Experimental Investigation of Mixing in a Stratified Fluid Due to Diffusion-Driven Flows in a Loosely-Packed Particle Layer

by

William B. Etheridge

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Signature of Author ....
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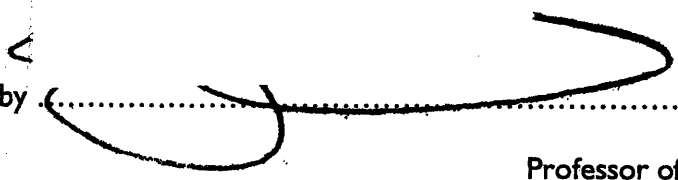
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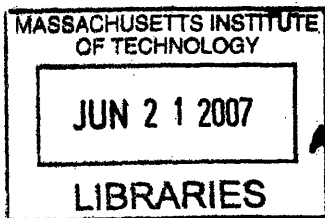Thomas Peacock
Atlantic Richfield Career Dev. Assistant Professor of Mechanical Engineering
Thesis Supervisor

Accepted by ....
John H. Lienhard V
Professor of Mechanical Engineering
Chairman, Undergraduate Thesis Committee

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ABSTRACT

An experimental study was undertaken to investigate if a loosely-packed particle layer can induce mixing due to diffusion-driven Phillips-Wunsch boundary flows in a quiescent stratified fluid. Diffusion-driven flows can exist along a sloped boundary in a fluid with a density gradient that varies with height, so these flows will also exist along the curved surface of a spherical boundary.

A particle layer was created using 6.35 mm diameter Polystyrene spheres in a salt-stratified fluid. A linear density stratification was pumped into an acrylic tank from below using a double-bucket system. A calibrated salinity probe, mounted on a computer-controlled traverse, was used to measure the evolution of the density profile in the tank over time.

The results of the density profiles showed a measurable change in the density profile in the tank over several weeks. The density profile in the area vertically adjacent to the particle layer was nonlinear due to mixing induced by the Phillips-Wunsch boundary flows created by the spherical particles.

Thesis Supervisor: Thomas Peacock
Title: Atlantic Richfield Career Dev. Assistant Professor of Mechanical Engineering

BACKGROUND

The Phillips-Wunsch diffusion-driven boundary flow occurs along a sloping boundary in a quiescent stratified fluid. This convective flow is due to the combined action of diffusion and gravity in the boundary layer along the sloping boundary. This phenomenon was first identified, independently and simultaneously, by Phillips[1] and Wunsch.[2]

In a salt-stratified fluid, with density that increases linearly with depth, the isopycnals (lines of constant density) are horizontal in the bulk fluid. In a container with vertical sidewalls, the fluid is in a very stable hydrostatic equilibrium. Boundary conditions in this situation require that there is zero salt flux across the bottom and free surface boundaries, which causes the normal density gradient at the surface to be gradually replaced by a region of constant density. This region expands until the fluid has reached diffusive equilibrium. This diffusion occurs over a timescale, τ, of the order

$$\tau \sim \frac{H^2}{\kappa},$$ (1)

where H is the height of the fluid and κ is the molecular diffusion coefficient. For salt-water in a 40 cm deep tank, this time scale is on the order of several years.

If the fluid contacts an inclined, insulated boundary sidewall, the isopycnals bend from the horizontal to meet the wall at right angles, as shown in Figure 1. The sidewall is impermeable, so the normal density gradient vanishes because there can be no salt flux through the wall. This creates a boundary layer along the wall that is no longer in hydrostatic equilibrium. The fluid in this layer is buoyant in comparison to fluid at the

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same height in the bulk fluid. This buoyant fluid then moves up along the wall,

generating a convective flow that reaches a steady state when viscous forces balance the
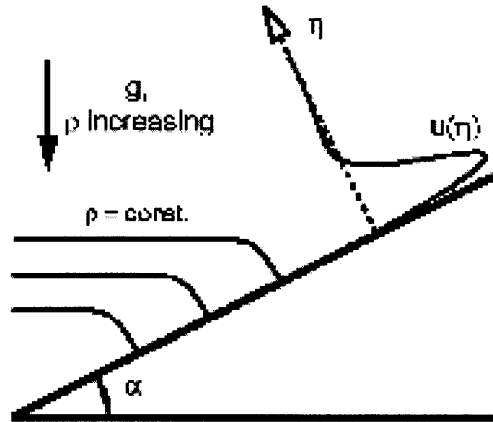
hydrostatic pressure.

Figure 1: Schematic of diffusion-driven flow. The isopycnals of the bulk fluid bend to meet the inclined wall, forming a boundary layer along the wall. The velocity profile, $u(\eta)$, travels up the slope due to the Phillips-Wunsch flow. [Graphic is reprinted from Peacock[4]]

The characteristic buoyancy layer thickness, δ, of the Phillips-Wunsch flow is

$$\delta = \left(\frac{N^2 \sin^2 \alpha}{4 \nu \kappa} \right)^{-\frac{1}{4}}, \tag{2}$$

where

$$N^2 = -\frac{g}{\rho_o} \frac{\partial \rho}{\partial z} \tag{3}$$

is the stability frequency in the interior of the fluid, ν is the kinematic viscosity, g is the

acceleration due to gravity, ρ is the density, α is the angle of inclination of the wall, ρ_o is

the average density, and z is the vertical coordinate (as measured from the bottom of the

container).

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In a container with a slanted sidewall we can expect the stratification to mix, as a function of height and time, as

$$\frac{\partial \rho}{\partial t} = \frac{\kappa}{A}\frac{\partial}{\partial z}\left(A\,\frac{\partial \rho}{\partial z}\right) \qquad (4)$$

where A is the cross sectional area of the container. In the case of a constant density gradient, the Phillips-Wunsch flow along a slanted sidewall will not cause the interior fluid to mix more rapidly than the timescale determined by molecular diffusion alone.[3]

The magnitude of the Phillips-Wunsch flow has been shown to be dependent on the angle of inclination of the wall. Flows can be expected to be found for most angles between fully horizontal and fully vertical, but the flows for angles less than 45° are much larger than for those above 45°. Peacock[4] found the angle of maximum velocity to be 2.8°± 1.0°.

If the sloping sidewall is replaced by a slanted plate that is suspended in a container with vertical sidewalls, without touching the upper or lower surfaces of the water, a Phillips-Wunsch flow will develop on both sides of the plate. The flow on the top side of the plate will flow upwards, while the flow on the bottom of the plate will flow downwards. The upward flow from the edge of the top side will flow into the downwards flow on the bottom side of the plate, creating a net circulation around the plate.[3]

An interesting variation on the Phillips-Wunsch flow, as suggested by Woods,[3] arises in the situation of a loosely packed gravel bed. The gravel bed can be approximated

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as a group of spherical particles, spaced such that the interparticle spacing is greater than the buoyancy layer thickness and also so that all the fluid is in communication at all heights. The top hemisphere of a particle will act like the topside of the aforementioned plate, driving fluid upwards because of its upward slope. The bottom hemisphere will similarly create downward flows. Woods[3] predicts that the mixing due to these particles will create a density profile that is determined by

$$\frac{\partial \rho}{\partial t} = \frac{\kappa}{A\phi} \frac{\partial}{\partial z}\left(A\phi \frac{\partial \rho}{\partial z}\right) \tag{5}$$

where ϕ is the liquid fraction as a function of height.

The Phillips-Wunsch flows from the gravel bed will create a nonlinear density profile in the fluid. We will attempt to experimentally observe this nonlinear density profile due to the Phillips-Wunsch flows from a layer of spherical particles in a salt-stratified quiescent fluid.

PROCEDURE

The particles used in this investigation were ¼ inch (6.35mm) diameter high-impact Polystyrene balls. The diameter of the particles was chosen such that the diameter would be much larger than the buoyancy layer thickness of the Phillips-Wunsch flow. For angles from 10 to 90 degrees, the buoyancy layer thickness ranged from about 0.5 mm to about 0.2 mm. The average density of the spheres was found to be approximately 1050 g/cc. The polystyrene spheres were coated with a spray-on polyurethane coating. This coating helped to eliminate the formation of air bubbles on the surface of the spheres. A total of 1600 polystyrene spheres was used for this experiment.

The experiment was performed in a 40 cm x 30 cm x 40 cm acrylic tank. A linear stratification was set up using a double bucket system to fill the tank with salt water. The first tank of the double bucket system had a density of 1123.5 g/cc and pumped to the second tank at a rate of 0.4 liters per minute. The second tank was filled with 30 liters of salt water with an initial density of 1016.4 g/cc and pumped to the acrylic tank at a rate of 0.8 liters per minute. A recirculating pump mixed the water in the second tank to ensure an even salinity as water was added from the first tank. The acrylic tank was filled from below, with the salt water passing through a diffuser as it entered to minimize mixing.

The spheres were placed in the bottom of the tank before the filling began. The tank was then filled with salt water until the layer of spheres was near the vertical midpoint of the water depth. At this point filling was stopped and a lid was placed over the tank. Without a lid covering the tank, the effects of the diffusion-driven flows are

overpowered by currents arising from either evaporative cooling on the free surface of the water or from external air currents. The water was allowed one hour to settle the rotational currents induced during filling before any measurements were taken. The spheres were initially distributed throughout a 1.5 cm layer that covered the cross-sectional area of the tank.

A profile of the density in the tank was taken using a calibrated PME salinity probe mounted on a Parker-Daedal traverse. The probe was calibrated using two sets of salt water samples that were placed in a Neslab RTE-140 temperature-controlled bath. Densities of the salt water samples were measured using an Anton Paar DMA38 density meter. A Matlab routine was used to convert the probe conductivity and temperature output voltages to calibrated density measurements. The tip of the probe was covered with a plastic mesh, with about 3 mm openings, to prevent the spheres from damaging the probe sensors. The traverse was computer controlled using a National Instruments LabVIEW motion control and data capture package. The velocity of the probe was set to 0.635 mm per second downwards, with measurements taken every 250 milliseconds.

A density profile was taken each day over the course of a week to measure the change in density profile due to the sphere layer over time. Unfortunately, further analysis of the resulting profiles would reveal that the voltage signals from the probe were too noisy to resolve any changes in profile. The excessive noise was due to the traverse used to move the probe.

The Parker traverse was swapped out for a LinTech traverse, which was known to cause less signal noise. We decided to keep the same stratification with the spheres and see if we could measure the long-term result of the sphere layer on the density profile. The time elapsed from the beginning of the experiment was 14 days.

It is important to note that the spheres had a tendency to gather together in clumps within the layer in the tank, as seen in Figure 2. The clumps grew in such a way that by the time that the following measurements were taken, the spheres in the layer had all gathered together in one half of the tank. About 200 of the spheres had air bubbles form on their surface by this time, causing them to float upwards out of the layer. These particles were very spread out, so any effects from them were assumed to be negligible.

The probe was set to move at 0.5 mm per second, with a measurement taken every 125 milliseconds. A measurement of the density profile was taken along the full height of the tank. Local profiles of the sphere layer were then measured. The tip of the probe was initially placed 3 cm above the sphere layer, and a density profile was taken to 3 cm below the sphere layer. Local profiles were first taken through areas of dense particle concentration. Density profiles were then measured through areas clear of particles but adjacent to the dense particle area.
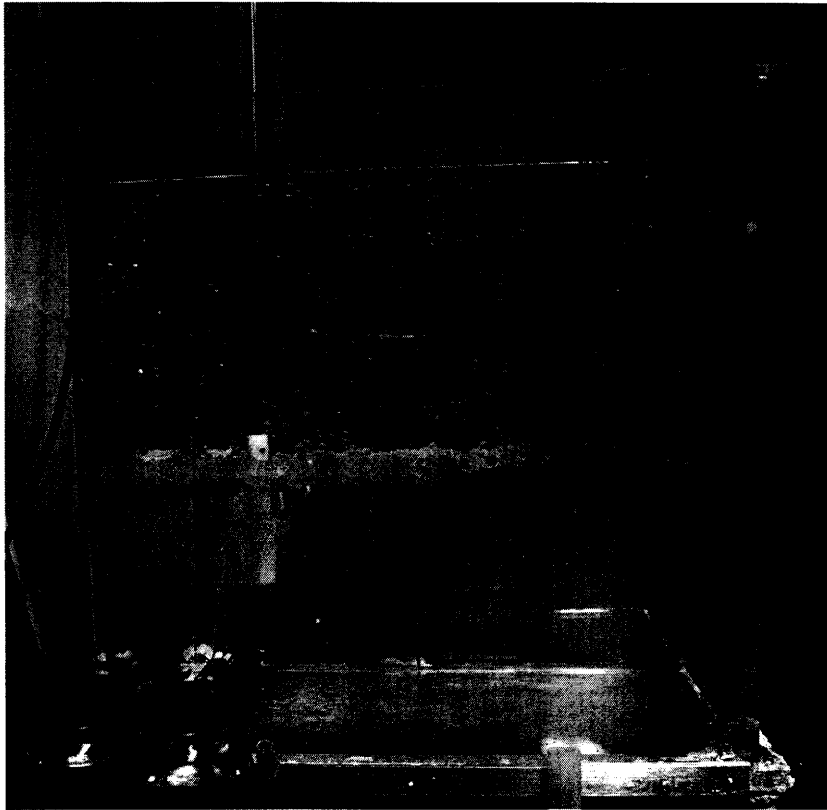
Figure 2: Image of tank during density measurement. The density probe is taking a density profile through the spherical particle layer. The tip of the probe is covered with a plastic mesh to prevent particles from damaging the sensors. Some of the spheres have floated upwards due to air bubbles forming on their surfaces, but the overall effect of these particles is assumed to be negligible.

RESULTS

The density profile taken one hour after filling ended is shown in Figure 3. The

plot has excessive noise due to the traverse used to move the probe, but the overall shape
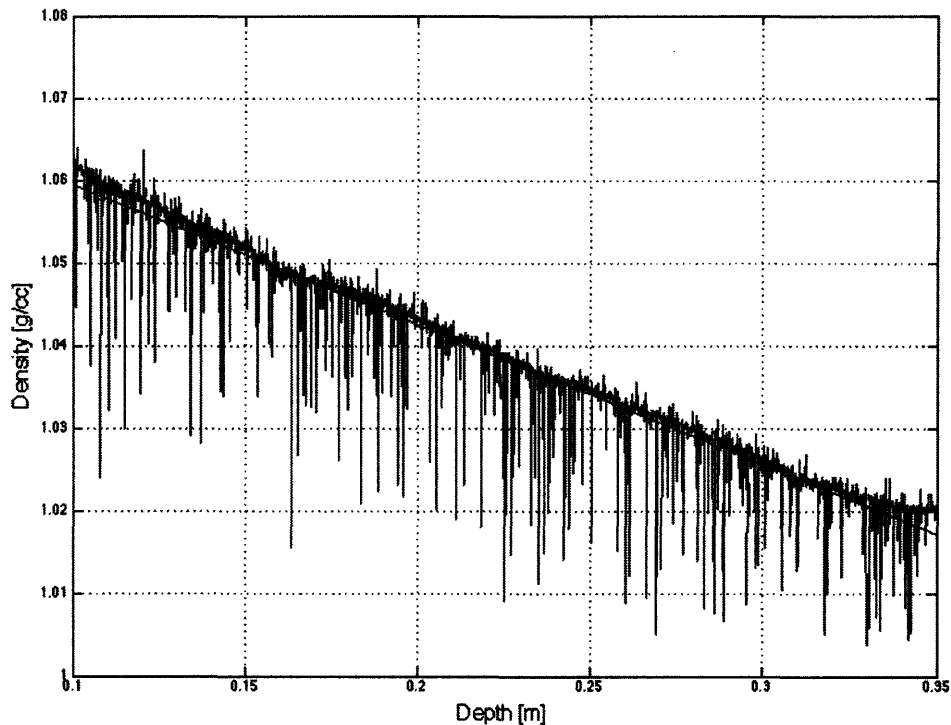
and slope of the plot is clear.

Figure 3: Initial Density Profile. Profile was taken one hour after the tank was filled. The depth is measured from the bottom of the tank.

The density profile taken after the spherical particles had been in the tank for 14

days is shown in Figure 4. This profile was taken using the LinTech traverse, which

greatly reduced the amount of signal noise. The nonlinearity in the top portion of the

tank is three times greater than was seen in the initial density profile. This 10 cm section

was possibly mixed by accident when taking measurements in the tank, although some

mixing is probably due to diffusion. The profile was not taken all the way to the bottom

of the tank due to limitations on the travel length of the traverse, so we cannot compare

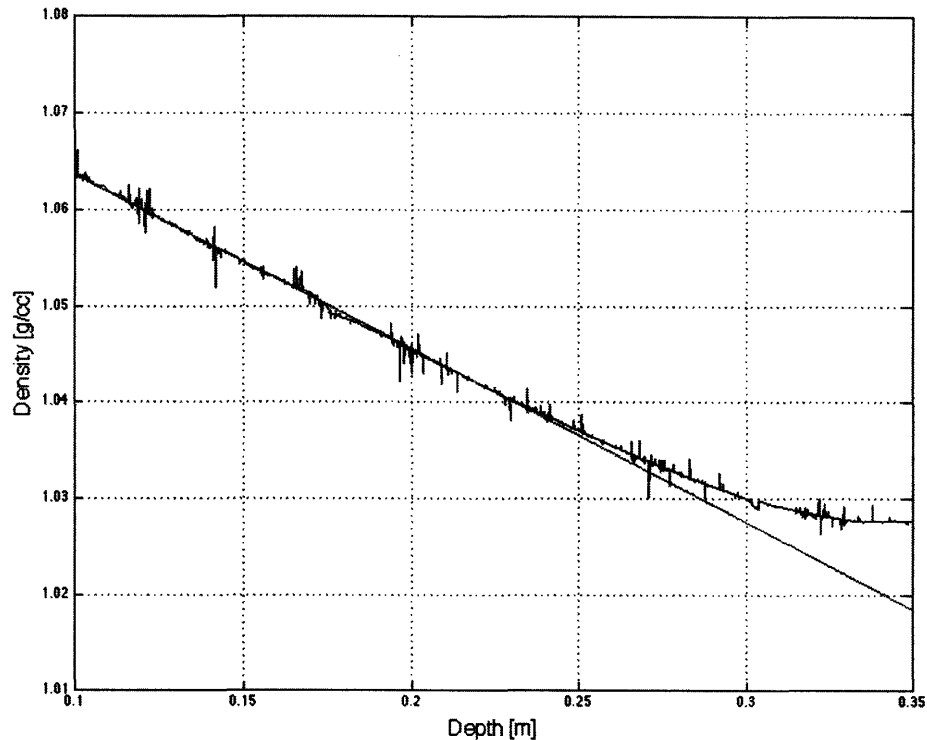diffusion mixing on the bottom surface of the tank.

Figure 4: Density Profile after 14 Days. Effect of spherical particle layer is visible around 0.17 m height. The nonlinearity of the density profile in the top 10 cm of the tank is about three times greater than in the initial density profile (see Figure 2).

A small nonlinearity in the density profile in Figure 4 can be seen around 0.17 m

depth. This nonlinear section was measured as the probe passed through the particle

layer in the tank. A more precise profile, seen in Figure 5, was then taken through the

particle layer. Mixing of the fluid can be seen between 15 cm and 17 cm, with a sharp

jump in density from 1.050 g/cc to 1.049 g/cc between 16 cm and 16.5 cm height.

The local density profile taken through an area of no spheres, 1 cm away from the

particle layer, shown in Figure 6, shows no visible effects of the Phillips-Wunsch flow.

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The effects of the particle layer appear to only extend in the vertical direction, with very little mixing in the lateral direction, as expected by the direction of the flow.
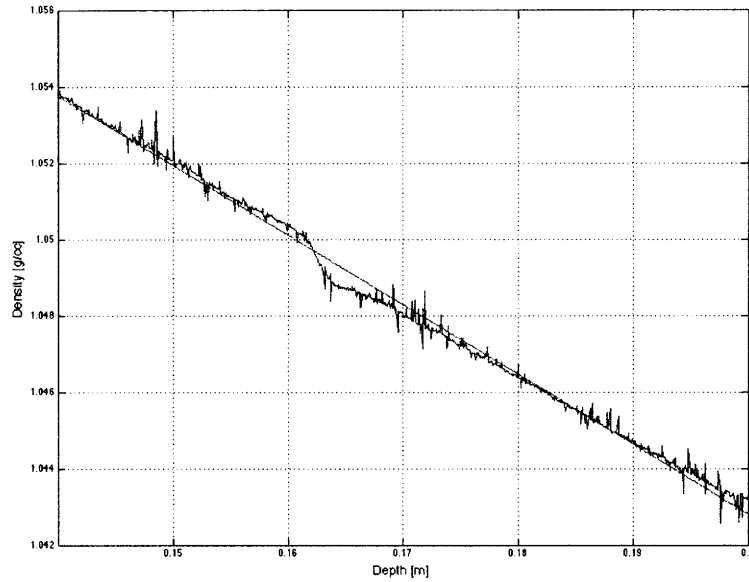

Figure 5: Local density profile through dense sphere layer. Between 15 cm and 17 cm in height, the density profile is nonlinear, including a sharp change in density between 1.050 g/cc and 1.049 g/cc.
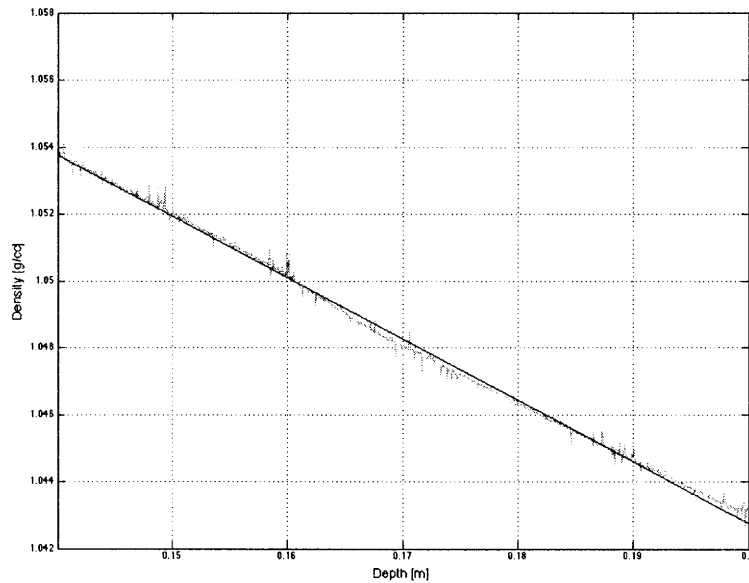

Figure 6: Local density profile through area of no particles. Particle layer was 1 cm away from measurement location. No visible effect of the Phillips-Wunsch flow is present away from the particle layer.

CONCLUSIONS

We have successfully observed a nonlinear density profile due to diffusion-driven Phillips-Wunsch flows resulting from a loosely-packed particle layer. The mixing due to this spherical particle layer had a measured effect in the region above and below the particle layer, but it had a negligible effect in the lateral direction.

Future work on this topic should attempt to measure the evolution of the density profile over time, and then compare this result to an analytical solution of Equation (5). It would be additionally useful to restrict the motion of the particles in the tank so that they would maintain even spacing between them and therefore not clump together over time.

ACKNOWLEDGMENTS

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