"THE LATERAL FAILURE OF SPARS"

A THESIS

Submitted to the

DEPARTMENT OF AERONAUTICAL ENGINEERING,

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

COURSE IX-B

by

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As Part of the Requirements for the Degree of

BACHELOR OF SCIENCE

1924

ACKNOWLEGEMENTS

To Professor Edward P. Warner we express thanks for his many suggestions which have made this thesis possible.

To Professor I.H.Cowdry we owe much of the general layout of the apparatus we used. He also furnished many valuable hints pertaining to the correction of the results obtained.

Mr.J.H.Zimmerman of the Testing Materials Laboratories also offored many valuable suggestions regarding the testing of the specimens.

Professor William A.Johnston aided materially our consideration of the theoretical and mathematical treatment of the problem.

May, 1924

SYNOPSIS

From twenty-one tests on the effect of depth-breadth ratio, twenty-four on the effect of span, and nine on the effect of distributing the load, performed with spruce, the results of which have been corrected to a standard condition of moisture content, grain slope, and specific gravity of the specimens, we have concluded principally that airplane spars designed with an allowable modulus of rupture of 10,500 pounds per square inch, may, under the conditions of loading now common, safely have a depth-breadth ratio of ten, and that this depth-breadth ratio, if the section is rectangular, will give the best strength-weight ratio.

A TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

*indicates photograph

LIST OF TABLES

I

INTRODUCTION

Introduction

Airplane wings of the sectional forms and sizes approved by modern design permit the use of spars often greater in depth than is necessary for sufficient strength, provided that the maximum limit of the ratio of spar depth to spar breadth be assumed to be four, as it is at present.

Among the many assumptions and limitations of the beam theory, and therefore of the formulas derived therefrom, which must be remembered by all who design structures by their application, is this: That the section shall be of reasonable dimensions.

The above mentioned limit of spar depth to spar breadth, four, is at present considered to be the maximum which will give reasonable dimensions, and the fiber stress in sections of this or smaller depth-breadth ratios, but not unreasonably flat, is considered to be given by the fundamental equation of the beam theory,

$$f = \frac{M\,y}{I},$$

where f = the maximum stress intensity on the section,

M = the bending moment at the section,

y = the distance of the most stressed fiber from the neutral axis, and

I = the moment of inertia about the neutral axis of the section.

For rectangular sections, which are the simplest, this becomes

$$f = M \frac{h/2}{\frac{bh^3}{12}} = M \frac{6}{bh^2} \quad ,$$

where b = spar breadth, and

d = spar depth.

It is, then, apparent that for any given material, and therefore for any given value of f, the bending moment that can be carried varies as the square of the spar depth.

Since the weight of a spar varies as its cross-sectional area and therefore, for spars of rectangular section, as the first power of the spar depth, whereas we have just seen that the strength varies as the square of the spar depth, it is important that the spar be designed with a cross section of as great a depth-breadth ratio as is possible in the wing section chosen, provided that the strength is not impaired more than enough to compensate for the gain in diminished weight. In other words, in the construction of airplanes it is important to use spars of the depth-breadth ratio which will give the maximum value of the ratio of strength to weight.

Furthermore, span is a dimension which may be unreasonable just as either of the others. It is well known that the strength of a column bears a relation to the ratio of its length to its smallest diameter. So a wing spar, which in biplane and other wing combinations, may act partially as a column under compressive loads,should be designed under rules governing the ratio of its length to smallest cross-sectional dimension. And,more obscurely,so the portion of a spar which in bending receives a compressive load should be designed under rules governing the ratio of its length to smallest cross-sectional dimension.

The failure of a spar in bending, when, for instance, its depth is unreasonably great in proportion to its breadth,appears as follows: As the load is applied the spar acts as any beam up to a certain amount of load, which may vary from practically zero up to the full load as figured by the beam formula,

$$f = \frac{My}{I} ,$$

depending upon the amount of the ratio of the spar dimensions,loading, et cetera.The portion of the spar under compression from the bending then begins to buckle as a column,and, in addition to this lateral deflection,the application of more load produces vertical deflection more rapidly than before the lateral deflection appeared.Finally the load, in terms of the reaction of the spar, reaches a maximum, below the load calculated for the spar by the beam formula above. The reaction of the spar, if the vertical deflection is increased, falls off again somewhat, while the lateral deflection is further increased.

The fiber of the spar under maximum tension from the bending remains straight, just as it does in a spar of reasonable dimensions. Further, every section of the spar seems to remain a plane section.

If the stress-strain diagram be plotted for this operation, that is if the load reaction of the spar be plotted against its vertical deflection, the accompanying characteristic diagram is obtained.

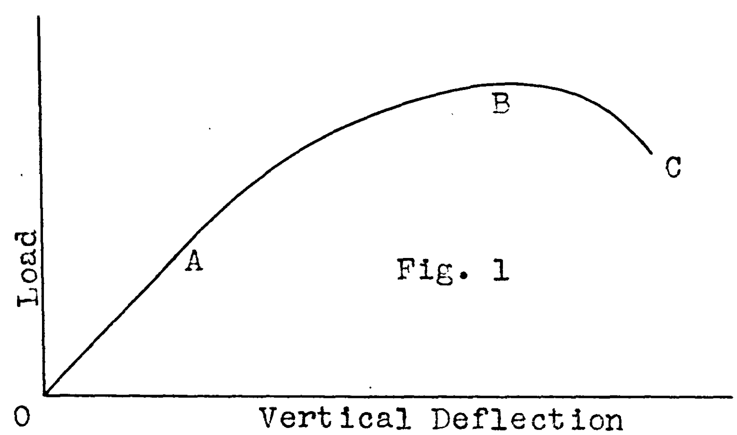


Fig. 1

From O to A the curve is as though the spar were of reasonable proportions. At A lateral deflection begins and continues until the specimen fails absolutely in tension from either the primary bending or a combination of this and the lateral bending, or in compression, or,- and this is the most likely, until the excessive amount of deflection imposed upon secondary structural members causes them to fail and the structure to disintegrate.

II

PREVIOUS RESEARCH

Previous Research

So far as is known there have been but three previous attempts to fix reasonable limits to the dimensions of beams for the application of the bending theory or to discover what corrections are necessary to the theory's promises for beams of unreasonable dimensions.

The first of these is a thesis by S.H.Goodman M.I.T.1922, entitled "Lateral Failure of Wing Spars" and Number 43 in the files of the Mechanical Engineering Department. The second is a thesis by Lucien Alchalel and Atahualpa Guimaraes,entitled "Lateral Failure of Airplane Wing Spars" and Number 4 in the files of the Mechanical Engineering Department.The third is a note published in Flight,May 30,1918, page 590,by J.Prescott, M.A.,D.Sc., entitled "The Sideways Buckling of Loaded Beams of Deep Section."

Goodman tested some thirty specimens,all of rectangular section, of section modulus of about 0.3 cubic inches, and of various breadth-depth ratios. In conclusion he offered three suggestions, two of which are definite concerning reasonable dimensions. The first suggestion is, "For maximum strength use cross sections of breadth-depth ratios of 1 to 1.625 to 1 to 2.250." The second definite suggestion is, "To avoid lateral collapse a beam must not have a ratio (breadth to depth) greater than one third if supported or one fourth if fixed." By "supported"

Goodman meant that the ends of the beam were free to rotate about an axis parallel to the direction of loading,and by "fixed" he meant that the ends were constrained from moving about any axis except so as to permit the usual vertical deflection under the load.

Alchalel and Guimaraes repeated much of the work of Goodman, recorded the magnitude of lateral deflections and took into account in their calculations the effect of span, which greatly complicated their results, until they,themselves, admitted in their report that their results seemed to be of little practical use. They,further,investigated the properties of some I-sections.

The details of Prescott's work are not available. The brief article in "Flight" sheds no light on his methods except to say,"The buckling load depends on the flexural rigidity for sideways bending, and on the torsional rigidity of the beam. It is clear that the torsional rigidity has something to do with the question because the beam could not buckle without twisting." The method indicated seems to be more of a mechanical analysis of the problem than any direct experimentation.

Prescott did however publish the following very interesting formulas,in which

E is Young's modulus,the modulus of Elasticity;

I,the smallest moment of inertia of the section;

N, the modulus of rigidity;

KN, the torsional rigidity;

L, the length of the beam;

and G, a couple which may be acting at its ends.

Case 1.Beam acted on by couples only:

$$GL = \pi\sqrt{EINK}$$

Case 2. Same but clamped at the ends:

$$GL = \pi 2\sqrt{EINK}$$

Case 3. Cantilever, end load of P

$$PL^2 = 4.01\sqrt{EINK}$$

Case 4. Simple beam,center concentrated load of P:

$$PL^2 = 16.94\sqrt{EINK}$$

Case 5. Same as case 4 but fixed at the ends:

$$PL^2 = 25.86\sqrt{EINK}$$

Case 6. Simple beam,total load of W uniformly distributed:

$$WL^2 = 28.3\sqrt{EINK}$$

Case 7. Same as case 6 but with cantilever:

$$WL^2 = 12.86\sqrt{EINK}$$

Prescott considered the load applied at the center line of the beam.

The value of K he used was that from the theory of torsion of prisms and condensed down to

$$K = \frac{3 \cdot b^3 \cdot d^3}{10 \cdot (b^2 + d^2)}$$

where b represented the breadth of the beam; and d its depth.

In 1913 Prescott published a book, "Mechanics of Particles and Rigid Bodies" (Longmans, Green & Co.) in Which, however, this subject was not treated.

III

OBJECTS

Objects

The objects of this thesis are:

1. To study lateral deflection and failure.
2. To study the tendency of various sections of high depth-breadth ratio to fail laterally.
3. To study the effect of span on the tendency to fail laterally.
4. To determine, if possible, what corrections must must be applied to the results obtained from the beam theory to cover the possibility of lateral failure.
5. To determine, if possible, what relations of span, depth, and breadth will give spars of the highest strength-weight ratio.
6. To determine, if possible, whether the tendency to fail laterally or to possess strength less than that given by the beam formula is influenced by or varies with any of the following properties of a section:
    a. Section modulus,
    b. Modulus of elasticity,
    c. Grain of the wood,
    d. Percentage of summer growth of wood,
    e. Percentage of moisture of wood,
    f. Rate of growth of the wood,
    g. Specific Gravity of the wood.

As has been pointed out in the introduction the actual modulus of rupture of a speciman may be lowered by the depth-breadth ratio of a section being either too large or too small. It is the purpose of this thesis to consider only those sections whose depth-breadth ratio seems too large.

A recent work of the Forest Products Laboratory derived a formula for the calculation of what is called a form factor, which when multiplied by the strength of a spar of rectangular cross section gives the strength of a spar of the sectional shape for which the form factor was calculated. It was also the purpose of this thesis to attempt to find a formula by which another such factor could be calculated to allow for the excessive dimensional ratios or tendency toward lateral failure which a section might have.

Only rectangular sections are considered in this thesis.

In view of the small amount of data previously gathered it was realized that in the time allotted only the surface of the problem could be touched. Therefore as complete a record as has been possible has been kept, much data being preserved and presented herein quite unnecessarily,it seems at present.

IV

METHOD OF ATTACK

Method of Attack

Two methods were considered as offering possible solutions to the problem.

The first was that which Prescott evidently used, mathematical analysis. No attempt was made to derive independently the formulas which he produced, for it seems on the face of the matter that if the load is assumed to be on the central plane through the beam and the beam is homogeneous, isotropic material,then there can be no lateral deflection,- that is,there can be no deflection in any plane other than the plane of loading, for there would be no lateral forces.It seems quite obvious that lateral deflection is purely the result of the line of action of each element of the load not passing through the center of gravity of that section of the beam on which it acts, in other words lateral deflection is a function of the dissymetry of the loading. The only alternatives, analytically,were those analagous to the long column formulas, and one in which no lateral deflection at all might be assumed

The experimental attack of the problem, the second method considered, was planned as simple as possible and yet be comprehensive of all the factors which pertain to such a material as wood.In view of the difficulties encountered by Alchalel and Guimares it was decided to make three separate sets of tests: (a) to find the effect of depth-breadth ratio, (b) to find the effect of span,and (c) to find the effect of distributing the load.

Mathematical Analysis

Efforts to accomplish any of the desired results by mathematical analysis have been futile,perhaps because of the small amount of time which could be so allotted. A report of the reasoning followed seems essential,however.

Only the simplest loading was considered,- a beam supported at the ends and having a concentrated load at the center.

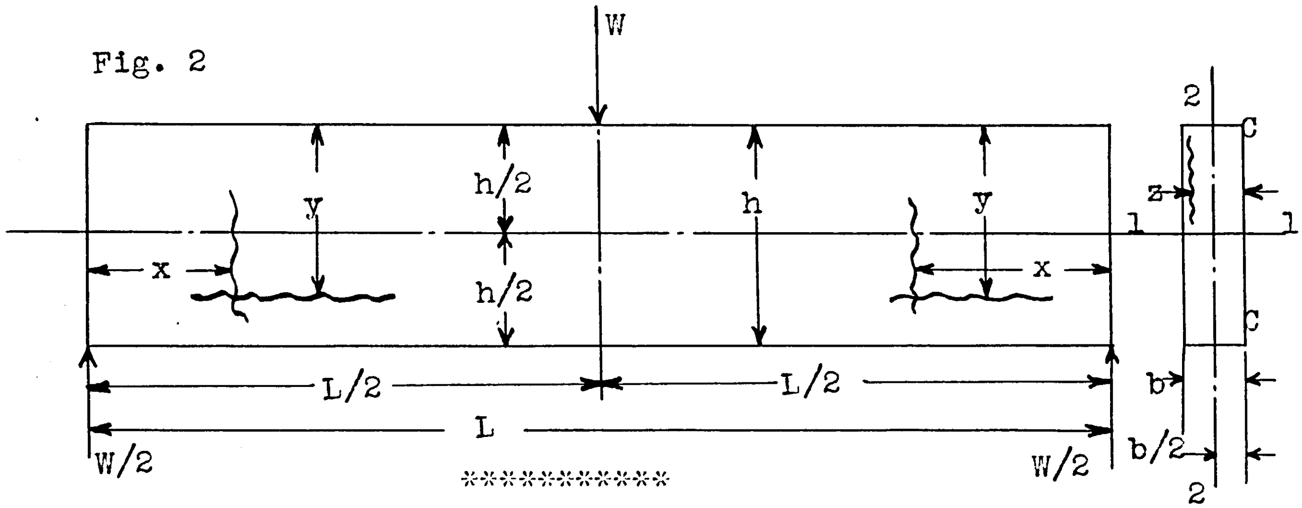
An attempt was first made to derive a formula much as the Gordon formulas have been developed for long columns.(See page 354 and following,Vol.II, "Applied Mechanics" by Fuller and Johnston published by John Wiley and Sons,Inc.,1919.) Here it was necessary to secure some expression for the lateral deflection,an impossibility in applying the method to beams it is believed. In the derivation of the Gordon formulas the lateral deflection was assumed proportional to the square of the column length. Such an assumption here would be erroneous due to the fact that the shearing forces which act between the elementary columns into which the beam may be considered divided, must be taken into account,as will be shown later in detail.

The Euler formulas suggested the next possible method. It may be here noted that by them a critical column load is deduced. It was believed that a beam of the dimensions which would produce lateral failure also possessed a critical load, and that if this critical load could be found it might safely be assumed that it would be equivalent of the maximum load allowable on the specimen considered.

The difficulty encountered in following this reasoning came in the form of an expression impossible to integrate mathematically. The authors believe that by means of graphical integration and the expenditure of considerable time this method might give results. The solution as far as we have been able to carry it is given in later pages.

Mathematical Derivation Analogous To
The Gordon Formulas

Fig. 2



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Consider the beam sketched above, and let

f be the apparent stress, given by the beam theory;

f' be the true stress including that due to lateral deflection;

v be the maximum lateral deflection;

A be the area of the section(A-bh);

I_{1-1} be the ~~the~~ moment of inertia about 1-1, $I_{1-1} = bh^3/12$;

I_{2-2} be the moment of inertia about 2-2, $I_{2-2} = hb^3/12$;

CC be the side thrown into compression by the lateral deflection.

****

$$f = \frac{Wx}{2}(y - h/2)\left(\frac{12}{b\,h^3}\right)$$

(1) $$= \frac{6W}{bh^3}\left(xy - \frac{x\,h}{2}\right)$$

***

Also for any long column under a load of P

$$f' = -\frac{P}{A} - \frac{Pvy}{I} \qquad (2)$$

where y/I is the section modulus of the column about the axis about which bending occurs.

In this case, considering the beam to be composed of a series of elementary layers each acting as a column under a load varying along its length, we may rewrite (2)

$$f' = -\left(\frac{P}{A} \pm \frac{Pv(z-b/2)}{I_{2-2}}\right)$$

Reducing and combining with (1), letting $f = \frac{P}{A}$

$$f' = f\left(1 - \frac{12vz}{b^2} - 6\frac{v}{b}\right) \qquad (3)$$

Here the only unknown is v. To complete the solution v must be expressed as a function of the properties of the beam.

In the Gordon derivation it is assumed that

$$v = k\frac{L^2}{c},$$

for columns free to turn at the ends, where $k = \pi^2$ and c = the smallest cross - sectional dimension of the column. Here, then, in any layer, or for any given value of y,

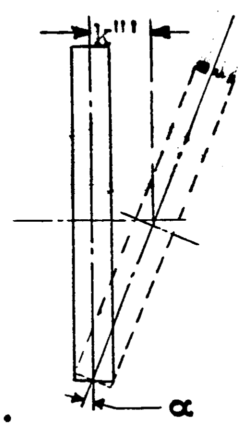
$$v = \pi^2 \frac{(L/2)^2}{b/2} = \frac{\pi^2 L^2}{2b} \qquad (4)$$

It is quite evident also that when y = h, v is zero, and that when v is a maximum y is zero, and that v varies directly as the first power of (h-y) for values between, since it is found by experiment that in lateral failure, due to the loading here considered, the fibers in maximum tension remain in their original vertical plane and that at any given section at b/2 distance from the axis 2-2 all the fibres lie in the same straight line. We may then write that for any given value of x

(5) $v = k'(h-y)$.

Also if we neglect the effect of true transverse curvature, that is the thickening of the section in compression and the shortening of all sectional dimensions in tension, it is evident that for any given values of y and x that v is a maximum on one side of the beam and a minimum on the other, and varies proportionally to the first power of z for values in between. We may then write that for any given values of x and y

(6) $v = k''z - k'''$



Now, if α, the angle through which the section has deflected, be small k''' may be expressed exactly as v in (4) and (5) with the addition of b/2 to the right hand member of each.

This leaves k' and k" to be determined before the calculation of v for (3) is possible. The determination of these two constants has not been possible to the present authors.

Mathematical Analysis Analagous to the Euler Formulas

Consider the beam previously sketched divided as in the Gordon method into layers each dy thick, b wide, and L/2 long. Each of these in compression acts as a free ended Euler column, restrained from buckling in a vertical plane, but, as long as there is no deflection, free to deflect in a horizontal plane.

Select one of these elementary layers at y distance from the neutral axis, which will be assumed to be the geometrical axis. Let x be measured as before, positive from each end toward the center. The differential compression on the section of this elementary layer, due to the vertical bending of the spar, is

$$dC = f(b\ dy)$$

$$= (b\ dy)\ M\ \frac{y}{I}$$

$$= \frac{bW}{2I}\ xy\ dy,$$ where the values of b, w, x, and y are as before and $I = I_{1-1}$

(1) $$\frac{d^2C}{dxdy} = \frac{bW}{2I}\ y$$

*****

For any given value of y, that is, in any layer $dC = kx$, where k is a constant.

Each elementary layer will deflect, if lateral deflection sets in, in the elastic curve which is produced for this loading, which must now be derived.

*****

Let u be the deflection at any point in the elastic curve. Let H be a constant of the curve. Then

$$\frac{d^2u}{dx^2} = \frac{(dC)}{H}$$

$$= k'x$$

$$\frac{du}{dx} = k'\frac{x}{2} + c'$$

$$u = k'\frac{x^3}{6} + c'x + c''$$

When x is zero, u is zero, therefore c" is zero.

When x is L/2, u is zero, therefore

$$c' = -k'\frac{L^2}{6\cdot 4}$$

Therefore

$$\frac{du}{dx} = k'\frac{x^2}{2} - k'\frac{L^2}{24}$$

Setting this equal to zero and solving for the value of x at which u will be a maximum

(3) $$x = \frac{L}{2}\sqrt{\frac{1}{3}} = 0.5773\ L/2$$

Also from the above

$$\frac{du}{dx} = k'\left(\frac{x^2}{2} - \frac{L^2}{24}\right) \quad \text{and}$$

(4) $u = k'\left(\frac{x^3}{6} - \frac{L^2 x}{24}\right)$, which is the equation of the elastic curve desired, that is the curve of the centerline of the beam after lateral deflection begins.

Each of the elements previously defined must now be divided into two parts, one extending from the end of the beam to a point 0.5773 of the distance to the center of the beam (that is to the point where u is a maximum) and the other part extending from this point to the center of the beam, its distance being

$$L/2 - 0.5773\ L/2 = 0.4227\ L/2$$

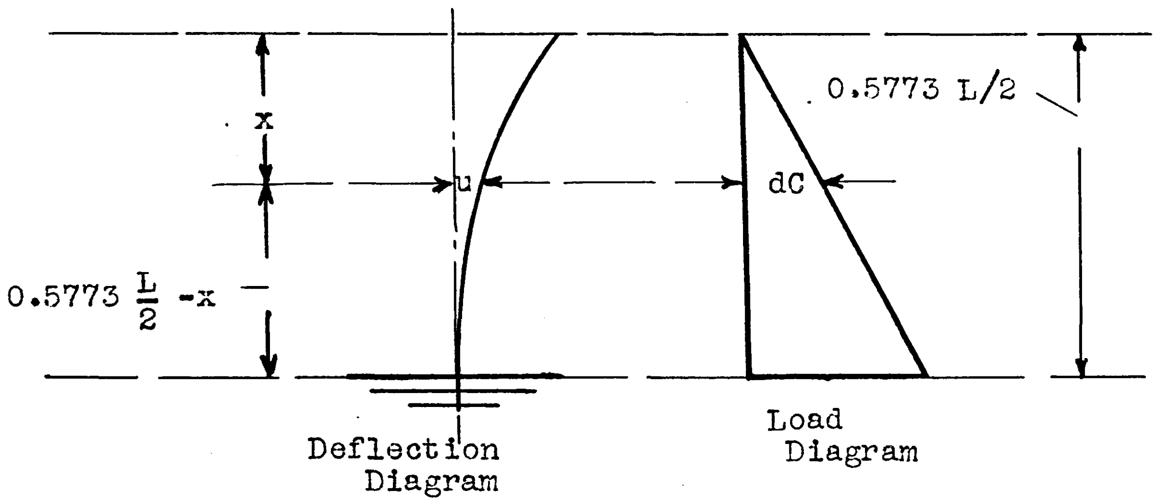
The first and longer of these elementary columns may be considered as fixed at one end, free at the other, and having a uniformly varying load increasing as the fixed end is approached.

The other also may be considered as fixed at one end and free at the other, but having a combined load consisting of a uniform end load equal to the reaction of the first part, and a uniformly varying load decreasing as the fixed end is approached.

Each of these may now be treated in the manner employed in the deduction of the Euler formulas, and it is conceivable that the critical differential compression in each may be determined. It would then be necessary to integrate along y to complete the colution.

As previously stated the authors have not been able in the time allotted for this thesis to carry out the rather lengthy graphical solution which seems necessary for some of the integrals encountered in the treatment suggested in the preceeding paragraph. The derivation of the first of these integrals will be given, however.

Fig. 4



Rewriting the condensed form of the original equation of this method, we have

$$dC = \frac{bW}{2I} xy\ dy.$$

If we consider this equality applied to any layer, that is for any constant value of y, it may be written better as

$$dC = \frac{bWydy}{2\ I}\ x$$

where bWydy/2I is a constant.

*****

Let us now solve the equation of the elastic curve

(4) $$u = k'\left(\frac{x^3}{6} - \frac{L^2x}{24}\right)$$

for x in terms of u so that we may substitute in (5) in order to get the bending moment acting on the elementary column at its base, that is when

$$x = 0.5773\ L/2$$

$$6u = k'(x^3 - \frac{L^2}{4} x)$$

$$x^3 = (\frac{L^2}{4}) x + (\frac{6u}{k'})$$

The solution of this (See "Mechanical Engineers' Handbook", edited by L.S.Marks, McGraw-Hill Book Co., page 117) provided that $L^6/1728$ be greater than $9u^2/(k')^2$, which is reasonable, since we have decided that u must remain very small in order that the elementary columns may be free to buckle in the plane of b, is

$$x = 2(\frac{L}{\sqrt{12}}) \cos \frac{\cos^{-1}(\frac{3u}{k'} \frac{L\sqrt[3]{L}}{\sqrt[3]{144}})}{3}$$

$$= 0.5773\ L \cos \frac{\cos^{-1} \frac{uL\sqrt[3]{L}}{1.747k'}}{3}$$

*****

We may therefore write

$$dC = \frac{bWydy}{2\ I} (0.5773\ L \cos \frac{\cos^{-1} \frac{u\ L\sqrt[3]{L}}{1.747k'}}{3})$$

$$= 0.2887 \frac{bLWydy}{I} \cos \frac{\cos^{-1} \frac{uL\sqrt[3]{L}}{1.747k'}}{3} .$$

Here, now, if we actually permit no lateral deflection both u and k' are zero and the expression is indeterminate, but if a small deflection be assumed, say 0.01 inch, it seems that it should then be possible to compute both u and k' for a series of values of x and thus to derive an expression for dC, and by integrating again derive the bending moment in the part of the elementary column which we are now considering, and so on through the computations analagous to those used in deriving the Euler formulas. The process must then be repeated for the second part of the elementary column. Then the two parts must be joined exactly as the Euler formula is deduced for a column with, for example, fixed ends. And lastly the critical differential compression resulting must be integrated over the entire range of y.

Experimental Work

The experimental work, which was the major task of this thesis, was divided into three sets of tests as previously stated.

For the determination of the effect of the depth-breadth ratio the specimens in the machine were supported at their ends so that they were free to deflect in their own plane (to allow for vertical deflection) and comparatively free to deflect laterally. The ends were supported on rollers so that there could be no horizontal external forces applied to the beam.

For the determination of the effect of span only three specimens were used. The apparatus was the same used in determining the effect of depth-breadth ratio. As soon as a specimen failed at one span the span was shortened by moving both end yokes toward the center and then retesting. No tests were made in a specimen after any permanent distortion had occurred.

For the determination of the effect of distributing the load specimens not damaged by the tests for the determination of the effect of depth-breadth ratio were retested with the load applied at the third points. The apparatus was the same as in the other two cases with the addition of an I-beam and pin described under "Apparatus".

The wood chosen for the tests was western spruce, since that wood is most frequently used for airplane spars, the design of which encouraged this thesis.

The sizes selected were such as would conveniently fit the apparatus available (described later). Three specimens of each size were considered sufficient.The depth-breadth ratios were selected to give both lateral failures and tensil and compressive failures. They were also so selected as to fall roughly into as few groups,each group of constant section modulus,as possible, since it was believed at first that the section modulus had a very important relation to lateral failure.

For the complete record of the characteristics of each specimen see the section headed "Specimens."

V

APPARATUS

Apparatus

For all three sets of tests which were made, the same apparatus, with minor adaptations in each case, was used.

The testing machine used was the old Olsen, 50,000 lb., hand operated machine in room 1-210 of the Massachusetts Institute of Technology.

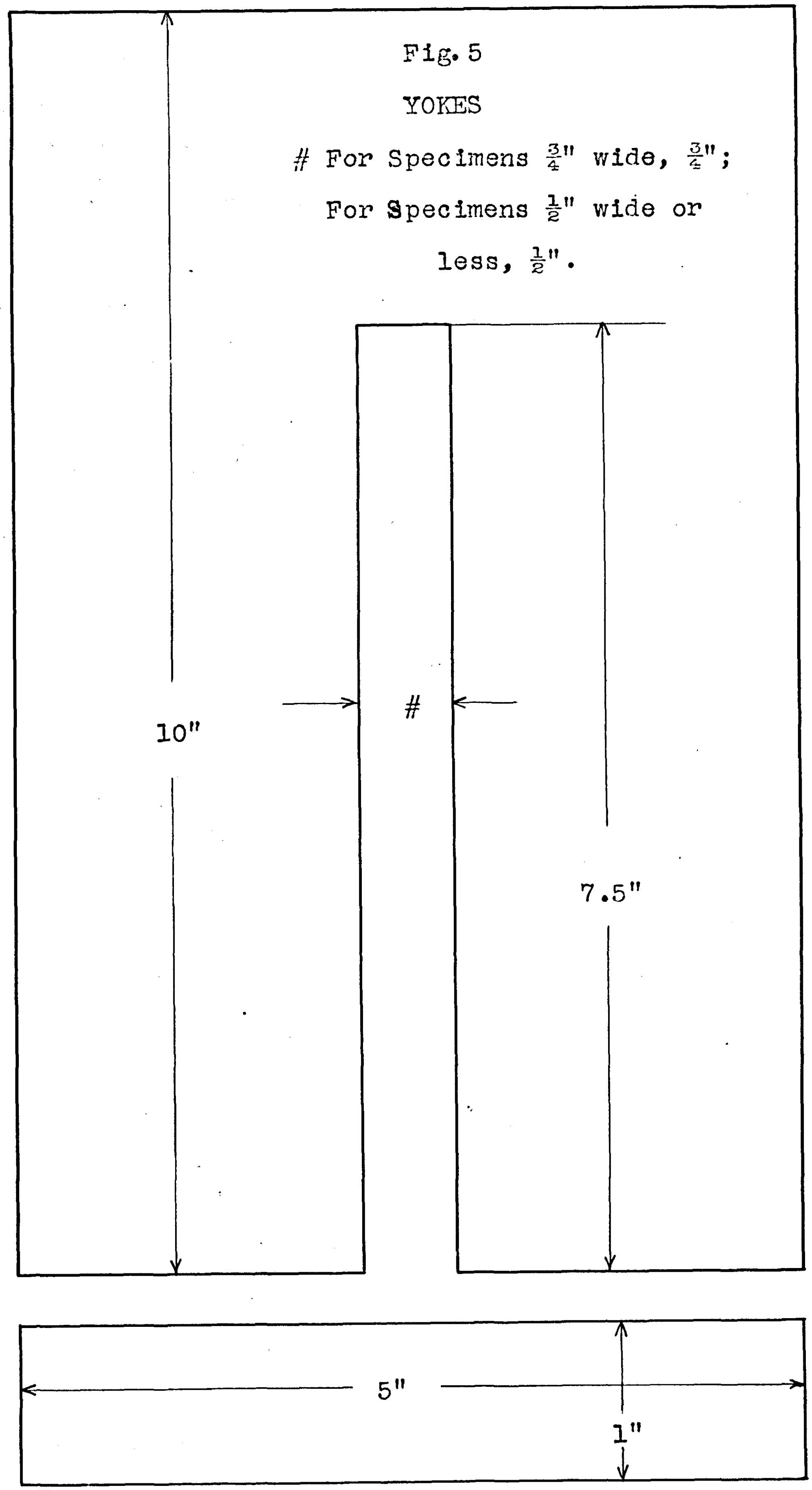
The general arrangement of the apparatus is best shown in Fig. 15 . A four inch steel I-beam, about five feet in length, was laid on the bed of the machine. On it was placed the assembly of apparatus containing the following groups in addition to the specimen: (a) the yokes and attachments, (b) the support assemblies, and (c) the head assembly.

Yokes, described further in Figure five were affixed to the specimen at the points of support and at the points of loading. Figs. 6 , 7 , and 8 , show how the yokes were attached to the specimen. At the points of loading, as shown best in Fig. 6 , the load which was applied through the yoke was transmitted to the specimen through in order, a steel bar and a wooden block. The blocks are further described in Fig. 9 and its table. .With the exception of a few cases it was found unnecessary to distribute the load at the points of support. Therefore no bars or blocks were, in general, used there. The yokes were fastened rigidly to the specimen by making them fit well by inserting shims made from common detail drawing paper, and in

Fig. 5

YOKES

\# For Specimens $\frac{3}{4}$" wide, $\frac{3}{4}$";
For Specimens $\frac{1}{2}$" wide or
less, $\frac{1}{2}$".



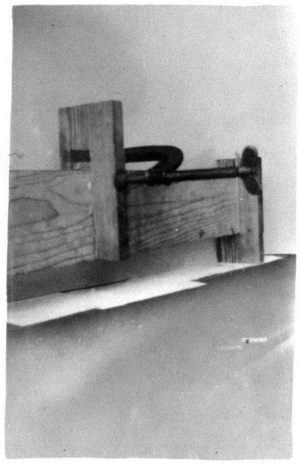


Fig. 6. Method of clamping yoke to the specimen, showing the steel bar and wooden block for distributing the load into the specimen to prevent crushing. The paper shims may also be seen.

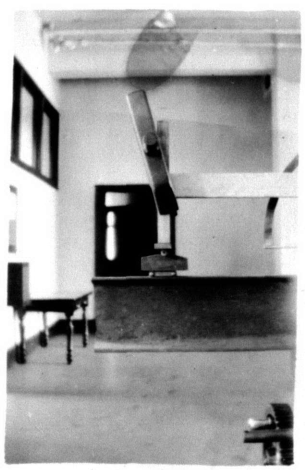


Fig. 7. Assembly at the end support, showing yoke resting, successively, on support bar, pin, support block, rollers, and I-beam.

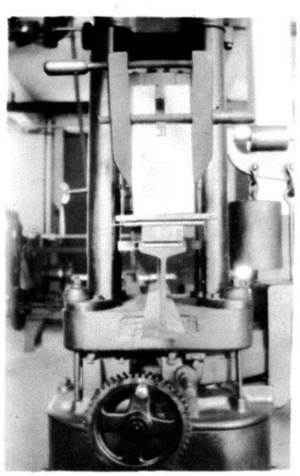
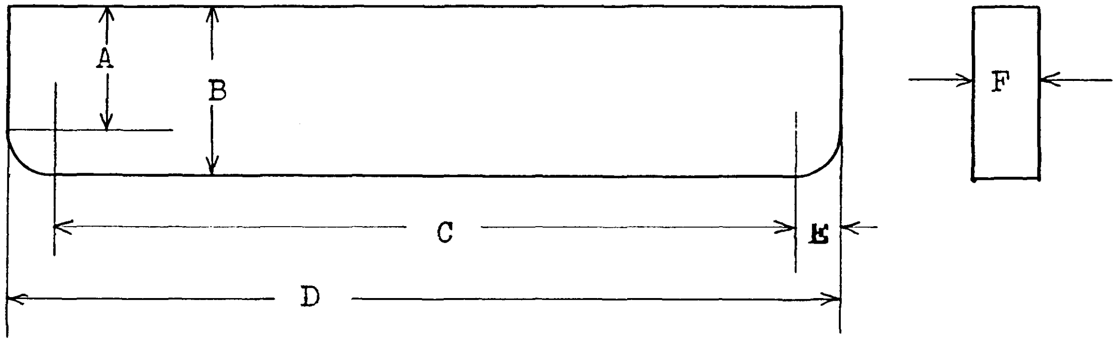


Fig. 8. End view of the same assembly shown by Fig. 7, above.

Fig. 9

LOAD BLOCKS



Block	A	B	C	D	E	F	B-A
1	.58	.65	2.90	3.27	.19	.50	.07
2	.63	.68	2.84	3.15	.16	.75	.05
1A	.55	.63	2.66	2.88	.11	.49	.08
2A	.52	.62	4.10	4.30	.10	.73	.10
2B	.52	.62	3.00	4.30	.58	.73	.10
1B	.40	.54	3.80	4.30	.25	.50	.14
32	.40	.52	2.30	2.85	.28	.53	.12
33	.40	.50	3.60	4.00	.20	.50	.10
34	.40	.50	5.70	6.00	.15	.47	.10

All dimensions in inches.

Table 1.

some cases from thin strips of wood,between the sides of the specimen and the yoke,and clamping the whole as shown in the photographs. It may be noted that it was necessary to use a slightly different sort of clamp at the loading points which came directly under the head of the machine on account of the lack of space there.

The support assemblies,which held the end yokes in position,received the load from the yokes (as shown in Figs. 7 and 8) through,in order, a support bar,a pin, a support block, and rollers which rested on the I-beam. The purpose of the support bar was to prevent the pin from sinking into the yoke when the deflections were being measured for obtaining the modulus of elasticity.

The head assembly varied, depending upon whether the test was made for the determination of the effect of load distribution, or for the determination of the effect of span or depth-breadth ratio. In the former case as shown in Fig. 21 ,a two inch I-beam was laid across the yokes at the points of loading, and the load transmitted to this I-beam at a point midway between the yokes from the head through a pin, as shown in Fig. 22 . In the latter case the load was applied direct from the head to the yoke as may be seen in Fig. 15 .

In all the tests the head of the machine was left free to adjust itself, the two little half-collars not being in place.

Before each test, before the head of the machine was brought down into contact with the assembly of apparatus just described, the scale was balanced. This balance also included the weight to the deflectometer used to measure the vertical deflection to be used in calculating the modulus of elasticity, the deflectometer being kept on the bed of the machine after it had been removed from in under the specimen.

The deflectometers used are shown in Fig. 12 .

For the determination of moisture the oven and scales shown in Fig, 10 ,were used. The oven was heated by means of a Bunsen burner.

For determining the weights to be used in the calculation of the specific gravity the scales shown in Fig. 11 were used.

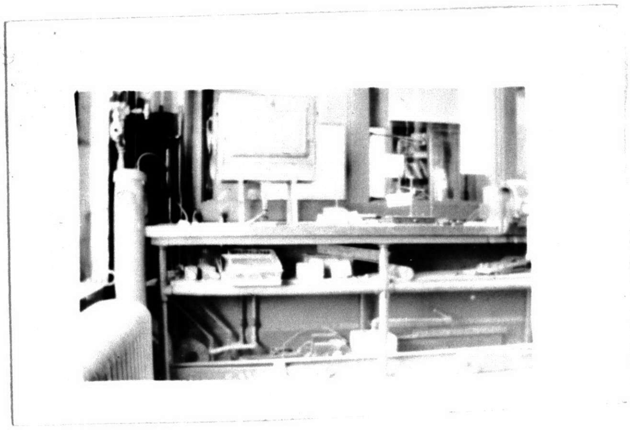


Fig.10. Oven and scales for finding moisture content of the specimens

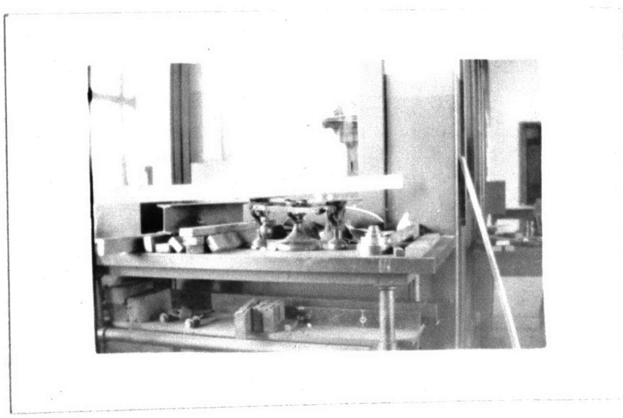


Fig.11. Scales used for finding specific gravity of specimens.

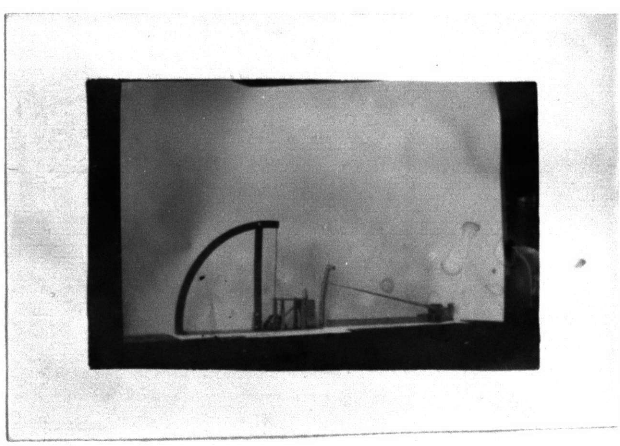


Fig.12. Deflectometers used for finding deflections from which the modulus of elasticity was calculated.

VI

SPECIMENS

Specimens

The specimens used were purchased from the Pigeon Brothers' lumber yard in East Boston, Mass., and were declared by them to be of an average grade of kiln dried western or sitka spruce, which had been stored under cover (the specimens were purchased in the months of December through March), and all from the same shipment and apparently from the same tree.

This similarity of the specimens has been considered an advantage in this case, whereas it is usually felt that the best average for a set of tests is obtained when the specimens are from as great a number of trees as possible. Here, however, where the object was a matter of comparison it is believed that specimens all from the same tree should improve the accuracy.

In all, the twenty-seven specimens afforded fifty-four tests. The specimens were composed into nine groups of three each, each group being composed of specimens of the same approximate dimensions. Each specimen was marked with a number and a letter, the number being that of the group to which it belonged and the letter, A, B, or C, distinguishing it from the other specimens in that group.

All the specimens were 48 inches in length, except those in group 9 which were originally 58 inches in length.

On pages 34,35, and 36,are sketches of the specimens. These, together with Table 2 ,page 40,comprise the record of the specimens which has been kept.

Wavy or curley grain has been indicated on the sketches by wavy lines. Specimen 1C', for instance, showed wavy grain along about half its breadth.

Sap wood has been indicated by roughly cross-hatching in red.

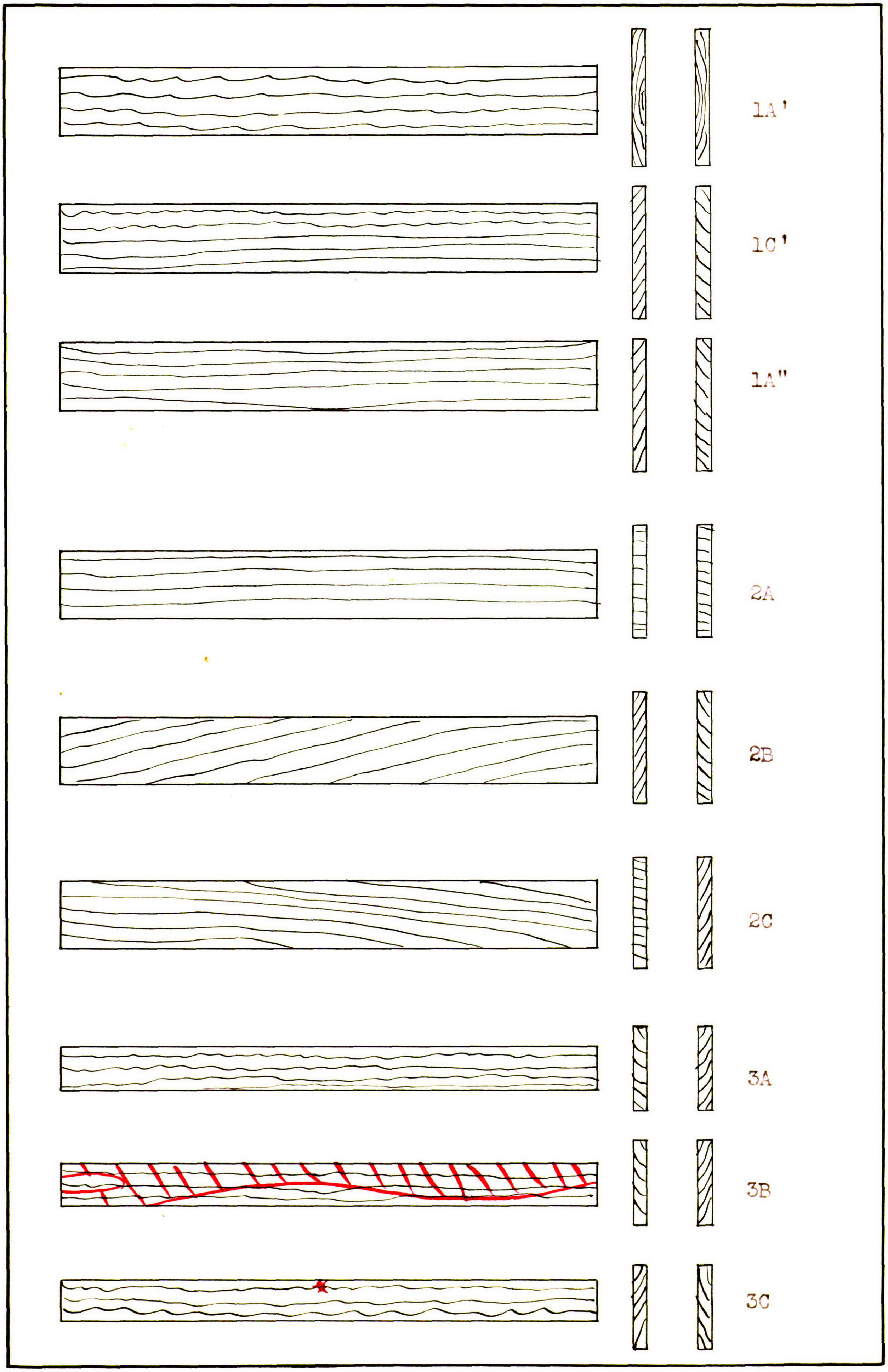
On one specimen a knot indicated by the red mark in the sketch was under compression during the test.

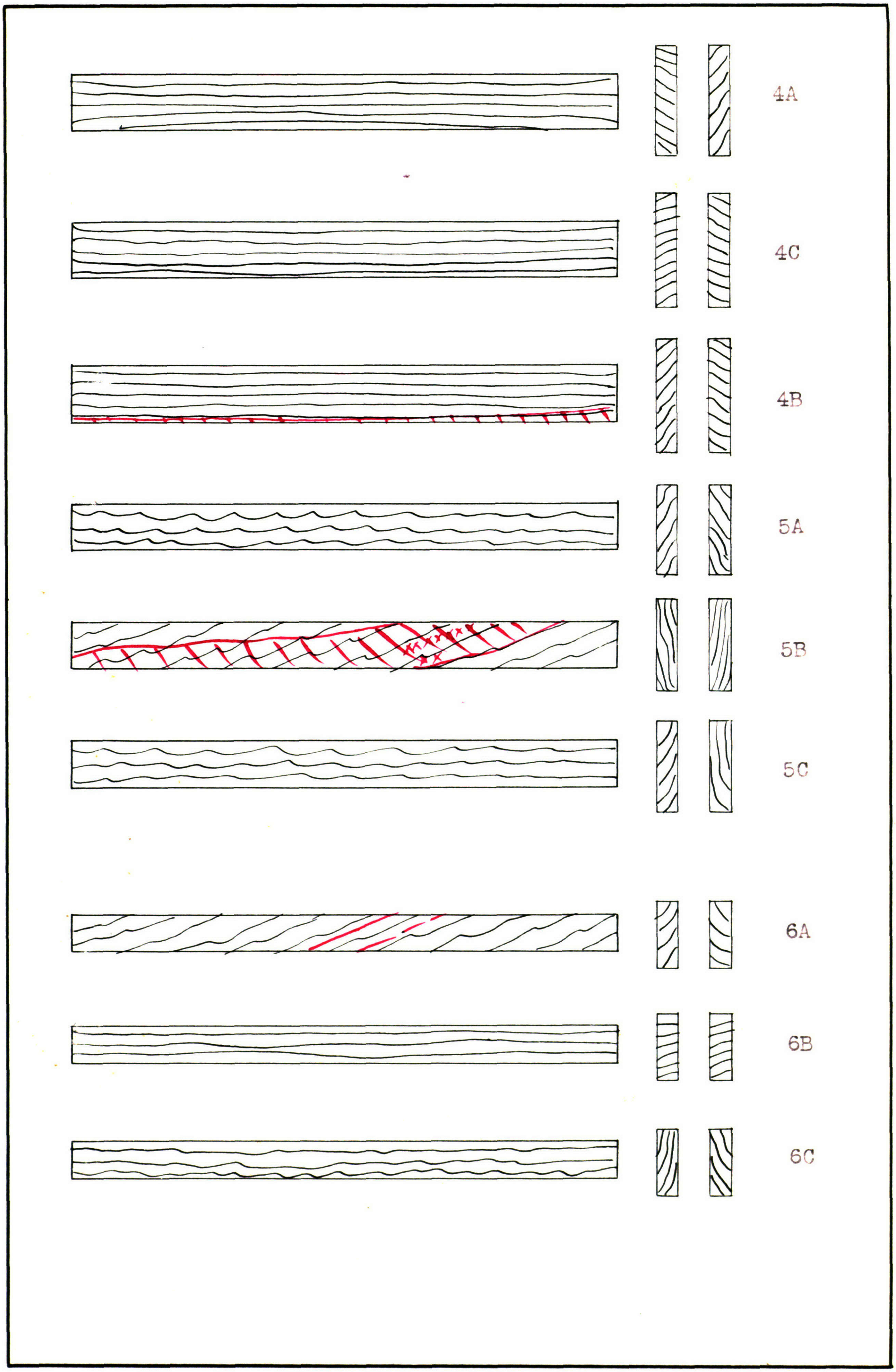
The series of red crosses on the sketch of specimen 5B represents the position of the fracture which was caused by a tension break, originating on the lower side of the specimen as sketched,the side under compression in the machine.

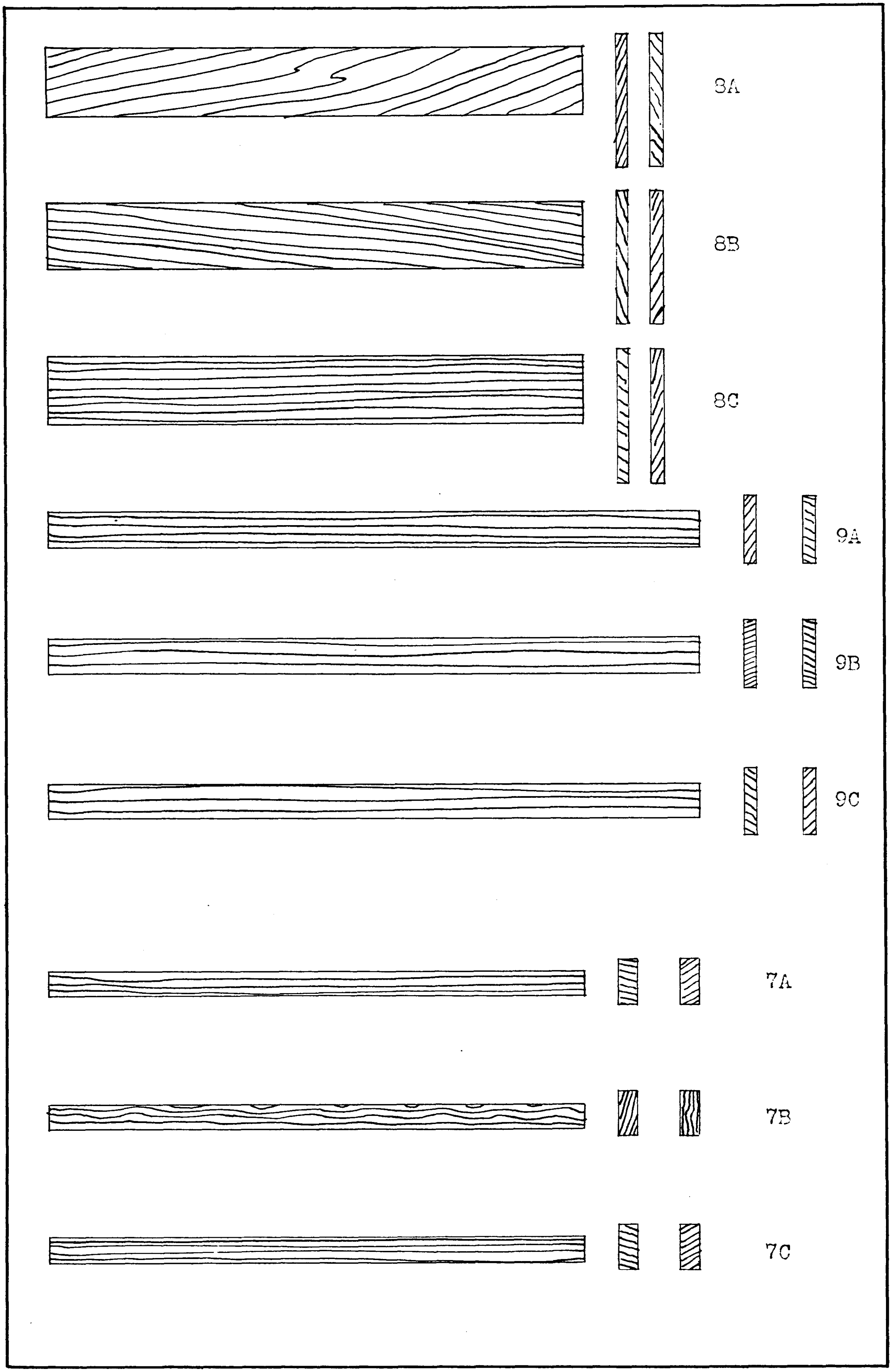
The full red line in the sketch of specimen 6A represents the position of a slight crack, perhaps due to checking,originally in the specimen. The dashed line approximately parallel to it represents the position of the fracture.

In Table 2 , page 40, b and h mean values in inches measured with an engineers' scale, from which h/b, y/I, and I were calculated. I and y/I are in inch units.

Determination of the grain slope was made in this manner: Make an ink of a solution of pitch in xylol. With a sharp pointed pen dipped in this ink prick the


1A'
1C'
1A"
2A
2B
2C
3A
3B
3C


4A
4C
4B
5A
5B
5C
6A
6B
6C


8A
8B
8C
9A
9B
9C
7A
7B
7C

surface of the wood of which the grain slope is desired. The ink will run by capillary action along the grain, for an eighth of an inch or so.Then with a fresh pen of ink again prick the wood at the point where the ink ceased to run further,and so on until a line of several inches has been established. By extending this parallel to itself the direction of the grain may be measured. The authors were surprised at the accuracy of this method compared with the slope determined by the slope of tension fractures in the wood under test. This method is the same used by the Forest Products Laboratory in their Project 228-4 from which the data for the correction of the results of this thesis for grain slope were taken.The slopes tabulated are the number of inches along the length of the specimen per inch of rise of the grain. For instance a slope of 25 corresponds to a slope of 0.80 inches in 20 as it is sometimes recorded.

Percent summer growth was established while viewing the ends of the specimen.

Percent moisture is based on the dry weight. It was determined in this manner: With a saw the specimen was cut across the grain into strips about a quarter of an inch wide. Twenty grams exactly from each specimen in the form of these strips were dried to a constant weight in an oven,Fig 10, and the constant weight recorded.This drying required about one and three-quarters to two hours. The temperatture of the oven was within a degree or two of 212°F. during the process.Though at this temperature other constituents than the moisture in the wood are driven off it is assumed that only the moisture is removed.The final constant weight divided into 100 times the difference between the final weight and the original twenty grams gives the percent moisture. It may be noted that in general the moisture content is such as to indicate that the specimens were kiln dried timber as ordered. This method was also used by the Forest Products Laboratory in the preparation of their Bulletin 70 on which our moisture corrections are based.

Rate of growth was taken by measuring the annual rings on the ends of the specimens. As good an average as possible with the limited area over which to measure was recorded.

Specific Gravity was calculated in the following manner: The specimen was weighed and the density calculated in pounds per cubic foot. This,the density as tested, multiplied by the fractional part of dry wood in the specimen (determined from the moisture content) divided by 62.5,the density of standard water,gave the specific gravity recorded.The formula for specific gravity is , therefore,

$$SG = \frac{100\ W}{62.5(100-\%M)\ b\ h\ L}$$

$$= \frac{1.6\ W}{b\ h\ L(100+\%M)},$$

where W is the weight of the specimen tested in pounds;

b, the depth of the specimen in feet;

h, the depth of the specimen in feet;

L, the length of the specimen in feet;and

%M ,the percent moisture determined by the method previously outlined, based on dry weight.

Specific Gravity was determined in this way by the Forest Products Laboratory in their "Notes Bearing on the Use of Spruce in Airplane Construction",and other publications from which our data for specific gravity corrections were taken.

TABLE 2 CHARACTERISTICS OF THE SPECIMENS

Specimen	b	h	h/b	y/I	I	Slope	%SG	%M	RG	SG
1A'	.53	6.00	11.32	.315	9.54	50	40	10.20	18	.397
1C'	.50	5.88	11.76	.347	8.47	200	25	5.26	8	.373
1A"	.53	5.98	11.27	.316	9.46	100	60	11.11	28	.382
2A	.51	4.97	9.75	.476	5.22	50	35	5.15	7	.362
2B	.51	4.94	9.69	.482	5.12	30.3	50	8.23	24	.396
2C	.50	4.90	9.80	.500	4.89	21.8	50	5.26	30	.415
3A	.48	3.72	7.75	.893	2.06	15.9	30	6.39	25	.412
3B	.47	3.70	7.87	.933	1.99	11.0	40	5.82	28	.433
3C	.48	3.71	7.73	.908	2.04	6.9	50	6.05	33	.425
4A	.71	5.00	7.04	.338	7.40	71.7	15	7.07	9	.405
4B	.72	5.00	6.94	.333	7.50	33.3	40	8.94	18	.380
4C	.73	4.99	6.84	.330	7.55	25	50	8.70	12	.392
5A	.73	3.99	5.32	.503	3.97	9.1	20	6.84	30	.387
5B	.74	3.98	5.38	.512	3.89	10.5	40	6.61	32	.380
5C	.75	3.98	5.31	.505	3.94	8.3	40	6.61	39	.384
6A	.75	2.92	3.90	.936	1.56	10.5	30	11.11	40	.384
6B	.75	2.95	3.93	.917	1.61	50	45	6.38	40	.390
6C	.74	2.91	3.94	.957	1.52	8.0	50	6.83	28	.402
7A	.74	2.00	2.70	2.03	.494	33.3	40	6.38	7	.364
7B	.75	2.01	2.68	1.99	.506	18.2	60	13.62	14	.452
7C	.76	2.03	2.67	1.92	.528	100	60	11.72	28	.418
8A	.35	5.88	16.8	.497	5.92	66.7	30	5.26	10	.391
8B	.35	5.90	16.8	.492	6.01	66.7	30	5.26	9	.385
8C	.35	5.89	16.8	.495	5.95	200	30	5.54	10	.399
9A	.37	3.00	8.12	1.80	.833	200	25	6.95	18	.391
9B	.40	3.00	7.70	1.67	.900	100	30	6.95	21	.391
9C	.38	3.00	8.01	1.78	.843	67	25	6.95	22	.407

Symbol	Significance
b	Breadth of the Specimen
h	Depth of the Specimen
h/b	Depth-Breadth Ratio
y/I	Section Modulus
I	Moment of inertia of the section, $bh^3/12$
Slope	Number of inches for 1 inch rise of grain
%SG	Percent Summer Growth
%M	Percent Moisture
RG	Rate of Growth
SG	Specific Gravity

VII

THE TESTS

The Tests

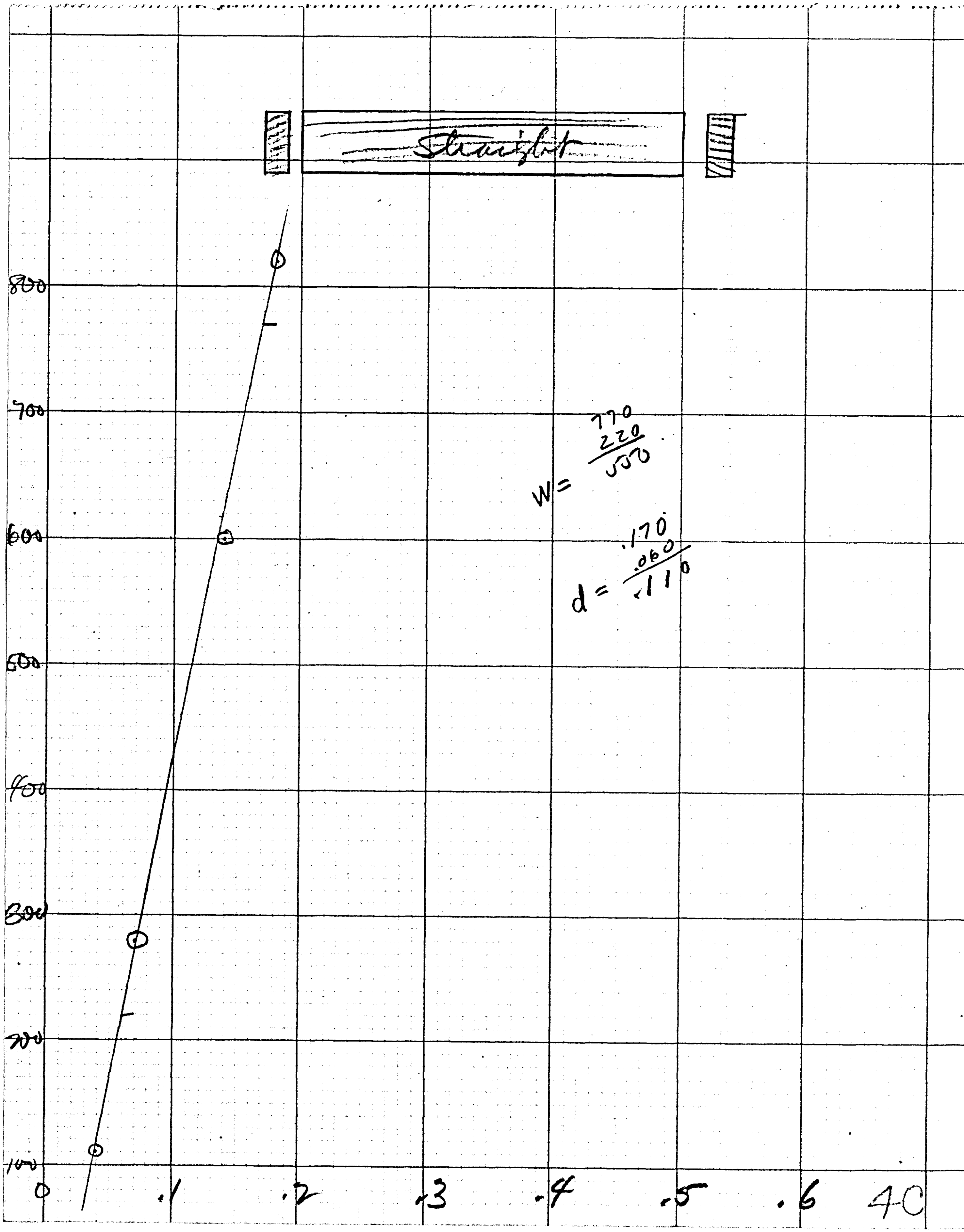
In the test of a specimen in the machine, in general, two things were desired: first, data to calculate the modulus of elasticity; and second, data on the ultimate strength, what the ultimate strength was, and the manner in which the failure occurred.

To secure the data from which the modulus of elasticity might be calculated a plot was kept for each specimen of the vertical deflections at a series of loads well below the maximum which it was assumed the specimen would carry. This plot gave the characteristic straight line of the stress-strain diagram below the elastic limit, the slope of which indicates the modulus of elasticity.

Points on the plot were obtained in this way: A deflectometer (See page 31, Fig.12) was placed under the center of the span to record the maximum vertical deflection. A small load was applied and read when the beam was in balance, and then plotted against the deflection indicated. This was repeated until five or more points defining the line had been obtained. A typical plot, part of the original data, is herewith included as Fig. 13, page 42.

As may be noticed, two points on the straight line on this plot where the line made good intersections with the coordinate lines of the paper have been checked. The vertical and horizontal distances to scale between these two points have been marked on the plot as w and d, w being the load required in pounds to

Fig. 13


Straight
800
700
600
500
400
300
200
100
0
.1
.2
.3
.4
.5
.6
40
W = 770 220 550
d = .170 .060 .110

produce the deflection d in inches. Where the load is a concentrated load applied at the center of a span, the formula for the modulus of elasticity is

$$E = \frac{wL^3}{48\ d\ I},$$

where E is the modulus of elasticity in lbs./sq.in.;

L, the span in inches;

I, the moment of inertia in inches to the fourth power; and

w and d are as above.

In the calculation of the modulus of elasticity from this formula using the values of w and d obtained from the plot the value of I used was that noted in Table 2 page 40, and the value of L that may be termed the effective span.

The effective span in each case is one inch less than the length of the specimen, since the yokes holding the ends of the specimen were each one inch in thickness, had their extreme faces flush with the ends of the specimen, and were centered above the pin on which the bar on which they rested was placed. The change of span with deflection is, of course, neglected. Thus, for the specimens 48 inches long the span used in the formula was 47 inches.

The moduli so obtained are included in Tables 3 and 4 in thousands of pounds per square inch units.

After sufficient points had been obtained on the load vs. deflection plot, the deflectometer was removed and placed on the bed of the machine so as not to disturb the balance of the beam. Then the load was applied while the beam was kept in balance and the failure observed.

If lateral deflection set in it was continued until the beam dropped and failed to rise on the application of more deflection.This maximum load was shown by the position of the rider on the beam is the one recorded,along the notation of lateral failure. When the maximum had been determined the load was released and the specimen removed and examined for permanent set. Specimens used later for tests requiring loading at the third points showed no permanent set after the test using central loading.

If the specimen failed in tension it was so recorded and the load of failure as shown by the position of the rider noted.

If the specimen began to show sign of a crushing failure application of deflection was continued until either a maximum load was reached or until the specimen failed in tension. The load noted is the maximum reading on the scale obtained for the specimen,and the manner of failure noted is the manner which appeared most directly to cause the load to reach the maximum,

Figs.14 and 15 show specimens under center loads. Fig.16 shows a specimen having been so loaded and broken in tension.

TABLE 3

ORIGINAL TEST DATA

Specimen	Failure Load	Failure Manner	Apparent E/1000	Apparent f	Block	h/b
1A'	1660	lat.	1062	6140	1A	11.32
1C'	1450	lat.	873	5920	1B	11.76
1A"	1925	lat.	1043	7150	1B	11.27
2A	1060	lat.	1220	5900	1A	9.75
2B	1515	lat.	1310	8600	1A	9.69
2C	1565	lat.	1230	9200	1A	9.80
3A	830	lat.	1330	8700	1A	7.75
3B	830	lat.	1280	9100	1A	7.87
3C	770	lat.	1220	8200	1A	7.73
4A	2240	ten.	1360	8900	2B	7.04
4B	2360	com.	1280	9300	2A	6.94
4C	2320	com.	1440	9000	2B	6.84
5A	1580	ten.	1190	9300	2A	5.32
5B	1500	ten.	1300	9050	2A	5.38
5C	1660	ten.	1240	9850	2B	5.31
6A	870	ten.	1470	9570	2B	3.90
6B	690	ten.	1342	7420	2B	3.93
6C	930	ten.	1356	10410	2B	3.94
7A	450	com.	1570	10730	2B	2.70
7B	460	com.	1990	10760	2B	2.68
7C	420	com.	1310	9490	2B	2.67
8A	600	lat.	1220	3500	1A	16.8
8B	510	lat.	1125	2950	1A	16.8
8C	720	lat.	1383	4190	1A	16.8

Load is maximum scale reading in pounds

lat. signifies lateral failure
com. signifies compression failure
ten. signifies tension failure

E is Modulus of Elasticity calculated from plot made as the specimen was loaded.

f is apparent modulus of rupture figured from the load given here.

Block noted is the one used to distribute the load and prevent crushing at the center of the span.

h/b is the depth-breadth ratio of the specimen,

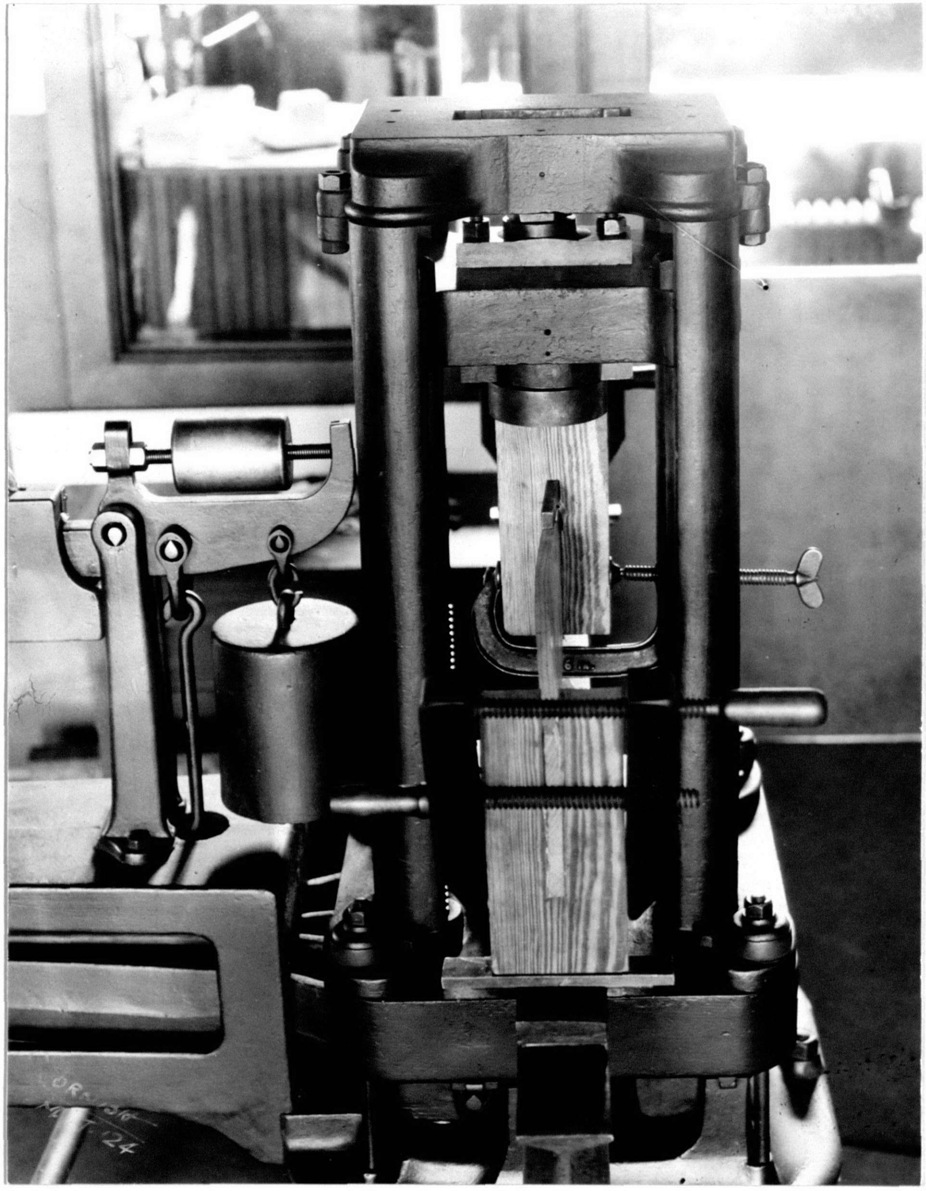


Fig. 14. Showing a specimen under a central concentrated load having failed laterally. The half of the span which may be seen has deflected to the left. Note that the tension edge remains in its own plane, and that the section is held vertical at the yokes.

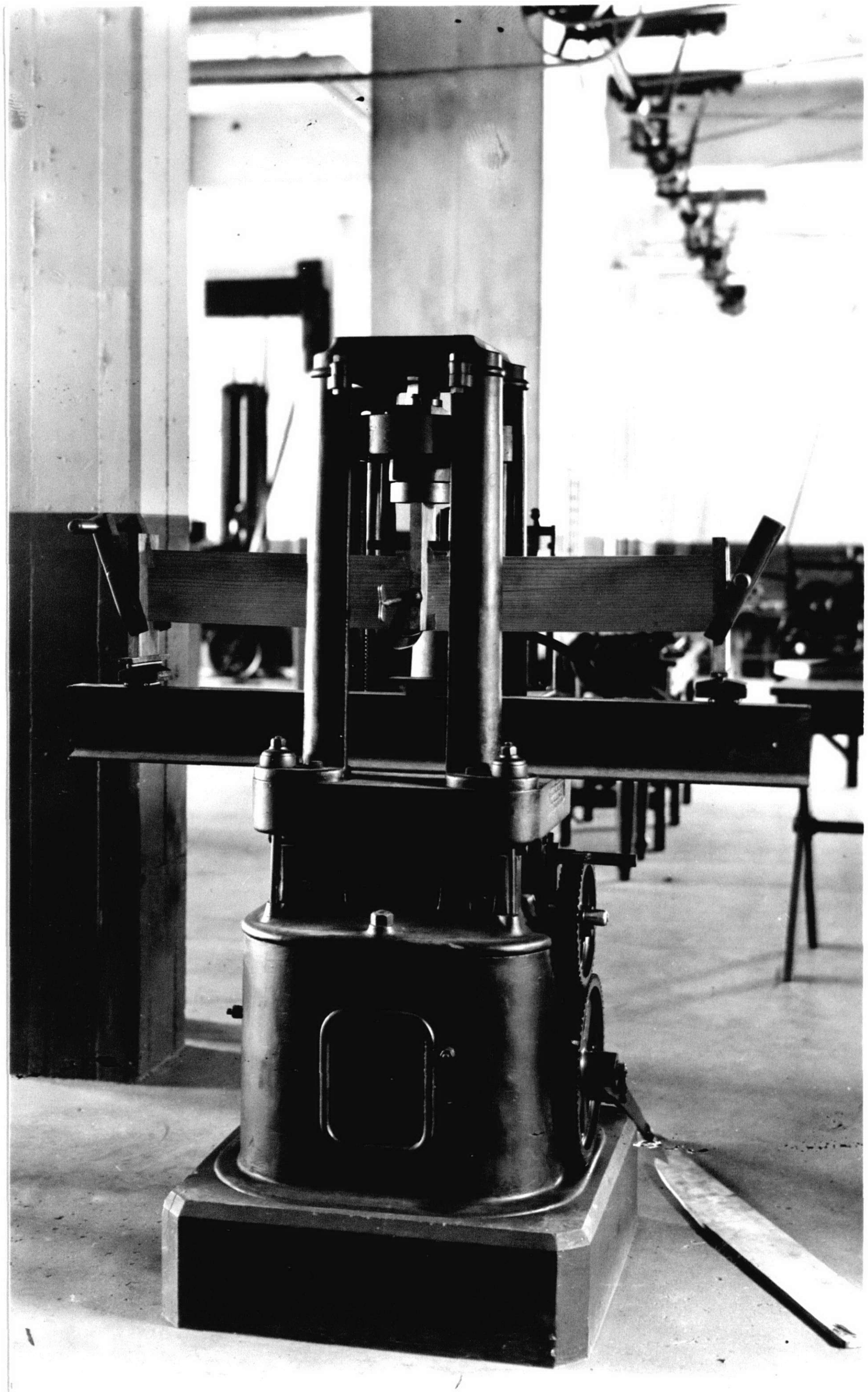


Fig. 15. The same as in Fig.14, taken from another angle before the load was removed. The specimen is still deflected laterally. Note the vertical deflection, and the general arrangement of the apparatus.

Tests for the effect of span were begun in exactly the same way as for the effect of depth-breadth ratio. Of course each of the specimens failed laterally at the long span. Unfortunately it was not realized that the modulus of elasticity of the specimen might vary with span, and the load-deflection curve was plotted only for the 57 inch span. After the test on this span had been made it was shortened progressively to 51, 45, 40, 35, and 30 inches effective span. Lateral failure occurred on all of these but the 30 inch span for all three specimens and produced no permanent distortions which could be observed. A length was then cut from the specimen 26 inches from end to end and tested with an effective span of 25 inches. From two of the specimens, 9A and 9C, 31 inch lengths were also cut, and the tests at the 30 inch lengths run over. This was done because the first 30 inch span tests on these specimens did not seem to be very accurate. The loads recorded by the second tests on this length were much higher and agreed better with that from 9B.

The tests to determine the effect of distributing the load were few and as a result the data obtained is rather incomplete. Here again, unfortunately, it was not realized that the modulus of elasticity might vary with the distribution of the load and the load-deflection charts were not plotted at all. Using the apparatus previously described the tests were run off in the usual way. They were considered complete on a specimen as soon as the ultimate load had been reached.

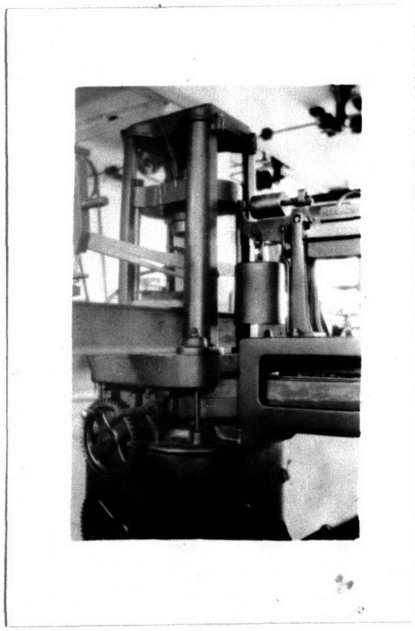


Fig.16. Specimen of low depth-breadth ratio, having been loaded at the center and broken in tension.

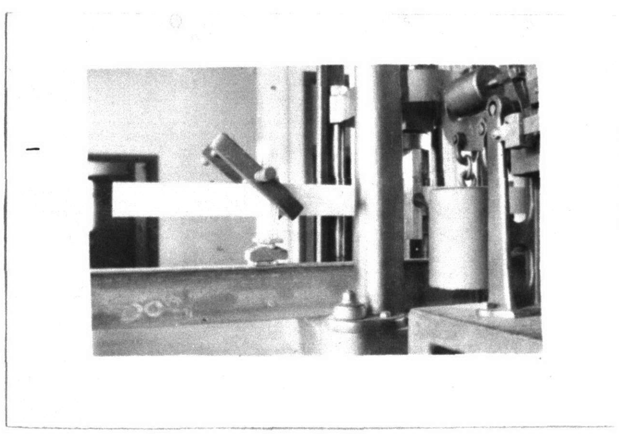


Fig.17. A span test, showing the manner of shortening the span after the test on the next longer span had shown lateral failure without permanent set.

TABLE 4

ORIGINAL TEST DATA

Span	Failure Load	Failure Manner	Apparent E/1000	Apparent f	Block	h/b	Specimen
57	230	lat.	1830	5910	1A	8.12	9A
57	260	lat.	1750	6180	1A	7.50	9B
57	245	lat.	2055	6210	1A	8.01	9C
51	270	lat.		6200	1A	8.12	9A
51	290	lat.		6170	1A	7.50	9B
51	290	lat.		6580	1A	8.01	9C
45	370	lat.		7480	1A	8.12	9A
45	440	lat.		8250	1A	7.50	9B
45	375	lat.		7510	1A	8.01	9C
40	440	lat.		7920	1A	8.12	9A
40	520	lat.		8670	1A	7.50	9B
40	490	lat.		8720	1A	8.01	9C
35	570	lat.		8970	1A	8.12	9A
35	680	lat.		9920	1A	7.50	9B
35	600	lat.		9360	1A	8.01	9C
30	810	lat.		10930	1B	8.12	9A
30	960	lat.		12000	1B	7.50	9B
30	910	lat.		12150	1B	8.01	9C
25	970	com.		10900	1B	8.12	9A
25	1120	com.		11670	1B	7.50	9B
25	1025	com.		11400	1B	8.01	9C

Load is the maximum scale reading in poinds.

Lateral failure is signified by lat.
Compression " " " com.

E is the modulus of elasticity in pounds per square inch calculated from plot made as the specimen was loaded with 57 inch span.

f is the apparent modulus of rupture, figured from the load given here.

Block noted is the one used to distribute the load and prevent crushing at the center of the span.

h/b is the depth-breadth ratio of the specimen.

Crushing across the grain where the end yokes transmitted the supporting vertical forces into the specimen appeared in some of the heavier specimens to disturb the accuracy of the tests. To overcome this crushing it was necessary to place what have here, for the sake of distinction, been called chips between the yoke and the specimen in order to distribute the load.

The chips have been tabulated in table 5. Of course the chips went in pairs, one for each end of the specimen.

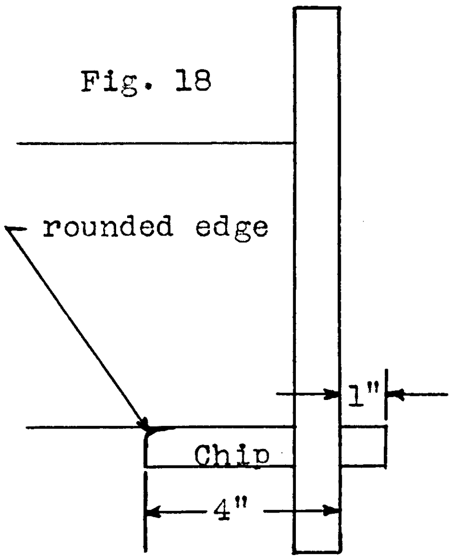
Chips 1-1 were small pieces of wood $\frac{1}{2}$" x 3" x 1/16" thick, and very flexible. It was felt that their use distributed the load just enough to prevent crushing.

Chips 2-2 were of wood, similar to chips 3-3, but were themselves crushed the first time they were used.

Chips 3-3 were of steel 1" x 5" x $\frac{1}{2}$". One corner was rounded off slightly. They were placed as shown in Figs. 18 and 19

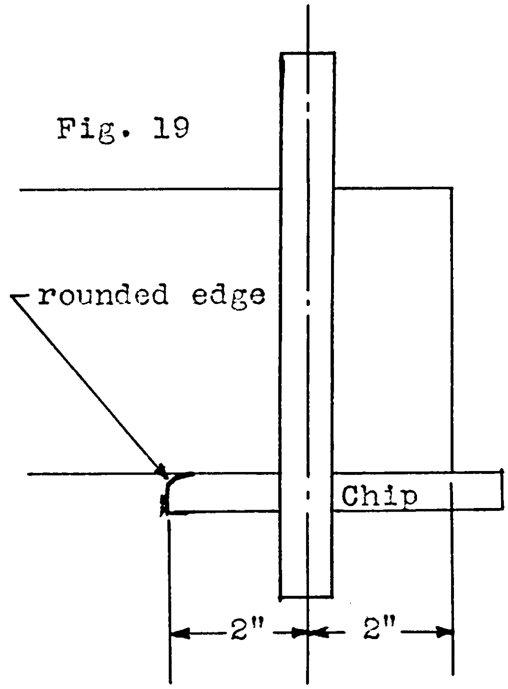
The details of the use of these chips may be found in the History of the Distributed Load Tests on pages 61 and 62.

Fig. 18



First Position

Fig. 19



Second Position

****

Figures 20, 21, and 22 show something of the way in which these tests were carried out.

Throughout this thesis the term "crushing" has been considered to mean such failure of the grain structure as occasioned the use of chips, whereas a failure in "compression" refers to the crushing of the grain by excessive compressive forces on the compression side of the neutral axis.

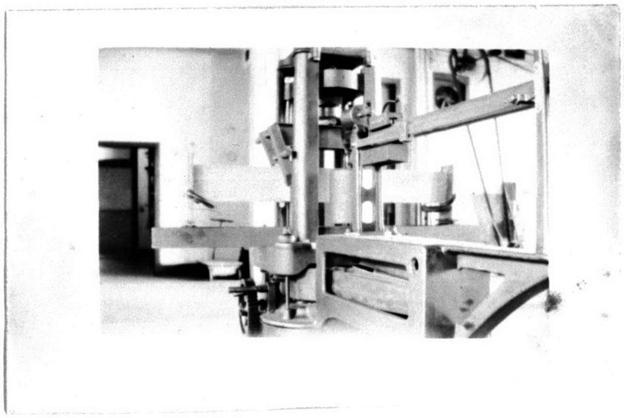


Fig. 20. A general view of the machine and arrangement of apparatus for the test of a specimen loaded at the third points.

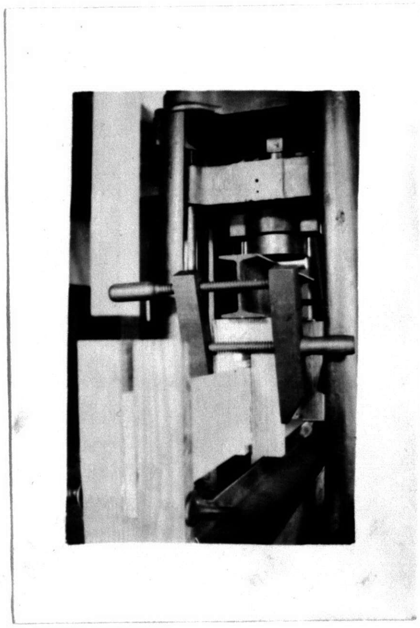


Fig.21. Showing a little more in detail the position of the short I-beam in the head assembly.

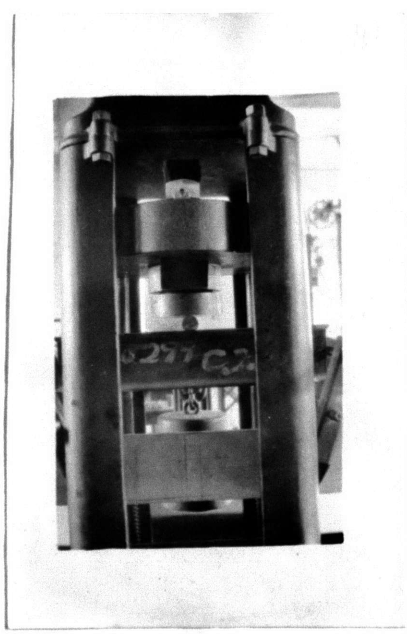


Fig.22. An end view, looking towards and parallel to the weighing beam, showing an end view of the pin which transmitted the deflections from the head to the short I-beam.

TABLE 5

ORIGINAL TEST DATA

Specimen	Failure Load	Manner	Apparent f	Block	h/b	Span	a	Chips
8A	910	lat.	3540	32	16.8	47	15.67	
8B	950	lat.	3660	32	16.8	47	15.67	
8C	950	lat.	3690	32	16.8	47	15.67	
3A	1065	lat.	7450	32	7.75	47	15.67	
3B	1160	lat.	8490	32	7.87	47	15.67	
3C	870	*	6200	32	7.73	47	15.67	
2A	1770	lat.	6600	32	9.75	47	15.67	1-1
1A'	3380	lat.	7540	33	11.32	44	14.17	3-3
1C'	2560	ten.	6300	34	11.76	44	14.17	3-3

*tension at a knot.

Load is the maximum scale reading in pounds.

lat. signifies lateral failure.
ten. signifies tension failure.

f is the apparent modulus of rupture, figured from the loads given here.

Block noted is the one used to distribute the load and prevent crushing at the center of the span.

h/b is the depth-breadth ratio.

Chips noted are the ones used to prevent crushing at the supports.

a is the arm used in computing the moment in calculating the modulus of rupture.

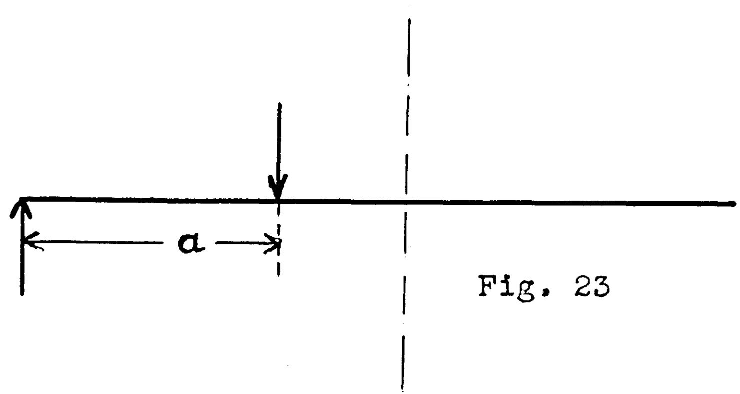


Fig. 23

HISTORY OF THE TESTS ON DEPTH-BREADTH RATIO
(In chronological order)

1A Tested first with the loads and reactions direct through the yokes. Failure by crushing of the grain under the load due to excessive bearing pressure. Specimen inverted in yokes and retested with a pin between head of the machine and the center yoke. Failure by shear at 1580 lbs. load.

1B Tested first as second test on 1A. Failure by crushing. Tested second with Block 1 to prevent crushing and failed laterally at 1350 pounds load. Tested again with the pin removed from the head assembly and failed at 1440 by splitting, but only after a decided lateral deflection.

1C On first test Block 1 which was used failed in shear. With Block 1A specimen split in tension at 1080 pounds load, caused possibly by a local failure from clamping the center yoke too tightly.

NOTE: The above three specimens were not considered to have given reliable tests. In addition to the above data the following information regarding them has been preserved:

Specimen	b	h	h/b	y/I	I	Slope	%SG	%M	RG
1A	.50	5.96	11.92	.338	8.84	23	40	7.0	25
1B	.50	5.88	11.76	.346	8.46	11	25	7.0	21
1C	.52	5.87	11.29	.331	8.75	19	50	5.0	18

Specimen	Failure Load	Failure Manner	Apparent E/1000	Block	Density(lbs/cu.ft.) (as tested)
1A	1585	Crush	980	1	25.1
1B	1350	Crush	970	1	28.3
1C	1080	Split	1180	1A	30.2

2A Lateral failure, max. load was 1060.

3A Lateral failure, max. load was 830.

4A With Block 2 the specimen showed slight crushing under a load of 1500 and some crushing at the supports, also. Specimen was not badly damaged and was inverted and tested with block 2A. A slight crushing was noted at 1730. The load was released and the specimen removed from the machine. Tested again later.

4B This specimen was tested next because of its apparent better ratio of spring to summer growth. Tested with Block 2A with sap wood on tension side. Failed at 2360 by splitting on tension side but showed signs of compression failures also.

5A Compression failure noted at 1500. Ultimate failure in tension at 1580.

2B First sign of failure at 1300, crushing under the load. Failed laterally at 1515. Permanent set from lateral deflection in one half the span only. Section was 0.03" greater in depth on the side on which set occurred.

3B Lateral failure, max. load was 830.

2C Lateral failure, max. load was 1565, but there was also marks of a compression failure.

3C Lateral failure, max.load was 770.

5B Crack due to tension at 1500.

4A Block 2A was rounded off to give new dimensions , and used. Specimen set as in first test on it. Went in tension at 2240.

4C Tension at 2320. Showed signs of excessive bearing under load which may have affected strength.

5C Tension at 1660.

6A)
6B(— All went as indicated on the table of the tests.
6C)

1A' Lateral deflection noted at 1600, maximum at 1660.

1B' Crushed under the load at 1150. This specimen was then thrown out of the tests. Data on it includes: b,.49; h, 5.99; E, 983,000; slope, 67; %SG, 20; %M, 4.5; R.G., 8; and density as tested, 22.4.

7A Went in Compression at 450.

8A)
8B(- All showed signs of lateral failure between
8C)

300 and 400; and reached the ultimate loads noted in the table. 8A was warped slightly.

1C' Block 1B. Specimen showed lateral deflection at 1200 and a maximum at 1450.

1A" Crushed on both ends and very slightly under the yoke. Slight compression failure, but at 1925 it failed laterally.

7C)
7B)- Nothing unusual.

HISTORY OF THE TESTS ON SPAN
(In chronological order)

9C Block 1A under the load. Tests for spans of 57, 51, 45, 40, and 35 inches were all lateral failures and without incident. On the first test of the specimen at an effective span of 30 inches there appeared to be a lateral failure at 575 pounds. This however was at a load only 90 pounds more than on the 35 inch span, and therefore at a lower maximum bending moment. Evidently, then, at the 35 inch span the specimen failed not only laterally but also in compression, the mark of which was not noticed before the 30 inch span test was run.

One end of the specimen was therefore cut off and the second 30 inch span test made with it. This is the one recorded. It showed,also, a compression failure, but the ultimate load was due to a lateral failure. The 25 inch test was without incident.

9A The tests on this specimen went in the same way. Here too a second test was necessary on the 30 inch span, at which the ultimate load was due to lateral failure, but in which compression participated.

9B This specimen acted the same as the other two above. Failure on the 35 inch span went at the same time in both lateral and compressive failures. It was impossible to tell which caused the load to reach a maximum.

On the 30 inch span the specimen seemed to go worst in compression, but whether or not the maximum load was due to this or not is not surely known. This test also is unique in the production of the only example we obtained of a compressive failure of the grain due to the lateral deflection. It was not necessary to carry the deflection past the point of maximum load to secure this phoenomenon. Measurements made on the specimen after it had been removed from the machine showed a compressive failure mark extending from one side of the specimen to the other at a distance of two inches from the center of the span, which was due to the vertical bending; and another compressive failure mark, not so large but nevertheless very definite on one side of the specimen only , the side which was in compression from the lateral bending, at a distance from the center of the span of four inches. The lateral bending did not appear during the test until after the compression failure due to the vertical bending had begun.

The 25 inch span test for this specimen showed lateral deflection, but went ultimately in compression.

HISTORY OF THE TESTS ON LOAD DISTRIBUTION
(In chronological order)

8A Specimen split yoke on one end after showing lateral deflection between 400 and 900. On a second test the double curvature caused by the lateral deflection displaced the loading yoke to one side (load was 980) and the lateral deflection took the form as under a single point load. Tested again later.

8B Tested in the same way as 8A but with more careful allignment of the yokes. Maximum load at 950 pounds. The radius of curvature between the points of loading seemed to be less than between an end and loading yokes.

8A Retested as 8B was tested. This is the result recorded.

8C Same way at 950.

3B Reached a maximum of 1160 after lateral deflection.

3C Knot on the tension side started a split. This test is worthless.

3A Without incident.

2A At 1600 pounds load showed crushing at end yoke. Chips 1-1 were used and the specimen retested. Lateral deflection appeared at 1700, max. at 1720.

1A' Crushed under the end yokes and slightly under the load blochs. Retested, inverted, using blocks 33, and chips 2-2. Crushed chips 2-2. Chips 3-3 used in first position, Fig. 18; then as in second position, Fig.19. Specimen naturally twisted in the yoke in the first position. This second position shortened the span to 44 inches from 47 inches, and since the distance between the loads was kept the same the moment varied only along the span between each end yoke's centerline and a point 14.17 inches from there toward the centerline of the spacimen, where the point of loading occurred. See the figure accompanying table 5 . Usual lateral deflection noticed at 3300 and a maximum load at 3380.

1C' Tension break not caused by any visible imperfections, except that the wood in which it occurred had a reddish tinge, was a sort of sap wood perhaps, and that the rate of growth there was very rapid, about 6 rings per inch.

VIII

CORRECTION OF THE DATA

Correction of the Data

It is a well known fact that the strength of wood is a function of the amount of moisture it contains, that the drier the wood, in general, the stronger it is. It is therefore essential that before the apparent moduli of rupture from the tests can be compared that they should be corrected to allow for the differences in the moisture contents of the specimens.

Not only moisture content affects strength. It is definitely known that grain slope and specific gravity affect it also. And it may be that the rate of growth and quite probably the percent of summer growth are other variables which must be considered.

In the correction of the data obtained from tests made for this thesis the following assumptions have been made to enable correction of the data:

(a) That specific gravity is a function of percent summer growth and rate of growth; and that therefore any correction for specific gravity will include correction for these two variables.

(b) That given any two specimens exactly alike except for moisture content, grain slope, or specific gravity that only their modulus of rupture and not their tendency to fail laterally is affected. That is to say, for example, that the mere drying of a specimen which would fail laterally at a certain load in the moist condition will not cause it to fail in any other way than laterally, but that when dry it will still fail laterally, though perhaps at a higher modulus of rupture. This assumption means also that the data determining whether or not a specimen will fail laterally or not are:(i) the dimensions of the specimen;(ii) the manner of loading;(iii) the end grain. It does not mean that these are the only data considered in the strength of a specimen which fails laterally.

(c) That moisture, specific gravity, and grain slope have the same effect on modulus of rupture whether or not the specimen fails laterally.

Corrections were also made in the moduli of elasticity in exactly the same manner.

After the corrected moduli of rupture had been obtained a corrected maximum bending moment was calculated representing the bending moment which the specimen would have withstood if it had been of what was adopted as the standard wood. This value, M_c, was further corrected to a standard cross-section by multiplying by the three-halves power of the ratio of the cross-sectional areas of the specimen and the standard. This new corrected moment is called M_c'.

Specimen Correction.

Consider specimen 3C.

From table 2 of the "Characteristics of the Specimens", page 40, %M is 6.05. The standard moisture to which all specimens were corrected is 7.36% (chosen because it made as small as possible the average correction). Correction must therefore be made for -1.31 % of moisture. From Forest Service Bulletin 70, Fig. 6, the strength was found to vary in this region 360 pounds per square inch modulus of rupture per percent. We therefore have a moisture correction of 1.31 x 360, or 472 pounds per square inch to subtract from the modulus of rupture given in table 3 of the"Original Test Data," page 45.

From the table of"Characteristics of the Specimens" the grain slope is found to be 1 inch in 6.9 inches. From Project 228-4 of the Forest Products Laboratory, Fig.2, we find that for slopes of one in forty or less there is no appreciable correction for grain slope, and we therefore correct to that value, an amount of 3950 pounds per square inch which must be added to bring the specimen up to the standard.

From the table of "Characteristics of the Specimens" the specific gravity is .425 . From the Forest Products Laboratory's "Notes Bearing on the Use of Spruce in Airplane Construction", Chart 6309M, we find that 795 pounds per square inch must be subtracted from the apparent modulus to correct to a standard specific gravity of .396 which was chosen in order to keep the correction small. There is then to be applied a total correction of -472 + 3950 - 795, or 2683 pounds per square inch to be added. From the Original Test Data, page 45, the apparent modulus of rupture is 8200. This plus 2683 gives a corrected modulus of rupture of 10,880 pounds per square inch since the scale reading was good only to the tens place. This value is denoted by f_c.

The value of y/I for this specimen was .908, giving a corrected moment of 10,880/.908 or 12,000 inch pounds the specimen would have carried had it been of standard wood.

The average area of these specimens was 2.46 square inches. This figure was adopted as a standard cross-sectional area. The area of specimen 3C was 1.78 square inches. The ratio of these areas is 1.382, which to the three-halves power is 1.486. Multiplying 12.000 by 1.486 we get a value of M_c' of 17,800 inch pounds.

A corrected modulus of elasticity has been obtained for each specimen in exactly the same way as the corrected modulus of rupture, using for the moisture correctio 40,000 pounds per square inch per percent moisture (from Fig.14 page 725, Mills, "Materials of Construction"), the slope corrections from Fig.3, Project 228-4 of the Forest Products Laboratory, and the specific gravity corrections from plot (d) page 2 Forest Service Bulletin 676. The corrected value is E_c in the tables.

CORRECTIONS FOR THE MODULUS OF RUPTURE

Specimen	Corrections for Moisture	Grain	Sp.Gr.	Total Correction
1A'	+ 852		- 27	+ 825
1C'	- 756		+632	- 124
1A"	+1125		+384	+1509
2A	- 799		+933	- 134
2B	+ 261	+ 350		+ 611
2C	- 756	+ 700	-521	- 577
3A	- 349	+1100	-439	+ 312
3B	- 554	+1850	-1013	+ 283
3C	- 472	+3950	-795	+2683
4A	- 104		-243	- 347
4B	+ 474	+ 140	+439	+1053
4C	+ 402	+ 550	+110	+1062
5A	- 187	+2350	+243	+2406
5B	- 270	+1950	+439	+2119
5C	- 270	+2750	+329	+3809
6A	+1125	+1950	+329	+3404
6B	- 353		+164	- 189
6C	- 191	+2800	-164	+2445
7A	- 353	+ 140	+878	+ 665
7B	+1878	+ 860	-1536	+1202
7C	+1308		-604	+ 704
8A	- 756		+137	- 619
8B	- 756		+302	- 454
8C	- 656		- 82	- 738
9A	- 140		+130	- 10
9B	- 140		+130	- 10
9C	- 140		-310	-450

The corrections are the amounts in pounds per square inch which must be added or subtracted as indicated to the apparent modulus of rupture as given in table to get the corrected modulus of rupture.

TABLE 6

CORRECTIONS FOR THE MODULUS OF ELASTICITY

Specimen	Corrections for Moisture	Grain	Sp.Gr.	Total Correction
1A'	+114	----	- 3	+111,000
1C'	- 84		+ 69	- 15,000
1A"	+150		+ 42	+192,000
2A	- 88		+102	+ 14,000
2B	+ 35	+ 50	-	+ 35,000
2C	- 84	+130	- 57	- 11,000
3A	- 99	+175	- 48	+ 28,000
3B	- 62	+290	-111	+117,000
3C	- 52	+540	- 87	+401,000
4A	+ 12		- 27	- 15,000
4B	+ 63	+ 50	+ 48	+161,000
4C	- 54	+110	+ 12	+ 68,000
5A	- 21	+360	+ 27	+366,000
5B	- 30	+310	+ 48	+328,000
5C	- 30	+430	+ 36	+436,000
6A	+150	+310	+ 36	+496,000
6B	- 39		+ 18	- 21,000
6C	- 21	+440	- 18	-401,000
7A	- 39	+ 50	+ 96	+107,000
7B	+251	+150	-168	+233,000
7C	+173		- 66	+107,000
8A	- 84		+ 15	- 69,000
8B	- 84		+ 33	- 51,000
8C	- 73		- 9	- 82,000
9A	- 16		- 15	- 31,000
9B	- 16		- 15	- 31,000
9C	- 16		+ 33	+ 17,000

The corrections are the amounts in pounds per square inch which must be added or subtracted as indicated to the apparent modulus of elasticity as given in tables 3 and 4. to get the corrected modulus of elasticity.

TABLE 7

DEPTH-BREADTH TESTS CORRECTED VALUES

Specimen	f_c	M_c	M_c'	E_c	h/b
1A'	6970	22100	15000	1173000	11.32
1C'	5800	16700	12800	858000	11.76
1A"	8660	27400	18500	1235000	11.27
2A	5770	12100	11600	1234000	9.75
2B	9210	18700	18000	1345000	9.69
2C	8620	17200	17400	1219000	9.80
3A	9010	10100	14900	1358000	7.75
3B	9380	10100	15500	1397000	7.87
3C	10880	12000	17800	1621000	7.73
4A	8550	25300	14600	1345000	7.04
4B	10350	30900	17500	1441000	6.94
4C	10060	30500	17000	1508000	6.81
5A	11710	23300	17400	1556000	5.32
5B	11170	21900	16700	1628000	5.38
5C	12660	25100	18800	1676000	5.31
6A	12970	13400	15500	1966000	3.91
6B	7230	7900	9000	1321000	3.93
6C	12860	13400	15800	955000	3.94
7A	11400	5600	10400	1677000	2.70
7B	11960	6000	10900	1757000	2.68
7C	10190	5300	9400	1417000	2.67
8A	2880	5800	7200	1151000	16.8
8B	2500	5100	6300	1074000	16.8
8C	3450	7000	8700	1301000	16.8

f_c is the corrected modulus of rupture in pounds per square inch, the sum of the apparent modulus of rupture from table 3 and the corrections from table 5 .

M_c is the maximum bending moment calculated from f_c. in pound-inches

M_c' is M_c corrected to a constant sectional area of 2.46 square inches, in pound-inches

Ec is the corrected modulus of elasticity.

h/b depth breadth ratio

TABLE 8

SPAN TESTS CORRECTED VALUES

Specimen	f_c	M_c	M_c'	E_c	Span	h/b
9A	5900	3280	10070	1799000	57	8.12
9B	6170	3700	10700	1719000	57	7.50
9C	5760	3230	9790	2072000	57	8.01
9A	6190	3440	10550		51	
9B	6160	3690	10650		51	
9C	6130	3440	10470		51	
9A	7470	4150	12720		45	
9B	8240	4933	14250		45	
9C	7060	3960	12000		45	
9A	7910	4390	13500		40	
9B	8660	5190	15000		40	
9C	8270	4640	14070		40	
9A	8960	4980	15300		35	
9B	9910	5940	17150		35	
9C	8910	5000	15150		35	
9A	10920	6070	18600		30	
9B	11990	7180	20700		30	
9C	11700	6570	20200		30	
9A	10890	6050	18600		25	
9B	11660	6980	20150		25	
9C	10950	6150	18650		25	

f_c is the corrected modulus of rupture in pounds per square inch, the sum of the apparent modulus of rupture from table 4 and the corrections from table 5 .

M_c is the maximum bending moment in pound inches calculated from f_c.

M_c' is M_c corrected to a constant sectional area of 2.46 square inches, in pound inches.

E_c is the corrected modulus of elasticity.

h/b depth-breadth ratio

TABLE 9

Two Point Loading Tests

~~DISTRIBUTED LOAD TESTS~~ CORRECTED VALUES

Specimen	f_c	M_c	M_c'	h/b
8A	2920	5870	7290	16.8
8B	3210	6530	8120	16.8
8C	2950	5950	7400	16.8
3A	7760	8700	11950	7.75
3B	8770	9400	13400	7.87
2A	6470	13600	13200	9.75
1A'	8370	26600	20600	11.32
1C'	6180	19200	16050	11.76

f_c is the corrected modulus of rupture in pounds per square inch, the sum of the apparent modulus of rupture from table 5 and the corrections from table 6 .

M_c is the maximum bending moment in inch pounds calculated from f_c.

M_c' is M_c corrected to a constant sectional area of 2.46 square inches, in inch pounds.

h/b depth-breadth ratio

TABLE 10

IX

RESULTS

Results

Interpretation of the corrected data so as to make it applicable to the six objectives of this thesis, previously enumerated, will now be attempted.

In the plots which have been made all the points to the left of the red, vertical line which may be drawn thereon indicate values from specimens which failed in either tension or compression. Points to the right of the red line indicate specimens whose failure was lateral. There were no overlappings.

Fig. 24 is a plot of corrected modulus of rupture from the first set of tests (single concentrated load at the center of a constant span; variable depth-breadth ratio) against the depth-breadth ratio. The low point at depth-breadth ratio of about 4 indicates specimen 6B. It may be noticed that there was nothing unusual about this specimen or its test except that the end grain ran approximately parallel to the breadth, whereas for 6A and 6C which gave the higher values the end grain was approximately parallel to the depth. This same condition holds true for the specimens of depth-breadth ratio of about 10, where the low point, 2A, had also a rate of growth about four times as fast as 2B and 2C plotted above it. The rate of growth difference alone might cause the low modulus in this case, but the fact that 2A and 6B, the only specimens markedly low, both show the same end grain characteristics compared to the

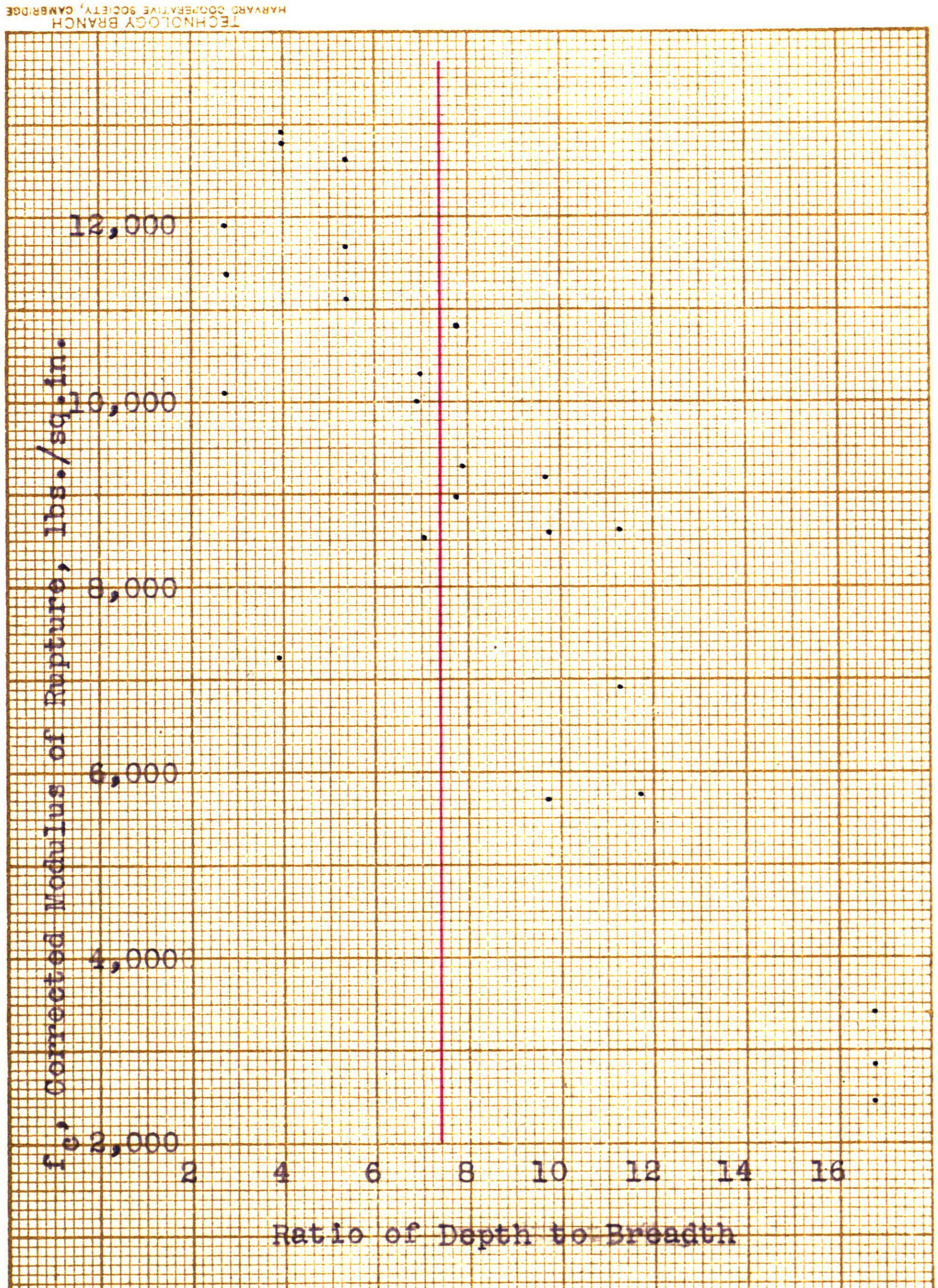


Fig. 24 . Plot of Depth-Breadth ratio against Corrected Modulus of Rupture.

two other specimens of approximately the same dimensions, seems to show that it would be well to chose spars with end grain parallel to the depth. This fact is not contradicted by any of the other data from this thesis, nor from anywhere else so far as is known.

The authors believe that the lines which would best represent the points on this plot would be two straight lines; one at constant modulus of rupture at about 10,500 pounds per square inch in the region to the left of the red line, and the other sloping downward to the right from the intersection of the first line with the red line through the average value of the three points at depth-breadth ratio of 16.8. It must be noticed, however, that the transition from compressive and tension failures to lateral is not as abrupt as these lines might indicate.

A similar plot, Fig. 25, has been made for the span tests. It is believed that two similar lines would best represent the points in this plot, and the thoughts in the preceding paragraph are generally applicable here also.

It may be noticed that the points in the lateral failure region in this plot appear to be arranged along a line slightly concave upwards. They have been replotted in logarithmic paper and the slope of the most representative line appears to be at forty-five degrees to the axes, indicating the straight line relation.

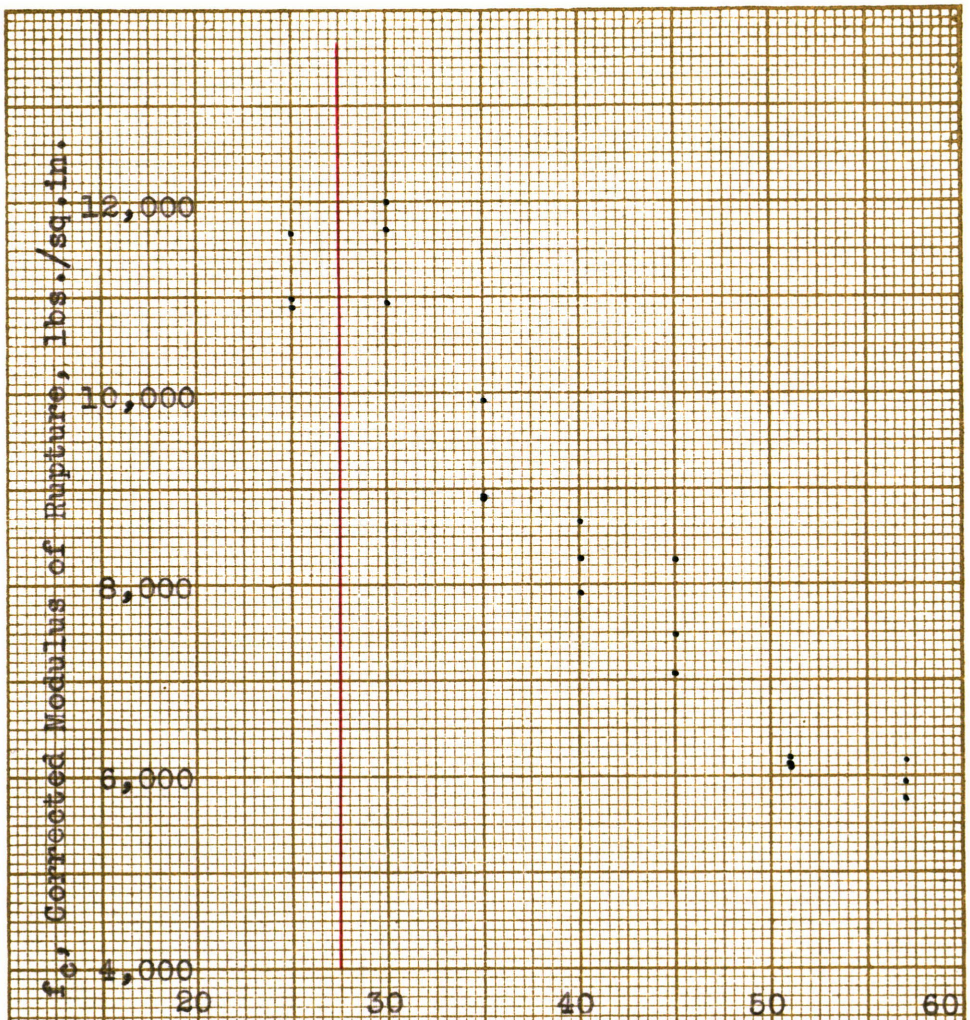


Fig. 25 Plot of Corrected Modulus of Rupture

against

Span

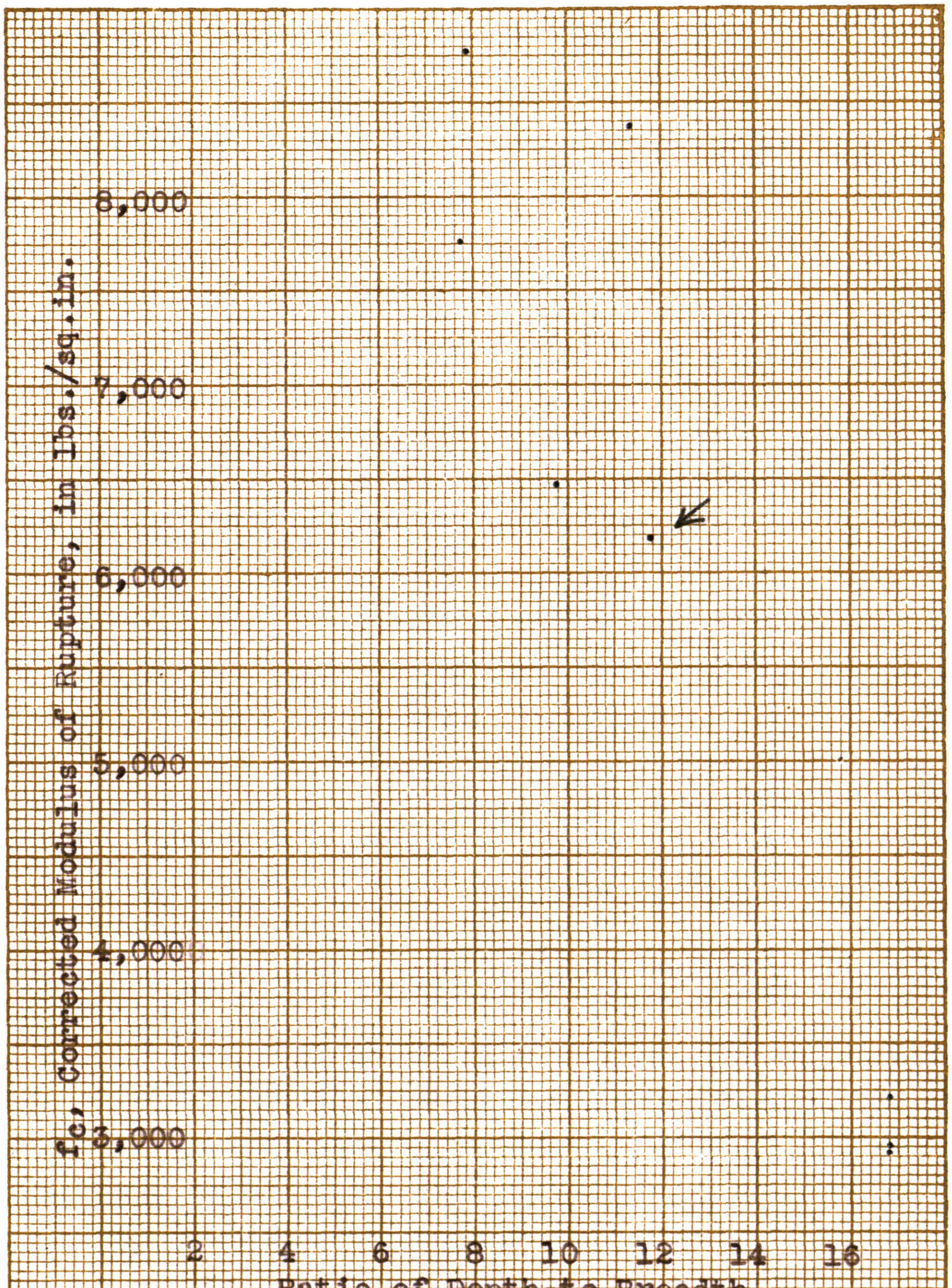


Fig.26 Plot of the depth-breadth ratio against Corrected Modulus of Rupture (Two Point Loading)

Only eight reliable points were available for the plot of modulus of rupture against depth-breadth ratio under two point loading. These have been plotted in Fig. 26 and show in general the same characteristics as the other two modulus of rupture plots. All the points indicate lateral failures except the point indicated by the arrow. This represents specimen 1C'. This specimen failed in tension. The only explaination that can be offered for this overlapping of the failures is that specimen 1C' in the region in which the failure occurred seemed to be of a slightly reddish wood, indicative of sap wood, which evidently must have failed at a stress reached before lateral deflection was induced.

Likewise three curves have been plotted for the corrected bending moments reduced to a constant sectional area (2.46 sq.in.). These have been plotted in Figs. 27, 28, and 29. Considering Fig. 27 first, it is very apparent that there is a maximum value for the bending moment at a depth-breadth ratio of about 12. That there should be a maximum is quite logical. If the modulus of rupture were constant and we consider still only specimens of the same cross-sectional area whose maximum bending moments we have in M_c', then the bending moment must decrease as the section modulus increases, and increas as the depth-breadth ratio increases. But we have seen in Fig. 26 that the modulus of rupture is not constant (considering as we are in the case of these two point loading tests only data in the region of lateral failure)

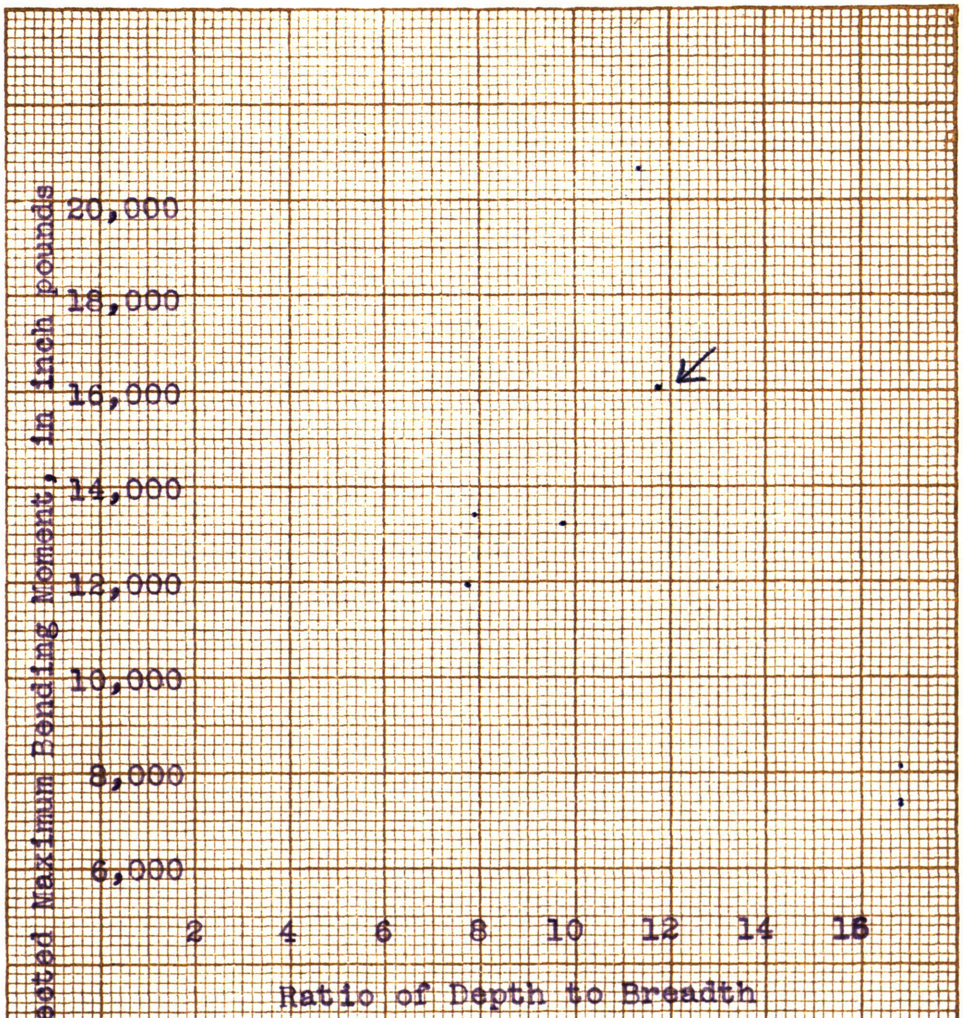


Fig.27 Plot of Depth-Breadth Ratio

against

Corrected Maximum Bending Moment

(Two point loading)

but that it decreases as the depth-breadth ratio increases. This might mean, when combined with the previous statement that the bending moment plotted against depth-breadth ratio (for a given area and a modulus of rupture decreasing as the depth-breadth ratio increases) would give either a straight line or a curve either concave or convex upwards. But, and this is the point, it follows from the theory of the matter that the bending moment varies directly as a power greater than one of the depth-breadth ratio; whereas, as has been pointed out before, the points on the modulus of rupture plot, though generally best represented by a negatively sloped straight line in the region of lateral failure, must actually lie on a line whose slope approached zero as the region of tension and compressive failures is approached. Thus in the lateral failure region at low depth-breadth ratios the moment is increased faster by increasing the depth-breadth ratio(whose rate of increase is taken constant) than it is decreased by the changing modulus of rupture, which is decreasing slowly in this region.

The bending moment varies directly as a power greater than one of the depth-breadth ratio, beacuse at constant modulus of rupture and constant sectional area the bending moment varies inversely as the section modulus, or directly as the depth; and since the area is constant the depth-breadth ratio will vary faster than the depth, or the bending moment will vary as a power of the depth-breadth ratio greater than one.

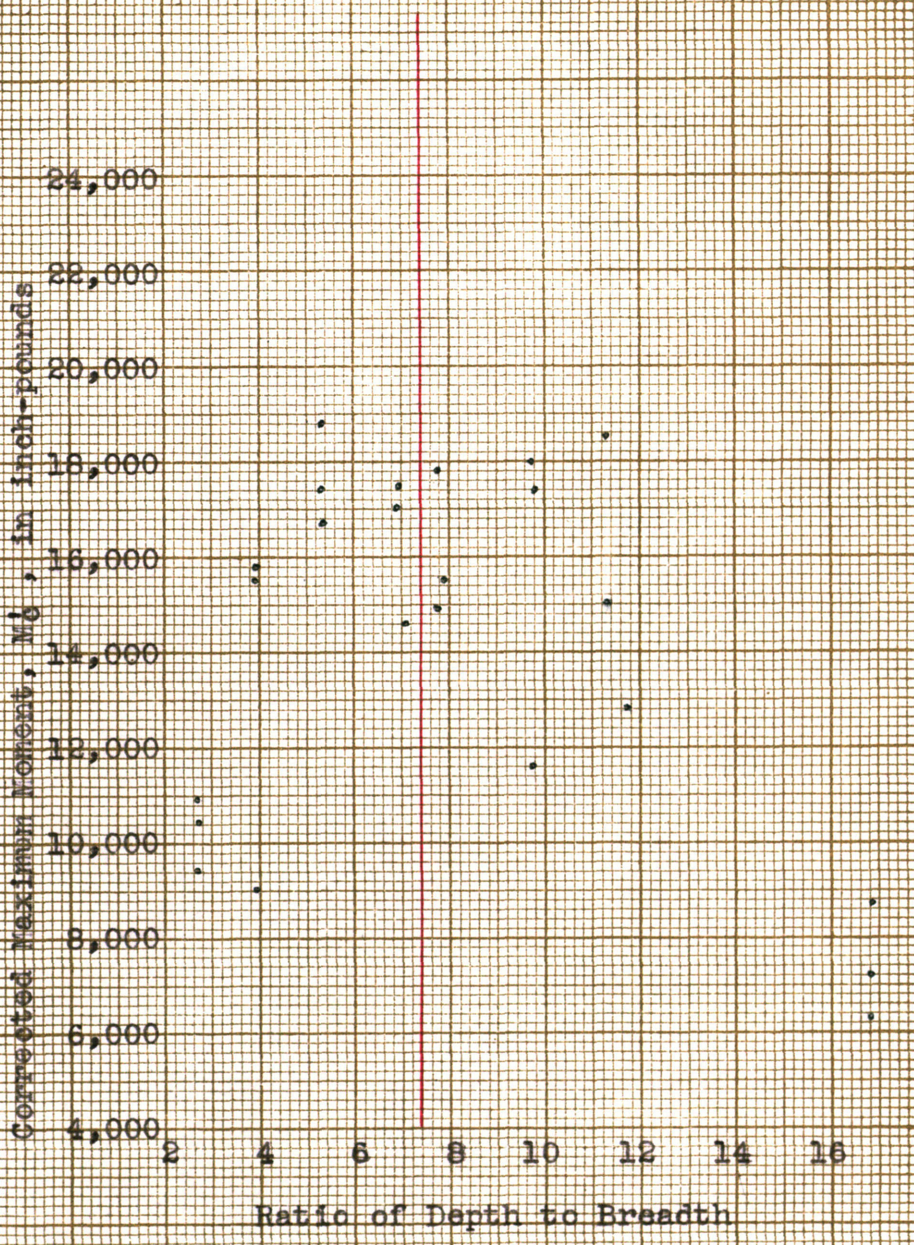


Fig. 28 Plot of Depth-Breadth Ratio
against
Corrected Maximum Moment

This same reasoning may be rehearsed to show that the points to the right of the red line on Fig. 28 should lie on a similar curve, though here it is quite evident from inspection that the maximum lies very close to the red line. Why the bending moment should again drop as the depth-breadth ratio passes from six toward four is not understood.

Considering Fig. 29 we may say that with identical section the moment varies directly as the modulus of rupture, so that were it not for the corrections to a constant sectional area this plot would be a replica of Fig. 25 with the scales changed. As it is however the small effect of the difference in sectional area of the three specimens has been introduced and the curve plotted to make the series of plots complete.

Fig. 30 is a plot of corrected modulus of elasticity against depth-breadth ratio for the first series of tests. It is believed that a straight line sloping downwards to the right would best represent the points in this plot. This negative slope is not understood. It would seem that st should slope positively, the stiffness increasing with depth-breadth ratio.

If we combine all the single point load tests, both for the effect of span and for the effect of depth-breadth ratio,on one plot of length-breadth ratio against depth-breadth ratio we get Fig. 31.

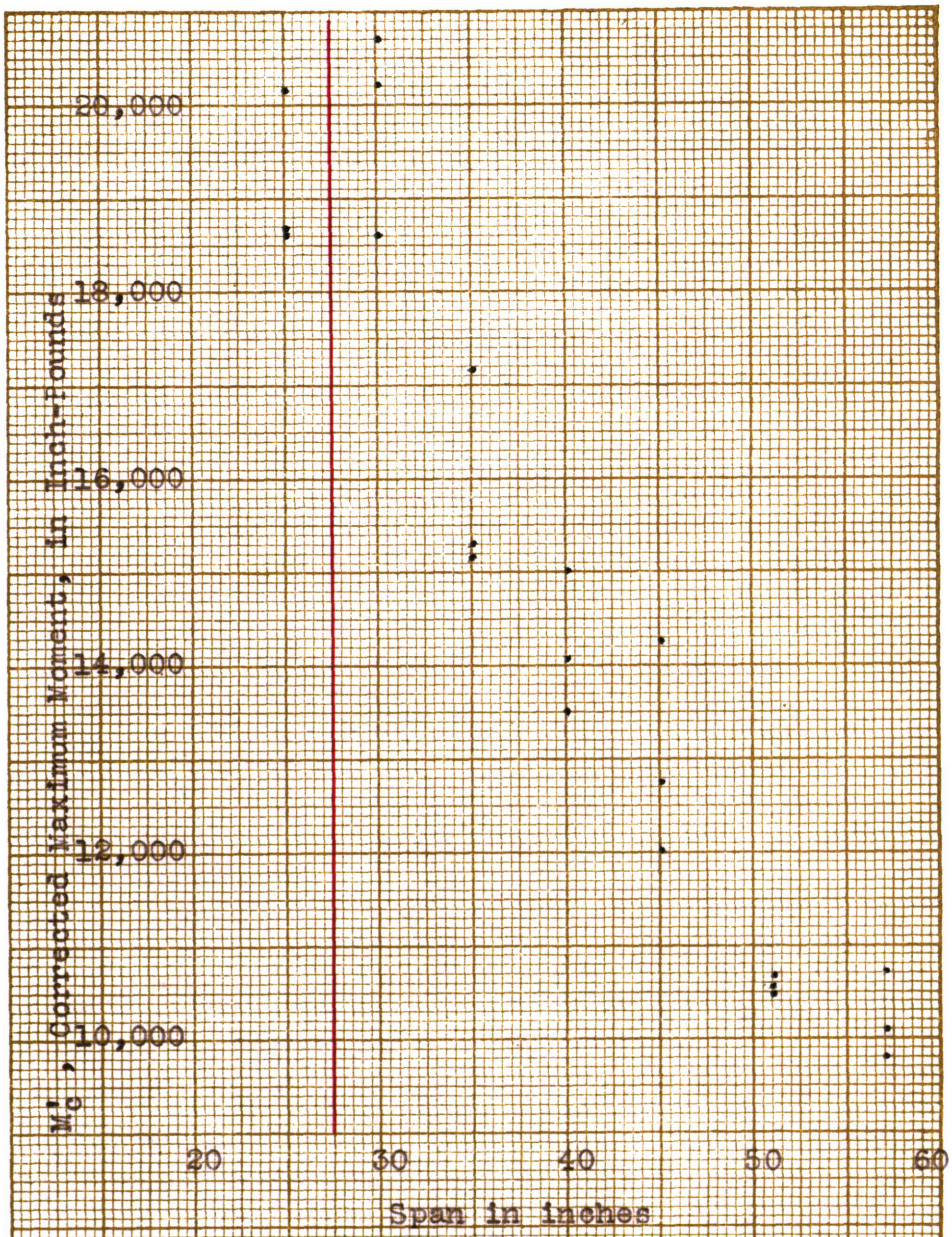


Fig. 29 Plot of M'_c, corrected maximum Bending Moment in inch-pounds against Span in inches

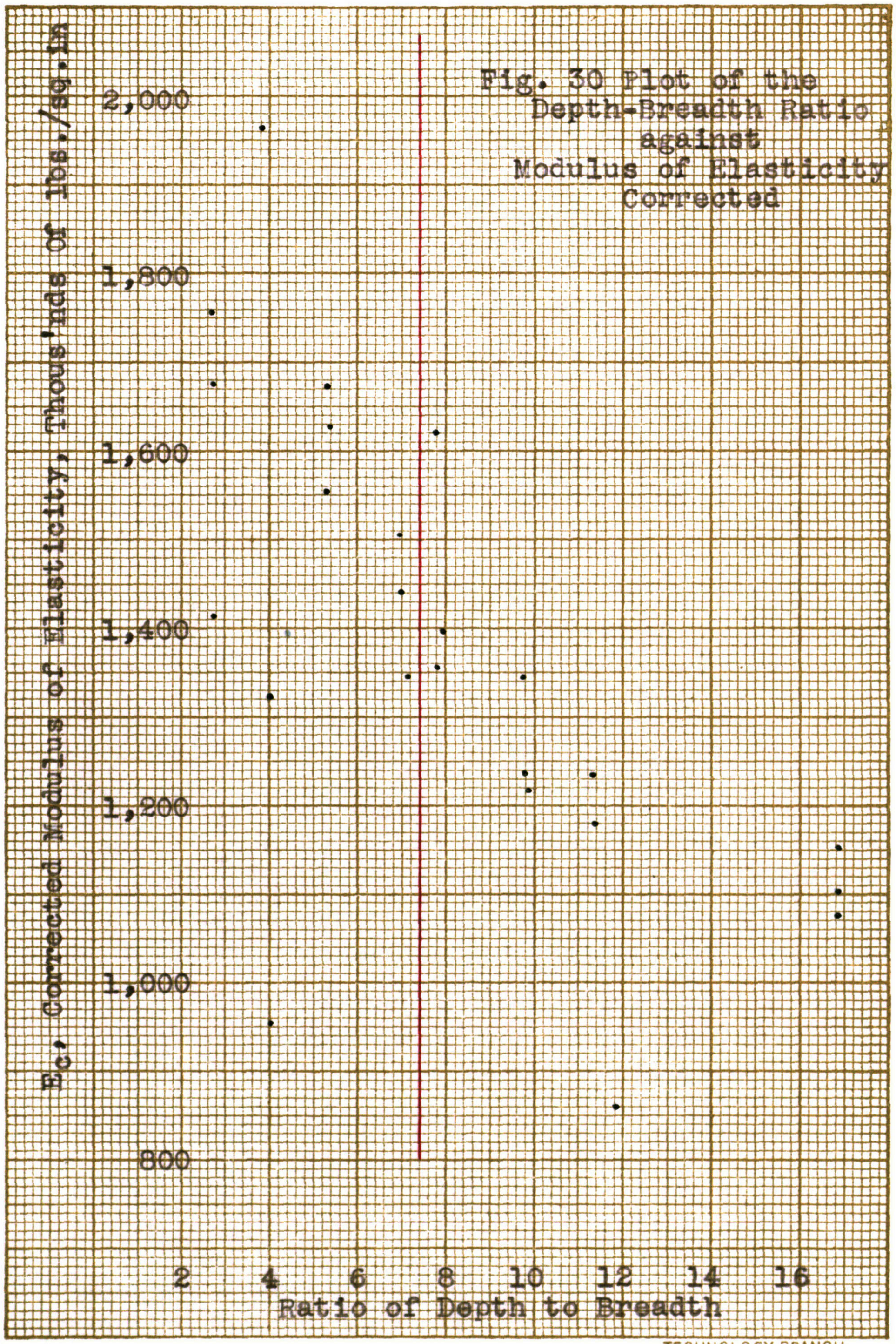


Fig. 30 Plot of the Depth-Breadth Ratio against Modulus of Elasticity Corrected
Ec, Corrected Modulus of Elasticity, Thous'nds of lbs./sq.in
2,000
1,800
1,600
1,400
1,200
1,000
800
2
4
6
8
10
12
14
16
Ratio of Depth to Breadth

The number adjacent to each point represents the approximate thousands of pounds per square inch modulus of rupture of that specimen. One of these points has a red line drawn through it. That is specimen 9B at a span of 30 inches, in the test of which the cause of failure could be attributed equally well to either compressive or lateral failure. Points above and to the right of this point all failed laterally. The others gave no lateral failures. Thus the red line of demarkation should pass through this point and slope upward to the left.

Table 11 was compiled to aid in plotting these points.

The effect of distributing the load has been further studied by compiling table 12 and plotting figures 32 and 33. It may be noted that the greatest increase in both modulus of rupture and in bending moment due to distributing the load occurs at a depth-breadth ratio of about 12, Furthermore, at most depth-breadth ratios distribution of the load tends to weaken the specimen, provided of course we assume that the specimen is of such sort as will fail laterally. This is just the opposite of the truth regarding beams which do not fail laterally.

TABLE 11

DIMENSIONAL RATIOS

Specimen	L/b	h/b	Specimen	Span	L/b	h/b
1A'	88.7	11.32	9A	57	154	8.12
1C'	94.0	11.76	9B	57	142	7.50
1A"	88.7	11.27	9C	57	152	8.01
2A	92.1	9.75	9A	51	138	8.12
2B	92.1	9.69	9B	51	127	7.50
2C	94.0	9.80	9C	51	136	8.01
3A	97.9	7.75	9A	45	122	8.12
3B	100.0	7.87	9B	45	113	7.50
3C	97.9	7.73	9C	45	120	8.01
4A	66.2	7.04	9A	40	108	8.12
4B	65.3	6.94	9B	40	100	7.50
4C	64.4	6.84	9C	40	107	8.01
5A	62.7	5.32	9A	35	94	8.12
5B	63.5	5.38	9B	35	87.5	7.50
5C	62.7	5.31	9C	35	93.4	8.01
6A	62.7	3.90	9A	30	81.2	8.12
6B	62.7	3.93	9B	30	75.0	7.50
6C	63.5	3.94	9C	30	80.0	8.01
7A	63.5	2.70	9A	25	67.6	8.12
7B	62.7	2.68	9B	25	62.5	7.50
7C	61.8	2.67	9C	25	66.7	8.01
8A	134.3	16.8				
8B	134.3	16.8				
8C	134.3	16.8				

L/b is the ratio of span to breadth.

h/b is the ratio of depth to breadth.

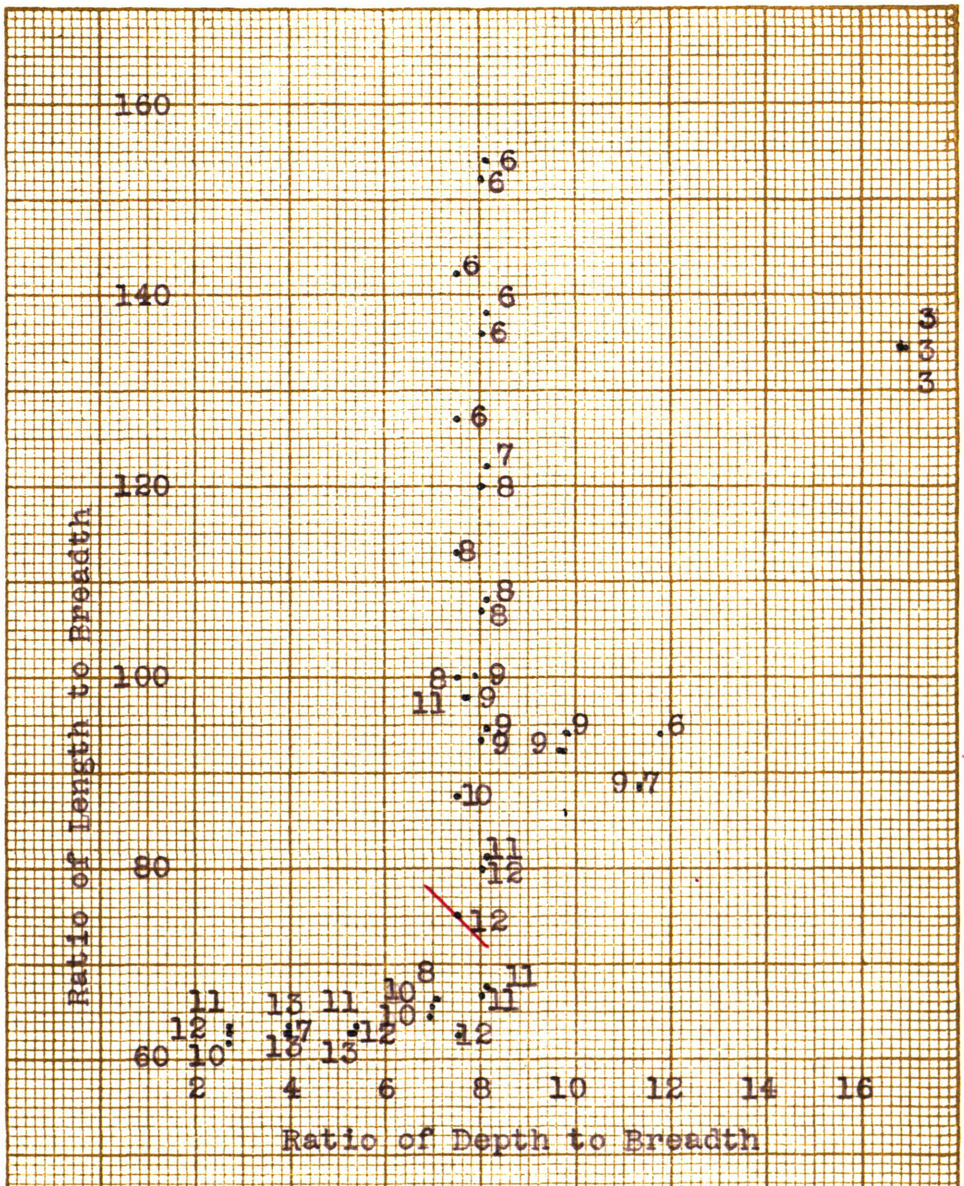


Fig.31 Plot of Depth-Breadth Ratio
against
Length-Breadth Ratio

Figures indicate approx. Thousands of Pounds
Corrected Modulus of Rupture.

TABLE 12

EFFECT OF LOAD DISTRIBUTION

Specimen	Increase in f	%Increase in f	h/b	Increase in M'_c	%Increase in M'_c
8A	40	1.1	16.8	90	1.3
8B	710	24.1	16.8	1820	28.9
8C	-500	-11.9	16.8	-1300	-14.9
3A	-1250	-14.4	7.75	-2950	-19.8
3B	-610	- 6.7	7.87	-2100	-13.5
2A	700	11.9	9.75	1600	13.8
1A'	1400	22.8	11.32	5600	37.4
1C'	1380	23.3	11.76	3250	26.2

Increase in f is the increase in pounds per square inch in apparent modulus of rupture under two point loading over single loading. See tables and for values of f of which this column is the differences.

%Increase in f is based on f for single point loading tests; values in table .

Increase in M'_c is increase in inch pounds in corrected maximum bending moment under two point loading over single loading. See tables and for values of M'_c of which this column is the differences.

%Increase in M'_c is based on M'_c for single point loading tests; for values see table.

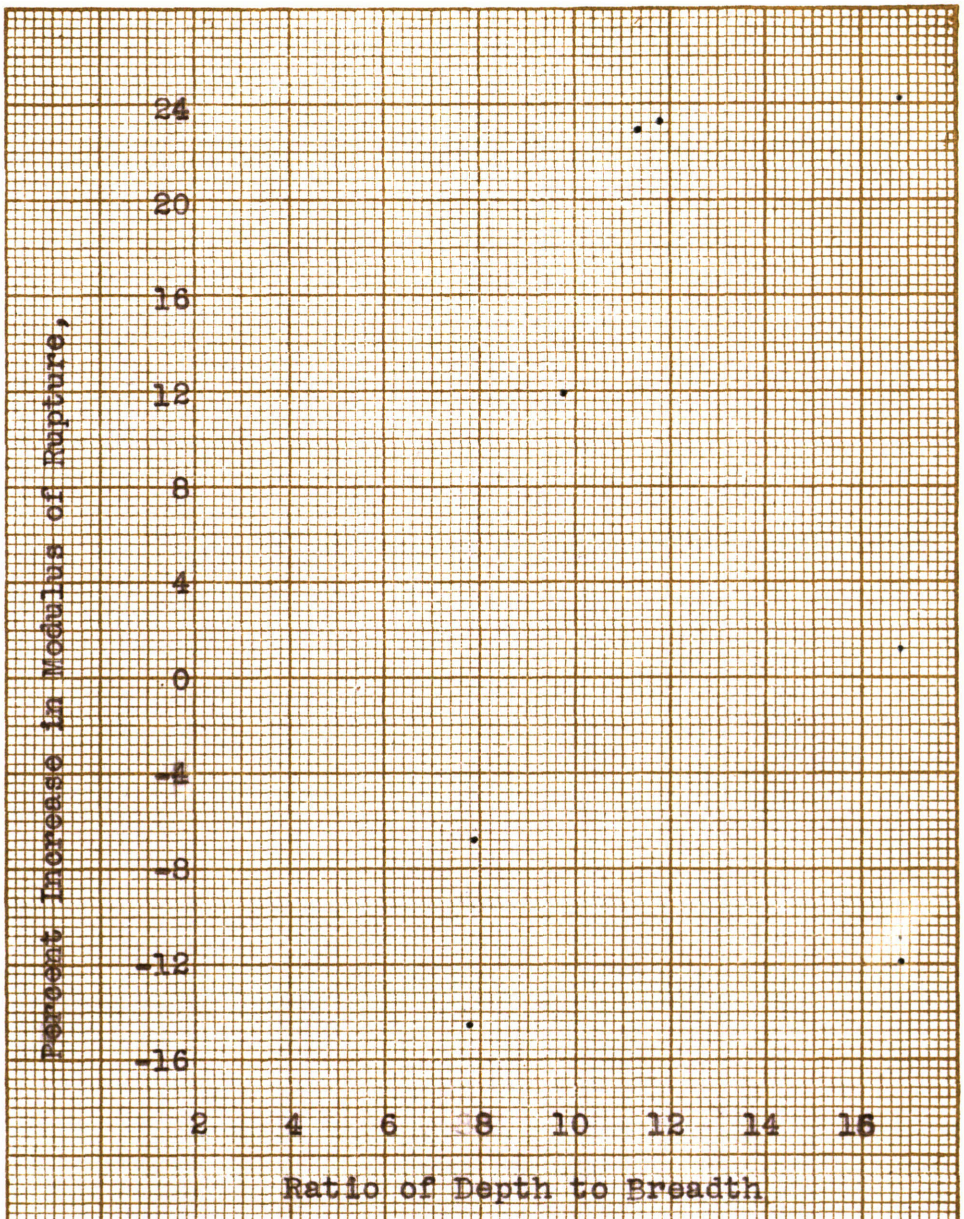


Fig. 32 Plot of Percent Increase in Apparent Modulus of Rupture with Two Point Loading against Depth-Breadth Ratio

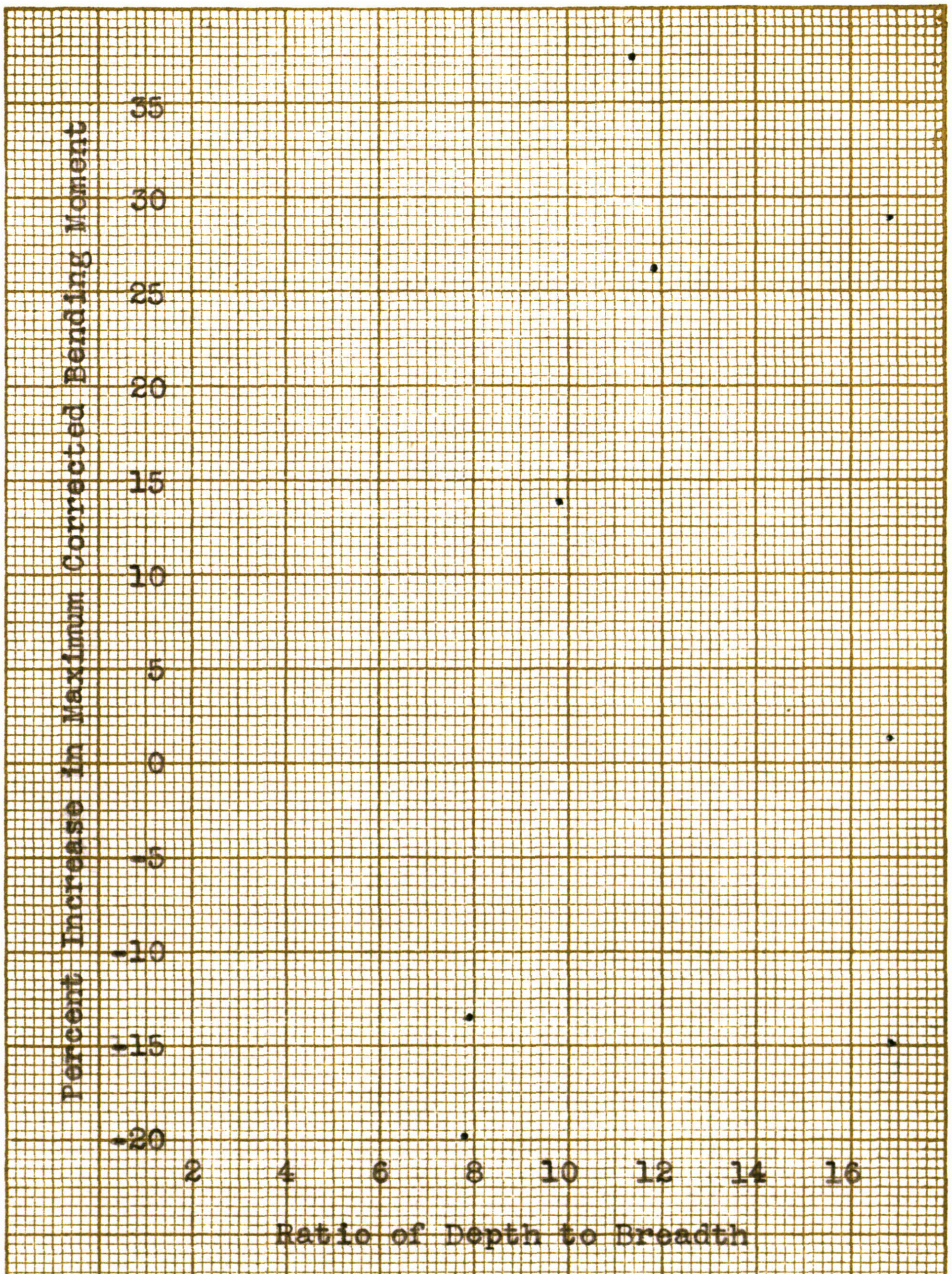


Fig. 33 Plot of Percent increase in Corrected Maximum Bending Moment with Two Point Loading against Depth-Breadth Ratio

X

AGREEMENT WITH PREVIOUSLY OBTAINED DATA

Agreement with Previously Obtained Data

As has been already noted there have been three works on this subject prior to this present one. Before we attempt to draw any conclusions it may be well to compare our results with those of these others.

Goodman's thesis will be considered first. He presented a curve from thirty points for the modulus of rupture against depth-breadth ratio. It is almost identical with Fig. 24, the same plot of our data, except that it is shifted about two units of depth-breadth ratio to the left and rises to a maximum of about 14,000 pounds per square inch, and then falls off again. The ordinates on Goodman's curve, however, were not corrected for moisture, etc. His span was somewhat shorter than the 47 inches selected for the points in this present work, the end supports bearing over 4 inches on each end though his specimens were of the same length, 48 inches. This slight discrepancy in span should not shift the curve so far, it seems in the light of our own span tests.

Now, regarding Goodman's conclusions, he suggests using depth-breadth ratios of 4.5 to 6 for maximum strength. Obviously now we can go higher than this. His critical depth-breadth ratio of 3 is quite clearly in error, also. For as we have seen, even on a span of 47 inches the critical ratio is somewhat over seven.

Lastly, we cannot consider Goodman's points as possible of being plotted with our own because he failed to preserve the necessary data on moisture, etc., of his specimens. His thesis is valuable,however,in that it checks very well the theory,in which we concur, that fixing the ends rigid increases the strength of the specimen.

Alchalel and Guimaraes, who began we may say where Goodman left off, have left results a little more tangible in the shape of formulas, and in the shape of critical values of what they call L/R, where L is the length of the specimen in feet,and R is the breadth-depth ratio, the reciprocal of the ratio we have used.

Alchalel and Guimaraes say the critical value of specimens such as we have tests under single point load should be 15 or 16, and they interpret Goodman's tests to show that it should be between 16 and 20. We have two sets of data which we feel sure should give a very reliable computation of this critical value, and to check these previous statements have worked it out as follows, the result being in one case a very close agreement with their predictions.

From the history of our tests on span it is quite evident that at the depth-breadth ratio of the specimens in the 9 group there is a critical span at about 30 inches or a little under, say 28 inches, or 2.33 feet.

The average critical L/R then is 2.33 x 8, or 18.67, which certainly compares well with Goodman's results of 16 to 20 and with Alchalel and Guimaraes' figures of 15 and 16.

Similarly from the history of the first set of our tests we know that on a 47 inch span the critical ratio is about 7.5. This would give a critical value of L/R of 29.4 which does not check very well.

Alchalel and Guimaraes suggest the following formula for the modulus of rupture:

$$f = 12,500 - 250\ L/R,$$

when h/b is 6 and L/R is not less than 10 and the load is a central concentrated load on a rectangular section with free ends. From our tests we find that at a 47 inch span and a depth-breadth ratio of six, for instance, the modulus of rupture is 10,500 pounds per square inch (See Fig. 24) whereas these values for span and h/b substituted in their formula give 6620 pounds per square inch, which is not a good check.

We have also attempted to check their other formula:

$$f = 12,500 - 700\ L/R$$

where h/b is 3.75 and L/R is not less than 14 but the results disagree still further.

For a depth-breadth ratio of six we tested at only one span. Hence we can compare the values given by the first of Alchalel and Guimaraes' formulas but once. We are certain of our value of f_c for depth-breadth ratios under 7.5 and can safely say that in this instance the formula does not check within fifty percent.

An attempt has also been made to check Prescott's formulas. Representative tests from our work were chosen for a beam simply loaded and failing by lateral deflection. Prescott's formula for this case is:

$$P_p = \frac{16.94}{L^2}\sqrt{EINK}$$

where the symbols are those explained on the following page. The actual corrected values from our tests are also tabulated. It was noted in all three cases that P_p was much larger, ranging from twice as large as P for the highest value to five times as large for the lowest value. This would indicate a very large discrepancy between Prescott's formula and our tests. We therefore say that we are unable to agree with Prescott's formulas, choosing as we have three very representative points from the many we have.

TABLE 13

Specimen	L^2	$N/10^6$	K	P_p	P
1C'	2209	.09	.218	2930	1450
8A	2209	.09	.0755	1700	600
9A	3249	.09	.0449	1300	230

L is span in inches

N is modulus of rigidity taken from British Advisory Committee's R. & M. 528

KN is the torsional rigidity where $K = 0.3 \frac{b^3 d^3}{b^2 + d^2}$

P_p is ultimate load by Prescott's formula.

P is ultimate load from tests.

XI

CONCLUSIONS

Conclusions

Under the heading of "Objects" have been listed six problems, the solution of which we have attempted to find. We now desire to give, as well as we are able, the answers.

It seems best now to discuss them in an order quite different from that in which they are listed because obviously a complete discussion of the first would cover all the rest.

Starting then with the last we may inspect our data and plots to discover the effect of:

(a) Section Modulus. This is nil. It will be remembered that the specimens were designed to fall into groups of section moduli. For instance, specimens in the 2 and 5 groups all have a section modulus of about .5 inches cubed, yet all those in the 2 group failed laterally while none of those in the 5 group showed any tendency to do this. This conclusion is borne out in all our tests in just the same manner.

(b) The Modulus of Elasticity. We have seen in our results that at constant span the modulus of elasticity varies inversely as about the first power of the depth-breadth ratio. (This from Fig. 30, the plot being considered to indicate a straight line). We also have reason to believe from our measurements during the span tests that it varies but little with span, since our measurements showed it to be about normal (perhaps a little high), averaging over 1,800,000 at the 57 inch span. Since buth span and breadth-depth ratio alike cause lateral failure it is evident that modulus of elasticity has nothing to do with it. It seems more logical to say that the modulus of elasticity is a function of the depth-breadth ratio and varies inversely with it.

(c) Grain Slope. This must be subdivided. We assumed on the basis of tests at the Forest Products Laboratory that the grain as it is usually taken on the side of a specimen has a definite effect on the modulus of rupture, and therefore on the maximum bending moment also, and made a correction in the apparent modulus of rupture from our tests to cover this. But as yet we have not dealt with the end grain except to point out in our results that end grain parallel to the breadth weakened the specimens. We now say that this is exactly what might be expected if the specimens were considered made up of a series of layers alternately dense (the summer wood) and spongy(the spring wood). Treating each specimen then as a composite beam it is at once apparent that the strength is greatest when the layers are parallel to the depth. The conclusion is therefore that for maximum strength a specimen must have straight side grain(under 1:40) and an end grain parallel to the depth, but that neither seems to affect the tendency to fail laterally.

(d) The Percentage of Summer Growth. In no instance have we succeeded in getting any data on this ourselves. From the reports of the Forest Products Laboratory we believe it should be considered only along with specific gravity.

(e) The Moisture Content. We have treated this the same as the side grain, making the corrections elsewhere explained.

(f) Rate of Growth. The tests on the 2 group, especially that test on 2A, indicates a weakening of strength by rapid growth. This checks well the work of H.L.Goodwin and W.H.Preston reported in M.E. Dept. Thesis 38 for 1920 at M.I.T. in which it is stated, "That the strength increases with the number of annual rings in the cross section."

(g) Specific Gravity. We have treated this also the same as the side grain, making the corrections elsewhere explained.

The second and third objects, the effect of depth-breadth ratio and the effect of span, may best be treated together. From Fig. 24 it is apparent that the modulus of rupture, which is considered to be a criterion of the strength in this instance, varies inversely as the depth-breadth ratio. From Fig. 25 it is apparent that span has the same effect.

Fig. 26 simply emphasizes Fig. 24. On the first two of these three plots the critical value of the abcissae has been marked.

We have seen in the comparison of our tests with those of Alchalel and Guimaraes that the critical value of L/R at the 30 inch span is 18 or 19 whereas at the 47 inch span it is 29 or 30. In other words the critical value of L/R increases with span. Then there is no reason to believe that it might not vary with either depth or breadth also. So we must banish the idea that there is such a thing as a critical L/R applicable in all cases alike, although many more tests might show that there is a critical value of this ratio which is a function of the three dimensions.

Fig. 31 is a plot of the length-breadth ratio against the depth-breadth ratio. In the discussion of our results concerning this we have mentioned a "red line of demarkation" on it which separates the lateral from the tension and compression failures. A similar line might be drawn through the points having a modulus of rupture of 7,000 or 8,000 in the lateral failure region. Though there are not here enough points to make the slope definite, it seems that it would have a larger negative value as the modulus of rupture of the line decreased; it certainly is not the same for all moduli of rupture. We interpret the slope of the critical red line of demarkation to mean what we have just shown regarding the L/R ratio, that it varies with the dimensions. That the slope of the other lines is a variable we interpret to mean that the tendency to fail laterally does not bear a constant relation to the modulus of rupture which the specimen possesses.

Other than this the only conclusions can be that in general, after the critical span or depth-breadth ratio has been reached, the modulus of rupture varies inversely as the first power of the depth-breadth ratio and of the span.

We feel we have a good answer to the fifth object of this thesis, the determination of the dimensional relations for best strength-weight ratio. In the dimensional relations which we chose lateral deflection is quite probable before the maximum stress is reached. Since lateral deflection is due to the compression induced by bending, we believe that dimensions chosen as best from the standpoint of the moment they will sustain will also prove best if a compressive load is added, and since wood is weaker in compression than in tension we believe they will also hold good if a tension is added to the bending.

We have therefore determined the dimensions which will give a minimum weight of spar capable of sustaining a bending moment. This has been done as follows: In our results we showed that the plots of moment carried on a given sectional area against depth-breadth ratio indicate the existence of a maximum somewhat advanced into the lateral failure region. From Figs. 27 and 28 we find that at a depth-breadth ratio of 10 there is very little decrease due to the abnormally high ratio.
Thus we are sure that on a span of 47 inches at a depth-breadth ratio of 10 we are not sacrificing anything in the moment which can be carried.

This is true despite the fact that the modulus of rupture has been reduced at this depth-breadth ratio due to lateral deflection.

This applies to a single concentrated load at the center of the span. Fortunately Figs. 32 and 33 show us that distributing the load as it is distributed along an airplane wing spar at a depth-breadth ratio of 10 increases not only the bending moment which can be sustained but also the modulus of rupture at least enough to compensate for the decrease in modulus of rupture due to the high depth-breadth ratio.

Since the direction of the lateral deflection is alternate between successive supports (which not only is to be expected from theory, but has been conclusively proven from our two point loading tests) we believe the rib spacing along a spar will have a much greater bearing on its lateral failure than its distance between supports, between strut points, for example. From an inspection of our plots and data as well as from this fact we conclude that for usual rib spacings and usual unsupported spar lengths the decrease in maximum bending moment due to increase in span will be well counterbalanced by the diminished distance between lateral supports, assuming of course that the ribs do furnish adequate lateral support to the spars.

Therefore a depth-breadth ratio of 10 is not only permissible but it will give what appears to be the maximum strength-weight ratio.

It may be, for this is something concerning which we have no knowlege at hand that the ribs necessary to furnish the needed lateral support would be so heavy that the gain in lightness of spar from using the high depth-breadth ratio would be overbalanced by the rib weight, but we doubt this. An investigation of the torsion exerted on the yokes during lateral deflection we believe would prove worthwhile.

Another consideration is that it is present practice to use I sections for spars. The depth-breadth ratio of an I section is usually spoken of as the ratio of total depth to flange width. Obviously it would not be fair to expect a section routed into I form to follow the same rules regarding lateral deflection and highest strength-weight ratio as an unrouted section. What correction may be necessary we cannot say, except to quote Alchalel and Guimaraes as saying that I sections which they have tested were only one third as strong as the equivalent area in a rectangular section. Since we tested no I sections we cannot verify this statement.

Certain it is, however, that the depth-breadth ratio of the web of an I section can be over 10, probably as much as 15, because there is a great deal of resistance to lateral deflection in the flanges.

The fourth result, and with it the factor analagous to the form factor which we explained in our "Objects" we desired to derive, we have been unable to obtain. We simply say that we conclude from our tests that use of the full modulus of rupture is permissible on sections of depth-breadth ratio of 10 loaded as wing spars at rib spacings now common and at unsupported lengths between strut points now common.

There remain a few conclusions regarding lateral deflection and failure which we will present in answer to the first of our objects. In the first place the description of lateral failure given in the introduction proved correct. Secondly, in general the higher the modulus of elasticity the more nearly the load at which lateral deflection begins coincides with the maximum load. Thirdly, the compression failure on specimen 9B at the 30 inch span due to lateral deflection indicates that the theory that places the maximum compression at 0.5773 of the distance from the end support to the center load is at least in a measure correct. And finally, that the strength of wooden beams which fail laterally is affected by all those variables which ordinarily affect the strength of beams but that lateral failure itself is mostly due to the dimensions of the specimen and the type of loading.

APPENDICES

APPENDIX A

Time Distribution

For this thesis 150 hours each were allotted, a total of 300 man-hours. Time has been spent as follows:

	Man-Hours
Preliminary reading and planning	10
Collection of Apparatus and Specimens	12
Testing specimens for strength	142
Testing specimens for properties	10
Theory	20
Calculations	34
Plotting	22
Compiling Tables and Writing	48
Drawings	6
Total	304

APPENDIX B

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