

RISK AND RETURNS IN COMMERCIAL REAL ESTATE:
AN EXPLORATION OF SOME FUNDAMENTAL RELATIONSHIPS

by

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ABSTRACT

This thesis applies some basic tools from modern financial economic theory to gain some insight into the nature of commercial real estate valuation, return risk, and risk premia, relevant for the analysis and evaluation of construction projects in the private sector. The basic motivation for the thesis is the fact that, while risk in the returns to such projects is potentially quite important in their evaluation, it is difficult to study such risk because regular and frequent time series of returns to real properties cannot be observed, due to infrequent trading of such assets.

The thesis is divided into two parts. Part I develops a multi-period cash flow based valuation model, explicitly incorporating the use of long-term leases as is common in much commercial real estate. This model is then used to derive insights regarding the nature of the return risk, based upon the nature of the observable cash flow or rental market risk. The effect of lease term on return risk, and the accuracy of the widely employed "simple cap rate valuation" method, are explored using this model, as well as the question of to what extent use of long-term leases may make some commercial real estate more "like a bond" than "like a stock".

Part II focuses on the use of appraisal based returns time series in the study of the nature of real estate return risk. Behavioral models of the appraisal process are developed which provide insight regarding the extent to which such time series may be "smoothed", that is, display less risk than is present in the true (unobservable) market value based returns. An empirical based approach to approximately correct for such smoothing is presented, and applied to a small sample using some widely cited indices of aggregate commercial real estate values. This analysis

indicates considerable smoothing, and also reveals that systematic risk defined with respect to national consumption (as suggested by the Consumption based Capital Asset Pricing Model) is much greater than systematic risk defined with respect to the stock market (as is usually done in applications of the CAPM to financial securities).

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Finally, I could not have done it without the support of my wife, Debby, my children, Jonathan and Nathaniel, and my Parents.

I alone am responsible for any errors that remain in the thesis.

David Geltner
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Chapter 1: Introduction

Approximately half of the value of the marketable assets in the United States fall into the category of real estate. A large fraction of this is commercial real estate, that is, office, residential, retail and industrial space and farmland which produces regular income streams for its owners. This commercial real estate is therefore very well described by the classical model of a long-lived "capital asset", deriving its present value not from its contribution to present consumption, but from its ability to provide for future consumption. As property "lives forever", and no one knows exactly what the future cash flows from any property will be for all years into the future, commercial real estate is certainly a "risky asset".

This risk is apparently important in the valuation of properties, at least if one is to judge by common parlance and the attention paid to "risk" in the commercial valuation literature. In the 1984 special issue on valuation published by the Journal of the American Real Estate and Urban Economics Association (AREUEA), editor Kenneth Lusht surveyed 71 professional appraisers asking them what valuation topics they considered most important for research. The second most frequently cited response was: "Estimating risk and determining the proper discount rate".

Though risk seems to be an important issue in the real world of real estate, there is a current drift in the academic real estate literature suggesting that "investment grade" (ie, high quality) commercial real estate may be almost riskless (at least in terms of systematic risk), and that the expected return premium in unsecuritized commercial real estate is not attributable to risk in the asset return, but rather to other factors, such as illiquidity. This view has come from studies of the growing amount of returns time series data available on commercial real estate from institutional portfolios. Virtually all of this data is based on appraised valuations, rather than actual market value returns which cannot be observed for unsecuritized assets that trade infrequently. Others have suggested that the cash flow fundamentals also may support the notion that high quality commercial real estate is virtually riskless, since one would expect corporate rental payments to the landlords of their office space to be much less volatile than the corporate earnings which underlie the risky returns to industrial corporations traded on the stock market.

(Gyourko & Linneman)

In short, there is no clear consensus on how "risky" commercial real estate properties are, even on a relative basis compared to common stocks, and there seems to be

considerable confusion even regarding how to measure or think about this issue. We seem to know much less about risk in real estate returns than we do about risk in stock market returns, even though real estate is of comparable magnitude to the stock market in capitalized value, and in some ways real estate assets are much simpler and easier to understand than modern industrial corporations. There seems to be a great need to better explore the fundamentals of the question, and the time for such an inquiry seems to be ripe. The appraisal industry is moving toward more formalization and institutionalization, vast sums of institutional capital (both foreign and domestic) are seeking real estate investments, while deregulation and innovation in financial markets are stimulating the possibilities for both securitization and de-securitization. This, then, is the motivation for this thesis.

1.1 Overview:

This thesis is an attempt to use some of the basic tools and techniques of modern financial economic theory to explore some questions of interest in the field of real estate risk and performance analysis. In particular, the Capital Asset Pricing Model (CAPM) from capital market general equilibrium theory is employed together with some techniques from multi-period capital budgeting theory, to

examine the nature of risk in the returns to unsecuritized commercial real estate assets or portfolios.

While I have attempted to bring more unity and integration into this thesis than would be present in a simple collection of essays, it should be noted at the outset that, as suggested by its title, this thesis has more than one focus, and does not seek to be fully comprehensive in its treatment of the question of the nature and determinants of real estate return risks and expected return premia.

Nevertheless, there are a couple of unifying themes in the thesis. One is the effort to make use of empirically observable (or potentially observable) data on unsecuritized real estate in order to study the risk characteristics of the unobservable returns on such assets. Returns on such assets are inherently unobservable (at least at the frequent and regular intervals necessary for studying risk characteristics) because they are only rarely and sporadically traded. Yet it is the risk in the return (that is, the risk in the value appreciation of the asset as well as in its income, as a fraction of the cost of the asset) which should matter to investors.

While we cannot observe useful time series of true returns on unsecuritized assets, we can, at least in principle, observe two types of relevant data. First, we can observe the cash flow time series of the assets. Since fundamentally it is these cash flows (together with the capitalization rate or discount rate) which determines asset value, there should be a relationship between risk in the asset's cash flows and risk in the asset's returns, which might enable us to draw conclusions about the nature of the return risk, given the nature of the cash flow risk.

Second, we may be able to observe a long and regular time series of appraised values of the asset, and from this derive a series of appraisal-based returns. While such a series is subject to appraisal error and smoothing, there should be some relation between appraisal-based returns and true returns, and therefore between the risk apparent in appraisal-based returns and the risk in the true returns. Considering the nature of the appraisal process and the stochastic characteristics of appraisal-based returns series as compared to what we would expect in a true returns series, we may be able to characterize this relationship and conclude something about the true returns risks from the appraisal-based returns data.

While the inference of unobservable true return risk characteristics from theoretically observable data is a major theme of this thesis, it is important to note that this is not primarily an empirical thesis. Although a brief empirical analysis is presented at the end of Part II, the major contribution of the thesis is intended to be conceptual. The basis upon empirically observable data sources such as cash flows and appraisal returns is used here more for the purpose of developing our intuition about the nature of real estate return risk (by starting from data about which we may have more initial intuition), rather than for the purpose of conducting an empirical analysis.

In part, this is because the empirical data available to me at this time are quite limited, rendering extensive formal empirical analysis of questionable value. I also feel, however, that the first step in any empirical analysis should be a careful development and exploration of the a priori theory and intuition, so as to better guide and interpret empirical findings in a world of "noisy" data. The model developed in Part I of the thesis, in particular, is really designed as a conceptual tool for the purpose of developing intuition, rather than as a framework for empirical analysis. This thesis should be viewed as only a first step, but a step which is useful in its own right.

Another unifying theme in the thesis is its basis upon the paradigm of efficient markets and symmetric information. As one runs into some flack in the academic as well as the practicing real estate community for adopting this paradigm, I feel I should say a few words about why this thesis is based upon it.

The reason is not because I am under any illusions that the world of real estate or other capital markets is perfectly described by the classical paradigm. Indeed, to a large extent, the frontier of mainstream financial economic theoretical research has moved beyond this model, because it was found to be flawed or incomplete in some respects, even with regard to applications in the financial securities markets for which the paradigm was first developed and where it probably fits best.

But it seems to me that though the paradigm is not absolutely true, it contains an important part of the truth, even in the real estate markets. It cannot lead us all the way there, but it can help us along a useful amount of the distance. This is certainly true in financial securities markets, or the classical paradigm would not have held sway over such a long and productive period in the history of financial economics. Advancements beyond this paradigm in mainstream finance are generally built upon the foundation

laid using the efficient market/symmetric information model, and are pursuing directions indicated by first trying the classical model, and then seeing where, how, and to what extent it failed.

In contrast, it seems to me that the classical paradigm has not yet been adequately applied to the study of real estate markets. In the words of Kenneth Lusht in his Presidential Address to the 1987 AREUEA Annual Meeting: "The state-of-the-art with respect to pricing real estate is similar to that with respect to pricing securities just prior to the development of the CAPM."

For whatever reasons (lack of data no doubt being a major one), the principle theoretical and technical developments arising from the classical paradigm in mainstream financial economics have to date not been very vigorously applied to real estate. While some would have us leap over this phase in the development of our knowledge of real estate markets (arguing that the classical paradigm is fundamentally much more flawed for real estate markets than it is for financial securities -- ie, it is not "just a data problem"), I question whether we can or should try such a leap. I suspect that we can get a lot more mileage out of the classical financial economic tools applying them to real estate than what has been thus far obtained. We can learn from applying

these tools and finding that they work, and we can also learn from applying them and finding that they don't work. But we must try to apply them, and we must try hard.

The thesis is organized as follows. The body of the thesis is presented in Chapters 2 through 5, divided into Parts I and II. Part I (Chapters 2 & 3) is an analysis of cash flow based fundamentals, while Part II (Chapters 4 & 5) focuses on the information contained in appraisal-based returns data.

Since cash flows (present or expected in the future) underlie all commercial real estate value, Part I is the more fundamental of the two parts of the thesis, and therefore perhaps more useful in developing our intuition. It is also relevant to several questions, some of which are explored in Part I, in addition to the systematic return risk issue which is the primary focus of Part II. The major conceptual development in Part I is the extension of the classical multi-period certainty-equivalent DCF valuation model to include riskless long-term leases, which characterize much of commercial real estate. This allows us to explore some of the fundamental relationships which characterize commercial real estate return risk (particularly where long-term, relatively riskless leases are the norm, such as the office building sector). These

relations, such as the relation between return risk and observable cash flow risk, are primarily useful for building intuition, though they may have some potential for empirical application.

Though cash flow data are fundamental, the appraisal based return data upon which Part II is based are, in a sense, "one step closer" to the true returns we are interested in learning about. The appraisers have (in one way or another) already conducted the discounted cash flow type valuation modelled in Part I, incorporating their perceptions of the market's expectation not only of the future cash flows but of the discount rate as well. It is the fact that error may be introduced in the returns series by the appraisal process which motivates Part II. The objective of Part II is to help develop an understanding of the way appraisal behavior may affect the apparent risk characteristics of the returns series.

The last part of Chapter 5 in Part II contains a brief empirical analysis, using appraisal-based returns data from the FRC Index and PRISA Index of unsecuritized institutional real estate portfolios. The focus of this analysis is the relative ability of the Consumption-based CAPM, as compared to the traditional stock market based CAPM, to explain observed risk premia in unsecuritized real estate. A related

issue is the question of the size of the "illiquidity" premium in unsecuritized real estate's expected return, as compared to its risk premium. It should be emphasized that this empirical analysis is not presented as a formal "test" of any theory, but rather for the purpose of gaining some "feeling" for the real world of investment grade unsecuritized real estate.

In both Part I and II general qualitative and numerical relationships between observable risk and true return risk are developed, based (in Part I) on the cash flow fundamentals or (in Part II) on the appraisal behavior characteristics. Finally, Part III (Chapter 6) draws some overall conclusions.

1.2 A Threshold Question: Why Not Use REIT Returns?...

Since this thesis is motivated by a desire to increase our understanding of the nature of true return risk in commercial real estate properties, the question naturally arises as to why not simply do an empirical study of the returns to securitized real estate portfolios, such as the REITs which trade on the stock exchanges. Securitized real estate presents regular and frequent true (ie, market value or transactions price -- hence, "opportunity cost" -- based) returns data, based on stock prices and dividends, and so

presents a more direct and theoretically accurate source of information on the subject this thesis is exploring.

A short answer to this question is that this thesis is not primarily empirical, and I am seeking to understand the nature and determininants of real estate return risk rather than to document what that risk has historically been ex post. But, as noted, there is an empirical part of this thesis, and potential further empirical applications of some of the models and methodology presented in this thesis would be of interest. So the question of the relevance of the REIT returns is an important one.

Of course, a number of studies have already analyzed REIT returns, and REIT data does provide an important source of empirical information about real estate risk and returns. [See, for example, Smith & Shulman, and Burns & Epley.] In general, these studies have found that REIT returns behave much like typical common stock returns, similar in particular to stocks of public utility companies. REITs generally have higher than average yields and lower than average volatility. REIT betas with respect to the stock market are smaller than average, but significantly positive, and REIT returns are highly correlated with the overall stock market return. Over the past two decades REITs have

generally had positive but not statistically significant alphas, not unlike many low-beta stocks.

But there are several reasons why it seems undesirable to base our empirical knowledge of real estate risk and return only on analysis of REIT data. First, there are not very many Equity REITs (REITs that hold all or mostly real estate equity, as opposed to mortgages or real estate debt assets), and many of them have small and changing portfolios, and/or have not existed or been publicly traded for very long. Others have changed their investment policies, such as going from diversified to specialized portfolios or from Mortgage REITs to Equity REITs. So it is difficult to obtain clear and specific information about real estate return risk by studying REITs alone.

Perhaps more serious is the perception, widely held among both real estate academics and practitioners, that "REITs are not Real Estate", in the sense that REIT risk and return characteristics are perceived to differ significantly from those of unsecuritized real estate and (presumably) even from those of the real property portfolios which underlie the REIT securities' values. Various explanations are offered to account for this difference, ranging from arguments that the "stock market is inefficient" (being subject to "investor irrationality" and "waves" or "herds"),

to arguments that REIT return risk reflects intangible REIT management risk more than the risk in the tangible assets which the REIT currently owns. Another possibility is that real estate assets are very heterogeneous, such that even seemingly large, diversified portfolios can differ significantly in their risk and return determinants.

It is not clear whether these explanations can really account for the differences between REIT and unsecuritized real estate returns, but those differences do seem to be apparent at least superficially between such unsecuritized portfolios as the FRC and PRISA Indices on the one hand, and the REITs on the other. This is shown in Table 1.1, which shows mean quarterly returns, standard deviations, betas, and contemporaneous cross-correlations among REITs and unsecuritized real estate and other key financial and economic indicators, over the past 15 years.

In Table 1.1, RRNARQ represents the real return (total nominal return less the 3-month T-bill rate) to the NAREIT Equity REIT Index. This is an index of virtually all exchange-traded REITs having more than two-thirds of their assets in real estate equities. RRSURQ is the real return to a portfolio of the five surviving "pure" equity REITs (ie, those trading continuously during the 15-year period, and which generally held more than 80 percent of their

assets in real estate equity). RRPRU and RRFC are the (appraisal-based) real returns to the PRISA and FRC Indices, respectively.

It is clear in Table 1.1 that the Equity REITs seem to behave one way, similar to the rest of the stock market, and the unsecuritized real estate portfolios seem to behave another way, rather different from the stock market, at least on the basis of their appraisal-based returns. While FRC and PRISA mean returns are about the same as the S&P500 over the 15-year period shown here, and a bit less than the REIT mean returns (keep in mind that RRSURQ has a survivor bias), FRC and PRISA total risk (standard deviation of return) and systematic risk (beta) are much less than the REITs and S&P500. Indeed, while REIT betas are quite statistically significantly positive, FRC and PRISA betas are virtually zero and even leaning toward the negative. In the correlation matrix we note that FRC and PRISA real returns are not well correlated with the REIT returns, and also seem to behave differently from REITs with respect to nominal interest rates.

Some of this apparent difference between REITS and FRC/PRISA may be due to the effects of appraisal-smoothing in the unsecuritized returns series. (This is particularly true with regard to the difference in apparent total risk.)

But analysis later in this thesis reveals that it is hard to account for some of the key differences observed in Table 1.1 simply by the appraisal smoothing model.

How ever curious and interesting is the difference observed in Table 1.1 between large diversified portfolios of securitized versus unsecuritized real estate assets, it is not the subject of this thesis. The question of securitization of real estate could be another thesis in its own right. This issue is raised here only as evidence why it is of interest to study unsecuritized real estate returns, even though such study faces difficult data problems (in both quantity and quality of data available), and even though much real estate return data which is in some sense "cleaner" is easily available from REITs traded on the stock market.

Table 1.1 Comparison of REIT vs Unsecuritized Real Estate Risk and Returns Characteristics

(Quarterly Real Returns from 73.3 to 87.4^{*})

	NAREIT Equity RRNARQ	Five Surviv RRSURQ	PRISA Index RRPRU	FRC Index RRFRC	S&P500 Index RRSP	T-Bill Nominal RF	CPI Chg INFL	Real Chg GNP
Mean	.022	.041	.013	.015	.014	.021	.017	.006
Std.Dev	.081	.085	.013	.013	.095	.007	.010	.011
Beta ^{**}	0.66	0.66	-0.01	-0.00	1.00			
(t-stat)	(9.26)	(8.18)	(0.31)	(0.07)				

Correlation Matrix:

RRNARQ	1.00	.90	.11	.16	.78	-.20	-.37	.02
RRSURQ		1.00	-.03	.08	.74	-.22	-.37	.11
RRPRU			1.00	.50	-.04	.31	-.00	.17
RRFRC				1.00	-.01	.23	-.33	.01
RRSP					1.00	-.20	-.32	.03
RF						1.00	.40	.10
INFL							1.00	.13
GNP								1.00

^{*}Except FRC, which is from 78.1 to 87.4, and RF, which is nominal, not real

^{**}Beta with respect to S&P500 Index

Table Key:

RRNARQ = Quarterly Real Total Return NAREIT Equity REIT Index
 RRSURQ = Quarterly Real Total Return 5 Survivor "Pure Equity" REITS
 RRPRU = Quarterly Real Total Return PRISA Index
 RRFRC = Quarterly Real Total Return FRC Index (78.1-87.4)
 RRSP = Quarterly Real Total Return S&P500 Index
 RF = Quarterly Nominal Interest Rate on 3-month Treasury Bills
 INFL = Quarterly Change in Consumer Price Index
 GNP = Quarterly Change in Real GNP

PART I: CASH FLOW BASED ANALYSIS

Part I presents and applies the major conceptual development of the thesis, a cash flow based multiperiod valuation model incorporating rental market risk and the use of long-term riskless leases. This model has application to, or affords interesting insights regarding, several questions of interest in real estate analysis and valuation, not limited only to the return risk issues explored elsewhere in the thesis. The questions we can explore using the multi-period model developed here include: The relation between cash flow risk and return risk; The relation between lease term and both cash flow risk and return risk; The accuracy of the use of simple valuation techniques such as the use of a cap rate multiplier; Real estate "duration" or sensitivity to inflation effects on interest rates; as well as other questions of interest. Chapter 2 presents the model itself, while Chapter 3 presents a numerical analysis and discussion of the implications and insights obtained from the model.

Chapter 2: A Conceptual Model of Long-lived Asset Value Under Uncertainty With Long-term Riskless Leases

This Chapter presents a conceptual model of long-lived asset value explicitly incorporating both cash flow risk and the presence of riskless leases of varying maturity. The model is based on the discrete-time multi-period capital budgeting models developed in the late 1970's by Myers & Turnbull and Bhattacharya. Extending these previous models to incorporate riskless long-term leases is of interest to real estate analysts in particular, because much commercial real estate, particularly office space, is typically rented out under 5 to 10 year leases that in many cases could be considered as being approximately riskless. (We here ignore lease default risk.)

The "approximately riskless" characterization of investment grade commercial property lease cash flow arises from the fact that building operating expenses are usually small compared to rental revenue (and may also be contractually fixed), and from the fact that commercial leases are anyway often "net" (that is, most of the expenses associated with operating and maintaining the building are born by the tenant) and/or contain adjustment provisions which allow inflation to be at least partly passed through to the tenant via changes in the lease-specified rental rate. Thus, over the period of the

lease, the landlord or owner of the building has very nearly riskless cash flows, possibly even in inflation-adjusted terms.

However, the landlord is exposed to cash flow risk based on the rental market risk at the time of lease expiration, and certainly the asset itself cannot be said to be riskless, since most of its value is derived from expected future cash flows in the "outyears", that is, beyond the expiration of existing leases.

The objective of this Chapter is to develop a model of property value based on expected future cash flows. Among other things, this model is intended to allow a representation of the relationship between risk in the property cash flows and risk in the property's total return. It is the latter risk which matters to the investor. But cash flows may be more easily and accurately observable than returns for unsecured real estate properties, and analysts may have better intuition and knowledge concerning cash flows or market rental prices than concerning the property's total returns.

2.1 Overview and Basic Concepts

Considering the presence of multi-period leases, one way to attack the problem of valuing a commercial office property would be to view the property as a portfolio consisting of some short to medium term bond-like assets (possibly with some inflation protection) plus a forward contract on a stock-like asset. The former are the existing leases, the latter is the property itself apart from its existing lease contracts. The present certainty-equivalent (or market) value of this portfolio is just the risk-adjusted discounted value of the expected future cash flow stream. Though risk may enter the picture both in the cash flows and in the discount rate, in this Chapter we will focus only on cash flow risk, and assume that the discount rate is riskless (but not necessarily constant).

The bond-plus-forward-stock model suggests that one way to value the office building would be to risklessly discount the contractual cash flows under the current leases, and then discount expected outyear cash flows at a higher risk-adjusted rate. But this method is a bit crude in its treatment of the outyears, since cash flows then also will presumably be contracted under long-term leases (they just haven't been signed yet). More appropriate would be a model which applies a high discount rate only when leases expire and a low rate between lease expirations. But, in addition to being "messy", such a

model would obfuscate the relationship between lease term, expected rent levels, and cash flow risk. We wish to use the model to clarify and explore this very relationship.

To solve this problem, the approach taken in this Chapter is to "short-cut" the problem of the different risk regimes (between the pre-leased versus not-yet-leased periods) by looking behind the lease payments to the rental market which underlies both the lease agreements and the building value itself. The model developed in this Chapter thus makes use of three conceptual "levels" of "cash flow". Each level is characterized by a different degree of empirical observability and/or experience-based familiarity to an analyst, in a market where long-term leases are the norm.

The most basic level of cash flow, which underlies the others and effectively determines building value, is the underlying opportunity cost of the space. Designated by $\{x_t\}$, this series represents the net rental price that would prevail in the market if there were no long-term leases, and all space rented out every period in short-term (single-period) leases. That is, x_t is the "spot" market equilibrium price of space at time t in the market in which the building is situated. The $\{x_t\}$ series is therefore the series of fundamental economic opportunity costs of the building's space at each point t in

time. In a long-term lease covering periods $t=1,2,\dots,T$, the building owner forgoes the opportunity to earn the $\{x_t\}$ cash flows during those time periods and in return accepts the long-term lease payments. In a market where there is little or no short-term spot rental of space (such as most office markets) we cannot directly observe $\{x_t\}$. But we can nevertheless use it as a conceptual construct for property valuation. Note that $\{x_t\}$ reflects both the vacancy rate and effective rental price components of the rental market risk, since x_t is defined to be the price at which landlords are willing to lease all their space on the short-term spot market.

The next level of cash flow related series defined in the model is the series of new-lease rental prices, $\{y_t\}$. This is the rental rate agreed to by the tenant and landlord entering into a riskless long-term lease agreement at time t . This series will be a function of the lease term, T , and so may be expressed more fully as $\{y(t,T)\}$. While observable in principle, new-lease rental price historical time series data are difficult to find, and may not be very meaningful, as they represent brokers' estimates of what "typical", "nominal" rents were, not counting special concessions which tenants or landlords might have obtained reflecting current market condi-

tions. However, analysts familiar with a market may have a good intuition or "feeling" for the nature of the $\{y_t\}$ series.

In this regard, we might note that while in principle the $\{x_t\}$ series underlies the $\{y_t\}$ series, in practice one may develop intuition about $\{x_t\}$ from knowledge of $\{y_t\}$. One way to operationally define x_t is that x_t is the price which just leaves the landlord indifferent between renting his space out for T years at the long-term riskless rate of $y(t,T)$ per year, versus instead renting it out one year at a time starting this year at the rate of x_t , with future spot rates uncertain.

The third level, the series $\{CF_t\}$ or, more fully, $\{CF(t,T)\}$, is the actual cash flow of the building in period t, including cash from existing vintage leases as well as cash from new leases just signed. This third level should be the most reliably and objectively observable type of cash flow data. Building owners and managers may have good understanding and intuition concerning this level of cash flows, while brokers and investors may have better understanding of the market rental price data represented by the $\{y_t\}$ or $\{x_t\}$ series. (Of course, CF_t , like y_t and x_t , is normalized per square foot per year, or some other common unit of space and time measurement.)

By establishing relationships between the $\{x_t\}$ series, which is independent of lease term, and the $\{y_t\}$ and $\{CF_t\}$ series, which are a function of lease term, the model developed in this Chapter enables analysis of the effects of lease term on rental prices and cash flow levels and risk characteristics. As noted, the model also short-cuts the "different risk regimes" problem in building valuation, and establishes a relationship between unobservable total return risk and observable cash flow or rental price risk.

In the remainder of this Chapter, Section 2.2 describes and discusses the simplifying assumptions which underlie the model and make it tractable, followed by Section 2.3 which develops the multi-period valuation model itself. Numerical analysis and discussion of the model's implications is left for Chapter 3.

2.2 The Assumptions Underlying the Model of Building Value

The model of property value is built on four assumptions, labelled:

- A.1: The Opportunity Cost Stochastic Process Assumption
- A.2: The One-Period Asset Pricing Model Assumption
- A.3: The Zero-NPV Lease Assumption
- A.4: The Riskless Lease and Constant Rent Assumption

As with all models, these assumptions simplify and abstract reality in order to allow us to focus on a few fundamental issues of interest, in this case revolving around the relationship between rate of return risk and cash flow risk and lease term. The hope, however, is that our abstractions from reality are not so gross as to miss the main essence of the truth as it relates to our subject. Each assumption is described below, along with some discussion of its justification and limitations.

2.2.1 The Opportunity Cost Stochastic Process Assumption

The first step in developing the multi-period cash flow valuation model is to identify a stochastic process (or more exactly, a "family" of such processes) which we will assume governs the realizations of the underlying opportunity cost time series $\{x_t\}$. We will assume under Assumption (A.1) that the opportunity cost cash flows $\{x_t\}$ are described by the following stochastic process:

Assumption (A.1):

$$x_t = (1+u_t)E_{t-1}[x_t] \tag{A.1.1}$$

$$E_{t-1}[x_t] = (1+g)(1+\alpha u_{t-1})E_{t-2}[x_{t-1}] + (1+g)b(X_{t-1} - E_{t-2}[x_{t-1}]) \tag{A.1.2}$$

where $b \geq 0$ and:

$$X_t = X_0(1+g)^t = (1+g)X_{t-1}, \quad \text{all } t \quad (\text{A.1.3})$$

$\{u_t\}$ is white noise with zero mean:
 $\text{cov}[u_t, u_{t-1}] = 0, E[u_t] = 0, \text{ all } L \ \& \ t, \text{ and} \quad (\text{A.1.4})$

$$\text{cov}_{t-1}[u_t, I_t] = \text{cov}_{t-1}[X_t, I_t] / E_{t-1}[X_t] = \sigma, \quad (\text{A.1.5})$$

where σ is a constant for all t

In (A.1), $E_{t-1}[\dots]$ refers to the expectation as of time $t-1$, that is, the optimal forecast conditional upon the knowledge of x_{t-1}, x_{t-2}, \dots , etc. The upper case X_t refers to the "central tendency" or long-run mean (trend) to which the cash flow series $\{x_t\}$ tends to revert (provided b is greater than g). The index I_t in (A.1.3) refers to the CAPM index, such as the return on the market portfolio, as will be described in Section 2.2.2 below.

Let us consider the nature of the cash flow process Assumption (A.1) for a moment. For one thing, (A.1) assumes "constant proportional risk". That is, the standard deviation of the one-period forecast errors in the $\{x_t\}$ series are constant proportions of $E_{t-1}[x_t]$ rather than constant absolute values as is more typical in "Box-Jenkins" models of univariate stochastic processes. Thus, if x_t grows large, the standard error of the forecast will grow large in absolute terms, but remain constant in proportional terms, and vice versa if x_t becomes small. This seems more plausible than to keep

assuming the same absolute magnitude of forecast error no matter what the size of x_t . Furthermore, if the traditional Box-Jenkins constant-absolute-error version were used here, we would obtain a valuation model, after applying Assumption (A.2), that could give negative valuations and valuations which decline as the lifetime of the building increases ceteris paribus. Thus, the constant proportional risk assumption embedded in (A.1) would seem to be quite sensible.

Let us now turn to a consideration of the meaning of the parameters in (A.1). The parameter g represents the deterministic geometric growth rate tendency in the underlying cash flows. If g includes inflation, then we are measuring cash flows in nominal terms. If g is net of inflation, then we are measuring cash flows in real terms. In principle, any deterministic trend pattern over time can be applied to the expected future cash flows, not just simple exponential growth or decay as represented by g . We only sacrifice algebraic simplicity. For example, in the numerical analysis in Chapter 3 a sensitivity analysis is performed in which it is assumed that the deterministic trend is cyclical, to reflect predictable cyclicity in the real estate rental market.

The parameters α and b play important roles in the relation between cash flow risk and asset return risk. As noted, b is

the rate of mean reversion. It gives the proportion of the distance between the current cash flow and its central tendency which we would expect to be closed each period. In other words, if $b=.50$ and x_t differs from X_t , then, barring other perturbations, we would expect x_t to move halfway back toward X_t each period. The parameter α is the "elasticity of expectations" of the cash flow process. It tells the sensitivity of future cash flow expectations to the present cash flow realization. If a one-unit change in x_t causes a one-unit change in $E_t[x_{t+1}]$, then $(1+g)\alpha=1$. More formally:

$$\alpha = \frac{\partial E_{t-1}[x_t]}{\partial x_{t-1}} / (1+g)$$

While the elasticity of expectations is often thought of as lying between 0 and 1, Fama describes the economic basis of this elasticity in the context of the multi-period asset valuation model we are using here. His analysis suggests that α ought typically to be near unity, but could either be less than or greater than 1. In the multi-period risky cash flow valuation model, α reflects the "smoothness" with which new information about the magnitude of a future cash flow x_t is revealed over time prior to t . If the same amount of uncertainty about the value of x_t is resolved (eliminated) between each two periods of time, then $\alpha=1$. If more uncertainty is resolved in the last period prior to t than in

each previous period, then $\alpha < 1$, while if less uncertainty is resolved between $t-1$ and t than between previous periods, then $\alpha > 1$. In most cases, therefore, it would seem reasonable to assume $\alpha = 1$, unless the analyst has reason to assume otherwise.

Now suppose $g \leq 0$ or we work with detrended values of $\{x_t\}$ to eliminate g . Then $b=0$ signifies that the underlying cash flow process $\{x_t\}$ has no central tendency, and can be characterized as a "random walk". If $\alpha=1$, then the cash flows follow a "pure" random walk, in the sense that $E_t[x_{t+1}]$ would simply equal x_t , no matter what the previous values of x prior to t had been. If $0 < \alpha < 1$ we still have a random walk, but now there is some "smoothing" in the conditional expectations, since $E_t[x_{t+1}]$ will be influenced by previous realizations of x prior to t . The value $(1-\alpha)$ can be thought of as the amount of smoothing. If $\alpha > 1$, then there is, in a sense, "negative smoothing", since the conditional expectation magnifies the current deviation from the prior expectation. No matter what the value of α , as long as $b=0$, the $\{x_t\}$ series is a random walk. If $b > 0$, then α still has the same interpretation, as the elasticity of expectations, and should still be generally assumed to lie near unity, but the cash flow process is no longer a random walk, since it tends to revert ultimately toward X_0 (after detrending to eliminate g). This has the effect of greatly reducing the amount of risk that is in the

long-run future cash flows, and hence, that is in the property's market value and total return. [Note that if we are not working with detrended cash flow values and $g > 0$, then b must exceed g in order for the underlying cash flow process to be mean-reverting.]

2.2.2 The Risk Pricing Model Assumption

The Risk Pricing Model Assumption consists of two assumptions. The first is to assume that a particular risk value model holds in a one-period world. The second is to assume that this one-period risk value model holds, one period at a time, each period during the life of the property.

The one-period risk value model is an assumption about how risk is priced in assets such as the property we are valuing in a one-period world. In such a world, there is no difference between cash flow or the income component of return and capital appreciation or the price component of return, since the asset must be liquidated and the proceeds consumed at the end of the period (there being no future period).

The one-period risk value model we shall use is presented below in the form of a generalized version of the Capital Asset Pricing Model (CAPM):

$$E[r(i)] = r + \mu\sigma_r(i) \quad (\text{A.2.1})$$

where $E[r(i)]$ is the expected total return (at the beginning of the period) on asset i ; r is the riskfree interest rate; μ is the "market price of return risk":

$$\mu = (E[r(m)] - r) / \text{cov}[r(m), I] \quad (\text{A.2.2})$$

and $\sigma_r(i)$ is the rate of return risk in asset i :

$$\sigma_r(i) = \text{cov}[r(i), I] \quad (\text{A.2.3})$$

where $r(m)$ is the total return to the "market portfolio", a broadly diversified portfolio of market-valued risky assets, and I is the "CAPM Index".

For example, in most traditional applications of the CAPM to securitized assets, the index I is taken to be the return to the market portfolio, which is usually proxied by the stock market. Thus, $I=r(m)$.

However, there is widespread belief among both practitioner and academic real estate analysts that this traditional version of the CAPM does not well apply to unsecuritized real

estate assets. [See, for example, Lusht, or Ibbotson & Siegel.] Also, recall Table 1.1 in Chapter 1, which indicates that, based on appraisal returns anyway, unsecuritized real estate portfolios seem to have negative risk if we define "I" to equal the stock market return, yet these portfolios display positive risk premia.]

This suggests that the traditional definition of $I=r(m)$ would lead to a serious underestimation of the risk premium μ_0 , in real estate. A more general version of the CAPM, such as the Consumption-based CAPM (Breedon), may be more appropriate for application to unsecuritized real estate assets. (In Part II of this thesis, the intuitive basis of the Consumption CAPM is presented along with some empirical evidence that indicates that the Consumption-based CAPM may indeed be able to explain unsecuritized real estate's return risk premium much better than the traditional stock market based CAPM.) For the Consumption-based CAPM the "CAPM Index" would be the (unexpected) percentage change in real national per capita consumption. Labelling this change C, we would thus have $I=C$ for the Consumption-based version of the CAPM.

The CAPM interpretation of (A.2.1) presented above is consistent with modern applied financial economic theory, in that it is based on a general equilibrium model of asset prices in

the capital markets. There is, however, another way in which the same equation (A.2.1) could be defined, more in line with traditional real estate practice (as taught, for example, in widely used textbooks, such as Jaffe & Sirmans, or Pyhrr & Cooper). This is essentially the "decision analysis" approach to risk valuation, assuming undiversified risk-averse investors, such that the asset return risk represented by $\sigma_r(1)$ is given by the asset's total risk or volatility. In effect, under this approach we would define $I=r(1)$ in (A.2.3), and let μ represent the market price of this total risk. Thus, μ in this case is not defined by (A.2.2), but is instead the marginal investor's willingness to pay (in the form of foregone expected return premium) to avoid return volatility in his investment in property i . While this definition of (A.2.1) is not based on general equilibrium analysis, it may be of interest to those familiar with the traditional real estate practice to know that the multi-period valuation model developed in this Chapter, and all the insights and implications derived from it, hold also under this traditional approach to real estate risk valuation, as well as under the CAPM approach. Henceforth in this thesis, however, unless stated otherwise, the CAPM approach to risk valuation will be assumed to hold.

Finally, the valuation model developed in this chapter is also amenable to incorporating the main theoretical general

equilibrium argument advanced to date in the real estate literature to explain real estate return premia in view of real estate's apparent lack of risk from a traditional stock market based CAPM perspective. This is the "New Equilibrium Theory" (NET) put forth by Ibbotson and others (see Ibbotson & Siegel, and Ibbotson, Diermier and Siegel). According to this theory, non-risk attributes of real estate investments which are disliked by most investors cause real estate values to be discounted relative to stocks (for an equivalent amount of risk), leading to a non-risk return premium in real estate. The fact that this premium is apparently observed only in unsecuritized real estate leads one to speculate that this premium (to the extent it exists) consists of an "illiquidity premium" and/or "information & transaction cost premium". Since this expected return premium is independent of risk in the asset, it would be reflected in the valuation model being developed here, as an additional term in eqn.(A.2.1), akin to the riskfree rate r , which would reflect the one-period non-risk return premium associated with property i . Thus (A.2.1) would be modified to become:

$$E[r(i)] = r + \lambda + \mu\sigma_r(i) \quad (\text{A.2.1a})$$

where λ is the non-risk expected return premium as per the NET. We assume in this thesis that λ is non-negative, but may be zero. [To the extent that illiquidity interacts with return risk to matter to investors, this would be represented

in the model by larger values of the parameter μ , that is, μ would include not only the market price of return risk defined in (A.2.2), but also another term representing the market price of illiquidity as it interacts with return risk.]

As noted, the second part of Assumption (A.2) is that the one-period risk value model described by eqn.(A.2.1) holds, one period at a time, in all periods. Thus, Assumption (A.2) is expressed as:

Assumption (A.2):

$$E_{t-1}[r(i)_t] = r + \lambda + \mu \text{cov}_{t-1}[r(i)_t, I_t] , \quad \text{all } t \quad (\text{A.2})$$

where $E_{t-1}[r(i)_t]$ is the expectation as of time $t-1$ (ie, based on knowledge of all events occurring through time $t-1$) of the return next period to asset i ; and $\text{cov}_{t-1}[\dots]$ is similarly the conditional covariance given information through time $t-1$.

Implicit in Assumption (A.2) are the commonly employed simplifications: that the riskfree rate, r , and the market price of return risk, μ , are both constant over time, and known by all investors. We assume similarly for the "illiquidity premium", λ .

2.2.3 The Zero-NPV Lease Assumption

Here we assume that lease signings are zero-NPV transactions as of the time when they are signed, at least for the landlord. This means simply that the value of the lease equals its opportunity cost based upon the foregone $\{x_t\}$ payments over the term of the lease. (Recall from section 2.1 that x_t represents the lease opportunity cost accruing to the landlord during period t if he had signed, during or prior to t , a lease covering his space during period t .)

To state the Zero-NPV Lease Assumption more formally, suppose a T -period lease is signed at the beginning of period s , and the first rent payment under the lease due and received by the landlord upon lease signing, and the remaining $T-1$ payments are received at the beginning of periods $t=s+1$, $t=s+2$, ..., $t=s+T-1$, giving the tenant occupancy rights starting at time s and going through the end of period $s+T-1$. [In other words, the rental payments are due at the beginning of each period.]

Then the opportunity cost of the lease to the landlord, as of the time of lease signing (that is, time $t=s$), is the cost of foregoing the stream $\{x_t\}$ where t goes from s to $s+T-1$. Defining $L_s[T]$ as the time s (market) value of a T -period

lease signed at time s (including the first rental payment), and letting $PCEV_s[.]$ designate the present certainty-equivalent value operator on a cash flow stream giving value as of time s , the zero-NPV lease signing assumption is expressed as:

Assumption (A.3):

$$\begin{aligned} L_s[T] &= PCEV_s[\{x_t\}], \quad t = s, s+1, \dots, s+T-1 \\ &= x_s + PCEV_s[\{x_t\}], \quad t = s+1, \dots, s+T-1 \end{aligned} \quad (A.3.1)$$

Thus, the value at time s of the lease signed at time s is equal to x_s plus the ex dividend value of an asset which gives its holder the right to obtain the spot market net rental payments the building could receive through time $s+T-1$.

In the valuation context relevant here, this zero-NPV lease assumption could simply be viewed as a version of the Miller-Modigliani Theorem of asset value invariance with respect to the financing method, since riskless leases can be risklessly traded for debt financing. The original M-M "Proposition I" (which asserted value invariance apart from the effect of taxes) is relevant here, because any effect multiperiod leasing would have on the property owner's taxes would be already reflected in the $\{x_t\}$ stream of opportunity costs. (Recall that x_t can be defined as the single-period lease rent value

that leaves the landlord indifferent between single and multiperiod leasing.)

2.2.4 The Riskless Lease & Constant Rent Assumption

This assumption consists of two parts: that rent payments specified in the lease contract are riskfree; and that rent payments are identical in each period under the lease.

The first assumption amounts to assuming: (i) that there is no risk that tenants will default on their lease obligations; (ii) that leases do not contain landlord participation in tenant revenues, or other features which might impart systematic risk to the rental payments under the lease; and (iii) that rent payments are net of any risky expenses and taxes which would otherwise cause the landlord's net cash flows to be different from and more risky than his rental revenue. In other words, for algebraic simplicity, we are assuming in our building value model that rental revenue is fixed in amount and timing under each lease (as described in section 2.2.3) and equals net cash flow to the landlord.

While this assumption may fairly accurately describe many leases, especially in office buildings, it is true that many real world leases are much more complex than this simplified

model, and are not actually riskless or completely "net" of expenses to the landlord. This assumption is therefore a simplification which is necessary for the tractability of the model to be developed.

The second assumption contained in Assumption (A.4) is that the lease rent per period is the same for all the periods under the lease. Conceptually, it would be very easy to let the rent change deterministically over the time covered by the lease (and this would not even greatly complicate the algebra if we let the rent grow at some constant rate), but this would appear to add little of substance to the valuation model developed in this Chapter.

The constant rent assumption is less restrictive than it first appears, since the values used in the model (for example, the x_t and the returns) can be expressed either in nominal or real terms, as long as we are consistent. Thus, "constant rent" can either mean constant in real terms or constant in nominal terms, two cases which more or less bracket the interesting real world cases of non-constant rents.

Expressed formally, the riskless lease and constant rent assumption is represented as:

Assumption (A.4):

$$\begin{aligned} y_t(s,T) &= y(s,T), \text{ all } t = s, s+1, \dots, s+T-1 \\ &= 0, \text{ otherwise,} \end{aligned} \tag{A.4.1}$$

where $y_t(s,T)$ is the rental payment received at time t under a T -period lease signed at time s , and $y(s,T)$ is therefore the (observable) market (new-lease) rent (per period) at time s , for T -period leases signed at time s .

2.3 The Multi-period Valuation Model

In this section we use the four assumptions described above to construct a multi-period valuation model which can be used to relate return risk to cash flow and underlying risk and to the lease term.

2.3.1 The Underlying Valuation Model

The first step is to derive a multi-period valuation model when there are only one-period leases. This is equivalent to a more general capital budgeting valuation model taking x_t to be the net cash flows from the "project" or "asset" being considered for construction or purchase. This model is the same as that used by Bhattacharya, only with the slight generalization that we allow for a geometric growth tendency in the cash flows.

The technique for deriving this model is to start from some terminal period, τ , and work back recursively one period at a time applying Assumptions (A.1) and (A.2), until you get to the present. Each step involves only algebra and some use of basic probability or stochastic processes theory and definitions. As this recursion procedure is well described in Myers & Turnbull, the details are be omitted here. The resulting

valuation formula is given below, where $V_s(\tau=T)$ equals the time s ex dividend value of an asset with T periods of cash flows remaining (ie, the first cash flow will be received at the beginning of period $s+1$):

$$V_s(\tau=T) = \theta \frac{(1-Z^T)}{(1-Z)} E_s[x_{s+1}] + \theta b S_T X_{s+1} \quad (1)$$

where:

$$\theta = (1-\mu\sigma)/(1+r+\lambda)$$

$$Z = (1+g)(1-b-\alpha\mu\sigma)/(1+r+\lambda)$$

$$S_1 = 0, \quad S_2 = (1+g)/(1+r+\lambda), \quad \text{and for } T \geq 3:$$

$$S_T = \frac{1+g}{1+r+\lambda}^{(T-1)} + \frac{1+g}{1+r+\lambda}^{(T-2)}(1+Z) + \dots \\ \dots + \frac{1+g}{1+r+\lambda}(1+Z+\dots+Z^{T-2})$$

and all the other parameters are defined as in Assumptions (A.1) and (A.2) described in sections 2.2.1 and 2.2.2. Note in particular that while the risk value model (A.2) is defined in terms of return risk, σ_r , the valuation formula here is defined in terms of the underlying cash flow risk, σ , introduced in the cash flow process assumption (A.1).

For the case of the perpetuity, where $T=\infty$, (1) simplifies to:

$$V_s(\tau=\infty) = \theta E_s[x_{s+1}] + \theta b(1+g)X_{s+1}/(r+\lambda-g) \quad (1a)$$

where:

$$\theta = (1-\mu\sigma)/[r+\lambda-g+(1+g)(b+\alpha\mu\sigma)]$$

Note that the first term on the RHS of (1) is stochastic over time, as the conditional expectation $E_s[x_{s+1}]$ depends on information revealed over time up through time s . This makes asset value stochastic over time, which imparts risk into the appreciation return component of the investment. But the second term on the RHS of (1) is non-stochastic in this model, depending only on parameters that are known constants, and on the central tendency, $X_t = X_0(1+g)^t$, which is also known with certainty.

Some intuition for formula (1) can be obtained by considering the nature of the short range and long range optimal univariate forecast of the asset's cash flows, using the cash flow process (A.1). For the short range forecast, consider the forecast of cash flows two periods hence:

$$E_{t-2}[x_t] = (1-b)(1+g)E_{t-2}[x_{t-1}] + bX_t$$

The two-period forecast is seen to have two components. The first term is simply the forecast of cash flow one period hence, modified to include the expected growth trend, while the second component is the central tendency as of two periods hence. The latter component is weighted by the factor b , the mean reversion rate, while the former component is weighted by the factor $(1-b)$.

The long range forecast L periods hence is given by:

$$E_{t-L}[x_t] = [(1-b)(1+g)]^{L-1}E_{t-L}[x_{t-L+1}] + [1-(1-b)^{L-1}]x_t$$

which at $L=\infty$ with $b > \max\{g, 0\}$ simply equals x_t exactly. (That is why x_t is called the central tendency of the cash flow.)

Thus, with some simplification, we can say that short range cash flow forecasts have roughly a proportion b which is fixed and non-stochastic (hence, riskless), while very long range forecasts are virtually entirely riskless if b is greater than g .

Now we note that the second term on the RHS of (1a) is just the factor θb times the next period's central tendency value x_{s+2} divided by the capitalization rate appropriate for a riskless constant growth perpetuity which starts out at the level of x_{s+2} and grows forever at the rate of g . The factor θ would normally be greater than one unless b is quite large, so the second term on the RHS of (1a) will be somewhat greater than the proportion b times the capitalized value of the riskless central tendency, reflecting the fact that for cash flow expectations beyond two periods hence, a proportion greater than b will generally be riskless.

To this capitalization of the riskless portion of the expected future cash flows, we add the first term on the RHS of (1a), the amount $\theta E_s[x_{s+1}]$, which is the capitalization of the risky portion of the expected future cash flows. We note that θ is smaller the larger is σ , α , and μ , which makes intuitive sense, given the meaning of each of these parameters as described in Section 2.2. θ is also smaller with larger b , since larger b implies that a larger proportion of the expected future cash flows are riskless (due to the central tendency) and are thus capitalized in the second term. Though θ will usually be greater than one, it will generally be much smaller than the riskless perpetuity capitalization factor $1/(r+\lambda-g)$.

Based upon this intuitive splitting of the valuation formula into stochastic and non-stochastic parts, let us define two terms which will help simplify and clarify other formulas we will develop later in this Chapter:

$$H[T] = \theta(1-Z^T)/(1-Z)$$

$$\implies H[\infty] = \theta$$

and:

$$K[T] = \theta b S_T$$

$$\implies K[\infty] = \theta b(1+g)/(r+\lambda-g)$$

Thus, $H[n]$ is the ex dividend present certainty-equivalent value multiplier for the stochastic component of a sequence of n expected cash flows, and $K[n]$ is the ex dividend present

value multiplier for the non-stochastic component of a sequence of n expected cash flows.

2.3.2 Including Long-term Riskless Leases

The next step in deriving the asset valuation model is to introduce riskless multi-period leases using Assumptions (A.3) and (A.4).

2.3.2.1 The Valuation Model and the New-lease Rental Price

By our zero-NPV lease signing assumption (A.3), the ex dividend value at time s of a property that has no continuing lease but will be leased at $s+1$ is always given by formula (1a) above, no matter what the term of the lease under which the property will be rented. This asset value invariance with respect to the lease term could also be viewed as a manifestation of the fact that the mere signing of the lease does not eliminate any risk, but merely transfers the exposure to some risk from the landlord to the tenant. As the transfer occurs at market prices and the market value of this risk is the same no matter who bears it, the landlord does not "get something for nothing", and his property value is left unchanged by the transfer of risk.

This implies that with multiperiod leasing the rental price, y_s , should generally be less than the underlying opportunity cost, x_s . Since the landlord's cash flows are derived from the $\{y_s\}$ series, and are less risky with multiperiod leasing, the expected cash flows based on $\{y_s\}$ must be discounted at a lower rate than would be applied to the expectations of the $\{x_s\}$ series in the absence of multiperiod leasing. To keep property value the same, the cash flow expectations based on $\{y_s\}$ must therefore be less than those based on $\{x_s\}$.

This makes sense, because the use of multiperiod leasing effectively transfers a risk burden from the landlord to the tenant. The contractual cash flow commitments represented by multiperiod leases are (effectively) like positive debt obligations on the books of the tenant, and like negative debt (or positive loan assets) in the landlord's books, thereby increasing the tenants leverage and decreasing the landlord's leverage (ceteris paribus), thus increasing the tenant's business risk and decreasing the landlord's.

Another way to see this is to note that while both the tenant and landlord face the opportunity cost represented by the $\{x_t\}$ series, the risk in this opportunity cost is positive to the landlord, and negative to the tenant. (By the value additivity principle, risk in inflows or revenues is "bad" in

that it reduces the market value of the owner of the cash flows, while risk in outflows or expenses is "good" in that it increases the market value of the owner of or obligee to the cash flows.) Thus, it is the landlord who must pay the tenant (in reduced rent) to get the tenant to sign the multiperiod lease and remove this risk for the remaining time covered by the lease.

In a sense, by observing that we can still use formula (1a) to value the property even with multiperiod leasing, we have solved the valuation problem with long-term leases. This is why the definition of the underlying spot price opportunity cost construct, $\{x_t\}$, is a kind of "short cut". But we do not yet have in (1a) a formula which is useful either for practical valuation or for conceptual analysis of the relation between cash flow risk, lease term, and return risk. Formula (1a) is expressed only in terms of the underlying market opportunity cost cash flows, and the risk of those opportunity costs, which are unobservable in markets where there is no short-term spot market for space. For the purposes of our analysis, we would like to relate formula (1a) to values we can observe, such as the market rental price of T-period leases, $y(t,T)$, where T is the typical length of time covered by leases in the market in question.

To do this, first consider the value of the lease to the landlord at the time of its signing. By eqn.(A.3.1) of our zero-NPV lease signing assumption we have that the lease value is just the present certainty-equivalent value (PCEV) of the opportunity cost cash flows x_t from time s through $s+T-1$. Formula (1) now gives us a way to specify this PCEV. So the value to the landlord at time s of the T -period lease signed at time s equals:

$$L_s[T] = x_s + V_s(T-1) \quad (2a)$$

where $V_s(T-1)$ is given by formula (1) with $\tau=T-1$.

But we know also that by Assumption (A.4) this lease has the value of a T -period riskless annuity paying the rental amount $y(s,T)$ each year for T years starting with the first payment now at time s . (Recall from Section 2.1 that $y(s,T)$ is defined as the new-lease rental price in a T -period lease signed at time s .) Thus, $L_s[T]$ is also given by:

$$L_s[T] = (1+r)a[T]y(s,T) \quad (2b)$$

where $a[T]$ is the present value of the T -period riskless annuity (starting one period hence, which is why we must multiply by $(1+r)$ in the formula above).

$$a[T] = [1-1/(1+r)^T]/r$$

Putting eqn.(2a) and (2b) together and expanding $V_s(T-1)$ by formula (1) we can solve for the following relationship

between the new-lease rental price, $y(s,T)$, and the underlying cash flow opportunity costs:

$$\begin{aligned}
 y(s,T) &= x_s / [(1+r)a[T]] + V_s(T-1) / [(1+r)a[T]] \\
 &= \frac{x_s}{(1+r)a[T]} + \frac{H[T-1] E_s[x_{s+1}]}{(1+r) a[T]} + \frac{K[T-1] X_{s+1}}{(1+r)a[T]} \quad (3)
 \end{aligned}$$

Note that only the first two terms in the RHS of (3) are stochastic.

There is some intuition in formula (3). Note that $V_s(T-1)$ is a risky cash flow present value multiplier with $T-1$ cash flows in it, based on the $\{x_t\}$ series, whereas $a[T]$ is a riskless cash flow multiplier with T cash flows in it. Thus, $V_s(T-1)/E_s[x_{s+1}]$ will usually be less than $a[T]-1$ (unless x_s is enough below the central tendency X_s and b is large enough). At $T=1$, $(1+r)a[T]=1$ and $V_s(T-1)=0$, which implies $y(s,T)=x_s$, as it should. The annuity multiplier $a[T]$ is slightly less than T for typical values of r and T . For $T > 1$, $y(s,T)$ will be somewhat less than x_s , unless x_s happens to be considerably below the central tendency X_s (which could cause the relevant expected future values of x to considerably exceed x_s if b is large enough). As noted above, it makes sense for $y(s,T)$ to usually be smaller than x_s , because of the direction of the transfer of the risk burden involved in the multiperiod leasing transaction.

2.3.2.2 Relation Between Rent Price and Underlying Opportunity Cost Levels

Equation (3) enables us to specify the relationship between the expected value of the (observable) new-lease rental price and the (unobservable) underlying cash flow opportunity cost.

Using the relationship from (A.1) that:

$$E_s[x_{s+1}] = (1+g)[\alpha x_s + (1-\alpha-b)E_{s-1}[x_s] + bXs]$$

and taking time $s-1$ conditional expectations of both sides of (3), yields the ratio of $E_{s-1}[Y(s,T)]$ over $E_{s-1}[x_s]$:

$$\frac{E_{s-1}[Y(s,T)]}{E_{s-1}[x_s]} = \frac{1 + [1 - (1 - R_s)b](1+g)H[T-1]}{(1+r)a[T]} + \frac{(1+g)K[T-1]}{(1+r)a[T]} R_s \quad (4)$$

where R_s is defined to be the current (time $s-1$) ratio of the next period's central tendency value over the next period's conditional expected value: $R_s = X_s/E_{s-1}[x_s]$. This ratio reflects the current deviation above or below the central tendency. The unconditional expected value of R_s is one. Thus, the ratio of the unconditional or long-run average value of y to x is just the above relation (4) with $R_s=1$, which is obviously less than one.

2.3.2.3 Relationship Between Rent Price Risk and Underlying Opportunity Cost Risk

Similar reasoning applied to eqn.(3) and using (A.1) as above allows us to establish the relation between the risk in the new-lease rental price series, labelled σ_y , and the underlying risk σ . Defining σ_y as follows:

$$\sigma_y = \text{cov}_{s-1}[Y(s,T), I_s] / E_{s-1}[Y(s,T)]$$

We obtain the following relation between σ_y and σ :

$$\sigma_y / \sigma = \frac{1 + \alpha H[T-1](1+g)}{1 + [1 - (1-R_s)b]H[T-1](1+g) + K[T-1](1+g)R_s} \quad (5)$$

Note that at $T=1$ all the terms are zero and the ratio is unity, as it should be. If b is positive (underlying cash flows are mean-reverting), this ratio is stochastic over time, depending on the current deviation of the rental market around its central tendency. However, at the average value of $R_s=1$, (5) reduces to:

$$\sigma_y / \sigma [R_s=1] = (1 + \alpha H[T-1](1+g)) / (1 + H[T-1](1+g) + K[T-1](1+g))$$

Since for $T > 1$ and $b > 0$ all the terms are positive, this implies that σ_y / σ with $R_s=1$ is less than unity, unless α is enough greater than one to offset the additional mean-reversion term in the denominator, which would be unlikely. Thus, with multi-period riskless leases, the new-lease rental price will usually and on average be less risky than the underlying

spot market opportunity cost (at least, if there is either significant mean-reversion or the elasticity of expectations is less than unity). This makes sense, because the multi-period lease rent is based not only on the current rental market conditions, but on expectations of the market in the future periods covered by the lease. From the perspective of any point in time, mean-reversion tendencies and smoothing in the conditional expectations due to inelastic expectations will act to reduce the risk in the expected future rental market and thereby tend to smooth the rental price series.

Note that this is not the same thing as the reduction in risk faced by the landlord as a result of multi-period leasing, in the sense that contractual cash flows under any given lease are, by assumption, riskless. This can be seen by considering the case where underlying cash flows are not mean-reverting with $b=0$ (therefore, $K[T-1]=0$ for any T) and the underlying elasticity of expectations equals or exceeds unity ($\alpha \geq 1$). In this case, the new-lease rental price series is at least as risky as the underlying spot market opportunity cost series, even though the landlords reduce their risk and face riskless cash flows within each lease. The $\{y(t,T)\}$ series is not the same as the $\{CF_t\}$ cash flow series obtained by the landlords, for only a fraction of each building (or a fraction of all buildings) is exposed to the new-lease rental price

$y(t,T)$ in each period t . The actual cash flow series $\{CF_t\}$ will indeed be less risky than the underlying spot market with multi-period leases, reflecting the smoothing caused by riskless cash flows within each lease (see Section 2.3.3).

2.3.2.4 Relation Between Rent Price Elasticity of Expectations and Underlying Opportunity Cost Elasticity of Expectations

Equation (3) together with the cash flow process assumption (A.1) also allows us to derive the relation between the elasticities of expectations of the observable rental price and the underlying opportunity costs. First, we define the rental price elasticity of expectations, α_y , analogous to that of the underlying opportunity costs:

$$\alpha_y = \frac{\partial E_{t-1}[y(t,T)]}{\partial y(t-1,T)} / (1+g)$$

Using the chain rule we can expand this derivative:

$$(1+g)\alpha_y = \frac{\frac{\partial E_{t-1}[y(t,T)]}{\partial E_{t-1}[x_t]} \frac{\partial E_{t-1}[x_t]}{\partial y(t-1,T)}}{\partial E_{t-1}[x_t]}$$

Then, using (3) and (A.1) we can specify both of the derivatives on the RHS above, to obtain the following relationship between α_y and α :

$$\alpha_y = \frac{1 + (1-b)H[T-1](1+g)}{1 + \alpha H[T-1](1+g)} \alpha \quad (6)$$

The intuition for relationship (6) can be developed as follows. First consider the non-mean-reverting case where

$b=0$. At $T=1$, $H[T-1]=0$, and $\alpha_y=\alpha$, as it should. For multi-period leases ($T \geq 2$), α_y will be less than α if and only if $\alpha < 1$, α_y will be greater than α if and only if $\alpha > 1$, and if $\alpha=1$, α_y will equal α , no matter how large T is. This makes sense, since with multi-period leases the rental price series $\{y(t,T)\}$ is based not only on the current opportunity cost but on expectations of future opportunity costs over the life of the lease. These future expectations are less (or more) sensitive to present realizations, depending on whether the underlying α is less (or greater) than unity.

This also has an implication regarding the relationship of lease term on α_y in the non-mean-reversion case. If underlying α is less (greater) than unity, then α_y will increase (decrease) with lease term. Note that this is opposite to the relationship between α and σ_y with respect to lease term. That is, if α is less (greater) than unity, σ_y decreases (increases) with lease term. Since α_y and σ_y interact multiplicatively in the total return risk (that is, as we shall see: $\sigma_r \sim \alpha_y \sigma_y$, if $b=0$), this results in return risk being less sensitive than one might think to the lease term, if cash flows are non-mean-reverting.

Consideration of the likely magnitude of $H[T-1]$ can tell us even more about α_y . For $b=0$, $H[T-1]$ will be the ex dividend

present certainty-equivalent value multiplier for a stream of T-1 risky cash flows. So $H[T-1]$ will be less than T-1, but not much less if the risk is not too great and T is not too large. For T in the neighborhood of 5 to 10 years, which is typical of office space lease terms, $H[T-1]$ should be roughly 20 or 30 percent less than T-1. This means that for typical values of T, α_y will be very near to unity, over a broad range of reasonable values for the underlying α , provided $b=0$. For example, for $H[T-1](1+g)=5$, (6) becomes: $\alpha_y = 6\alpha/(1+5\alpha)$, which gives the following relation between α_y and α :

α	α_y
0.50	0.86
0.75	0.95
1.00	1.00
1.25	1.03
1.50	1.06

The intuitive reason why α_y should approximately equal one for the non-mean-reverting case no matter what α is can be easily understood. With multi-period leases, the $\{y(t,T)\}$ series of rental prices is like a weighted moving average of the underlying $\{x_t\}$ series, based on the PCEV formula represented by $H[T-1]$. The longer the lease term, T, the longer is the moving average. For illustration, ignore growth, risk, illiquidity, and the time value of money, and think of $y(t,T)$ as equal to $(1/T)$ times the sum of the expectations as of time t of $x_t, x_{t+1}, x_{t+2}, \dots, x_{t+T-1}$. The elasticity of expectations

of the rental price, α_y , equals the change in $E_{t-1}[y(t,T)]$ caused by a one unit change in $y(t-1,T)$. Under the above simplifications, this is given by:

$$\begin{aligned} \alpha_y &= \frac{\Delta E_{t-1}[y(t,T)]}{\Delta y(t-1,T)} = \frac{\Delta E_{t-1}[(1/T)(x_t + x_{t+1} + \dots + x_{t+T-1})]}{\Delta\{(1/T)E_{t-1}[(x_{t-1} + x_t + \dots + x_{t+T-2})]\}} \\ &= \frac{E_{t-1}[\Delta(x_t + x_{t+1} + \dots + x_{t+T-1})]}{E_{t-1}[\Delta(x_{t-1} + x_t + \dots + x_{t+T-2})]} \end{aligned} \quad (7)$$

It is clear that the sum that is changing in the numerator is almost identical to the sum that is changing in the denominator, except that the sum in the numerator includes x_{t+T-1} and excludes x_{t-1} , as the moving average "moves" one unit in time. But all the other $T-2$ terms ranging from x_t through x_{t+T-2} are included in both the numerator and denominator.

The random walk assumption on the $\{x_t\}$ series implies that all the time $t-1$ expectations of terms from x_t through x_{t+T-1} will be identically affected by the realization of x_{t-1} . For example, a one unit change in x_{t-1} would cause a change of α units in each of the subsequent expected values, $E_{t-1}[x_t]$, $E_{t-1}[x_{t+1}]$, etc.... Therefore, any realization of x_{t-1} which causes a one unit change in the denominator of would also cause a very similar change in the numerator, at least if T and/or α

were large enough so that the weight of the x_{t-1} term in the total value of the denominator is small.

That is, we can rewrite the above ratio in terms of a change in the $t-1$ realization of x , as follows:

$$\alpha_y = \frac{T\alpha(\Delta x_{t-1})}{[1+(T-1)\alpha](\Delta x_{t-1})} = \frac{\alpha T}{\alpha T + 1 - \alpha}$$

From which it is clear that for reasonably large values of T , α_y must be near unity. Although this analysis ignores risk, growth, illiquidity, and the time value of money, it is not hard to believe that these factors do not have a tremendous effect on this approximation.

Now consider the case of mean-reversion, where $b > \max(g, 0)$. (Note that b not only appears directly in (6), but indirectly as well via its effect on $H[T-1]$.) In general, α_y must be less with mean-reversion than it is without mean reversion, because a portion of the expected future cash flows is non-stochastic, governed by the central tendency. Recall from the discussion in Section 2.3.1 that even in short-range forecasts of x_{t+l} with mean-reversion, at least approximately the portion b of the expected cash flows is non-stochastic. This portion will not be influenced by changes in the realized value of x_{t-1} , thus dampening the sensitivity of $E_{t-1}[y(t, T)]$ to $y(t-1, T)$,

and reducing α_y below the level it would obtain without mean-reversion. Indeed, using reasoning analogous to that presented above for the random walk case, we would expect α_y with mean-reversion to approximately equal $(1-b)$, for a broad range of reasonable values of α . From formula (6) it is obvious that this would be the case for large values of $H[T-1]$. However, with mean-reversion, $H[T-1]$ will tend to be much smaller than it is without mean-reversion, as $H[T-1]$ essentially represents the present value only of the stochastic part of the future cash flow stream.

This reflects another side of the difference between mean-reversion and random walks. The future values of x_{t+H} are not nearly so equally weighted in the stochastic part of the moving average of the $\{E_t[x_{t+H}]\}$ sequence which composes the $\{y(t,T)\}$ series, with only the near-term $E_t[x_{t+H}]$ values being very susceptible to stochastic change. This is effectively like reducing the number of x_{t+H} terms in the sums in both the numerator and denominator of the illustrative equation (7) above. This effect partly offsets the effect of the non-stochastic portion of $\{E_t[x_{t+H}]\}$, raising α_y slightly above the $(1-b)$ level for most values of α , and also making α_y a bit more sensitive to α than it is without mean reversion. Nevertheless, α_y remains fairly insensitive to the underlying α , and generally smaller than what it would be under the non-

mean-reversion case. For example, assuming $H[T-1](1+g)=2$, and $b=.25$, formula (6) becomes $\alpha_y = 2.5\alpha/(1+2\alpha)$, which gives:

α	α_y
0.50	0.63
0.75	0.75
1.00	0.83
1.25	0.89
1.50	0.94

2.3.2.5 Mean Reversion in the Rental Price Series and the Underlying Series

It seems fairly intuitive that the rate of mean reversion would be the same in the rental price series $\{y(t,T)\}$ as it is in the underlying series $\{x_t\}$. This intuition is confirmed as follows. Define b_y to be the rate at which $\{y(t,T)\}$ tends to revert to its central tendency, which we label $Y(t,T)$. [This central tendency for $\{y(t,T)\}$ is related in principle to that of the $\{x_t\}$ series using equation (4).] Just as b can be defined formally from (A.1) by the relation:

$$b = \frac{E_t[x_{t+1}] - E_t[x_{t+2}]/(1+g)}{E_t[x_{t+1}] - X_{t+1}}$$

So, we can analogously define b_y the relation:

$$b_y = \frac{E_t[y(t+1,T)] - E_t[y(t+2,T)]/(1+g)}{E_t[y(t+1,T)] - Y(t+1,T)}$$

Using equation (3) and (A.1) we can expand this second ratio and relate it to the first to reveal that:

$$\begin{aligned}
(1+g)(1-b_y) &= \frac{(1+(1-b)H[T-1](1+g))(E_t[x_{t+2}]-X_{t+2})/[(1+r)a[T]]}{(1+(1-b)H[T-1](1+g))(E_t[x_{t+1}]-X_{t+1})/[(1+r)a[T]]} \\
&= (1+g)(1-b)
\end{aligned}$$

which confirms the intuitive result that $b_y=b$.

2.3.2.6 Summarizing up to Here...

At the outset of Section 2.3.2 we obtained a valuation model for a building with multi-period leases, but that model was specified entirely in terms of the unobservable underlying parameters, $E_s[x_{s+1}]$, σ , α , and b . In the following four subsections we used the underlying cash flow stochastic process assumption (A.1) and the valuation formula (1) itself, to develop relationships between these underlying parameters and the corresponding observable parameters based on the new-lease rental price time series, $\{y(s,T)\}$. We saw that these relationships were reasonably intuitive, at least at a rough qualitative level. Thus, all of the unobservable variables and parameters in our original valuation formula (1a) can now be expressed in terms of variables and parameters which are at least potentially observable.

This means that formula (1a) may have some practical role aiding actual property valuations in some situations. But more importantly for the purpose of this thesis, we can use

the relationships developed above to build our intuition about real estate return risk and its relationship to rental market risk and lease term, by exploring the conceptual relationships implied in the above analysis. Before we can do this, however, we must introduce one more level of complexity into the model, for we have not yet considered the effect of existing vintage leases in the buildings we are evaluating, and the relationship to actual observed cash flow, the $\{CF_t\}$ series, which reflects these existing leases. This is the subject of the next section of the Chapter.

Summary of Assumptions...

Assumption (A.1): The Opportunity Cost Stochastic Process...

$$x_t = (1+u_t)E_{t-1}x_t \quad (A.1.1)$$

$$E_{t-1}[x_t] = (1+g)(1+\alpha u_{t-1})E_{t-2}[x_{t-1}] + (1+g)b(X_{t-1} - E_{t-2}[x_{t-1}]) \quad (A.1.2)$$

where $b \geq 0$ and:

$$X_t = X_0(1+g)^t = (1+g)X_{t-1}, \quad \text{all } t \quad (A.1.3)$$

$$\{u_t\} \text{ is white noise with zero mean:} \\ \text{cov}[u_t, u_{t-1}] = 0, \quad E[u_t] = 0, \quad \text{all } L \text{ \& } t \quad (A.1.4)$$

$$\text{cov}_{t-1}[u_t, I_t] = \text{cov}_{t-1}[x_t, I_t] / E_{t-1}[x_t] = \sigma, \\ \text{where } \sigma \text{ is a constant for all } t \quad (A.1.5)$$

Assumption (A.2): The Risk Value Model...

$$E_{t-1}[r(i)_t] = r + \lambda + \mu \text{cov}_{t-1}[r(i)_t, I_t], \quad \text{all } t \quad (A.2)$$

Assumption (A.3): Zero-NPV Lease Signing Transaction...

$$L_s[T] = \text{PCEV}_s[\{x_t\}], \quad t = s, s+1, \dots, s+T-1 \\ = x_s + \text{PCEV}_s[\{x_t\}], \quad t = s+1, \dots, s+T-1 \quad (A.3.1)$$

Assumption (A.4): Riskless Constant-Rent Leases...

$$y_t(s, T) = y(s, T), \quad \text{all } t = s, s+1, \dots, s+T-1 \\ = 0, \quad \text{otherwise,} \quad (A.4.1)$$

Summary of Valuation Model...

$$V_s(\tau=T) = H[T]E_s[x_{s+1}] + K[T]X_{s+1} \quad (1) \text{ and } (1a)$$

where:

$$H[T] = \Phi(1-Z^T)/(1-Z)$$

$$\implies H[\infty] = \theta$$

and:

$$K[T] = \Phi b S_T$$

$$\implies K[\infty] = \theta b(1+g)/(r+\lambda-g)$$

and...

$$\theta = (1-\mu\sigma)/[r+\lambda-g+(1+g)(b+\alpha\mu\sigma)]$$

$$\Phi = (1-\mu\sigma)/(1+r+\lambda)$$

$$Z = (1+g)(1-b-\alpha\mu\sigma)/(1+r+\lambda)$$

$$S_1 = 0, \quad S_2 = (1+g)/(1+r+\lambda), \text{ and for } T \geq 3:$$

$$S_T = \left(\frac{1+g}{1+r+\lambda}\right)^{T-1} + \left(\frac{1+g}{1+r+\lambda}\right)^{T-2}(1+Z) + \dots \\ \dots + \left(\frac{1+g}{1+r+\lambda}\right)(1+Z+\dots+Z^{T-2})$$

2.3.3 Return Risk and Cash Flows Including Existing Leases

Our objective in this section is to develop the relationship between asset return risk, σ_r , and underlying risk, σ , so that we can numerically analyze not only the σ_r/σ relationship, but also other relationships of interest. In particular, since we must relate the actual cash flow series $\{CF_t\}$ to the underlying opportunity cost series $\{x_t\}$ to obtain the formula for σ_r/σ , the development in this Section will also allow us to specify the relation between the observable cash flow risk, σ_{CF} , and both the return risk and underlying opportunity cost risk. Together with the relationship σ_y/σ already specified in Section 2.3.2, this will enable us to numerically analyze the relationships among all three levels of cash flow risk, as well as between these cash flow risks and the total return risk. Also, since $CF(t,T)$ and $y(t,T)$ are functions of lease term T , specifying the valuation formula in terms of CF and y instead of in terms of x enables us to explore the relationship between risk and lease term.

2.3.3.1 Taking Account of Existing Leases

The first thing we must do is to take account of the fact that a property may have existing vintage or newly-signed leases which have not expired as of the present. These conti-

ning leases reflect the market rental price during past periods, that is, $y(t,T)$ values for times t prior to the present. They give the building some future cash flows which are riskless (from which comes the "bond-like" part of the property's value discussed at the outset of this Chapter).

We begin by imagining a building all of whose space rents out under a single lease. Formula (1) gives the ex dividend building value at time s if s happens to be a time when the lease has just expired. But suppose that there are n periods left in the building's current lease (not counting the present period, for which rent has already been received). Note that n can range from 0, if the lease is expiring in the current period, to $T-1$ if we have a new lease that was just signed at the beginning of the current period. At any time s , the building value, V_s , will be given by:

$$V_s = B_s + S_s \quad (8)$$

where B_s is the "bond part" value and S_s is the "stock part" value, as follows:

$$B_s = a[n]y(s+n+1-T,T)$$

$$S_s = \text{PCEV}_s[V_{s+n}]$$

Recall that $y(s+n+1-T,T)$ is the new-lease rental price prevailing for T -period leases signed $T-n-1$ periods prior to time s , and $a[n]$ is the present (ex dividend) value of a riskless unit annuity with n payments left. $\text{PCEV}_0[V_n]$ must be

determined in the same way that equation (1a) of Section 2.3.1 was derived (ie, by recursion). The simple breakdown of building value in (8) can now be used to develop the formula for total return risk as a function of underlying risk.

2.3.3.2 Risk in Total Returns

By definition, the total return to asset 1 in period s is given by:

$$r(1)_s = (CF_s + V_s) / V_{s-1} - 1$$

Since the total return has both a cash flow component and an appreciation component, the return risk will also have these two components:

$$\sigma_r = \text{cov}_{s-1}[CF_s, I_s] / V_{s-1} + \text{cov}_{s-1}[V_s, I_s] / V_{s-1} \quad (9)$$

2.3.3.2.1 Intuition & Qualitative Relationships...

Let us now use the above definitions and the analysis in Section 2.3.3.1 to develop some intuition about the nature of the return risk, σ_r .

First, consider the relative magnitudes of the two components in (9), the cash flow component and the appreciation component. In long-lived assets, the cash flow in any one year is generally a small fraction of the value of the asset. For example, in investment grade commercial real estate, as

represented by the PRISA and FRC indices, the income return component or cash yield averages about 8 percent, or less than one-tenth the asset ex dividend value. Thus, as with any long-lived asset, we would generally expect most of the risk in the total return, the bulk of σ_r , to come from the second term on the RHS of (9), the appreciation component.

Now consider a peculiarity of the return risk when there are multi-period riskless leases. From Section 2.3.3.1, it is clear that the cash flow component of (9) will exist only when $n=0$ at time $s-1$, that is, only when the building's lease expires at the end of the current ($s-1$) period, such that a new lease must be signed at time s . In that case, the entire next period cash flow of the single-lease building is risky as of time $s-1$. Otherwise, there is no risk in the next period cash flow, and the first term in (9) is zero.

Also from Section 2.3.3.1, we see that the second term in (9), the appreciation component of the risk, has two parts: a bond value part based on B_s , and a stock value part based on S_s . While the bond value part is always riskless once the lease is signed, it is not riskless in the period prior to the signing of a new lease. The bond value part will thus also only be risky $(1/T)$ th of the time, when $n=0$ at time $s-1$.

The stock value part, S_s , is always risky ex ante, but its magnitude will tend to decrease (with the PCEV formula) as n gets larger (ie, the "forward" claim on the stock part gets farther removed from the present). This discounting effect will be more pronounced when there is mean reversion in the cash flows. In that case the expectation of cash flows in the future rapidly becomes almost riskless as one looks farther into the future where the cash flow expectation is dominated by the deterministic central tendency. This causes the covariance in the forward stock value part of the numerator of (9) to diminish rapidly as time remaining on the existing lease increases.

Thus, the numerator (the covariance) in (9) will always include the stock value part, but only in $(1/T)$ th of the years will it include a cash flow part and a bond value part. On the other hand, the denominator in (9), V_{s-1} , will never include the $s-1$ cash flow (since V_{s-1} is ex dividend), but will always include the bond value part as well as the stock value part of asset value. (Since the denominator of (9) is a value, not a covariance, we include the total value of V_{s-1} , not just the ex ante risky component).

What does this imply for the relation between return risk and lease term? Since, in the numerator of (9), multi-period

riskless leases cause the cash flow component and the bond part of the appreciation component of total return covariance to be attenuated by the factor $(1/T)$, while, in the denominator of (9), building value is (by our assumption A.3) left essentially unchanged by the presence of multi-period leases, it would appear that return risk is at least slightly reduced by the presence of multi-period riskless leases.

While the tendency of return risk to decrease with lease term is intuitively appealing, it is not clear, a priori, how strong this relationship will be. We noted that the cash flow component of return risk for long-lived assets is a small fraction of the total return risk, so the fact that this component is attenuated by the multi-period leases will not have much effect on the total return risk. The bond part of the appreciation component of the total return risk is also attenuated by the $(1/T)$ factor, but with lease terms in the neighborhood of 5 to 10 years, this bond part will be a fairly small proportion of the total asset value, representing the value of somewhere between 0 and $T-1$ years of cash flow opportunities in an asset which derives its total value from a perpetuity of cash flow opportunities. Thus, the appreciation component of the total return risk may also not be too greatly reduced by the use of multi-period leases. Furthermore, at least if cash flows follow a random walk, offsetting factors may come into play between the underlying risk, the observable

cash flow risk, and the return risk. Recall, for example, that if underlying elasticity of expectations exceeds unity and there is no mean reversion, then rental price risk actually increases with lease term. Even if underlying elasticity of expectations is less than unity, if there is no mean reversion then the riskless lease effect could be diluted by the fact that the rental price elasticity of expectations will in that case increase with lease term. Therefore, at least for non-mean-reverting cash flows, the tendency of return risk to decrease with lease term may not be too strong.

Formula (9) and the analysis of the valuation formula in Section 2.3.1 can also be used to reveal the effect of mean reversion on the magnitude of the return risk, independent from any lease term effect. The presence of mean reversion in the cash flows introduces another difference between the numerator and denominator in (9), which turns out to be quite important. Recall that with mean-reversion, the valuation formula (1) has a non-stochastic component as well as a stochastic component, the non-stochastic component reflecting the central tendency. This non-stochastic component of property value will of course be included in the value in the denominator of (9), but not in the terms in the numerator, since being non-stochastic it has no covariance. As the non-stochastic component of asset value can be quite large, this can result in a great reduction in return risk, compared to the random

walk cash flow case, regardless of lease term. This is merely a reflection of the fact that most of the value of a long-lived asset is derived from the expectation of medium to long-term future cash flows, which with mean reversion is almost riskless, due to the dominance of the deterministic central tendency in such future cash flows.

2.3.3.2.2 Quantitative Formulae...

The qualitative analysis in the preceding section offers some insights, but does not allow us to verify the quantitative significance of the relationships noted. To do this, we must specify formula (9) in terms of the underlying cash flow risk, σ .

To define σ_r/σ in a manner amenable to numerical analysis, the approach used here is to model a stylized building, $(1/T)$ th of whose space becomes available as a result of expiring leases each period. We will also assume a stylized prior cash flow history, in which cash flows prior to the present period are assumed to be all on the central tendency or (in the random walk case) all on the expected growth (or "ungrowth") path backwards from the current cash flow. These assumptions merely serve to facilitate the conceptual numerical analysis which is the objective of this inquiry, and do

not represent fundamental constraints on the multi-period cash flow valuation model.

Using relation (3) between $y(t,T)$ and x_t , and the stylized simple cash flow history noted above, both B_s and S_s can be expanded and expressed in terms of $E_s[x_{s+1}]$. This enables one to express the $\{V_s\}$ series in terms of the underlying $\{x_s\}$ series and, more to the point, in terms of the $\{u_s\}$ series of underlying forecast errors. The observable cash flow series $\{CF_s\}$ is also easily expressed in terms of the $\{y(s,T)\}$ series, and from this via (3) and the stylized cash flow history assumption, in terms of the $\{x_s\}$ and therefore the $\{u_s\}$ series. We can thus derive the risk characteristics of both the $\{V_s\}$ and $\{CF_s\}$ series in terms of the underlying risk, σ , for each $(1/T)$ th part of the stylized building. (These T parts have, respectively, $n=T-1$, $n=T-2$, ..., $n=0$ periods remaining on their leases.) Using the value additivity principle to put these parts together, we thus value the entire property. Though the algebra is rather lengthy and messy, the derivations are completely straightforward.

While the stylized cash flow history assumption does not permit the numerical examination of the effects of deviations from the central tendency in prior periods (this would just be too messy, and add little insight), it does permit us to con-

sider the effects of deviations from the central tendency in the current period. To do this, it is convenient to consider time $s-1$ to be the current period, and to define a "numeraire" to normalize the numerical valuation formula. The numeraire is defined by:

$$E_{s-1}[x_s : x_{s-1} = X_{s-1}] = 1$$

where $E_{s-1}[x_s : x_{s-1} = X_{s-1}]$ refers to the time $s-1$ expectation of x_s , given that x_{s-1} equals the central tendency value X_{s-1} (or, in the random walk case, x_{s-1} can be any arbitrary value since there is no central tendency).

With this definition of a numeraire, it is convenient to express the normalized deviation from the central tendency in the current period by the factor D :

$$D = x_{s-1} / X_{s-1}$$

Thus, $D=1$ implies the current underlying cash flow is just on its central tendency. With this definition of D , assumption (A.1) gives the current expectation of the next period's underlying opportunity cost cash flow in normalized value as $[1+(D-1)\alpha]$, which means that the next period deviation ratio R_s defined in Section 2.3.2 is given by:

$$R_s = X_s / E_{s-1}[x_s] = 1 / [1+(D-1)\alpha]$$

With these definitions and assumptions in mind, the relationship between return risk and underlying risk can now be

quantified. We first define some terms to simplify and clarify the formula:

- C1 = Cash flow covariance = $\text{cov}_{s-1}[\text{CF}_s, I_s]$
- C2 = Bond part of appreciation covariance
= $\text{cov}_{s-1}[B_s, I_s]$
- C3 = Forward stock part of appreciation covariance
= $\text{cov}_{s-1}[S_s, I_s]$
- C4a = Bond part of asset value for lease just signed in present (s-1) period = (1/T)th fraction of B_{s-1} with $n=T-1$
- C4b = Bond part of asset value for leases signed prior to s-1 = Components of B_{s-1} for the T-1 (1/T)th fractions of the property with $n=1$ to T-2
- C5 = Forward stock part of time s-1 asset value = S_{s-1} including each of T fractions with $n=0$ to T-1

Formula (9) thus expands to:

$$\sigma_r = \frac{C1 + C2 + C3}{C4a + C4b + C5} \sigma \quad (10)$$

where the component terms are defined as follows:

$$C1 = \frac{1 + \alpha H[T-1](1+g)}{(1+r)a[T]} [1+(D-1)\alpha] \quad (10a)$$

$$C2 = a[T-1] \frac{1 + \alpha H[T-1](1+g)}{(1+r)a[T]} [1+(D-1)\alpha] \quad (10b)$$

$$C3 = \sum_{n=0}^{T-1} \left\{ \frac{(1+g)^n}{(1+r+1)^n} (Q^n) \alpha H[\infty](1+g)[1+(D-1)\alpha] \right\} \quad (10c)$$

$$C4a = a[T-1] \frac{D + H[T-1](1+g)[1+(D-1)\alpha] + K[T-1](1+g)}{(1+r)a[T](1+g)} \quad (10d)$$

$$C4b = \sum_{n=1}^{T-2} \left\{ a[n] \frac{1 + H[T-1](1+g) + K[T-1](1+g)}{(1+r)a[T](1+g)^{(T-n)}} \right\} \quad (10e)$$

$$C5 = \sum_{n=0}^{T-1} \frac{(1+g)^n}{(1+r+1)^n} \left\{ (Q^n)H[\infty][1+(D-1)\alpha] \right. \\ \left. + \left[1 + \frac{(1-Q^n)/(1-Q)}{(1+g)/(r+1-g)} \right] K[\infty] \right\} \quad (10f)$$

where: $Q = 1-b-\alpha\mu\sigma$.

Although messy, these formulas are all fairly intuitive if you study them for awhile, given the discussion in Section 2.3.3.2.1 and the definitions of the terms described in Section 2.3.2. The only mystery terms are perhaps those with the Q factor in them. These terms result from the recursion of the forward stock value part of V_s and V_{s-1} . Note that for $n=0$ these terms all disappear or reduce to the value they would have without being multiplied by Q .

In the case where there are no leases: $T=1$, $(1+r)a[T]=1$, $H[T-1]=K[T-1]=0$, and (10) reduces to the formula relating return risk to cash flow risk found by Myers & Turnbull if $b=0$ or to the corresponding formula found by Bhattacharya if $b>0$ (and $g=0$).

2.3.3.3 Observable Cash Flow Risk

The same analysis and conventions as were used above to obtain the σ_r/σ relation can be used to obtain the relation between observable cash flow risk and underlying opportunity cost risk, σ_{cf}/σ . First, we define the observable risk analogous to the other risk definitions:

$$\sigma_{cf} = \frac{\text{cov}_{s-1}[\text{CF}_s, I_s]}{E_{s-1}[\text{CF}_s]}$$

Expansion of the above definition in terms of x_s and then u_s using assumption (A.1), eqn.(3), and the stylized cash flow history assumption noted previously, reveals the following relationship between cash flow risk and underlying risk:

$$\sigma_{cf} = \frac{[1+\alpha H[T-1](1+g)][1+(D-1)\alpha]}{C6 + C7 + C8} \sigma \quad (11)$$

where:

- C6 = Current (at time s-1) expected cash flow next period (s) from the new lease to be signed next period, times $(1+r)a[T]$
- C7 = Cash flow to be received next period from the lease just signed this period (s-1), times $(1+r)a[T]$
- C8 = Cash flow to be received next period (s) from the leases signed in prior periods (s-2 to s-(T-1)), times $(1+r)a[T]$

and these terms in the denominator are expanded as follows:

$$C6 = [1+(1-b)H[T-1](1+g)][1+(D-1)\alpha] + bH[T-1](1+g) + K[T-1](1+g) \quad (11a)$$

$$C7 = \{D + H[T-1](1+g)[1+(D-1)\alpha] + K[T-1](1+g)\}/(1+g) \quad (11b)$$

$$C8 = (1 + H[T-1](1+g) + K[T-1](1+g)) \sum_{j=s-2}^{s+1-T} 1/(1+g)^{(s-j)} \quad (11c)$$

2.4 Summary and Relation to Volatility

This Chapter has presented a valuation model for a long-lived asset under uncertainty with multi-period riskless leases. The purpose of the model is to allow numerical analysis of the relationships between return risk, cash flow risk, rental market risk, and lease term. This analysis and some of its implications will be pursued in the next Chapter. Before moving to that analysis, however, we should note a generalization which is possible concerning the relationships developed in this Chapter.

While the definition of return risk used in this Chapter is the systematic risk or covariance of returns with a CAPM-type white noise index (as defined in assumption (A.2)), the mathematical relationships found here between return risk and the risk in the rental market or cash flow will also hold for the corresponding relationships regarding the one-period conditional standard deviations. In other words, the relationships identified in this chapter can be applied to volatility as well as to systematic risk. For example:

$$\frac{\text{cov}_{t-1}[r(i)_t, I_t]}{\text{cov}_{t-1}[y(t, T), I_t]/E_{t-1}[y(t, T)]} = \frac{\text{SD}_{t-1}[r(i)_t]}{\text{SD}_{t-1}[y(t, T)]/E_{t-1}[y(t, T)]}$$

and similarly for the other levels of cash flow, $\{x_t\}$ and $\{CF_t\}$; where $\text{SD}_{t-1}[\dots]$ is the standard deviation conditional on knowledge through time $t-1$.

This holds because both $r(i)_t$ as well as all the levels of cash flow are deterministic affine functions of the underlying cash flow one-period proportional forecast error, u_t :

$$\begin{aligned} r(i)_t &= E_{t-1}[r(i)_t] + G_r u_t \\ x_t &= E_{t-1}[x_t] + G_x u_t \\ y_t &= E_{t-1}[y(t, T)] + G_y u_t \\ CF_t &= E_{t-1}[CF(t, T)] + G_{CF} u_t \end{aligned}$$

All of the terms and parameters on the RHS above are known constants as of time $t-1$ except for u_t . Thus, the covariance of the LHS with I_t equals G times the covariance of u_t with I_t , and similarly the standard deviation of the LHS equals G times the standard deviation of u_t . So the LHS covariances with I_t and their standard deviations are related by the same ratio, the ratio of the corresponding G parameters (eg, $\sigma_y/\sigma = G_y/G_x = \text{SD}[y]/\text{SD}[x]$). The focus of the analysis in this Chapter has been to quantify the G parameters in such a way that numerical analysis can be carried out to explore these relationships.

Summary of Valuation Model with Leases...

$$V_{s-1} = (C4a + C4b + C5)/T$$

where...

C4a = Bond part of asset value for lease just signed in present (s-1) period = (1/T)th fraction of B_{s-1} with $n=T-1$

C4b = Bond part of asset value for leases signed prior to s-1 = Components of B_{s-1} for the T-1 (1/T)th fractions of the property with $n=1$ to T-2

C5 = Forward stock part of time s-1 asset value = S_{s-1} including each of T fractions with $n=0$ to T-1

and more specifically...

$$C4a = a[T-1] \frac{D + H[T-1](1+g)[1+(D-1)\alpha] + K[T-1](1+g)}{(1+r)a[T](1+g)} \quad (10d)$$

$$C4b = \sum_{n=1}^{T-2} \left\{ a[n] \frac{1 + H[T-1](1+g) + K[T-1](1+g)}{(1+r)a[T](1+g)^{(T-n)}} \right\} \quad (10e)$$

$$C5 = \sum_{n=0}^{T-1} \frac{(1+g)^n}{(1+r+1)^n} \left\{ (Q^n)H[\infty][1+(D-1)\alpha] + \left[1 + \frac{(1-Q^n)/(1-Q)}{(1+g)/(r+1-g)} \right] K[\infty] \right\} \quad (10f)$$

where: $Q = 1-b-\alpha\mu\sigma$.

$$D = x_{s-1}/X_{s-1}$$

and the value is normalized to the numeraire:

$$E_{s-1}[x_s : x_{s-1} = X1] = 1.$$

Chapter 3: Some Quantitative and Practical Implications of the Valuation Model

In this Chapter we use the valuation model and relationships developed in the previous Chapter to explore some quantitative and practical implications of interest to real estate analysts. Section 3.1 presents a numerical analysis to develop understanding and intuition regarding the nature of commercial real estate return risk. Section 3.2 presents some applications of the model to topics of interest.

3.1 Numerical Analysis

In this Section we quantitatively explore: (3.1.1) the effect of lease term on risk; (3.1.2) the relations between cash flow risk, rental market risk, and return risk; and (3.1.3) the sensitivity of these findings to various factors, such as interest rates, growth and cyclicity in the cash flows.

3.1.1 Lease Term Effect on Risk

In the discussion in Chapter 2 it was suggested that, holding underlying risk σ constant, both observable cash flow risk σ_{CF} and total return risk σ_r should generally fall with the length of the lease term, but by how much, or how significant

this effect would be was difficult to gauge from just looking at the algebraic formulas. The Tables 3.1a-b (located at the end of Section 3.1) quantify this effect, and allow us to develop a feeling for the lease term effect.

The Tables show the ratio of the expected new-lease rental price to the expected underlying spot rent, $E_t[y_{t+1}]/E_t[x_{t+1}]$, as well as the new-lease rental price elasticity of expectations, α_y , and the ratios σ_r/σ , σ_y/σ , and σ_{CF}/σ , for a range of plausible underlying α , and b values, for lease terms ranging from 1 to 10 years. The Tables also assume base case interest and growth rates of .03 and 0, respectively, a market price of return risk times underlying risk factor of $\mu\sigma=.05$, no illiquidity premium ($\lambda=0$), and that the rental market is at its long-run average or previously expected balance between supply and demand ($D=1$). (Sensitivity of results to these parameters is explored in Section 3.1.3.) Table 3.1a shows the ratios for the non-mean-reverting case with $b=0$, while Table 3.1b presents the mean-reverting case with $b=.25$. (Sensitivity over the range of plausible b values is explored in Section 3.2.1, where we will see that $b=.25$ is well representative of the mean-reversion case.)

The level and pattern of the rental price expectations and elasticity of expectations are as anticipated in the discus-

sion in Sections 2.3.2.2 and 2.3.2.4 of the previous Chapter. Over the range of α considered here, we note that α_y is always near unity for large T (or a bit less with mean reversion), and that it is not very sensitive to lease term, is increasing in α (but varies much less than α), and is decreasing in b . (We shall see later that the fact that α_y is near unity for $b=0$ has important implications regarding the magnitude of the return risk relative to the observable cash flow risk for the case without mean reversion.)

$E_t[y_{t+1}]$ is less than $E_t[x_{t+1}]$ for $T > 1$, reflecting the lower risk the landlord bears, and higher risk the tenant bears, with multi-period leases. This effect increases with lease term, but not greatly, as cash flows under existing leases give only a fraction of the total value of the (infinite-lived) property (making it impossible even for multi-year leases to eliminate most of the risk that matters to the landlord), and because the time-value-of-money (riskless interest rate) enters into the lease valuation equation as well as risk discounting. (Indeed, for the longer-term leases in the case of mean-reversion, it is the time-value-of-money that dominates the lease valuation picture, since mean-reversion eliminates most of the risk in longer term leases. This is why the rent price ratio E_y/E_x does not fall with increasing lease

term nearly as much as the return risk σ_r does, under mean reversion.)

The observable rental market risk, represented by the new-lease rental price time series risk, σ_y , is seen in Table 3.1a to be not very sensitive to the lease term in the non-mean-reversion case, but slightly decreasing over lease term if $\alpha < 1$ and slightly increasing over lease term if $\alpha > 1$. Mean reversion is seen in Table 3.1b to introduce much more sensitivity to lease term, with σ_y generally decreasing with lease term even when $\alpha > 1$. (The exception is with $\alpha > 1$ and very short lease term.) This pattern of relationship between σ_y and T makes sense, based on the discussion in Section 2.3.2.3.

Recall that market rental price risk is dampened below the underlying risk by two factors: inelastic expectations, and mean reversion in the expected future underlying opportunity costs, with both factors operating over the remaining lease term period when the new lease is signed (which is $T-1$ years). Both of these effects, but particularly the mean-reversion effect, are more important the longer into the future the lease extends. While the two effects operate in the same direction if $\alpha < 1$, if $\alpha > 1$ then the elasticity of expectations effect is opposite to the mean reversion effect (if there is any). In this case the elasticity of expectations effect

causes expectations of future opportunity costs to be more volatile than past realizations, which influences lease rental prices to be more risky with longer lease term. In the case of very short term leases and $\alpha > 1$, there is not time under the lease for mean reversion to have much effect, so the mean-reversion effect is dominated by the elastic expectations effect, which is why we see $\sigma_y > \sigma$ only for $T \leq 3$ in Table 3.1b.

Unlike σ_y , observable cash flow risk σ_{CF} is seen in the Tables to be quite sensitive to lease term, and always decreasing over lease term, even when $\alpha > 1$ and without mean-reversion. This makes sense, as multi-period leasing essentially eliminates all but the fraction $(1/T)$ th of the cash flow risk. Indeed, with $\alpha = 1$ and no mean-reversion, this is exactly the fraction of cash flow risk that is eliminated. In other cases, the relationship between $\{y_t\}$ and $\{x_t\}$ enters the picture in a manner which causes a and b to influence the cash flow risk, since cash flow $\{CF_t\}$ is based on rental price $\{y_t\}$, which in turn is based on leases which involve multi-period opportunity cost valuation of future values in the $\{x_t\}$ series. The result is that observable cash flow risk will usually fall off faster with T than the fraction $(1/T)$.

Holding underlying opportunity cost risk σ constant, Tables 3.1 reveal that total return risk σ_r also falls off with lease

term, even when $\alpha > 1$ and there is no mean reversion, but σ_r is much less sensitive to lease term than is observable cash flow risk. If $T=10$, for example, σ_{cf} is generally about one-tenth or less the value it would have with no long-term leasing, while σ_r falls off only to about two-thirds the value it would have with no long-term leases in the case of no mean reversion, or to about one-third the value it would have with no long-term leases in our mean reversion case.

As noted in the discussion in Section 2.3.3.2, it makes sense for total return risk to be less sensitive to lease term than cash flow risk is, since the cash flow component itself is only a small fraction of total return risk, and the bulk of the remainder of the return risk comes from the "stock part" of the property value, which is not very sensitive to lease term.

This relative insensitivity of total return risk to lease term may have some practical value. Since total return risk is the risk that matters for estimating the expected return risk premium $\mu\sigma_r$, the relative insensitivity of σ_r over plausible ranges of T makes the task of estimating this premium easier. In this regard, note that though the fall in return risk with lease term is larger in relative terms under mean reversion, it is smaller in absolute terms, simply

because there is very little return risk even with no long-term leasing when cash flows are mean-reverting. Of course, it is the absolute amount of return risk (measured here in normalized units equal to the underlying risk, σ) which matters for purposes of estimating the return risk premium.

3.1.2 Return Risk Related to Observable Rental Market and Cash Flow Risk

The relationships described in the previous section between, on the one hand σ_r , σ_y , σ_{CF} , and on the other hand the underlying risk σ , contain implicit within them the relationships between return risk and the observable rental market risk and cash flow risk: σ_r/σ_y , and σ_r/σ_{CF} . But as the underlying opportunity cost risk σ is unobservable, explicitly quantifying the σ_r/σ_y and σ_r/σ_{CF} ratios will help to develop our intuition about the nature of commercial real estate return risk. Tables 3.2a-b display these ratios for lease terms of 1, 5, and 10 years under the same parameter value assumptions as were represented in Tables 3.1 of the previous section.

First we note that in all cases the ratio of return risk to observable cash flow risk is very well approximated by the value T times the ratio of the return risk to the rental market risk:

$$\sigma_r/\sigma_{CF} \sim T(\sigma_r/\sigma_y) \quad \text{approx. (1)}$$

This makes sense, because it is just a reflection of the fact that: $\sigma_{CF} \sim \sigma_y/T$, cash flow risk is about (1/T)th times the rental price risk, since (1/T)th of the average building is up for new lease signing and hence exposed to the rental market risk in each year.

The result, as the numbers in the Tables show, is that, without mean reversion, return risk can be much greater than observable cash flow risk with multi-period leases, and with mean reversion, return risk is a substantial fraction of observable cash flow risk. With mean-reversion at the rate of $b=.25$, we see that for typical values of T for office buildings, return risk is generally between roughly one-half (at $T=5$) and more than nine-tenths (at $T=10$) of the cash flow risk under the base case parameter assumptions. [The role and importance of expected cash flow mean reversion in determining return risk will be discussed further in Section 3.2.1.]

Since time series data on the $\{CF_t\}$ cash flow series should at least in principle be observable, the quantification of the σ_r/σ_{CF} ratio described here in principle allows empirical study of some important aspects of the nature of unsecuritized real estate return risk, using only cash flow data, without recourse to the use of appraisal-based real estate returns

series. The main conceptual difficulty with such approach which appears in the above analysis, however, is that the relationship between return risk and cash flow risk seems to be quite sensitive to the rate of mean reversion, or to the fraction of cash flows which is viewed as being susceptible to mean reversion. While the importance of this sensitivity is itself a useful insight, the difficulty of pinning down empirically or conceptually the correct value to use for this fraction may make it difficult to draw precise empirical conclusions regarding return risk, based on studies of cash flow risk alone. [This will be discussed further in Section 3.2.1].

If we view σ_r/σ_{CF} as being well approximated by $T(\sigma_r/\sigma_y)$, then the relation between return risk and rental market risk becomes the more fundamental relationship for building our intuition about the nature of commercial real estate return risk. Here we note that while σ_r/σ_y tends to decrease with lease term, over the relevant range of T values σ_r/σ_y is not very sensitive to lease term, so a general statement about the approximate relationship between σ_r and σ_y can be made independent of the exact value of the lease term. Examination of the values of σ_r/σ_y in Tables 3.2a-b and of the values of α_y in Table 3.1a of the previous Section leads to the following approximation:

$\sigma_r/\sigma_y \sim \alpha_y$, with no mean reversion ($b \leq \max\{0, g\}$)

$\sigma_r/\sigma_y =$ "small", with significant mean reversion ($b \geq .2$ or so)

where "small" means about one-tenth or less.

approx.(2)

Approximation (2) holds pretty well over the plausible values of the underlying α , σ , and T parameters, and so may be viewed as a rather robust and therefore potentially useful result. The intuition in approx.(2) can be seen as follows.

First consider the case of significant mean-reversion ($b \geq .2$). Here, property value is nearly riskless, no matter what is going on in the current rental market (ie, even though there may be considerable risk from year to year in the rental market), since reversion to the deterministic long-run mean of the opportunity cost renders the current expectation of medium-to-long-run future cash flows insensitive to these "fleeting" ups and downs in the rental market. Most of the value of the (infinite-lived) property, and therefore most of the potential for return risk, comes from the present value of these medium-to-long-run opportunity cost forecasts, which mean-reversion renders almost riskless from the perspective of current valuation.

Now consider the case without mean reversion in the opportunity costs. With no central tendency, current ups and downs

in the rental market may signal permanent changes in the market and the property's opportunities. This is particularly true with long-term leases, since new-lease rental prices themselves reflect market expectations of opportunities over the next $T-1$ years, and so already directly reflect some medium-term forecasting. However, expectations of future rental prices, $\{E_t[y_{t+l}]\}$, are less (or more) sensitive to current realizations of $\{y_t\}$, the smaller (or larger) is the elasticity of expectations in the rental market, α_y . With a smaller α_y expected future cash flows will be less sensitive to current market ups and downs, rendering property value less sensitive to these innovations in the $\{y_t\}$ series, and hence rendering the property total return less risky.

To see more specifically and quantitatively why the approximation $\sigma_r \approx \alpha_y \sigma_y$ holds without mean reversion, consider that most of the risk in the total return comes from the appreciation return component, and that the bulk of the appreciation component will generally be in the "stock part" of property value (the value from the "outyears" cash flows beyond expiration of the current leases), rather than from the "bond part". In other words, using the notation from Chapter 2:

$$\begin{aligned} \sigma_r &= \text{cov}_{t-1}[r(i)_t, I_t] = \text{cov}_{t-1}[(CF_t + V_t), I_t] / V_{t-1} \\ &\sim \text{cov}_{t-1}[S_t, I_t] / S_{t-1} \quad \text{approx. (3)} \end{aligned}$$

where: $V_t = B_t + S_t$, property (i)'s total value is the value of the bond part plus the stock part.

The value of the stock part S_t is based on the expected future value of the new-lease rental price, $E_t[y(t+L, T)]$, $L=1, 2, \dots, \infty$. In fact, to a commonly-used approximation:

$$S_t \approx E_t[y(t+1, T)]/\Omega_S \quad \text{approx. (4)}$$

where Ω_S is a deterministic "cap rate" or inverse price/earnings multiplier estimate. Thus, approx.(3) can be written in terms of the expected new-lease rental price:

$$\begin{aligned} \sigma_r &= \frac{\text{cov}_{t-1}[E_t[y(t+1, T)], I_t]/\Omega_S}{E_{t-1}[y(t, T)]/\Omega_S} = \frac{\text{cov}_{t-1}[E_t[y_{t+1}], I_t]}{E_{t-1}[y_t]} \\ &= \frac{(1+g)\text{cov}_{t-2}[E_{t-1}[y_t], I_{t-1}]}{E_{t-1}[y_t]} = \frac{(1+g)\text{cov}_{t-2}[\Delta E_{t-1}[y_t], I_{t-1}]}{E_{t-1}[y_t]} \\ &= \frac{(1+g)\text{cov}_{t-2}[(\Delta E_{t-1}[y_t]/\Delta y_{t-1})\Delta y_{t-1}, I_{t-1}]}{E_{t-1}[y_t]} \\ &= \frac{(1+g)\text{cov}_{t-2}[(1+g)\alpha_y \Delta y_{t-1}, I_{t-1}]}{E_{t-1}[y_t]} \\ &= \frac{(1+g)^2 \alpha_y \text{cov}_{t-2}[\Delta y_{t-1}, I_{t-1}]}{E_{t-1}[y_t]} \end{aligned}$$

where $\Delta[...]$ indicates the difference operator, and we have made use of the definition of α_y from Chapter 2. Since it is only the difference in y_t that is subject to conditional covariance from year to year, $\text{cov}_{t-2}[\Delta y_{t-1}, I_{t-1}] = \text{cov}_{t-2}[y_{t-1}, I_{t-1}]$.

Therefore, using the definition of σ_y , we can continue to manipulate approx.(3) above as follows:

$$\begin{aligned} \sigma_r & \sim \frac{(1+g)^2 \alpha_y \text{cov}_{t-2}[Y_{t-1}, I_{t-1}]}{E_{t-1}[Y_t]} \sim \frac{(1+g) \alpha_y \text{cov}_{t-1}[Y_t, I_t]}{E_{t-1}[Y_t]} \\ & = (1+g) \alpha_y \sigma_y \sim \alpha_y \sigma_y \end{aligned}$$

which is the part of approximation (2) we were trying to demonstrate.

The reason why this approximation does not hold well for the case of mean-reversion is that the cap rate multiplier, Ω_s in approx.(4), does not remain invariant over time as $E_t[Y_{t+1}]$ deviates from its deterministically known central tendency (at least if approx.(4) is to be very accurate), so that we cannot think of Ω_s in approx.(4) as being independent of the value of $E_t[Y_{t+1}]$. Rather, Ω_s changes so as to reduce the sensitivity of S_t to changes in the value of $E_t[Y_{t+1}]$.

Note in Tables 3.1a and 3.2a that σ_r/σ_y differs from α_y in a way that is predictable considering what is left out of approx.(3) above. We ignored the contribution of cash flow risk to total return risk. This contribution is larger the larger is the cash flow risk, in other words, the smaller is the lease term. This causes approx.(2) to be biased too small for small values of T , that is, σ_r is actually larger than $\alpha_y \sigma_y$

for small T . On the other hand, the bond part of property value was also left out of approx.(3), which considered only the risk in the stock part. As the bond part is less risky than the stock part (particularly for large T), and as the bond part of total property becomes more important as T increases, this causes $\alpha_y \sigma_y$ to be biased too large. For lease terms greater than about $T=3$, this bond part omission effect dominates over the offsetting cash flow risk omission effect, so that $\alpha_y \sigma_y$ overstates σ_r the more so as T increases beyond about $T=3$. Thus we obtain the pattern of σ_r/σ_y falling with lease term, but rather gently.

When we combine the $\sigma_r \sim \alpha_y \sigma_y$ approximation with the fact that without mean reversion $\alpha_y \sim 1$ (noted previously in Section 2.3.2.4 of Chapter 2, and Section 3.1.1 above), we obtain a somewhat cruder approximation which says that for the case of non-mean-reverting opportunity costs, $\sigma_r \sim \sigma_y$: return risk roughly equals rental market risk when there is no mean reversion. This is an intuitive result. Combining this result with the previous point that $\sigma_{cf} \sim \sigma_y/T$, we obtain as a crude approximation that return risk is equal to observable cash flow risk times the lease term: $\sigma_r \sim T\sigma_{cf}$. This also is intuitive, since, in the absence of mean-reversion, a shock or innovation in the observable multiperiod lease rental market should more or less proportionately affect property value

(because it affects all of the outyear cash flow forecasts), but such shock or innovation only affects the $(1/T)$ th portion of the current cash flows associated with the lease that just expired. Examination of the σ_r/σ_{cf} values in Table 3.2a reveals that this intuition does hold approximately, but as T gets larger the actual σ_r/σ_{cf} ratio falls considerably below the lease term, for the reasons described above.

Summary of Basic Relationships...

It is perhaps fairly intuitive, at least in retrospect, that return risk in real estate should approximately equal the risk in the observable long-term rental market if there is no mean reversion in opportunity costs, or that return risk should be very small if there is significant mean reversion no matter what is the risk in the rental market. But the model developed in Chapter 2 and the numerical analysis presented in Tables 3.1 and 3.2 enable us to confirm this intuition and to quantify it more precisely, as well as to examine the effect of lease term.

To see that σ_r/σ_y is a gently falling convex function over lease term (ie, σ_r/σ_y declines at a lower rate as T increases - - after all, the ratio cannot go less than zero) also now seems rather intuitive. Combining these results with the $\sigma_{cf} \sim$

σ_y/T relationship, we can summarize the major result of this section. With multiperiod leases, the ratio of return risk to observable cash flow risk σ_r/σ_{cf} increases with lease term, and for lease terms typical of investment quality office buildings (ie, 5-10 years) return risk can be much greater than cash flow risk with no mean reversion, or, if there is mean reversion then σ_r/σ_{cf} can still be a large fraction or perhaps even above unity for office buildings. Mean reversion thus looms as a crucial factor in governing the relation between return risk and cash flow risk, as it should.

3.1.3 Sensitivity Analysis

In this section we examine the sensitivity of the results reported in the previous two sections to the parameter values describing the growth (g), the time-value-of-money (r), the illiquidity premium (λ), and the amount and value of underlying risk ($\mu\sigma$), as well as to the possibility of a predictable cyclical tendency in the expected cash flows (in the case of non-mean-reversion) or of current deviations from the deterministic central tendency (in the case of mean-reversion). In general, we find that the relationships and approximations described in Sections 3.1.1 and 3.1.2 are robust over plausible ranges of growth, interest rates, illiquidity premia, and underlying $\mu\sigma$ values. Cyclicalities and deviations from the

central tendency will tend to perturb the relationships described above away from the values suggested in Sections 3.1.1 and 3.1.2, but this occurs generally in a rather modest or symmetric manner, so that when the rental market is in its "normal" state, and on average over time, the relationships described in the previous Sections will hold.

3.1.3.1 Underlying Risk and Market Price of Return Risk

Tables 3.3 and 3.4 show sensitivity to the underlying risk and value of risk assumption, $\mu\sigma$. The Tables show the ratios and α_y values discussed in the preceding two sections for $a=1$; $b=0$ and $.25$; and T values of $1, 5$ and 10 years; for $\mu\sigma$ values of $.01$, and $.10$. Recall that the base case assumption for $\mu\sigma$ in the previous Tables was $\mu\sigma=.05$. The other parameters are held the same as before ($g=0$, $r=.03$, $\lambda=0$, $D=1$).

We see in Tables 3.3 and 3.4 that most of the ratios and α_y are quite insensitive to this range of $\mu\sigma$, particularly in the case of mean reversion. A slight exception is the observable rental price to underlying opportunity cost ratio: $E[y]/E[x]$, which is reduced, the more so for larger T , as $\mu\sigma$ increases. This is intuitive, as greater underlying risk causes long-term leasing to save the landlord more risk disutility.

We also note, primarily in the non-mean-reversion case, that σ_r falls off more rapidly with lease term as $\mu\sigma$ increases. This makes sense because the stock value part of the property value (being a perpetuity) is more affected by underlying risk than is the bond value part, so increases in $\mu\sigma$ cause the stock value to fall relative to the bond value. Since return risk is primarily derived from risk in the stock value component of the appreciation return, this causes return risk to fall with $\mu\sigma$, the more so for larger T since bond value is more important (and stock value less important) as T increases.

Similarly, we see that for most of the relevant range of T , total return risk per unit of underlying risk (ie, the ratio σ_r/σ) falls with increases in $\mu\sigma$. But for very small values of T , this ratio increases slightly with $\mu\sigma$. This pattern also makes sense when we consider the components of total return risk, as described in formula (10) of Section 2.3.3.2 of Chapter 2. That formula is a ratio whose denominator consists of the entire bond value plus stock value part of the property ex dividend value, and whose numerator consists of three components: the next period's cash flow risk, the next period's appreciation risk in the bond value part, and the

one-period risk in the stock value part of the appreciation in total property value.

Since only part (approx. $1/T$) of the bond value part appears in the numerator, while the entire bond value part appears in the denominator, the stock value component is relatively larger in the numerator of the total return risk formula than it is in the denominator. Thus, to the extent we can ignore the cash flow component of total return risk, parameters which reduce property ex dividend value by discounting the stock value part proportionately more than the bond value part will reduce total return risk. This is what happens with the μ_0 parameter when T is large enough.

But the cash flow component of the numerator is larger (by roughly the factor $1/T$) the smaller is the lease term, making it less accurate to ignore the cash flow effect the shorter is the lease term. Since the cash flow is relatively less sensitive to the market price of return risk (μ) and does not appear in the denominator, this can make the numerator of the total risk formula fall less than the denominator with increases in μ_0 if T is small enough. This dominance of a cash flow risk effect over the bond part of the appreciation return risk effect when T is small is akin to the effect in

σ_r/σ_y we noted in Section 3.1.2 in the case of no mean reversion.

3.1.3.2 Effect of Growth on Risk

Tables 3.5 and 3.6 report the sensitivity to the deterministic growth trend in the market, g . With all else as before, the Tables show values for $g=-.02$ and $g=+.02$. The relationships described in Sections 3.1.1 and 3.1.2 still hold (with sensitivity to lease term dampened slightly with increases in g). Note also that, ceteris paribus, for multi-period leases, return risk increases with growth under non-mean-reversion (even with α less than 1, which is different in this case from the effect of growth with $T=1$), but decreases with growth under mean-reversion. This sensitivity of return risk to opportunity cost growth is not very intuitive, and is different from that found by Myers & Turnbull (except in the $T=1$, $b=0$ case in which the present model collapses to exactly the case examined by Myers & Turnbull). We can develop some understanding of this effect of growth on risk by once again examining formula (10) in Chapter 2.

With mean reversion, the effect of growth on the value of the riskless central tendency capitalization in the denomina-

tor of the total return formula dominates all other effects of growth, so growth reduces return risk. Recall that the capitalization of the central tendency appears in the denominator of the total return risk formula, but not in the numerator, since this deterministic value component is part of the property ex dividend value, but contains no risk. Since the central tendency is a perpetuity subject to the market growth trend, increases in g increase this already large component of total property value rather markedly. The capitalized value of the central tendency represented by the value $K[\infty]$ in formula (10) of Chapter 2 has value related to growth roughly by the factor $1/(r+\lambda-g)$.

Without mean reversion the picture is more complicated. If $T=1$, the cash flow component of risk in the numerator of the total return is unaffected by growth (by definition, since by (A.1.1) the growth trend is already included in the one-period conditional expectation), while the denominator (asset ex dividend value) increases with growth expectations. (By A.1.2, multi-period forecasts explicitly reflect growth.) Thus, the cash flow risk effect alone falls with growth. But there is a potentially countervailing effect in the appreciation return component. Both the expected asset ex dividend value next period (upon which the appreciation return component is based, in the numerator of the total return) and the asset ex dividend value this period (in the denominator of the

total return) increase with growth in the expected future cash flows. But the risk in the numerator of this appreciation return component is magnified by the fact that there is one more year of growth in its first cash flow, and by the elasticity of expectations. The latter effect can cut either way relative to the denominator, depending on whether α is less than or greater than one.

Thus, the numerator of the return risk formula suffers conflicting effects from growth. The appreciation risk in the numerator is affected by growth proportionately by the factor $\alpha(1+g)$ compared to the denominator, while the cash flow risk in the numerator is unaffected, or effectively reduced compared to the denominator of the formula. The result is that, if α is less than one, the cash flow effect dominates and return risk falls with growth in the absence of multi-period leases.

However, with multi-period leases cash flow risk does increase with growth (due to the capitalization, in the value of the lease that will be signed next year, of some growth in expected cash), and the bond component of the present asset value in the denominator of the return risk formula decreases with growth. This latter effect is because positive growth implies that the average past values of the rental prices (which are capitalized in the bond part of the asset present

value) are lower relative to present rental prices than they would be in the absence of a growth trend in expected values. (Positive growth forward in time implies negative growth backward in time.) The result is that growth tends to increase risk with multi-period leases in the absence of mean reversion.

3.1.3.3 Time Value of Money and Illiquidity Premium

Tables 3.7 and 3.8 show the sensitivity to the time-value of money assumption, or the riskless interest rate, r . Values for $r=.01$ and $.05$ are reported. The basic intuitive approximations described in Sections 3.1.1 and 3.1.2 continue to hold over this range of interest rates. Sensitivity to lease term increases moderately with r , as it does with $\mu\sigma$, and for essentially the same reasons. Indeed, the sensitivity to r is in general much like the sensitivity to $\mu\sigma$, and also like the sensitivity to g only in the opposite direction. For the most part r and g appear together inversely in the formulas as $(1+g)/(1+r+\lambda)$ terms raised to various powers.

Increasing interest rates thus increases the return risk under mean reversion and decreases the return risk under non-mean-reversion, except that for small values of T return risk slightly increases with r for $T \leq 3$ or so even without mean reversion. The reason is essentially the same as described

above for the growth sensitivity and μ_0 sensitivity. For small values of T , a cash flow risk effect dominates over the appreciation return risk effect.

Tables 3.9 and 3.10 show sensitivity to the illiquidity premium, λ . The Tables show values for $\lambda=0$ and $\lambda=.02$. Sensitivity to λ is almost identical to sensitivity to r , for the obvious reason that r and λ usually appear together additively as $r+\lambda$ in the formulas.

We note that, because of the dominant role played by the capitalization of the riskless central tendency in the denominator of the return risk formula, in the case of mean-reversion, though return risk always remains small in absolute terms compared to the non-mean-reverting case, return risk is relatively speaking much more sensitive to the time-value-of-money and the illiquidity premium (as well as to growth, in the opposite way) under mean-reversion than it is without mean-reversion. Also, because the riskless central tendency reduces the risky portion of the stock value part of next period's expected asset value which appears in the numerator of the appreciation return risk component (thereby increasing the importance in return risk of the highly lease-term-sensitive cash flow and bond value components), return risk is relatively speaking more sensitive to lease term under mean-reversion than without mean-reversion.

When we put these effects together, the result is that, over the range of T values typical of investment quality office properties, high values of r or λ , and/or a low value of g, can cause return risk to equal or exceed cash flow risk even when mean reversion is quite strong and when for lower values of T the return risk would be much smaller than the cash flow risk. For example, if $r=.05$, $\lambda=.02$, and $g=-.02$ (all of which would seem to be within plausible bounds, in real terms), then even with $b=.50$, we have (at $\alpha=1$ and other base case assumptions as before): $\sigma_r/\sigma_{cf} = 0.84$ at $T=5$, and $\sigma_r/\sigma_{cf} = 1.41$ at $T=10$; while at $T=1$ we would have only $\sigma_r/\sigma_{cf} = 0.25$.

3.1.3.4 Cyclicalities and Deviation from the Central Tendency

Tables 3.11 through 3.14 show sensitivity to the presence of cyclicalities in the expectations in the case of non-mean-reversion (Tables 3.11-12), or to the deviation of the current opportunity cost realization from its central tendency in the case of mean-reversion (Tables 3.13-14).

Consider first the effect of predictable cyclicalities in the opportunity cost trend. It has been suggested that, due to the lead time required for supply of constructed facilities to respond to the demand, and due to the inelasticity of supply

on the downward side (ie, supply can only very slowly fall with natural depreciation and attrition of buildings), a rather predictable "real estate cycle" can be observed in most commercial real estate markets. [See, eg, Wheaton.] The implication is that instead of a constant expected growth trend indicated by the parameter g in the foregoing analysis, the growth in the expected future series of underlying opportunity costs would cycle around some long-term average rate.

While the imposition of such a cyclical tendency on the underlying opportunity cost expectations is conceptually very straightforward, the algebraic formulas and numerical analysis becomes much more complicated, as we must add a cycle phase multiplier term and we can no longer take advantage of the simple closed-form constant-growth perpetuity formula in the underlying valuation equation (1a) of Chapter 2. As a result, we cannot model the perpetual property literally. But we can approximate perpetual property rights very closely simply by taking the valuation terminal period, τ in formula (1) of Chapter 2, to be very large.

In Tables 3.11 and 3.12 the terminal period of property value is taken to be $\tau=50$ years, and a generalized sine function over time takes the place of the $(1+g)$ factor in eqn.(A.1.2) of the underlying opportunity cost stochastic process assumption (A.1) of Chapter 2. Thus, we have:

$$\frac{E_{t-1}[x_t]}{(1+\alpha u_{t-1})E_{t-2}[x_{t-1}]} = \frac{1 + A\sin[2\pi(t-B)/P]}{1 + A\sin[2\pi(t-1-B)/P]}$$

where A is the relative amplitude of the sine wave (as a fraction of the current $x_{t-1}=1$), P is the complete cycle period in years, and B is the current (t-1) cycle phase in years.

In the Tables, $\alpha=1$ and the other base case parameters are as before. The amplitude A is set at 0.20, and the sensitivity results are shown at quarter-cycle points for periods of P=5 and P=10 years. While the historical macroeconomic business cycle of the national economy is on the order of 5 years, Wheaton (op cit) finds evidence that, at least in the office market, the real estate cycle may be considerably longer, perhaps on the order of 10 years. For comparison to the no-cycle case, the P=1, B=0 case is also shown (which is equivalent to no cycles, since $\sin[2\pi t] = \sin[2\pi(t-1)]$).

We see in the Tables that the risk relationships described in Sections 3.1.1 and 3.1.2 are generally not very sensitive to the phase and period of the cycle, and their average values over time are virtually identical to their no-cycle values. The expected rental price to underlying opportunity cost ratio E_y/E_x , and the elasticity of rental price expectations α_y cycle more widely around their average values, which are similar to their no-cycle values. Thus, it seems that predictable

cyclicality in the underlying rental market does not significantly affect the conclusions previously reached, particularly if we are concerned with average values over time.

Let us turn now to Tables 3.13 and 3.14, which show sensitivity to the parameter D , which measures the deviation of the current (time $t-1$) opportunity cost from the unconditional mean or long-run central tendency value at $t-1$. As described in Section 2.3.3.2.2 of Chapter 2, $D = x_{t-1}/X_{t-1}$, so that $D=1$ corresponds to the present opportunity cost being right on its central tendency. The Tables show sensitivity to D values ranging from $D=0.6$ to $D=1.4$, with $b=.25$, $\alpha=1$, and other base case parameter values as before.

Similar to the effect of cyclicality, average values of the ratios of interest over the range of D values approximately equal the values of these ratios taken at the average D value ($D=1$) used in the previous analyses. Unlike the cyclicality effect without mean-reversion, however, most of the values and ratios in Tables 3.13-14 are fairly sensitive to the current deviation from the central tendency. An exception is the rental price elasticity of expectations, which in a sense has already accounted for deviations from the central tendency since it is defined as the change in expectations per unit change in current realization.

It is perhaps interesting to note that the underlying property price/earnings multiplier, V_{t-1} normalized to $E_{t-1}[x_t : x_{t-1} = X_{t-1}] = 1$, is rather insensitive to the current D value, as is the ratio of total return risk to observable cash flow risk. V_{t-1} is insensitive to current cash flow deviations because with mean-reversion property value is dominated by the invariant riskless central tendency, with current deviations of cash flow above or below that tendency having only a small fleeting effect in the capitalization of the expected future cash flow stream.

The σ_r/σ_{cf} ratio is relatively insensitive to the D values because its numerator and denominator both vary in the same way over D . Both σ_{cf}/σ and σ_r/σ increase with D , reflecting the increasing magnitude of the current cash flows and hence of the current cash flow risk. But σ_r increases proportionately more with D than σ_{cf} does, because most of the bond value component of the current asset ex dividend value in the denominator of the return risk formula is insensitive to D , being based on rental prices established prior to the current period when D was realized.

3.1.4 Summary of Numerical Analysis

In general, the numerical analysis presented in Sections 3.1.1 and 3.1.2 confirmed the qualitative intuition developed in Chapter 2, and filled out this intuition by specifying relationships quantitatively. The sensitivity analysis in Section 3.1.3 found these relationships to be robust, at least on average over time, and when the underlying market is at its "normal" ($D=1$) level.

As noted at the end of Section 3.1.2, there seem to be no major surprises or counter-intuitive findings in this analysis, at least after one thinks about them for awhile. But the relationships studied here are quite fundamental to the nature of risk in real estate assets, and may be useful in furthering our understanding and knowledge of real estate asset return risk and property valuation. Perhaps the most difficult conceptual issue raised in the above analysis is the difference between the case of mean-reverting versus non-reverting cash flows. The next Section of this Chapter will expand some of these points.

Table 3.1a Lease Term & Risk: No Mean Reversion (b=0)

(r=.03, g=0, $\mu\sigma$ =.05, λ =0, D=1)

Lease Term (T)	E[y]/E[x]	α_y	σ_y/σ	σ_{cf}/σ	σ_r/σ
$\alpha = 0.75:$					
1	1.000	.750	1.000	1.000	.821
2	.975	.852	.880	.440	.785
3	.956	.893	.840	.280	.756
4	.938	.915	.820	.205	.731
5	.921	.929	.808	.162	.707
6	.904	.938	.800	.133	.685
7	.889	.945	.794	.113	.664
8	.874	.950	.790	.099	.645
9	.859	.954	.786	.087	.626
10	.845	.957	.784	.078	.608
$\alpha = 1.00:$					
1	1.000	1.000	1.000	1.000	1.084
2	.975	1.000	1.000	.500	1.041
3	.952	1.000	1.000	.333	1.001
4	.929	1.000	1.000	.250	.963
5	.908	1.000	1.000	.200	.928
6	.887	1.000	1.000	.167	.895
7	.867	1.000	1.000	.143	.864
8	.848	1.000	1.000	.125	.835
9	.830	1.000	1.000	.111	.807
10	.813	1.000	1.000	.100	.781
$\alpha = 1.25:$					
1	1.000	1.250	1.000	1.000	1.347
2	.975	1.116	1.120	.560	1.297
3	.948	1.078	1.159	.386	1.243
4	.921	1.060	1.179	.295	1.191
5	.895	1.050	1.191	.238	1.142
6	.870	1.043	1.199	.200	1.097
7	.846	1.038	1.204	.172	1.054
8	.824	1.035	1.208	.151	1.015
9	.802	1.032	1.211	.135	.977
10	.782	1.030	1.214	.121	.942

Table 3.1b Lease Term & Risk: Mean Reversion ($b=.25$)

($r=.03, g=0, \mu\sigma=.05, \lambda=0, D=1$)

Lease Term (T)	$E[y]/E[x]$	α_y	σ_y/σ	σ_{CF}/σ	σ_r/σ
$\alpha = 0.75:$					
1	1.000	.750	1.000	1.000	.116
2	.975	.750	.880	.440	.098
3	.956	.750	.779	.260	.084
4	.940	.750	.695	.174	.073
5	.927	.750	.625	.125	.063
6	.916	.750	.565	.094	.056
7	.907	.750	.515	.074	.050
8	.900	.750	.472	.059	.045
9	.894	.750	.436	.048	.040
10	.889	.750	.405	.041	.037
$\alpha = 1.00:$					
1	1.000	1.000	1.000	1.000	.144
2	.975	.880	1.000	.500	.126
3	.952	.848	.919	.306	.108
4	.932	.834	.834	.208	.093
5	.916	.827	.756	.151	.081
6	.903	.822	.687	.115	.072
7	.892	.820	.628	.090	.064
8	.882	.818	.578	.072	.057
9	.875	.817	.534	.059	.052
10	.868	.816	.497	.050	.047
$\alpha = 1.25:$					
1	1.000	1.250	1.000	1.000	.173
2	.975	.982	1.120	.560	.153
3	.948	.921	1.053	.353	.132
4	.924	.896	.971	.243	.114
5	.905	.882	.885	.177	.099
6	.889	.875	.808	.135	.087
7	.876	.870	.740	.106	.077
8	.865	.867	.681	.085	.069
9	.856	.865	.631	.070	.062
10	.849	.864	.587	.059	.057

Table 3.2a Return Risk Related to Observable Rental Market Risk and Cash Flow Risk: No Mean Reversion Case (b=0)

(r=.03, g=0, $\mu\sigma=.05$, $\lambda=0$, D=1)

Lease Term (T)	σ_r/σ_y	σ_r/σ_{CF}
$\alpha = 0.75:$		
1	.821	.821
5	.876	4.379
10	.776	7.762
$\alpha = 1.00:$		
1	1.084	1.084
5	.928	4.640
10	.781	7.812
$\alpha = 1.25:$		
1	1.347	1.347
5	.959	4.797
10	.776	7.764

Table 3.2b Return Risk Related to Observable Rental Market Risk and Cash Flow Risk: Mean Reversion Case (b=.25)

(r=.03, g=0, $\mu\sigma=.05$, $\lambda=0$, D=1)

Lease Term (T)	σ_r/σ_y	σ_r/σ_{CF}
$\alpha = 0.75:$		
1	.116	.116
5	.102	.508
10	.091	.907
$\alpha = 1.00:$		
1	.144	.144
5	.108	.539
10	.094	.944
$\alpha = 1.25:$		
1	.173	.173
5	.112	.560
10	.096	.964

Table 3.3 Lease Term & Risk: Sensitivity to $\mu\sigma$

($r=.03, g=0, \lambda=0, \alpha=1, D=1$)

Lease Term (T)	$E[y]/E[x]$	α_y	σ_y/σ	σ_{CF}/σ	σ_r/σ
$\mu\sigma = .01:$					
b=0:					
1	1.000	1.000	1.000	1.000	1.040
5	.981	1.000	1.000	.200	.963
10	.959	1.000	1.000	.100	.877
b=.25:					
1	1.000	1.000	1.000	1.000	.139
5	.983	.821	.756	.151	.084
10	.972	.809	.497	.050	.050
$\mu\sigma = .10:$					
b=0:					
1	1.000	1.000	1.000	1.000	1.144
5	.824	1.000	1.000	.200	.888
10	.668	1.000	1.000	.100	.687
b=.25:					
1	1.000	1.000	1.000	1.000	.152
5	.839	.834	.756	.151	.079
10	.759	.825	.499	.050	.044

**Table 3.4 Return Risk Relation to Observable Risk:
Sensitivity to $\mu\sigma$**

($r=.03, g=0, \lambda=0, \alpha=1, D=1$)

Lease Term(T)	σ_r/σ_y	σ_r/σ_{CF}
$\mu\sigma = .01:$		
b=0:		
1	1.040	1.040
5	.963	4.813
10	.877	8.772
b=.25:		
1	.139	.139
5	.110	.552
10	.101	1.006
$\mu\sigma = .10:$		
b=0:		
1	1.144	1.144
5	.888	4.440
10	.687	6.873
b=.25:		
1	.152	.152
5	.105	.524
10	.088	.879

Table 3.5 Lease Term & Risk: Sensitivity to Growth

($r=.03, \mu\sigma=.05, \lambda=0, \alpha=1, D=1$)

Lease Term (T)	$E[y]/E[x]$	α_y	σ_y/σ	σ_{CF}/σ	σ_r/σ
g = -.02:					
b=0:					
1	1.000	1.000	1.000	1.000	1.084
5	.875	1.000	1.000	.192	.891
10	.753	1.000	1.000	.091	.714
b=.25:					
1	1.000	1.000	1.000	1.000	.227
5	.883	.829	.762	.146	.125
10	.801	.819	.516	.047	.071
g = .02:					
b=0:					
1	1.000	1.000	1.000	1.000	1.084
5	.942	1.000	1.000	.208	.965
10	.878	1.000	1.000	.109	.850
b=.25:					
1	1.000	1.000	1.000	1.000	.051
5	.951	.824	.749	.156	.030
10	.942	.813	.477	.052	.017

Table 3.6 Return Risk Relation to Observable Risk:
Sensitivity to Growth

($r=.03, \mu\sigma=.05, \lambda=0, \alpha=1, D=1$)

Lease Term(T)	σ_r/σ_y	σ_r/σ_{CF}
g = -.02:		
b=0:		
1	1.084	1.084
5	.891	4.640
10	.714	7.835
b=.25:		
1	.227	.227
5	.164	.854
10	.137	1.500
g = .02:		
b=0:		
1	1.084	1.084
5	.965	4.640
10	.850	7.792
b=.25:		
1	.051	.051
5	.039	.189
10	.036	.331

Table 3.7 Lease Term & Risk: Sensitivity to Time-value of Money

(g=0, $\mu\sigma=.05$, $\lambda=0$, $\alpha=1$, D=1)

Lease Term (T)	E[y]/E[x]	α_y	σ_y/σ	σ_{cf}/σ	σ_r/σ
r = .01:					
b=0:					
1	1.000	1.000	1.000	1.000	1.063
5	.906	1.000	1.000	.200	.944
10	.806	1.000	1.000	.100	.826
b=.25:					
1	1.000	1.000	1.000	1.000	.051
5	.914	.824	.749	.150	.029
10	.865	.813	.478	.048	.017
r = .05:					
b=0:					
1	1.000	1.000	1.000	1.000	1.105
5	.909	1.000	1.000	.200	.913
10	.819	1.000	1.000	.100	.741
b=.25:					
1	1.000	1.000	1.000	1.000	.228
5	.917	.829	.762	.152	.126
10	.872	.819	.516	.052	.072

Table 3.8 Return Risk Relation to Observable Risk:
Sensitivity to Time-value of Money

(g=0, $\mu\sigma=.05$, $\lambda=0$, $\alpha=1$, D=1)

Lease Term(T)	σ_r/σ_y	σ_r/σ_{CF}
r = .01:		
b=0:		
1	1.063	1.063
5	.944	4.720
10	.826	8.264
b=.25:		
1	.051	.051
5	.039	.197
10	.036	.360
r = .05:		
b=0:		
1	1.105	1.105
5	.913	4.566
10	.741	7.411
b=.25:		
1	.228	.228
5	.165	.825
10	.139	1.388

Table 3.9 Lease Term & Risk: Sensitivity to Illiquidity Premium

($r=.03, \mu\sigma=.05, g=0, \alpha=1, D=1$)

Lease Term (T)	$E[y]/E[x]$	α_y	σ_y/σ	σ_{CF}/σ	σ_r/σ
l = 0:					
b=0:					
1	1.000	1.000	1.000	1.000	1.084
5	.908	1.000	1.000	.200	.928
10	.813	1.000	1.000	.100	.781
b=.25:					
1	1.000	1.000	1.000	1.000	.144
5	.916	.827	.756	.151	.081
10	.868	.816	.497	.050	.047
l = .02:					
b=0:					
1	1.000	1.000	1.000	1.000	1.105
5	.876	1.000	1.000	.200	.913
10	.756	1.000	1.000	.100	.744
b=.25:					
1	1.000	1.000	1.000	1.000	.228
5	.884	.829	.762	.152	.126
10	.804	.819	.516	.052	.072

**Table 3.10 Return Risk Relation to Observable Risk:
Sensitivity to Illiquidity Premium**

($r=.03$, $\mu\sigma=.05$, $g=0$, $\alpha=1$, $D=1$)

Lease Term(T)	σ_r/σ_y	$\sigma_r/\sigma_{\text{cr}}$
l = 0:		
b=0:		
1	1.084	1.084
5	.928	4.640
10	.781	7.812
b=.25:		
1	.144	.144
5	.108	.539
10	.094	.944
l = .02:		
b=0:		
1	1.105	1.105
5	.913	4.566
10	.744	7.444
b=.25:		
1	.228	.228
5	.165	.825
10	.139	1.392

Table 3.11 Lease Term & Risk: Sensitivity to Cyclicity
($r=.03, \mu\sigma=.05, g=0, \alpha=1, b=0, T=5$)

Cycle Phase (B)	$E[y]/E[x]$	α_y	σ_y/σ	σ_{CF}/σ	σ_r/σ
A=0, P=1:					
0	.907	1.000	1.000	.200	.925
A=.2, P=5:					
0	.771	.939	1.000	.202	.925
1.25	.975	1.238	1.000	.194	.916
2.50	1.108	1.090	1.000	.198	.925
3.75	.848	.789	1.000	.206	.934
-----	-----	-----	-----	-----	-----
Avg.	.926	1.014	1.000	.200	.925
A=.2, P=10:					
0	.916	1.065	1.000	.217	.919
2.50	1.110	1.119	1.000	.197	.922
5.00	.897	.918	1.000	.182	.932
7.50	.762	.914	1.000	.203	.929
-----	-----	-----	-----	-----	-----
Avg.	.921	1.004	1.000	.200	.925

Table 3.12 Return Risk Relation to Observable Risk:
Sensitivity to Cyclicity
($r=.03, \mu\sigma=.05, g=0, \alpha=1, b=0, T=5$)

Cycle Phase (B)	σ_r/σ_y	σ_r/σ_{CF}
A=0, P=1:		
0	.925	4.626
A=.2, P=5:		
0	.925	4.573
1.25	.916	4.722
2.50	.925	4.680
3.75	.934	4.526
-----	-----	-----
Avg.	.925	4.625
A=.2, P=10:		
0	.919	4.238
2.50	.922	4.677
5.00	.932	5.125
7.50	.929	4.571
-----	-----	-----
Avg.	.925	4.653

Table 3.13 Lease Term & Risk: Sensitivity to Current Deviation from Central Tendency (assuming mean reversion)

($r=.03$, $\mu\sigma=.05$, $g=0$, $\alpha=1$, $b=.25$)

Deviation (D)	$E[y]/E[x]$	α_y	σ_y/σ	σ_{CF}/σ	σ_r/σ
T = 1:					
0.6	.600	1.000	1.000	1.000	.090
0.8	.800	1.000	1.000	1.000	.118
1.0	1.000	1.000	1.000	1.000	.144
1.2	1.200	1.000	1.000	1.000	.170
1.4	1.400	1.000	1.000	1.000	.194
---	-----	-----	-----	-----	-----
Avg.	1.000	1.000	1.000	1.000	.143
T = 5:					
0.6	.687	.827	.604	.102	.050
0.8	.801	.827	.691	.128	.066
1.0	.916	.827	.756	.151	.081
1.2	1.030	.827	.806	.172	.096
1.4	1.145	.827	.846	.191	.111
---	-----	-----	-----	-----	-----
Avg.	.916	.827	.741	.149	.081

Table 3.14 Return Risk Relation to Observable Risk:
Sensitivity to Current Deviation from Central
Tendency

($r=.03, \mu\sigma=.05, g=0, \alpha=1, b=.25$)

Deviation (D)	σ_r/σ_y	σ_r/σ_{α}	V_{t-1}
T = 1:			
0.6	.090	.090	25.72
0.8	.118	.118	26.29
1.0	.144	.144	26.87
1.2	.170	.170	27.44
1.4	.194	.194	28.02
---	-----	-----	-----
Avg.	.143	.143	26.87
T = 5:			
0.6	.083	.495	26.06
0.8	.096	.517	26.47
1.0	.108	.539	26.88
1.2	.119	.560	27.29
1.4	.131	.581	27.70
---	-----	-----	-----
Avg.	.107	.538	26.88

3.2 Some Practical Implications and Insights

This Section presents three applications of the valuation model and risk relationships developed previously in this Chapter and Chapter 2. We begin in Section 3.2.1 with an analysis of the expected return risk premium in unsecuritized real estate, including consideration of the issue of cash flow mean reversion, which is quite important in this regard. We next consider the accuracy of the traditional simple cap rate method of valuation. Finally, we turn in Section 3.2.3 to a somewhat informal consideration of real estate's "duration", or the question of to what extent do multi-period leases make real estate more "like a bond" instead of "like a stock" in its sensitivity to nominal interest rate changes. These applications are designed primarily to further build intuition regarding the nature of real estate return risk, though there may be some fairly direct practical implications as well.

3.2.1 Implications Regarding Real Estate's Expected Return Risk Premium: The Mean-Reversion Issue

Perhaps the most direct and obvious application of the relationships developed above, indeed, the application which primarily motivated their development, is to help build our intuition regarding the expected return risk premium for unsecuritized real estate assets. According to the fundamental

risk value model presented as Assumption (A.2) in Section 2.2.2 of Chapter 2, the total expected return each period to an unsecuritized real estate property is equal to the riskfree rate, plus a possible illiquidity premium, plus a risk premium:

$$E[r(i)] = r + \lambda + \mu\sigma_r(i)$$

The focus here will be on the risk premium component of this total: $E[r(i)] - r - \lambda = \mu\sigma_r(i)$. As noted in the introduction to this thesis, the magnitude or even existence of this premium is currently a somewhat controversial topic. While nobody denies that there is some sort of fairly large expected total return premium, some would attribute all or nearly all of the difference between $E[r(i)]$ and r to what is here being labelled the illiquidity premium λ . Here, we will use the findings from Section 3.1 to try to shed some conceptual light on this question. —

The basic idea is very simple. Expressed as $\mu\sigma_r$, it is hard to get much intuition about the size of the risk premium, because we do not have much intuition about the size of unsecuritized real estate's return risk, σ_r . But by algebraically expanding $\mu\sigma_r$, we can express the risk premium in terms of observable cash flow risk and the relationships developed in the previous section, about which we may have more intuition.

Since $\mu\sigma_r$ is in principle an ex ante expectation, such intuition may have fairly direct application to real world problems, such as property valuation and capital budgeting, for which the total expected return is a key input. (In practice, an analyst would have a specific property or portfolio of properties in mind when applying this technique; here we shall speak more generally of "office properties" and "other properties", the primary distinction being that the multi-period riskless lease concept is most relevant to the office sector.)

3.2.1.1 Expansion of the Risk Premium...

Using the definitions in Section 2.2.2 of Chapter 2 it is straightforward to expand the risk premium formula into a product of four factors:

$$\mu\sigma_r(i) = \frac{(E[r(m)]-r)}{SD[r(m)]} \cdot \frac{SD[CF(i)]}{corr[r(m),I]} \cdot \frac{corr[r(i),I]}{corr[r(m),I]} \cdot (\sigma_r/\sigma_{CF})^i \quad (1)$$

where: $(E[r(m)]-r)/SD[r(m)]$ is the stock market's expected return risk premium divided by its return volatility; $SD[..]$ is the one-period conditional (ex ante) standard deviation; $corr[..]$ is the correlation coefficient; I is the CAPM index; $(\sigma_r/\sigma_{CF})^i$ is the return risk to observable cash flow risk ratio for real estate asset i , as described in the previous section;

and $CF(i)$ refers to real estate asset i 's unanticipated deviations from expected cash flow:

$$SD[CF(i)] = SD_{t-1}[(CF_t - E_{t-1}[CF_t])/E_{t-1}[CF_t]], \text{ for asset } i.$$

[In Chapter 2 it was noted in Section 2.2.2 that Assumption (A.2) could be defined in such a way that the risk premium depended only on the asset's own total risk, rather than the CAPM-based systematic risk, consistent with much traditional real estate practice (but inconsistent with general equilibrium in efficient markets). Here, however, we are applying only the general equilibrium based CAPM representation of Assumption (A.2).]

Note that the four factors in eqn.(1) include: one characteristic of the stock market, a kind of "risk premium coefficient of variation", $(E[r(m)] - r)/SD[r(m)]$; two characteristics of the real estate asset being studied, the cash flow forecast error volatility $SD[CF(i)]$ and the return risk to cash flow risk ratio $(\sigma_r/\sigma_{CF})^1$; and one ratio which is a comparison between a characteristic of the return risk of the real estate asset and the same characteristic for the stock market. In theory, all of these factors should be quantified in terms of their ex ante expected values held by investors in the real estate market in which the asset is traded. Let us try to

build our intuition regarding the nature of real estate assets' risk premia by briefly considering each of these factors.

3.2.1.2 Developing Intuition for Real Estate Risk Premia

The first factor in the RHS of eqn.(1) is the stock market risk premium coefficient of variation. In the numerator of this ratio, the stock market expected return risk premium is an often-used and rather familiar number to most analysts. Historically, this premium has been in the neighborhood of 8 or 9 percent per year, and this is a range often employed in practice to estimate ex ante expectations. The stock market return volatility $SD[r(m)]$ is often approximated at about 20 percent per year, which seems to be a fairly stable figure. Thus, the first factor in (1) is approximately 0.40.

The next factor in (1) is the real estate asset's cash flow volatility, $SD[CF(i)]$. This is a factor for which it might be possible in many cases to dig up some relevant historical data, although use of historical data for this purpose is a bit tricky, and subject to statistical estimation error as well as the "ex post/ex ante" problem. Since $SD[CF(i)]$ is supposed to be the volatility in unanticipated cash flow percentage changes, the relevant historical statistic to use

would be the standard deviation of the (white noise) residual from a forecasting model of the cash flows.

Another way to approach an estimation of the $SD[CF(1)]$ factor is to use the $\sigma_{cf} \approx \sigma_y/T$ approximation from the previous Section. One might have an idea that the long-term lease rental market volatility (vacancy rate volatility compounded by the effective new-lease rental price volatility) is on the order of, for example, 20 percent per year. Then, if lease terms range from 5 to 10 years, the observable cash flow volatility $SD[CF(1)]$ could be estimated at around 3 percent per year. As it will be helpful for the purpose of demonstrating the point we are making in this section to continue to quantify the factors in eqn.(1), let us take a range of 1 to 5 percent per year as a plausible order of magnitude estimate for the $SD[CF(1)]$ factor in the case of office properties. For non-office properties (assumed here to be effectively without multi-period riskless leasing) this factor would be much larger, perhaps in the 5 to 25 percent range.

(To base these numbers on some empirical evidence, an analysis was done of the Frank Russell Co (FRC) Index. This index consists of a broadly diversified portfolio of investment grade commercial properties, a large fraction of which is office properties. The cash flows of the FRC Index were

reconstructed from the income and appreciation return series, and these cash flows were adjusted for inflation, and then modelled using a univariate forecasting model. The white noise residuals from this model indicated unanticipated real cash flow volatility in the FRC Index of slightly over 7 percent per year, over the 1978-87 period. A similar exercise conducted on Prudential's PRISA Index revealed an annual unanticipated real cash flow volatility of some 23 percent per year over the 1971-87 period.)

Using these range estimates for $E[CF(1)]$ and our previous 0.40 estimate, our running total product on the RHS of eqn.(1) is a range of .004 to .02 for office properties and .02 to .10 for other properties.

We come now to the third factor on the RHS of eqn.(1), the ratio of the return correlations with the CAPM index for our asset as compared to the stock market. The likely value for this ratio for most real estate assets will depend crucially on the version of the CAPM being used. If one uses the traditional single-period CAPM with the stock market taken as the market proxy, then the denominator of this ratio is by definition equal to unity, the maximum possible correlation. In the numerator on the other hand, there is no reason to suppose that the correlation between a typical real estate asset and the stock market is particularly high. Indeed, while we might

suppose that many information shocks or innovations would tend to affect all capital assets in the economy the same way (eg, news relevant to real interest rates, for example), the empirical evidence noted in Chapter 1 (based on appraisal returns for real estate) indicates that much real estate is virtually uncorrelated or even negatively correlated with the stock market. Thus, if we are using the traditional single-period CAPM with stock market as proxy, the ratio which composes the third term in eqn.(1) is likely very small, perhaps even zero or negative. This would naturally imply a very small or even negative risk premium for most properties.

On the other hand, if we use the theoretically more general Consumption-based CAPM, a different intuition would seem logical regarding the sign and magnitude of the ratio of the cross-correlations. [Note that the traditional CAPM can be derived as a special case of the CCAPM by assuming (unrealistically) either that there are only two points in time, or that all state variables -- that is, all factors relevant to individuals' economic welfare (utility) -- follow white noise processes.] A priori there would seem to be little reason to believe that most real estate assets should be generally affected in an opposite manner to the stock market by unanticipated changes in consumption.

When consumption is higher than anticipated, that often indicates a stronger than anticipated economy, and therefore better than previously anticipated performance for most capital assets in the economy, both real estate and stocks. To the extent that higher than anticipated consumption is "bad news" for capital assets (eg, possible implications for future real interest rates or inflation), there would seem to be little fundamental reason for real estate assets to react opposite to stocks to this news. So, unless we have some specific reason to assume otherwise, the ratio of the correlations using the CCAPM should at least be positive.

It is more difficult to have any a priori insight regarding the magnitude of the cross-correlation ratio. One thing we can say is that, in contrast to the traditional CAPM with stock market as proxy, under the CCAPM it is possible for this ratio to exceed unity. Some real estate assets might be more highly correlated with consumption than the stock market is.

Clearly, the denominator of this ratio under the CCAPM is nowhere near unity. For example, over the 1971-87 period the S&P500 had a correlation coefficient with quarterly real per capita changes in consumption of only 0.18. While this figure may be artificially reduced by smoothing in government consumption numbers, such smoothing should affect both the numerator and denominator of the cross-correlation ratio in equal

proportions, leaving the ratio unchanged. Furthermore, according to CCAPM theory, it is the conditional correlation, or correlation with unanticipated changes in consumption which should be used in the cross-correlation ratio in eqn.(1). As the stock market is often used as a leading indicator of the economy, much of the positive relation between the stock market and the economy may already be incorporated in investor expectations, leaving much less positive correlation between unanticipated consumption changes and the stock market than we see in the correlation with consumption itself.

Let us assume for the sake of argument that under the Consumption CAPM the cross-correlation ratio term in eqn.(1) is in the range of 0.5 to 1.0 for most typical real estate properties (office as well as others). This then gives us a running total product on the RHS of (1) in the range of .002 to .02 for office properties, and in the range of .01 to .10 for other properties.

This brings us to the last factor on the RHS of (1) which must be incorporated to arrive at an estimate of the real estate risk premium, the return risk to observable cash flow risk ratio, $(\sigma_r/\sigma_{cf})^1$. This ratio was extensively discussed in Section 3.1, where some intuition and quantitative appreciation of this ratio was developed.

First, we noted that this ratio is increasing with the lease term. Indeed, inspection of the σ_r/σ_{CF} ratio over the lease term values in the Tables shown in Section 3.1 reveal that σ_r/σ_{CF} increases almost linearly with T, so office properties with long-term leases would have their $(\sigma_r/\sigma_{CF})^1$ factor in eqn.(1) much larger than non-office properties that do not use long-term riskless leases. If we assume office properties have lease terms in the 5 to 10 year range, and non-office properties have no multi-period riskless leases, then the $(\sigma_r/\sigma_{CF})^1$ factor for office properties will be in the neighborhood of 5 times larger than that for non-office properties, regardless of the underlying α , b , and σ parameters. Since our running total risk premium product coming into this last factor is about five times greater for non-office properties than it is for office properties, this implies that the risk premium estimate will now be about equal between office and non-office properties. The higher return risk to cash flow risk ratio with multi-period leasing offsets the lower cash flow risk we assumed for office properties.

If office properties are to have a lower risk premium estimate than non-office properties, then we must either go back and change one of our previous estimates of likely values for the eqn.(1) factors either for office or non-office proper-

ties, or we must focus our attention on the other main determinant of the magnitude of the σ_r/σ_{cf} ratio, the mean-reversion parameter, b .

Indeed, we see clearly at this point how important the mean reversion issue is in determining the likely magnitude of real estate expected return risk premia. As described in Section 3.1, σ_r/σ_{cf} is highly sensitive to the extent in which asset i 's cash flows are viewed as being mean reverting over the long run. If the cash flows are not mean reverting, then σ_r/σ_{cf} roughly equals T , the lease term, but a bit less for markets characterized by long-term leases (over 3 years). On the other hand, if the cash flows are mean reverting, then even for large T the $(\sigma_r/\sigma_{cf})^l$ ratio could be a fairly small fraction, though it could also range up to and fractionally exceeding unity if expected growth is low and/or the riskless interest rate or illiquidity premium is high. (Recall also that these results regarding the size of the σ_r/σ_{cf} ratio were found in Section 3.1.3 to be quite robust over the plausible ranges of the underlying parameters.)

Summarizing up to here...

In summary, the intuition reflected in the running product numbers we have been generating in this Section suggest the following conclusion. If we use the Consumption CAPM as

opposed to the traditional stock market based CAPM, and if we view cash flows as not mean-reverting, then, though the range of our estimate is quite broad, it certainly seems possible for real estate risk premia to be fairly large, for both office and non-office properties. The range estimate of 1 to 10 percent certainly makes a risk premium in the neighborhood of 3 to 4 percent seem quite plausible. As the spread between T-bills and the FRC Index return is in the 3 to 4 percent range, this would seem to imply that the role may be quite minor for the "illiquidity premium" in the total expected return for unsecuritized real estate.

On the other hand, if we either use the traditional CAPM instead of the CCAPM, or we view real estate cash flows as tending to revert over the long run to a deterministically known mean, then the real estate risk premium is almost certainly very small. On the basis of the analysis in Section 3.1, the σ_r/σ_{CF} ratio has a value roughly equal to $T/10$ if there is significant mean reversion, implying that our range estimate of the return risk premium would be in the neighborhood of 0.1 to about 1.5 percent per year, implying that most or almost all of a 4 percent spread over the riskless interest rate must be attributable to the illiquidity premium rather than to risk.

It should be noted that the crucial importance of the expected cash flow mean reversion issue in determining the magnitude of the risk premium is not unique to real estate that makes use of multi-period riskless leases, such as office buildings. The return risk to cash flow risk ratio is as sensitive to mean reversion at T=1 as it is with multi-period leases. Indeed, the importance of mean reversion in this context is not unique to real estate at all. The same point could be applied to the stock market, for example.

3.2.1.3 Discussion of the Mean Reversion Issue

In view of its importance in determining return risk, it may be worthwhile to consider the mean reversion issue as it relates to real estate assets in a little more depth. As a preliminary, note that although mean reversion is measured by a continuous parameter (b can assume any value from 0 to 1), for practical purposes the implication of mean reversion on return risk can be viewed without much loss of accuracy as pretty much an "either/or" issue. This is seen in the almost "knife-edge" picture of the σ_r/σ_{cf} ratio over the range of b values shown in Table 3.15 (located at the end of Section 3.2.1), for T=1 and T=5. Mean reversion greatly reduces return risk even for small values of b . For values of b in excess of about .15, return risk is such a small fraction of

underlying cash opportunity risk that, at least for real estate, the resulting risk premium would be quite small, and the further reductions in larger values of b would not make much practical difference.

As b represents the annual percentage rate at which cash flows are expected to close the gap between their current level and their central tendency or long run mean, values of b in excess of .15 would seem to be quite likely, to the extent that we view cash flows as mean-reverting at all. (For example, $b=0.25$ implies that after 5 years still only some three-quarters of this gap will be closed: $(1-.75^5)=0.763$.)

With this in mind, we may characterize the $b=0$ no-mean-reversion assumption and the $b>.15$ significant-mean-reversion assumption as being polar extremes, each of which is probably unrealistic taken literally. The $b=0$ assumption implies that cash flows could "wander forever". Though the proportional error feature of (A.1) prevents negative cash flow expectations, apart from that, $b=0$ implies literally that we expect cash flows eventually, with certainty, to depart any neighborhood of values (even after detrending by the growth rate, g). Even though this wandering away from any level need not occur in any finite amount of time, this feature of the $b=0$ assumption seems unrealistic. On the other hand, the mean reversion assumption implies that we think we know, with certainty, what

the long-run trend of the cash flows is. While it may indeed seem plausible for cash flows have some mean-reversion tendency, it is difficult to imagine that we know exactly with certainty what the long-run trend line is over the entire future. Thus, both the mean-reverting model and the non-mean-reverting model of cash flows are unrealistic and a bit extreme.

The usefulness of these models lies in the fact that each captures a part of the truth, and they are unrealistic in opposite ways. One models cash flow as having less determinism than it probably really has, while the other models cash flow as having more determinism than it probably really has. The two assumptions thus bracket the truth, in some sense. Crudely, we might take a mid-point between the implications of the mean-reversion case and the non-mean-reversion case, to get a more realistic picture of the nature of real estate risk premia.

In this regard, it may be useful to consider what would be the effect of altering the mean-reversion model so that the trend line, X_t , was risky rather than deterministic. While the math involved in formally modelling such a "reversion-to-a-stochastic-trend" process is beyond the scope of this thesis, we can get a good idea of the effect of such a modifica-

tion, using the intuition developed in Chapter 2 regarding the return risk formula.

A review of formula (10) in Chapter 2 and the intuition and derivation of that formula suggests that adding even a fairly small amount of risk to the long-run cash flow trend line would considerably increase the return risk in the asset. Return risk is so small under mean reversion primarily because the riskless central tendency term (the part of C_5 with $K[\infty]$ in it) is quite large, and it appears in the denominator of eqn.(10) but not in the numerator. But if the central tendency X_t were risky, then this term would not be so large, and further, it would also appear in the numerator (though possibly reduced by some elasticity of expectations type dampening factor).

This also suggests that the true return risk implications of the cash flow fundamentals of real estate lie somewhere between those of the mean-reverting and non-mean-reverting cases described in the previous section. Yet another route to this same conclusion is suggested by an examination of the economics underlying the mean-reversion model.

As sketched by Bhattacharya in his original development of the mean-reverting cash flow capital budgeting model, mean

reversion is fundamentally an effect of equilibrium in the real capital market. The idea is that long-run equilibrium should cause the input factors used to produce a capital asset to earn neither more nor less than a fair compensation, in the long run. Viewing the cash thrown off by an asset as this compensation, this should imply that capital asset cash flows will tend to revert toward a long run mean. If a capital asset earns (in the form of cash flow) more or less than the long-run equilibrium "fair" level, then entry or exit into the market (of real assets) by competitors should drive this cash flow yield back to the fair level in the long run. This assumes, of course, that the physical asset can be reproduced by competitors, either exactly or in the form of a close substitute.

Underlying this long-run equilibrium argument for mean-reversion in cash flows is the idea that the reproduction cost of the physical asset is itself mean-reverting over time. This seems plausible to the extent that the asset is a machine or structure whose primary physical inputs are labor and commodities, and as long as technological changes do not severely alter the basic asset design or production process. Market wages and commodity prices are generally viewed as mean-reverting over time (to some exponentially growing trend level, perhaps). So an asset whose inputs consist fundamentally primarily of wages and commodity costs will then have a

reproduction cost that is mean-reverting. Asset cash flows would then be expected to at least tend to revert toward some fraction of the current cost of reproducing the asset (in its current possibly depreciated condition, as it gets older).

However, an important difference between real estate assets and other physical capital is the extent to which land is a major input production factor in the asset, in addition to labor and commodities. While the structure or building on a piece of property may be easily reproduced using inputs whose cost follows a mean-reverting path over time, the land underneath the structure is unique and perpetual. Land cannot be either created or destroyed, and is probably best viewed in this context as a kind of pure capital asset, whose current market value reflects all currently available information about its future value, and whose returns would therefore be expected to be well approximated as a white noise process. This implies that the land value component of the real estate asset reproduction cost could well follow a non-mean-reverting or random walk type process.

Thus, we may conceptually view the reproduction cost of a real estate asset as consisting of two components. One is the reproduction cost of the structure currently in place on the property. The other is the current value of the land. The former may be well viewed as mean-reverting over time, while

the latter may be better modelled as non-mean-reverting. It therefore makes sense conceptually to think of real estate cash flows (in their role as the compensation for the cost of the asset) as consisting of two components, one of which compensates the presently existing structural input on the land, and the other of which compensates the land input. While long run equilibrium would drive the former cash flow component to be mean-reverting, it would not have this effect on the latter component. Equilibrium would drive the land component of the cash flow toward a fixed fraction of the (non-mean-reverting) land value, so this component would therefore be non-mean-reverting.

If one looked at a historical time series of ex post cash flows during a period when no new major structure was built on the property, the cash flow series might well appear to be mean-reverting, since a part of the total is mean-reverting. But a part of the future cash flow stream expected by investors could nevertheless be non-mean-reverting.

The operative question then becomes: what fraction of the property's current value is attributable to the presently-existing structure on it, and what fraction is attributable to the land. If we can answer this question, then we can get some idea where we lie in the range between the return risk implications of mean-reversion versus non-mean-reversion. We

can think of the real estate asset as a portfolio of land plus structure. As the systematic risk (and hence, the expected return risk premium) of a portfolio is just the value-weighted average of that of its components, this will give an idea how much to weight the mean-reversion implications versus the non-mean-reversion implications in our return risk analysis.

But this question should be fairly easy to answer, conceptually at least, on average over time. Buildings wear out and eventually are replaced (not necessarily by the same type and size of new structure). When a building is just new, its value probably represents almost all of the property value, with land value apart from the building being only a minor part of the total. But when a building is old or near to the time when it will be replaced, very little of the current property value is attributable to the building, and almost all of the current property value is effectively land value. This should be empirically apparent by noticing that land parcels with structures that are physically or economically obsolete on them sell for prices very little above the price of otherwise similar vacant lots in the same area.

There are two key implications of this analysis. First, if (real physical/economic, as opposed to "accounting") depreciation is "straight-line", then on average over time, 50 percent of property value is the land component, and 50 percent is the

existing-structures component. This would imply that the return risk implications and expected return risk premium are on average over time about halfway between those implied by the mean-reversion assumption and those implied by the non-mean-reversion assumption. Second, we should expect, cet. par., the return risk of a property to increase over time as the structure on the property ages (and/or becomes more economically obsolete, due possibly as much to changes in neighboring land use patterns as to physical characteristics of the building itself). This second implication is intuitively appealing.

Table 3.15 The Effect of Mean Reversion on Return Risk

($r=.03$, $\mu\sigma=.05$, $g=0$, $\lambda=0$, $\alpha=1$, $D=1$)

b	σ_r/σ	σ_r/σ_y	σ_r/σ_{CF}
T=1:			
0	1.084	1.084	1.084
.05	.426	.426	.426
.10	.274	.274	.274
.15	.207	.207	.207
.20	.169	.169	.169
.25	.144	.144	.144
.30	.127	.127	.127
.35	.115	.115	.115
.40	.105	.105	.105
.45	.097	.097	.097
.50	.091	.091	.091
T=5:			
0	.928	.928	4.640
.05	.333	.353	1.764
.10	.196	.220	1.102
.15	.136	.162	.809
.20	.103	.129	.644
.25	.081	.108	.539
.30	.067	.093	.466
.35	.056	.083	.413
.40	.048	.075	.373
.45	.042	.068	.342
.50	.038	.063	.317

3.2.2 Accuracy of Simple Cap Rate Valuation

A common practice in real estate valuation is to estimate the current property value (ex dividend) as the current expectation of the next year's cash flow divided by a "cap rate". Obviously the mathematics of this procedure are such that there must exist some value of the cap rate, labeled Ω , which will give the correct current property value. Defining the current time to be $t-1=0$ and the current correct property value to be V_0 , we have:

$$\begin{aligned} \Omega &= E_0[CF_1]/V_0 \\ \langle \text{---} \rangle & \quad V_0 = E_0[CF_1]/\Omega \end{aligned} \tag{2}$$

(Note that this implies that the cap rate which gives the correct property value equals the current expected income component of the property's return.)

In practice, however, the true V_0 is not known a priori. The inputs to the value estimation process are $E_0[CF_1]$ and the cap rate, the latter of which the analyst must estimate. If the analyst knows the true current expected total return on the property (eg, by knowledge of the return risk premium and the interest rate) and he knows the underlying expected rental price growth trend, then since real estate assets exist in perpetuity it is reasonable to estimate the cap rate as a constant-growth perpetuity capitalization rate. This suggests

that the analyst would employ an estimated cap rate, labelled Ω^* , defined as the correct current expected total return to the property (given by Assumption A.2 of Chapter 2) minus the expected underlying cash flow growth trend:

$$\begin{aligned}\Omega^* &= E_0[r(i)_1] - g \\ &= r + \lambda + \mu\sigma_r(i)_0 - g\end{aligned}\tag{3}$$

where the terms are all as defined in Assumption (A.2) of Chapter 2.

In this Section the valuation model and formulas developed in Chapter 2 are used to examine the question: When will this estimated cap rate give the correct current property value? That is, when will $\Omega^* = \Omega$?...

This question is similar to that addressed by Myers & Turnbull and subsequently by Bhattacharya for capital budgeting problems (see citations in bibliography). In those papers the authors considered when discounted cash flow analysis (DCF) using a constant risk-adjusted discount rate ("cost of capital") would yield an accurate project valuation. DCF using a constant discount rate is identical to the simple cap rate valuation method considered here in the special case where expected cash flows are a constant-growth perpetuity. Myers & Turnbull assumed that cash flows followed the non-

mean-reverting process modelled by the $b=0$ case here, and found "good news" in the sense that the simple DCF procedure yielded valuation that was very nearly correct, and exactly correct in the case of the perpetuity, provided the correct risk-adjusted discount rate (equal to the true expected total return) was used. However, Bhattacharya found that in the case where cash flows were mean-reverting (corresponding to our $b > \max\{0, g\}$ assumption here) the simple DCF procedure gave results that could be substantially biased, even in the case of the perpetuity.

These earlier studies therefore suggest that one factor which may be important in the relationship between Ω and Ω^* is whether or not cash flows are mean reverting. Another factor which we want to consider here which was not addressed in those earlier studies is the effect of multi-period leasing and the presence of existing vintage leases in the property. Intuitively, this should affect the relationship between Ω and Ω^* because the presence of existing leases introduces a "bond part" component into the asset value, which suggests that a constant discount rate model may be too simple. The presence of existing leases may affect cash flow risk (which enters into σ_r and hence into Ω^*) differently than expected cash flows (which enters into Ω).

To address these questions it will be convenient to define a measure of the current conditions in the real estate market, which we label D , the current deviation from the average or previously expected market condition. Consistent with the definition in Chapter 2, D is defined as:

$$1+u_0 = x_0/E_{-1}[x_0] = D = x_0/X_0 \quad (4)$$

where the last equality is relevant only in the case of mean-reversion. (Note that, implicit in this definition is the assumption, which is necessary for tractability of the analysis, that in previous periods ($t < 0$) the real estate market was at its long-run average or previously expected condition.) Thus, $D=1$ represents the situation where the real estate market is at its unconditional expectation or long-run average level. It is natural to consider this long-run "normal" $D=1$ level of the market as a kind of "base case", but we also need to consider how, as D may vary over time, the relationship between Ω and Ω^* may vary with it, as V_0 , $E_0[CF_1]$, and σ_r all may vary with D in different ways.

The basic qualitative relationship between Ω and Ω^* can be perceived algebraically. Using the fact that the total expected return consists of the expected cash flow return component plus the expected value appreciation return component:

$$E_0[r(i)_1] = \frac{E_0[CF_1]}{V_0} + \frac{E_0[V_1] - V_0}{V_0}$$

we can use the definitions of Ω and Ω^* to relate the true and estimated cap rate values:

$$\Omega^* = \Omega + \frac{E_0[V_1] - V_0}{V_0} - g \quad (5)$$

The estimated cap rate equals the true cap rate plus the current expected appreciation return component, minus the underlying long-run expected opportunity growth rate. In the case of a perpetual asset like real estate, one would expect the appreciation return component, on average over time, to equal the underlying opportunity growth rate. Formula (5) therefore suggests that the estimated cap rate may be accurate on average over time. As formula (5) did not require either the assumption that there is no mean reversion in the cash flows or that there is no multi-period leasing, this suggests that the simple cap rate valuation procedure may be more robust than first appears. We can expand and confirm this finding using a more in-depth algebraic analysis, reported in the following section, and using numerical simulation, reported in Section 3.2.2.2.

3.2.2.1 Algebraic Analysis

According to (5), Ω^* will equal Ω if and only if $E_0[V_1] = (1+g)V_0$. It turns out that this condition will hold without mean reversion ($b=0$) no matter what is the current market condition (ie, for all D), provided $T=1$. This is the case Myers & Turnbull examined. If $T>1$ (multi-period leasing), then even without mean-reversion $E_0[V_1]$ will not exactly equal $(1+g)V_0$, and Ω will not exactly equal Ω^* , unless $D=1$. With mean reversion, Ω will equal Ω^* if and only if $D=1$, no matter what the lease term (ie, even for $T=1$). [This is because, with mean-reversion, expected rental prices are constant-growth if and only if $D=1$, and the formula for Ω^* is predicated upon expected rental prices being constant-growth.] Thus, at the long-run "normal" ($D=1$) market condition (which should be representative of the average over time), $\Omega=\Omega^*$, and "simple cap rate valuation" is accurate, no matter whether cash flows are mean-reverting or not, and whether or not there are long-term leases.

These results can be seen by the following analysis. Recall that the property ex dividend value at any time consists of the "bond value" part (present value of existing vintage leases) plus the "stock value" part. Therefore:

and:

$$V_0 = B_0 + S_0$$

$$E_0[V_1] = E_0[B_1] + E_0[S_1]$$

Thus, the necessary and sufficient condition for $\Omega = \Omega^*$ is that both $E_0[B_1] = (1+g)B_0$ and $E_0[S_1] = (1+g)S_0$. The conditions when these equalities will be met can be seen most easily by a simple example. Let $T=3$, and define the current market underlying opportunity cost to be $x_0 = D$. This implies that:

$$E_{-2}[x_{-1}] = 1/(1+g) = X_{-1} \quad (6a)$$

$$E_{-1}[x_0] = 1 = X_0 \quad (6b)$$

$$E_0[x_1] = (1+g)[1+(D-1)\alpha] , \quad X_1 = (1+g) \quad (6c)$$

$$E_0[x_2] = bX_2 + (1-b)(1+g)E_0[x_1] \quad (6d)$$

$$= b(1+g)^2 + (1-b)(1+g)^2[1+(D-1)\alpha] , \quad X_2 = (1+g)^2 \quad (6e)$$

Consider first the stock part of the property value, S_0 , and the expectation of this component next year, $E_0[S_1]$. This is the component of property value that is due to cash flow opportunities beyond the expiration of the existing leases, in perpetuity. Based on formula (1a) of Chapter 2 and the reasoning that led to the development of formula (10) in that Chapter, S_0 in our three-lease building is given by the following formula:

$$\begin{aligned}
S_0 = & (1/3) \{ H[\infty]E_0[x_1] + K[\infty]X_1 \\
& + \frac{1+g}{1+r} [Q H[\infty] E_0[x_1] + (K[\infty]+bH[\infty])X_1] \\
& + \frac{(1+g)^2}{(1+r)^2} [Q^2H[\infty]E_0[x_1] + (K[\infty]+bH[\infty])\frac{1-Q^2}{1-Q}X_1] \} \quad (7)
\end{aligned}$$

where all the terms are defined in Chapter 2 (in particular, recall that $H[\infty]$ is the capitalization factor for the risky component of the cash flows and $K[\infty]$ is the capitalization factor for the riskless central tendency component of the cash flows, and $Q=1-b-\alpha\mu\sigma$ is a discount factor that is introduced in the recursion of the forward value of the stock part back to the present value).

Note that each line in (7) gives the value of the stock part of one-third of the building. The first line is the third whose lease expires this period and therefore the exact cash flow from which is unknown even next period (and so this third has no bond part in its value). The other two lines are the forward stock part of the values of the parts of the building that do have bond parts in their value.

In the same manner, our current expectation of the value of the stock part of the property next year is:

$$\begin{aligned}
E_0[S_1] &= (1/3) \{ H[\infty]E_0[x_2] + K[\infty]X_2 \\
&\quad + \frac{1+g}{1+r} [Q H[\infty] E_0[x_2] + (K[\infty]+bH[\infty])X_2] \\
&\quad + \frac{(1+g)^2}{(1+r)^2} [Q^2 H[\infty]E_0[x_2] + (K[\infty]+bH[\infty]\frac{1-Q^2}{1-Q})X_2] \} \quad (8)
\end{aligned}$$

By inspection we see that (8) is identical to (7) except that the terms in (7) which multiply $E_0[x_1]$ now multiply $E_0[x_2]$ in (8), and the terms in (7) which multiply X_1 now in (8) multiply X_2 . But of course, by definition, $X_2=(1+g)X_1$, and observe from (6c) and (6e) that if either $b=0$ (no mean reversion) or $D=1$ (long-run "normal" market) then $E_0[x_2] = (1+g)E_0[x_1]$. Thus, if either $b=0$ or $D=1$, then $E_0[S_1] = (1+g)S_0$. In this example, $T=3$, but it is obvious by induction that this result would obtain for any value of T . The stock part of asset value is expected to grow at the underlying rate g no matter what the current market condition if there is no mean reversion, or if the market is at its long-run average if there is mean reversion, whether or not there are multi-period leases. However, this same result does not obtain for the bond value part of asset total value.

Consider now when $E_0[B_1]$ will equal $(1+g)B_0$. Since the bond part is just the risklessly discounted present value of the existing lease rental commitments, it is helpful to consider

the relationship between the new-lease rental prices and expected future prices over time.

From formula (3) of Chapter 2 and formulas (6) above, we see that the current new-lease rental price is:

$$\begin{aligned}
 y(0,3) &= \frac{x_0 + H[T-1]E_0[x_1] + K[T-1]X_1}{(1+r)a[T]} \\
 &= \frac{D + H[T-1](1+g)[1+(D-1)\alpha] + K[T-1](1+g)}{(1+r)a[T]}
 \end{aligned}$$

and the expected value of next year's rental price is:

$$\begin{aligned}
 E_0[y(1,3)] &= \frac{E_0[x_1] + H[T-1]E_0[x_2] + K[T-1]X_2}{(1+r)a[T]} \\
 &= \frac{(1+g)[1+(D-1)\alpha]}{(1+r)a[T]} \\
 &\quad + \frac{H[T-1]\{b(1+g)^2 + (1-b)(1+g)^2[1+(D-1)\alpha]\}}{(1+r)a[T]} \\
 &\quad + \frac{K[T-1](1+g)^2}{(1+r)a[t]}
 \end{aligned}$$

Note that the relationship between $E_0[y(1,3)]$ and $y(0,3)$ depends on D , but if $D=1$, then the above formulas simplify to:
at $D=1\dots$

$$y(0,3) = \frac{1 + H[T-1](1+g) + K[T-1](1+g)}{(1+r)a[T]}$$

$$E_0[y(1,3)] = \frac{(1+g) + H[T-1](1+g)^2 + K[T-1](1+g)^2}{(1+r)a[t]}$$

$$= (1+g)y(0,3)$$

At D=1 the expected next year rental is exactly (1+g) times this year's rental. (This makes sense, since the market is unperturbed on its expected track, so there is no reason why we should not expect it to continue to remain on that track.) Similarly, we can see for the past values of the rental price:

$$y(-j,3) = \frac{1/(1+g)^j + H[T-1]/(1+g)^{(j-1)} + K[T-1]/(1+g)^{(j-1)}}{(1+r)a[T]}$$

for j=1,2,...

Thus, if and only if D=1 we will have:

$$y(0,3) = y(-j,3)(1+g)^j$$

Let us relate this now to the bond value part of our T=3 example. The bond value part equals the riskless present value of the lease payments to be received under the two leases that do not expire this period. Since each lease covers (1/3) of the building:

$$B_0 = (1/3) \left\{ \begin{array}{l} y(0,3)/(1+r) \quad + \quad y(0,3)/(1+r)^2 \\ + \\ y(-1,3)/(1+r) \end{array} \right\}$$

(The other third of the building has no bond part but only the stock part in its value.)

Similarly the expectation of next year's bond part value is:

$$E_0[B_1] = (1/3) \left\{ \begin{array}{l} E_0[y(1,3)]/(1+r) + E_0[y(1,3)]/(1+r)^2 \\ + \\ y(0,3)/(1+r) \end{array} \right\}$$

But since, if and only if $D=1$, we have: $E_0[y(1,3)] = (1+g)y(0,3)$, and $y(0,3) = (1+g)y(-1,3)$, we therefore have: $E_0[B_1] = (1+g)B_0$ if and only if $D=1$. This obviously holds whether or not there is mean-reversion, and although in this example $T=3$, it is clear that this result will hold for any value of T . In the special case of $T=1$ examined by Myers & Turnbull and by Bhattacharya, there is no bond part of asset value, so the relationship between V_0 and $E_0[V_1]$ depends entirely on the stock part.

We have thus confirmed the results stated at the outset of this Section, which are summarized in the following "matrix":

	b=0 no mean reversion	b>0 mean reversion
T=1 No Long-term Leases	D=1: $\Omega = \Omega^*$ D<>1: $\Omega = \Omega^*$	D=1: $\Omega = \Omega^*$ D<>1: $\Omega <> \Omega^*$
T>1 Long-term Leases	D=1: $\Omega = \Omega^*$ D<>1: $\Omega <> \Omega^*$	D=1: $\Omega = \Omega^*$ D<>1: $\Omega <> \Omega^*$

While the algebraic analysis allows us to establish the above qualitative relationships, numerical analysis is required to give us some idea how much Ω^* will differ from Ω

in those cases where it is different. This is pursued in the next Section.

3.2.2.2 Numerical Analysis

Table 3.16 shows the results of a numerical analysis comparing Ω and Ω^* under both non-mean-reverting and mean-reverting cash flows, over a range of current market conditions ($D=0.6$ to 1.4), and under varying assumptions about underlying elasticity of expectations ($\alpha=0.75$ to 1.25). The Table reports the Ω and Ω^* values for $T=1$ and for $T=5$. Table 3.17 reports some resulting property valuations using the estimated cap rate versus the true rate.

In these Tables, Ω^* is quantified using definition (3) above, with σ_r quantified using formula (10) from Chapter 2, as in all the numerical simulations in Section 3.1 of this Chapter. The true cap rate Ω is quantified in the Tables using definition (2) above, with V_0 quantified as the denominator of Chapter 2's formula (10) and $E_0[CF_1]$ quantified as the denominator of Chapter 2's formula (11). These formulas all collapse to equal the relevant corresponding (perpetuity) formulas in Myers & Turnbull's ($T=1, b=0$) article and Bhattacharya's ($T=1, g=0, b \geq 0$) article. Thus, the results reported here should be consistent with the previous literature.

The qualitative relationships found in the algebraic analysis are, of course, confirmed, and three main quantitative conclusions emerge from Table 3.16 (located at the end of Chapter 3). First, with multi-period lease terms typical of office properties, the deviation of Ω^* from Ω can be rather severe in the non-mean-reversion case, but appears to be rather minor in the mean-reversion case. For example, with no mean reversion and $T=5$ with $\alpha=1$, deviation of x_0 by ± 20 percent around its previously expected value causes deviations in Ω^* from Ω of only some ± 10 basis points with $b=.25$, but in the neighborhood of 70 to 110 basis points without mean-reversion.

Given the typical level of the total cap rate (7 to 15 percent), this could cause substantial mis-estimation of property value, in the neighborhood of 10 percent or more (without mean-reversion). During times when the rental market is above average ($D>1$), without mean-reversion property values would be underestimated using Ω^* , while during times when the market is below average the opposite would occur. However, in practice, the significance of this error may be reduced, at least for small $T>1$, by the fact that the error with mean reversion is opposite in direction to the error without mean reversion (and considering the argument in Section 3.2.1 that actual cash

flow expectations behavior probably lies somewhere between the cases modelled here as $b=0$ and $b=.25$).

A second major quantitative conclusion emerges from the pattern of deviation of $\hat{\Omega}$ around Ω across the values of D . As indicated in Table 3.17, which shows correct versus $\hat{\Omega}$ -based estimated property values, the pattern in $\hat{\Omega}$ errors results (after the inverse operation) in property values estimated using $\hat{\Omega}$ being quite accurate on average over time. That is, the mis-estimates of property value caused by use of $\hat{\Omega}$ instead of Ω are very nearly symmetric around the true property value, over values of D .

The third major conclusion from the quantitative analysis is that property values estimated using $\hat{\Omega}$ will show different variance over time than will the true property values. In particular, in the case of non-mean-reversion (with multi-period leases), use of the simple cap rate valuation procedure with the cap rate based on the correct current total expected return will cause smoothing in the estimated property value series over time, relative to the true property value series. The estimated property value varies with D in the same direction as the true value, but not as much. Under mean-reversion, on the other hand, while the estimated value still varies with changes in D in the same direction as the true

value, with $b=.25$ it tends to vary more than the true value (unless the lease term is very long). [So, with $T>1$ and $D<>1$, the estimated property value under $b=0$ and the estimated value under $b=.25$ are generally on opposite sides of the true value, except at high values of T .]

With lease terms typical of office buildings ($T\geq 4$ or so), while the smoothing appears to be rather severe in the case of no mean reversion, the "unsmoothing" seems to be rather minor in the case of mean reversion. Therefore, on the basis of the conclusion from Section 3.2.1 regarding the integration of the implications of mean-reversion and non-mean-reversion, this suggests that, on the whole, for office buildings, use of $\hat{\Omega}$ instead of the (unobservable true) Ω tends to introduce some smoothing into the estimated property value series. (The opposite might be the case for properties without multi-period leases.)

One way to understand all of the above results is to recognize that there are two potential sources of error in the estimated cap rate $\hat{\Omega}$. One is the effect of the existing vintage leases. This source of error, which is more important when there is no mean-reversion, causes bias in the direction of overestimating property value when $D<1$ and underestimating it when $D>1$, the more so the longer the lease term (if there

is not mean-reversion). When either $D=1$ or $T=1$, there is no error from this source. The second source of error in Ω^* is the non-constant-growth rental prices effect. The accuracy of Ω^* as defined by (3) is predicated upon expected rents being a constant-growth perpetuity. But if rents are mean-reverting, then their expected future values will not be constant-growth if $D \neq 1$. This second source of error thus applies only when $b > 0$ and $D \neq 1$, but it applies whether or not there is multi-period leasing. The direction of the bias from this source of error is opposite to that of the vintage lease effect, causing underestimation of property value when $D < 1$ and overestimation when $D > 1$. Both sources of bias are present in mean-reverting cash flows with $T > 1$, and since the vintage lease effect increases with lease term and goes opposite to the non-constant-growth effect, the bias in Ω^* under mean-reversion falls with lease term, at least up to a point. [After a high enough T , the vintage lease effect dominates even in mean-reverting cash flows, causing bias to increase with further increases in T .]

It may be that the bias introduced into Ω^* when $D \neq 1$ is not too significant in practice because property appraisers and analysts may realize how the current market conditions differ from the long-run norm, and they may be aware of the bias this introduces if they do not adjust their cap rate appropriately.

In effect, good analysts may modify formula (3) in the direction of (2). However, in practice it may be difficult to recognize how far the current market is from its (ex ante expected) long-run normal level.

Use of more explicit multi-period cash flow forecasting in a DCF framework as opposed to the simple cap rate valuation procedure can help eliminate at least the non-constant-growth source of bias, but only to the extent that the analyst knows the "correct" central tendency and rate of mean reversion, factors about which it may be difficult to have much reliable knowledge. Furthermore, as the above analysis has pointed out, there could still remain an important vintage lease effect which could bias the valuation. Perhaps more importantly, we saw in the above analysis that with multi-period leases the vintage leases and the non-constant-growth sources of bias offset each other to some degree, at least over moderate lease terms (and to the extent that mean-reversion plays an important role). This implies that the elimination of one source of bias (by use of multi-period forecasts and DCF) could make the overall valuation more biased! (Of course, an analyst diligent enough to do mean-reverting multi-period cash flow forecasts may also be diligent enough to separately value the bond and stock parts of the property value using different discount rates, possibly avoiding the vintage lease bias effect as well.)

3.2.2.3 Conclusions Regarding Simple Cap Rate Valuation

There is certainly some "good news" in the above analysis. Simple cap rate valuation appears to be quite accurate on average over time or when the rental market is near its long-run average level of balance between supply and demand. This result is quite similar to the result obtained by Myers & Turnbull, but differs from the finding of Bhattacharya. (At $D=1$, expected future cash flows are constant-growth even with mean-reversion, so simple cap rate valuation is equivalent to constant cost-of-capital DCF valuation for an infinite-maturity project.) The result obtained here that simple cap rate valuation is accurate at $D=1$ for all lease terms (including $T=1$) applies to the mean-reversion case as well as to the non-mean-reversion case, and thus appears to contradict Bhattacharya's conclusion.

As noted, in the $T=1$ and $g=0$ case, Bhattacharya's formulas are the same as those used here. Bhattacharya's impression that the use of simple constant-discount-rate valuation would be biased even in the case of the $D=1$ perpetuity appears to come from his failure to eliminate some terms which cancel out at $D=1$ in his perpetuity formula. These cancellable terms obfuscate the equality of the constant-discount-rate DCF valu-

ation and the true valuation for the $D=1$ perpetuity. Bhattacharya's false impression about bias in this case was probably reinforced by his numerical analysis, which was limited to finite maturity projects, and showed bias increasing as maturity increased to the maximum horizon he considered, which was 40 years. Had he continued his analysis to longer maturities, however, he would indeed have found the bias shrinking toward zero at infinite maturity (with $D=1$), consistent with the result found above.

Where the results here differ from the Myers & Turnbull finding, bringing some "bad news", is in the effect of multi-period leases when the rental market is not at its long-run normal level. Thus, at any given point in time, simple cap rate valuation may be biased, even without mean reversion, although the bias may be small particularly if lease terms are not very long. If we believe that actual expectations behavior lies somewhere between the mean-reversion and non-mean-reversion cases, then the situation is helped by the fact that mean-reversion implies opposite direction of bias than is implied by non-mean-reversion (at least over moderate lease terms). The absolute magnitude of bias under non-mean-reversion increases with lease term (from zero bias for any D at $T=1$) because it is caused purely by the vintage lease effect (cash flow expectations are still constant-growth). In contrast, the magnitude of bias under mean-reversion decreases

with lease term (at least up to a point), because the non-constant-growth cash flow effect under mean-reversion is opposite to the vintage lease effect, and exceeds it, but by less as the lease term increases). For long-term leases, the overall effect of this bias is likely to cause some smoothing in the estimated property value time series as compared to the true value series.

3.2.3 Real Estate Sensitivity to Nominal Interest Rates: Is Real Estate More "Like a Bond" or "Like a Stock"?

One of the assumptions underlying the basic property valuation model developed in Chapter 2, noted in Assumption (A.2), is that the interest rate, represented by r , is constant. Of course, in reality this is not true, and it is interesting to ask how sensitive is property value to changes in interest rates. This question is related to the question of what is real estate's "duration", and is of interest to portfolio managers.

In this Section we use the property valuation model to examine real estate sensitivity to a particular type of change in interest rates, namely, a change in the nominal interest rate holding all else constant (including the real interest rate and the real underlying opportunity growth expectation relevant for cash flows beyond current contractual cash flow commitments). In times of important uncertainty about inflation, this type of sensitivity should be a major source of return risk in bonds (where all of the cash flows are contractually fixed in nominal terms) but should have no effect on the risk and value of stocks (since real values are by assumption held constant). Thus, the question examined in this Section boils down to the question of to what extent do long-term nominally riskless leases render real estate assets more

"like a bond" as opposed to "like a stock" with respect to sensitivity to nominal interest rate changes.

While our property valuation model, by its assumption of a constant interest rate, lacks the ability to explore this question in a completely rigorous manner, we can gain some insight and appreciation for this question by simply examining the sensitivity of property value to a change in the nominal interest rate holding all else constant in our model. This sensitivity is measured by the nominal "duration" of the asset, labelled δ , defined as:

$$\delta = -(1+i)(\partial V/\partial i)/V \quad (9)$$

where i represents here the inflation rate, and V is current property value (in "real" or constant-purchasing-power terms). The greater is δ , the greater is the sensitivity of the asset's real return to changes in nominal interest rates, or to inflation (holding real underlying expectations and real interest rates constant).

This question was recently examined in a study by Hartzell et al (Salomon Bros. 1987). However, they used a different valuation model, effectively a DCF model in which all expected cash flows are discounted at the same risk-adjusted rate. They estimated real estate duration to range from 0 (by definition) with a lease term of $T=1$ year (ie, no multi-period leases, and hence, no "bond part" of asset value) to a duration of 4.2

years with a lease term of $T=21$ years, with intermediate values of 0.6 and 1.8 years with lease terms of 6 or 11 years, respectively. (In effect, they assume a property with a single newly-signed lease, with $T-1$ years remaining on the lease.)

The Hartzell et al paper examines the effect of "inflation pass-throughs" in the leases, that is, recognizing that many real estate leases allow some pass-through of inflation from the landlord to the tenant, so that the lease cash flow is not fixed in nominal terms. (In the extreme case of complete pass-through, the lease cash flows are fixed in real terms.) Here, we consider only the no pass-through case, as this puts an upper bound on real estate's duration, and we shall see that even this upper bound is quite low. (With pass-throughs, real property value should be less sensitive to inflation, and duration accordingly smaller.)

To quantify the above-defined duration measure using our valuation model from Chapter 2, we treat the underlying opportunity cost cash flows $\{x_t\}$ as being measured in nominal dollars, and continue to use Assumption (A.4) as stated in Chapter 2, namely that the rental payments under a T -period lease signed at time s , designated $y_t(s,T)$, are fixed at the level of $y(s,T)$ for $t=s, s+1, \dots, s+T-1$. Thus, we now interpret

both r and g as nominal parameters, including inflation. To be more specific, with i representing the rate of inflation:

$$r = (1+r^R)(1+i)^{-1} \quad , \quad \text{and:} \quad g = (1+g^R)(1+i)^{-1}$$

where r^R and g^R are the real rates.

We must also adjust the "numeraire" expected cash flows for the next period, such that $E_0[x_1 : x_0 = X_0] = (1+i)$, instead of unity. [This has the effect of multiplying the S_0 "stock part" of current property value by $(1+i)$ times the value given by the C5 component in the denominator of Chapter 2's formula (10) using the nominal values of r and g .]

With these definitions and adjustments in mind, the current property value V_0 is given by the denominator of formula (10) in Chapter 2, such that:

$$V_0 = B_0 + (1+i)S_0$$

where the stock value component is completely insensitive to inflation: $\partial[(1+i)S_0]/\partial i = 0$, as it should be given the assumptions and focus of the analysis. Thus, $\partial V_0/\partial i = \partial B_0/\partial i$, and we can rewrite (9) in terms of the percentage sensitivity of the bond value component to inflation:

$$\delta = -(1+i) \frac{\partial B_0/\partial i}{B_0} \frac{B_0}{[B_0 + (1+i)S_0]} \quad (10)$$

In other words, real estate nominal duration equals the nominal duration of the bond value component, times the proportion of bond value in total value, which is a very intuitive relation given the assumptions and focus of this analysis.

By definition, the bond value component has nominally fixed cash flows, and so its duration is the well known present-value-weighted average time until the bond cash flows will be received. Consider a "monolithic" building that has only one single lease covering all its space, where there are n periods left in the lease. In this case the cash flows (rental payments from existing lease) are all equal, so the bond duration formula is simply:

$$-(1+i)[\partial B_0/\partial i]/B_0 = \frac{1/(1+r) + 2/(1+r)^2 + \dots + n/(1+r)^n}{1/(1+r) + 1/(1+r)^2 + \dots + 1/(1+r)^n} \quad (11)$$

Under the stylized market history assumption described in Chapter 2 in the derivation of formula (10) in that Chapter, we have bond value and stock value components as follows (with n periods left on the lease in a market where T -period leases are the norm at signing):

$$\begin{aligned} B_0 &= a[n] y(n-T+1, T) \\ &= a[n] \frac{[1 + H[T-1](1+g) + K[T-1](1+g)]}{(1+r)a[T](1+g)^{(T-n)}} \end{aligned} \quad (12)$$

$$S_0 = \frac{(1+g)^n}{(1+r)^n} \left\{ (Q^n)H[\infty] + \left(1 + \frac{(1-Q^n)/(1-Q)}{(1+r)/(r-g)}\right)K[\infty] \right\} \quad (13)$$

where all the terms are as defined in Chapter 2, and we have assumed here that $D=1$, that is, we are getting long-run average duration or duration when the rental market is at its normal balance.

Combining (11), (12) and (13) into (10), we obtain a quantitative formula for the duration as a function of the number of years remaining on the lease. It is clear by inspection of these formulas that for lease terms ranging up to 10 years or so δ will be a very small number, similar to the duration of a very short-term bond, or even less. The duration factor (11) will be less than half the lease term ($T/2$) even when the maximum $n=T-1$ years remain on the lease, and this factor will fall down to zero when the current lease expires at the end of the current period (no bond component). The bond part of total property value will also be a rather small fraction, always if lease terms (T) are small, but whenever remaining years on the lease (n) is small even if T is large.

Using the above formulas, Table 3.18 reports typical quantitative values of the duration δ and the bond value component as a fraction of the total property value (B_0/V_0), for n ranging between 0 and 10 years and $T=11$ (a fairly extreme

case). We see that the duration even at $n=10$ is only 2.64 years without mean reversion or 1.25 years under $b=.25$ mean reversion (with 5 percent inflation and other base case parameters as before). Similar numbers are obtained under other plausible parameter assumptions.

These numbers bracket the finding of Hartzell et al, and imply that for the typical property whose leases are in the 5-10 year range, and therefore where the average remaining years (n) is in the 0-5 year range, the duration would be on the order of 0.15 to 0.3 years depending on whether mean-reversion is assumed or not. Either way, this is very little duration, making real estate in this respect much more like a stock than a bond. The reason is only partly because the stock part tends to dominate over the bond part in total asset value. Another factor is that the lease cash flows, though they may originally cover a period as long as the maturity of a medium-term bond, do not contain a large "balloon" payment of principle at maturity. Another contributing factor is that the average time remaining on a lease is only about half its original term.

Table 3.16 Estimated versus True Cap Rate
 (r=.03, $\mu\sigma$ -.05, g=0, λ =0)

D	b=0:		b=.25:	
	Ω	Ω^*	Ω	Ω^*
T=1:				
α =.75:				
0.6	.071	.071	.026	.034
0.8	.071	.071	.031	.035
1.0	.071	.071	.036	.036
1.2	.071	.071	.041	.037
1.4	.071	.071	.045	.037
α =1.00:				
0.6	.084	.084	.023	.035
0.8	.084	.084	.030	.036
1.0	.084	.084	.037	.037
1.2	.084	.084	.044	.038
1.4	.084	.084	.050	.040
α =1.25:				
0.6	.097	.097	.020	.035
0.8	.097	.097	.030	.037
1.0	.097	.097	.039	.039
1.2	.097	.097	.047	.041
1.4	.097	.097	.055	.042
T=5:				
α =.75				
0.6	.079	.064	.031	.032
0.8	.071	.065	.032	.033
1.0	.065	.065	.033	.033
1.2	.061	.066	.034	.034
1.4	.057	.066	.035	.034
α =1.00:				
0.6	.101	.074	.031	.033
0.8	.086	.075	.033	.033
1.0	.076	.076	.034	.034
1.2	.070	.077	.035	.035
1.4	.065	.078	.037	.036
α =1.25:				
0.6	.127	.082	.031	.033
0.8	.101	.085	.033	.034
1.0	.087	.087	.035	.035
1.2	.078	.088	.037	.036
1.4	.072	.089	.038	.037

Table 3.17 Estimated versus True Valuation
 ($r=.03, \mu\sigma=.05, g=0, \lambda=0$)

D	b=0:		b=.25:	
	v	v*	v	v*
T=1:				
$\alpha=.75:$				
0.6	9.85	9.85	27.03	20.47
1.0	14.07	14.07	27.93	27.93
1.4	18.30	18.30	28.82	34.84
---	-----	-----	-----	-----
Avg.	14.07	14.07	27.93	27.75
$\alpha=1.00:$				
0.6	7.13	7.13	25.72	17.38
1.0	11.88	11.88	26.87	26.37
1.4	16.63	16.63	28.02	35.27
---	-----	-----	-----	-----
Avg.	11.88	11.88	26.87	26.60
$\alpha=1.25:$				
0.6	5.14	5.14	24.50	14.47
1.0	10.27	10.27	25.89	25.89
1.4	15.41	15.41	27.27	35.47
---	-----	-----	-----	-----
Avg.	10.27	10.27	25.89	25.28
T=5:				
$\alpha=.75$				
0.6	10.16	12.53	27.28	26.21
1.0	14.08	14.08	27.94	27.94
1.4	18.01	15.70	28.60	29.62
---	-----	-----	-----	-----
Avg.	14.08	14.10	27.94	27.92
$\alpha=1.00:$				
0.6	7.54	10.32	26.06	25.05
1.0	11.88	11.88	26.88	26.88
1.4	16.22	13.57	27.70	28.62
---	-----	-----	-----	-----
Avg.	11.88	11.92	26.88	26.85
$\alpha=1.25:$				
0.6	5.66	8.80	24.93	24.09
1.0	10.27	10.27	25.90	25.90
1.4	14.88	12.00	26.86	27.60
---	-----	-----	-----	-----
Avg.	10.27	10.36	25.90	25.86

Table 3.18 Nominal Duration in a Single-lease Property

($r=.0815$, $\mu\sigma=.05$, $g=.05$, $\lambda=0$, $D=1$, $T=11$, $i=.05$)

Yrs. Remaining On Lease (n)	Nominal Duration(δ)	Bond Part Fraction(B_0/V_0)
b=0:		
0	0	0
1	.049	.049
2	.150	.101
3	.304	.156
4	.511	.213
5	.769	.270
6	1.073	.328
7	1.419	.385
8	1.800	.440
9	2.208	.493
10	2.638	.543
-----	-----	-----
Avg.	.993	.271
b=.25:		
0	0	0
1	.023	.023
2	.069	.047
3	.139	.071
4	.232	.097
5	.348	.122
6	.487	.149
7	.647	.175
8	.829	.203
9	1.030	.230
10	1.251	.257
-----	-----	-----
Avg.	.460	.125

PART II: APPRAISAL BASED RETURNS ANALYSIS

While the cash flow based analysis of Part I gives us an insight into some of the fundamentals which determine the nature of real estate return risk, that analysis was based on simplifying assumptions about the nature of investors' cash flow expectations. Data on real estate returns themselves could provide more direct information about the nature of risk in these returns. Although we cannot obtain true market value based returns for the vast majority of commercial real estate because it is unsecuritized, there is a growing body of time series data on returns of unsecuritized real estate based on appraised values. This data comes largely from institutional portfolios of properties, often known as CREFS (Comingled Real Estate Funds).

The problem with these appraisal based data is that they do not present us with the true returns, in the sense of the opportunity costs (ie, market transactions based returns), which are relevant from an economic perspective. More to the point, analysts have long believed that these appraisal based returns series are "smoothed", that is, show less risk than is really there. However, there has been very little attempt to quantify this smoothing, either empirically or by analyzing conceptually the nature of the appraisal process to see how

and to what extent smoothing might occur. This is therefore the purpose of Part II of this thesis.

Like Part I, Part II is viewed primarily as a conceptual contribution, focusing on the nature of the appraisal process to develop insights regarding how much smoothing might be present, and the nature of this smoothing and how it might be corrected. Chapter 4 considers the appraisal process at the disaggregate or individual property level, while Chapter 5 considers how additional smoothing may be introduced at the aggregate (portfolio or index) level. Part II also contains a brief empirical analysis at the end of Chapter 5, in which a simple smoothing correction procedure is applied to estimate the systematic risk of some commonly cited unsecuritized commercial real estate indices, using both the traditional stock market based CAPM and the Consumption-based CAPM.

Chapter 4: Modelling Risk Smoothing In Disaggregate Level Appraisal Based Returns Series

The purpose of this Chapter is to develop some insight as to how smoothing may occur, and how much smoothing may be present, in disaggregate level (ie, individual property) appraisal based returns series. The primary concern here is smoothing that causes systematic risk (that is, covariance with the CAPM Index) to be underestimated in the appraisal based returns series.

The Chapter is organized into four Sections. Section 4.1 discusses a basic assumption which underlies both this Chapter and Chapter 5. Section 4.2 presents the appraisal return models. Section 4.3 presents some quantitative implications, and a simplified summary model. Finally, Section 4.4 describes a simple empirical technique for correcting estimates of the systematic risk in returns series to adjust for appraisal smoothing.

4.1 A Basic Assumption: White Noise True Returns...

In this Chapter and the next, it is assumed that true asset returns are "white noise", that is, uncorrelated over time. While this assumption is probably not perfectly true, it is widely employed as an acceptable approximation of the truth in studies of the financial securities markets. The rationale

for this assumption in the present context is discussed below, along with another simplification that will be used: that in studying risk in returns we ignore the income return component to focus entirely on the appreciation component.

Consider for simplicity a real estate asset that pays no dividends (eg, vacant land), in a world where the ex ante expected return to this asset is always the same, say, equal to a constant riskfree rate plus a risk premium plus an "illiquidity premium". The risk that matters to an investor is related to how the actual return to this asset may differ in any given period from this expected return. In this case (with no cash flows), the actual return is just the change in value of the asset between two points in time, divided by the value of the asset at the first point in time. To be more precise, the value that matters in this return from an economic point of view is the "opportunity cost" of the asset, which is given by its market price, or "transaction price", that is, the price at which it would be sold if an actual sales transaction were to take place.

By the time a transaction price is agreed upon by a buyer and a seller, it is reasonable to assume that both parties will have availed themselves of as much information as possible relevant to the current and future value of the asset. Thus, the market transaction price of the asset at time t

reflects all information available as of time t relevant to the value of the asset. Similarly, a transaction at time $t+1$ would occur at a price reflecting all relevant information available as of time $t+1$. Between time t and time $t+1$, only two things can therefore cause the asset value to change. One is the previously-expected deterministic trend in the asset value; and the other is the "arrival" of new information relevant to the value of the asset. The former is associated with the ex ante expected return, and is not risky. It is only the latter, the arrival of new information, which causes the return to be risky.

Thus, at least in this simple world, risk in the asset return is caused purely by the arrival of new information affecting asset value. Since the arrival and nature of new information is, by definition, unpredictable, the unexpected changes in asset value must be uncorrelated over time. Hence, the risky component of the asset return (that is, its deviation from its prior expected value) should be uncorrelated over time, that is, "white noise".

Note that this does not imply that the variables which affect asset value must be uncorrelated over time. For example, vacancy rates in the local rental market certainly are relevant to asset value, and may well be highly autocorrelated, and hence quite predictable. But autocorrelation in

such variables can and would be used by both the buyer and seller to forecast the future cash flows and capitalized value of the asset, and their current time- t valuation of the asset would reflect such forecasts as well as the ex ante expected return on the asset. In general, to the extent that information arrivals after time t are forecastable, these forecasts will be embedded in asset values as of time t , so that only the uncorrelated residuals from such forecasting (that is, the "news", or "innovations" in the information relevant to asset values) will cause unexpected changes in asset value.

This argument is not changed by the fact that the asset in question may pay dividends, except that the white noise argument now applies to the total return, not necessarily to the appreciation return component alone. However, in practice, almost all of the risk in the total return is found in the appreciation component, so in studying the nature of total return risk one does not lose much accuracy by considering only the appreciation component, and treating that component alone as a white noise process. Tables 4.1-3 illustrate these points (located at the end of the Chapter).

Table 4.1 shows the quarterly mean return and volatility for the total return as well as the appreciation and income components separately, for the S&P500 Index, the NAREIT Index of Equity REITs, the FRC Index and the PRISA Index. The first

two of these series are securitized, and hence represent true transaction price based returns. Indeed, as noted in Chapter 1, the real estate securities such as the Equity REITs represent the only true returns series that we have for real estate assets. The last two series are appraisal based indices of unsecuritized commercial real estate assets held in CREFs. The period covered is 1978-1987. We note that, while a large share of the mean return in the real estate assets is in the income component, almost all of the volatility is derived from the appreciation component alone. This holds both for the true returns and the appraisal based returns.

Tables 4.2 and 4.3 show statistics relevant to the white noise assumption for the same four portfolios. Table 4.2 applies to the total returns, while Table 4.3 applies to the appreciation component only. The Q statistic is the "Bartlett's Q" value for 10 lags, a widely used statistic for describing the amount of autocorrelation in a series and testing the null hypothesis of white noise. The Q statistic is distributed Chi-square with degrees of freedom equal to the number of lags. Thus, we reject the white noise null hypothesis with 90 percent confidence if Q exceeds 15.99, or with 95 percent confidence if Q exceeds 18.31. Neither of the two true returns series comes very close to being able to reject the white noise null hypothesis even at the 90 percent level. In contrast, the two appraisal based returns series easily

reject the white noise hypothesis at the 95 percent level. The same picture is presented by the second column in the Tables, which gives the highest single autocorrelation coefficient (and its lag), compared to the standard error of the autocorrelation coefficient (which is .158). Note that the picture is very similar whether we use the total return or just the appreciation component.

Although the white noise assumption seems to be a pretty good approximation in the only true returns series we have, it may be argued that "illiquidity" in unsecuritized real estate markets as compared to stock markets introduces the possibility of more autocorrelation in the true returns of such real estate assets. While the unexpected component of the return is, as described above, white noise, the expected return may not be constant as assumed in the above argument. Even in the case of securitized assets, which have no illiquidity premium, autocorrelated changes in the riskfree interest rate and in the asset's ex ante expected risk premium could cause the expected return to not be constant and possibly to display autocorrelation, which would then introduce autocorrelation into any empirical series of the true total return. If the illiquidity premium component of the expected return is significant, and if investors' concerns about illiquidity change significantly over time in a manner which is autocorrelated and not offset by simultaneous changes in the other components

of the expected return, then this could cause unsecuritized real estate assets' true returns to be more autocorrelated over time than securitized assets' returns.

As we cannot observe true returns series that could be used to test this proposition for unsecuritized assets, it is very difficult to know how important this argument is. In this circumstance, it would seem interesting to at least see where the white noise assumption leads us, particularly since analyzing smoothing in any sort of quantitative way would be extremely difficult if not impossible without this assumption. The insights and implications derived from combining the white noise assumption with a model of appraisal behavior can then be checked for intuitive or empirical plausibility, which may in turn shed some light on the reasonableness of the white noise assumption. It is in this spirit that the analysis in this Chapter and the next is offered. In the interest of analytical tractability, the analysis will also make use of the previously noted approximation that the risk in the appreciation component is practically the same as the risk in the total return.

4.2 A Model of Appraisal Based Returns

In this Section we will use the white noise and appreciation return assumptions described above together with charac-

terizations of the appraisal process to develop quantitative models of the relationship between appraisal based returns and the corresponding true economic returns, which will enable us to quantify the difference in risk displayed by the two series.

4.2.1 True Returns and Risk: Some Basic Definitions

Based on Section 4.1, we assume that the entire return consists only of the appreciation component, and that this is a white noise process. (This leaves out a large part of the return, which would be of concern if we were trying to study the expected return, but this model is designed only to study risk in the return.) Thus, the true property value follows a geometric random walk, so the log of the true property value follows an arithmetic random walk. (We ignore here any deterministic trend in asset value since this does not affect the risk smoothing issue, implying that our asset value random walk is driftless and that the expected appreciation return is zero.) To simplify notation, define: $v_t = \log(\underline{V}_t)$, where \underline{V}_t is the true property value at time t . Therefore, the true period t market based return for property i , that is, the return from time $t-1$ to time t , labelled $r(i)_t$, is given by:

$$r(i)_t = v(i)_t - v(i)_{t-1} = \sum_{k=1}^N z(i)_k \quad (1)$$

where the $z(i)_k$ are white noise increments occurring sequentially at N intermediate points in time between $t-1$ and t . (Think of $z(i)_1$ as being realized at $1/N$ th of the time between $t-1$ and t , $z(i)_N$ as being realized at time t exactly.)

We assume that the return risk is stationary, and define two risk measures of interest based on (1). The systematic return risk of property i is labelled $S(i)$:

$$S(i) = \text{cov}[r(i)_t, I_t] = N \text{cov}[z(i), I] \quad (1a)$$

and the total risk or volatility is labelled $\sqrt{v(i)}$:

$$\sqrt{v(i)} = \sqrt{\text{var}[r(i)_t]} = \sqrt{N} \sqrt{\text{var}[z(i)]} \quad (1b)$$

In the following two Sections, formal models of the appraisal process, based on simple characterizations of appraisal behavior, will be developed that will enable a quantitative relationship to be derived between the above-defined true returns and the empirically observable appraisal based returns. This relationship will then enable us to quantify the relationship between the above risk measures for the appraisal returns as compared to the true returns, in other words, to quantify, at a conceptual level at least, the amount of smoothing. As there are two major methods of appraisal used to estimate the value of commercial property, two separate models are developed. Section 4.2.2 considers the

"Income Method", while Section 4.2.3 considers the "Market Method".

4.2.2 The Income Method of Appraisal

Under the "Income Method" of appraisal, the appraiser forecasts future net cash flows of the property, and discounts these expected cash flows to present value using some discount rate and capitalization procedure (eg, this could be the "simple cap rate valuation" procedure discussed in Part I, or it could be a more detailed DCF analysis). Thus, the appraiser is trying to go through the same fundamental valuation calculation that both buyers and sellers in the market would do, and thereby to estimate the market price of the property.

In principle, the appraiser could be "exactly right" every time using this method. He could use the same (subjective) cash flow forecast and the same (subjective) discount rate and capitalization procedure as the "market" (ie, the successfully transacting buyer and seller) would use at each point t in time. But the crux of the smoothing problem with the Income Method of appraisal is that, even if the appraiser were right every time, he could not know for sure that he was right. Commercial properties are generally fairly unique (in size, age, design, type of use, lease structure, property manager,

tenant type, and location, location, location). It is therefore unlikely that a very similar property will have been bought at the same time as of which the appraiser is valuing the property. So there is generally no way for him to verify with certainty, using "hard" (that is, objective, market-based) evidence, how right or wrong his income-based valuation is. (Valuation by comparison to transactions prices of similar properties is the other major appraisal technique, the "Market Method", which will be considered in the next section.)

The appraiser therefore has an income-based estimate, about which he has some doubt or uncertainty, of the current value of the property. In this circumstance, it is natural for a sort of "tyranny of past appraisals" to take hold. The appraiser will probably have available to him the previous appraised value of the property, an appraisal which might have been done by himself or some other appraiser. In the case of CREF returns, the appraiser will know that the past appraisal was accepted by the same manager for whom he is now doing the current appraisal. It would therefore be natural for the appraiser to at least consider this past appraised value (probably adjusted for inflation), and to possibly modify his own current independent estimate somewhat in the direction of this past appraised value.

Indeed, although such an "averaging" of his current valuation and the last appraised value has been presented here as a kind of behavioral model, it also makes some formal statistical sense, to the extent that real property value does not change over time. In that case, if appraisal errors are less than perfectly positively correlated across time, then "two estimates are better than one", as the standard error of the appraisal is reduced. (Of course, the smoothing problem is introduced precisely because property value can change over time, apart from inflation and deterministic trends, and to ignore this possibility is to ignore the very risk which we are trying to study in this thesis.)

Considering the foregoing, the following model of Income Method based appraised value is proposed:

$$V_t^* = a(V_t + \delta_t) + (1-a)V_{t-1}^* \quad (2)$$

where V_t^* is the (log of the) appraised value as of time t ; V_t is the (log of the) true value; δ_t is the difference between the appraiser's initial current income-based (log) valuation of the property and its current true (log) value; and the parameter "a" is the appraiser's "confidence factor", the relative weight he puts on his initial current valuation as opposed to the previous appraised value. The confidence fac-

tor can range in principle from zero to one, but in practice it is not likely to lie at either extreme of this range. We assume that the error term, δ_t , has a zero expected value, is uncorrelated across time, and has zero contemporaneous covariance with the CAPM index. [Note that because we are working in log values, the zero-mean assumption for δ_t implies that the initial valuation is biased on average, but this bias in practice would be very slight.]

Since we are working in log values, we have returns related to values by simple subtraction:

$$\begin{aligned} r_t &= V_t - V_{t-1} \\ r_t^* &= V_t^* - V_{t-1}^* \end{aligned} \tag{3}$$

where r_t is the true return at time t , and r_t^* is the appraisal based return.

Substituting (2) into (3) and expanding, we obtain the relation between true and appraisal based returns:

$$\begin{aligned} r_t^* &= ar_t + a(1-a)r_{t-1} + a(1-a)^2r_{t-2} + \dots \\ &\quad + A[\{\delta_t\}] \end{aligned} \tag{4}$$

where:

$$\begin{aligned} A[\{\delta_t\}] &= a(\delta_t - \delta_{t-1}) + a(1-a)(\delta_{t-1} - \delta_{t-2}) \\ &\quad + a(1-a)^2(\delta_{t-2} - \delta_{t-3}) + \dots \end{aligned} \tag{4a}$$

Under the Income Method model, therefore, the appraisal return at time t equals an exponentially declining weighted infinite moving average of the true return, plus an appraisal error term which has zero expected value, and which would tend to diversify away in a large portfolio if appraisal errors are contemporaneously less than perfectly correlated across properties. Since the weights on the past true returns sum to unity, the appraisal return is unbiased (thanks to our assumption that appraisal log valuation is unbiased). But risk is smoothed.

To quantify the smoothing based on the Income Method model, define $S^*(i)$ to be the appraisal based systematic risk of property i , and $\sqrt{v^*(i)}$ to be the appraisal based total risk. From (4) we see that:

$$\begin{aligned}
 S^*(i) &= \text{cov}[r^*(i)_t, I_t] \\
 &= \text{cov}\left[\sum_{j=0}^{\infty} a(1-a)^j r(i)_{t-j}, I_t\right] \\
 &= \text{cov}[ar(i)_t, I_t] = a \text{cov}[r(i)_t, I_t] \\
 &= aS(i)
 \end{aligned}
 \tag{4a}$$

where the third line above results because the CAPM index, $\{I_t\}$, is an unpredictable process. (According to CAPM theory, it is only the unexpected changes, or deviations from conditional expected values, of the market portfolio or the aggre-

gate consumption, which define the relevant risk in the CAPM index. Therefore, $\{I_t\}$ is a white noise process and by its definition: $\text{cov}[r_{t-j}, I_t] = 0$ for all $j > 0$.)

Similarly, the total risk is given by:

$$\begin{aligned}
 \sqrt{v^*(i)} &= \sqrt{\text{var}[r^*(i)_t]} \\
 &= \sqrt{ \{ [a^2 + a^2(1-a)^2 + a^2(1-a)^4 + \dots] \text{var}[r(i)_t] \\
 &\quad + [a^2 + a^4 + a^2[(1-a)-(1-a)^2]^2 + \dots] \text{var}[\delta(i)_t] \} } \\
 &= \sqrt{ \frac{a \text{var}[r(i)_t] + 2a^2 \text{var}[\delta(i)_t]}{2 - a^2} } \qquad (4b)
 \end{aligned}$$

The smoothing factor for the systematic risk, defined as the ratio of the true over the appraisal based systematic risk, is seen to be the inverse of the confidence factor:

$$S(i)/S^*(i) = 1/a$$

Since the confidence factor is a fraction of one, systematic risk in appraisal returns is less than the true systematic risk, the more so as the appraiser's "confidence" decreases or as the "tyranny of past appraisals" increases. Note, however, that the sign of the appraisal based systematic risk is the same as the true risk, since "a" is always positive. (This is at the theoretical level. Since smoothing reduces the absolute difference between the systematic risk and zero, it becomes more likely that statistical estimation error could

cause an empirical estimate of systematic risk to have the wrong sign.)

The relationship between appraised and true total risk is more complicated, as the appraisal errors, the δ_t 's, come into play. The smoothing factor for total risk at the disaggregate level under the Income Method is:

$$\sqrt{v(i)}/\sqrt{v^*(i)} = 1/\sqrt{\{ a/(2-a^2) + [2a^2/(2-a^2)][\text{var}[\delta]/\text{var}[r]]\}}$$

If it were not for the $\text{var}[\delta]$ term, this ratio would clearly exceed one for $0 < a < 1$. But at the disaggregate level, it is at least conceptually possible for appraisal error to be large enough to cause appraisal based returns to display more volatility than the true returns, if appraisal errors are large enough and true returns do not have much volatility.

However, it is important to note that the $A[\{\delta_t\}]$ term in (4) will tend to diversify away in aggregate level (portfolio) returns, so that the $\text{var}[\delta]$ term would disappear in the smoothing factor at the aggregate level, making appraisal based volatility clearly less than true volatility in aggregate returns.

4.2.3 The Market Method of Appraisal

Let us now consider the smoothing implications of the other principle method used in commercial property appraisal, the "Market Method", in which the current value of the subject property is estimated by reference to actual transactions prices of other similar properties recently sold in the same or similar markets. Formalizing this method of appraisal involves somewhat more messy algebra, and as we shall see, does not give the simple moving average relation between true and appraisal based returns found above. However, the risk smoothing implications are similar.

We can model the Market Method appraisal process in the following way. The appraiser is assumed to estimate V_t at time t by looking at other similar properties which sold during the period between $t-1$ and t . (In fact, he might reach back beyond $t-1$ or he might use sales only from the last part of period t , but we will discuss the implications of this later.) Suppose one such property was sold at each of the intermediate points $1, 2, \dots, n, \dots, N$. Then the appraiser has a sample of N sales of similar properties. Suppose the (log of the) market value (transaction price) of each of the similar properties sold was equal to that of our subject property except for a random unobservable error amount,

$$e_1, e_2, \dots, e_n, \dots, e_N$$

where these errors have a zero mean and also are uncorrelated, both over time with themselves and contemporaneously with the CAPM index.

The log of the price of the first similar property sold is thus: $V_{t-1} + z_1 + e_1$. Where V_{t-1} is the (log of the) true market value of our subject property as of time $t-1$. The log of the price of the second similar property sold is: $V_{t-1} + z_1 + z_2 + e_2$. And so on, until for the N th similar property the log price is: $V_{t-1} + e_N + \sum z_k$, where the summation runs from $k=1$ to $k=N$.

In principle, the appraiser could take that last sale, the one at time t exactly, and appraise the value of our property to be equal to that sale price. But he would be ignoring all the information contained in the other $N-1$ data points in his reference sample. His error would be e_N which, though it has an expected value of zero, has a variance N times greater than the variance his appraised value will have (in logs) if he uses the entire sample of N data points. The conscientious appraiser should use the whole sample to appraise the (log of) the value of our property as the arithmetic mean of the observed log prices, each of which is observed to be

$$V_{t-1} + e_n + z_1 + z_2 + \dots + z_n.$$

Thus, as of time t , the conscientious appraiser's estimate of the log of the value of our property is:

$$\begin{aligned} V_t^* &= \frac{1}{N} \sum_{n=1}^N \left(V_{t-1} + e_n + \sum_{k=1}^n z_k \right) \\ &= V_{t-1} + \sum_{n=1}^N \sum_{k=1}^n z_k / N + \sum_{n=1}^N e_n / N \\ &= V_{t-2} + \sum_{k=1}^N z_{k,t-1} + \sum_{n=1}^N \sum_{k=1}^n z_{k,t} / N + \sum_{n=1}^N e_n / N \end{aligned}$$

where the $z_{k,t}$ represent the true increments during period t (that is, from point $t-1$ to t in time) and $z_{k,t-1}$ are the true increments from the previous period between $t-2$ and $t-1$.

The appraisal based return in period t , r_t^* , is just the above value minus the corresponding value from the previous period. The result is that r_t^* is a weighted sum of the z increments from $t-2$ to t , plus the difference between the similarity errors in the two periods:

$$r_t^* = \left\{ \left[\sum_{k=1}^N (k-1) z_{k,t-1} \right] + \left[\sum_{k=1}^N (N-k+1) z_{k,t} \right] \right\} / N + (\delta_t - \delta_{t-1})$$

where δ_t is defined by:

$$\delta_t = \left(\sum_{n=1}^N e_{n,t} \right) / N$$

The same steps of reasoning can be used to obtain a more general representation of r_t^* under the Market Method which allows the N similar property sales used by the appraiser in his reference sample to occur uniformly throughout the last T periods (instead of only during the last single period), or indeed, to occur during the last T fraction of the current period. [Note that T here in Part II has nothing to do with the T which stood for lease term in Part I.] This gives the following formula for appraisal based returns under the Market Method:

$$r_t^* = \left(\sum_{L=0}^M \sum_{k=1}^{N/T} a_{k,t-L} z_{k,t-L} \right) / N + (\delta_t - \delta_{t-T}), \text{ for all } T, \quad (5)$$

where: the z_k are the intermediate true return increments within periods; and the $a_{k,t-L}$ weights are defined as follows:

For $T \leq 1$, $M=1$ and:

$$\begin{aligned} a_{k,t-L} &= N/T - k + 1, \text{ for } L=0 \text{ and } 1 + (1-T)N/T \leq k \leq N/T \\ &= N, \text{ for } L=0 \text{ and } 1 \leq k \leq (1-T)N/T \\ &= k - [1 + (1-T)N/T], \text{ for } L=1 \text{ and } 1 + (1-T)N/T \leq k \leq N/T \\ &= 0, \text{ otherwise} \end{aligned}$$

For $T > 1$, $M=T$ and:

$$\begin{aligned} a_{k,t-L} &= (N - k + 1) / T, \text{ for } L=0 \text{ and } 1 \leq k \leq N \\ &= N/T, \text{ for } 1 \leq L \leq T-1 \text{ and } 1 \leq k \leq N \\ &= (k-1) / T, \text{ for } L=T \text{ and } 1 \leq k \leq N \\ &= 0, \text{ otherwise} \end{aligned}$$

Note that the above definitions of the $a_{k,t-l}$ weights implies that each of these weights is non-negative, and that all of these weights summed over all k and all L equal N^2 in the case of $T \geq 1$, or $(N/T)^2$ in the case of $T \leq 1$.

Unlike the case with the Income Method, under the Market Method it is not possible to express the appraisal based return series $\{r_t^*\}$ defined by (5) as a deterministic function of the true return series $\{r_t\}$. For example, r_t^* is not a moving average of current and past values in $\{r_t\}$ as it was under the Income Method. However, deterministic relationships do exist between the risk characteristics of r_t^* and r_t . Taking the appropriate moments of (5) and (1), we find:

$$S^*(1) = K_1 S(1) \tag{5a}$$

$$\sqrt{v^*(1)} = \sqrt{\{K_2 + 2(\text{var}[\delta]/\text{var}[r])\}} \sqrt{v(1)} \tag{5b}$$

where the factors K_1 and K_2 are deterministic functions of T and N as follows:

Letting:

$$S_1 = 1 + 2 + \dots + N-1$$

$$S_2 = 1^2 + 2^2 + \dots + (N-1)^2$$

we have...

For $T < 1 \dots$

$$K_1 = 1 + [S_1/N^2 - (N-1)/N]T \quad \text{---> } 1-T/2 \text{ as } N \text{---> } \infty$$

$$K_2 = 1 - [(N-1)/N]T + 2TS_2/N^3 \quad \text{---> } 1-T/3 \text{ as } N \text{---> } \infty$$

For $T > 1 \dots$

$$K_1 = (N+S_1)/(TN^2) \quad \text{---> } 1/(2T) \quad \text{as } N \text{---> } \infty$$

$$K_2 = (1/T^2)[2S_2/(N^3) + 1/N + T - 1] \\ \text{---> } 2/(3T^2) + 1/T - 1/T^2 \quad \text{as } N \text{---> } \infty$$

The factors K_1 and K_2 are always positive and less than unity. As N approaches infinity, these factors approach simple limits. For example, if $T=1$, K_1 approaches $(1/2)$ and K_2 approaches $(2/3)$. These limits are approached rapidly even for small N .

As with the Income Method, in the case of the systematic risk, the fact that K_1 is less than one implies that there will be smoothing in the systematic risk, and the fact that K_1 is always positive means that the sign of the appraisal based systematic risk will be the same as that of the true risk, at least at the theoretical level.

Also like the Income Method model, the smoothing factor at the disaggregate level here is more complicated for the total risk, as the "similarity errors" variance term, $\text{var}[\delta]$, enters

the formula for $\sqrt{v^*}(i)$. Since this term is positive, it is possible for appraisal based total risk at the disaggregate level to exceed true total risk if $\text{var}[\delta]$ is large enough and $\text{var}[r]$ is small enough.

As an example of this Market Method model of appraisal return risk smoothing, suppose the appraiser uses reference sales covering the last half of the most recent period, so $T=1/2$. The true systematic risk will be approximately $(4/3)$ times the appraisal based systematic risk. The true variance will be $1/[(5/6)+2(\text{var}[\delta]/\text{var}[r])]$ times the appraisal based variance. If the appraiser uses reference sales from the previous two periods, then the systematic risk smoothing factor $1/K_1$ will approach 4 in the limit, while the total risk smoothing factor will approach $1/[(5/12) + 2(\text{var}[\delta]/\text{var}[r])]$. According to the assumptions in this model, the $\text{var}[\delta]$ terms should become unimportant when studying the appraisal based returns of a large diversified portfolio, as the similarity errors should diversify away across properties.

In applying the Market Method model described here to develop our intuition about the nature and magnitude of appraisal smoothing, the definitions of the parameters N and T need not be taken too literally. Appraisal is ultimately a subjective process, an "art" more than a "science". An expli-

cit reference sample may not literally be used by the appraiser, but his valuation will be strongly influenced by what he knows about recent actual transactions of similar properties in the market. The parameters N and T should be interpreted as the "effective" sample size and the "effective" reference period used by the appraiser in this subjective process.

It should also be noted that the Market Method model presented above assumes that the appraiser suffers no "lack of confidence" in his Market Method valuation, so that there is no averaging of his current valuation with a past appraised value, as there was in the Income Method model. This may be a plausible assumption when the Market Method is applied literally, since this approach is more objective than the Income Method, being based on "hard" transactions price data, which themselves cover a period of past time. However, to the extent that the appraiser still lacks perfect confidence in his Market Method appraisal, there may still be a "tyranny of past appraisals" effect, leading to additional smoothing beyond what is modelled here. In such circumstances, the exponential moving average relationship which resulted from the lack of confidence modelled in Section 4.2.2 could compound the smoothing inherent already in the Market Method modelled here in Section 4.2.3.

4.3 Summary of the Nature of Disaggregate Level Smoothing

The models presented in the previous Section are "pure types", in the sense that the Income Method model of Section 4.2.2 assumes that the appraiser uses only the Income Method, and the Market Method model of Section 4.2.3 assumes that the appraiser uses only the Market Method. In reality, most appraisals use both. The Income Method analysis is used to inform the Market Method analysis, and vice versa. One result of this integration of the two types of valuation analyses is that the appraiser's "confidence factor" [parameter "a" in (4)] can be larger than it otherwise would be, and the period of time covered by the reference sample of similar sales [parameter "T" in (5)] can be smaller than it otherwise would be (and also, the appraisal errors, measured by $\text{var}[\delta]$, may be smaller than they otherwise would be). Another result of this is likely to be that the form of the mathematical relationship between r_t^* and r_t is likely not to be exactly either the exponential moving average relationship of (4) or the more complicated intermediate return increments relationship of (5), but rather something "in between", some weighted average of the two relationships.

This does not pose as great a problem in developing our intuition about the magnitude and nature of appraisal risk

smoothing as might first appear, however, because the two models of appraisal returns are fairly similar in their implications in this regard. In this Section we will explore those implications numerically, and then suggest a simple "summary model" which will capture most of the essence of both methods of appraisal as far as disaggregate level risk smoothing is concerned.

4.3.1 Numerical Analysis

The similarity of the risk smoothing implications of the two main appraisal methods is seen in Tables 4.4 and 4.5, which show numerical values for the smoothing factors implied by the Income Method and the Market Method respectively, over a plausible range of values for the appraisers' behavioral parameters ("a" in the case of the Income Method, or T and N in the case of the Market Method).

Note that in the Tables, the total risk smoothing factor, $\sqrt{v(i)}/\sqrt{v^*(i)}$, is shown assuming the $\text{var}[\delta]$ appraisal error term is zero, which means that the figures represent upper bounds for the amount of total risk smoothing at the disaggregate level. The $\sqrt{v(i)}/\sqrt{v^*(i)}$ figures in the Tables are relevant, however, for considering the effect of disaggregate level smoothing at the aggregate level in

portfolio returns, since most of the appraisal error will be diversified away in aggregate returns. (Possible additional smoothing introduced at the aggregate level will be considered in the next Chapter.)

The range of smoothing factor values is similar between Tables 4.4 and 4.5. The main difference between the Market Method and Income Method appears to be that the Market Method introduces a bit more smoothing into the systematic risk, particularly at the larger values of T.

The numbers in the Tables are also interesting for the insight they provide on the nature and magnitude of appraisal return risk smoothing at the disaggregate level. We note first of all that the systematic risk appears to be more smoothed than the total risk. This is because of the lack of non-contemporaneous cross-correlation with the CAPM index. The result is that in the systematic risk only the current period's true return component in the appraisal return is revealed in the contemporaneous covariance between the appraisal return and the CAPM index. The total risk, on the other hand, is less smoothed, even ignoring the potential "unsmoothing" effect of appraisal error at the disaggregate level, because all of the lagged components of the true return do enter the unconditional appraisal risk measurement, only each component is attenuated by the smoothing.

Another interesting feature of the smoothing factors shown in the Tables is their sensitivity to the appraisal behavioral parameters. The systematic risk smoothing is more sensitive to these parameters than is the total risk smoothing. The sensitivity of the smoothing factors to the confidence factor (a) and the period covered by the reference sample (T) provides some interesting insight. Most appraisals in CREFs are made either quarterly or annually. Other things being equal, one would expect that more frequent appraisals (eg, quarterly) would be associated with greater "tyranny of past appraisals", since the past appraisals are so recent. Thus, more frequent appraisals should be associated with lower "a" factors and greater smoothing. The same implication occurs under the Market Method. More frequent appraisals mean shorter inter-appraisal periods, which means (cet. par.) that the time period covered by the reference sample of similar sales will cover a larger fraction of the inter-appraisal period or more inter-appraisal periods prior to the current period (hence, larger T , which implies more smoothing). It seems plausible that values of T in excess of 1, and values of "a" below 0.5, would most likely be associated with quarterly appraisals, whereas lower values of T and higher values of "a" would be associated with annual appraisals.

4.3.2 A Simple Summary Model of Disaggregate Level Smoothing

If we think of the actual appraisal process as a kind of mixture of the Income Method modelled by (4) and the Market Method modelled by (5), then we can capture most of the essence of this process, as far as the risk smoothing issue is concerned, in a simplified summary model, expressing the appraisal return as a more generalized weighted moving average of the true return:

$$r_t^* = w_0 r_t + w_1 r_{t-1} + w_2 r_{t-2} + \dots, \text{ where } \sum_{k=0}^{\infty} w_k = 1 \quad (6)$$

In this model we have dropped the $\text{var}[\delta]$ appraisal error terms, because our interest in using this simplified model is primarily for representing the contribution of disaggregate level smoothing in aggregate portfolio returns.

In the case of the pure Income Method, the w_j weights in model (6) are given by: $w_j = a(1-a)^j$, which makes (4) a special case of (6), except for the $\text{var}[\delta]$ term. In the case of the pure Market Method model (5), there exist no values of the w_j weights in (6) that will cause (6) to be exactly equivalent to (5). However, the risk smoothing factors implied by (5) when the $\text{var}[\delta]$ term is ignored, namely,

$S(i)/S^*(i)=1/K_1$, and $\sqrt{v(i)}/\sqrt{v^*(i)}=1/\sqrt{K_2}$, are obtained by setting the w_j weights in (6) as follows:

For $T \leq 1$:

$$w_0 = 1 - T/2, \quad w_1 = T/2, \quad w_j = 0 \text{ all } j > 1$$

For $T > 1$:

$$\begin{aligned} w_j &= 1/(2T), \quad j=0 \text{ and } j=T \\ w_j &= 1/T, \quad j=2,3,\dots,T-1 \\ w_j &= 0, \quad j > T \end{aligned}$$

Thus, as far as the risk smoothing we are interested in studying here is concerned, the Market Method can be well represented by the general moving average model (6), which has the advantage of being notationally much simpler than (5).

If actual appraisals use a mixture of the Market Method and the Income Method, so that the actual relationship between true and appraisal based returns is a kind of average between eqns.(4) and (5), then, as far as risk smoothing analysis is concerned, this average can be represented by (6), with the w_j weights defined as values between those implied by the pure Income Method and those implied by the Market Method.

The simple summary model (6) implies the following smoothing factors:

$$\begin{aligned} S(i)/S^*(i) &= 1/w_0 \\ \sqrt{v(i)}/\sqrt{v^*(i)} &= 1/\sqrt{(w_0^2+w_1^2+w_2^2+\dots)} \end{aligned}$$

Both of these factors will always be greater than one, implying smoothing, but smoothing in total risk will be less than

smoothing in systematic risk, and the sign of the theoretical systematic risk will be correct.

4.4 A Simple Empirical Based Correction Method

The above conceptual analysis gives some interesting insight into the nature and magnitude of appraisal based smoothing at the disaggregate level, and its relationship to appraisal behavior. However, the range in values of the smoothing factor implied by the range of intuitively plausible behavioral parameters is fairly broad, particularly in the case of the systematic risk. Fortunately, the relationships between true and appraisal based returns implied by both the Income Method model (4) and the Market Method model (5) suggest a simple method to empirically adjust the estimation of systematic risk using appraisal based returns data, to correct the smoothing. [Once a smoothing factor for the systematic risk is estimated, one can derive an approximate estimate also for the total risk smoothing factor (apart from the appraisal error effect) using the numerical relationship between the two factors suggested in Tables 4.4 and 4.5.]

The correction technique involves estimating the normalized systematic risk ("beta", defined as $\beta(i) = S(i) / \text{var}[I]$) as the sum of the contemporaneous plus lagged coefficients in a

multivariate regression of r_t^* on $I_t, I_{t-1}, \dots, I_{t-j}$, etc. simultaneously, instead of just as the coefficient on the contemporaneous index in the bivariate regression.

We can see the way this correction procedure works as follows. (Here, the correction is demonstrated using the moving average relationship (6), but the procedure also works for the Market Method relationship (5), as shown in Appendix A to this Chapter.)

Normally, beta is estimated empirically using the simple contemporaneous regression (on appraisal based returns):

$$r_t^* = \alpha + \beta^* I_t \quad (7)$$

This gives the unadjusted apparent beta from the appraisal returns, β^* .

Now suppose we regress r^* on lagged values of the exogenous index:

$$r_t^* = \alpha + \beta_0^* I_t + \beta_1^* I_{t-1} + \beta_2^* I_{t-2} + \dots \quad (8)$$

Each β_l^* estimate in regression (8) is a partial regression coefficient, and as such it is the partial derivative of the LHS dependent variable with respect to a change in one RHS variable holding the others constant.

Since the $\{I_t\}$ series is white noise, the RHS variables I_t , I_{t-1} , I_{t-2} , ... in (8) are uncorrelated. Thus, the partial regression coefficients in (8) are the same as simple regression coefficients in a series of bivariate regressions of r_t^* on the lagged I variables:

$$r_t^* = \alpha_0 + \beta_0^* I_t \quad (8.0)$$

$$r_t^* = \alpha_1 + \beta_1^* I_{t-1} \quad (8.1)$$

$$r_t^* = \alpha_2 + \beta_2^* I_{t-2} \quad (8.2)$$

$$\begin{aligned} & \vdots \\ & \vdots \\ r_t^* &= \alpha_k + \beta_k^* I_{t-k} \quad (8.k) \end{aligned}$$

The definition of the simple regression coefficient is just (asymptotically) the covariance of the RHS variable with the RHS variable, divided by the variance of the RHS variable:

$$\beta_L^* = \text{cov}[r_t^*, I_{t-L}] / \text{var}[I_{t-L}] \quad (9)$$

Substituting from (6) for r_t^* in (9) implies:

$$\beta_L^* = \text{cov}[w_0 r_t + w_1 r_{t-1} + \dots + w_L r_{t-L} + \dots, I_{t-L}] / \text{var}[I_{t-L}] \quad (10)$$

But we know that by the white noise true returns assumption and the definition of the CAPM index: $\text{cov}[r_{t-j}, I_{t-L}] = 0$ for all $j < L$. Therefore, (10) implies:

$$\beta_L^* = w_L \text{cov}[r_{t-L}, I_{t-L}] / \text{var}[I_{t-L}] \quad (11)$$

and by stationarity (constant risk), (11) implies:

$$\beta_L^* = w_L \text{cov}[r, I] / \text{var}[I] = w_L \beta \quad (12)$$

And from (6), the w_L sum to unity over the $L=0, 1, \dots$ lags. So:

$$\sum_{L=0}^{\infty} \beta_L^* = \sum_{L=0}^{\infty} w_L \beta = \beta \sum_{L=0}^{\infty} w_L = \beta \quad (13)$$

which demonstrates the lagged regression correction procedure.

Of course, in practice one would cut off the summation after a reasonable number of lags. In determining this cut-off, the a priori intuition regarding the magnitude of the w_j weights gained from the behavioral models presented in Section 4.2 can be useful. It is clear, for example, since the w_j weights are all non-negative for both the Income Method and the Market Method models, that the summation over the lags which represents the corrected beta estimate should in theory be monotonically increasing (in its absolute value) as the number of lags in the regression increases. Thus, if the corrected beta starts falling in absolute value (or if its statistical significance starts decreasing) as one continues to add lags to the regression, this may be an indication that one has gone too far, particularly if the number of lags is already at or near the number where for reasonable values of the behavioral parameters ("a", T) one would expect the w_j weights to be very small.

In applying this correction procedure, it is important to consider that statistical estimation error can throw off the beta estimate, and the lagged regression correction procedure

tends to have more estimation error than the traditional bivariate regression, because more parameters must be estimated. If the true betas are small to begin with, then this can cause mis-estimates of sign as well as magnitude, especially in the uncorrected beta estimate. This makes it somewhat difficult to obtain a reliable estimate of the smoothing factor ratio $S(i)/S^*(i)$ by using this empirical correction technique. It therefore makes sense to combine the empirical evidence from this correction procedure with the a priori intuition obtained using the behavioral models and numerical analysis presented in Sections 4.2 and 4.3 above, in arriving at a conclusion regarding the magnitude of smoothing and the true value of the beta.

Finally, it should be noted that, although this lagged regression correction procedure has been demonstrated here at the disaggregate level, it will be argued in the next Chapter that the same procedure can also correct for the additional smoothing introduced at the aggregate level. The procedure will be applied at that level in the next Chapter to examine systematic risk in the FRC and PRISA Indices.

Table 4.1 Mean & Standard Deviation by Return Component: Four Portfolios

(Quarterly nominal returns 1978-87)

Portfolio	Total Return		Appreciation		Income	
	Mean	Std.Dev	Mean	Std.Dev	Mean	Std.Dev.
S&P500	.039	.085	.028	.086	.012	.002
NAREIT(Eq)	.044	.068	.023	.066	.021	.006
FRC	.031	.014	.012	.013	.019	.002
PRISA	.033	.020	.012	.018	.020	.003
T-bills	.023	.007	NA	NA	NA	NA

Table 4.2 Total Returns Autocorrelation Statistics: Four Portfolios

(Quarterly nominal returns 1978-87)

Portfolio	Bartlett's Q	Ratio
		max autocorr/std.err. (lag)
S&P500	7.93	1.53 (L=3)
NAREIT(Eq)	5.46	1.09 (L=3)
FRC	39.56	3.20 (L=1)
PRISA	59.42	4.22 (L=2)

Table 4.3 Appreciation Returns Autocorrelation Statistics: Four Portfolios

(Quarterly nominal returns 1978-87)

Portfolio	Bartlett's Q	Ratio
		max autocorr/std.err. (lag)
S&P500	7.76	1.50 (L=3)
NAREIT(Eq)	6.32	1.20 (L=3)
FRC	28.89	2.83 (L=1)
PRISA	44.57	3.85 (L=2)

Table 4.4: Income Method Values of the Smoothing Factors for plausible values of the confidence factor (a)...

	a =	.90	.75	.50	.25
1/a	=	1.11	1.33	2.00	4.00
$\sqrt{[(2-a^2)/a]}$	=	1.15	1.38	1.87	2.78

Definitions for Table 4.4:

a = Weight placed on current initial valuation = "Confidence Factor"

1/a = $S(1)/S^*(1)$ = Systematic Risk Smoothing Factor

$\sqrt{[(2-a^2)/a]}$ = $\sqrt{v(1)}/\sqrt{v^*(1)}$ = Total Risk Smoothing Factor (exclu var[δ] term)

Table 4.5: Market Method Values of the Smoothing Factors for plausible values of N and T...

T=		.25	.50	1.00	2.00	3.00	4.00
N=5:							
1/K ₁	=	1.11	1.25	1.67	3.33	5.00	6.67
$\sqrt{(1/K_2)}$	=	1.04	1.09	1.21	1.54	1.83	2.09
N=20:							
1/K ₁	=	1.14	1.31	1.90	3.80	5.71	7.63
$\sqrt{(1/K_2)}$	=	1.04	1.10	1.22	1.55	1.84	2.09
N=100:							
1/K ₁	=	1.14	1.33	1.98	3.95	5.95	7.94
$\sqrt{(1/K_2)}$	=	1.04	1.10	1.22	1.55	1.84	2.09

Definitions for Table 4.5:

T = Periods (or fraction of period) covered by the reference sample of similar property sales

N = Number of properties in the reference sample

1/K₁ = $S(1)/S^*(1)$ = Systematic Risk Smoothing Factor

$\sqrt{(1/K_2)}$ = $\sqrt{v(1)}/\sqrt{v^*(1)}$ = Total Risk Smoothing Factor (exclu var[δ] term)

**CHAPTER 4 APPENDIX A:
THE LAGGED REGRESSION CORRECTION TECHNIQUE APPLIED TO THE
MARKET METHOD MODEL...**

Substituting from The Market Method model (5) for r_t^* in (8) implies:

$$\beta_L^* = \text{cov}[(1/N) \sum_{k=0}^N \sum_{L=0}^T a_{k,t-L} z_{k,t-L}, I_{t-L}] / \text{var}[I_{t-L}] \quad (9a)$$

Then the white noise returns assumption and CAPM index definition imply:

$$\beta_L^* = \text{cov}[(1/N) \sum_{k=0}^N a_{k,t-L} z_{k,t-L}, I_{t-L}] / \text{var}[I_{t-L}] \quad (10a)$$

and stationarity implies:

$$\beta_L^* = (1/N) \sum_{k=0}^N a_{k,t-L} \text{cov}[z_k, I] / \text{var}[I] \quad (11a)$$

Recalling that by definition (1a) in section 4.2:

$$\text{cov}[r, I] = N \text{cov}[z_k, I],$$

and $\beta = \text{cov}[r, I] / \text{var}[I]$, so that (11) implies:

$$\begin{aligned} \beta_L^* &= (1/N) \sum_{k=0}^N a_{k,t-L} (1/N) \beta \\ &= (1/N^2) \sum_{k=0}^N a_{k,t-L} \beta \end{aligned} \quad (12a)$$

Now according to model (5) the $a_{k,t-L}$ weights sum over k and L to N^2 . (This is equivalent simply to assuming that the appraisal returns are unconditional unbiased). So, if we sum

all the β_L^* coefficients over all the L lags in regression (8),
we get:

$$\begin{aligned}
 \sum_{L=0}^T \beta_L^* &= (1/N^2) \sum_{L=0}^T \sum_{k=0}^N a_{k,t-L} \beta = \beta \\
 &= (1/N^2) N^2 \beta \\
 &= \beta \qquad \qquad \qquad (13)
 \end{aligned}$$

which demonstrates the correction procedure using the Market
Method model.

**Chapter 5:
Estimating Real Estate Systematic Risk From Aggregate Level
Appraisal Based Returns**

The purpose of this Chapter is to extend the behavioral based analysis of systematic risk smoothing in appraisal returns data to the level of aggregate (ie, portfolio, or index) returns. We shall see that it is quite possible that considerable additional smoothing is introduced at the aggregate portfolio level, compounding the disaggregate level smoothing discussed in the preceding Chapter. However, this additional layer of smoothing also can be corrected, in theory, using the lagged regression correction technique presented in Chapter 4. The last part of this Chapter therefore presents an empirical examination of the systematic risk observable in appraisal based unsecuritized commercial real estate returns, using the FRC and PRISA Indices. In addition to shedding some light on the smoothing issue, this analysis presents an interesting comparison of the systematic risk observed in such real estate indices as defined by the Consumption based CAPM as opposed to the traditional stock market based CAPM. (Recall that this was an issue which appeared possibly significant our analysis in Part I of this thesis in gaining an understanding of the nature of real estate return risk and risk premia.)

5.1 Aggregate Level Smoothing of Systematic Risk

In this Section we shall consider two sources of additional smoothing which may be introduced at the aggregate level. Section 5.1.1 considers the "appraisal timing effect" assuming consistent appraisals. Section 5.1.2 considers the effect of a certain type of inconsistency in appraisals over time, which may be common in CREF data, which we shall refer to as the "inside appraisal effect". Section 5.1.3 then summarizes the analysis.

[It should be noted that although the analysis in Chapter 4 considered smoothing in total risk as well as in systematic risk, in this Chapter we narrow our focus to systematic risk alone. Consideration of total risk smoothing at the aggregate level requires analysis of appraisal effects on intra-portfolio covariances and appraisal bias in the apparent "heterogeneity" or diversification effects of the portfolio. Such analysis is beyond the scope of the present thesis.]

Before we begin to consider aggregate level smoothing effects, we must start by positing a model relating appraisal returns to true returns at the disaggregate level. The model used in this Chapter is the "simple summary model" suggested at the end of the previous Chapter [eqn.(6) in that Chapter].

According to this model, appraisal based returns are a moving average of true returns:

$$r^*(i)_t = w_0 r(i)_t + w_1 r(i)_{t-1} + w_2 r(i)_{t-2} + \dots + w_T r(i)_{t-T} \quad (1)$$

where: $\sum_{k=0}^T w_k = 1$, and $0 \leq w_k \leq 1$ for all k ,

and $r^*(i)_t$ is the appraisal based return to property i during period t , and $r(i)_t$ is the true return to i during period t .

This model was extensively discussed in the previous Chapter, where some intuition was developed regarding the appraisal behavioral determinants of the w_k weights and the plausible range of likely values for these weights. The model is used here because of its notational simplicity and because it captures well the essence of the return risk smoothing issue. It will be convenient to recall the systematic risk smoothing relationship implied by this model:

$$S(i)/S^*(i) = \text{cov}[r(i), I] / \text{cov}[r^*(i), I] = 1/w(i)_0 \quad (1a)$$

5.1.1 Aggregate Returns with Consistent Appraisals

For simplicity, in this section we shall assume both longitudinal and cross-sectional consistency in appraisals. Longitudinal consistency means that the w_k weights remain the same through time, and cross-sectional consistency means that each

property in the portfolio is subject to the same w_k weights [$w(i)_k = w_k$, all i]. With regard to systematic risk, relaxation of these consistency assumptions will generally only have the effect of replacing w_k with its expectation, $E[w_k]$, taken over time and over the properties constituting the portfolio. [In section 5.1.2 we will relax this consistency assumption in a special way.]

The true return to the portfolio is the value-weighted average of the true returns to each property in the portfolio. And the true systematic risk of the portfolio is the value-weighted average systematic risk of the individual properties. If the same could be said about appraisal based portfolio returns, then the smoothing factor (1a) would hold at the aggregate level of portfolio returns just as it does at the disaggregate level of individual asset returns. But whether and to what extent this averaging property applies to appraisal based portfolio returns depends on the times during the current period at which the individual properties are appraised and their returns aggregated into the portfolio.

Suppose first that all properties in the portfolio are appraised as of the same date each period. For example, all properties are appraised as of the last day of each quarter. Then, the averaging property will apply, and the appraisal

based portfolio returns will simply be the weighted average of the appraisal based returns for each individual property. The portfolio's appraisal based return will be well modelled by eqn.(1) at the aggregate level:

$$r_t^* = w_0 r_t + w_1 r_{t-1} + \dots$$

where the r_t are the true portfolio returns; and the smoothing factor will be given by the disaggregate ratio (1a): $1/w_0$.

Now suppose that the properties in the portfolio are appraised as of different times within the current period. In other words, the appraised value reported as the value of a given property for the current period may in fact be the appraised value of that property as of some point in time during, but not necessarily at the end of, the current period. In this case the systematic risk apparent in the reported portfolio returns will equal $(1 - F/2)$ times w_0 times the true systematic risk of the portfolio, where F is the fraction of the period during which properties are appraised. This is seen in the following manner.

Consider a portfolio consisting of three properties: $j=a,b,c$. Let $r(1-12)_j$ represent the annual true return to property j during year t , that is, the return over the period from one to 12 months ago from the perspective of the end of year t , also labelled $r(j)_t$. Similarly, $r(13-24)_j$ represents

the annual true return to property j during year $t-1$, that is, the return over the period from 13 to 24 months ago, also labelled $r(j)_{t-1}$. The monthly return to property j during the s th month prior to the end of year t is represented as $r(s)_j$.

For simplicity (with very little loss of substantive accuracy), assume now that annual returns are just the sum of the monthly returns:

$$\begin{aligned} r(j)_t &= r(1-12)_j = r(1)_j + r(2)_j + \dots + r(12)_j \\ r(j)_{t-1} &= r(13-24)_j = r(13)_j + r(14)_j + \dots + r(24)_j \end{aligned} \quad (2)$$

and so on...

The value weights of the properties in the portfolio are given by c_j , where the c_j sum to unity and, for tractability we assume that the c_j are constant through time. The true portfolio return in year t is given by:

$$\begin{aligned} r_t &= c_a r(a)_t + c_b r(b)_t + c_c r(c)_t \\ &= c_a r(1-12)_a + c_b r(1-12)_b + c_c r(1-12)_c \\ &= c_a [r(1)_a + \dots + r(12)_a] \\ &\quad + c_b [r(1)_b + \dots + r(12)_b] \\ &\quad + c_c [r(1)_c + \dots + r(12)_c] \\ &= r(1) + \dots + r(12) \end{aligned} \quad (3)$$

where $r(s)$ is the true monthly portfolio return in the s th month counting back before the end of year t .

Annual appraisal returns for each property follow the disaggregate model (1):

$$r^*(j)_t = w_0 r(j)_t + w_1 r(j)_{t-1} + \dots, \text{ all } j$$

where the period index, t , is defined with respect to the time when the property is appraised which may or may not be the same as the end of the calendar period referred to in the first line of (3).

Now suppose each property is appraised once every 12 months, but at different times during the year. Property (a) was just appraised as of the end of calendar year t , property (b) was appraised as of the end of the month before, and property (c) was appraised as of the end of the month before that. Thus, redefining $r^*(j)_t$ as property j 's reported appraisal based return for calendar period t , that is, with the period index t defined with respect to a common period the same for the entire portfolio, and expanding (1) in terms of the monthly returns, we have:

$$\begin{aligned} r^*(a)_t &= w_0 r(1-12)_a + w_1 r(13-24)_a + \dots \\ r^*(b)_t &= w_0 r(2-13)_b + w_1 r(14-25)_b + \dots \\ r^*(c)_t &= w_0 r(3-14)_c + w_1 r(15-26)_c + \dots \end{aligned} \quad (4)$$

In other words, the appraisal return on property a covers the true calendar year from 1 to 12 months ago; the appraisal return on property b covers the year ending one month before

the true calendar year, that is, from 2 to 13 months ago; and so on.

Let the reported appraisal based portfolio return for year t be given by r_t^* , which (assuming the appraisal value weights are true) equals:

$$r_t^* = c_a r^*(a)_t + c_b r^*(b)_t + c_c r^*(c)_t \quad (5)$$

Expanding (5) in terms of monthly true returns, we have:

$$\begin{aligned} r_t^* = & w_0 [c_a r(1-12)_a + c_b r(2-13)_b + c_c r(3-14)_c] \\ & + w_1 [c_a r(13-24)_a + c_b r(14-25)_b + c_c r(15-26)_c] \\ & + \dots \end{aligned} \quad (6)$$

Further expanding (6) in components of actual yearly returns to calendar year t , to make (6) more directly comparable to (3), we see:

$$\begin{aligned} r_t^* = & w_0 [c_a r(1-12)_a + c_b r(2-12)_b + c_c r(3-12)_c] \\ & + w_0 [c_b r(13)_b + c_c r(13-14)_c] \\ & + w_1 [c_a r(13-24)_a + c_b r(14-25)_b + c_c r(15-26)_c] \\ & + \dots \end{aligned} \quad (7)$$

We are interested in comparing $S = \text{cov}[r, I] = \text{cov}[r_t, I_t]$ with $S^* = \text{cov}[r^*, I] = \text{cov}[r_t^*, I_t]$, where I_t is an unpredictable white noise annual exogenous index. Thus, I_t also can be expressed as a sum of monthly white noise increments:

$$I_t = I(1-12) = I(1) + I(2) + \dots + I(12) \quad (8)$$

Considering the white noise characteristic of the true returns and the CAPM index, we see that this covariance at the individual property level is given by:

$$\begin{aligned}
 \text{cov}[r(j)_t, I_t] &= \text{cov}[r(1-12)_j, I(1-12)] \\
 &= \text{cov}[r(1)_j + \dots + r(12)_j, I(1) + \dots + I(12)] \\
 &= \text{cov}[r(1)_j, I(1)] + \dots + \text{cov}[r(12)_j, I(12)] \\
 &= 12\text{cov}[r(s)_j, I(s)] \tag{9}
 \end{aligned}$$

where $\text{cov}[r(s), I(s)]$ is the monthly covariance, or monthly systematic risk, in property j 's true return.

The third equality in (9) comes from the fact that true returns and the CAPM index have zero non-contemporaneous covariance [by the assumption described at the beginning of Chapter 4]. The fourth equality in (9) comes from the stationarity assumption of constant risk.

In the same manner, $\text{cov}[r(3-12)_j, I_t]$ for example would equal:

$$\begin{aligned}
 \text{cov}[r(3-12)_j, I(1-12)] &= \text{cov}[r(3)_j + \dots + r(12)_j, I(1) + \dots + I(12)] \\
 &= \text{cov}[r(3)_j + \dots + r(12)_j, I(3) + \dots + I(12)] \\
 &= 10\text{cov}[r(s)_j, I(s)] \\
 &= (10/12)\text{cov}[r(j)_t, I_t] \\
 &= (10/12)\text{cov}[r(j), I] \tag{10}
 \end{aligned}$$

Thus, while the true portfolio systematic risk is the weighted average of the systematic risks of the individual properties:

$$\text{cov}[r, I] = c_a \text{cov}[r(a), I] + c_b \text{cov}[r(b), I] + c_c \text{cov}[r(c), I] \tag{11}$$

we see from the year t component of r_t^* shown in the first line of the RHS of (7) that the reported appraisal based systematic risk of the portfolio is:

$$\begin{aligned}
 \text{cov}[r^*, I] &= \text{cov}[r_t^*, I_t] \\
 &= w_0 \text{cov}[c_a r(1-12)_a + c_b r(2-12)_b + c_c r(3-12)_c, I(1-12)] \\
 &= c_a w_0 (12/12) \text{cov}[r(a), I] \\
 &\quad + c_b w_0 (11/12) \text{cov}[r(b), I] \\
 &\quad + c_c w_0 (10/12) \text{cov}[r(c), I] \tag{12}
 \end{aligned}$$

The same type of reasoning reveals by induction that if properties in a portfolio are appraised as of different times during the last F fraction of the period, with uniform value fractions of the portfolio appraised at each point in time within this "appraising window", then

$$\text{cov}[r^*, I] = w_0 (1-F/2) \text{cov}[r, I] \tag{13}$$

and therefore that:

$$(S/S^*) = 1/[w_0(1-F/2)]$$

where S and S^* are, respectively, the true and appraisal based systematic risk of the portfolio. The aggregate level smoothing factor, (S/S^*) , is greater than the disaggregate level smoothing factor by the ratio $1/(1-F/2)$.

Notice from equation (6) or (7) above that the appraisal based portfolio return r_t^* is not a weighted average of the

present and past true portfolio returns, such as is the case in the disaggregate model (1). In other words, the analog at the portfolio level of the moving average model (1) does not hold (unless all the properties' returns are perfectly correlated, or all properties are appraised as of the same time such that $F=0$). Thus, corrective procedures based purely on the amount of autocorrelation in the $\{r_t^*\}$ series under the assumption that equation (1) holds at the aggregate level (such as the method employed by Ross & Zisler), may underestimate the amount of smoothing, due to the reduction in autocorrelation caused by lack of perfect correlation across the cohorts of properties that are appraised at different times.

Observe from the second line on the RHS of (7), however, that the covariance of r_t^* with the CAPM index lagged one period, I_{t-1} , picks up the fraction of true covariance missed in the contemporaneous covariance in (12):

$$\begin{aligned}
 \text{cov}[r_t^*, I_{t-1}] &= \text{cov}[w_0[c_b r(13)_b + c_c r(13-14)_c], I(13-24)] \\
 &= w_0[c_b \text{cov}[r(13)_b, I(13)] + c_c \text{cov}[r(13-14)_c, I(13-14)]] \\
 &= c_b w_0 (1/12) \text{cov}[r(b), I] \\
 &\quad + c_c w_0 (2/12) \text{cov}[r(c), I]
 \end{aligned} \tag{14}$$

Thus:

$$\text{cov}[r_t^*, I_t] + \text{cov}[r_t^*, I_{t-1}] = w_0 \left[\sum_{j=a}^c c_j \text{cov}[r(j), I] \right]$$

which means that the lagged regression method of empirical correction to the estimation of systematic risk from appraisal based returns, as described in Chapter 4 for disaggregate level returns, will also work here at the aggregate level.

In summary, the effect of aggregating properties into a portfolio in which different properties are appraised as of different points in time causes the reported portfolio returns to be more smoothed (ie, to understate systematic risk more) than occurs at the disaggregate level. Let us label this the "appraisal timing effect". If $1/w_0$ is the smoothing factor at the disaggregate level described in (1), then at the portfolio level the smoothing factor becomes $1/[(1-F/2)w_0]$. As $(1-F/2)w_0$ is necessarily positive, the systematic risk in the reported appraisal based portfolio returns still correctly indicates the sign of the true systematic risk. [That is, in theory. As noted in Chapter 4, in empirical application statistical estimation error can possibly throw off the sign estimate.]

5.1.2 Aggregate Returns with Inconsistent Appraisals

Thus far, we have assumed longitudinal consistency, that is, the appraisal based return for each property at the disaggregate level is described by equation (1) at each point in time. Now, we will relax this assumption in a special way.

Suppose that the quarterly appraisal of properties in a large institutional portfolio is characterized by the following procedure. For each property, once per year an "outside" or independent appraiser is hired by the portfolio manager to appraise the property. The other three quarterly appraisals are done by "inside" appraisers who work permanently for the portfolio manager. The inside appraisals tend to just retain the last outside appraiser's value for the property (possibly adjusted deterministically for inflation, so the value remains constant in real terms).

In the extreme, this means that three out of the four quarterly appraisals per year give appreciation return components that are virtually deterministic, while in the one outside appraisal all of the appreciation return stochasticity for the entire year (not just the one previous quarter) shows up.

With this characterization of quarterly appraisal based returns in an institutional portfolio, equation (1) applies one fourth of the time, but with the period of return taken to be one year (annual returns series), and the other three quarters of the time model (1) does not apply, or, in effect, as far as any risk in the returns is concerned, the w_k weights are all zero in those three quarters.

This type of longitudinal inconsistency in appraisal might be termed the "inside appraisal effect", and it adds yet another level of smoothing (compounding any smoothing from appraisal timing described in Section 5.1.1) to the aggregate portfolio level quarterly return series.

To see this, suppose a portfolio consists of four properties, $i=1,2,3,4$. The true return to property i in each quarter t is given by $r(i)_t$, and the true return to the portfolio is r_t :

$$r_t = c_1 r(1)_t + c_2 r(2)_t + c_3 r(3)_t + c_4 r(4)_t \quad (15)$$

where the c_i are the value weights of the four properties in the portfolio, with the c_i summing to unity.

To clarify the effect we are focusing on here, suppose the annual outside appraisal is exactly accurate (ie, in the context of (1), $w_0=1$ for all the properties whenever the outside appraisal is done), and all appraisals are done at the end of each quarter (ie, in the context of Section 5.1.1, $F=0$).

The appraisal based return to property i in quarter t is $r^*(i)_t$, which equals zero if the property was appraised by an inside appraiser this quarter, and equals the sum of the last four true quarterly returns (ie, equals the true annual

return) if the property was appraised by an outside appraiser this quarter:

$$r^*(i)_t = \begin{cases} 0, & \text{if inside appraisal} \\ r(i)_t + r(i)_{t-1} + r(i)_{t-2} + r(i)_{t-3}, & \text{if outside appraisal} \end{cases} \quad (16)$$

Suppose each quarter one of the properties is appraised by the outside appraiser and the rest are appraised by inside appraisers (ie, outside appraisals are evenly spread throughout the year). Then the reported appraisal based quarterly return on the entire portfolio is:

$$r^*_t = \sum_{i=1}^4 c_i r^*(i)_t = c_j r^*(j)_t = c_j [r(j)_t + r(j)_{t-1} + r(j)_{t-2} + r(j)_{t-3}], \quad (17)$$

based on (16), where j is the one property that happened to have its outside appraisal in quarter t .

Obviously, (17) does not equal the true portfolio return (16); nor is (17) a weighted average of present and past true portfolio returns (unless all properties returns are perfectly correlated). So the analog of the moving average model eqn.(1) at the portfolio level does not hold.

Note also that if the value weights on the outside appraisal time cohorts, the c_j , are not all about equal (ie, $c_j = 1/4$ for all j), then (17) implies that seasonality will be introduced into the appraisal based returns time series for the

portfolio. For example, if most properties receive their outside appraisal in the fourth quarter, then c_j for $j=4$ will be near unity, while the other c_j will be much less than $(1/4)$. This will give the portfolio's fourth quarter return a much higher level on average, near to an annual return instead of a quarterly return, while the returns in the other three quarters will on average be smaller than typical true quarterly returns.

The systematic risk displayed by the appraisal based portfolio returns is S_t^* , the value-weighted average of the appraisal based systematic risk of each property:

$$S_t^* = \text{cov}[r_t, I_t] = c_j \text{cov}[r(j)_t, I_t] \quad (18)$$

The equality in (18) holds because of relationship (17) and because the true returns, like the exogenous CAPM index, are unpredictable white noise. Since all but property j have deterministic returns in quarter t , the only covariance comes from property j , which is a fraction c_j of the portfolio value.

The appraisal based systematic risk is a function of the quarter of the year, since different properties' risk appears in the portfolio returns each quarter. Define S^* to be the annual average systematic risk displayed by the appraisal

based portfolio returns (ie, not conditioned upon what quarter it is):

$$\begin{aligned}
 S^* &= E_t[S_t] = E_t[c_j \text{cov}[r(j)_t, I_t]] \\
 &= E_t[c_j] E_t[\text{cov}[r(j)_t, I_t]] \\
 &= E_j[c_j] E_j[\text{cov}[r(j)_t, I_t]] \\
 &= (1/4) E_j[\text{cov}[r(j)_t, I_t]] \\
 &= (1/4) S
 \end{aligned} \tag{19}$$

where $E_t[...]$ indicates expectation taken over the year, $E_j[...]$ indicates value-weighted expectation taken over properties, and S is the true portfolio systematic risk. (19) results from the fact that when we take the expectation over time, there is an equal one-fourth probability that we will be observing the S_t at any one of the four annual quarters, and from the fact that each annual quarter t corresponds on a one-to-one basis with a particular value j , that is, with a cohort of properties all with their outside appraisals occurring in this quarter t .

By (19), the appraisal based systematic risk in the portfolio is only one-fourth the true systematic risk, even though outside appraisals are exactly true at each point in time, and all properties are appraised as of the same point in time each quarter.

Result (19) is no doubt an overstatement of the amount of smoothing introduced by the "inside appraisal effect", since inside appraisals are not completely deterministic, and some of the risk in the total return will come from risk in the income component (which is ignored in this model). Also, some portfolio managers may employ outside appraisers more frequently than once per year, especially on large properties. But this analysis serves to put an order of magnitude and an upper bound on the inside appraisal effect. Note that this inside appraisal smoothing effect is compounded on top of any smoothing at the disaggregate level and on top of any aggregate level smoothing introduced by the "appraisal timing effect" described in the previous Section.

Empirically, the inside appraisal effect discussed here will be evidenced by some autocorrelation introduced into the portfolio returns series to the extent that the returns to the properties in the different outside appraisal cohorts are correlated. To the extent that this cross-correlation among property cohorts is less than perfect, the amount of autocorrelation apparent in the appraisal based portfolio quarterly returns series will understate the degree of smoothing from the inside appraisal effect. For example, suppose the outside appraisal cohorts are contemporaneously uncorrelated: $\text{cov}[r(i)_t, r(j)_t] = 0$. Then the inside appraisal phenomenon will

introduce no autocorrelation at all into the appraisal based portfolio returns. Yet the smoothing implied by eqn.(19) will still be present to the same degree. This implies that risk adjustment procedures based solely on autocorrelation may understate the amount of smoothing and adjustment necessary. However, the lagged regression correction procedure mentioned previously should in principle still work. This is due to the same reason described in Section 5.1.1, namely, the covariance left out of the contemporaneous regression (that is, the covariance of the three cohorts without outside appraisals in any given quarter) is picked up in the coefficients of the lagged regression.

Another approach to correcting the inside appraisal effect would be to use annual returns rather than quarterly returns. The annual returns should not show any inside appraisal effect, but they will show more appraisal timing aggregation smoothing of the type described in Section 5.1.1. If outside appraisals are evenly distributed at the end of each quarter, then $F=1$ for the appraisal timing effect in the annual returns, and the (S/S^*) smoothing ratio (assuming inside appraisals are deterministic) would be approximately $2/w_0$ in the annual returns [from eqn.(13) of Section 5.1.1], as opposed to $4/w_0$ in the quarterly returns [from eqn.(19)

above], where $1/w_0$ represents the disaggregate level smoothing.

Another empirical sign that the inside appraisal effect may be operating is the presence of seasonality in the appraisal based quarterly returns. True returns cannot display such seasonality, but as noted before, if more outside appraisals are done in one season of the year, and the inside appraisal effect is operating, the quarter when most of the outside appraisals occur will show a "spike" of greater than average return, in most years.

5.1.3. Implications of the Aggregation Analysis

It appears from the preceding analysis is that aggregate appraisal based returns could be significantly more smoothed regarding systematic risk than are the disaggregate returns which underlie the aggregate data. Aggregation, unless it is done with longitudinally consistent appraisals all conducted at the same time each period, adds additional smoothing into the series.

The analysis in the preceding sections enables us to get a "ball-park" idea of the degree to which aggregate appraisal based returns series such as the FRC or PRISA Indices may

understate true systematic risk, under some assumptions about how appraisals might typically be done. Suppose, for example, that at the disaggregate level annual outside appraisals are conducted with a w_0 weight in model (1) equal to $(2/3)$. Suppose further that properties are appraised quarterly, but with deterministic inside appraisals three out of four quarters. This introduces an inside appraisal effect factor of 4 into the aggregate level quarterly smoothing, as described in Section 5.1.2. Finally, suppose properties are appraised "as of" dates which are uniformly distributed over the last four weeks prior to the end of each quarter. This implies $F=(1/3)$ from Section 5.1.1, which introduces an appraisal timing effect factor of $1/(1-1/6)=(6/5)$ on top of the other two factors. The overall smoothing factor is thus $(S/S^*) = (3/2)*4*(6/5) = 7.2$. True systematic risk would be more than seven times that indicated by the unadjusted covariance between the appraisal based returns and the CAPM index.

Even if the smoothing factor of 4 from the inside appraisal effect is overstated by twice (as noted in Section 5.1.2, the factor of 4 is an extreme case) such that the actual inside appraisal effect smoothing factor is only 2 instead of 4, the aggregate level smoothing is still $S/S^*=3.6$, in this example.

This type of heuristic numerical analysis using intuitively plausible values for the behavioral parameters ("a" and T from Chapter 4 at the disaggregate level to determine w_0 , and F and the inside appraisal effect from Chapter 5 at the aggregate level), can shed light on the amount of systematic risk smoothing we can expect to find in aggregate appraisal based returns series, such as the FRC and PRISA Indices. From the preceding "ball-park" analysis, it would seem plausible to expect the (S/S^*) smoothing factor to lie roughly in the range of 3 to 8. This compares to a smoothing factor of 3 estimated by Ross & Zisler in their study of volatility smoothing in the FRC Index. Their estimate was based on a correction procedure which assumes that eqn.(1) holds at the aggregate level so that the smoothing factor can be estimated purely from the amount of autocorrelation in the returns data. In Section 5.2 we will apply the lagged regression correction procedure described in Chapter 4 to empirically estimate the systematic risk smoothing in the FRC and PRISA Indices. As argued previously, this correction procedure should work even though the moving average relationship (1) does not hold at the aggregate level, and even though all of the aggregate level smoothing may not be reflected in additional autocorrelation.

5.2 Empirical Evidence Regarding Smoothing and Real Estate Systematic Risk

Up till now, this thesis has avoided empirical analysis in order to concentrate on developing fundamental conceptual relationships and a priori intuition regarding the nature of real estate return risk. However, it would seem to be of interest at this point to try to apply some of the conceptual analysis to some "real" returns data, by examining the FRC and PRISA Indices of institutionally held commercial real estate assets. The purpose of this Section is not to attempt to formally "test" any theory or hypothesis, nor to derive an empirical estimate of the expected return risk premium for these unsecuritized real estate indices. The available data, and the relevant theory, do not allow such analysis at this time. However, since these aggregate level return indices are available to the public and are widely cited in the real estate literature, a brief empirical analysis would seem useful to gain some feeling for what these data indicate regarding the systematic risk measurement issue.

The PRISA and FRC Indices are among the oldest and most widely cited appraisal based returns indices for commercial real estate. The PRISA Index indicates the performance since 1971 of the Prudential Realty Investment Separate Account, a diversified portfolio of "investment grade" commercial proper-

ties managed by the Prudential Insurance Co. for institutional investors. The portfolio composition changes over time, and includes some levered as well as unlevered properties, and some holdings of mortgages and cash as well as real estate equity.

Though not as old as PRISA, the FRC Index is more widely cited in the real estate literature, and is probably the closest thing that presently exists to a "standard" indicator of institutionally held unsecuritized commercial real estate performance. The FRC Index is compiled by the Frank Russell Company for the National Council of Real Estate Investment Fiduciaries (NCREIF). The properties which compose the Index are contributed from the 45 CREF managers which are members of NCREIF. The FRC Index began at the end of 1977 with less than 300 individual properties with total appraised value slightly over \$700 Million, and now includes slightly over 1000 properties appraised at almost \$12 Billion. The FRC is a "pure" real estate equity index, in the sense that the only assets in the Index are unlevered real estate properties. (However, the composition of the index has changed over time.) As with PRISA, the FRC portfolio has always been broadly diversified both geographically and by property type. (Currently, the FRC is rather heavily weighted toward office properties, which compose some 45 percent of the total value.)

In this Section, the lagged regression technique described in Chapter 4 will be applied to the FRC and PRISA Indices to estimate the amount of systematic risk in the unsecuritized real estate properties composing the these indices. Our purpose here is partly just to demonstrate this smoothing correction technique. But this exercise also presents a good opportunity to compare the traditional stock market based CAPM with the Consumption based CAPM discussed previously in Part I of this thesis. For the reader not familiar with the Consumption based CAPM, a brief description of the intuition behind this model is provided below.

5.2.1 Intuitive Explanation of the Consumption CAPM

The Consumption CAPM (CCAPM) is a more general version of the traditional CAPM, theoretically valid in a multi-period context with stochastic investment opportunities over time. The CCAPM also has the advantage of not depending on an unobservable construct, namely, the "market portfolio" of all risky assets, as the index in the definition of systematic risk. National consumption is reported quarterly by the U.S. Bureau of Economic Analysis.

As with any general equilibrium model of asset prices under uncertainty, the rigorous derivation of the CCAPM is based upon some simplifying assumptions which are not strictly

realistic. These include the assumption of time-additive utility, and either the assumption of complete markets or homogeneous expectations on the part of all investors. Because of such assumptions, we would not expect the CCAPM to hold perfectly in reality. But the logic in the following heuristic explanation of the model shows why it is reasonable to expect that it might hold to a useful approximation.

The basic intuition behind the CCAPM is that when national consumption is greater than expected, most investors' consumption is greater than expected, making them "better off" than they had expected to be (in an economic sense, utility being classically defined on consumption). Similarly, when national consumption is down, most investors are likely to be "worse off". An asset that, cet. par., pays off more (in the sense of having a higher return) when investors are already better off anyway and less when they are worse off is going to add to the ex ante risk (variation) in investors' future consumption time-paths, and therefore reduce the expected utility of their future consumption. Similarly, an asset that pays off more when investors are worse off (national consumption down) and less when they are better off anyway (consumption up) will reduce the risk in their consumption. It will act as a kind of "hedge". Investors should be willing to pay a premium for such an asset, driving its expected return down in equilibrium.

In essence, both the traditional CAPM and the CCAPM are trying to offer a way to distinguish and quantify an asset's relative contribution to the risk that matters to investors. But the traditional CAPM focuses only on the risk in investors' wealth, and as that model is usually applied, it implicitly assumes that all investors' wealth is invested only in stocks. Portfolio returns add to wealth, but from an economic perspective, wealth is just a means to the end of consumption, and this holds no matter where one's wealth is invested. The CCAPM therefore focuses directly on the risk in consumption over time.

This is why the CCAPM is valid in a multi-period world with changing investment opportunities while the traditional CAPM is not. Investors allocate their wealth between consumption and investment considering not only the current level of their wealth but also future consumption plans and the nature of perceived investment opportunities. Thus, the level of consumption reflects all of these factors. Equilibrium (which includes expected utility maximization) in a dynamic multiperiod context requires that the expected indirect utility of the marginal dollar of wealth (investment) equal the direct utility of the marginal dollar spent on consumption, at each point in time. So consumption is the index in a multiperiod CAPM for essentially the same reason

that wealth is the index in the two-period traditional Sharp-Lintner-Mossin model. It determines the marginal utility of (and therefore, the willingness to pay for) future returns from any asset. Asset return covariance with consumption gives the contribution of the asset to the investor's current expected utility of future consumption, and hence, the investor's current willingness to pay for the asset.

5.2.2 Estimation of the Betas

The lagged regression correction technique involves estimating the true systematic risk by regressing the appraisal based returns on the lagged CAPM index. In the absence of smoothing one can normally estimate a security's beta by regressing its return on the contemporaneous CAPM index. The beta estimate is then the regression coefficient on the CAPM index. If the security's returns are smoothed, however, one needs to regress its returns not only on the contemporaneous CAPM index but also on the lagged CAPM index. The true beta estimate is then the sum of the partial regression coefficients on all of the lagged (as well as contemporaneous) CAPM index terms.

[Note that the "beta" is defined as $\beta(1) = S(1)/\text{var}[I]$, so that beta is simply a normalized version of the systematic risk we have been working with in this and previous Chapters,

and the smoothing factor S/S^* is the same as the beta smoothing factor: β/β^* .]

As with any statistical estimation procedure, it is important to realize that the lagged regression correction procedure will in empirical practice be subject to estimation error. Since more parameters must be estimated to calculate the corrected beta estimate when returns are smoothed, in general more estimation error is likely to be introduced than would be the case if we could use the simple contemporaneous regression. Based on the behavioral models of appraisal smoothing at both the disaggregate and aggregate levels discussed in the preceding sections, it is clear that the theoretical accuracy of the correction increases (or at least does not decrease) as the number of lags included in the correcting regression increases. This means that one must decide how many lags to include in practice.

This decision is one of judgement, but judgement that can be informed by the relevant theory. Although theoretical accuracy of the correction increases, the likelihood of spurious correlations in the sample and estimation error also increases as one adds to the number of lags. Ideally, one would like to include as many lags as possible without including spurious or erroneous coefficients.

This is where the appraisal behavioral analysis can help us. That analysis suggested that, at least when working with quarterly returns data at the aggregate level, one should probably include at least a full year of lags (lags of $L=0,1,2,3$), to cover the inside appraisal effect at the aggregate level and possible similar-sales reference samples (T) exceeding one period in the Market Method at the disaggregate level. The behavioral analysis also indicated that the theoretical smoothed beta of the appraisal series (without correction) would be of the same sign as the true (corrected) beta (though estimation error could throw this off). We also observed that the smoothing correction (the sum of lagged coefficients) should in theory be non-decreasing (in absolute value) as the number of lags used in the correcting regression increases. However, at least after a certain reasonable number of lags, the behavioral analysis implied that the further inclusion of any more lags should add very little if any additional value to the corrected beta estimate.

Considering the foregoing, the procedure adopted in this analysis was to include at least four lags (up to $L=3$) in the correcting regression, and to add more lags until the absolute value and t -statistics (of the corrected beta) began to no longer increase. In general, this point occurred at five lags, or a bit longer in the case of the FRC Index. For the

sake of simplicity and consistency in the numbers reported here, the convention has been adopted of showing all results taken with five lags included in the correcting regression. (Sensitivity analysis reveals that very similar beta estimates are achieved with 8 lags, only with lower t-statistics.)

It is interesting to note that both the FRC and PRISA returns show seasonality, with a total return spike in the fourth quarter. As noted in Section 5.1, such seasonality could reflect the inside appraisal effect, with most "outside" appraisals occurring in the fourth quarter, and "inside" appraisals tending to retain the valuations reported in the last outside appraisal. This seasonality, which cannot be present in true economic returns, is most pronounced in the FRC Index. To control for this seasonality, a dummy variable is employed in the RHS of the CAPM regression, labelled SEAS4, which assumes a value of 1 if it is the fourth quarter, zero otherwise.

The results of the regression runs are summarized in Tables 5.1a-b, for the FRC Index and PRISA Index respectively. Regressions labelled I include no lagged coefficients, and so estimate the beta unadjusted for smoothing. Regressions labelled II include four lags on the CAPM index, to estimate the adjusted beta correcting for the smoothing in the appraisal based aggregate returns. In these regressions the

sum of the I_{t-L} coefficients ($L=0,1,2,3,4$) is reported together with the t-ratio for the null hypothesis that the true value of this sum equals zero. This sum is the adjusted beta estimate correcting for the smoothing. The regressions labelled (S) take the real return to the stock market as the CAPM index I_t ("traditional CAPM"), while the regressions labelled (C) take the unexpected change in real per capita national consumption as the CAPM index I_t ("Consumption based CAPM").

To model these unexpected changes in consumption, the actual consumption series was converted into a white noise series. This was done using the residuals from a forecasting model of the percent change in quarterly real national consumption expenditures per capita. A white noise null hypothesis can be rejected for the consumption changes themselves, but not for the residuals from this forecasting model. The model employed is a univariate ARIMA(2,1,1) model of the real per capita consumption changes over the 20-year period from 1968 through 1988. The residuals from this model have a correlation coefficient of .795 with the actual consumption changes, but while the latter have a Bartlett's Q value (at 10 lags) of 23.03, the former have $Q = 8.841$ (compared to rejection of the white noise null hypothesis at 90 percent confidence if $Q > 15.99$, or at 95 percent confidence if $Q > 18.31$).

It should also be noted that, consistent with underlying CAPM theory, real (inflation adjusted) returns have been used in all of the regressions. Also, both the unadjusted and adjusted regressions reported in Table 5.1 were transformed where necessary (namely, in the PRISA returns) to get rid of excessive autocorrelation in the regression residuals, using the iterative Cochrane-Orcutt procedure.

5.2.3 Interpretation of the Empirical Findings

Several points are apparent in the systematic return risk estimates shown in Tables 5.1a-b. First, there seems to be evidence of considerable smoothing, at least in the case of the consumption betas. In the one case where the smoothed and corrected betas are statistically significant (namely, the FRC consumption betas), we see that the smoothing ratio S/S^* equals $1.368/0.411 = 3.33$, which is within the range of 3 to 8 suggested by the informal numerical analysis in Section 5.1.3 (and almost identical to the ratio found by Ross and Zisler for volatility, using a different method, on the same Index).

As noted previously, if there is much smoothing, or if the true beta is quite small in absolute value, then statistical estimation error is likely to cause the smoothed (uncorrected)

beta estimate to be of the wrong sign or to differ by a large proportional amount from the theoretical smoothed beta. This may account for the lack of statistical significance in most of the estimates of the smoothed betas, and hence the difficulty in obtaining reliable indications of the smoothing factor in any except the FRC Consumption beta case. For example, if the true real estate beta with respect to the stock market is zero, then the theoretical smoothing factor is $S/S^*=0/0$, which is undefined, no matter how much or little smoothing there is. [Something like this appears to be going on in the case of the traditional CAPM regressions on the stock market return.]

This points to the second major implication of the results shown in the Tables. Even after correcting for smoothing, the unsecuritized real estate stock market betas appear to be extremely small in absolute value, virtually zero in fact. (The very small standard errors on the beta estimates are due to the very small volatility of the appraisal based returns series, as compared to typical stock market returns series. This permits a rather precise estimation of the beta.) This confirms the conventional wisdom that unsecuritized real estate covaries negatively or very little with the stock market, and stands in sharp contrast to results obtained for securitized real estate such as REITs. This inconsistency

between unsecuritized and securitized real estate return behavior with respect to the stock market raises potentially very interesting questions, either about the nature of REITs, or the accuracy of appraisals (apart from smoothing), or the efficiency of the stock market.

A third important indication in the Tables is that the real estate consumption betas appear to be much higher than the real estate stock market betas. This suggests that the expected return risk premium which would be predicted by the Consumption based CAPM would be quite a bit higher than that predicted by the traditional stock market based CAPM.

Greater power on the part of the Consumption CAPM to explain a sizable positive expected return premium in unsecuritized real estate is suggested more clearly in Table 5.2. In this Table the normalized betas, that is, the real estate betas divided by the beta of the stock market, are presented. Of course, the traditional stock market beta with respect to the stock market is 1.00 by definition. But the stock market's (ex post) consumption beta over the entire 1971.1-1987.4 period studied was 1.880. Dividing the real estate consumption betas by this stock market consumption beta, we see in Table 5.2 the real estate systematic risk as a proportion of the stock market's systematic risk. After adjusting for smoothing, both the PRISA and FRC consumption betas imply that

these portfolios should have expected return risk premia that are a substantial fraction (0.73 for FRC, 0.43 for PRISA) of that of the stock market, according to the Consumption CAPM. This result is quite different from what is implied by the traditional CAPM and the stock market beta.

The stock market risk premium (spread over T-bills) has historically been around 8 or 9 percent per year, while the real estate portfolios studied here have shown an average spread over T-bills of only some 3 to 4 percent per year. Therefore, the ratios shown in Table 5.2 seem consistent with the notion that a sizeable fraction, at least, of the observed spread between these real estate portfolios and T-bills can be explained as a "risk premium", provided that: (i) one defines "systematic risk" (ie, the return risk which "should matter" to diversified investors) on the basis of national consumption rather than on the basis of the stock market return; and (ii) one corrects for appraisal smoothing. (Indeed, in the case of the FRC Index, even without the smoothing correction, about half the ex post return premium can be explained by systematic risk and the CCAPM.)

Thus, the implications of this (admittedly very limited) empirical analysis appear to allow unsecuritized real estate risk and returns to fit more consistently in the mainstream financial economic paradigm which explains risky asset return

spreads primarily in terms of general equilibrium effects of investor risk aversion and asset return risk with diversified investors. As noted, however, the data used in this analysis consist only of a very small (and changing over time, and not necessarily very representative) sample of unsecuritized real estate returns (compared to the universe of such assets in the economy). Also, of course, the data reflect only ex post realizations, rather than the ex ante expectations which matter in theory. Therefore, the betas in Table 5.2 should not be taken too literally or precisely, but only as being suggestive of what could be going on in the relation between commercial real estate risk and returns.

Table 5.1a: FRC Index Regression Results (40 obs., 78.1-87.4):

Variable:	Coefficients (t-stats) from Regression Run:			
	(Stock Mkt Beta)		(Consumption Beta)	
	I(S)	II(S)	I(C)	II(C)
Constant	.010 (5.31)	.010 (4.58)	.010 (5.68)	.009 (5.95)
SEAS4	.020 (5.54)	.019 (5.19)	.021 (6.08)	.021 (6.64)
I_t	-.000 (0.02)	-.006 (0.34)	.411 (2.11)	.425 (2.26)
I_{t-1}		.019 (0.88)		-.072 (0.36)
I_{t-2}		-.013 (0.60)		.266 (1.39)
I_{t-3}		-.015 (0.67)		.354 (1.85)
I_{t-4}		.024 (1.08)		.395 (2.04)
Sum of β coefs:		<u>.009 (0.20)</u>		<u>1.368 (2.68)</u>
R^2	.453	.494	.512	.617
D-W	1.709	1.664	1.558	1.933

[All data are quarterly real (inflation adjusted) returns]

I = $r(m)$ = S&P500 return for Stock Mkt Beta regressions (S)

I = C = Change in national per capita consumption for Consumption Beta regressions (C)

Table 5.1b: PRISA Index Regression Results (68 obs., 71.1-87.4):

Variable:	Coefficients (t-stats) from Regression Run:			
	(Stock Mkt Beta)		(Consumption Beta)	
	I(S)	II(S)	I(C)	II(C)
Constant	.012 (3.39)	.011 (2.98)	.011 (3.34)	.011 (3.00)
SEAS4	.005 (2.02)	.005 (2.01)	.005 (2.01)	.006 (2.43)
I _t	-.006 (0.41)	-.010 (0.69)	.067 (0.43)	.125 (0.79)
I _{t-1}		.027 (1.62)		-.199 (1.23)
I _{t-2}		-.012 (0.69)		.367 (2.14)
I _{t-3}		.018 (1.17)		.177 (1.09)
I _{t-4}		.009 (0.54)		.332 (2.09)
Sum of β coefs:		<u>.032 (0.68)</u>		<u>.802 (1.62)</u>
R ²	.277	.324	.277	.392
D-W	2.078	2.054	2.056	2.121

[All data are quarterly real (inflation adjusted) returns]

I = r(m) = S&P500 return for Stock Mkt Beta regressions (S)

I = C = Change in national per capita consumption for Consumption Beta regressions (C)

Table 5.2: Normalized Adjusted versus Unadjusted Beta Estimates**:

FRC Index (78.1-87.4):	β_S	β_C
(I) Unadjusted (β^*)	-.0004	.219
(II) Adjusted (β)	.009	.728
PRISA Index (71.1-87.4):	β_S	β_C
(I) Unadjusted (β^*)	-.006	.036
(II) Adjusted (β)	.032	.427

 ** β_S is with respect to stock market real return risk; β_C is with respect to consumption risk. The betas in this table are normalized on the stock market by dividing by the S&P500's ex post beta over the 71.1-87.4 period, which was 1.00 with respect to the stock market (by definition) and 1.880 with respect to consumption: $\beta_S = \text{cov}[r, r_M] / \text{var}[r_M]$; $\beta_C = \text{cov}[r, C] / \text{cov}[r_M, C] = \text{cov}[r, C] / \text{var}[C] / 1.88$, where r_M is the real return to the S&P500, and C is the unexpected (white noise) change in consumption (modelled by the residuals from a univariate time series model of the real per capita consumption series).

Chapter 6: Summary and Conclusions from Parts I and II

This thesis is an attempt to apply some basic tools from the modern financial economic theory of general equilibrium in efficient capital markets with symmetric information to gain some insight into the nature of real estate valuation, return risk and risk premia. Although these tools, such as the Consumption-based CAPM and multi-period certainty-equivalent valuation, have been around for a decade or more, they have not yet been applied to address real estate issues specifically.

While real estate markets, like other capital markets, are obviously not perfectly efficient or symmetric in information availability, it is the author's opinion that, in the study of real estate markets, we have not yet gotten as much "mileage" or useful insights as we can from this paradigm and the basic tools derived from it. In particular, one motivation of the thesis was to use these traditional finance tools to examine more carefully the currently fairly widespread impression that unsecuritized real estate has very little systematic risk, and that real estate return premia are therefore either evidence of disequilibrium between capital market segments or largely explained in terms of illiquidity or other non-risk penalties.

The thesis is divided into two parts. Part I focuses on the fundamentals relating real estate cash flows to valuation and return risk. A multi-period cash-flow-based valuation model which incorporates multi-period riskless leases is developed and applied at a conceptual level. Part II focuses on the nature of appraisal based returns time series data for unsecuritized real estate, and on how to correct for "smoothing" in the apparent risk observed in such series. The Consumption-based CAPM is compared to the traditional stock market based CAPM in a brief empirical analysis in Part II.

The valuation model developed in Part I incorporates multi-period riskless leases, to reflect practice in many commercial real estate markets -- office buildings in particular. Basic insights from the model include the following: (i) While return risk decreases with the use of multi-period riskless leases (holding underlying opportunity risk constant), the ratio of return risk to observable cash flow risk increases, to the point where it is likely that at lease term lengths prevailing in many real estate markets return risk equals or even possibly substantially exceeds observable cash flow risk; (ii) Real estate return risk, like that of any risky cash flow capital asset, is very sensitive to the extent to which investors regard the asset's cash flows as tending to revert to a known deterministic trend line ("mean-reversion"). In any

asset or portfolio, significant mean reversion implies that there is very little cash-flow-based return risk.

The valuation model is also useful for exploring questions related to the extent to which the use of multi-period riskless leases makes some types of real estate assets more "like a bond" rather than "like a stock". In general, it is found here that real estate is much more like a stock than like a bond, even where lease cash flows are fixed in nominal dollars and lease terms are relatively long, for example, in the neighborhood of 10 years. Thus, real estate's "nominal duration", or sensitivity to changes in nominal interest rates (holding real interest rates and real underlying rental market opportunity expectations constant) is very small, more like a stock's than like a long or medium term bond's. This is in part because much or most of the asset value is in the "stock part", the present value of cash flows expected beyond the expiration of existing leases.

On the other hand, the bond part of real estate value is great enough to disrupt the accuracy of the constant-discount-rate based "simple cap rate valuation" procedure in which the next year expected cash flow is divided by a cap rate consisting of the current expected total return minus the underlying rental price growth rate. This commonly used procedure is, however, found to be accurate on average over time, or when

the rental market is at its long-run normal level. In contrast to previous capital budgeting literature, this latter finding appears to hold even with mean-reverting cash flows.

The analysis in Part II of the thesis uses the idea that true asset returns should be at least well approximated as uncorrelated over time, white noise. In combination with simple models of appraisal behavior ("Income Method" and "Market Method" at the disaggregate individual property level) and aggregation characteristics at the portfolio level, this white noise assumption enables a quantitative conceptual characterization of the amount of risk smoothing introduced by using unadjusted appraisal based returns to estimate real estate risk. An empirical correction procedure is also suggested by this analysis, using lagged regression and summing the lagged beta coefficients to estimate the portfolio beta.

This correction procedure is applied to data from the FRC Index and PRISA Index of unsecuritized commercial real estate returns. This empirical analysis tends to support the conceptual analysis, which indicated that appraisal based returns series of unsecuritized portfolios could be considerably smoothed, under plausible characterizations of appraiser behavior. The empirical analysis also seems to indicate that real estate has much more systematic risk with respect to national consumption than it does with respect to the stock

market, and that consumption risk in unsecuritized real estate may be sufficient to account for at least a large fraction of the observed risk premium.

In summary, to return to one of my main original motivations: What does the thesis have to contribute to the debate about the importance of the mainstream financial concept of systematic risk in determining the expected return premium in real estate assets? It seems to me that one useful contribution of the thesis in this regard is to help to clarify or more precisely delineate the arguments which one must use in order to support the position that systematic return risk in unsecuritized real estate assets is generally very small and does not play a major role in defining the expected return premium.

Considering both Parts I and II, I believe the thesis suggests that in order to take this position, one must argue one or the other of two points. One approach is to argue that it is the traditional stock market oriented CAPM which defines the relevant systematic risk that matters to investors, instead of the more general Consumption-based CAPM. The other approach is to accept the Consumption-based CAPM but then to argue two other points. First, considering the analysis in Part I, one must argue that investors view real estate cash flows as being significantly mean-reverting while they do not

view stock market cash flows as being mean-reverting. Second, in view of the empirical evidence in Part II, one must also argue that appraisal based returns data are not significantly smoothed, and/or that the relatively strong positive correlations observed ex post in the FRC and PRISA Indices in recent years between real estate returns and national consumption (as compared to correlation between stock market returns and consumption) are not representative of broadly held investor ex ante expectations.

In summary, I view this thesis as generally supporting the idea that systematic risk may after all be a very important factor in determining the expected return risk premia in unsecuritized commercial real estate assets. In this sense, at least, real estate may not be as different from securitized financial assets as some have previously thought. My reason for this opinion is that I do not particularly care for either of the two arguments noted in the previous paragraph, which the thesis suggests must underlie the counter-argument against systematic risk playing a major role. The traditional stock market CAPM is in theory just a special (and in principle less realistic) case of the Consumption CAPM. As for the other argument, while mean-reversion in cash flow expectations certainly reduces real estate return risk below what it would otherwise be, I question whether this can eliminate enough return risk to render such risk unimportant in establishing

expected total returns. I say this considering the nature and role of land value in real estate asset value (as discussed in Part I), and considering the rather extreme expectations behavioral implications of the mean-reversion model (ie, that the long-run mean cash flow level is known with certainty). Also, the appraisal based returns empirical evidence in Part II of the thesis seems to support an intuitively reasonable notion that real estate returns probably tend to have about as much positive correlation with national consumption as stock market returns do, perhaps even more. If this is true, and there is significant smoothing in appraisal returns, then systematic risk can account for at least half of the observed risk premium in the FRC and PRISA returns.

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