A Military Effectiveness Analysis and Decision Making Framework for Naval Ship Design and Acquisition

By

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B.S. Naval Architecture and Marine Engineering
Webb Institute of Naval Architecture, 2001

Submitted to the Department of Ocean Engineering
in Partial Fulfillment of the Requirements for the Degrees of

Master of Science in Naval Architecture and Marine Engineering

and

Master of Science in Ocean Systems Management

at the
Massachusetts Institute of Technology

June 2003

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ABSTRACT

This research develops a new framework for performing military effectiveness analyses and design tradeoff decisions. It provides an extensive survey of literature for effectiveness analysis and multi-criteria decision making to develop a single consistent philosophy for such analyses.

This philosophy is applied to a requirements and effectiveness analysis case study of a conventional submarine that is performed using Response Surface Methods to facilitate design space visualization and decision maker interaction. Measures of Merit are developed and applied to the case study. The resulting requirements space and methods to visualize and explore it in a decision making context are presented and discussed

Lastly, a framework is proposed that would facilitate the concurrent consideration of requirements and effectiveness analyses with design and technology forecasting to create a Unified Tradeoff Environment that would provide decision makers with pertinent information to facilitate better informed requirements derivation and design selection.

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ACKNOWLEDGMENTS

This thesis would not have been possible without the help and support of Dr. Cliff Whitcomb who introduced me to this area of research and provided invaluable academic advice throughout my two years at MIT.

I thank Ms. Katherine Drew of the Office of Naval Research (Code 334, Ship Systems and Structure Division) for the financial support of this research. Dr. Hank Marcus was instrumental in this work and the completion of my graduate studies by accommodating and supporting my academic interests and providing helpful guidance.

I would also like to thank Kostas Psallidas for blazing the trail for the use of the computer programs used in this work and his willingness to answer my many questions. Lastly, thanks go to my family and friends for their support while I have been at MIT.

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NOMENCLATURE

Department of Defense Acquisition Instructions	DODI
Mission Tasks	MTs
Dimensional Parameters	DPs
Measures of Performance	MOPs
Measures of Effectiveness	MOEs
Measures of Force Effectiveness	MOFEs
Measures of System Effectiveness	MOSEs
Overall Measure of Effectiveness	OMOE
Measures of Merit	MOM
Analysis of Alternatives	AoA
Mission Tasks	MTs
Military Operations Research Society	MORS
Multi-Criteria Decision Making	MCDM
Weighted Sum	WS
Hierarchical Weighted Sum	HWS
Analytical Hierarchy Process	AHP
Multi-Attribute Utility	MAU
Figure of Merit	FOM
Rational Decision Making	RDM
Cumulative Prospect Theory	CPT
Aerospace Systems Design Laboratory	ASDL
Design of Experiments	DOE
Response Surface Methods	RSM
Optimal Deadrise Hull	ODH
Conventional (Non-Nuclear) Submarine	SSK
Probabilistic System of Systems Effectiveness Methodology	POSSEM
Survivability of a Random Search	SRS
Survivability of Suspected Target Search at the End of Burst	STS-EB
Survivability of Suspected Target Search at the End of Search	STS-ES
Mission Capability - Area Denial	MC-AD
Mission Capability - Strike	MC-S
Positive Detection Swath	PDS
Air Independent Propulsion	AIP
Burst Speed	Vmax
STS Evasion Endurance Speed	VEES
Time at Burst Speed	Tburst
AIP Balance Speed	Vbalance
AIP Endurance	TAIPendur
Submerged Endurance on Battery	Tbatt
Submerged Battery Loiter Speed	Vloiter
Unified Tradeoff Environment	UTE
Integrated Theater Engagement Model	ITEM

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CHAPTER 1: INTRODUCTION

PURPOSE

The design of an effective system rests upon understanding how to measure system effectiveness, how to draw an appropriate boundary to define the extent of the system to include in the analysis, how to clearly and accurately represent this and other design information to decision-makers, and how to make rational design decisions. Dr. Dean Rains, one of the most prolific authors on the subject of military effectiveness analysis for naval ship design notes that:

> Combatant ship design is a series of tradeoffs often made with little knowledge of the impact of the decisions, except on ship size or displacement. However, many other considerations, such as combat effectiveness, survivability, and initial cost may be equally important in the design process. [Rains, 1984]

These other considerations range from those stated above to other areas such as operational availability and systems reliability. A vital component of the design of these systems is the ability to measure these characteristics, which is a difficult task. As Zink *et al* observes:

> Measures and targets that [drive] these studies are dependent on the subjective opinion of the customer/user, i.e. the requirements. These requirements are often ambiguous and typically change over time. *Therefore, understanding the simultaneous impact of requirements, product design variables, and emerging technologies during the concept formulation and development stages is critically important, and until now elusive.* [Zink *et al*, 2000]

In order to gain a firm understanding of the simultaneous impacts that Zink *et al* describes, the ship designer must be introduced to subjects that have traditionally been beyond the designer's purview. Further, to design a modern, highly complex engineering system, the designer must understand what external factors are most important to the design, the interaction of these multiple, competing design factors, how the system relates to its environment, and frameworks that decision makers use to evaluate the system. Therefore, this research has four primary goals:

1. To provide a survey of literature for systems effectiveness analysis.
2. To provide a survey of literature of Multi-Criteria Decision making models.
3. To synthesize competing theories of each survey into consistent philosophies to approach the problem of requirements and effectiveness analysis for naval ship design.
4. To perform a requirements and effectiveness analysis on a case study of design tradeoffs in terms of requirements and effectiveness.

A Systems Perspective

During the first half of the cold war, "ship level requirements, rather than the ship's contribution to the performance of the task force, drove the design process" [Rains, 1999]. The International Council on Systems Engineering (INCOSE) recognizes a general problem associated with this approach:

> Organizations focused on the optimization of their products often lost sight of the overall system. Each organization perceived that their part must be optimal, using their own disciplinary criteria, and failed to recognize that all parts of a system do not have to be optimal for the system to perform optimally. [INCOSE, 2000]

Beginning in the late 1970s and early 1980s naval engineers realized that it was important to look at the collective whole of how a vehicle or weapon was assembled, which led to the use of systems engineering concepts in a naval systems context, which leads to two primary questions: what is a system? and what is systems engineering?

Recognizing the importance of systems engineering, the Department of Defense established the Defense Systems Management College, which provides the following definitions [DSMC, 2000]:

- System – a system is an integrated composite of people, products, and processes that provide a capability to satisfy a stated need or objective.

- Systems Engineering – a logical sequence of activities and decisions that transforms an operational need into a description of system performance parameters and a preferred system configuration.

The application of systems engineering to naval engineering has been discussed extensively by Tibbitts *et al*, who describe it as "a process which transforms an operational need into a description of system parameters and integrates those parameters to optimize the overall system effectiveness" [Tibbitts *et al,* 1993].

Thus it is clear that engineers must consider how the system that they are designing interacts with the environment it operates in and the other systems it operates with. This expansion of scope was coined the 'supersystem,' which includes everything outside the ship that either affects it or is affected by it. As defined by Hockberger, the supersystem is "the system that is just big enough to include everything that must be taken into account in determining the optimal (most cost-effective) ship for the mission requirements" [Hockberger, 1996]. Having briefly introduced some ship design and systems engineering concepts, two key considerations have arisen: systems effectiveness and requirements.

EFFECTIVENESS AND REQUIREMENTS ANALYSES

To evaluate systems in the supersystem context, appropriate metrics must be applied. These are generally called measures of effectiveness and they are generally considered to be "inherent in the *mission* and are *external* to the ship" [Hockberger, 1996]. Hockberger goes further to stress the importance of evaluating effectiveness in a mission context:

> The ship's effectiveness has to do with the *change* in the military situation that results from its involvement in the engagement, which is a matter of *outcomes*, and Measures of Effectiveness can thus be seen as *outputs* of an engagement…[thus] it is the synergism between the new ship or system and the rest of the task force that is at issue, and it is the task force effectiveness and attainment of mission Measures of Effectiveness that must be used as the

> basis for assessing and comparing the performance of each alternative. [Hockberger, 1996]

In the case of torpedo design research, Frits *et al* observed that the use of effectiveness analysis existed, but it was virtually decoupled from the design process. The analysis appeared in series with the design work, leading to an iterative cycle in which fleet operators developed torpedo tactics, had a torpedo built, and then re-developed tactics to better suit the torpedo that was delivered. They noted that the "lack of interaction between the warfare analyst and the weapon designer prevents the weapon system from reaching its greatest potential effectiveness" [Frits *et al,* 2002]. Thus, Frits *et al* found complete disconnects between the weapons analysts, designers, and requirement setters. Hollingsworth and Mavris noted that the:

> Most commonly used approaches to conceptual design today start with a fixed set of requirements, and synthesize and size various concepts, using either deterministic or probabilistic methods, to achieve the final optimal vehicle design. This approach, however, does not always yield the most affordable vehicle. In many cases, the final performance and affordability of a given aircraft is predetermined the moment the system requirements are defined and accepted. Further, it is often the case that the design requirements are not fixed but rather evolve through the development life of the vehicle. [Hollingsworth and Mavris, 2000]

A similar perspective was echoed in a Government Accounting Office report on best practices in weapon systems procurement. It demonstrated that the current practice of setting requirements prior to the designation of funds to conduct systems engineering denies decision makers and designers of "the knowledge needed to match wants with resources before starting a program…to evaluate the sufficiency of available resources – knowledge, time, money, and capacity…in time to help identify and make critical trade-offs that proceed the formalization of requirements." [GAO, 2001].

Therefore, Frits *et al* advocates a shift of design philosophies that would lead to the development of:

> an environment in which the effects of changes in engineering parameters are analyzed to determine their impact on overall…effectiveness. This process is accomplished by linking a conceptual…design program with a [simulation] program. Thus, the linkages between design variables, weapon performance, and tactics can be more thoroughly understood, and a vehicle with the greatest overall effectiveness can be created. [Frits *et al,* 2002]

Such concurrent development of effectiveness models and engineering analysis is required to optimize a system and provide decision makers with pertinent information to facilitate better informed requirements derivation.

PROCEDURE

This discussion will begin with a literature review section discussing performance and effectiveness measures. The section will establish a base of ground rules that provide clear definitions and guidelines for the development of appropriate systems measures for use in a military effectiveness analysis.

Then, fundamental aspects of decision making will be studied through a second literature review. Psychological, mathematical, and practical implications and applications of the methodologies will be discussed, and a method for use in this research will be selected. This section will also provide a brief introduction to the role of uncertainty in decision making and how it will be addressed in this analysis.

The next section will introduce the method that will be used to facilitate tradeoff studies. It will specifically address the application of the methodology to performing requirements based tradeoffs. Then, the discussion will turn to the subject of uncertainty, and its role in tradeoff studies.

Next, the discussion will examine a case study that will apply what has been learned from the previously mentioned literature reviews. A design case study for a conventionally powered submarine will be discussed and appropriate systems measures will be developed. This section will also discuss a hierarchy for aggregating the systems measures with the decision making model chosen earlier.

Finally, the results of applying this tradeoff methodology to the models developed will be presented. The discussion will finish with important conclusions and recommendations for future work.

Chapter 2: Measures of Merit

Overview

In a major work that studied the varying styles in strategy and analysis of the military services, Builder demonstrated that the modern military is dependent upon many types of analyses, such as operations, systems, requirements, cost effectiveness, programming, and budgeting analyses. Thus, Builder noted, "analysis has become the language of institutional advocacy for ideas and things in the military bureaucracies" [Builder, 1989].

Builder specifically characterized each military branch's styles and attitudes, noting that the Navy has traditionally had "little tolerance of analysis for planning or evaluating the Navy, by either requirements or systems analyses" [Builder, 1989]. Unlike the Army and Air Force, the Navy "has never relied on analysis for requirements – qualitative or quantitative. Navy requirements come from its experience and traditions, and from the quality thinking of its people, well steeped in both" [Builder, 1989].

In the Navy's defense, Builder states that institutional Navy skepticism of requirements analysis is not necessarily uncalled for, but it may be overdone:

> The Navy knows, correctly I think, that results or outcomes in war are largely incalculable…walking the balance between the analysis of war outcomes and the analysis of relationships in war is tricky. The Navy needs not use analysis to *determine* its force requirements or effectiveness; but it could benefit from the use of analysis to *understand* what may end up driving its force requirements and effectiveness, even within the vast uncertainties of war. [Builder, 1989]

Builder completed this study in 1989, prior to a DOD-wide realization that such a shift in thinking was necessary.

Much changed during the 1990s due to the end of the Cold War and the introduction of Acquisition Reform. Department of Defense Acquisition Instructions (DODI) 5000.2 specifies that programs must "select measures of effectiveness that relate directly to a system's performance characteristics and to mission accomplishment. Decision makers need to know the contribution of the system to the outcome of battle, not just how far it can shoot or how fast it can fly" [Ito, 1995]. These instructions are currently under review for revision, and it is not known what the new versions will require.

However, it is clear that the reason for performing analyses such as an "effectiveness analysis is to determine the military worth of the alternatives in performing mission tasks (MTs)" [OAS, 2000]. Thus, as Builder suggests, the Navy can gain great insight into requirements relationships and alternatives by pursuing more mature effectiveness analyses.

In order to gain this insight, the system under study must be understood; as Mason notes, "a thorough understanding of the boundaries for any system must be accomplished within the context of the analysis at hand" [Mason, 1995]. Therefore a brief discussion of specific terms used in the effectiveness analysis process is necessary at this point.

DEFINITIONS

While there is no consensus on specific definitions, the following definitions will serve as the baseline for this work:

Effectiveness – "Effectiveness is the condition of achieving a requirement" [Hockberger, 1996].

System Effectiveness – System effectiveness is the "ability of a system to accomplish a mission, and achieve a favorable battle outcome" [Brown, 1995]. Some references include optimization in this definition, but it will be left out of the definition used in this work. Optimization, in general, will be discussed later in this chapter.

Dimensional Parameters (DPs) – "DPs are the properties or characteristics of the physical entities whose values determine system behavior and the structure under consideration even when at rest" [Green and Johnson, 2002].

Measures of Performance (MOPs) – MOPs are "related to inherent parameters (physical and structural) but measure attributes of system behavior" [Green and Johnson, 2002]. MOPs are generally "non-probabilistic measures of performance, where 'the MOP class provides for the collection of metrics…that are not probabilities of successful outcomes of functions.' Thus MOPs are the 'consequence' of specific configurations of physical elements." [Brown, 1995]

Measures of Effectiveness (MOEs) – MOEs are a "measure of how the system performs its functions within an operational environment" [Green and Johnson, 2002]. MOEs are metrics that measure "the *degree* of effectiveness attained in a achieving a requirement" [Hockberger, 1996].

Measures of Force Effectiveness (MOFEs) – MOFEs are a "measure of how the system, and the force of which it is a part, performs its missions" [Green and Johnson, 2002]. MOFEs are may also be referred to as Measures of System Effectiveness (MOSEs), or as an Overall Measure of Effectiveness (OMOE).

Measures of Merit (MOMs) – MOMs are a general term for all measures that characterize a system under analysis, they "subsume all measures that characterize a…system" [Green and Johnson, 2002]. In this study, MOMs will collectively refer to MOPs, MOEs, and MOFEs.

As the definitions indicate, MOMs develop in a very hierarchical manner. An Air Force Analysis of Alternatives (AoA) guidebook states that "MOEs are often supported by one or more MOPs…[and that] MOEs may support other MOEs as well as Mission Tasks (MTs); [however], when using hierarchical MOEs, a clear rollup methodology should be described[1]"[OAS, 2000]. To help visualize these relationships, an example MOM hierarchy is shown in Figure 1:

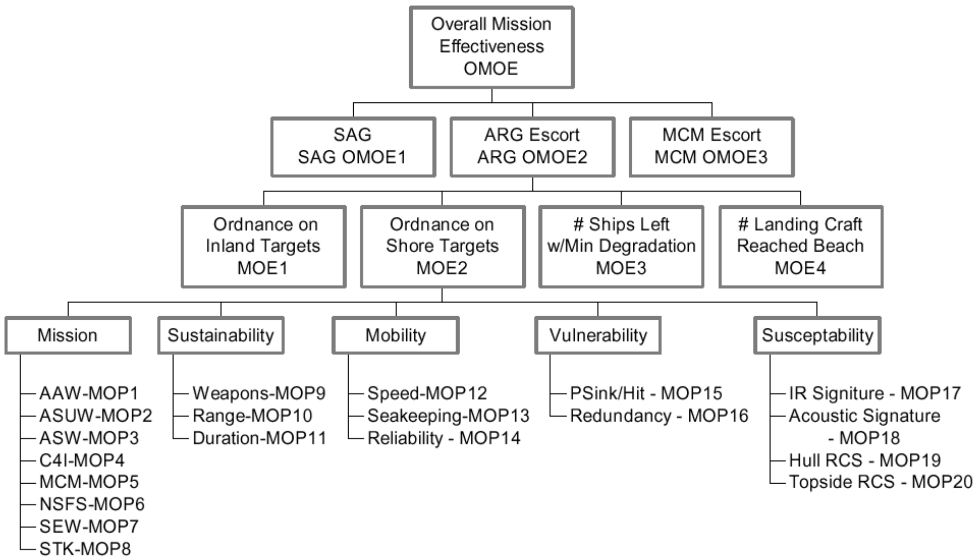

Figure 1: Sample MOM Heirarchy [Brown and Salcedo, 2002]

MEASURES OF MERIT

After defining the key terms used to describe MOMs, the varying theories of what constitutes a MOP or MOE can be discussed. The most structured and significant work towards a unified theory of MOMs appears to be from weapons and combat systems designers [Tibbitts *et al,* 1993] and the Military Operations Research Society (MORS). One of the most prolific authors from this constituency is Green, who discusses the importance of bounding the system in terms of internal and external attributes early in the process of developing MOMs. This is a crucial and often overlooked step because "a change in the boundaries changes the parameter set and the resulting system behavior and performance" [Green, 2001a].

---

[1] This will be discussed in a later section.

A useful method to visualize this is a series of concentric rings, similar to a sliced onion or tree, as shown in Figure 2:

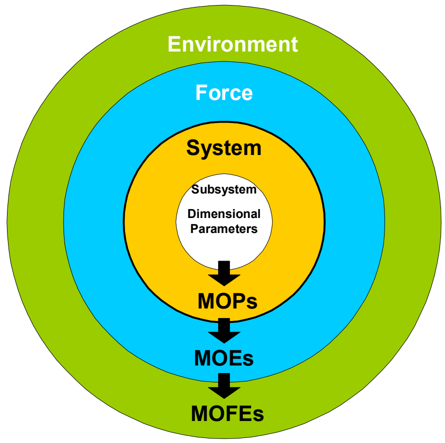

Figure 2: System Boundary Levels [Green and Johnson, 2002]

Green begins by specifying the DPs and MOPs as characteristics that are measured within subsystems and the system, "whereas MOEs and MOFEs are specified and measured external to the boundary" in relation to associated forces or environments [Green and Johnson, 2002]. In discussing models used for effectiveness analysis Leite and Mensh specify two groups of metrics, similar to Green's system boundary levels: those related to the model and its internal operation, and MOMs for the "system performance as a function of its intended operational employment" [Leite and Mensh, 1999].

Green describes a process model that begins with four inputs: the mission, the expected threat, the environment, and potential system concepts. The description begins by stating that "candidate systems [should be] evaluated in the Mission Context for performance" [Green, 2001b]. The majority of the literature reviewed supported the approach that "the first step in developing MOEs and task force mission analysis is to select the missions and define them in quantitative terms" [Rains, 1999]. Tibbetts *et al* also encourages the use of "battle overviews [which] form the basis for establishing measures of effectiveness and set the stage for later mission effectiveness studies."[Tibbitts *et al,* 1993] In the case where a ship is the system under analysis, Green recommends "viewing the ship as a weapons system [to keep] these performance goals in context with the assigned missions" [Green, 2001b]. This implies that MOEs should be developed in parallel with the system requirements, and Hockberger stresses that this needs to be done because: it can be done, they help formulate requirements, and it helps make the design process more efficient [Hockberger, 1996].

To be able to conduct such a mission analysis, a model of the system under development and its warfighting environment must be developed. Leite and Mensch directly address this topic and provide a step-by-step process for developing the model as shown in Figure 3:

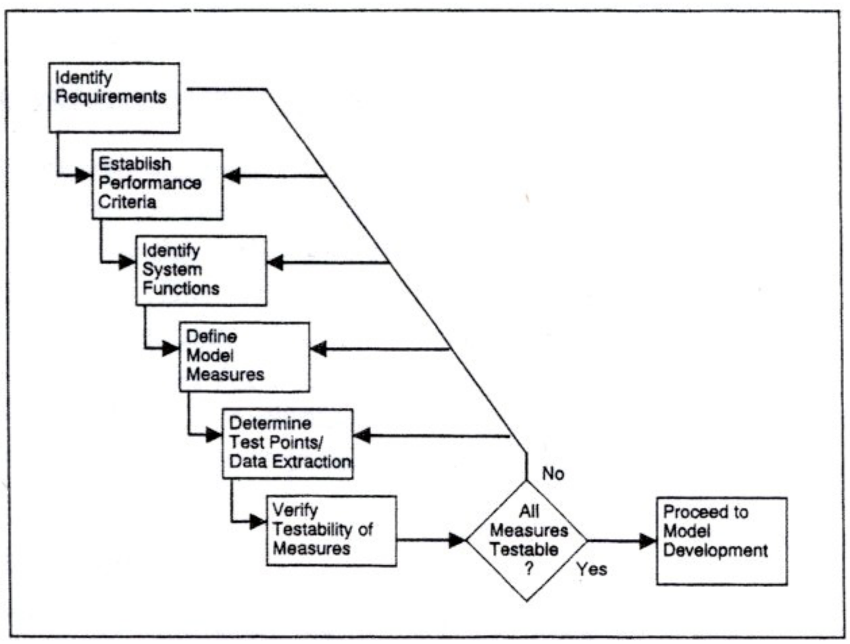

Figure 3: Model Development Process [Leite and Mensh, 1999]

After developing an appropriate system model, the outputs of the scenario are used as inputs to metrics for representing the previously defined MOMs as shown in Figure 4:

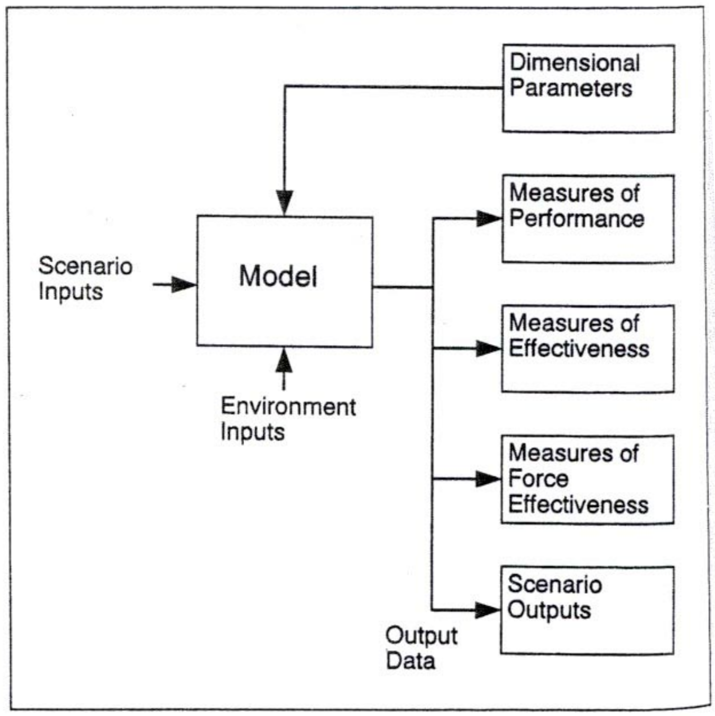

Figure 4: Relation of Models to MOMs [Leite and Mensh, 1999]

Returning to Green's process model, it continues to develop by focusing on what are generally called the 'ilities.'[2] Green keeps the focus of the work on mission and system solutions by relating "operational availability, reliability, survivability, and weapons systems performance…to their subsequent impact on ship design" [Green, 2001b]. Similar to Green, Brown develops a MOM hierarchy from a 'Cycle of Mission Accomplishment' composed of: Availability, Reliability, Survivability, and Capability [Brown, 1995].

While developing the MOMs, the literature stresses that the measures "must be independent at the level of analysis under evaluation" [Green, 2001a]. The Air Force AoA guidebook advises that "MOEs should not be strongly correlated with one another (to avoid

[2] The 'ilities' include system performance characteristics such as affordability, performability, standardability, producibility, deliverability, riskability, reliability, and maintainability. [Keane *et al*, 1996], [Shupp, 2003]

overemphasizing particular aspects of the alternatives)…[and that] MOEs must be independent of the nature of the alternatives, as all alternatives are evaluated using all MOEs" [OAS, 2000].

Green proposes that the result of such an approach is a balance "between those elements, both combat systems and ship systems, that are required for mission success [and that the] process model focuses on the mission goals rather than starting with a set of constraints that accept degradation in the performance of these goals as a price that must be paid" [Green, 2001b]. More specifically, Malerud *et al* describes four steps for developing MOMs [Malerud *et al*, 2000]:

1. Define high-level properties through a qualitative, top-down approach
2. Outline MOPs by first identifying DPs that characterize identified high-level properties
3. Develop MOEs as metrics to judge system performance against user requirements
4. MOFEs present a more unique challenge as they are often "more qualitative…[requiring] military and analyst judgment."

While developing a process model for an analysis, Green and Johnson recommend the following general characteristics of successful MOMs be observed:

Table 1: Characteristics of MOMs [Green and Johnson, 2002]

Characteristics	Definition
• Mission oriented	• Relates to force/system.
• Discriminatory	• Identifies real difference between alternatives.
• Measurable	• Can be computed or estimated.
• Quantitative	• Can be assigned numbers or ranked.
• Realistic	• Relates realistically to the C2 system and associated uncertainties.
• Objective	• Defined or derived, independent of subjective opinion (it is recognized that some measures cannot be objectively defined).
• Appropriate	• Relates to acceptable standards and analysis objectives.
• Sensitive	• Reflects changes in system variables.
• Inclusive	• Reflects those standards required by the analysis objectives.
• Independent	• Mutually exclusive with respect to other measures.
• Simple	• Easily understood by the user.

Green also advocates that "expressing MOPs, MOEs, and MOSEs as a probability allows us to determine if a parametric change is statistically significant" [Green, 2001a]. Further, Green insists that the MOMs developed for use in analyses must be "efficient in the statistical sense (small variance/reasonable accuracy)." [Green and Johnson, 2002] Lastly, Green concludes with the advice that "if it can't be expressed as a probability it probably is not an effectiveness measure." [Green, 2001a]

Mason also advocated the use of probabilistic terms, specifically citing the work of Girard and Elele whose definitions of MOEs are much more mathematically rigorous because they are expressed in probabilistic terms.

> In Girard's terms, an MOE is the probability of the successful accomplishment of a function, where all probabilities are conditional, and are derived from MOPS and lower level (or prior) MOEs, and where a function is a process relating in an outcome. Thus 'an MOE defined by an objective function at an upper level is a dependent variable, and is a mathematical function of the MOEs defined by objective functions at a lower level.' Ultimately, an 'audit trail' equation is generated, linking the conditional upper level MOE to measurable MOPs. Elele uses Baye's Rule to develop a similar probability based MOE definition. [Mason, 1995]

The idea of cost effectiveness is central to making tradeoffs; however, the literature overwhelmingly advocates that cost should not be included in the development of MOMs. The Air Force AoA guidebook states that "because MTs are tasks, cost is never a MT or a MOE, and cost is never considered in the effectiveness analysis" [OAS, 2000]. It goes on to emphasize that MOMs should be very transparent:

> Ideally, MOEs should normally represent raw quantities like numbers of something or frequencies of occurrence. Attempts to disguise these quantities through a mathematical transformation (for example, through normalization), no matter how well meaning, reduce the information content and may be regarded as "tampering with the data." This same reasoning applies to the use of MOEs defined as ratios; a ratio essentially "hides" both quantities. [OAS, 2000]

Willard summarized the Defense Acquisition University's point of view on this issue as follows:

> Cost-effectiveness should not be represented as a ratio, giving values with meaningless signs or values (infinities when division by zero occurs). Rather,

> one plots points on a graph, with Delta-MOE on the vertical (y) axis and Delta-cost on the horizontal one (x), using the pairs of numbers for the different candidates. Now two options with the same effectiveness will be at equal altitudes, whatever their costs, and two with equal cost, whatever their MOEs, will lie above one another. The informational value one desires of a ratio is there without the confusion; and it is thus unnecessary to limit the scope of the analysis to constant cost or constant MOE. [Willard, 2002]

In naval engineering publications, Rains appears to be the most prolific author to tackle the issue of MOMs in ship design, defining MOEs as "numerical indicators which directly relate performance to cost" [Rains, 1999], stressing that MOEs must include cost to "tempers results, making lower cost systems with good performance possibly the most effective for the money required."[Rains, 1994]. This philosophy is reflected in an example MOE from Rains' work: percent of mission completed per dollars invested in the effort. This MOE is calculated by determining the fraction of ships available to perform the mission at the culmination of effort and dividing it by the total cost of the effort and ships [Rains, 1994]. This theory is in direct conflict with much of the literature reviewed, and will not be used in this research.

In fact, the Air Force AoA guidebook expressly advises against the use of ratios (cost/kill, kills/sortie, etc.) similar to Rains "because they frequently hide necessary information" [OAS, 2000]. The guidebook provides the following example:

> As an example, suppose that one alternative kills 0.01 targets per sortie and a second alternative kills 0.1 targets per sortie. The second alternative is ten times better than the first, right? That sounds significant, but is it…? The truth is, we can't tell from the ratio alone. If there are 10 targets to be killed, the answer is likely to be a resounding yes -- 100 sorties may be acceptable, but probably not 1,000. However, if there are 1,000 targets to be killed, the answer is almost certainly no, for we are looking at very large numbers of sorties even for the better alternative. By using the ratio instead of the numbers of sorties required, there has been a loss of understanding without a corresponding gain of any sort. [OAS, 2000]

Another consideration when choosing MOMs is their long-range applicability. These effectiveness measurements are not constrained to the early stages of design. As the system design progresses, it is constantly measured, and ultimately must prove that its performance meets its requirements prior to delivery and acceptance by the military. The Air Force AoA guidebook states that "if possible, MOEs should be chosen to provide suitable assessment criteria for use during later developmental and operational testing. This "linking" of the AoA to testing is valuable to the test community and the decision-maker" [OAS, 2000].

Lastly, Leibowitz provides some less theoretical and more practical considerations for MOE development. First, Leibowitz recognizes that MOM development is not an exact art and that value judgments are inherent at some stage of the process. "A measure of effectiveness resembles a moral principle in that its validity cannot be established by reason alone…we must make a value judgment" [DARCOM, 1979]. Leibowitz also reminds the reader that MOMs are not just metrics from analytical models. They must also incorporate the preferences of the decision-maker and customer. An interesting passage from the Army's Handbook for Weapon Systems Analysis reads:

> In the dynamic compromise process (1) we make use of our limited understanding of the supersystem to obtain an approximate measure of the system's effectiveness, (2) adjust this measure so that it becomes possible to relate it to the system's elements, (3) we readjust the measure until it is satisfactory to the decision maker, and (4) we re-readjust it until the projected study does not exceed the time-and-effort deadline.
>
> We are not quite finished. We must examine the resulting fourth-order approximation to see if it is close enough to the 'true' measure of effectiveness to make the study worthwhile. This can only be done by 'feel.' If we decide that the approximate measure is too far off, then, depending on the situation, we have five courses of action: (1) learn more about the supersystem, (2) learn more about the system itself, (3) talk the decision-maker into reversing his interpretation, (4) suggest an extension of the scope of the study, or (5) call the whole study off. However, in most cases, this last drastic step should not be necessary.
>
> The point is that regardless of how you finally select a measure of effectiveness, this measure must be reasonably close to representing the true purpose of the system. If it is not, then all the linear programming and all the game theory in the world will not save us form optimizing auto assembly lines so as to provide the maximum number of coffee breaks per hour. And, then

> we would soon find that no one was willing to sponsor (such) an operations-research study…. [DARCOM, 1979]

EXAMPLE MEASURES OF MERIT

While there is no "magic list of canned effectiveness measures" [Green, 2001a] for early stage development, there have been many studies performed in the past, and many examples of MOMs can be drawn from these. These examples can either be applied directly to the problem at hand, or serve as a springboard for developing more appropriate MOMs.

For example, the Mine Warfare Center uses 28 MOPs with four functional categories (sense, engage, control, and logistics) that were chosen to be applicable to all of their mine countermeasures studies [Mine Warfare Center, A-2G-2758]. More specifically, Liete and Mensh listed many successful MOMs from their work and experience, and these are summarized in Table 2:

Table 2: Sample MOMs [Leite and Mensh, 1999]

DPs	MOPs	MOEs
size	gain	probability of detection
weight	throughput	reaction time
aperture size	error rate	targets designated
capacity	signal to noise ratio	probability of kill
location/orientation	fragment size/pattern	
firing arcs / cutouts		

The Air Force AoA Guidebook provides guidance on determining system worth, but places the most emphasis on the military worth of the system. It includes "a small set of highly significant measures of military performance that are used most frequently at mission and campaign levels" [OAS, 2000]. Similarly, Hockberger cites a number of performance categories as well. These two sets of performance measures and categories are included in Table 3:

Table 3: Sample Performance Categories [Hockberger, 1996] & [OAS, 2000]

[Hockberger, 1996]	[OAS, 2000]
Mission Support (sensors, weapons, vehicles, etc.)	Time to accomplish high level objectives
Readiness (manning, RMA, facilities, endurance, etc.)	Targets placed at risk
Survivability (signatures, damage resistance/control)	Targets negated
Mobility (speed, seakeeping, maneuverability, stability)	Level of collateral damage
C4 (Command, Control, Communications, Computers, navigation)	Friendly survivors
Human Support (safety, health, habitability, recreation)	Numbers and types of resources used

As mentioned previously, Green advocated developing a probabilistic framework to perform effectiveness analysis. In developing this framework, the following list of mission success factors was included [Green, 2001b]:

- Availability of System for Mission
- Platform Performance Parameters
- Target Acquisition Capabilities
- Weapons Set
- C4ISR Capabilities
- Platform Signature and Countermeasures
- Operational Environment
- Survivability

Including these factors into an analysis, Green proposed the following Mission Success Formula for naval ship design effectiveness evaluation, as shown in Equation 1:

Equation 1: Green's Mission Success Formula [Green, 2001b]

$$\text{Mission Success} = A_O * R_M * S * MAM$$

Where:

- A_O = mission availability
- R_M = mission reliability
- S = survivability = probability of ship loss
- MAM = mission attainment measure
    - $MAM=WSE=P_K*P_D*P_C*P_E*P_{WK}$
    - P_K = Ship killability (a function of vulnerability and susceptability)
    - P_D = Probability of detection
    - P_C = Probability of control (correct identification, one track per target, etc.)
    - P_E = Probability of engagement (the ability to guide the weapon to within its acquisition cone)
    - P_{WK} = Probability of weapon kill (the ability of the weapon to achieve the desired level of kill)

As Rains notes, his analyses include an underlying assumption that “probability results are useful and meaningful” [Rains, 1994]. A probabilistic approach such as Green’s does not calculate discrete numbers, rather it results in a fractional system, which can lead to some initial confusion. For instance, it is not immediately clear what it means to lose fractions of ships, missiles, or capability. However, such an approach is more suitable to modeling thus making analyses easier and it smoothes effectiveness results.

Lastly, Crary developed a fleet effectiveness model that "not only measures the performance of a fleet of ships, but also illustrates how surface combatant mission capabilities affect fleet performance" [Crary, 1999]. The overall fleet MOE developed is defined as the probability that a fleet will win the war. The model is a function of three factors that are summed from sequential phases of the total scenario under consideration [Crary, 1999]:

- Phase Weight – a simple weight for the length of time of the phase under evaluation in comparison to the total length of time of the operation
- Mission Importance – an expert opinion weighting of the military value of components during specific phases of the operation
- Mission Effectiveness – a function of the capability of assets assigned to a phase, degradation to effectiveness due to logistics constraints, and synergy of platforms involved in the phase

MEASURES OF MERIT PHILOSOPHY

Given this review of literature on the subject of MOMs, this section will develop a single, consistent description of a MOMs system for application to ship concept design, a so called MOM Philosophy.

1. The definitions of DPs, MOPs, MOEs, and MOMs stated earlier in this chapter are adopted. To constrain the discussion and analysis, no MOFEs will be considered, though the definition is still supported. In summary, the definitions and hierarchy (from most system specific to least) are as follows:
    a. DPs are physical characteristics that drive system behavior.
    b. MOPs are non-probabilistic measures of specific configurations of DPs, calculated from DPs.
    c. MOEs are preferably probabilistic measures of the operational performance of the system, calculated from MOPs. The system boundary generally separates MOEs from MOPs.
    d. MOMs will be used as a phrase to refer to MOPs and MOEs in general.

2. The majority view that cost should be excluded from the effectiveness analysis is accepted for this Philosophy. It is important to stress that cost cannot be excluded from the complete design tradeoff analysis.
3. MOMs will be made as quantitative and probabilistic as possible. It is understood that such a format does not capture every important aspect of the effectiveness analysis; thus, the Philosophy will allow non-probabilistic, but quantitative MOMs.
4. MOMs will be developed following the steps that Malerud *et al* described [Malerud *et al,* 2000]:
    a. Define high-level properties (DPs) through a qualitative, top-down approach.
    b. Outline MOPs by first identifying DPs that characterize identified high-level properties.
    c. Develop MOEs as metrics to judge system performance against user requirements.
5. Normalization and ratio schemes will not be used.

Lastly, a brief discussion of the term "optimal" (to include variants 'optimized,' 'optimum,' etc.) is necessary. During the course of the literature review this term came up very often, in many different contexts, with vague and varying definitions.

As can be seen from the MOM Philosophy detail above, and the examples from earlier in this chapter, it is possible to have multiple MOEs, and even MOFEs (hereafter called 'top-level MOMs'). Thus, it is improper to use the term 'optimal' too loosely, because the optimization of multiple, competing attributes is a much more difficult problem than that of the optimization of a single attribute. Therefore, when multiple top-level MOMs are in use, multi-criteria decision making methods must be used to accurately and objectively model and determine system effectiveness.

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CHAPTER 3: MULTI-CRITERIA DECISION MAKING

OVERVIEW

A ship is composed of many systems: propulsion, electrical, weapons, mechanical, and environmental to name a few. Many of these systems are complicated in their own right, but their interactions can be even more so. Further, due to these interactions, it is entirely possible that the integration of optimized subsystems into a ship design will not create an optimized ship system. Therefore, it is clear that a ship design is a multi-criteria decision problem by its very nature, composed of multiple, competing objectives.

Thus, the determination of an 'optimized' design is not one that can be approached from traditional, analytical optimization methods such as objective function definition and use of different gradient methods, knee-of-the-curve, or Kuhn-Tucker necessary and sufficient conditions. Rather, the presence of multiple criteria must be considered and Multi-Criteria Decision Making (MCDM) methods must be used. This can lead to the determinations of many optimums requiring the use of Pareto analysis, which will be described in this chapter as well.

As Chapter 2 demonstrated, MOMs can be composed of many characteristics. Therefore, the primary goal of this discussion is to examine existing MCDM methods and discuss methodologies to aggregate DPs, MOPs, and MOEs into one or more top-level MOMs.

Further, design decisions of these types are rarely made unilaterally, so decision processes in teams must be examined. This examination has the potential to lack some of the mathematical rigor that MOM development demonstrated because it more directly involves multiple stakeholder interaction and conflicting preferences. Because stakeholders draw knowledge from personal experience, knowledge, and preferences, it is very important that any preferred methodology that is identified be internally consistent and rational to prevent natural biases from skewing the MCDM process.

This discussion of MCDM will begin by providing descriptions of differing MCDM models and three examples. Next, the subjects of Rational Decision Making (RDM) and groups will be discussed, followed by a discussion of the modeling of uncertainty in decision making. The section will conclude by describing a MCDM Philosophy for use in this research.

MULTI-CRITERIA DECISION MAKING MODELS

There are many methods that can be used to model MCDM, but this work will introduce only the most prevalent, to include: weighted sum (WS), hierarchical weighted sum (HWS), analytical hierarchy process (AHP), multi-attribute utility (MAU) analysis.

The WS method is the simplest, and most commonly implemented of the methods to be discussed. This method is implemented by summing the product of objective weights and attribute levels (MOEs in effectiveness analysis) to arrive at a figure of merit (FOM) [Whitcomb, 1998a]. Whitcomb notes that this method has been proven to be highly inconsistent and has a number of concerns that should be addressed prior to use [Whitcomb, 1998a]:

- Objective definitions are only defined at a single level, which impedes transparency of relationships
- The method does not attempt to mitigate or eliminate dependence between attributes
- Risk is assessed in an over simplistic manner

The remaining three MCDM models are all similar in one way because they are all based on a hierarchical approach, somewhat analogous to the discussion on MOMs. This approach eliminates the first concern with the WS model, and greatly aides in realizing the second concern. According to Whitcomb, three major advantages of the use of hierarchical relationships are that they [Whitcomb, 1998a]:

- Refine the ability to define appropriate aspects of each MOE.
- Show objective function relationships to each other.
- Organize the evaluation.

The simplest model that uses a hierarchy is the hierarchical weighted sum. This method is a "modification of the weighted sum method, using the objective hierarchy versus the single level objective sum of products formulation" that the WS method used [Whitcomb, 1998a]. A byproduct of the straightforward nature of this method is its ease of use and easy implementation with spreadsheet models.

The Analytical Hierarchy Process is similar to the HWS, except it reflects customer or decision maker preferences and priorities [Saaty, 1988]. The key to this method is the use of pairwise comparisons of every attribute at each level of the hierarchy. By performing these pairwise comparisons, a relative importance scale is developed for each attribute. Oliver *et al* provides a succinct description of the results of the pairwise comparison process:

> The results are summarized in a matrix, and the principal eigenvector of the matrix provides the values for the priorities. If all of the effectiveness measures can be computed analytically, then these priorities are used directly as weighting factors…[however], some of the effectiveness measures may be of the type that are matters of user preference. In this case the designs are considered in pairs for each of the effectiveness measures by the individuals participating. The results are combined with the weighting factors to yield a preference for each design. [Oliver *et al*, 1997]

Whitcomb notes that a benefit of this method is that it inherently provides a consistency check of the pairwise comparisons. However, as the number of attributes under consideration "becomes large, approximately greater than seven, decision makers may have trouble keeping the criteria straight" [Whitcomb, 1998a].

Similarly, Islam notes that the use of large numbers of pairwise comparisons in the AHP model can be a major drawback because of the amount of work involved. Thus, his work attempted to prove "Saaty's suggestion of clustering alternatives into groups according to a common attribute" [Islam, 1997]. With the use of an aerospace example, Isalm showed that "in the clustering procedure, the number of comparisons required is much less than is required in the unified approach and the rankings that result are sufficiently close to the standard AHP with all the pairwise comparisons" [Islam, 1997]. When using the AHP, Islam's method should be considered.

The final major MCDM model to be discussed is multi-attribute utility analysis, which is almost solely grounded in customer or decision-maker preferences and priorities; however, it also includes other characteristics such as uncertainty and risk [Keeny and Raiffa, 1976]. Whitcomb notes that the MAU analysis does not directly use the heirarcy developed earlier, but it can play a vital role in ensuring the independence of the attribute in the analysis.

This model is based on the utility function, which is "a specific type of value function in that the units are based on an ordered metric scale and is developed under the condition of risk."[Whitcomb, 1998b]. Because complex decisions have numerous attributes, this method combines the individual utilities into a single function, the MAU function. These are analytic functions, thus "the use of an ordered metric scale allows utility to be defined with respect to any two points on the scale, which are then assigned any convenient value. The quantities for the worst and best decision outcomes can be defined, forming the basis for actual measurement of utility" [Whitcomb, 1998b].

Unfortunately, such a method returns to "the fundamental problem in group decision making, that combining preferences across markets to form a group utility function is likely to violate Arrow's Impossibility Theorem" [Whitcomb, 1998b]. However, in practical application, Whitcomb notes that a major benefit of the MAU method is "the ability to incorporate the decision maker's nonlinear preferences towards each of the objectives into the decision process" [Whitcomb, 1998a]

In an attempt to mitigate this, some RAND studies use the 'Delphi Method,' which is a technique for obtaining expert guidance and judgment from groups. The Delphi Method has the following three key features that are "intended to minimize the effects of dominant individuals, irrelevant communications, and group pressure encouraging conformity" [Don, 2002]:

1. Group opinion is defined as an appropriate statistical aggregate of the individual opinions in the final round.
2. The opinions of the members of the group are obtained in such a way that the responses are anonymous.
3. Iterations are obtained by conducting systematic controlled feedback between decision rounds.

However, the first point highlights one problem inherent in the Delphi method. By aggregating group opinion, it is easily possible that the result will not please any of the decision makers. This is a prime example of Arrow's Impossibility Theorem, which will be introduced later in this chapter.

Aggregation in general is not a bad solution to simplify MCDM problems. As the foregoing discussion has shown, the aggregation of lower levels of the hierarchy is vitally important to most of the methods. The Air Force AoA Guidebook refers to aggregation of MOMs as 'Rolling Up the Results,' which allows decision maker to compare the alternatives with a smaller number of measures; however, the "advantage of having a smaller number of measures carries the obvious disadvantage: information, and along with it potential insight, is lost in the roll up process" [OAS, 2000].

They propose only using aggregation when it is firmly grounded in sound logic and meets the following conditions [OAS, 2000]:

- The aggregation arises naturally from relationships among the MOEs

- The significance of the aggregates is clear
- The aggregates tell a clearer story than the individual MOEs

In the process of rolling the MOMs up, the Guidebook also addresses the topic of weighting the MOEs. The Guidebook states that:

> Weighting assigns different values (weights) to different MOEs. It is a seductive idea: clearly not all MOEs are created equal. A difficulty with weighting, however, is that an analyst's weights may not be a decision-maker's weights. By weighting, the analyst is proclaiming judgment superior to that of the decision-maker. Weighting is strongly discouraged. Almost invariably, weighting is an attempt, conscious or otherwise, to avoid thinking through alternative methods of presenting the results in a clearer manner. Better presentations almost always can be found; take the time to look for them. [OAS, 2000]

[deNeufville, 1990] also provides an excellent example of the problems with weighted methods.

Further, DODI 5000.2 warns against methods that lead to customer or preferential weighting of different attributes:

> Never use schemes in which several measures of effectiveness are weighted and combined into an overall score. Weighting schemes are sometimes helpful, but they must be clearly explained in the analysis so that their results can be interpreted correctly. [Brown, 1995]

It is interesting to notice the contradiction between the official guidance and the more mature and useful methods of MCDM that all involve some form of weighting. If weighting is avoided, then the decision-maker will be presented with much more information than they either want or can be reasonably expected to handle, or both. Therefore, perhaps Hockberger's comments on the subject strike a reasonable compromise:

> Lower level MOEs should be calculated and combined within the model or simulation, which can determine the way each MOP of an alternative concept contributes to achieving them and how they combine to produce higher level MOEs. Human judgment and weights are only required for going the rest of the way up the tree, combining the MOEs the model yields in order to produce the overall composite MOE. [Hockberger, 1996]

This compromise still leaves the decision maker with the task of performing one or more tradeoffs, but at least of far fewer competing options. A widely accepted method for visualizing these various alternatives in relation to one another is the Pareto plot.

To generate the Pareto plot, the decision maker plots, for example, two competing MOMs (say MOM 1 and MOM 2) for a point design with one MOM each on the abscissa and ordinate. The decision maker can continue to plot the remaining, competing point designs on the Pareto plot. A useful method for doing this is to scale the values between a 'Good' and 'Marginal' value where the Ideal is achieved at point (1,1) and least ideal at (0,0). Implementing this method will result in a plot similar to Figure 5:

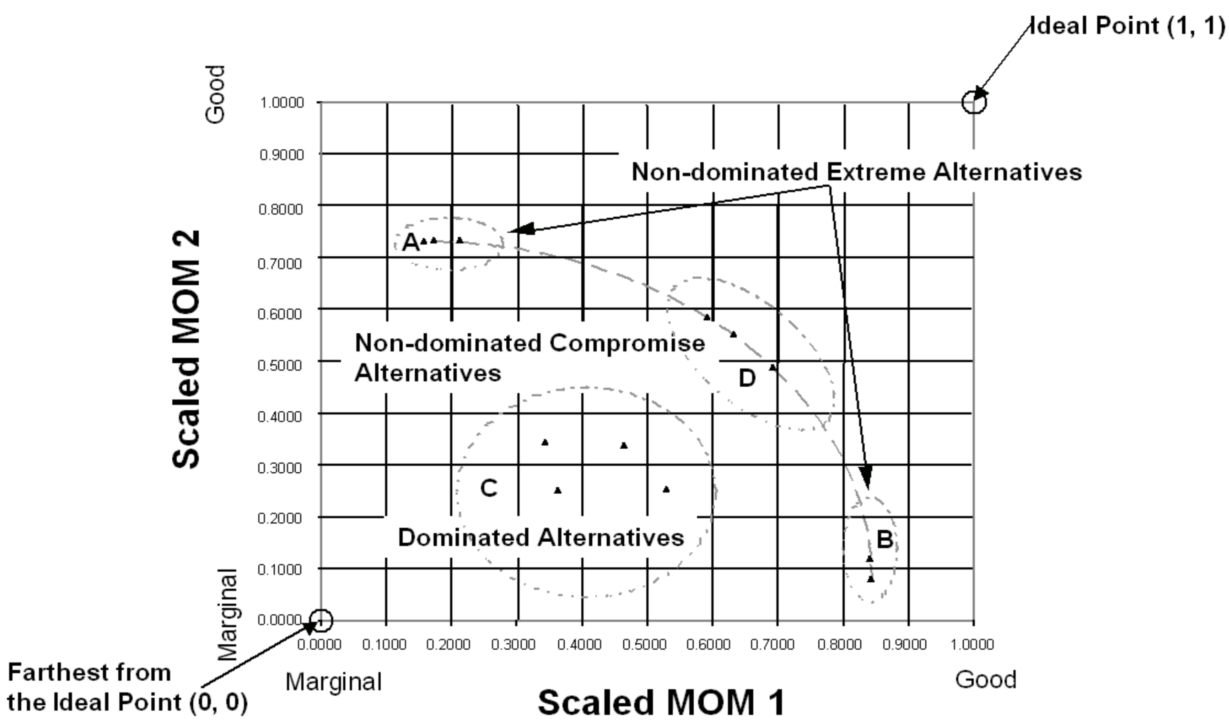

Figure 5: Example Pareto Plot [XIII-A, 2001]

By populating a plot such as this, the decision maker can clearly begin to see a Pareto Frontier (the curved, dashed line) emerge if enough point designs are plotted. The points may be considered Pareto optimal if, by moving away from the point, one MOM cannot be improved without degrading the value of the second MOM. It is also important to note that the Pareto frontier is not necessarily linear, convex, or of any specific form, as shown in Figure 6:

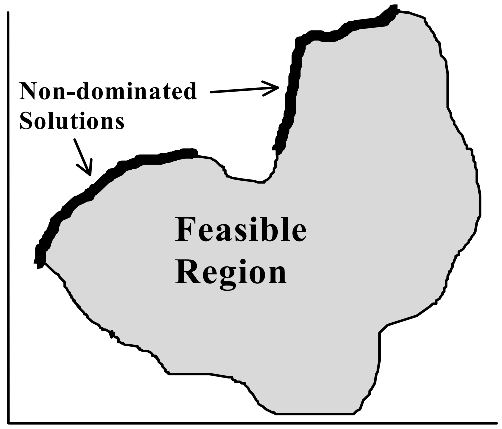

Figure 6: Non-Convex Pareto Frontier [Brown and Salcedo, 2002]

The Pareto frontier represents 'non-inferior' or 'non-dominated' solutions to the MOM 1 versus MOM 2 problem. These solutions are "the conceptual equivalents, in multiobjective problems, of a technically efficient solution in a single objective problem" [deNeufville, 1990], and are represented in Figure 5 by regions A (representing the extreme Pareto optimums), and B (representing the compromise Pareto optimums). All point designs that do not fall on the frontier are considered dominated by those on the frontier and are thus inferior designs, as represented by region C. While the Pareto plot cannot identify a single 'optimal' solution, it reveals equally efficient designs that can be concentrated on for a final series of tradeoffs

This discussion can only introduce these methods. For a more thorough discussion of the above MCDM models with respect to ship design consult [Whitcomb, 1998a] and with respect to complex systems in general consult [deNeufville, 1990].

MULTI-CRITERIA DECISION MAKING EXAMPLES

Prior to leaving the subject of MCDM, it is useful to examine some examples of the application of these methods in actual research. To begin, an example of hierarchy will be discussed. Whitcomb provides an excellent example of a hierarchy for use with either a HWS or AHP model in Figure 7:

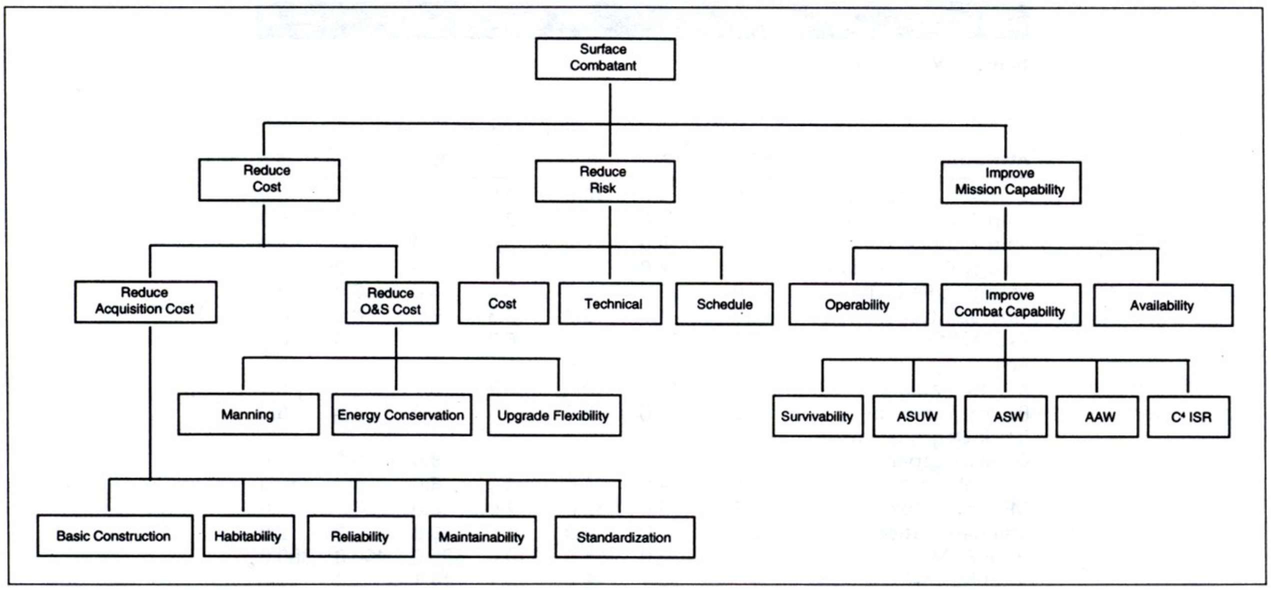

Figure 7: Whitcomb's Heirarchy Structure [Whitcomb, 1998a]

As another example, Mustin developed the "dendritic" to aid in the determination of data required for his studies:

> The purpose of the dendritic is to refine tasks to the point where data explicative of performance can be gathered. The dendritic is formed by focusing on the overall intent of related joint tasks across levels of war and determining a questions whose-data supported answer will define this intent….Similarly, corresponding functional areas form critical subordinate issues that generally reflect the level at which MOEs are developed. Specific task requirements within each of the functional areas serve to formulate another level of sub issues that may determine underlying MOPs. Continued refinement of task requirements into more specific and lower levels of aggregation ultimately leads to the point where data can be gathered. [Mustin, 1996]

An example of Mustin's dendritic for Force Protection is included as Figure 8:

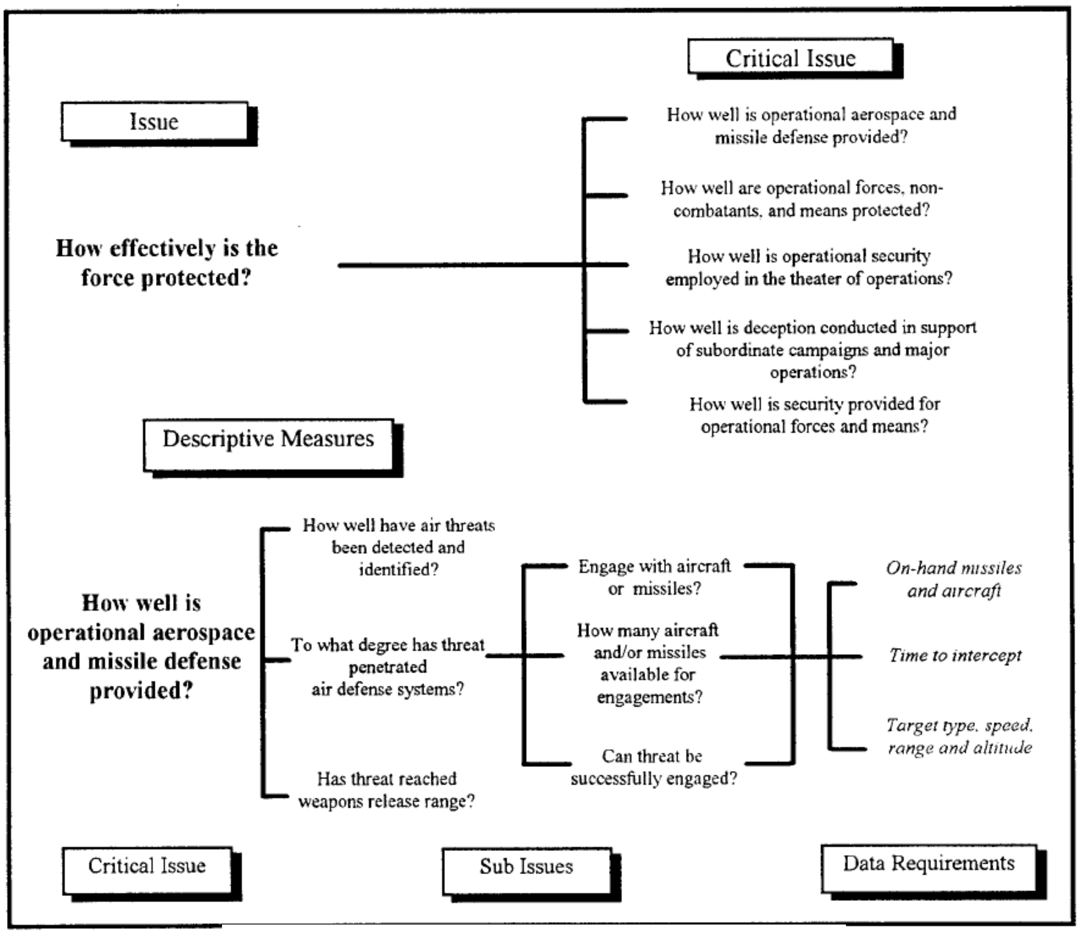

Figure 8: Mustin's Dendritic [Mustin, 1996]

Lastly, Crary adds an interesting twist to an AHP model in his work on surface combatant fleet effectiveness. Crary developed a traditional AHP by eliciting expert opinion from 15 individuals with a mission importance survey. However, the data was not averaged because Crary contends that doing so would lose any information that is valuable from differences in expert opinion. Therefore, "to capture these differences, we treat the 15 sets of weights as samples from a large population, and estimate probability distributions for mission importance by phase of the war" [Crary, 1999]. Crary used a Dirichlet distribution to model the AHP weights/expert opinion. Thus, "by treating mission importance weights as random, FMOE [Fleet MOE] for a given fleet of ships also becomes random with a distribution" [Crary, 1999].

Now, with a firm grounding in some of methods of MCDM, it is important to gain an understanding of the psychology of rational decision making (RDM) and the more humanistic considerations of group decision making.

RATIONAL DECISION MAKING AND GROUPS

The study of RDM is outside the field of engineering; however, it plays a vital role in all engineering decisions. Therefore, it is important that factors influencing such decisions be identified and considered. Two Nobel Prize winning researchers in the area of RDM are Kahneman and Tversky. Through decades of research, they have repeatedly demonstrated cases in everyday life where people do not behave logically and that these departures from rational logic occur in systematic patterns

Their research has identified "psychological principles that govern the perception of decision problems and the evaluation of options…[leading to situations] in which people systematically violate the requirements of consistency and coherence" [Tversky and Kahneman, 1981].

Kahneman and Tversky are most well known for the development of Prospect Theory, which is an "alternative theory of choice…in which value is assigned to gains and losses rather than to final assets and in which probabilities are replaced by decision weights" [Kahneman and Tversky, 1979].

One of the key elements of Prospect Theory is the 'certainty effect,' which is the natural tendency of people to "overweight outcomes that are considered certain, relative to outcomes which are merely probable" [Kahneman and Tversky, 1979]. They note that:

> In the positive domain [positive outcomes, i.e. gains], the certainty effect contributes to a risk averse preference for a sure gain over a larger gain that is merely probable. In the negative domain [negative outcomes, i.e. loses], the

> same effect leads to a risk seeking preference for a loss that is merely probable over a smaller loss that is certain. [Kahneman and Tversky, 1979].

This led to the conclusion that there is a fourfold pattern of risk attitudes. People exhibit "risk aversion for gains and risk seeking for losses of high probability…[and] risk seeking for gains and risk aversion for losses of low probability" [Tversky and Kahneman, 1992].

They also identified the 'reflection effect,' which was the realization, that by reflecting positive prospects (gambles) about zero, thereby making them losses instead of gains, reverses the preference order. This implies that "risk aversion in the positive domain is accompanied by risk seeking in the negative domain" [Kahneman and Tversky, 1979].

Lastly, in a departure from conventional decision theory at the time, they proposed that decisions are better modeled as being reference dependent.

> The carriers of value are changes in wealth or welfare, rather than final states. This assumption is compatible with basic principles of perception and judgment. Our perceptual apparatus is attuned to the evaluation of changes or differences rather than to the evaluation of absolute magnitudes. [Kahneman and Tversky, 1979]

Kahneman and Tversky revisited their theory in 1991, revising it to what they called the 'Cumulative Prospect Theory' (CPT). The original Prospect Theory included a mathematical formulation to model the behavior they observed, and CPT updated that formulation to be a continuous model. This model is composed of "a value function that is concave for gains, convex for losses, and steeper for losses than for gains…[and] a nonlinear transformation of the probability scale, which overweights small probabilities and underweights moderate and high probabilities" [Tversky and Kahneman, 1992].

This value function, which is a "means of ranking the order of relative preference between sets of consequences" [Whitcomb, 1998b], exhibits the three essential characteristics of their theory [Tversky and Kahneman, 1991]:

1. Reference Dependence – "the carriers of value are gains or losses defined relative to a reference point"
2. Loss Aversion – "the function is steeper in the negative than in the positive domain; losses loom larger than corresponding gains"
3. Diminishing Sensitivity – "the marginal value of both gains and losses decreases with their size"

The first two of these characteristics have been discussed, but diminishing sensitivity has not. The diminishing sensitivity characteristic:

> Entails that the impact of a given change in probability diminishes with its distance from the boundary. For example, an increase of .1 in the probability of winning a given prize has more impact when it changes the probability of winning from 0.9 to 1.0 or from 0 to 0.1 than when it changes the probability of winning from 0.3 to 0.4 or from 0.6 to 0.7. [Tversky and Kahneman, 1991]

Therefore, this characteristic drives the weighting function to be more concave near zero and more convex near one.

The three properties mentioned above are clearly seen in the following figure, which represents their value function:

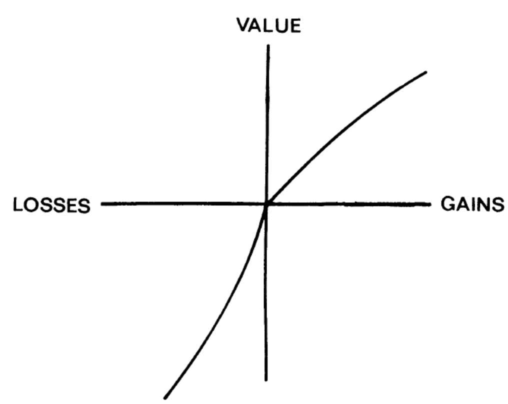

Figure 9: Prospect Theory Value Function [Kahneman and Tversky, 1979].

Mathematical expressions were developed to model this value function, as well as the weighting function and can be found in [Tversky and Kahneman, 1992].

One of the areas of psychology that Tversky and Kahneman studied was the subject of heuristics, the formulations that individuals develop to serve as personal guides while considering and solving a problem. They concluded, "people rely on a limited number of heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations" [Tversky and Kahneman, 1974].

Their work identified the following primary heuristics: representativeness (insensitivity to prior probability of outcomes, predictability, and sample size), availability (biases of retrievability of circumstances, imaginability, and illusory correlation), and adjustment and anchoring (insufficient adjustment "usually employed in numerical prediction when a relevant value is available") [Tversky and Kahneman, 1974]. They conclude by noting that many of their test subjects fail "to infer from lifelong experience such fundamental statistical rules as regression toward the mean, or the effect of sample size on sampling variability" [Tversky and Kahneman, 1974].

Kahneman and Tversky's research addressed individuals in decision making situations, but the design of a system is not generally decided by a single person, rather it is a group decision. This adds another dynamic to RDM: group decision making. For decision making purposes, a group is defined as "a collection of individuals with non-commensurate and conflicting preferences" [Whitcomb, 1998b]

Whitcomb goes further to note that "in general, the fundamental problem with group decision making is that there is no way to define a group utility function, either by combining individual utilities or by assessing group preference as a whole, as shown by Arrow's Impossibility Theorem" [Whitcomb, 1998b].

Arrow's Theorem puts forth two axioms and five conditions that describe conditions that an ideal group decision situation should satisfy. The two axioms are stated as [Sage, 1977]:

Axiom 1: Any two alternatives must be comparable, i.e., between alternatives x_1 and x_2 either $x1$ is preferred over x_2, or x_2 is preferred over x_1, or both x_1 and x_2 are equally acceptable.

Axiom 2: All comparisons between alternatives x_1, x_2 and x_3 are transitive, that is, given x_1 is not preferred over x_2 and x_2 is not preferred over x_3, then x_1 is not preferred over x_3.

And the five conditions can be summarized as [French, 1988] [Sage, 1977]:

1. Basic conditions
2. Positive association of social and individual values
3. Independence of irrelevant alternatives
4. Condition of citizens' sovereignty
5. Condition of nondictatorship

Unfortunately, Arrow shows that these axioms and conditions prove to be mutually exclusive, thus preventing the determination of a utility function that satisfies all stakeholders when more than one decision maker is involved.

UNCERTAINTY CONSIDERATIONS

The probabilistic approach to MOMs advocated by Crary and Green among others raises an important factor that must be considered in concept design: uncertainty. Uncertainty plays a role in the development of the MOM metrics, on decision weights and probabilities, and in MCDM and RDM. Thus, Zanini notes in the case of decision weights, it is important to emphasize that "given the subjective and abstract nature of [decision] weights, there is no attempt to seek a definitively "right" set of weights, but rather to explore how different assumptions and weightings affect the relative ranking of options" [Zanini, 2002]. This applies to MOMs as well as probabilities used in the analysis as well

This can be further generalized to the whole concept design framework to show that the objective is not to develop a single absolute optimum, rather it is elicit relationships for determining what characteristics have the greatest impact on the design, why they do, and how these relationship can be better exploited to lead to a better design

As Ito notes, one common method to introduce uncertainty into the analysis is to use Monte Carlo simulations:

> In reality, input variables such as PKSS, time delay, [and] initial detection range [are] not exactly the expected values. They include uncertainty in nature, which could be represented by a certain probability density function (pdf). To assess the effect of stochastic events, Monte Carlo simulation can be used. [Ito, 1995]

A Monte Carlo simulation randomly selects values for selected variables. The uncertainty is introduced by giving each variable of interest a probability distribution over a specified range. The generator then performs at least 1,000 to 10,000 simulations with values chosen at a frequency consistent with the probability distribution to simulate the probability distribution well [Crystal Ball, 2000].

Rains also noted that a “discrete analysis [versus the continuous results of a probabilistic analysis] would probably require a Monte Carlo technique to perform the needed calculations” [Rains, 1994]. Prior to the advent of high power desktop computers, Monte Carlo simulations were very resource intensive; therefore in earlier work, Rains used probabilistic analyses to avoid the high computation needs of Monte Carle methods stating, “the underlying assumption in all of the analyses presented in this paper is that probability results are useful and meaningful” [Rains, 1994]. As a byproduct of this approach, a probabilistic approach such as Rains and Green’s is not based in discrete numbers, rather it results in a fractional system, which can lead to some initial confusion. For instance, it is not immediately clear what it means to lose fractions of ships, missiles, or capability. However, such an approach is more suitable to modeling thus making analyses easier and it smoothes effectiveness results. Now, the power of desktop computers can easily handle Monte Carlo simulations with commercial software packages, as will be discussed later in this work.

One final method for analyzing the role of uncertainty in decisions is a real options approach, which was applied by Gregor to naval ship design and acquisition in 2003. "Real options involve the 'right but not the obligation' to take a course of action" [Gregor, 2003] Therefore, they provide a means of reevaluation as uncertainties are resolved. Gregor's research provided a first cut at determining "the value of these options and…the best types and amount of flexibility to design into naval systems in order to maximize the value of the system over time under uncertain conditions" [Gregor, 2003].

MULTI-CRITERIA DECISION MAKING PHILOSOPHY

Given this review of literature on the subject of MCDM, this section will develop a single, consistent description of a MCDM system for application to ship concept design, a so called MCDM Philosophy. Unfortunately, none of the methods discussed in this section are ideal. However, many key characteristics of the MCDM Philosophy have been illustrated.

1. The MCDM and MOM hierarchies should be identical.
2. Subjective judgments should be minimized and involve extensive dialogue between the technologists and decision makers.
3. Weighting schemes should be avoided when used with top-level MOMs. However, weighting methods for rolling-up lower level MOMs can be used when applied with AHP and Pareto analysis.
    a. The AHP model is chosen because it works well with the hierarchy and due to its inherent consistency check.
    b. The WS and HWS models are too simplistic to begin to accurately model the MCDM problem.
    c. The MAU analysis is not chosen because it can be cumbersome and is not prescriptive.

4. Kahneman and Tversky showed that decisions are often made in surprisingly irrational manners. Therefore, every effort should be made to make the MCDM methodology as independent of subjectivity as possible; however, this cannot be avoided when examining the top-level MOMs. Therefore, when performing trades of top-level MOMs, new methods must be used to visualize and perform these tradeoffs. These will be discussed in Chapter 6.
5. Uncertainty analysis should be performed.

At this point, various methodologies for performing effectiveness analyses and MCDM have been discussed, and guiding philosophies have been developed for each. Next, a methodology for implementing these two philosophies must be introduced.

CHAPTER 4: TRADEOFF METHODOLOGY

OVERVIEW

The concept design process of a naval combatant has traditionally been accomplished using rules of thumb, heuristics, accumulated experience, and parametric data, thus making it difficult to find an optimized solution. Further, due to the rapid increases in technological options available for ships, and a steady trend of shrinking defense budgets over the past few decades, the complexity and difficulty of design optimization has been ever increasing.

The identical situation has also occurred in the aerospace industry over the past few decades, and sophisticated optimization methods have been developed to meet this challenge in aircraft design. The aerospace industry has been developing an increasingly popular method for concept exploration coupling Design of Experiments and Response Surface Methods (DOE/RSM) techniques. These two statistical techniques identify the design variables that have the greatest impact on the design, and with appropriate software, lead to easily manipulable equations which can be used to define the design space, conduct tradeoff studies, and facilitate better informed decision making.

Builder characterized systems analysis as a discipline that "seeks to find and compare complex alternatives about which too little is known or knowable. [Therefore,] assumptions or theories or models may have to substitute for facts or real-world data" [Builder, 1989]. The previous two chapters have laid the foundation for developing models to represent the assumptions that will be required for an effectiveness analysis.

This chapter will address a tradeoff methodology called Response Surface Methods (RSM) that can be used to manage the MOM models and ship designs in order to populate a design space. The example that will be presented in Chapter 6 will demonstrate that the application of a statistical method such as this RSM is efficient, cost effective, and not overly complicated.

PREVIOUS RESEARCH

The Aerospace Systems Design Laboratory (ASDL) at the Georgia Institute of Technology was organized in 1992, and two of its primary areas of research are Probabilistic Design Methodology and Multi-Attribute Decision Making. The fruit of eleven years of research in these two areas has been the ASDL's successful application of DOE/RSM to concept design for aerospace systems. Examples of ASDL work range from examining the design of a single aircraft to exploring the direct and indirect relations of an aircraft on the single unit level, mission level, and campaign level. Research has also examined limited applications of DOE/RSM, such as the optimized selection of an engine for a jet. The ASDL has invested a great amount of time and effort in furthering the science of design space visualization and analysis for aerospace systems.

In 2000, Professor Whitcomb in the Naval Construction and Engineering (XIIIA) Program at MIT began an Office of Naval Research sponsored effort to translate aerospace DOE/RSM techniques to the field of naval combatant design. This first application led to successful research in submarine concept exploration by Goggins in 2001 [Goggins, 2001]. In this work, Goggins "generated a response surface for cost, submerged displacement, length, submerged speed, and OMOE" [Goggins, 2001]. This work used an OMOE that was a function of test depth, submerged speed, and modular payload length.

Price built upon Goggins' work in 2002, using DOE/RSM to investigate the impacts and propagation of design parameter uncertainty at the concept design stage [Price, 2002]. This work recognized that:

> The complexity of the ship design process leads to numerous assumptions and a great deal of uncertainty in the point designs during the concept exploration phase. While it is not feasible to eliminate this uncertainty, it is useful to explore how it affects the overall design. An analysis of the uncertainty associated with each point design provides the designer with additional information for comparing designs. [Price, 2002]

Addressing a much more specific design issue, Whalen's research in 2002 used DOE/RSM to:

> develop an Optimal Deadrise Hull (ODH) that reduces mechanical shock where it first enters the boat, at the hull-sea interface. Planing boat hydrodynamics were reviewed and the mechanical shock environment was evaluated. The ODH analysis is performed on the MkV Special Operations Craft in order to determine the effects of hull deadrise on vertical acceleration. Finally, the results of the ODH analysis are used to perform a design space study of planing hulls in order to optimize the overall design for vertical acceleration based on hull deadrise, cruise speed, and payload weight. [Whalen, 2002]

Lastly, Psallidas' research in 2003 applied DOE/RSM to assessing the impact of forecasted technological improvements on system performance [Psallidas, 2003]. Psallidas' work sought to:

> aid the decision maker in projecting the performance of future vessel concepts and in allocating the resources for technological research and development in an optimum way. The impact of technology [is] assessed through the use of technology k-factors that [are] introduced into a mathematical synthesis model [that] modify technical characteristics or cost parameters of the design. These modifications will result in changes of the technical metrics to simulate the hypothetical improvement or degradation associated with the new technology. [Psallidas, 2003]

DESIGN OF EXPERIMENTS

Design of Experiments (DOE) is a method by which a designer can examine numerous design parameters (DPs) and quantitatively understand the effect that each of these factors has on the overall design (also called "response") [JMP, 2002]. This is a method that is used prior to the

Response Surface Methods, and is commonly called a "screening" experiment because it identifies which design factors are statistically significant to the response.

Given a set of *k* input variables (factors) to the overall design problem, a small set of designs is developed by linearly selecting two factor values over a significant range of each factor's value. The result is a set of *n* designs determined as Equation 2:

Equation 2: Required Number of DOE Designs

$$n = 2^k$$

These designs are then developed, and the designer can use statistical techniques to determine the individual and interactive effects each factor has with the overall design [JMP, 2002]. Thus the designer can determine a smaller set *m* of the *k* factors that have the most statistically significant impact on the ship design.

RESPONSE SURFACE METHODS

Response Surface Methods (RSM) focus on the *m* factors identified by the DOE screening experiment. Similar to DOE, RSM linearly varies the values of the *m* factors; however, at least three values of each are generally used: a threshold (minimum value), goal (maximum value), and middle (mean of threshold and goal values).

Next, point designs are developed to satisfy either the Box-Behnken or Central Composite models of the chosen design space represented by the extreme threshold and goal values. Examples of the two models discussed here are provided in Figure 10, representing a three factor design (*m*=3):

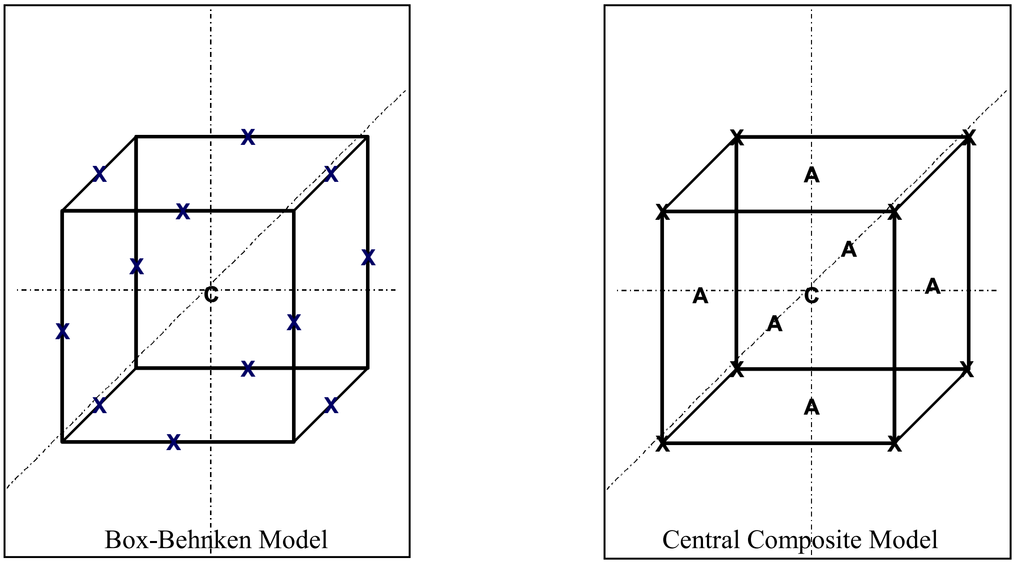

Figure 10: Three Variable Design Models

The boxes in Figure 10 represent the design space that it is believed a desirable solution lies in. Thus, the Box-Behnken model avoids point designs at corners of the design space because the designer believes that the corners do not represent feasible designs. The model is then populated with 13 point designs, 12 of which lie between corner points, and the last at the center of the design space. Conversely, the Central Composite model places point designs at the corners of the design space because the designer believes these represent feasible alternatives. This model incorporates 15-point designs: eight at the corners, 6 in the middle of the sides of the design space, and one at the center or the design space.

After the designs are developed and the appropriate model is populated, a statistical software package called "JMP" is used to develop the response surfaces [JMP, 2002]. The "response surface" is essentially a multi-dimensional surface fit to the model by JMP. The response surface is defined by a second order interpolation as shown in Equation 3:

Equation 3: Response Surface Equation

$$y = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{ii} x_i^2 + \sum_{i=1}^{k} \sum_{j=i+1}^{k} b_{ij} x_i x_j + \varepsilon$$

Where the $b_{0,i,ii,iii}$ terms represent constants of regression, ε represents error, and the summations represent linear, quadratic, and interaction terms respectively [JMP, 2002]. This equation defines the response surface, and if it is determined to have a statistically accurate fit, represents all feasible concept designs. Thus, JMP can develop an infinite number of variations by using the response surface equations to interpolate between the point designs.

At this point, JMP's graphical interfaces can be used to visualize the design space and assess all feasible design variants; therefore, JMP's response equations have the potential of creating a virtually infinite number of variations of m design variables. As stated earlier, the addition of DOE/RSM modeling to the concept exploration process frees the designer from the finite number of designs that have traditionally been used.

At this point, a naval architect can change one or all of the m design factors through simple manipulations in JMP's user interface or extract the equations into another application. For example, if one of the m factors is cost, an upper limit (threshold) cost value can be input to JMP, which then processes all of the equations and interpolates a boundary surface within the design space (one of the boxes described before). This allows the designer to begin to define the "feasible design space," essentially an area that constrains the investigation to a region that will produce a design that costs less than the cost threshold.

The goal and threshold values of the remaining m factors are input in a similar manner, further reducing the size of the feasible design space until all of the m factors have been included. This final feasible design space is generally a much smaller subset of the initial design space. By virtue of this process the systems designer knows that all of the concept designs inside those boundaries are feasible. The visualization and interpretation of the results of a response surface model will be discussed in Chapter 6.

CHAPTER 5: CONVENTIONAL SUBMARINE DESIGN CASE STUDY

OVERVIEW

Now that background information on the three primary areas of investigation of this research has been discussed, a case study that ties their application together can be developed and investigated. The subject of this case study will be a conventional (non-nuclear) submarine (SSK) design problem.

This discussion will begin by examining the role that mission analysis plays in requirements and effectiveness analysis. Then, MOMs for a SSK will be developed following the MOM Philosophy from Chapter 2. Following this, the results of the application of these MOMs to the case study will be presented in Chapter 6 using the MCDM Philosophy developed in Chapter 3 and the Response Surface tradeoff methodology from Chapter 4.

THE ROLE OF MISSION ANALYSIS

One of the primary conclusions from the discussion of MOMs is that a system should be evaluated as it relates to a supersystem. This requires the analysis of factors internal and external to system boundaries, which Soban and Mavris characterize as a system of systems approach that "is based on existing probabilistic methodologies that define [the system, and the] extrapolation of these methods to the theater level…redefining the system as the total warfighting environment" [Soban and Mavris, 2000a].

ASDL is developing a framework to facilitate such an analysis called the Probabilistic System of Systems Effectiveness Methodology (POSSEM), which provides a linked analysis environment that is fully probabilistic from the system to theater and campaign levels. Such a framework is well suited for RSM analysis "because there is a clear analysis path from the campaign code all the way back to the [DP level], [thus] transparency is enhanced and a proper assessment may be conducted" [Soban and Mavris, 2001].

To perform such an analysis, the system must be modeled in an operational context, creating the need for operational analysis. ASDL does exactly this in many of their papers; "the aircraft is sized according to the primary mission and subsequently 'flown' on the secondary mission to record the fallout performance" [Soban and Mavris, 2001]. Therefore, this analysis will do exactly the same.

The SSK in this design problem will be evaluated in two operational contexts. Its primary mission will be an area denial mission and its secondary mission will be a strike and special operations force insertion mission. These two missions are pictorially described in Figure 11:

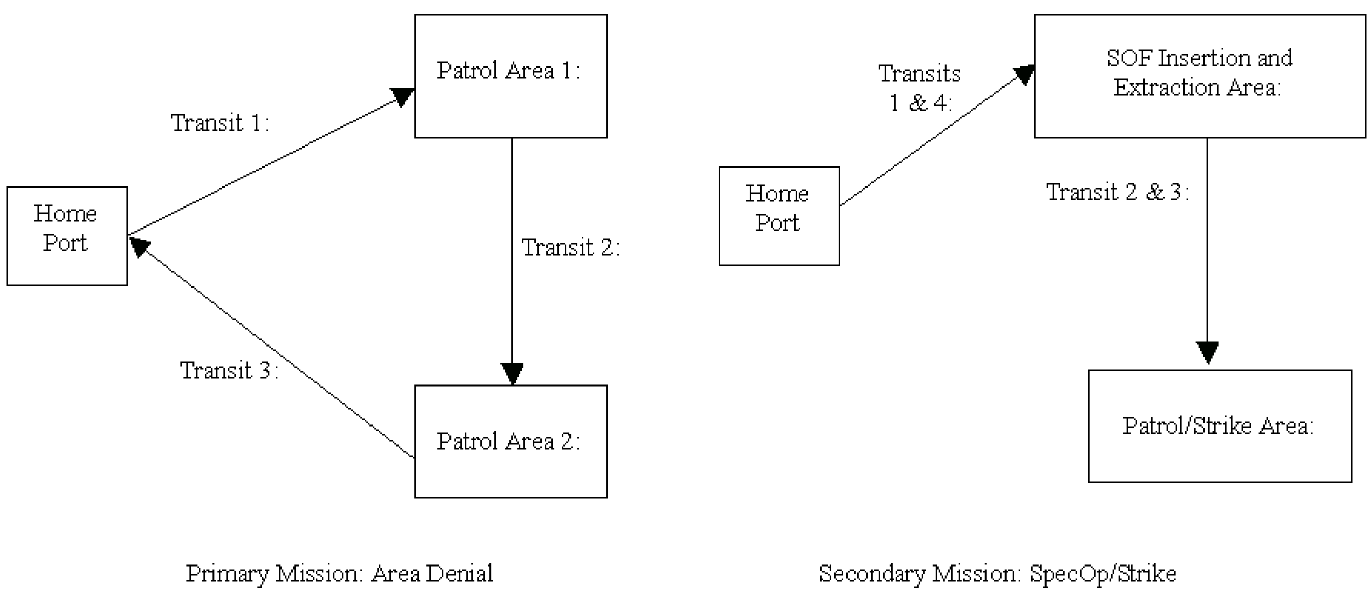

Figure 11: Notional Mission Scenarios

As mentioned in Chapter 1, it must be stressed that systems engineering and mission analysis based operational effectiveness should play a significant role in requirements derivation. As Shupp states, "mission analysis explores and exposes the boundaries of a system's behavior" [Shupp, 2003].

MEASURES OF MERIT DEVELOPMENT

With an understanding of the importance of MOMs and mission analysis in requirements derivation, specific MOMs can be developed for the SSK case study. Kowalski *et al* presents a simple, but very useful framework for doing so, called the 'Goal-Question-Metric' format as shown in Figure 12:

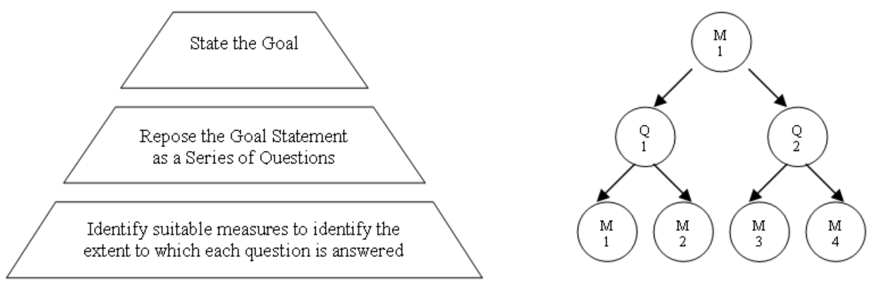

Figure 12: The Goal-Question-Metric Format [Kowalski *et al*, 1998]

Following a format such as this is consistent with the MOM Philosophy developed in Chapter 2. Also, since the goals will be related to a system requirement, it will satisfy Leite and Mensh's requirement that "all metrics must be traceable to requirements and all requirements must be associated with metrics" [Leite and Mensh, 1999].

The first step from Figure 12 is to 'State the Goal.' The ultimate goal of a system is to complete its mission, and in the case of the SSK, its two missions are defined above. Unfortunately, this will not suffice because this mission attainment must be quantified in some manner. To do this, the second step 'Repose the Goal Statement as a Series of Questions' is used. This leads to two primary questions:

1. What is the probability of the SSK avoiding detection?
2. How well can the SSK perform its primary and secondary missions?

These two questions can be answered by following step three: 'identify suitable measures to identify the extent to which each question is answered.' Five MOEs were identified to answer those questions as follows:

1. Probability of the SSK avoiding detection?
    a. SRS – Survivability of a Random Search
    b. STS-EB – Survivability of Suspected Target Search at the End of Burst
    c. STS-ES – Survivability of Suspected Target Search at the End of Search
2. SSK mission performance?
    a. MC-AD – Mission Capability - Area Denial
    b. MC-S – Mission Capability - Strike

Explicit probabilistic formulas can be found to answer the first question about detection by borrowing from the field of operations research. Unfortunately the second question is more difficult to quantify in probabilistic terms, but this will be addressed later.

The remainder of this section will provide brief discussions of each of the five MOEs; however, detailed derivations and explanations of their respective formulae are provided in Appendix 1. It is important to stress that these are rough order of magnitude estimates based off of simplified data. Many technical factors, ranging from environmental to design and operational, impact this analysis and are not being considered in order to simplify calculations. Values of constants in all of the formulae are provided at the beginning of Appendix 1.

SRS – Survivability of a Random Search

This metric must relate the patrolling SSK to a platform and sensor searching for it. Since this situation has a moving searcher seeking a moving target, a ‘perfect’ search, in which the target is stationary, should not be used. Therefore, the primary tool for conducting this analysis will be a “random” search. A “random” search is clearly not the best way to conduct a deliberate search; however, it is generally considered to be a good lower bound for detection probability, and “often provides accurate answers” [Washburn, 1996].

In this application of a random acoustic search, the sensor performing the search will be treated as a ‘cookie cutter;’ that is, the sensor will sweep out a path at a given speed and for a given time with a width of twice the range of the sensor. The range of the sensor is considered to be a “positive detection range,” so that if a target is outside the range it will not be detected, and if it comes within that range, it will be detected. For the purposes of this study, a “positive detection swath” (PDS) term is created, which is a weighted average of snorkel and Air Independent Propulsion (AIP) operation detection distances based on the submarine’s indiscretion rate. The formula for MOE SRS is shown as Equation 4:

Equation 4: Survivability of Random Search Equation

$$SRS = e^{\left(\frac{-24 \cdot N_S \cdot V \cdot PDS \cdot Patrol\_Duration}{A}\right)}$$

The MOPs and DPs that are used in this equation are defined in Appendix 1. The only DPs that feed into this MOE and will be examined in the RSM analysis are the submarine’s AIP endurance and balance speed.[3]

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[3] For an excellent, contemporary discussion of AIP technology see [Psallidas, 2003].

STS-EB – Survivability of Suspected Target Search at the End of Burst

This metric models a SSK fleeing a datum. It is based upon the assumption that the SSK has been detected by a distant searcher who has to dispatch an air asset to conduct the search for the SSK. The formula used is a random search formula; however, it has been altered to reflect the increase in search area over time as the sub flees the datum. The formula for MOE STS-EB is shown as Equation 5:

Equation 5: Survivability of Suspected Target Search at the End of Burst

$$STS - EB = e^{\left[\frac{-W \cdot V}{\pi \cdot V_{Max}^{2}} \cdot \left(\frac{1}{t_o} - \frac{1}{t_B}\right)\right]}$$

The MOPs and DPs that are used in this equation are defined in Appendix 1. The RSM factors are burst speed and burst endurance.

This formula includes the assumption that the searcher is not at the datum at the time of detection; therefore, the SSK has a head start on the searcher. This is called the 'time late.' It also assumes that the search stops at the time that the SSK ends its burst (high speed for escape situations with correspondingly low endurance)

STS-ES – Survivability of Suspected Target Search at the End of Search

This metric is very similar to STS-EB, except it is based on the more realistic assumption that the searcher searches longer than the submarine can burst. Therefore, it must include two speeds for the submarine, the burst speed and a slower evasion speed. The formula for MOE STS-ES is a modified version of Equation 5 as shown in Equation 6:

Equation 6: Survivability of Suspected Target Search at the End of Search

$$STS - ES = e^{\left[\frac{-W \cdot V}{\pi} \cdot \left[\frac{1}{V_{Max}^{2} \cdot t_o} - \frac{1}{V_{Max}^{2} \cdot t_B} + \frac{t - t_B}{V_{Max} \cdot t_B \cdot \left(V_{EES} \cdot t - V_{EES} \cdot t_B + V_{Max} \cdot t_B\right.}\right]\right]}$$

The MOPs and DPs that are used in this equation are defined in Appendix 1. The RSM factors in this equation are burst speed, burst endurance, and STS evasion endurance speed.

MC-AD – Mission Capability-Area Denial

This metric is not as easy to quantify in probabilistic terms as the detection avoidance metrics were; however, Whitcomb and McHugh developed a metric that models the effective area of influence of a SSK. This metric is based upon the SSK's range and the range, quantity, and mix of weapons it carries [Whitcomb and McHugh, 1999]. By adding the distance that the SSK can travel on AIP and battery to the range of the weapon, a radius is created. This radius is used to circumscribe a circle that represents the feasible area of influence of the SSK, which is multiplied by the number of weapons that it carries. The formula for MC-AD is presented as Equation 7:

Equation 7: Mission Capability-Area Denial

$$MC = \pi \cdot (\text{AIP\_Range} + \text{Bat\_Range} + \text{Torp\_Range})^2 \cdot \text{Number\_Torps} \cdot \text{Torp\_Mission\_Value} + \\ + \pi \cdot (\text{AIP\_Range} + \text{Bat\_Range} + \text{CM\_Range})^2 \cdot \text{Number\_CMs} \cdot \text{CM\_Mission\_Value}$$

The MOPs and DPs that are used in this equation are defined in Appendix 1. The RSM factors in this equation are the AIP endurance and balance speed, submerged endurance on battery, submerged battery loiter speed, and loadout package.

The loadout package represents the weapons that the SSK is carrying. It is assumed that a maximum loadout is 16 torpedoes, 16 torpedo-tube-launched land attack cruise missiles, or a mix of both that sums to 16 weapons. The packages available for tradeoff analysis are shown in Table 4:

Table 4: Summary of Loadout Packages

Loadout Package #	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
# of Cruise Missiles	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
# of Torpedoes	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Lastly, to show a measurable difference between each loadout in each mission scenario, the two weapons are given a 'mission value.' The analysis is built upon the assumption that torpedoes provide military utility to the area denial mission, and that the cruise missiles will have no military utility because they are for land attack, which is not part of the mission. Therefore, the SSK design is considered a single-role platform. In practical operation there would be utility to a mix; however, that is not necessary to demonstrate this methodology. Appendix 1 has more discussion of this scheme.

It should be noted that the numerical result of this MOE might not seem intuitively useful. This is true in absolute terms; however, MOEs can provide valuable information by illustrating relative assessments of effectiveness, as it does in this case.

MC-S – Mission Capability-Strike

This metric is exactly like MC-AD except it reverses the 'mission value' variable, crediting the cruise missiles with military utility, and removing it from the torpedoes.

SUMMARY OF MEASURES OF MERIT

Now that the five MOEs have been developed they need to be related to RSM and requirements analysis. In RSM terms, the factors are the input variables and the responses are the MOEs. Zink *et al* provides some guidance with respect to the nomenclature for requirements analysis, stating that [Zink *et al,* 2000]:

- **Requirements** are thresholds on performance…that must be satisfied.
- **Desirements** are metrics that are desired to be maximized or minimized to delineate between competing alternatives, which satisfy the requirements.

During the process of developing the MOEs, a conscious effort was made to restrict the factors chosen to DPs or MOPs that would serve as natural requirements in the design process, such as AIP and burst endurance, balance and burst speeds, as well as weapon mix. Therefore, to facilitate a RSM tradeoff analysis, these requirements were given a range of factor values from a threshold to a goal value. Correspondingly, the MOEs (RSM responses) are clearly seen as desirements to be maximized. Table 5 summarizes these characteristics of this analysis:

Table 5: Factors and Responses for RSM Analysis

	Requirements		
	Factors		
MOE	**MOP/DP**	**Threshold**	**Goal**
Survivability of Suspected Target Search	Burst Speed "V_{max}" (knots)	15	25
	STS Evasion Endurance Speed "V_{EES}" (knots)	1	4
	Time at Burst Speed "T_{burst}" (hrs)	0.5	2
Survivability of Random Search	AIP Balance Speed "$V_{balance}$" (knots)	2	8
	AIP Endurance "$T_{AIPendur}$" (days)	5	25
Mission Capability	Submerged Endurance on Battery "T_{batt}" (hours)	50	100
	Submerged Battery Loiter Speed "V_{loiter}" (knots)	2	6
	Loadout Package	0	16

Desirements
Responses
MOEs
Survivability of Suspected Target Search - End of Burst "STS-EB"
Survivability of Suspected Target Search - End of Search "STS-ES"
Survivability of Random Search "RS"
Mission Capability - Area Denial "MC-AD"
Mission Capability - Strike "MC-S"

Lastly, these five MOEs will be considered top-level MOEs for two primary reasons consistent with the MCDM Philosophy developed in Chapter 3:

- The rolling up of these MOEs will obscure valuable insight during the tradeoff visualization in the next chapter
- This limited example is intended to simply show the tradeoff methodology; therefore the complexity of a hierarchy of MOEs is not necessary.

CHAPTER 6: RESULTS

The previous five chapters have set the stage for performing a military effectiveness tradeoff analysis for naval ship design and acquisition. The need for such an analysis firmly grounded in the principles of systems engineering and requirements analysis was established. Then Measures of Merit and methods of Multi-Criteria Decision Making were discussed to create a rigorous framework for the analysis. Next, Response Surface Methods were introduced to facilitate the performance of the actual tradeoff studies. Finally, a notional case study for a conventional submarine was presented and top-level MOMs were derived.

Prior to describing the steps involved in performing the effectiveness and tradeoff analysis, a brief note on the technical nature of this study must be made. This analysis is first and foremost a warfighting analysis. As such, it is decoupled from engineering models that verify the feasibility of every concept it will generate. This is deemed acceptable because this research represents only one-third of the framework that provides such verification. This framework, that integrates an engineering model into the process, will be described in Chapter 7.

Therefore, this chapter will proceed to wrap effectiveness analysis, multi-criteria decision making, and response surface methods into one part of the advocated effectiveness and framework and methodology. This will be achieved by first calculating the top-level MOMs Then, the design space will be presented and sample tradeoffs will be made. Lastly, an uncertainty analysis will be performed on the design space and will be discussed.

IMPLEMENTATION OF EFFECTIVENESS ANALYSIS

Having defined the number of factors, their range, and the responses of interest in the previous chapter, the effectiveness analysis can be implemented. The first step in this process is to develop a factor matrix. JMP will perform this automatically, which saves a great deal of time with an eight-factor analysis.

After inputting the factor ranges into JMP a Central Composite design was chosen because its use of corner points allows the best coverage of the factor ranges of interest. JMP then created 145 variants from combinations of the goal, threshold, and middle values of the factors. This is not a full factorial design; however, in this application it is sufficiently large to populate the design space to achieve a statistically accurate model. A sample of the input factor matrix is provided in Table 6; the full table is included in Appendix 2.

Table 6: Sample of Input Factor Matrix

		Factors/Requirements							
Variant	Pattern	Burst Speed "Vmax" (knots)	Evasion Endurance Speed "VEES" (knots)	Time at Burst Speed "Tburst" (hrs)	AIP Balance Speed "Vbalance" (knots)	AIP Endurance "TAIPendur" (days)	Endurance on Battery "Tbatt" (hours)	Battery Loiter Speed "Vloiter" (knots)	Loadout Package
1	---+--++	15	1	0.5	8	5	50	6	16
2	+-+-+++-	25	1	2	2	25	100	6	0
3	++-+++--	25	4	0.5	8	25	100	2	0
4	++----+-	25	4	0.5	2	5	50	6	0
5	---++--+	15	1	0.5	8	25	50	2	16
46	-+-++-++	15	4	0.5	8	25	50	6	16
47	00000000	20	2.5	1.25	5	15	75	4	8
48	-+--++--	15	4	0.5	2	25	100	2	0
49	--+--++-	15	1	2	2	5	100	6	0
50	000000a0	20	2.5	1.25	5	15	75	2	8
141	-+---++-	15	4	0.5	2	5	100	6	0
142	+--+-+++	25	1	0.5	8	5	100	6	16
143	++++--+-	25	4	2	8	5	50	6	0
144	+--+++-+	25	1	0.5	8	25	100	2	16

The 'pattern' column describes the mix of factors for the variant, where a '-' or 'a' represents the threshold value, the '0' the middle value, and a '+' or 'A' the goal value. JMP automatically creates the table with each variant's corresponding factor values.

The resulting factor matrix was copied out of JMP and inserted into a spreadsheet that applied the top-level MOM formulas. The resulting MOM values were then copied from the spreadsheet and inserted into JMP to represent the response values. A sample of the response matrix is provided in Table 7; the full table is included in Appendix 2.

Table 7: Sample of Response Matrix

		Desirements/Responses				
Variant	Pattern	Survivability of Suspected Target Search - End of Burst "STS-EB"	Survivability of Suspected Target Search - End of Search "STS-ES"	Survivability of Random Search "SRS"	Mission Capability - Area Denial "MC-AD"	Mission Capability - Strike "MC-S"
1	---+--++	0.754	0.261	0.543	0.804	0.000
2	+-+-+++-	0.837	0.826	0.683	0.000	2.895
3	++-+++--	0.903	0.681	0.618	0.000	15.763
4	++----+-	0.903	0.681	0.661	0.000	0.653
5	---++--+	0.754	0.261	0.618	12.093	0.000
46	-+-++-++	0.754	0.411	0.618	13.100	0.000
47	00000000	0.775	0.719	0.641	1.114	1.832
48	-+--++--	0.754	0.411	0.683	0.000	2.011
49	--+--++-	0.609	0.589	0.661	0.000	1.042
50	000000a0	0.775	0.719	0.641	0.961	1.634
141	-+---++-	0.754	0.411	0.661	0.000	1.042
142	+--+-+++	0.903	0.591	0.543	1.231	0.000
143	++++--+-	0.837	0.827	0.543	0.000	1.739
144	+--+++-+	0.903	0.591	0.618	12.592	0.000
145	+-++-+-+	0.837	0.826	0.543	0.682	0.000

Now that the factor and response matrices have been created, JMP can perform multi-dimensional regressions on the data. The results of these regressions are the response surface equations for each top-level MOM.

JMP has a number of response surface exploration and visualization tools. The one that captures the broadest picture is called the 'Surface Plot,' which displays a three-dimensional plot of a MOM as a function of two variables. An example of these is included as Figure 13:

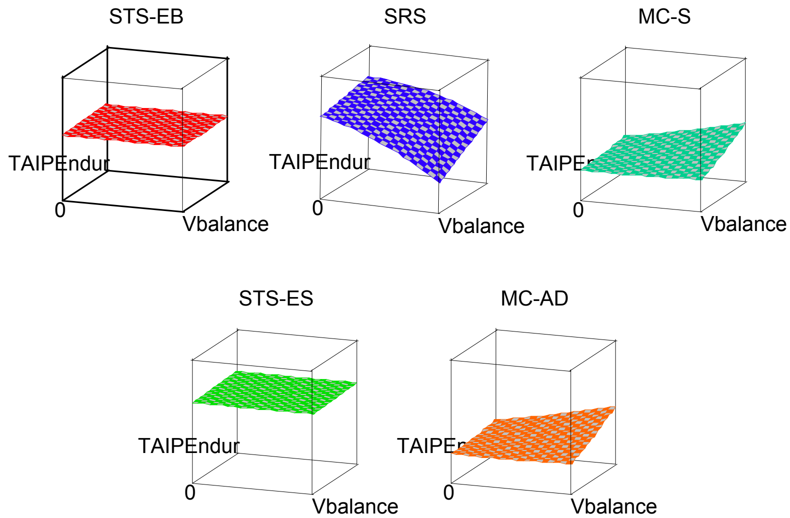

Figure 13: Examples of Individual Response Surfaces

These plots illustrate the response surfaces of all five MOMs as a function of AIP balance speed and endurance. From a brief look at these plots, it is clear that neither factor impacts STS-EB or STS-ES; however, they have a significant impact on SRS with less significant, but similar, impacts on MC-AD and MC-S. These plots will change if any of the other factors are varied; however, it is not possible to easily visualize higher dimensional problems. Fortunately, JMP has other plots that can accomplish this.

However, before studying the design space, the response surfaces must be found to be statistically accurate. JMP performs a number of tests to determine this, but the three best indicators are the R squared, mean, and F ratio values of the regression. The first two of these are found by using the Actual by Predicted plot of the response surface analysis. An example of this plot for STS-ES is provided as Figure 14:

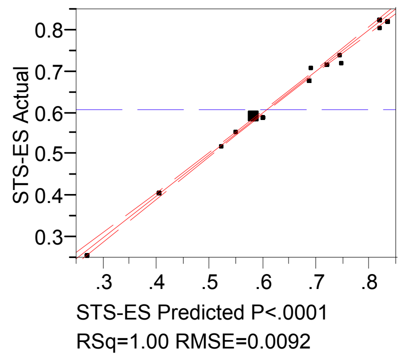

Figure 14: Actual by Predicted Plot for STS-ES

This plot shows how well the regression fits the data supplied in the response matrix. The R squared test represents "the proportion of the variation in the response that can be attributed to terms in the model rather than to random error" [JMP, 2002]. In this case, the R squared value is 1.00, which indicates a very accurate fit. Even though it is difficult to tell, all 145 variants are accounted for in the figure.

Another test involves the mean of the regression. The solid diagonal red line represents the regression, and the hashed diagonal red lines closely surrounding it represent the 95th percentile confidence region. The horizontal hashed blue line represents the mean of the regression. The fit is further confirmed to be statistically valid if the mean line is not enclosed in the 95th percentile confidence region.

The last test is the result of the F ratio, which can be found in the 'Analysis of Variance' output of JMP.

> The F ratio is a statistical tool to test the hypothesis that all coefficients in [Equation 3] are zero. If the hypothesis is not true, i.e. at least one coefficient is non-zero, then the F Ratio will be large. The "Prob > F"…is the probability of obtaining a greater F Ratio by chance alone if the specified model fits no better than the overall response mean. Significance probabilities of 0.05 or less are often considered evidence that there is at least one significant regression factor in the model. [JMP, 2002]

From the JMP output data in Appendix 3, it is clear that the response surface fits for the five top-level MOMs used in this analysis have adequate R squared, mean, and F ratio values to be considered statistically acceptable fits. Now that the response surfaces have been created and verified as statistically accurate, the design space can be explored to show potential design tradeoffs.

Design Space Analysis

Comprehending the visualization of a complete design space in JMP is not difficult, but it can be better understood by first examining the many responses that it represents individually. JMP creates a 'prediction profiler' that isolates the impact of every factor for every response as shown in Figure 15:

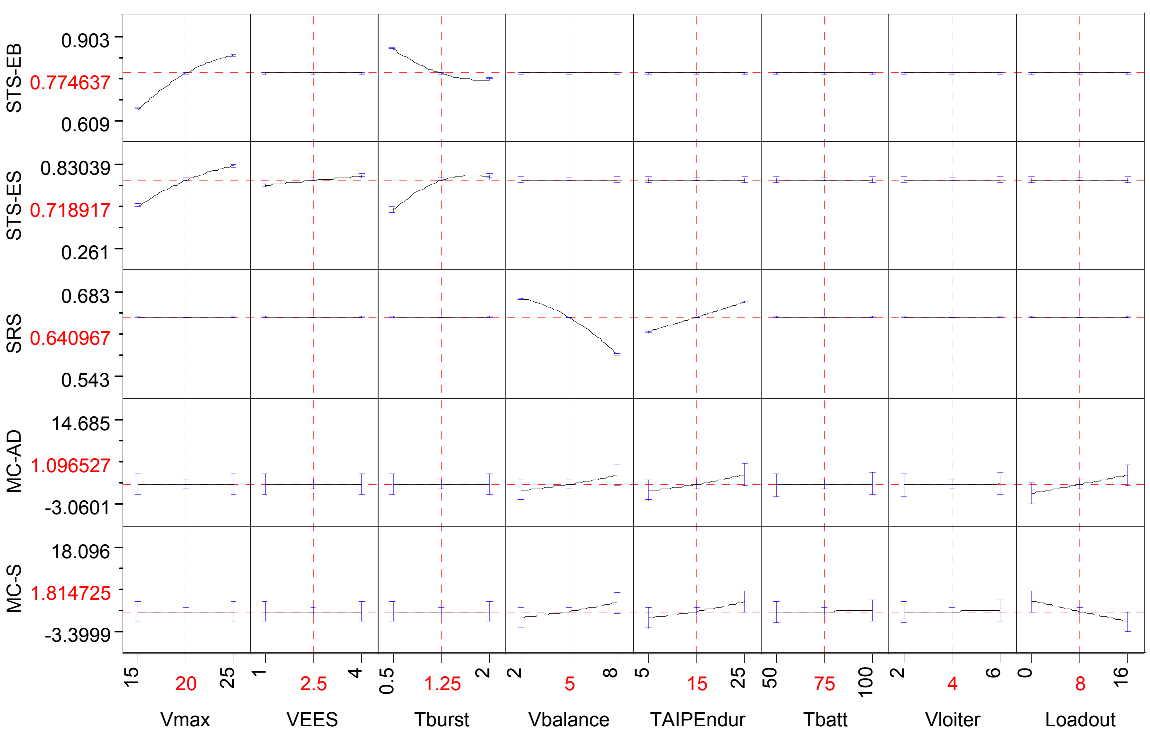

Figure 15: JMP Prediction Profiler for Top-Level MOMs

This interactive plot is not JMP's most elegant method of presenting information, but it is one of the most informative ones. The prediction profiler displays 'prediction traces' (predicted responses as one factor is changed while holding the others constant, represented by the black lines in each box) for each factor along the abscissa. As a factor is changed, JMP recalculates the prediction traces to show the impact of the change on the responses.

A flat line, near zero slope, indicates that a particular factor has no interaction with a certain response. This is expected in many of these cases because of the way the MOM formulas were created, for instance, the only factors in the SRS equation are balance speed[4] and endurance, therefore it is logical that the other six factors would not impact the SRS response. The threshold and goal values of each factor are reflected as the extreme values in each box, and the current value is displayed between them in red. The same applies for the responses on the ordinate, except their extremes have been calculated by JMP. By moving the red, hashed crosshair along any prediction trace, corresponding changes in factors and responses can be seen.

The capability to manipulate the factors in this manner can illustrate the relationships between each variable to allow a better understanding of what factors truly drive the responses. For instance, the inverse role that burst endurance plays in STS-EB and STS-ES is not intuitively clear at first glance, but it is accurate. The random search equation is exponential in character, and the datum search version of it includes an area factor that increases with time.

The inverse relationship of STS-EB and burst endurance is a factor of the time-late[5] and is partly an artifact of the simplicity of the analysis. For an example, take the extreme case of the burst lasting for the duration of the time-late. Following the assumption that the search ends

[4] The balance speed is "the speed at which the maximum AIP power is equal to the submarine power requirements for hotel load and propulsion" [Psallidas, 2003].
[5] The time-late is the delay between the detection of the SSK by a surface ship, and the arrival of an air asset to locate the SSK.

when the burst ends, the searcher never even gets to start looking for the SSK. This is not realistic, but explains the behavior of the equation. As burst endurance increases, the search rate of the searcher overtakes the area created by the time-late decreasing the impact of burst endurance.

A more realistic example is seen by the reverse trend in STS-ES because, following the burst, the SSK is operating at its slow, evasion endurance speed, adding much less area to the search as time goes by. Since the total search time is constant, burst endurance determines the amount of time that the searcher (who's search rate is constant) has to search while the SSK is at the much slower speed. This increases the probability that the searcher has of detecting the target. Therefore, in the STS-ES case, survivability is driven by the burst speed and endurance.

This creates an interesting case of competing demands that requires a compromise solution. JMP can provide one solution by utilizing desirability functions, which are functions that tell JMP which responses to maximize and minimize. These functions can be seen in the column that has been added to the right side of Figure 16:

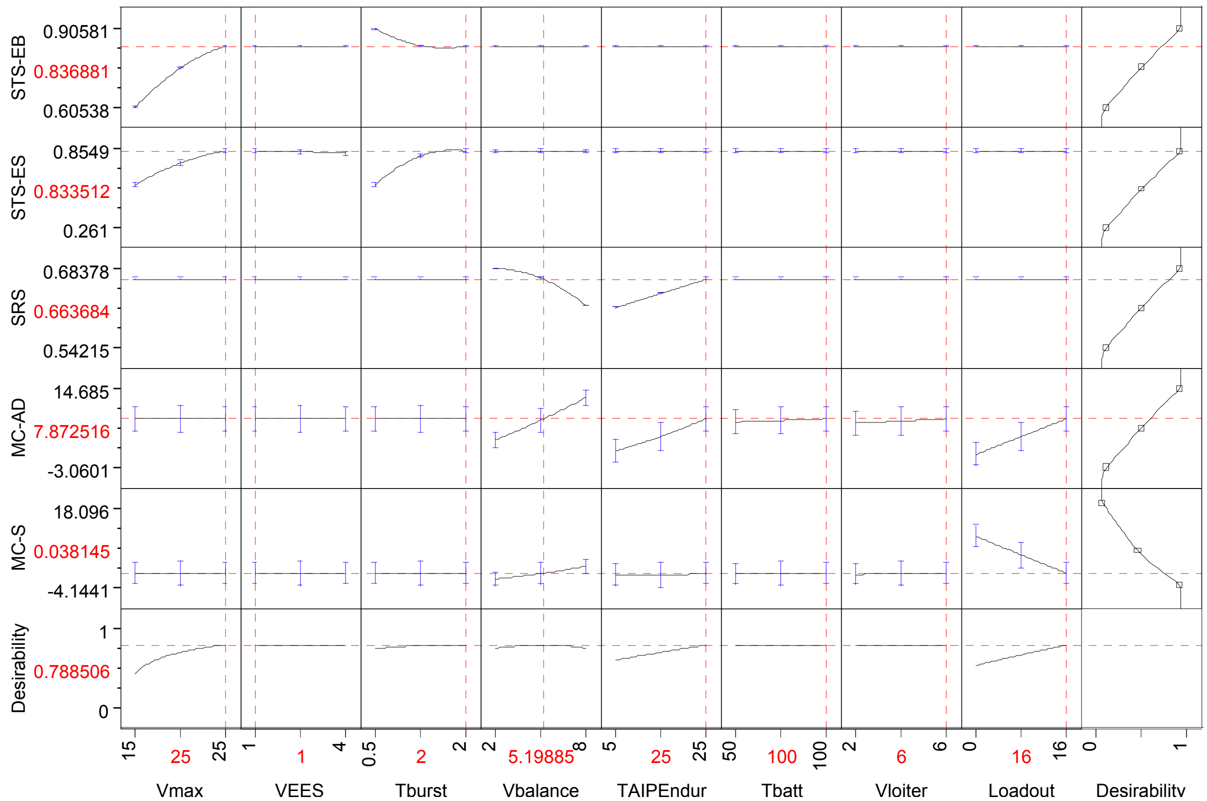

Figure 16: Desirability Functions and Maximized/Minimized Responses

This example shows a desirability analysis for the area denial mission. Therefore all of the search MOMs as well as the MC-AD MOM have high desirability and the MC-S has low desirability. JMP defaults to a linear desirability scale, as seen in all of the responses. In this situation, JMP is attempting to maximize all of the desirabilities, resulting in a compromise situation. Modifying the desirability curve for each response to emphasize or de-emphasize any particular MOMs takes little effort on the part of the user or JMP.

Considering these two examples, it is clear that the prediction profiler is a very powerful tool for an analyst or designer, but may provide too cluttered of a picture for use by decision makers. Fortunately, JMP has another graphical interface that presents the actual response surfaces and is very suitable for use in tradeoff discussions with decision makers.

The contour plot is a visualization tool in JMP that can simultaneously show the response surfaces with respect to two competing factors. For instance, from the prediction profiler it is clear that evasion endurance speed does not have a major impact on either STS-EB or STS-ES. Therefore, tradeoffs between these two responses should focus on burst endurance and speed.

With the aid of contour plots, contours of values of each MOM can be seen in relation to their factors, similar to a topographic map. Figure 17 shows incremental contours of STS-EB and STS-ES:

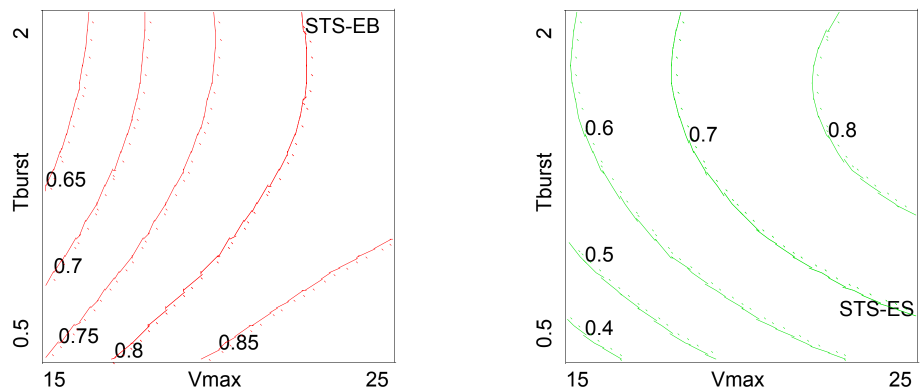

Figure 17: Contour Plot of STS-EB (left) and STS-ES (right)

These contours represent feasible and infeasible regions with respect to the two variables. The feasible side is the side of each solid line with the dots. To gain further insight, these contours can be plotted simultaneously as shown in Figure 18:

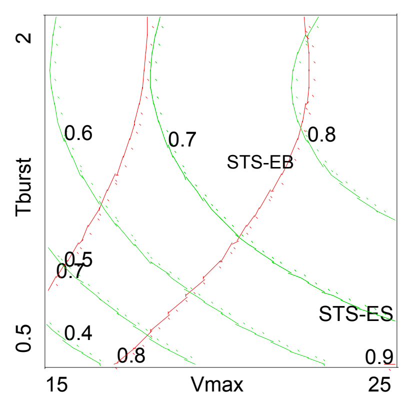

Figure 18: Simultaneous Plot of Contours of STS-EB and STS-ES

To better represent an analysis where requirements are being discussed, regions of the contour plot can be excluded from the design space by setting low and high limits of acceptability for the responses. For instance, if the threshold value of STS-ES is 0.6 and its goal is 0.8, and STS-EB's threshold is 0.7 and goal is 0.8, the resulting contour plots are Figure 19:

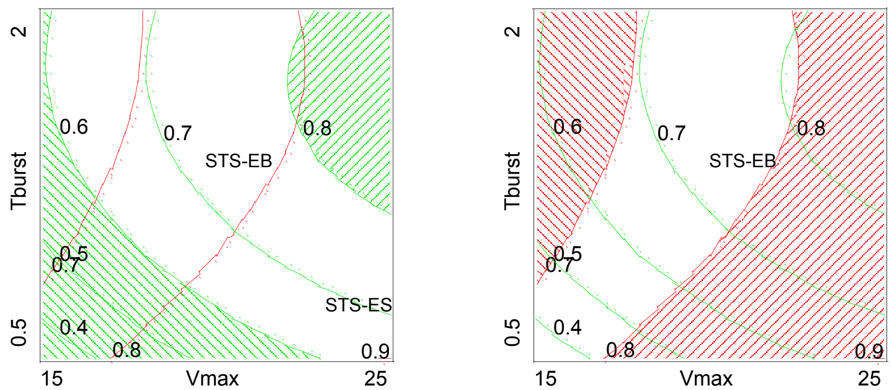

Figure 19: Example Threshold and Goal Limits on Contour Plots

The feasible design space in each of these is the white region that is not shaded. If these two requirements were imposed simultaneously, the plots could be laid on top of each other as shown in Figure 20:

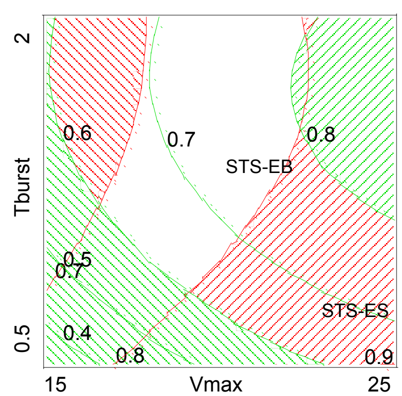

Figure 20: Compromise Design Space

The contour plot now shows the feasible region that is a compromise of these two competing MOMS.

Visualization such as this is possible because of the multi-dimensionality of RSM, which JMP captures. As mentioned earlier, Figure 13 depicted the response surfaces of all five MOMs as a function of AIP balance speed and endurance. This is one of the primary tradeoffs that should be considered in SSK design; therefore, some contour plots will be produced to discover some relationships.

To analyze the area denial mission, it is clear from the prediction profiler that the only MOMs of interest that are driven by these two factors are SRS and MC-AD. From the prediction profiler, and based on the fact that the mission under consideration is the area denial mission, it is clear that the loadout package should be all torpedoes.

Now, the design space for this scenario can be visualized, starting with Figure 21:

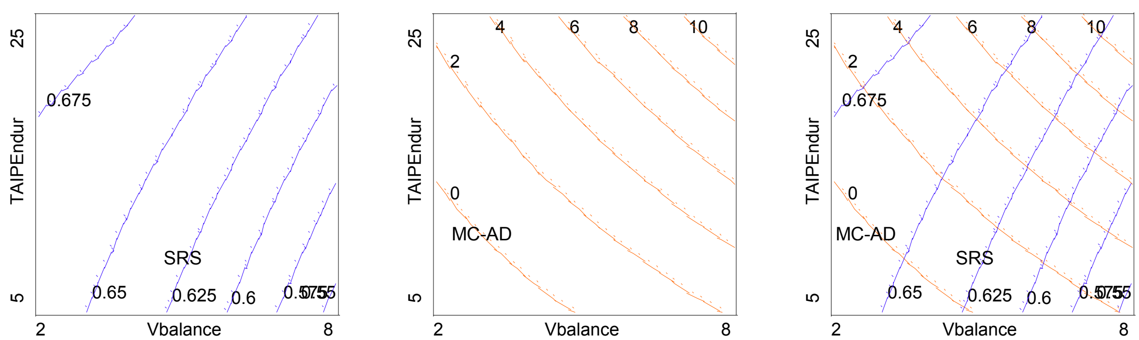

Figure 21: Contours for SRS (left), MC-AD (middle), and both together (right)

Again, placing thresholds on the MOMs will begin to define a requirements space. For instance, if a threshold of SRS=0.675 and MC-AD=6.0 is used, Figure 22 represents the design space:

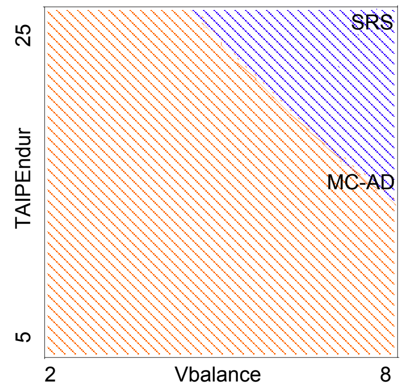

Figure 22: Contour Plot for Thresholds: SRS=0.6 and MC-AD=6.0

Unfortunately, this contour plot does not have an open area; therefore, there is no feasible design space because the two thresholds are mutually exclusive. However, if the SRS threshold was decreased to SRS=0.6, the design space opens up to show the feasible region in Figure 23:

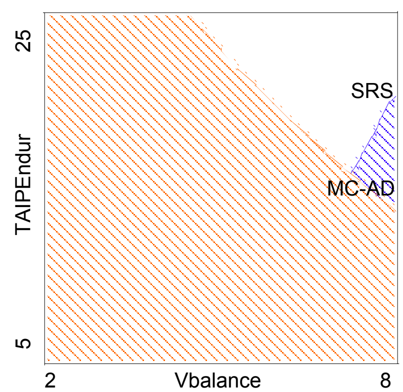

Figure 23: Feasible Space for Thresholds: SRS=0.6 and MC-AD=6.0

Now that the design space has been identified, an interactive decision making process can begin. Groups of decision makers can explore the boundaries and interiors of design spaces with the ease of moving a cursor and a few slider bars in the JMP interface to continue to create contour plots to perform tradeoffs. In the process of doing this, decision makers can begin to understand the constraints that mutually conflicting attributes place on the military effectiveness of the system. Thus, an evaluation of technologically grounded alternatives is easily integrated into a requirements analysis to create a requirements space.

APPLICATION OF UNCERTAINTY ANALYSIS

As discussed in Chapter 3, a key factor in decision making is uncertainty. Two methods for introducing uncertainty into the decision making process were identified and discussed: Monte Carlo simulation and Real Options. Due to the relative immaturity and difficulty of a real options approach and the fact that the response surface equations created by the previously discussed JMP analysis are readily applicable to a Monte Carlo analysis, a Monte Carlo simulation will be discussed in this section.

The five response surface equations from JMP are functions of the eight input factors. JMP stores the constants of regression for these equations by Equation 3's individual terms (intercept, linear, quadric and interaction), and then sums them to determine the response. This data is extracted as 'Parameter Estimates' via a data table for each response modeled in JMP. These values can be easily integrated into a spreadsheet that can calculate all five MOM responses. One important note about this process is that the response surface equations do not use the actual factor values. They must be scaled between their threshold and goal values to fit a –1 (threshold) to +1 (goal) scale.

Once the response equations have been modeled in a spreadsheet, a program called Crystal Ball is used to perform the Monte Carlo simulation[6] [Crystal Ball, 2000]. The user must then select a probability distribution to model each factor, choosing the distribution shape, extreme values, and most likely value. Then, Crystal Ball performs simulations (5000 was

[6] The use of Crystal Ball and application of Monte Carlo simulation will only be described in general terms in this discussion. For a detailed discussion of this process, consult [Psallidas, 2003].

chosen for this case) with values randomly selected at a frequency that will simulate the probability distribution well.

While the program is doing this, the response surface equations simultaneously calculate their values based on the randomly picked factors, and the resulting responses are compiled by Crystal Ball. Once all of the simulations have been run, the program reports frequency distributions, cumulative plots, reverse cumulative plots, and statistical information on each of the responses.

A Monte Carlo simulation was performed on the response surface equations for the five top-level MOMs developed for this analysis, using the probability distributions on the eight input factors as shown in Table 8:

Table 8: Monte Carlo Factor Distribution Information

Factor	Threshold	Goal	Likeliest	Distribution
Burst Speed "V_{max}" (knots)	15	25	20	Triangle
STS Evasion Endurance Speed "V_{EES}" (knots)	1	4	2.5	Triangle
Time at Burst Speed "T_{burst}" (hrs)	0.5	2	1	Triangle
AIP Balance Speed "$V_{balance}$" (knots)	2	8	5	Triangle
AIP Endurance "$T_{AIPendur}$" (days)	5	25	15	Triangle
Submerged Endurance on Battery "T_{batt}" (hours)	50	100	80	Triangle
Submerged Battery Loiter Speed "V_{loiter}" (knots)	2	6	4	Triangle
Loadout Package	0	16	16	Triangle

The results of most interest from an analysis such as this are the reverse cumulative charts, which show the probability distribution of forecasted MOM values based on the predicted probability distributions placed upon their respective factors.

For instance, based upon the assumed distributions of burst speed, burst endurance, and evasion endurance speed, STS-ES has a 100% probability of a MOM value of 0.48 and a virtually 0% probability of achieving a MOM value of 0.83. The reverse cumulative shows the middle ground between these extremes in Figure 24:

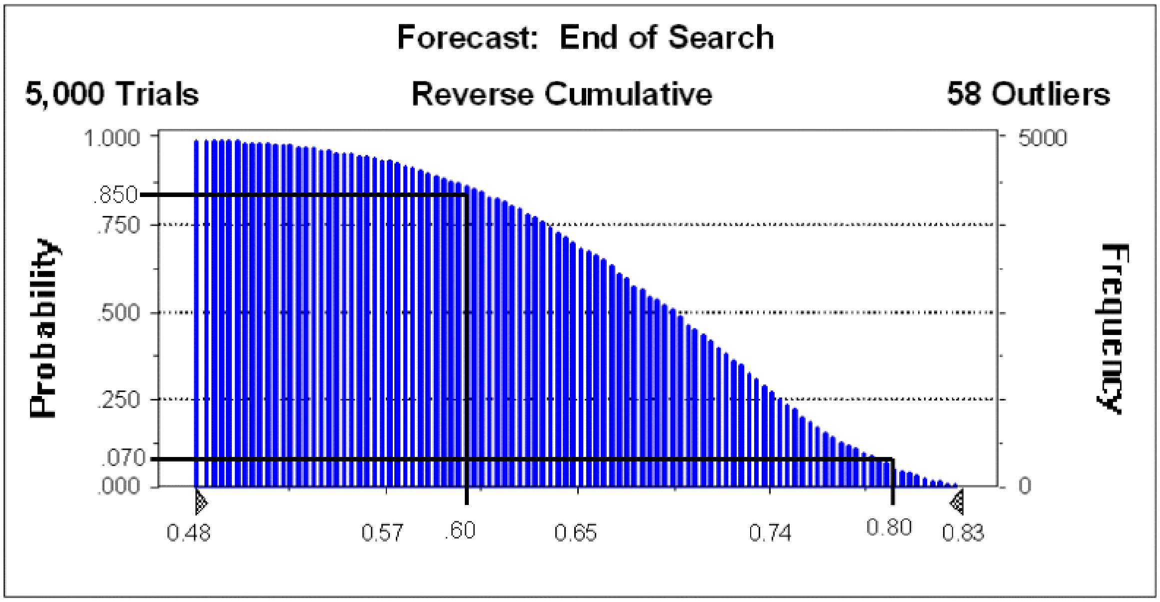

Figure 24: Reverse Cumulative Distribution of STS-ES

For instance, if the threshold value of STS-ES is 0.6 and its goal is 0.8 as in Figure 20, then the probabilities of achieving the threshold value is 85% and the goal value is 7%.

While the addition of uncertainty analyses may make the consideration of multiple criteria more difficult, it allows the decision makers to make a more informed decision. The results of the Monte Carlo analysis for all five top-level MOMs are included in Appendix 4, along with a description of extracting the response surface equations from JMP.

CHAPTER 7: APPLICATIONS FOR IMPLEMENTATION

The case study created in Chapter 5 and examined in Chapter 6 illustrated a versatile, decision maker-friendly methodology for exploring the impact of design requirements on the effectiveness of a SSK. With that case study in mind, a sophisticated framework for the implementation of an expanded version of the analysis will be discussed.

UNIFIED TRADEOFF ENVIRONMENT

Prior to describing an improved framework for the supersystem, the system must be revisited. As mentioned earlier, this analysis did not involve an engineering model to validate the variants that were developed. Further, the methodology did not integrate any consideration of the impact of future advances in technological capability, such as improved propulsion systems.

These two oversights were intentional for this analysis, but are essential for achieving a balanced understanding of, and design for, the system under consideration. To do so, the response surface methodology must incorporate three groups of factors: concept design variables, requirements, and technology K-factors. The first two are intuitively clear, but the technology K-factor is less clear. This K-factor is a factor that is inserted into the engineering model to represent a predicted notional degradation or improvement to various technologies based on future research and development. By introducing these factors, the analysis integrates the impact of future advances in technological capability.

The simultaneous combination of the design variables, requirements, and K-factors creates what the ASDL terms the 'Unified Tradeoff Environment' (UTE). A convenient way to visualize the UTE is to place three prediction profilers side by side, as in Figure 25:

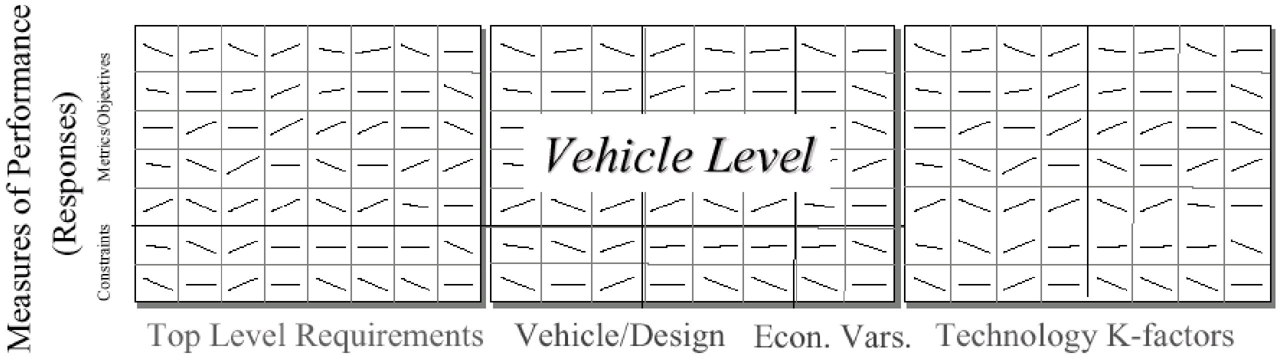

Figure 25: The Unified Tradeoff Environment [Soban and Mavris, 2000a]

Examination of the design problem in this manner allows the simultaneous consideration of the effects of each of the three factor sets on system constraints and objective responses.

Mavris and DeLaurentis provide an overview of how the UTE is developed. First, a baseline set of each of the factors is determined. Then, the requirements space is developed with the design variables and K-factors held constant at their baseline. Likewise, when the design variable space is developed, requirements and K-factors are held at baseline, and a similar method is used when developing the K-factor space. This results in three sets of response surface equations that can be manipulated as follows:

> The three sets of regression equations are then aggregated into an overall expression for changes in desirements as a function of requirements, design/economic variables, and technology improvements….For the purposes of visibility and creation of decision-support tools, it is assumed that the three sets of RSE inputs are independent (and thus un-correlated) from each other. Thus, their contributions are considered to be additive. However, subsequent confirmation testing is employed to check the validity of this assumption. If some variables are dependent, one possible solution is to identify mixes of design variables, requirements, and technology factors that are independent and then create three "mixed" sets of RSEs. [Mavris and DeLaurentis, 2000].

Another example of the flexibility and application of the UTE equations is demonstrated by the following statement:

> equation sets can be interchanged and subsequently fed to a non-linear, simultaneous equation solver to determine if solutions exist in the aspiration

> space….For example, one could fix the requirements and conduct a search over evolutionary technologies and design variables to achieve the goals. Alternatively, the design variables can be fixed while the search is over the requirements and technology levels. [Mavris and DeLaurentis, 2000].

The characterization of the design, requirements, and technology spaces into a Unified Tradeoff Environment introduces a much more rigorous analysis into the traditional design process.

Further, the UTE can play an important role in the process of requirements tradeoff and definition "where the requirements study can be used to determine which specific point in a requirements space the system is to fall. This can be performed using Integrated Product and Process Development" methods [Hollingsworth and Mavris, 2000], grounded in a sound group decision making strategy developed from the MCDM Philosophy developed in Chapter 3.

EXPANDED EFFECTIVENESS ANALYSIS

The creation of the UTE will play a key role in the development of an expanded effectiveness analysis because it brings more information to the analysis process. The effectiveness models developed for this study are extremely crude and elementary ones. They focused primarily on the single platform under consideration, but did make the necessary steps to fully place the SSK into an operational context. As discussed in the MOM Philosophy, this is a key factor for a proper effectiveness analysis.

Unfortunately, the models used examined operational circumstances in an independent manner: a long-term search, a datum search, and mission capability. In reality, these are not independent, and there are many more considerations. For the method developed so far, practical application is key. To do so, the response surface methods must be linked to a more mature effectiveness analysis hierarchy.

As mentioned earlier, the ASDL is developing a framework to facilitate such an analysis called the Probabilistic System of Systems Effectiveness Methodology (POSSEM), which provides a linked analysis environment that is fully probabilistic from the system to theater and campaign levels. Such a framework is well suited for RSM analysis "because there is a clear analysis path from the campaign code all the way back to the [DP level], transparency is enhanced and a proper assessment may be conducted" [Soban and Mavris, 2001].

An integral part of this expansion of the effectiveness analysis is the use of a mature mission and campaign level analysis program. The ASDL has partnered with Johns Hopkins to use their Integrated Theater Engagement Model (ITEM) to conduct aircraft effectiveness assessments. An example of the use of ITEM in this process is provided as Figure 26:

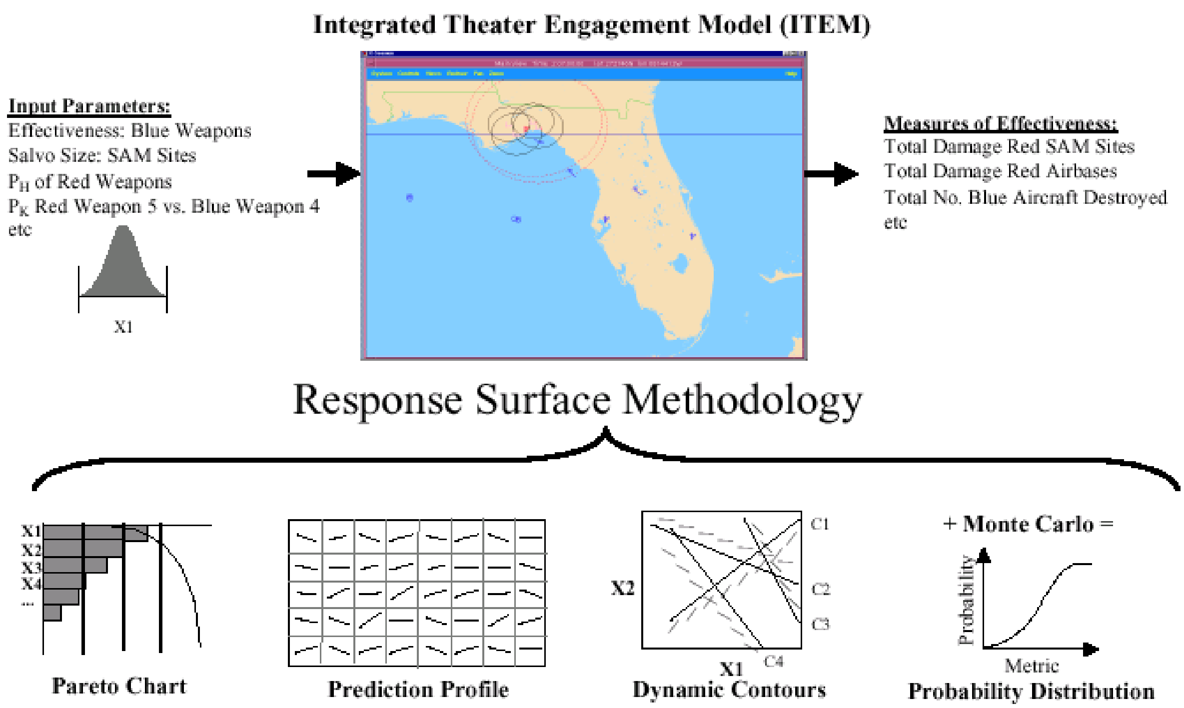

Figure 26: Integration of Campaign Effectiveness Analysis Code [Soban and Mavris, 2001]

By integrating the ITEM program with the response surface methodology, the ASDL was able to map system level MOPs to mission level MOEs. This integrates the use of prediction and contour profilers with uncertainty analysis and allows for responses to be developed from mature models.

This approach requires the analysis of factors internal and external to system boundaries, creating a system of systems approach that "is based on existing probabilistic methodologies that define the aircraft as the system…[and the] extrapolation of these methods to the theater level…redefining the system as the total warfighting environment" [Soban and Mavris, 2000a]. Thus, a virtual response surface hierarchy can be created as shown in Figure 27:

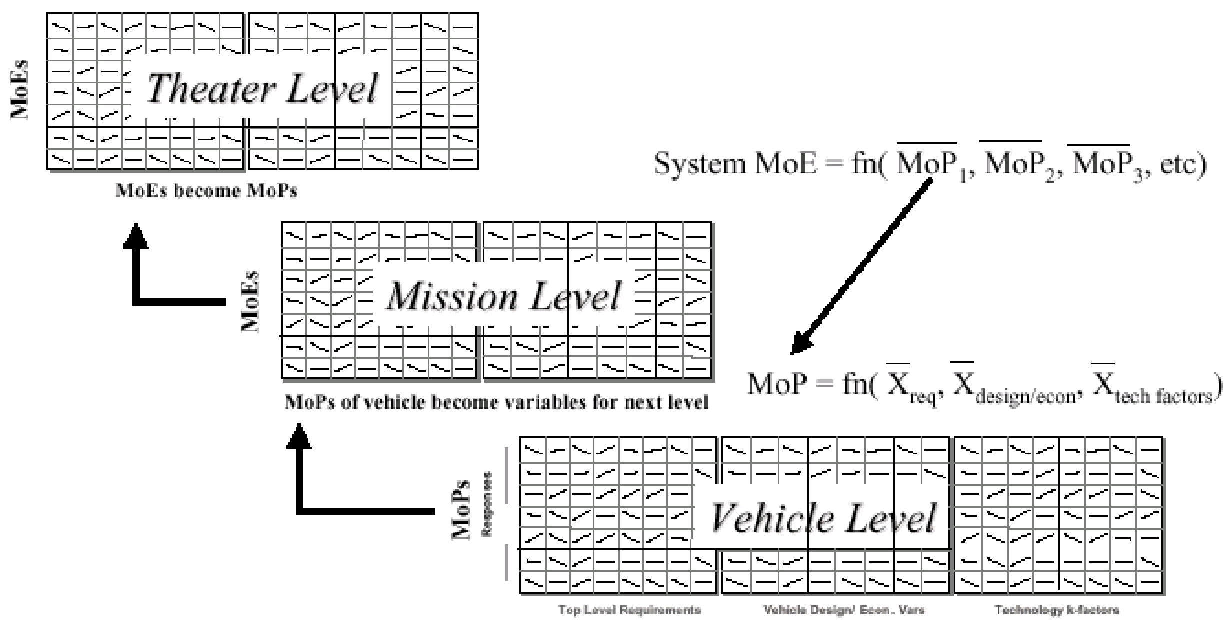

Figure 27: System of Systems Approach [Soban and Mavris, 2000a]

By using the probabilistic System of Systems approach grounded on a solid MOM and MCDM Philosophy, better systems can be designed. Instead of designing the system "to its own pre-defined performance and mission constraints, [it] can now be optimized to fulfill theater level goals and objectives" [Soban and Mavris, 2000a].

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CHAPTER 8: CONCLUSIONS

SUMMARY

This research has set the stage for performing a military effectiveness tradeoff analysis for naval ship design and acquisition. The need for such an analysis firmly grounded in the principles of systems engineering and requirements was established. The Unified Tradeoff Environment framework and effectiveness tradeoff methodology advocated by this research facilitates an informed negotiation of requirements, desirements, and design parameters by decision makers. This process allows vehicle design and mission requirements, "when optimized to maximize the overall effectiveness of the system, [to] become the requirements to which the vehicles are then designed."[Soban and Mavris, 2000a]

This represents a profound improvement over traditional, ad hoc tradeoff methodologies, which rely on a limited number of point designs and data. The design space meta model visualized in JMP provides a continuous, interactive design space examination tool that can be used in real time by decision makers to explore and negotiate the "simultaneous impact of requirements, product design variables, and emerging technologies during the concept formulation and development stages"[Zink *et al*, 2000] to reach compromise design solutions.

In performing this research, two significant philosophies were developed to guide the development of Measures of Merit (MOM) and facilitate rational, Multi-Criteria Decision Making (MCDM). The MOM philosophy is summarized as follows:

1. The definitions and hierarchy of MOMs (from most system specific to least) are as follows:
    a. DPs are physical characteristics that drive system behavior.

    b. MOPs are non-probabilistic measures of specific configurations of DPs, calculated from DPs.
    c. MOEs are preferably probabilistic measures of the operational performance of the system, calculated from MOPs. The system boundary generally separates MOEs from MOPs.
    d. MOMs will be used as a phrase to refer to MOPs and MOEs in general.
2. Cost should be excluded from the effectiveness analysis but must not be excluded from the complete design tradeoff analysis.
3. MOMs should be as quantitative and probabilistic as possible.
4. MOMs should be developed as follows:
    a. Define high-level properties (DPs) through a qualitative, top-down approach.
    b. Outline MOPs by first identifying DPs that characterize identified high-level properties.
    c. Develop MOEs as metrics to judge system performance against user requirements.
5. Normalization and ratio schemes should not be used.

Applying this MOM Philosophy to an effectiveness analysis provides a logical method to organize an analysis and ensures the traceability of synthesis model design parameters to MOMs. This MOM Philosophy is complemented by the following, corresponding MCDM Philosophy:

1. MCDM and MOM hierarchies should be identical.
2. Subjective judgments should be minimized and involve extensive dialogue between the technologists and decision makers.
3. Weighting schemes should be avoided when used with top-level MOMs. However, weighting methods for rolling-up lower level MOMs can be used when applied with AHP and Pareto analysis.
4. Decisions are often made in surprisingly irrational manners; thus, every effort should be made to make the MCDM mythology as independent of subjectivity as possible. Therefore, when performing trades of top-level MOMs, interactive decision making methods such as Response Surfaces must be used to visualize and perform these tradeoffs.
5. Uncertainty analysis should be performed.

Applying a coherent MCDM Philosophy such as this, with an organized effectiveness analysis, provides the decision maker with valuable information.

By integrating this information into a Unified Tradeoff Environment whose visualization is facilitated by Response Surface Methods, requirements and effectiveness analysis is much more efficiently coupled with design and technology insertion analysis. Further, as Frits *et al* notes, "instead of giving fixed performance requirements to the weapon designer, it is desirable to step back a level, giving the designer a [MOE] requirement and access to an [effectiveness] model. This new process allow[s] for more design freedom and flexibility in the development of future…systems. Including these…parameters opens up the design space, creating additional options in the decision-maker's quest to design a reliable, yet effective, weapon at low cost" [Frits *et al,* 2002].

RECOMMENDATIONS FOR FUTURE WORK

To further improve the MCDM Philosopy, further research into meta model methods of interaction and negotiation for design, effectiveness, and requirements tradeoffs should be explored. This should be conducted as an investigation to determine the state of the art of such methods in both naval and non-naval industries and organizations. Application of more mature methods to allow real-time 'what if' excursions will further facilitate informed decision processes and more effective designs.

Secondly, significant improvement in the effectiveness analysis can be achieved by integrating more mature warfighting simulation and evaluation codes. Implementation of such codes creates a need for a time dependent version of response surfaces analysis as described by Soban and Mavris:

> Instead of the response being a function of a set of variables, the response would be a function of a vector of variables. Each vector would represent the set of decisions that could be made at each decision node. Another advantage of this formulation is that probability distributions could be applied to each possible path at each node. In this way, the human decision maker can be modeled. [Soban and Mavris, 2001]

Finaly, this research provided only one-third of the total UTE framework. The next significant step will be to integrate the work from this thesis with technology insertion analysis as done by [Psallidas, 2003] and design analysis similar to [Goggins, 2001].

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APPENDICES

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APPENDIX 1:

MOM DESCRIPTIONS

Given Values for MOM Constants

MOE Type	Constant	Constant Value
STS	PDS STS (nm)	1
	Speed of Searcher (kts)	100
	Time Late (hrs)	0.25
	Total Search Time (hrs)	3
RS	*Towed Array Detection Distances (nm)*	
	Operating Condition	*Deep Detection Distance (nm)*
	Snorkel	18.6
	Battery	2.7
	Number of Searching Ships	3
	Speed of Searching Ships (kts)	10
	Patrol Duration (days)	45
	Search Area (nm^2)	500000
MC	Torpedo Range (nm)	5
	Cruise Missile Range (nm)	600
	Torp_Mission_Value - Area Denial	1
	CM_Mission_Value - Area Denial	0
	Torp_Mission_Value - Strike	0
	CM_Mission_Value - Strike	1

Towed Array Detection Distances estimated from Miasnikov's work:
Miasnikov, Eugene, "Can Russian Strategic Submarines Survive at Sea? The Fundamental Limits of Passive Acoustics," Science and Global Security, Volume 4, 1994.

Derivation of Formula for MOE "SRS"

SRS is the SSK's expected Survivability of a Random Search

Factors:

AIP_Endur - AIP Endurance (days)

$V_{balance}$ - AIP Balance Speed (kts)

Response:

SRS

Givens:

N_s - Number of Searchers

V - Speed of Searchers (kts)

A - Total Search Area(nm^2)

$TADD_{Snorkel}$ - Towed Array Detection Distance on Snorkel (nm)

$TADD_{Battery}$ - Towed Array Detection Distance on Battery (nm)

IR_{AIP} - AIP Indiscretion Rate

Patrol_Duration (days)

First, Indiscretion Rates are determined. This is simple in the case of AIP, since it is zero. In the case of traditional diesel electric operation, it is not difficult either. As a reference point, Stenard's thesis was used to develop a IR versus speed curve for a typical SSK. Regression of this data resulted in the following formula:

$$IR_{Battery\_Snorkel} = 0.0004 \cdot V_{balance}^{3} - 0.0038 \cdot V_{balance}^{2} + 0.0224 \cdot V_{balance} + 0.0018$$

Due to the fact that the AIP system cannot run for the entire patrol, a simple composite IR is developed:

$$IR_{Composite} = IR_{AIP} \cdot \frac{AIP\_Endur}{Patrol\_Duration} + IR_{Battery\_Snorkel} \frac{Remaining\_Patrol\_Endurance}{Patrol\_Duration}$$

Where:

$$Remaining\_Patrol\_Endurance = Patrol\_Duration - AIP\_Endur$$

Now that a formula for IR has been developed, data from Miasnikov for notional towed array de water detection (TADD) distances are used to find a notional detection distance for this analysis

$$DD\_Deep = IR_{Composite} \cdot TADD_{Snorkel} + \left(1 - IR_{Composite}\right) \cdot TADD_{Battery}$$

To simplify the random search equation and avoid the use of probability distributions, a Positive Detection Swath is used, where:

$$PDS = 2 \cdot DD\_Deep$$

Which can be substituted directly into Washburn's Random Search Equation:

$$P_{detect\_RS} = 1 - e^{\left(\frac{-24 \cdot N_s \cdot V \cdot PDS \cdot Patrol\_Duration}{A}\right)}$$

Giving a SRS of:

$$SRS = 1 - P_{detect\_RS}$$

Derivation of Formulae for MOEs "STS-EB" and "STS-ES"

STS is the SSK's expected survivability of a Suspected Target Search (STS)

Factors:

- t_B - Burst Endurance (hrs - inclusive of t_o)
- V_{Max} - Burst Speed of SSK (kts)
- V_{EES} - Evasion Endurance Speed of SSK (kts)

Givens:

- W - PDS for Sonobuoys (nr
- V - Speed of Searchers (kts
- t_o - Time Late (hrs)
- t - Search Time (hrs)

Response:

- STS_EB - expected survivability at the end of SSK's burst
- STS_ES - expected survivability at the end of a three hour search

This type of search (STS) is generally referred to as a "datum search" because the SSK is flee reference datum. Washburn provides a general formula for datum searches:

$$P_{detect\_STS} = 1 - e^{\left[\frac{-W \cdot V}{\pi \cdot U^2} \cdot \left(\frac{1}{t_o} - \frac{1}{t_o + t}\right)\right]}$$

Note: In this formula, U represents speed of the evade

This version of the formula can be applied directly to STS_EB:

$$P_{detect\_STS\_EB} = 1 - e^{\left[\frac{-W \cdot V}{\pi \cdot V_{Max}^2} \cdot \left(\frac{1}{t_o} - \frac{1}{t_B}\right)\right]}$$

Where:

$$STS\_EB = 1 - P_{detect\_STS\_EB}$$

Unfortunately, STS_ES is not as straightforward. This analysis will assume that the burst endurance is always less than the search time. Further, it will assume that after the burst endurance is reached, the SSK will reduce speed to V_{EES}, its Evasion Endurance Speed, for which it will have enough battery endurance to complete a search of three hours in total duration. Therefore, due to the speed change, Washburn's equation cannot be applied directly. So, following a procedure similar to the development of his equation, the following modified equation was derived (see following page for derivation):

$$P_{detect\_STS\_ES} = 1 - e^{\left[\frac{-W \cdot V}{\pi} \cdot \left[\frac{1}{V_{Max}^2 \cdot t_o} - \frac{1}{V_{Max}^2 \cdot t_B} + \frac{t - t_B}{V_{Max} \cdot t_B \cdot \left(V_{EES} \cdot t - V_{EES} \cdot t_B + V_{Max} \cdot t_B\right)}\right]\right]}$$

Where:

$$STS\_ES = 1 - P_{detect\_STS\_ES}$$

It should be noted that this analysis assumes that, if detected, the probability of kill is 1

Derivation of Revised STS Equation

Based off of Washburn's derivation on pages 2-1, 2-2, and 2-7:

If:

$$\gamma(u) = \text{detection\_rate}$$

Solution of the differential equation:

$$\frac{d}{dt}q(t) = -q(t)\cdot\gamma(t)$$

Yields:

$$q(t) = e^{-n(t)}$$

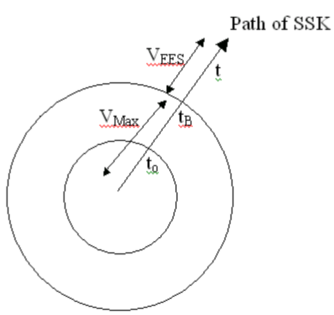
Where:

$$n(t) = \int_0^t \gamma(u)\,du$$

Which results in

$$\text{detection\_probability} = 1 - q(t) = 1 - e^{-n(t)}$$

Graphically, the situation described in the MOE derivation section looks like



So, the impact of the change from V_{Max} to V_{EES} on the increasing seacrh area must be modeled as follows:

$$n = \int_{t_o}^{t_B} \gamma_1\,du + \int_{t_B}^{t} \gamma_2\,du$$

Where:

$$\gamma_1 = \frac{V\cdot W}{\pi\cdot V_{Max}^2\cdot u^2} \qquad \text{and} \qquad \gamma_2 = \frac{V\cdot W}{\pi\cdot\left[V_{EES}\cdot\left(u - t_B\right) + V_{Max}\cdot t_B\right]^2}$$

Applying the intergral to find t_B from above yields:

$$P_{detect\ \ STS\ \ ES} = 1 - e^{\left[\frac{-W\cdot V}{\pi}\cdot\left[\frac{1}{V_{Max}^2\cdot t_o} - \frac{1}{V_{Max}^2\cdot t_B} + \frac{t - t_B}{V_{Max}\cdot t_B\cdot\big(V_{EES}\cdot t - V_{EES}\cdot t_B + V_{Max}\cdot t_B}\right]\right]}$$

Derivation of Formula for MOEs "MC"

MC is an expression of the SSK's mission capability

Factors: Response:

AIP_Endur - AIP Endurance (days) MC

$V_{balance}$ - AIP Balance Speed (kts)

Time_endurance_batt - Battery Endurance (days)

$V_{loiter\_battery}$ - Submerged Battery Loiter Speed (knots)

Number_CMs - Number of Cruise Missiles

Number_Torps - Number of Torpedos

Givens:

Torp_Range - Torpedo Range (nm)

CM_Range - Cruise Missile Range (nm)

Torp_Mission_Value - Torpedo Mission Value

CM_Mission_Value - Torpedo Mission Value

This MOE provides a sense of the total area that the SSK can influence based solely on its weapons systems ranges and AIP/battery endurance. This formula will be used as a MOE for two missions. The first will be an area denial mission, for which the preferred weapon is a MK-48 torpedo loadout. Therefore, the Torp_Mission_Value will equal 1 and CM_Mission_Value will equal 0. The second will be a strike mission, for which the preferred weapon is a Tomahawk cruise missile loadout. Therefore, the Torp_Mission_Value will equal 0 and CM_Mission_Value will equal 1.

The general MC metric is similar to the one used by Whitcomb and McHugh in 1999:

$$\mathrm{MC} = \pi \cdot (\mathrm{AIP\_Range} + \mathrm{Bat\_Range} + \mathrm{Torp\_Range})^2 \cdot \mathrm{Number\_Torps} \cdot \mathrm{Torp\_Mission\_Value} + \ldots$$

$$\ldots + \pi \cdot (\mathrm{AIP\_Range} + \mathrm{Bat\_Range} + \mathrm{CM\_Range})^2 \cdot \mathrm{Number\_CMs} \cdot \mathrm{CM\_Mission\_Value}$$

Where:

$$\mathrm{AIP\_Range} = \mathrm{AIP\_endur} \cdot V_{balance}$$

$$\mathrm{Bat\_Range} = \mathrm{Time\_endurance\_bat} \cdot V_{loiter\_battery}$$

It should be noted that the two missions should not be compared to each other. Rather, if the area denial mission is being analyzed, ignore the strike mission version of the metric. This metric does not represent multi-mission scenarios.

Lastly, the values of MC will be divided by 10^8 to simplify their presentation and manipulation in . and Crystal Ball.

APPENDIX 2:

JMP FACTORS AND RESPONSES

		Requirements/Factors							
Variant	Pattern	Burst Speed "Vmax" (knots)	STS Evasion Endurance Speed "V_{EES}" (knots)	Time at Burst Speed "Tburst" (hrs)	AIP Balance Speed "Vbalance" (knots)	AIP Endurance "TAIPendur" (days)	Submerged Endurance on Battery "Tbatt" (hours)	Submerged Battery Loiter Speed "Vloiter" (knots)	Loadout Package
1	---+--++	15	1	0.5	8	5	50	6	16
2	+-+-+++-	25	1	2	2	25	100	6	0
3	++-+++--	25	4	0.5	8	25	100	2	0
4	++----+-	25	4	0.5	2	5	50	6	0
5	---++--+	15	1	0.5	8	25	50	2	16
6	--+-+-+-	15	1	2	2	25	50	6	0
7	++-++--+	25	4	0.5	8	25	50	2	16
8	+-+--+--	25	1	2	2	5	100	2	0
9	--+++---	15	1	2	8	25	50	2	0
10	++-----+	25	4	0.5	2	5	50	2	16
11	--+++-++	15	1	2	8	25	50	6	16
12	----+---	15	1	0.5	2	25	50	2	0
13	++-+-+-+	25	4	0.5	8	5	100	2	16
14	+----+-+	25	1	0.5	2	5	100	2	16
15	++++-+--	25	4	2	8	5	100	2	0
16	+--++---	25	1	0.5	8	25	50	2	0
17	-++--+++	15	4	2	2	5	100	6	16
18	+---++--	25	1	0.5	2	25	100	2	0
19	-+-+-+--	15	4	0.5	8	5	100	2	0
20	+--+--+-	25	1	0.5	8	5	50	6	0
21	+++-++++	25	4	2	2	25	100	6	16
22	-+++-++-	15	4	2	8	5	100	6	0
23	-++----+	15	4	2	2	5	50	2	16
24	-+++-+-+	15	4	2	8	5	100	2	16
25	+++-++--	25	4	2	2	25	100	2	0
26	00a00000	20	2.5	0.5	5	15	75	4	8
27	+-+++--+	25	1	2	8	25	50	2	16
28	-----+++	15	1	0.5	2	5	100	6	16
29	---+++--	15	1	0.5	8	25	100	2	0
30	--++++-+	15	1	2	8	25	100	2	16
31	+---+-+-	25	1	0.5	2	25	50	6	0
32	+-+---+-	25	1	2	2	5	50	6	0
33	-+----++	15	4	0.5	2	5	50	6	16
34	--++-+--	15	1	2	8	5	100	2	0
35	-+--++++	15	4	0.5	2	25	100	6	16
36	+-+-+-++	25	1	2	2	25	50	6	16
37	++--++-+	25	4	0.5	2	25	100	2	16
38	000a0000	20	2.5	1.25	2	15	75	4	8
39	+---++++	25	1	0.5	2	25	100	6	16
40	-+-+-+++	15	4	0.5	8	5	100	6	16
41	A0000000	25	2.5	1.25	5	15	75	4	8
42	-++-+++-	15	4	2	2	25	100	6	0
43	-+-+++-+	15	4	0.5	8	25	100	2	16
44	-+-++---	15	4	0.5	8	25	50	2	0

45	+++-+-+-	25	4	2	2	25	50	6	0
46	-+-++-++	15	4	0.5	8	25	50	6	16
47	00000000	20	2.5	1.25	5	15	75	4	8
48	-+--++--	15	4	0.5	2	25	100	2	0
49	--+--++-	15	1	2	2	5	100	6	0
50	000000a0	20	2.5	1.25	5	15	75	2	8
51	00A00000	20	2.5	2	5	15	75	4	8
52	+-+++-+-	25	1	2	8	25	50	6	0
53	-+++----	15	4	2	8	5	50	2	0
54	0000000a	20	2.5	1.25	5	15	75	4	0
55	+-++----	25	1	2	8	5	50	2	0
56	-+---+-+	15	4	0.5	2	5	100	2	16
57	+-------	25	1	0.5	2	5	50	2	0
58	+--+---+	25	1	0.5	8	5	50	2	16
59	---++-+-	15	1	0.5	8	25	50	6	0
60	+++-+--+	25	4	2	2	25	50	2	16
61	+--+-+--	25	1	0.5	8	5	100	2	0
62	-+--+-+-	15	4	0.5	2	25	50	6	0
63	00000A00	20	2.5	1.25	5	15	100	4	8
64	+-+-+---	25	1	2	2	25	50	2	0
65	+-++--++	25	1	2	8	5	50	6	16
66	++-+--++	25	4	0.5	8	5	50	6	16
67	--+++++-	15	1	2	8	25	100	6	0
68	++--+---	25	4	0.5	2	25	50	2	0
69	++-++-+-	25	4	0.5	8	25	50	6	0
70	+-----++	25	1	0.5	2	5	50	6	16
71	0a000000	20	1	1.25	5	15	75	4	8
72	0000A000	20	2.5	1.25	5	25	75	4	8
73	++++-+++	25	4	2	8	5	100	6	16
74	--++---+	15	1	2	8	5	50	2	16
75	00000a00	20	2.5	1.25	5	15	50	4	8
76	------+-	15	1	0.5	2	5	50	6	0
77	+--++++-	25	1	0.5	8	25	100	6	0
78	-------+	15	1	0.5	2	5	50	2	16
79	++++---+	25	4	2	8	5	50	2	16
80	++--+++-	25	4	0.5	2	25	100	6	0
81	--+---++	15	1	2	2	5	50	6	16
82	-+++--++	15	4	2	8	5	50	6	16
83	000000A0	20	2.5	1.25	5	15	75	6	8
84	-+------	15	4	0.5	2	5	50	2	0
85	-----+--	15	1	0.5	2	5	100	2	0
86	--++-+++	15	1	2	8	5	100	6	16
87	+++--+-+	25	4	2	2	5	100	2	16
88	-++-+---	15	4	2	2	25	50	2	0
89	----+++-	15	1	0.5	2	25	100	6	0
90	-+-+--+-	15	4	0.5	8	5	50	6	0
91	-++++--+	15	4	2	8	25	50	2	16
92	--+-++++	15	1	2	2	25	100	6	16
93	0A000000	20	4	1.25	5	15	75	4	8
94	0000000A	20	2.5	1.25	5	15	75	4	16
95	+-+----+	25	1	2	2	5	50	2	16

96	+---+--+	25	1	0.5	2	25	50	2	16
97	+--++-++	25	1	0.5	8	25	50	6	16
98	----++-+	15	1	0.5	2	25	100	2	16
99	++++++-+	25	4	2	8	25	100	2	16
100	+++++++-	25	4	2	8	25	100	6	0
101	++---+++	25	4	0.5	2	5	100	6	16
102	+++--++-	25	4	2	2	5	100	6	0
103	-++++-+-	15	4	2	8	25	50	6	0
104	-++---+-	15	4	2	2	5	50	6	0
105	--+-----	15	1	2	2	5	50	2	0
106	+-++-++-	25	1	2	8	5	100	6	0
107	a0000000	15	2.5	1.25	5	15	75	4	8
108	-+++++--	15	4	2	8	25	100	2	0
109	--+-+--+	15	1	2	2	25	50	2	16
110	+++---++	25	4	2	2	5	50	6	16
111	---+++++	15	1	0.5	8	25	100	6	16
112	-++--+--	15	4	2	2	5	100	2	0
113	-+--+--+	15	4	0.5	2	25	50	2	16
114	-+-+---+	15	4	0.5	8	5	50	2	16
115	+++++-++	25	4	2	8	25	50	6	16
116	++-+-++-	25	4	0.5	8	5	100	6	0
117	--+-++--	15	1	2	2	25	100	2	0
118	---+-+-+	15	1	0.5	8	5	100	2	16
119	----+-++	15	1	0.5	2	25	50	6	16
120	--++--+-	15	1	2	8	5	50	6	0
121	++-+++++	25	4	0.5	8	25	100	6	16
122	+-+--+++	25	1	2	2	5	100	6	16
123	000A0000	20	2.5	1.25	8	15	75	4	8
124	+-++++++	25	1	2	8	25	100	6	16
125	-++-++-+	15	4	2	2	25	100	2	16
126	++--+-++	25	4	0.5	2	25	50	6	16
127	-+++++++	15	4	2	8	25	100	6	16
128	--+--+-+	15	1	2	2	5	100	2	16
129	+----++-	25	1	0.5	2	5	100	6	0
130	++---+--	25	4	0.5	2	5	100	2	0
131	++-+----	25	4	0.5	8	5	50	2	0
132	+++-----	25	4	2	2	5	50	2	0
133	+-+-++-+	25	1	2	2	25	100	2	16
134	0000a000	20	2.5	1.25	5	5	75	4	8
135	-+-++++-	15	4	0.5	8	25	100	6	0
136	---+----	15	1	0.5	8	5	50	2	0
137	-++-+-++	15	4	2	2	25	50	6	16
138	---+-++-	15	1	0.5	8	5	100	6	0
139	+-++++--	25	1	2	8	25	100	2	0
140	+++++---	25	4	2	8	25	50	2	0
141	-+---++-	15	4	0.5	2	5	100	6	0
142	+--+-+++	25	1	0.5	8	5	100	6	16
143	++++--+-	25	4	2	8	5	50	6	0
144	+--+++-+	25	1	0.5	8	25	100	2	16
145	+-++-+-+	25	1	2	8	5	100	2	16

		Desirements/Responses				
Variant	Pattern	Survivability of Suspected Target Search - End of Burst "STS-EB"	Survivability of Suspected Target Search - End of Search "STS-ES"	Survivability of Random Search "SRS"	Mission Capability - Area Denial "MC-AD"	Mission Capability - Strike "MC-S"
1	---+--++	0.754	0.261	0.543	0.804	0.000
2	+-+-+++-	0.837	0.826	0.683	0.000	2.895
3	++-+++--	0.903	0.681	0.618	0.000	15.763
4	++----+-	0.903	0.681	0.661	0.000	0.653
5	---++--+	0.754	0.261	0.618	12.093	0.000
6	--+-+-+-	0.609	0.589	0.683	0.000	2.217
7	++-++--+	0.903	0.681	0.618	12.093	0.000
8	+-+--+--	0.837	0.826	0.661	0.000	0.544
9	--+++---	0.609	0.589	0.618	0.000	15.205
10	++-----+	0.903	0.681	0.661	0.060	0.000
11	--+++-++	0.609	0.589	0.618	13.100	0.000
12	----+---	0.754	0.261	0.683	0.000	1.815
13	++-+-+-+	0.903	0.681	0.543	0.682	0.000
14	+----+-+	0.903	0.591	0.661	0.100	0.000
15	++++-+--	0.837	0.827	0.543	0.000	1.557
16	+--++---	0.903	0.591	0.618	0.000	15.205
17	-++--+++	0.609	0.591	0.661	0.359	0.000
18	+---++--	0.903	0.591	0.683	0.000	2.011
19	-+-+-+--	0.754	0.411	0.543	0.000	1.557
20	+--+--+-	0.903	0.591	0.543	0.000	1.739
21	+++-++++	0.837	0.827	0.683	1.638	0.000
22	-+++-++-	0.609	0.591	0.543	0.000	2.345
23	-++----+	0.609	0.591	0.661	0.060	0.000
24	-+++-+-+	0.609	0.591	0.543	0.682	0.000
25	+++-++--	0.837	0.827	0.683	0.000	2.011
26	00a00000	0.853	0.523	0.641	1.114	1.832
27	+-+++--+	0.837	0.826	0.618	12.093	0.000
28	-----+++	0.754	0.261	0.661	0.359	0.000
29	---+++--	0.754	0.261	0.618	0.000	15.763
30	--++++-+	0.609	0.589	0.618	12.592	0.000
31	+---+-+-	0.903	0.591	0.683	0.000	2.217
32	+-+---+-	0.837	0.826	0.661	0.000	0.653
33	-+----++	0.754	0.411	0.661	0.149	0.000
34	--++-+--	0.609	0.589	0.543	0.000	1.557
35	-+--++++	0.754	0.411	0.683	1.638	0.000
36	+-+-+-++	0.837	0.826	0.683	1.139	0.000
37	++--++-+	0.903	0.681	0.683	0.992	0.000
38	000a0000	0.775	0.719	0.672	0.264	0.660
39	+---++++	0.903	0.591	0.683	1.638	0.000
40	-+-+-+++	0.754	0.411	0.543	1.231	0.000
41	A0000000	0.850	0.808	0.641	1.114	1.832
42	-++-+++-	0.609	0.591	0.683	0.000	2.895
43	-+-+++-+	0.754	0.411	0.618	12.592	0.000
44	-+-++---	0.754	0.411	0.618	0.000	15.205

45	+++-+-+-	0.837	0.827	0.683	0.000	2.217
46	-+-++-++	0.754	0.411	0.618	13.100	0.000
47	0	0.775	0.719	0.641	1.114	1.832
48	-+--++--	0.754	0.411	0.683	0.000	2.011
49	--+--++-	0.609	0.589	0.661	0.000	1.042
50	000000a0	0.775	0.719	0.641	0.961	1.634
51	00A00000	0.757	0.743	0.641	1.114	1.832
52	+-+++-+-	0.837	0.826	0.618	0.000	16.331
53	-+++----	0.609	0.591	0.543	0.000	1.385
54	0000000a	0.775	0.719	0.641	0.000	3.664
55	+-++----	0.837	0.826	0.543	0.000	1.385
56	-+---+-+	0.754	0.411	0.661	0.100	0.000
57	+-------	0.903	0.591	0.661	0.000	0.444
58	+--+---+	0.903	0.591	0.543	0.570	0.000
59	---++-+-	0.754	0.261	0.618	0.000	16.331
60	+++-+--+	0.837	0.827	0.683	0.856	0.000
61	+--+-+--	0.903	0.591	0.543	0.000	1.557
62	-+--+-+-	0.754	0.411	0.683	0.000	2.217
63	00000A00	0.775	0.719	0.641	1.222	1.970
64	+-+-+---	0.837	0.826	0.683	0.000	1.815
65	+-++--++	0.837	0.826	0.543	0.804	0.000
66	++-+--++	0.903	0.681	0.543	0.804	0.000
67	--+++++-	0.609	0.589	0.618	0.000	18.096
68	++--+---	0.903	0.681	0.683	0.000	1.815
69	++-++-+-	0.903	0.681	0.618	0.000	16.331
70	+-----++	0.903	0.591	0.661	0.149	0.000
71	0a000000	0.775	0.713	0.641	1.114	1.832
72	0000A000	0.775	0.719	0.662	2.745	3.823
73	++++-+++	0.837	0.827	0.543	1.231	0.000
74	--++---+	0.609	0.589	0.543	0.570	0.000
75	00000a00	0.775	0.719	0.641	1.010	1.699
76	------+-	0.754	0.261	0.661	0.000	0.653
77	+--++++-	0.903	0.591	0.618	0.000	18.096
78	-------+	0.754	0.261	0.661	0.060	0.000
79	++++---+	0.837	0.827	0.543	0.570	0.000
80	++--+++-	0.903	0.681	0.683	0.000	2.895
81	--+---++	0.609	0.589	0.661	0.149	0.000
82	-+++--++	0.609	0.591	0.543	0.804	0.000
83	000000A0	0.775	0.719	0.641	1.278	2.041
84	-+------	0.754	0.411	0.661	0.000	0.444
85	-----+--	0.754	0.261	0.661	0.000	0.544
86	--++-+++	0.609	0.589	0.543	1.231	0.000
87	+++--+-+	0.837	0.827	0.661	0.100	0.000
88	-++-+---	0.609	0.591	0.683	0.000	1.815
89	----+++-	0.754	0.261	0.683	0.000	2.895
90	-+-+--+-	0.754	0.411	0.543	0.000	1.739
91	-++++--+	0.609	0.591	0.618	12.093	0.000
92	--+-++++	0.609	0.589	0.683	1.638	0.000
93	0A000000	0.775	0.723	0.641	1.114	1.832
94	0000000A	0.775	0.719	0.641	2.227	0.000
95	+-+----+	0.837	0.826	0.661	0.060	0.000

96	+---+--+	0.903	0.591	0.683	0.856	0.000
97	+--++-++	0.903	0.591	0.618	13.100	0.000
98	----++-+	0.754	0.261	0.683	0.992	0.000
99	++++++-+	0.837	0.827	0.618	12.592	0.000
100	+++++++-	0.837	0.827	0.618	0.000	18.096
101	++---+++	0.903	0.681	0.661	0.359	0.000
102	+++--++-	0.837	0.827	0.661	0.000	1.042
103	-++++-+-	0.609	0.591	0.618	0.000	16.331
104	-++---+-	0.609	0.591	0.661	0.000	0.653
105	--+-----	0.609	0.589	0.661	0.000	0.444
106	+-++-++-	0.837	0.826	0.543	0.000	2.345
107	a0000000	0.636	0.559	0.641	1.114	1.832
108	-+++++--	0.609	0.591	0.618	0.000	15.763
109	--+-+--+	0.609	0.589	0.683	0.856	0.000
110	+++---++	0.837	0.827	0.661	0.149	0.000
111	---+++++	0.754	0.261	0.618	14.685	0.000
112	-++--+--	0.609	0.591	0.661	0.000	0.544
113	-+--+--+	0.754	0.411	0.683	0.856	0.000
114	-+-+---+	0.754	0.411	0.543	0.570	0.000
115	+++++-++	0.837	0.827	0.618	13.100	0.000
116	++-+-++-	0.903	0.681	0.543	0.000	2.345
117	--+-++--	0.609	0.589	0.683	0.000	2.011
118	---+-+-+	0.754	0.261	0.543	0.682	0.000
119	----+-++	0.754	0.261	0.683	1.139	0.000
120	--++--+-	0.609	0.589	0.543	0.000	1.739
121	++-+++++	0.903	0.681	0.618	14.685	0.000
122	+-+--+++	0.837	0.826	0.661	0.359	0.000
123	000A0000	0.775	0.719	0.579	2.550	3.591
124	+-++++++	0.837	0.826	0.618	14.685	0.000
125	-++-++-+	0.609	0.591	0.683	0.992	0.000
126	++--+-++	0.903	0.681	0.683	1.139	0.000
127	-+++++++	0.609	0.591	0.618	14.685	0.000
128	--+--+-+	0.609	0.589	0.661	0.100	0.000
129	+----++-	0.903	0.591	0.661	0.000	1.042
130	++---+--	0.903	0.681	0.661	0.000	0.544
131	++-+----	0.903	0.681	0.543	0.000	1.385
132	+++-----	0.837	0.827	0.661	0.000	0.444
133	+-+-++-+	0.837	0.826	0.683	0.992	0.000
134	0000a000	0.775	0.719	0.621	0.206	0.565
135	-+-++++-	0.754	0.411	0.618	0.000	18.096
136	---+----	0.754	0.261	0.543	0.000	1.385
137	-++-+-++	0.609	0.591	0.683	1.139	0.000
138	---+-++-	0.754	0.261	0.543	0.000	2.345
139	+-++++--	0.837	0.826	0.618	0.000	15.763
140	+++++---	0.837	0.827	0.618	0.000	15.205
141	-+---++-	0.754	0.411	0.661	0.000	1.042
142	+--+-+++	0.903	0.591	0.543	1.231	0.000
143	++++--+-	0.837	0.827	0.543	0.000	1.739
144	+--+++-+	0.903	0.591	0.618	12.592	0.000
145	+-++-+-+	0.837	0.826	0.543	0.682	0.000

		Intermediate Calcs - SRS			Intermediate Calcs - STS		Intermediate Formulas - MC			
Variant	Pattern	$IR_{Battery\_Snorkel}$	$IR_{Composite}$	DD_Deep	Pdetect_STS_EB	Pdetect_STS_ES	Ifs for load packages 0-7	Ifs for load packages 8-15	If for 16, also show's total loadout of torpedos	Total loadout of CMs
1	---+--++	0.14	0.127	4.72	0.246	0.739	0	0	16	0
2	+-+-+++-	0.03	0.015	2.94	0.163	0.174	0	0	0	16
3	++-+++--	0.14	0.063	3.71	0.097	0.319	0	0	0	16
4	++----+-	0.03	0.031	3.19	0.097	0.319	0	0	0	16
5	---++--+	0.14	0.063	3.71	0.246	0.739	0	0	16	0
6	--+-+-+-	0.03	0.015	2.94	0.391	0.411	0	0	0	16
7	++-++--+	0.14	0.063	3.71	0.097	0.319	0	0	16	0
8	+-+--+--	0.03	0.031	3.19	0.163	0.174	0	0	0	16
9	--+++---	0.14	0.063	3.71	0.391	0.411	0	0	0	16
10	++-----+	0.03	0.031	3.19	0.097	0.319	0	0	16	0
11	--+++-++	0.14	0.063	3.71	0.391	0.411	0	0	16	0
12	----+---	0.03	0.015	2.94	0.246	0.739	0	0	0	16
13	++-+-+-+	0.14	0.127	4.72	0.097	0.319	0	0	16	0
14	+----+-+	0.03	0.031	3.19	0.097	0.409	0	0	16	0
15	++++-+--	0.14	0.127	4.72	0.163	0.173	0	0	0	16
16	+--++---	0.14	0.063	3.71	0.097	0.409	0	0	0	16
17	-++--+++	0.03	0.031	3.19	0.391	0.409	0	0	16	0
18	+---++--	0.03	0.015	2.94	0.097	0.409	0	0	0	16
19	-+-+-+--	0.14	0.127	4.72	0.246	0.589	0	0	0	16
20	+--+--+-	0.14	0.127	4.72	0.097	0.409	0	0	0	16
21	+++-++++	0.03	0.015	2.94	0.163	0.173	0	0	16	0
22	-+++-++-	0.14	0.127	4.72	0.391	0.409	0	0	0	16
23	-++----+	0.03	0.031	3.19	0.391	0.409	0	0	16	0
24	-+++-+-+	0.14	0.127	4.72	0.391	0.409	0	0	16	0
25	+++-++--	0.03	0.015	2.94	0.163	0.173	0	0	0	16
26	00a00000	0.07	0.046	3.43	0.147	0.477	0	8	8	8
27	+-+++--+	0.14	0.063	3.71	0.163	0.174	0	0	16	0
28	-----+++	0.03	0.031	3.19	0.246	0.739	0	0	16	0
29	---+++--	0.14	0.063	3.71	0.246	0.739	0	0	0	16
30	--++++-+	0.14	0.063	3.71	0.391	0.411	0	0	16	0
31	+---+-+-	0.03	0.015	2.94	0.097	0.409	0	0	0	16
32	+-+---+-	0.03	0.031	3.19	0.163	0.174	0	0	0	16
33	-+----++	0.03	0.031	3.19	0.246	0.589	0	0	16	0
34	--++-+--	0.14	0.127	4.72	0.391	0.411	0	0	0	16
35	-+--++++	0.03	0.015	2.94	0.246	0.589	0	0	16	0
36	+-+-+-++	0.03	0.015	2.94	0.163	0.174	0	0	16	0
37	++--++-+	0.03	0.015	2.94	0.097	0.319	0	0	16	0
38	000a0000	0.03	0.023	3.07	0.225	0.281	0	8	8	8
39	+---++++	0.03	0.015	2.94	0.097	0.409	0	0	16	0
40	-+-+-+++	0.14	0.127	4.72	0.246	0.589	0	0	16	0
41	A0000000	0.07	0.046	3.43	0.150	0.192	0	8	8	8
42	-++-+++-	0.03	0.015	2.94	0.391	0.409	0	0	0	16
43	-+-+++-+	0.14	0.063	3.71	0.246	0.589	0	0	16	0
44	-+-++---	0.14	0.063	3.71	0.246	0.589	0	0	0	16

45	+++-+-+-	0.03	0.015	2.94	0.163	0.173	0	0	0	16
46	-+-++-++	0.14	0.063	3.71	0.246	0.589	0	0	16	0
47	0	0.07	0.046	3.43	0.225	0.281	0	8	8	8
48	-+--++--	0.03	0.015	2.94	0.246	0.589	0	0	0	16
49	--+--++-	0.03	0.031	3.19	0.391	0.411	0	0	0	16
50	000000a0	0.07	0.046	3.43	0.225	0.281	0	8	8	8
51	00A00000	0.07	0.046	3.43	0.243	0.257	0	8	8	8
52	+-+++-+-	0.14	0.063	3.71	0.163	0.174	0	0	0	16
53	-+++----	0.14	0.127	4.72	0.391	0.409	0	0	0	16
54	0000000a	0.07	0.046	3.43	0.225	0.281	0	0	0	16
55	+-++----	0.14	0.127	4.72	0.163	0.174	0	0	0	16
56	-+---+-+	0.03	0.031	3.19	0.246	0.589	0	0	16	0
57	+-------	0.03	0.031	3.19	0.097	0.409	0	0	0	16
58	+--+---+	0.14	0.127	4.72	0.097	0.409	0	0	16	0
59	---++-+-	0.14	0.063	3.71	0.246	0.739	0	0	0	16
60	+++-+--+	0.03	0.015	2.94	0.163	0.173	0	0	16	0
61	+--+-+--	0.14	0.127	4.72	0.097	0.409	0	0	0	16
62	-+--+-+-	0.03	0.015	2.94	0.246	0.589	0	0	0	16
63	00000A00	0.07	0.046	3.43	0.225	0.281	0	8	8	8
64	+-+-+---	0.03	0.015	2.94	0.163	0.174	0	0	0	16
65	+-++--++	0.14	0.127	4.72	0.163	0.174	0	0	16	0
66	++-+--++	0.14	0.127	4.72	0.097	0.319	0	0	16	0
67	--+++++-	0.14	0.063	3.71	0.391	0.411	0	0	0	16
68	++--+---	0.03	0.015	2.94	0.097	0.319	0	0	0	16
69	++-++-+-	0.14	0.063	3.71	0.097	0.319	0	0	0	16
70	+-----++	0.03	0.031	3.19	0.097	0.409	0	0	16	0
71	0a000000	0.07	0.046	3.43	0.225	0.287	0	8	8	8
72	0000A000	0.07	0.031	3.19	0.225	0.281	0	8	8	8
73	++++-+++	0.14	0.127	4.72	0.163	0.173	0	0	16	0
74	--++---+	0.14	0.127	4.72	0.391	0.411	0	0	16	0
75	00000a00	0.07	0.046	3.43	0.225	0.281	0	8	8	8
76	------+-	0.03	0.031	3.19	0.246	0.739	0	0	0	16
77	+--++++-	0.14	0.063	3.71	0.097	0.409	0	0	0	16
78	-------+	0.03	0.031	3.19	0.246	0.739	0	0	16	0
79	++++---+	0.14	0.127	4.72	0.163	0.173	0	0	16	0
80	++--+++-	0.03	0.015	2.94	0.097	0.319	0	0	0	16
81	--+---++	0.03	0.031	3.19	0.391	0.411	0	0	16	0
82	-+++--++	0.14	0.127	4.72	0.391	0.409	0	0	16	0
83	000000A0	0.07	0.046	3.43	0.225	0.281	0	8	8	8
84	-+------	0.03	0.031	3.19	0.246	0.589	0	0	0	16
85	-----+--	0.03	0.031	3.19	0.246	0.739	0	0	0	16
86	--++-+++	0.14	0.127	4.72	0.391	0.411	0	0	16	0
87	+++--+-+	0.03	0.031	3.19	0.163	0.173	0	0	16	0
88	-++-+---	0.03	0.015	2.94	0.391	0.409	0	0	0	16
89	----+++-	0.03	0.015	2.94	0.246	0.739	0	0	0	16
90	-+-+--+-	0.14	0.127	4.72	0.246	0.589	0	0	0	16
91	-++++--+	0.14	0.063	3.71	0.391	0.409	0	0	16	0
92	--+-++++	0.03	0.015	2.94	0.391	0.411	0	0	16	0
93	0A000000	0.07	0.046	3.43	0.225	0.277	0	8	8	8
94	0000000A	0.07	0.046	3.43	0.225	0.281	0	0	16	0
95	+-+----+	0.03	0.031	3.19	0.163	0.174	0	0	16	0

96	+---+--+	0.03	0.015	2.94	0.097	0.409	0	0	16	0
97	+--++-++	0.14	0.063	3.71	0.097	0.409	0	0	16	0
98	----++-+	0.03	0.015	2.94	0.246	0.739	0	0	16	0
99	++++++-+	0.14	0.063	3.71	0.163	0.173	0	0	16	0
100	+++++++-	0.14	0.063	3.71	0.163	0.173	0	0	0	16
101	++---+++	0.03	0.031	3.19	0.097	0.319	0	0	16	0
102	+++--++-	0.03	0.031	3.19	0.163	0.173	0	0	0	16
103	-++++-+-	0.14	0.063	3.71	0.391	0.409	0	0	0	16
104	-++---+-	0.03	0.031	3.19	0.391	0.409	0	0	0	16
105	--+-----	0.03	0.031	3.19	0.391	0.411	0	0	0	16
106	+-++-++-	0.14	0.127	4.72	0.163	0.174	0	0	0	16
107	a0000000	0.07	0.046	3.43	0.364	0.441	0	8	8	8
108	-+++++--	0.14	0.063	3.71	0.391	0.409	0	0	0	16
109	--+-+--+	0.03	0.015	2.94	0.391	0.411	0	0	16	0
110	+++---++	0.03	0.031	3.19	0.163	0.173	0	0	16	0
111	---+++++	0.14	0.063	3.71	0.246	0.739	0	0	16	0
112	-++--+--	0.03	0.031	3.19	0.391	0.409	0	0	0	16
113	-+--+--+	0.03	0.015	2.94	0.246	0.589	0	0	16	0
114	-+-+---+	0.14	0.127	4.72	0.246	0.589	0	0	16	0
115	+++++-++	0.14	0.063	3.71	0.163	0.173	0	0	16	0
116	++-+-++-	0.14	0.127	4.72	0.097	0.319	0	0	0	16
117	--+-++--	0.03	0.015	2.94	0.391	0.411	0	0	0	16
118	---+-+-+	0.14	0.127	4.72	0.246	0.739	0	0	16	0
119	----+-++	0.03	0.015	2.94	0.246	0.739	0	0	16	0
120	--++--+-	0.14	0.127	4.72	0.391	0.411	0	0	0	16
121	++-+++++	0.14	0.063	3.71	0.097	0.319	0	0	16	0
122	+-+--+++	0.03	0.031	3.19	0.163	0.174	0	0	16	0
123	000A0000	0.14	0.095	4.21	0.225	0.281	0	8	8	8
124	+-++++++	0.14	0.063	3.71	0.163	0.174	0	0	16	0
125	-++-++-+	0.03	0.015	2.94	0.391	0.409	0	0	16	0
126	++--+-++	0.03	0.015	2.94	0.097	0.319	0	0	16	0
127	-+++++++	0.14	0.063	3.71	0.391	0.409	0	0	16	0
128	--+--+-+	0.03	0.031	3.19	0.391	0.411	0	0	16	0
129	+----++-	0.03	0.031	3.19	0.097	0.409	0	0	0	16
130	++---+--	0.03	0.031	3.19	0.097	0.319	0	0	0	16
131	++-+----	0.14	0.127	4.72	0.097	0.319	0	0	0	16
132	+++-----	0.03	0.031	3.19	0.163	0.173	0	0	0	16
133	+-+-++-+	0.03	0.015	2.94	0.163	0.174	0	0	16	0
134	0000a000	0.07	0.061	3.67	0.225	0.281	0	8	8	8
135	-+-++++-	0.14	0.063	3.71	0.246	0.589	0	0	0	16
136	---+----	0.14	0.127	4.72	0.246	0.739	0	0	0	16
137	-++-+-++	0.03	0.015	2.94	0.391	0.409	0	0	16	0
138	---+-++-	0.14	0.127	4.72	0.246	0.739	0	0	0	16
139	+-++++--	0.14	0.063	3.71	0.163	0.174	0	0	0	16
140	+++++---	0.14	0.063	3.71	0.163	0.173	0	0	0	16
141	-+---++-	0.03	0.031	3.19	0.246	0.589	0	0	0	16
142	+--+-+++	0.14	0.127	4.72	0.097	0.409	0	0	16	0
143	++++--+-	0.14	0.127	4.72	0.163	0.173	0	0	0	16
144	+--+++-+	0.14	0.063	3.71	0.097	0.409	0	0	16	0
145	+-++-+-+	0.14	0.127	4.72	0.163	0.174	0	0	16	0

APPENDIX 3:

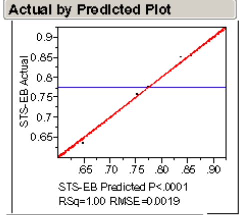
JMP RSE DATA

final- Fit Least Squares

Least Squares Fit

Response STS-EB

Actual by Predicted Plot



Summary of Fit

RSquare	0.999768
RSquare Adj	0.999666
Root Mean Square Error	0.00191
Mean of Response	0.775634
Observations (or Sum Wgts)	145

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	44	1.5741309	0.035776	9809.719
Error	100	0.0003647	0.000004	Prob > F
C. Total	144	1.5744956		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.7746368	0.000525	1475.2	<.0001
Vmax(15,25)&RS	0.0944462	0.000167	563.88	<.0001
VEES(1,4)&RS	2.405e-15	0.000167	0.00	1.0000
Tburst(0.5,2)&RS	-0.052677	0.000167	-314.5	<.0001
Vbalance(2,8)&RS	2.624e-15	0.000167	0.00	1.0000
TAIPEndur(5,25)&RS	2.624e-15	0.000167	0.00	1.0000
Tbatt(50,100)&RS	3.279e-15	0.000167	0.00	1.0000
Vloiter(2,6)&RS	2.624e-15	0.000167	0.00	1.0000
Loadout(0,16)&RS	2.624e-15	0.000167	0.00	1.0000
Vmax(15,25)*VEES(1,4)	1.332e-15	0.000169	0.00	1.0000
Vmax(15,25)*Tburst(0.5,2)	0.01975	0.000169	117.01	<.0001
VEES(1,4)*Tburst(0.5,2)	2.22e-15	0.000169	0.00	1.0000
Vmax(15,25)*Vbalance(2,8)	2.22e-15	0.000169	0.00	1.0000
VEES(1,4)*Vbalance(2,8)	2.22e-15	0.000169	0.00	1.0000
Tburst(0.5,2)*Vbalance(2,8)	2.22e-15	0.000169	0.00	1.0000
Vmax(15,25)*TAIPEndur(5,25)	2.22e-15	0.000169	0.00	1.0000
VEES(1,4)*TAIPEndur(5,25)	3.109e-15	0.000169	0.00	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	2.665e-15	0.000169	0.00	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	2.22e-15	0.000169	0.00	1.0000
Vmax(15,25)*Tbatt(50,100)	1.776e-15	0.000169	0.00	1.0000
VEES(1,4)*Tbatt(50,100)	3.553e-15	0.000169	0.00	1.0000
Tburst(0.5,2)*Tbatt(50,100)	2.665e-15	0.000169	0.00	1.0000
Vbalance(2,8)*Tbatt(50,100)	2.22e-15	0.000169	0.00	1.0000
TAIPEndur(5,25)*Tbatt(50,100)	2.22e-15	0.000169	0.00	1.0000
Vmax(15,25)*Vloiter(2,6)	2.22e-15	0.000169	0.00	1.0000
VEES(1,4)*Vloiter(2,6)	1.776e-15	0.000169	0.00	1.0000
Tburst(0.5,2)*Vloiter(2,6)	2.665e-15	0.000169	0.00	1.0000
Vbalance(2,8)*Vloiter(2,6)	2.22e-15	0.000169	0.00	1.0000
TAIPEndur(5,25)*Vloiter(2,6)	2.22e-15	0.000169	0.00	1.0000
Tbatt(50,100)*Vloiter(2,6)	1.332e-15	0.000169	0.00	1.0000
Vmax(15,25)*Loadout(0,16)	1.332e-15	0.000169	0.00	1.0000
VEES(1,4)*Loadout(0,16)	3.553e-15	0.000169	0.00	1.0000
Tburst(0.5,2)*Loadout(0,16)	2.22e-15	0.000169	0.00	1.0000
Vbalance(2,8)*Loadout(0,16)	3.553e-15	0.000169	0.00	1.0000
TAIPEndur(5,25)*Loadout(0,16)	2.22e-15	0.000169	0.00	1.0000
Tbatt(50,100)*Loadout(0,16)	2.665e-15	0.000169	0.00	1.0000
Vloiter(2,6)*Loadout(0,16)	2.665e-15	0.000169	0.00	1.0000
Vmax(15,25)*Vmax(15,25)	-0.031611	0.001265	-24.99	<.0001
VEES(1,4)*VEES(1,4)	0.0003891	0.001265	0.31	0.7591
Tburst(0.5,2)*Tburst(0.5,2)	0.0303891	0.001265	24.02	<.0001
Vbalance(2,8)*Vbalance(2,8)	0.0003891	0.001265	0.31	0.7591
TAIPEndur(5,25)*TAIPEndur(5,25)	0.0003891	0.001265	0.31	0.7591
Tbatt(50,100)*Tbatt(50,100)	0.0003891	0.001265	0.31	0.7591
Vloiter(2,6)*Vloiter(2,6)	0.0003891	0.001265	0.31	0.7591
Loadout(0,16)*Loadout(0,16)	0.0003891	0.001265	0.31	0.7591

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Vmax(15,25)&RS	1	1	1.1596099	317965.7	<.0001
VEES(1,4)&RS	1	1	0.0000000	0.0000	1.0000
Tburst(0.5,2)&RS	1	1	0.3607316	98912.81	<.0001
Vbalance(2,8)&RS	1	1	0.0000000	0.0000	1.0000
TAIPEndur(5,25)&RS	1	1	0.0000000	0.0000	1.0000
Tbatt(50,100)&RS	1	1	0.0000000	0.0000	1.0000
Vloiter(2,6)&RS	1	1	0.0000000	0.0000	1.0000
Loadout(0,16)&RS	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*VEES(1,4)	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*Tburst(0.5,2)	1	1	0.0499280	13690.29	<.0001
VEES(1,4)*Tburst(0.5,2)	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*Vbalance(2,8)	1	1	0.0000000	0.0000	1.0000
VEES(1,4)*Vbalance(2,8)	1	1	0.0000000	0.0000	1.0000
Tburst(0.5,2)*Vbalance(2,8)	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*TAIPEndur(5,25)	1	1	0.0000000	0.0000	1.0000
VEES(1,4)*TAIPEndur(5,25)	1	1	0.0000000	0.0000	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	1	1	0.0000000	0.0000	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*Tbatt(50,100)	1	1	0.0000000	0.0000	1.0000
VEES(1,4)*Tbatt(50,100)	1	1	0.0000000	0.0000	1.0000
Tburst(0.5,2)*Tbatt(50,100)	1	1	0.0000000	0.0000	1.0000
Vbalance(2,8)*Tbatt(50,100)	1	1	0.0000000	0.0000	1.0000
TAIPEndur(5,25)*Tbatt(50,100)	1	1	0.0000000	0.0000	1.0000

final- Fit Least Squares

Least Squares Fit

Response STS-EB

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Vmax(15,25)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
VEES(1,4)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
Tburst(0.5,2)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
Vbalance(2,8)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
TAIPEndur(5,25)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
Tbatt(50,100)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
VEES(1,4)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
Tburst(0.5,2)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
Vbalance(2,8)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
TAIPEndur(5,25)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
Tbatt(50,100)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
Vloiter(2,6)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*Vmax(15,25)	1	1	0.0022770	624.3676	<.0001
VEES(1,4)*VEES(1,4)	1	1	0.0000003	0.0946	0.7591
Tburst(0.5,2)*Tburst(0.5,2)	1	1	0.0021044	577.0347	<.0001
Vbalance(2,8)*Vbalance(2,8)	1	1	0.0000003	0.0946	0.7591
TAIPEndur(5,25)*TAIPEndur(5,25)	1	1	0.0000003	0.0946	0.7591
Tbatt(50,100)*Tbatt(50,100)	1	1	0.0000003	0.0946	0.7591
Vloiter(2,6)*Vloiter(2,6)	1	1	0.0000003	0.0946	0.7591
Loadout(0,16)*Loadout(0,16)	1	1	0.0000003	0.0946	0.7591

Response Surface

Coef

	Vmax(15,25)	VEES(1,4)	Tburst(0.5,2)	Vbalance(2,8)	TAIPEndur(5,25)	Tbatt(50,100)	Vloiter(2,6)	Loadout(0,16)	STS-EB
Vmax(15,25)	-0.031611	1.332e-15	0.01975	2.22e-15	2.22e-15	1.776e-15	2.22e-15	1.332e-15	0.0944462
VEES(1,4)	.	0.0003891	2.22e-15	2.22e-15	3.109e-15	3.553e-15	1.776e-15	3.553e-15	2.405e-15
Tburst(0.5,2)	.	.	0.0303891	2.22e-15	2.665e-15	2.665e-15	2.665e-15	2.22e-15	-0.052677
Vbalance(2,8)	.	.	.	0.0003891	2.22e-15	2.22e-15	2.22e-15	3.553e-15	2.624e-15
TAIPEndur(5,25)	.	.	.	.	0.0003891	2.22e-15	2.22e-15	2.22e-15	2.624e-15
Tbatt(50,100)	.	.	.	.	.	0.0003891	1.332e-15	2.665e-15	3.279e-15
Vloiter(2,6)	.	.	.	.	.	.	0.0003891	2.665e-15	2.624e-15
Loadout(0,16)	.	.	.	.	.	.	.	0.0003891	2.624e-15

Solution

Variable	Critical Value
Vmax(15,25)	28.010069
VEES(1,4)	2.5
Tburst(0.5,2)	1.5096974
Vbalance(2,8)	5
TAIPEndur(5,25)	15
Tbatt(50,100)	75
Vloiter(2,6)	4
Loadout(0,16)	8

Solution is a SaddlePoint

Critical values outside data range

Predicted Value at Solution 0.8411723

Canonical Curvature

Eigenvalues and Eigenvectors

Eigenvalue	0.0319	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	-0.0331
Vmax(15,25)	0.15358	0.00000	0.00000	0.00000	-0.00000	0.00000	0.00000	0.98814
VEES(1,4)	0.00000	0.06301	0.46682	-0.40407	0.24602	-0.46572	0.58088	-0.00000
Tburst(0.5,2)	0.98814	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	-0.15358
Vbalance(2,8)	0.00000	0.04750	0.44123	0.28705	-0.70927	0.28530	0.36905	-0.00000
TAIPEndur(5,25)	0.00000	0.99347	-0.11317	0.00176	-0.00885	0.00056	-0.01139	-0.00000
Tbatt(50,100)	0.00000	0.04756	0.42819	-0.54592	0.15962	0.64117	-0.28253	-0.00000
Vloiter(2,6)	0.00000	0.04558	0.35554	0.66561	0.61647	0.20631	0.07664	-0.00000
Loadout(0,16)	0.00000	0.04952	0.51461	0.11518	-0.17563	-0.49806	-0.66374	-0.00000

Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.7746368	0.000525	1475.19	<.0001
Vmax(15,25)&RS	0.0944462	0.000167	563.88	<.0001
VEES(1,4)&RS	2.405e-15	0.000167	0.00	1.0000
Tburst(0.5,2)&RS	-0.052677	0.000167	-314.50	<.0001
Vbalance(2,8)&RS	2.624e-15	0.000167	0.00	1.0000
TAIPEndur(5,25)&RS	2.624e-15	0.000167	0.00	1.0000
Tbatt(50,100)&RS	3.279e-15	0.000167	0.00	1.0000
Vloiter(2,6)&RS	2.624e-15	0.000167	0.00	1.0000
Loadout(0,16)&RS	2.624e-15	0.000167	0.00	1.0000
Vmax(15,25)*VEES(1,4)	1.332e-15	0.000169	0.00	1.0000
Vmax(15,25)*Tburst(0.5,2)	0.01975	0.000169	117.01	<.0001
VEES(1,4)*Tburst(0.5,2)	2.22e-15	0.000169	0.00	1.0000
Vmax(15,25)*Vbalance(2,8)	2.22e-15	0.000169	0.00	1.0000
VEES(1,4)*Vbalance(2,8)	2.22e-15	0.000169	0.00	1.0000
Tburst(0.5,2)*Vbalance(2,8)	2.22e-15	0.000169	0.00	1.0000
Vmax(15,25)*TAIPEndur(5,25)	2.22e-15	0.000169	0.00	1.0000
VEES(1,4)*TAIPEndur(5,25)	3.109e-15	0.000169	0.00	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	2.665e-15	0.000169	0.00	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	2.22e-15	0.000169	0.00	1.0000
Vmax(15,25)*Tbatt(50,100)	1.776e-15	0.000169	0.00	1.0000
VEES(1,4)*Tbatt(50,100)	3.553e-15	0.000169	0.00	1.0000
Tburst(0.5,2)*Tbatt(50,100)	2.665e-15	0.000169	0.00	1.0000
Vbalance(2,8)*Tbatt(50,100)	2.22e-15	0.000169	0.00	1.0000
TAIPEndur(5,25)*Tbatt(50,100)	2.22e-15	0.000169	0.00	1.0000
Vmax(15,25)*Vloiter(2,6)	2.22e-15	0.000169	0.00	1.0000
VEES(1,4)*Vloiter(2,6)	1.776e-15	0.000169	0.00	1.0000
Tburst(0.5,2)*Vloiter(2,6)	2.665e-15	0.000169	0.00	1.0000
Vbalance(2,8)*Vloiter(2,6)	2.22e-15	0.000169	0.00	1.0000
TAIPEndur(5,25)*Vloiter(2,6)	2.22e-15	0.000169	0.00	1.0000
Tbatt(50,100)*Vloiter(2,6)	1.332e-15	0.000169	0.00	1.0000
Vmax(15,25)*Loadout(0,16)	1.332e-15	0.000169	0.00	1.0000
VEES(1,4)*Loadout(0,16)	3.553e-15	0.000169	0.00	1.0000
Tburst(0.5,2)*Loadout(0,16)	2.22e-15	0.000169	0.00	1.0000
Vbalance(2,8)*Loadout(0,16)	3.553e-15	0.000169	0.00	1.0000
TAIPEndur(5,25)*Loadout(0,16)	2.22e-15	0.000169	0.00	1.0000
Tbatt(50,100)*Loadout(0,16)	2.665e-15	0.000169	0.00	1.0000
Vloiter(2,6)*Loadout(0,16)	2.665e-15	0.000169	0.00	1.0000

final- Fit Least Squares

Least Squares Fit

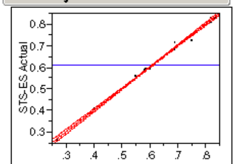
Response STS-EB

Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob>|t|
Vmax(15,25)*Vmax(15,25)	-0.031611	0.001265	-24.99	<.0001
VEES(1,4)*VEES(1,4)	0.0003891	0.001265	0.31	0.7591
Tburst(0.5,2)*Tburst(0.5,2)	0.0303891	0.001265	24.02	<.0001
Vbalance(2,8)*Vbalance(2,8)	0.0003891	0.001265	0.31	0.7591
TAIPEndur(5,25)*TAIPEndur(5,25)	0.0003891	0.001265	0.31	0.7591
Tbatt(50,100)*Tbatt(50,100)	0.0003891	0.001265	0.31	0.7591
Vloiter(2,6)*Vloiter(2,6)	0.0003891	0.001265	0.31	0.7591
Loadout(0,16)*Loadout(0,16)	0.0003891	0.001265	0.31	0.7591

Response STS-ES

Actual by Predicted Plot



STS-ES Predicted P<.0001
RSq=1.00 RMSE=0.0092

Summary of Fit

RSquare	0.998089
RSquare Adj	0.997248
Root Mean Square Error	0.009173
Mean of Response	0.609724
Observations (or Sum Wgts)	145

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	44	4.3943221	0.099871	1186.835
Error	100	0.0084149	0.000084	Prob > F
C. Total	144	4.4027370		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.7189175	0.002522	285.02	<.0001
Vmax(15,25)&RS	0.1339769	0.000805	166.52	<.0001
VEES(1,4)&RS	0.0299846	0.000805	37.27	<.0001
Tburst(0.5,2)&RS	0.1111077	0.000805	138.10	<.0001
Vbalance(2,8)&RS	-1.53e-15	0.000805	-0.00	1.0000
TAIPEndur(5,25)&RS	-1.53e-15	0.000805	-0.00	1.0000
Tbatt(50,100)&RS	-1.53e-15	0.000805	-0.00	1.0000
Vloiter(2,6)&RS	-1.31e-15	0.000805	-0.00	1.0000
Loadout(0,16)&RS	-1.53e-15	0.000805	-0.00	1.0000
Vmax(15,25)*VEES(1,4)	-0.007625	0.000811	-9.40	<.0001
Vmax(15,25)*Tburst(0.5,2)	-0.015875	0.000811	-19.58	<.0001
VEES(1,4)*Tburst(0.5,2)	-0.029625	0.000811	-36.54	<.0001
Vmax(15,25)*Vbalance(2,8)	-4.88e-15	0.000811	-0.00	1.0000
VEES(1,4)*Vbalance(2,8)	-3.55e-15	0.000811	-0.00	1.0000
Tburst(0.5,2)*Vbalance(2,8)	-3.55e-15	0.000811	-0.00	1.0000
Vmax(15,25)*TAIPEndur(5,25)	-4e-15	0.000811	-0.00	1.0000
VEES(1,4)*TAIPEndur(5,25)	-5.33e-15	0.000811	-0.00	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	-4.88e-15	0.000811	-0.00	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	-4.44e-15	0.000811	-0.00	1.0000
Vmax(15,25)*Tbatt(50,100)	-4.44e-15	0.000811	-0.00	1.0000
VEES(1,4)*Tbatt(50,100)	-4.44e-15	0.000811	-0.00	1.0000
Tburst(0.5,2)*Tbatt(50,100)	-4.44e-15	0.000811	-0.00	1.0000
Vbalance(2,8)*Tbatt(50,100)	-4.88e-15	0.000811	-0.00	1.0000
TAIPEndur(5,25)*Tbatt(50,100)	-4.44e-15	0.000811	-0.00	1.0000
Vmax(15,25)*Vloiter(2,6)	-4.44e-15	0.000811	-0.00	1.0000
VEES(1,4)*Vloiter(2,6)	-4.44e-15	0.000811	-0.00	1.0000
Tburst(0.5,2)*Vloiter(2,6)	-4e-15	0.000811	-0.00	1.0000
Vbalance(2,8)*Vloiter(2,6)	-4.88e-15	0.000811	-0.00	1.0000
TAIPEndur(5,25)*Vloiter(2,6)	-4.44e-15	0.000811	-0.00	1.0000
Tbatt(50,100)*Vloiter(2,6)	-4.44e-15	0.000811	-0.00	1.0000
Vmax(15,25)*Loadout(0,16)	-4e-15	0.000811	-0.00	1.0000
VEES(1,4)*Loadout(0,16)	-4.44e-15	0.000811	-0.00	1.0000
Tburst(0.5,2)*Loadout(0,16)	-4.88e-15	0.000811	-0.00	1.0000
Vbalance(2,8)*Loadout(0,16)	-4.44e-15	0.000811	-0.00	1.0000
TAIPEndur(5,25)*Loadout(0,16)	-5.33e-15	0.000811	-0.00	1.0000
Tbatt(50,100)*Loadout(0,16)	-4.88e-15	0.000811	-0.00	1.0000
Vloiter(2,6)*Loadout(0,16)	-5.33e-15	0.000811	-0.00	1.0000
Vmax(15,25)*Vmax(15,25)	-0.035412	0.006077	-5.83	<.0001
VEES(1,4)*VEES(1,4)	-0.000912	0.006077	-0.15	0.8811
Tburst(0.5,2)*Tburst(0.5,2)	-0.085912	0.006077	-14.14	<.0001
Vbalance(2,8)*Vbalance(2,8)	0.0000884	0.006077	0.01	0.9884
TAIPEndur(5,25)*TAIPEndur(5,25)	0.0000884	0.006077	0.01	0.9884
Tbatt(50,100)*Tbatt(50,100)	0.0000884	0.006077	0.01	0.9884
Vloiter(2,6)*Vloiter(2,6)	0.0000884	0.006077	0.01	0.9884
Loadout(0,16)*Loadout(0,16)	0.0000884	0.006077	0.01	0.9884

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Vmax(15,25)&RS	1	1	2.3334761	27730.29	<.0001
VEES(1,4)&RS	1	1	0.1168800	1388.965	<.0001
Tburst(0.5,2)&RS	1	1	1.6048395	19071.4	<.0001
Vbalance(2,8)&RS	1	1	0.0000000	0.0000	1.0000
TAIPEndur(5,25)&RS	1	1	0.0000000	0.0000	1.0000
Tbatt(50,100)&RS	1	1	0.0000000	0.0000	1.0000
Vloiter(2,6)&RS	1	1	0.0000000	0.0000	1.0000
Loadout(0,16)&RS	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*VEES(1,4)	1	1	0.0074420	88.4384	<.0001
Vmax(15,25)*Tburst(0.5,2)	1	1	0.0322580	383.3438	<.0001
VEES(1,4)*Tburst(0.5,2)	1	1	0.1123380	1334.989	<.0001

final- Fit Least Squares

Least Squares Fit

Response STS-ES

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Vmax(15,25)*Vbalance(2,8)	1	1	0.0000000	0.0000	1.0000
VEES(1,4)*Vbalance(2,8)	1	1	0.0000000	0.0000	1.0000
Tburst(0.5,2)*Vbalance(2,8)	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*TAIPEndur(5,25)	1	1	0.0000000	0.0000	1.0000
VEES(1,4)*TAIPEndur(5,25)	1	1	0.0000000	0.0000	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	1	1	0.0000000	0.0000	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*Tbatt(50,100)	1	1	0.0000000	0.0000	1.0000
VEES(1,4)*Tbatt(50,100)	1	1	0.0000000	0.0000	1.0000
Tburst(0.5,2)*Tbatt(50,100)	1	1	0.0000000	0.0000	1.0000
Vbalance(2,8)*Tbatt(50,100)	1	1	0.0000000	0.0000	1.0000
TAIPEndur(5,25)*Tbatt(50,100)	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
VEES(1,4)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
Tburst(0.5,2)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
Vbalance(2,8)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
TAIPEndur(5,25)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
Tbatt(50,100)*Vloiter(2,6)	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
VEES(1,4)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
Tburst(0.5,2)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
Vbalance(2,8)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
TAIPEndur(5,25)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
Tbatt(50,100)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
Vloiter(2,6)*Loadout(0,16)	1	1	0.0000000	0.0000	1.0000
Vmax(15,25)*Vmax(15,25)	1	1	0.0028575	33.9578	<.0001
VEES(1,4)*VEES(1,4)	1	1	0.0000019	0.0225	0.8811
Tburst(0.5,2)*Tburst(0.5,2)	1	1	0.0168190	199.8722	<.0001
Vbalance(2,8)*Vbalance(2,8)	1	1	0.0000000	0.0002	0.9884
TAIPEndur(5,25)*TAIPEndur(5,25)	1	1	0.0000000	0.0002	0.9884
Tbatt(50,100)*Tbatt(50,100)	1	1	0.0000000	0.0002	0.9884
Vloiter(2,6)*Vloiter(2,6)	1	1	0.0000000	0.0002	0.9884
Loadout(0,16)*Loadout(0,16)	1	1	0.0000000	0.0002	0.9884

Response Surface

Coef

	Vmax(15,25)	VEES(1,4)	Tburst(0.5,2)	Vbalance(2,8)	TAIPEndur(5,25)	Tbatt(50,100)	Vloiter(2,6)	Loadout(0,16)	STS-ES
Vmax(15,25)	-0.035412	-0.007625	-0.015875	-4.88e-15	-4e-15	-4.44e-15	-4.44e-15	-4e-15	0.1339769
VEES(1,4)	.	-0.000912	-0.029625	-3.55e-15	-5.33e-15	-4.44e-15	-4.44e-15	-4.44e-15	0.0299846
Tburst(0.5,2)	.	.	-0.085912	-3.55e-15	-4.88e-15	-4.44e-15	-4e-15	-4.88e-15	0.1111077
Vbalance(2,8)	.	.	.	0.0000884	-4.44e-15	-4.88e-15	-4.88e-15	-4.44e-15	-1.53e-15
TAIPEndur(5,25)	.	.	.	.	0.0000884	-4.44e-15	-4.44e-15	-5.33e-15	-1.53e-15
Tbatt(50,100)	.	.	.	.	.	0.0000884	-4.44e-15	-4.88e-15	-1.53e-15
Vloiter(2,6)	.	.	.	.	.	.	0.0000884	-5.33e-15	-1.31e-15
Loadout(0,16)	.	.	.	.	.	.	.	0.0000884	-1.53e-15

Solution

Variable	Critical Value
Vmax(15,25)	29.12334
VEES(1,4)	1.6281731
Tburst(0.5,2)	1.6837003
Vbalance(2,8)	5
TAIPEndur(5,25)	15
Tbatt(50,100)	75
Vloiter(2,6)	4
Loadout(0,16)	8

Solution is a SaddlePoint

Critical values outside data range

Predicted Value at Solution 0.8645603

Canonical Curvature

Eigenvalues and Eigenvectors

Eigenvalue	0.0018	0.0001	0.0001	0.0001	0.0001	0.0001	-0.0343	-0.0897
Vmax(15,25)	-0.06677	0.00000	-0.00000	-0.00000	-0.00000	-0.00000	0.98580	0.15409
VEES(1,4)	0.98480	-0.00000	0.00000	-0.00000	0.00000	0.00000	0.04029	0.16895
Tburst(0.5,2)	-0.16034	0.00000	-0.00000	0.00000	-0.00000	-0.00000	-0.16303	0.97350
Vbalance(2,8)	-0.00000	0.38632	-0.57541	0.52378	0.48517	0.09960	0.00000	0.00000
TAIPEndur(5,25)	-0.00000	-0.00911	-0.00541	0.00644	-0.20695	0.97827	0.00000	0.00000
Tbatt(50,100)	-0.00000	-0.35977	0.62532	0.48383	0.48531	0.09959	0.00000	0.00000
Vloiter(2,6)	-0.00000	-0.60846	-0.39360	-0.46473	0.49874	0.10072	0.00000	0.00000
Loadout(0,16)	-0.00000	0.59247	0.35061	-0.52491	0.48736	0.11401	0.00000	0.00000

Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.7189175	0.002522	285.02	<.0001
Vmax(15,25)&RS	0.1339769	0.000805	166.52	<.0001
VEES(1,4)&RS	0.0299846	0.000805	37.27	<.0001
Tburst(0.5,2)&RS	0.1111077	0.000805	138.10	<.0001
Vbalance(2,8)&RS	-1.53e-15	0.000805	-0.00	1.0000
TAIPEndur(5,25)&RS	-1.53e-15	0.000805	-0.00	1.0000
Tbatt(50,100)&RS	-1.53e-15	0.000805	-0.00	1.0000
Vloiter(2,6)&RS	-1.31e-15	0.000805	-0.00	1.0000
Loadout(0,16)&RS	-1.53e-15	0.000805	-0.00	1.0000
Vmax(15,25)*VEES(1,4)	-0.007625	0.000811	-9.40	<.0001
Vmax(15,25)*Tburst(0.5,2)	-0.015875	0.000811	-19.58	<.0001
VEES(1,4)*Tburst(0.5,2)	-0.029625	0.000811	-36.54	<.0001
Vmax(15,25)*Vbalance(2,8)	-4.88e-15	0.000811	-0.00	1.0000
VEES(1,4)*Vbalance(2,8)	-3.55e-15	0.000811	-0.00	1.0000
Tburst(0.5,2)*Vbalance(2,8)	-3.55e-15	0.000811	-0.00	1.0000
Vmax(15,25)*TAIPEndur(5,25)	-4e-15	0.000811	-0.00	1.0000
VEES(1,4)*TAIPEndur(5,25)	-5.33e-15	0.000811	-0.00	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	-4.88e-15	0.000811	-0.00	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	-4.44e-15	0.000811	-0.00	1.0000
Vmax(15,25)*Tbatt(50,100)	-4.44e-15	0.000811	-0.00	1.0000
VEES(1,4)*Tbatt(50,100)	-4.44e-15	0.000811	-0.00	1.0000
Tburst(0.5,2)*Tbatt(50,100)	-4.44e-15	0.000811	-0.00	1.0000
Vbalance(2,8)*Tbatt(50,100)	-4.88e-15	0.000811	-0.00	1.0000
TAIPEndur(5,25)*Tbatt(50,100)	-4.44e-15	0.000811	-0.00	1.0000
Vmax(15,25)*Vloiter(2,6)	-4.44e-15	0.000811	-0.00	1.0000

final- Fit Least Squares

Least Squares Fit

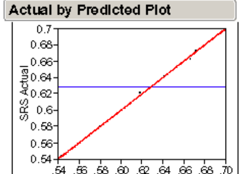
Response STS-ES

Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob>|t|
VEES(1,4)*Vloiter(2,6)	-4.44e-15	0.000811	-0.00	1.0000
Tburst(0.5,2)*Vloiter(2,6)	-4e-15	0.000811	-0.00	1.0000
Vbalance(2,8)*Vloiter(2,6)	-4.88e-15	0.000811	-0.00	1.0000
TAIPEndur(5,25)*Vloiter(2,6)	-4.44e-15	0.000811	-0.00	1.0000
Tbatt(50,100)*Vloiter(2,6)	-4.44e-15	0.000811	-0.00	1.0000
Vmax(15,25)*Loadout(0,16)	-4e-15	0.000811	-0.00	1.0000
VEES(1,4)*Loadout(0,16)	-4.44e-15	0.000811	-0.00	1.0000
Tburst(0.5,2)*Loadout(0,16)	-4.88e-15	0.000811	-0.00	1.0000
Vbalance(2,8)*Loadout(0,16)	-4.44e-15	0.000811	-0.00	1.0000
TAIPEndur(5,25)*Loadout(0,16)	-5.33e-15	0.000811	-0.00	1.0000
Tbatt(50,100)*Loadout(0,16)	-4.88e-15	0.000811	-0.00	1.0000
Vloiter(2,6)*Loadout(0,16)	-5.33e-15	0.000811	-0.00	1.0000
Vmax(15,25)*Vmax(15,25)	-0.035412	0.006077	-5.83	<.0001
VEES(1,4)*VEES(1,4)	-0.000912	0.006077	-0.15	0.8811
Tburst(0.5,2)*Tburst(0.5,2)	-0.085912	0.006077	-14.14	<.0001
Vbalance(2,8)*Vbalance(2,8)	0.0000884	0.006077	0.01	0.9884
TAIPEndur(5,25)*TAIPEndur(5,25)	0.0000884	0.006077	0.01	0.9884
Tbatt(50,100)*Tbatt(50,100)	0.0000884	0.006077	0.01	0.9884
Vloiter(2,6)*Vloiter(2,6)	0.0000884	0.006077	0.01	0.9884
Loadout(0,16)*Loadout(0,16)	0.0000884	0.006077	0.01	0.9884

Response SRS

Actual by Predicted Plot



SRS Predicted P<.0001 RSq=1.00
RMSE=0.0005

Summary of Fit

RSquare	0.999923
RSquare Adj	0.999889
Root Mean Square Error	0.000537
Mean of Response	0.627772
Observations (or Sum Wgts)	145

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	44	0.37375069	0.008494	29493.01
Error	100	0.00002880	0.000000	Prob > F
C. Total	144	0.37377949		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.640967	0.000148	4343.6	<.0001
Vmax(15,25)&RS	-2.19e-16	0.000047	-0.00	1.0000
VEES(1,4)&RS	0	0.000047	0.00	1.0000
Tburst(0.5,2)&RS	2.186e-16	0.000047	0.00	1.0000
Vbalance(2,8)&RS	-0.045762	0.000047	-972.2	<.0001
TAIPEndur(5,25)&RS	0.0241923	0.000047	513.98	<.0001
Tbatt(50,100)&RS	-2.19e-16	0.000047	-0.00	1.0000
Vloiter(2,6)&RS	-2.19e-16	0.000047	-0.00	1.0000
Loadout(0,16)&RS	0	0.000047	0.00	1.0000
Vmax(15,25)*VEES(1,4)	1.776e-15	0.000047	0.00	1.0000
Vmax(15,25)*Tburst(0.5,2)	1.776e-15	0.000047	0.00	1.0000
VEES(1,4)*Tburst(0.5,2)	2.22e-15	0.000047	0.00	1.0000
Vmax(15,25)*Vbalance(2,8)	3.109e-15	0.000047	0.00	1.0000
VEES(1,4)*Vbalance(2,8)	3.109e-15	0.000047	0.00	1.0000
Tburst(0.5,2)*Vbalance(2,8)	2.665e-15	0.000047	0.00	1.0000
Vmax(15,25)*TAIPEndur(5,25)	1.776e-15	0.000047	0.00	1.0000
VEES(1,4)*TAIPEndur(5,25)	1.776e-15	0.000047	0.00	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	2.22e-15	0.000047	0.00	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	0.01325	0.000047	279.33	<.0001
Vmax(15,25)*Tbatt(50,100)	2.665e-15	0.000047	0.00	1.0000
VEES(1,4)*Tbatt(50,100)	3.553e-15	0.000047	0.00	1.0000
Tburst(0.5,2)*Tbatt(50,100)	3.109e-15	0.000047	0.00	1.0000
Vbalance(2,8)*Tbatt(50,100)	2.22e-15	0.000047	0.00	1.0000
TAIPEndur(5,25)*Tbatt(50,100)	2.665e-15	0.000047	0.00	1.0000
Vmax(15,25)*Vloiter(2,6)	2.665e-15	0.000047	0.00	1.0000
VEES(1,4)*Vloiter(2,6)	3.553e-15	0.000047	0.00	1.0000
Tburst(0.5,2)*Vloiter(2,6)	1.776e-15	0.000047	0.00	1.0000
Vbalance(2,8)*Vloiter(2,6)	2.22e-15	0.000047	0.00	1.0000
TAIPEndur(5,25)*Vloiter(2,6)	2.22e-15	0.000047	0.00	1.0000
Tbatt(50,100)*Vloiter(2,6)	1.332e-15	0.000047	0.00	1.0000
Vmax(15,25)*Loadout(0,16)	2.665e-15	0.000047	0.00	1.0000
VEES(1,4)*Loadout(0,16)	3.553e-15	0.000047	0.00	1.0000
Tburst(0.5,2)*Loadout(0,16)	3.109e-15	0.000047	0.00	1.0000
Vbalance(2,8)*Loadout(0,16)	2.22e-15	0.000047	0.00	1.0000
TAIPEndur(5,25)*Loadout(0,16)	3.553e-15	0.000047	0.00	1.0000
Tbatt(50,100)*Loadout(0,16)	1.332e-15	0.000047	0.00	1.0000
Vloiter(2,6)*Loadout(0,16)	3.997e-15	0.000047	0.00	1.0000
Vmax(15,25)*Vmax(15,25)	0.0000354	0.000356	0.10	0.9209
VEES(1,4)*VEES(1,4)	0.0000354	0.000356	0.10	0.9209
Tburst(0.5,2)*Tburst(0.5,2)	0.0000354	0.000356	0.10	0.9209
Vbalance(2,8)*Vbalance(2,8)	-0.015465	0.000356	-43.50	<.0001
TAIPEndur(5,25)*TAIPEndur(5,25)	0.0005354	0.000356	1.51	0.1352
Tbatt(50,100)*Tbatt(50,100)	0.0000354	0.000356	0.10	0.9209
Vloiter(2,6)*Vloiter(2,6)	0.0000354	0.000356	0.10	0.9209
Loadout(0,16)*Loadout(0,16)	0.0000354	0.000356	0.10	0.9209

final- Fit Least Squares

Least Squares Fit

Response SRS

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Vmax(15,25)&RS	1	1	0.00000000	0.0000	1.0000
VEES(1,4)&RS	1	1	0.00000000	0.0000	1.0000
Tburst(0.5,2)&RS	1	1	0.00000000	0.0000	1.0000
Vbalance(2,8)&RS	1	1	0.27223539	945223.1	<.0001
TAIPEndur(5,25)&RS	1	1	0.07608481	264172.5	<.0001
Tbatt(50,100)&RS	1	1	0.00000000	0.0000	1.0000
Vloiter(2,6)&RS	1	1	0.00000000	0.0000	1.0000
Loadout(0,16)&RS	1	1	0.00000000	0.0000	1.0000
Vmax(15,25)*VEES(1,4)	1	1	0.00000000	0.0000	1.0000
Vmax(15,25)*Tburst(0.5,2)	1	1	0.00000000	0.0000	1.0000
VEES(1,4)*Tburst(0.5,2)	1	1	0.00000000	0.0000	1.0000
Vmax(15,25)*Vbalance(2,8)	1	1	0.00000000	0.0000	1.0000
VEES(1,4)*Vbalance(2,8)	1	1	0.00000000	0.0000	1.0000
Tburst(0.5,2)*Vbalance(2,8)	1	1	0.00000000	0.0000	1.0000
Vmax(15,25)*TAIPEndur(5,25)	1	1	0.00000000	0.0000	1.0000
VEES(1,4)*TAIPEndur(5,25)	1	1	0.00000000	0.0000	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	1	1	0.00000000	0.0000	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	1	1	0.02247200	78024.58	<.0001
Vmax(15,25)*Tbatt(50,100)	1	1	0.00000000	0.0000	1.0000
VEES(1,4)*Tbatt(50,100)	1	1	0.00000000	0.0000	1.0000
Tburst(0.5,2)*Tbatt(50,100)	1	1	0.00000000	0.0000	1.0000
Vbalance(2,8)*Tbatt(50,100)	1	1	0.00000000	0.0000	1.0000
TAIPEndur(5,25)*Tbatt(50,100)	1	1	0.00000000	0.0000	1.0000
Vmax(15,25)*Vloiter(2,6)	1	1	0.00000000	0.0000	1.0000
VEES(1,4)*Vloiter(2,6)	1	1	0.00000000	0.0000	1.0000
Tburst(0.5,2)*Vloiter(2,6)	1	1	0.00000000	0.0000	1.0000
Vbalance(2,8)*Vloiter(2,6)	1	1	0.00000000	0.0000	1.0000
TAIPEndur(5,25)*Vloiter(2,6)	1	1	0.00000000	0.0000	1.0000
Tbatt(50,100)*Vloiter(2,6)	1	1	0.00000000	0.0000	1.0000
Vmax(15,25)*Loadout(0,16)	1	1	0.00000000	0.0000	1.0000
VEES(1,4)*Loadout(0,16)	1	1	0.00000000	0.0000	1.0000
Tburst(0.5,2)*Loadout(0,16)	1	1	0.00000000	0.0000	1.0000
Vbalance(2,8)*Loadout(0,16)	1	1	0.00000000	0.0000	1.0000
TAIPEndur(5,25)*Loadout(0,16)	1	1	0.00000000	0.0000	1.0000
Tbatt(50,100)*Loadout(0,16)	1	1	0.00000000	0.0000	1.0000
Vloiter(2,6)*Loadout(0,16)	1	1	0.00000000	0.0000	1.0000
Vmax(15,25)*Vmax(15,25)	1	1	0.00000000	0.0099	0.9209
VEES(1,4)*VEES(1,4)	1	1	0.00000000	0.0099	0.9209
Tburst(0.5,2)*Tburst(0.5,2)	1	1	0.00000000	0.0099	0.9209
Vbalance(2,8)*Vbalance(2,8)	1	1	0.00054498	1892.199	<.0001
TAIPEndur(5,25)*TAIPEndur(5,25)	1	1	0.00000065	2.2678	0.1352
Tbatt(50,100)*Tbatt(50,100)	1	1	0.00000000	0.0099	0.9209
Vloiter(2,6)*Vloiter(2,6)	1	1	0.00000000	0.0099	0.9209
Loadout(0,16)*Loadout(0,16)	1	1	0.00000000	0.0099	0.9209

Response Surface

Coef

	Vmax(15,25)	VEES(1,4)	Tburst(0.5,2)	Vbalance(2,8)	TAIPEndur(5,25)	Tbatt(50,100)	Vloiter(2,6)	Loadout(0,16)	SRS
Vmax(15,25)	0.0000354	1.776e-15	1.776e-15	3.109e-15	1.776e-15	2.665e-15	2.665e-15	2.665e-15	-2.19e-16
VEES(1,4)	.	0.0000354	2.22e-15	3.109e-15	1.776e-15	3.553e-15	3.553e-15	3.553e-15	0
Tburst(0.5,2)	.	.	0.0000354	2.665e-15	2.22e-15	3.109e-15	1.776e-15	3.109e-15	2.186e-16
Vbalance(2,8)	.	.	.	-0.015465	0.01325	2.22e-15	2.22e-15	2.22e-15	-0.045762
TAIPEndur(5,25)	.	.	.	.	0.0005354	2.665e-15	2.22e-15	3.553e-15	0.0241923
Tbatt(50,100)	.	.	.	.	.	0.0000354	1.332e-15	1.332e-15	-2.19e-16
Vloiter(2,6)	.	.	.	.	.	.	0.0000354	3.997e-15	-2.19e-16
Loadout(0,16)	.	.	.	.	.	.	.	0.0000354	0

Solution

Variable	Critical Value
Vmax(15,25)	20
VEES(1,4)	2.5
Tburst(0.5,2)	1.25
Vbalance(2,8)	-0.312641
TAIPEndur(5,25)	8.1996451
Tbatt(50,100)	75
Vloiter(2,6)	4
Loadout(0,16)	8

Solution is a SaddlePoint

Critical values outside data range

Predicted Value at Solution 0.6732603

Canonical Curvature

Eigenvalues and Eigenvectors

Eigenvalue	0.0029	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0179
Vmax(15,25)	0.00000	0.04169	0.45198	0.02494	0.70682	-0.50970	0.18430	-0.00000
VEES(1,4)	0.00000	0.98042	-0.13459	0.00025	0.00445	-0.02685	0.01469	-0.00000
Tburst(0.5,2)	0.00000	0.04907	0.44574	0.36028	-0.61460	-0.26544	0.47001	-0.00000
Vbalance(2,8)	0.33898	-0.00000	-0.00000	-0.00000	0.00000	-0.00000	0.00000	0.94079
TAIPEndur(5,25)	0.94079	-0.00000	-0.00000	-0.00000	0.00000	-0.00000	0.00000	-0.33898
Tbatt(50,100)	0.00000	0.06872	0.37514	0.66916	0.18334	0.49667	-0.35565	-0.00000
Vloiter(2,6)	0.00000	0.07089	0.44462	-0.51589	0.06541	0.60857	0.39561	-0.00000
Loadout(0,16)	0.00000	0.07195	0.49042	-0.39453	-0.29113	-0.22802	-0.67964	-0.00000

Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.640967	0.000148	4343.57	<.0001
Vmax(15,25)&RS	-2.19e-16	0.000047	-0.00	1.0000
VEES(1,4)&RS	0	0.000047	0.00	1.0000
Tburst(0.5,2)&RS	2.186e-16	0.000047	0.00	1.0000
Vbalance(2,8)&RS	-0.045762	0.000047	-972.23	<.0001
TAIPEndur(5,25)&RS	0.0241923	0.000047	513.98	<.0001
Tbatt(50,100)&RS	-2.19e-16	0.000047	-0.00	1.0000
Vloiter(2,6)&RS	-2.19e-16	0.000047	-0.00	1.0000
Loadout(0,16)&RS	0	0.000047	0.00	1.0000
Vmax(15,25)*VEES(1,4)	1.776e-15	0.000047	0.00	1.0000
Vmax(15,25)*Tburst(0.5,2)	1.776e-15	0.000047	0.00	1.0000
VEES(1,4)*Tburst(0.5,2)	2.22e-15	0.000047	0.00	1.0000
Vmax(15,25)*Vbalance(2,8)	3.109e-15	0.000047	0.00	1.0000
VEES(1,4)*Vbalance(2,8)	3.109e-15	0.000047	0.00	1.0000

final- Fit Least Squares

Least Squares Fit

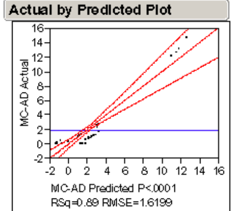
Response SRS

Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob>|t|
Tburst(0.5,2)*Vbalance(2,8)	2.665e-15	0.000047	0.00	1.0000
Vmax(15,25)*TAIPEndur(5,25)	1.776e-15	0.000047	0.00	1.0000
VEES(1,4)*TAIPEndur(5,25)	1.776e-15	0.000047	0.00	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	2.22e-15	0.000047	0.00	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	0.01325	0.000047	279.33	<.0001
Vmax(15,25)*Tbatt(50,100)	2.665e-15	0.000047	0.00	1.0000
VEES(1,4)*Tbatt(50,100)	3.553e-15	0.000047	0.00	1.0000
Tburst(0.5,2)*Tbatt(50,100)	3.109e-15	0.000047	0.00	1.0000
Vbalance(2,8)*Tbatt(50,100)	2.22e-15	0.000047	0.00	1.0000
TAIPEndur(5,25)*Tbatt(50,100)	2.665e-15	0.000047	0.00	1.0000
Vmax(15,25)*Vloiter(2,6)	2.665e-15	0.000047	0.00	1.0000
VEES(1,4)*Vloiter(2,6)	3.553e-15	0.000047	0.00	1.0000
Tburst(0.5,2)*Vloiter(2,6)	1.776e-15	0.000047	0.00	1.0000
Vbalance(2,8)*Vloiter(2,6)	2.22e-15	0.000047	0.00	1.0000
TAIPEndur(5,25)*Vloiter(2,6)	2.22e-15	0.000047	0.00	1.0000
Tbatt(50,100)*Vloiter(2,6)	1.332e-15	0.000047	0.00	1.0000
Vmax(15,25)*Loadout(0,16)	2.665e-15	0.000047	0.00	1.0000
VEES(1,4)*Loadout(0,16)	3.553e-15	0.000047	0.00	1.0000
Tburst(0.5,2)*Loadout(0,16)	3.109e-15	0.000047	0.00	1.0000
Vbalance(2,8)*Loadout(0,16)	2.22e-15	0.000047	0.00	1.0000
TAIPEndur(5,25)*Loadout(0,16)	3.553e-15	0.000047	0.00	1.0000
Tbatt(50,100)*Loadout(0,16)	1.332e-15	0.000047	0.00	1.0000
Vloiter(2,6)*Loadout(0,16)	3.997e-15	0.000047	0.00	1.0000
Vmax(15,25)*Vmax(15,25)	0.0000354	0.000356	0.10	0.9209
VEES(1,4)*VEES(1,4)	0.0000354	0.000356	0.10	0.9209
Tburst(0.5,2)*Tburst(0.5,2)	0.0000354	0.000356	0.10	0.9209
Vbalance(2,8)*Vbalance(2,8)	-0.015465	0.000356	-43.50	<.0001
TAIPEndur(5,25)*TAIPEndur(5,25)	0.0005354	0.000356	1.51	0.1352
Tbatt(50,100)*Tbatt(50,100)	0.0000354	0.000356	0.10	0.9209
Vloiter(2,6)*Vloiter(2,6)	0.0000354	0.000356	0.10	0.9209
Loadout(0,16)*Loadout(0,16)	0.0000354	0.000356	0.10	0.9209

Response MC-AD

Actual by Predicted Plot



Summary of Fit

RSquare	0.888534
RSquare Adj	0.839489
Root Mean Square Error	1.619853
Mean of Response	1.823869
Observations (or Sum Wgts)	145

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	44	2091.6197	47.5368	18.1167
Error	100	262.3923	2.6239	Prob > F
C. Total	144	2354.0121		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1.0965273	0.445409	2.46	0.0155
Vmax(15,25)&RS	8.745e-16	0.14207	0.00	1.0000
VEES(1,4)&RS	0	0.14207	0.00	1.0000
Tburst(0.5,2)&RS	0	0.14207	0.00	1.0000
Vbalance(2,8)&RS	1.5703231	0.14207	11.05	<.0001
TAIPEndur(5,25)&RS	1.6546077	0.14207	11.65	<.0001
Tbatt(50,100)&RS	0.1095692	0.14207	0.77	0.4424
Vloiter(2,6)&RS	0.1612077	0.14207	1.13	0.2592
Loadout(0,16)&RS	1.8955923	0.14207	13.34	<.0001
Vmax(15,25)*VEES(1,4)	1.776e-15	0.143176	0.00	1.0000
Vmax(15,25)*Tburst(0.5,2)	1.776e-15	0.143176	0.00	1.0000
VEES(1,4)*Tburst(0.5,2)	3.553e-15	0.143176	0.00	1.0000
Vmax(15,25)*Vbalance(2,8)	3.553e-15	0.143176	0.00	1.0000
VEES(1,4)*Vbalance(2,8)	3.553e-15	0.143176	0.00	1.0000
Tburst(0.5,2)*Vbalance(2,8)	3.553e-15	0.143176	0.00	1.0000
Vmax(15,25)*TAIPEndur(5,25)	1.776e-15	0.143176	0.00	1.0000
VEES(1,4)*TAIPEndur(5,25)	1.776e-15	0.143176	0.00	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	1.776e-15	0.143176	0.00	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	1.4133125	0.143176	9.87	<.0001
Vmax(15,25)*Tbatt(50,100)	1.776e-15	0.143176	0.00	1.0000
VEES(1,4)*Tbatt(50,100)	1.776e-15	0.143176	0.00	1.0000
Tburst(0.5,2)*Tbatt(50,100)	3.553e-15	0.143176	0.00	1.0000
Vbalance(2,8)*Tbatt(50,100)	0.0543125	0.143176	0.38	0.7052
TAIPEndur(5,25)*Tbatt(50,100)	0.0603125	0.143176	0.42	0.6745
Vmax(15,25)*Vloiter(2,6)	1.776e-15	0.143176	0.00	1.0000
VEES(1,4)*Vloiter(2,6)	1.776e-15	0.143176	0.00	1.0000
Tburst(0.5,2)*Vloiter(2,6)	0	0.143176	0.00	1.0000
Vbalance(2,8)*Vloiter(2,6)	0.0814375	0.143176	0.57	0.5708
TAIPEndur(5,25)*Vloiter(2,6)	0.0905625	0.143176	0.63	0.5285
Tbatt(50,100)*Vloiter(2,6)	0.0604375	0.143176	0.42	0.6738
Vmax(15,25)*Loadout(0,16)	1.776e-15	0.143176	0.00	1.0000
VEES(1,4)*Loadout(0,16)	1.776e-15	0.143176	0.00	1.0000
Tburst(0.5,2)*Loadout(0,16)	3.553e-15	0.143176	0.00	1.0000
Vbalance(2,8)*Loadout(0,16)	1.577	0.143176	11.01	<.0001
TAIPEndur(5,25)*Loadout(0,16)	1.660625	0.143176	11.60	<.0001
Tbatt(50,100)*Loadout(0,16)	0.109625	0.143176	0.77	0.4457

final- Fit Least Squares

Least Squares Fit

Response MC-AD

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Vloiter(2,6)*Loadout(0,16)	0.16125	0.143176	1.13	0.2628
Vmax(15,25)*Vmax(15,25)	0.0187207	1.073066	0.02	0.9861
VEES(1,4)*VEES(1,4)	0.0187207	1.073066	0.02	0.9861
Tburst(0.5,2)*Tburst(0.5,2)	0.0187207	1.073066	0.02	0.9861
Vbalance(2,8)*Vbalance(2,8)	0.3117207	1.073066	0.29	0.7720
TAIPEndur(5,25)*TAIPEndur(5,25)	0.3802207	1.073066	0.35	0.7238
Tbatt(50,100)*Tbatt(50,100)	0.0207207	1.073066	0.02	0.9846
Vloiter(2,6)*Vloiter(2,6)	0.0242207	1.073066	0.02	0.9820
Loadout(0,16)*Loadout(0,16)	0.0182207	1.073066	0.02	0.9865

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Vmax(15,25)&RS	1	1	0.00000	0.0000	1.0000
VEES(1,4)&RS	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)&RS	1	1	0.00000	0.0000	1.0000
Vbalance(2,8)&RS	1	1	320.56889	122.1716	<.0001
TAIPEndur(5,25)&RS	1	1	355.90446	135.6383	<.0001
Tbatt(50,100)&RS	1	1	1.56070	0.5948	0.4424
Vloiter(2,6)&RS	1	1	3.37843	1.2875	0.2592
Loadout(0,16)&RS	1	1	467.12513	178.0254	<.0001
Vmax(15,25)*VEES(1,4)	1	1	0.00000	0.0000	1.0000
Vmax(15,25)*Tburst(0.5,2)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*Tburst(0.5,2)	1	1	0.00000	0.0000	1.0000
Vmax(15,25)*Vbalance(2,8)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*Vbalance(2,8)	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)*Vbalance(2,8)	1	1	0.00000	0.0000	1.0000
Vmax(15,25)*TAIPEndur(5,25)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*TAIPEndur(5,25)	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	1	1	0.00000	0.0000	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	1	1	255.67388	97.4395	<.0001
Vmax(15,25)*Tbatt(50,100)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*Tbatt(50,100)	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)*Tbatt(50,100)	1	1	0.00000	0.0000	1.0000
Vbalance(2,8)*Tbatt(50,100)	1	1	0.37758	0.1439	0.7052
TAIPEndur(5,25)*Tbatt(50,100)	1	1	0.46561	0.1774	0.6745
Vmax(15,25)*Vloiter(2,6)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*Vloiter(2,6)	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)*Vloiter(2,6)	1	1	0.00000	0.0000	1.0000
Vbalance(2,8)*Vloiter(2,6)	1	1	0.84890	0.3235	0.5708
TAIPEndur(5,25)*Vloiter(2,6)	1	1	1.04980	0.4001	0.5285
Tbatt(50,100)*Vloiter(2,6)	1	1	0.46754	0.1782	0.6738
Vmax(15,25)*Loadout(0,16)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*Loadout(0,16)	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)*Loadout(0,16)	1	1	0.00000	0.0000	1.0000
Vbalance(2,8)*Loadout(0,16)	1	1	318.32691	121.3172	<.0001
TAIPEndur(5,25)*Loadout(0,16)	1	1	352.98245	134.5247	<.0001
Tbatt(50,100)*Loadout(0,16)	1	1	1.53826	0.5862	0.4457
Vloiter(2,6)*Loadout(0,16)	1	1	3.32820	1.2684	0.2628
Vmax(15,25)*Vmax(15,25)	1	1	0.00080	0.0003	0.9861
VEES(1,4)*VEES(1,4)	1	1	0.00080	0.0003	0.9861
Tburst(0.5,2)*Tburst(0.5,2)	1	1	0.00080	0.0003	0.9861
Vbalance(2,8)*Vbalance(2,8)	1	1	0.22143	0.0844	0.7720
TAIPEndur(5,25)*TAIPEndur(5,25)	1	1	0.32943	0.1256	0.7238
Tbatt(50,100)*Tbatt(50,100)	1	1	0.00098	0.0004	0.9846
Vloiter(2,6)*Vloiter(2,6)	1	1	0.00134	0.0005	0.9820
Loadout(0,16)*Loadout(0,16)	1	1	0.00076	0.0003	0.9865

Response Surface

Coef

	Vmax(15,25)	VEES(1,4)	Tburst(0.5,2)	Vbalance(2,8)	TAIPEndur(5,25)	Tbatt(50,100)	Vloiter(2,6)	Loadout(0,16)	MC-AD
Vmax(15,25)	0.0187207	1.776e-15	1.776e-15	3.553e-15	1.776e-15	1.776e-15	1.776e-15	1.776e-15	8.745e-16
VEES(1,4)	.	0.0187207	3.553e-15	3.553e-15	1.776e-15	1.776e-15	1.776e-15	1.776e-15	0
Tburst(0.5,2)	.	.	0.0187207	3.553e-15	1.776e-15	3.553e-15	0	3.553e-15	0
Vbalance(2,8)	.	.	.	0.3117207	1.4133125	0.0543125	0.0814375	1.577	1.5703231
TAIPEndur(5,25)	.	.	.	.	0.3802207	0.0603125	0.0905625	1.660625	1.6546077
Tbatt(50,100)	.	.	.	.	.	0.0207207	0.0604375	0.109625	0.1095692
Vloiter(2,6)	.	.	.	.	.	.	0.0242207	0.16125	0.1612077
Loadout(0,16)	.	.	.	.	.	.	.	0.0182207	1.8955923

Solution

Variable	Critical Value
Vmax(15,25)	20
VEES(1,4)	2.5
Tburst(0.5,2)	1.25
Vbalance(2,8)	3.3304934
TAIPEndur(5,25)	9.2085302
Tbatt(50,100)	65.727452
Vloiter(2,6)	3.9160153
Loadout(0,16)	6.0657119

Solution is a SaddlePoint

Predicted Value at Solution -0.072416

Canonical Curvature

Eigenvalues and Eigenvectors

Eigenvalue	1.7999	0.0477	0.0187	0.0187	0.0187	-0.0079	-0.3615	-0.7231
Vmax(15,25)	0.00000	0.00000	0.01601	0.59890	0.80067	-0.00000	-0.00000	0.00000
VEES(1,4)	0.00000	0.00000	0.99937	-0.03490	0.00611	-0.00000	-0.00000	-0.00000
Tburst(0.5,2)	0.00000	0.00000	0.03160	0.80006	-0.59908	-0.00000	-0.00000	-0.00000
Vbalance(2,8)	0.57754	-0.06110	0.00000	0.00000	0.00000	0.01101	0.72548	-0.36916
TAIPEndur(5,25)	0.60697	-0.05682	0.00000	0.00000	0.00000	0.00989	-0.68821	-0.39323
Tbatt(50,100)	0.03672	0.69507	-0.00000	-0.00000	-0.00000	0.71734	0.00282	-0.03065
Vloiter(2,6)	0.05396	0.71400	-0.00000	-0.00000	0.00000	-0.69656	0.00461	-0.04548
Loadout(0,16)	0.54202	0.01057	-0.00000	-0.00000	-0.00000	-0.00206	-0.00299	0.84029

Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1.0965273	0.445409	2.46	0.0155
Vmax(15,25)&RS	8.745e-16	0.14207	0.00	1.0000
VEES(1,4)&RS	0	0.14207	0.00	1.0000

final- Fit Least Squares

Least Squares Fit

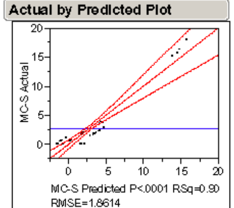
Response MC-AD

Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob>|t|
Tburst(0.5,2)&RS	0	0.14207	0.00	1.0000
Vbalance(2,8)&RS	1.5703231	0.14207	11.05	<.0001
TAIPEndur(5,25)&RS	1.6546077	0.14207	11.65	<.0001
Tbatt(50,100)&RS	0.1095692	0.14207	0.77	0.4424
Vloiter(2,6)&RS	0.1612077	0.14207	1.13	0.2592
Loadout(0,16)&RS	1.8955923	0.14207	13.34	<.0001
Vmax(15,25)*VEES(1,4)	1.776e-15	0.143176	0.00	1.0000
Vmax(15,25)*Tburst(0.5,2)	1.776e-15	0.143176	0.00	1.0000
VEES(1,4)*Tburst(0.5,2)	3.553e-15	0.143176	0.00	1.0000
Vmax(15,25)*Vbalance(2,8)	3.553e-15	0.143176	0.00	1.0000
VEES(1,4)*Vbalance(2,8)	3.553e-15	0.143176	0.00	1.0000
Tburst(0.5,2)*Vbalance(2,8)	3.553e-15	0.143176	0.00	1.0000
Vmax(15,25)*TAIPEndur(5,25)	1.776e-15	0.143176	0.00	1.0000
VEES(1,4)*TAIPEndur(5,25)	1.776e-15	0.143176	0.00	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	1.776e-15	0.143176	0.00	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	1.4133125	0.143176	9.87	<.0001
Vmax(15,25)*Tbatt(50,100)	1.776e-15	0.143176	0.00	1.0000
VEES(1,4)*Tbatt(50,100)	1.776e-15	0.143176	0.00	1.0000
Tburst(0.5,2)*Tbatt(50,100)	3.553e-15	0.143176	0.00	1.0000
Vbalance(2,8)*Tbatt(50,100)	0.0543125	0.143176	0.38	0.7052
TAIPEndur(5,25)*Tbatt(50,100)	0.0603125	0.143176	0.42	0.6745
Vmax(15,25)*Vloiter(2,6)	1.776e-15	0.143176	0.00	1.0000
VEES(1,4)*Vloiter(2,6)	1.776e-15	0.143176	0.00	1.0000
Tburst(0.5,2)*Vloiter(2,6)	0	0.143176	0.00	1.0000
Vbalance(2,8)*Vloiter(2,6)	0.0814375	0.143176	0.57	0.5708
TAIPEndur(5,25)*Vloiter(2,6)	0.0905625	0.143176	0.63	0.5285
Tbatt(50,100)*Vloiter(2,6)	0.0604375	0.143176	0.42	0.6738
Vmax(15,25)*Loadout(0,16)	1.776e-15	0.143176	0.00	1.0000
VEES(1,4)*Loadout(0,16)	1.776e-15	0.143176	0.00	1.0000
Tburst(0.5,2)*Loadout(0,16)	3.553e-15	0.143176	0.00	1.0000
Vbalance(2,8)*Loadout(0,16)	1.577	0.143176	11.01	<.0001
TAIPEndur(5,25)*Loadout(0,16)	1.660625	0.143176	11.60	<.0001
Tbatt(50,100)*Loadout(0,16)	0.109625	0.143176	0.77	0.4457
Vloiter(2,6)*Loadout(0,16)	0.16125	0.143176	1.13	0.2628
Vmax(15,25)*Vmax(15,25)	0.0187207	1.073066	0.02	0.9861
VEES(1,4)*VEES(1,4)	0.0187207	1.073066	0.02	0.9861
Tburst(0.5,2)*Tburst(0.5,2)	0.0187207	1.073066	0.02	0.9861
Vbalance(2,8)*Vbalance(2,8)	0.3117207	1.073066	0.29	0.7720
TAIPEndur(5,25)*TAIPEndur(5,25)	0.3802207	1.073066	0.35	0.7238
Tbatt(50,100)*Tbatt(50,100)	0.0207207	1.073066	0.02	0.9846
Vloiter(2,6)*Vloiter(2,6)	0.0242207	1.073066	0.02	0.9820
Loadout(0,16)*Loadout(0,16)	0.0182207	1.073066	0.02	0.9865

Response MC-S

Actual by Predicted Plot



Summary of Fit

RSquare	0.903193
RSquare Adj	0.860598
Root Mean Square Error	1.861359
Mean of Response	2.542338
Observations (or Sum Wgts)	145

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	44	3232.4617	73.4650	21.2041
Error	100	346.4656	3.4647	Prob > F
C. Total	144	3578.9273		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1.8147254	0.511816	3.55	0.0006
Vmax(15,25)&RS	-8.75e-16	0.163252	-0.00	1.0000
VEES(1,4)&RS	-8.75e-16	0.163252	-0.00	1.0000
Tburst(0.5,2)&RS	0	0.163252	0.00	1.0000
Vbalance(2,8)&RS	1.8933154	0.163252	11.60	<.0001
TAIPEndur(5,25)&RS	2.0134923	0.163252	12.33	<.0001
Tbatt(50,100)&RS	0.1394385	0.163252	0.85	0.3951
Vloiter(2,6)&RS	0.2060231	0.163252	1.26	0.2099
Loadout(0,16)&RS	-2.614092	0.163252	-16.01	<.0001
Vmax(15,25)*VEES(1,4)	-8.88e-15	0.164522	-0.00	1.0000
Vmax(15,25)*Tburst(0.5,2)	-7.11e-15	0.164522	-0.00	1.0000
VEES(1,4)*Tburst(0.5,2)	-5.33e-15	0.164522	-0.00	1.0000
Vmax(15,25)*Vbalance(2,8)	-5.33e-15	0.164522	-0.00	1.0000
VEES(1,4)*Vbalance(2,8)	-5.33e-15	0.164522	-0.00	1.0000
Tburst(0.5,2)*Vbalance(2,8)	-3.55e-15	0.164522	-0.00	1.0000
Vmax(15,25)*TAIPEndur(5,25)	-5.33e-15	0.164522	-0.00	1.0000
VEES(1,4)*TAIPEndur(5,25)	-7.11e-15	0.164522	-0.00	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	-5.33e-15	0.164522	-0.00	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	1.6285625	0.164522	9.90	<.0001
Vmax(15,25)*Tbatt(50,100)	-7.11e-15	0.164522	-0.00	1.0000
VEES(1,4)*Tbatt(50,100)	-7.11e-15	0.164522	-0.00	1.0000
Tburst(0.5,2)*Tbatt(50,100)	-3.55e-15	0.164522	-0.00	1.0000
Vbalance(2,8)*Tbatt(50,100)	0.0543125	0.164522	0.33	0.7420
TAIPEndur(5,25)*Tbatt(50,100)	0.0603125	0.164522	0.37	0.7147
Vmax(15,25)*Vloiter(2,6)	-8.88e-15	0.164522	-0.00	1.0000

final- Fit Least Squares

Least Squares Fit

Response MC-S

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
VEES(1,4)*Vloiter(2,6)	-3.55e-15	0.164522	-0.00	1.0000
Tburst(0.5,2)*Vloiter(2,6)	-1.07e-14	0.164522	-0.00	1.0000
Vbalance(2,8)*Vloiter(2,6)	0.0815	0.164522	0.50	0.6214
TAIPEndur(5,25)*Vloiter(2,6)	0.0905	0.164522	0.55	0.5835
Tbatt(50,100)*Vloiter(2,6)	0.075375	0.164522	0.46	0.6478
Vmax(15,25)*Loadout(0,16)	-3.55e-15	0.164522	-0.00	1.0000
VEES(1,4)*Loadout(0,16)	-8.88e-15	0.164522	-0.00	1.0000
Tburst(0.5,2)*Loadout(0,16)	-7.11e-15	0.164522	-0.00	1.0000
Vbalance(2,8)*Loadout(0,16)	-1.9	0.164522	-11.55	<.0001
TAIPEndur(5,25)*Loadout(0,16)	-2.0195	0.164522	-12.27	<.0001
Tbatt(50,100)*Loadout(0,16)	-0.1395	0.164522	-0.85	0.3985
Vloiter(2,6)*Loadout(0,16)	-0.206063	0.164522	-1.25	0.2133
Vmax(15,25)*Vmax(15,25)	0.0185085	1.233051	0.02	0.9881
VEES(1,4)*VEES(1,4)	0.0185085	1.233051	0.02	0.9881
Tburst(0.5,2)*Tburst(0.5,2)	0.0185085	1.233051	0.02	0.9881
Vbalance(2,8)*Vbalance(2,8)	0.3120085	1.233051	0.25	0.8008
TAIPEndur(5,25)*TAIPEndur(5,25)	0.3805085	1.233051	0.31	0.7583
Tbatt(50,100)*Tbatt(50,100)	0.0210085	1.233051	0.02	0.9864
Vloiter(2,6)*Vloiter(2,6)	0.0240085	1.233051	0.02	0.9845
Loadout(0,16)*Loadout(0,16)	0.0185085	1.233051	0.02	0.9881

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
Vmax(15,25)&RS	1	1	0.00000	0.0000	1.0000
VEES(1,4)&RS	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)&RS	1	1	0.00000	0.0000	1.0000
Vbalance(2,8)&RS	1	1	466.00361	134.5021	<.0001
TAIPEndur(5,25)&RS	1	1	527.03967	152.1189	<.0001
Tbatt(50,100)&RS	1	1	2.52760	0.7295	0.3951
Vloiter(2,6)&RS	1	1	5.51792	1.5926	0.2099
Loadout(0,16)&RS	1	1	888.35222	256.4041	<.0001
Vmax(15,25)*VEES(1,4)	1	1	0.00000	0.0000	1.0000
Vmax(15,25)*Tburst(0.5,2)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*Tburst(0.5,2)	1	1	0.00000	0.0000	1.0000
Vmax(15,25)*Vbalance(2,8)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*Vbalance(2,8)	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)*Vbalance(2,8)	1	1	0.00000	0.0000	1.0000
Vmax(15,25)*TAIPEndur(5,25)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*TAIPEndur(5,25)	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	1	1	0.00000	0.0000	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	1	1	339.48362	97.9848	<.0001
Vmax(15,25)*Tbatt(50,100)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*Tbatt(50,100)	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)*Tbatt(50,100)	1	1	0.00000	0.0000	1.0000
Vbalance(2,8)*Tbatt(50,100)	1	1	0.37758	0.1090	0.7420
TAIPEndur(5,25)*Tbatt(50,100)	1	1	0.46561	0.1344	0.7147
Vmax(15,25)*Vloiter(2,6)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*Vloiter(2,6)	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)*Vloiter(2,6)	1	1	0.00000	0.0000	1.0000
Vbalance(2,8)*Vloiter(2,6)	1	1	0.85021	0.2454	0.6214
TAIPEndur(5,25)*Vloiter(2,6)	1	1	1.04835	0.3026	0.5835
Tbatt(50,100)*Vloiter(2,6)	1	1	0.72722	0.2099	0.6478
Vmax(15,25)*Loadout(0,16)	1	1	0.00000	0.0000	1.0000
VEES(1,4)*Loadout(0,16)	1	1	0.00000	0.0000	1.0000
Tburst(0.5,2)*Loadout(0,16)	1	1	0.00000	0.0000	1.0000
Vbalance(2,8)*Loadout(0,16)	1	1	462.08000	133.3696	<.0001
TAIPEndur(5,25)*Loadout(0,16)	1	1	522.03267	150.6737	<.0001
Tbatt(50,100)*Loadout(0,16)	1	1	2.49091	0.7189	0.3985
Vloiter(2,6)*Loadout(0,16)	1	1	5.43510	1.5687	0.2133
Vmax(15,25)*Vmax(15,25)	1	1	0.00078	0.0002	0.9881
VEES(1,4)*VEES(1,4)	1	1	0.00078	0.0002	0.9881
Tburst(0.5,2)*Tburst(0.5,2)	1	1	0.00078	0.0002	0.9881
Vbalance(2,8)*Vbalance(2,8)	1	1	0.22184	0.0640	0.8008
TAIPEndur(5,25)*TAIPEndur(5,25)	1	1	0.32993	0.0952	0.7583
Tbatt(50,100)*Tbatt(50,100)	1	1	0.00101	0.0003	0.9864
Vloiter(2,6)*Vloiter(2,6)	1	1	0.00131	0.0004	0.9845
Loadout(0,16)*Loadout(0,16)	1	1	0.00078	0.0002	0.9881

Response Surface

Coef

	Vmax(15,25)	VEES(1,4)	Tburst(0.5,2)	Vbalance(2,8)	TAIPEndur(5,25)	Tbatt(50,100)	Vloiter(2,6)	Loadout(0,16)	MC-S
Vmax(15,25)	0.0185085	-8.88e-15	-7.11e-15	-5.33e-15	-5.33e-15	-7.11e-15	-8.88e-15	-3.55e-15	-8.75e-16
VEES(1,4)	.	0.0185085	-5.33e-15	-5.33e-15	-7.11e-15	-7.11e-15	-3.55e-15	-8.88e-15	-8.75e-16
Tburst(0.5,2)	.	.	0.0185085	-3.55e-15	-5.33e-15	-3.55e-15	-1.07e-14	-7.11e-15	0
Vbalance(2,8)	.	.	.	0.3120085	1.6285625	0.0543125	0.0815	-1.9	1.8883154
TAIPEndur(5,25)	.	.	.	.	0.3805085	0.0603125	0.0905	-2.0195	2.0134923
Tbatt(50,100)	.	.	.	.	.	0.0210085	0.075375	-0.1395	0.1394385
Vloiter(2,6)	.	.	.	.	.	.	0.0240085	-0.206063	0.2060231
Loadout(0,16)	.	.	.	.	.	.	.	0.0185085	-2.614092

Solution

Variable	Critical Value
Vmax(15,25)	20
VEES(1,4)	2.5
Tburst(0.5,2)	1.25
Vbalance(2,8)	3.0883664
TAIPEndur(5,25)	8.6483354
Tbatt(50,100)	60.345659
Vloiter(2,6)	3.701337
Loadout(0,16)	9.7569239

Solution is a SaddlePoint

Predicted Value at Solution 0.2287551

Canonical Curvature

Eigenvalues and Eigenvectors

final- Fit Least Squares

Least Squares Fit

Response MC-S

Response Surface

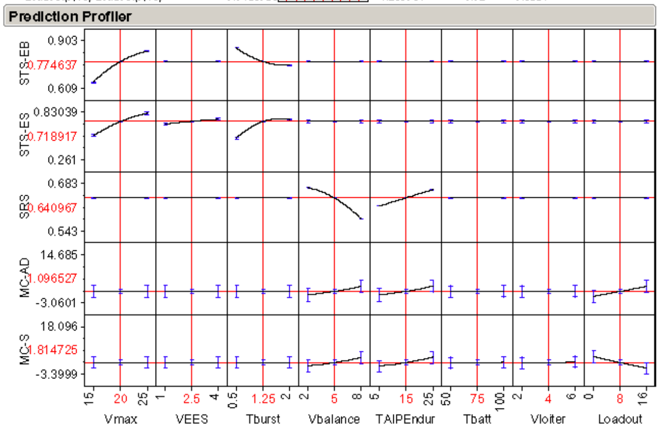
Canonical Curvature

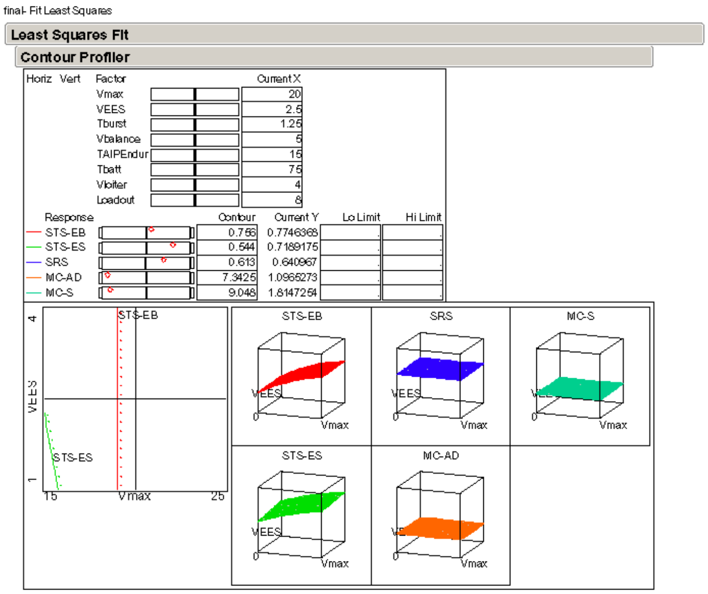
Eigenvalue	2.0977	0.0561	0.0185	0.0185	0.0185	-0.0153	-0.4685	-0.9139
Vmax(15,25)	-0.00000	-0.00000	0.99880	-0.00535	0.04864	-0.00000	0.00000	0.00000
VEES(1,4)	-0.00000	-0.00000	-0.03839	-0.70211	0.71103	0.00000	-0.00000	0.00000
Tburst(0.5,2)	-0.00000	-0.00000	-0.03034	0.71205	0.70148	-0.00000	-0.00000	0.00000
Vbalance(2,8)	0.57133	-0.06395	-0.00000	-0.00000	-0.00000	0.01191	0.73219	0.36502
TAIPEndur(5,25)	0.60006	-0.05988	-0.00000	-0.00000	-0.00000	0.01091	-0.68082	0.41558
Tbatt(50,100)	0.03582	0.69763	0.00000	0.00000	0.00000	0.71465	0.00351	0.03579
Vloiter(2,6)	0.05261	0.71085	0.00000	-0.00000	0.00000	-0.69928	0.00554	0.05389
Loadout(0,16)	-0.55628	-0.01812	-0.00000	0.00000	-0.00000	0.00389	0.01835	0.83058

Scaled Estimates

Term	Scaled Estimate	Std Error	t Ratio	Prob>|t|
Intercept	1.8147254	0.511816	3.55	0.0006
Vmax(15,25)&RS	-8.75e-16	0.163252	-0.00	1.0000
VEES(1,4)&RS	-8.75e-16	0.163252	-0.00	1.0000
Tburst(0.5,2)&RS	0	0.163252	0.00	1.0000
Vbalance(2,8)&RS	1.8933154	0.163252	11.60	<.0001
TAIPEndur(5,25)&RS	2.0134923	0.163252	12.33	<.0001
Tbatt(50,100)&RS	0.1394385	0.163252	0.85	0.3951
Vloiter(2,6)&RS	0.2060231	0.163252	1.26	0.2099
Loadout(0,16)&RS	-2.614092	0.163252	-16.01	<.0001
Vmax(15,25)*VEES(1,4)	-8.88e-15	0.164522	-0.00	1.0000
Vmax(15,25)*Tburst(0.5,2)	-7.11e-15	0.164522	-0.00	1.0000
VEES(1,4)*Tburst(0.5,2)	-5.33e-15	0.164522	-0.00	1.0000
Vmax(15,25)*Vbalance(2,8)	-5.33e-15	0.164522	-0.00	1.0000
VEES(1,4)*Vbalance(2,8)	-5.33e-15	0.164522	-0.00	1.0000
Tburst(0.5,2)*Vbalance(2,8)	-3.55e-15	0.164522	-0.00	1.0000
Vmax(15,25)*TAIPEndur(5,25)	-5.33e-15	0.164522	-0.00	1.0000
VEES(1,4)*TAIPEndur(5,25)	-7.11e-15	0.164522	-0.00	1.0000
Tburst(0.5,2)*TAIPEndur(5,25)	-5.33e-15	0.164522	-0.00	1.0000
Vbalance(2,8)*TAIPEndur(5,25)	1.6285625	0.164522	9.90	<.0001
Vmax(15,25)*Tbatt(50,100)	-7.11e-15	0.164522	-0.00	1.0000
VEES(1,4)*Tbatt(50,100)	-7.11e-15	0.164522	-0.00	1.0000
Tburst(0.5,2)*Tbatt(50,100)	-3.55e-15	0.164522	-0.00	1.0000
Vbalance(2,8)*Tbatt(50,100)	0.0543125	0.164522	0.33	0.7420
TAIPEndur(5,25)*Tbatt(50,100)	0.0603125	0.164522	0.37	0.7147
Vmax(15,25)*Vloiter(2,6)	-8.88e-15	0.164522	-0.00	1.0000
VEES(1,4)*Vloiter(2,6)	-3.55e-15	0.164522	-0.00	1.0000
Tburst(0.5,2)*Vloiter(2,6)	-1.07e-14	0.164522	-0.00	1.0000
Vbalance(2,8)*Vloiter(2,6)	0.0815	0.164522	0.50	0.6214
TAIPEndur(5,25)*Vloiter(2,6)	0.0905	0.164522	0.55	0.5835
Tbatt(50,100)*Vloiter(2,6)	0.075375	0.164522	0.46	0.6478
Vmax(15,25)*Loadout(0,16)	-3.55e-15	0.164522	-0.00	1.0000
VEES(1,4)*Loadout(0,16)	-8.88e-15	0.164522	-0.00	1.0000
Tburst(0.5,2)*Loadout(0,16)	-7.11e-15	0.164522	-0.00	1.0000
Vbalance(2,8)*Loadout(0,16)	-1.9	0.164522	-11.55	<.0001
TAIPEndur(5,25)*Loadout(0,16)	-2.0195	0.164522	-12.27	<.0001
Tbatt(50,100)*Loadout(0,16)	-0.1395	0.164522	-0.85	0.3985
Vloiter(2,6)*Loadout(0,16)	-0.206063	0.164522	-1.25	0.2133
Vmax(15,25)*Vmax(15,25)	0.0185085	1.233051	0.02	0.9881
VEES(1,4)*VEES(1,4)	0.0185085	1.233051	0.02	0.9881
Tburst(0.5,2)*Tburst(0.5,2)	0.0185085	1.233051	0.02	0.9881
Vbalance(2,8)*Vbalance(2,8)	0.3120085	1.233051	0.25	0.8008
TAIPEndur(5,25)*TAIPEndur(5,25)	0.3805085	1.233051	0.31	0.7583
Tbatt(50,100)*Tbatt(50,100)	0.0210085	1.233051	0.02	0.9864
Vloiter(2,6)*Vloiter(2,6)	0.0240085	1.233051	0.02	0.9845
Loadout(0,16)*Loadout(0,16)	0.0185085	1.233051	0.02	0.9881

Prediction Profiler





final- Fit Least Squares
Least Squares Fit
Contour Profiler
Horiz Vert Factor Current X
Vmax 20
VEES 2.5
Tburst 1.25
Vbalance 5
TAIPEndur 15
Tbatt 75
Vloiter 4
Loadout 8
Response Contour Current Y Lo Limit Hi Limit
STS-EB 0.756 0.7746368
STS-ES 0.544 0.7189175
SRS 0.613 0.640967
MC-AD 7.3425 1.0965273
MC-S 9.048 1.8147254
STS-EB
STS-ES
VEES
Vmax
15
25
4
1
SRS
MC-S
MC-AD

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APPENDIX 4:

UNCERTAINTY ANALYSIS

MOE	MOP	Threshold	Goal	Desired
Survivability of Suspected Target Search	Burst Speed "V_{max}" (knots)	15	25	*20*
	STS Evasion Endurance Speed "V_{EES}" (knots)	1	4	*2.5*
	Time at Burst Speed "T_{burst}" (hrs)	0.5	2	*1.166667*
Survivability of Random Search	AIP Balance Speed "$V_{balance}$" (knots)	2	8	*5*
	AIP Endurance "$T_{AIPendur}$" (days)	5	25	*15*
Mission Capability	Submerged Endurance on Battery "T_{batt}" (hours)	50	100	*76.66667*
	Submerged Battery Loiter Speed "V_{loiter}" (knots)	2	6	*4*
	Loadout Package	0	16	*11*

Resulting MOE Values (from RSEs)

Survivability of Suspected Target Search	**End of Burst**	0.781
	End of Search	0.706
Survivability of Random Search	**Random Search**	0.641
Mission Capability	**Area Denial**	1.820
	Strike	0.843

Baseline assumptions

1. Non-AIP patrol speed will be same as AIP Balance speed
2. Mission scenario lasts 45 days
3. Sub's total endurance can support assumed mission scenario duration

Factors	0-1 Scaled Value	Variable Name	-1 to +1 Scaled Value
Burst Speed "Vmax" (knots)	0	Vmax	-1
STS Evasion Endurance Speed "VEES" (knots)	0	VEES	-1
Time at Burst Speed "Tburst" (hrs)	-0.111111111	Tburst	-1.222222222
AIP Balance Speed "Vbalance" (knots)	0	Vbalance	-1
AIP Endurance "TAIPendur" (days)	0	TAIPendur	-1
Submerged Endurance on Battery "Tbatt" (hours)	0.066666667	Tbatt	-0.866666667
Submerged Battery Loiter Speed "Vloiter" (knots)	0	Vloiter	-1
Loadout Package	0.375		-0.25

This table interpolates desired factor values into the scaled values necessary to compute the responses.The desired factor values are pulled from the italicized 'Desired' column with green cells on the previous page. They are then scaled from 0 to 1 based on the Goal and Threshold values from the previous page. Then, they are transformed to the -1 to +1 scale that the RSE's are based off of. The -1 to +1 Scaled Value column is fed to RSE Table 1, which performs the RSE calculations using the constant parameter estimates from RSE Table 2. Lastly, the response values are returned to the five blue cells on the previous page.

RSE Table 1	Desirements/Responses				
RSE Factors	Survivability of Suspected Target Search - End of Burst "STS-EB"	Survivability of Suspected Target Search - End of Search "STS-ES"	Survivability of Random Search "RS"	Mission Capability - Area Denial "MC-AD"	Mission Capability - Strike "MC-S"
Intercept	0.775	0.719	0.641	1.097	1.815
Vmax(15,25)&RS	0.000	0.000	0.000	0.000	0.000
VEES(1,4)&RS	0.000	0.000	0.000	0.000	0.000
Tburst(0.5,2)&RS	0.006	-0.012	0.000	0.000	0.000
Vbalance(2,8)&RS	0.000	0.000	0.000	0.000	0.000
TAIPEndur(5,25)&RS	0.000	0.000	0.000	0.000	0.000
Tbatt(50,100)&RS	0.000	0.000	0.000	0.007	0.009
Vloiter(2,6)&RS	0.000	0.000	0.000	0.000	0.000
Loadout(0,16)&RS	0.000	0.000	0.000	0.711	-0.980
Vmax(15,25)*VEES(1,4)	0.000	0.000	0.000	0.000	0.000
Vmax(15,25)*Tburst(0.5,2)	0.000	0.000	0.000	0.000	0.000
VEES(1,4)*Tburst(0.5,2)	0.000	0.000	0.000	0.000	0.000
Vmax(15,25)*Vbalance(2,8)	0.000	0.000	0.000	0.000	0.000
VEES(1,4)*Vbalance(2,8)	0.000	0.000	0.000	0.000	0.000
Tburst(0.5,2)*Vbalance(2,8)	0.000	0.000	0.000	0.000	0.000
Vmax(15,25)*TAIPEndur(5,25)	0.000	0.000	0.000	0.000	0.000
VEES(1,4)*TAIPEndur(5,25)	0.000	0.000	0.000	0.000	0.000
Tburst(0.5,2)*TAIPEndur(5,25)	0.000	0.000	0.000	0.000	0.000
Vbalance(2,8)*TAIPEndur(5,25)	0.000	0.000	0.000	0.000	0.000
Vmax(15,25)*Tbatt(50,100)	0.000	0.000	0.000	0.000	0.000
VEES(1,4)*Tbatt(50,100)	0.000	0.000	0.000	0.000	0.000
Tburst(0.5,2)*Tbatt(50,100)	0.000	0.000	0.000	0.000	0.000
Vbalance(2,8)*Tbatt(50,100)	0.000	0.000	0.000	0.000	0.000
TAIPEndur(5,25)*Tbatt(50,100)	0.000	0.000	0.000	0.000	0.000
Vmax(15,25)*Vloiter(2,6)	0.000	0.000	0.000	0.000	0.000
VEES(1,4)*Vloiter(2,6)	0.000	0.000	0.000	0.000	0.000
Tburst(0.5,2)*Vloiter(2,6)	0.000	0.000	0.000	0.000	0.000
Vbalance(2,8)*Vloiter(2,6)	0.000	0.000	0.000	0.000	0.000
TAIPEndur(5,25)*Vloiter(2,6)	0.000	0.000	0.000	0.000	0.000
Tbatt(50,100)*Vloiter(2,6)	0.000	0.000	0.000	0.000	0.000
Vmax(15,25)*Loadout(0,16)	0.000	0.000	0.000	0.000	0.000
VEES(1,4)*Loadout(0,16)	0.000	0.000	0.000	0.000	0.000
Tburst(0.5,2)*Loadout(0,16)	0.000	0.000	0.000	0.000	0.000
Vbalance(2,8)*Loadout(0,16)	0.000	0.000	0.000	0.000	0.000
TAIPEndur(5,25)*Loadout(0,16)	0.000	0.000	0.000	0.000	0.000
Tbatt(50,100)*Loadout(0,16)	0.000	0.000	0.000	0.003	-0.003
Vloiter(2,6)*Loadout(0,16)	0.000	0.000	0.000	0.000	0.000
Vmax(15,25)*Vmax(15,25)	0.000	0.000	0.000	0.000	0.000
VEES(1,4)*VEES(1,4)	0.000	0.000	0.000	0.000	0.000
Tburst(0.5,2)*Tburst(0.5,2)	0.000	-0.001	0.000	0.000	0.000
Vbalance(2,8)*Vbalance(2,8)	0.000	0.000	0.000	0.000	0.000
TAIPEndur(5,25)*TAIPEndur(5,25	0.000	0.000	0.000	0.000	0.000
Tbatt(50,100)*Tbatt(50,100)	0.000	0.000	0.000	0.000	0.000
Vloiter(2,6)*Vloiter(2,6)	0.000	0.000	0.000	0.000	0.000
Loadout(0,16)*Loadout(0,16)	0.000	0.000	0.000	0.003	0.003
MOE Values	**0.781**	**0.706**	**0.641**	**1.820**	**0.843**

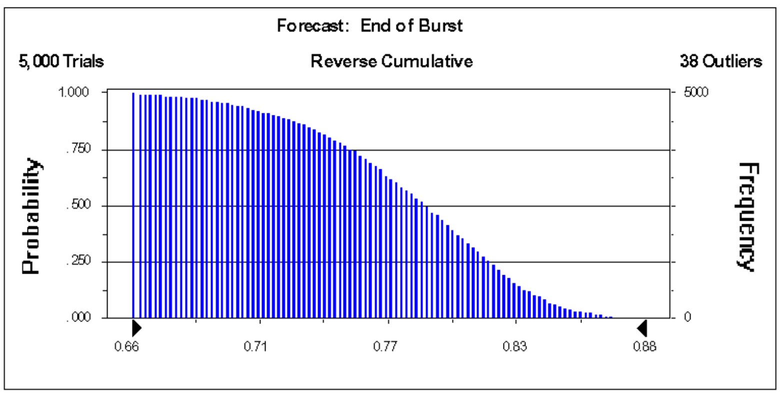
RSE Table 2	Desirements/Responses				
RSE Factors	Survivability of Suspected Target Search - End of Burst "STS-EB"	Survivability of Suspected Target Search - End of Search "STS-ES"	Survivability of Random Search "RS"	Mission Capability - Area Denial "MC-AD"	Mission Capability - Strike "MC-S"
Intercept	0.77463685	0.71891746	0.640967	1.0965273	1.814725
Vmax(15,25)&RS	0.09444615	0.13397692	-2.19E-16	8.75E-16	-8.75E-16
VEES(1,4)&RS	2.40E-15	0.02998462	0.00E+00	0.00E+00	-8.75E-16
Tburst(0.5,2)&RS	-5.27E-02	1.11E-01	2.19E-16	0	0
Vbalance(2,8)&RS	2.62E-15	-1.53E-15	-0.045762	1.5703231	1.893315
TAIPEndur(5,25)&RS	2.62E-15	-1.53E-15	2.42E-02	1.6546077	2.013492
Tbatt(50,100)&RS	3.28E-15	-1.53E-15	-2.19E-16	0.1095692	0.139438
Vloiter(2,6)&RS	2.62E-15	-1.31E-15	-2.19E-16	0.1612077	0.206023
Loadout(0,16)&RS	2.62E-15	-1.53E-15	0.00E+00	1.90E+00	-2.614092
Vmax(15,25)*VEES(1,4)	1.33E-15	-0.007625	1.78E-15	1.78E-15	-8.88E-15
Vmax(15,25)*Tburst(0.5,2)	1.98E-02	-1.59E-02	1.78E-15	1.78E-15	-7.11E-15
VEES(1,4)*Tburst(0.5,2)	2.22E-15	-2.96E-02	2.22E-15	3.55E-15	-5.33E-15
Vmax(15,25)*Vbalance(2,8)	2.22E-15	-4.89E-15	3.11E-15	3.55E-15	-5.33E-15
VEES(1,4)*Vbalance(2,8)	2.22E-15	-3.55E-15	3.11E-15	3.55E-15	-5.33E-15
Tburst(0.5,2)*Vbalance(2,8)	2.22E-15	-3.55E-15	2.66E-15	3.55E-15	-3.55E-15
Vmax(15,25)*TAIPEndur(5,25)	2.22E-15	-4.00E-15	1.78E-15	1.78E-15	-5.33E-15
VEES(1,4)*TAIPEndur(5,25)	3.11E-15	-5.33E-15	1.78E-15	1.78E-15	-7.11E-15
Tburst(0.5,2)*TAIPEndur(5,25)	2.66E-15	-4.89E-15	2.22E-15	1.78E-15	-5.33E-15
Vbalance(2,8)*TAIPEndur(5,25)	2.22E-15	-4.44E-15	1.33E-02	1.4133125	1.628563
Vmax(15,25)*Tbatt(50,100)	1.78E-15	-4.44E-15	2.66E-15	1.78E-15	-7.11E-15
VEES(1,4)*Tbatt(50,100)	3.55E-15	-4.44E-15	3.55E-15	1.78E-15	-7.11E-15
Tburst(0.5,2)*Tbatt(50,100)	2.66E-15	-4.44E-15	3.11E-15	3.55E-15	-3.55E-15
Vbalance(2,8)*Tbatt(50,100)	2.22E-15	-4.89E-15	2.22E-15	0.0543125	0.054313
TAIPEndur(5,25)*Tbatt(50,100)	2.22E-15	-4.44E-15	2.66E-15	0.0603125	0.060313
Vmax(15,25)*Vloiter(2,6)	2.22E-15	-4.44E-15	2.66E-15	1.78E-15	-8.88E-15
VEES(1,4)*Vloiter(2,6)	1.78E-15	-4.44E-15	3.55E-15	1.78E-15	-3.55E-15
Tburst(0.5,2)*Vloiter(2,6)	2.66E-15	-4.00E-15	1.78E-15	0	-1.07E-14
Vbalance(2,8)*Vloiter(2,6)	2.22E-15	-4.89E-15	2.22E-15	0.0814375	8.15E-02
TAIPEndur(5,25)*Vloiter(2,6)	2.22E-15	-4.44E-15	2.22E-15	0.0905625	9.05E-02
Tbatt(50,100)*Vloiter(2,6)	1.33E-15	-4.44E-15	1.33E-15	0.0604375	7.54E-02
Vmax(15,25)*Loadout(0,16)	1.33E-15	-4.00E-15	2.66E-15	1.78E-15	-3.55E-15
VEES(1,4)*Loadout(0,16)	3.55E-15	-4.44E-15	3.55E-15	1.78E-15	-8.88E-15
Tburst(0.5,2)*Loadout(0,16)	2.22E-15	-4.89E-15	3.11E-15	3.55E-15	-7.11E-15
Vbalance(2,8)*Loadout(0,16)	3.55E-15	-4.44E-15	2.22E-15	1.58E+00	-1.9
TAIPEndur(5,25)*Loadout(0,16)	2.22E-15	-5.33E-15	3.55E-15	1.66E+00	-2.0195
Tbatt(50,100)*Loadout(0,16)	2.66E-15	-4.89E-15	1.33E-15	1.10E-01	-0.1395
Vloiter(2,6)*Loadout(0,16)	2.66E-15	-5.33E-15	4.00E-15	1.61E-01	-2.06E-01
Vmax(15,25)*Vmax(15,25)	-0.0316109	-0.0354116	3.537E-05	0.0187207	0.018508
VEES(1,4)*VEES(1,4)	0.00038909	-0.0009116	3.537E-05	0.0187207	0.018508
Tburst(0.5,2)*Tburst(0.5,2)	0.03038909	-0.0859116	3.537E-05	0.0187207	0.018508
Vbalance(2,8)*Vbalance(2,8)	0.00038909	0.00008843	-0.015465	0.3117207	0.312008
TAIPEndur(5,25)*TAIPEndur(5,25)	0.00038909	0.00008843	0.0005354	0.3802207	0.380508
Tbatt(50,100)*Tbatt(50,100)	0.00038909	0.00008843	3.537E-05	0.0207207	0.021008
Vloiter(2,6)*Vloiter(2,6)	0.00038909	0.00008843	3.537E-05	0.0242207	0.024008
Loadout(0,16)*Loadout(0,16)	0.00038909	0.00008843	3.537E-05	0.0182207	0.018508

Crystal Ball Report
Simulation started on 4/17/03 at 13:52:11
Simulation stopped on 4/17/03 at 13:52:42

Forecast: End of Burst **Cell: C12**

Summary:
Display Range is from 0.66 to 0.88
Entire Range is from 0.62 to 0.89
After 5,000 Trials, the Std. Error of the Mean is 0.00

Statistics:	Value
Trials	5000
Mean	0.78
Median	0.79
Mode	---
Standard Deviation	0.04
Variance	0.00
Skewness	-0.48
Kurtosis	2.96
Coeff. of Variability	0.06
Range Minimum	0.62
Range Maximum	0.89
Range Width	0.27
Mean Std. Error	0.00



Forecast: End of Burst (cont'd) **Cell: C12**

Percentiles:

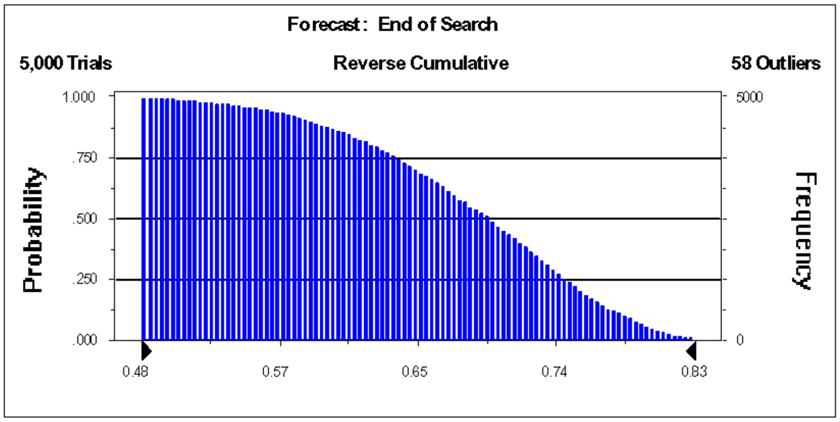
Percentile	Value
0%	0.62
10%	0.72
20%	0.74
30%	0.76
40%	0.77
50%	0.79
60%	0.80
70%	0.81
80%	0.82
90%	0.84
100%	0.89

End of Forecast

Forecast: End of Search **Cell: C13**

Summary:
Display Range is from 0.48 to 0.83
Entire Range is from 0.37 to 0.84
After 5,000 Trials, the Std. Error of the Mean is 0.00

Statistics:	Value
Trials	5000
Mean	0.69
Median	0.70
Mode	---
Standard Deviation	0.08
Variance	0.01
Skewness	-0.52
Kurtosis	2.95
Coeff. of Variability	0.11
Range Minimum	0.37
Range Maximum	0.84
Range Width	0.47
Mean Std. Error	0.00



Forecast: End of Search (cont'd) **Cell: C13**

Percentiles:

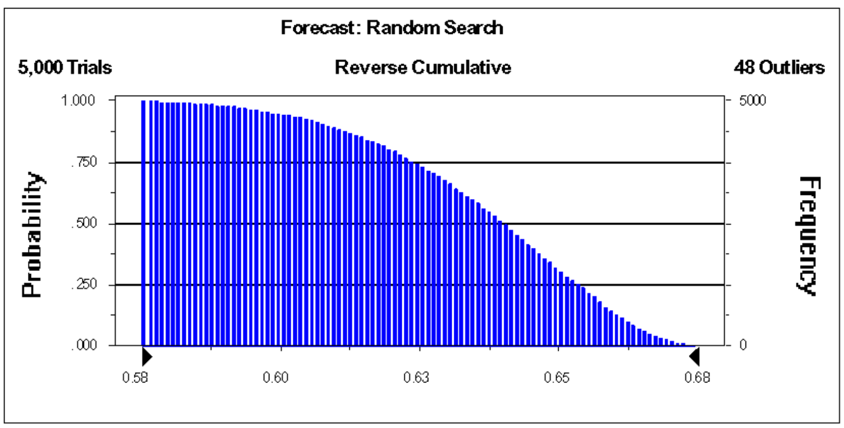
Percentile	Value
0%	0.37
10%	0.58
20%	0.62
30%	0.65
40%	0.67
50%	0.70
60%	0.72
70%	0.74
80%	0.76
90%	0.78
100%	0.84

End of Forecast

Forecast: Random Search **Cell: C14**

Summary:
- Display Range is from 0.58 to 0.68
- Entire Range is from 0.56 to 0.68
- After 5,000 Trials, the Std. Error of the Mean is 0.00

Statistics:	Value
Trials	5000
Mean	0.64
Median	0.64
Mode	---
Standard Deviation	0.02
Variance	0.00
Skewness	-0.61
Kurtosis	2.98
Coeff. of Variability	0.03
Range Minimum	0.56
Range Maximum	0.68
Range Width	0.12
Mean Std. Error	0.00



Forecast: Random Search (cont'd) **Cell: C14**

Percentiles:

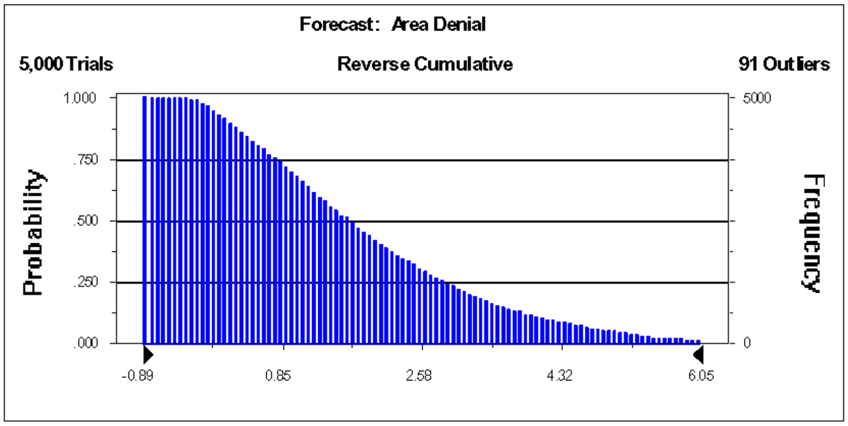
Percentile	Value
0%	0.56
10%	0.61
20%	0.62
30%	0.63
40%	0.64
50%	0.64
60%	0.65
70%	0.65
80%	0.66
90%	0.66
100%	0.68

End of Forecast

Forecast: Area Denial **Cell: C15**

Summary:
Display Range is from -0.89 to 6.05
Entire Range is from -0.89 to 9.35
After 5,000 Trials, the Std. Error of the Mean is 0.02

Statistics:	Value
Trials	5000
Mean	1.93
Median	1.65
Mode	---
Standard Deviation	1.58
Variance	2.48
Skewness	0.94
Kurtosis	3.79
Coeff. of Variability	0.82
Range Minimum	-0.89
Range Maximum	9.35
Range Width	10.24
Mean Std. Error	0.02



Forecast: Area Denial (cont'd) **Cell: C15**

Percentiles:

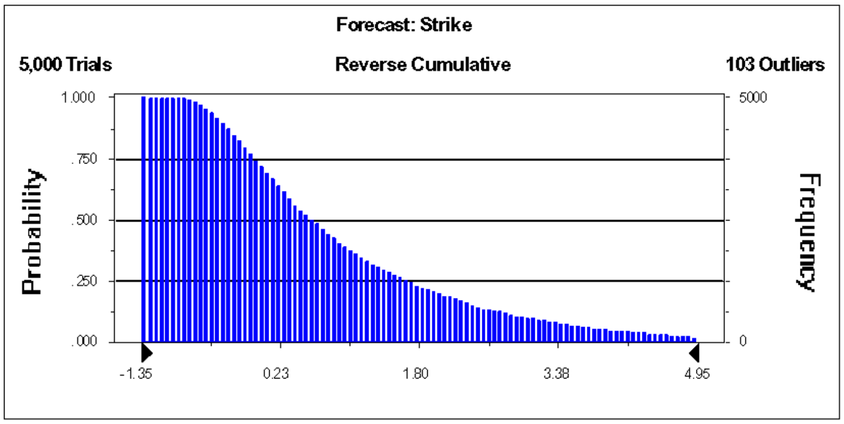
Percentile	Value
0%	-0.89
10%	0.13
20%	0.52
30%	0.91
40%	1.26
50%	1.65
60%	2.06
70%	2.55
80%	3.18
90%	4.13
100%	9.35

End of Forecast

Forecast: Strike **Cell: C16**

Summary:
Display Range is from -1.35 to 4.95
Entire Range is from -1.35 to 8.92
After 5,000 Trials, the Std. Error of the Mean is 0.02

Statistics:	Value
Trials	5000
Mean	0.95
Median	0.54
Mode	---
Standard Deviation	1.47
Variance	2.15
Skewness	1.45
Kurtosis	5.44
Coeff. of Variability	1.54
Range Minimum	-1.35
Range Maximum	8.92
Range Width	10.27
Mean Std. Error	0.02



Forecast: Strike (cont'd) **Cell: C16**

Percentiles:

Percentile	Value
0%	-1.35
10%	-0.48
20%	-0.23
30%	0.02
40%	0.26
50%	0.54
60%	0.88
70%	1.34
80%	1.96
90%	3.00
100%	8.92

End of Forecast

Assumptions

Assumption: Desired **Cell: E2**

Burst Speed "V_{max}" (knots)

Triangular distribution with parameters:

Minimum	15.00
Likeliest	20.00
Maximum	25.00

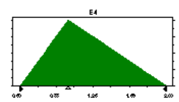
Selected range is from 15.00 to 25.00

Assumption: E4 **Cell: E4**

Time at Burst Speed "T_{burst}" (hrs)

Triangular distribution with parameters:

Minimum	0.50
Likeliest	1.00
Maximum	2.00



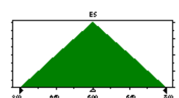
Selected range is from 0.50 to 2.00

Assumption: E5 **Cell: E5**

AIP Balance Speed "$V_{balance}$" (knots)

Triangular distribution with parameters:

Minimum	2.00
Likeliest	5.00
Maximum	8.00



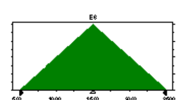
Selected range is from 2.00 to 8.00

Assumption: E6 **Cell: E6**

AIP Endurance "$T_{AIPendur}$" (days)

Triangular distribution with parameters:

Minimum	5.00
Likeliest	15.00
Maximum	25.00



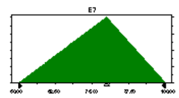
Selected range is from 5.00 to 25.00

Assumption: E7 **Cell: E7**

Submerged Endurance on Battery "T_{batt}" (hours)

Triangular distribution with parameters:

Minimum	50.00
Likeliest	80.00
Maximum	100.00



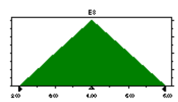
Selected range is from 50.00 to 100.00

Assumption: E8 **Cell: E8**

Submerged Battery Loiter Speed "V_{loiter}" (knots)

Triangular distribution with parameters:

Minimum	2.00
Likeliest	4.00
Maximum	6.00



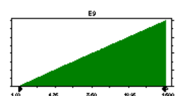
Selected range is from 2.00 to 6.00

Assumption: E9 **Cell: E9**

Loadout Package

Triangular distribution with parameters:

Minimum	1.00
Likeliest	16.00
Maximum	16.00



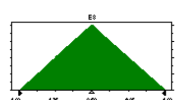
Selected range is from 1.00 to 16.00

Assumption: E3 **Cell: E3**

STS Evasion Endurance Speed "V_{EES}" (knots)

Triangular distribution with parameters:

Minimum	1.00
Likeliest	2.50
Maximum	4.00



Selected range is from 1.00 to 4.00

End of Assumptions

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