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MODELLING OF THE FLOW AND DEFORMATION FIELDS AT THE PROFILES ROLLING BY FIELD LINES METHOD

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ABSTRACT

This paper presents a method for modeling of the rolling process based at the deformable continuous medium mechanics, the theory of field lines. The rolling of the profiles and plates may be evaluated as the plane strain state process. Using the equations of the continuous medium and the initial conditions and the limits conditions we solved the speed field, the strain rate field, the strain field. Applying an adequate computation program we obtained the values of the field factors of modeling process. The results are showed into this paper.

KEYWORDS: rolling process, field lines, continuous medium

1. Introduction

At the rolling of plates and profiles the deformation in a direction may be neglected, respectively in the lateral direction (Figure 1).

The lateral dimension of body (h_{i-1}) is appropriated of the dimension b_i of the deformation form of the rolls. Thus, the strain in this direction may be neglected [2].

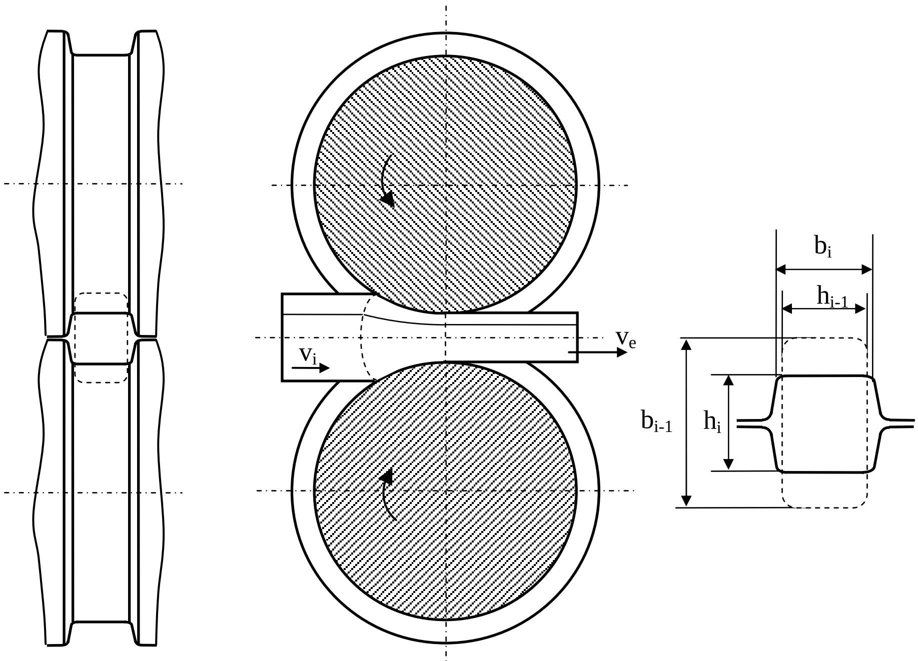

Fig. 1 *Scheme of profile rolling process and deformation conditions*

We consider the rolled body as a deformable continuous medium. The volume occupied by the continuous medium, at the really moment is divided in three domains (Figure 2) [2,4,6]:

- the domain D_1 before the entrance of medium between the rolls (rigid plastic medium),

- the domain D_2, deformation domain, the medium is between the rolls. In this domain is developed the deformation process,
- the domain D_3 at the exit of material between the rolls (rigid plastic, too).

2. Defining of the field line equation

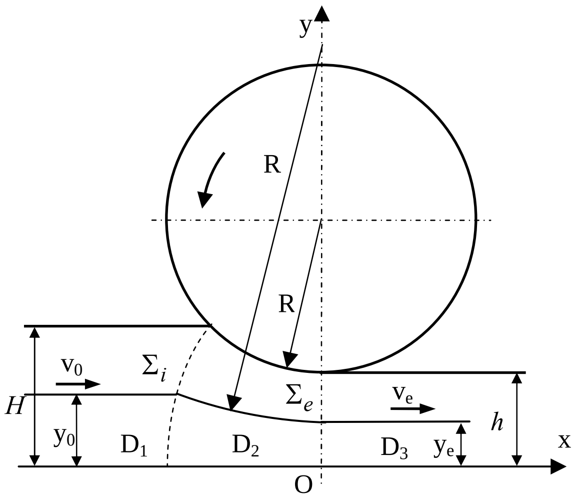

Fig. 2 *The deformation domain and field line*

In the conditions of a stationary regime of the continuous medium flow the field lines coincide to the movement trajectories of the material particles. In the Figure 2 we represent the field line of the material particles that are situated to y_0 of the axe Ox, the symmetry axe of the body.

The equations of the current line of parameter y_0 are so defined:

$y = y_0$, in D_1
respectively, (1)
$y = y_e$, in D_3

In the deformation domain D_2 the field line may be expressed by circle arch what satisfy the following conditions:

- circle center is on the axe Oy, the axe of the rolling cylinder centers,
- the circle radius respects the conditions:

$$R(y) = R = \begin{cases} R_0 & \text{for} \quad y = H \\ \infty & \text{for} \quad y = 0 \end{cases} \tag{2}$$

H is the initial semi-thickness of the body.

The conditions (2) are accomplished by the equation [2]:

$$R(y) = \frac{H}{y_0} \cdot R_0 \tag{3}$$

Consequently the field line equation is:

$$x^2 + \left(y - \frac{H}{y_0} R_0 - y_e \right)^2 = R^2 \tag{4}$$

We admit the hypothesis of the proportional repartitions of the deformation to the thickness of body. In this condition we have:

$$y_e = y_0 \cdot \frac{h}{H}$$

and (4) becomes [6]:

$$x^2 + \left(y - \frac{H}{y_0} R_0 - y_0 \frac{h}{H} \right)^2 - \left(\frac{H}{y_0} \cdot R_0 \right)^2 = 0 \tag{5}$$

The surface Σ_i is defined by the points of coordinates:

$$\begin{aligned} & y = y_0 \\ & x = x_0 = -\sqrt{\left(\frac{H}{y_0} \cdot R_0 \right)^2 - \left(y_0 - \frac{H}{y_0} R_0 - y_0 \frac{h}{H} \right)^2} \end{aligned} \tag{6}$$

3. Defining of the speed field

In the D_1 and D_3 domains we have:

$$\begin{matrix} v_x = v_i \\ v_y = 0 \end{matrix} \quad \text{and} \quad \begin{matrix} v_x = v_e \\ v_y = 0 \end{matrix} \text{, respectively}$$

In the domain D_2 we have the following conditions [1,3,5]:

- the continuity equation to incompressible medium
  $div(\vec{v}) = 0$

For plain strain state we have:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{7}$$

- the field line equation:

$$\frac{dx}{v_x} = \frac{dy}{v_y} \tag{8}$$

From (5), (7) and (8) we obtain:

$$\frac{v_x}{v_y} = -\frac{y - a}{x} \tag{9}$$

We denote

$$a = \frac{H}{y_0} R_0 + y_0 \frac{h}{H}$$

$y = y_0$, in D_1

Then we have:

$$\frac{v_x}{v_y} = -\frac{y-a}{x} \tag{10}$$

and

$$v = v_x \sqrt{1 + \left(\frac{x}{y-a}\right)^2} \tag{11}$$

The continuity condition is defined by the constant material flux in the long of the field line.

We consider the elementary volume in the long of the field line (fig.3) [4,6].

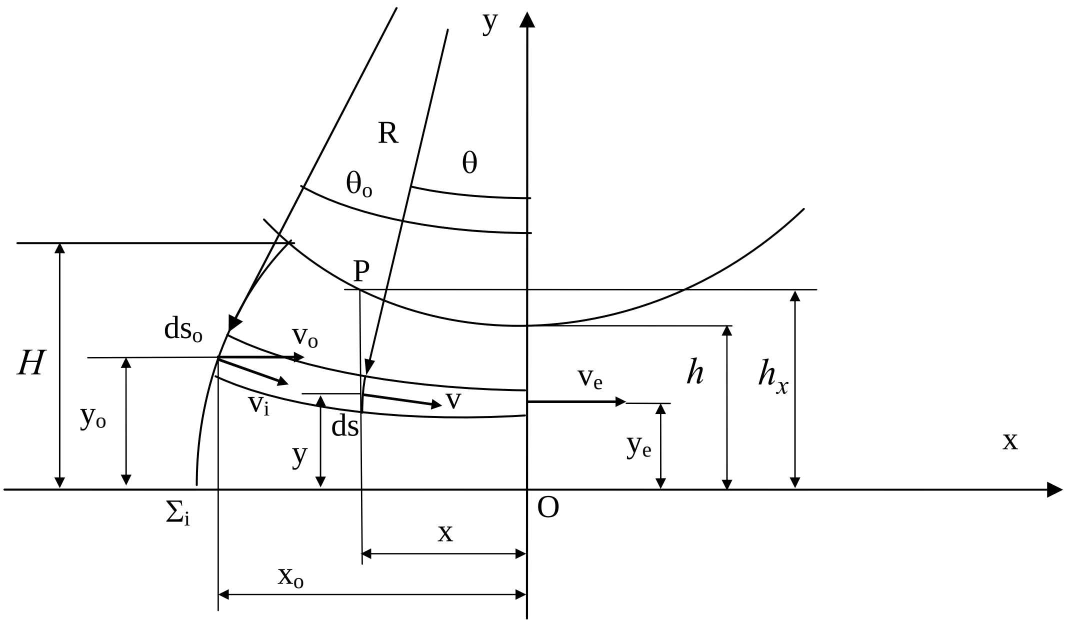

Fig. 3 *Scheme for establishing of continuity equation*

At the elementary surface $b \times ds_o$ of the surface Σ_i a discontinuity of material particle speed is appeared. Thus, we have:

$$v_i = v_o \cdot \cos\theta_o$$
$$\Delta v = v_t = v_o \cdot \sin\theta_o \tag{12}$$

where

$$\theta_o = \arcsin\left(\frac{x_o}{R}\right) = \arcsin\left(\frac{x_o \cdot y_o}{H \cdot R_o}\right) \tag{13}$$

Applying the hypothesis of the plane strain state we may write:

$$v_i \cdot ds_o = v \cdot ds = v_e \cdot ds_e$$

At the same time having the hypothesis of uniform repartition of the deformation on the thickness of the body we have:

$$v = v_e \cdot \frac{h}{h_x} \tag{14}$$

In this relation we have h_x the thickness of the body according to coordinate x of the point of the field line and v_e is the speed of material particle at the exit from the deformation domain.

We have:

$$h_x = R_o + h - \sqrt{R_o^2 - x^2} \tag{15}$$

and

$$v_e = v_i \cdot \frac{H}{h} \tag{16}$$

Using the relations (10), (11), (14 and (15) we obtain:

$$v_x = v_e \cdot \frac{h}{a_o - \sqrt{R_o^2 - x^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{x}{y-a}\right)^2}}$$

$$v_y = v_e \cdot \frac{h}{a_o - \sqrt{R_o^2 - x^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{x}{y-a}\right)^2}} \cdot \frac{x}{y-a} \qquad (17)$$

4. Defined of the strain rate field

The components of the strain rate tensor are defined by equations:

$$\dot{\varepsilon}_{xx} = \frac{\partial v_x}{\partial x}; \quad \dot{\varepsilon}_{yy} = \frac{\partial v_y}{\partial y}; \quad \dot{\varepsilon}_{xy} = \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

Using the function (17) for the speed components we obtain:

$$\dot{\varepsilon}_{xx} = -\dot{\varepsilon}_{yy} = -\frac{v_e \cdot h \cdot x}{(a_o - \sqrt{R_o^2 - x^2})\sqrt{1+\left(\frac{x}{y-a}\right)^2}} \left[\frac{1}{(y-a)^2 \cdot \left[1+\left(\frac{x}{y-a}\right)^2\right]} + \frac{1}{(a_o - \sqrt{R_o^2 - x^2}) \cdot \sqrt{R_o^2 - x^2}}\right] \qquad (18)$$

$$\dot{\varepsilon}_{xy} = \frac{v_e \cdot h}{2(a_o - \sqrt{R_o^2 - x^2})(y-a)\sqrt{1+\left(\frac{x}{y-a}\right)^2}} \left[\frac{2x^2}{(y-a)^2 \cdot \left[1+\left(\frac{x}{y-a}\right)^2\right]} + \frac{x^2}{\sqrt{R_o^2 - x^2}} + 1\right] \qquad (19)$$

The strain rate intensity is defined by the relation [1]:

$$\bar{\dot{\varepsilon}} = \frac{2}{3}\sqrt{\dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{yy}^2 + 2\dot{\varepsilon}_{xy}^2} \qquad (20)$$

For the numerical calculus we use the following normalized coordinates:

$$\eta = \frac{x}{x_o} \quad \text{respectively,} \quad \varphi = \frac{y_o}{H} \qquad (21)$$

and obtain:

$$v_x = v_e \cdot \frac{h}{a_o - \sqrt{R_o^2 - (\eta x_o)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{\eta x_o}{y(\eta)-a}\right)^2}}$$

$$v_y = v_e \cdot \frac{h}{a_o - \sqrt{R_o^2 - (\eta x_o)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{\eta x_o}{y(\eta)-a}\right)^2}} \cdot \frac{\eta x_o}{y(\eta)-a} \qquad (22)$$

$$\dot{\varepsilon}_{xx} = -\dot{\varepsilon}_{yy} = -\frac{v_e \cdot h \cdot \eta \cdot x}{(a_o - \sqrt{R_o^2 - (\eta x)^2})\sqrt{1+\left(\frac{\eta x}{y-a}\right)^2}} \left[\frac{1}{(y-a)^2 \cdot \left[1+\left(\frac{\eta x}{y-a}\right)^2\right]} + \right.$$
$$\left. + \frac{1}{(a_o - \sqrt{R_o^2 - (\eta x)^2}) \cdot \sqrt{R_o^2 - (\eta x)^2}} \right] \quad (23)$$

$$\dot{\varepsilon}_{xy} = \frac{v_e \cdot h}{2(a_o - \sqrt{R_o^2 - (\eta x)^2})(y-a)\sqrt{1+\left(\frac{\eta x}{y-a}\right)^2}} \left[\frac{2(\eta x)^2}{(y-a)^2 \cdot \left[1+\left(\frac{\eta x}{y-a}\right)^2\right]} + \right.$$
$$\left. + \frac{(\eta x)^2}{\sqrt{R_o^2 - (\eta x)^2}} + 1 \right] \quad (24)$$

The field of strain intensity is defined by the expression [4]:

$$\bar{\varepsilon} = \int_0^t \bar{\dot{\varepsilon}} \cdot dt \quad (25)$$

For the numerical solve we will use the following principle (fig.4):

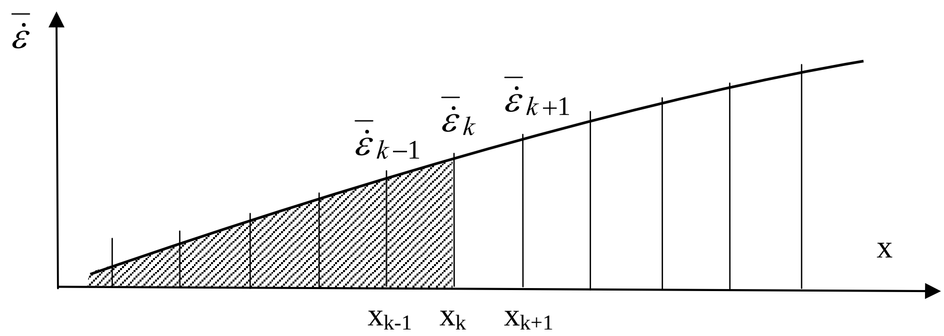

Fig. 4 *The calculus scheme of the strain intensity*

We defined the time differential as:

$$dt = \frac{dx}{v_x}$$

The numerical expression of the equation (25) is:

$$\bar{\varepsilon}_k = \bar{\varepsilon}_{k-1} + \left(\bar{\dot{\varepsilon}}_{k-1} + \bar{\dot{\varepsilon}}_k\right) \cdot \frac{x_k - x_{k-1}}{v_{x_k} + v_{x_{k-1}}} \quad (26)$$

In this expression the index k is defined in function of the index i as $k=n-i$, where i is the division operator in the long of the field line ($i=1,2,...,n$).

For initialization of the values of $\bar{\varepsilon}$ we consider what at the $k=0$, respectively, $i=n$, that is in the point of surface Σ_i the deformation is defined by the rotation of speed vector of angle θ_o.

Thus the initial strain is:

$$\bar{\varepsilon}_0 = \mathrm{tg}\,\theta_0 \quad (27)$$

Using the calculus algorithm described above we developed a computation program. The results of the calculus program we will present in the future paper.

5. Conclusions

The solving of the deformation process is possible using various methods. The field (or flow) line is one of these.

First we must defined clearly the domain that, at the real moment, is occupied of the body and the initial and limit conditions. The body is considered as deformable continuous medium. Then, we must define the equation of the flow line.

Using the equations of the mechanics of the deformable continuous medium, applying the initial conditions and the conditions at the limits we obtained, in the analytical form, the expressions of the components of the speed, and, derived by these, the components of the strain rate tensor.

If the components of strain rate tensor are defined, that is the field of strain rate tensor is defined, we can calculate the strain rate intensity, and finally, the strain intensity. Thus the analyze of cinematic process is solved.

From this level we can develop the analyze of process dynamic for establishing the data of the evaluation of the rolling process. The solving of this action we will show in the next paper.

References

[1] **Moussy F., Franciosi P**. *Physique et mecanique de la mise en forme des meteaux. Presses du CNRS*, Paris, 1990, ISBN 2-87682-023-4

[2] **Dumitrescu A.T.** *Contributii la modelarea laminarii in calibre.* Teza de doctorat, Institutul Politehnic Bucuresti, 1986.

[3] **Adrian M., Badea S.** *Bazele teoretice ale proceselor de deformare plastica*. Editura tehnica, Bucuresti, 1983.

[4] **Cananau N.** *Teoria deformarii plastice.* Universitatea Dunarea de Jos din Galati, vol.2, 1995.

[5] **Dragan I.** T*ehnologia deformarii plastice*. Editura Didactica si Pedagogica, Bucuresti, 1976.

[6] **Corobete G**. C*ontributii la cercetarea procesului de laminare a sarmelor din otel cu caracteristici mecanice superioare*. Referat de doctorat, Universitatea Dunarea de Jos din Galati, 2006.