

Essays on Trades and Security Prices

by

Anna A. Obizhaeva

Submitted to the The Alfred P. Sloan School of Management
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy


at the

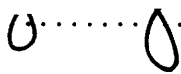
MASSACHUSETTS INSTITUTE OF TECHNOLOGY


June 2007

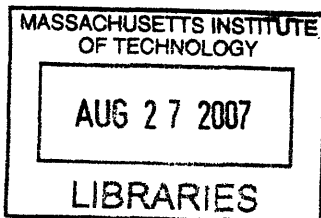
© Anna A. Obizhaeva, MMVII. All rights reserved.

The author hereby grants to MIT permission to reproduce and
distribute publicly paper and electronic copies of this thesis document
in whole or in part.

Author 
The Alfred P. Sloan School of Management
May 2, 2007

Certified by 
Jiang Wang
Mizuho Professor of Finance
Thesis Supervisor

Accepted by 
Birger Wernerfelt
Professor of Management Science



ARCHIVES

Essays on Trades and Security Prices

by

Anna A. Obizhaeva

Submitted to the The Alfred P. Sloan School of Management
on May 2, 2007, in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

Abstract

This thesis consists of three chapters that investigate the complex relation between security prices and trades of market participants.

In the first chapter, I study the evolution of stock prices after trades with different underlying motives using a novel data set of portfolio transitions. Institutional specifics allow me to identify portfolio transition purchases and sales as most likely induced by information-related and liquidity-related factors, respectively. I find that purchases permanently shift stock prices to new levels; moreover, these price changes are more significant after large trades, trades in stocks with a high degree of information asymmetry, and trades that reflect new rather than stale information. At the same time, sales trigger only temporary price pressure effects that are reversed in the following weeks. Thus, my findings provide supporting evidence for a long-standing tenet of market microstructure stating that information-motivated and liquidity-motivated transactions generate different price dynamics.

In the second chapter, I analyze the price dynamics in response to trades in more detail; in particular, I focus on the properties of price impact. I explore the following questions: (1) how the price impact coefficients relate to various stock characteristics and differ across trading venues; (2) how they evolve during execution of multi-trade "packages"; and (3) what functional form best describes price impact functions. Regarding most of these questions, there exists an extensive theoretical literature which provides interesting insights. Using a unique data set of portfolio transition trades, I document a number of empirical facts about price impact, some of which can not be easily explained by existing models. For instance, the price impact coefficients relate positively to the market capitalization and to the amount of noise trading; they increase during buy "packages" and decrease during sell "packages"; finally, total price impact is concave in trade size, fitting well the square-root specification, however, surprisingly, its permanent component is also non-linear.

In the last chapter, based on joint work with Jiang Wang, we study how security prices affect trading strategies. The supply/demand of a security in the market is an intertemporal, not a static, object and its dynamics is crucial in determining market participants' trading behavior. We show that the dynamics of the supply/demand, rather than its static properties, is of critical importance to the optimal trading strategy of a given order. Using a limit-order-book market, we develop a simple framework to model the dynamics of supply/demand and its impact on execution cost. We demonstrate that the optimal execution strategy involves both discrete and continuous trades, not only continuous trades as previous work suggested. The cost savings from the optimal strategy over the simple continuous strategy can be substantial. We also show that the predictions about the optimal trading behavior can have interesting implications on the observed behavior of intraday volume, volatility and prices.

Thesis Supervisor: Jiang Wang
Title: Mizuho Professor of Finance

Acknowledgments

Throughout the preparation of my thesis, I have been blessed to be surrounded by wonderful people without whom its completion would have been impossible.

My advisors and professors at Sloan helped me find my calling and mature professionally. They are the best people I could ever hope to learn from. I am especially grateful to Professor Jiang Wang, under whose thoughtful and kind supervision I have evolved from a rookie Ph.D. student to a stand-alone researcher. I extend a very special thanks to Professor Steve Ross who sets for me the example of a man with amazing intuition both in research and in life. I thank also Professor Jun Pan for her valuable advices, and Professors Anna Pavlova and Dimitri Vayanos for their encouragement and strong belief in my success.

I am very much indebted to Mark Krizman who has helped me find a fascinating research area and who has introduced me to the curious world of the finance industry. I am also grateful to my colleagues at Windham Capital for their support.

I thank my fellows at the Sloan Ph.D. program with whom I have been learning about finance and making my first steps into the professional world: Jack Bao, Alex Edmans, Li He, Isil Irel, Jiro Kondo, Adam Kolasinski, Ioanid Rosu and Oleg Rytchkov, among others. Our assistants, especially Sharon Cayley and Svetlana Sussman, made my stay at MIT run smoothly, and I am grateful to them.

Thanks to the remarkable environment of MIT, the recent years have been a stimulating and exciting period of my life both in a professional and a social perspective. I am grateful to all those who contributed to creating and maintaining this unique academic world in conformity with the highest standards. Interaction with amazing people of various origins, backgrounds and interests (a lot of whom have become my dearest friends) has left its profound mark on my way of thinking.

There also exists another world to which I feel very attached. It is my home - Moscow. My Russian friends, my relatives, my school teachers, and my professors at MSU have contributed to shaping my personality at the earliest stages. A mere knowledge that these people believed in me made the process of writing this thesis

much less painful.

Also, I especially thank Olivier. Throughout this endeavor, his support and optimism have greatly helped me during hard times; he was also the first to share with me the joy of delightful moments.

Finally, I dedicate this work to my wonderful family: my parents, my sister Olya and my brother Nikola.

Contents

1	Information vs Liquidity: Evidence from Portfolio Transition Trades	15
1.1	Introduction	15
1.2	Hypotheses Development	20
1.2.1	Institutional Details	20
1.2.2	Hypotheses	22
1.3	Data Description	26
1.3.1	Data set of Portfolio Transitions	26
1.3.2	Supplementary Data	28
1.3.3	Summary Statistics: Portfolio Transitions	28
1.3.4	Summary Statistics: Transition Orders	30
1.4	Event Study	31
1.4.1	Design of Tests	31
1.4.2	Results	32
1.5	Discussion and Extensions	39
1.5.1	Alternative Explanations	40
1.5.2	Price Dynamics and Trading Venues	41
1.6	Conclusion	43
2	The Price Impact Puzzles	65
2.1	Introduction	65
2.2	Main Variables	70
2.3	Data Description	73
2.3.1	Data set of Portfolio Transitions	73

2.3.2	Summary Statistics	75
2.3.3	Supplementary Data	75
2.4	The Cross-Sectional Analysis	76
2.4.1	Motivation	76
2.4.2	Price Impact and Stock Size	77
2.4.3	Price Impact and Spread	80
2.4.4	Regression Analysis	80
2.5	Price Impact Dynamics	84
2.5.1	Motivation	84
2.5.2	Test Design and Results	85
2.5.3	Discussion	87
2.6	Price Impact Functions	90
2.6.1	Motivation	90
2.6.2	Test Design and Results	91
2.6.3	Permanent and Temporary Price Impact	92
2.6.4	Non-Linear Price Impact Coefficients	94
2.7	Conclusion	96
3	Optimal Trading Strategy and Supply/Demand Dynamics (joint work with Jiang Wang)	109
3.1	Introduction	109
3.2	Statement of the Problem	115
3.3	Limit Order Book and Supply/Demand Dynamics	116
3.3.1	Limit Order Book (LOB)	116
3.3.2	Limit Order Book Dynamics	119
3.3.3	Execution Cost	122
3.4	Conventional Models As A Special Case	123
3.4.1	Conventional Setup	124
3.4.2	The Continuous-Time Limit	125

3.4.3	Temporary Price Impact	126
3.4.4	A Special Case of Our Framework	128
3.5	Discrete-Time Solution	129
3.6	Continuous-Time Solution	133
3.7	Optimal Execution Strategy and Cost	135
3.7.1	Properties of Optimal Execution Strategy	135
3.7.2	Minimum Execution Cost	139
3.7.3	Empirical Implications	143
3.8	Extensions	144
3.8.1	Time Varying LOB Resiliency	145
3.8.2	Different Shapes for LOB	145
3.8.3	Risk Aversion	146
3.9	Conclusion	149
3.10	Appendix	150
3.10.1	Proof of Proposition 1	150
3.10.2	Proof of Proposition 2	151
3.10.3	Proof of Propositions 3 and 4	153

4

List of Figures

1-1	Price Response to Large Transition Orders, Equally-Weighted Case . . .	61
1-2	Price Response to Large Transition Orders, Principle-Weighted Case . . .	62
1-3	Price Response to Large Transition Orders, Value-Weighted Case . . .	63
1-4	Price Response to All Transition Orders, Equally-Weighted Case	64
3-1	The limit order book and its dynamics.	118
3-2	Optimal execution strategy with N discrete trading intervals.	132
3-3	Profiles of the optimal execution strategy and ask price.	137
3-4	Optimal execution strategy vs. strategy from the conventional models.	143
3-5	Optimal execution strategies for different coefficients of risk aversion.	148

List of Tables

1.1	Identification Assumption: Target vs. Legacy Portfolios	50
1.2	Summary Statistics for Portfolio Transitions	51
1.3	Portfolio Transition Orders with Large OMT Trades	52
1.4	The Cumulative Average Abnormal Returns (CAARs)	53
1.5	The CAARs and Information Asymmetry	54
1.6	Correlation Matrices	55
1.7	The CAARs: Purchases, Double Sort on Size and <i>ANUM</i>	56
1.8	The CAARs: Purchases, Double Sort on Size and <i>SPREAD</i>	57
1.9	The CAARs: Purchases, Double Sort on Size and <i>PIN</i>	58
1.10	The CAARs and Past Returns	59
1.11	The CAARs and Trading Venues	60
2.1	Summary Statistics	101
2.2	Price Impact and Market Capitalization	102
2.3	Price Impact and Percentage Spread	103
2.4	The Regression Analysis, Purchases	104
2.5	The Regression Analysis, Sales	105
2.6	Price Impact Dynamics	106
2.7	Price Impact Functions	107
2.8	Temporary and Permanent Price Impact	108
3.1	Profiles of the optimal execution strategy for different LOB resiliency.	141
3.2	Cost savings by the optimal execution strategy from the simple strategy.	142

Chapter 1

Information vs Liquidity: Evidence from Portfolio Transition Trades

1.1 Introduction

Different stock price patterns are expected in response to trades with different information content. Information-motivated trades shift security prices permanently to new fundamental levels. In contrast, liquidity-motivated trades bring about only temporary stock price deviations, which are expected to be promptly reversed.¹ Though being at the very heart of market microstructure, these two patterns are difficult to study. The main reason is that researchers usually do not observe the information content behind transactions. In this paper, I study portfolio transition trades, among which I identify the ones that are most likely driven either by private information or by liquidity needs. I show that the price dynamics after these trades indeed crucially depend on information content behind them.

This paper is one of the first academic studies to examine a phenomenon of port-

¹The differential impact of these trades on return dynamics is emphasized in Wang (1994) and Llorente et al. (2002). Similar intuition underlies the empirical models of Hasbrouck (1988, 1991). Many theoretical studies analyze uninformed trading and return dynamics (e.g., Grossman and Miller (1988), De Long et al.(1990), Campbell, Grossman and Wang (1993), Wurgler and Zhuravskaya (2002), Barberis and Shleifer (2003), Chordia and Subrahmanyam (2004), Barberis, Shleifer, and Wurgler (2005), and Greenwood (2005a, 2005b), Andrade et al.(2006)). Other papers study the impact of informed trading on stock prices (e.g. Glosten and Milgroom (1985), Kyle (1985), Easley and O'Hara (1987)).

folio transitions.² Portfolio transitions are sizable and expensive transferring of funds from legacy (i.e. existing) to target portfolios. They are undertaken by various money management entities such as pension plans, insurance funds, endowments, and foundations. The main reasons for these transactions are strategic changes in global asset allocation, replacements of fund managers, and large cash inflows and outflows. They are typically delegated to professional managers who help execute them in a cost-efficient way. The study is based on a novel data set of transitions that are carried out by the leading provider of portfolio transition services.

This unique data allows me to precisely identify large supply/demand shocks caused by portfolio transitions and also to classify their information content. In particular, based on the institutional details of how securities in legacy and target portfolios are selected, I argue that on average, portfolio transition purchases are likely to be information-motivated trades whereas portfolio transition sales are liquidity-motivated ones.

I find that transition purchases change stock prices permanently upwards and that prices remain at new levels over several subsequent months; for some classes of stocks, even the price continuation is observed. In contrast, only temporary price pressure effects are induced by transition sales, and initial price deviations are adjusted over the following weeks. Extending my analysis, I show that the permanent price changes are especially pronounced after purchases of stocks with a high degree of information asymmetry, in which informed agents, by definition, have particularly substantial information advantage. Furthermore, transition purchases containing new rather than stale information, as indicated by past returns, have also stronger permanent effects on the stock prices. These findings provide solid support to my main conclusion: stock prices are indeed affected permanently by information-motivated trades and only temporarily by liquidity-motivated ones.

My main identification assumption is based on the double-selection mechanism

²Although portfolio transition management is an important part of modern financial markets, there are no academic papers that analyze this business per se. However, several papers study the selection and termination of investment management firms by institutional investors, which might induce portfolio transitions (Parwada and Yang (2004), Goyal and Wahal (2006)).

that underlies portfolio transitions: transitioned stocks are selected by fund managers that are, in turn, chosen by asset owners. Of course, transition trades are explicitly determined by hired and terminated fund managers, who choose securities into target and legacy portfolios based on their assessment of stocks' favorable perspectives. Thus, sales of legacy portfolios do not reveal any negative information about stocks. At the same time, purchases of target portfolios might be informative, since they reflect the managers' choices out of numerous alternatives in the market.³ What further strengthens the identification assumption are decisions of asset owners who hire and terminate fund managers, thus implicitly affecting the composition of legacy and target portfolios. There is evidence that asset owners might have abilities to select knowledgeable managers. For instance, in my sample, target portfolios outperform legacy portfolios in post-transition months. Moreover, several studies emphasize that asset owners usually rely on fairly sophisticated tools in selecting fund managers.⁴ Finally, Busse, Goyal, and Wahal (2006) point out that the predictability in performance of institutional winners portfolios for up to one year enables asset owners to benefit from picking winners.⁵ Consequently, transition purchases are most likely determined by positive signals of skilled fund managers whereas transition sales are merely uninformative liquidations of unskilled managers' positions.

In this paper, identification of supply and demand shocks is not subject to various concerns that are frequently relevant in other studies. First, the trading intentions of transition managers are set in advance; this contrasts with other studies in which *ex ante* orders cannot be identified, and realized trades might be influenced by a variety of investment styles and order-placement strategies. Second, the trading direction is explicitly given, whereas it is often necessary to apply heuristic and not perfectly

³Yet, this argument is weakened in a light of inconclusive evidence on the stock-picking abilities of fund managers. For instance, Lakonishok, Shleifer, and Vishny (1992), Coggin, Fabozzi, and Rahman (1993), and Ibbotson and Kaplan (2000) offer evidence in favor of different opinions about this issue.

⁴These choices of asset owners are thoroughly described in Del Guercio and Tkac (2002), Dishi, Gallagher, and Parwada (2006), Goyal and Wahal (2006), Heisler et al.(2006), and Parwada and Faff (2005).

⁵The performance persistence of institutional investment managers was also documented by Goyal and Wahal (2006), Ferson and Khang (2002), and Tonks (2005).

precise algorithms for inferring a trade side. Third, the magnitude of shocks is observed; however, researchers frequently have to find instruments for actual magnitude of shocks. Fourth, the execution time of trades is specified at daily frequencies; in contrast, additional assumptions about timing often have to be made. Fifth, transition trades occur at typical days, and prices dynamics are not affected by abnormal market conditions, release of new information about stock fundamentals or changes in stock characteristics. Sixth, a large cross-section of stocks is available for the analysis. The uniqueness of this data is obvious when it is compared to other studies of supply/demand shocks, for instance, to the index inclusions/deletions studies. In these studies, researchers find instruments for the shocks based on the stock index weights and industry size, assume that all the rebalancing is done during events, and hope that the results of tests for stocks that undergo index deletion/addition can be reliably extrapolated onto the normal trading days.

This paper contributes to a strand of empirical literature that studies price dynamics in response to trades with different information content. Demand and supply shocks are hard to identify, since only unsigned trading volume is usually observed. It is even a more challenging task to distinguish between informative and uninformative shocks. However, many studies have succeeded in overcoming these difficulties.

This work is related to the extensive empirical literature that analyzes how markets absorb uninformative shocks. These studies can be categorized into several groups. The first group examines security prices around rare events that are believed to trigger uninformed trading.⁶ The second group uses observed variables, i.e., large unsigned trading volume or signed order imbalances, as indicators of times with substantial liquidity trading.⁷ The third group focuses on prices after trades of particular agents who are likely being uninformed.⁸ Finally, other studies, ones most similar to my

⁶The examples are large-scale events, i.e., index inclusions and deletions (Garry and Goetzmann (1986), Harris and Gurel (1986), Shleifer (1986), Kaul, Mehrotra and Morck (2000), Wurgler and Zhuravskaya (2002), Chen, Noronha and Singhal (2004), Greenwood (2005a, 2005b) among others), or small-scale events, i.e., catastrophic events (Froot and O'Connell (1999)).

⁷See Campbell, Grossman and Wang (1993), Llorente et al.(2002), Avramov, Chordia and Goyal (2006) and Chordia, Roll and Subrahmanyam (2002, 2005), Chordia and Subrahmanyam (2004), respectively.

⁸The examples include trades of margin traders (Andrade, Chang and Seasholes (2005)) or trades

work, investigate specific trades of market participants that are not driven by private information.⁹ Although many of these studies find that initial price deviations are partially or completely reversed in time, the cases of permanent price changes or even price continuation are documented as well. This inconclusive evidence might be explained by various limitations inherent to most of the studies, such as either rough identification of liquidity shocks, small cross-section of stocks, or atypical time periods. As mentioned above, this paper avoids most of the concerns and analyzes the price responses to liquidity shocks that are identified in a precise manner.

Less is known about the market's responses to information-motivated trades. Several groups of market participants are believed to have information advantage over the rest of the market. However, the analysis of how prices respond to their trades, while often hindered by scarcity and limitations of available data, provides only mixed evidence. For instance, stock prices respond only mildly to insider trading (Lakonishok and Lee (2001)). Also, professional investment managers are often considered being fairly sophisticated. Yet, the comprehensive analysis of the relationship between their trades and security prices is fairly problematic: in the U.S., institutional holdings are disclosed in SEC 13-F filings only on a quarterly basis, thus making it difficult to disentangle whether institutional trades follow past returns, induce temporary price pressure effects, or move prices permanently.

The results of this paper are consistent with findings on institutional trading that are based on high frequency proprietary data sets. These studies often show that institutional purchases are followed by permanent price increases whereas institutional sales create only transitory effects.¹⁰ Certainly, institutional buys are more likely driven by information, and institutional sells are usually triggered by liquidity needs. Moreover, the purchased securities reveal stronger signals of private information than

of individual investors (Kaniel, Saar, Titman (2005)).

⁹For instance, fire sales of fund managers (Coval and Stafford (2005)) or mergers-induced trading of arbitrageurs (Mitchell, Pulvino and Stafford (2002)).

¹⁰The examples include studies of block trading (Kraus and Stoll (1972), Holthausen, Leftwich, and Mayers (1987, 1990), Keim and Madhavan (1996), Gemmill (1996)) and of institutional trading (Chan and Lakonishok (1993, 1995), Keim and Madhavan (1997)). Somewhat similar patterns are found in Campbell et al. (2005), who draw their conclusions based on both intraday TAQ and 13-F data.

the sold ones do: the former are chosen as the best securities out of numerous alternatives available in the market, whereas the latter are selected as the worst among the limited sample of securities in the current portfolios. Nevertheless, information content of institutional trades is still hard to purge in an accurate manner. Thus, more evidence on stock prices dynamics after information- and liquidity-motivated trades would be valuable.

The paper is structured as follows. Section 2 states the hypotheses. Section 3 describes portfolio transitions data. Section 4 explains tests and provides the evidence that supports the hypotheses. Section 5 discusses alternative explanations and extends the results. Section 6 concludes.

1.2 Hypotheses Development

Researchers can rarely observe motivations behind trades. Portfolio transitions are natural large-scale experiments that allow me to examine sizable demand and supply shocks which, with a high probability, can be identified as either informative or uninformative trades. Differences in their information content are expected to lead to differences in the triggered price dynamics. I start this section with a discussion of the portfolio transition industry that helps justify my main identification assumption: on average, transition purchases are information-motivated trades, whereas transition sales are liquidity-motivated ones. I then state my hypotheses that link portfolio transition trades and subsequent return dynamics.

1.2.1 Institutional Details

The current academic knowledge about portfolio transitions is fairly limited. Since my main identification argument is based on their complicated mechanisms, I briefly discuss here interesting details of portfolio transition management industry.

Portfolio transitions are undertaken by institutional investors, such as corporate and public pension plans, endowments, union plans, and foundations. In the present study, these investors are referred to as asset owners. As professionals in the area of

investment management, they make asset allocation decisions on behalf of their plans' beneficiaries. Typically, asset owners do not participate in the investment process directly but choose to delegate their funds to external fund managers, who run large funds and in turn have many institutional clients.¹¹ For most of them, managers hold portfolios with essentially the same composition of securities.¹² Interestingly, asset owners closely monitor and frequently reshuffle their fund managers, thus creating a fair amount of hiring and firing activity in the industry. Transferring of funds from terminated managers to newly hired managers is the most common reason for portfolio transitions. Also, transitions might be necessary when asset owners change the asset mix, deploy new cash inflows, or disburse funds; most of these events trigger large transfers of funds from legacy, i.e. existing, portfolios to target portfolios.

Every portfolio transition is unique. However, a typical portfolio transition is executed along the following lines. First, the manager to be hired is informed a few days or weeks in advance about the decision of the asset owner. Second, the manager to be terminated is notified a day or two before the transition that funds will be withdrawn from his management. To avoid front-running and to verify his actual holdings, he is instructed to stop trading. Since portfolio transitions is a very complicated task and, if poorly executed, might harm plans's excess annual returns, it is typically delegated to the professional transition managers. These transition managers are either pre-selected by the asset owner in advance or chosen through a bidding process. Just before transition, they get to know the list of securities of terminated manager, which is verified by a custodian, and the "wish" list of securities of hired manager. Transition managers design, coordinate and execute portfolio transitions; upon the completion, they issue a detailed post-trade report for their clients.

To minimize transaction costs, transition managers can use several different trading venues. These venues include internal crossing, external crossing, and open market trading. Some trades are crossed internally against the stocks from other transition

¹¹In general, an asset owner employs about 10 fund managers; larger plans with over \$1 billion in assets might hire more than 20 fund managers. The average mandate size is around \$50-\$100 million, and fund managers have about 10-20 clients.

¹²This industry is a mirror image of retail investors and their mutual fund managers, but in institutional setting.

mandates or against requests, submitted by an affiliated passive investment management team. Some trades are carried out through various crossing networks such as POSIT, LiquidNet, or Pipeline; these networks facilitate less expensive execution, which is, however, uncertain and prone to information leakage. The rest are traded directly through traditional markets.

Several details are worth mentioning. Terminated managers cannot use portfolio transitions to get rid of the stocks with negative perspectives; the composition of legacy portfolios is fixed with their current positions. At the same time, hired managers have discretion to use portfolio transitions to tilt their positions towards stocks with favorable perspectives; however, in practice, they usually recommend portfolios that are very similar to the portfolios of their current clients.

1.2.2 Hypotheses

In the market, there exists a complicated relationship between returns, trades, and underlying information. Market participants trade for a variety of reasons, and their trades can be broadly classified as either information-motivated or liquidity-motivated ones. These two are expected to generate different market's dynamics. On one hand, information-motivated trades reveal new information and shift stock prices permanently.¹³ On the other hand, liquidity-driven trades induce temporary price pressure effects: stock prices change in order to attract risk-averse counterparties, and as liquidity providers are compensated for their services, these deviations reverse. The asymmetry is emphasized in the theoretical work of Wang (1994) and Llorente et al.(2002). It also underlies the empirical models of Hasbrouck (1988, 1991). However, to design a study about these patterns is a challenging task, since information content behind trades is typically unknown to researchers.

Remarkably, portfolio transition purchases can be classified as information-motivated trades and portfolio transition sales - as liquidity-motivated ones. To justify this claim, I consider separately the decisions of market participants who originate these

¹³If information is only partially impounded into prices, then even the return continuation can be observed.

transactions: fund managers and asset owners.

First, terminated and hired fund managers explicitly determine legacy and target portfolios by choosing securities out of numerous alternatives, available in the market. Consequently, the composition of both portfolios reflects their positive assessment of stocks' fundamentals. This implies that sales of legacy portfolios do not carry any negative information.¹⁴ Moreover, on average, purchases of target portfolios indicate favorable perspectives of stocks which are chosen by hired managers among numerous alternatives. Informativeness of purchases is further enforced by managers' ability to use portfolio transitions to tilt their current portfolios towards better stocks. Yet, their informativeness might be questioned, since there is no conclusive evidence about the stock-picking abilities of fund managers.

Second, asset owners implicitly affect the composition of target and legacy portfolios; they decide how to reshuffle their funds between various managers and asset classes. As I discuss next, asset owners seem to have the manager-picking skills: they delegate their funds to skilled managers and withdraw their funds from managers with no abilities. Consequently, transition purchases are purchases of stocks, selected by skilled hired managers; these are information-motivated trades. At the same time, transition sales are liquidation of portfolios of unskilled terminated managers; these are liquidity-motivated transactions.

The portfolio transition data set provides an opportunity to demonstrate that asset owners might have the manager-picking abilities, which are at heart of my identification assumption. I implement the following procedure. I select 1,639 two-sided transitions, representing shifts of funds between legacy and target portfolios. Based on their snapshots right before and after transitions, I compare their performance in post-transition period; more precisely, I analyze the differences between target and legacy cumulative raw returns during six months, starting with the transition month.

Results in Table 1.1 show that, on average, target portfolios outperform legacy portfolios in post-transition period. In the transition month, the difference between

¹⁴Similar argument can be found in Coval and Stafford (2005), who argue that fire sales of mutual fund managers are unlikely be driven by information reasons.

target and legacy portfolios is about 30 bps, partially reflecting the triggered price pressure effects. These initial deviations reverse in the subsequent months; however, the average difference remains above 20 bps for up to half a year. I also split my sample into small and large portfolio transitions and find that these patterns are more pronounced for the larger ones. The reason might be that either asset owners, responsible for allocation of larger funds, are more knowledgeable and spend more resources when hiring new managers, or fund managers, responsible for managing larger portfolios, are more skillful.

It is worth mentioning several observations. First, this difference in performance is largely attributed to the price deviations in the transition month, when information, known to hired managers, is being revealed to the market. If the transition month is skipped, then no statistically significant difference between legacy and target portfolios is observed.¹⁵ Second, the discrepancy between legacy and target portfolios seems to be fairly small to justify the costly changes in fund management (on average, total transaction costs are about 25-30 bps). However, my analysis is based only on snapshots of holdings during transitions, and does not take in account possible portfolio rebalancing in the post-transition period.

Several additional rationales support my identification assumption. The level of expertise of asset owners in choosing fund managers is much higher than that of retail investors in selecting mutual funds (Del Guercio and Tkac (2002), Heisler et al. (2006)). For instance, the anecdotal evidence suggests that asset owners heavily rely on the recommendations of professional consultants and spend lot of their resources to evaluate and select fund managers. The hiring process usually includes several stages, starting with an initial screening, which is based on past performance and its consistency, and ending with several rounds of personal interviews, which help eval-

¹⁵Using a matched sample of firing and hiring decisions of asset owners, Goyal and Wahal (2006) conclude that if asset owners had stayed with fired investment managers, their excess returns would be larger than those actually delivered by newly hired managers. However, Goyal and Wahal (2006) study the differences between the performance of hired and fired managers at much longer horizons. Contradictive results might be also attributed to the nature of data, used in our studies: while this study is based on the precise information about legacy and target portfolios, Goyal and Wahal (2006) rely on the voluntarily disclosed information, public sources, and imperfect matching between hired and fired managers.

uate qualitative characteristics of managers. Furthermore, Busse, Goyal and Wahal (2006) point out that the tendency of asset owners to pick past winners might help them to benefit from persistency of performance of institutional fund managers, at least in the short-run; this short-run persistency was also documented in Goyal and Wahal (2006), Ferson and Khang (2002), and in Tonks (2005).¹⁶

My identification assumption leads to the following hypothesis:

Hypothesis 1: *On average, transition purchases are information-motivated trades, and they shift stock prices permanently upwards. At the same time, transition sales are liquidity-motivated trades, and they affect stock prices only temporarily.*

Some transition purchases are expected to be especially informative. By definition, these are purchases of stocks with a high degree of information asymmetry, in which informed traders have particularly substantial advantage over other market participants. Thus, formally, my second hypothesis states the following:

Hypothesis 2: *Transition purchases of stocks with a higher degree of information asymmetry contain more information and lead to more substantial permanent price adjustments.*

By the nature of portfolio transitions, choices of stocks to be purchased are bound to the current positions of hired managers. Some stocks in their portfolios were bought recently based on information that has not been fully incorporated into stock prices; the others were acquired long time ago based on the positive signals that have been already fully reflected in the market. Consequently, transition purchases might contain either new or stale information. It is important to disentangle these two cases, since only unexpected components reveal new information and shift security prices permanently (see Hasbrouck (1988, 1991)).

Certainly, the complete trading history of hired managers could reveal which of the current holdings might be attributed either to stale or to new information. However,

¹⁶These findings contrast with the well-known lack of persistence in retail mutual funds industry, and can be partially explained by more sticky and lumpy capital flows of institutional investors.

this data is unavailable. To get around this problem, I suggest using recent past returns that might help identify these two groups of observations. Indeed, if past risk-adjusted returns are high, then most likely favorable information has been already revealed to other market participants and this has been reflected in the recent upward price dynamics. In contrast, if past risk-adjusted returns are low, then information content of transition purchases seems to be unknown to the market. This observation leads to my next hypothesis, which relates transition purchases to past returns and states the following:

Hypothesis 3: *Transition purchases of stocks with high past returns contain only latent information, and their permanent price impact is insignificant. In contrast, transition purchases of stocks with low past returns contain new information, and their permanent price impact is large.*

To summarize, in this section I discuss several hypotheses about portfolio transition trades and the price dynamics that they are expected to generate. Next, I describe the data set of portfolio transitions and design of my tests.

1.3 Data Description

1.3.1 Data set of Portfolio Transitions

In this study, I use a novel database of portfolio transitions.¹⁷ Data is made available by a leading provider of portfolio transition services, who supervises more than 30% of stock transitions executed in the U.S. More precisely, after various filters, I consider

¹⁷There are several recent studies about selections and termination of fund managers by institutional investors, which are also closely related to portfolio transitions. In comparison with this paper, other studies employ much less accurate and comprehensive data sets. Indeed, Parwada and Yang (2004), Parwada and Faff (2005), and Dishy, Gallagher, and Parwada (2006) analyze data set on the allocations and withdrawals of Australian pension plan investment management mandates that is constructed based on the voluntary disclosures, public domain sources, pension plan reports, and the investigation by a team of financial reporters. Similarly, Goyal and Wahal (2006) rely on the data set of hiring and firing decisions that are voluntarily disclosed by fund managers to Mercer Investment Consulting and Institutional Investor Publications, as well as articles published in Pensions and Investments.

2,234 portfolio transitions with a trading volume of roughly \$400 billion executed on behalf of U.S. institutions from January 2001 to December 2005.

The portfolio transition data set is exceptionally detailed and clean. It is constructed from the actual post-transition reports built by transition managers for their clients on a case-by-case basis; these reports were thoroughly discussed during portfolio transitions. The data was further checked manually to avoid any potential typographical errors and inaccuracies. As a result, it represents a reliable picture of actual processes.

For each transition, I observe a post-trade execution report. Each report contains two types of data: the general information and the more detailed description of executed trades. The general information includes starting date, ending date, and the base currency. Starting (ending) date denotes the day of the first (last) transaction. Base currency is the trading currency of an asset owner. However, at the time when this data set was constructed, mainly data on the transitions of U.S. clients was available.

The information on executed trades specifies the number of shares traded, the execution prices, the benchmark prices, as well as information on various transaction costs. Since the implementation shortfall methodology is used for quality control, pre-trade benchmarks are often employed. The execution prices and shares traded are provided in aggregated form; all trade records are clustered at daily levels. Moreover, they are further grouped according to execution methods. There are three different execution methods: internal crossing, external crossing, and open market trading. Thus, for each transition, each stock, each trading day and each trading venue, the number of shares traded, the average execution price, and the pre-trade benchmark price are specified. Also, in-kind transactions, or assets being part of both legacy and target portfolios, are observed; availability of these transactions allows me to reconstruct legacy and target portfolios.

1.3.2 Supplementary Data

I use standard CRSP database to get additional stock information: prices, returns, volume, and shares outstanding. My sample includes ordinary common stocks (with CRSP share codes of 10 and 11) listed on the New York Stock Exchange (NYSE), the American Stock Exchange (Amex), and NASDAQ in the period of January 2001 through December 2005. Any derivative securities, such as ADRs, REITS, or closed-end funds are excluded. The estimates of probability of information-motivated trading, PIN, are downloaded from Soeren Hvidkjaer's web site. I also use unadjusted data from the Institutional Brokers Estimates System(I/B/E/S) to get the number of analysts who follow stocks.

In order to eliminate records with potential errors and unrepresentative trades, I have filtered out transitions with obviously wrong information or typographical errors due to the storing and collecting methods. The stocks with missing CRSP information, necessary to construct variables for the tests, are removed as well. Moreover, I exclude low-priced stocks with prices below \$1.

After filtering, I consider about 682,481 transition orders. About 274,244 orders are at least partially executed through open markets; for 79,132 of these orders, the fraction directed to open markets is larger than 1% of the average daily volume in the previous month. Among them, there are 36,544 buy orders and 42,588 sell orders.

1.3.3 Summary Statistics: Portfolio Transitions

In this section, I describe a playing field and general characteristics of transitions, which are analyzed in the paper.

Panel A of Table 1.2 provides a general description of portfolio transitions in the sample. These transactions are large. For instance, during a typical transition, about \$50 million or more than a million shares in 133 various stocks are traded. The distribution of transition size is skewed by several large observations; large transitions involve several billions of dollar trading volume, and the maximum number of stocks, traded in a particular transition, is as high as 3,669.

Panel B of Table 1.2 shows the distribution of transitions across the market capitalization groups.¹⁸ While about 70% of dollar value is executed in large stocks, only 1.5% is traded in small stocks; this disparity is less extreme, if instead the number of stocks is considered, since trades in large stocks tend to be more sizable. The observed dominance of large stocks in the sample is the result of numerous regulatory restrictions, faced by institutional investors.

Portfolio transition orders are often split over several days. Usually, the execution horizon is negotiated between transition managers, who suggest the optimal trading time as a part of their pre-trade analysis, and their clients, who might have additional constraints that require faster trading.¹⁹ Panel C of Table 1.2 describes the transition durations, defined as the number of days between the first and the last trades in a given transition. Interestingly, it takes from one up to 18 days to complete transitions. About 56% of transitions are executed in one day, while most others are spread over a week. If trading volume is considered, then only 26% of transitions are completed in one day; this is expected because more complex transitions typically require more time to be executed.

Also, legacy portfolios are sold slightly faster than target portfolios are acquired (results are not reported). These patterns are due to the agency-based approach, followed by the transition team: it does not provide any additional capital but rather finance purchases of target portfolios with the proceeds from sales of legacy portfolios.²⁰

To summarize, portfolio transitions are large and complicated transactions. A typical size of a transition is about \$50 million. It involves more than 100 stocks, most of which are the large ones. Typically, transition orders are spread over different trading venues and over time. It takes from one to 18 days to complete a transition; about 56% of them are finished in one day, and 96% are executed over a week.

¹⁸Thresholds for size groups are based on the capitalization distribution of the NYSE stocks in 2003.

¹⁹Often fund managers ask for quicker execution because they want to minimize the blackout period during which the ability to trade is temporarily suspended.

²⁰When quick turnaround are necessary or when costs must be known in advance, the principle-based approach is used: the entire portfolio is sold to a broker who takes all risks of selling stocks for a fixed pre-determined price.

1.3.4 Summary Statistics: Transition Orders

In this section, I describe the transition orders in individual securities: their main characteristics and how they are split over time and different trading venues. I consider a sample of transition orders, for which more than 1% of the average daily volume is traded through open markets; the effects on security prices of disregarded small trades is difficult to detect statistically.

Panel A of Table 1.3 shows summary statistics for the selected sample. Transition purchases and sales have similar characteristics. Their average size is typically about \$1 million: \$0.1 million is common for both legacy and target portfolios, \$0.5 million is traded through the open market, finally, \$0.2 million is crossed externally and internally, respectively. Definitely, these significant values might be influenced by a few large orders. Transition orders account, on average, for about 6 bps of the shares outstanding or for more than 14% of the average daily volume. At the same time, the median values are much lower: roughly 2.8% of the average daily volume and 1.5 bps of the shares outstanding, most of which is executed through the open market trading.²¹

Transition orders are frequently split over several days. Panel B of Table 1.3 presents information on the distribution of trades over time. For each day relative to the starting day of transition and trading venue, Panel B exhibits the average trade size, normalized by the average daily volume in the previous month. Clearly, most transition orders are executed during the first week, and especially during the first two days. This is important for interpreting the results, presented in the next section. Also, portfolio transition orders are executed more quickly than routine institutional strategies.²² This observation highlights that portfolio transitions represent a unique subset of institutional trades.

²¹To compare, the average daily turnover is about 40 bps at NYSE in 2005.

²²For example, acknowledging a usual practice of splitting institutional orders, Chan and Lakonishok (1995) reconstruct an ex ante institutional order looking at trading packages, or a sequence of trades which is followed by a 5-day period when a manager stays out of the market. They show that only about 20% of the value of institutional trading is executed in one day; meanwhile trading packages, which take four and more days to complete, account for about a half of all institutional trades. Related evidence can be found in Keim and Madhavan(1995).

1.4 Event Study

In this section, I test the hypotheses of Section 1.2.2 by analyzing the behavior of stock prices after portfolio transition trades. I start with a detailed description of the tests and then present the results.

1.4.1 Design of Tests

I follow the event-study approach to investigate the dynamics of prices after portfolio transitions trades. First, I define the risk-adjusted returns, $r_{i,t}^{adj}$. The specification of a correct risk-adjustment model is always problematic. I chose to adjust returns for their exposure to three risk factors, as suggested in Fama and French (1992). The composition of target and legacy portfolios might depend on the past returns; therefore I augment the model by the momentum factor (see Carhart (1997)). For each security and each month, I estimate the factor loadings using five pre-event years of data with at least 24 monthly return observations. To correct for potential biases due to non-synchronous trading, I apply a standard methodology: I regress monthly excess stock returns on the contemporaneous market value-weighted excess return, a size factor, a B/M factor and a momentum factor with their lags (see Dimson (1979)). The size factor is a return on a portfolio that is long in small stocks and short in large stocks. The B/M factor is a return on a portfolio that is long in stocks with high B/M and short in stocks with low B/M. The momentum factor is a return on a portfolio that is long in stocks with high past returns and short in stocks with low past returns. Sensitivity to a factor is a sum of coefficients on the factor and its lagged value.

Second, I calculate the cumulative abnormal returns, CARs:

$$CAR_{i,T} = \sum_{t=0}^T r_{i,t}^{adj} \quad (1.1)$$

where $r_{i,t}^{adj}$ is the risk-adjusted return of stock i at day t and T is a horizon. Both short-term horizons, such as several days and weeks, as well as longer horizons such as one, two, and three months are considered. The use of cumulative returns rather

than buy-and-hold returns is advocated by Fama (1998), who argues that the former is subject to less severe biases.

Third, for each horizon T , the cumulative average abnormal returns, CAARs, and their t -statistics are calculated following the Fama-McBeth procedure for data grouped at monthly levels. This procedure allows one to correct for the cross-sectional correlations between stock returns. Cross-sectional interdependence might be induced either by general market dynamics or by portfolio trading; in both situations, contemporaneously traded stocks share similar characteristics. The example is a portfolio transition of an asset owner who transfers funds from the large-stock into growth-stock portfolios. T -statistics are further adjusted with the Newey-West procedure to correct for intertemporal correlations. The number of lags is chosen by the automatic bandwidth selection procedure.²³²⁴

The first hypothesis emphasizes the distinctive patterns of the CAARs after transition purchases and sales. For purchases, the CAARs are expected to be shifted upwards during transitions and then remain significantly positive at longer horizons. For sales, the CAARs are predicted to deviate downwards but then reverse and become insignificant at longer horizons. The results, presented below, support this hypothesis.

1.4.2 Results

Test of Hypothesis 1

Figure 1-1 plots the cumulative average abnormal returns, CAARs, after large transition orders.²⁵ Panel A of Table 1.4 quantifies the results. Returns are weighted equally; horizons for up to three months are considered. Two main patterns are

²³Results are similar, if other numbers of lags are used.

²⁴Pooled regressions with clustering at time and security levels (or only time level) produce very similar estimates of standard errors; if only clustering at security level is considered, then standard errors are much smaller. Further discussion on panel data estimation can be found in Petersen (2007).

²⁵This sample includes orders during which more than 1% of the average daily volume in the previous month is traded through open markets; the effects of these trades on security prices can be more easily detected.

clearly observed. First, trades certainly affect the contemporaneous stock returns: on average, during the first week, purchases are accompanied by 0.43%-increase in stock prices, whereas sales coincide with 0.36%-decline in stock prices. Second, for purchases and sales, price signatures exhibit a surprising asymmetry at longer horizons. Typically, sales induce only temporary price pressure effects: the initial price decline reverses and disappears in several weeks. In contrast, purchases move prices permanently, and the gap between the CAARs after purchases and sales is about 0.40%. By and large, this finding supports the first hypothesis.

Figure 1-2 depicts the principle-weighted CAARs or, more precisely, returns weighted with the size of open market trades, which are normalized by the average daily volume in the previous month.²⁶ This weighting scheme puts more weight onto potentially more informative trades and leads to even stronger evidence of asymmetry: the difference between principle-weighted CAARs for purchases and sales is roughly 0.80% in three months after portfolio transitions, which is twice as high as for the equally-weighted case.

Panel B of Table 1.4 presents the CAARs' estimates for returns, weighted with their market capitalizations in the previous month; Figure 1-3 graphically illustrates the results. In this case, the price responses are quite different. First, the initial price reaction is less significant than for the equally-weighted case; stock price increases by only 0.17% for the stocks, acquired as part of target portfolios, and decreases by 0.25% for the stocks, sold as part of legacy portfolios. Second, at longer horizons, no asymmetry between prices of sold and purchased stocks is observed: initial price changes reverse and come back to the original pre-transition levels after a few weeks. These patterns are comprehensible, since large and transparent securities are weighted more heavily in this case and, while trading these securities, hired managers do not have information advantage over other market participants.

Figure 1-4 presents the average price responses to the sample of all orders without any restrictions. Again, although the temporary price deviations are observed, the difference between sold and bought stocks at longer horizons is insignificant. These

²⁶For each month, 1% of extreme observation of weights is truncated.

patterns are due to the numerous small trades that get the same weight but do not affect security prices significantly.

It is worth mentioning that price pressure effects, which are created by transition trades, are too significant to be explained by bid-ask bounce. The magnitude of contemporaneous price impact is about 50bps for both purchases and sales (Figure 1-1). In fact, for stocks in this study, the typical percentage spread is roughly equal to 20bps. Thus, only about 20% of temporary price deviation might be attributed to bid-ask bounce.

Interestingly, the price pressure effects, created by trades of a particular market participant rather than a segment of investors, might cause a substantial price impact not only at daily but also at weekly and monthly frequencies. Since these shocks represent only a small fraction of trading volume, this implies that the observed intensive trading, which is largely attributed to noise trading, can not eliminate the effects of one-sided order imbalances on securities prices.²⁷

The evidence of long-lasting price pressure caused by portfolio transition sales is related to several studies of institutional trades. For example, Coval and Stafford (2005) have documented that mutual fund sales/purchases, triggered by flows in and out of funds, create patterns of price pressure, which do not vanish for several quarters. Jin and Scherbina (2006) point out that for three months following the managerial change, sell-offs of recently hired fund managers might result in substantial underperformance of the inherited losers relative to other momentum losers. Also, Subrahmanyam (2006) shows that monthly order imbalances, constructed from intraday data, might affect stock returns for up to two months.²⁸

In summary, the study of stock prices around transition trades shows that the contemporaneous stock prices deviate from the pre-trade levels. Moreover, on average, during transition sales, stock prices change only temporarily and reverse to the original levels over the next several weeks; during transition purchases, stock prices

²⁷In contrast, Chordia, Roll and Subrahmanyam (2005) document that even the aggregated intraday order imbalances are usually accommodated within the subsequent hour.

²⁸However, Mitchell, Pulvino and Stafford (2002) describe the impact of the merger arbitrageurs' trades on the stock prices; however, their price pressure effects are relatively short-lived.

are shifted to new levels permanently. These patterns clearly support *Hypothesis 1*.

Test of Hypothesis 2

Hypothesis 2 states that transition purchases in stocks with higher levels of information asymmetry are more informative and induce larger permanent price changes. In order to test this hypothesis, I consider transition trades in stocks with high and low information asymmetry and then examine their CAARs separately.

The information asymmetry of individual stocks is not directly observed. There is no agreement on how to define its best proxy. For robustness, I use several measures suggested in the previous literature. For instance, one of its proxies might be the market capitalization: exposure of large stocks to information asymmetry is expected to be much lower than that of small stocks. Empirical literature provides evidence consistent with this claim; for example, Lo and McKinlay (1990) find that prices of the stocks with smaller capitalization do not follow random walks, which would be expected for the securities with no significant informed trading. The different patterns of the equally-weighted and value-weighted CAARs, discussed in the previous section, suggest that permanent price impact indeed tend to be more pronounced for small stocks rather than for large ones. Further, in this section, I focus on other proxies of information asymmetry.

The first proxy is the number of analysts who follow stocks or, in other words, the number of individuals who produce information about firms, *ANUM*. I construct these measures based on the monthly number of analysts who provide I/B/E/S with their end-of-fiscal-year earnings forecast. The large number of analysts following stocks enhances transparency regarding their perspectives and lessen the information asymmetry. For instance, Brennan and Subrahmanaym (1995) and Easley, O'Hara and Paperman (1998) argue that trading in stocks with the greater number of investment analysts is associated with the lower adverse selection costs; also, the initiations of analyst coverage usually improve liquidity.

The second measure of information asymmetry is the percentage spread, *SPREAD*. According to insights of the theoretical work, this proxy reflects the assessment of mar-

ket participants about how probable the trading against informed traders is. Thus, stocks with the higher information asymmetry tend to have the larger percentage spread.

The third measure of information asymmetry is the probability of information-motivated trading, PIN, which is constructed based on the structural model of Easley and O'Hara (1987, 1992). Easley et al. (1996) suggest the algorithm to estimate these measures using intraday data. The higher the probability of information-motivated trading is, the larger the degree of information asymmetry is expected. Consistent with this claim, Easley et al. (1996) show that PIN is closely related to spread, which is in turn tightly linked to the adverse selection costs of trading.

The suggested measures are noisy proxies for unobserved information asymmetry, they are far from being perfect. Indeed, the strength and the exact functional form of their cross-sectional relation with the degree of asymmetry are unknown. Moreover, there are several additional concerns. For instance, Boehmer et al. (2006) show that inaccuracies in trade classification lead to downward biases in the estimates of PIN. The number of analysts, *ANUM*, is frequently not identified for the stocks with potentially high information asymmetry, since these stocks are not regularly followed by analysts. Nevertheless, I use these proxies to test *Hypothesis 2*.

To examine how the price dynamics depend on information asymmetry, I slightly modify the event-study test, described in Section 1.4.1: each month, observations are additionally sorted into three groups based on their information asymmetry, and afterwards their CAARs are analyzed.

Table 1.5 shows the CAARs for two groups of stocks: ones with low and high information asymmetry. For all three proxies, similar patterns are observed. First, consistent with the previous results, after transition sales, the prices tend to come back to pre-transition levels regardless of information asymmetry. Second, after transition purchases, the price dynamics differ for the low and high information asymmetry groups: for the former, only temporary price deviations are observed whereas for the later, the initial price deviations are permanent and more significant. Interestingly, in the latter case, stock prices even continue to increase in the post-event period, thus

indicating that information, originally known to hired managers, is being gradually revealed to other market participants.

Table 1.6 shows that proxies of information asymmetry are highly correlated with the market capitalization. For instance, the correlation between the probability of information-motivated trading, *PIN*, and stock size is about -0.60. Also, fewer analysts follow smaller stocks, and larger spreads are typically observed in their markets. In order to clearly disentangle the effects of information asymmetry from the size effects, I apply a double-sorting procedure. Each month, observations are first sorted into three groups based on their stock capitalization. Within each size group, observations are further clustered into two sets according to the associated degree of information asymmetry. Then, I test whether the permanent price impact patterns differ across stocks within size groups.

Results in Table 1.7 confirm that the number of analysts, *ANUM*, is directly linked to the long-run price dynamics even after controlling for stock size. First, consistent with the previous findings, transition purchases induce permanent price changes in small stocks but not in large stocks. More interestingly, if only small stocks are considered, then long-run price changes are much more significant for stocks with low *ANUM* than for those with high *ANUM*. For instance, in two and three months after transitions in the former, the CAARs increase to about 1.96% and 2.21%, being significant at 1% and 5% levels, respectively; at the same time, the corresponding CAARs for the latter are only 0.73% and 0.42%, being statistically indistinguishable from zero. Similar patterns are observed for the medium-size stocks. Thus, the *ANUM*-sorting reveals the variations in the post-transition price dynamics that is unrelated to size.²⁹

Table 1.8 shows the results for a double sorting on size and the percentage spread, *SPREAD*. Clearly, *SPREAD* is related to the stock price dynamics even after controlling for stock size: indeed, for small stocks, purchases of stocks with large *SPREAD* trigger permanent price shifts, at the same time, they bring about much

²⁹The results for sales are not presented. As before, their CAARs are not statistically different from zero at long horizons.

less significant price deviations for stocks with small *SPREAD*. Table 1.9 demonstrates that a double sorting on size and *PIN* leads to the very similar conclusions as well.

In this section, I show that transition purchases of stocks, traditionally considered as being prone to high risk of information asymmetry, are associated with significant and permanent upward shifts in prices; moreover, increasing price dynamics often continue even in post-transition periods. The robustness of these patterns is checked for various proxies of information asymmetry. Thus, the documented evidence is in favor of *Hypothesis 2*.

Test of Hypothesis 3

In this section, I test *Hypothesis 3*: transition purchases that reflect stale information (as shown by large abnormal past returns) have less significant permanent effects on stock prices, whereas the opposite is expected to hold for transition purchases that contain new information (as shown by low abnormal past returns).

To test this claim, I first sort observations into three groups based on their risk-adjusted returns in the previous three months, $R_{-3m,-1}^{adj}$. For instance, group "Low" includes stocks with the bottom 33% of negative risk-adjusted past returns, and group "High" includes stocks with the top 33% of positive risk-adjusted past returns. Then, for each of these groups, I estimate the CAARs and their statistics following the procedure, described in Section 1.4.1.³⁰

Table 1.10 shows that, in fact, permanent effects of transition buy orders are much more significant for the stocks with low past returns than for the ones with high past returns. For the former, the CAARs are as high as 1.61% and 1.47% in two and three months after portfolio transitions, respectively; at the same time, for the latter, the triggered price effects are short-lived, and stock prices revert to their pre-transition levels during the following weeks. Thus, only unanticipated trades reveal new information and shift security prices permanently. At the same time,

³⁰If returns in the previous six months, $R_{-6m,-1}^{adj}$, are considered or if stocks are split into three groups according to their past returns, then the results are qualitatively similar.

for transition sales, the CAARs tend to decline for stocks with high pre-transition returns and stay close to zero for stocks with low past returns. This is consistent with information hypothesis, since the latter situations are more likely to occur during replacement of fund managers which trigger uninformative redemptions of legacy portfolios.

It is also worth mentioning the intriguing short-run patterns of the CAARs after transition sales. Although their CAARs for both groups are insignificant in the long run, the short-run market's dynamics are very different: for stocks with low past returns, the sales have almost no significant effects on the stock prices; in contrast, the price pressure effects triggered by the sales of stocks with high past returns last for about a two weeks. These effects might be partially explained by the arguments that are unrelated to specifics of portfolio transitions. For instance, Saar (2001) claims that the information content of sales might depend on the past prices: their permanent price impact increases after a long run-up in stock prices.³¹ If the market cannot ex ante distinguish the portfolio transition trades from other transactions, then this reasoning is applicable for the documented short-run price dynamics.

In this section, I present the supportive evidence for *Hypothesis 3*. Indeed, if stocks are selected by hired managers based on the new information, then their purchases create permanent price changes. However, if stocks are chosen by hired managers for matching their current positions rather than for information reasons, then their purchases do not change the long-run price dynamics.

1.5 Discussion and Extensions

In this section, I first discuss alternative explanations for the documented asymmetry of price responses after transition purchases and sales and conclude that these explanations are less plausible than the primary information hypothesis. Also, I extend my

³¹In his model, informed agents are short-sale constrained and, typically, cannot sell stocks even if negative signals are received. Thus, information component of sales is small and demand only insignificant price changes. However, a long run-up in the stock prices implies that informed agents acquired stocks and are currently able to sell them; thus, the information content and permanent price impact of sales increase.

analysis and demonstrate the price dynamics after transition orders executed through the trading venues, other than the open market.

1.5.1 Alternative Explanations

Many effects are associated with portfolio transition trades and might contribute to the following-up price dynamics.

First, if no perfect substitutes are available for securities, then their excess demand and supply curves are not flattened by risk-averse arbitrageurs and trades induce permanent effects on the prices.³² However, this could explain asymmetry between purchases and sales only if it were harder to find a hedge for a short rather than for a long position. Hence, the substitution hypothesis cannot explain the documented patterns.

A second explanation might be that transition trades change the composition of investors who hold the security: large purchases increase the institutional ownership. Boehmer and Kelley (2005) show that stocks with greater institutional ownership are priced more efficiently. In fact, the increase in the number of institutions that hold the stock might increase the number of analysts following it and induce competition among the informed traders. Both effects enhance efficiency, decrease the future trading costs, and shift up the stock price following the transition purchases. Also, large purchases might create block holders who closely monitor the firm and contribute to the increase of its value. However, the transition trades analyzed in this study are fairly insignificant to trigger these effects. Table 1.3 shows that the average value of orders traded through the open market is about 3 bps of the shares outstanding. In comparison, to find the effects on market efficiency, Boehmer and Kelley (2005) analyze much larger 60-bp changes in the institutional holdings. Also, it is unclear why the opposite effects are not observed for the transition sales. Hence, the institutional ownership hypothesis is unlikely to explain the documented buy/sell asymmetry either.

³²See Garry and Goetzmann(1986), Shleifer(1986), Kaul, Mehrotra, Morck(2000), Wurgler and Zhuravskaya(2002), among others.

Finally, another explanation might be that large transition trades change the severity of short-sale constraints: purchases make the ownership more concentrated, and also asset owners are usually reluctant to be engaged into short selling. These two mechanisms might aggravate the short-sale constraints. Miller (1977) notes that prices of constrained stocks may be inflated beyond fundamental levels since only beliefs of optimistic investors are incorporated.³³ Thus, the change in the severity of short-sale constraints can explain the permanent price impact asymmetry after transition buys and sells. However, the transition trades seem to be too small to make this explanation plausible.

1.5.2 Price Dynamics and Trading Venues

In the main part, I analyze the price dynamics after transition trades executed through traditional exchanges (open markets). In this section, I examine how security prices evolve after the trades carried out through alternative trading venues: internal and external crossing networks as well as in-kind transactions.

To minimize transaction costs, managers use various trading platforms to implement portfolio transitions. Some securities in legacy and target portfolios overlap; these securities are transferred as in-kind transactions, obviously without any cost. Other securities are traded in open markets; in the preceding sections, the effect of these transactions on security prices is discussed in detail. Finally, some orders are executed through non-conventional trading venues, internal and external crossing networks, which help alleviate or completely remove transaction costs.

If transition companies succeed in attracting many transitions or, alternatively, are affiliated with large passive asset management firms, then they have access to internal pools of liquidity, against which they may cross fraction of their orders. These internal crosses have neither transaction cost nor information leakage and therefore might boost the competitive advantage of transition companies. Furthermore, some orders

³³Duffie, Garleanu and Pedersen (2002) show that prices of constrained stocks can be inflated even higher than the marginal valuation of the most optimistic investors, if they take into account the potential profit to be made by lending stocks in the future.

might be directed to external crossing networks such as ITG's POSIT, LiquidNet, or PipeLine. Ever since being introduced, these networks are valuable sources of liquidity for institutional investors. Although they come in various forms and have different clienteles, their primary goal is typically to facilitate efficient execution of large orders. In these systems, orders are matched against each others at the prices derived from primary markets. Though a full anonymity is not guaranteed, crossing networks substantially reduce information leakage and help avoid bid-ask spread.³⁴

Next, I examine the price dynamics triggered by transition orders which are executed through different trading systems. More precisely, I consider three groups of orders: the orders executed entirely as internal crosses (39,639 orders), the orders executed entirely through external networks (59,944 orders), and the orders transferred entirely as in-kind transactions (27,449 orders). These sets are based from the original sample that includes only large orders accounting for more than 1% of the average trading volume in the previous month.

Table 1.11 shows the CAARs after transition purchases and sales executed through either internal and external crossing networks (Panel A and B, respectively) or as in-kind transactions (Panel C). First, regardless of trading venue, the estimated CAARs after both purchases and sales are statistically insignificant in post-transition periods. Perhaps, these transactions tend to reflect common benchmarks rather than managers' bets on individual securities or mostly affect large stocks, in which hired managers do not have information advantage over other market participants. Yet, consistent with information hypothesis, security prices shift upwards after transition purchases carried out through external crossing networks, though these changes are not statistically significant too.

Second, the interesting patterns are observed after transactions in external crossing networks in the short-run: on average, sales are associated with decline in stock prices, whereas purchases are accompanied by the statistically significant price increase. These pattern are surprising given that these networks have no price discov-

³⁴See Ramistella (2006) for the recent overview of crossing networks.

ery mechanisms.³⁵ Several explanations might be suggested. Certainly, these findings might be explained by a selection bias: when prices move against the trading direction, the larger fraction of orders is directed to open markets, since the transaction costs are not a concern anymore, if pre-transition benchmarks are used. Thus, external crosses are mostly used during adverse price changes and mechanically exhibit price impact. Alternatively, external crossing networks can be subject to information leakage, which is often brought up by practitioners during discussions of crossing networks. However, it is difficult to differentiate the underlying mechanisms in data.

To summarize, the price behavior after trades executed through different trading venues is broadly consistent with information hypothesis.

1.6 Conclusion

The main goal of this paper is to investigate how the market learns new information and how it accommodates liquidity shocks. A unique data set of portfolio transitions allows me to identify information-motivated and liquidity-motivated trades; more precisely, I argue that transition purchases contain private information whereas transition sales are uninformative changes in supply. Consistently with market microstructure beliefs, the analysis of price dynamics, triggered by these transactions, reveals the asymmetric price signatures after the trades with different underlying motives. On one hand, transition purchases permanently shift prices; moreover, information is often not fully revealed, and security prices continue to increase in the post-transition period. These patterns are especially pronounced after large trades, trades in stocks with high degree of information asymmetry, and trades that reflect new rather than stale information. On the other hand, transition sales induce only temporary price pressure effects that reverse in the following weeks.

Asymmetric price dynamics after purchases of target portfolios and sales of legacy portfolios allows for an alternative interpretation. Given that price dynamics reflect

³⁵To remind, these orders are executed entirely through crossing networks and, consequently, the price pressure effects can not be attributed to contemporaneous open market trades.

the information content of trades, these results show that a subset of money managers, who manage target portfolios, do have the stock-picking abilities and can deliver abnormal returns, at least in the short run after portfolio transitions. Moreover, these results suggest that asset owners might be able to differentiate skilled managers from the others. This is the novel and important evidence on investment strategies of the largest institutional investors as well as investment managers. So far, the behavior of these market participants has remained largely unexplored, mostly due to the scarcity and limitations of available data; however, both the extent of assets under their jurisdiction and their social importance are good reasons to learn more about this subject.

The unanswered question is how the market learns information content of portfolio transition trades or, more generally, of any other trades. Certainly, the execution of these complicated transactions requires extensive communication between market participants, which might be unobserved by researchers but crucial for information dissemination. More detailed investigation of price dynamics during these transactions might shed a light onto these issues.

References

- Andrade, Sandro, Charles Chang and Mark Seasholes, 2006, Trading imbalances, predictable reversals and cross-stock effects, *Working paper*.
- Avramov, Doron, Tarun Chordia, and Amit Goyal, 2006, Liquidity and autocorrelations in individual stock returns, *Journal of Finance* 61, 2365–2394.
- Barberis, Nicholas and Andrei Shleifer, 2003, Style investing, *Journal of Financial Economics* 68, 161–199.
- Barberis, Nicholas, Andrei Shleifer, and Jeff Wurgler, 2005, Comovement, *Journal of Financial Economics* 75, 283–317.
- Boehmer, Ekkehart, Joachim Grammig, and Erik Theissen, 2006, Estimating the probability of informed trading - Does trade misclassification matter?, *Working Paper*.
- Boehmer, Ekkehart, and Eric Kelley, 2005, Institutional investors and the informational efficiency of prices, *Working Paper*.
- Brennan, Michael, and Avanidhar Subrahmanyam, 1996, Market microstructure and asset pricing: On the compensation for illiquidity in stock returns, *Journal of Financial Economics* 41, 441–464.
- Busse, Jeffrey, Amit Goyal, and Sunil Wahal, 2006, Performance Persistence in Institutional Investment Management, *Working Paper*.
- Cahart, Mark, 1997, On persistence in mutual fund performance, *Journal of Finance* 52, 57-82.
- Campbell, John, Sanford, Grossman, and Jiang Wang, 1993, Trading volume and serial correlation in stock returns, *Quarterly Journal of Economics* 108(4), 905–939.
- Campbell, John, Ramadorai, Tarun and Tuomo Vuolteenaho Tuomo, 2005, Caught on tape: Institutional order flow and stock returns, *NBER Working Paper No. W11439*.
- Chan, Louis, and Josef Lakonishok, 1993, Institutional trades and intra-day stock price behavior, *Journal of Financial Economics* 33, 173–199.
- Chan, Louis, and Josef Lakonishok, 1995, The behavior of stock prices around institutional trades, *Journal of Finance* 50(4), 1147–1174.
- Chen, Honghui, Gregory Noronha, and Vijay Singhal, 2004, The price response to S&P 500 Index additions and deletions: evidence of asymmetry and a new explanation, *Journal of Finance* 59, 1901-1929.

- Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam, 2002, Order imbalance, liquidity, and market returns, *Journal of Financial Economics* 65 (1), 111–130.
- Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam, 2005, Liquidity and market efficiency, *Working paper, Emory University*.
- Chordia, Tarun and Avanidhar Subrahmanyam, 2004, Order imbalance and individual stock returns: theory and evidence, *Journal of Financial Economics* 72 (3), 485–518.
- Coggin, Daniel, Frank Fabozzi, Shafiqur Rahman, 1993, The investment performance of US equity pension fund managers: An empirical investigation, *Journal of Finance* 48, 1039–55.
- Coval, Joshua, and Erik Stafford, 2005, Asset fire sales (and purchases) in equity markets, *Harvard Business School Working Paper*, No. 05–077.
- De Long, Bradford, Andrei Shleifer, Lawrence Summers and Robert Waldmann, 1990, Noise trader risk in financial markets, *Journal of Political Economy* 98(4), 703–738.
- Del Guercio, Diane, and Paula Tkac, 2002, The determinants of the flow of funds of managed portfolios: Mutual funds versus pension funds, *Journal of Financial and Quantitative Analysis* 37, 523–557.
- Dimson, Elroy, 1979, Risk measurement when shares are subject to infrequent trading, *Journal of Financial Economics* 5, 263–278.
- Dishi, Elor, David Gallagher, and Jerry Parwada, 2006, Institutional investment flows and fund manager turnover: Evidence from pension plan mandate, *Working Paper*.
- Duffie, Darrell, Nicolae Garleanu, and Lasse Pedersen, 2002, Securities lending, shorting, and pricing *Journal of Financial Economics* 66, 307–339.
- Easley, David, and Maureen O’Hara, 1987, Price, trade size, and information in securities markets, *Journal of Financial Economics* 19, 69–90.
- Easley, David, and Maureen OHara, 1992, Time and the process of security price adjustment, *Journal of Finance* 47, 577–606.
- Easley, David, Nick Kiefer, Maureen OHara, Maureen, and Joseph Paperman, 1996, Liquidity, information, and infrequently traded stocks, *Journal of Finance* 51, 1405–1436.
- Easley, David, Maureen OHara, and Joseph Paperman, 1998, Financial analysts and information-motivated trade, *Journal of Financial Markets* 1, 175–201.

- Fama, Eugene, 1998, Market efficiency, long-term returns, and behavioral finance, *Journal of Financial Economics* 49, 283–306.
- Fama, Eugene, and Kenneth French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47(2), 427–465.
- Fama, Eugene, and James MacBeth, 1973, Risk, return, and equilibrium: empirical tests, *Journal of Political Economy* 81, 607–636.
- Ferson, Wayne, and Kenneth Khang, 2002, Conditional performance measurement using portfolio weights: Evidence for pension funds, *Journal of Financial Economics*, 65, 249–282.
- Froot, Kenneth, and Paul O’Connell, 1999, The pricing of US catastrophe reinsurance, *The Financing of Catastrophe Risk*, 195–232.
- Garry, Mark, and William Goetzmann, 1986, Does delisting from the S&P 500 affect stock price? *Financial Analysts Journal* 42, 64–69.
- Gemmill, Gordon, 1996, Transparency and liquidity: A study of block trades in the London Stock Exchange under different publication rules, *Journal of Finance* 51, 1765–1790.
- Glosten, Lawrence and Paul Milgrom, 1985, Bid, ask and transaction prices in a specialist market with heterogeneously informed traders, *Journal of Financial Economics* 14(1), 71–100.
- Goyal, Amit, and Sunil Wahal, 2006, The selection and termination of investment management firms by plan sponsors, *Working Paper*.
- Greenwood, Robin, 2005a, Short- and long-term demand curves for stocks: theory and evidence on the dynamics of arbitrage, *Journal of Financial Economics* 75(3), 607–649.
- Greenwood, Robin, 2005b, A cross-sectional analysis of the excess comovement of stock returns, *Harvard Business School Working Paper*, No. 05–069.
- Grossman, Sanford, and Merton Miller, 1988, Liquidity and market structure, *Journal of Finance* 43(3), 617–637.
- Harris, Lawrence, and Eitan Gurel, 1986, Price and volume effects associated with changes in the SP500: New evidence for the existence of price pressure, *Journal of Finance* 41, 851–60.
- Hasbrouck, Joel, 1988, Trades, quotes, and information, *Journal of Financial Economics* 22, 229–252.
- Hasbrouck, Joel, 1991, Measuring the information content of stock trades, *Journal of Finance* 46(1), 179–207.

- Heisler, Jeffrey, Christopher Knittel, John Neumann, and Scott Stewart, 2006, Why do institutional plan sponsors fire their investment managers? *Working paper, Boston University*.
- Holthausen, Robert, Richard Leftwich, and David Mayers, 1987, The effect of large block transactions on security prices: A cross-sectional analysis, *Journal of Financial Economics* 19(2), 237–67.
- Holthausen, Robert, Richard Leftwich, and David Mayers, 1990, Large-block transactions, the speed of response, and temporary and permanent stock-price effects, *Journal of Financial Economics* 26(1), 71–95.
- Ibbotson, Roger, and Paul Kaplan, 2000, Does asset allocation policy explain 40%, 90%, or 100 % of performance?, *The Financial Analysts Journal* 56(1), 26–33.
- Jin, Li, and Anna Scherbina, 2005, Inheriting losers, *Harvard Business School Working Paper*.
- Kaniel, Ron, Gideon Saar, and Sheridan Titman, 2004, Individual investor sentiment and stock returns, *Working paper, Duke University*.
- Kaul, Aditya, Vikas Mehorta, and Randall Morck, 2000, Demand curves for stocks do slope down: New evidence from an index weights adjustment, *Journal of Finance* 55, 893–912.
- Keim, Donald, and Madhavan, Ananth, 1995, Anatomy of the trading process empirical evidence on the behavior of institutional traders, *Journal of Financial Economics* 37(3), 371–398.
- Keim, Donald, and Ananth Madhavan, 1996, The upstairs market for large-block transactions: Analysis and measurement of price effects, *Review of Financial Studies* 9(1), 1–36.
- Keim, Donald, and Ananth Madhavan, 1997, Transactions costs and investment style: an inter-exchange analysis of institutional equity trades, *Journal of Financial Economics* 46(3), 265–292.
- Kraus, Alan, and Hans Stoll, 1972, Price impacts of block trading on the New York Stock Exchange, *Journal of Finance* 27, 569–588.
- Kyle, Albert, 1985, Continuous auctions and insider trading, *Econometrica* 53(6), 1315–1336.
- Lakonishok, Josef, and Inmoo Lee, 2001, Are insider trades informative?, *Review of Financial Studies*, 14(1) 79–111.
- Lakonishok, Josef, Andrei Shleifer, and Robert Vishny, 1992, The structure and performance of the money management industry, *Brookings Papers: Microeconomics*, 339–391.

- Llorente, Guillermo, Roni Michaely, Gideon Saar, and Jiang Wang, 2002, Dynamic volume-return relation of individual stocks, *Review of Financial Studies* 15, 1005–1047.
- Lo, Andrew, and Craig MacKinlay, 1988, Stock market prices do not follow random walks: Evidence from a simple specification test, *Review of Financial Studies*, 1, 41–66.
- Miller, Edward, 1977, Risk, uncertainty, and divergence of opinion, *Journal of Finance* 32, 1151–1168.
- Mitchell, Mark, Todd Pulvino, and Erik Stafford, 2004, Price pressure around mergers, *Journal of Finance* 59, 31–63.
- Parwada, Jerry, and Wenling Joey Yang, 2004, Stock market effects of institutional investment flows: Evidence from Australian pension plan mandate changes, *Working Paper*.
- Parwada, Jerry, and Robert Faff, 2005, Pension plan investment management mandates: An empirical analysis of manager selection, *Journal of Financial Services Research*, 77–98.
- Petersen, Mitchell, 2007, Estimating standard errors in finance panel data sets: Comparing approaches, *Working Paper*.
- Ramistella, Alex, 2006, Crossing networks: Bringing back large block trades to institutional trading, *TowerGroup Working Paper*.
- Saar, Gideon, 2001, Price impact asymmetry of block trades: An institutional trading explanation, *Review of Financial Studies* 14, 1153–1181.
- Shleifer, Andrei, 1986, Do demand curves for stocks slope down? *Journal of Finance* 41, 579–590.
- Subrahmanyam, Avanidhar, 2006, Lagged order flows and returns: A longer-term perspective, *Working Paper*.
- Tonks, Ian, 2005, Performance persistence of pension-fund managers,” *Journal of Business*, 78, 1917–1942.
- Wang, Jiang, 1994, A Model of Competitive Stock Trading Volume, *Journal of Political Economy*, 102(1), 127–168.
- Wurgler, Jeffrey, and Ekaterina Zhuravskaya, 2002, Does arbitrage flatten demand curves for stocks?, *Journal of Business* 75(4), 583–608.

Table 1.1: Identification Assumption: Target vs. Legacy Portfolios

Horizon	All Transitions		Large Transitions		Small Transitions	
	Δ	t-stat	Δ	t-stat	Δ	t-stat
Month 0	0.31**	(4.58)	0.30**	(4.33)	0.33**	(2.77)
Month 1	0.25**	(2.65)	0.27**	(2.82)	0.23	(1.40)
Month 2	0.22 [†]	(1.78)	0.29*	(2.45)	0.15	(0.71)
Month 3	0.21	(1.45)	0.22	(1.60)	0.19	(0.78)
Month 4	0.26	(1.51)	0.22	(1.46)	0.29	(0.96)
Month 5	0.20	(1.00)	0.24	(1.40)	0.17	(0.46)
Month 6	0.25	(1.18)	0.26	(1.47)	0.24	(0.64)
# Obs	1639		819		820	

Table 1.1 shows the average difference between cumulative raw returns of target and legacy portfolios for two-sided transitions, Δ . Returns are cumulated starting the month of portfolio transitions. Six subsequent months are considered. Estimates for small and large transitions are shown separately. Small and large transitions are defined based on the total size of legacy and target portfolios. The threshold is about \$100 million. T-statistics are presented in parentheses. The sample ranges from January 2001 to December 2005. **is significance at 1% level, *is significance at 5% level, [†]is significance at 10% level.

Table 1.2: Summary Statistics for Portfolio Transitions

Panel A: Summary Statistics for Transitions

	<u>\$Volume (000)</u>	<u>#Shares (000)</u>	<u>#Stocks</u>
Mean	184,504	6,048	268
Median	53,359	1,837	133
25th	16,597	548	65
75th	149,513	5,152	314
Max	24336,138	881,040	3,669

Panel B: Split over Cap Quintiles

<u>Cap Qnt</u>	<u>\$Volume</u>	<u>#Stocks</u>
1(Small)	1.46%	8.60%
2	5.17%	14.30%
3	9.28%	16.97%
4	14.29%	20.66%
5(Large)	69.80%	39.46%

Panel C: Split over Time

<u>Duration</u>	<u>#Trans</u>	<u>\$Volume</u>
1 day	55.71%	25.66%
2 days	18.70%	18.98%
3 days	10.22%	16.68%
4 days	5.51%	8.76%
5 days	3.12%	7.70%
6–10 days	5.63%	18.41%
11–18 days	1.12%	3.82%

Table 1.2 presents general information on the portfolio transitions: summary statistics in Panel A, distribution of transition trades across different capitalization quintiles in Panel B, distribution of transition trades across time in Panel C. Panel A shows average, median, standard deviation, maximum value and minimum value for the dollar volume (in thousands), the number of shares traded (in thousands) and the number of different stocks in transitions. Panel B shows the distribution of the dollar volume and the number of stocks in transitions across different capitalization groups. Panel C presents the duration of transitions and their dollar volume. Capitalization thresholds are calculated based on NYSE stock capitalization quintiles. The sample ranges from January 2001 to December 2005.

Table 1.3: Portfolio Transition Orders with Large OMT Trades

Panel A: Summary Statistics of Transition Orders

		Buy			Sell			In-Kind
		OMT	EC	IC	OMT	EC	IC	-
\$ Volume (000)	Mean	512.37	208.57	219.52	529.08	242.49	207.69	105.12
	Median	150.80	0.00	0.00	150.22	0.00	0.00	0.00
# Shrs (000)	Mean	20.55	9.42	8.38	21.15	10.29	7.68	3.40
	Median	7.33	0.00	0.00	7.50	0.00	0.00	0.00
# Shrs/ADV (%)	Mean	6.96	3.56	2.39	8.11	5.60	2.59	0.61
	Median	2.83	0.00	0.00	2.67	0.00	0.00	0.00
# Shrs/ShrOut (bp)	Mean	2.89	1.46	1.07	3.18	1.85	1.09	0.28
	Median	1.51	0.00	0.00	1.46	0.00	0.00	0.00

Panel B: Distribution of Transition Trades over Time

		Time Horizon T								
		0d	1d	2d	3d	4d	1w	2w	>2w	All
Buy	OMT	1.06	2.43	1.53	0.77	0.56	0.26	0.32	0.04	6.96
	EC	0.68	1.07	0.67	0.42	0.32	0.16	0.20	0.04	3.56
	IC	0.37	0.67	0.51	0.38	0.25	0.07	0.14	0.01	2.39
Sell	OMT	1.50	2.21	1.54	0.96	0.66	0.39	0.69	0.15	8.11
	EC	0.82	1.11	0.91	0.79	0.65	0.34	0.61	0.37	5.60
	IC	0.52	0.78	0.46	0.31	0.22	0.11	0.13	0.07	2.59
In-Kind	-	0.61								0.61

Table 1.3 presents descriptive statistics for portfolio transition orders. Panel A shows summary statistics on portfolio transition orders and their distribution over various trading venues: open market (OMT), external crossing (EC) and internal crossing (IC). For transition orders, the means and medians of the following characteristics are reported: the dollar value (in thousands), the number of shares (in thousands), the number of shares as a fraction of shares outstanding (in bps), the number of shares as a fraction of average daily volume (ADV). Panel B shows the distribution of the portfolio transition trades over time. For each day relative to starting day of transition, Table exhibits the average trade size normalized by the average daily volume in the previous month (in percents). In Panel B, Open market trades (OMT), external crossing trades (EC), and internal crossing (IC) trades are considered separately. Data is presented for buy and sell sides as well as in-kind transactions. All statistics are calculated based on pooled data from January 2001 to December 2005. Only transition orders during which execution more than 1% of average daily volume is traded through open market trading are included in the sample.

Table 1.4: The Cumulative Average Abnormal Returns (CAARs)

Panel A: Stocks Weighted Equally, EW

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Buy	0.22** (4.97)	0.35** (7.49)	0.43** (7.08)	0.30** (2.78)	0.42* (2.41)	0.41 [†] (1.75)	0.52 (1.19)
Sell	-0.18** (-3.94)	-0.39** (-5.32)	-0.36** (-6.69)	-0.10 (-0.95)	0.02 (0.09)	-0.10 (-0.44)	0.12 (0.38)
Δ	0.41** (4.58)	0.75** (6.60)	0.82** (8.51)	0.43** (3.70)	0.44** (2.67)	0.56* (2.51)	0.39 (1.46)

Panel B: Stocks Weighted by Market Capitalization, VW

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Buy	0.03 (0.95)	0.07 [†] (1.99)	0.17* (2.03)	0.00 (-0.03)	0.03 (0.28)	-0.02 (-0.13)	0.30 (0.93)
Sell	-0.13** (-5.46)	-0.25** (-8.43)	-0.25** (-3.24)	0.02 (0.19)	-0.07 (-0.43)	-0.08 (-0.30)	-0.07 (-0.15)
Δ	0.17** (5.88)	0.34** (16.28)	0.45** (7.74)	0.03 (0.21)	0.12 (0.57)	0.08 (0.34)	0.35 (1.02)

Table 1.4 presents the cumulative average abnormal returns, $CAAR_T$, for the acquired (Buy) and sold (Sell) stocks for various horizons T starting the event day 0 and up to three months. Returns are adjusted with the 4-factor model, which includes three Fama-French factors and the momentum factor. Estimates are calculated using the Fama-McBeth method for data grouped at monthly frequencies. Equally-weighted and value-weighted schemas are considered. T-statistics are adjusted with the Newey-West procedure and presented in parentheses. The sample ranges from January 2001 to December 2005. **is significance at 1% level, *is significance at 5% level, [†]is significance at 10% level.

Table 1.5: The CAARs and Information Asymmetry

<i>Panel A: Groups by Number of Analysts, ANUM</i>							
	0d	1d	1w	2w	1m	2m	3m
Buy Low	0.32** (4.20)	0.52** (6.30)	0.53** (3.55)	0.54** (4.23)	0.61* (2.01)	1.12** (2.67)	1.01 [†] (1.89)
High	0.13** (4.20)	0.21** (5.54)	0.22** (2.98)	0.18 (1.02)	-0.01 (-0.02)	-0.15 (-0.56)	-0.10 (-0.21)
Sell Low	-0.22** (-3.34)	-0.42** (-3.83)	-0.35* (-2.41)	-0.06 (0.14)	0.11 (0.97)	0.00 (0.44)	0.17 (1.01)
High	-0.13** (-3.16)	-0.26** (-5.35)	-0.25** (-3.27)	-0.13 (-0.40)	-0.17 (-0.19)	-0.07 (0.02)	0.15 (0.35)
<i>Panel B: Groups by Percentage Spread, SPREAD</i>							
	0d	1d	1w	2w	1m	2m	3m
Buy Low	0.13** (4.44)	0.20** (7.55)	0.30* (2.35)	0.12 (0.78)	0.01 (0.07)	-0.06 (-0.20)	-0.02 (-0.06)
High	0.39** (3.86)	0.69** (4.73)	0.76** (3.39)	0.79** (3.23)	0.81* (2.22)	1.31* (2.43)	1.32 (1.47)
Sell Low	-0.13** (-3.36)	-0.28** (-5.37)	-0.32** (-4.99)	-0.13 (-0.40)	-0.20 (-0.78)	-0.23 (-0.66)	-0.25 (-0.80)
High	-0.26** (-3.22)	-0.61** (-4.13)	-0.58** (-4.97)	-0.10 (-0.33)	0.45 [†] (1.80)	0.52 (1.15)	0.60 (1.44)
<i>Panel C: Groups by Probability of Informed Trading, PIN</i>							
	0d	1d	1w	2w	1m	2m	3m
Buy Low	0.05 (1.26)	0.09 [†] (1.95)	0.09 (0.67)	0.36* (2.19)	0.41 [†] (1.79)	0.19 (0.62)	0.39 (0.94)
High	0.29** (5.09)	0.47** (6.07)	0.62** (4.80)	0.74** (3.96)	0.74** (2.99)	1.01** (3.42)	0.67 (1.46)
Sell Low	-0.07 (-1.57)	-0.14** (-3.99)	-0.04 (-0.44)	0.09 (0.86)	0.16 (1.12)	0.14 (0.69)	0.49 (0.91)
High	-0.20** (-3.56)	-0.49** (-5.44)	-0.50** (-3.77)	0.00 (-0.19)	0.08 (0.18)	0.30 (0.29)	0.83 (1.09)

Table 1.5 shows the cumulative average abnormal returns, $CAAR_T$, following transition orders for the stock with different degrees of information asymmetry. In Panel A, stocks are sorted based on the number of analysts following them. In Panel B and Panel C, stocks are sorted based on the percentage spread and the probability of informed trading, respectively. The "Low" ("High") group includes stocks with bottom (top) 33% of ranked stocks. The estimates are calculated following the Fama-McBeth procedure. T-statistics are adjusted with the Newey-West methodology and presented in parentheses. Returns are adjusted for risk with the 4-factor model. The sample ranges from January 2001 to December 2005. **is significance at 1% level, *is significance at 5% level, [†]is significance at 10% level. 54

Table 1.6: Correlation Matrices

Panel A: Correlation Matrix for Transition Purchases

	LN Cap	SPREAD	PIN	ANUM
LN Cap	1	-0.50	-0.63	0.67
SPREAD		1	0.45	-0.34
PIN			1	-0.46

Panel B: Correlation Matrix for Transition Sales

	LN Cap	SPREAD	PIN	ANUM
LN Cap	1	-0.52	-0.62	0.66
SPREAD		1	0.46	-0.35
PIN			1	-0.45

Table 1.6 shows the correlations between stock characteristics. These characteristics include the natural logarithm of the market capitalization (in millions), the percentage spread, the probability of informed trading, and the number of analysts following stocks. All estimates are calculated following the Fama-McBeth procedure with observations grouped at monthly levels. Samples of transition purchases and sales are considered separately (Panel A and B). The sample ranges from January 2001 to December 2005.

Table 1.7: The CAARs: Purchases, Double Sort on Size and ANUM

Panel A: Small Size Stocks

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Low ANUM	0.45** (3.78)	0.72** (5.26)	0.76** (3.38)	0.89* (2.33)	1.19* (2.62)	1.96** (2.70)	2.21* (2.20)
High ANUM	0.30** (3.45)	0.50** (4.00)	0.89** (3.55)	0.85* (2.17)	0.72* (2.11)	0.73 (1.02)	0.42 (0.40)

Panel B: Medium Size Stocks

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Low ANUM	0.25** (4.96)	0.40** (5.98)	0.73** (7.26)	0.69** (7.83)	0.85** (5.40)	0.87** (3.54)	0.79 (1.55)
High ANUM	0.15* (2.48)	0.16* (2.11)	0.27** (2.80)	0.48* (2.35)	0.53 (1.56)	0.15 (0.29)	0.10 (0.14)

Panel C: Large Size Stocks

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Low ANUM	0.09* (2.33)	0.13* (2.23)	0.10 (0.92)	-0.25 (-1.01)	-0.21 (-0.78)	-0.11 (-0.31)	0.02 (0.05)
High ANUM	0.04 (1.16)	0.13* (2.13)	0.18 (1.53)	0.13 (1.57)	-0.01 (-0.04)	0.14 (0.39)	0.41 (0.75)

Table 1.7 shows the cumulative average abnormal returns, $CAAR_T$, following transition buys for the stocks sorted by their market capitalization and the number of analysts following them, ANUM. Returns for small stocks are presented in Panel A, for medium size stocks in Panel B, and for large stocks in Panel C. Inside each size group, stocks are sorted by ANUM into two groups, "Low ANUM" and "High ANUM". The estimates of CAARs are calculated following the Fama-McBeth procedure. T-statistics are adjusted with the Newey-West methodology and presented in parentheses. Returns are adjusted for risk with the 4-factor model. The sample ranges from January 2001 to December 2005. **is significance at 1% level, *is significance at 5% level, †is significance at 10% level.

Table 1.8: The CAARs: Purchases, Double Sort on Size and *SPREAD*

Panel A: Small Size Stocks

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Low SPREAD	0.33** (3.49)	0.48** (3.93)	0.69** (2.68)	0.68† (1.99)	0.48† (1.89)	0.44 (0.78)	0.18 (0.18)
High SPREAD	0.43** (3.61)	0.80** (4.11)	0.78* (2.65)	0.89† (1.98)	1.02† (1.96)	2.06* (2.65)	2.40† (1.93)

Panel B: Medium Size Stocks

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Low SPREAD	0.25** (3.69)	0.22** (3.89)	0.41** (3.48)	0.41* (2.62)	0.34 (1.31)	0.20 (0.46)	0.28 (0.51)
High SPREAD	0.19** (3.29)	0.43** (4.57)	0.66** (5.07)	0.79** (4.51)	0.87** (3.65)	0.51 (1.26)	0.63 (1.04)

Panel C: Large Size Stocks

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Low SPREAD	0.01 (0.21)	0.09 (1.55)	0.07 (0.66)	0.23** (2.98)	0.15 (1.15)	0.39 (1.28)	0.57 (1.57)
High SPREAD	0.14** (4.92)	0.22** (3.70)	0.15 (0.99)	-0.14 (-0.63)	-0.07 (-0.55)	-0.07 (-0.21)	0.01 (0.01)

Table 1.9 shows the cumulative average abnormal returns, $CAAR_T$, following transition buys for the stocks sorted by their market capitalization and their percentage spread, *SPREAD*. Returns for small stocks are presented in Panel A, for medium size stocks in Panel B, and for large stocks in Panel C. Inside each size group, stocks are sorted by *SPREAD* into two groups, "Low *SPREAD*" and "High *SPREAD*". The estimates of CAARs are calculated following the Fama-McBeth procedure. T-statistics are adjusted with the Newey-West methodology and presented in parentheses. Returns are adjusted for risk with the 4-factor model. The sample ranges from January 2001 to December 2005. **is significance at 1% level, *is significance at 5% level, †is significance at 10% level.

Table 1.9: The CAARs: Purchases, Double Sort on Size and *PIN*

Panel A: Small Size Stocks

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Low PIN	0.35** (2.74)	0.57** (3.56)	0.85** (3.01)	1.17* (2.25)	1.37* (2.32)	1.40 (1.63)	1.21 (1.04)
High PIN	0.36** (3.85)	0.55** (4.60)	0.80** (4.26)	0.87** (3.03)	0.86* (2.57)	1.36* (2.54)	1.68** (3.13)

Panel B: Medium Size Stocks

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Low PIN	0.06 (0.82)	0.09 (1.26)	0.21** (2.75)	0.07 (0.47)	0.03 (0.07)	-0.22 (-0.36)	-0.52 (-0.60)
High PIN	0.13* (2.59)	0.22* (2.25)	0.22 (1.38)	0.45* (2.10)	0.53 [†] (1.72)	0.33 (0.67)	-0.38 (-0.48)

Panel C: Large Size Stocks

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Low PIN	0.03 (0.86)	0.05 (0.97)	0.13 (0.90)	0.31 [†] (2.05)	0.50* (2.51)	0.60 (1.52)	0.54 (1.30)
High PIN	0.02 (0.39)	0.08 (1.36)	0.09 (0.56)	-0.24 (-0.84)	-0.24 (-0.73)	0.13 (0.27)	0.74 (1.16)

Table 1.9 shows the cumulative average abnormal returns, $CAAR_T$, following transition buys for the stocks sorted by their market capitalization and the probability of informed trading, *PIN*. Returns for small stocks are presented in Panel A, for medium size stocks in Panel B, and for large stocks in Panel C. Inside each size group, stocks are sorted by *PIN* into two groups, "Low *PIN*" and "High *PIN*". The estimates of CAARs are calculated following the Fama-McBeth procedure. T-statistics adjusted with the Newey-West methodology and presented in parentheses. Returns are adjusted for risk with the 4-factor model. The sample ranges from January 2001 to December 2005. **is significance at 1% level, *is significance at 5% level, [†]is significance at 10% level.

Table 1.10: The CAARs and Past Returns

Panel A: Transition Purchases

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Low $R_{-3m,-1}^{adj}$	0.37** (4.95)	0.59** (4.91)	0.87** (4.58)	0.95** (3.38)	1.22** (3.04)	1.61* (2.15)	1.47 (1.59)
High $R_{-3m,-1}^{adj}$	0.10† (1.79)	0.24** (2.79)	0.22 (1.58)	0.14 (0.66)	0.06 (0.23)	0.35 (1.07)	0.36 (0.53)

Panel B: Transition Sales

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Low $R_{-3m,-1}^{adj}$	-0.17* (-2.27)	-0.44** (-2.72)	-0.24 (-1.42)	0.26 (1.03)	0.79† (1.77)	0.98 (1.35)	1.33 (1.33)
High $R_{-3m,-1}^{adj}$	-0.20* (-2.42)	-0.50** (-4.32)	-0.60** (-4.37)	-0.19 (-1.32)	-0.02 (-0.23)	-0.22 (-0.93)	-0.01 (-0.39)

Table 1.10 shows the cumulative average abnormal returns, CAARs, following transition trades for stock grouped by their past returns. Stocks are sorted into two groups based on their risk-adjusted past returns $R_{-3m,-1}^{adj}$ in the previous three months. The "Low" group includes stocks with bottom 33% of ranked stocks that have negative past returns. The "High" group includes stocks with top 33% of ranked stocks that have positive past returns. Purchases and sales are considered in Panel A and B, respectively. The estimates are calculated following the Fama-McBeth procedure. T-statistics are adjusted with the Newey-West methodology and presented in parentheses. Returns are adjusted for risk with the 4-factor model. The sample ranges from January 2001 to December 2005.

Table 1.11: The CAARs and Trading Venues

Panel A: Internal Crosses

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Buy	-0.06 (-0.92)	0.01 (0.08)	0.28 (1.32)	0.06 (0.17)	-0.21 (-0.35)	0.06 (0.08)	0.49 (0.41)
Sell	-0.01 (-0.09)	0.05 (0.45)	0.15 (0.74)	1.03 (1.37)	0.81 (1.15)	0.46 (0.42)	0.79 (0.52)
Δ	-0.05 (-0.65)	-0.04 (-0.23)	0.14 (0.58)	-0.84 (-1.49)	-1.00 (-1.63)	-0.37 (-0.35)	-0.31 (-0.24)

Panel B: External Crosses

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
Buy	0.14* (2.22)	0.29** (4.06)	0.33* (2.51)	0.23 (1.10)	0.36 (1.32)	0.04 (0.10)	-0.14 (-0.27)
Sell	-0.14** (-3.19)	-0.20** (-3.84)	-0.04 (-0.20)	-0.01 (-0.04)	-0.16 (-0.47)	-0.26 (-0.49)	-0.06 (-0.10)
Δ	0.27** (3.72)	0.48** (6.88)	0.37 (1.50)	0.24 (0.61)	0.52 (1.15)	0.30 (0.46)	-0.09 (-0.17)

Panel C: In-kind Transactions

	Time Horizon T						
	0d	1d	1w	2w	1m	2m	3m
In-Kind	0.02 (0.56)	0.02 (0.47)	0.06 (0.44)	0.22 (1.32)	0.12 (0.40)	0.21 (0.47)	0.23 (0.43)

Table 1.11 shows the cumulative average abnormal returns, CAARs, following transition orders executed through different trading venues. In Panel A, the orders executed entirely through internal crossing networks are included. In Panel B, the orders executed entirely through external crossing network are considered. In Panel C, the orders entirely transferred as in-kind transactions are included. Purchases and sales are considered separately. The estimates are calculated following the Fama-McBeth procedure. T-statistics are adjusted with the Newey-West methodology and presented in parentheses. Returns are adjusted for risk with the 4-factor model. The sample ranges from January 2001 to December 2005.

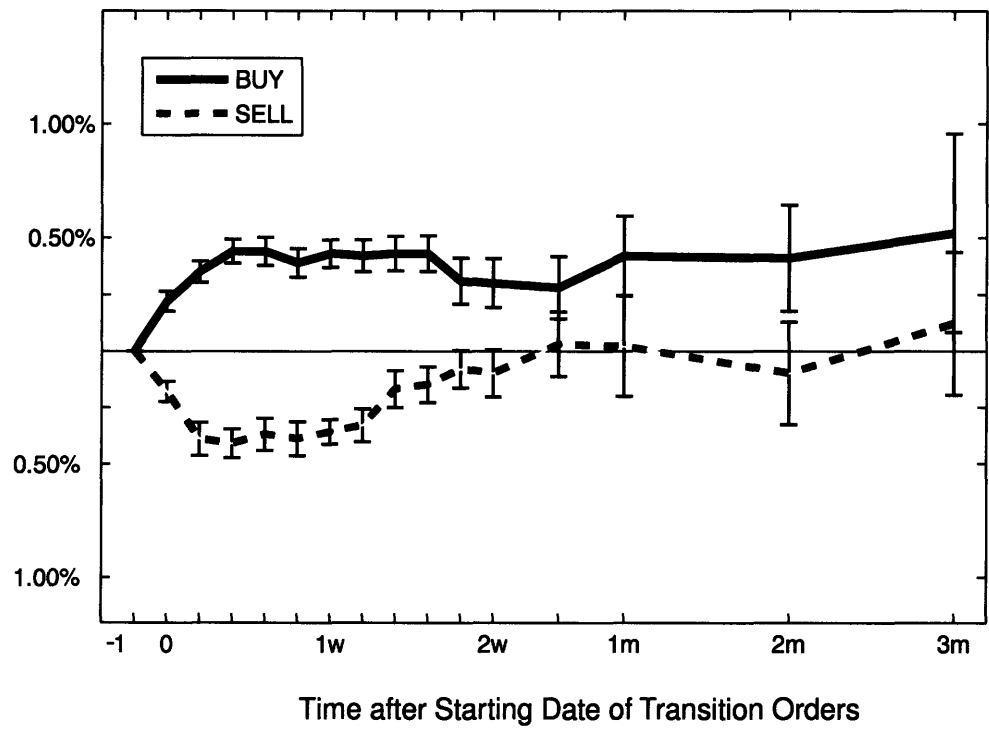


Figure 1-1: Price Response to Large Transition Orders, Equally-Weighted Case

Figure shows the cumulative average abnormal returns (in percents), $CAAR_T$, following large portfolio transition buys and sells. The day 0 is the starting date of transition trades. Horizons T up to three months are considered. Equally-weighted returns are calculated. Means and standard errors are calculated using the Fama-McBeth procedure for observations grouped at monthly level. Standard errors are shown as error bars. The sample ranges from January 2001 to December 2005.

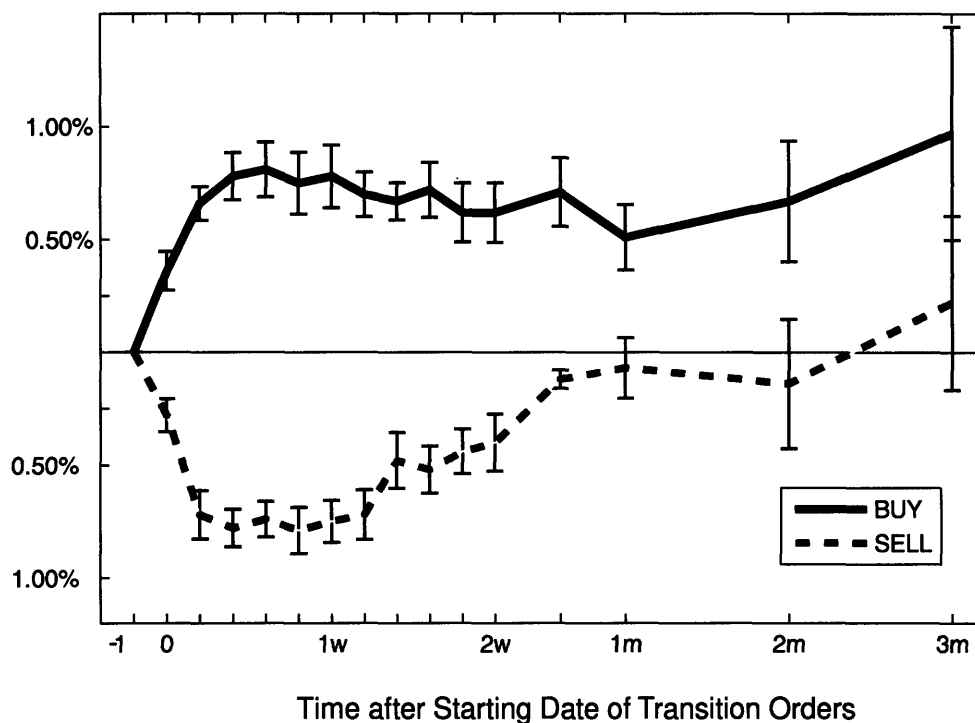


Figure 1-2: Price Response to Large Transition Orders, Principle-Weighted Case

Figure shows the cumulative average abnormal returns (in percents), $CAAR_T$, following large portfolio transition buys and sells. The day 0 is the starting date of transition trades. Horizons T up to three months are considered. Principle-weighted returns are calculated; namely, observations are weighted by the size of trades normalized by average daily volume in the previous month. Means and standard errors are calculated using the Fama-McBeth procedure for observations grouped at monthly level. Standard errors are shown as error bars. The sample ranges from January 2001 to December 2005.

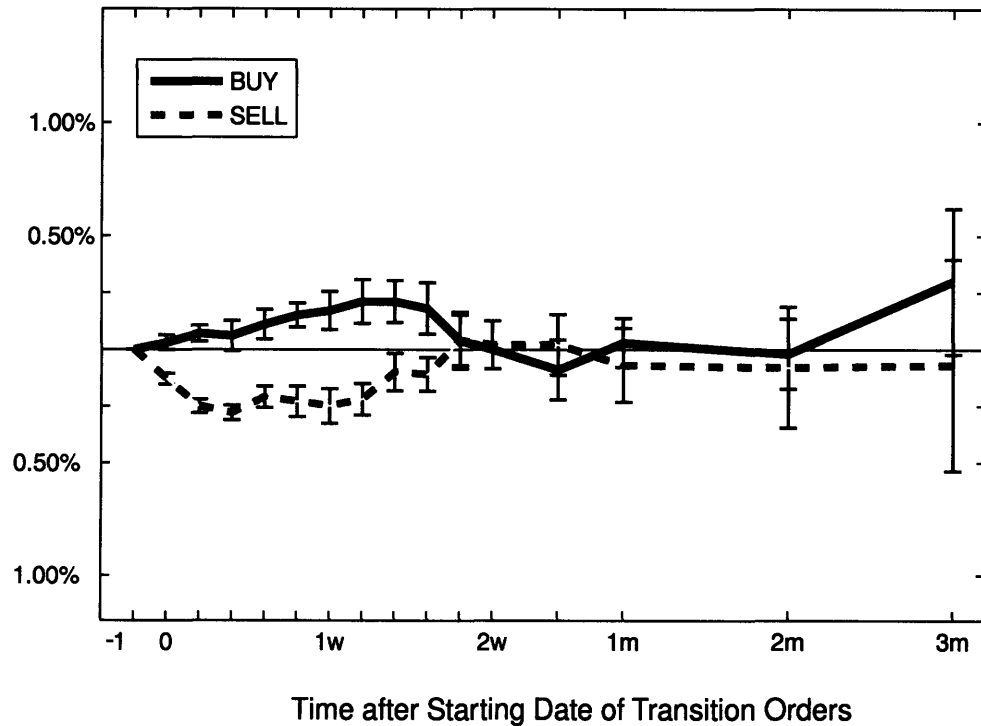


Figure 1-3: Price Response to Large Transition Orders, Value-Weighted Case

Figure shows the cumulative average abnormal returns (in percents), $CAAR_T$, following large portfolio transition buys and sells. The day 0 is the starting date of transition trades. Horizons T up to three months are considered. Value-weighted returns are calculated. Means and standard errors are calculated using the Fama-McBeth procedure for observations grouped at monthly level. Standard errors are shown as error bars. The sample ranges from January 2001 to December 2005.

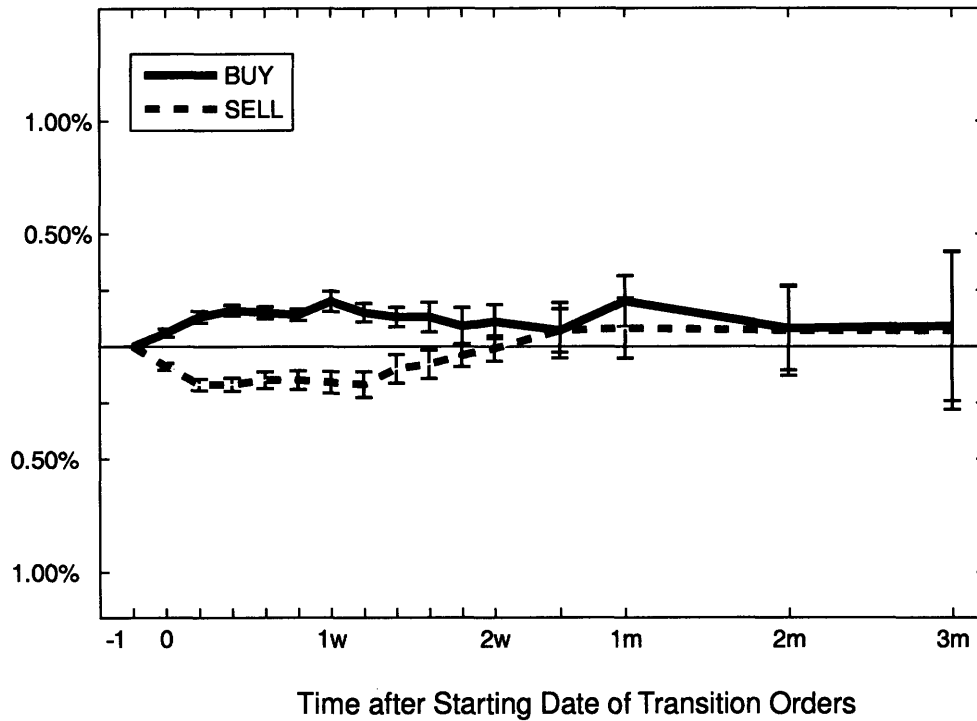


Figure 1-4: Price Response to All Transition Orders, Equally-Weighted Case

Figure shows the cumulative average abnormal returns (in percents), $CAAR_T$, following portfolio transition buys and sells. All transition trades are included. The day 0 is the starting date of transition trades. Horizons T up to three months are considered. Equally-weighted returns are calculated. Means and standard errors are calculated using the Fama-McBeth procedure for observations grouped at monthly level. Standard errors are shown as error bars. The sample ranges from January 2001 to December 2005.

Chapter 2

The Price Impact Puzzles

2.1 Introduction

Price impact is an important variable that is of interest to both researchers and practitioners. Researchers are interested in learning about price impact because it reflects a complicated and not fully understood interplay between main blocks of market microstructure: trades, prices and information. Practitioners are keen to know about price impact because it accounts for the largest part of their transaction costs. Many theoretical studies provide interesting insights about price impact and its properties. However, price impact is notoriously difficult to measure empirically; existing evidence about this variable is fairly limited and is usually subject to various assumptions.

I use a unique data set of portfolio transition trades that allows me to construct the estimates of price impact and its coefficients with great precision. Moreover, these estimates are available for a large cross-section of stocks.¹ The main goal of the study is to investigate several interesting questions about price impact using these estimates: (1) how its coefficients are related to various stock characteristics and differ across

¹Formally, the price impact coefficients quantify the changes in prices in response to trades equivalent to 1% of the average daily volume. They are defined in percents of standard deviation of daily returns.

trading venues; (2) how they evolve during execution of multi-trade "packages"; and (3) what functional form best describes price impact functions. I document a number of empirical findings, some of which can not be easily explained within the existing theoretical frameworks.

First, I show that the price impact coefficients relate positively to market capitalization: transactions in larger stocks tend to be associated with more substantial price impact. This finding contradicts the conventional intuition that the trading in larger stocks involves less risk of information asymmetry and, consequently, demands slighter price concessions. I suggest several explanations for a positive relation between stock size and price impact. My analysis shows that it might be partially induced by concavity of price impact functions.² An alternative explanation relies on the following argument. For large stocks, trading volume is often dominated by small orders of noise traders and is the assembly of numerous two-sided transactions, buys alternates with sells; although turnover of these stocks is substantial, overall order imbalances are small. The opposite is true for small stocks.³ Thus, the information content of one-sided trades equivalent to 1% of the average daily volume (or the price impact coefficients) might be more substantial for large rather than for small stocks.

Second, the price impact coefficients tend to increase with the amount of noise trading or, more precisely, with its proxy, i.e. the monthly turnover.⁴ Again, the conventional market microstructure models predict the opposite effects (e.g., Kyle (1985)). However, the second explanation, suggested above for a positive relation between stock size and price impact, is consistent with this evidence.

Third, the link between two measures of liquidity, the price impact coefficients and the percentage spread, is found to be weaker than expected. The price impact coefficients increase with the spread for a sample of sell orders, however, no explicit

²Typically, trades in larger stocks are smaller in terms of a percentage of the average daily volume. Thus, concavity of price impact functions mechanically leads to the documented results.

³These patterns are thoroughly examined in Chordia, Huh, and Subrahmanyam (2007).

⁴They are also positively related to the dispersion of analysts' forecasts.

patterns between these variables can be statistically detected for a sample of buy orders. Consequently, more careful investigation of different facets of liquidity and how they relate to each other would be valuable.

Fourth, the cross-sectional regression analysis reveals that the price impact coefficients are, on average, higher for transactions executed on Nasdaq rather than on NYSE/Amex exchanges. The corresponding coefficients by the Nasdaq dummy variable are positive across various regression specifications, yet not statistically significant.

Fifth, this paper provides novel findings about interesting dynamics of price impact that are observed in response to multi-trade "packages". During buy "packages", the price impact coefficients of individual trades increase as more and more trades are executed. In contrast, they decrease when sale "packages" are carried out. These findings are fairly robust: they are documented for various size groups, for different time periods and specifications of price impact. It is worth mentioning that this evidence is only partially consistent with the theoretical predictions of how security prices are expected to evolve in response to trading "packages". Indeed, theoretical models about multi-period trading strategies suggest that the dynamics of the price impact coefficients should be always non-increasing.⁵

Finally, I explore what functional form describes price impact functions in the best way. I find that total price impact is definitely concave. More precisely, it can be well approximated by the square-root specifications, which are popular in the literature on transaction costs. Yet, the evidence on the permanent and temporary components of price impact is much less conclusive. For instance, the permanent price impact does not seem to satisfy a linear specification, which would be expected based on the "no-arbitrage" arguments of the theoretical literature (see Huberman and Stanzl (2004)).

⁵The effects of multi-period information-related trading on prices is modeled in Kyle (1985) and its extensions; the effects of multi-period liquidity-related trading on prices is considered in Vayanos (1999, 2001).

Aforementioned empirical results are based on the database of portfolio transition trades. Portfolio transitions are large and expensive transfers of funds from legacy, i.e., existing, to target portfolios. These transitions are undertaken by various money management institutions such as pension plans, insurance funds, endowments, and foundations. They often occur when these institutions change the global asset allocations, replace the fund managers, or accommodate large cash inflows and outflows. This paper is one of the first academic works that studies this interesting phenomenon.⁶

This dataset allows me to identify supply/demand shocks and to estimate price impact in a very precise manner. In fact, this study benefits from various institutional features of portfolio transitions and the structure of data set. First, the trading intentions of transition managers are set in advance. This contrasts with other studies in which *ex ante* orders cannot be identified and realized trades might be influenced by a variety of investment styles and order-placement strategies. Second, the trading direction is given, whereas it is often necessary to apply heuristic and not perfectly precise algorithms for inferring a trade side. Third, the magnitude and timing of trades are observed. Fourth, the pre-trade benchmark and execution prices are explicitly recorded in the data set. Fifth, transition trades occur during typical days and stock price dynamics are not affected by abnormal market conditions. Sixth, a large cross-section of stocks is available and adds credibility to the analysis of the price impact patterns across stocks with different characteristics.

To the best of my knowledge, the price impact dynamics during trading "packages" has not been analyzed in the academic literature. However, the other questions under consideration have been addressed in the previous empirical work. In a closely related paper, Almgren et al. (2005) focuses on the analysis of the functional form

⁶The size of portfolio transition business is comparable to the size of hedge fund industry. Yet, only a few academic studies are available on this topic, e.g. Obizhava (2007). Somewhat related, a few papers analyze the selection and termination of fund managers by institutional investors which often lead to portfolio transitions, e.g. Parwada and Yang (2004) and Goyal and Wahal (2006).

of price impact functions, which incorporates the elements of cross-sectional analysis. Extracted from the trading records of Citigroup, their data set has somewhat similar structure, yet their results are quite different. For instance, they conclude that the permanent price impact is linear and that the temporary price impact is concave with the value of curvature constant equal to $3/5$.

This paper is related to the strand of literature that examines the cross-sectional patterns of the price impact coefficients. However, researchers often use datasets with incomplete information and, consequently, rely on various approximate algorithms and assumptions. For instance, in most cases, they analyze the price impact coefficients using publicly available TAQ data, which requires inaccurate classification of buyer-initiated and seller-initiated trades, among other assumptions.⁷ As discussed in Section 2.4, they usually draw conclusions that vary significantly and typically depend on the specifications of the price impact coefficients. Related papers focus on the cross-sectional patterns of price impact rather than those of the price impact coefficients. For instance, Chan and Lakonishok (1995) study price impact using a proprietary dataset of institutional trades that contains fairly detailed trading records (including buy/sell indicators) but still lacks information on the ex ante trading intentions and actual "packages". Another example is the paper of Keim and Madhavan (1997) who examine price impact based on Plexus data, which is well-suited for this type of analysis.

This study contributes to the empirical literature on the functional form of price impact functions.⁸ Consistently with the paper, this literature comes to a conclusion that, by and large, price impact functions are concave. However, their exact specification as well as the functional forms of its permanent and temporary components

⁷The recent examples of these studies include Breen, Hodrick and Korajczyk (2002), Chen, Stanzl and Watanabe (2001), and Lillo, Farmer and Mantegna (2003).

⁸The examples are Hasbrouck (1991a), Hausman, Lo, and MacKinlay (1992), Keim and Madhavan (1996), Barra (1997), Chen, Stanzl and Watanabe (2001), Lillo, Farmer and Mantegna (2003), Almgren et al.(2005).

are still debated. In this study, I provide additional evidence on these issues, which is derived from the portfolio transition data.⁹

The paper is structured as follows. Section 2 defines the price impact variables, which are the focus of this study. Section 3 describes portfolio transitions data, which is used to estimate price impact. Section 4 presents the cross-sectional analysis of the price impact coefficient. Section 5 explores the dynamics of the price impact during execution of multi-trade "packages". Section 6 focuses on the functional form of price impact functions. Section 7 concludes.

2.2 Main Variables

Security prices change in response to trades: they tend to increase upon buy orders and to fall upon sell orders. The price impact coefficients, or the strength of relation between trades and price changes, depend on the fundamental questions of market microstructure, i.e., how information is incorporated and how liquidity is provided. Two mechanisms might be instigated: first, trades might reveal new information and induce the corresponding price adjustments; second, if supply and demand are inelastic, trades might deplete existing liquidity and mechanically trigger the price pressure effects.

The goal of this study is to examine the price impact coefficients, associated with portfolio transition orders. I start by defining of a price impact variable, PI . For each order,

$$PI = \frac{1}{\sigma} (P_{exec}/P_{bch} - 1) \times \mathbb{I}_{BS} \times 100\%, \quad (2.1)$$

where P_{exec} is the trade-weighted average execution price, P_{bch} is the benchmark price, σ is the standard deviation of daily returns in the previous month, and \mathbb{I}_{BS}

⁹The price impact is also analyzed in Glosten and Harris(1988), Hasbrouck (1991 a,b), Holthausen, Leftwich and Mayers (1987, 1990), Madhavan, Richardson, and Roomans (1997), Du-four and Engle (2000), among others.

is an indicator function equal to 1 for buy orders and -1 for sell orders. In case of portfolio transitions, the benchmarks are agreed upon in advance and set at pre-transition levels. As in Almgren et al. (2005), the variable PI is expressed not in raw percentage terms but rather as a percentage of "normal" daily motion of price, defined by the standard deviation. It is also worth mentioning that PI stands for the price impact of a total transition order, regardless of how it is split into individual trades and over time.¹⁰

Total price impact (2.1) consists of two parts: the permanent and temporary components. The former captures shifts in the market's assessment of stock fundamentals, whereas the latter reflects the short-lived liquidity effects. I disentangle these two components of price impact, PI_{perm} and PI_{temp} , in the following way:

$$PI_{perm} = \frac{1}{\sigma}(P_{base}/P_{bch} - 1) \times \mathbb{I}_{BS} \times 100\%, \quad (2.2)$$

$$PI_{temp} = PI - PI_{perm} = \frac{1}{\sigma}(P_{exec} - P_{base})/P_{bch} \times \mathbb{I}_{BS} \times 100\%, \quad (2.3)$$

where P_{exec} , P_{bch} , σ , \mathbb{I}_{BS} are defined as before, and P_{base} is the price at "base" time, or time by which presumably information has been fully incorporated into security prices and the temporary price pressure effects disappear. For robustness, several "base" times are considered: the close at the last transaction day, the close at the next day, and in one or two weeks after execution.

In this study, I also focus on the price impact coefficients, the market microstructure measures associated with the price-trade relation. Since there is no agreement on what is the best way to estimate these coefficients, I next discuss my specification in more detail. For each portfolio transition order, the price impact coefficient λ_{PI} is defined as,

¹⁰However, only the part of transition orders executed as open market trades is considered; trades executed in external and internal crossing networks are disregarded in this study.

$$\lambda_{PI} = \frac{(P_{exec}/P_{bch} - 1)/\sigma}{X/1\%ADV} \times \mathbb{I}_{BS} \times 100\%, \quad (2.4)$$

where P_{exec} is the trade-weighted execution price, P_{bch} is the (pre-trade) benchmark price, σ is the standard deviation of daily returns in the previous month, \mathbb{I}_{BS} is a buy/sell indicator function, X is the size of transition order in shares, and ADV is the average trading daily volume in the previous month. Variable λ_{PI} reflects how strongly security prices change in response to trades; more precisely, it quantifies how significant is the price change (in percent of the standard deviation of daily returns) in response to orders that are equivalent to 1% of the average daily volume.¹¹

This specification is inspired by a measure of information asymmetry, first introduced by Kyle (1985) and defined as the price change over the trade size. However, in order to make these variables more meaningful for a cross-section of stocks, it is natural to use scaled rather than absolute values. Therefore, I normalize trade size by a fraction of the average daily volume as well as returns by the standard deviation of daily returns. Thus, a certain fraction of daily trading volume is expected to trigger a certain level of "normal" price motion.¹²

This scaling procedure assures the proper specification of the price impact coefficients for the stocks with the same market capitalization but with different numbers of shares outstanding and, consequently, being traded in the market. Also, this measure is stable with respect to splits. The following example illustrates why scaling is appropriate. Consider two identical firms A and B with stocks A and B, respectively. Firm A has Z shares outstanding, out of which $Z/365$ shares are traded daily at price \$100. Firm B has $2Z$ shares outstanding, out of which $2Z/365$ shares are traded daily

¹¹Following Keim and Madhavan (1997), Conrad et al. (2003) among others, I do not adjust (2.4) for risk. Slightly positive average market return over 2001-2005 might induce bias into price impact coefficients; however, these biases are too small to drive the results. For robustness check, I redo my analysis for two time periods from January 2001 to June 2003 and from July 2003 to December 2005, during which downward and upward market dynamics have prevailed, respectively.

¹²Similar arguments can be found in Almgren et al.(2005).

at price \$50. Given the same standard deviation of returns for both stocks, the price impact coefficients, λ_{PI} 's, are equal in the following, formally identical, situations: (1) for price change from \$100 to \$110 associated with $Z/365$ shares of stock A and (2) for price change from \$50 to \$55 associated with $2Z/365$ shares of stock B. By analogy, the normalization of numerator in (2.4) by the standard deviation of returns allows me to compare the price impact coefficients across securities with various volatilities.

Similarly, Breen, Hodrick, Korajczyk (2002) strongly advocate the use of scaled variables X , however, they suggest shares outstanding as a normalizer.¹³ In my opinion, the average trading volume is a more natural scaling factor, since it more precisely represents "normal" trading activity. At the same time, a part of shares outstanding can be frozen on the accounts of some market participants and do not participate in the daily turnover. Consequently, it is more reasonable to define price impact for 1% of the average daily volume than the shares outstanding.

To summarize, in this section, I construct the main variables under investigation in this study. In the next section, I describe the unique data set of portfolio transition trades that I use to estimate them.

2.3 Data Description

2.3.1 Data set of Portfolio Transitions

In this study, I use a proprietary data set of portfolio transition trades. Data is made available by the leading provider of portfolio transition services, who supervises more than 30% of stock transitions executed in the United States. The sample covers 2,234 transitions executed on behalf of institutions from January 2001 to December 2005. Most of these institutions are corporate and public pension plans; however, there are

¹³Other related examples include the price impact specifications in Korajczyk and Sadka (2004) and Keim and Madhavan (1997) among others.

also endowments, foundations, mutual funds, and insurance companies.

Portfolio transition are transfers of funds from legacy into target portfolios, carried out on behalf of these institutional investors. Transitions might be triggered by various events such as replacement of fund managers, changes in the global asset allocation, or large-scale restructuring. These transactions are very large and complicated. Their typical size is about \$50-\$100 million, and they usually involve thousands of trades in more than 100 different stocks. These challenging tasks are frequently delegated to professional transition managers, who help execute them in a cost efficient way: they design and coordinate the execution of portfolio transitions, often spreading transition orders over several days and channeling them into different trading platforms. When portfolio transitions are completed, managers prepare post-trade reports for their clients with a detailed execution analysis. This study is based on the data set that is constructed from a collection of actual post-trade reports; since these reports are thoroughly discussed by transition managers and their clients, the data is exceptionally clean and comprehensive.¹⁴

For each portfolio transition, the starting date and the ending date are specified. Moreover, detailed information on implemented trades is provided. It includes the number of shares traded, the execution price, and the pre-trade benchmark price as well as various transaction costs estimates. All data is aggregated at daily levels and is further grouped according to execution methods: internal crossing, external crossing, and open market trading. In this study, I consider only open market trades and focus on the associated price impact.¹⁵

In the following analysis, I exclude small trades, since they are not associated with any significant price changes and might bias the results; more precisely, I include orders with more than 1,000 shares traded through open markets and representing

¹⁴Description of transition management industry and this data can be found in Obizhaeva (2007).

¹⁵Some orders can be crossed internally against orders from other portfolio transitions or affiliated index management unit. Others can be sent into crossing networks such as POSIT, LiquidNet, or PipeLine.

more than 0.10% of the average trading volume in that stock. Also, in order to eliminate records with potential errors and unrepresentative trades, I have filtered out entries with obviously wrong information or typographical errors. Stocks with the missing CRSP information are removed as well. Finally, I exclude "penny" stocks with prices below 1\$.

After filtering, there are 72,299 buy orders and 87,132 sell orders in the sample. The number of orders executed per month ranges from 481 to 7,208 and is slightly increasing over time.

2.3.2 Summary Statistics

Panel A of Table 2.1 shows a descriptive statistics for the size of transition orders in the sample. All statistics are calculated from pooled data. Transition purchases and sales have very similar characteristics. On average, transition buy and sell orders, executed in open markets, represent about 1.21 and 1.15 bps of the shares outstanding, or 1.63% and 1.56% of the average daily volume, respectively. These values are slightly skewed, and their medians are smaller. Clearly, transition orders are quite large, and they are expected to induce significant price changes.

Panel B of Table 2.1 presents descriptive statistics for the individual stocks that are either bought or sold during portfolio transitions. In particular, it includes the market capitalization, the annual turnover, the percentage spread, the number of analysts who cover these stocks, the dispersion of analysts forecasts, and the standard deviation of daily returns. Apparently, most stocks in the sample are large, liquid with high turnover, and with a large number of analysts following them.

2.3.3 Supplementary Data

I also use the standard CRSP database to get additional stock information: prices, returns, trading volumes, and shares outstanding. My sample includes ordinary com-

mon stocks (with CRSP share codes of 10 and 11) listed on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), or NASDAQ in the period of January 2001 through December 2005. Any derivative securities, such as ADRs, REITS, or closed-end funds are excluded. Furthermore, I use unadjusted data from the Institutional Brokers Estimates System(I/B/E/S) to get the number of analysts who follow stocks together with their end-of-fiscal-year earnings forecast.

2.4 The Cross-Sectional Analysis

2.4.1 Motivation

The theoretical literature has long recognized that the price impact coefficients should vary significantly across securities. Moreover, it has provided valuable insights on what factors might influence their magnitude. First, a degree of information asymmetry is a key determinant of the cross-sectional price impact patterns: the larger and the more likely information asymmetry is, the higher compensation is required by liquidity providers, and the greater price impact coefficients are associated with the trades. Second, the market-making cost is obviously an important issue for the price impact coefficients as well: the easiness of finding a hedge or reallocating securities to other market participants is expected to mitigate price impact.

The empirical literature has provided ample evidence that price impact indeed exhibits substantial variation across stocks; though, it is still far from saying the last word on how in particular the price impact coefficients are related to various stock characteristics. The main obstacle is to reliably measure the price impact coefficients; moreover, for the trustworthy cross-sectional studies, these estimates should be available for a large sample of stocks with different characteristics. For instance, even the direction of trades is rarely observed by researchers, needless to mention other numerous caveats in such studies like unknown trading motives and

order placement strategies, absence of contemporaneous changes in fundamentals or in the overall market, among others.

Luckily, the data set of portfolio transitions provides me with a good opportunity to estimate and to examine the price impact coefficients, estimated in a precise manner for a large sample of securities. Indeed, *ex ante* portfolio transition orders and their direction are observed, their motives are clearly defined, pre-transition benchmark and execution prices are explicitly recorded. Moreover, although portfolio transitions tend to affect larger stocks to a greater extent, they still involve numerous securities with fairly different characteristics. Thus, this data is perfectly suited for identification of supply and demand shocks and for estimation of their price impact coefficients both with a great precision and for a large sample of securities. Next, I present the results of the cross-sectional analysis, which is based on these estimates.

2.4.2 Price Impact and Stock Size

Historically, the relation between the price impact coefficients and the market capitalization has been studied first. It is commonly believed that small stocks are subjects to information asymmetry. Numerous examples support this claim. For instance, Lo and McKinlay (1990) find that the prices of stocks with small capitalization do not follow random walks, which would be expected for the securities with insignificant informed trading. Easley, Hvidkjaer and O'Hara (2002) show that the correlation between the probability of informed trading, PIN, and the stock size is negative and large in absolute terms. Consequently, the market microstructure literature unanimously conjectures that the price impact coefficients have to decrease with market capitalization.

To investigate how the price impact coefficients depend on market capitalization, I implement the following test. First, I split all transition orders into five groups

according to their corresponding market capitalization.¹⁶ Since orders of large stocks prevail in my samples, disproportionately more observations are attributed to the large-cap group. Second, for each group, I estimate the means, the medians and the standard errors of the price impact coefficients, λ_{PI} 's, associated with orders in this group. To avoid influence of extreme values, I winsorize 10% of top and bottom observations of λ_{PI} .

Results are reported in Table 2.2. Striking patterns are clearly observed. In contrast to aforementioned intuition, the coefficients λ_{PI} 's tend to increase with the market capitalization; and the greatest price impact coefficients are associated not with the small stocks but rather with the large ones. These patterns are observed both for the mean and median values regardless of trade direction. For instance, for buy orders executed on NYSE/Amex, the median price impact coefficients gradually increase from 1.95% for small stocks to 12.79% for large stocks.¹⁷ Similarly, for sell orders executed on NYSE/Amex, the coefficients change from 2.44% for small stocks to 8.51% for large stocks. Likewise, the positive relation between λ_{PI} and the market capitalization is found for the universe of Nasdaq-traded stocks.

The positive relation between the price impact coefficients and the stock size contradicts widely accepted beliefs. However, Breen, Hodrick, Korajczyk (2002) draw a similar conclusion without emphasizing it. They mention that the price impact coefficients, estimated with data from TAQ database, also increases with the market capitalization, if trade size in its definition is scaled by the shares outstanding and, consequently, these coefficients are defined for a percent of turnover. However, if instead trade size is not normalized and mere number of executed shares is used in denominator, then the relation reverses and becomes negative. Similar negative relation between unscaled variables is documented, for instance, in Hasbrouck (1991b)

¹⁶Capitalization thresholds are calculated based on NYSE capitalization quintiles in 2003.

¹⁷Variable λ_{PI} quantifies price impact of virtual trades equivalent to 1% of the average daily volume, measured in percents of the return standard deviations.

and Chen, Stanzl and Watanabe (2005), among others. Thus, the specification of the price impact coefficients seems to be crucial for this issue.

As I argued before, the specification (2.4) is a reasonable definition of the price impact coefficients. Then, what might explain the observed counter-intuitive relation between price impact and stock size? Certainly, it can be explained by non-linearity of temporary price impact (e.g., Almgren et al. (2005)). Indeed, as shown in Table 2.2, the average magnitude of transition orders (as a percentage of the average daily volume) is much smaller for larger stocks. Together with concavity of price impact functions, these differences might induce the documented patterns of the price impact coefficients. I will come back to the non-linearity issues in the last section, and show that, indeed, the adjustment for non-linearity changes these patterns in some cases.

Another explanation is the following. Large stocks are traded very intensively. However, the significant part of their trading volume is usually attributed to noise trading, which is typically two-sided: buys alternates with sells. Consistently, for example, Chordia, Huh, and Subrahmanyam (2007) find that the stock size is positively related to the monthly turnover and negatively to the absolute order imbalances, which capture net selling or buying pressure. Consequently, for large stocks, a one-sided trade equivalent to 1% of the average daily volume might be a very strong signal of private information. At the same time, the opposite argument is true for small stocks: though they are not traded much, a large fraction of their trading volume comes from one-sided orders. Consequently, for small stocks, the information content of a one-sided trade, equivalent to 1% of the average trading volume, might be less significant. This argument further implies that the price impact coefficients, defined in (2.4), increase with the market capitalization.

2.4.3 Price Impact and Spread

I also examine the relation between the price impact coefficients and the percentage spread. These variables are often used as alternative measures of liquidity. Yet, they describe different facets of liquidity, and the relation between them might be more complicated than a one-to-one correspondence.¹⁸ Presumably, stocks with large percentage spreads are traded with the large price impact coefficients, and vice versa. Indeed, Breen, Hodrick and Korajczyk (2002) conclude that the bid/ask spread and the price impact coefficients are related; however, the former is not a sufficient statistic for the price impact.¹⁹ Next, I empirically examine their relation based on the evidence from portfolio transitions.

The testing procedure is very similar to the one described in Section 2.4.2. The only difference is that groups are formed based on the percentage spread as opposed to stock size. As before, for each group, the summary statistics for the price impact coefficients is reported. Presented in Table 2.3, the results are surprising. For transition purchases, the price impact coefficients tend to decrease with the bid/ask spread; for transition sales, their mean and median values exhibit u-shaped patterns across spread groups. The split of the sample into NYSE/Amex and Nasdaq-traded stocks reveals that both groups demonstrate similar interdependence. These counter-intuitive results might be partially attributed to a manifestation of relation between price impact and market capitalization, discussed in the previous section. I address this issue in the next section.

2.4.4 Regression Analysis

Certainly, the aforementioned results might be contaminated by the influence of various stock characteristics. To examine the incremental impact of specific variables, I

¹⁸Also, the price impact coefficients might mechanically picking up the effects of spreads.

¹⁹See also Hasbrouck (1991a)

study the cross-sectional variation in the price impact coefficients within a regression framework.

More specifically, I regress variable λ_{PI} on the following stock characteristics:

- *LN Cap*: The first variable is the logarithm of the market capitalization (in billions). The results of Section 2.4.2 reveal that contrary to the conventional intuition, stock size relates positively to λ_{PI} . Their robustness is checked in the regression analysis.
- *SPREAD*: The second variable is the percentage spread (in percents). As discussed in Section 2.4.3, two measures of liquidity, spread and λ_{PI} , are expected to be closely related to each other.
- *ANUM*: The third variable is the number of analysts following stocks. It is constructed based on the number of analysts who provide I/B/E/S with their end-of-fiscal-year earnings forecast. By and large, the greater number of investment analysts is associated with the more transparent trading and the lower adverse selection costs, which are in turn directly linked to λ_{PI} .
- *LN Turn*: The fourth variable is the logarithm of the monthly turnover. Often, it is thought of as a good proxy for the amount of noise trading. In fact, the substantial stock turnover is a sign of the active trading of "noninformational" traders (e.g., Campbell, Grossman and Wang (1993)). Therefore, reflecting the intensity of uninformed trading and information content behind typical trades, this variable might affect λ_{PI} .
- *DISP*: The fifth variable is the dispersion of analysts' forecasts. It is defined as the standard deviation of the end-of-fiscal-year earnings forecast normalized by its mean. This variable usually increases with earnings uncertainty. Its high values imply that information asymmetry between informed agents and

other market participants is large and, consequently, trades are associated with significant price impact, λ_{PI} (e.g., Sadka and Scherbina (2007)). Substantial analyst disagreement might also foster two-sided noise trading.

- σ : The sixth variable is the standard deviation of daily returns. It also might be related to earnings uncertainty, which potentially leads to significant information advantage of informed agents and high λ_{PI} . Also, high volatility of stock returns might be a sign of intensive trading of noise traders.
- *Nasdaq*: The last variable is the Nasdaq dummy variable, which is equal to 1 for all Nasdaq-traded securities and 0 for the others. It captures potential dependence of λ_{PI} on the specifics of different trading venues.

To avoid any spurious association between corresponding contemporaneous characteristics and λ_{PI} , all explanatory variables are as of the end of the previous month.

Different specifications of regressions are considered. They are estimated with the slightly modified Fama-McBeth procedure for data grouped at monthly levels. Namely, for each calendar month, I run the cross-sectional regressions of λ_{PI} 's on the stock characteristics and report the average regression coefficients. Since the number of observations varies significantly across months, I weight the monthly regression estimates by their precision (see Fama (1998)). The t-statistics are further adjusted with the Newey-West procedure to correct for intertemporal correlations.²⁰ This procedure allows one to correct for the cross-sectional correlations between stock returns.²¹ Table 2.4 and Table 2.5 demonstrate the results for different regression specifications based on the samples of buy and sell orders, respectively.

First, to check the robustness of conclusions in Section 2.4.2 and Section 2.4.3, I examine the regression results for the market capitalization and the percentage spread.

²⁰The number of lags is chosen by the automatic bandwidth selection procedure.

²¹The interdependence is of a concern, since contemporaneously traded stocks might share similar characteristics, moreover, the estimates of the price impact coefficients in each month might be contaminated by the overall market dynamics.

For both purchases and sales, the coefficients by the market capitalization are positive and statistically significant for various regression specifications. These results provide supportive evidence for the patterns, discussed in Section 2.4.2: transactions in larger stocks tend to be associated with more significant price impact coefficients.

The coefficients by the percentage spread are much less homogeneous: for purchases, they are, with a few exceptions, negative but statistically insignificant, whereas for sales, they are positive and statistically significant. Thus, the results in Section 2.4.3 can be attributed to the differences in the market capitalization across groups. Besides, two measures of liquidity, the spread and λ_{PI} , seem to relate positively to each other.

Second, the price impact coefficients relate positively to the monthly turnover, which proxies the amount of noise trading.²² Indeed, the corresponding estimates of coefficients are positive and statistically significant for most specifications. This evidence is consistent with the suggested explanation for the positive relation between stock size and λ_{PI} (see Section 2.4.2): stocks, in which two-sided noise trading accounts for the largest part of trading volume, experience larger price impact per virtual trades equivalent to one percent of the average daily volume.

Third, the price impact coefficients seem to be higher for Nasdaq-traded stocks: the coefficients by Nasdaq dummies are all positive. However, their t-statistics are fairly small and, consequently, can not represent statistically strong evidence. Furthermore, contribution of characteristics *ANUM*, the numbers of analysts who follow stocks, and *STD*, the standard deviation of returns, to explanation of the cross-sectional differences in λ_{PI} is statistically insignificant. Also, the price impact coefficients tend to increase with the dispersion of analysts' forecasts, which might proxy the intensity of noise trading as well as the extent of potential information advantage

²²Also, variable λ_{PI} tends to increase with the dispersion of analysts' forecasts, which might be associated with the amount of noise trading as well. The relation between λ_{PI} and the volatility is not statistically significant.

attributed to informed market participants.

To summarize, the price impact coefficients of portfolio transition orders vary substantially across securities with different characteristics. The implemented cross-sectional analysis reveals the following patterns: (1) they tend to increase with the market capitalization and spread; (2) they are positively related to the amount of noise trading and the degree of information advantage of informed traders; (3) they seem to be slightly higher for Nasdaq-traded than NYSE/Amex-traded securities.

2.5 Price Impact Dynamics

2.5.1 Motivation

Market participants often split their large orders into smaller trades and execute them over time. This practice is common among traders who wish to minimize transaction costs while trading for their liquidity needs and traders who seek to alleviate information leakage while trading on their private information. Transition managers are not an exception. Helping implement transition in a cost effective way, they also tend to stretch execution of large orders over time. Thus, transition orders usually consist of several consecutive trades. In this section, I study the dynamics of the price impact coefficients during these multi-trade "packages".

In the theoretical literature, it has long been recognized that traders often split their orders to minimize transaction costs. Theoretical analysis of price impact, associated with multi-period trading strategies, provides interesting insights about the complicated dynamic relation between individual trades and returns (e.g., Kyle (1985), Vayanos (1999, 2001)). Yet, to the best of my knowledge, the dynamics of the price impact coefficients over the course of multi-trade "packages" has not been examined empirically. Indeed, empirical literature has mostly focused on the price responses to individual trades (e.g., Chan and Lakonishok (1993), Kraus and Stoll

(1972), Keim and Madhavan (1996)). Also, a number of empirical studies point out that it might be misleading to consider individual trades as a unit of price impact analysis; they examine the effects on prices of entire "packages", however, leaving unanswered the question of how the price impact coefficient actually evolve during these "packages" (e.g., Chan and Lakonishok (1995), Keim and Madhavan (1997)). To fill the gap, I study the evolution of the price impact coefficients associated with sequences of trades that constitute portfolio transition orders.

It is worth mentioning that portfolio transitions provide a unique laboratory to study the market response to the sequence of trades. First, since most transition orders are fairly large, transition managers often split them over the course of several days. Indeed, about half of all orders take more than one day to be executed, and the majority of them are completed during a week. Second, the trading intentions of transition managers are set in advance; this contrasts with other studies in which ex ante orders cannot be easily identified: realized trades might be influenced by unobserved specifics of investment styles and order-placement strategies. Thus, the data set of portfolio transitions allows me to observe a large number of ex ante orders and how they are split over several days.

2.5.2 Test Design and Results

To investigate the evolution of price impact, I study the average price impact coefficients during the first and the second halves of transition orders. I design and implement the testing procedure in the following way.

First, I select all transition orders that take more than two days to be completed. Second, for each of them, individual open market trades are grouped into two sets: trades that are executed during the first half and trades that are executed during the second half of trading strategies.²³ Third, for each transition order, I compute

²³For instance, if transition order is executed in four days, then trades of the first two days belong

Δ , the pairwise difference between the average price impact coefficients of individual trades in the first and the second groups, $\bar{\lambda}_{PI,1} - \bar{\lambda}_{PI,2}$. The price impact coefficients of individual trades are defined as in (2.4); however, instead of the pre-transition benchmark prices, P_{bch} , the close prices at days, previous to trades, are used. Fourth, 10% top and 10% bottom values of Δ 's are winsorized. Lastly, the means and the medians of Δ 's are estimated. I next present my conclusions which are based on the sample of 35,289 observations.

Table 2.6 shows the means and the medians of Δ 's along with p-values of their t-tests and Wilcoxon signed-rank tests, respectively. Interestingly, very different patterns are observed for transition buy and sell orders.

On one hand, during buy orders, the price impact coefficients of individual trades tend to increase. Indeed, the results in Table 2.6 demonstrate that Δ 's are typically negative, i.e. the average price impact coefficients in the first halves of order execution, $\bar{\lambda}_{PI,1}$'s, are significantly lower than those in the second halves, $\bar{\lambda}_{PI,2}$'s. For instance, the average gap between them is about -18% for NYSE/Amex stocks or -3%, if the medians are considered.²⁴ These differences are statistically significant. Moreover, similar patterns are observed for stocks traded on Nasdaq and stocks with different market capitalization.

On the other hand, during sell orders, the price impact coefficients of individual trades tend to decrease over time. According to Table 2.6, the differences between $\bar{\lambda}_{PI,1}$ and $\bar{\lambda}_{PI,2}$ tend to be positive. During execution of sell orders, the price impact coefficients drop, on average, by about 20% or by 9%, if the median values are considered. These differences are also statistically significant. As before, they are observed for different trading venues and for different market capitalization quantiles.

To check the robustness of these patterns, I re-examine the dynamics of the price

to the first group and trades in the last two days belong to the second group.

²⁴As a reminder, the price impact coefficients are measured as a percentage of the standard deviations of daily returns and denote the price impact of trades equivalent to 1% of the average daily volume.

impact coefficients separately for two time periods: January 2001 through June 2003 and July 2003 through December 2005. The market is declining in the first time span and rising in the second one. Panel C of Table 2.6 shows that despite distinctive market conditions, similar patterns prevail in both cases for buy and sell orders.

2.5.3 Discussion

How are these findings related to existing theoretical literature? The theoretical papers on multi-period strategies and security prices can be broadly divided into two groups, according to the trading motives: either information-related or liquidity-related ones.

First, some traders may have private information about the future payoff distribution and, consequently, trade for information-related motives. Kyle (1985) analyzes the dynamic trading of a risk-neutral informed insider in this situation. He shows that an insider trades on his private information slowly and that the price impact coefficients are approximately constant over time.²⁵ In various extensions of this classical model, insiders typically trade more aggressively at the beginning; their behavior leads to the increase of the price impact coefficients during the first periods (when information content of order flow is high) and its decrease later (when informed insiders have already exploited their private information and their contribution to the trading volume is insignificant). For instance, these patterns are found in extensions in which an insider is risk-averse (e.g., Baruch (2002)) or when there are multiple strategic informed traders (e.g., Holden and Subrahmanyam (1992, 1994)). Thus, theoretical literature predicts that the price impact coefficients are non-increasing over time during execution of information-motivated orders.

Second, some market participants can initiate trading because of liquidity-related

²⁵In fact, non-constant functions are inconsistent with equilibrium. If price impact increased, then insider would prefer to incorporate all his private information into prices immediately. If price impact decreased, then insider could implement destabilizing schemas and earn unbounded profits.

motives such as risk sharing, portfolio rebalancing, accommodation of exogenous inflows and outflows. In these situations, they do not possess any private information about stocks' fundamentals. However, formally they have information advantage over other market participants as well: they know that they will be executing large orders and that future stock prices will be changing adversely.²⁶ Vayanos (2001) studies this interesting form of information asymmetry. In his model, information about stock future payoff is public, and a large risk-averse trader with liquidity needs implements the multi-period trading strategy.²⁷ Interestingly, if this trader does not manipulate the market, the model predicts decrease of the price impact coefficients over time: the market maker learns about his intentions, and unexpected order flows gradually decrease. To summarize implications of the theoretical literature, the price impact coefficients associated with any multi-period orders (including portfolio transition orders) are expected to be non-increasing over time regardless of motives behind them.

Before proceeding, it is worth mentioning another detail. On average, portfolio transition purchases and sales are expected to have different information contents. Typically, the purchases of stocks from target portfolios are information-motivated trades, whereas the sales of stocks from legacy portfolios are liquidity-motivated ones. This interesting dichotomy of transition trades is highlighted and discussed in Obizhaeva (2007). In fact, target portfolio usually outperform legacy portfolios in the post-transition periods for up to half a year; this observation implies that hired managers are better than terminated managers in generating abnormal returns. Thus, on one hand, target portfolios are selected by skilled managers; purchases of these portfolios reflect their private information and correspond to information-motivated trades. On the other hand, the sales of legacy portfolios are just liquidations of

²⁶For example, if pension fund is liquidating its position in a given stock to meet obligations before its participants, it has private information that the stock prices will drop in the future. This is somewhat similar to the situation when insiders observe the negative signals about future payoffs.

²⁷See also Vayanos (1999) for a variation of this model.

portfolios of unskilled managers; they represent uninformative shocks to supply.²⁸

The price impact patterns, found for transition sales, support the prediction of Vayanos (2001) about price dynamics during non-informative trading. As predicted, the price impact coefficients are much higher for the first halves of portfolio transitions than for the second halves. However, as shown in Table 2.5, the drop in $\bar{\lambda}_{PI}$ is more significant for larger stocks; if information is fully revealed by the end of transitions, this fact contradicts the intuition: ex ante information asymmetry is more substantial for smaller stocks and, consequently, they are expected to experience the larger decrease in the price impact coefficients.

At the same time, the patterns of the price impact coefficients, associated with transition purchases, can not be easily explained by the existing models about information-driven trading sequences. These models predict the non-increasing price impact dynamics; however, the opposite patterns are observed. These patterns may occur in the situations when a market maker gradually learns about the active presence of informed traders in the market.

To summarize, I show that the price impact coefficients increase over time when stocks from target portfolios are being purchased and decrease when stocks from legacy portfolios are being sold. These patterns are only partially consistent with the intuition of theoretical models. Although these results are being obtained for the sample of portfolio transition orders, they may have implications for a broader class of institutional trades, which are often split over several days.

²⁸More detailed discussion can be found in Obizhaeva (2007); similar arguments are presented in Coval and Stafford (2005).

2.6 Price Impact Functions

2.6.1 Motivation

There is no consensus on whether price impact functions are concave, linear, or convex and, furthermore, what the reasonable estimates of their parameters are.

Linear price impact functions are frequently used in the literature. For instance, the theoretical literature often relies on this parametric form because it greatly simplifies the models. Also, linear specification serves as a basis for a large number of empirical studies. There are reasons to believe that price impact functions are indeed linear: Huberman and Stanzl (2000) argue that non-linear price impact would lead to arbitrage.

However, several factors might induce concavity of price impact functions by mitigating the transaction costs of large orders. First, block traders might credibly signal the liquidity-related motives behind their trades (see Seppi (1990)). Second, information behind large orders might be leaking while they are being 'shopped', thus, lessening their realized effects on security prices. Finally, there are concerns about the endogeneity of observed trades: since traders can place their orders strategically, most of large costly orders will be split into smaller trades and executed over time (see Bertsimas and Lo (1998)). Consistently with aforementioned arguments, the empirical literature often finds examples in favor of concave price impact functions; the examples include Almgren et al. (2005), Hasbrouck (1991a), Hausman, Lo, and MacKinlay (1992) among many others. At the same time, there is no agreement yet on what the reasonable values of their curvature parameters are: many papers advocate the square-root specification (see Barra (1997)), whereas others claim that other functional forms are more appropriate (see Almgren et al (2005)).

To provide additional evidence on these controversial issues, I estimate the price impact functions using the dataset of portfolio transition trades. Taking advantage of

clear distinction between sell and buy orders in this data set, I analyze price impact functions separately for these two groups. Additionally, I explore how their functional form changes across different market quintiles.

2.6.2 Test Design and Results

I assume that the price impact might be reasonably described by the power-law functions and estimate their parameters. More precisely, I run pooled regressions of the following form:

$$PI_i = \beta \left(\frac{X_i}{1\%ADV_i} \right)^\gamma + \epsilon \quad (2.5)$$

where PI_i is the total price impact of order i , defined in (2.1), X_i is the size of transition order in shares, and ADV_i is the average daily volume in the previous month.

I next estimate (2.5) using the entire sample of portfolio transition orders.²⁹ Parameters β and γ are parameters of interest. Parameter β stands for the magnitude of price impact; more specifically, it quantifies the price impact for trades equivalent to 1% of the average daily volume and is measured as a fraction of the return standard deviations. At the same time, parameter γ controls the curvature of price impact functions, i.e. if γ is close to 1 or 1/2, than linear or square-root specifications of these functions are validated, respectively.³⁰

Table 2.7 presents the results of the calibration exercise: the estimates of parameters with their standard errors, t-statistics as well as 95%-confidence intervals. For transition buys, the point estimates of parameters are $\hat{\beta} = 0.14$ and $\hat{\gamma} = 0.31$ (Panel

²⁹To adjust for cross-correlation within observations of the same months, I cluster observations at monthly levels. Also, I winsorize the top and bottom 10% of variable PI and filter out 1% of observations with very large values of $X/1\%ADV$ (approximately corresponding to values larger than 50).

³⁰The price impact coefficients, λ_{PI} 's, discussed in the previous sections, correspond to parameter β , if a linear specification with $\gamma = 1$ is implicitly assumed.

A); for transition sales, the analogous estimates are $\hat{\beta} = 0.09$ and $\hat{\gamma} = 0.47$ (Panel B). Clearly, the standard errors of $\hat{\beta}$ imply that β 's are positive. As for the curvature parameters, the hypothesis $\gamma = 1$ can be strongly rejected both for transition buys and sells. These results confirm that price impact functions are concave. Moreover, the popular square-root model seems to be a good description of actual price impact functions, although these functions tend to be slightly flatter for transition purchases.

Table 2.7 also shows the results for different size groups. These tests are similar to the ones implemented in Chen, Stanzl and Watanabe (2001): all stocks are sorted into five size groups according their market capitalization and then price impact functions (2.5) are estimated for each of these groups. Similar patterns are observed. Point estimates of β 's range from 0.05 to 0.16, and the ones of γ 's are from 0.23 to 0.58. For all groups, the magnitude parameters, β 's, are significantly positive and the curvature parameters, γ 's, are fairly close to 1/2. The exception is price impact of large purchases that tends to have slightly lower $\hat{\gamma}$'s: the market reaction to these orders depends on their magnitude to a less extent.

To summarize, in this section I calibrate price impact functions on the data set of portfolio transition trades. The results suggest that price impact is concave and well described by the square-root functions. Lastly, it is worth mentioning that estimated price impact functions are not attributes of individual stocks but rather represent aggregated functions for entire sample or various size groups.

2.6.3 Permanent and Temporary Price Impact

The total price impact is often decomposed into two components: permanent and temporary ones. The former is the permanent price shift due to information that is partially revealed through a trade, whereas the latter is the transitory price change due to liquidity effects that are induced by quick execution and limited elasticity of demand and supply. A number of papers highlight the difference between these

two parts (e.g., Holthausen, Leftwich, and Mayers (1987), Gemmill (1996), Keim and Madhavan (1996), Almgren et al (2005)). In this section, I calibrate the permanent and temporary price impact functions separately.

More precisely, I re-estimate equation (2.5) but substitute the left-hand-side variable PI with PI_{perm} and PI_{temp} , defined in (2.2) and (2.3), for permanent and temporary components, respectively. Of course, there is some ambiguity of how to disentangle these two parts or, in other words, how to define an appropriate "base" time by which all liquidity effects dissipate. Therefore, for robustness, I consider several specifications of a "base" time: the close at the last day of order execution, the close at the next day and one or two weeks after execution. As before, transition purchases and sales are examined separately.

Panel A of Table 2.8 presents the point estimates of parameters with their statistics for portfolio transition purchases. As I argued before, these transactions are expected to contain private information. Indeed, their permanent price impact tend to be positive: for most specifications, estimates $\hat{\beta}$'s are statistically significant and locate above zero. However, most of $\hat{\gamma}$'s can not be statistically separated from zero. This finding implies that the functional form of permanent price impact is far from being linear, furthermore, it suggests that its magnitude is almost unrelated to a trade size. Similar invariance has been also documented in the literature on block trades (e.g., Keim and Madhavan (1996)).³¹ At the same time, for temporary component, the estimates of all parameters are indistinguishable from zero; thus, no strong statements can be made about its functional form.

Panel B of Table 2.8 demonstrates the results of calibration tests for portfolio transition sales. These results are clearly very different from those for purchases. The dissimilarity is expected and explained by the differences in their information content

³¹This independence might be attributed to specifics of the sample, which is dominated by trades in large stocks. Also, the true relationship might be obscured by noise, which is fairly substantial as shown by very small R^2 's.

- transition sales are, by and large, liquidity-motivated transaction (see Obizhaeva (2007)). Consequently, they should not contain a permanent component. In fact, for the most of "base" times, the curvature estimates $\hat{\gamma}$'s are indistinguishable from zero; similarly, the magnitude estimates $\hat{\beta}$'s are also close to zero or even slightly negative for "base" times of one and two weeks. Meanwhile, temporary price impact has significantly positive $\hat{\beta}$'s and $\hat{\gamma}$'s, with the latter typically lower than 1/2. Thus, transition sales induce price impact which is transitory in its nature.

To summarize, the following results of this section can be highlighted. First, price impact functions are probably non-linear. Second, they can be well approximated by square-root functions. Third, the functional form of their temporary and permanent components is more difficult to purge; however, the permanent price impact does not satisfy a linear specification.

2.6.4 Non-Linear Price Impact Coefficients

In the previous section, I have shown that price impact functions are concave. Therefore, I slightly modify (2.4) and consider the following specification of the price impact coefficients, adjusted for non-linearity:

$$\tilde{\lambda}_{PI} = \frac{(P_{exec}/P_{bch} - 1)/\sigma}{\sqrt{(X/1\%ADV)}} \times \mathbb{I}_{BS} \times 100\% \quad (2.6)$$

which is different from (2.4) only by a radical sign in its denominator. This modification is chosen because price impact functions are found to be well described by a square-root specification. Next, I redo all tests in this paper for variable $\tilde{\lambda}_{PI}$. For brevity, I present below only a quick summary of the results.³²

What is the relation between the market capitalization and $\tilde{\lambda}_{PI}$? I study this question with tests, described in Section 2.4. By and large, the price impact coeffi-

³²Full version of results can be provided by the author at request.

cients exhibit either u-shaped or increasing in stock size patterns. Typically, stocks from the largest size quintile display $\tilde{\lambda}_{PI}$'s, which are higher than those for stocks of other sizes, perhaps only with an exception for the smallest stocks. These patterns are observed for the mean and median values of $\tilde{\lambda}_{PI}$, for purchases and sales, and for different trading venues. At the same time, the regression analysis leads to weaker results. For purchases, the coefficients by stock size are always positive but statistically insignificant; for sales, they do not exhibit any unambiguous patterns. Overall, the results show that the positive relation between the market capitalization and the price impact coefficients in Section 2.4 might be partially attributed to concavity of price impact functions; however, this evidence is quite inconclusive.

What is the relation between the percentage spread and $\tilde{\lambda}_{PI}$? After redoing the tests of Section 2.4, I conclude the following. For buy orders, the relation between $\tilde{\lambda}_{PI}$ and the spread is fairly weak. For sell orders, $\tilde{\lambda}_{PI}$'s tend to increase across the spread groups (or their relation is slightly u-shaped); this positive relation is also found in the regression analysis: the coefficients by the spread variables are positive and statistically significant for all specifications. Thus, adjustment for non-linearity does not affect the conclusion of Section 2.4: the price impact coefficients tend to relate positively to the percentage spread.

Also, $\tilde{\lambda}_{PI}$ is usually higher for Nasdaq-traded stocks, however, these results are not statistically significant. Moreover, $\tilde{\lambda}_{PI}$ tend to relate positively to the amount of noise trading: the regression coefficients by turnover are positive for all specifications, but their t-statistics are fairly small. The regression coefficients by the number of analysts, the dispersion of analysts' forecasts, and the volatility are statistically insignificant. These results are based on the analysis similar to the one in Section 2.4.4.

I also re-examine the dynamics of the price impact coefficients, when they are adjusted for non-linearity, and I redo the tests of Section 2.5 for the specification (2.6). Interestingly, all findings are the same: $\tilde{\lambda}_{PI}$ increases during execution of buy

orders and decreases during that of sell orders. Thus, the patterns of $\tilde{\lambda}_{PI}$ dynamics in response to multi-trade "packages" are robust to the adjustment for potential non-linearity.

On the whole, the properties of the price impact coefficients, adjusted for non-linearity, are quite similar to the properties of unadjusted ones. However, the cross-sectional patterns become slightly weaker; this fact implies that some of them might be partially attributed to the concavity of price impact functions.

2.7 Conclusion

This study is a step towards building a comprehensive understanding of the price impact patterns. I focus here on the price impact coefficients. Formally, these variables quantify the magnitude of price changes (defined in percents of the standard deviation of daily returns) in response to virtual trades, which are equivalent to 1% of the average daily volume. Proprietary data set of portfolio transitions helps estimate them in a very precise manner for a large cross-section of stocks.

Using these estimates, I document interesting empirical findings about the cross-sectional patterns of the price impact coefficients, their dynamics during implementation of multi-period strategies, and the functional form of price impact. Some of these results can not be easily explained within the existing models and are briefly highlighted below.

The first surprising result is the positive relation between the price impact coefficients and the market capitalization. This finding clearly challenges the conventional intuition, according to which market participants, who trade in large stocks, do not face significant information asymmetry and, consequently, their trades induce only small price changes. Related to these patterns is the documented positive relation between the price impact coefficients and the amount of noise trading; again, the

classical models predict the opposite patterns. In this paper, I provide a sketch of explanation that can justify these two findings. However, more work on this topic is required and other explanations might be suggested.

The second interesting finding, which is difficult to place in the context of existing models, is the evolution of the price impact coefficients during execution of multi-trade "packages". This study shows that they increase in response to sequences of buy trades and decrease, if multiple sell trades are carried out. The theoretical literature has long been studying the price dynamics during multi-period strategies, executed either by informed or by uninformed traders. However, to the best of my knowledge, neither of these models predict the increasing signature of the price impact coefficients. Thus, it is difficult to reconcile the observed dynamics with the theoretical literature.

Finally, the analysis of price impact functions reveals that a linear specification can be rejected for its permanent components with a high level of confidence. This finding is puzzling given that, in theory, non-linear functions for permanent price impact would allow for arbitrage. Certainly, all these findings call for more extensive empirical analysis and theoretical explanations.

References

- Almgren, Robert, Chee Thum, Emmanuel Hauptmann, and Hong Li, 2005, Direct estimation of equity market impact, *Working paper*.
- Baruch, Shmuel, 2002, Insider trading and risk aversion, *Journal of Financial Markets*, 5, 451–464.
- Bertsimas, Dimitri, and Andrew Lo, 1998, Optimal control of execution costs, *Journal of Financial Markets*, 1, 1–50.
- Breen, William, Laurie Hodrick, and Robert Korajczyk, 2002, Predicting Equity Liquidity, *Management Science*, 48, 470–483.
- Campbell, John, Sanford, Grossman, and Jiang Wang, 1993, Trading volume and serial correlation in stock returns, *Quarterly Journal of Economics* 108(4), 905–939.
- Chan, Louis, and Josef Lakonishok, 1993, Institutional trades and intra-day stock price behavior, *Journal of Financial Economics* 33, 173–199.
- Chan, Louis, and Josef Lakonishok, 1995, The behavior of stock prices around institutional trades, *Journal of Finance* 50(4), 1147–1174.
- Chen, Zhiwu, Werner Stanzl, and Masahiro Watanabe, 2001, Price impact costs and the limit of arbitrage, *ICF Working Paper* No. 00–66.
- Chordia, Tarun, Sahn-Wook Huh, and Avanidhar Subrahmanyam, 2007, The cross-section of expected trading activity, *Review Financial Studies*.
- Conrad, Jennifer, Kevin Johnson, and Sunil Wahal, 2003, Institutional trading and alternative trading systems, *Journal of Financial Economics*, 70, 99–134.
- Coval, Joshua, and Erik Stafford, 2005, Asset fire sales (and purchases) in equity markets, *Harvard Business School Working Paper*, No. 05–077.
- Holden, Craig, and Avanidhar Subrahmanyam, 1992, Long-Lived Private Information and Imperfect Competition, *Journal of Finance*, 47, 247–270.
- Holden, Craig, and Avanidhar Subrahmanyam, 1994, Risk aversion, imperfect competition, and long-lived private information, *Economic Letters*, 44, 181–190.
- Dufour, Alfonso and Robert Engle, 2000, Time and the price impact of a trade, *Journal of Finance*, 55(6), 2467–2498.
- Easley, David, Soeren Hvidkjaer and Maureen O’Hara, 2002, Is information risk a determinant of asset returns? *Journal of Finance*, 57, 2185–2221.

- Fama, Eugene, and James MacBeth, 1973, Risk, return, and equilibrium: empirical tests, *Journal of Political Economy* 81, 607–636.
- Gemmill, Gordon, 1996, Transparency and liquidity: A study of block trades in the London Stock Exchange under different publication rules, *Journal of Finance* 51, 1765–1790.
- Glosten, Lawrence, and Lawrence Harris, 1988, Estimating the components of the bid/ask spread, *Journal of Financial Economics*, 21(1), 123–142.
- Goyal, Amit, and Sunil Wahal, 2006, The selection and termination of investment management firms by plan sponsors, *Working Paper*.
- Hasbrouck, Joel, 1991[a], Measuring the information content of stock trades, *Journal of Finance*, 46, 179–207.
- Hasbrouck, Joel, 1991[b], The summary informativeness of stock trades: An econometric analysis, *Review of Financial Studies*, 4, 571–595.
- Hausman, Jerry, Andrew Lo, and Craig MacKinley, 1992, An ordered probit analysis of transaction stock prices, *Journal of Financial Economics*, 31, 319–379.
- Holthausen, Robert, Richard Leftwich, and David Mayers, 1987, The effect of large block transactions on security prices: A cross-sectional analysis, *Journal of Financial Economics* 19(2), 237–67.
- Holthausen, Robert, Richard Leftwich, and David Mayers, 1990, Large-block transactions, the speed of response, and temporary and permanent stock-price effects, *Journal of Financial Economics* 26(1), 71–95.
- Huberman, Gur, and Werner Stanzl, 2004, Price manipulation and quasi-arbitrage, *Econometrica*, 74(4), 1247–1276.
- Keim, Donald, and Ananth Madhavan, 1995, Anatomy of the trading process empirical evidence on the behavior of institutional traders, *Journal of Financial Economics* 37(3), 371–398.
- Keim, Donald, and Ananth Madhavan, 1996, The upstairs market for large-block transactions: Analysis and measurement of price effects, *Review of Financial Studies* 9(1), 1–36.
- Keim, Donald, and Ananth Madhavan, 1997, Transactions costs and investment style: an inter-exchange analysis of institutional equity trades, *Journal of Financial Economics* 46(3), 265–292.
- Kraus, Alan, and Hans Stoll, 1972, Price impacts of block trading on the New York Stock Exchange, *Journal of Finance* 27, 569–588.
- Korajczyk, Robert, and Ronnie Sadka, 2004, Are momentum profits robust to trading costs? *The Journal of Finance*, 59(3), 1039–1082.

- Kyle, Albert, 1985, Continuous auctions and insider trading, *Econometrica* 53(6), 1315–1336.
- Lillo, Fabrizio, Doyme Farmer, and Rosario Mantegna, 2003, Master curve for price-impact function, *Nature*, 421, 129–130.
- Lo, Andrew, and Craig MacKinlay, 1988, Stock market prices do not follow random walks: Evidence from a simple specification test, *Review of Financial Studies*, 1, 41–66.
- Madhavan, Ananth, Matthew Richardson, Mark Roomans, 1997, Why do security prices change? A transaction-level analysis of NYSE stocks, *Review of Financial Studies*, 10(4), 1035–1064.
- Obizhaeva, Anna, 2007, Information vs. liquidity: Evidence from portfolio transitions, *Working paper*.
- Parwada, Jerry, and Wenling Joey Yang, 2004, Stock market effects of institutional investment flows: Evidence from Australian pension plan mandate changes, *Working Paper*.
- Sadka, Ronnie, and Anna Scherbina, Analyst disagreement, mispricing and Liquidity, 2007, *Journal of Finance*, forthcoming.
- Seppi, Duane, 1990, Equilibrium block trading and asymmetric information, *The Journal of Finance*, 45, 73–94.
- Torre, Nicolo, 1997, BARRA Market impact model handbook, *BARRA, Berkeley, California*.
- Vayanos, Dimitri, 1999, Strategic trading and welfare in a dynamic market, *Review of Economic Studies*, 66, 219–254.
- Vayanos, Dimitri, 2001, Strategic trading in a dynamic noisy market, *Journal of Finance*, 56, 131–171.

Table 2.1: Summary Statistics

<i>Panel A: Summary Statistics for Transition Orders:</i>				
	BUY		SELL	
	Mean	Median	Mean	Median
# Shrs/ShrOut (bp)	1.21	0.66	1.15	0.59
# Shrs/ADV (%)	1.63	1.27	1.56	1.19

<i>Panel B: Summary Statistics for Stocks:</i>				
	BUY		SELL	
	Mean	Median	Mean	Median
Cap (x 10 ⁹)	6.07	1.69	6.84	1.83
Turn (%)	209.40	152.32	211.55	154.24
SPREAD (%)	0.25	0.13	0.24	0.12
ANUM	10.61	9.00	11.24	10.00
DISP (%)	11.36	2.16	11.48	2.47
σ	0.02	0.02	0.02	0.02

Table 2.1 presents summary statistics. Panel A shows the properties of portfolio transition orders, such as the size of orders as the fraction of shares outstanding (in basis points) and of average daily volume (in percents). Panel B shows the characteristics of stocks. These characteristics include the market capitalization (in billions), *Cap*, the annual turnover (in percents), *Turn*, the percentage spread (in percents), *SPREAD*, the number of analysts following stocks *ANUM*, the dispersion of analysts' forecasts (in percents of average forecasts), *DISP*, and the standard deviation of daily returns, σ . Purchases and sales are considered separately. All statistics are calculated based on pooled data from January 2001 to December 2005.

Table 2.2: Price Impact and Market Capitalization

Panel A: NYSE/AMEX Stocks

		Cap Quintiles					
		1 (Small)	2	3	4	5 (Large)	All
BUY	PI Mean	6.73	8.33	10.95	19.07	23.53	17.75
	PI Median	1.95	3.10	4.00	7.28	12.79	5.47
	PI SEs	1.41	1.18	1.23	1.25	1.19	0.64
	Avg(σ)	0.03	0.03	0.02	0.02	0.02	0.02
	Avg(%ADV)	17.87	6.27	3.20	1.94	0.96	2.87
	# Obs	1,692	4,931	7,925	11,305	17,994	43,847
	SELL	PI Mean	4.63	5.56	4.03	6.62	13.63
	PI Median	2.44	2.48	2.12	3.66	8.51	3.80
	PI SEs	1.47	1.23	1.11	1.15	1.07	0.60
	Avg(σ)	0.03	0.03	0.02	0.02	0.02	0.02
	Avg(%ADV)	29.68	8.07	3.74	1.82	0.81	3.26
	# Obs	1,743	5,116	9,688	13,819	22,581	52,948

Panel B: Nasdaq Stocks

		Cap Quintiles					
		1 (Small)	2	3	4	5 (Large)	All
BUY	PI Mean	7.69	9.81	15.20	21.41	25.71	13.85
	PI Median	2.52	3.52	4.91	6.91	10.93	3.79
	PI SEs	1.02	1.18	1.62	2.51	3.48	0.76
	Avg(σ)	0.04	0.03	0.03	0.02	0.02	0.03
	Avg(%ADV)	11.46	4.36	2.35	1.53	0.76	4.51
	# Obs	4,839	7,853	6,244	3,498	2,245	24,679
	SELL	PI Mean	7.40	9.28	6.77	8.51	13.24
	PI Median	3.06	3.27	2.29	4.36	5.42	3.12
	PI SEs	1.06	1.15	1.47	2.19	2.95	0.71
	Avg(σ)	0.04	0.03	0.03	0.02	0.02	0.03
	Avg(%ADV)	16.92	4.68	2.74	1.74	0.71	5.67
	# Obs	5,674	8,657	7,514	4,555	3,097	29,497

Table 2.2 shows the means, medians and standard errors of the price impact coefficients for purchases and sales and different capitalization groups. The price impact coefficients are defined in (2.4). Additionally, the average standard deviation of daily returns, $Avg(\sigma)$, the average size of trades as a percentage of average daily volume in the previous month, $Avg(\%ADV)$, and the number of observations are presented. Data is reported separately for NYSE/AMEX and Nasdaq. Capitalization thresholds are calculated based on NYSE stock capitalization quintiles. The sample ranges from January 2001 to December 2005.

Table 2.3: Price Impact and Percentage Spread

		Percentage Spread Quintiles					
		1 (Small)	2	3	4	5 (Large)	All
<i>Panel A: NYSE/AMEX Stocks</i>							
BUY	PI Mean	21.41	22.31	15.55	15.33	14.07	17.75
	PI Median	9.95	9.68	5.04	4.58	3.41	5.47
	PI SE	1.63	1.53	1.45	1.36	1.20	0.64
	Spr Median	0.04	0.06	0.10	0.16	0.37	0.10
	# Obs	8,747	8,764	8,757	8,756	8,745	43,847
	SELL						
SELL	PI Mean	11.20	7.60	5.66	8.96	11.44	8.96
	PI Median	5.87	3.85	2.46	3.61	3.96	3.80
	PI SE	1.49	1.41	1.34	1.27	1.13	0.60
	Spr Median	0.04	0.06	0.10	0.16	0.35	0.10
	# Obs	10,563	10,598	10,555	10,573	10,571	52,948
	<i>Panel B: Nasdaq Stocks</i>						
		Percentage Spread Quintiles					
		1 (Small)	2	3	4	5 (Large)	All
BUY	PI Mean	20.67	17.25	16.45	9.95	4.69	13.85
	PI Median	7.95	6.17	5.44	3.22	2.02	3.79
	PI SE	2.11	1.88	1.69	1.53	1.13	0.76
	Spr Median	0.06	0.12	0.20	0.34	0.84	0.20
	# Obs	4,914	4,927	4,912	4,928	4,920	24,679
	SELL						
SELL	PI Mean	8.12	7.15	6.44	10.26	10.96	8.57
	PI Median	3.42	2.64	2.44	3.77	3.11	3.12
	PI SE	1.94	1.75	1.59	1.47	1.09	0.71
	Spr Median	0.06	0.12	0.19	0.31	0.79	0.19
	# Obs	5,871	5,882	5,873	5,877	5,881	29,497

Table 2.3 shows the means, and median, of the price impact coefficients with their standard errors, for purchases and sales and different groups of stocks based on their percentage spread. The price impact coefficients are defined in (2.4). Also, the median values of the percentage spread, *Spr* Median, in percents is shown for each group. Data is presented separately for NYSE/AMEX and Nasdaq. Capitalization thresholds are calculated based on NYSE stock capitalization quintiles. The sample ranges from January 2001 to December 2005.

Table 2.4: The Regression Analysis, Purchases

<i>Regressions of Price Impact on Explanatory Variables, Purchases</i>							
<i>Regression Specifications</i>							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
const	13.46** (4.00)	13.83** (3.72)	11.36** (3.42)	13.36** (2.84)	14.22** (3.54)	11.99* (2.50)	12.94* (2.55)
LN Cap	3.56* (5.68)	3.36* (4.91)	3.72* (5.62)	3.78* (6.26)	3.25* (5.32)	2.83* (3.17)	3.44* (6.17)
SPREAD		-1.03 (-0.52)			-2.02 (-0.81)		-0.78 (-0.59)
ANUM						0.24 [†] (1.70)	
LN Turn			1.29 [†] (1.68)			1.35 [†] (1.74)	1.60 [†] (1.99)
DISP					0.74 (1.23)		0.80 (0.97)
σ				45.60 (0.58)		-68.30 (-0.89)	-45.31 (-0.57)
Nasdaq	3.23 (1.34)	3.18 (1.30)	2.26 (1.06)	3.37 (1.54)	2.83 (1.08)	1.94 (0.85)	1.82 (0.78)
Avg R^2	0.01	0.02	0.02	0.02	0.02	0.03	0.03
# Obs							72,299

Table 2.4 shows the regression analysis of the price impact coefficients, defined in (2.4), on various stocks' characteristics for the sample of purchases. These characteristics are *LN Cap*, the logarithm of capitalization (in billions); *SPREAD*, the percentage spread (in percents); *ANUM*, the number of analysts following stocks; *LN Turn*, the logarithm of turnover; *DISP*, the dispersion of analysts' forecasts defined as a standard deviation of forecasts normalized by its means; σ , the standard deviation of daily returns, and *Nasdaq*, the exchange constant equal to 1 for Nasdaq-traded stocks. The estimated are calculated following the Fama-McBeth procedure for data grouped at a monthly level and weighted with a precision level. T-statistics are adjusted with the Newey-West procedure and presented in parentheses. ** is significance at 1% level, * is significance at 5% level, [†] is significance at 10% level.

Table 2.5: The Regression Analysis, Sales

<i>Regressions of Price Impact on Explanatory Variables, Sales</i>							
<i>Regression Specifications</i>							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
const	9.29** (2.72)	7.18 [†] (1.89)	5.32 (1.68)	4.42 (1.37)	4.80 (1.19)	5.41 (1.50)	-0.33 (-0.07)
LN <i>Cap</i>	1.70 [†] (1.79)	2.26 [†] (1.91)	1.74 [†] (1.85)	1.89 [†] (1.82)	2.86* (2.30)	2.84* (2.28)	3.03* (2.43)
SPREAD		4.26 [†] (1.95)			5.73* (2.31)		7.74** (3.16)
ANum						-0.20 (-1.41)	
LN <i>Turn</i>			1.17 (1.54)			1.28 [†] (1.79)	1.47 [†] (1.99)
DISP					2.17 [†] (1.83)		2.09 [†] (1.75)
σ				100.60 (1.42)		16.77 (0.32)	-48.51 (-0.93)
Nasdaq	2.80 (1.64)	3.11 [†] (1.77)	0.62 (0.64)	1.75 (1.16)	3.72 (1.98)	1.00 (0.68)	1.25 (0.90)
Avg R^2	0.01	0.02	0.02	0.02	0.02	0.02	0.02
# Obs							87,132

Table 2.5 shows the regression analysis of the price impact coefficients, defined in (2.4), on various stocks' characteristics for the sample of sales. These characteristics are LN *Cap*, the logarithm of capitalization (in billions); *SPREAD*, the percentage spread (in percents); ANUM, the number of analysts following stocks; LN *Turn*, the logarithm of turnover; *DISP*, the dispersion of analysts' forecasts defined as a standard deviation of forecasts normalized by its means; σ , the standard deviation of daily returns, and *Nasdaq*, the exchange constant equal to 1 for Nasdaq-traded stocks. The estimated are calculated following the Fama-McBeth procedure for data grouped at a monthly level and weighted with a precision level. T-statistics are adjusted with the Newey-West procedure and presented in parentheses. **is significance at 1% level, *is significance at 5% level, [†]is significance at 10% level.

Table 2.6: Price Impact Dynamics

Panel A: NYSE/AMEX Stocks

		Cap Quintiles					
		1 (Small)	2	3	4	5 (Large)	All
BUY	Mean	-12.67	-2.24	-9.73	-62.37	-3.36	-18.14
	p-val	(0.10)	(0.72)	(0.19)	(0.00)	(0.76)	(0.00)
	Median	-1.05	-3.64	-2.64	-16.80	10.50	-3.35
	p-val	(0.13)	(0.03)	(0.04)	(0.00)	(0.00)	(0.00)
SELL	Mean	1.58	20.06	12.60	38.12	30.25	24.84
	p-val	(0.85)	(0.01)	(0.09)	(0.00)	(0.00)	(0.00)
	Median	2.94	6.73	8.21	11.21	20.91	8.97
	p-val	(0.00)	(0.00)	(0.00)	(0.00)	(0.93)	(0.00)

Panel B: Nasdaq Stocks

		Cap Quintiles					
		1 (Small)	2	3	4	5 (Large)	All
BUY	Mean	-12.55	-39.90	-52.74	-84.59	-16.93	-39.33
	p-val	(0.03)	(0.00)	(0.00)	(0.00)	(0.59)	(0.00)
	Median	-0.47	-3.52	-8.98	-43.53	0.88	-3.43
	p-val	(0.05)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
SELL	Mean	-8.38	2.18	24.39	67.27	72.75	19.32
	p-val	(0.17)	(0.75)	(0.01)	(0.00)	(0.00)	(0.00)
	Median	2.46	9.18	11.55	32.96	66.03	8.69
	p-val	(0.05)	(0.00)	(0.00)	(0.00)	(0.87)	(0.00)

Panel C: All Stocks, Different Time Periods

	BUY				SELL			
	Mean	p-val	Median	p-val	Mean	p-val	Median	p-val
2001-2003	-10.16	(0.06)	-0.93	(0.00)	52.16	(0.00)	10.54	(0.00)
2003-2005	-39.32	(0.00)	-4.94	(0.00)	7.83	(0.04)	8.27	(0.00)

Table 2.6 shows the means and medians of pairwise differences between the average price impacts coefficients in the first and the second halves, $\Delta = \bar{\lambda}_{PI,1} - \bar{\lambda}_{PI,2}$, along with p-values of their t-tests and Wilcoxon signed-rank tests. The price impact coefficient of individual trades are defined in (2.4), but instead of P_{bch} the close price in the previous to a trade day is used. 10% of bottom and top observations are winsorized. P-values are presented in parentheses. Data is presented for NYSE/AMEX and Nasdaq (Panel A and B). The sample ranges from January 2001 to December 2005. In Panel C, results for periods Jan 2001 through June 2003 and July 2003 through Dec 2005 are presented.

Table 2.7: Price Impact Functions

<i>Panel A: Purchases, Price Impact = $\beta (X/1\%ADV)^\gamma + \epsilon$</i>							
	Parameter β			Parameter γ			R^2
	$\hat{\beta}$	SE	95% CI	$\hat{\gamma}$	SE	95% CI	
All Stocks	0.14	0.02	[0.09..0.19]	0.31	0.05	[0.21..0.40]	0.04
Cap Quintile							
1 (Small)	0.13	0.03	[0.06..0.20]	0.35	0.08	[0.20..0.51]	0.09
2	0.10	0.03	[0.04..0.17]	0.42	0.10	[0.22..0.62]	0.05
3	0.11	0.03	[0.05..0.18]	0.40	0.10	[0.19..0.60]	0.03
4	0.15	0.03	[0.09..0.22]	0.25	0.08	[0.09..0.40]	0.04
5 (Large)	0.16	0.03	[0.10..0.21]	0.23	0.07	[0.08..0.38]	0.03
<i>Panel B: Sales, Price Impact = $\beta (X/1\%ADV)^\gamma + \epsilon$</i>							
	Parameter β			Parameter γ			R^2
	$\hat{\beta}$	SE	95% CI	$\hat{\gamma}$	SE	95% CI	
All Stocks	0.09	0.02	[0.05..0.12]	0.47	0.08	[0.32..0.62]	0.03
Cap Quintile							
1 (Small)	0.11	0.03	[0.05..0.17]	0.46	0.08	[0.29..0.62]	0.09
2	0.10	0.03	[0.04..0.16]	0.35	0.14	[0.06..0.64]	0.03
3	0.05	0.02	[0.01..0.09]	0.58	0.13	[0.32..0.84]	0.02
4	0.08	0.02	[0.03..0.13]	0.56	0.12	[0.31..0.80]	0.02
5 (Large)	0.12	0.03	[0.06..0.17]	0.55	0.11	[0.34..0.76]	0.02

Table 2.7 shows the calibration of price impact functions, Price Impact = $\beta (X/1\%ADV)^\gamma + \epsilon$, where X is the size of transition order, and ADV is the average trading daily volume. Price impact is defined as $\frac{1}{\sigma}(P_{exec}/P_{bch} - 1) \times \mathbb{I}_{BS}$, where P_{exec} is the execution price, P_{bch} is the benchmark price, σ is the standard deviation of daily returns in the previous month, and \mathbb{I}_{BS} is a buy/sell indicator. Point estimates, their standard errors, t-statistics and 95%-confident intervals are presented. All observations are clustered at monthly levels. Data is presented separately for transition purchases and sales (Panel A and B) as well as different capitalization quintiles. Capitalization thresholds are calculated based on NYSE stock capitalization quintiles. The sample ranges from January 2001 to December 2005.

Table 2.8: Temporary and Permanent Price Impact

<i>Panel A: Purchases, Price Impact = $\beta(X/1\%ADV)^\gamma + \epsilon$</i>								
Base Day	PI	Parameter β			Parameter γ			R^2
		$\hat{\beta}$	SE	95% CI	$\hat{\gamma}$	SE	95% CI	
t=+0	Perm	0.16	0.03	[0.10..0.22]	0.30	0.06	[0.18..0.41]	0.03
	Temp	-0.03	0.01	[-0.05..0.00]	0.19	0.23	[-0.27..0.65]	0.00
t=+1	Perm	0.18	0.04	[0.09..0.27]	0.13	0.07	[-0.02..0.27]	0.02
	Temp	-0.03	0.04	[-0.10..0.04]	-0.56	0.61	[-1.78..0.67]	0.00
t=+5	Perm	0.15	0.10	[-0.04..0.35]	0.13	0.18	[-0.23..0.50]	0.01
	Temp	0.01	0.09	[-0.18..0.19]	0.60	2.71	[-4.83..6.03]	0.00
t=+10	Perm	0.28	0.17	[-0.05..0.61]	-0.05	0.22	[-0.50..0.39]	0.01
	Temp	-0.12	0.18	[-0.49..0.25]	-0.35	0.75	[-1.85..1.15]	0.00

<i>Panel B: Sales, Price Impact = $\beta(X/1\%ADV)^\gamma + \epsilon$</i>								
Base Day	PI	Parameter β			Parameter γ			R^2
		$\hat{\beta}$	SE	95% CI	$\hat{\gamma}$	SE	95% CI	
t=+0	Perm	0.05	0.03	[-0.01..0.10]	0.59	0.19	[0.20..0.97]	0.01
	Temp	0.04	0.01	[0.01..0.07]	0.27	0.13	[0.01..0.54]	0.01
t=+1	Perm	0.01	0.02	[-0.04..0.05]	0.99	0.83	[-0.67..2.64]	0.00
	Temp	0.09	0.03	[0.04..0.15]	0.23	0.11	[0.00..0.46]	0.01
t=+5	Perm	-0.15	0.09	[-0.32..0.02]	-0.07	0.27	[-0.61..0.46]	0.00
	Temp	0.25	0.08	[0.10..0.40]	0.20	0.11	[-0.02..0.43]	0.02
t=+10	Perm	-0.28	0.15	[-0.58..0.02]	0.16	0.15	[-0.14..0.45]	0.01
	Temp	0.37	0.15	[0.08..0.67]	0.25	0.11	[0.03..0.47]	0.02

Table 2.8 shows the calibration of permanent and temporary price impact functions, $\text{Price Impact} = \beta(X/1\%ADV)^\gamma + \epsilon$, where X is the size of transition order, and ADV is the average trading daily volume. Permanent price impact is defined as $\frac{1}{\sigma}(P_{base}/P_{bch} - 1) \times \mathbb{I}_{BS}$, and temporary price impact is defined as $\frac{1}{\sigma}(P_{exec} - P_{base})/P_{bch} \times \mathbb{I}_{BS}$, where P_{base} is the close price for the "base" day, P_{bch} is the benchmark price, P_{exec} is the execution price, σ is the standard deviation of daily returns in the previous month, and \mathbb{I}_{BS} is a buy/sell indicator. Several "base" days are considered: the close at the last day of execution, the close at the next day and one or two weeks after execution. Point estimates, their standard errors, t-statistics and 95%-confident intervals are presented. All observations are clustered at monthly levels. Data is presented separately for transition purchases and sales (Panel A and B) as well as different capitalization quintiles. Capitalization thresholds are calculated based on NYSE stock capitalization quintiles. The sample ranges from January 2001 to December 2005.

Chapter 3

Optimal Trading Strategy and Supply/Demand Dynamics (joint work with Jiang Wang)

3.1 Introduction

It has been well documented that the supply/demand of a security in the market is not perfectly elastic.¹ The limited elasticity of supply/demand or liquidity can significantly affect how market participants trade, which in turn will influence security prices through the changes in their supply/demand.² Thus, to study how market participants trade is important to our understanding of how securities markets function, how liquidity is provided and consumed, and how it affects the behavior of security

¹See, for example, Holthausen, Leftwich and Mayers (1987, 1990), Shleifer (1986), Scholes (1972). For the more recent work, see also Greenwood (2004), Kaul, Mehrotra and Morck (2000), Wugler and Zhuravskaya (2002). There is also extensive theoretical work in justifying an imperfect demand/supply in securities market based on market frictions and asymmetric information. See, for example, Grossman and Miller (1998), Kyle (1985) and Vayanos (1999, 2001).

²Many empirical studies have shown that this is a problem confronted by institutional investors who need to execute large orders and often break up trades in order to manage the trading cost. See, for example, Chan and Lakonishok (1993, 1995, 1997), Keim and Madhavan (1995, 1997).

prices.³ In the paper, we approach this problem by focusing on the optimal strategy to execute a given order, leaving aside its underlying motive. This is also referred to as the optimal execution problem. We show that it is the dynamic properties of supply/demand such as its time evolution after trades, rather than its static properties such as the instantaneous price impact function, that are central to the cost of trading and the optimal strategy.

We consider a limit-order-book market, in which the supply/demand of a security is represented by the limit orders posted to the “book,” i.e., a trading system and trade occurs when buy and sell orders match. We propose a simple framework to describe the limit-order-book and how it evolves over time. By incorporating several salient features of the book documented empirically, we attempt to capture the dynamics of supply/demand a trader faces. We show that the optimal trading strategy crucially depends on how the limit-order book responds to a sequence of trades and it involves complex trading patterns including both discrete and continuous trades.

In particular, the optimal strategy consists of an initial discrete trade, followed by a sequence of continuous trades. The initial discrete trade is aimed at pushing the limit order book away from its steady state in order to attract new orders onto the book. The size of the initial trade is chosen to draw sufficient new orders at desirable prices. The subsequent continuous trades will then pick off the new orders and keep the inflow coming. A discrete trade finishes off any remaining order at the end of trading horizon when future demand/supply is no longer of concern. The combination of discrete and continuous trades for the optimal execution strategy is in sharp contrast to simple strategies of splitting a order into small trades as suggested in the literature. Moreover, we find that the optimal strategy and the cost saving depends primarily on the dynamic properties of supply/demand and is not

³For example, Kyle (1985) and Wang (1993) examine the behavior of traders with superior information and how it affects liquidity and asset prices and Vayanos (1999, 2001) considers the trading behavior of large traders with risk-sharing needs and its impact on market behavior.

very sensitive to the instantaneous price-impact function, which has been the main focus in previous work. Especially, the speed at which the limit order book rebuilds itself after being hit by a trade, which is also referred to as the resilience of the book, plays a critical role in determining the optimal execution strategy and the cost it saves.

Our predictions about optimal trading strategies lead to interesting implications about the behavior of trading volume, liquidity and security prices. For example, it suggests that the trading behavior of large institutional traders may contribute to the observed U-shaped patterns in intraday volume, volatility and bid-ask spread. It also suggests that these patterns can be closely related to institutional ownership and the resilience of the supply/demand of each security.

The problem of optimal execution takes the order to be executed as given. Ideally, we should consider both the optimal size of an order and its execution, taking into account the underlying motives to trade (e.g., return and risk, preferences and constraints) and the costs to execute trades.⁴ The diversity in trading motives makes it difficult to tackle such a problem as a general level. Given that in practice the execution of trades is often separated from the decisions on the trades, in this paper we focus on the execution problem as an important and integral part of the general problem of optimal trading behavior.

Several authors have studied the problem of optimal execution. For example, Bertsimas and Lo (1998) propose a linear price impact function and solve for the optimal execution strategy to minimize the expected cost of executing a given order. Almgren and Chriss (1999, 2000) include risk considerations in a similar setting using a mean-variance objective function.⁵ The framework adopted in these papers share

⁴For example, many authors have considered the problem of optimal portfolio choices in the presence of transactions costs, e.g., Constantinides (1986), Davis and Norman (1990), and Leland (2000).

⁵See also, Almgren (2003), Dubil (2002), Huberman and Stanzl (2005), Subramanian and Jarrow (2001), among others.

two main features. First, it uses a discrete-time setting so that the times to trade are fixed at given intervals. Second, it relies on price impact functions to describe how a sequence of trades affects prices at which trades are executed. A discrete-time setting is clearly undesirable for the execution problem because the timing of trades is an important choice variable and should be determined optimally. A natural way to address this issue would be to take a continuous-time limit of the discrete-time formulation. But such a limit leads to degenerate solutions with the simple price impact functions considered previously. In particular, Lo and Bertsimas (1998) consider the permanent price impact by assuming a static, linear impact function. As a result, the price impact of a sequence of trades depends only on their total size and is independent of their distribution over time. In this case, the execution cost becomes strategy independent in the continuous-time limit. Almgren and Chris (1999, 2000) and Huberman and Stanzl (2005) also allow temporary price impact, which depends on the pace of trades. Introducing temporary price impact adds a dynamic element to the price impact function by penalizing speedy trades. But it restricts the execution strategy to continuous trades in the continuous-time limit, which is in general sub-optimal.

The simple price impact functions used in previous work do not fully capture the intertemporal nature of supply/demand in the market. In particular, it limits the extent to which the allocation of trades over time, given their sizes, influences current and future supply/demand and the resulting execution cost. Yet, it is clear that how to allocate trades over time is at the heart of the problem. Thus, modelling the intertemporal properties of supply/demand is essential in analyzing the optimal execution strategy. Taking these considerations into account, our framework attempts to capture these intertemporal aspects of the supply/demand by directly modelling the liquidity dynamics in a limit-order-book market. We show that when the timing of trades is chosen optimally, the optimal execution strategy differs significantly from

those suggested in earlier work and yields substantial cost reduction. It involves a mixture of discrete and continuous trades. Moreover, the characteristics of the optimal execution strategy are mostly determined by the dynamic properties of the supply/demand rather than its static properties as described by the price impact function.

In modelling the supply/demand dynamics, we choose the limit-order-book market mainly for concreteness. Our description of the limit-order-book dynamics relies on an extensive empirical literature.⁶ We choose the shape of the limit-order-book to yield a linear price-impact function, which is widely adopted in previous work. More importantly, we explicitly model the resilience of the book, which several empirical studies document as an important property of the book (see, e.g., Biais, Hillion and Spatt (1995) and Harris (1990)).

Our analysis is partial equilibrium in nature. We take the dynamics of the limit-order-book as given and do not attempt to provide an equilibrium justification for the specific limit-order-book dynamics used in the paper. Nonetheless, it is worth pointing out that in addition to the empirical motivation mentioned above, the supply/demand dynamics we consider is also consistent with several equilibrium models (e.g., Kyle (1985) and Vayanos (1999, 2001)). In particular, Vayanos (2001) analyzes the optimal trading behavior of a large trader who trades with a set of competitive market makers for risk sharing. He shows that the price impact of the large trader is linear in his trades and the supply/demand by the market makers exhibits certain form of resilience. Although his analysis relies on specific assumptions on traders' trading motives and preferences, it does provide additional theoretical basis for the qualitative properties of supply/demand dynamics we consider.

⁶See, for example, Ahn, Bae and Chan (2000) for a study on the Hong Kong Stock Exchange, Biais, Hillion and Spatt (1995) on the Paris Bourse, Chung, Van Ness and Van Ness (1999) on the NYSE, Hasbrouck and Saar (2002) on the Island ECN, Hollfield, Miller and Sandas (2003) on the Stockholm Stock Exchange and Griffiths, Smith, Turnbull and White (2000) on the Toronto Stock Exchange.

Several authors have also considered equilibrium models for the limit-order-book market, including Foucault, Kadan and Kandel (2004), Goettler, Parlour and Rajan (2005) and Rosu (2005). For tractability, the set of order-placement strategies allowed in studies are severely limited to obtain an equilibrium. For example, Foucault, Kadan and Kandel (2004) and Rosu (2005) only allow orders of a fixed size. Goettler, Parlour and Rajan (2005) focus on one-shot strategies. These simplifications are helpful when we are interested in certain properties of the book, but quite restrictive when analyzing the optimal trading strategy. A more general and realistic equilibrium model must allow general strategies. From this perspective, our analysis, namely to solve the optimal execution strategy under general supply/demand dynamics, is an unavoidable step in this direction.

The rest of the paper is organized as follows. Section 3.2 states the optimal execution problem. Section 3.3 introduces the limit-order-book market and a model for the limit order book dynamics. In Section 3.4, we show that the conventional setting in previous work can be viewed as a special case of our limit-order-book framework. We also explain why the stringent assumptions in the conventional setting lead to its undesirable properties. In Section 3.5, we solve the discrete-time version of the problem within our framework. We also consider its continuous-time limit and show that it is economically sensible and properly behaved. Section 3.6 provides the solution of the optimal execution problem in the continuous-time setting. In Section 3.7, we analyze the properties of the optimal execution strategy and their dependence on the dynamics of the limit order book. We also compare it with the strategy predicted by the conventional setting. In addition, we examine the empirical implications of the optimal execution strategy. Section 3.8 discusses possible extensions of the model. Section 3.9 concludes. All proofs are given in the appendix.

3.2 Statement of the Problem

The problem we are interested in is how a trader optimally executes a given order. To fix ideas, let us assume that the trader has to buy X_0 units of a security over a fixed time period $[0, T]$. Suppose that the trader ought to complete the order in $N + 1$ trades at times t_0, t_1, \dots, t_N , where $t_0 = 0$ and $t_N = T$. Let x_{t_n} denote the trade size for the trade at t_n . We then have

$$\sum_{n=0}^N x_{t_n} = X_0. \quad (3.1)$$

A strategy to execute the order is given by the number of trades, $N + 1$, the set of times to trade, $\{0 \leq t_0, t_1, \dots, t_{N-1}, t_N \leq T\}$ and trade sizes $\{x_{t_0}, x_{t_1}, \dots, x_{t_N} : x_{t_n} \geq 0 \forall n \text{ and } (3.1)\}$. Let Θ_D denote the set of these strategies:

$$\Theta_D = \left\{ \{x_{t_0}, x_{t_1}, \dots, x_{t_N}\} : 0 \leq t_0, t_1, \dots, t_N \leq T; x_{t_n} \geq 0 \forall n; \sum_{n=0}^N x_{t_n} = X_0 \right\}. \quad (3.2)$$

Here, we have assumed that the strategy set consists of execution strategies with finite number of trades at discrete times. This is done merely for easy comparison with previous work. Later we will expand the strategy set to allow uncountable number of trades over time.

Let \bar{P}_n denote the average execution price for trade x_{t_n} . We assume that the trader chooses his execution strategy to minimize the expected total cost of his purchase:

$$\min_{x \in \Theta_D} E_0 \left[\sum_{n=0}^N \bar{P}_n x_n \right]. \quad (3.3)$$

For simplicity, we have assumed that the trading horizon T is fixed and the trader is risk-neutral who cares only about the expected value not the uncertainty of the

total cost. We will incorporate risk considerations later (in Section 3.8), which also allows us to endogenize the trading horizon.

The solution to the trader's optimal execution strategy crucially depends on how his trades impact the prices. It is important to recognize that the price impact of a trade has two key dimensions. First, it changes the security's *current* supply/demand. For example, after a purchase of x units of the security at the current price of \bar{P} , the remaining supply of the security at \bar{P} in general decreases. Second, a change in current supply/demand can lead to evolutions in future supply/demand, which will affect the costs for future trades. In other words, the price impact is determined by the full dynamics of supply/demand in response to a trade. Thus, in order to fully specify the optimal execution problem, we need to model the supply/demand dynamics.

3.3 Limit Order Book and Supply/Demand Dynamics

The actual supply/demand of a security in the market place and its dynamics depend on the actual trading process. From market to market, the trading process varies significantly, ranging from a specialist market or a dealer market to a centralized electronic market with a limit order book. In this paper, we consider the limit-order-book market, which is arguably the closest, at least in form, to the text-book definition of a centralized market.

3.3.1 Limit Order Book (LOB)

A limit order is an order to trade a certain amount of a security at a given price. In a market operated through a limit-order-book, thereafter LOB for short, traders post

their supply/demand in the form of limit orders to a electronic trading system.⁷ A trade occurs when an order, say a buy order, enters the system at the price of an opposite order on the book, in this case a sell order, at the same price. The collection of all limit orders posted can be viewed as the total demand and supply in the market.

Let $q_A(P)$ be the density of limit orders to sell at price P and $q_B(P)$ the density of limit orders to buy at price P . The amount of sell orders in a small price interval $[P, P+dP)$ is $q_A(P)(P+dP)$. Typically, we have

$$q_A(P) = \begin{cases} +, & P \geq A \\ 0, & P < A \end{cases} \quad \text{and} \quad q_B(P) = \begin{cases} 0, & P > B \\ +, & P \leq B \end{cases}$$

where $A \geq B$ are the best ask and bid prices, respectively. We define

$$V = (A+B)/2, \quad s = A-B \tag{3.4}$$

where V is the mid-quote price and s is the bid-ask spread. Then, $A = V + s/2$ and $B = V - s/2$. Because we are considering the execution of a large buy order, we will focus on the upper half of the LOB and simply drop the subscript A .

In order to model the execution cost for a large order, we need to specify the initial LOB and how it evolves after been hit by a series of buy trades. Let the LOB (the upper half of it) at time t be $q(P; F_t; Z_t; t)$, where F_t denotes the fundamental value of the security and Z_t represents the set of state variables that may affect the LOB such as past trades. We will consider a simple model for the LOB, to capture its dynamic nature and to illustrate their importance in analyzing the optimal execution problem, and return to its extensions to better fit the empirical LOB dynamics later. In

⁷The number of exchanges adopting an electronic trading system with posted orders has been increasing. Examples include NYSE's OpenBook program, Nasdaq's SuperMontage, Toronto Stock Exchange, Vancouver Stock Exchange, Euronext (Paris, Amsterdam, Brussels), London Stock Exchange, Copenhagen Stock Exchange, Deutsche Borse, and Electronic Communication Networks such as Island. For the fixed income market, there are, for example, eSpeed, Euro MTS, BondLink and BondNet. Examples for the derivatives market include Eurex, Globex, and Matif.

particular, we assume that the fundamental value the security F_t follows a Brownian motion, reflecting the fact that in absence of any trades, the mid-quote price may change due to news about the fundamental value of the security. Thus, $V_t = F_t$ in absence of any trades and the LOB maintains the same shape except that the mid-point, V_t , is changing with F_t . In addition, we assume that the only set of relevant state variables is the history of past trades, which we denote by $x_{[0, t]}$, i.e., $Z_t = x_{[0, t]}$.

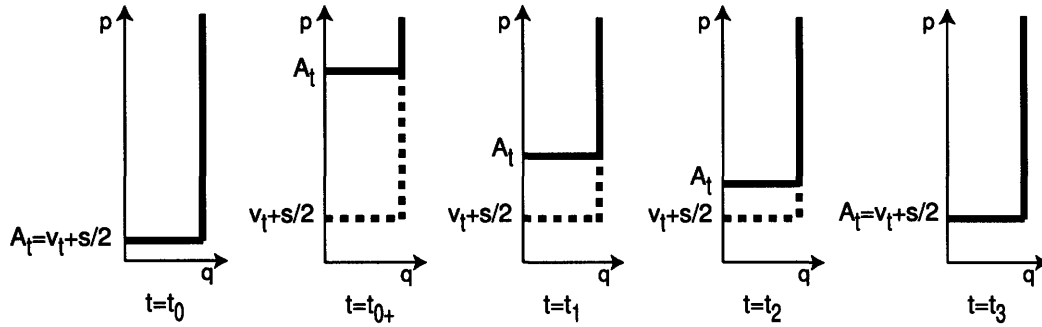


Figure 3-1: The limit order book and its dynamics.

This figure illustrates how the sell side of limit order book evolves over time in response to a sale trade. Before the trade at time $t_0 = 0$, the limit order book is full at the ask price $A_0 = V_0 + s/2$, which is shown in the first panel from the left. The trade of size x_0 at $t = 0$ “eats off” the orders on the book with lowest prices and pushes the ask price up to $A_{0+} = (F_0 + s/2) + x_0/q$, as shown in the second panel. During the following periods, new orders will arrive at the ask price A_t , which fill up the book and lower the ask price until it converges to its new steady state $A_t = F_t + \lambda x_0 + s/2$, as shown in the last panel on the right. For clarity, we assume that there are no fundamental shocks.

At time 0, we assume that the mid-quote is $V_0 = F_0$ and LOB has a simple block shape

$$q_0(P) \equiv q(P; F_0; 0; 0) = q 1_{\{P \geq A_0\}}$$

where and $A_0 = F_0 + s/2$ is the initial ask price and $1_{\{z \geq a\}}$ is an indicator function:

$$1_{\{z \geq a\}} = \begin{cases} 1, & z \geq a \\ 0, & z < a \end{cases}$$

In other words, q_0 is a step function of P with a jump from zero to q at the ask price $A_0 = V_0 + s/2 = F_0 + s/2$. The first panel in Figure 3-1 shows the shape of the book at time 0.

Now we consider a trade of size x_0 at $t = 0$. The trade will “eat off” all the sell orders with prices from $F_0 + s/2$ up to A_{0+} , where A_{0+} is given by

$$\int_{F_0 + s/2}^{A_{0+}} q dP = x_0$$

or $A_{0+} = F_0 + s/2 + x_0/q$. The average execution price is $\bar{P} = F_0 + s/2 + x_0/(2q)$, which is linear in the size of the trade. Thus, the shape of the LOB we propose is consistent with the linear price impact function assumed in previous work. This is also the main reason we adopted it here.

Right after the trade, the limit order book becomes:

$$q_{0+}(P) \equiv q(P; F_0; Z_{0+}; 0_+) = q 1_{\{P \geq A_{0+}\}}.$$

$A_{0+} = F_0 + s/2 + x_0/q$ is the new ask price. Orders at prices below $A_{0+} = (F_0 + s/2) + x_0/q$ have all been executed. The book is left with limit sell orders at prices above (including) A_{0+} . The second panel of Figure 3-1 plots the limit order book right after the trade.

3.3.2 Limit Order Book Dynamics

What we have to specify next is how the LOB evolves over time after being hit by a trade. Effectively, this amounts to describing how the new sell orders arrive to fill in the gap in the LOB eaten away by the trade. First, we need to specify the impact of the trade on the mid-quote price, which will determine the prices of the new orders. In general, the mid-quote price will be shifted up by the trade. We assume that the

shift in the mid-quote price will be linear in the size of the total trade. That is,

$$V_{0+} = F_0 + \lambda x_0$$

where $0 \leq \lambda \leq 1/q$ and λx_0 gives the permanent price impact the trade x_0 has. If there are no more trades after the initial trade x_0 at $t = 0$ and there are no shocks to the fundamental, the limit order book will eventually converge to its new steady state

$$q_t(P) = q 1_{\{P \geq A_t\}}$$

where t is sufficiently large, $A_t = V_t + s/2$ and $V_t = F_0 + \lambda x_0$. Next we need to specify how the limit order book converges to its steady-state. Note that right after the trade, the ask price is $A_{0+} = F_0 + s/2 + x_0/q$, while in the steady-state it is $A_\infty = F_0 + s/2 + \lambda x_0$. The difference between the two is $A_{0+} - A_\infty = x_0(1/q - \lambda)$. We assume that the limit order book converges to its steady state exponentially:

$$q_t(P) = q 1_{\{P \geq A_t\}} \tag{3.5}$$

where

$$A_t = V_t + s/2 + x_0 \kappa e^{-\rho t}, \quad \kappa = 1/q - \lambda \tag{3.6}$$

and $\rho \geq 0$ gives the convergence speed and $V_t = V_{0+}$ in absence of new trades and changes in F_t , which measures the “resilience” of the LOB.⁸

Equations (3.5) and (3.6) imply that after a trade x_0 , the new sell orders will start coming in at the new ask price A_t at the rate of $\rho q(A_t - V_t - s/2)$. For convenience,

⁸A number of empirical studies documented the existence of the resiliency of LOB. See, for example, Biais, Hillion and Spatt (1995), Hamao and Hasbrouck (1995), Coppejans, Domowitz and Madhavan (2001), and Ranaldo (2004). Moreover, the idea of liquidity being exhausted by a trade and then replenished as traders take advantage of profit opportunities is behind most of the dynamic equilibrium frameworks of LOB. See, for example, Foucault, Kadan and Kandel (2004), Goettler, Parlour and Rajan (2005), Parlour (1998), Rosu (2005).

we define

$$D_t = A_t - V_t - s/2 \quad (3.7)$$

which stands for the deviation of current ask price A_t from its steady state level $V_t + s/2$.

We can easily extend the LOB dynamics described above for a single trade to allow multiple trades and shocks to the fundamental value. Let $n(t)$ denote the number of trades during interval $[0, t)$, $t_1, \dots, t_{n(t)}$ the times for these trades, and x_{t_i} their sizes, respectively. Let X_t be the remaining order to be executed at time t , before trading at t . We have

$$X_t = X_0 - \sum_{t_n < t} x_{t_n}. \quad (3.8)$$

with $X_{T_+} = 0$. Let

$$V_t = F_t + \lambda(X_0 - X_t) = F_t + \lambda \sum_{i=0}^{n(t)} x_{t_i} \quad (3.9)$$

where $X_0 - X_t$ is the total amount of purchase during $[0, t)$. The ask price at any time t is

$$A_t = V_t + s/2 + \sum_{i=0}^{n(t)} x_{t_i} \kappa e^{-\rho(t-t_i)} \quad (3.10)$$

and the limit order book at any time t is given by (3.5). Panels 2 to 5 in Figure 3-1 illustrates the time evolution of the LOB after a trade. We can easily extend the above description to include sell orders which may occur in the mean time and can shift the mid-quote V_t . If not predictable, they are not important to our analysis. Thus, we omit them here.

Before we go ahead with the LOB dynamics and examines its implications on execution strategy, several comments are in order. We note that the simple LOB dynamics described above is assumed to be given, without further economic justification. Presumably, it is driven by the optimizing behavior of those who submit the

orders and thus provide liquidity to the market.⁹ In addition, the LOB dynamics may be further affected by the strategic interactions among market participants (see, for example, Vayanos (1999, 2001)). To describe the actual LOB dynamics will require an equilibrium framework. However, for any equilibrium analysis, we first need to study the optimal trading strategy under general LOB dynamics. Thus, our analysis can be viewed as a necessary step along this direction. Clearly, our setting is general enough for this purpose. It should be emphasized that the goal of this paper is to demonstrate the importance of supply/demand dynamics in determining the optimal trading strategy. The specific model we use mainly helps us to make the point in a simple and revealing way. Its partial equilibrium nature as well as its quantitative features are not crucial to our main conclusions.

3.3.3 Execution Cost

Given the above description of the LOB dynamics, we can now describe the total cost of an execution strategy for a given order X_0 . Let x_{t_n} denote the trade at time t_n and A_{t_n} the ask price at t_n prior to the trade. The evolution of ask price A_t as given in (3.10) is not continuous. For clarity, A_t always denotes the left limit of A_t , $A_t = \lim_{s \rightarrow t^-} A_s$, i.e., the ask price before the trade at t . The same convention is followed for V_t . The cost for x_{t_n} is then

$$c(x_{t_n}) = \int_0^{x_{t_n}} P_{t_n}(x) dx \quad (3.11)$$

⁹Several recent work show that traders do use the rich information revealed by the books when deciding on their order submission strategies. See, for example, Cao, Hansch, and Wang (2003), Harris and Panchapagesan (2005), Bloomfield, O'Hara, and Saar (2003), Ranaldo (2004), among others. Many authors have developed models for optimal order placement in markets with limit orders. See Foucault (1999), Foucault, Kadan, and Kandel (2001), Glosten (1994), Goettler, Parlou and Rajan (2005), Harris(1998), Parlour (1998), Parlour and Seppi(2003), Rock (1996), Rosu (2005), Sandas (2001), and Seppi (1997). However, as mentioned earlier, most of these models impose strong restrictions on the strategies allowed.

where $P_t(x)$ is defined by equation

$$x = \int_{A_t}^{P_t(x)} q_t(P) dP. \quad (3.12)$$

For block-shaped LOB given in (3.5), we have

$$P_t(x) = A_t + x/q$$

and

$$c(x_{t_n}) = [A_{t_n} + x_{t_n}/(2q)] x_{t_n}. \quad (3.13)$$

The total cost is $\sum_{n=0}^N c(x_{t_n})$. Thus, the the optimal execution problem (3.3) now reduces to

$$\min_{x \in \Theta_D} E_0 \left[\sum_{n=0}^N [A_{t_n} + x_{t_n}/(2q)] x_{t_n} \right] \quad (3.14)$$

under our dynamics of the limit order book given in (3.9) and (3.10).

3.4 Conventional Models As A Special Case

Previous work on optimal execution strategy uses a discrete-time setting with fixed time intervals and relies on a specific price-impact function to describe supply/demand (e.g., Bertsimas and Lo (1998) and Almgren and Chriss (1999, 2000)). Such a setting, however, avoids the question of optimal trading times. In this section, we briefly describe the setting used in previous work and its limitations. We then show that the conventional setting can be viewed as a special case of our framework with specific restrictions on the LOB dynamics. We further point out why these restrictions are unrealistic when the timing of trades is determined optimally.

3.4.1 Conventional Setup

We first consider the setup proposed by Bertsimas and Lo (1998). We adopt a simple version of their framework which captures the basic features of the models used in earlier work.

In a discrete-time setting, the trader trades at fixed time intervals, $n\tau$, where $\tau = T/N$ and $n = 0, 1, \dots, N$ are given. Each trade will have an impact on the price, which will affect the total cost of the trade and future trades. Most models assume a linear price-impact function of the following form:

$$\bar{P}_n = \bar{P}_{n-1} + \lambda x_n + u_n = (F_n + s/2) + \lambda \sum_{i=0}^n x_i \quad (3.15)$$

where the subscript n denotes the n -th trade at $t_n = n\tau$, \bar{P}_n is the average price at which trade x_n is executed with $\bar{P}_{0-} = F_0 + s/2$, λ is the price impact coefficient and u_n is i.i.d. random variable, with a mean of zero and a variance of $\sigma^2\tau$.¹⁰ In the second equation, we have set $F_n = F_0 + \sum_{i=0}^n u_i$. Clearly, λ captures the permanent price impact a trade has. The trader who has to execute an order of size X_0 solves the following problem:

$$\min_{\{x_0, x_1, \dots, x_N\}} \mathbb{E}_0 \left[\sum_{n=0}^N \bar{P}_n x_n \right] = (F_0 + s/2)X_0 + \lambda \sum_{n=0}^N X_n (X_{n+1} - X_n). \quad (3.16)$$

where \bar{P}_n is defined in (3.15) and X_n is a number of shares left to be acquired at time t_n (before trade x_{t_n}) with $X_{N+1} = 0$.

As Bertsimas and Lo (1998) show, given that the objective function is quadratic in x_n , it is optimal for the trader to split his order into small trades of equal sizes and

¹⁰Huberman and Stanzl (2004) have argued that in the absence of quasi-arbitrage, permanent price-impact functions must be linear.

execute them at regular intervals over the fixed period of time:

$$x_n = \frac{X_0}{N+1} \tag{3.17}$$

where $n = 0, 1, \dots, N$.¹¹

3.4.2 The Continuous-Time Limit

Although the discrete-time setting with a linear price impact function gives a simple and intuitive solution, it leaves a key question unanswered, namely, what determines the time-interval between trades. An intuitive way to address this question is to take the continuous-time limit of the discrete-time solution, i.e., to let N goes to infinity. However, as Huberman and Stanzl (2005) point out, the solution to the discrete-time model (3.16) does not have a well-defined continuous-time limit. In fact, as $N \rightarrow \infty$, the cost of the trades as given in (3.16) approaches the following limit:

$$(F_0 + s/2)X_0 + (\lambda/2)X_0^2$$

which is strategy-independent. Thus, for a risk-neutral trader, the execution cost with continuous trading is a fixed number and any continuous strategy is as good as another. Therefore, the discrete-time model as described above does not have a well-behaved continuous-time limit.¹² For example, without increasing the cost the trader can choose to trade intensely at the very beginning and complete the whole order in an arbitrarily small period. If the trader becomes slightly risk-aversion, he will choose to finish all the trades right at the beginning, irrespective of their price

¹¹If the trader is risk averse, he will trade more aggressively at the beginning, trying to avoid the uncertainty in execution cost in later periods.

¹²In taking the continuous-time limit, we have held λ constant. This is, of course, unrealistic: for different τ , λ can be different. However, the problem remains as long as λ has a finite limit when $\tau \rightarrow 0$.

impact.¹³ Such a situation is clearly undesirable and economically unreasonable.

This problem has led several authors to propose different modifications to the conventional setting. He and Mamaysky (2001), for example, directly formulate the problem in continuous-time and impose fixed transaction costs to rule out any continuous trading strategies. Similar to the more general price impact function considered by Almgren and Chriss (1999, 2000), Huberman and Stanzl (2005) proposes a temporary price impact of a particular form to penalize high-intensity continuous trading. Both of these modifications limit us to a subset of feasible strategies, which is in general sub-optimal. Given its closeness to our paper, we now briefly discuss the modification with temporary price impact.

3.4.3 Temporary Price Impact

Almgren and Chriss (1999, 2000) include a temporary component in the price impact function, which can in general depend on the trading interval τ . The temporary price impact gives additional flexibility in dealing with the continuous-time limit of the problem. In particular, they specify the following dynamics for the execution prices of trades:

$$\hat{P}_n = \bar{P}_n + G(x_n/\tau) \quad (3.18)$$

where \bar{P}_n is the same as given in (3.15), $\tau = T/N$ is the time between trades, and $G(\cdot)$ describes a temporary price impact, which reflects temporary price deviations from “equilibrium” caused by trading. With $G(0) = 0$ and $G'(\cdot) > 0$, the temporary

¹³As $N \rightarrow \infty$, the objective function to be minimized for a risk-averse trader with a mean-variance preference approaches the following limit

$$C(x_{[0, T]}) = \mathbb{E} \left[\int_0^T P_t dX_t \right] + \frac{1}{2} a \text{Var} \left[\int_0^T P_t dX_t \right] = (F_0 + s/2)X_0 + (\lambda/2)X_0^2 + \frac{1}{2} a \sigma^2 \int_0^T X_t^2 dt$$

where $a > 0$ is the risk-aversion coefficient and σ is the price volatility of the security. The trader cares not only about the expected execution cost but also its variance, which is given by the last term. Only the variance of the execution cost depends on the strategy. It is easy to see that the optimal strategy is to choose an L-shaped profile for the trades, i.e., to trade with infinite speed at the beginning, which leads to a value of zero for the variance term in the cost function.

price impact penalizes high trading volume per unit of time, x_n/τ . Using a linear form for $G(\cdot)$, $G(z) = \theta z$, it is easy to show that as N goes to infinity the expected execution cost approaches to

$$(F_0 + s/2)X_0 + (\lambda/2)X_0^2 + \theta \int_0^T \left(\frac{dX_t}{dt} \right)^2 dt$$

(see, e.g., Grinold and Kahn (2000) and Huberman and Stanzl (2005)). Clearly, with the temporary price impact, the optimal execution strategy has a continuous-time limit. In fact, it is very similar to its discrete-time counterpart: It is deterministic and the trade intensity, defined by the limit of x_n/τ , is constant over time.¹⁴

The temporary price impact reflects an important aspect of the market, the difference between short-term and long-term supply/demand. If a trader speeds up his buy trades, as he can do in the continuous-time limit, he will deplete the short-term supply and increase the immediate cost for additional trades. As more time is allowed between trades, supply will gradually recover. However, as a heuristic modification, the temporary price impact does not provide an accurate and complete description of the supply/demand dynamics, which leads to several drawbacks. First, the temporary price impact function in the form considered in Almgren and Chriss (2000) and Huberman and Stanzl (2005) rules out the possibility of discrete trades. This is not only artificial but also undesirable. As we show later, in general the optimal execution strategy does involve both discrete and continuous trades. Moreover, introducing the temporary price impact does not capture the full dynamics of supply/demand.¹⁵ Also, simply specifying a particular form for the temporary price impact function says little about the underlying economic factors that determine it.

¹⁴If the trader is risk-averse with a mean-variance preference, the optimal execution strategy has a decreasing trading intensity over time. See Almgren and Chriss (2000) and Huberman and Stanzl (2005).

¹⁵For example, two sets of trades close to each other in time versus far apart will generate different supply/demand dynamics, while in Huberman and Stanzl (2005) they lead to the same dynamics.

3.4.4 A Special Case of Our Framework

In the conventional setting, the supply/demand of a security is described by a price impact function at fixed times. This is inadequate when we need to determine the optimal timing of the execution strategy. We show in Section 3.3, using a simple limit order book framework, that the supply/demand is an intertemporal object which exhibits rich dynamics. The simple price impact function, even with the modification proposed by Almgren and Chriss (1999, 2000) and Humberman and Stanzl (2005), misses important intertemporal aspects of the supply/demand that are crucial to the determination of optimal execution strategy.

We can see the limitations of the conventional model by considering it as a special case of our general framework. Indeed, we can specify the parameters in the LOB framework so that it will be equivalent to the conventional setting. First, we set the trading times at fixed intervals: $t_n = n\tau$, $n = 0, 1, \dots, N$. Next, we make the following assumptions on the LOB dynamics as described in (3.5) and (3.9):

$$q = 1/(2\lambda), \quad \lambda = \lambda, \quad \rho = \infty \quad (3.19)$$

where the second equation simply states that the price impact coefficient in the LOB framework is set to be equal to its counterpart in the conventional setting. These restrictions imply the following dynamics for the LOB. As it follows from (3.10), after the trade x_n at t_n ($t_n = n\tau$) the ask price A_{t_n} jumps from $V_{t_n} + s/2$ to $V_{t_n} + s/2 + 2\lambda x_n$. Over the next period, it comes all the way down to the new steady state level of $V_{t_n} + s/2 + \lambda x_n$ (assuming no fundamental shocks from t_n to t_{n+1}). Thus, the dynamics of ask price A_{t_n} is equivalent to dynamics of \bar{P}_{t_n} in (3.15).

For the parameters specified in (3.19), the cost for trade x_{t_n} , which can be calcu-

lated as $c(x_{t_n}) = [A_{t_n} + x_{t_n}/(2q)] x_{t_n}$, becomes

$$c(x_{t_n}) = [F_{t_n} + s/2 + \lambda(X_0 - X_{t_n}) + \lambda x_{t_n}] x_{t_n}$$

which is the same as the trading cost in the conventional model (3.16). Thus, the conventional model is a special case of LOB framework for parameters in (3.19).

The main restrictive assumption we have to make to obtain the conventional setup is that $\rho = \infty$ and the limit order book always converges to its steady state before the next trading time. This is not crucial if the time between trades is held fixed. But if the time between trades is allowed to shrink, this assumption becomes unrealistic. It takes time for the new limit orders to come in to fill up the book again. The shape of the limit order book after a trade depends on the flow of new orders as well as the time elapsed. As the time between trades shrinks to zero, the assumption of infinite recovery speed becomes less reasonable and it gives rise to the problems in the continuous-time limit of the conventional model.

3.5 Discrete-Time Solution

We now return to our general framework and solve the model for the optimal execution strategy when trading times are fixed, as in the conventional model. We then show that in contrast to the conventional setting, our framework is robust for studying convergence behavior as time between trades goes to zero. Taking the continuous-time limit we examine the resulting optimal execution strategy which turns out to include both discrete and continuous trading.

Suppose that trade times are fixed at $t_n = n\tau$, where $\tau = T/N$ and $n = 0, 1, \dots, N$. We consider the corresponding strategies $x_{[0, T]} = \{x_0, x_1, \dots, x_n\}$ within the strategy set Θ_D defined in Section 3.2. The optimal execution problem, defined in (3.3), now

reduces to

$$\begin{aligned}
J_0 = & \min_{\{x_0, \dots, x_N\}} \mathbb{E}_0 \left[\sum_{n=0}^N [A_{t_n} + x_n/(2q)] x_n \right] \\
& \text{s.t.} \quad A_{t_n} = F_{t_n} + \lambda(X_0 - X_{t_n}) + s/2 + \sum_{i=0}^{n-1} x_i \kappa e^{-\rho\tau(n-i)}
\end{aligned} \tag{3.20}$$

where F_t follows a random walk. This problem can be solved using dynamic programming. We have the following result:

Proposition 1 *The solution to the optimal execution problem (3.20) is*

$$x_n = -\frac{1}{2}\delta_{n+1} [D_{t_n}(1 - \beta_{n+1}e^{-\rho\tau} + 2\kappa\gamma_{n+1}e^{-2\rho\tau}) - X_{t_n}(\lambda + 2\alpha_{n+1} - \beta_{n+1}\kappa e^{-\rho\tau})] \tag{3.21}$$

with $x_N = X_N$, where $D_t = A_t - V_t - s/2$. The expected cost for future trades under the optimal strategy is

$$J_{t_n} = (F_{t_n} + s/2)X_{t_n} + \lambda X_0 X_{t_n} + \alpha_n X_{t_n}^2 + \beta_n D_{t_n} X_{t_n} + \gamma_n D_{t_n}^2 \tag{3.22}$$

where the coefficients α_{n+1} , β_{n+1} , γ_{n+1} and δ_{n+1} are determined recursively as follows

$$\alpha_n = \alpha_{n+1} - \frac{1}{4}\delta_{n+1}(\lambda + 2\alpha_{n+1} - \beta_{n+1}\kappa e^{-\rho\tau})^2 \tag{3.23a}$$

$$\beta_n = \beta_{n+1}e^{-\rho\tau} + \frac{1}{2}\delta_{n+1}(1 - \beta_{n+1}e^{-\rho\tau} + 2\kappa\gamma_{n+1}e^{-2\rho\tau})(\lambda + 2\alpha_{n+1} - \beta_{n+1}\kappa e^{-\rho\tau}) \tag{3.23b}$$

$$\gamma_n = \gamma_{n+1}e^{-2\rho\tau} - \frac{1}{4}\delta_{n+1}(1 - \beta_{n+1}e^{-\rho\tau} + 2\kappa\gamma_{n+1}e^{-2\rho\tau})^2 \tag{3.23c}$$

with $\delta_{n+1} = [1/(2q) + \alpha_{n+1} - \beta_{n+1}\kappa e^{-\rho\tau} + \gamma_{n+1}\kappa^2 e^{-2\rho\tau}]^{-1}$ and terminal condition

$$\alpha_N = 1/(2q) - \lambda, \quad \beta_N = 1, \quad \gamma_N = 0. \tag{3.24}$$

Proposition 1 gives the optimal execution strategy when we fix the trade times at a certain interval τ . But it is only optimal among strategies with the same fixed trading interval. In principle, we want to choose the trading interval to minimize the execution cost. One way to allow different trading intervals is to take the limit $\tau \rightarrow 0$, i.e., $N \rightarrow \infty$, in the problem (3.20). Figure 3-2 plots the optimal execution strategy $\{x_n, n = 0, 1, \dots, N\}$ for $N = 10, 25, 100$, respectively. Clearly, it is very different from the strategy given in (3.17) and obtained previously when the dynamics of demand/supply is ignored. Moreover, as N becomes large, the strategy splits into two parts, large trades at both ends of the horizon (the beginning and the end) and small trades in between.

The next proposition describes the continuous-time limit of the optimal execution strategy and the expected cost:

Proposition 2 *In the limit of $N \rightarrow \infty$, the optimal execution strategy becomes*

$$\lim_{N \rightarrow \infty} x_0 = x_{t=0} = \frac{X_0}{\rho T + 2} \quad (3.25a)$$

$$\lim_{N \rightarrow \infty} x_n/(T/N) = \dot{X}_t = \frac{\rho X_0}{\rho T + 2}, \quad t \in (0, T) \quad (3.25b)$$

$$\lim_{N \rightarrow \infty} x_N = x_{t=T} = \frac{X_0}{\rho T + 2} \quad (3.25c)$$

and the expected cost is

$$J_t = (F_0 + s/2)X_t + \lambda X_0 X_t + \alpha_t X_t^2 + \beta_t X_t D_t + \gamma_t D_t^2$$

where coefficients $\alpha_t, \beta_t, \gamma_t$ are given by

$$\alpha_t = \frac{\kappa}{\rho(T-t)+2} - \frac{\lambda}{2}, \quad \beta_t = \frac{2}{\rho(T-t)+2}, \quad \gamma_t = -\frac{\rho(T-t)}{2\kappa[\rho(T-t)+2]}. \quad (3.26)$$

The optimal execution strategy given in Proposition 2 is different from those

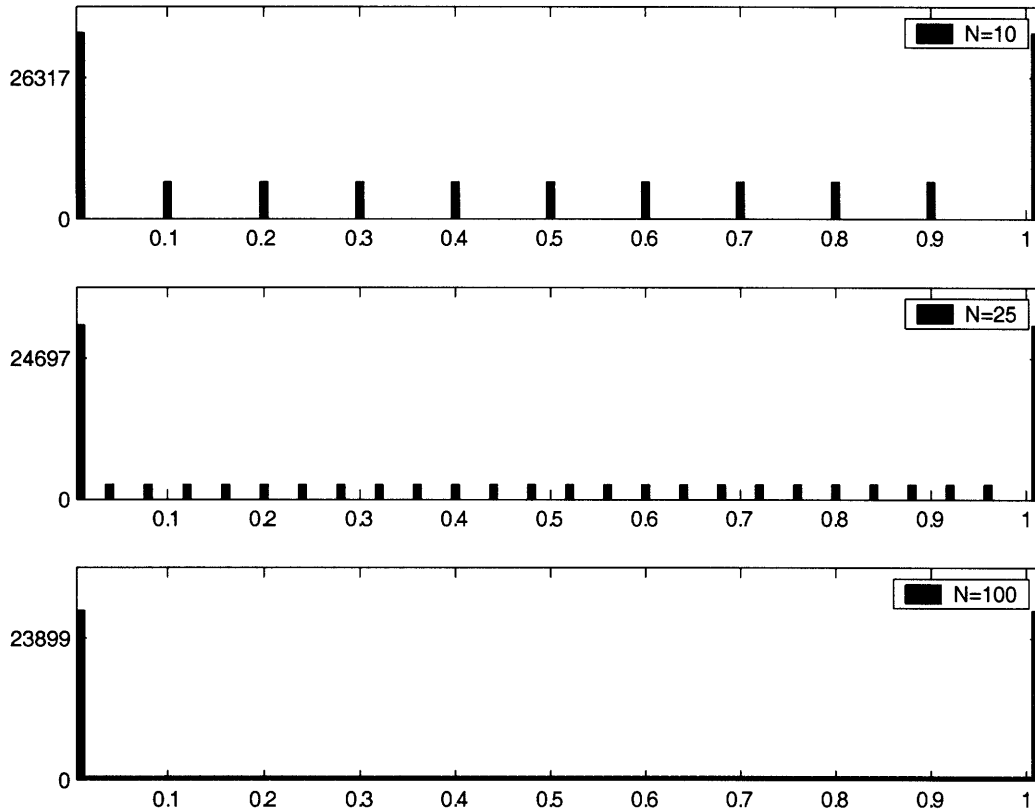


Figure 3-2: Optimal execution strategy with N discrete trading intervals.

This figure plots the optimal trades for N fixed intervals, where N is 10, 25 and 100 for respectively the top, middle and bottom panels. The initial order to trade is set at $X_0 = 100,000$ units, the time horizon is set at $T = 1$ day, the market depth is set at $q = 5,000$ units, the price-impact coefficient is set at $\lambda = 1/(2q) = 10^{-4}$ and the resiliency coefficient is set at $\rho = 2.231$.

obtained in the conventional setting. In fact, it involves both discrete and continuous trades. This clearly indicates that the timing of trades is a critical part of the optimal strategy. It also shows that ruling out discrete or continuous trades ex ante is in general suboptimal. More importantly, it demonstrates that both the static and dynamic properties of supply/demand, which are captured by the LOB dynamics in our framework, are important in analyzing the optimal execution strategy. We return in Section 3.7 to examine in more detail the properties of the optimal execution strategy and their dependence on the LOB dynamics.

3.6 Continuous-Time Solution

The nature of the continuous-time limit of the discrete-time solution suggests that limiting ourselves to discrete strategies can be suboptimal. We should in general formulate the problem in continuous-time setting and allow both continuous and discrete trading strategies. In this section, we present the continuous-time version of the LOB framework and derive the optimal strategy.

The uncertainty in model is fully captured by fundamental value F_t . Let $F_t = F_0 + \sigma Z_t$ where Z_t is a standard Brownian motion defined on $[0, T]$. \mathcal{F}_t denotes the filtration generated by Z_t . A general execution strategy can consist of two components, a set of discrete trades at certain times and a flow of continuous trades. A set of discrete trades is also called an “impulse” trading policy.

Definition 1 Let $N_+ = \{1, 2, \dots\}$. An impulse trading policy $(\tau_k, x_k) : k \in N_+$ is a sequence of trading times τ_k and trade amounts x_k such that: (1) $0 \leq \tau_k \leq \tau_{k+1}$ for $k \in N_+$, (2) τ_k is a stopping time with respect to \mathcal{F}_t , and (3) x_k is measurable with respect to \mathcal{F}_{τ_k} .

The continuous trades can be defined by a continuous trading policy described by the intensity of trades $\mu_{[0, t]}$, where μ_t is measurable with respect to \mathcal{F}_t and $\mu_t dt$ gives the trades during time interval $[t, t + dt)$. Let us denote \hat{T} the set of impulse trading times. Then, the set of admissible execution strategies for a buy order is

$$\Theta_C = \left\{ \mu_{[0, T]}, x_{\{t \in \hat{T}\}} : \mu_t, x_t \geq 0, \int_0^T \mu_t dt + \sum_{t \in \hat{T}} x_t = X_0 \right\} \quad (3.27)$$

where μ_t is the rate of continuous buy trades at time t and x_t is the discrete buy trade for $t \in \hat{T}$. The dynamics of X_t , the number of shares to acquire at time t , is

then given by the following equation:

$$X_t = X_0 - \int_0^t \mu_s ds - \sum_{s \in \hat{T}, s < t} x_s.$$

Now let us specify the dynamics of ask price A_t . Similar to the discrete-time setting, we have $A_0 = F_0 + s/2$ and

$$A_t = A_0 + \int_0^t [dV_s - \rho D_s ds - \kappa dX_s] \quad (3.28)$$

where $V_t = F_t + \lambda(X_0 - X_t)$ as in (3.9) and $D_t = A_t - V_t - s/2$ as in (3.7). The dynamics of A_t captures the evolution of the limit order book, in particular the changes in V_t , the inflow of new orders and the continuous execution of trades.

Next, we compute the execution cost, which consists of two parts: the costs from continuous trades and discrete trades, respectively. The execution cost from t to T is

$$C_t = \int_t^T A_s \mu_s ds + \sum_{s \in \hat{T}, t \leq s \leq T} [A_s + x_s/(2q)] x_s. \quad (3.29)$$

Given the dynamics of the state variables in (3.9), (3.28), and cost function in (3.29), the optimal execution problem now becomes

$$J_t \equiv J(X_t, A_t, V_t, t) = \min_{\{\mu_{[0, T]}, \{x_{t \in \hat{T}}\}\} \in \Theta_C} E_t [C_t] \quad (3.30)$$

where J_t is the value function at t , the expected cost for future trades under the optimal execution strategy. At time T , the trader is forced to buy all of the remaining order X_T , which leads to the following boundary condition:

$$J_T = [A_T + 1/(2q)X_T] X_T.$$

The next proposition gives the solution to the problem:

Proposition 3 *The value function for the optimization problem (3.30) is*

$$J_t = (F_t + s/2)X_t + \lambda X_0 X_t + \alpha_t X_t^2 + \beta_t D_t + \gamma_t D_t^2$$

where $D_t = A_t - V_t - s/2$. The optimal execution strategy is

$$x_0 = x_T = \frac{X_0}{\rho T + 2}, \quad \mu_t = \frac{\rho X_0}{\rho T + 2} \quad \forall t \in (0, T) \quad (3.31)$$

where the coefficients α_t , β_t , and γ_t are the same as given in Proposition 2.

Obviously, the solution we obtained with the continuous-time setting is identical to the continuous-time limit of the solution in the discrete-time setting. The optimal strategy consists of both continuous and discrete trades.

3.7 Optimal Execution Strategy and Cost

In contrast with previous work, the optimal execution strategy includes discrete and continuous trading. We now analyze the properties of the optimal execution strategy in more detail. Interestingly, while it does not depend on parameters λ and q , which determine static supply/demand, it crucially depends on parameter ρ , which describes the LOB dynamics, and the horizon for execution T . Further in this section we quantify the cost reduction which the optimal execution strategy brings and discuss its empirical implications.

3.7.1 Properties of Optimal Execution Strategy

The first thing to notice is that the execution strategy does not depend on λ and q . Coefficient λ captures the permanent price impact of a trade. Given the linear form,

the permanent price impact gives an execution cost of $(F_0+s/2)X_0 + (\lambda/2)X_0^2$, which is independent of the execution strategies. This is a rather striking result given that most of the previous work focus on λ as the key parameter determining the execution strategy and cost. As we show earlier, λ affects the execution strategy when the times to trade are exogenously set at fixed intervals. When the times to trade are determined optimally, the impact of λ on execution strategy disappears. Given the linear form of the price impact function, λ fully describes the instantaneous supply/demand, or the static supply/demand. Our analysis clearly shows that the static aspects of the supply/demand does not fully capture the factors that determining the optimal execution strategy.

Coefficient q captures the depth of the market. In the simple model for the limit order book we have assumed, market depth is constant at all price levels above the ask price. In this case, the actual value of the market depth does not affect the optimal execution strategy. For more general (and possibly more realistic) shapes of the limit order book, the optimal execution strategy may well depend on the characteristics of the book.

The optimal execution strategy depends on two parameters, the resilience of the limit order book ρ and the horizon for execution T . We consider these dependencies separately.

Panel (a) of Figure 3-3 plots the optimal execution strategy, or more precisely the time path of the remaining order to be executed. Clearly, the nature of the optimal strategy is different from those proposed in the literature, which involve a smooth flow of small trades. When the timing of trades is determined optimally, the optimal execution strategy consists of both large discrete trades and continuous trades. In particular, under the LOB dynamics we consider here, the optimal execution involves a discrete trade at the beginning, followed by a flow of small trades and then a discrete terminal trade. Such a strategy seems intuitive given the dynamics of the limit order

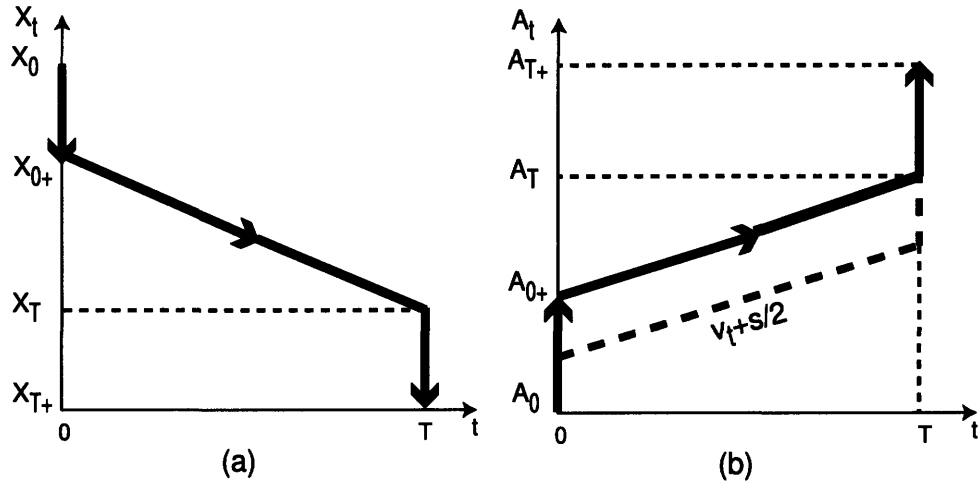


Figure 3-3: Profiles of the optimal execution strategy and ask price.

Panel (a) plots the profile of optimal execution policy as described by X_t . Panel (b) plots the profile of realized ask price A_t . After the initial discrete trade, continuous trades are executed as a constant fraction of newly incoming sell orders to keep the deviation of the ask price A_t from its steady state $V_t + s/2$, shown with grey line in panel (b), at a constant. A discrete trade occurs at the last moment T to complete the order.

book. The large initial trade pushes the limit order book away from its stationary state so that new orders are lured in. The flow of small trades will “eat up” these new orders and thus keep them coming. At the end, a discrete trade finishes the remaining part of the order. The final discrete trade is determined by two factors. First, the order has to be completed within the given horizon. Second, the evolution of supply/demand afterwards no longer matters. In practice, both of these two factors can take different forms. For example, the trading horizon T can be endogenously determined rather than exogenously given. We consider this extension in Section 3.8.

The size of the initial trade determines the prices and the intensity of the new orders. If too large, the initial trade will raise the average prices of the new orders. If too small, an initial trade will not lure in enough orders before the terminal time. The trade off between these two factors largely determines the size of the initial trade.

The continuous trades after the initial trade are intended to maintain the flow of new orders at desirable prices. To see how this works, let us consider the path of the ask price A_t under the optimal execution strategy. It is plotted in panel (b) of

Figure 3-3. The initial discrete trade pushes up the ask price from $A_0 = V_0 + s/2$ to $A_{0+} = V_0 + s/2 + X_0/(\rho T + 2)/q$. Afterwards, the optimal execution strategy keeps $D_t = A_t - V_t - s/2$, the deviation of the current ask price A_t from its steady state $V_t + s/2$, at a constant level of $\kappa X_0/(\rho T + 2)$. Consequently, the rate of new sell order flow, which is given by ρD_t , is also maintained at a constant level. The ask price A_t goes up together with $V_t + s/2$, the steady-state “value” of the security, which is shown with the grey line in Figure 3-3(b). As a result, from (3.28) with $dA_t = dV_t$ for $0 < t < T$, we have $\rho D_t = \kappa \mu_t$ or $\mu_t = (1/\kappa)\rho D_t$. In other words, under the optimal execution strategy a constant fraction of $1/\kappa$ of the new sell orders is executed to maintain a constant order flow.

Our discussion above shows that the dynamics of the limit order book, which is captured by the resilience parameter ρ , is the key factor in determining optimal execution strategy. In order to better understand this link, let us consider two extreme cases, when $\rho = 0$ and ∞ . When $\rho = 0$, we have no recovery of the limit order book after a trade. In this case, the cost of execution will be strategy independent and it does not matter when and at what speed the trader eats up the limit order book. This result is also true in a discrete setting with any N and in its continuous-time limit. When $\rho = \infty$, the limit order book rebuilds itself immediately after a trade. As we discussed in Section 3.4, this corresponds to the conventional setting. Again, the execution cost becomes strategy independent. It should be pointed out that even though in the limit of $\rho \rightarrow 0$ or ∞ , the optimal execution strategy given in Proposition 3 converges to a pure discrete strategy or a pure continuous strategy, other strategies are equally good given the degeneracy in these two cases.

When $0 < \rho < \infty$, the resiliency of the limit order book is finite, the optimal strategy is a mixture of discrete and continuous trades. The fraction of the total order executed through continuous trades is $\int_0^T \mu_t dt / X_0 = \rho T / (\rho T + 2)$, which increases with ρ . In other words, it is more efficient to use small trades when the limit order book is

more resilient. This is intuitive because discrete trades do less in taking full advantage of new order flows than continuous trades.

Another important parameter in determining the optimal execution strategy is the time-horizon to complete the order T . From Proposition 3, we see that as T increases, the size of the two discrete trades decreases. This result is intuitive. The more time we have to execute the order, the more we can use continuous trades to benefit from the inflow of new orders and to lower the total cost.

3.7.2 Minimum Execution Cost

So far, we have focused on the optimal execution strategy. We now examine how important the optimal execution is, as measured by the execution cost it saves. For this purpose, we use the strategy obtained in the conventional setting and its cost as the benchmark. The total expected execution cost of a buy order of size X_0 is equal to its fundamental value $(F_0 + s/2)X_0$, which is independent of the execution strategy, plus the extra cost from the price impact of trading, which does depend on the execution strategy. Thus, we will only consider the execution cost, net of the fundamental value, or the net execution cost.

As shown in Section 3.4, the strategy from the conventional setting is a constant flow of trades with intensity $\mu_\infty = X_0/T$, $t \in [0, T]$. Under this simple strategy, we have $V_t = F_t + \lambda(t/T)X_0$, $D_t = [\kappa X_0/(\rho T)](1 - e^{-\rho t})$ and $A_t = V_t + D_t + s/2$. The expected net execution cost for the strategy with constant rate of execution μ_∞ is given by

$$\tilde{J}_0^{\text{CM}} = \mathbb{E}_0 \left[\int_0^T (A_t - F_t - s/2)(X_0/T) dt \right] = (\lambda/2)X_0^2 + \kappa \frac{\rho T - (1 - e^{-\rho T})}{(\rho T)^2} X_0^2$$

where the superscript stands for the ‘‘Conventional Model’’. From Proposition 3, the

expected net cost under the optimal execution strategy is

$$\tilde{J}_0 = J_0 - (F_0 + s/2)X_0 = (\lambda/2)X_0^2 + \frac{\kappa}{\rho T + 2}X_0^2$$

(note that at $t = 0$, $D_0 = 0$). Thus, the improvement in expected execution cost by the optimal strategy is $J_0^{\text{CM}} - J_0$, which is given by

$$\tilde{J}_0^{\text{CM}} - \tilde{J}_0 = \kappa \frac{2\rho T - (\rho T + 2)(1 - e^{-\rho T})}{(\rho T + 2)(\rho T)^2} X_0^2$$

and is always non-negative. The relative gain can be defined as $\Delta = (\tilde{J}_0^{\text{CM}} - \tilde{J}_0)/\tilde{J}_0^{\text{CM}}$.

In order to calibrate the magnitude of the cost reduction by the optimal execution strategy, we consider some numerical examples. Let the size of the order to be executed be $X_0 = 100,000$ shares and the initial security price be $A_0 = F_0 + s/2 = \$100$. We choose the width of the limit order book, which gives the depth of the market, to be $q = 5,000$. This implies that if the order is executed at once, the ask price will move up by 20%. Without losing generality, we consider the execution horizon to be one day, $T = 1$.¹⁶ The other parameters, especially ρ , may well depend on the security under consideration. In absence of an empirical calibration, we with consider a range of values for them.

Table 3.1 reports the numerical values of the optimal execution strategy for different values of ρ . As discussed above, for small values of ρ , most of the order is executed through two discrete trades, while for large values of ρ , most of the order is executed through a flow of continuous trades as in the conventional models. For intermediate ranges of ρ , a mixture of discrete and continuous trades is used.

Table 3.2 reports the relative improvement in the expected net execution cost by

¹⁶Chan and Lackonishok (1995) documented that for institutional trades T is usually between 1 to 4 days. Keim and Madhavan (1995) found that the duration of trading is surprisingly short, with almost 57% of buy and sell orders completed in the first day. Keim and Madhavan (1997) reported that average execution time is 1.8 days for a buy order and 1.65 days for a sell order.

ρ	Half-life ($\log 2/\rho$)	Trade x_0	Trade over $(0, T)$	Trade x_N
0.001	693.15 day	49,975	50	49,975
0.01	69.31 day	49,751	498	49,751
0.5	1.39 day	40,000	20,000	40,000
1	270.33 min	33,333	33,334	33,333
2	135.16 min	25,000	50,000	25,000
4	67.58 min	16,667	66,666	16,667
5	54.07 min	14,286	71,428	14,286
10	27.03 min	8,333	83,334	8,333
20	13.52 min	4,545	90,910	4,545
50	5.40 min	1,921	96,153	1,921
300	0.90 min	331	99,338	331
1000	0.20 min	100	99,800	100
10000	0.03 min	10	99,980	10

Table 3.1: Profiles of the optimal execution strategy for different LOB resiliency.

The table reports values of optimal discrete trades x_0 and x_T at the beginning and the end of the trading horizon and the intensity of continuous trades in between for an order of $X_0 = 100,000$ for different values of the LOB resilience parameter ρ or the half-life of an LOB disturbance $\tau_{1/2}$, which is defined as $\exp\{-\rho\tau_{1/2}\} = 1/2$. The initial ask price is \$100, the market depth is set at $q = 5,000$ units, the (permanent) price-impact coefficient is set at $\lambda = 1/(2q) = 10^{-4}$, and the trading horizon is set at $T = 1$ day, which is 6.5 hours (390 minutes).

the optimal execution strategy over the simple strategy of the conventional setting. Let us first consider the extreme case in which the resilience of the LOB is very small, e.g., $\rho = 0.001$ and the half-life for the LOB to rebuild itself after being hit by a trade is 693.15 days. In this case, even though the optimal execution strategy looks very different from the simple execution strategy, as shown in Figure 3-4, the improvement in execution cost is minuscule. This is not surprising as we know the execution cost becomes strategy independent when $\rho = 0$. For a modest value of ρ , e.g. $\rho = 2$ with a half life of 135 minutes (2 hours and 15 minutes), the improvement in execution cost ranges from 4.32% for $\lambda = 1/(2q)$ to 11.92% for $\lambda = 0$. When ρ becomes large and the LOB becomes very resilient, e.g., $\rho = 300$ and the half-life of LOB deviation is 0.90 minute, the improvement in execution cost becomes small again, with a maximum of 0.33% when $\lambda = 0$. This is again expected as we know that the simple strategy is

ρ	Half-life	λ				0
		$\frac{1}{2q}$	$\frac{1}{10q}$	$\frac{1}{50q}$	$\frac{1}{100q}$	
0.001	693.15 day	0.00	0.01	0.02	0.02	0.02
0.01	69.31 day	0.08	0.15	0.16	0.16	0.17
0.5	1.39 day	2.82	5.42	5.99	6.06	6.13
1	270.33 min	3.98	8.16	9.14	9.26	9.39
2	135.16 min	4.32	9.97	11.51	11.71	11.92
4	67.58 min	3.19	9.00	11.05	11.35	11.65
5	54.07 min	2.64	8.07	10.21	10.53	10.86
10	27.03 min	1.13	4.58	6.65	7.01	7.41
20	13.52 min	0.37	1.98	3.54	3.89	4.31
50	5.40 min	0.07	0.49	1.24	1.50	1.88
300	0.90 min	0.00	0.02	0.08	0.13	0.33
1000	0.20 min	0.00	0.00	0.01	0.02	0.10
10000	0.03 min	0.00	0.00	0.00	0.00	0.09

Table 3.2: Cost savings by the optimal execution strategy from the simple strategy.

Relative improvement in expected net execution cost $\Delta = (\tilde{J}^{\text{CM}} - \tilde{J}_0)/\tilde{J}^{\text{CM}}$ is reported for different values of LOB resiliency coefficient ρ and the permanent price-impact coefficient. The order size is set at 100,000, the market depth is set at $q = 5,000$ and the horizon for execution is set at $T = 1$ day (equivalent of 390 minutes).

close to the optimal strategy when $\rho \rightarrow \infty$ (as in this limit, the cost becomes strategy independent).

In order to see the difference between the optimal strategy and the simple strategy obtained in conventional settings, we compare them in Figure 3-4. The solid line shows the optimal execution strategy of the LOB framework and the dashed line shows the execution strategy of the conventional setting. Obviously, the difference between the two strategies are more significant for smaller values of ρ .

Table 3.2 also reveals an interesting result. The relative savings in execution cost by the optimal execution strategy is the highest when $\lambda = 0$, i.e., when the permanent price impact is zero.¹⁷

¹⁷Of course, the magnitude of net execution cost becomes very small as λ goes to zero.

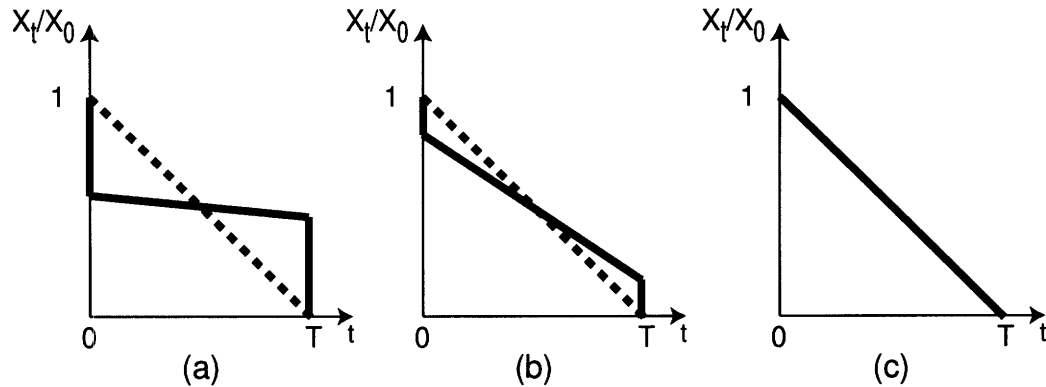


Figure 3-4: Optimal execution strategy vs. strategy from the conventional models. The figure plots the time paths of remaining order to be executed for the optimal strategy (solid line) and the simple strategy obtained from the conventional models (dashed line), respectively. The order size is set at $X_0 = 100,000$, the initial ask price is set at \$100, the market depth is set at $q = 5,000$ units, the (permanent) price-impact coefficient is set at $\lambda = 1/(2q) = 10^{-4}$, and the trading horizon is set at $T = 1$ day, which is assumed to be 6.5 hours (390 minutes). Panels (a), (b) and (c) plot the strategies for $\rho = 0.001, 2$ and $1,000$, respectively.

3.7.3 Empirical Implications

Optimality of discrete trades at the beginning and the end of the trading period leads to interesting empirical implications. It is well documented that there is a U-shaped pattern in the intraday trading volume, price volatility and average bid-ask spread.¹⁸ Several authors have proposed theoretical models that can help to explain the intraday price and volume patterns.¹⁹ Most of these models generate the intraday patterns from the time variation in information asymmetry and/or trading opportunities associated with market closures.

Our model suggests an alternative source for such patterns. Namely, they can be generated by the optimal execution of block trades. It is well known that large-block transactions have become a substantial fraction of the total trading volume for

¹⁸Intraday patterns in volume and prices in the U.S. markets have been documented by Jain and Joh(1988), Gerety and Mulherin (1992), Chan, Christie, Schultz (1995), among others. They are also present in other markets. See McInish and Wood(1991) for the Toronto Stock Exchange, Hamao and Hasbrouck (1995) for the Tokyo Stock Exchange, Niemayer and Sandas (1993) for the Stockholm Stock Exchange, and Kleidon and Werner (1996) for the London Stock Exchange.

¹⁹See, for example, Admati and Pfleiderer (1988), Back and Baruch (2004), Brock and Kleidon (1992), Foster and Viswanathan(1990, 1995), and Hong and Wang (2000).

common stocks. According to Keim and Madhavan(1996), block trades represented almost 54% of New York Stock Exchange share volume in 1993 while in 1965 the corresponding figure was merely 3%. Thus, the execution strategies of institutional traders can influence the intraday variation in volume and prices. It is often the case that institutional investors have daily horizons to complete their orders, for example to accommodate the inflows and outflows in mutual funds. For reasonable values of the LOB recovery speed ρ , our optimal execution strategy implies large trades at the beginning and at the end of trading period. If execution horizon of institutional traders coincides with a trading day, their trading can cause the increase in trading volume and bid-ask spread at the beginning and the end of a trading day.

Our model predicts higher variation in the optimal trading profile for stocks with lower ρ . This implies that stocks with low resilience in its LOB (low ρ) and high institutional holdings should exhibit more intraday volume variation. We leave the empirical tests of these predictions for future research.

3.8 Extensions

So far, we have used a parsimonious LOB model to analyze the impact of supply/dynamics on optimal execution strategy. Obviously, the simple characteristics of the model does not reflect the richness in the LOB dynamics observed in the market. However, the framework we developed is quite flexible to allow for extensions in various directions. In this section, we briefly discuss some of these extensions. First, we consider the case where the resilience of the LOB is time-varying. Next, we discuss the possibility of allowing more general shapes of the static limit order book. Finally, we include risk considerations in optimization problem. This also allows us to endogenize the trading horizon T , which is taken as given above.

3.8.1 Time Varying LOB Resiliency

Our model can easily incorporate time-variation in LOB resiliency. It has been documented that trading volume, order flows and transaction costs all exhibit a U-shaped intraday pattern, high at the opening of the trading day, then falling to lower constant levels during the day and finally rising again towards the close of trading day. This suggests that the liquidity in the market may well vary over a trading day. Monch (2004) has attempted to incorporate such a time-variation in implementing the conventional models.

We can easily allow time-variation in LOB and its dynamics in our model. In particular, we can allow the resilience coefficient to be time dependent, $\rho = \rho_t$ for $t \in [0, T]$. The results in Proposition 1, 2, 3 still hold if we replace ρ by ρ_t , ρT by $\int_0^T \rho_t dt$ and $\rho(T - t)$ by $\int_t^T \rho_t dt$.

3.8.2 Different Shapes for LOB

We have considered a simple shape for the LOB, which is a step function. As we showed in Section 3.3, this form of the LOB is consistent with the static linear price impact function widely used in the literature. Huberman and Stanzl (2005) have provided theoretical arguments in support of the linear price impact functions. However, empirical literature has suggested that the shape of the LOB can be more complex (see, e.g., Hopman (2003)). Addressing this issue, we can allow more general shapes of the LOB in our framework. For example, we may extend our analysis to LOB with a density of placed limit orders defined by power function. This will also make the LOB dynamics more complex. As a trade eats away the tip of the LOB, we have to specify how the LOB converges to its steady state. With a complicated shape for the LOB, this convergence process can take many forms which involves assumptions about the flow or new orders at a range of prices. For certain specifications of this convergence process, our model is still tractable. For brevity, we do not present these

cases here. But beyond certain point, closed form solutions become hard to find. Although the actual strategy can be quite complex and depends on the specifics of the LOB shape and its dynamics, we expect its qualitative features to be the same as that under the simple LOB dynamics we considered.

3.8.3 Risk Aversion

Let us consider the optimal execution problem for a risk-averse trader. For tractability, we assume that he has a mean-variance objective function with a risk-aversion coefficient of a . The optimization problem (3.30) now becomes

$$J_t \equiv J(X_t, A_t, V_t, t) = \min_{\{\mu_{[0, T]}, \{x_{t \in \hat{T}}\}\} \in \Theta_C} E_t [C_t] + \frac{1}{2} a \text{Var}_t [C_t] \quad (3.32)$$

with (3.9), (3.28), (3.29) and the same terminal condition $J_T = [A_T + 1/(2q)X_T] X_T$. Since the only source of uncertainty is F_t and only the trades executed in interval $[t, t+dt)$ will be subject to this uncertainty, we can rewrite (3.32) in a more convenient form:

$$J_t = \min_{\{\mu_{[0, T]}, \{x_{t \in \hat{T}}\}\} \in \Theta_C} E_t [C_t] + \frac{1}{2} a \int_t^T \sigma^2 X_s^2 ds. \quad (3.33)$$

At time T , the trader is forced to buy all of the remaining order X_T . This leads to the following boundary condition:

$$J_T = [A_T + 1/(2q)X_T] X_T.$$

The next proposition gives the solution to the problem for a risk averse trader:

Proposition 4 *The solution to the optimization problem (3.33) is*

$$\begin{aligned}
x_0 &= X_0 \frac{\kappa f'(0) + a\sigma^2}{\kappa\rho f(0) + a\sigma^2} \\
\mu_t &= \kappa x_0 \frac{\rho g(t) - g'(t)}{1 + \kappa g(t)} e^{-\int_0^t \frac{\kappa g'(s) + \rho}{1 + \kappa g(s)} ds}, \quad \forall t \in (0, T) \\
x_T &= X_0 - x_0 - \int_0^T \mu_s ds
\end{aligned}$$

and the value function is

$$J_t = (F_t + s/2)X_t + \lambda X_0 X_t + \alpha_t X_t^2 + \beta_t D_t + \gamma_t D_t^2$$

where $D_t = A_t - V_t - s/2$ and the coefficients are given by

$$\alpha_t = \frac{\kappa f(t) - \lambda}{2}, \quad \beta_t = f(t), \quad \gamma_t = \frac{f(t) - 1}{2\kappa}$$

and

$$\begin{aligned}
f(t) &= (v - a\sigma^2)/(\kappa\rho) + \left[-\frac{\kappa\rho}{2v} + e^{\frac{2\rho v}{2\kappa\rho + a\sigma^2}(T-t)} \left(\frac{\kappa\rho}{2v} - \frac{\kappa\rho}{v - a\sigma^2 - \kappa\rho} \right) \right]^{-1} \\
g(t) &= -\frac{f'(t) - \rho f(t)}{\kappa f'(t) + a\sigma^2}
\end{aligned}$$

with $v = \sqrt{a^2\sigma^4 + 2a\sigma^2\kappa\rho}$.

It can be shown that as risk aversion coefficient goes to 0 the coefficients α_t , β_t , and γ_t converge to the ones given in Proposition 2, which presents the results for the risk neutral trader.

The nature of the optimal strategy remains qualitatively the same under risk aversion: discrete trades at the two ends of the trading horizon with continuous trades in the middle. The effect of trader's risk aversion on his optimal trading profile is shown in Figure 3-5. The more risk averse is the trader, the larger the initial trade

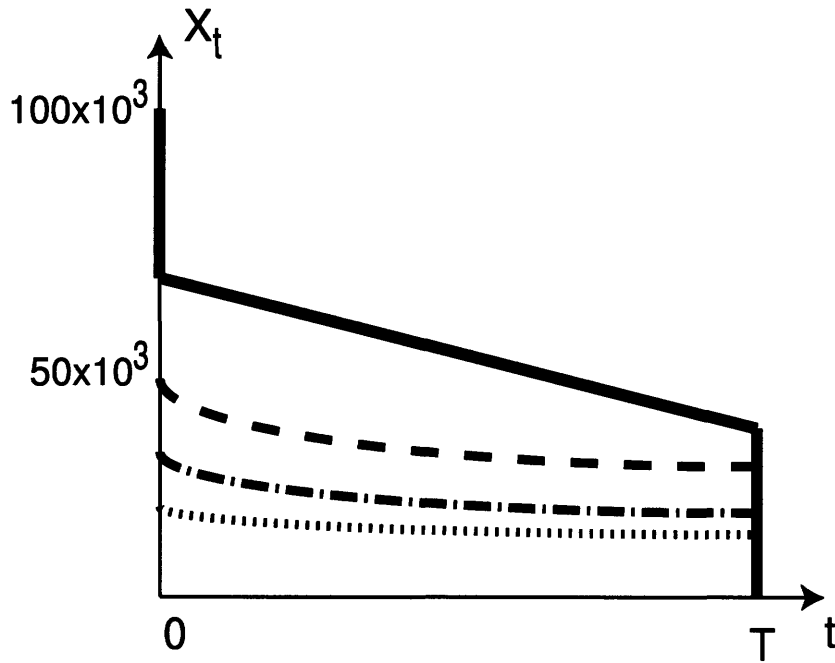


Figure 3-5: Optimal execution strategies for different coefficients of risk aversion. This figure shows the profiles of optimal execution policies X_t for the traders with different coefficients of risk aversion $a = 0$ (solid line), $a = 0.05$ (dashed line), $a = 0.5$ (dashed-dotted line) and $a = 1$ (dotted line), respectively. Variable X_t indicates how much shares still has to be executed before trading at time t . The order size is set at $X_0 = 100,000$, the market depth is set at $q = 5,000$ units, the permanent price-impact coefficient is set at $\lambda = 0$, and the trading horizon is set at $T = 1$, the resiliency coefficient is set at $\rho = 1$.

more trades he shifts to the beginning.

So far, we have assumed the execution horizon, $[0, T]$, to be exogenously given, and ignored any time preference for execution a trade. Risk aversion, however, introduces a natural preference for such a preference: Trading sooner reduces uncertainty in execution prices. Such a preference is clearly reflected in the optimal policy as shown in Figure 3-5. Such a time preference provides a mechanism to endogenize the execution horizon. For example, T is sufficiently large, when the trader is risk-averse enough, he may optimally finish the whole order soon before T .

3.9 Conclusion

In this paper, we analyze the optimal trading strategy to execute a large order. We show that the static price impact function widely used in previous work fails to capture the intertemporal nature of a security's supply/demand in the market. We construct a simple dynamic model for a limit order book market to capture the intertemporal nature of supply/demand and solve for the optimal execution strategy. We show that when trading times are chosen optimally, the dynamics of the supply/demand is the key factor in determining the optimal execution strategy. Contrary to previous work, the optimal execution strategy involves discrete trades as well as continuous trades, instead of merely continuous trades. This trading behavior is consistent with the empirical intraday volume and price patterns. Our results on the optimal execution strategy also suggest testable implications for these intraday patterns and provide new insight into the demand of liquidity in the market.

The specific model we used for the LOB dynamics is very simple since our goal is mainly to illustrate its importance. The actual LOB dynamics can be much more complex. However, the framework we developed is fairly general to accommodate rich forms of LOB dynamics. Moreover, with the current increase in the number of open electronic limit order books, our LOB model can be easily calibrated and used to address real world problems.

3.10 Appendix

3.10.1 Proof of Proposition 1

From (3.7), we have

$$D_{t_n} = A_{t_n} - V_{t_n} - s/2 = \sum_{i=0}^{n-1} x_{t_i} \kappa e^{-\rho r(n-i)} \quad (3.36)$$

From (3.36), the dynamics of D_t between trades will be

$$D_{t_{n+1}} = (D_{t_n} + x_{t_n} \kappa) e^{-\rho r} \quad (3.37)$$

with $D_0 = 0$. We can then re-express the optimal execution problem (3.20) in terms of variables X_t and D_t :

$$\min_{x \in \Theta_D} \mathbb{E}_0 \sum_{n=0}^N [(F_{t_n} + s/2) + \lambda(X_0 - X_{t_n}) + D_{t_n} + x_{t_n}/(2q)] x_{t_n}. \quad (3.38)$$

under dynamics of D_t given by (3.37).

First, by induction we prove that value function for (3.38) is quadratic in X_t and D_t and has a form implied by (3.22):

$$J(X_{t_n}, D_{t_n}, F_{t_n}, t_n) = (F_{t_n} + s/2)X_{t_n} + \lambda X_0 X_{t_n} + \alpha_n X_{t_n}^2 + \beta_n X_{t_n} D_{t_n} + \gamma_n D_{t_n}^2. \quad (3.39)$$

At time $t = t_N = T$, the trader has to finish the order and the cost is

$$J(X_T, D_T, F_T, T) = (F_T + s/2)X_T + [\lambda(X_0 - X_T) + D_T + X_T/(2q)]X_T.$$

Hence, $\alpha_N = 1/(2q) - \lambda$, $\beta_N = 1$, $\gamma_N = 0$. Recursively, the Bellman equation yields

$$J_{t_{n-1}} = \min_{x_{n-1}} \left\{ [(F_{t_{n-1}} + s/2) + \lambda(X_0 - X_{t_{n-1}}) + D_{t_{n-1}} + x_{n-1}/(2q)] x_{n-1} \right. \\ \left. + E_{t_{n-1}} J [X_{t_{n-1}} - x_{n-1}, (D_{t_{n-1}} + \kappa x_{n-1})e^{-\rho\tau}, F_{t_n}, t_n] \right\}.$$

Since F_{t_n} follows Brownian motion and value function is linear in F_{t_n} , it immediately follows that the optimal x_{n-1} is a linear function of $X_{t_{n-1}}$ and $D_{t_{n-1}}$ and the value function is a quadratic in $X_{t_{n-1}}$ and $D_{t_{n-1}}$ satisfying (3.39), which leads to the recursive equation (3.23) for the coefficients. Q.E.D.

3.10.2 Proof of Proposition 2

First, we prove the convergence of the value function. As $\tau = T/N \rightarrow 0$, the first order approximation of the system (3.23) in τ leads to the following restrictions on the coefficients:

$$\begin{aligned} \lambda + 2\alpha_t - \beta_t \kappa &= 0 \\ 1 - \beta_t + 2\kappa\gamma_t &= 0 \end{aligned} \tag{3.40}$$

and

$$\begin{aligned} \dot{\alpha}_t &= \frac{1}{4}\kappa\rho\beta_t^2 \\ \dot{\beta}_t &= \rho\beta_t - \frac{1}{2}\rho\beta_t(\beta_t - 4\kappa\gamma_t) \\ \dot{\gamma}_t &= 2\rho\gamma_t + \frac{1}{4\kappa}\rho(\beta_t - 4\kappa\gamma_t)^2. \end{aligned} \tag{3.41}$$

It is easy to verify that α_t , β_t and γ_t given in (3.26) are the solution of (3.41), satisfying (3.40) and the terminal condition (3.24). Thus, as $\tau \rightarrow 0$ the coefficients of the value function (3.23) converge to (3.26).

Next, we prove the convergence result for the optimal execution policy $\{x_t\}$. Substituting α_t , β_t , γ_t into (3.21), we can show that as $\tau \rightarrow 0$, the execution policy

converges to

$$x_t = \left\{ X_t \frac{1}{\rho(T-t) + 2} - D_t \frac{1 + \rho(T-t)}{\kappa[\rho(T-t) + 2]} \right\} \left[1 - \frac{1}{2}\rho^2(T-t)\tau \right] + \frac{1}{2}(\rho/\kappa)D_t\tau + o(\tau) \quad (3.42)$$

where $o(\tau)$ denotes terms to the higher order of τ . At $t = 0$, $D_0 = 0$ and we have $\lim_{\tau \rightarrow 0} x_0 = \frac{X_0}{\rho T + 2}$. Moreover, after the initial discrete trade x_0 all trades will be the continuous (except possibly at T) and equal to

$$x_t = \frac{1}{\kappa}\rho D_t\tau + o(\tau), \quad t = n\tau, \quad n = 1, \dots, N-1. \quad (3.43)$$

We prove this by induction. First, using (3.42), where $X_\tau = X_0 - x_0$ and $D_\tau = kx_0(1 - \rho\tau)$, it is easy to check that (3.43) holds for x_τ . Second, let us assume that (3.43) holds for some x_t , where $t = n\tau$, then we can show that $x_{t+\tau}$ will satisfy it as well. In fact, the dynamics of X_t and D_t is defined by

$$X_{t+\tau} = X_t - x_t, \quad D_{t+\tau} = (D_t + kx_t)(1 - \rho\tau), \quad t = n\tau, \quad n = 0, \dots, N-1. \quad (3.44)$$

Substituting these into (3.42) and using the induction assumption, we get that

$$x_{t+\tau} = (\rho/\kappa)D_{t+\tau}\tau + o(\tau).$$

Thus, after the discrete trade x_0 at time $t = 0$ all consequent trades will be the continuous. Moreover, (3.43) implies the following form of X_t and D_t dynamics:

$$X_{t+\tau} = X_t - \frac{1}{\kappa}\rho D_t\tau + o(\tau), \quad D_{t+\tau} = D_t + o(\tau). \quad (3.45)$$

Taking into account the initial condition right after the trade at time 0, we find that

$$D_t = D_\tau = \frac{kX_0}{\rho T + 2} + o(\tau).$$

Thus, from (3.43) as $\tau \rightarrow 0$ for any $t \in (0, T)$ trade x_t converges to $\frac{\rho X_0}{\rho T + 2} \tau$. Since all shares X_0 should be acquired by time T , it is obvious that $\lim_{\tau \rightarrow 0} x_T = \frac{X_0}{\rho T + 2}$. Q.E.D.

3.10.3 Proof of Propositions 3 and 4

We give the proof of Proposition 4 along with Proposition 3 as a special case. Let us first formulate problem (3.33) in terms of variables X_t and $D_t = A_t - V_t - s/2$ whose dynamics similar to (3.37) is

$$dD_t = -\rho D_t dt - \kappa dX_t \quad (3.46)$$

with $D_0 = 0$. If we write the cost of continuous and discrete trading as following:

$$dC_t^c = (F_t + s/2)\mu_t dt + \lambda(X_0 - X_t)\mu_t dt + D_t\mu_t dt \quad (3.47)$$

$$\Delta C_t^d = 1_{\{t \in \hat{T}\}} [(F_t + s/2)x_t + \lambda(X_0 - X_t)x_t + D_t x_t + x_t^2/(2q)]. \quad (3.48)$$

then (3.33) is equivalent to

$$\min_{\{\mu_{[0, T]}, \{x_{t \in \hat{T}}\}\} \in \Theta_C} \mathbb{E}_t \left[\int_0^t dC_t^c + \sum_{t \in \hat{T}} \Delta C_t^d \right] + (a/2) \int_t^T \sigma^2 X_s^2 ds \quad (3.49)$$

with (3.46), (3.47) and (3.48).

This is the optimal control problem with a single control variable X_t . We can now apply standard methods to find its solution. In particular, the solution will be characterized by three regions where it will be optimal to trade discretely, continu-

ously and do not trade at all. We can specify the necessary conditions for each region which any value function should satisfy. In fact, under some regularity conditions on the value function we can use Ito's lemma together with dynamic programming principle to derive Bellman equation associated with (3.49). For this problem, Bellman equation is a variational inequality involving first-order partial differential equation with gradient constraints. Moreover, the value function should also satisfy boundary conditions. Below we will heuristically derive the variational inequalities and show the candidate function which satisfies them. To prove that this function is a solution we have to check the sufficient conditions for optimality using verification principle.²⁰

We proceed with the proof of Proposition 4 in three steps. First, we heuristically define the variational inequalities (VI) and the boundary conditions for the optimization problem (3.49). Second, we show that the solution to the VI exists and implies a candidate value function and a candidate optimal strategy. Third, we verify that candidate value function and optimal strategy are indeed solution to optimization problem. Finally, we will discuss the properties of optimal strategies.

A. Variational Inequalities

Let $J(X_t, D_t, F_t, t)$ be a value function for our problem. Then, under some regularity conditions it has to satisfy the necessary conditions for optimality or Bellman equation associated with (3.49). For this problem, Bellman equation is a variational inequality involving first-order partial differential equation with gradient constraints, i.e.,

$$\min \left\{ J_t - \rho D_t J_D + \frac{1}{2} \sigma^2 J_{FF} + a \sigma^2 X_t^2, (F_t + s/2) + \lambda(X_0 - X_t) + D_t - J_X + \kappa J_D \right\} = 0.$$

Thus, the space can be divided into three regions. In the discrete trade (DT) region,

²⁰For detailed treatment of similar problems see Hindy, Huang and Zhu(1997), Shreve and Soner (1994), Eastham and Hastings(1988).

the value function J has to satisfy

$$J_t - \rho D_t J_D + \frac{1}{2} \sigma^2 J_{FF} + a \sigma^2 X_t^2 > 0, \quad (F_t + s/2) + \lambda(X_0 - X_t) + D_t - J_X + \kappa J_D = 0. \quad (3.50)$$

In the no trade (NT) region, the value function J satisfies:

$$J_t - \rho D_t J_D + \frac{1}{2} \sigma^2 J_{FF} + a \sigma^2 X_t^2 = 0, \quad (F_t + s/2) + \lambda(X_0 - X_t) + D_t - J_X + \kappa J_D > 0. \quad (3.51)$$

In the continuous trade (CT) region, the value function J has to satisfy:

$$J_t - \rho D_t J_D + \frac{1}{2} \sigma^2 J_{FF} + a \sigma^2 X_t^2 = 0, \quad (F_t + s/2) + \lambda(X_0 - X_t) + D_t - J_X + \kappa J_D = 0. \quad (3.52)$$

In addition, we have the boundary condition at terminal point T :

$$J(X_T, D_T, F_T, T) = (F_T + s/2)X_T + \lambda(X_0 - X_T)X_T + D_T X_T + X_T^2/(2q). \quad (3.53)$$

Inequalities (3.50)-(3.53) are the so called variational inequalities (VI's), which are the necessary conditions for any solutions to the problem (3.49).

B. Candidate Value Function

Basing on our analysis of discrete-time case we can heuristically derive the candidate value function which will satisfy variational inequalities (3.50)-(3.53). Thus, we will be searching for the solution in a class of quadratic in X_t and D_t functions. Note that it is always optimal to trade at time 0. Moreover, the nature of the problem implies that there should be no NT region. In fact, if we assume that there exists a strategy with no trading at period (t_1, t_2) , then it will be always suboptimal with respect to the similar strategy except that the trade at t_1 is reduced by sufficiently small amount ϵ and ϵ trades are continuously executed over period (t_1, t_2) . Thus, the candidate value function has to satisfy (3.52) in CT region and (3.50) in any other

region.

Since there is no NT region, $(F_t+s/2) + \lambda(X_0-X_t) + D_t - J_X + \kappa J_D = 0$ holds for any point (X_t, D_t, F_t, t) . This implies a particular form for the quadratic candidate value function:

$$\begin{aligned} J(X_t, D_t, F_t, t) &= (F_t+s/2)X_t + \lambda X_0 X_t \\ &+ [\kappa f(t) - \lambda]X_t^2/2 + f(t)X_t D_t + [f(t) - 1]D_t^2/(2\kappa) \end{aligned} \quad (3.54)$$

where $f(t)$ is a function which depends only on t . Substituting (3.54) into $J_t - \rho D_t J_D + \frac{1}{2}\sigma^2 J_{FF} + a\sigma^2 X_t^2 \geq 0$ we have:

$$(\kappa f' + a\sigma^2)X_t^2/2 + (f' - \rho f)X_t D_t + (f' + 2\rho - 2f)D_t^2/(2\kappa) \geq 0 \quad (3.55)$$

which holds with an equality for any point of the CT region.

Minimizing with respect to X_t , we show that the CT region is specified by:

$$X_t = -\frac{f' - \rho f}{\kappa f' + a\sigma^2} D_t. \quad (3.56)$$

For (X_t, D_t) in the CT region (3.55) holds with the equality. Thus, function $f(t)$ can be found from the Riccati equation:

$$f'(t)(2\rho\kappa + a\sigma^2) - \kappa\rho^2 f^2(t) - 2a\sigma^2 \rho f(t) + 2a\sigma^2 \rho = 0. \quad (3.57)$$

This guarantees that $J_t - \rho D_t J_D + \frac{1}{2}\sigma^2 J_{FF} + a\sigma^2 X_t^2$ is equal to zero for any points in CT region and greater then zero for any other points. Taking in account terminal condition $f(T) = 1$, we can solve for $f(t)$. As a result, if the trader is risk neutral and $a = 0$, then

$$f(t) = \frac{2}{\rho(T-t)+2}.$$

Substituting the expression for $f(t)$ into (3.54) we get the candidate value function of Proposition 3. If the trader is risk averse and $a \neq 0$, then

$$f(t) = \frac{1}{\kappa\rho}(v - a\sigma^2) - \left[\frac{\kappa\rho}{2v} + \left(\frac{\kappa\rho}{v - a\sigma^2 - \kappa} - \frac{\kappa\rho}{2v} \right) e^{\frac{2\rho v}{2\rho\kappa + a\sigma^2}(T-t)} \right]^{-1}$$

where v is the constant defined in Proposition 4. From (3.54) this results in the candidate value function specified in Proposition 4.

C. Verification Principle

Now we verify that the candidate value function $J(X_0, D_0, F_0, 0)$ obtained above is greater or equal to the value achieved by any other trading policy. Let $X_{[0, T]}$ be an arbitrary feasible policy from Θ_C and $V(X_t, D_t, F_t, t)$ be the corresponding value function. We have

$$X(t) = X(0) - \int_0^t \mu_t dt - \sum_{s \in \hat{T}, s < t} x_s$$

where $\mu_t \geq 0$ and $x_t \geq 0$ for $t \in \hat{T}$. For any τ and X_0 , we consider a hybrid policy which follows policy X_t on the interval $[0, \tau]$ and the candidate optimal policy on the interval $[\tau, T]$. The value function for this policy is

$$\begin{aligned} V_\tau(X_0, D_0, F_0, 0) = & \mathbb{E}_0 \left[\int_0^\tau [(F_t + s/2) + \lambda(X_0 - X_t) + D_t] \mu_t dt \right. \\ & \left. + \sum_{t_i < \tau, t_i \in \hat{T}} [(F_{t_i} + s/2)x_{t_i} + \lambda(X_0 - X_{t_i})x_{t_i} + D_{t_i}x_{t_i} + x_{t_i}^2/(2q)] + J(X_\tau, D_\tau, F_\tau, \tau) \right]. \end{aligned} \quad (3.58)$$

For any function, e.g., $J(X_t, D_t, F_t, t)$ and any (X_t, D_t, F_t, t) , we have

$$\begin{aligned}
J(X_t, D_t, F_t, t) = & J(X_0, D_0, F_0, 0) + \int_0^t J_s ds + \int_0^t J_X dX + \int_0^t J_D dD \\
& + \int_0^t J_F dF + \int_0^t \frac{1}{2} J_{FF} (dF)^2 + a\sigma^2 \int_0^t X_s^2 ds + \sum_{t_i < t, t_i \in \hat{T}} \Delta J. \quad (3.59)
\end{aligned}$$

Use $dD_t = -\rho D_t dt - \kappa dX_t$ and substitute (3.59) for $J(X_\tau, D_\tau, F_\tau, \tau)$ into (3.58), we have

$$\begin{aligned}
V_\tau(X_0, D_0, F_0, 0) = & J(X_0, D_0, F_0, 0) \\
& + E_0 \int_0^\tau \left[F_t + \frac{s}{2} + \lambda(X_0 - X_t) + D_t - J_X + \kappa J_D \right] \mu_t dt \\
& + E_0 \int_0^\tau \left(J_t - \rho D_t J_D + \frac{1}{2} \sigma^2 J_{FF} + a\sigma^2 X_t^2 \right) dt \\
& + E_0 \sum_{t_i < t, t_i \in \hat{T}} \left[\Delta J + \left(F_t + \frac{s}{2} + \lambda(X_0 - X_t) + D_t + x_{t_i}/(2q) \right) x_{t_i} \right] \\
= & J(X_0, D_0, F_0, 0) + I_1 + I_2 + I_3 \quad (3.60)
\end{aligned}$$

Now we are ready to show that for any arbitrary strategy X_t and for any moment τ it is true that

$$V_\tau(X_0, D_0, F_0, 0) \geq J(X_0, D_0, F_0, 0). \quad (3.61)$$

It is clear that VI (3.50)-(3.52) implies non-negativity of I_1 and I_2 in (3.60). Moreover, it implies that $I_3 \geq 0$. It is easy to be shown if you rewrite $\Delta J(X_{t_i}, D_{t_i}, F_{t_i}, t_i)$ as $J(X_{t_i} - x_{t_i}, D_{t_i} + \kappa x_{t_i}, F_{t_i} + \sigma Z_{t_i}, t_i) - J(X_{t_i}, D_{t_i}, F_{t_i}, t_i)$. This complete the proof of (3.61).

Use it for $\tau = 0$ to see that $J(X_0, D_0, F_0, 0) \leq V(X_0, D_0, F_0, 0)$. Moreover there is an equality if our candidate optimal strategy is used. This complete the proof of Proposition 3.

D. Properties of the Optimal Execution Policy

We now analyze the properties of optimal execution strategies. First, let us consider the risk neutral trader with $a = 0$. Substituting the established expression for $f(t)$ into (3.56), we find that the CT region is given by

$$X_t = \frac{\rho(T-t)+1}{\kappa} D_t.$$

This implies that after the initial trade $x_0 = \frac{X_0}{\rho T+2}$ which pushes the system from its initial state X_0 and $D_0 = 0$ into CT region, the trader trades continuously at the rate $\mu_t = \frac{\rho X_0}{\rho T+2}$ staying in CT region and executes the rest $x_T = \frac{X_0}{\rho T+2}$ at the end of trading horizon. In fact, this is the same solution as we had for continuous time limit of solution of problem (3.20).

If the trader is risk averse then the CT region is given by

$$X_t = g(t)D_t, \quad \text{where} \quad g(t) = -\frac{f'(t) - \rho f(t)}{f'(t)\kappa + a\sigma^2}.$$

This implies that after discrete trade $x_0 = X_0 \frac{\kappa f'(0) + a\sigma^2}{\rho \kappa f(0) + a\sigma^2}$ at the beginning which pushes the system from its initial state into CT region, the trader will trade continuously at the rate

$$\mu_t = \kappa x_0 \frac{\rho g(t) - g'(t)}{1 + \kappa g(t)} e^{-\int_0^t \frac{\kappa g'(s) + \rho}{1 + \kappa g(s)} ds}.$$

This can be shown taking in account the dynamics of D_t given in (3.37) and specification of CT region. At the end the trader finishes the order. Q.E.D.

References

- Admati, Anat R., and Paul C. Pfleiderer, 1988, A theory of intraday patterns: Volume and price variability, *Review of Financial Studies* 1, 3-40.
- Ahn, Hee-Joon, Kee-Hong Bae and Kalok Chan, 2001, Limit orders, depth, and volatility, *Journal of Finance*, 56, 767-788.
- Almgren, Robert, 2003, Optimal execution with nonlinear impact functions and trading-enhanced risk, *Applied Mathematical Finance* 10, 1-18.
- Almgren, Robert, and Neil Chriss, 1999, Value under liquidation, *Risk*, 12 (12).
- Almgren, Robert, and Neil Chriss, 2000, Optimal execution of portfolio transactions, *University of Toronto Working Paper*.
- Back, Kerry, and Shmuel Baruch, 2004, Information in securities markets: Kyle meets Glosten and Milgrom, *Econometrica*, 72, 433-465.
- Bertsimas, Dimitris, and Andrew W. Lo, 1998, Optimal control of execution costs, *Journal of Financial Markets*, 1, 1-50.
- Biais, Bruno, Pierre Hillion, and Chester S. Spatt, 1995, An empirical analysis of the limit order book and order flow in Paris Bourse, *Journal of Finance* 50, 1655-1689.
- Biais, Bruno, Christophe Bisiere, and Chester S. Spatt, 2003, Imperfect competition in financial markets, *Carnegie Mellon University Working Paper*.
- Bloomfield, Robert, Maureen O'Hara, and Gideon Saar, 2004, The "Make or Take" decision in an electronic market: Evidence on the evolution of liquidity, *Journal of Financial Economics*, forthcoming.
- Brock, William A., and Allan W. Kleidon, 1992, Periodic market closure and trading volume: A model of intraday bids and asks, *Journal of Economic Dynamics and Control*, 16, 451-489.
- Cao, Charles, Oliver Hansch and Xiaoxin Wang, 2003, The informational content of an open limit order book, *EFA 2004 Maastricht Meetings Paper No. 4311*.
- Chan, Kakeung, William Christie, and Paul Schultz, 1995, Market structure and the intraday pattern of bid-ask spreads for Nasdaq securities, *Journal of Business*, 68(1), 35-60.
- Chan, Louis K., and Josef Lakonishok, 1993, Institutional trades and intraday stock price behavior, *Journal of Financial Economics*, 33, 173-199.
- Chan, Louis K., and Josef Lakonishok, 1995, The behavior of stock prices around institutional trades, *Journal of Finance*, Vol.50, No.4, 1147-1174.

- Chan, Louis K., and Josef Lakonishok, 1997, Institutional equity trading costs: NYSE versus Nasdaq, *Journal of Finance*, Vol.52, No.2, 713–735.
- Chung, Kee H., Bonny Van Ness, and Robert Van Ness, 1999, Limit orders and the bid-ask spread, *Journal of Financial Economics*, Vol. 53, No 2, 255–287.
- Constantinides, George, 1986, Capital market equilibrium with transactions costs, *Journal of Political Economy*, 94, 842–862.
- Coppejans, Mark T., Ian H. Domowitz, and Anath Madhavan, 2001, Liquidity in an automated auction, *AFA 2002 Atlanta Meetings*.
- Davis, M. and A. Norman, 1990, Portfolio selection with transaction costs, *Mathematics of Operations Research*, 15, 676–713.
- Dubil, Robert, 2002, Optimal liquidation of large security holdings in thin markets, *Research in International Entrepreneurial Finance and Business Ventures at the Turn of the Third Millennium*, Editor: Yochanan Shachmurove, *Academy of Entrepreneurial Finance*.
- Easley, David, and Maureen O’Hara, 1992, Time and process of security price adjustment, *Journal of Finance*, 47, 577–605.
- Eastham, Jerome, and Kevin J. Hastings, 1988, Optimal impulse control of portfolios, *Mathematics of Operations Research*, 13(4), 588–605.
- Foster, Douglas, and S. Viswanathan, 1990, A theory of the interday variations in volume, variance, and trading costs in securities markets, *Review of Financial Studies*, 3, 592–624.
- Foster, Douglas, and S. Viswanathan, 1995, Can speculative trading explain the volume-volatility relation?, *Journal of Business and Economic Statistics*, 13, 379–396.
- Foucault, Thierry, 1999, Order flow composition and trading costs in a dynamic limit order market, *Journal of Financial Markets*, 2, 99–134.
- Foucault, Thierry, Ohad Kadan, and Eugene Kandel, 2001, Limit order book as a market for liquidity, *Review of Financial Studies*, forthcoming.
- Gerety, Mason, and Harold Mulherin, 1992, Trading halts and market activity: an analysis of volume at the open and the close, *Journal of Finance*, 47, 1765–1784.
- Glosten, Lawrence R., 1994, Is the electronic open limit order book inevitable?, *Journal of Finance*, Vol.49, No.4, 1127–1161.
- Goettler, Ron, Christine Parlour, Uday Rajan, 2005, Equilibrium in a dynamic limit order market, *Journal of Finance*, Vol.60, No. 5, 2149–2192.

- Greenwood, Robin, 2004, Short- and long-term demand curves for stock: theory and evidence on the dynamics of arbitrage, *Journal of Financial Economics*, forthcoming.
- Griffiths, Mark, and Brian Smith, Alasdair Turnbull and Robert W. White, 2000, The Cost and determinants of order aggressiveness, *Journal of Financial Economics* 56, 65–88.
- Grinold, Richard, and Ronald Kahn, 2000, *Active Portfolio Management*, McGraw-Hill, New York.
- Hamao, Yasushi, and Joel Hasbrouck, 1995, Securities trading in the absence of dealers: Trades and quotes on the Tokyo Stock Exchange, *The Review of Financial Studies*, Vol. 8, No. 3, 849–878.
- Handa, Puneet, and Robert Schwartz, 1996, Limit order trading, *Journal of Finance*, 51, 1835–1861.
- Harris, Lawrence, 1986, A transactions data study of weekly and intraday patterns in stock prices, *Journal of Financial Economics*, 16, 99–117.
- Harris, Lawrence, 1990, Liquidity, trading rules and electronic trading systems, *NYU Salomon Center Series in Finance and Economics, Monograph* 1990–4.
- Harris, Lawrence, 1998, Optimal dynamic order submission strategies in some stylized trading problems, *Financial Markets, Institutions and Instruments*, 7, 1–76.
- Harris, Lawrence, and Venkatesh Panchapagesan, 2005, The information-content of the limit order book: Evidence from NYSE specialist trading decisions, *Journal of Financial Markets*, 8, forthcoming.
- Hasbrouck, Joel, 1996, Order characteristics and stock price evolution: An application to program trading, *Journal of Financial Economics*, 41(1), 129–149.
- Hasbrouck, Joel, and Duane J. Seppi, 2001, Common factors in prices, order flows, and liquidity, *Journal of Financial Economics*, 59, 383–411.
- Hasbrouck, Joel and Gideon Saar, 2002, Limit orders and volatility in a hybrid market: the Island ECN, *NYU Working paper*.
- He, Hua, and Harry Mamaysky, 2001, Dynamic trading policies with price impact, *Yale ICF Working Paper* No. 00–64.
- Hindy, Ayman, Chi-fu Huang, and Steven Zhu, 1997, Optimal consumption and portfolio rules with durability and habit formation, *Journal of Economic Dynamics and Control* 21.
- Hollifield, Burton, Robert A. Miller, and Patrik Sandas, 2004, Empirical analysis of limit order markets, *Review of Economic Studies*, 71, 1027–1063.

- Holthausen, Robert W., Richard W. Leftwich, and David Mayers, 1987, The effect of large block transactions on security prices: a cross-sectional analysis, *Journal of Financial Economics*, 19, 237–267.
- Holthausen, Robert W., Richard W. Leftwich, and David Mayers, 1990, Large block transactions, the speed of response of temporary and permanent stock-price effect, *Review of Financial Studies*, 9, 71–95.
- Hong, Harrison and Jiang Wang, Trading and returns under periodic market closures, 2000, *Journal of Finance*, 55, 297–354.
- Huberman, Gur, and Werner Stanzl, 2004, Arbitrage-free price update and price-impact functions, *Econometrica*, 72, No 4, 1247–1275.
- Huberman, Gur, and Werner Stanzl, 2005, Optimal liquidity trading, *Review of Finance* 9, 165–200.
- Irvine, Paul J., George J. Benston, and Eugene Kandel, 2000, Liquidity beyond the inside spread: measuring and using information in the limit order book, *Working Paper*.
- Jain, Prem, and Gun-Ho Joh, 1992, The dependence between hourly prices and trading volume, *Journal of Financial and Quantitative Analysis* 23, 269–284.
- Kaul, Aditya, Vikas Mehrotra, and Randall Morck, 2000, Demand curves for stocks do slope down: new evidence from an index weights adjustment, *Journal of Finance* 55 (2), 893-912.
- Keim, Donald B., and Ananth Madhavan, 1995, Anatomy of the trading process: Empirical evidence on the behavior of institutional traders, *Journal of Financial Economics* 37, 371- 398.
- Keim, Donald B. and Ananth Madhavan, 1996, The upstairs market for large-block transactions, *Review of Financial Studies* 9, 1–36.
- Keim, Donald B., and Ananth Madhavan, 1997, Transactions costs and investment style: An inter-exchange analysis of institutional equity trades, *Journal of Financial Economics* 46, 265-292.
- Kleidon, Allan, and Ingrid Werner, 1996, UK and US trading of British cross-listed stocks: An intraday analysis of market integration, *Review of Financial Studies*, 9, 619–664.
- Koski, Jennifer, and Roni Michaely, 2000, Prices, liquidity, and the information content of trades, *Review of Financial Studies*, 13, 659–696.
- Leland, Hayne, 2000, Optimal portfolio implementation with transactions costs and capital gains taxes, *University of California at Berkeley Working Paper*.

- McInish, Thomas, and Robert Wood, 1991, Hourly returns, volume, trade size and number of trades, *Journal of Financial Research*, 14(4), 303–315.
- Monch, Burkart, 2004, Optimal liquidation strategies, *Goethe University Working Paper*.
- Parlour, Christine, 1998, Price dynamics in limit order markets, *Review of Financial Studies*, 11, 789–816.
- Parlour, Christine, and Duane Seppi, 2003, Liquidity-based competition for order flow, *Review of Financial Studies*, 16, 301–303.
- Perold, Andre, 1988, The implementation shortfall: Paper versus reality, *Journal of Portfolio Management*, 14, 4–9.
- O'Hara, Maureen, 1995, Market microstructure theory, *Blackwell Publishers, Cambridge, Mass.*
- O'Hara, Maureen, 2003, Liquidity and price discovery, *Journal of Finance*, 58, 1335–1354.
- Ranaldo, Angelo, 2004, Order aggressiveness in limit order book markets, *Journal of Financial Markets*, 7, 53–74.
- Rock, Kevin, 1996, The specialists order book and price anomalies, *Review of Financial Studies*.
- Rosu, Ioanid, 2005, A dynamic model of the limit order book, *University of Chicago Working Paper*.
- Sandas, Patrick, 2001, Adverse selection and competitive market making: evidence from a pure limit order book, *Review of Financial Studies*, 14, 705–734.
- Scholes, Myron, 1972, The market for securities: substitution versus price pressure and the effects of information on share price, *Journal of Business*, 45, 179–211.
- Seppi, Duane, 1997, Liquidity provision with limit orders and a strategic specialist, *Review of Financial Studies*, 10, 103–150.
- Shleifer, Andrei, 1986, Do demand curves for stocks slope down?, *Journal of Finance*, 41, 579–590.
- Shreve, Steven, and Halil Soner, 1994, Optimal investment and consumption with transaction costs, *Annals of Applied Probability*, No.4, 609–692.
- Subramanian, Ajai, and Robert A. Jarrow, 2001, The liquidity discount, *Mathematical Finance*, 11(4), 447–474.
- Vayanos, Dimitri, 1999, Strategic Trading and Welfare in a Dynamic Market, *Review of Economic Studies*, 66, 219–254.

- Vayanos, Dimitri, 2001, Strategic Trading in a Dynamic Noisy Market, *Journal of Finance*, 56, 131–171.
- Wurgler, Jeffrey, and Ekatherina Zhuravskaya, 2002, Does arbitrage flatten demand curves for stock?, *Journal of Business*, vol. 7, 583–608.