove and the remarks of Professors Chernavsky, Yang and Wróblewski. We believe that the Pomeron pole exists, and that it couples to inelastic processes. The question of how important this effect is at present accelerator energies is a quantitative one, and has been recently investigated by Chew and Snider and Sall and Marchesini. The general picture is that partial cross sections rise as a consequence of some threshold effect, then fall as some inverse power which corresponds to the exchange of a Regge pole with $\alpha \leqslant 0.5$, and finally, when Pomeron exchange is allowed, develop an approximately constant high energy tail. This behavior was beautifully exhibited in an example shown yesterday by Wróblewski in which the threshold behavior was divided out and the deviation from an inverse power developed after a linear fall of more than one decade in a $\log \sigma$ versus $\log s$ plot. On the other hand, I want to stress that it is not essential to have constant partial cross sections at high energy for the hypothesis of limiting fragmentation. One can construct models in which the constant high energy tail of each partial cross section is neglected and in which, nonetheless, the single particle distributions satisfy the hypothesis of limiting fragmentation.

B l o c h i n t s e v:

My comment concerns the applicability of the hydrodynamic description of events in cosmic rays. Hydrodynamic description can be applied when the average impulse, average energy and average mass in every element of volume exceed considerably the quantum fluctuations of these quantities, in these volumes.

In this connection, several years ago I paid attention to the necessity of being very careful in applying hydrodynamic description and multiple elementary particle production in collisions of hadrons, the conditions mentioned above not always are carried out.

C h e r n a v s k y:

The dependences of partial cross-sections $\sigma_n(s)$ on energy in the multiperiphere schemes with the light blocks and with heavy ones can differ considerably. In the first one, as it seems to me, $\sigma_n(s)$ will increase and decrease rapidly, in the second one, as I've already mentioned the diagram change will be slower.

REFERENCES

1. R. P. F e y n m a n, Phys. Rev. Lett. **23**, 1415 (1969), preprint 1969.
2. D. A m a t i, S. F u b i n i, S. S t a n g h e l l i n i, M. T o n i n, Nuovo Cim. **22**, 56 (1961); **26**, 896 (1962).
3. I. M. D r e m i n et al., JETP **48**, 952 (1965).
4. I. I. R o y z e n, D. S. C h e r n a v s k i i, JETP, **42**, 65 (1962).
5. A. A. L o g u n o v, L. D. S o l o v i e v, A. N. T a v k h e l i d s e. Phys. Lett. **24B**, 181 1967; K. I g i, S. M a t s u d a. Phys. Rev. Lett. **18**, 625 1967; R. D o l e n, D. H o r n, C. S c h m i d, Phys. Rev. Lett. **19**, 402 (1967); Phys. Rev. **166**, 1772 (1968).
6. K. A. T e r - M a r t i r o s y a n, JETP **44**, 341 (1963); I. A. V e r d i e v, A. M. P o p o v a, K. A. T e r - M a r t i r o s y a n, JETP **46**, 1295 (1964); T. M. B. K i b b l e, Phys. Rev. **131**, 2282 (1963).
7. G. F. C h e w, A. P i g n o t t i, Phys. Rev. Lett. **20**, 1078 (1968); Phys. Rev. **176**, 2112 (1968); N. F. B a l i, G. F. C h e w, A. P i g n o t t i; Phys. Rev. Lett. **19**, 614 (1967); Phys. Rev. **163**, 1572 (1967); L. C a n e s c h i, A. P i g n o t t i, Phys. Rev. Lett. **22**, 1219 (1969).
8. I. I. R o y z e n, in «Nucleons and Pions», Dubna, 1968; Phys. Lett. **29B**, 428 (1969).
9. G. F. C h e w, M. L. G o l d b e r g e r, F. E. L o w, Phys. Rev. Lett, **22**, 208 (1969).
10. I. M. D r e m i n, I. I. R o y z e n, D. S. C h e r n a v s k i i, paper 12—3.
11. I. M. D r e m i n, JETP Letters, **11**, 272 (1970). See also I. M. D r e m i n, D. S. C h e r n a v s k i i, JETP **45**, 1943, 1963.
12. H. M. C h a n, J. L o s k i e w i c z, A. A l l i s o n, Nuovo Cim. **57A**, 93 (1968).
13. G. T. Z a t s e p i n, DAN **67**, 933 (1949).
14. E. L. F e i n b e r g, D. S. C h e r n a v s k i i. DAN **81**, 795 (1951); **91**, 511 (1953).

15. D. I. Blokhintzev. Proc. of CERN Symposium, vol. 2, p. 155 (1956).
16. A. H. Mueller, I. Muzinich, paper 12—11.
17. V. N. Akimov, D. S. Chernavskii, I. M. Dremin, I. I. Royzen, Nucl. Phys. **B14**, 284 (1969).
18. V. A. Akimov et al., paper 12—2.
19. E. L. Feinberg, D. S. Chernavskii, Uspekhi **82**, 1 (1964).
20. V. N. Akimov, I. M. Dremin, preprint Lebedev Phys. Inst. A—62 (1966).
21. N. F. Bali, A. Pignotti, D. Steele, paper 12—10; N. F. Bali, et al., paper 12—13.
22. a) H. Piotrowska, paper 12—14.
b) Scandinavian Bubble Chamber Collaboration, paper 3a—31.
23. O. V. Kancheli, paper 12—29.
24. D. M. Tow. Phys. Rev., D**2**, 154, 1970.
25. a) M. Miesowiez, Rept. Progr. in Cosmic Ray Phys. **10**, 1070.
b) P. Ciok, et al., Nuovo Cim. **8**, 166; **10**, 741 (1958).
c) N. A. Dobrotin, S. A. Slavatinskii, Nucl. Phys. **35**, 152 (1962).
26. R. Rybicki, W. Wolter, Intern. Conf. C. R., Budapest (1969).
27. L. Van Hove, Nucl. Phys. **B9**, 331 (1969); Lecture of the Hawaii Conference, Ref. TH 1115 — CENR 1969.
28. J. Ballam et al., paper 12—24.
29. V. N. Akimov et al., paper 12—4.
30. V. N. Akimov et al., paper 3a—42.
31. A. Jurewicz et al., paper 12—9.
32. G. Bialkowski, R. Sosnowski, W. Wojcik, paper 12—25.
33. T. Hofmokl, paper 12—8.
34. T. Coghen et al., paper 3a—48.
35. V. A. Kolkunov et al., paper 12—39.
36. G. Ranft, J. Ranft, papers 12—20, 12—23.
37. V. A. Abramovsky, O. V. Kancheli, paper 12—35.
38. V. N. Gribov, A. A. Migdal, Sov. Nucl. Phys. **8**, 1002, 1212 (1968).
39. G. F. Chew, D. R. Shider, Preprint UCRL — 20033.
40. E. Fermi, Progr. Theor. Phys. **5**, 570 (1950).
41. I. Ya. Pomeranchuk, DAN, **78**, 889 (1951).
42. R. Hagedorn, Nuovo Cim. **25**, 1017 (1962); **35**, 216 (1965).
43. L. D. Landau, Bull. of the Acad. of Sci. of the USSR, Ser. Phys. **17**, 51 (1953).
44. S. R. Tuan, paper 12—21.
45. C. B. A. McCusker, I. Cairns, Phys. Rev. Lett, **23**, 658 (1969); I. Cairns et al., Phys. Rev. **186**, 1394 (1969).
46. I. N. Sissakyan, E. L. Feinberg, D. S. Chernavskii, Sov. JETR **52**, 545 (1967).
47. A. Böhm et al., paper 3a—43.
48. E. L. Feinberg, paper 12—6 (see also E. L. Feinberg, Preprint P. N. Lebedev Physical Institute N 44).
49. G. I. Marmer et al., Phys. Rev. Lett, 23, 1469, 1969. See also paper 12—19.
50. N. M. Pukhov, D. S. Chernavskii, Preprint of the Lebedev Phys. Inst. N 29, 1968.
51. H. W. Lewis, J. R. Oppenheimer, S. A. Wouthuysen, Phys. Rev. **73**, 127 (1942); N. Hu, Acta Physica Sinica **10**, 320, 1954.
52. H. Gemmel, H. A. Kastrup, Nucl. Phys. **B14**, 565 (1969);
Z. Physik **229**, 321 (1969).
53. V. A. Matveev, A. N. Tavkhelidze, paper 12—30.
54. B. M. Barbashov et al., paper 12—31.
55. E. S. Fradkin, Thesis Lebedev Phys. Inst. (1960).
56. E. S. Fradkin, Proc. of the Lebedev Phys. Inst., **29**, (1965);
Acta Phys. Hung. **19**, 176 (1965);
57. B. M. Barbashov, Sov. JETP **408**, 607 (1965).
58. L. Van Hove, Nuovo Cim. **28**, 798 (1963); Rev. Mod. Phys. **36**, 655 (1964).
59. H. Fukuda, C. Iso, Nuovo Cim. **43**, 43 (1956);
Z. Koba, M. Namiki, Nucl. Phys. **B8**, 413 (1968).
60. K. Zalewski, Nucleonika **13**, 9 (1968); L. Michejda, J. Turnau, A. Bialas, Nuovo Cim. **56A**, 241 (1968).
61. I. M. Dremin, Sov. Nucl. Phys. **10**, 818 (1969).
62. J. Benecke et al., Phys. Rev. **188**, 2159 (1969).
63. T. T. Chou, C. N. Yang, Talk, given by Prof. C. N. Yang at this Conference.
64. S. Takagi, Progr. tech. Phys., **7**, 123, 1952.
65. J. C. Vander Vedle, paper 12—22.
66. D. D. Reeder et al., paper 3a—55.
67. O. Czyzewski, K. Rybicki, paper 3a—24.
68. S. Koba, A. Krzywicki, preprint 363/VII, 369/VII **1963.**